BỘ CÔNG THƯƠNG
ĐẠI HỌC CÔNG NGHIỆP TP. HỒ CHÍ MINH

Bài giảng

KỸ THUẬT ĐIỆN – ĐIỆN TỬ

ELECTRICITY AND ELECTRONICS

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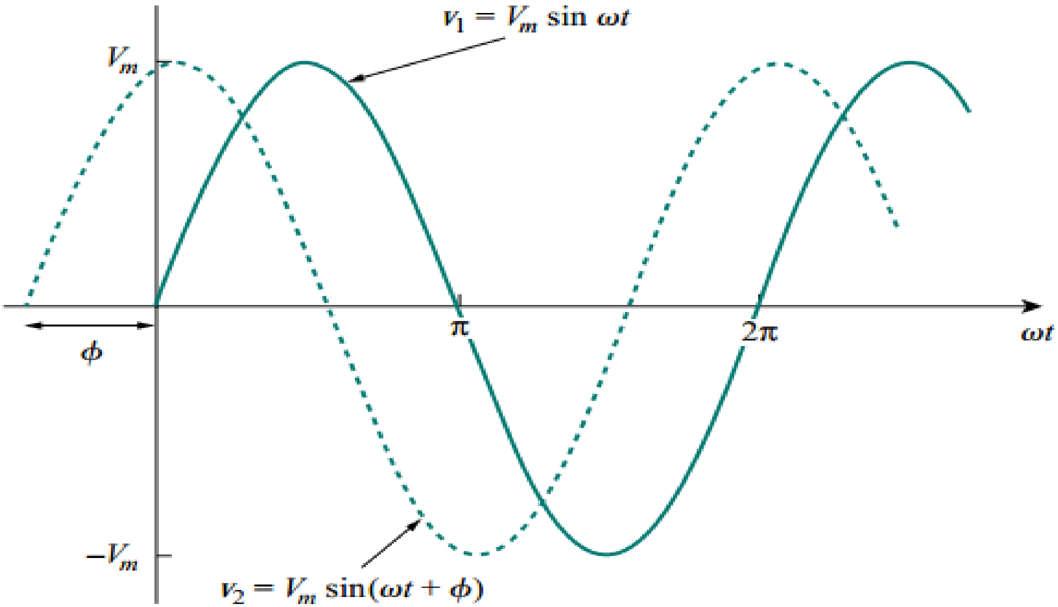
Chapter 4: Phân tích mạch AC

- Introduction of AC circuit
- Average Power, Apparent Power and Power Factor
- Complex Power
- Polyphase circuits
- Single-Phase Three-Wire Systems
- Three-Phase Y-Y Connection
- Power Measurement

4.1. Giới thiệu mạch AC

- Mạch điện mà nguồn áp hoặc nguồn dòng biến đổi theo thời gian được kích thích bởi một nguồn sine hoặc cosin thì được gọi là mạch xoay chiều AC.



$$\boxed{v(t) = V_m \sin \omega t}$$

V_m = the *amplitude* of the sinusoid

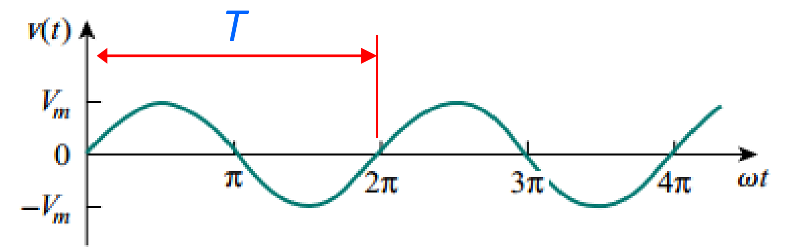
ω = the *angular frequency* in radians/s

ωt = the *argument* of the sinusoid

4.1. Giới thiệu mạch AC

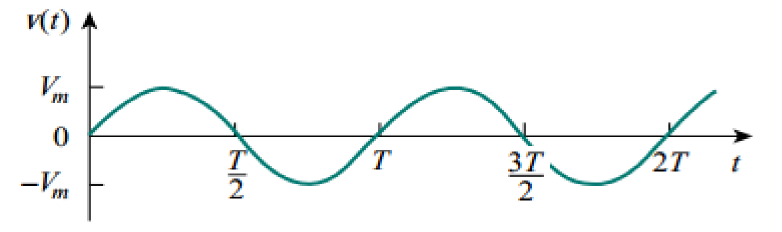
❖ **Biểu diễn tín hiệu điện áp hay dòng điện theo dạng hàm SINE**

$$\boxed{v(t) = V_m \sin \omega t}$$



T được gọi là chu kỳ (period)

$$T = \frac{2\pi}{\omega} \rightarrow \omega T = 2\pi$$



f (Hz) là tần số (*frequency*)

$$f = \frac{1}{T} \rightarrow \omega = 2\pi f$$

ω (rad/s); *ωt: argument; (ωt +Φ): argument & Φ in the phase*

$$v(t) = V_m \sin(\omega t + \phi)$$

v(t) repeats itself every *T* seconds is shown by replacing *t* by (*t + T)*

$$v(t+T) = V_m \sin \omega(t+T) = V_m \sin \omega\left(t + \frac{2\pi}{\omega}\right)$$

$$\boxed{v(t+T) = v(t)}$$

$$= V_m \sin(\omega t + 2\pi) = V_m \sin \omega t = v(t)$$

4.1. Introduction of AC circuit

❖ **Sức cản của các phần tử trong mạch điện (Trở kháng- IMPEDANCE)**

- Mối quan hệ giữa Điện áp – Dòng điện của các phần tử thụ động điện **passive elements là:**

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I}, \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

