BỘ CÔNG THƯƠNG
ĐẠI HỌC CÔNG NGHIỆP TP. HỒ CHÍ MINH

Bài giảng

KỸ THUẬT ĐIỆN – ĐIỆN TỬ

ELECTRICITY AND ELECTRONICS

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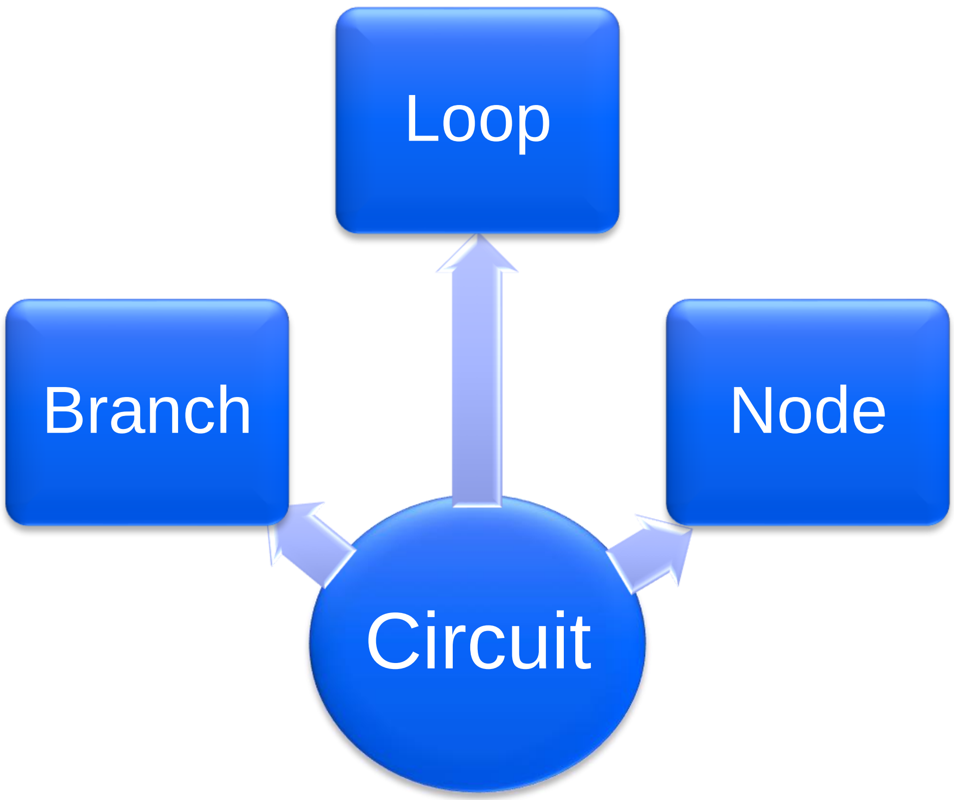
Chapter 2: DC circuit analysis methods

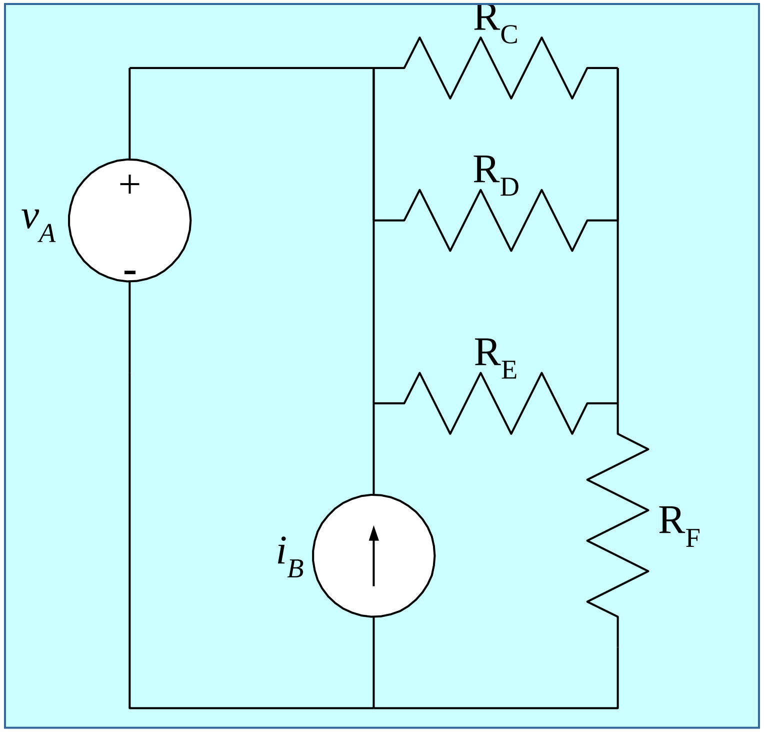
- Nodes, Loops, and Branches
- Kirchhoff's Current Law
- Kirchhoff's Voltage Law
- Series and Parallel Connected Sources
- Resistors in Series and Parallel
- Voltage and Current Division
- Methods of Circuit Analysis

Structure of an electrical circuit



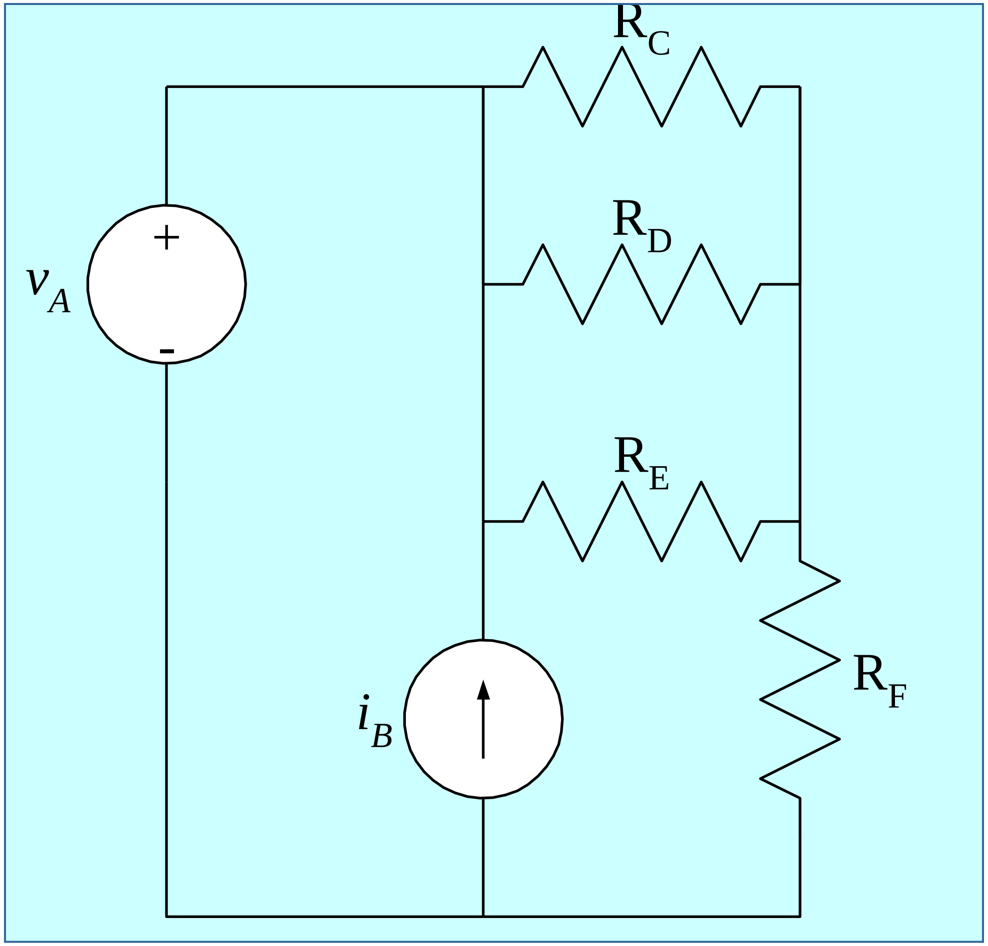
2.1 Nodes, Loops, and Branches

- **A node** is defined as a point where two or more components are connected.
- The key thing to remember is that we connect components with wires. It doesn't matter how many wires are being used; it only matters how many components are connected together.



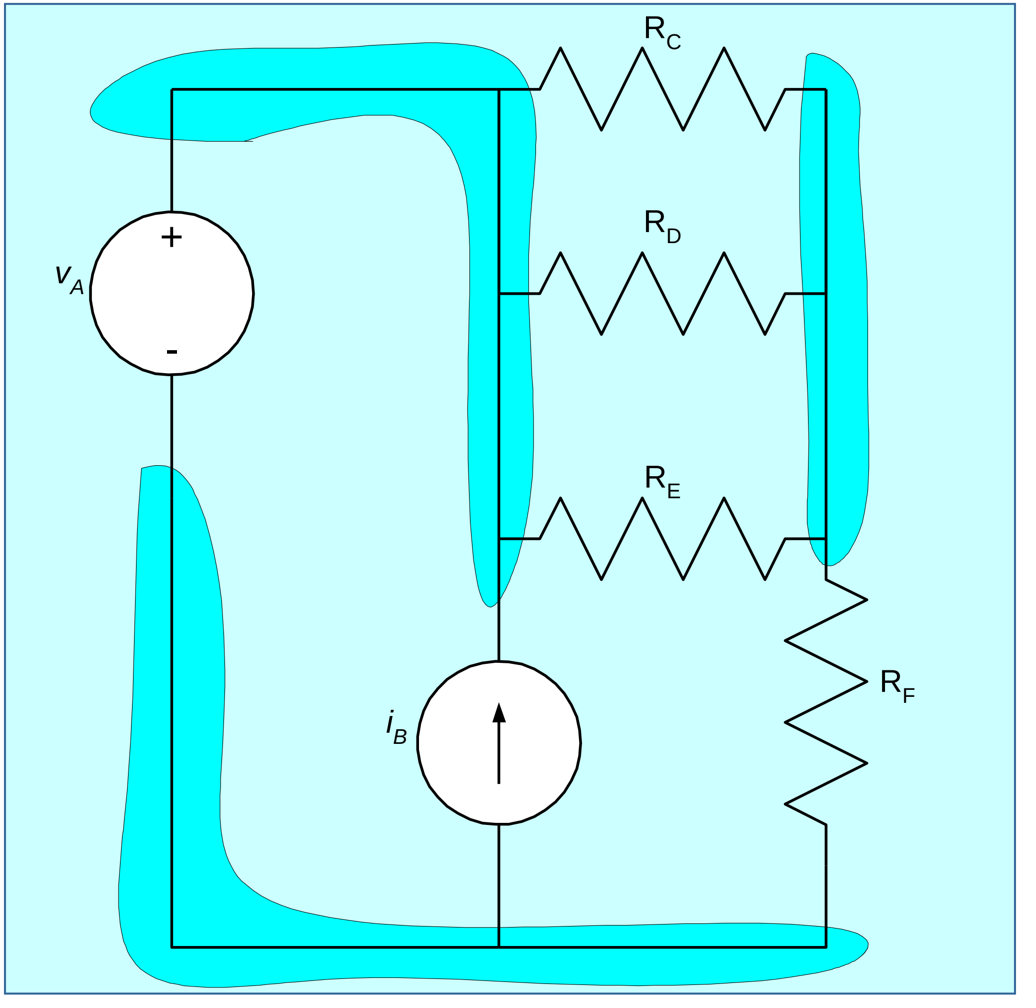
Example: How Many Nodes?

- To test understanding of nodes, let's look at the example circuit schematic given here.
- How many nodes are there in this circuit?



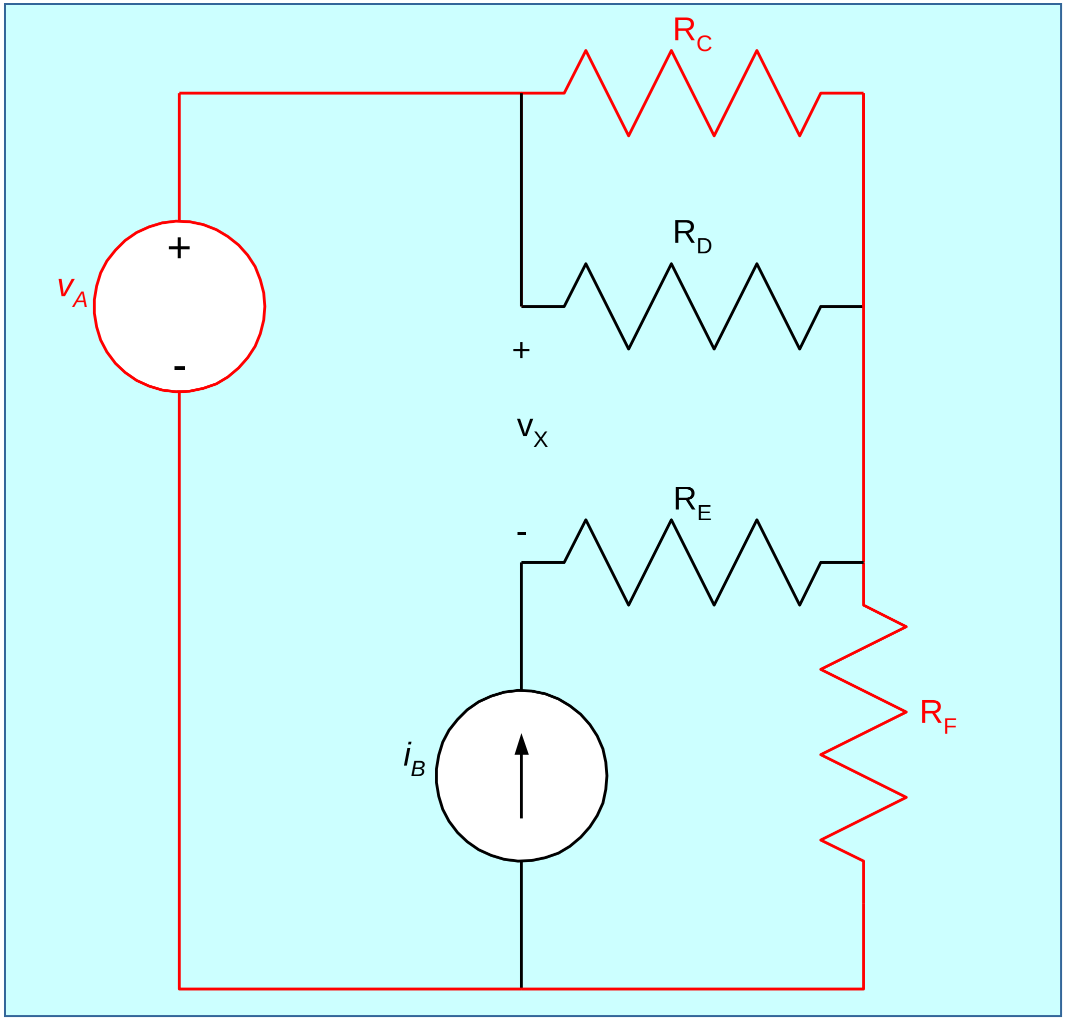
Example: How Many Nodes?

- In this schematic, there are **three** nodes. These nodes are shown in dark blue here.
- Some students count more than three nodes because they have considered two points connected by a wire to be two nodes.

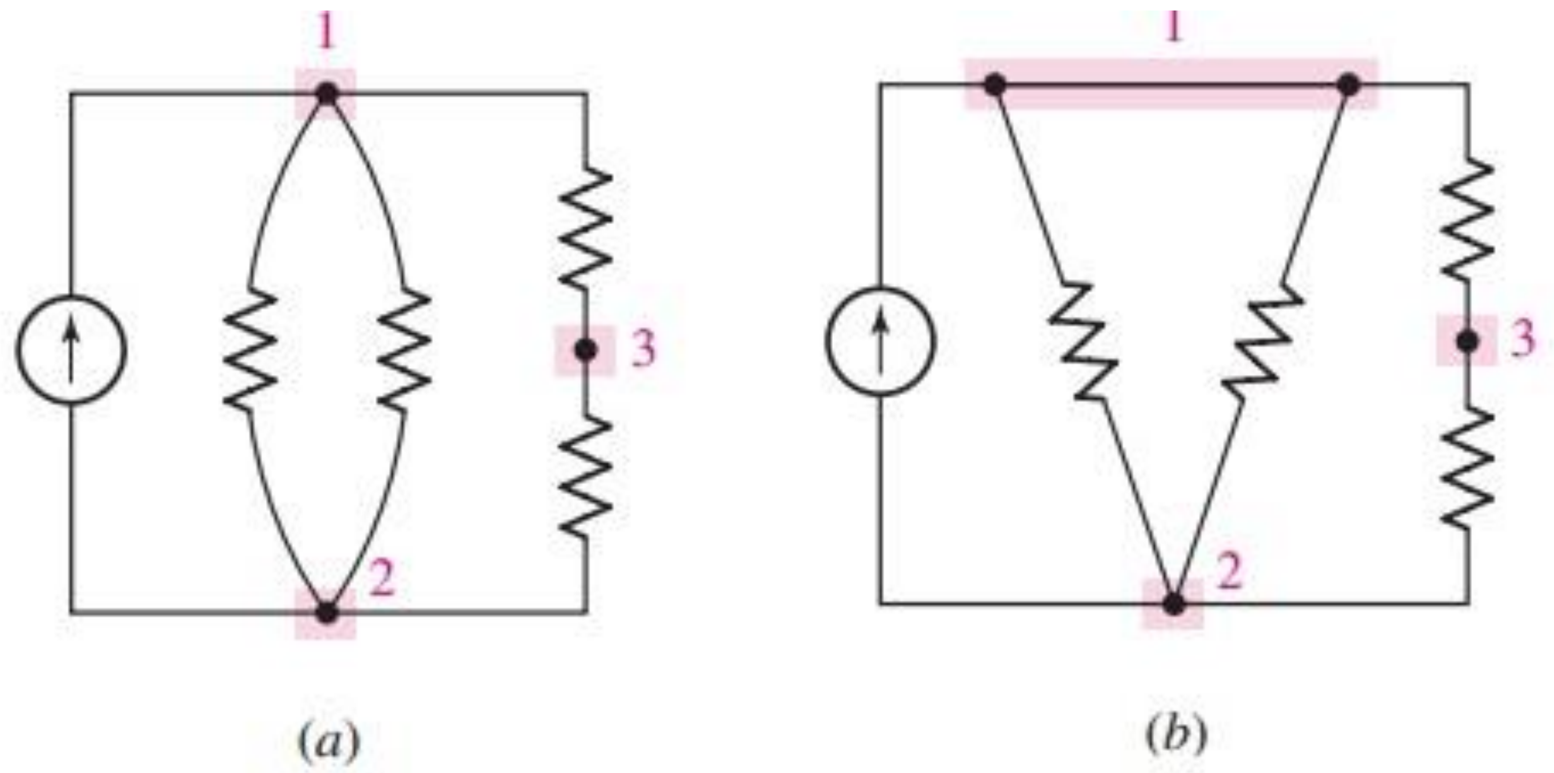


2.1 Nodes, Loops, and Branches

- **A Loop** can be defined in this way: Start at any node and go in any direction and end up where you start.
- Here is a loop we will call Loop #1. The path is shown in red.



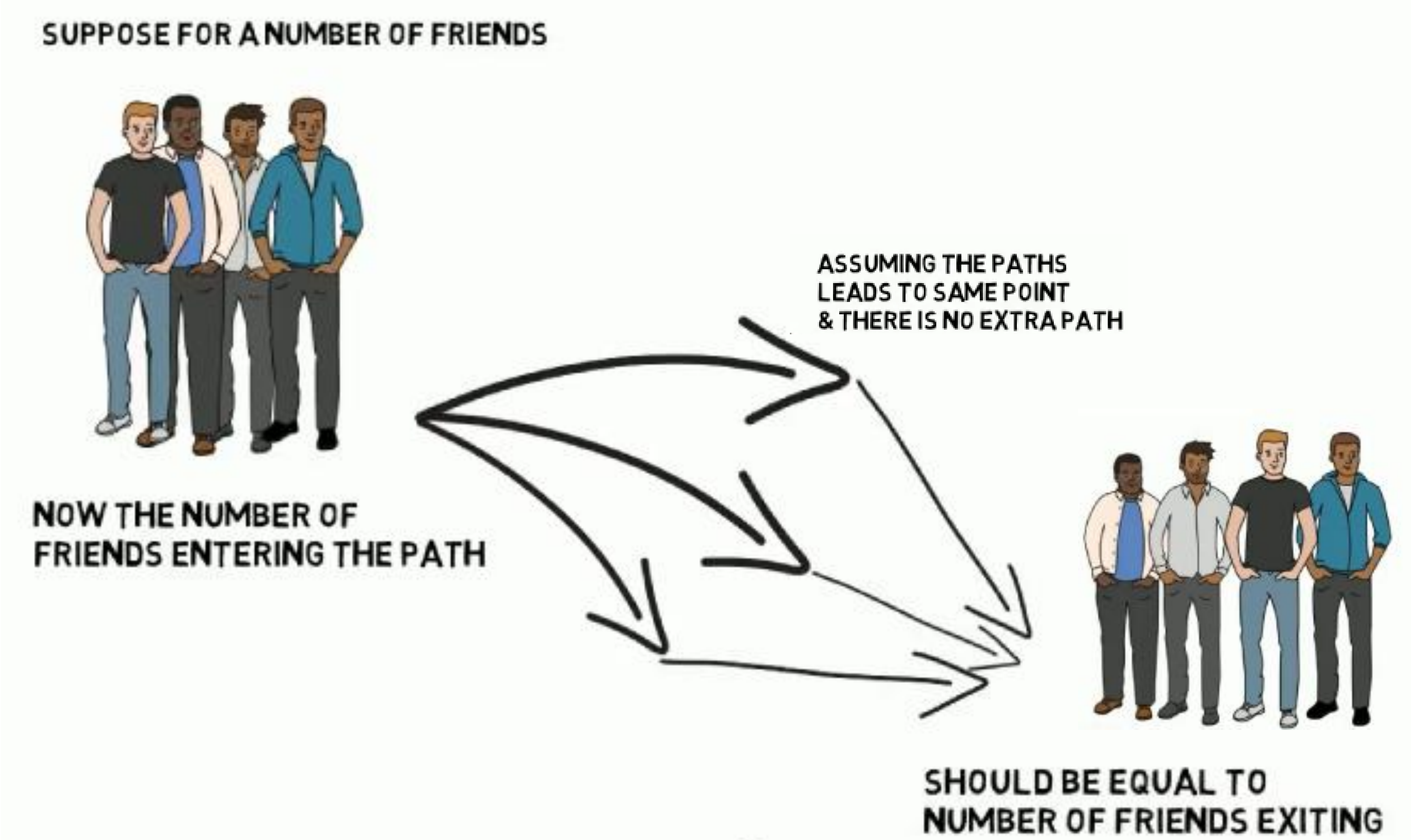
Example: Find nodes, loops and branches



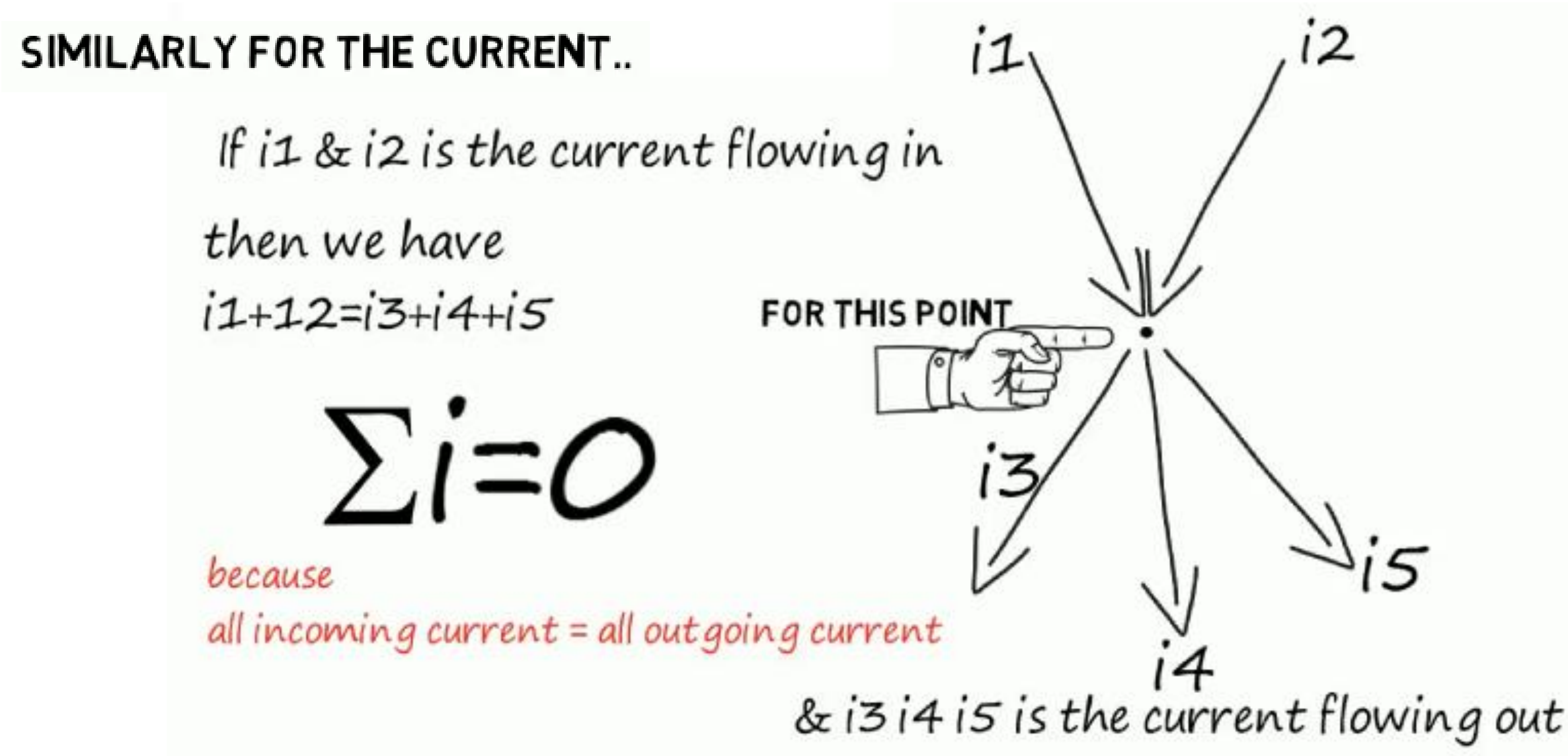
(*a*) A circuit containing three nodes and five branches. (*b*) Node 1 is redrawn to look like two nodes; it is still one node.

Movie: Kirchhoff’s current law

2.2 Kirchhoff's Current Law



2.2 Kirchhoff's Current Law

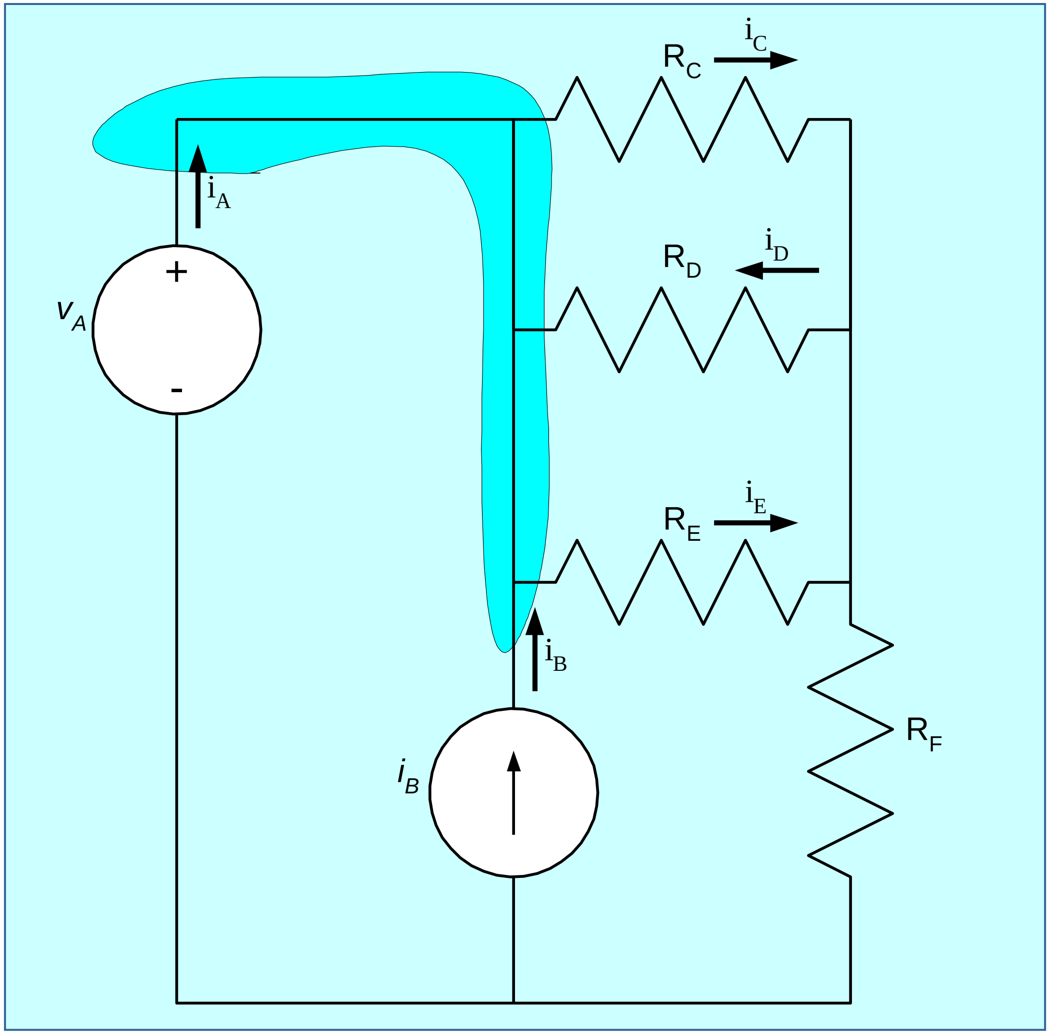


Kirchhoff's Current Law

2.2 Kirchhoff's Current Law

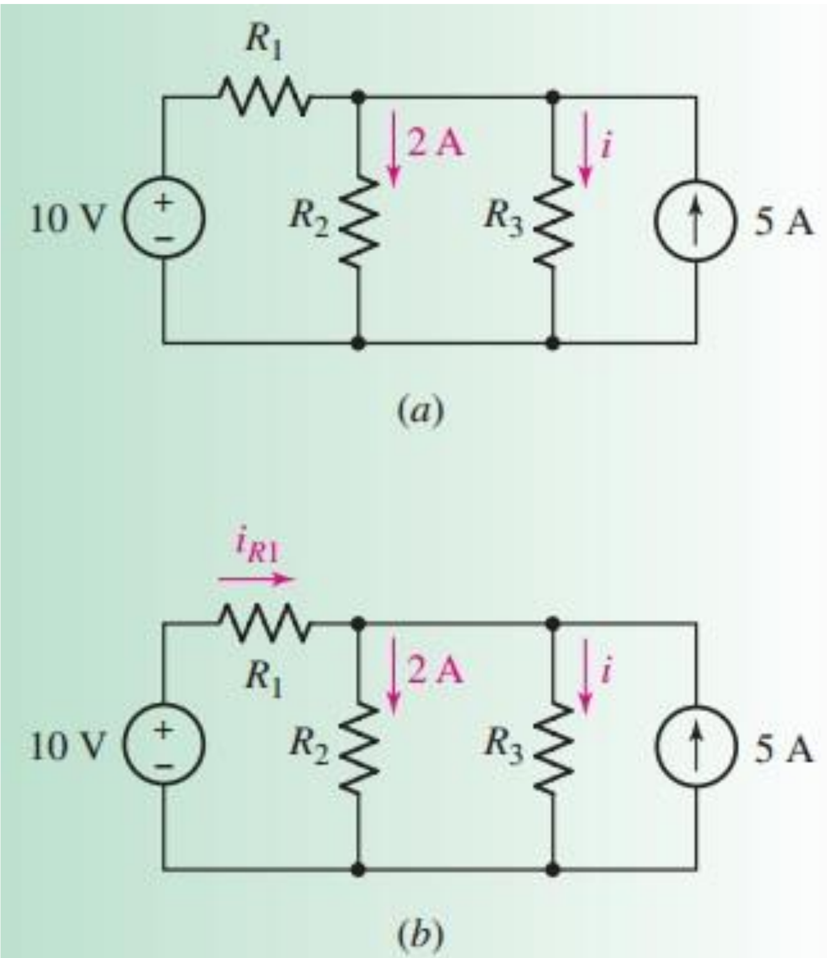
Example: assigned reference polarities for all of the currents for the nodes indicated in darker blue.
For this circuit, and using my rule, we have the following equation:

