IBM OPEN SCIENCE PRIZE 2021

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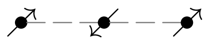
Introduction:

This paper provides an overview of IBM Open Science 2021.

Goal:

Simulating the evolution (or time evolving) of the state $|110\rangle$ to time $t = \pi$ under the XXX Heisenberg Hamiltonian H_{Heis} for a 3 particles of spin-$(1/2)$ aligned



We will use 3 qubits from the 7 qubits `ibmq_jakarta` computer $|q_6\, q_5\, q_4\, q_3\, q_2\, q_1\, q_0\rangle$ so

$$|110\rangle \rightarrow |0101000\rangle$$

The goal is to simulate the evolution of $|110\rangle$ with the best fidelity as possible using Trotterization with at least 4 steps $(n \geq 4)$.

Spin Model:

The XXX Heisenberg spin model

$$H_{\text{Heis}} = \sum_{\langle ij \rangle}^{N} J \left(\sigma_x^{(i)} \sigma_x^{(j)} + \sigma_y^{(i)} \sigma_y^{(j)} + \sigma_z^{(i)} \sigma_z^{(j)} \right). \tag{1}$$

We take $N = 3, J = 1$

How to simulate an interaction?

1. You must have the Hamiltonian describing this interaction.
2. Get its corresponding evolution operator.
3. Implementation: decomposition of U onto q-gates.

$$H_{\text{Heis}} \longrightarrow U_{\text{Heis}} \longrightarrow \mathbb{G} \tag{2}$$

4. Verifing the fidelity of the evolved state at time $t = \pi$.

The Evolution Operator:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H\, |\psi(t)\rangle \;; \quad |\psi(t)\rangle = U\, |\psi(t=0)\rangle$$

$$U_{\text{heis3}}(t) = e^{-itH_{\text{heis3}}} \tag{3}$$

Totterization:

With $H^{(0,1)}_{\text{Heis1}} = \sum_{k=1}^{3} \sigma_k^{(0)} \sigma_k^{(1)}$

$$U_{\text{Heis3}}(t) = \exp\left[-it\left(H^{(0,1)}_{\text{Heis2}} + H^{(1,2)}_{\text{Heis2}}\right)\right]$$

The two Hamlit. don't **comute** $\left[\, H^{(0,1)}_{\text{Heis2}} + H^{(1,2)}_{\text{Heis2}} \,\right] \neq 0 \;\Rightarrow\; e^{H^{(1,2)}_{\text{Heis2}} + H^{(0,1)}_{\text{Heis2}}} \neq e^{H^{(1,2)}_{\text{Heis2}}}\, e^{H^{(0,1)}_{\text{Heis2}}}$

$$U_{\text{Heis3}}(t) \approx \left[\exp\left(\frac{-it}{n} H^{(0,1)}_{\text{Heis2}}\right) \exp\left(\frac{-it}{n} H^{(1,2)}_{\text{Heis2}}\right)\right]^{n}. \tag{4}$$

Where n is the number of *Trotter* sterps[1]. We can than write

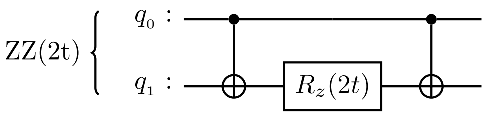
$$U_{\text{Heis3}}(t) \approx \left[XX\left(\frac{2t}{n}\right)^{(0,1)} YY\left(\frac{2t}{n}\right)^{(0,1)} ZZ\left(\frac{2t}{n}\right)^{(0,1)} XX\left(\frac{2t}{n}\right)^{(1,2)} YY\left(\frac{2t}{n}\right)^{(1,2)} ZZ\left(\frac{2t}{n}\right)^{(1,2)} \right]^{n} \tag{5}$$

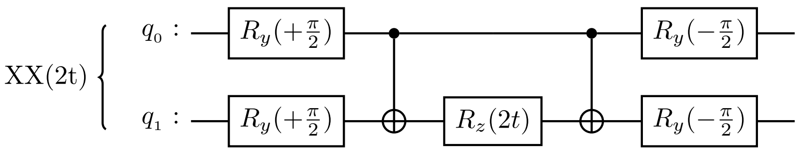
Where $KK(2t) = e^{-it\sigma_k \otimes \sigma_k}$, $k = x, y, z$. We have now to implement U_{heis3}:

