### Perceptron

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### Objectives

- To write a machine learning program recognizing a handwritten digit using Perceptron
- To explain neural network terminologies via examples



### Contents

- I. McCulloch-Pitts Neuron
- II. Perceptron
- III. Linear Threshold Unit
- IV. "Handwritten Digit Recognition" Example



### References

- Raul Rojas (1996). Neural Networks: A Systematic Introduction. Springer.
- Aurelien Geron (2017). Hands on Machine Learning with Scikit Learn and TensorFlow. O'Reilly Media.

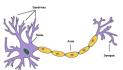


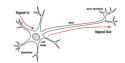
### Why Do We Smile?



### A Biological Neuron

- https://towardsdatascience.com/mcculloch-pitts-model-5fdf65ac5dd1
- Dendrite: Receives signals from other neurons
- Soma: Processes the information
- Axon: Transmits the output of
  this pourse.
- Synapse: Point of connection to other neurons



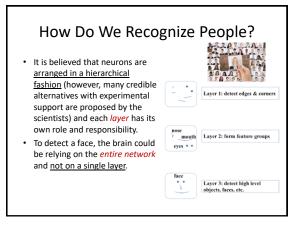


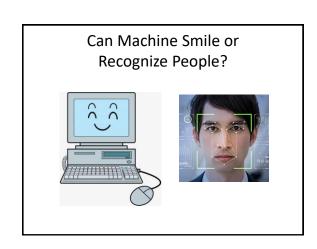
### An Overly Simplified Illustration Let's say you are watching Friends. Each neuron only fires when its intended criteria is met i.e., a neuron may perform a certain role to a certain stimulus. The "laugh or not" set of neurons that will help you make a decision on whether to laugh or not.

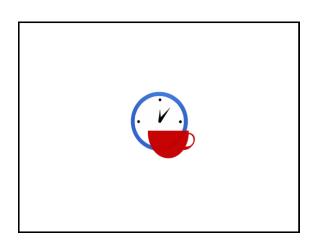
In reality, it is not just a couple of neurons which would do the decision

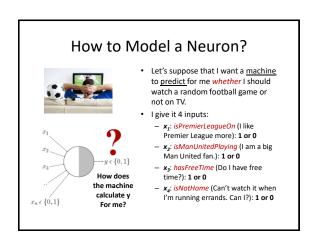
making.

# Neural Network There is a massively parallel interconnected network of 10<sup>th</sup> neurons (100 billion) in our brain. Now the sense organs pass the information to the first/lowest layer of neurons to process it. And the output of the processes is passed on to the next layers in a hierarchical manner, some of the neurons will fire and some won't and this process goes on until it results in a final response — in this case, laughter.

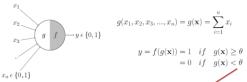








### McCulloch-Pitts Neuron: Mathematical Model of a Biological Neuron



- g(x) is just doing a sum of the inputs a simple aggregation.
- $x_1 = 1$ ,  $x_2 = 1$ ,  $x_3 = 1$ ,  $x_4 = 0$ . g(x) = ?
- theta here is called thresholding parameter. If I always watch the game when the sum turns out to be 2 or more, the theta is 2 here. This is called the thresholding logic.
- x₁= 1, x₂ = 1, x₃ = 1, x₄ = 0, θ = 2 ⇒ Should I what the football game?

### Telling a Machine To Laugh Using M-P Neuron



- In reality, I laugh when having 10 inputs and the sum turns out to be 4 or more
- Then I will code the machine with a function that sums 10 inputs, set theta
  to 4, and give it a "Laugh" icon.
- When I give the machine NEW input values, it will calculate the sum and if the sum turns out to be 4 or more, it will show the "Laugh" icon.

### Representing Boolean Functions Using M-P Neuron. Why?





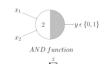
Input	Input	Output
A	В	Y
0	0	0
0	1	0
1	0	0
1	1	1



### Representing an AND Function

- In reality, I have <u>2 inputs</u> and want to calculate the AND value of these inputs.
- How do I tell a machine to do this task for me?

$$g(x) = x_1 + x_2 \ge ? \text{ (i.e. } y = 1)$$

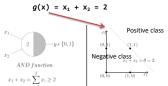


Input	Input	Output
Α	В	Υ
0	0	0
0	1	0
1	0	0
1	1	1

Find a threshold, then code the sum function and the threshold in the machine!