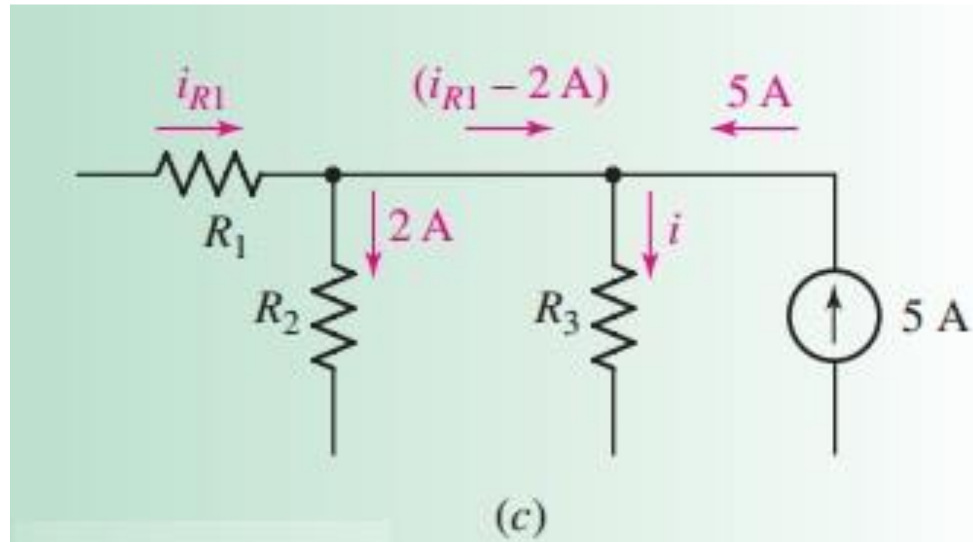
$$-i_A + i_C - i_D + i_E - i_B = 0$$



Example 1 of Kirchhoff’s Current Law-

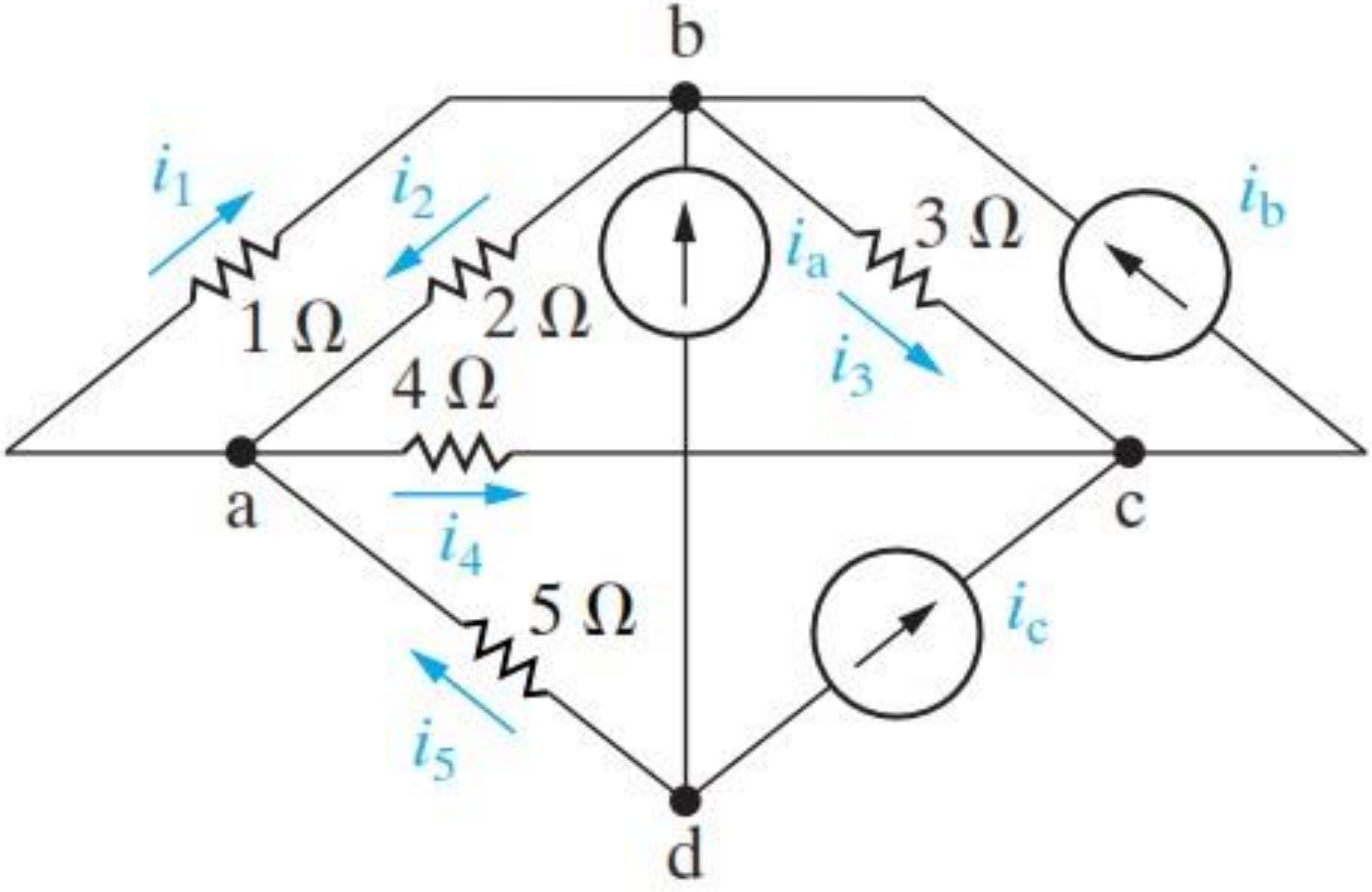
Find currents go across the paths





(*a*) Simple circuit for which the current through resistor R_3 is desired. (*b*) The current through resistor R_1 is labeled so that a KCL equation can be written. (*c*) The currents into the top node of R_3 are redrawn for clarity.

Example 2: Find current equations of Node a, b, c, d



2.2 Kirchhoff's Current Law

Solution of Example 2:

The current equations of Node a, b, c, d

node a $\quad i_1 + i_4 - i_2 - i_5 = 0,$

node b $\quad i_2 + i_3 - i_1 - i_b - i_a = 0,$

node c $\quad i_b - i_3 - i_4 - i_c = 0,$

node d $\quad i_5 + i_a + \mathrm{i}_c = 0.$

2.3 Kirchhoff's Voltage Law

Suoppose you go for a walk..

and at some point you meet a number of friends..

Now at end

total number of friends you met = number of friends at path

So KVL states that..

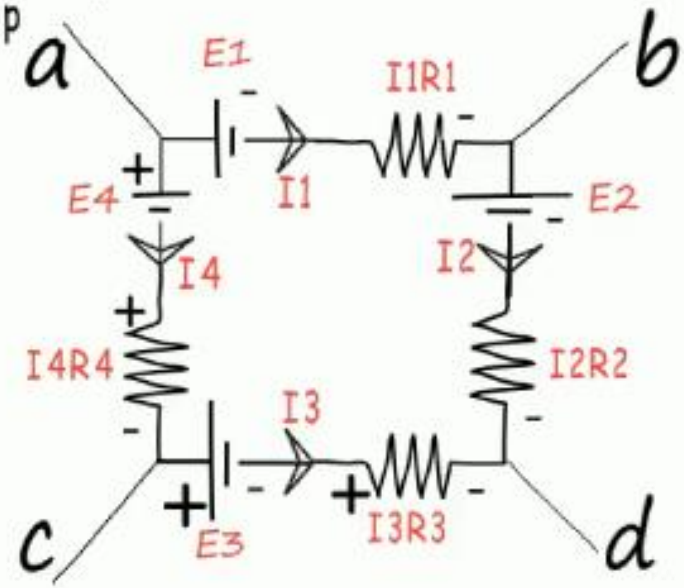
(KIRCHOFF'S VOLTAGE LAW)

In any closed electrical circuit algebric sum of product of current and resistance and the sum of EMF connected in it is equal to zero

2.3 Kirchhoff's Voltage Law

Example

Suppose you go from point a>b>c>d>a covering a whole loop



The first component you meet is>-E1

second component>-I1R1

third component >-E3

fourth component >-I2R2

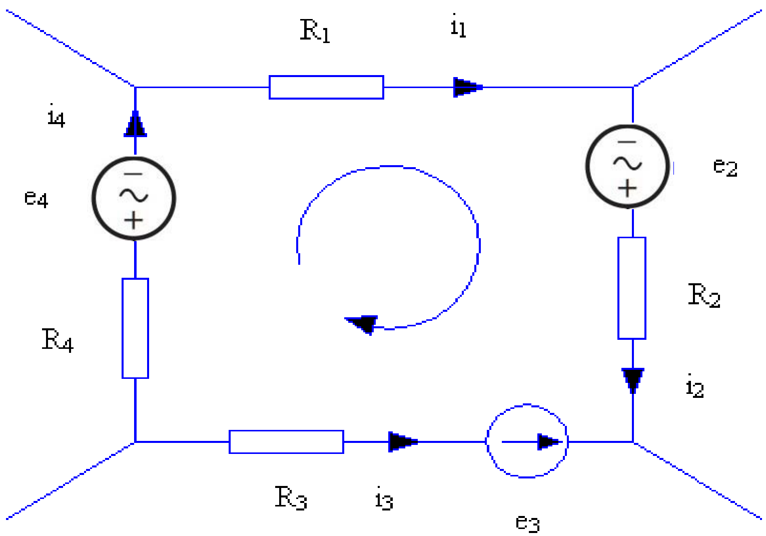
And so on..

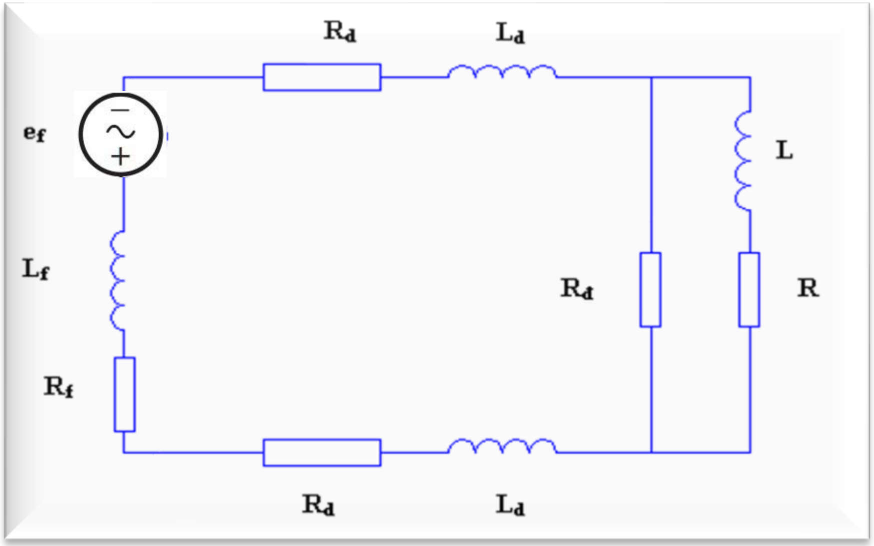
So adding all the individual components (with sign convection) and equating it to zero we get

$$-E1-I1R1-E2-I2R2+I3R3+E3+I4R4+E4=0$$

$$\sum \pm IR + \sum \pm E = 0$$

The algebraic sum of the voltages around any closed path is zero.

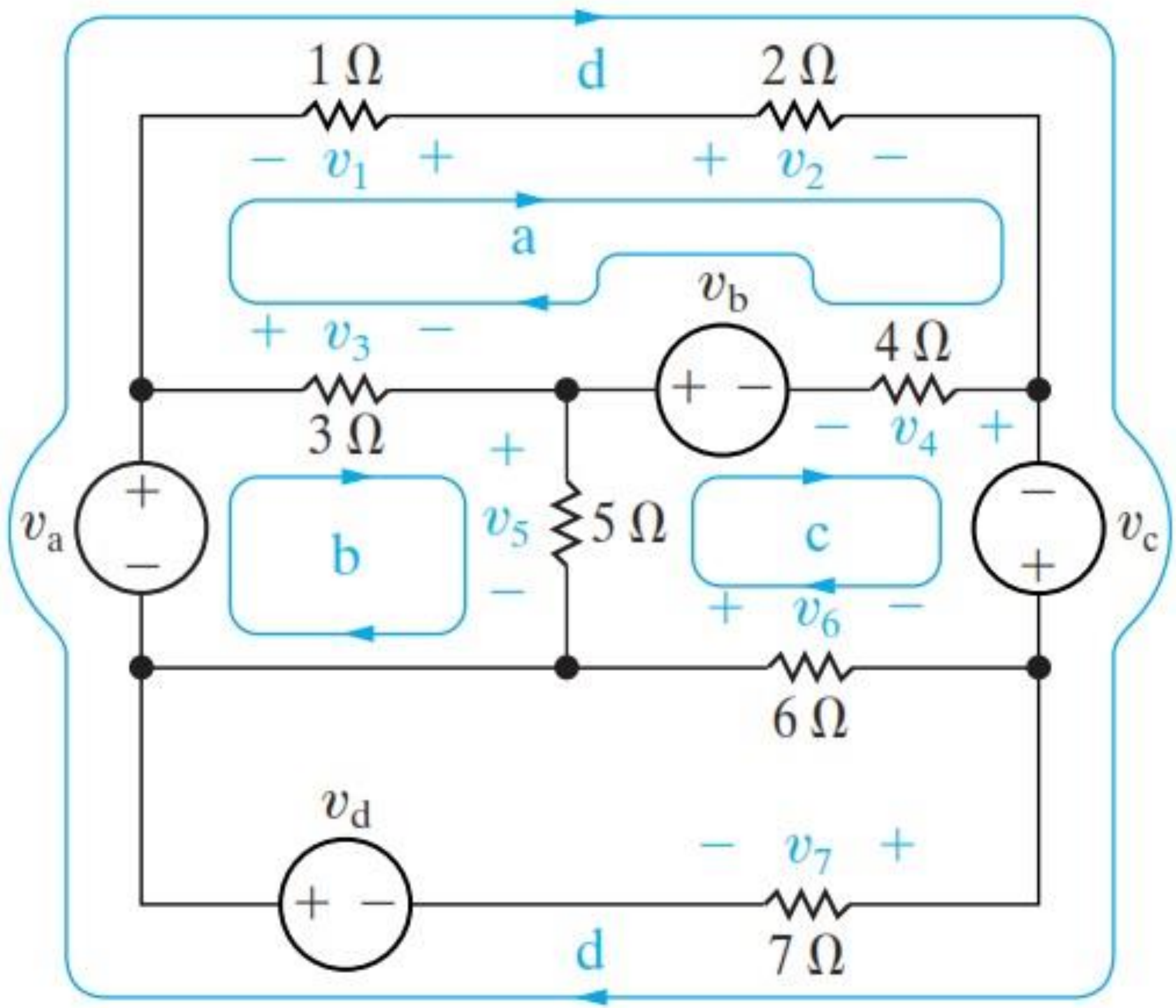




$\sum u = \sum e$

Equation of Kirchhoff's voltage law?

Example 3: Find voltage equations of Path a, b, c, d



Solution of Example 3:

The voltage equations of Path a, b, c, d

$$\text{path a} \qquad -v_1 + v_2 + v_4 - v_b - v_3 = 0,$$

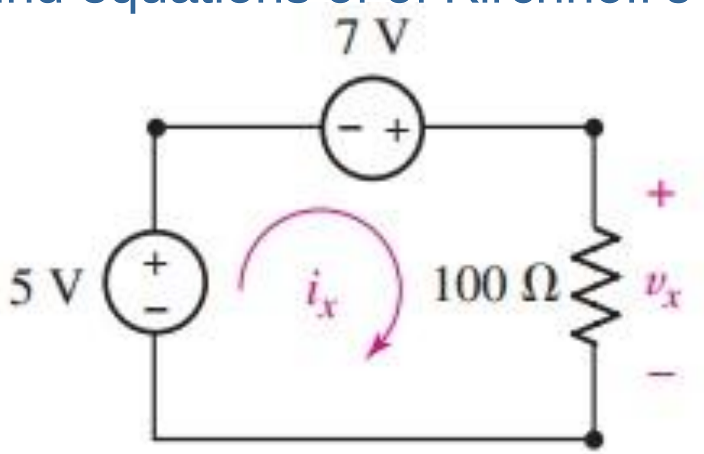
$$\text{path b} \qquad -v_a + v_3 + v_5 = 0,$$

$$\text{path c} \qquad v_b - v_4 - v_c - v_6 - v_5 = 0,$$

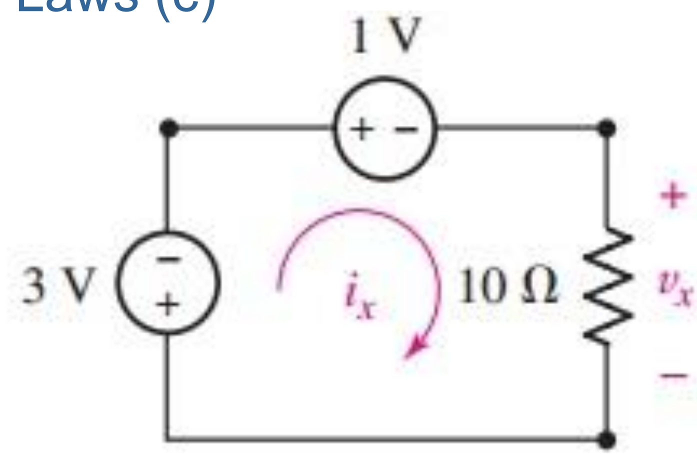
$$\text{path d} \qquad -v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$$

PROBLEM 1:

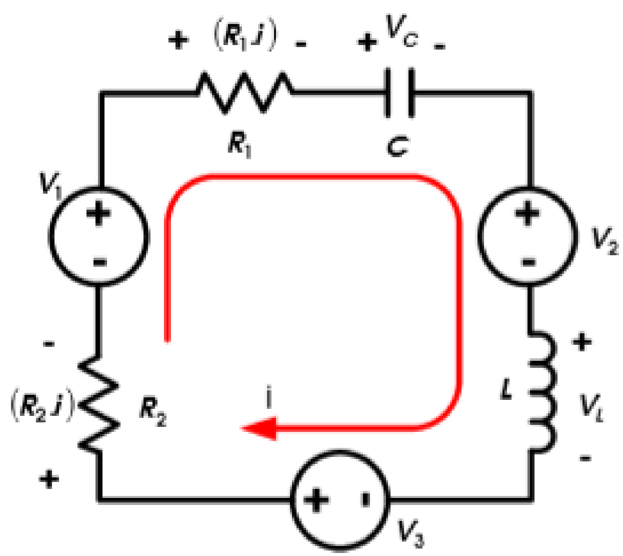
1. Find v_x (a & b) and Ix;
2. Find equations of of Kirchhoff's Voltage Laws (c)



(a)

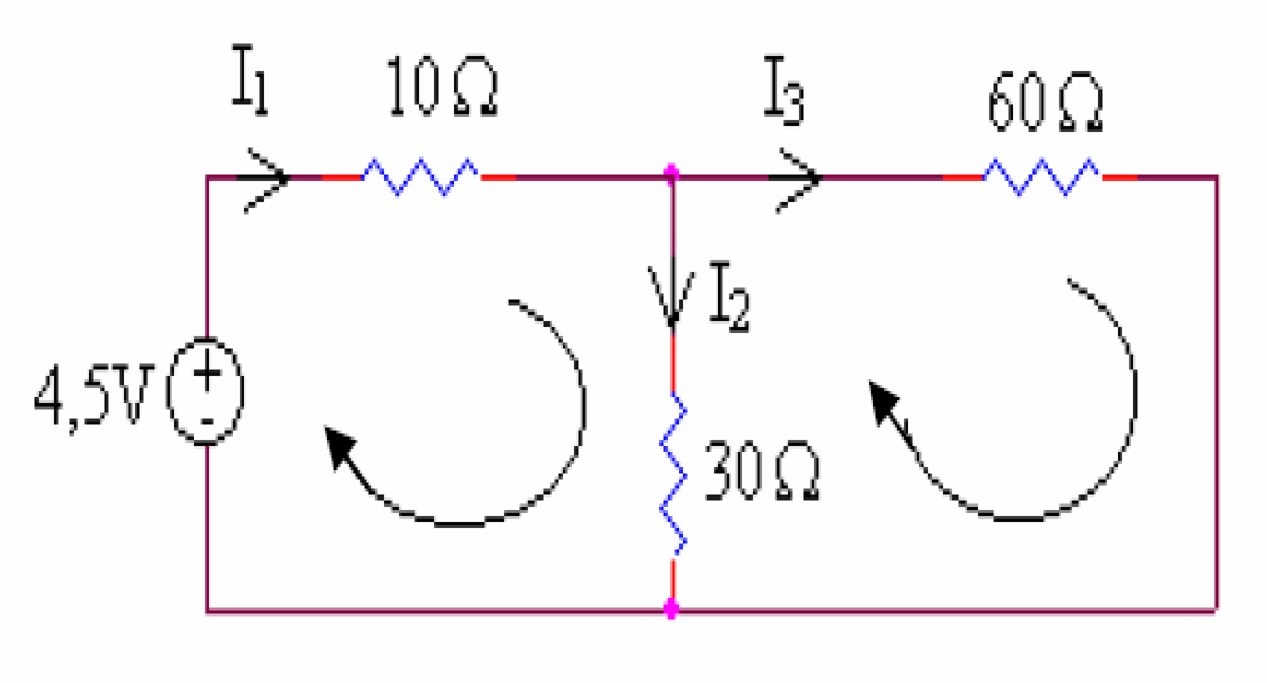


(b)

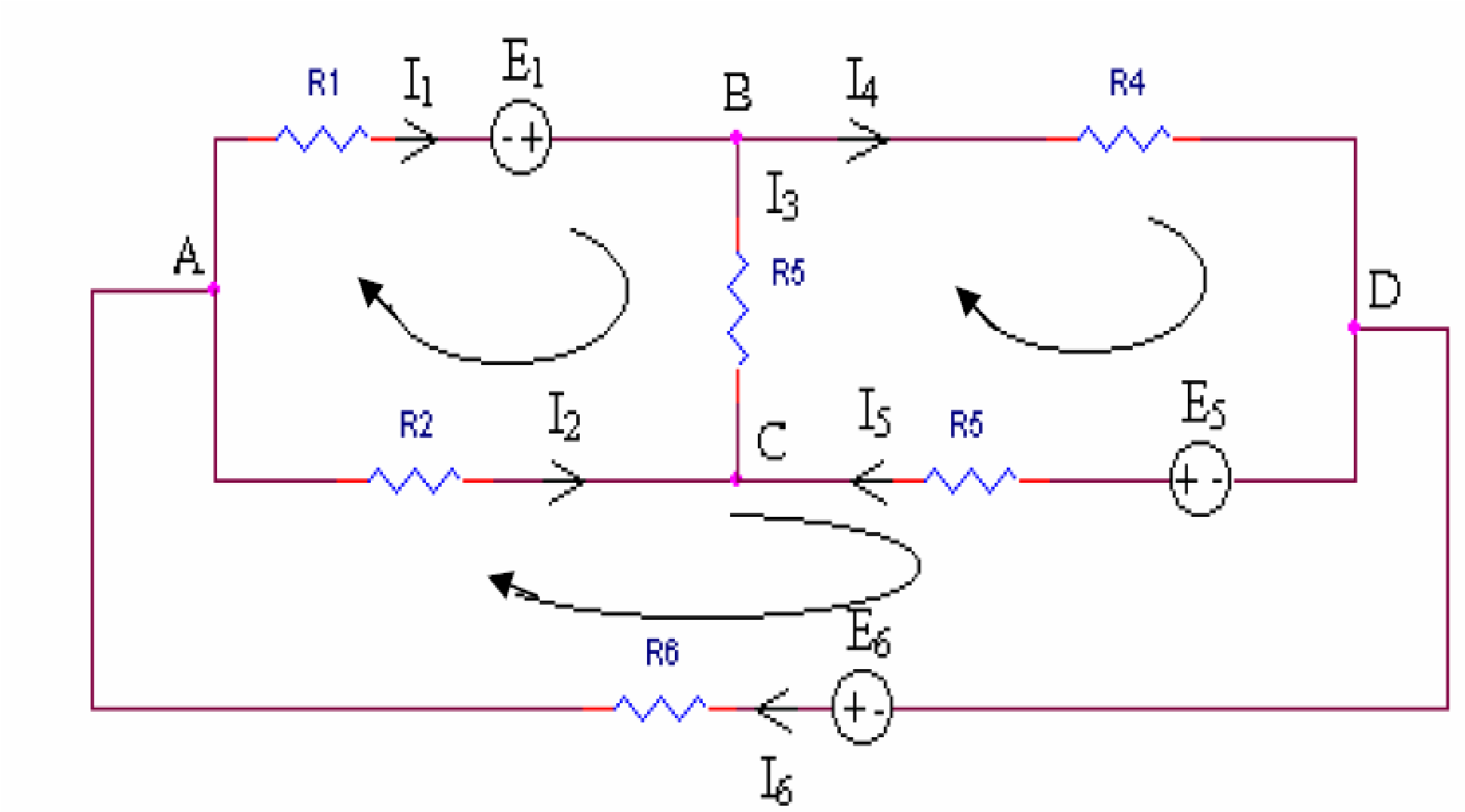


(c)

PROBLEM 2: Find current equations (I_1, I_2, I_3)



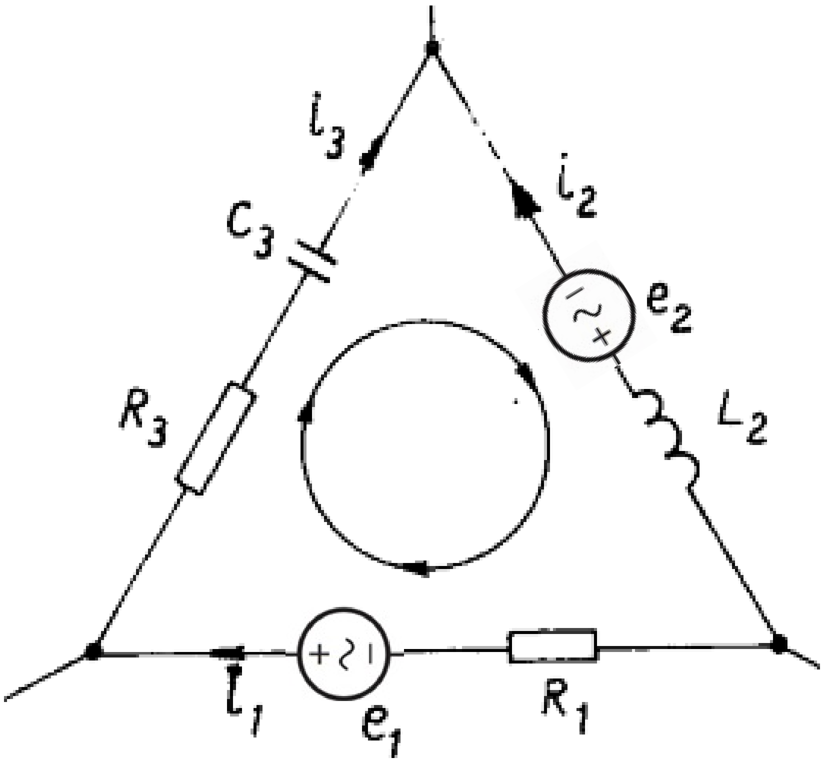
PROBLEM 3: Find current equations (I_1 -> I_6)



2.3 Kirchhoff’s Laws

PROBLEM 4:

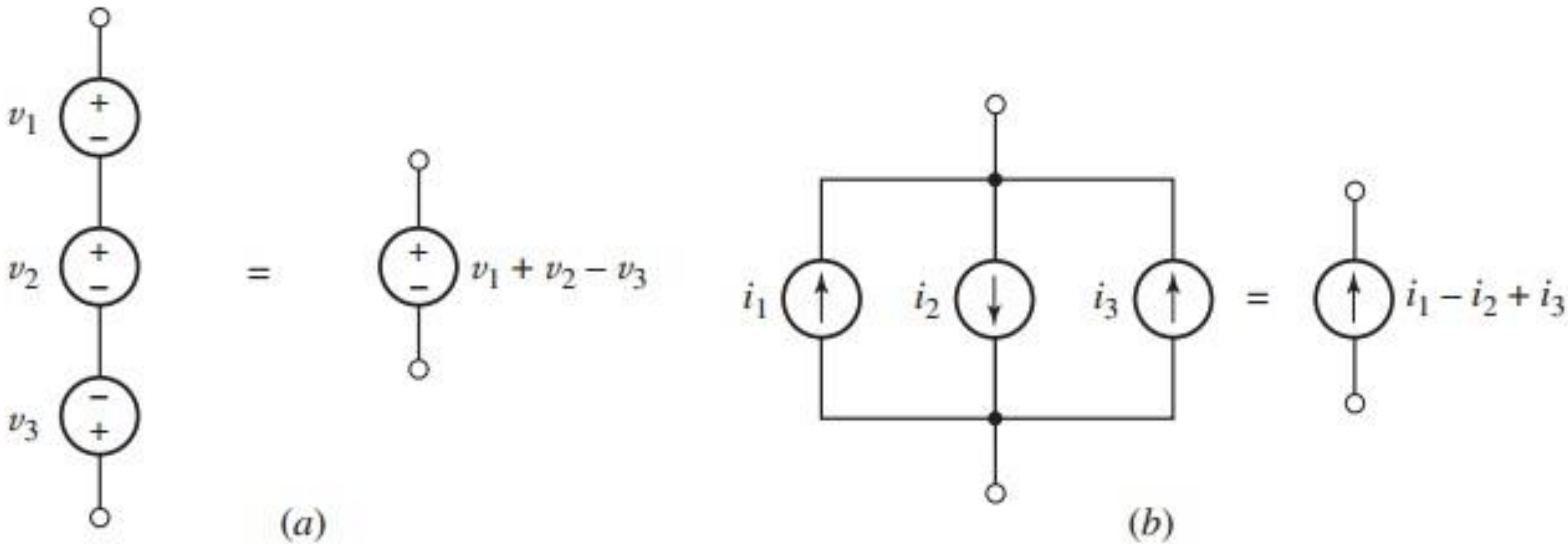
Find equations of of Kirchhoff’s Current and Voltage Laws



Chose the best solution?

A	$-R_3i_3 + \frac{1}{C_3}\int i_3dt - L_2\frac{di_2}{dt} + R_1i_1 = e_2 - e_1$
B	$R_3i_3 + \frac{1}{C_3}\frac{di_2}{dt} - L_2\int i_3dt + R_1i_1 = e_2 - e_1$
C	$R_3i_3 + \frac{1}{C_3}\int i_3dt - L_2\frac{di_2}{dt} + R_1i_1 = e_2 - e_1$
D	$R_3i_3 + C_3\int i_3dt - \frac{1}{L_2}\frac{di_2}{dt} + R_1i_1 = e_2 - e_1$
E	$R_3i_3 + \frac{1}{C_3}\int i_3dt + L_2\frac{di_2}{dt} + R_1i_1 = e_2 - e_1$

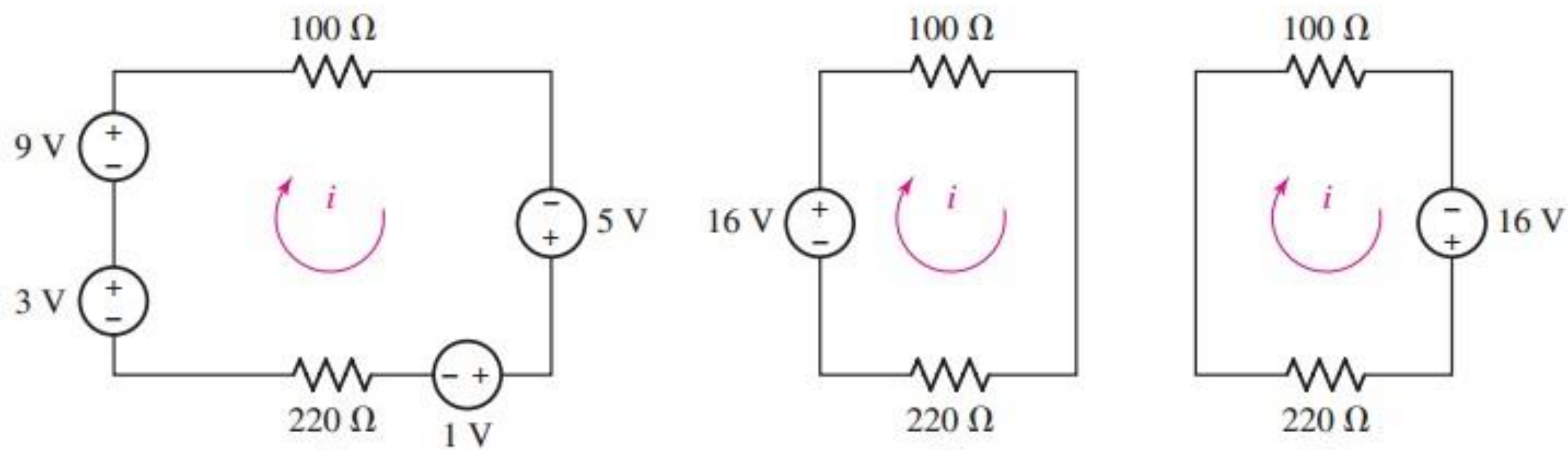
Equivalent voltage and current sources



(*a*) Series-connected voltage sources can be replaced by a single source. (*b*) Parallel current sources can be replaced by a single source.

