Midterm Project

Due: Oct 26 , 2015

Implementing a dynamic delta hedging strategy.

Delta-hedging is a particular hedging strategy that aims to replicate the value of an option written on a traded asset through dynamically buying (or selling) a proper number of shares of the underlying asset and borrowing from (or lending to) a bank.

Description of the delta-hedging process:

1. Let the hedging period be from a start-date t_0 to an end-date t_N. At start date t_0, assuming there is an initial cash position in the amount of \$500.
2. At t_0, we sell a European call option contract with expiration date T, strike price K. Assume the option contract is written on one share of stock and $t_0 < t_N \le T$.
3. To hedge the short position in the European call, we decide to buy Δ shares of the underlying stock at t_0, where $\Delta = \frac{\partial V}{\partial S}$ is the rate of change of option value V with respect to changes in the underlying price S.
4. As Δ changes during the hedging period, we need to re-balance our portfolio (buy/sell stocks) everyday to maintain a long position of Δ_i shares of stock for each date t_i, $i = 1, 2, ..., N$. Δ_i should be calculated using implied volatility for each date.
5. For each date $t_i, i = 1, 2, ..., N$, calculate the hedging error:
$$HE_i = \Delta_{i-1}S_i + B_{i-1}e^{r_{i-1}dt} - V_i$$
where $B_i = V_i - \Delta_i S_i$. S_i, V_i, r_i are stock price, option price, risk-free rate for date $t_i, i = 0, 1, ..., N$. dt represents 1 business day, which is $\frac{1}{252}$ year. Cumulative hedging error until t_n is $\sum_{i=1}^{n} HE_i$.
6. For each date t_i, the current total wealth = cash + wealth(long position in the stock) + wealth(short position in the call option). Assume no transaction costs.

Project tasks are:

1. Test your delta hedging implementation using the Black-Scholes model.
   - Use the following model to simulate the price series $\{S_0, S_{\Delta t}, S_{2\Delta t}, \cdots, S_T\}$:
   $$S_{t+\Delta t} = S_t + \mu S_t \Delta t + \sigma S_t \sqrt{\Delta t} Z_t.$$
   - Apply Black-Scholes formula to obtain the option price series $\{C_0, S_{\Delta t}, C_{2\Delta t}, \cdots, C_T\}$.
   - Assume $S_0 = 100$, $T = 0.4$, $\mu = 0.05$, $\sigma = 0.24$, $r = 0.025$, $N = 100$. Consider a European Call option on S_T with $K = 105$ and $T = 0.4$. Construct the delta-hedging portfolios for all N periods and report the hedging error.
2. Use the real market data given in the project data files to test the validity of Black-Scholes model by using the Black-Scholes formula to construct the delta-hedging portfolio.
   (a) For each date t_i, calculate the total wealth if we sell a call without putting on any hedge.
   (b) Given t_0, t_N, T and K, the program should output a file "result.csv" containing stock price, option price, implied volatility, delta, hedging error, cumulative hedging error, wealth, wealth(no hedge) :
   For example, if t_0=2011-07-05, t_N=2011-07-29, T=2011-09-17, $K = 500$, the output should be like:

date	S	V	...	wealth	wealth(no hedge)
2011-07-05	532.44	44.2			
2011-07-06	535.36	46.9			
...					
2011-07-28	610.94				
2011-07-29	603.69				

(c) Data files are provided as below. Data files should not be modified manually or by other tools.

- "interest.csv" contains daily risk-free rates in 2011. When calculating implied volatility and delta for each date, use the risk-free rate of corresponding date.
- "sec_GOOG.csv" contains adjusted closing stock prices in 2011. Assume there is no dividend.
- "op_GOOG.csv" contains option prices data in 2011. Option price=(**best_bid**+**best_offer**)/2. **cp_flag** is C for call option, P for put option. **exdate** is expiration date. For option at date t_i, time to expiry = (number of business days between t_i and T)/252.
  Note: Feel free to modify the sample codes in lab3 to make it work. You're free to design your own Option classes, but It's required to use OOP or generic programming style to finish this project. Using only functions or trivial classes without any member variables are not acceptable.