### Geometric Interpretation

 Here, all the input points that lie ON or ABOVE, just (1, 1), output 1 when passed through the M-P AND neuron.



- The M-P AND neuron learnt a linear decision boundary!
- The M-P AND neuron is splitting the <u>input sets</u> into <u>two classes</u> <u>positive</u> and <u>negative</u>.

### How About 3-Input AND Function?

- In reality, I have 3 inputs and want to calculate the AND value of these inputs.
- How do I tell a machine to do this task for me?

$$g(\mathbf{x}) = \mathbf{x_1} + \mathbf{x_2} + \mathbf{x_3} \ge ? \text{ (i.e. } \mathbf{y} = \mathbf{1})$$

$$x_1$$

$$x_2$$

$$y \in \{0, 1\}$$

$$\begin{cases}
A & B & C \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{cases}$$

Find a threshold, then code the sum function and the threshold in the machine!

### Representing an OR Function

- In reality, I have <u>2 inputs</u> and want to calculate the OR value of these inputs.
- How do I tell a machine to do this task for me?

$$g(x) = x_1 + x_2 \ge ? \text{ (i.e. } y = 1)$$

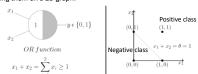


A	В	Output
0	0	0
0	1	1
1	0	1
1	1	1

Find a threshold, then code the sum function and the threshold in the machine!

### Geometric Interpretation

- The inputs are obviously boolean, so only 4 combinations are possible—(0,0), (0,1), (1,0) and (1,1).
- Plotting them on a 2D graph:



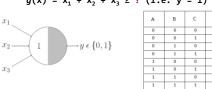
- The M-P OR neuron learnt a linear decision boundary!
- The M-P OR neuron is splitting the <u>input sets</u> into <u>two classes</u> <u>positive</u> and negative.

### How About 3-Input OR Function?

In reality, I have <u>3 inputs</u> and want to calculate the OR value of these inputs.

• How do I tell a machine to do this task for me?

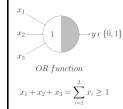
$$g(x) = x_1 + x_2 + x_3 \ge ?$$
 (i.e.  $y = 1$ )

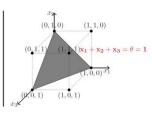


Find a threshold, then code the sum function and the threshold in the machine!

### Geometric Interpretation

All the points that lie ON or ABOVE that plane (positive half space)
will result in output 1 when passed through the OR function M-P
unit and all the points that lie BELOW that plane (negative half
space) will result in output 0.





### Representing a NOT Function

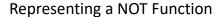
- In reality, I have <u>1 input</u> and want to calculate the NOT value of this input.
- How do I tell a machine to do this task for me?

$$g(x) = x_1 \ge ?$$



Input	Output
Α	Y
0	1
1	0

### **Inhibitory Inputs** Let's suppose that I want a $\underline{\text{machine}}$ to <u>predict</u> for me *whether* I should watch a random football game or not on TV. I give it 4 inputs: x<sub>1</sub>: isPremierLeagueOn (I like Premier League more): 1 or 0 x<sub>2</sub>: isManUnitedPlaying (I am a big These inputs can either be excitatory or Man United fan.): 1 or 0 inhibitory. Excitatory inputs are NOT the ones that will x3: hasFreeTime (Do I have free time?): 1 or 0 make the neuron fire on their own but they might fire it when combined together. Inhibitory inputs are those that have x4: isNotHome (Can't watch it when I'm running errands. Can I?): 1 or 0 maximum effect on the decision making irrespective of other inputs



In reality, I have 1 input and want to calculate the NOT value of this input. How do I tell a machine to do this task for me?

Take the input as an *inhibitory input*, i.e. set below rule

if  $x_1 = 1$  then y = 0

then set the thresholding parameter to ?,  $g(x) = x_1 \ge ?$ 

 $y \in \{0, 1\}$ 

Input	Output
Α	Y
0	1
1	0

Find inhibitory rules and a threshold, then code the inhibitory rules and the threshold in the machine!