Problem 5:

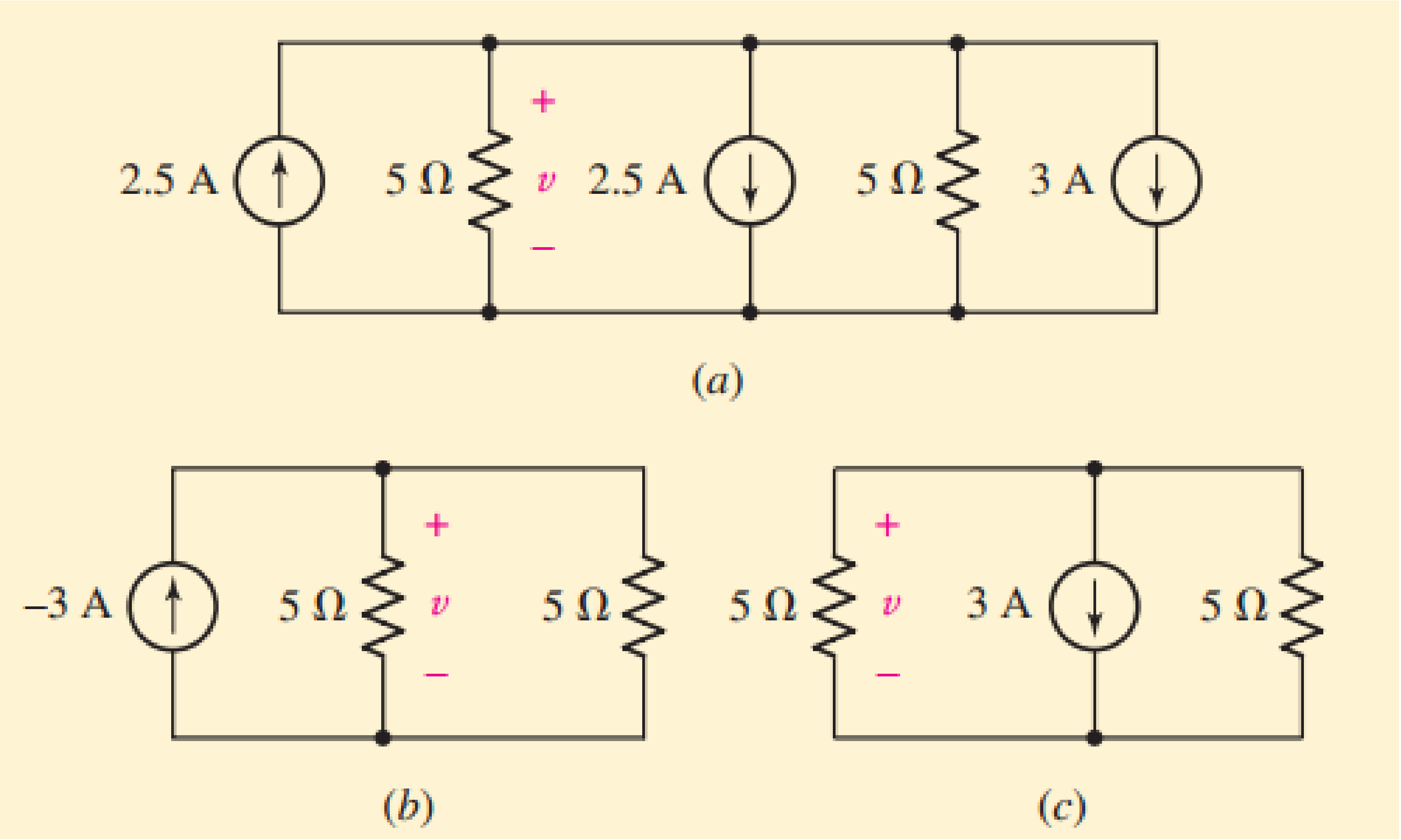
Find current?



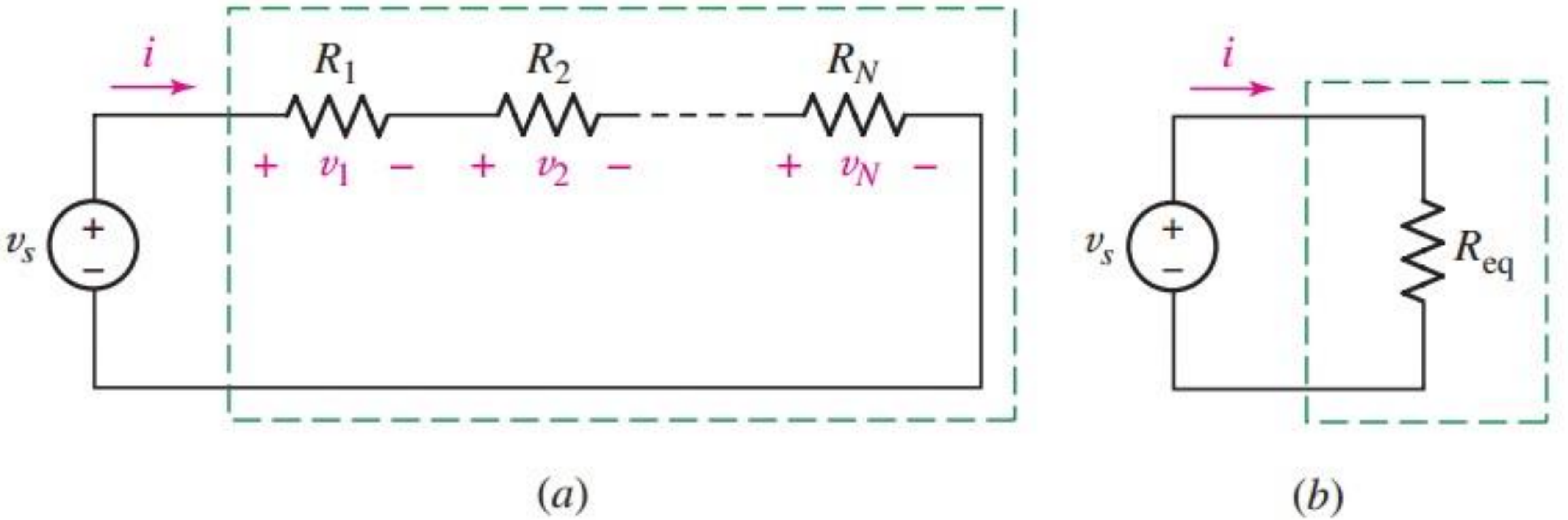
(a) (b) (c)

Problem 6:

Determine the voltage v the circuit by first combining the sources into a single equivalent current source?



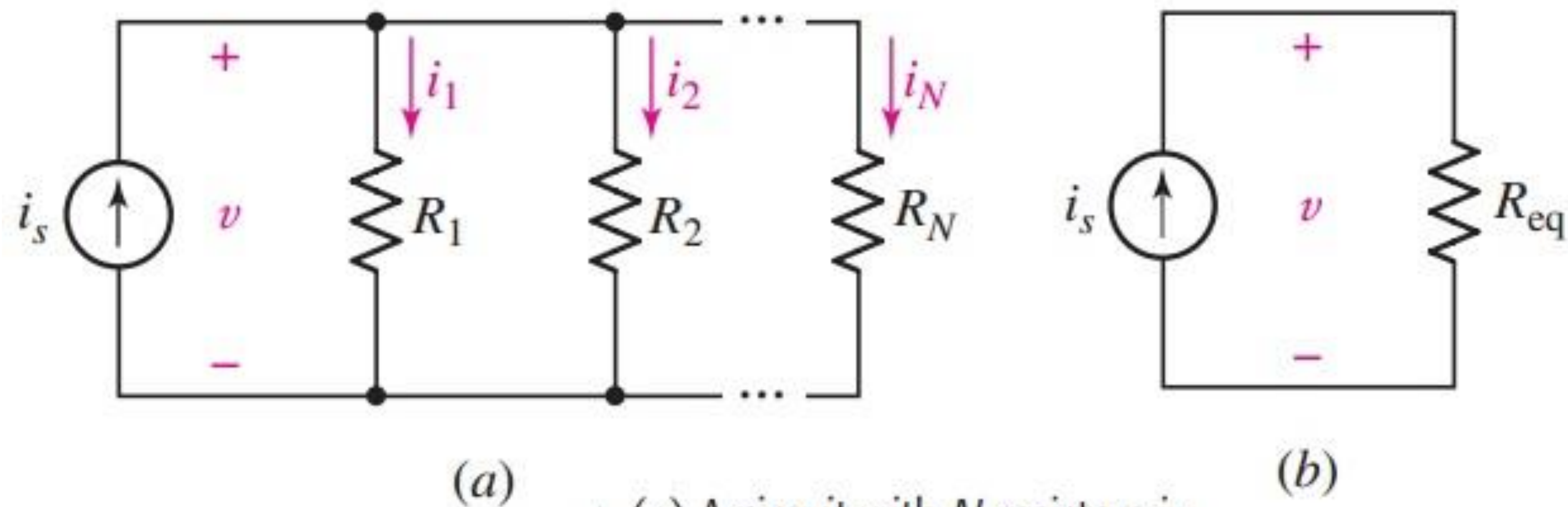
Resistors in Serials



(a) Series combination of N resistors. (b) Electrically equivalent circuit.

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N}$$

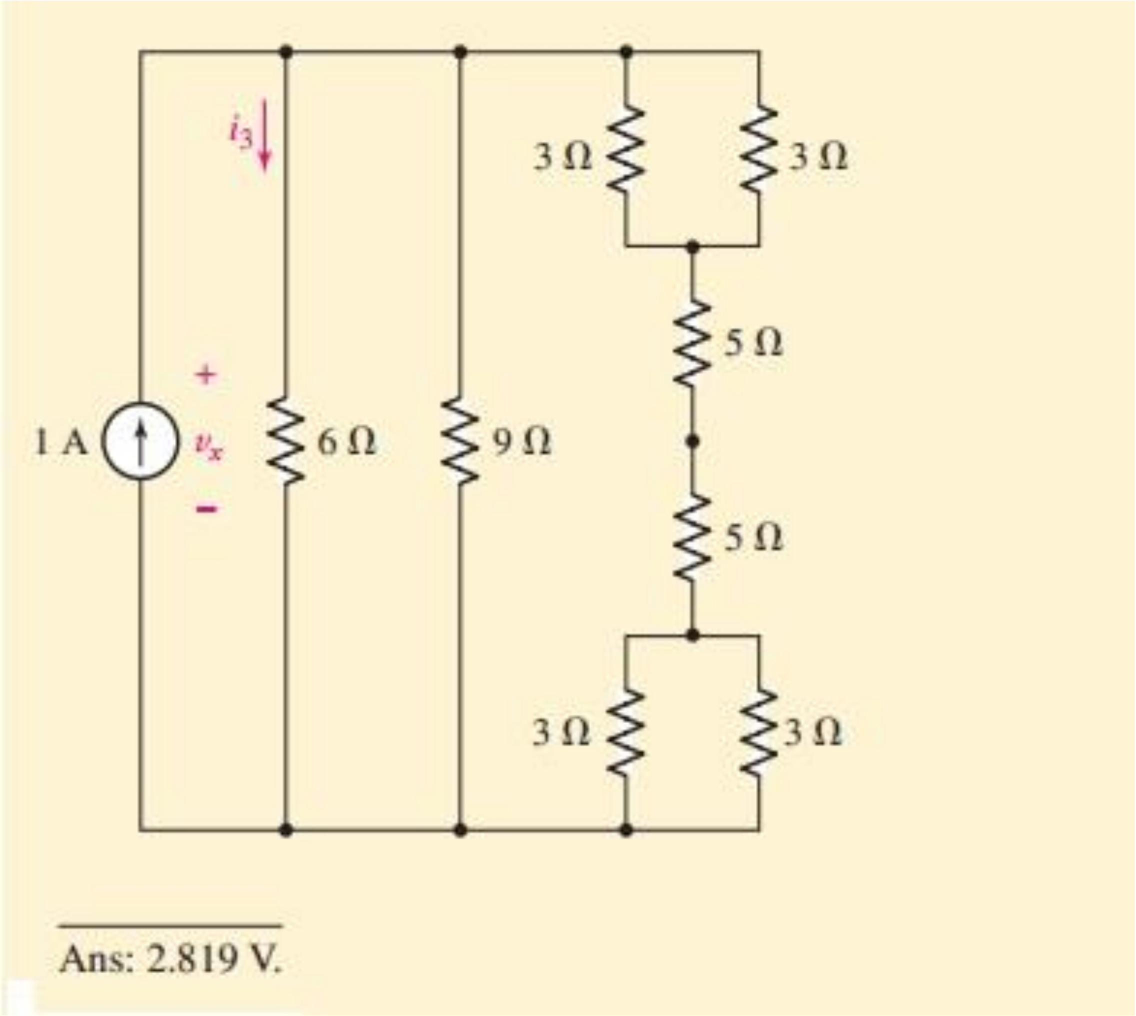
Resistors in parallel



(*a*) A circuit with *N* resistors in parallel. (*b*) Equivalent circuit.

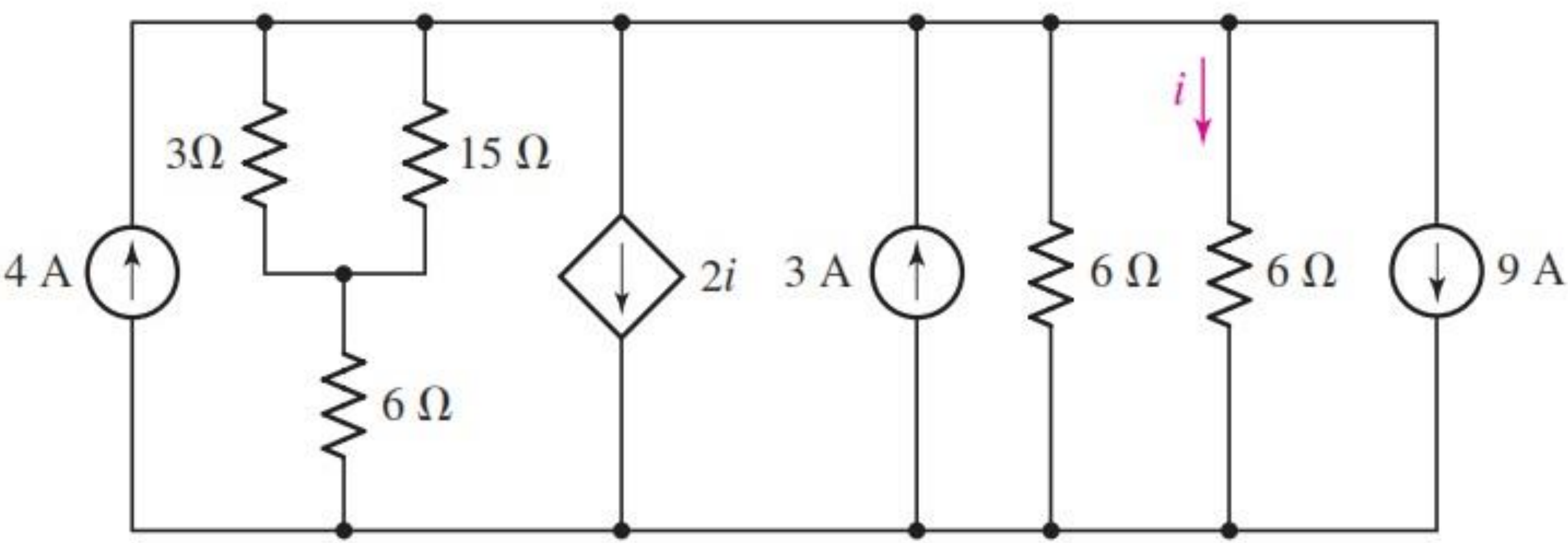
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

Problem 7: find v_x



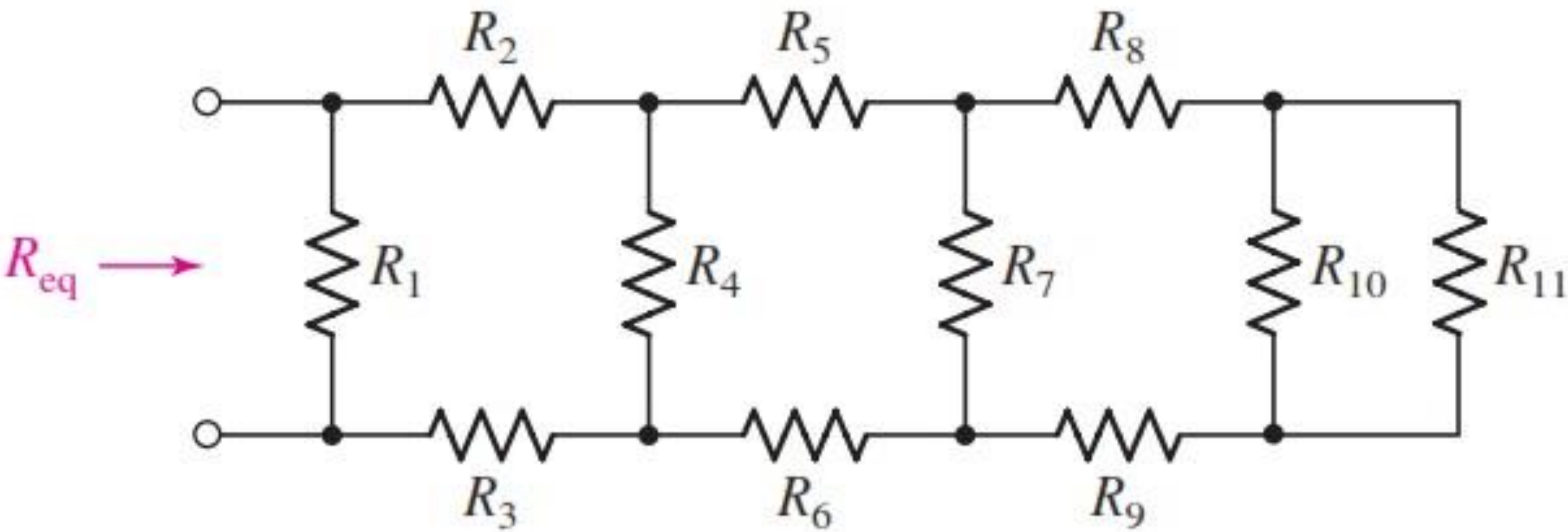
Problem 8

Determine the power absorbed by the 15 Ω resistor in the circuit

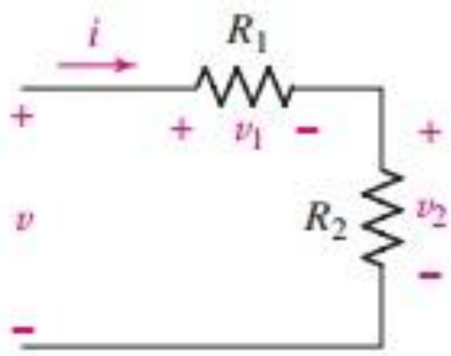


Problem 9

Calculate the equivalent resistance R_{eq} of the network $R_1 = 2R_2 = 3R_3 = 4R_4$ etc. and $R_{11} = 3\ \Omega$.



Voltage division



An illustration of voltage division.

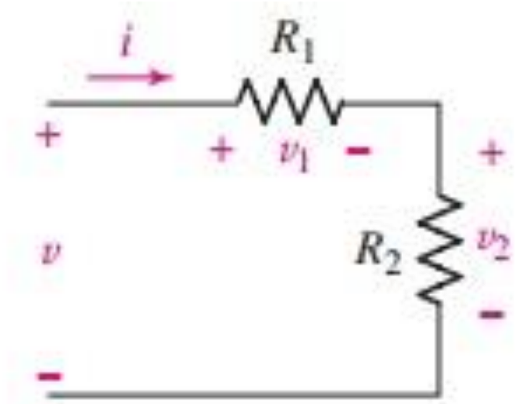
$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

removing R_2 and replacing

it with the series combination of $R_2, R_3, \ldots, R_N$, then we have the general result for voltage division across a string of N series resistors

$$\boxed{v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v}$$

Voltage division



Prove

- **Applying the Ohm Law:**

  V1=R1*I, v2=R2*I,

- **Applying Kirchoff's voltage law (K2):**

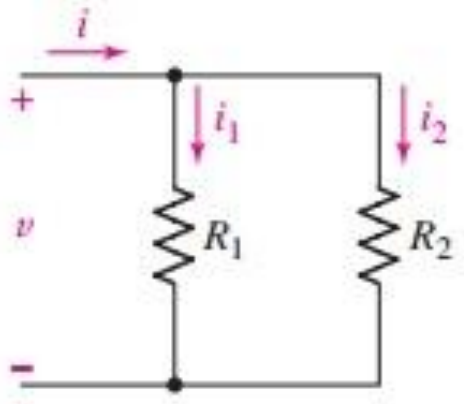
  V=V1+V2=(R1+R2)I,

  => I=V/(R1+R2)

$$\Rightarrow V_1 = \frac{R_1}{R_1 + R_2} V$$

$$\Rightarrow V_2 = \frac{R_2}{R_1 + R_2} V$$

Current division



An illustration of current division.

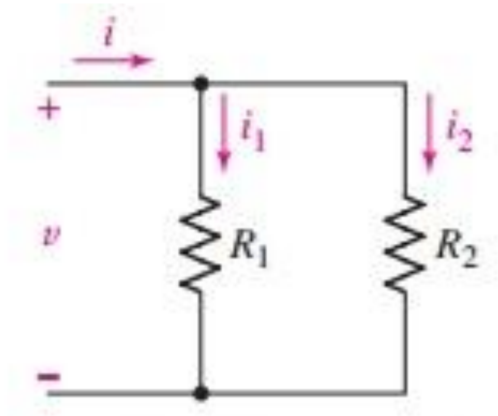
$$i_1 = i\frac{R_2}{R_1 + R_2}$$

$$i_2 = i\frac{R_1}{R_1 + R_2}$$

For a parallel combination of N resistors, the current through resistor R_k is

$$i_k = i\frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

Current division



Prove

- **Applying the Ohm Law:**

I1=V/R1; I2=V/R2

- **Applying Kirchoff's current law (K1):**

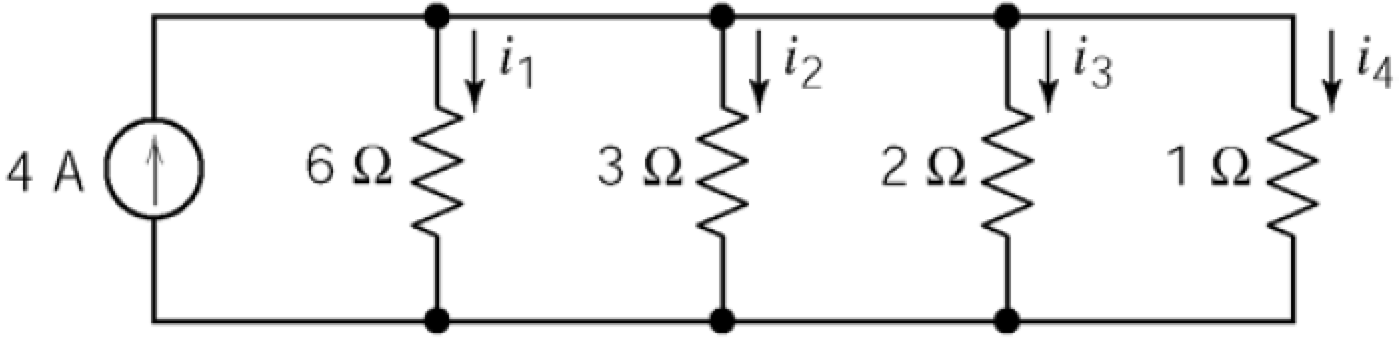
I=I1+I2 **=> I=(1/R1+1/R2)*V**

$$\Rightarrow V = \frac{R_1 \times R_2}{R_1 + R_2} \times I$$

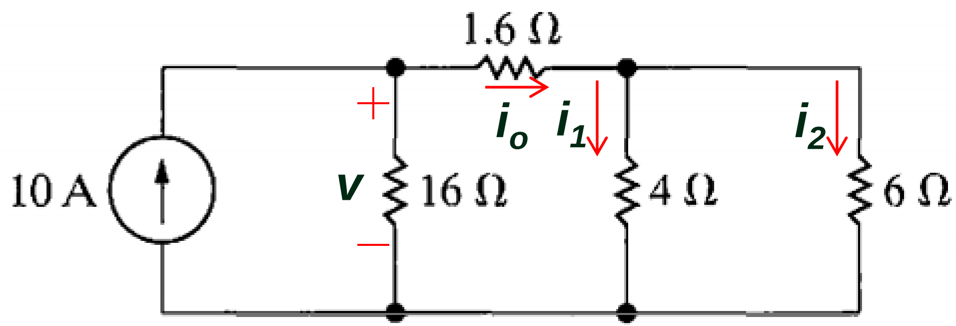
$$\Rightarrow I_1 = \frac{V}{R_1} = \frac{R_1 \times R_2}{R_1(R_1 + R_2)} V = \frac{R_2}{R_1 + R_2} V$$

$$\Rightarrow I_2 = \frac{V}{R_2} = \frac{R_1 \times R_2}{R_2(R_1 + R_2)} V = \frac{R_1}{R_1 + R_2} V$$

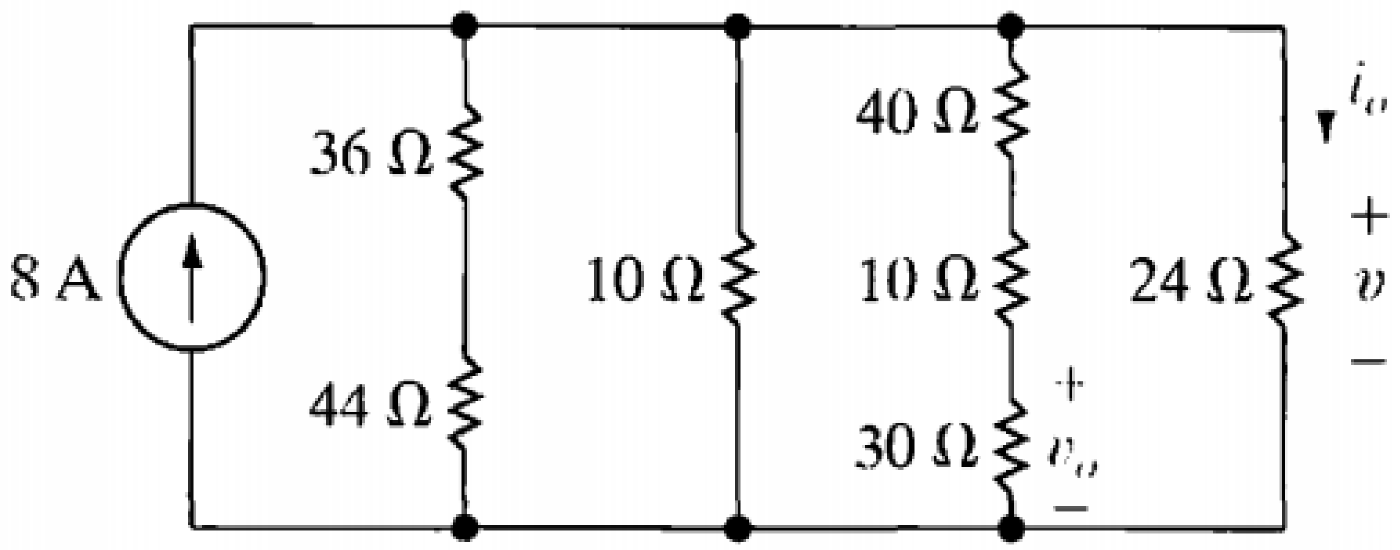
Problem 10: Find current i_1-i_4



Problem 11: Find the power dissipated in the 6 Ω resistor



Problem 12: Find i_o and v_o

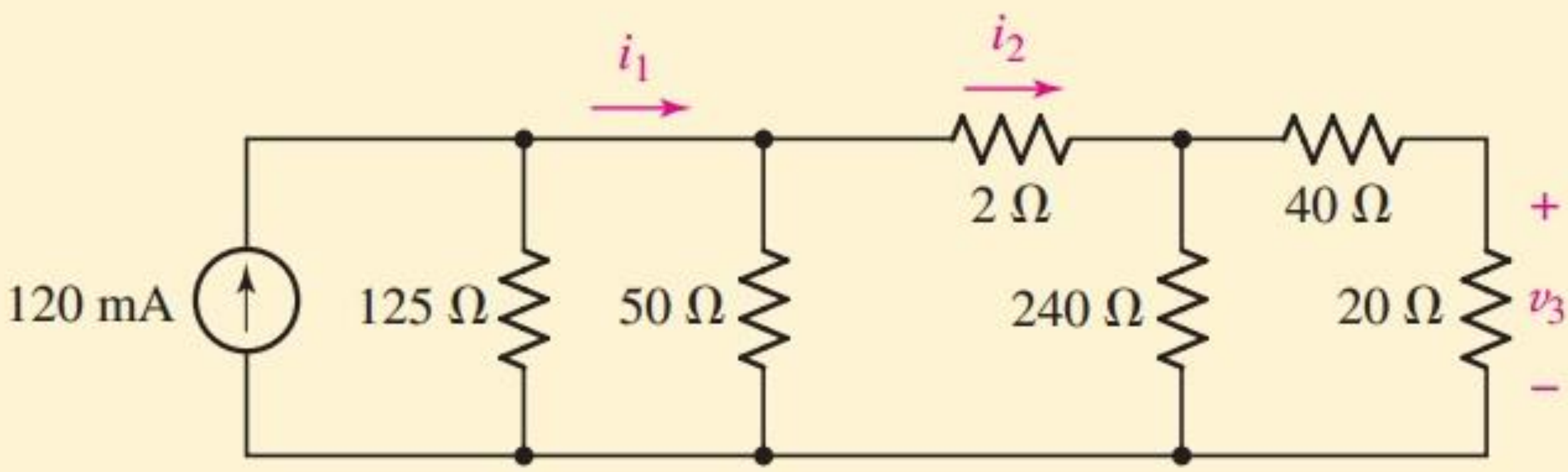


Problem 13:

Problem of voltage and current division

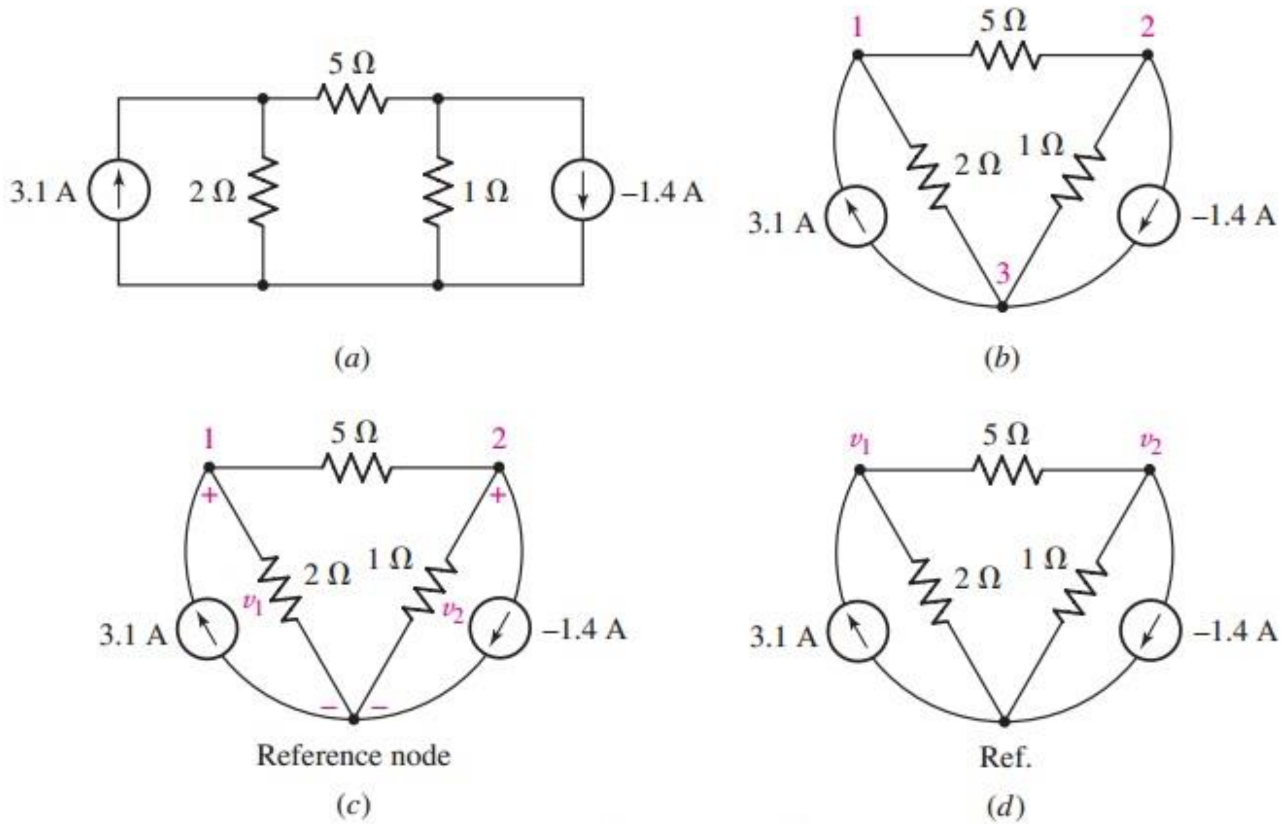
PRACTICE

In the circuit use resistance combination methods and current division to find i_1, i_2, and v_3.