- Viết lại theo dạng điện áp pha và dòng pha

$$\frac{\mathbf{V}}{\mathbf{I}} = \underbrace{R}_{Z_R}, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \underbrace{j\omega L}_{Z_L}, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \underbrace{\frac{1}{j\omega C}}_{Z_C}$$

- Biểu diễn định luật Ohm theo dạng pha cho các phần tử thụ động điện

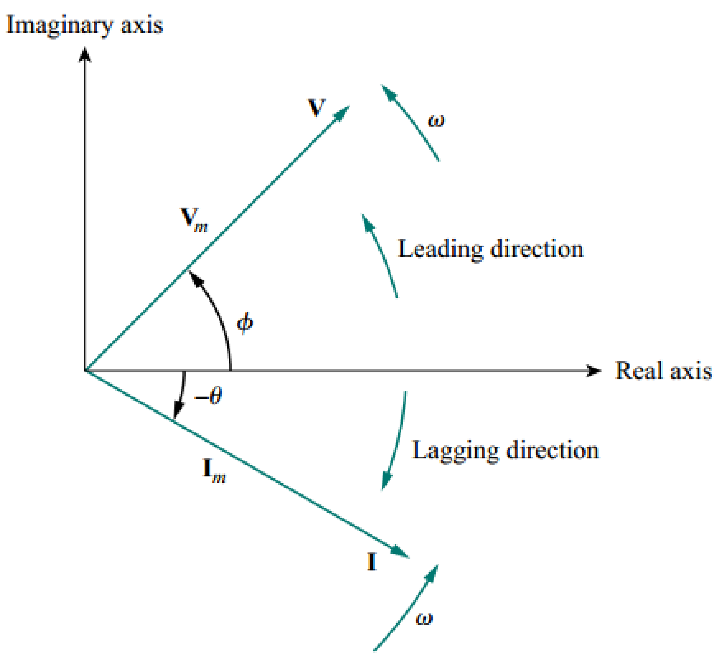
$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \qquad \text{or} \qquad \mathbf{V} = \mathbf{Z}\mathbf{I}}$$

Trong đó: **Z được gọi là trở kháng** (*impedance)*, đo bằng ohms.

❖ Cách biểu diễn tín hiệu điện áp theo miền thời gian và dạng pha

$$v(t) = V_m \cos(\omega t + \phi) \quad \Longleftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)



Sinusoid-phasor transformation.

Time-domain representation	Phasor-domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

4.1. Introduction of AC circuit

❖ BIỂU DIỄN TÍN HIỆU THEO DẠNG PHA

Sinusoids are easily expressed in ***terms of phasors***, which are more convenient to work with than sine and cosine functions.

> A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- A complex number *z* can be written in **rectangular** form as

$$\boxed{z = x + jy}$$

where $j = \sqrt{-1}$; x is the real part of z; y is the imaginary part of z.

- The complex number *z* can also be written in **polar** or **exponential** form as

$$\boxed{z = r\angle\phi = re^{j\phi}}$$

where *r* is the magnitude of *z*, and *Φ* is the phase of *z*.

4.1. Introduction of AC circuit

❖ BIỂU DIỄN IMPEDANCE THEO DẠNG PHA

- The impedance may be expressed in rectangular form as

Resistance (Trở kháng)

$$\boxed{\mathbf{Z} = R + jX}$$

Reactance (Điện kháng)

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

Where:

- R = Re $\mathbf{Z}$ is the *resistance* and X = Im $\mathbf{Z}$ is the *reactance*.
- The **reactance** X may be ***positive*** or ***negative***. (impedance is inductive when X is positive or capacitive when X is negative).
- Impedance $\mathbf{Z} = R + jX$ is said to be ***inductive*** or lagging since *current lags voltage*, while impedance $\mathbf{Z} = R - jX$ is ***capacitive*** or leading because *current leads voltage*.
- The impedance, resistance, and reactance are all measured in ohms

❖ BIỂU DIỄN IMPEDANCE THEO DẠNG PHA

Expression of Impedance

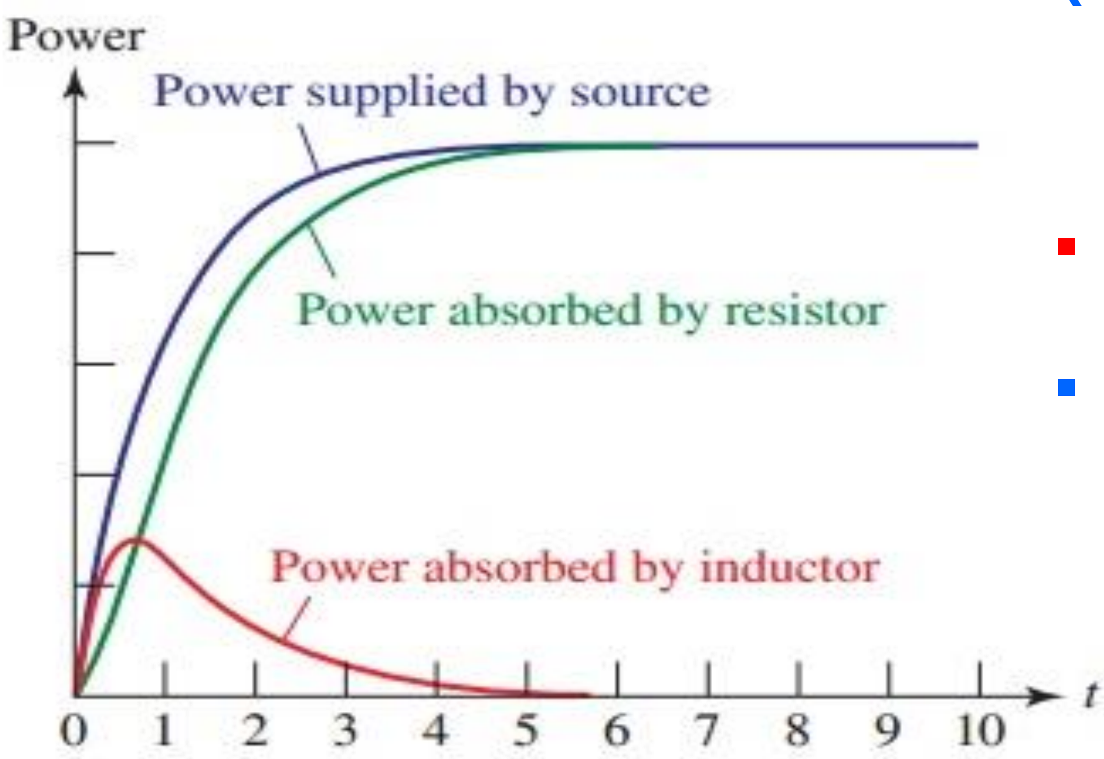
$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}|\underline{/\theta}}$$