### Representing NOR Function

- In reality, I have 2 inputs and want to calculate the NOR value of these inputs.
- How do I tell a machine to do this task for me?
- For a NOR neuron to fire, we want ALL the inputs 1 to be 0 so we take them all as inhibitory inputs, i.e. set below rule:

if  $x_1 = 1$  or  $x_2 = 1$  then y = 0

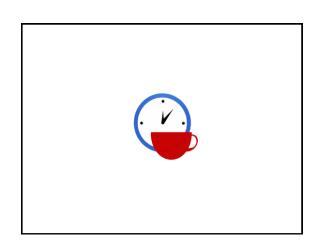
set the thresholding parameter to 0,		Inputs	
$g(x) = x_1 + x_1 \ge ?$	х	Υ	Z
	0	0	1
$y \in \{0, 1\}$	0	1	0
y * (0,2)	1	0	0
	1	1	0

Find inhibitory rules and a threshold, then code the inhibitory rules and the threshold in the machine!

### Limitations Of M-P Neuron

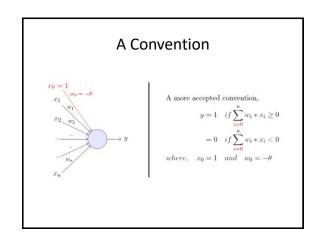
- What about non-boolean (say, real) inputs?
- Do we always need to hand code the threshold?
- Are all inputs equal? What if we want to assign more importance to some inputs?
- What about functions which are not linearly separable? Say XOR function.

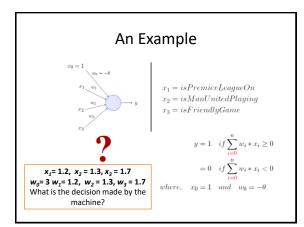


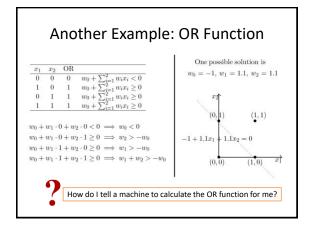


### Perceptron Rewriting the above, = 0 if $\sum_{i=1}^{n} w_i * x_i - \theta < 0$ Perceptron Model (Minsky-Papert in 1969) It overcomes some of the limitations of the M-P neuron by introducing the concept of *numerical weights* (a measure of importance) for inputs, and a mechanism for

learning those weights.







### Perceptron vs. McCulloch-Pitts Neuron

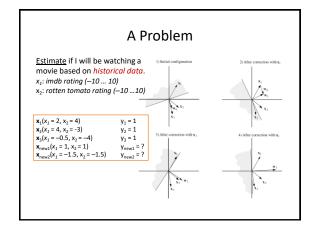
McCulloch Pitts Neuron (assuming no inhibitory inputs)

$$y = 1 \quad if \sum_{i=0}^{n} x_i \ge 0$$
$$= 0 \quad if \sum_{i=0}^{n} x_i < 0$$

$$y = 1 \quad if \sum_{i=0}^{n} \underline{w_i} * x_i \ge 0$$
$$= 0 \quad if \sum_{i=0}^{n} \underline{w_i} * x_i < 0$$

- A single Perceptron can only be used to implement linearly separable functions, just like the M-P neuron.
- Then what is the difference?
- Here, the weights, including the threshold can be learned and the inputs can be *real* values. What does this mean?





### Perceptron Learning Algorithm [1]

- The training set consists of two sets (containing m elements), P and N, in n+1 - dimensional extended input space:  $\mathbf{x} = [x_0, x_1, x_2, ..., x_n]$
- We *look for* a vector  $\mathbf{\omega} = [\omega_0, \omega_1, \omega_2, ..., \omega_n]$  capable of absolutely separating both sets, so that all vectors in P belong to the open positive half-space and all vectors in N to the open negative half-space of the linear separation.