Example 4: Finding node voltages in circuit (Figure a)

N-node circuit will need (N−1) voltages and (N−1) equations

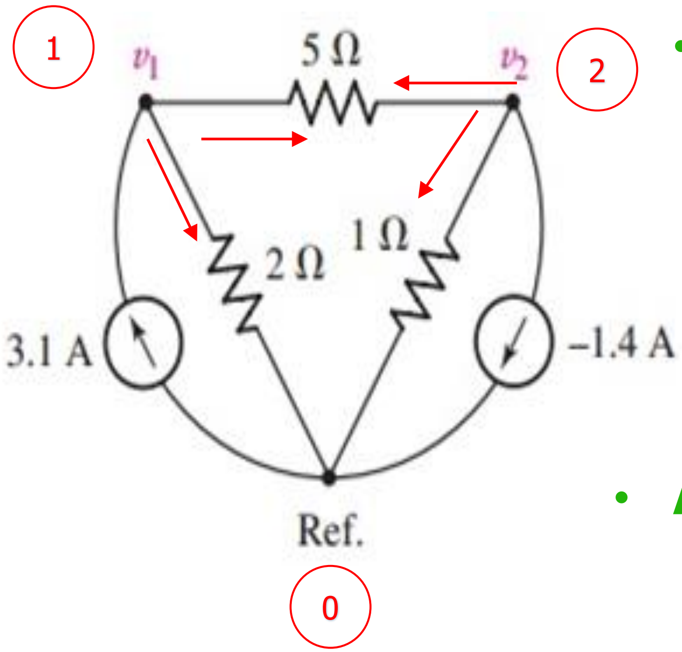


(a) A simple three-node circuit. (b) Circuit redrawn to emphasize nodes. (c) Reference node selected and voltages assigned. (d) Shorthand voltage references. If desired, an appropriate ground symbol may be substituted for "Ref."

Note: Applying the Kirchoff's current law (k1)

Finding node voltages :

Applying the Kirchoff's current law (k1)



- **At node 1 we obtain:** $$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$$

$$0.7v_1 - 0.2v_2 = 3.1 \quad (1)$$

- **At node 2 we obtain:** $$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4)$$

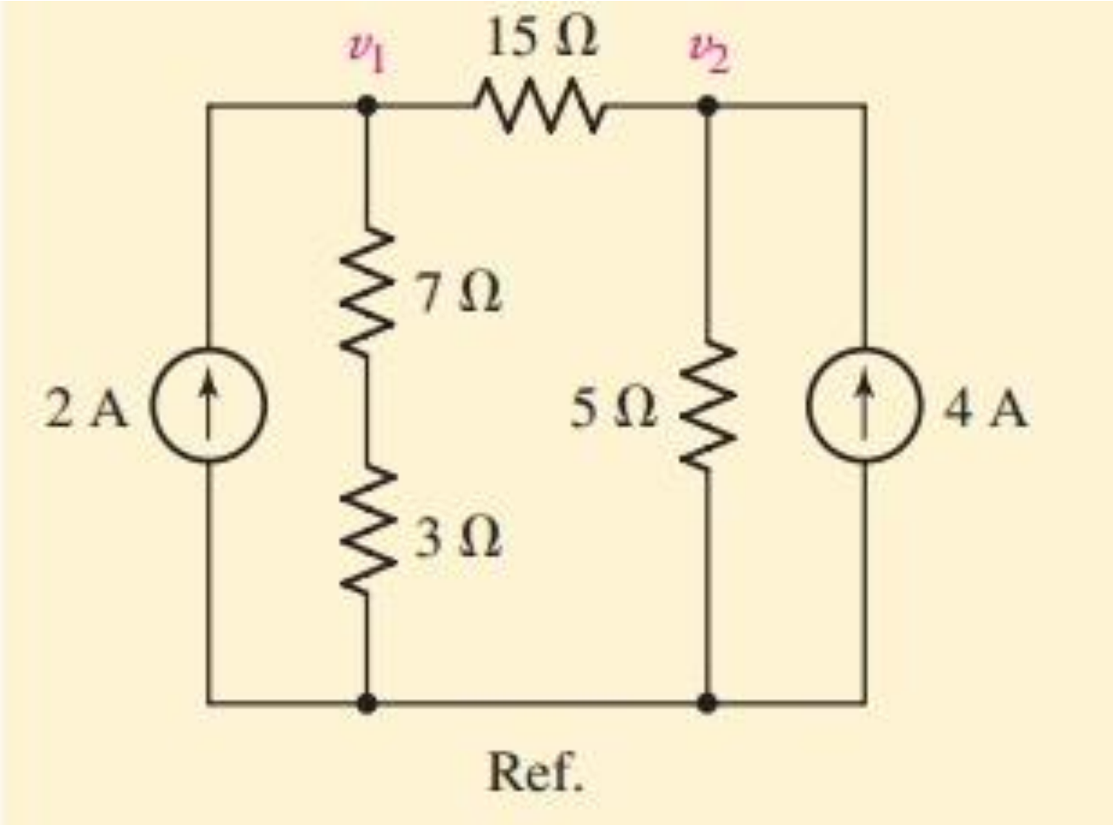
$$-0.2v_1 + 1.2v_2 = 1.4 \quad (2)$$

Summary of Basic Nodal Analysis Procedure

1. **Count the number of nodes** (N)
2. **Designate a reference node.** The number of terms in your nodal equations can be minimized by selecting the node with the greatest number of branches connected to it.
3. **Label the nodal voltages** (there are N-1 of them).
4. **Write a KCL equation for each of the non-reference nodes**. Sum the currents *flowing into a node* from sources on one side of the equation. On the other side, sum the currents flowing out of the node through resistors. Pay close attention to "-" signs.
5. **Express any additional unknowns such as currents or voltages other than nodal voltages in terms of appropriate nodal voltages**. This situation can occur if voltage sources or dependent sources appear in our circuit.
6. **Organize the equations**. Group terms according to nodal voltages.

Example 5:

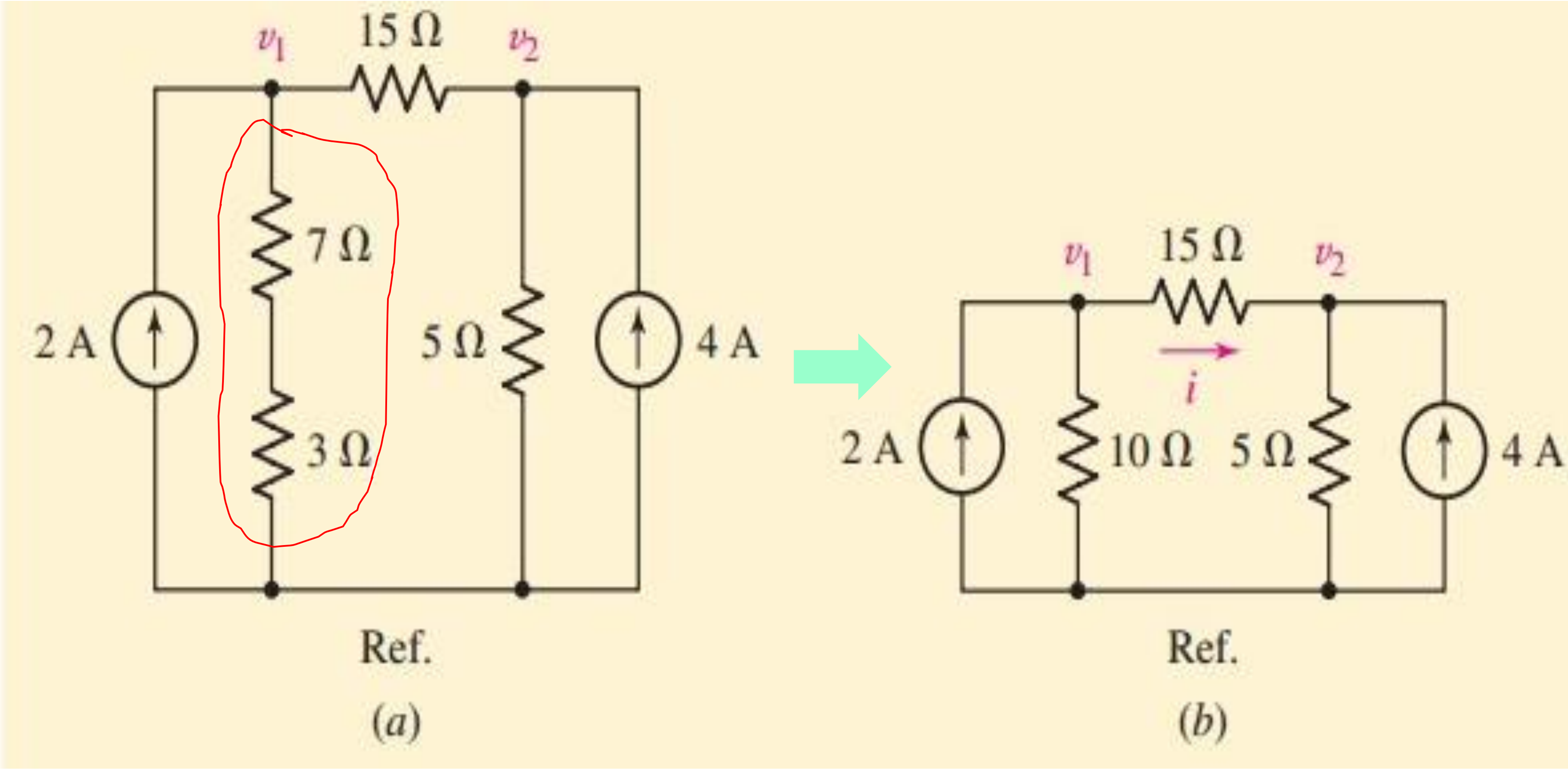
Determine the current flowing left to right through the 15Ω resistor.



(a)

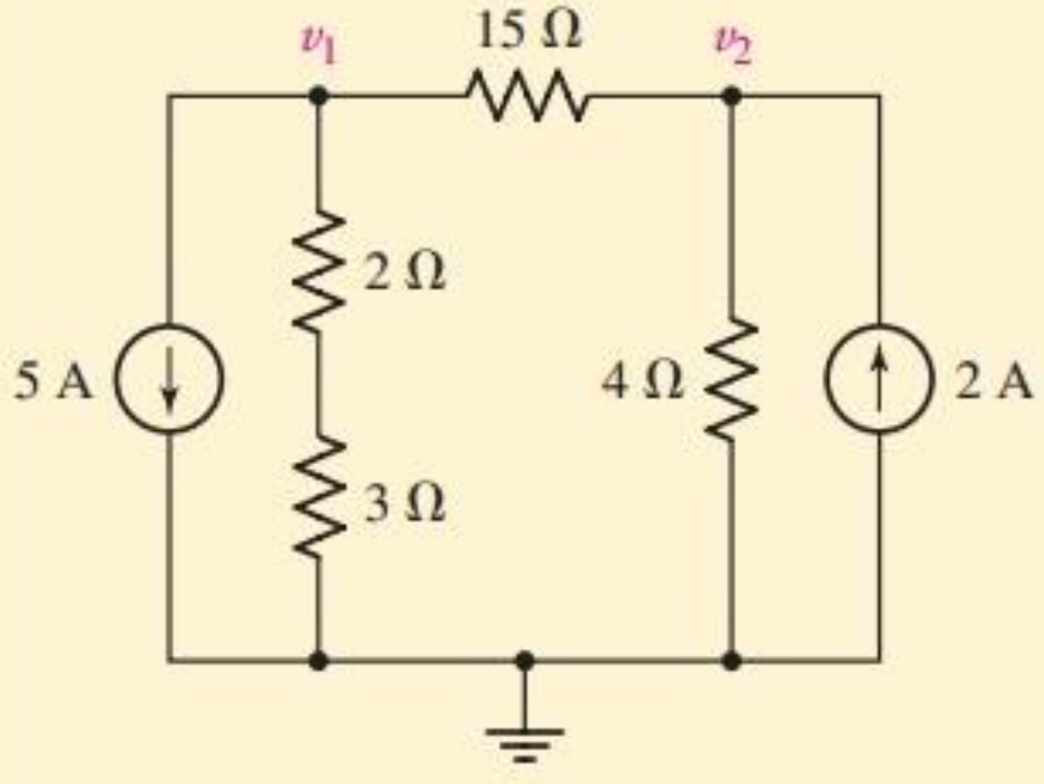
Solution of Example 5:

Determine the current flowing left to right through the 15Ω resistor.



Problem 16:

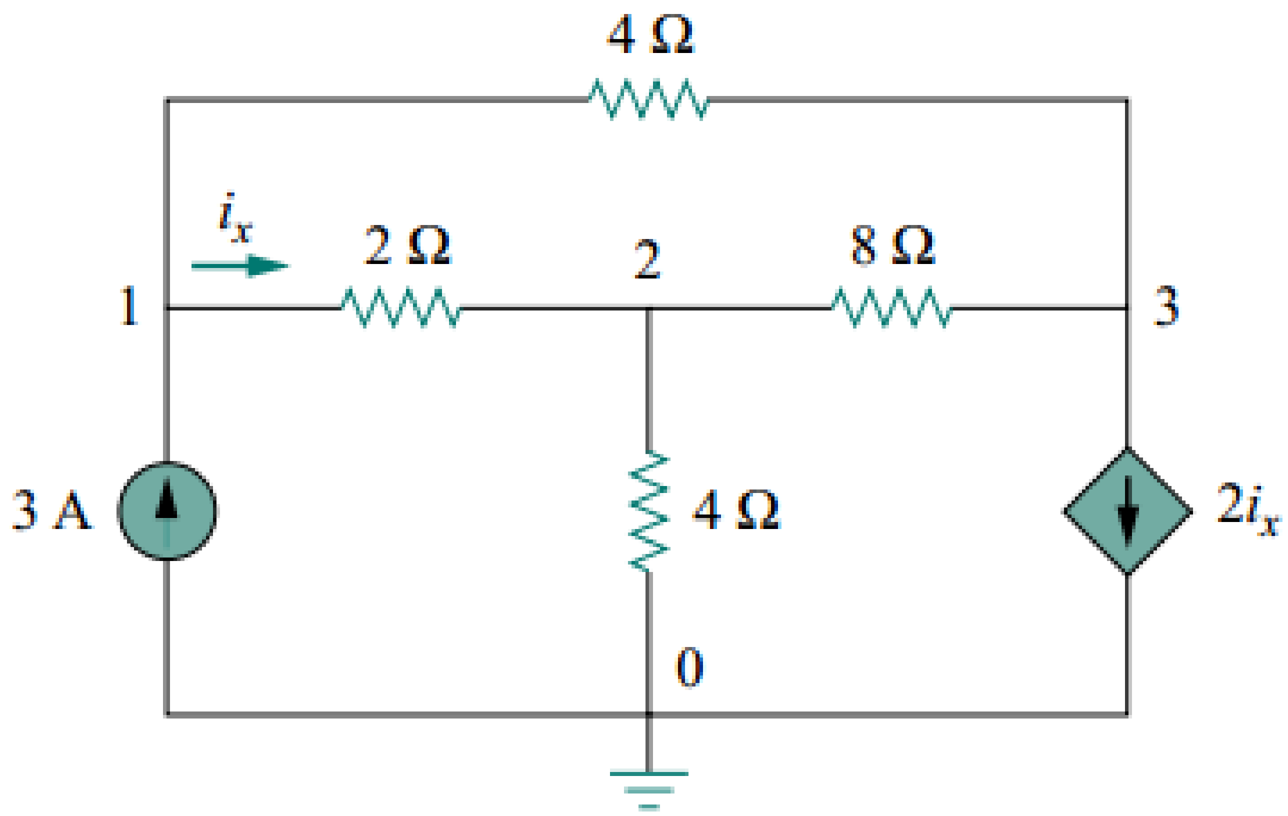
For the circuit determine the nodal voltages v_1 and v_2



Ans: $v_1 = -145/8$ V, $v_2 = 5/2$ V.

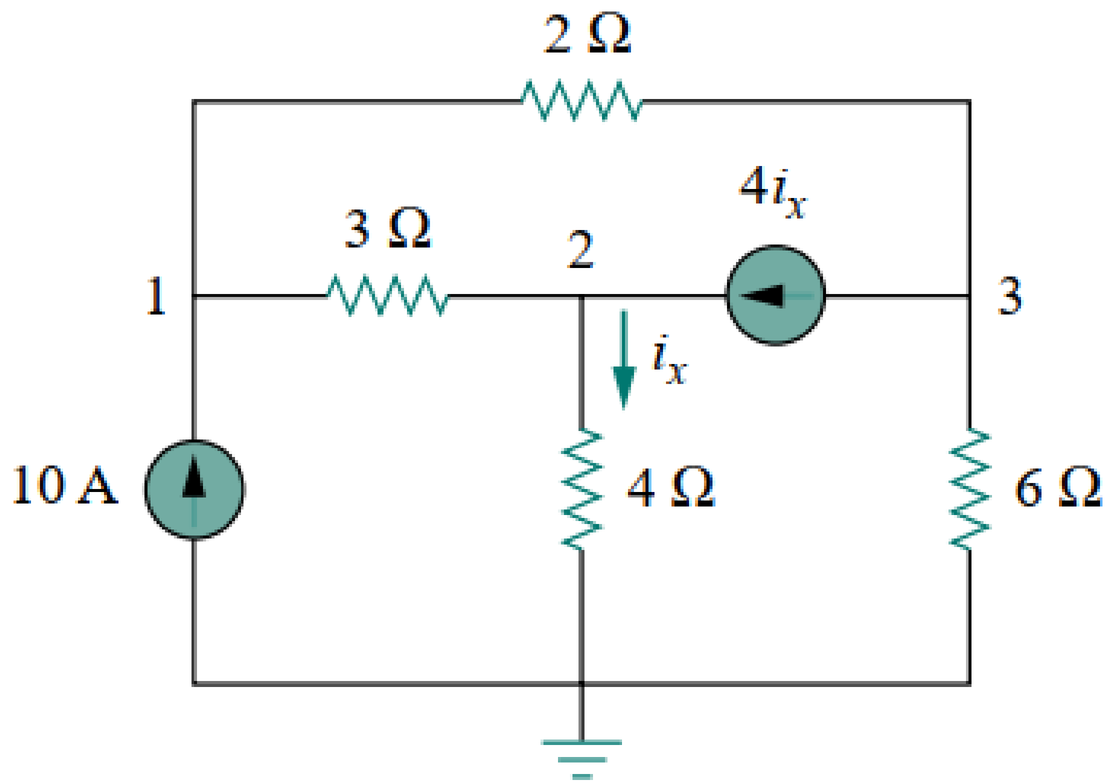
Problem 17:

Determine the voltages at the nodes in Figure



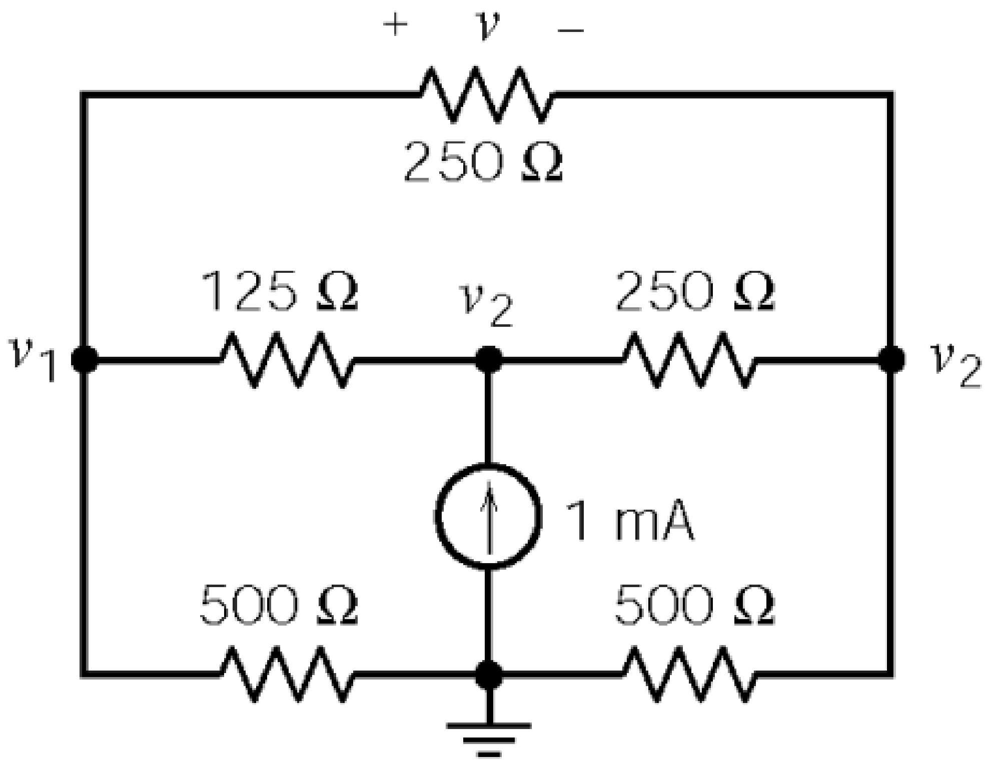
Problem 18:

Determine the voltages at the nodes in Figure



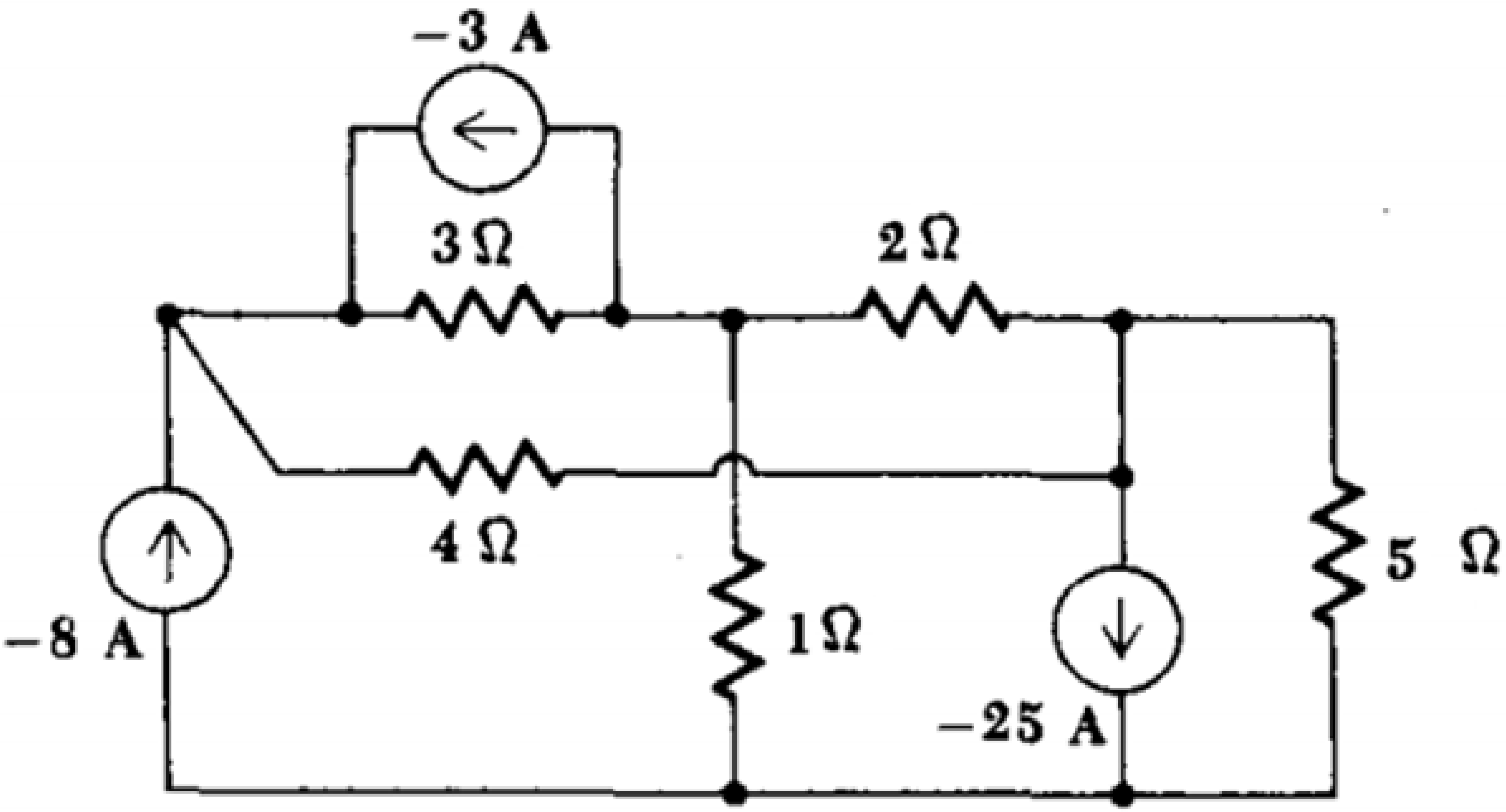
Answer: $v_1 = 80$ V, $v_2 = -64$ V, $v_3 = 156$ V.

HOMEWORK 1



2.7 Nodal-Voltage Method – Homework

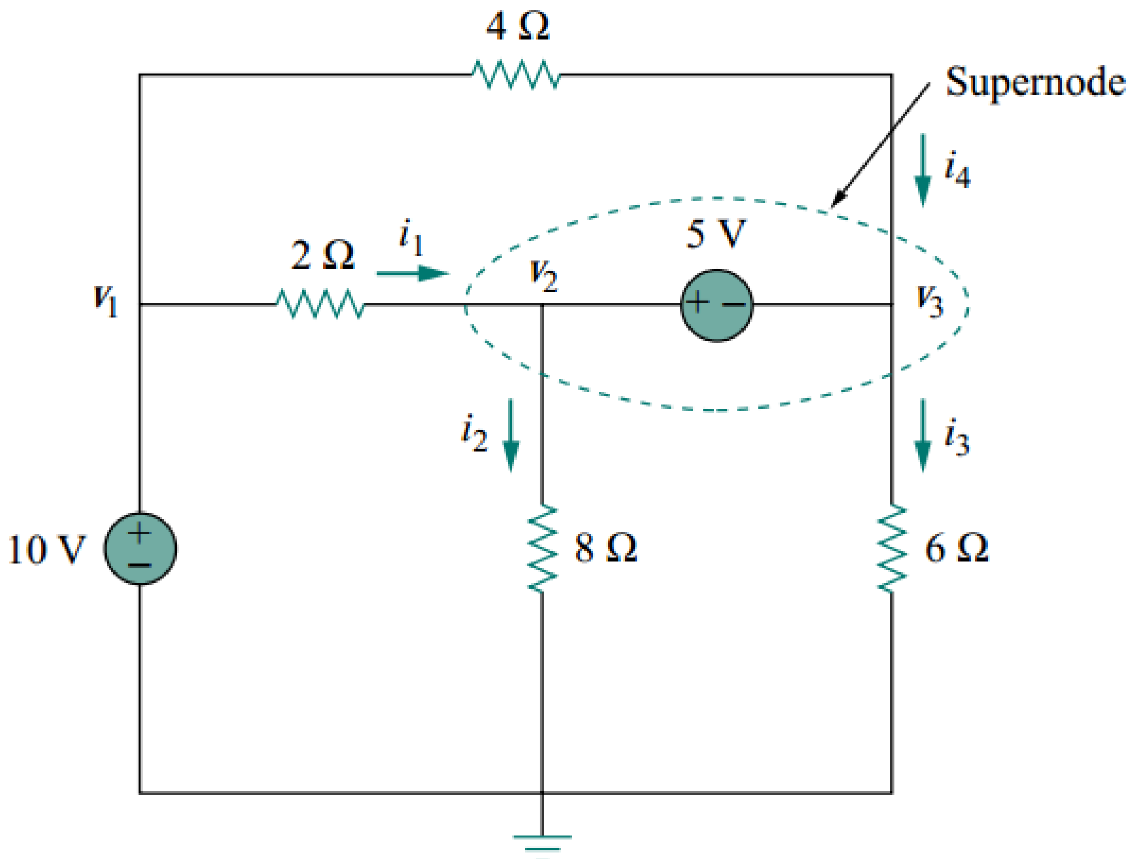
HOMEWORK 2



2.7 Nodal-Voltage Method: Supernode (Siêu nút)

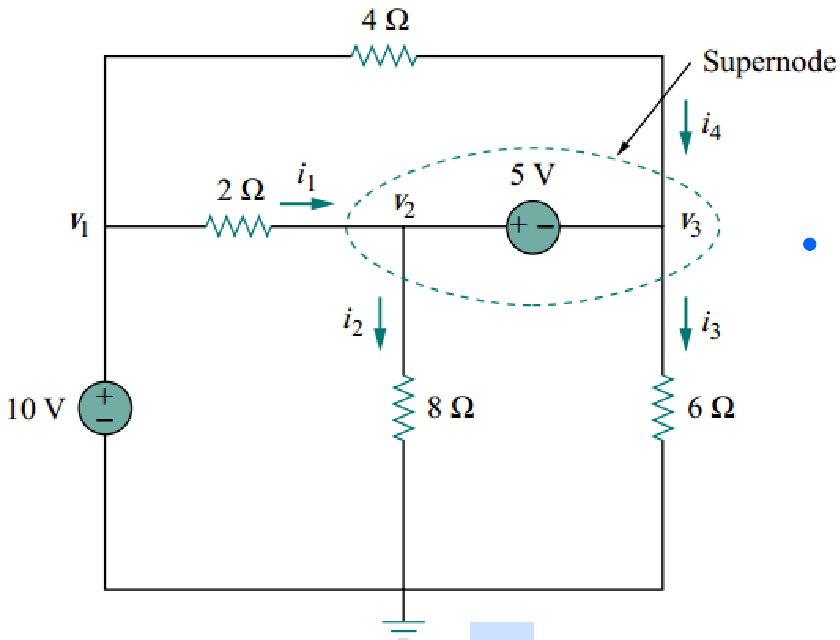
If the voltage source (dependent or independent) is connected between two ***nonreference*** nodes, the two *nonreference* nodes form a ***supernode***; we apply both **KCL** and **KVL** to determine the node voltages.

