**Appendix**

**Matlab Code**

% cubic spline interpolation

% set values

x = [0.639, 0.648, 0.657, 0.666];

y = [0.019944, 0.006335, -0.007263, -0.020850];

n=length(x);

h=diff(x);

v = diff(y);

% solve for Mi

b0(1)=0;

b0(n)=0;

A(1,1) = 1;

A(n,n) = 1;

for i=2:n-1

A(i,i) = 1/3 \* (h(i-1) + h(i));

A(i,i-1) = 1/6 \* h(i-1);

A(i,i+1) = 1/6 \* h(i);

b0(i)=(y(i+1) - y(i)) ./ h(i) - (y(i) - y(i-1)) ./ h(i-1);

end

m = A\b0'

% compute coefficients

a = y;

b = v ./ h - 1/2 \* h .\* m(1:n-1,1)' - 1/6 \* h .\* (diff(m))';

c = m / 2;

d = 1/6 \* diff(m)' ./ h;

% xx1 = 0.639:0.001:0.648;

% xx2 = 0.648:0.001:0.657;

% xx3 = 0.657:0.001:0.666;

% yy1 = a(1) + b(1) .\* (xx1-x(1)) + c(1) .\* (xx1-x(1)).^2 + d(1).\*(xx1-x(1)).^3;

% yy2 = a(2) + b(2) .\* (xx2-x(2)) + c(2) .\* (xx2-x(2)).^2 + d(2).\*(xx2-x(2)).^3;

% yy3 = a(3) + b(3) .\* (xx3-x(3)) + c(3) .\* (xx3-x(3)).^2 + d(3).\*(xx3-x(3)).^3;

% xx = [xx1,xx2,xx3];

% yy = [yy1,yy2,yy3];

%

% plot(x,y,'o',xx,yy)

% compute the root of PHI(x) = 0 using Newton's Method

s\_0 = 0;

s = 0.5;

while abs(s\_0 - s) > 0.0001

s\_0 = s;

fx = a(2) + b(2) .\* (s-x(2)) + c(2) .\* (s-x(2)).^2 + d(2).\*(s-x(2)).^3;

fxp = b(2) + 2\*c(2).\*(s - x(2)) + 3\*d(2).\*(s - x(2)).^2;

s = s - fx ./ fxp;

end

s

us = (1-s) - (a(2) + b(2) .\* (s-x(2)) + c(2) .\* (s-x(2)).^2 + d(2).\*(s-x(2)).^3);

usp = -1 - (b(2) + 2\*c(2).\*(s - x(2)) + 3\*d(2).\*(s - x(2)).^2);

usp2 = -(2\*c(2) + 6\*d(2).\*(s - x(2)));

% plug into BS equation

r = 0.05;

q = 0.02;

sigma = 0.2;

bs = 0.5 \* sigma^2 \* s^2 \* usp2 + (r - q) \* s \* usp - r \* us

**Result:**

