中山大学本科生期末考试

考试科目：《高等代数》（B卷）

学年学期：2019年第1学期　　姓　　名：________

学 院/系：数据院　　学　　号：________

考试方式：闭卷　　年级专业：________

考试时长：120分钟　　班　　别：________

警示 《中山大学授予学士学位工作细则》第八条："考试作弊者，不授予学士学位。"

------------以下为试题区域，共三道大题，总分100分，考生请在答题纸上作答------------

一、问答题（共 3 小题，共 20 分）

1) (5 分) For a matrix equation $A\vec{x}=\vec{b}$, where $\vec{x}$ is the solution vector, please tell (1) when there is no solution, (2) when there is a unique solution, and when there are infinitely many solutions.

2) (9 分) Detail the following matrix factorization: 1) LU factorization; 2) Diagonalization; 3) QR decomposition.

3) (6 分) What is the difference between orthogonal set and orthonormal set? What is orthogonal matrix?

二、计算题（共 5 小题，共 60 分）

1) (15 分) Compute the reduced echelon form for the following matrix

$$\begin{pmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{pmatrix}$$

Please point out the pivot positions. Is the above matrix invertible? If it is invertible, please compute its inverse.

2) (12 分) Compute the determinant of the following matrices (6% for each):

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} x & -1 & 0 & 0 \\ 0 & x & -1 & 0 \\ 0 & 0 & x & -1 \\ a_4 & a_3 & a_2 & x+a_1 \end{pmatrix}$$

3) (8 分) Are the following vectors linearly independent? Why?

$$\vec{a}_1=\begin{pmatrix} 1 \\ -2 \\ 4 \\ -8 \end{pmatrix}, \vec{a}_2=\begin{pmatrix} 1 \\ 3 \\ 9 \\ 27 \end{pmatrix}, \vec{a}_3=\begin{pmatrix} 1 \\ 4 \\ 16 \\ 64 \end{pmatrix}, \vec{a}_4=\begin{pmatrix} 3 \\ -3 \\ 3 \\ -3 \end{pmatrix}$$

4) (12 分) Given the following matrix

$$A=\begin{pmatrix}1 & 1 & 1\\ 3 & -5 & 1\\ -1 & 5 & -2\\ 3 & -7 & 8\end{pmatrix}$$

Please

(a) Describe its column space and a basis for it. (3%)
(b) Compute the rank of matrix A (3%)
(c) Find an orthogonal basis for the column space of matrix A. (6%)

5) (13 分) Given the following matrix

$$A=\begin{pmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 0 & 0 & 2\end{pmatrix}$$

Please

(a) Compute its eigenvalues and eigenvectors . (6%)
(b) Is matrix A diagonalizable? If it is, please diagonalize it? (5%)
(c) What is the determinant value of A? (2%)

三、证明题（共 3 小题，共 20 分）

1) (8 分) Suppose $\{\vec{w}_1,\cdots,\vec{w}_m\}$ is an orthogonal basis of a subspace W of R^n. Then

(a) (2 分) For any $\vec{y}$ in R^n, what is its orthogonal projection onto W?

(b) (6 分) Denote the orthogonal projection of $\vec{y}$ onto W by $\hat{\vec{y}}$. Please prove that there exist $\vec{z}\in W^{\perp}$ such that $\vec{y}=\hat{\vec{y}}+\vec{z}$. Please also prove this kind of decomposition, namely reconstructing any vector by addition of two vectors in W and the complementary $W^{\perp}$ respectively, is unique.

2) (6 分) Let $W=\{(x_1 \quad x_2 \quad \cdots \quad x_n)^T \mid x_i\in R, i=1,2,\cdots n; x_1+x_2+\cdots x_n=0\}$. Please prove that W is a subspace of R^n.

3) (6 分) Let $\vec{\eta}$ be a solution of matrix equation $A\vec{x}=\vec{b}$, where A is a matrix. Let $\{\vec{\xi}_1,\vec{\xi}_2,\cdots,\vec{\xi}_r\}$ be a basis of the null space of matrix A, and $\vec{b}$ is not zero. Please prove:

(a) (3 分) $\vec{\eta},\vec{\xi}_1,\vec{\xi}_2,\cdots,\vec{\xi}_r$ are linearly independent

(b) (3 分) $\vec{\eta},\vec{\eta}+\vec{\xi}_1,\vec{\eta}+\vec{\xi}_2,\cdots,\vec{\eta}+\vec{\xi}_r$ are linearly independent