Example 6



2.7 Nodal-Voltage Method: Supernode

Solution of Example 6



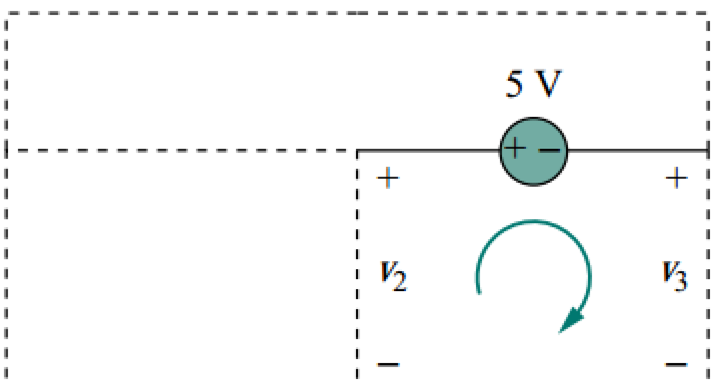
$$v_1 = 10 \text{ V} \qquad (1)$$

- KCL must be satisfied at a supernode like any other node

$$i_1 + i_4 = i_2 + i_3$$

or $$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

$$16v_1 - 15v_2 - 10v_3 = 0 \qquad (2)$$



- To apply Kirchhoff's voltage law to a supernode

$$-v_2 + 5 + v_3 = 0 \quad \Longrightarrow \quad v_2 - v_3 = 5 \qquad (3)$$

From Eqs. (1), (2), and (3) we obtain node voltages.

v_1 = 10 V, V2=8,4, V3=3,4

Summary of Supernode Analysis Procedure

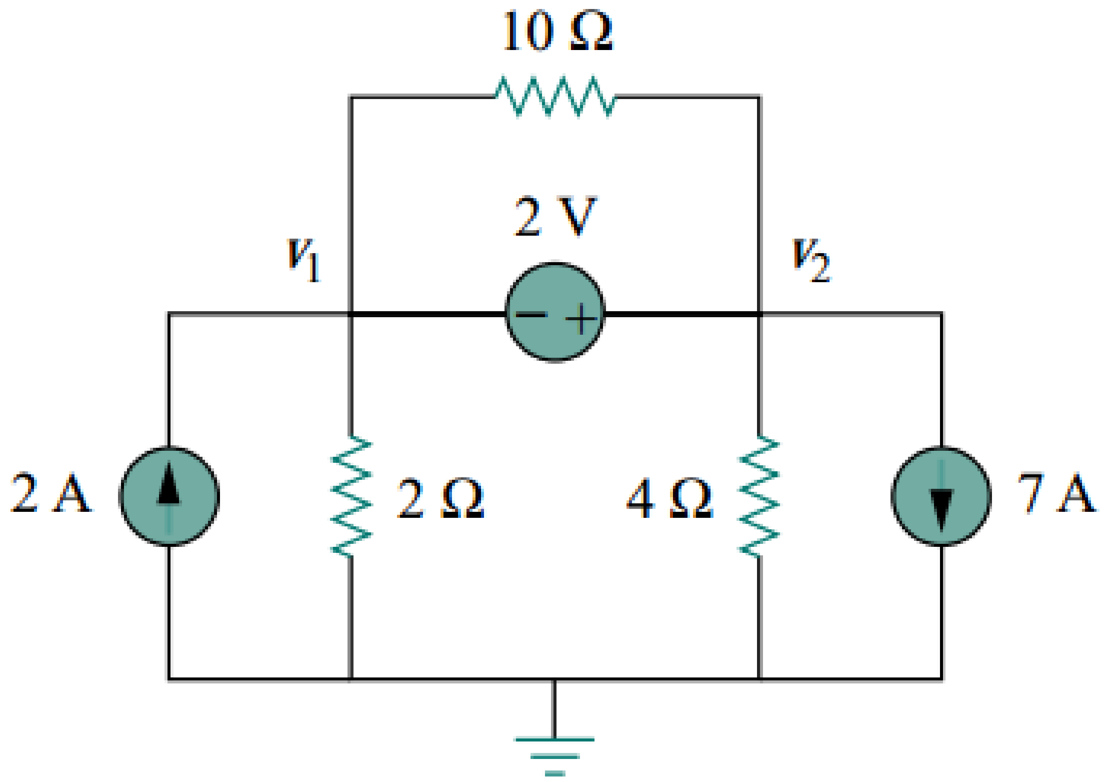
1. **Count the number of nodes** (N)
2. **Designate a reference node.** The number of terms in your nodal equations can be minimized by selecting the node with the greatest number of branches connected to it.
3. **Label the nodal voltages** (there are N-1 of them).
4. **If the circuit contains voltage sources**, **form a supernode about each one**. This is done by enclosing the source, its two terminals, and any other elements connected between the two ternicals within a broken-line enclosure.
5. **Write a KCL equation for each of the nonreference nodes and for each supernode *that does not contain the reference node***. Sum the currents *flowing into a node/supernode* from current sources on one side of the equation. On the other side, sum the currents flowing out of the node/supernode through resistors. Pay close attention to “-” signs.

Summary of Supernode Analysis Procedure

6. **Relate the voltage across each voltage source to nodal voltages.** This is accomplished by simple application of KVL; one such equation os needed for each suppernode defined.
7. **Express any additional unknowns (i.e., currents or voltages other than nodal voltages) in terms of appropriate nodal voltages**. This situation can occur if dependent sources appear in our circuit.
8. **Organize the equations**. Group terms according to nodal voltages.
9. **Solve the system of equations for the nodal voltages** (there will be N-1 of them).

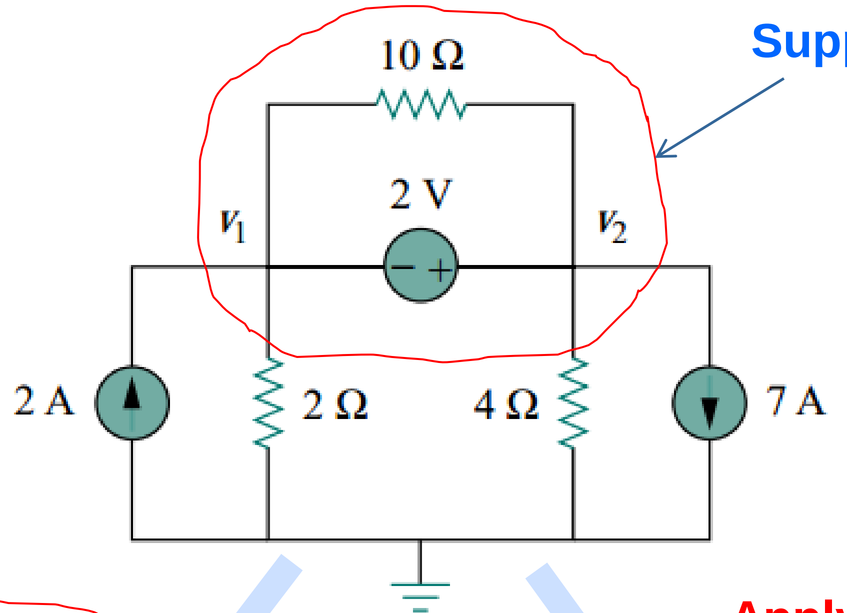
Problem 19:

Find the node voltages.

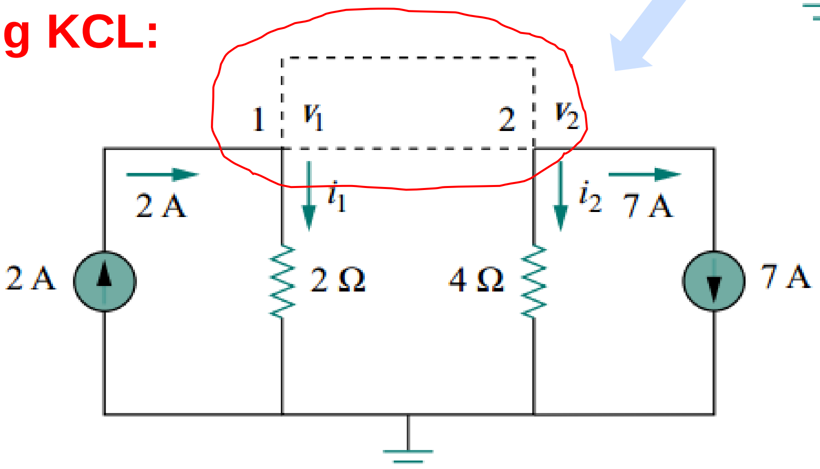


Solution problem 19:

Suppernode

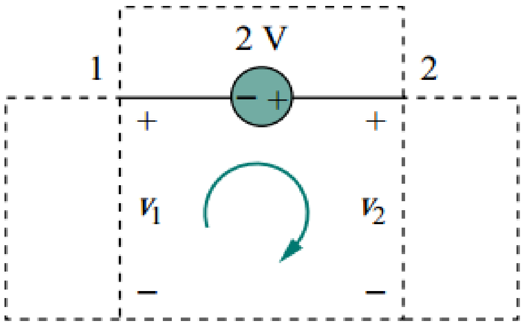


Applying KCL:



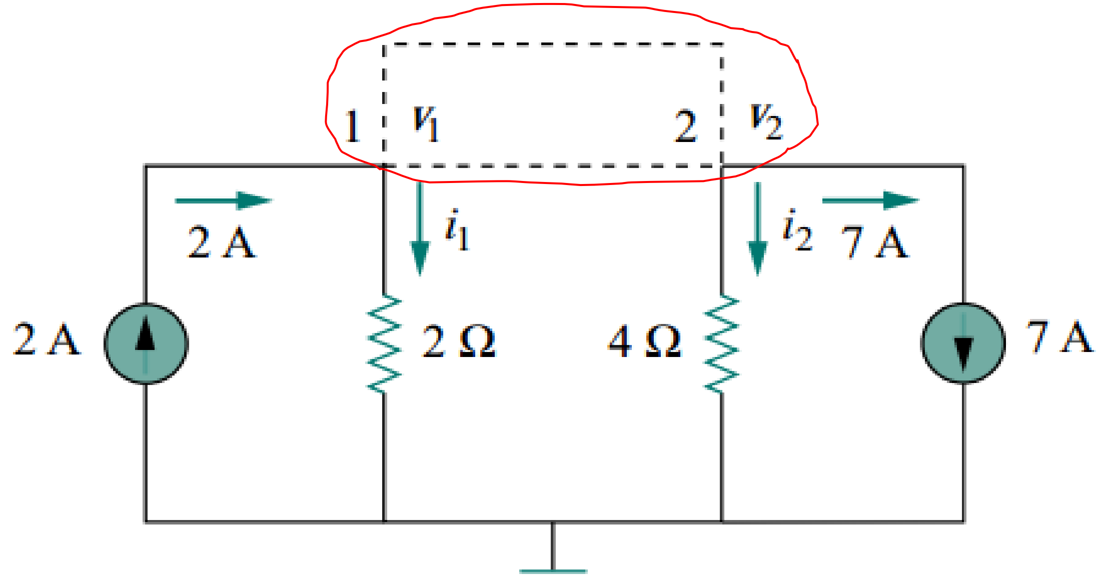
(a)

Applying KVL:



(b)

Solution problem 19:



Applying KCL to the supernode:

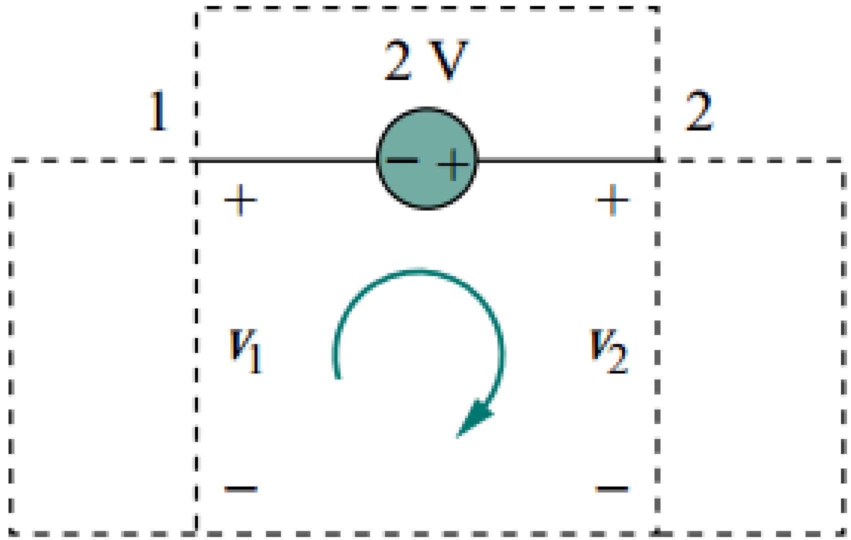
$$2 = i_1 + i_2 + 7 \quad (1)$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Longrightarrow \quad 8 = 2v_1 + v_2 + 28$$

or $$v_2 = -20 - 2v_1 \quad (2)$$

Solution problem 19:



Applying KVL to the circuit

$$-v_1 - 2 + v_2 = 0 \quad \Longrightarrow \quad v_2 = v_1 + 2 \qquad (3)$$

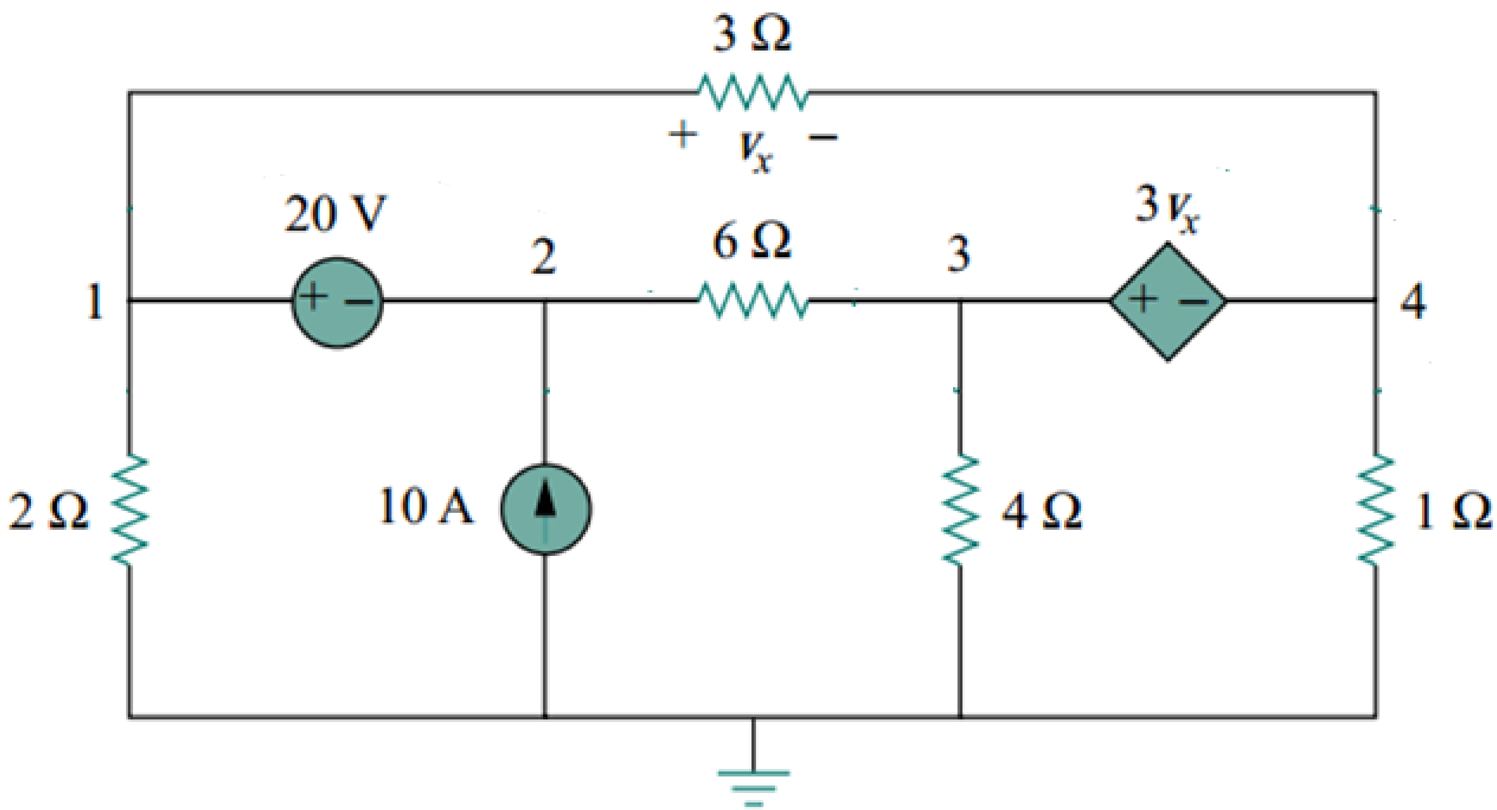
From Eqs. (2) and (3), we write $\quad v_2 = v_1 + 2 = -20 - 2v_1$

or $\quad 3v_1 = -22 \quad \Longrightarrow \quad v_1 = -7.333 \text{ V}$

V2 = -5.333V

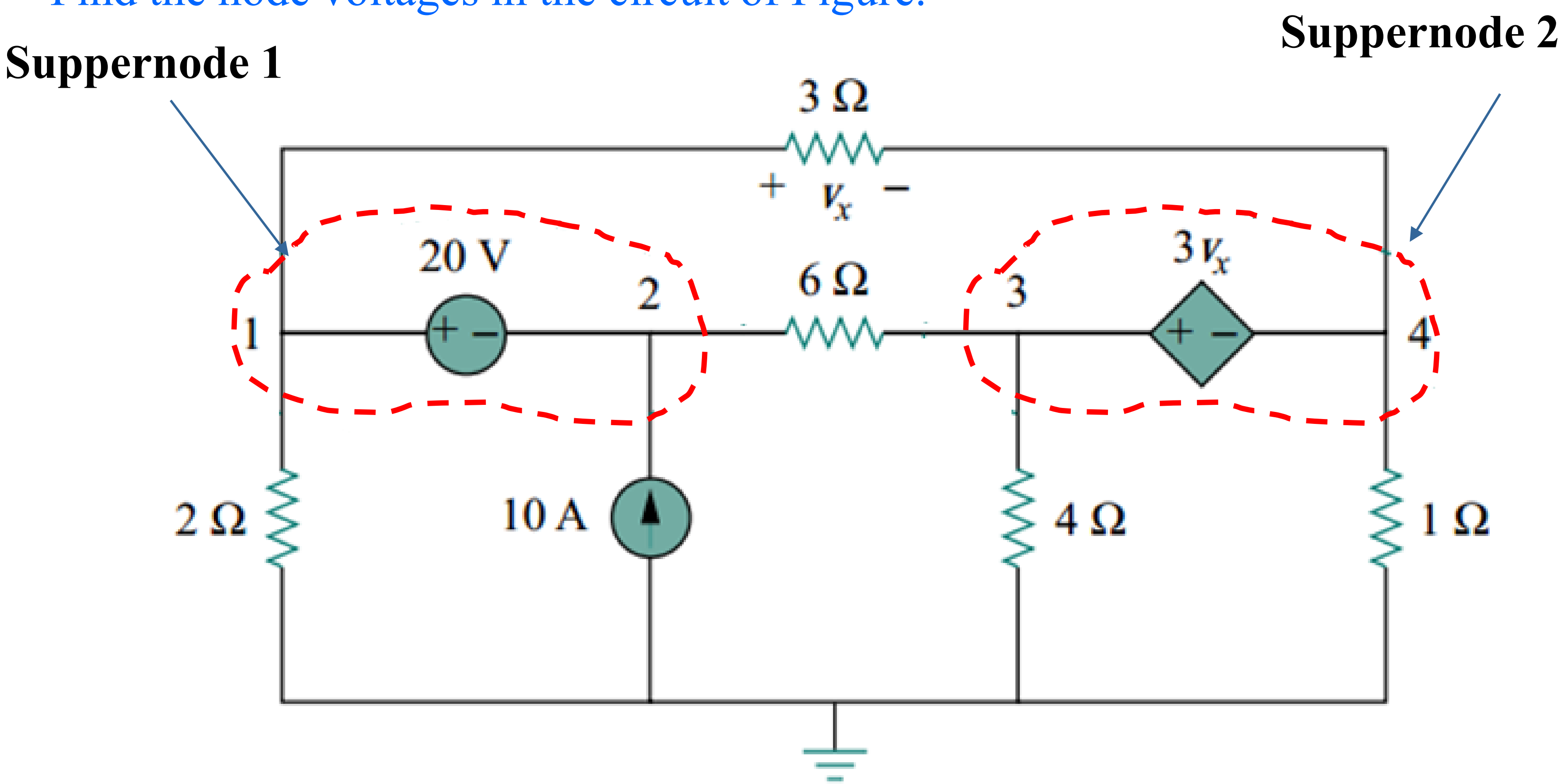
Problem 20:

Find the node voltages in the circuit of Figure.



Solution Problem 20:

Find the node voltages in the circuit of Figure.



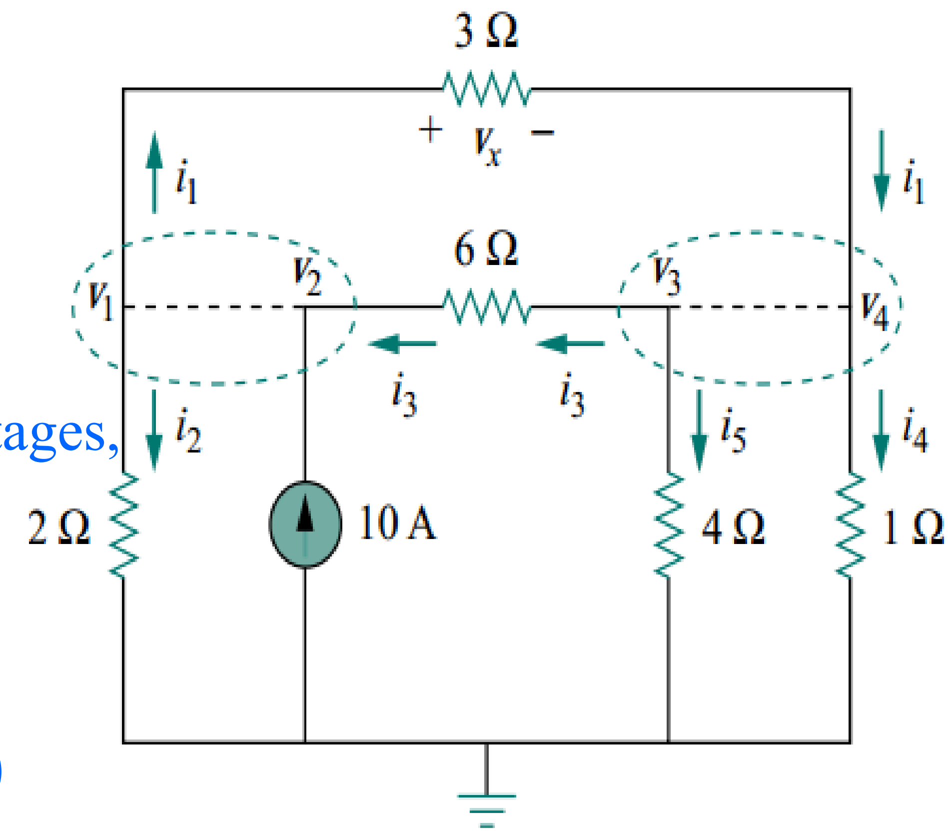
Solution Problem 20:

We apply KCL to the two supernodes

- At supernode 1-2:

$$i_3 + 10 = i_1 + i_2$$

Expressing this in terms of the node voltages,



or
$$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

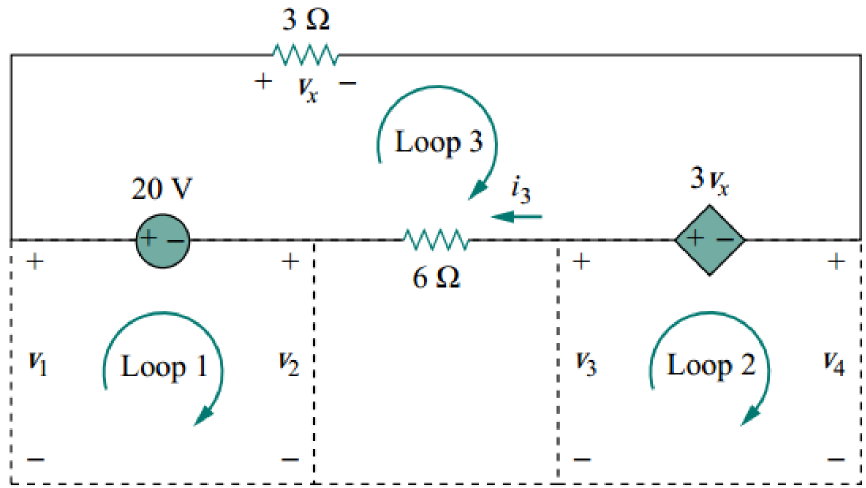
$$5v_1 + v_2 - v_3 - 2v_4 = 60 \quad (1)$$

- At supernode 3-4,

$$i_1 = i_3 + i_4 + i_5 \implies \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

or
$$4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \quad (2)$$

Solution Problem 20:



We now apply KVL to the branches involving the voltage sources

- For loop 1, $-v_1 + 20 + v_2 = 0 \implies v_1 - v_2 = 20$ (3)
- For loop 2, $-v_3 + 3v_x + v_4 = 0$ But $v_x = v_1 - v_4$ so that
  $3v_1 - v_3 - 2v_4 = 0$ (4)
- For loop 3 $v_x - 3v_x + 6i_3 - 20 = 0$ But $6i_3 = v_3 - v_2$ and $v_x = v_1 - v_4$. Hence
  $-2v_1 - v_2 + v_3 + 2v_4 = 20$ (5)

Solution Problem 20:

We can eliminate one node voltage so that we solve three simultaneous equations instead of four.

From Eq. (3), $v_2 = v_1 - 20$. Substituting this into Eqs. (1) and (2), respectively, gives

$$6v_1 - v_3 - 2v_4 = 80 \qquad (6)$$

and

$$6v_1 - 5v_3 - 16v_4 = 40 \qquad (7)$$

Equations (4), (6), and (7) can be cast in matrix form as

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

Solution Problem 20:

Using Cramer's rule,

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{vmatrix} = -18, \qquad \Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{vmatrix} = -480$$

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{vmatrix} = -3120, \qquad \Delta_4 = \begin{vmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{vmatrix} = 840$$

Thus, we arrive at the node voltages as

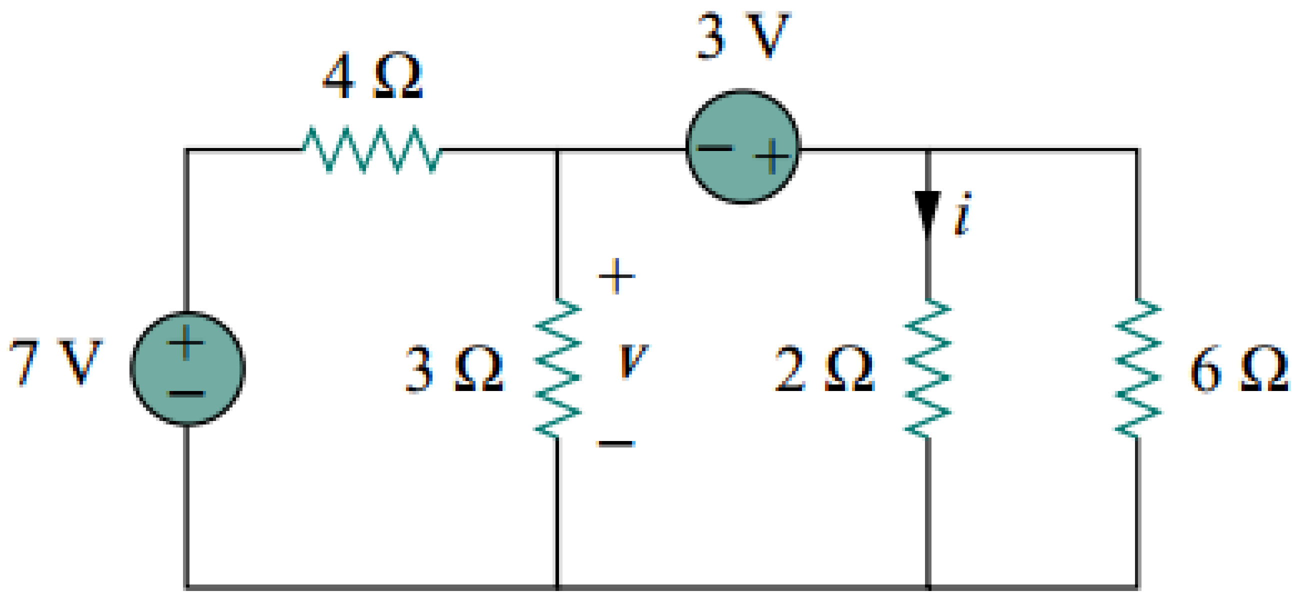
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.667 \text{ V}, \qquad v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.333 \text{ V}$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.667 \text{ V}$$

and $v_2 = v_1 - 20 = 6.667$ V. We have not used Eq. (5); it can be used to cross check results.

HOMEWORK 1

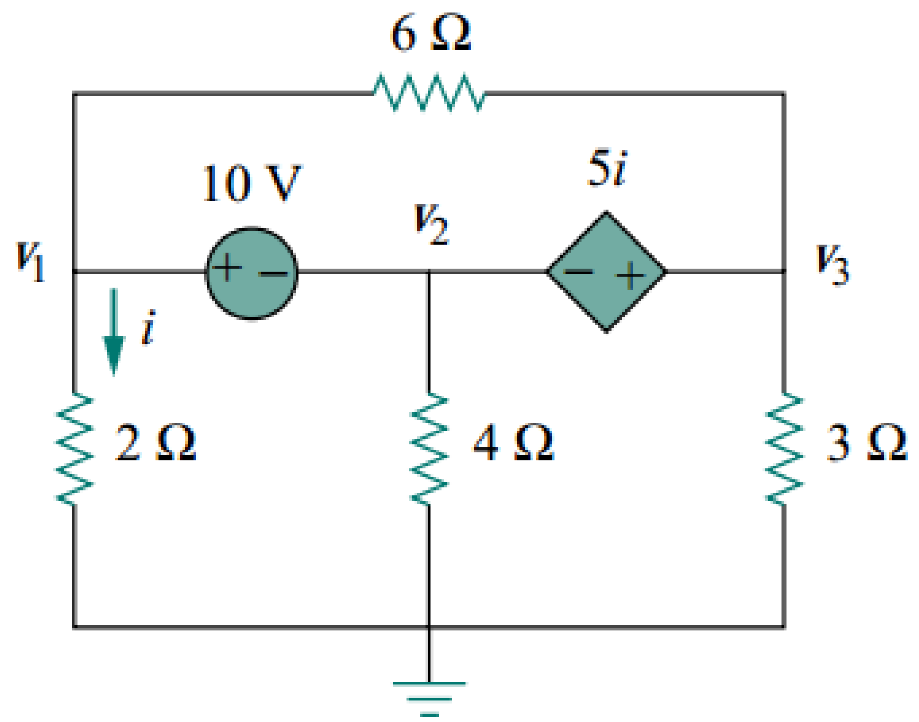
Find *v* and *i* in the circuit in Figure.



Answer: -0.2 V, 1.4 A.

HOMEWORK 2

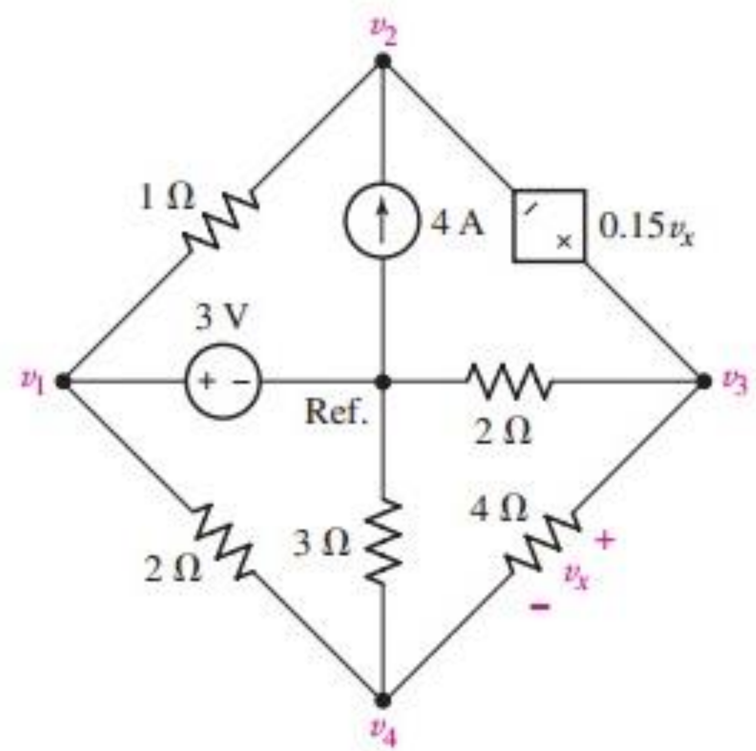
Find v_1, v_2, and v_3 in the circuit in Figure using nodal analysis.