Polar form $\quad \mathbf{Z} = |\mathbf{Z}|\underline{/\theta}$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta$$

Instantaneous Power (công suất tức thời)



- *p(t)* >0, power is absorbed by the circuit.
- *p(t)* <0, power is absorbed by the source;

Power is transferred from the circuit to the source. This is possible because of the storage elements (capacitors and inductors) in the circuit.

4.2. Instantaneous Power – Công suất tức thời

Instantaneous Power

$$\boxed{p(t) = v(t)i(t)} \qquad (3.1)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

where V_m and I_m are the amplitudes (or peak values), and θ_v and θ_i are the phase angles of the voltage and current, respectively.

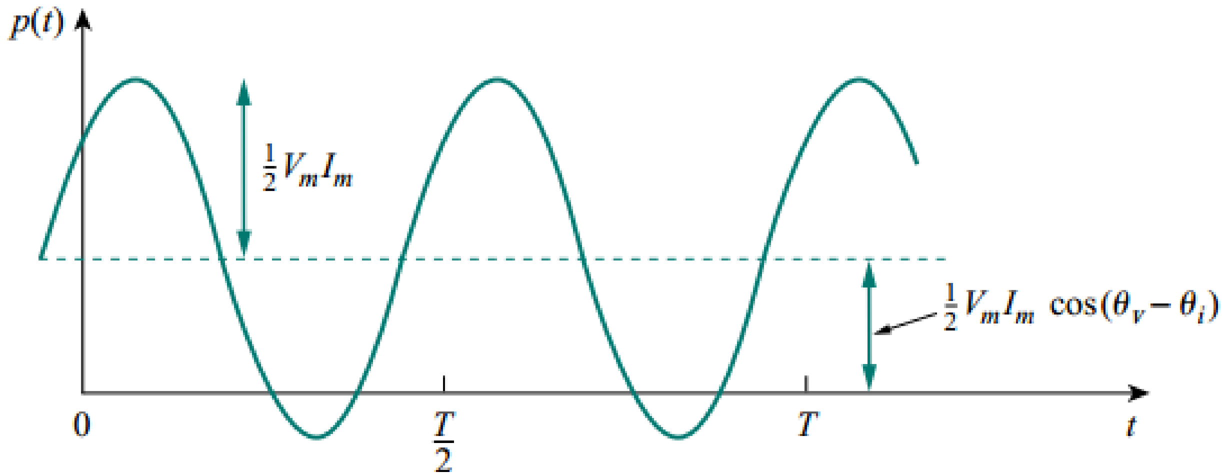
$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\rightarrow \quad p(t) = v(t)i(t) = V_m I_m \underbrace{\cos(\omega t + \theta_v)}_{A} \underbrace{\cos(\omega t + \theta_i)}_{B} \qquad (3.2)$$

$$\rightarrow \quad p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i) \qquad (3.3)$$

4.3 Average Power- công suất trung bình

- *Instantaneous Power:* $p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$



The instantaneous power changes with time and is therefore difficult to measure.=> The ***average power*** is more convenient to measure.

❑ Definition:

***The average* power is the average of the instantaneous power over one period**

The ***wattmeter***, the instrument for measuring ***average* power**

4.3 Average Power- công suất trung bình

❑ Demonstrate:

- *Instantaneous Power:* $p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$
- *Average power:* $P = \frac{1}{T}\int_0^T p(t)\,dt$

$$\Rightarrow P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$P = \underbrace{\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt}_{\text{integrand is constant}} + \underbrace{\frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt}_{\text{integrand is a sinusoid}}$$

- *Average power:* $\boxed{P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}$

4.3 Average Power- công suất trung bình

- *Average power:* $$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

- When $\theta_v = \theta_i$, the ***voltage*** and ***current*** are ***in phase***, *purely resistive circuit* or resistive load *R*

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |\mathbf{I}|^2 R$$

- When $\theta_v - \theta_i = \pm 90^o$, we have a ***purely reactive circuit***, and

$$P = \frac{1}{2} V_m I_m \cos 90^\circ = 0$$

A resistive load (R) absorbs power at all times, while a reactive load (*L* or *C*) absorbs zero average power.

Exercise 3.6

Given that $v(t) = 120\cos(377t + 45^{o})$ V and $i(t) = 10\cos(377t - 10^{o})$ A. Find the instantaneous power and the average power absorbed by the passive linear network.