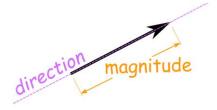
### Batch Perceptron

input: A training set  $(\mathbf{x}_1,y_1),\dots,(\mathbf{x}_m,y_m)$ initialize:  $\mathbf{w}^{(1)} = (0, \dots, 0)$ for  $t=1,2,\ldots$ if  $(\exists i \text{ s.t. } y_i \langle \mathbf{w}^{(t)}, \mathbf{x}_i \rangle \leq 0)$  then  $\mathbf{w}^{(t+1)} = \mathbf{w}^{(t)} + y_i \mathbf{x}_i$ else output  $\mathbf{w}^{(t)}$ 

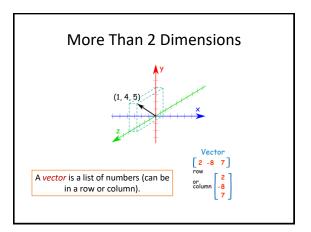


### **Vector Review**

- https://www.mathsisfun.com
- A vector has *magnitude* and *direction*:



# Vector a in Cartesian Coordinates y (x,y) 100,y 12 (12,5) 5 10 15 x



### **Dot Product**

- The Dot Product gives a number as an answer (a "scalar", not a vector).
- Method 1 (definition):

 $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$ 

Where:

 $|\mathbf{a}|$  is the magnitude (length) of vector  $\mathbf{a}$ 

- | b | is the magnitude (length) of vector b
- $\theta$  is the angle between  ${f a}$  and  ${f b}$
- Method 2 (definition):

 $\mathbf{a} \cdot \mathbf{b} = \mathbf{a}_{\mathsf{x}} \times \mathbf{b}_{\mathsf{x}} + \mathbf{a}_{\mathsf{y}} \times \mathbf{b}_{\mathsf{y}}$ 

So we multiply the x's, multiply the y's, then add.

### In A n+1 Dimensional Space

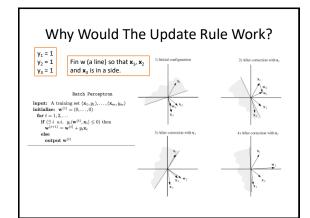
$$\mathbf{w}^T \mathbf{x} = \|\mathbf{w}\| \|\mathbf{x}\| \cos \alpha \qquad \cos \alpha = \frac{\mathbf{w}^T \mathbf{x}}{\|\mathbf{w}\| \|\mathbf{x}\|}$$

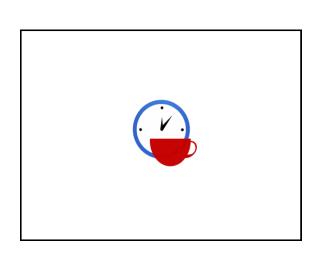
$$\mathbf{w} = [w_0, w_1, w_2, ..., w_n]$$

$$\mathbf{x} = [1, x_1, x_2, ..., x_n]$$

$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w}^{\mathrm{T}} \mathbf{x} = \sum_{i=1}^{n} w_i * x_i$$

 When the <u>dot product</u> of two vectors is 0, they are perpendicular to each other.





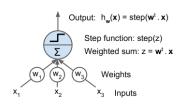
### Linear Threshold Unit [2]

- The LTU computes a weighted sum of its inputs:
  - $z = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n = \mathbf{w}^T \cdot \mathbf{x}$
- Then applies a step function to that sum and outputs the result:  $h_w(\mathbf{x}) = \text{step}(\mathbf{z}) = \text{step}(\mathbf{w}^T \cdot \mathbf{x}).$

Output:  $h_{\mathbf{w}}(\mathbf{x}) = \text{step}(\mathbf{w}^t \cdot \mathbf{x})$ Common step functions: Step function: step(z) Weighted sum: z = wt. x

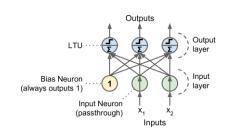
### Training An LTU

- Training an LTU means finding the right values for w<sub>0</sub>, w<sub>1</sub>, and w<sub>2</sub>
- · The training algorithm is discussed shortly.



### Perceptron With Multiple Outputs

• A Perceptron is simply composed of a single layer of LTUs.

