CAMP and Bootstrap

ISYE/MATH 6783 - Assignment4

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***Abstract*—CAPM is a widely used single factor pricing model which describes the relationship between individual assets and the market. In this report, CAPM is validated using the log return of ten stocks, market return and risk free rate. Nine out of ten stocks support CAPM. To improve accuracy, bootstrap is performed. With bootstrap, the regression results of all ten stocks indicate that the CAPM model is valid.**

I. Introduction

The stock market is extremely complex and unpredictable. But people never give up on find a universally applicable method to model stock prices. The Capital Asset Pricing Model (CAPM) is an empirical model that describes the relationship between price of an individual asset and market. The CAPM is popular due to its simplicity and utility in a variety of situations. The model is formulated as below.

$$R_{asset} = R_f + \beta * (R_{mkt} - R_f) + \alpha + \epsilon$$

CAPM states that if all investors follow certain assumptions, α should be zero. The assumptions are:

1. Aim to maximize economic utilities (Asset quantities are given and fixed).
2. Are rational and risk-averse.
3. Are broadly diversified across a range of investments.
4. Are price takers, i.e. they cannot influence prices.
5. Can lend and borrow unlimited amounts under the risk free rate of interest.
6. Trade without transaction or taxation costs.
7. Deal with securities that are all highly divisible into small parcels (All assets are perfectly divisible and liquid).
8. Have homogeneous expectations.
9. Assume all information is available at the same time to all investors.

The rest of this report is organized as follows. Section 2 will introduce the test results of ten stocks on CAPM and the confidence intervals of coefficients. CAPM with bootstrap will be done in Section 3 followed by conclusions in Section 4.

II. Validation of CAPM

A. Data Manipulation

Two datasets are given. One contains monthly returns of the S&P500 and the rates of the 3-month U. S. Treasury bill from January 1994 to December 2006. The other contains the monthly log returns of ten stocks. And the excess return is defined as below.

$$excess\_return = stock\_return - risk\_free\_rate$$

where the risk free rate is approximated by the rates of 3-month treasury bills. Also, since the units and time of these rates differs, all data is transformed in to monthly log returns before factor analysis is performed.

B. Linear regression

To fit the CAPM model, linear regression is performed. By estimating the coefficients of $stockreturn \sim marketreturn$, the CAPM model can be validated if the intercept term is zero. Besides coefficients, the 95% confidence intervals of alpha and beta are also calculated. The 95% confidence interval means the probability that a random sample falls into this range is 0.95.

The result of linear regressions for each stock is shown in Table I.

As shown in the table, all α are small and the estimated betas with\without alpha are slightly different.

The confidence intervals of α and β are shown in Table II.

The confidence intervals of α containing zero indicates that there is a high possibility that α is equal to zero. This is identical to the p values of linear regression which means the hypythesis $H_0 : \alpha = 0$ holds, indicating that CAPM is valid.

TABLE I
CAPM ALPHA AND BETA

Regression	Alpha	Beta	Beta w/o intercept
AAPL	0.004	1.3751	1.3723
ADBE	0.0049	1.5248	1.5215
ADP	0.001	0.8418	0.8411
AMD	-0.0003	2.313	2.3132
DELL	0.0091	1.6636	1.6573
GTW	-0.0051	2.2244	2.2279
HP	0.0021	0.8705	0.8691
IBM	0.0028	1.3397	1.3378
MSFT	0.0045	1.4489	1.4459
ORCL	0.0041	1.5621	1.5593

TABLE II
95% CONFIDENCE INTERVALS

Regression	95% CI of Alpha	95% CI of Beta
AAPL	(-0.0059, 0.0140)	(0.8185, 1.9317)
ADBE	(-0.0048, 0.0145)	(0.9865, 2.0632)
ADP	(-0.0028, 0.0047)	(0.6343, 1.0493)
AMD	(-0.0120, 0.0114)	(1.6617, 2.9643)
DELL	(0.001, 0.0172)	(1.211, 2.1161)
GTW	(-0.0163, 0.0060)	(1.6052, 2.8436)
HP	(-0.0051, 0.0093)	(0.4690, 1.2720)
IBM	(-0.0019, 0.0076)	(1.0753, 1.6041)
MSFT	(-0.0013, 0.0103)	(1.1261, 1.7718)
ORCL	(-0.0046, 0.0129)	(1.0733, 2.0509)