4.3 Average Power- công suất trung bình

Solution Ex 3.6:

$$\begin{cases} v(t) = 120\cos(377t + 45^\circ)\ \text{V} \\ i(t) = 10\cos(377t - 10^\circ)\ \text{A} \end{cases}$$

- The instantaneous power:

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$\rightarrow \quad p(t) = 344.2 + 600\cos(754t + 35^\circ)\ \text{W}$$

- The average power:

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45^\circ - (-10^\circ)]$$

$$= 600\cos 55^\circ = 344.2\ \text{W}$$

Exercise 3.7

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$

when a voltage $\mathbf{V} = 120\angle 0^\circ$ is applied across it.

Solution Ex3.7:

The current through the impedance is

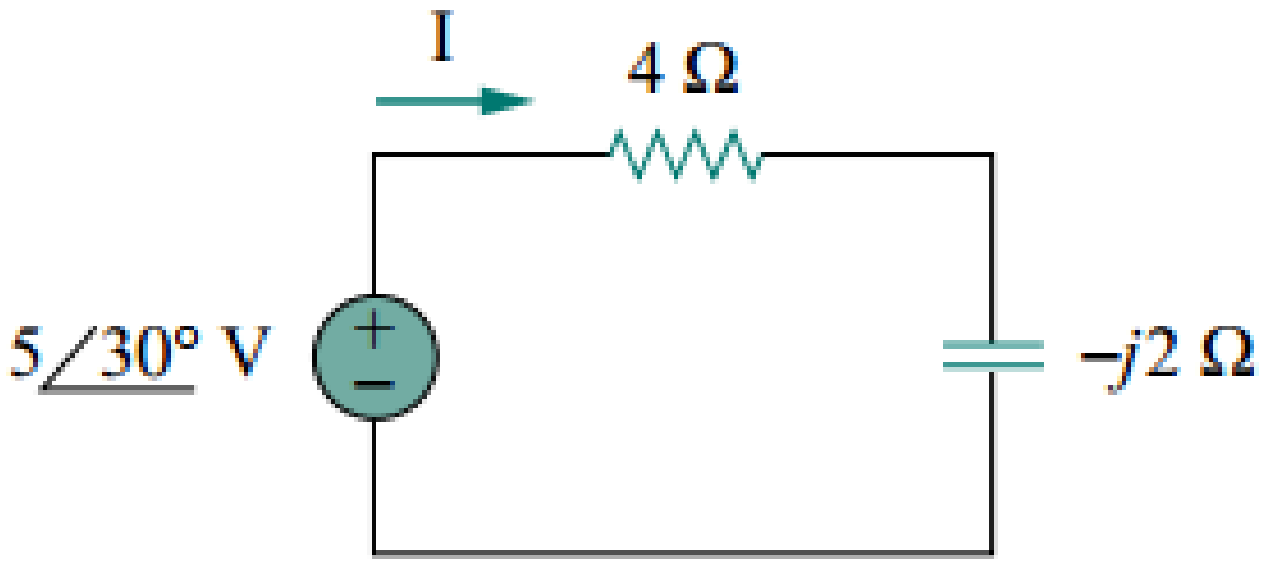
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\angle 0^\circ}{30 - j70} = \frac{120\angle 0^\circ}{76.16\angle -66.8^\circ} = 1.576\angle 66.8^\circ \text{ A}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ) = 37.24 \text{ W}$$

Exercise 3.8

For the circuit shown in Figure, find the average power supplied by the source and the average power absorbed by the resistor.



Solution:

The current $\mathbf{I}$ is given by

$$\mathbf{I} = \frac{5\angle 30^\circ}{4 - j2} = \frac{5\angle 30^\circ}{4.472\angle -26.57^\circ} = 1.118\angle 56.57^\circ \text{ A}$$

The average power supplied by the voltage source is

$$P = \frac{1}{2}(5)(1.118)\cos(30^\circ - 56.57^\circ) = 2.5 \text{ W}$$

The current through the resistor is

$$\mathbf{I} = \mathbf{I}_R = 1.118\angle 56.57^\circ \text{ A}$$

and the voltage across it is

$$\mathbf{V}_R = 4\mathbf{I}_R = 4.472\angle 56.57^\circ \text{ V}$$

The average power absorbed by the resistor is

$$P = \frac{1}{2}(4.472)(1.118) = 2.5 \text{ W}$$

which is the same as the average power supplied. Zero average power is absorbed by the capacitor.

4.3 Average Power- công suất trung bình

HOMEWORK 3.4

Calculate the instantaneous power and average power absorbed by the passive linear network of Fig.

$$v(t) = 80\cos(10t + 20^\circ)\ \text{V} \qquad \text{and} \qquad i(t) = 15\sin(10t + 60^\circ)\ \text{A}$$

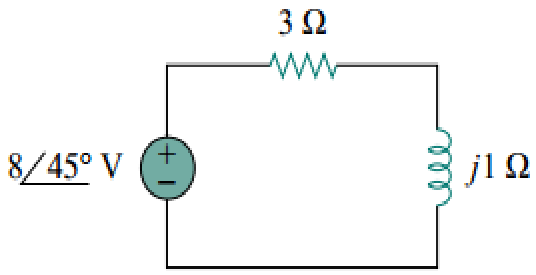
Answer: $385.7 + 600\cos(20t - 10^\circ)$ W, 385.7 W.

HOMEWORK 3.5

A current $\mathbf{I} = 10\underline{/30^\circ}$ flows through an impedance $\mathbf{Z} = 20\underline{/-22^\circ}\ \Omega$. Find the average power delivered to the impedance.

Answer: 927.2 W.