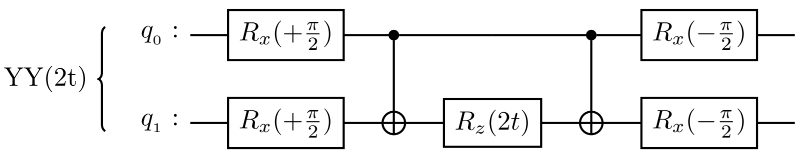
$$\mathbf{ZZ(2t)} = e^{-it\sigma_z \otimes \sigma_z} = \mathbb{CX}\,(\mathbb{I} \otimes \mathbb{R}_z(2t))\,\mathbb{CX}$$

$$\mathbf{XX(2t)} = (\mathbb{R}_y(\tfrac{\pi}{2})) \otimes \mathbb{R}_y(\tfrac{\pi}{2}))\;\mathbf{ZZ(2t)}\;(\mathbb{R}_y(-\tfrac{\pi}{2}) \otimes \mathbb{R}_y(-\tfrac{\pi}{2}))$$

$$\mathbf{YY(2t)} = (\mathbb{R}_x(\tfrac{\pi}{2})) \otimes \mathbb{R}_x(\tfrac{\pi}{2}))\;\mathbf{ZZ(2t)}\;(\mathbb{R}_x(-\tfrac{\pi}{2}) \otimes \mathbb{R}_x(-\tfrac{\pi}{2}))$$

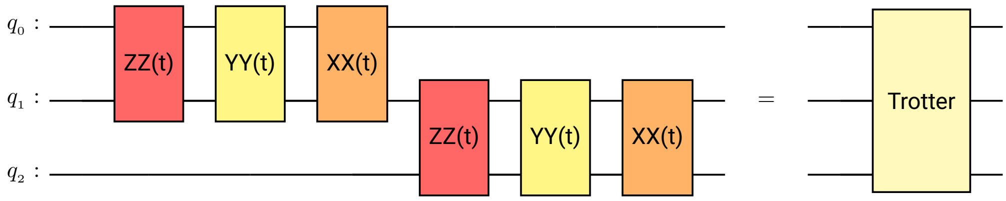




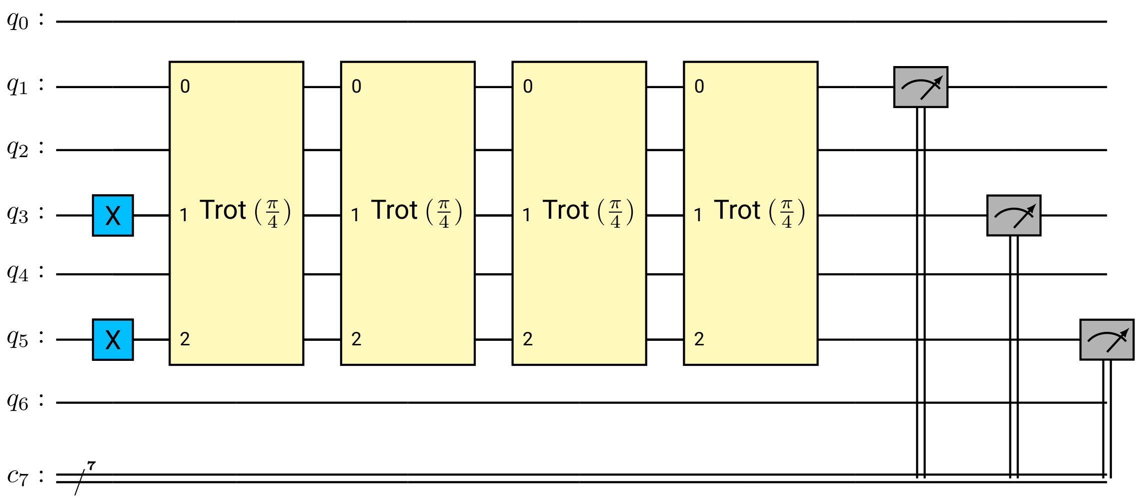


[1] This is not the unique way!
$\sigma_x = \mathbb{R}_y(\frac{\pi}{2})\,\sigma_z\,\mathbb{R}_y(-\frac{\pi}{2})$; $\quad \sigma_y = -(\mathbb{R}_x(\frac{\pi}{2})\,\sigma_z\,\mathbb{R}_x(-\frac{\pi}{2}))$.

Combine subcircuits into single gate representing one ($n = 1$) trotter step.



Time evolve the state $|110\rangle$ from $t = 0$ to $t = \pi$ under H_{heis3} using **4 trotter** steps:



State Tomography:

You have 3 qubits q_0, q_1, q_2, which states are respectively

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$$
$$|\psi\rangle_2 = \frac{1}{\sqrt{2}}(|0\rangle - e^{i\frac{\pi}{2}}\,|1\rangle)$$

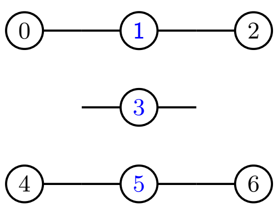
When we measure them we get 50% the state $|0\rangle$ and 50% the state $|1\rangle$. so we can't difere from them!

• The solution is to measure the state in **different basis** and this is called *"state tomography"*.

Appendix

The IBM Q Jakarta Characteristics

- Num. of Qubits: 7
- Quantum Volume: 16
- Qlops: 2.4k
- Avg. CNOT Error: 8.485e-3
- Avg. Readout Error: 3.591e-2



Info of: `january 28, 2022`.