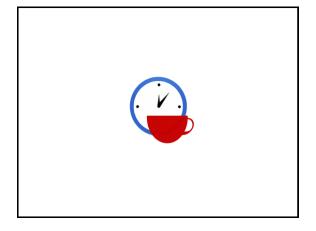


### How is a Perceptron Trained?

- The Perceptron is fed one training instance at a time, and for each instance it makes its predictions.
- For every output neuron that produced a wrong prediction, it reinforces the connection weights from the inputs that would have contributed to the correct prediction.
- Perceptron *learning rule* (weight update):

$$w_{i,j}^{\text{(next step)}} = w_{i,j} + \eta (\hat{y}_j - y_j) x_i$$

- $x_i$  is the  $i^{th}$  input value of the *current training instance*.
- $-\ \ \gamma_{j}$  is the  $\emph{target output}$  of the  $j^{th}$  output neuron for the current training instance.
- n is the learning rate.



### The MNIST Dataset

- The MNIST dataset, which is a set of 70,000 small images of digits students and employees of the US Census Bureau.
- Each image is labeled with the digit it represents.
- Scikit-Learn provides many helper functions to download popular datasets. MNIST is one of them.

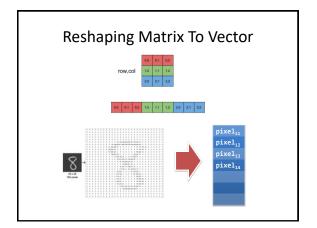
00000000000 /11/1////// handwritten by high school 22222222 3333333333 444444444 555555555 7777777777 888**88**88*888*8**8** 999999999

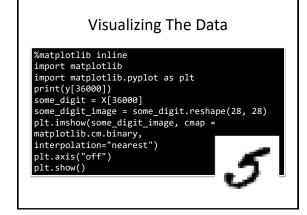
# import numpy as np from sklearn.datasets import fetch\_openml def sort\_by\_target(mnist): reorder\_train = np.array(sorted([(target, i) for i, target in enumerate(mnist.target[:60000])])];; 1 reorder\_test = np.array(sorted([(target, i) for i, target in enumerate(mnist.target[:60000])])]; 1 mist.tadat[:60000] = mnist.tadar[corder\_train] mist.tadat[:60000] = mnist.target[reorder\_train] mist.target[:60000] = mnist.target[reorder\_train] mist.target[:60000] = mnist.target[reorder\_test + 60000] try: mist = fetch\_openml('mnist\_784', version=1, cache=True) mist.target = mnist.target.astype(np.int8) # fetch\_openml() returns targets as strings sort\_by\_target(mnist) # fetch\_openml() returns an unsorted dataset from sklearn.datasets import fetch\_mldata mnist = fetch\_mldata('MNIST original')

```
Loading Data

X, y = mnist["data"], mnist["target"]
print('X.shape:', X.shape)
print('y.shape:', y.shape)

X.shape: (70000, 784)
y.shape: (70000,)
```





```
Preparing Training Data

The MNIST dataset is actually already split into a training set (the first 60,000 images) and a test set (the last 10,000 images):

X train, X_test, y_train, y_test = X[:60000], X[60000:], y[:60000], y[60000:]

# Shuffle the training set;
# this will guarantee that all cross-validation folds will be similar import numpy as np shuffle_index = np.random.permutation(60000)

X train, y_train = X train[shuffle_index], y_train[shuffle_index] print ('X_train.shape:', y_train.shape)
print ('X_train.shape:', y_train.shape)
print ('Y_test.shape:', y_test.shape)

X_train.shape: (60000, 784)
y_train.shape: (60000, 784)
y_test.shape: (10000, 784)
y_test.shape: (10000, 784)
y_test.shape: (10000, 784)
```

```
Normalizing Image Inputs

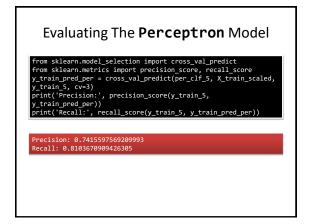
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler()
X train scaled =
scaler.fit_transform(X_train.astype(np.float64))
X_test_scaled = scaler.fit_transform(X_test.astype(np.float64))
print ('X_train_scaled.shape:', X_train_scaled.shape)
print ('X_test_scaled.shape:', X_test_scaled.shape)

X_train_scaled.shape: (60000, 784)
X_test_scaled.shape: (10000, 784)
```