HOMEWORK 3.6

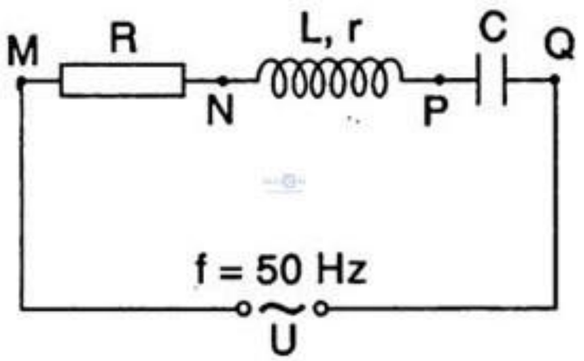


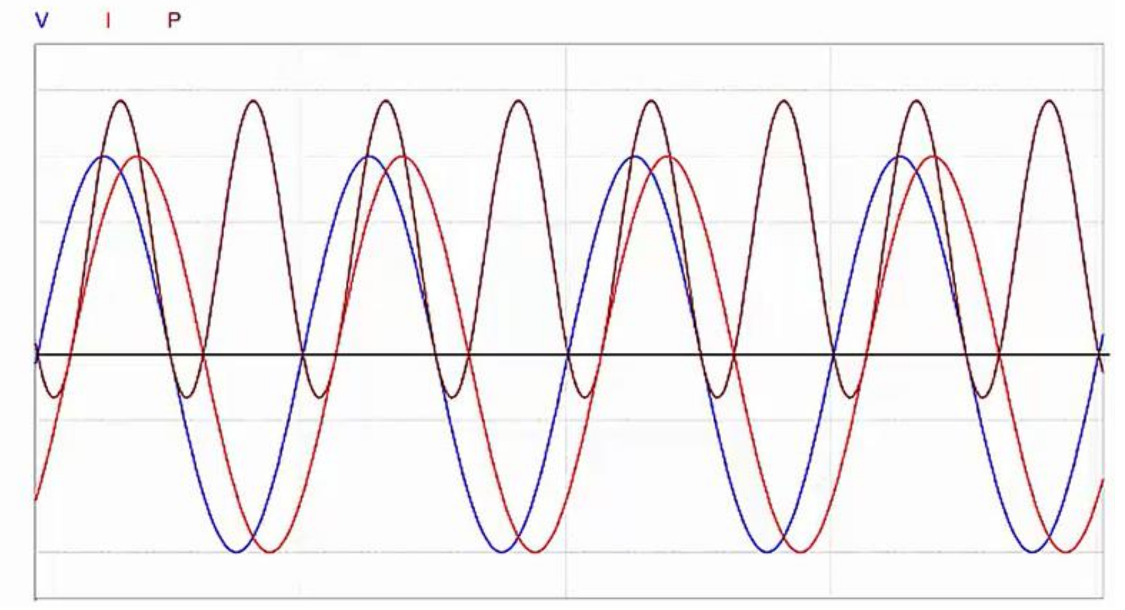
In the circuit of Fig. 11.4, calculate the average power absorbed by the resistor and inductor. Find the average power supplied by the voltage source.

Answer: 9.6 W, 0 W, 9.6 W.

REAL, REACTIVE, AND APPARENT POWER:

- *Power in system with resistive and reactive components*





- **REAL -> Resistive (R)**
- **REACTIVE -> L,C: Oscillates**

Apparent power (S): Công suất biểu kiến

Power Factor (PF): Hệ số công suất

4.4 Apparent Power and Power Factor

- ***Instantaneous Power:*** $p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$
- **Average power:** $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$\Rightarrow P = \frac{1}{\sqrt{2}} V_m \frac{1}{\sqrt{2}} I_m \cos(\theta_v - \theta_i) = \underbrace{\boxed{0.707 * V_m}}_{V_{rms}} * \underbrace{\boxed{0.707 * I_m}}_{I_{rms}} * \cos(\theta_v - \theta_i)$$

- **Other terms of the power values**

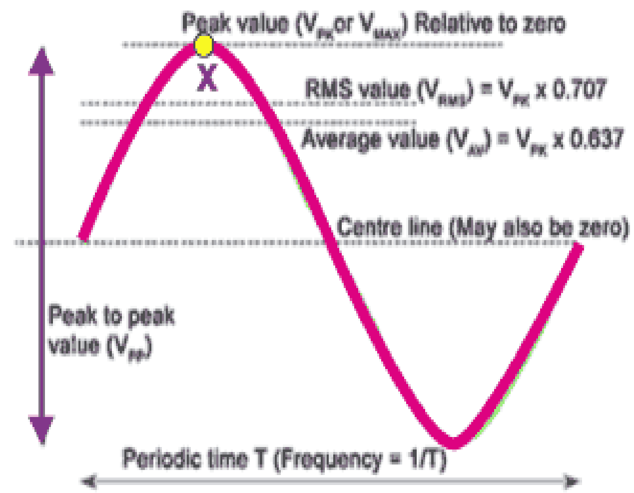
$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$$\boxed{S = V_{rms} I_{rms}}$$

Apparent power S

$\boxed{\cos(\theta_v - \theta_i)}$

power factor (pf)

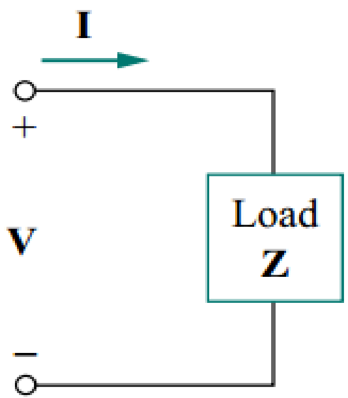


The apparent power (in VA) is the product of the rms values of voltage and current.

4.5 Complex Power- Công suất phức

- Complex power is important in **power analysis** because it contains all the information pertaining to the **power absorbed** by a **given load**.