TABLE III
CAPM ALPHA AND BETA WITH BOOTSTRAP

Bootstrap	Alpha	Std(Alpha)	Beta	Std(Beta)
AAPL	0.0037	0.0052	1.3765	0.3141
ADBE	0.0048	0.0047	1.5311	0.2622
ADP	0.001	0.0018	0.8438	0.1226
AMD	-0.0003	0.0058	2.3062	0.3692
DELL	0.0091	0.0042	1.6577	0.2575
GTW	-0.005	0.0055	2.266	0.4301
HP	0.0019	0.0035	0.8683	0.1856
IBM	0.0029	0.0024	1.34	0.1439
MSFT	0.0045	0.0029	1.4621	0.1691
ORCL	0.004	0.0044	1.5532	0.2673

III. CAPM WITH BOOTSTRAP

Bootstrap is a sampling technique that helps reduce estimation error by randomly picking sample from the original dataset with replacement. Then run regression on each sample dataset and take the mean of coefficients.

The result of CAPM with bootstrap is shown in Table III

The result looks similar compared to the coefficients without bootstrap. However, as shown in Table IV, the 99% confidence intervals indicate that the hypothesis $H_0 : \alpha = 0$ holds for all stocks.

TABLE IV
CONFIDENCE INTERVALS WITH BOOTSTRAP

Bootstrap	99% CI of Alpha	99% CI of Beta
AAPL	(-0.0095, 0.0177)	(0.6782, 2.3678)
ADBE	(-0.0075, 0.0164)	(0.8821, 2.1574)
ADP	(-0.0037, 0.0053)	(0.5349, 1.1722)
AMD	(-0.0150, 0.0141)	(1.3471, 3.3606)
DELL	(-0.0025, 0.0200)	(1.0626, 2.3475)
GTW	(-0.0190, 0.0081)	(1.3068, 3.5459)
HP	(-0.0066, 0.0114)	(0.3768, 1.3513)
IBM	(-0.0033, 0.0083)	(0.9803, 1.7085)
MSFT	(-0.0030, 0.0117)	(1.0525, 1.9340)
ORCL	(-0.0076, 0.0155)	(0.8210, 2.2215)

IV. CONCLUSION

Given the log return of ten stocks, market return and risk free rate, linear regression is perfermed to test CAPM. With bootstrap, result shows that α of all all stocks are zero which confirms the CAPM model is valid.

Appendix A
R code

```
# compute market excess log return
mkt < − read.csv("m_sp500ret_3mtcm.txt", header = T, sep = 'nt', skip = 1)
mkt$log_rf < − log(mkt$X3mTCM/100 + 1)/12
mkt$ex_logret < − log(mkt$sp500+1) - mkt$log_rf
mkt < − subset(mkt, select=c("Date", "ex_logret", "log_rf"))

# compute stock excess log return
stock < − read.csv("m_logret_10stocks.txt", header = T, sep = 'nt')
stock$Date < − NULL
stock < − na.omit(stock)
stock_ex < − stock - mkt$log_rf

alphas < − c()
betas < − c()
betas1 < − c()
ci < − c()

for (i in 1:10){
# regression
linreg < − lm(stock_ex[,i]   mkt$ex_logret)
alphas < − c(alphas, linreg$coefficients[1])
betas < − c(betas, linreg$coefficients[2])
#print(summary(linreg))

# confidence interval
interval.beta < − confint(linreg, level=0.95)
#print (interval.beta)
ci < − cbind(ci, interval.beta)

# regression without intercept
linreg < − lm(stock_ex[,i] ∼ mkt$ex_logret - 1)
betas1 < − c(betas1, linreg$coefficients[1])
}

# bootstrap
bootcapm< −function(x,y,B=500)
ind< −seq(1,length(x))
bootCoeff< −matrix(0,B,3)
for(b in 1:B){
bootind< −sample(ind,replace=T)
yb< −y[bootind]
xb< −x[bootind]
fitb< −lm(yb xb)
bootCoeff[b,]< −c(fitb$coef,mean(yb)/sd(yb))
}
return(bootCoeff)
}
```

```
alphas < − c()
betas < − c()
ci_alpha < − c()
ci_beta < − c()
std_alpha < − c()
std_beta < − c()

set.seed(12345)
for (i in 1:10){
boot < − bootcapm(mkt$ex_logret, stock_ex[,i], 1000)
alphas < − c(alphas, mean(boot[,1]))
betas < − c(betas, mean(boot[,2]))
ci_alpha < − c(ci_alpha, quantile(boot[,1], c(0.005, 0.995)))
ci_beta < − c(ci_beta, quantile(boot[,2], c(0.005, 0.995)))
std_alpha < − c(std_alpha, sd(boot[,1]))
std_beta < − c(std_beta, sd(boot[,2]))
```