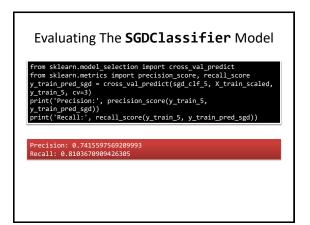
# # Only try to identify one digit-for example, the number 5 y\_train\_5 = (y\_train == 5) # True for all 5s, False for all other digits. y\_test\_5 = (y\_test == 5) print ('y\_train:', y\_train) print ('y\_train\_5:', y\_train\_5) y\_train: [2 1 7 ... 5 9 8] y\_train\_5: [False False False ... True False False]

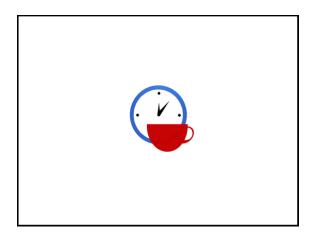
# Training a Perceptron for Number 5 from sklearn.linear\_model import Perceptron per\_clf\_5 = Perceptron(max\_iter = 1000, tol = 3, random\_state=42) per\_clf\_5.fit(X\_train\_scaled, y\_train\_5) Perceptron(alpha=0.0001, class\_weight=None, early\_stopping=False, eta0=1.0, fit\_intercept=True, max\_iter=1000, n\_iter=None, n\_iter\_no\_change=5, n\_jobs=None, penalty=None, random\_state=42, shuffle=True, tol=3, validation\_fraction=0.1, verbose=0, warm\_start=False)

# # The classifier guesses that this image does not represents a 5 (Correct). print ('Prediction:', per\_clf\_5.predict([X\_test\_scaled[4000]])) Prediction: [False]



### from sklearn.linear\_model import SGDClassifier # Stochastic Gradient Descent (SGD) classifier sgd\_clf\_5 = SGDClassifier(max\_iter = 1000, tol = 3, random\_state = 42, loss="perceptron", learning\_rate="constant", eta0=1, penalty=None) sgd\_clf\_5.fit(X\_train\_scaled, y\_train\_5) SGDClassifier(alpha=0.0001, average=False, class\_weight=None, early\_stopping=False, epsilon=0.1, eta0=1, fit\_intercept=True, ll\_ratio=0.15, learning\_rate='constant', loss='perceptron', max\_iter=1000, n\_iter=None, n\_iter\_no\_change=5, n\_jobs=None, penalty=None, powen\_t=0.5, random\_state=42, shuffle=True, tol=3, validation\_fraction=0.1, verbose=0, warm\_start=False)





# Perceptron from Scratch (I) import numpy as np class Perceptron(object): def \_\_init\_\_(self, n\_features, n\_iter=100, learning\_rate=0.01): self.n\_iter = n\_iter self.learning\_rate = learning\_rate self.weights = np.zeros(n\_features + 1) def fit(self, X, y): unique, counts = np.unique(y, return\_counts=True) total\_pos = counts[1] total\_nps = counts[0] for \_iter in range(1, self.n\_iter): true\_pos, true\_neg = self.update\_weights(X, y) self.display\_info(\_iter, true\_pos, true\_neg, total\_pos, total\_neg)

### 

```
Perceptron from Scratch (III)

def display.info(self, _iter, true_pos, true_neg, total_pos, total_neg):
    if 0 == iter % 10:
        print ('Iteration:', iter)
        print ('Iteration:', iter)
        print ('Iteration:', self.weights[0])
        print ('Itrue_nos/total_pos:', true_nog*100/total_pos)
        print ('Itrue_nos/total_pos:', true_neg*100/total_neg)

np_per_clf = Perceptron(784)
np_per_clf.frit(X_train_scaled, _train_5)
print ('Prediction:', np_per_clf.predict([some_digit]))

Iteration: 10
    weights[0]: -11.7299999999999999

true_pos/total_pos: 78.55484043534404
true_neg/total_neg: 97.85962599778505

treation: 20
    weights[0]: -12.20999999999984
true_pos/total_pos: 78.93377605607822
true_neg/total_neg: 97.893662599778505
```