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$$



$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle \theta_v - \theta_i$$

$$\mathbf{V}_{\text{rms}} = \mathbf{Z}\mathbf{I}_{\text{rms}}$$

$$\boxed{\mathbf{S} = I^2_{\text{rms}}\mathbf{Z} = \frac{V^2_{\text{rms}}}{\mathbf{Z}^*}}$$

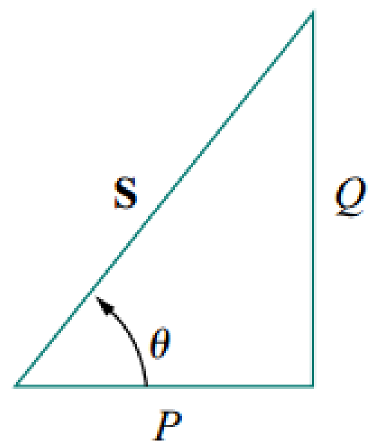
Since $\mathbf{Z} = R + jX$

$$\boxed{\mathbf{S} = I^2_{\text{rms}}(R + jX) = P + jQ}$$

- The **magnitude** of the **complex power** is the **apparent power**
- The **complex power** is measured in **volt-amperes** (VA)
- The **angle of the complex power** is the **power factor** angle.

4.5 Complex Power- Công suất phức

Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.



$$\boxed{\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ}$$

where *P* and *Q* are the *real* and *imaginary* parts of the complex power

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R$$

$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

- *P* is the average or **real power** and it depends on the **load's resistance *R***
- Q depends on the **load's reactance X** and is called the **reactive power**.

4.5 Complex Power- Công suất phức

- **Average power (P):** $P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i),$

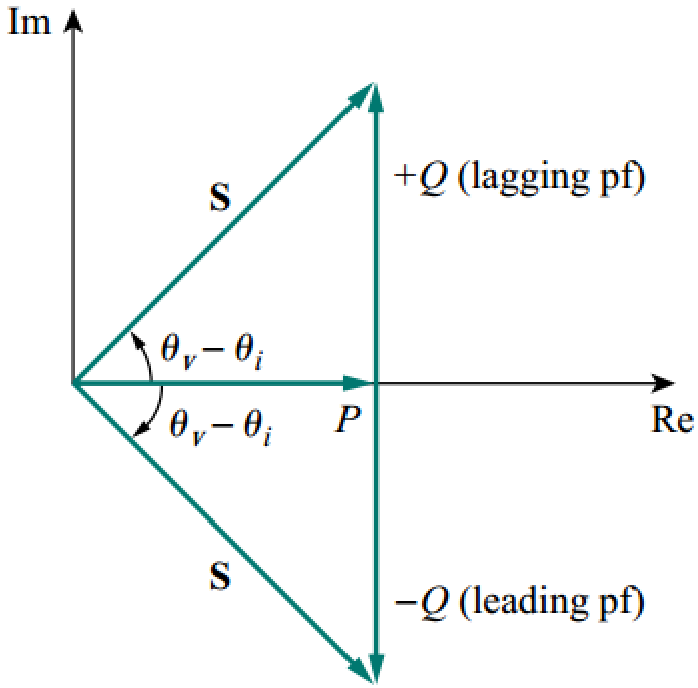
  It is real power (*P)* in watts delivered to a load; it is the only *useful power*. It is the actual power dissipated by the load.

- **Reactive power (Q):** $Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$

  - It is a measure of the energy exchange between the source and the reactive part of the load.
  - The unit of *Q* is the *volt-ampere reactive* (VAR) to distinguish it from the real power
  - The reactive power is being transferred back and forth between the load and the source. It represents a lossless interchange between the load and the source.

