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Homework 3

September 18, 2015

Due: September 28, 2015

1. Design a StdNormalCDF class to approximate the standard normal cumulative distribution function $\Phi(x), x \in \mathbb{R}$ following the algorithm in notes. Sample files are provided.

   Test your implementation by output $\Phi(-3)$, $\Phi(3)$ and check whether their summation is equal to 1.

2. Based on problem 1's StdNormalCDF class, implement the Black-Scholes formulas for European-style Vanilla Call and Put Options.

   Calculate the Black-Scholes price for a call option with strike $K = 100$, current price $S_0 = 100$, time to expiration $\tau = 0.5$, risk free rate $r = 0.01$, underlying volatility $\sigma = 0.2$ and dividend yield $q = 0$.

   Calculate a put option's Black-Scholes price with the same parameters and verify the put-call parity.

   Note: There are multiple ways to implement Black-Scholes formulas. It could be simple functions with many parameters, or member functions in class Option, or an independent BSCalculator class communicating with Option class. Although any implementation is acceptable, a design with object-oriented style is highly encouraged here.

3. Investigation of the near term behavior of option prices as S_0 and σ vary.

   Based on problem 2's implementation, output a csv-file "problem3.csv" containing a table of Black-Scholes prices for a call option with fixed $K = 100, \tau = 0.5, r = 0.01, q = 0$ and varying σ, S_0. σ is from 0.1 to 0.6 with increment 0.05, while S_0 is from 85 to 115 with increment 5.

   The expected output table is like below:

S\\σ	0.1	0.15	...	0.55	0.6
85					
90					
...					
110					
115					

4. Consider an option portfolio consisting of a long position in the call option in problem 3 with $\tau = 0.5$ and a short position in a call option with a different time to expiration $\tau' = 0.4$. Both call options have the same strike and the same underlying. (This position is called a "Call Calendar Spread".)

   Output a csv-file "problem4.csv" of the portfolio's value for varying σ and S_0. All the other parameters remains the same as problem 3.