Answer: $v_1 = 3.043$ V, $v_2 = -6.956$ V, $v_3 = 0.6522$ V

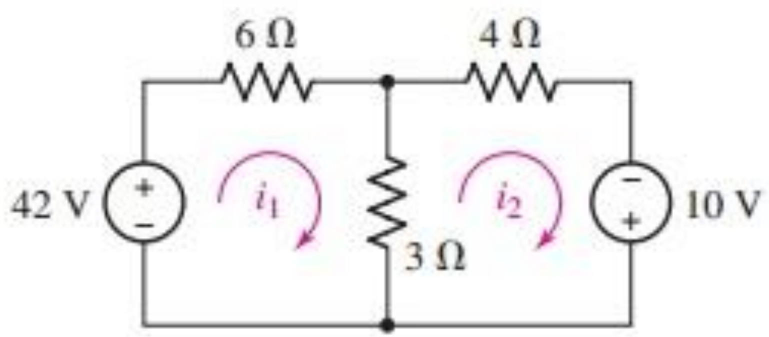
HOMEWORK 3

Determine the nodal voltages in the circuit



Ans: $v_1 = 3$ V, $v_2 = -2.33$ V, $v_3 = -1.91$ V, $v_4 = 0.945$ V.

- A mesh is a loop that does not contain any other loop within it.
- Nodal analysis applies KCL to find unknown voltages in a given circuit, *while mesh analysis applies KVL to find unknown currents.*



For the left-hand mesh,

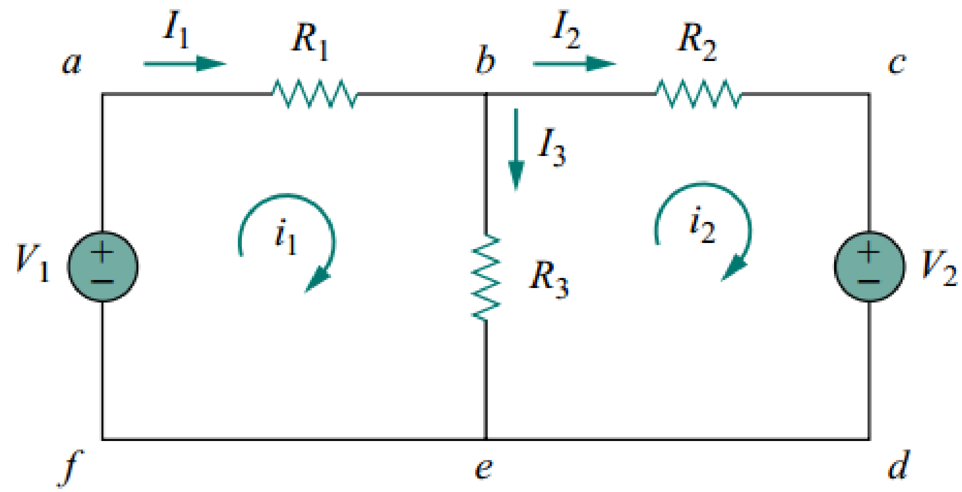
$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

while for the right-hand mesh,

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

Example 7:

Finding currents I_1 and I_2



Solution E7:

- Step 1: Assign mesh currents i_1, i_2 *in* to the *2* meshes
- Step 2: Applying KVL to 2 meshes:
  - Applying KVL to mesh 1, we obtain:

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0 \quad \text{or} \quad (R_1 + R_3)i_1 - R_3 i_2 = V_1 \qquad (1)$$

  - Applying KVL to mesh 2, we obtain:

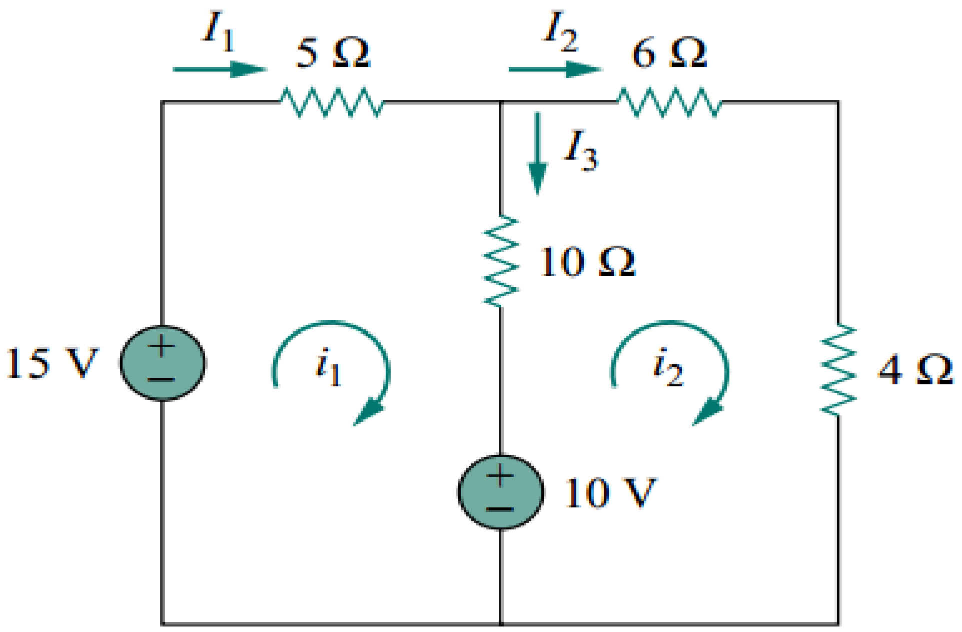
$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0 \quad \text{or} \quad -R_3 i_1 + (R_2 + R_3)i_2 = -V_2 \qquad (2)$$

- Step 3: Solving for the mesh currents.

2.8 Methods of Circuit Analysis: Mesh current

Example 8:

For the circuit in Figure, find the branch currents *I*1, *I*2, and *I*3 using mesh analysis.



Solution E8:

- Step 1: Assign mesh currents i_1, i_2 *in* to the *2* meshes
- Step 2: Applying KVL to 2 meshes:

For mesh 1: $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$ or $3i_1 - 2i_2 = 1$ (1)

For mesh 2: $6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$ or $i_1 = 2i_2 - 1$ (2)

To use Cramer's rule, we cast Eqs. (1) and (2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

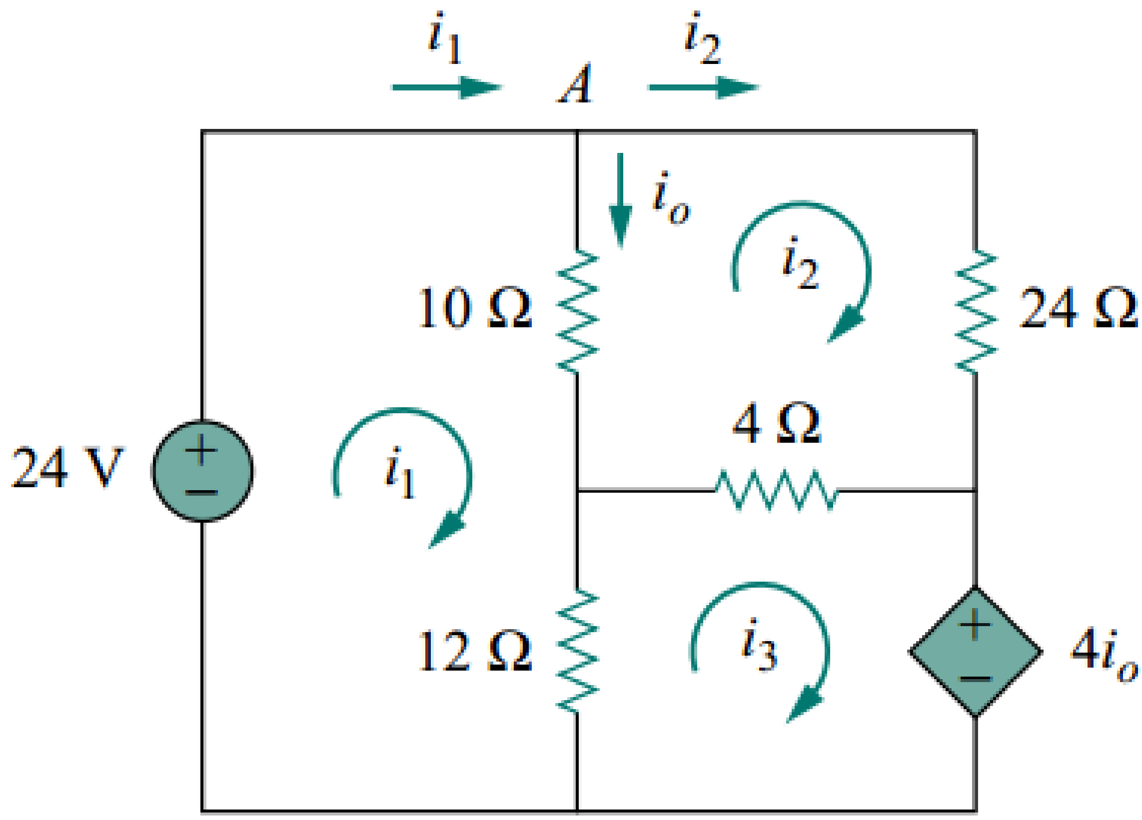
Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

as before.

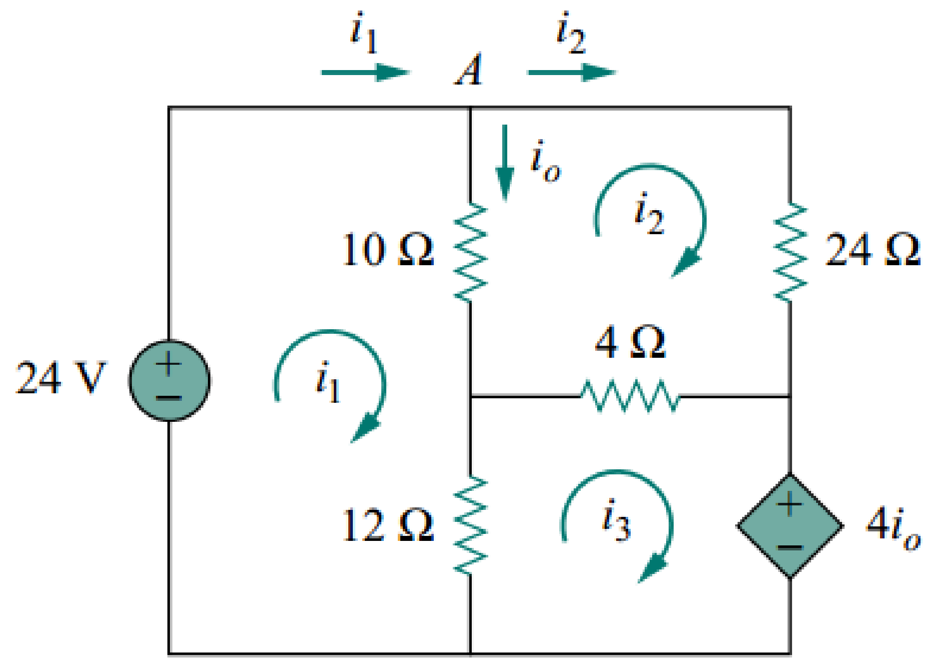
Example 9:

Use mesh analysis to find the current i_o in the circuit in Figure.



Solution E9:

We apply KVL to the three meshes:



- For mesh 1:

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or $$11i_1 - 5i_2 - 6i_3 = 12 \qquad (1)$$

- For mesh 2:

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or $$-5i_1 + 19i_2 - 2i_3 = 0 \qquad (2)$$

- For mesh 3:

$$4i_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $i_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or $$-i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

2.8 Methods of Circuit Analysis: Mesh current

To use Cramer's rule, we cast Eqs. (1), (2) and (2) in matrix form as

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{array}{c|ccc|c} & 11 & -5 & -6 & \\ & -5 & 19 & -2 & \\ & -1 & -1 & 2 & \\ - & 11 & -5 & -6 & + \\ - & -5 & 19 & -2 & + \\ - & & & & + \end{array}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{array}{c|ccc|c} & 12 & -5 & -6 & \\ & 0 & 19 & -2 & \\ & 0 & -1 & 2 & \\ - & 12 & -5 & -6 & + \\ - & 0 & 19 & -2 & + \\ - & & & & + \end{array} = 456 - 24 = 432$$

$$\Delta_2 = \begin{array}{c|ccc|c} & 11 & 12 & -6 & \\ & -5 & 0 & -2 & \\ & -1 & 0 & 2 & \\ - & 11 & 12 & -6 & + \\ - & -5 & 0 & -2 & + \\ - & & & & + \end{array} = 24 + 120 = 144$$

$$\Delta_3 = \begin{array}{c|ccc|c} & 11 & -5 & 12 & \\ & -5 & 19 & 0 & \\ & -1 & -1 & 0 & \\ - & 11 & -5 & 12 & + \\ - & -5 & 19 & 0 & + \\ - & & & & + \end{array} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A} \qquad i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $i_o = i_1 - i_2 = 1.5$ A.

2.8 Methods of Circuit Analysis: Mesh current

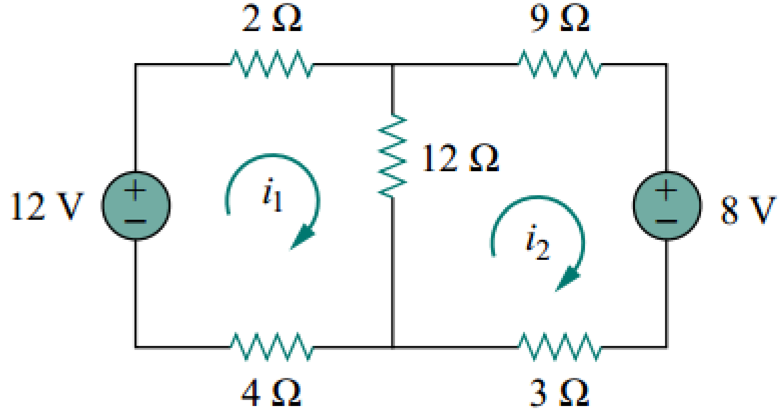
Summary of Basic Mesh Analysis Procedure

1. **Determine if the circuit is a planar circuit.** If not, perform nodal analysis instead.
2. **Count the number of meshes** (M). Redraw the circuit if necessary.
3. **Label each of the M mesh currents.** Generally, defining all mesh currents to flow clockwise results in a simpler analysis.
4. **Write a KVL equation around each mesh.** Begin with a convenient node and proceed in the direction of the mesh current. Pay close attention to "−" signs. If a current source lies on the periphery of a mesh, no KVL equation is needed and the mesh current is determined by inspection.
5. **Express any additional unknowns such as voltages or currents other than mesh currents in terms of appropriate mesh currents.** This situation can occur if current sources or dependent sources appear in our circuit.
6. **Organize the equations.** Group terms according to mesh currents.
7. **Solve the system of equations for the mesh currents** (there will be M of them).

2.8 Methods of Circuit Analysis: Mesh current

HOMEWORK 1

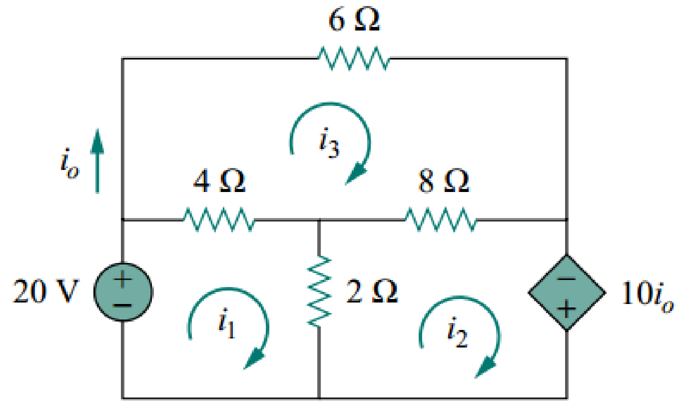
Calculate the mesh currents i_1 and i_2 in the circuit of Figure.



Answer: $i_1 = \frac{2}{3}$ A, $i_2 = 0$ A.

HOMEWORK 2

Using mesh analysis, find i_o in the circuit in Fig. 3.21.

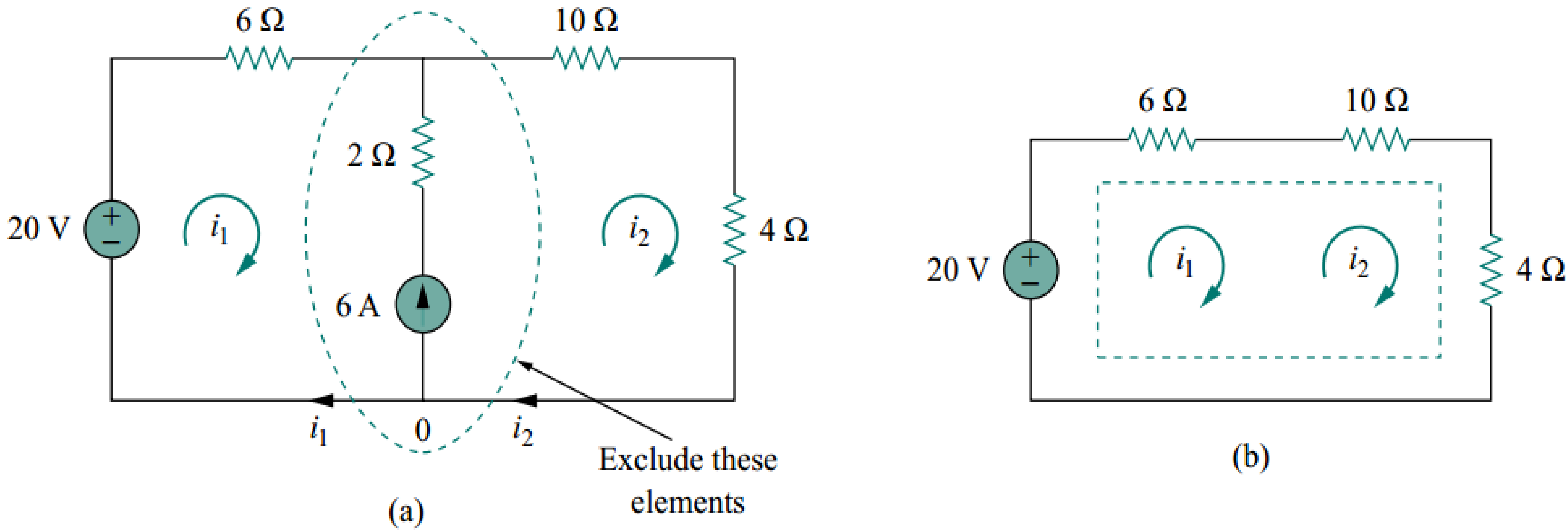


Answer: −5 A.

2.8 Methods of Circuit Analysis: Supermesh (Siêu nút)

When a current source exists between two meshes: Consider the circuit in Figure(a). We create a ***supermesh*** by excluding the current source and any elements connected in series with it, as shown in Figure (b). Thus,

A **supermesh** results when two meshes have a (dependent or independent) current source in common.



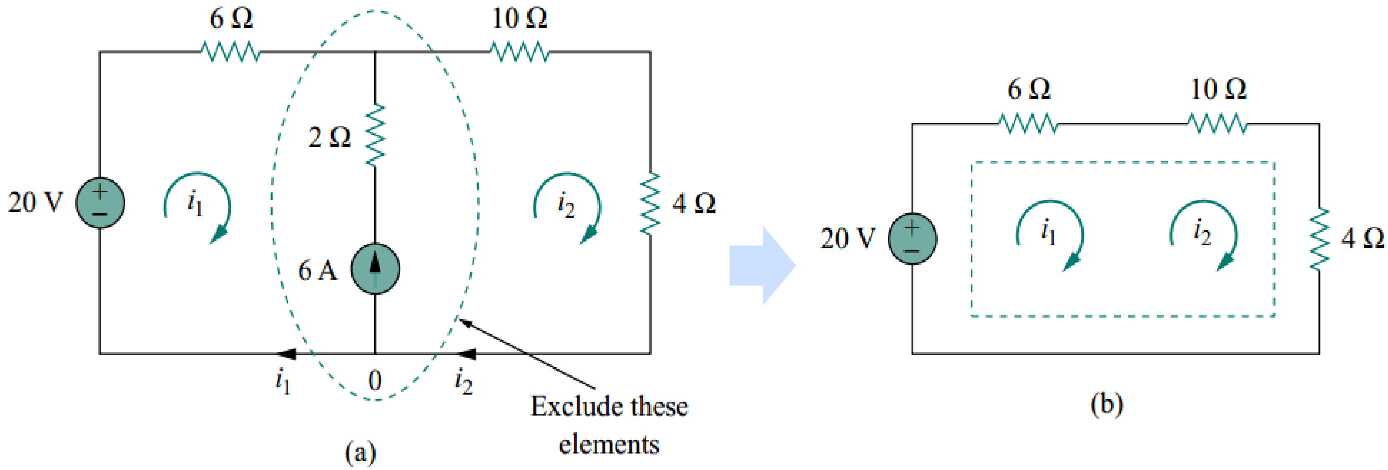
(a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

2.8 Methods of Circuit Analysis: Supermesh

Note the following properties of a supermesh:

1. The current source in the supermesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

Example 10: Use mesh analysis to find the current i_1 and i_2 the circuit in Figure.



Solution E10:

Applying KVL to the supermesh in Figure(b) gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0 \quad \text{or} \quad 6i_1 + 14i_2 = 20 \qquad (1)$$

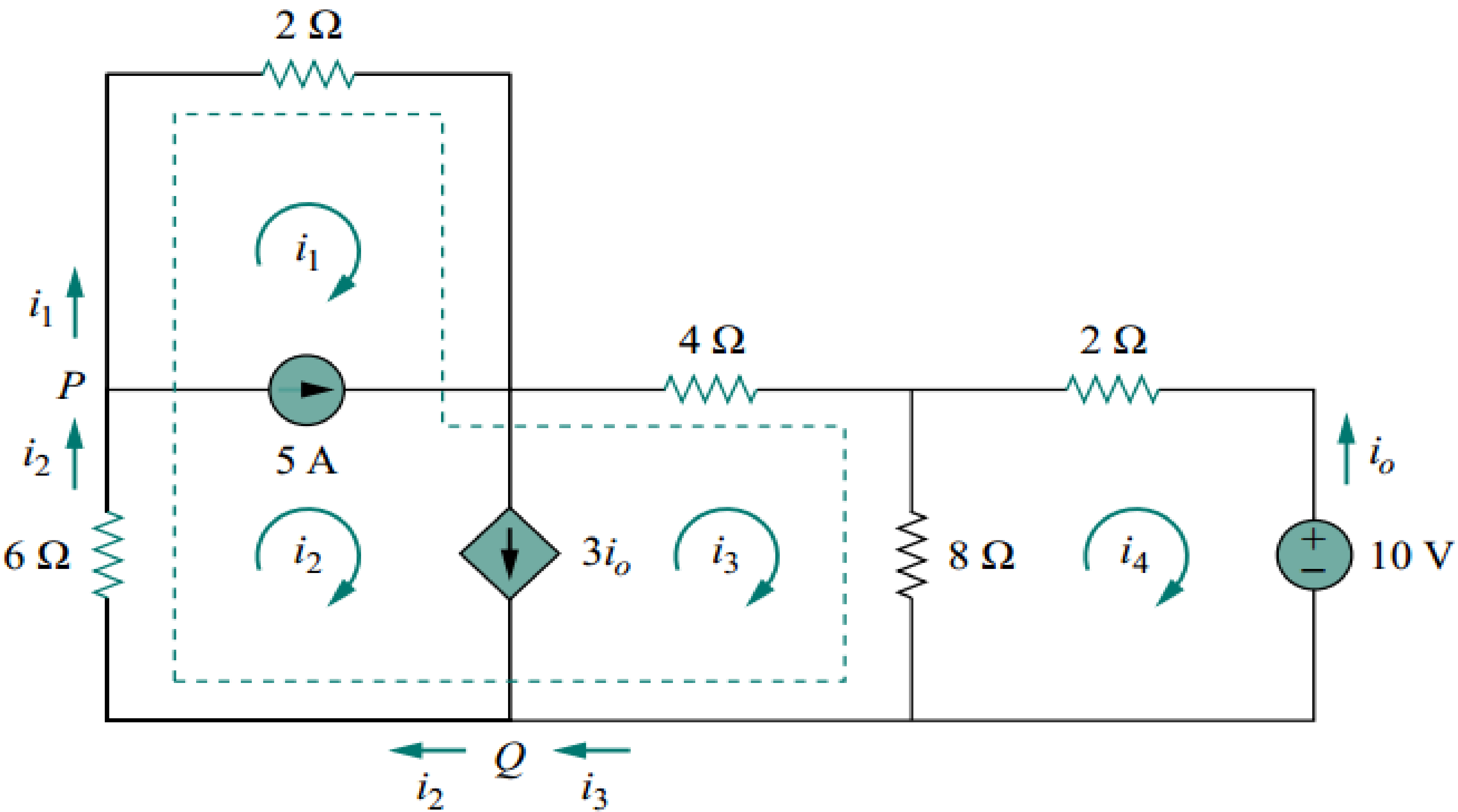
We apply KCL to a node in the branch where the two meshes intersect (node 0)

$$i_2 = i_1 + 6 \qquad (2)$$

Solving Eqs. (1) and (2), we get

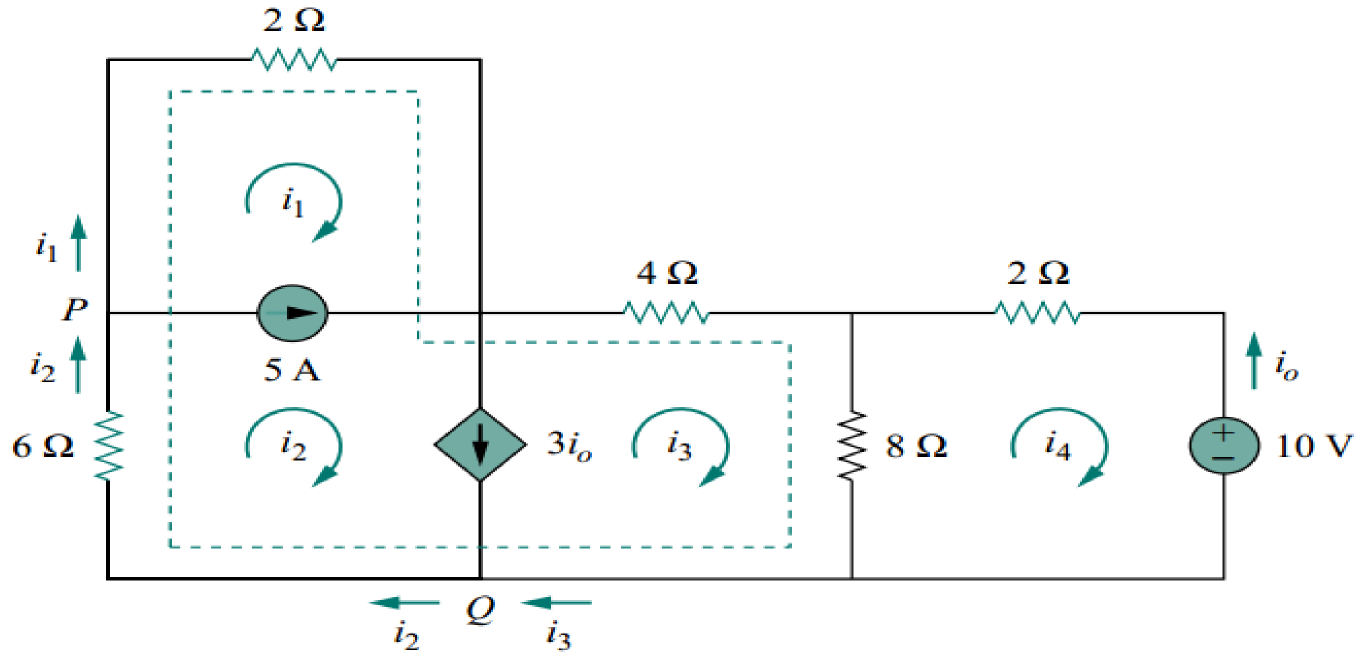
$$i_1 = -3.2 \text{ A}, \qquad i_2 = 2.8 \text{ A}$$

Example 11: For the circuit in Figure, find i_1 to i_4 using mesh analysis



2.8 Methods of Circuit Analysis: Supermesh

Solution E11: For the circuit in Figure, find i_1 to i_4 using mesh analysis



Applying KVL to the larger supermesh,

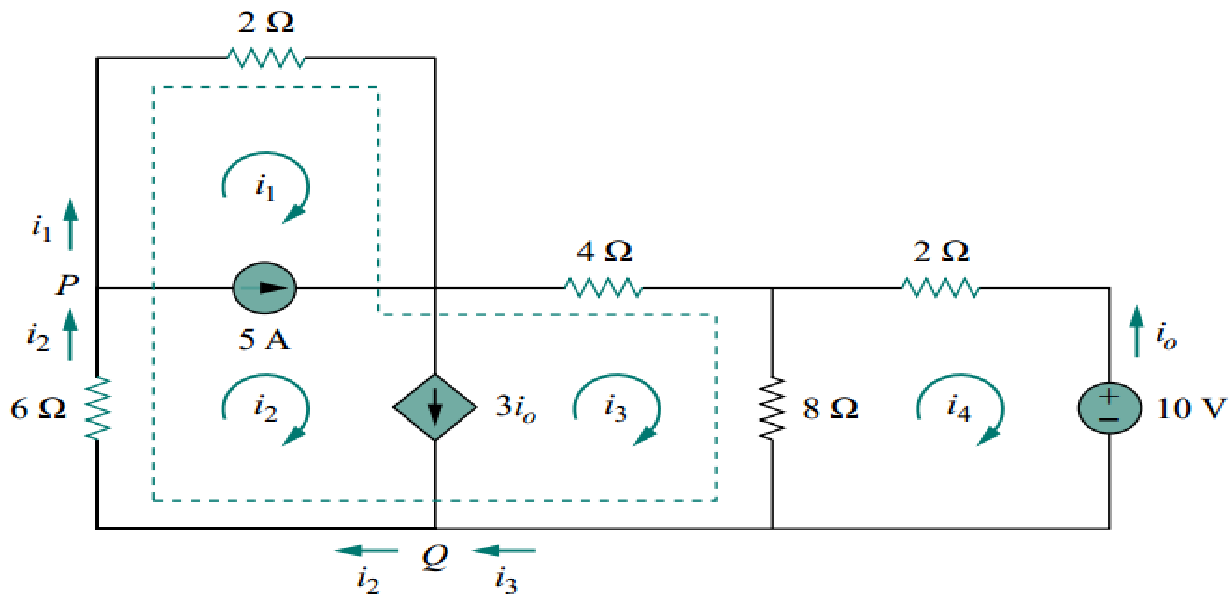
$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0 \quad \text{or} \quad i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad (1)$$

For the independent current source, we apply KCL to node *P* : $i_2 = i_1 + 5$ (2)

For the dependent current source, we apply KCL to node *Q*:

$$i_2 = i_3 + 3i_o \quad \text{But } i_o = -i_4\text{, hence,} \quad i_2 = i_3 - 3i_4 \quad (3)$$

Solution E11:



Applying KVL in mesh 4, $2i_4 + 8(i_4 - i_3) + 10 = 0$

or $5i_4 - 4i_3 = -5$ (4)

From Eqs. (1) to (4),

$$i_1 = -7.5 \text{ A}, \quad i_2 = -2.5 \text{ A}, \quad i_3 = 3.93 \text{ A}, \quad i_4 = 2.143 \text{ A}$$

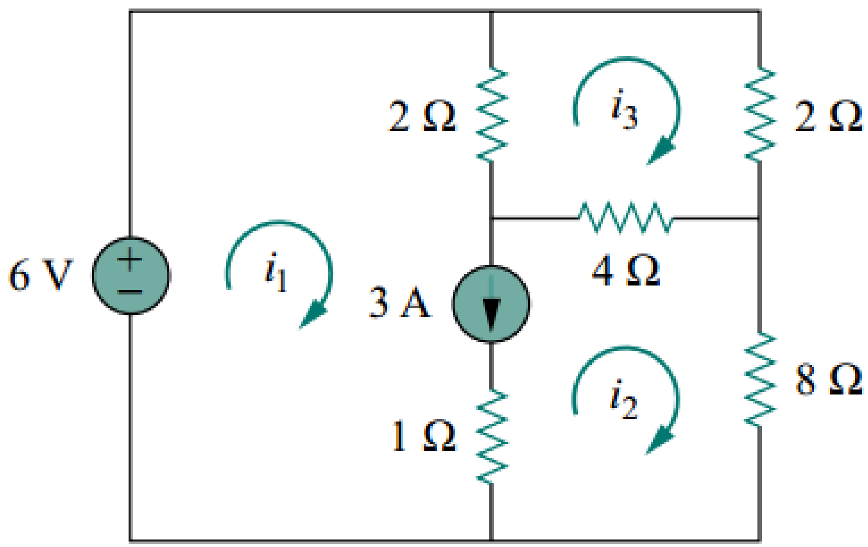
2.8 Methods of Circuit Analysis: Supermesh

Summary of Supermesh Analysis Procedure

1. **Determine if the circuit is a planar circuit.** If not, perform nodal analysis instead.
2. **Count the number of meshes** (M). Redraw the circuit if necessary.
3. **Label each of the M mesh currents.** Generally, defining all mesh currents to flow clockwise results in a simpler analysis.
4. **If the circuit contains current sources shared by two meshes, form a supermesh to enclose both meshes.** A highlighted enclosure helps when writing KVL equations.
5. **Write a KVL equation around each mesh/supermesh.** Begin with a convenient node and proceed in the direction of the mesh current. Pay close attention to "—" signs. If a current source lies on the periphery of a mesh, no KVL equation is needed and the mesh current is determined by inspection.
6. **Relate the current flowing from each current source to mesh currents.** This is accomplished by simple application of KCL; one such equation is needed for each supermesh defined.
7. **Express any additional unknowns such as voltages or currents other than mesh currents in terms of appropriate mesh currents.** This situation can occur if dependent sources appear in our circuit.
8. **Organize the equations.** Group terms according to nodal voltages.
9. **Solve the system of equations for the mesh currents** (there will be M of them).