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

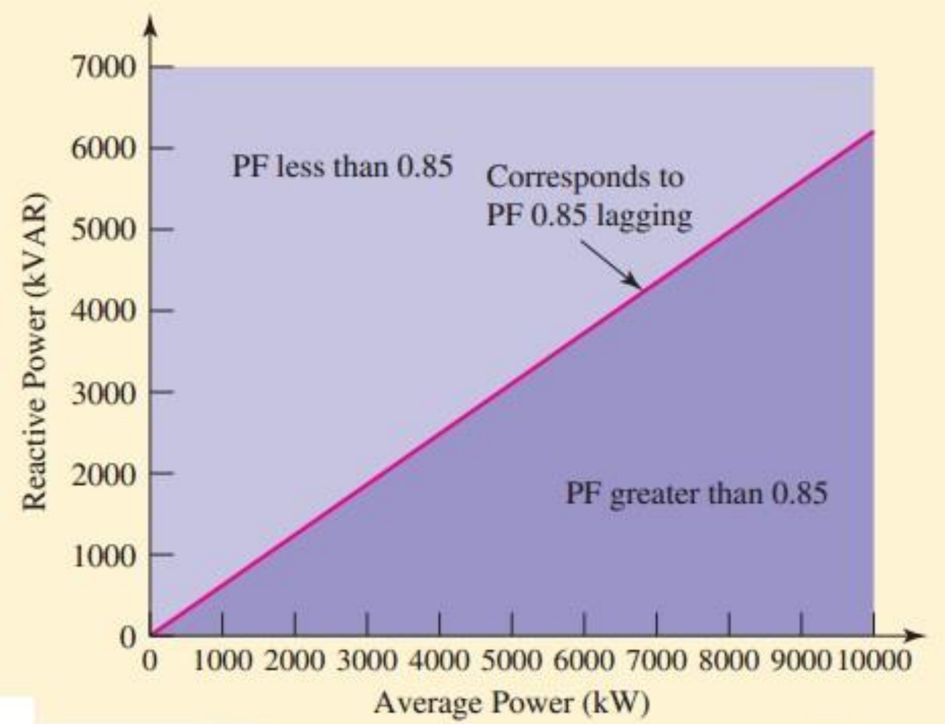
4.5 Complex Power- Công suất phức



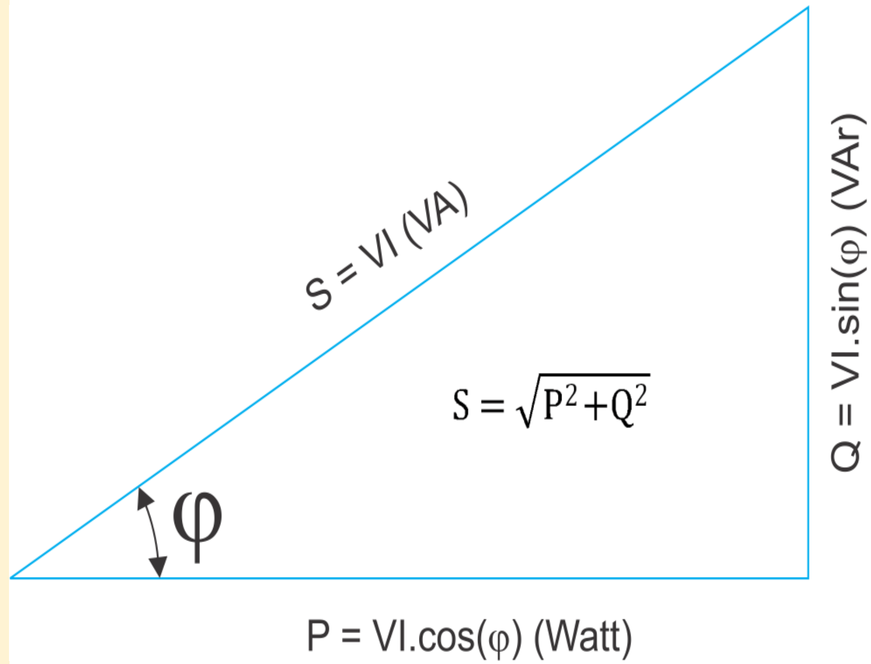
- When **S** lies in the first quadrant, we have an **inductive load** and a lagging PF
- When **S** lies in the fourth quadrant, the load is **capacitive** and the PF is leading

It is also possible for the complex power to lie in the second or third quadrant. This requires that the load impedance have a negative resistance, which is possible with active circuits.

4.5 Complex Power- Công suất phức



Plot showing acceptable ratio of reactive power to average power for power factor benchmark of 0.85 lagging.



Exercise 3.9

The voltage across a load is $v(t) = 60\cos(\omega t - 10^\circ)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A.

Find:

(a) the complex and apparent powers,

(b) the real and reactive powers,

(c) the power factor and the load impedance.

4.5 Complex Power- Công suất phức

Solution: (a) the complex and apparent powers:

$$\mathbf{V}_{\text{rms}} = \frac{60}{\sqrt{2}}\underline{/-10^\circ}, \qquad \mathbf{I}_{\text{rms}} = \frac{1.5}{\sqrt{2}}\underline{/+50^\circ}$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \left(\frac{60}{\sqrt{2}}\underline{/-10^\circ}\right)\left(\frac{1.5}{\sqrt{2}}\underline{/-50^\circ}\right) = 45\underline{/-60^\circ}\ \text{VA}$$

The apparent power is $S = |\mathbf{S}| = 45\ \text{VA}$

(b) the real and reactive powers:

$$\mathbf{S} = 45\underline{/-60^\circ} = 45[\cos(-60^\circ) + j\sin(-60^\circ)] = 22.5 - j38.97$$

$P = 22.5\ \text{W}$ (Real powers) $Q = -38.97\ \text{VAR}$ (Reactive powers)

(c) the power factor and the load impedance: $\text{pf} = \cos(-60^\circ) = 0.5\ \text{(leading)}$

It is leading, because the reactive power is negative. The load impedance is

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\underline{/-10^\circ}}{1.5\underline{/+50^\circ}} = 40\underline{/-60^\circ}\ \Omega$$

which is a capacitive impedance.

HOMEWORK 3.7

For a load, $\mathbf{V}_{\text{rms}} = 110\angle 85^\circ$ V, $\mathbf{I}_{\text{rms}} = 0.4\angle 15^\circ$ A. Determine: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Answer: (a) $44\angle 70^\circ$ VA, 44 VA, (b) 15.05 W, 41.35 VAR, (c) 0.342 lagging, $94.06 + j258.4\ \Omega$.

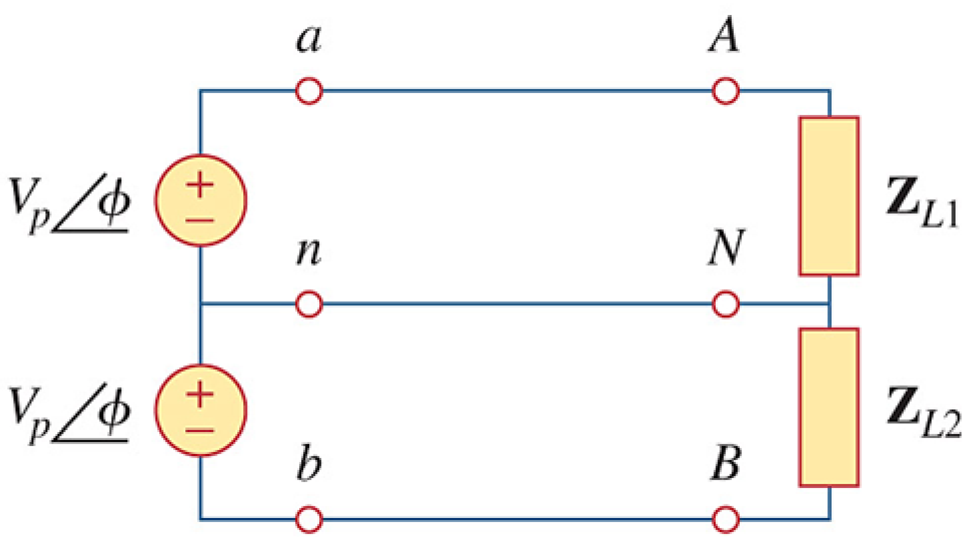
WHY THREE PHASE SYSTEM?

