中山大学本科生期末考试

考试科目：《 高 等 代 数 》（A 卷）

学年学期：**2017** 学年第 **1** 学期　　　　姓　　名：＿＿＿＿＿＿＿＿

学　院/系：数据科学与计算机学院　　　　学　　号：＿＿＿＿＿＿＿＿

考试方式：闭卷　　　　年级专业：＿＿＿＿＿＿＿＿

考试时长：120 分钟　　　　班　　别：＿＿＿＿＿＿＿＿

任课老师：

警示 《中山大学授予学士学位工作细则》第八条：“考试作弊者，不授予学士学位。”

------------以下为试题区域，共三道大题，总分 100 分，考生请在答题纸上作答------------

一、客观题（共 5 小题，每小题 3 分，共 15 分）

1） (3%) Each eigenvalue of A is also an eigenvalue of A^2. (True or False，justify)

2） (3%) If W is a subspace of $\mathbb{R}^n$, then W and $W^\perp$ have no vectors in common. (True or False，justify)

3） (3%) If $\boldsymbol{\alpha_1}=(1,1,0,0)$, $\boldsymbol{\alpha_2}=(0,0,1,1)$, $\boldsymbol{\alpha_3}=(1,0,1,0)$, $\boldsymbol{\alpha_4}=(1,1,1,1)$, then the maximal linearly independent set is

(A) $\boldsymbol{\alpha_1},\boldsymbol{\alpha_2}$　　(B) $\boldsymbol{\alpha_1},\boldsymbol{\alpha_2},\boldsymbol{\alpha_3}$　　(C) $\boldsymbol{\alpha_1},\boldsymbol{\alpha_2},\boldsymbol{\alpha_4}$　　(D) $\boldsymbol{\alpha_1},\boldsymbol{\alpha_2},\boldsymbol{\alpha_3},\boldsymbol{\alpha_4}$

4） (3%) The general solution of $A\mathbf{x}=\mathbf{0}$ is $\mathbf{x}=\boldsymbol{c_1}\begin{pmatrix}1\\0\\2\end{pmatrix}+\boldsymbol{c_2}\begin{pmatrix}0\\1\\-1\end{pmatrix}$, then A is

(A) $\begin{pmatrix}-2 & -1 & 1\end{pmatrix}$　　(B) $\begin{pmatrix}2 & 0 & -1\\0 & 1 & 1\end{pmatrix}$　　(C) $\begin{pmatrix}-1 & 0 & 2\\0 & 1 & -1\end{pmatrix}$　　(D) $\begin{pmatrix}-2 & 1 & 1\end{pmatrix}$

5） (3%) If A is a $n\times n$ matrix, $\boldsymbol{x_1},\boldsymbol{x_2}$are solutions of $A\boldsymbol{x}=\boldsymbol{\beta}$, and $\boldsymbol{x_1}\neq\boldsymbol{x_2}$, then the determinant of A is

(A) 0　　(B) 1　　(C) $\boldsymbol{x_1^T x_2}$　　(D) the determinant of A depends on whether $\boldsymbol{\beta}$ is $\mathbf{0}$.

二、计算题（共 6 小题，共 65 分）

1) Let $A=(\boldsymbol{a_1}\quad\boldsymbol{a_2}\quad\boldsymbol{a_3})=\begin{pmatrix}2 & 2 & -1\\2 & -1 & 2\\-1 & 2 & 2\end{pmatrix}$, $B=(\boldsymbol{b_1}\quad\boldsymbol{b_2})=\begin{pmatrix}1 & 4\\0 & 3\\-4 & 2\end{pmatrix}$,

(a) (5%) prove that $\boldsymbol{a_1},\boldsymbol{a_2},\boldsymbol{a_3}$ form a basis of $\mathbb{R}^3$.

(b) (5%) compute the coordinates of b_1,b_2 relative to this basis $\{\boldsymbol{a_1},\boldsymbol{a_2},\boldsymbol{a_3}\}$.

2) (10%) Compute the determinant of the following matrix:

$$D=\begin{vmatrix} a & b & c & d \\ a & a+b & a+b+c & a+b+c+d \\ a & 2a+b & 3a+2b+c & 4a+3b+2c+d \\ a & 3a+b & 6a+3b+c & 10a+6b+3c+d \end{vmatrix}$$

3) Let $A=\begin{pmatrix} -1 & 1 & 0 \\ -4 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$,

(a) (5%) compute the eigenvalues of the given matrix.

(b) (5%) compute the eigenvectors of the eigenvalues in (a).

4) Let $A=\begin{pmatrix} 1 & -2 & 2 \\ -2 & -2 & 4 \\ 2 & 4 & -2 \end{pmatrix}$, (a) (5%) diagonalize A. (b) (5%) compute A^n.

5) Given the Vandermonde determinant: $D=\begin{vmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \\ \dots & \dots & \dots & \dots \\ x_1^{n-1} & x_2^{n-1} & \dots & x_n^{n-1} \end{vmatrix}$

(a) (5%) when n = 2, compute D.

(b) (5%) determine the general form of the determinant.

6) Let $A=\begin{pmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{pmatrix}$,

(a) (5%) prove columns of A are linearly independent.

(b) (5%) find an orthogonal basis for *ColA*.

(c) (5%) find a QR factorization of A.

三、证明题（共 3 小题，共 20 分）

1) (5%) If $\boldsymbol{a_1}+\boldsymbol{a_2}, \boldsymbol{a_2}+\boldsymbol{a_3}, \boldsymbol{a_3}+\boldsymbol{a_4}, \boldsymbol{a_4}+\boldsymbol{a_5}, \boldsymbol{a_5}+\boldsymbol{a_1}$ are linearly independent, prove that $\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}, \boldsymbol{a_4}, \boldsymbol{a_5}$ are linearly independent.

2) (5%) Let A is $n \times n$ symmetric matrix, U,V are two subspaces of $\mathbb{R}^n$, and $U=\{\boldsymbol{x}\in\mathbb{R}^n | A\boldsymbol{x}=\boldsymbol{0}\}$, $V=\{A\boldsymbol{x} \mid \boldsymbol{x}\in\mathbb{R}^n\}$. Please prove that U is the orthogonal complement of V in $\mathbb{R}^n$.

3) Let A, B are $n \times n$ matrics，please prove:

(a) (5%) If $A^3=B^3, A^2B=B^2A,$ and A^2+B^2 is invertible, then $A=B$.

(b) (5%) If $A+B$ is invertible, then $A(A+B)^{-1}B=B(A+B)^{-1}A$.