HOMEWORK 1

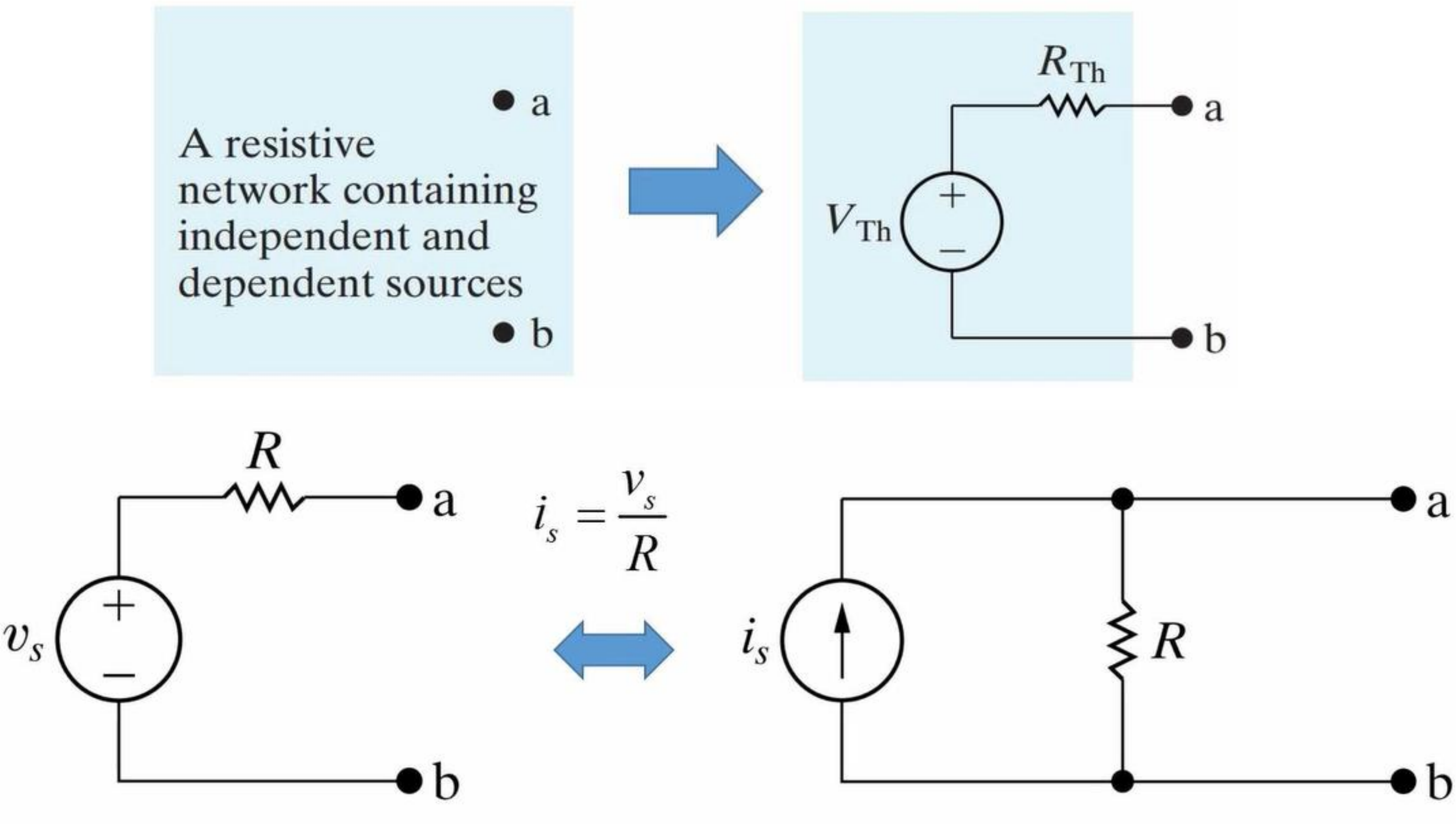
Use mesh analysis to determine i_1, i_2, and i_3 in Figure?



Answer: $i_1 = 3.474$ A, $i_2 = 0.4737$ A, $i_3 = 1.1052$ A.

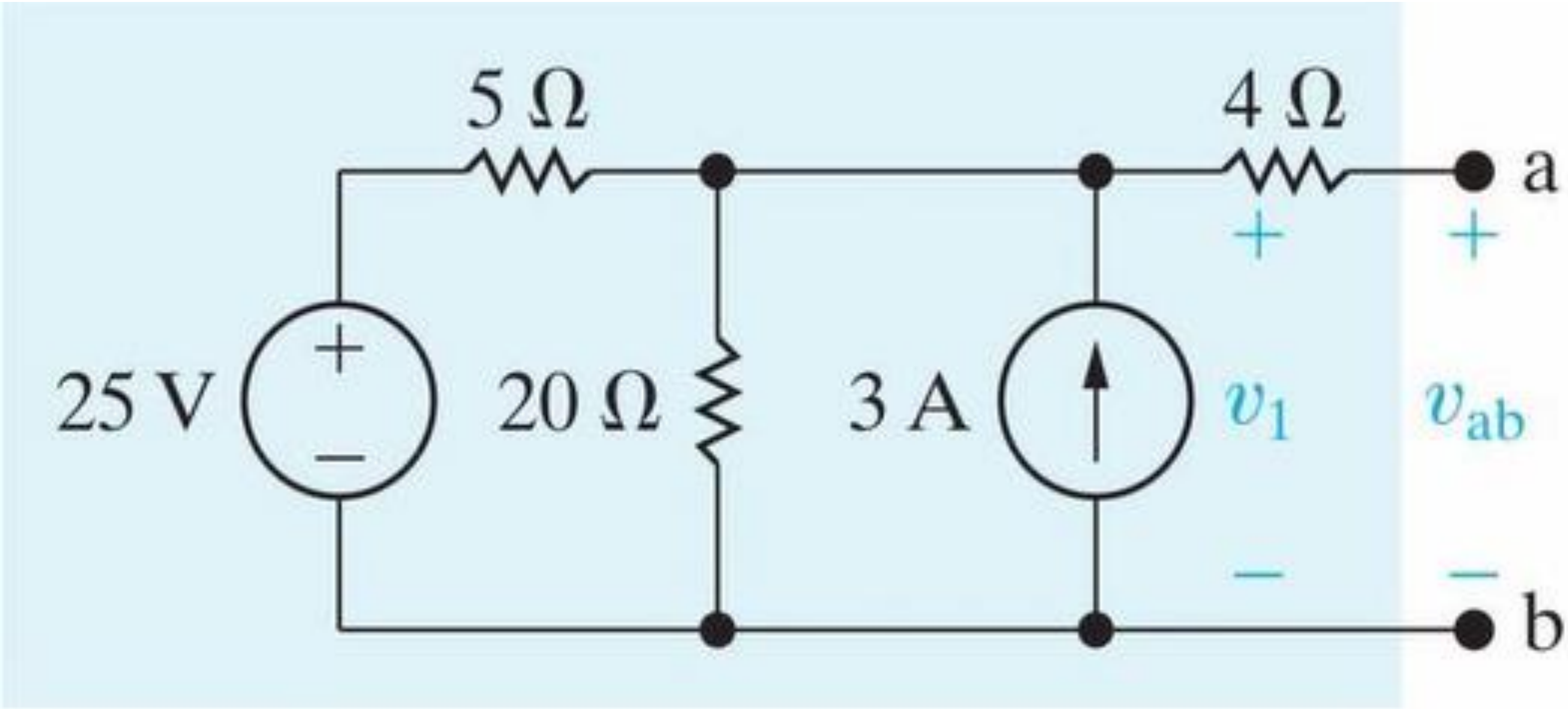
2.9 Thévenin-Norton Equivalents: Tương đương

Thévenin and Norton Equivalents



Example 12:

Finding the Thévenin Equivalent with a Independent Source



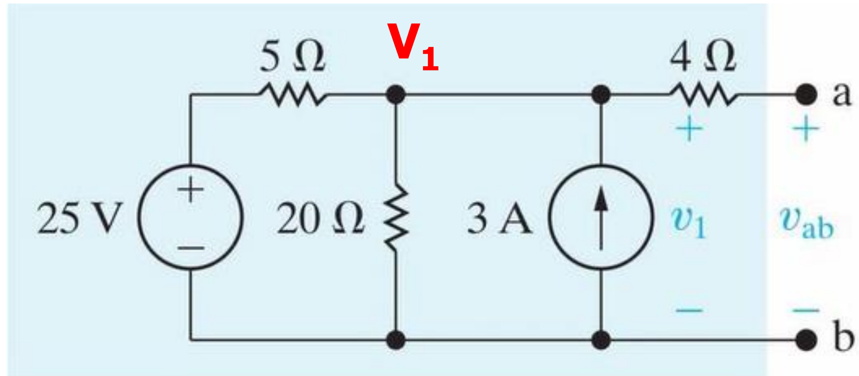
Solution E12: Finding V_{th} and R_{th}

Solution#1: a,b open

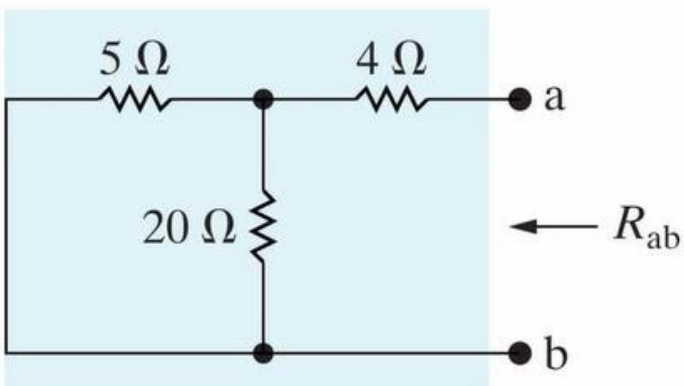
Find V_{th}

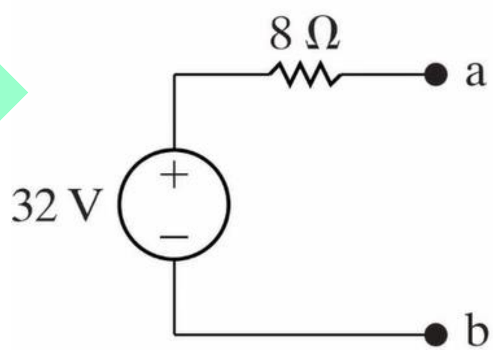
$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0$$

$$\Rightarrow v_1 = 32(V) = V_{Th}$$



Find R_{th}



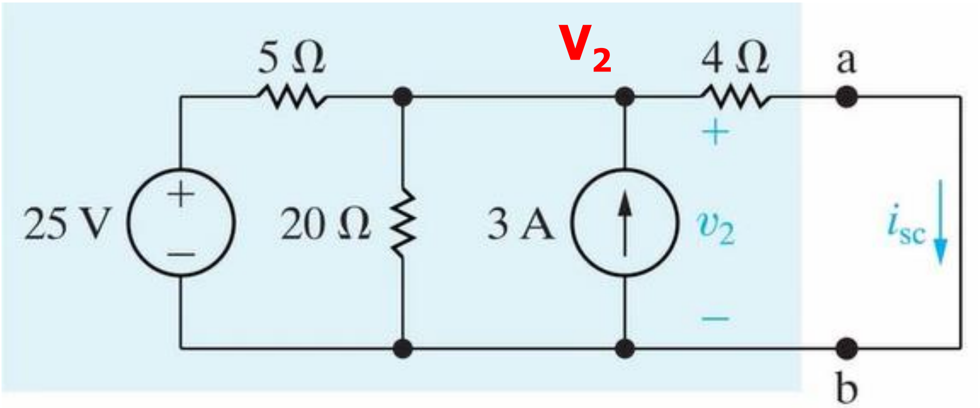


(5//20)nt4

$$R_{ab} = R_{Th} = 4 + \frac{5x20}{25} = 8(\Omega)$$

Solution#2: a,b close

Find R_{th}

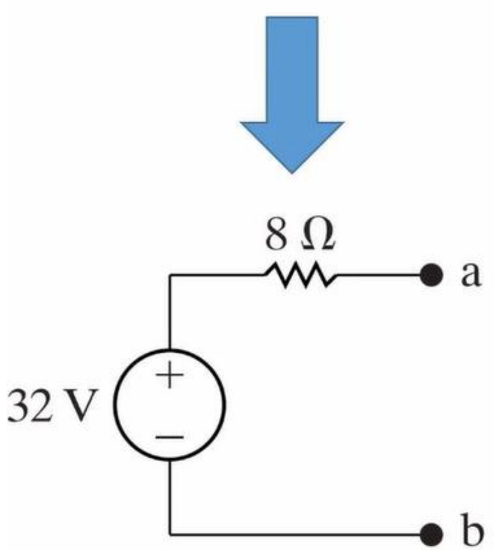


$$\frac{v_2-25}{5}+\frac{v_2}{20}+\frac{v_2}{4}-3=0$$

$$\Rightarrow v_2=16(V)$$

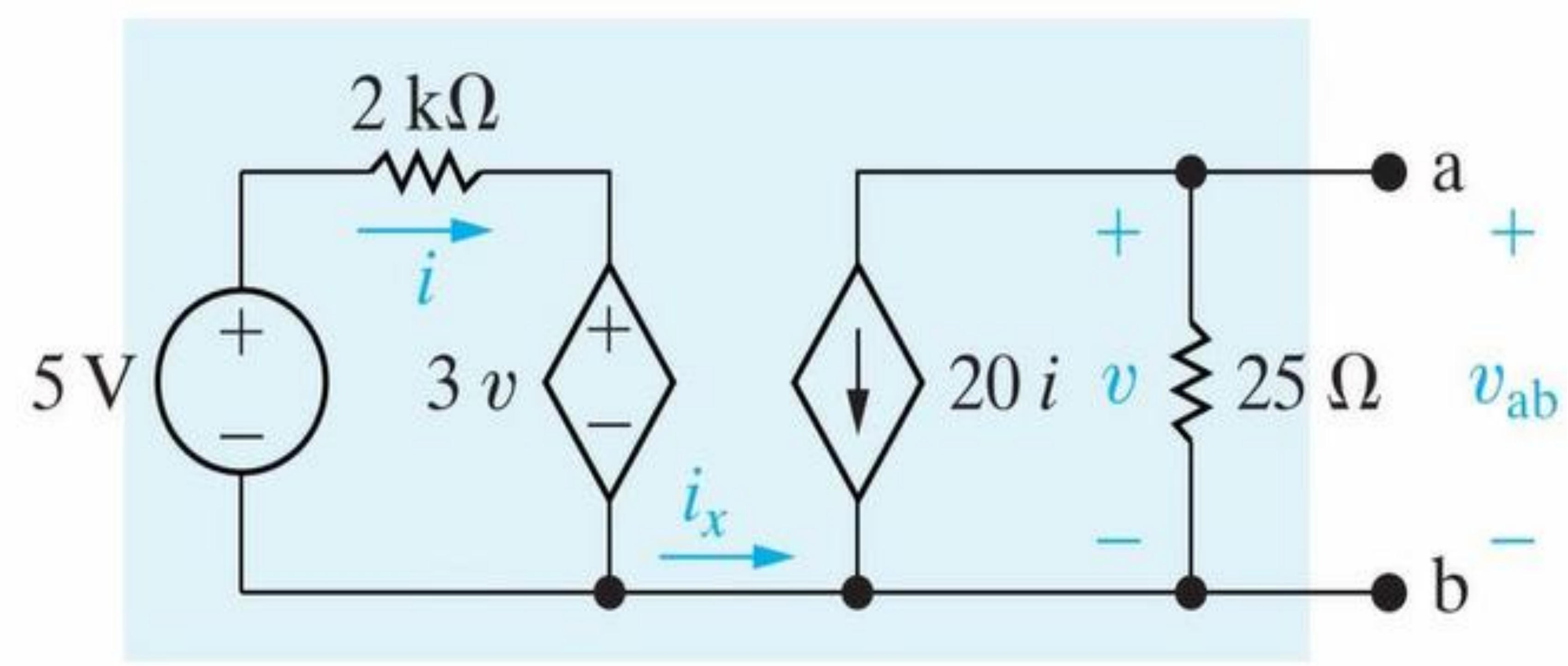
$$\Rightarrow i_{sc}=\frac{16}{4}=4(A)$$

$$\Rightarrow R_{Th}=\frac{V_{Th}}{i_{sc}}=\frac{32}{4}=8\Omega$$



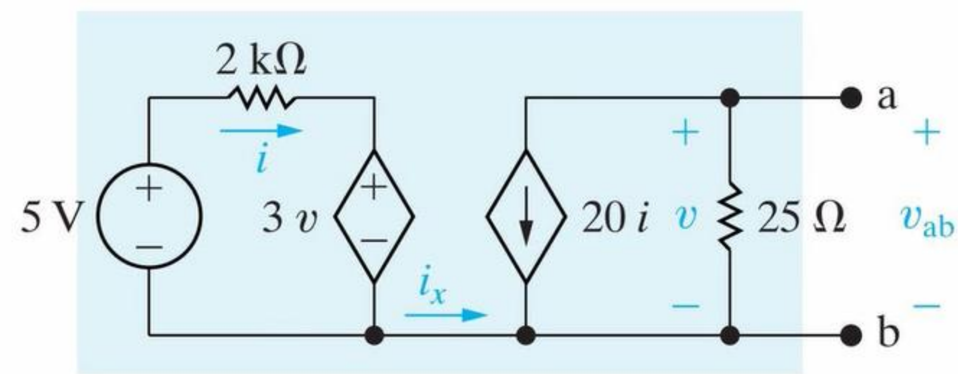
Example 13:

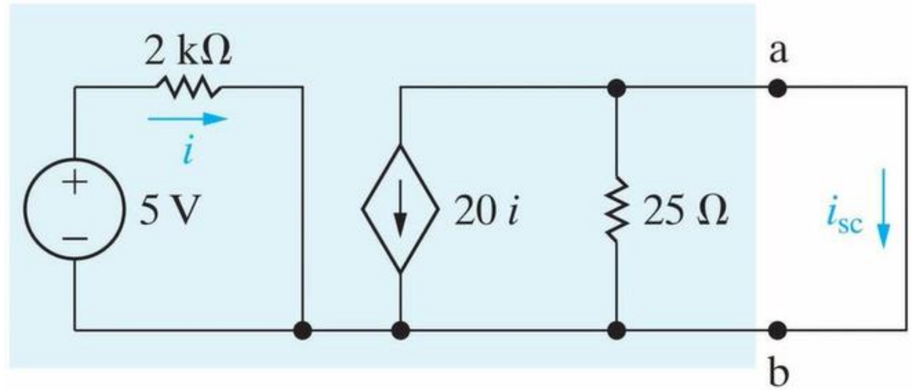
Finding the Thévenin Equivalent with a Dependent Source



2.9 Thévenin-Norton Equivalents

Solution of Example 13:



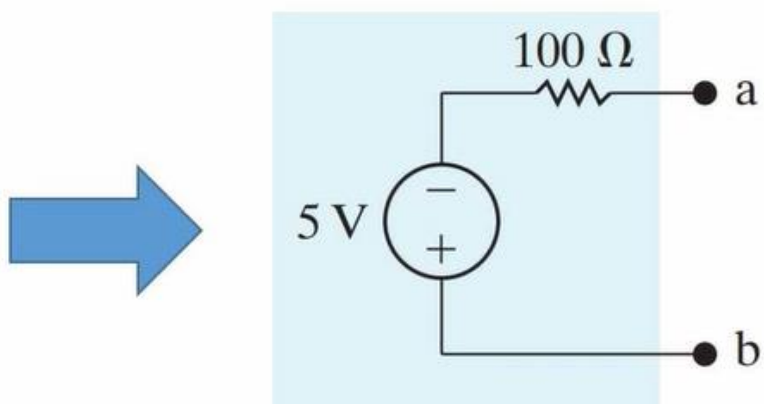


$$\begin{cases} V_{Th} = v_{ab} = (-20i)(25) = -500i \\ i = \dfrac{5-3v}{2000} = \dfrac{5-3V_{Th}}{2000} \end{cases}$$

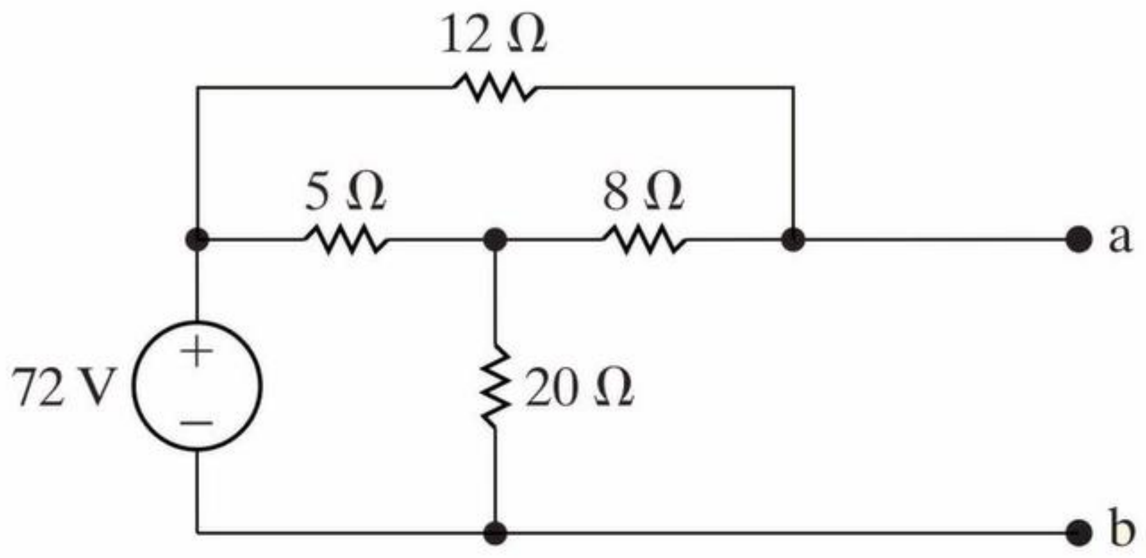
$$\Rightarrow V_{Th} = -5(V)$$

$$\begin{cases} i_{sc} = -20i \\ i = \dfrac{5}{2000} = 2.5(mA) \end{cases} \Rightarrow i_{sc} = -50(mA)$$

$$\Rightarrow R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-5}{-50} x10^3 = 100\Omega$$



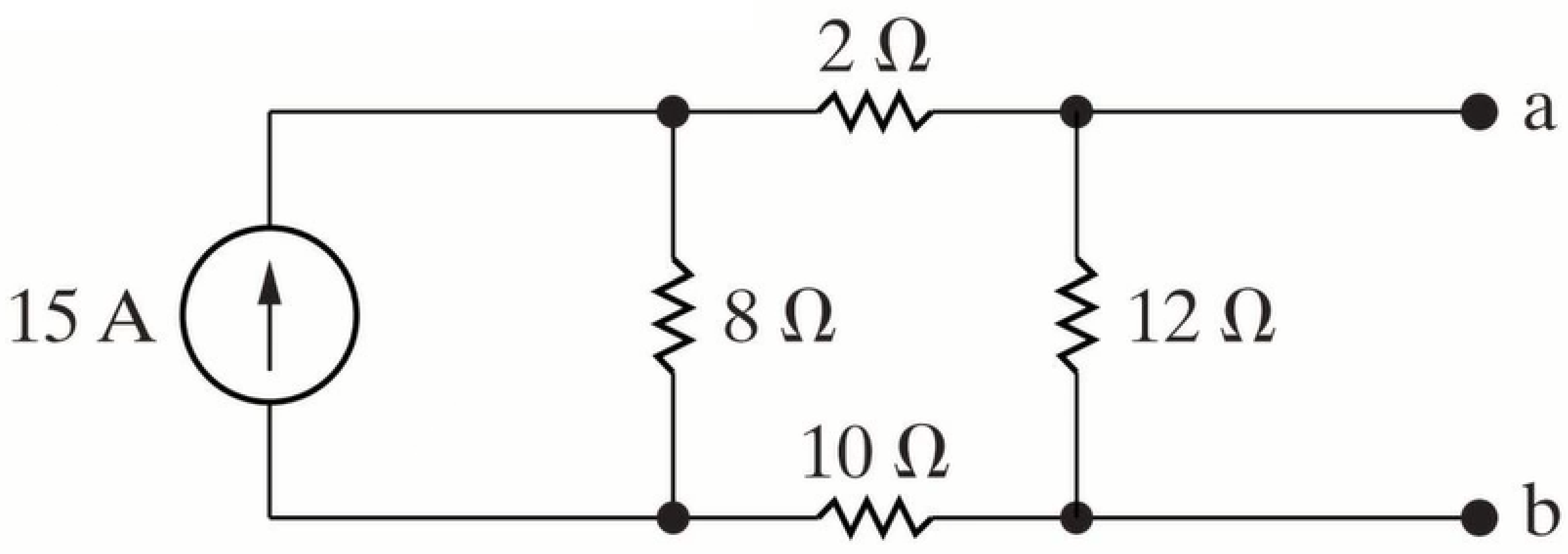
1) Find the Thévenin equivalent circuit?



Answer: $V_{ab} = V_{Th} = 64.8(V)$

$R_{Th} = 6(\Omega)$

2) *Find the Norton equivalent circuit?*

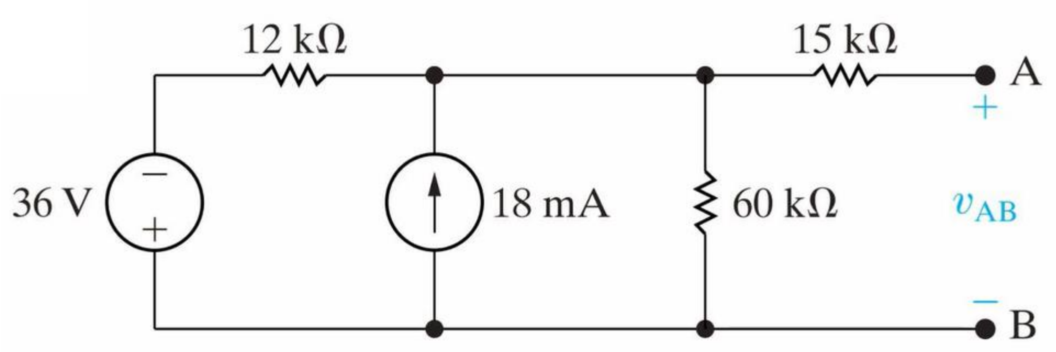


Answer: $I_N = 6(A)$

$R_N = 7.5(\Omega)$

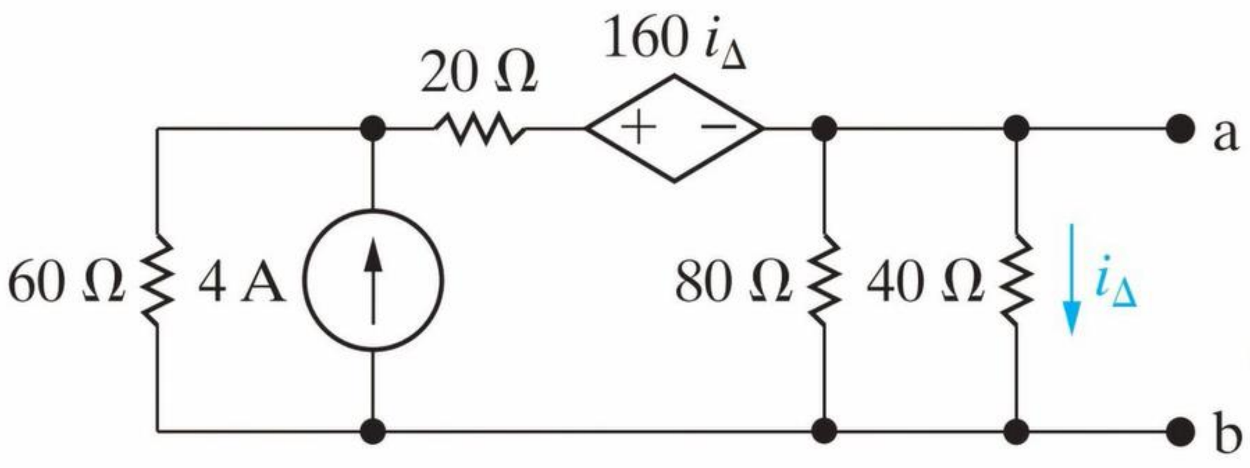
2.9 Thévenin-Norton Equivalents - HOMEWORK

3) *Find* v_{AB} ?



