Assignment on Risk Management

Due Date: Dec 3, 2015

Value-at-Risk (VaR) and Expected Shortfall (ES)

As one of the well-known risk measures, value-at-risk (VaR) measures the potential loss from extreme negative returns. VaR is often defined in dollars, denoted by \$VaR, so that the \$VaR loss is implicitly defined from the probability of getting an even larger loss as in

$$\Pr\left(\$Loss > \$VaR\right) = p$$

In the case with returns, a VaR is defined as

$$\Pr\left(r > -VaR\right) = p.$$

Thus, the -VaR is defined as the number so that we would get a worse return only with probability p. That is, we are $(1-p)100\%$ confident that we will get a return better than -VaR.

Expected shortfall is an alternative risk measure that is more sensitive to the shape of the loss distribution. Expected shortfall is also called conditional value-at-risk (CVaR) or TailVaR, and defined as

$$ES = -E\left[r | r < -VaR\right]$$

1. Assume that you invest \$500,000 in IBM (PERMNO = 12490) and \$500,000 in GE (PERMNO = 12060). Using daily returns in CRSP DSF dataset from *January 2001 to December 2006*, compute one-day 5% VaR, \$VaR, and expected shortfall of your portfolio. (*Hint.* Use a historical distribution of daily returns)

2. For the same portfolio, use daily returns from *January 2001 to December 2009* and compute one-day VaR, \$VaR, and expected shortfall of your portfolio. Explain how historical distributions and the risk measures are different from the answers in Question 1.

Volatility modeling

A simple RiskMetrics Model

JP Morgan's RiskMetrics variance model, also known as exponential smoother, considers the variance dynamic model as follows:

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)r_t^2 \tag{1}$$

That is, the RiskMetrics model's forecast for tomorrow's volatility can be seen as a weighted average of today's volatility and today's squared return. When estimating a parameter λ on a large number of assets, RiskMetrics found that the estimates were quite similar across assets, and they simply set $\lambda = 0.94$ for every asset for daily variance forecasting.

3. Compute variances (annualized) of IBM and GE using daily returns from January 1996 to December 2000 and use them as initial values for RiskMetrics model. That is, using equation (1) with $\lambda = 0.94$ and computed standard deviations as initial values, generate and plot time-series of variance (σ_{t+1}^2) for IBM and GE from *January 2001 to December 2009.*

GARCH (Generalized Autoregressive Conditional Heteroskedasticity)

The GARCH(h,k) model, introduced by Bollerslev (1986), is of the form

$$\sigma_{t+1}^2 = \omega + \sum_{i=0}^{h} \beta_i\sigma_{t-i}^2 + \sigma_{j=1}^{k}\alpha_j r_{t-j}^2. \tag{2}$$

The simplest GARCH model of dynamic variance (GARCH(1,1)) can be written as

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha r_t^2. \tag{3}$$

The RiskMetrics model can be seen as a special case of GARCH(1,1) if we set $\alpha = 1-\lambda$, $\beta = \lambda$, and $\omega = 0$

4. Using daily returns of IBM and GE from from January 1996 to December 2000, estimate parameters α and β for IBM and GE. (*Hint.* See PROC AUTOREG)

5. Using estimated parameters in Question 5 and sample variances of IBM and GE over the period January 1996 to December 2000, generate and plot time-series of variance (σ_{t+1}^2) for IBM and GE from *January 2001 to December 2009.* Compare the plot with the result of Question 3.