Answer: $v_{AB} = 120(V)$

4) *Find the Thévenin equivalent circuit?*

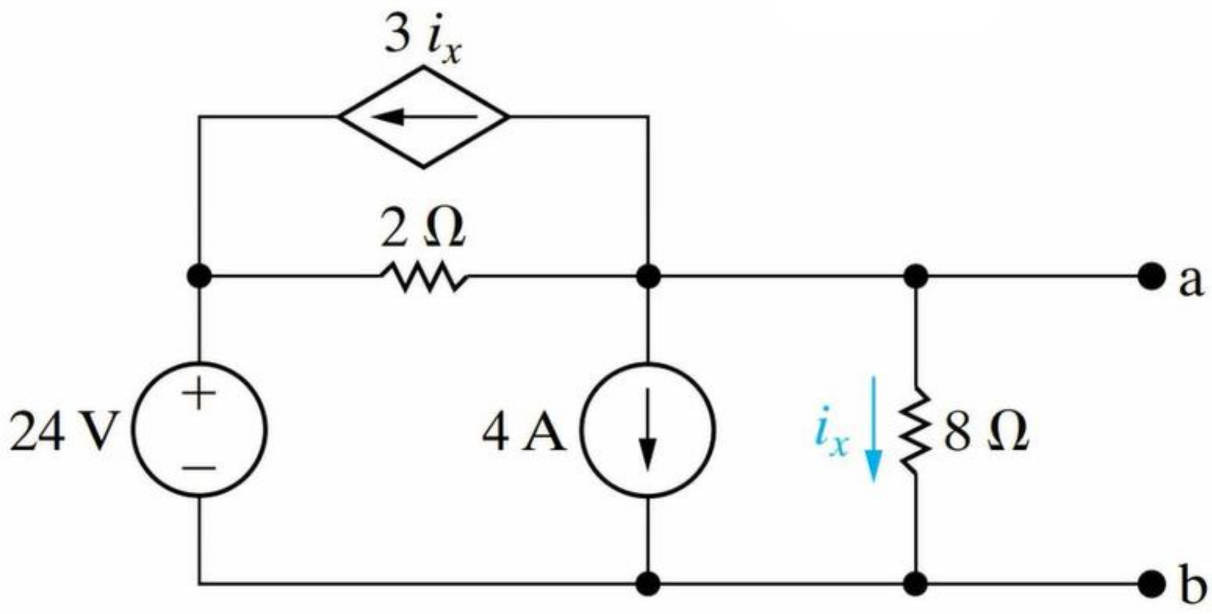


Answer: $V_{ab} = V_{Th} = 30(V)$

$R_{Th} = 10(\Omega)$

2.9 Thévenin-Norton Equivalents - HOMEWORK

5) *Find the Thévenin equivalent circuit?*



Answer: $V_{ab} = V_{Th} = 8(V)$

$R_{Th} = 1(\Omega)$

📖 **Phát biểu 1:**

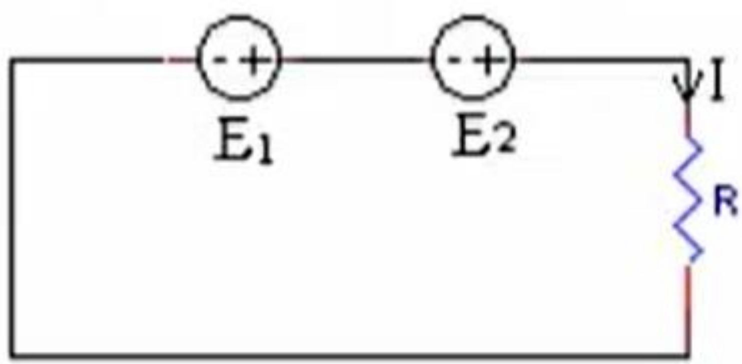
Đáp ứng của ***nhiều nguồn*** kích thích tác động ***đồng thời*** thì ***bằng tổng*** các đáp ứng ***tạo bởi mỗi nguồn kích thích*** tác động riêng lẻ

📖 **Cách phát biểu khác:**

Trong mạch gồm nhiều nguồn (Nguồn áp, dòng độc lập) dòng điện qua một nhánh bằng tổng đại số các dòng điện qua nhánh đó do tác dụng riêng rẻ của từng nguồn, các nguồn khác xem như bằng 0.

☞ **Lưu ý**: Nhiều nguồn kích thích (có thể áp hoặc dòng) ***độc lập***. Nguồn dòng thì cho ***hở mạch*** còn nguồn áp thì ***ngắn mạch***

Ví dụ 1:

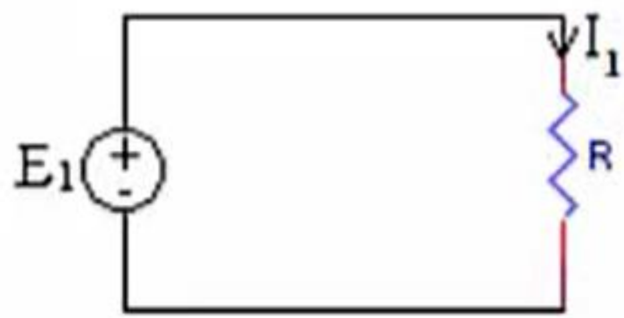


$$E_1 + E_2 = I.R$$

Solution:

Cho từng nguồn tác động:

- **Bước 1**: E_1 tác động: $E_2=0$



$$E_1 = I_1 . R$$

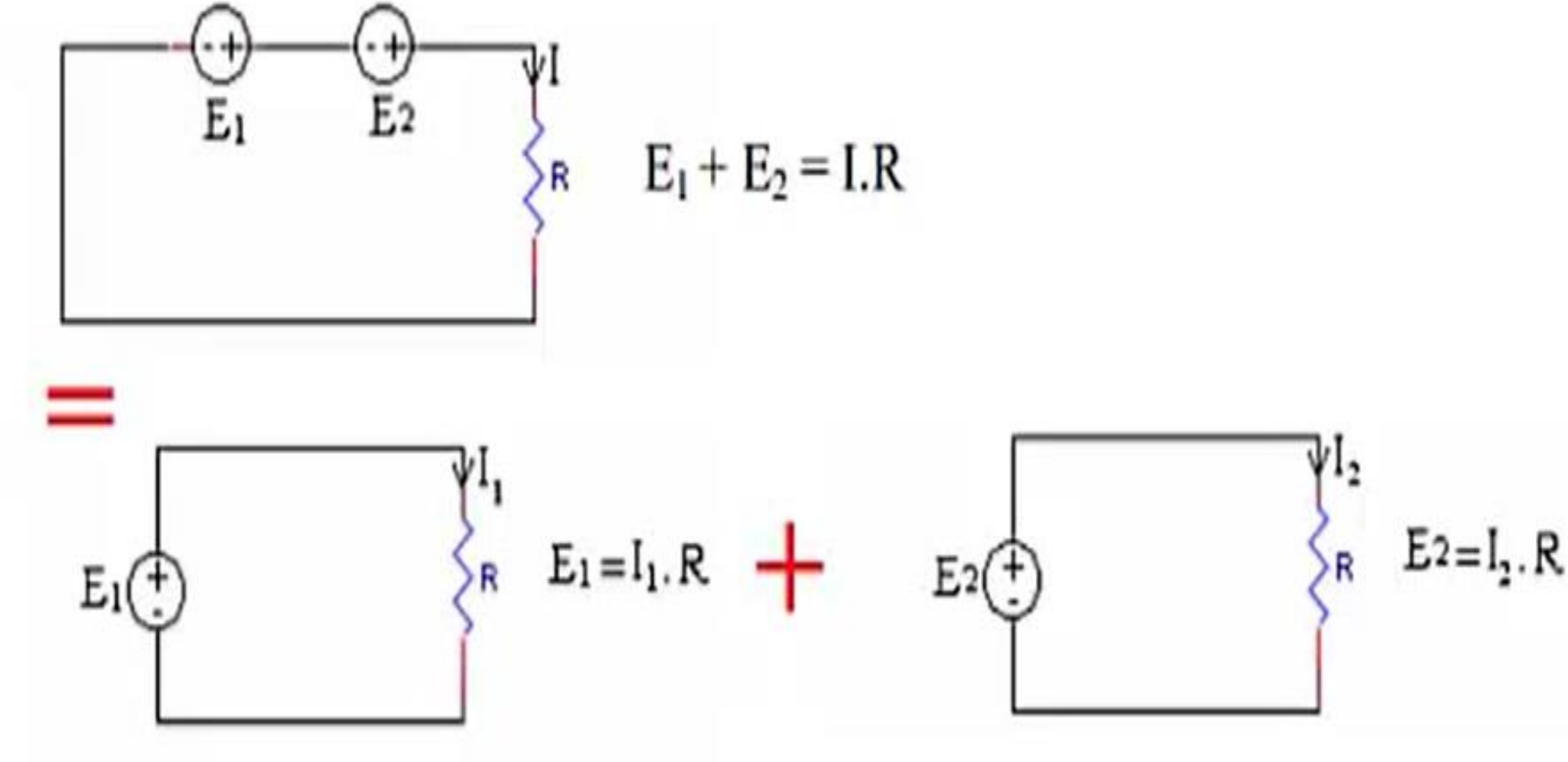
- **Bước 2**: E_2 tác động: $E_1=0$

$$E_2 = I_2 . R$$

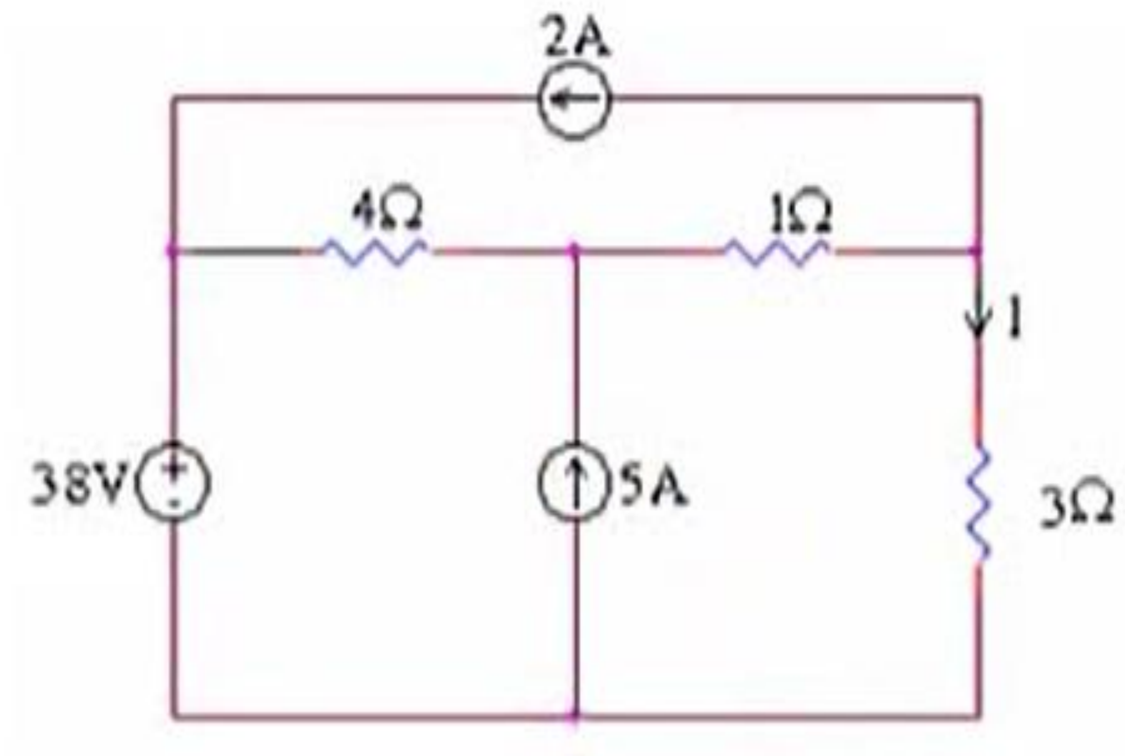
$$E_1 + E_2 = R.(I_1 + I_2)$$

2.10 Principle of Superposition – Nguyên lý xếp chồng

Vậy có thể tóm tắt như sau:

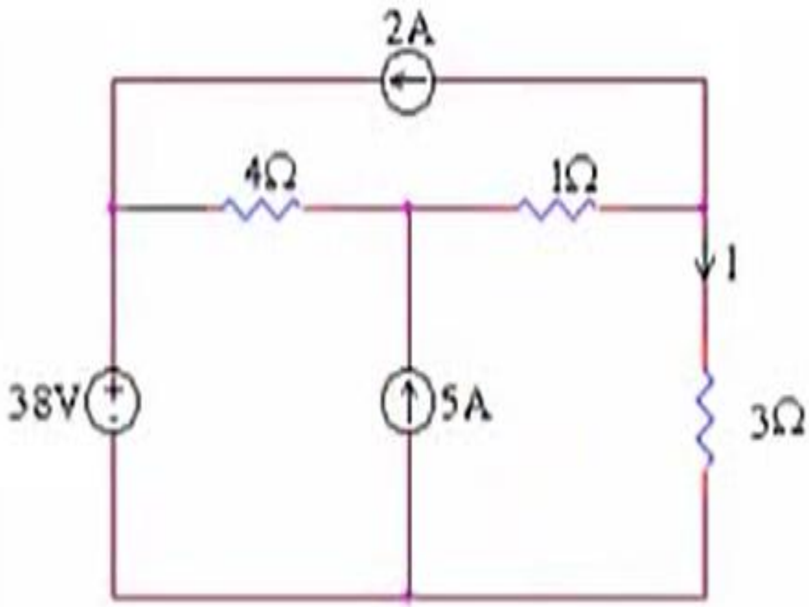


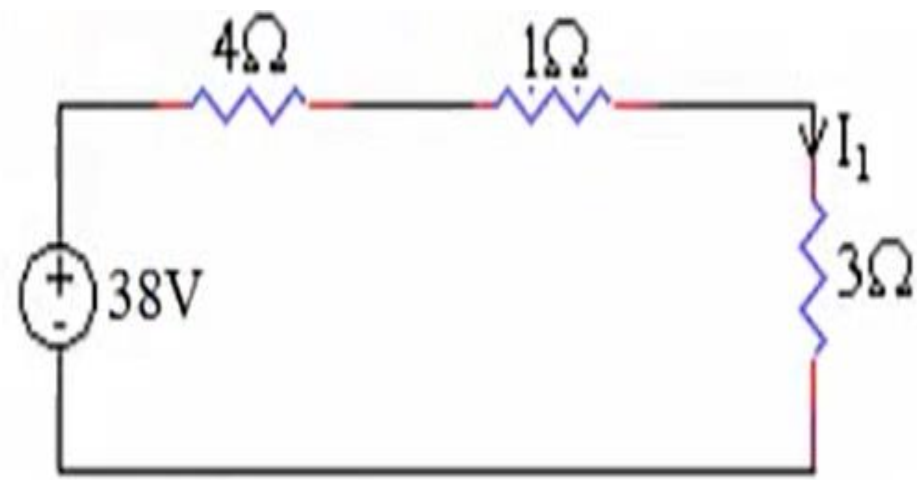
Ví dụ 2: Dùng phương pháp xếp chồng tìm dòng điện I?



Giải Ví dụ 2:

❖ **TH1: Xét nguồn 38V tác động** (Các nguồn còn lại cho bằng 0)

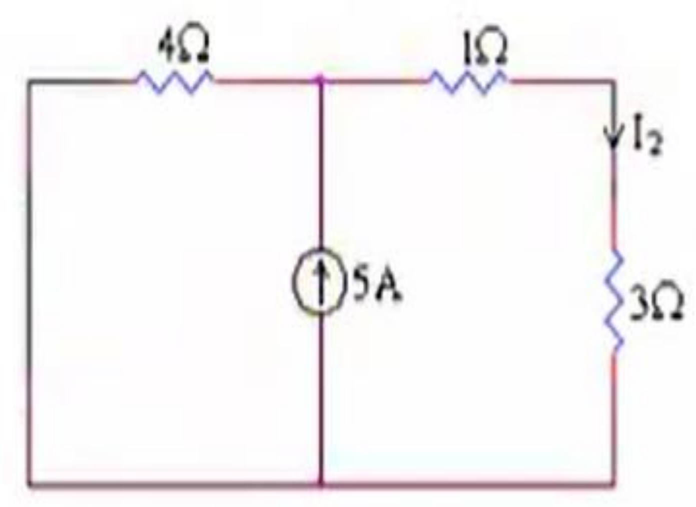


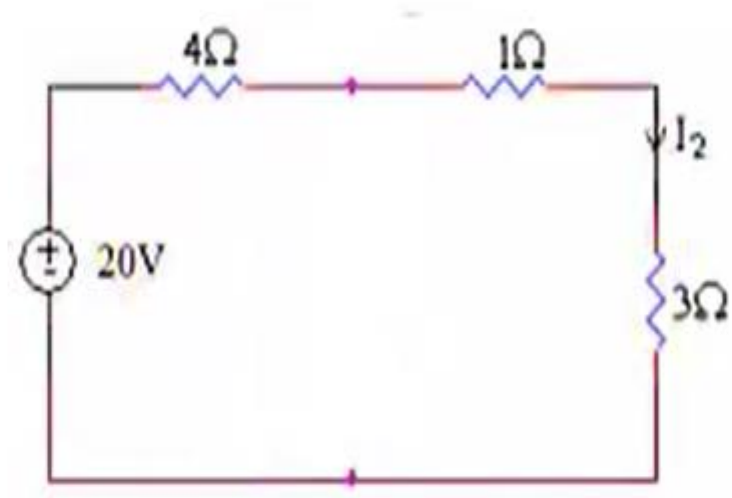


$$\Rightarrow I_1 = \frac{38}{8} A$$

Giải Ví dụ 2:

❖ **TH2: Nguồn 5A tác động** (Các nguồn còn lại cho bằng 0)

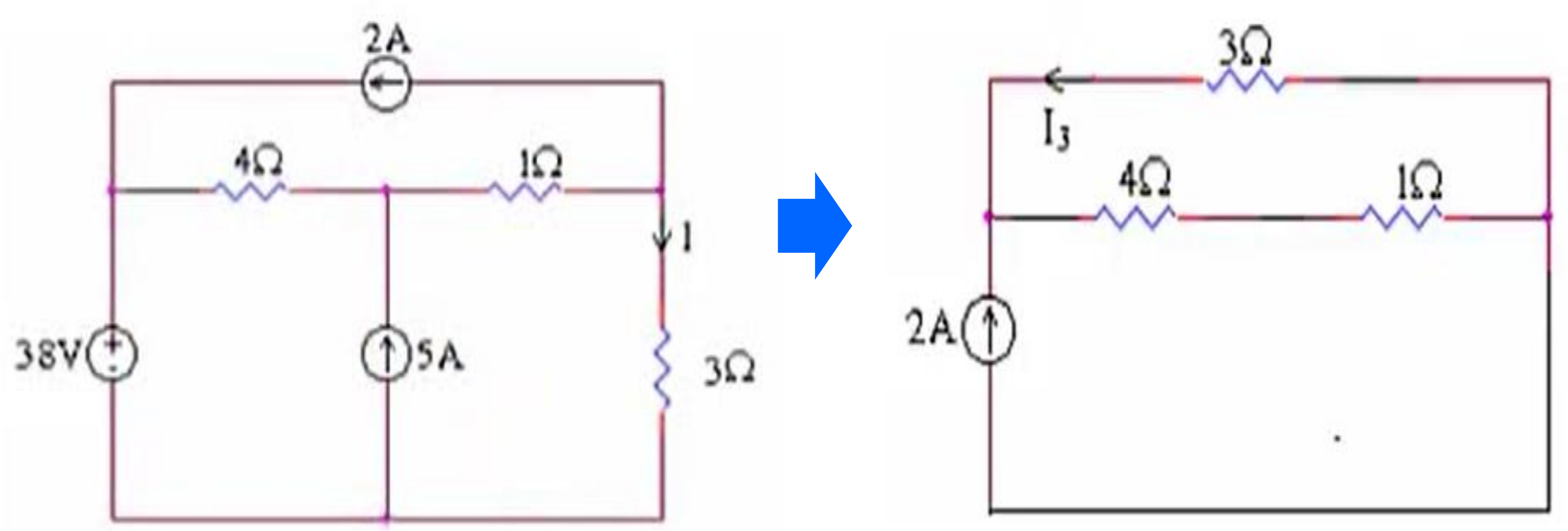




$$\Rightarrow I_2 = \frac{20}{8} A$$

Giải Ví dụ 2:

❖ **TH3: Nguồn 2A tác động** (Các nguồn còn lại cho bằng 0)

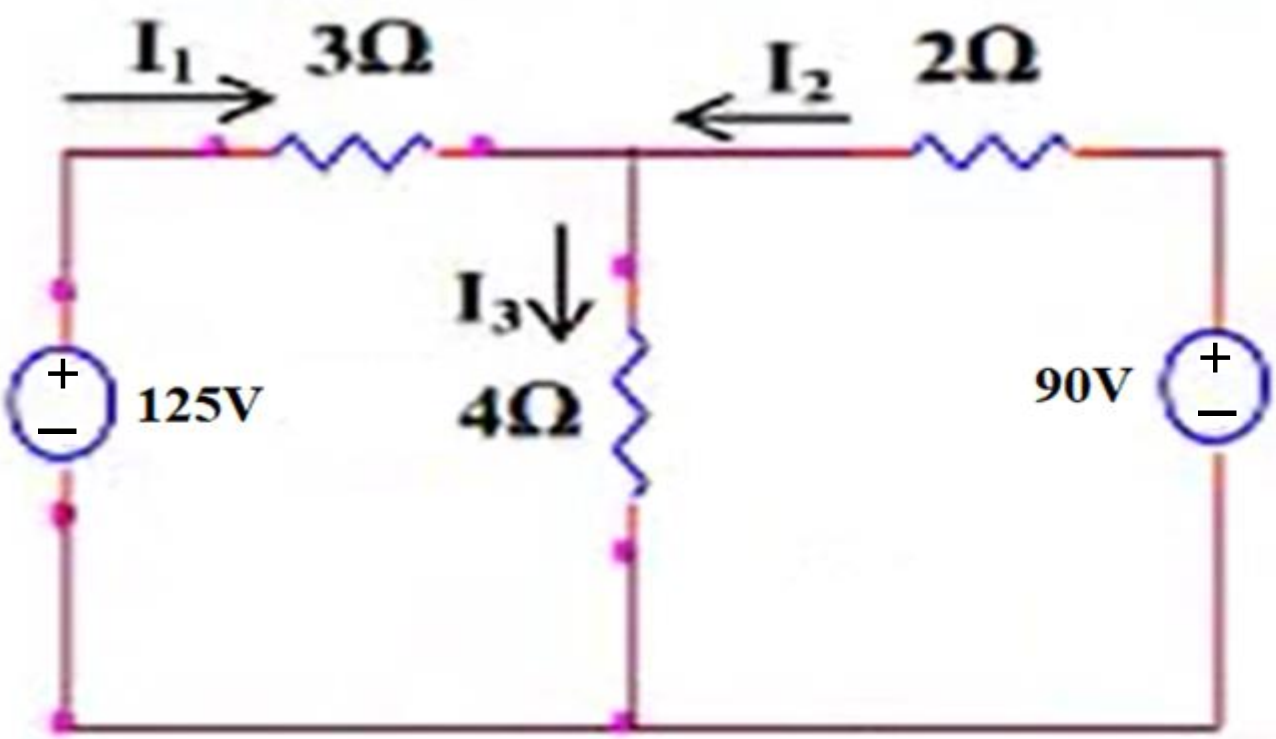


$$I_3 = 2A.\frac{5}{3+5} = \frac{10}{8}$$ (Áp dụng định luật chia dòng)

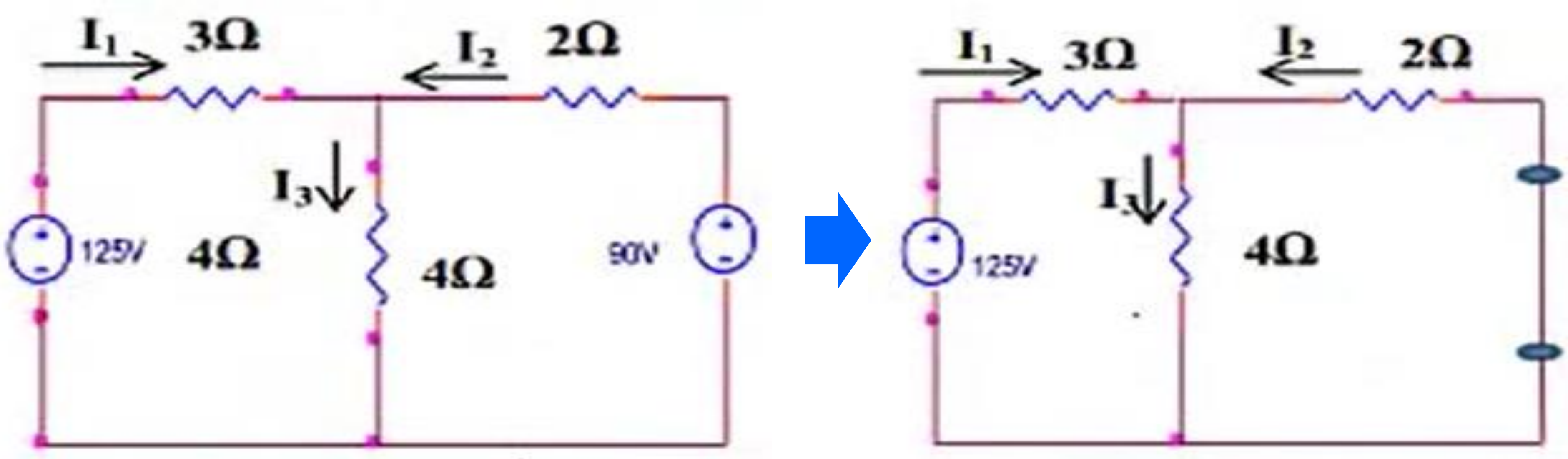
Vậy $I = I_1+I_2+I_3 = 38/8 + 20/8 - 10/8 = 48/8 = 6A$

Ví dụ 3:

Cho mạch điện như hình vẽ. Tìm dòng điện các nhánh và điện áp trên điện trở 4Ω.



- **TH1**: Mạch điện chỉ có nguồn 125V tác động hủy nguồn 90V



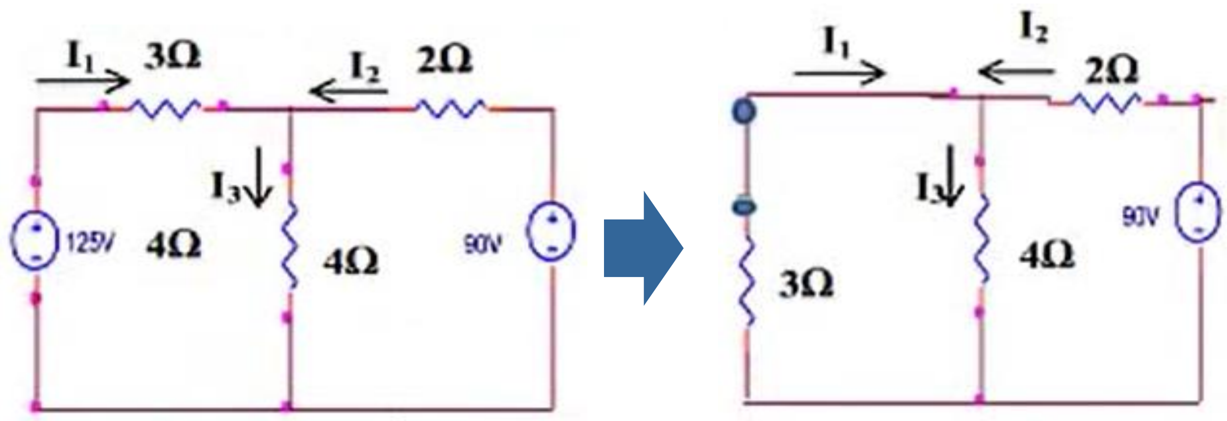
$$4\Omega // 2\Omega = \frac{4\times 2}{4+2} = 1,33\Omega \qquad I_1^{'} = \frac{125}{3+1,33} = 28,87A$$

Dùng công thức chia dòng, ta có:

$$I_3^{'} = \frac{2}{4+2} I_1^{'} = 9,62A \qquad I_2^{'} = I_1^{'} - I_3^{'} = 28,87 - 9,62 = 19,25A$$

2.10 Principle of Superposition – Nguyên lý xếp chồng

- **TH2**: Mạch điện chỉ có nguồn 90V tác động hủy nguồn 125V



$$4\Omega // 3\Omega = \frac{4\times 3}{4+3} = 1,71\Omega$$

$$I_2^{"} = \frac{90}{2+1,71} = 24,26A$$

Dùng công thức chia dòng, ta có: $$I_1^{"} = \frac{4}{4+3} I_2^{"} = 13,86A$$

$$I_3^{"} = I_2^{"} - I_1^{"} = 24,26 - 13,86 = 10,4A$$

KL: Dòng điện trong các nhánh khi có cả 2 nguồn làm việc:

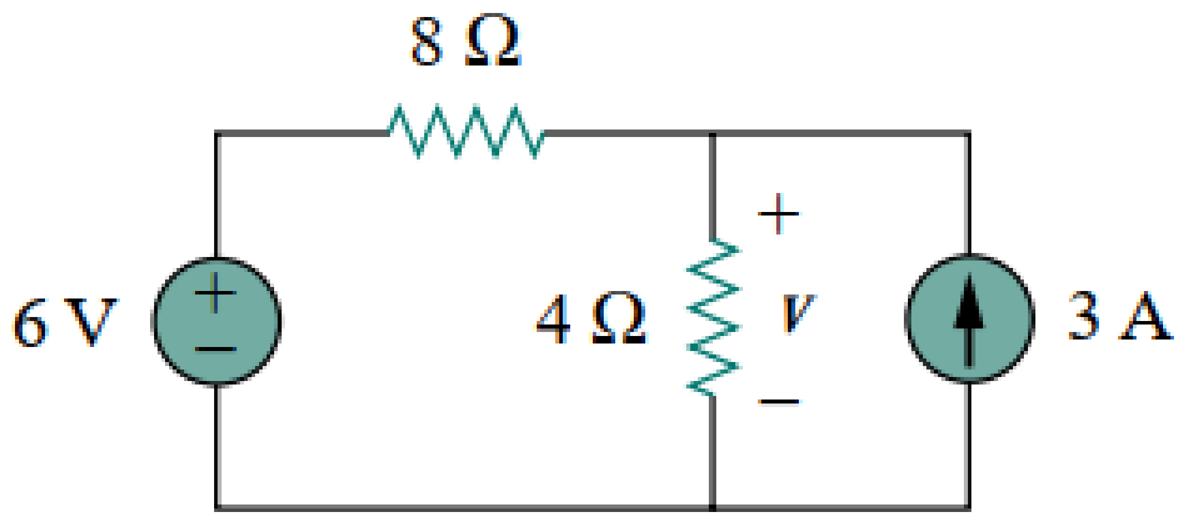
$$I_1 = I_1^{'} - I_1^{"} = 28,7 - 13,86 = 15A$$

$$I_2 = I_2^{"} - I_2^{'} = 24,26 - 19,25 = 5A$$

$$I_3 = I_1 - I_2 = 15 + 5 = 20A$$

HOMEWORK 1

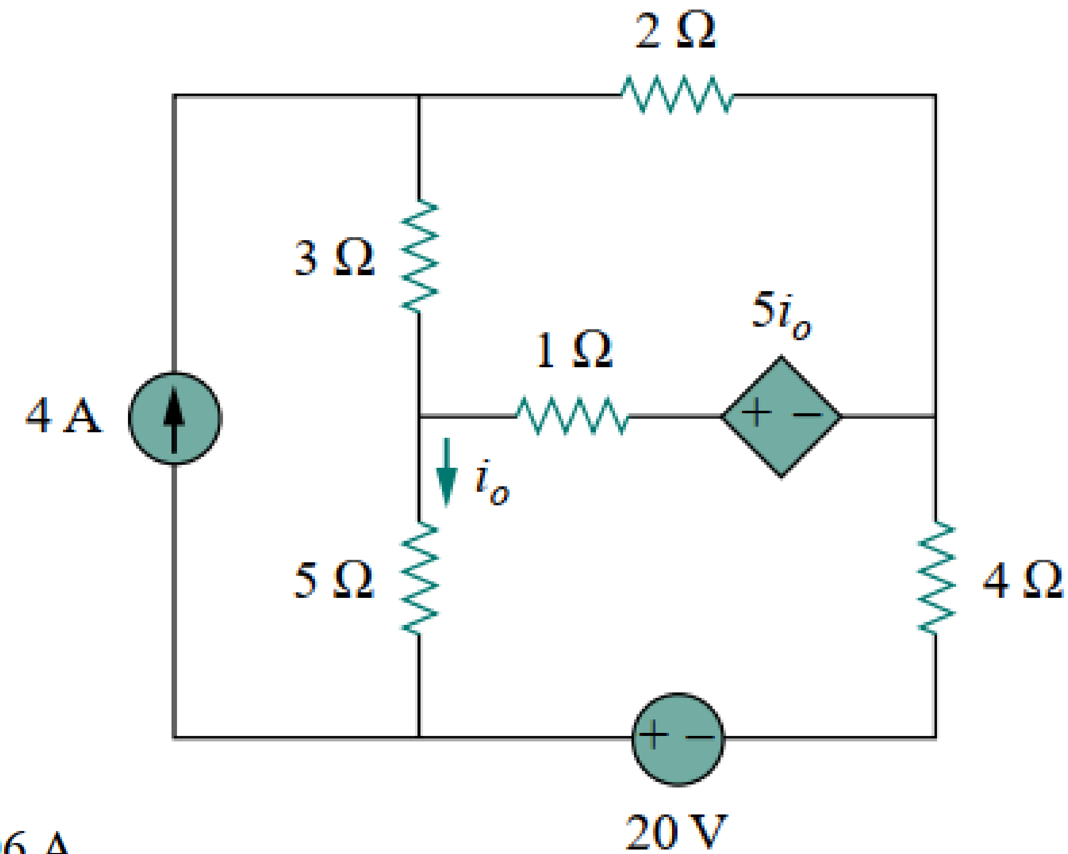
Use the superposition theorem to find *v* in the circuit



Answer: V=10 V

HOMEWORK 2

Use the superposition theorem to find Find i_O in the circuit



Answer:

$$i_o = -\frac{8}{17} = -0.4706 \text{ A}$$

THANK YOU