- ALL electric power system in the world used 3-phase system to **GENERATE**, **TRANSMIT** and **DISTRIBUTE**
- Instantaneous power is constant – thus smoother rotation of electrical machines
- More economical than single phase – less wire for the same power transfer

Balanced 3-phase systems

Single-phase two-wire system:

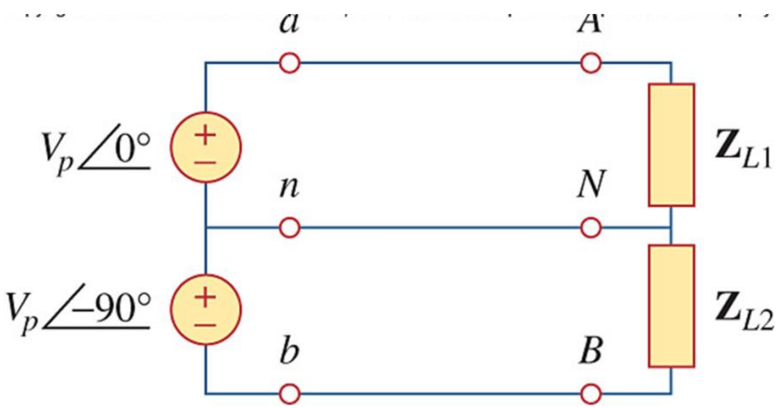
- Single source connected to a load using two-wire system



Single-phase three-wire system:

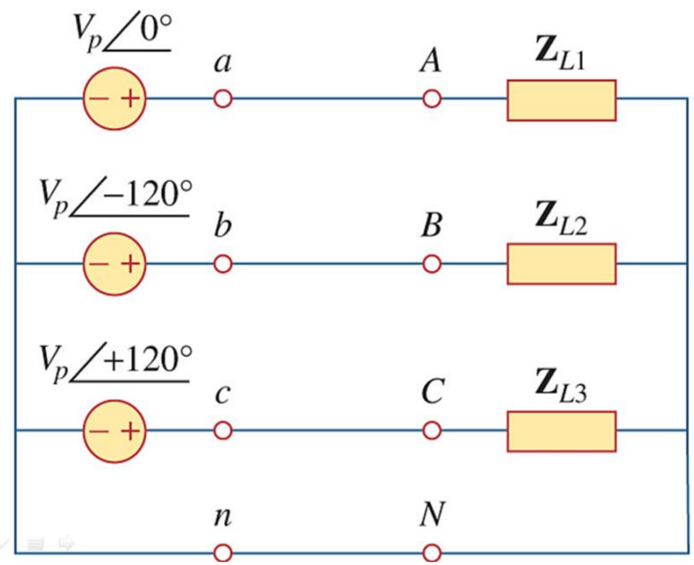
- Two sources connected to two loads using three-wire system
- Sources have EQUAL magnitude and are IN PHASE

4.7 Single-Phase Three-Wire Systems



Balanced Two-phase three-wire system:

- Two sources connected to two loads using three-wire system
- Sources have EQUAL frequency but DIFFERENT phases

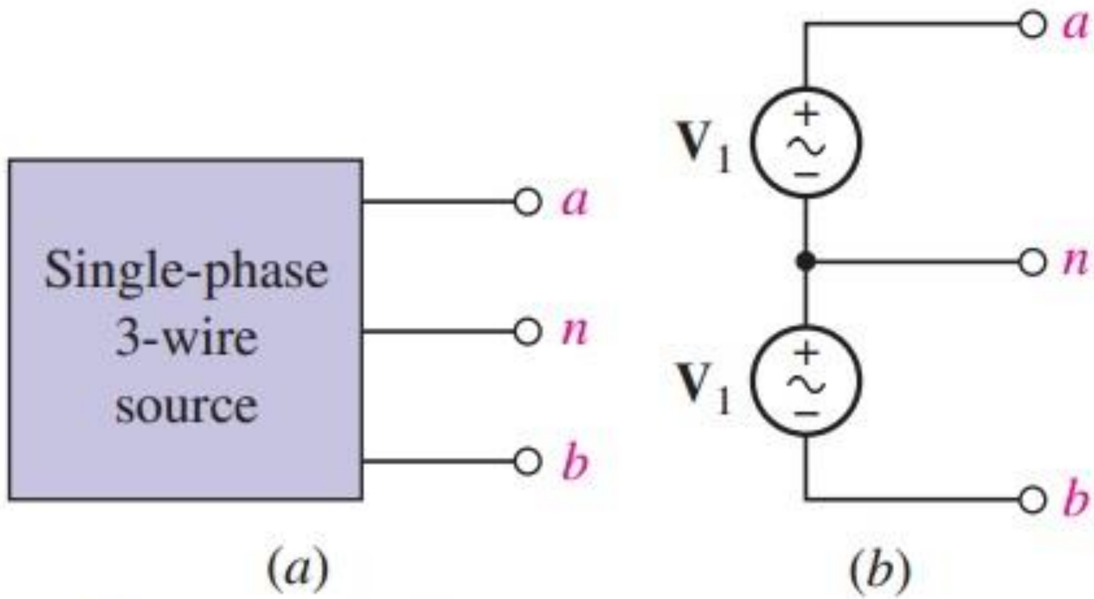


Balanced Three-phase four-wire system:

- Three sources connected to 3 loads using four-wire system
- Sources have EQUAL frequency but DIFFERENT phases

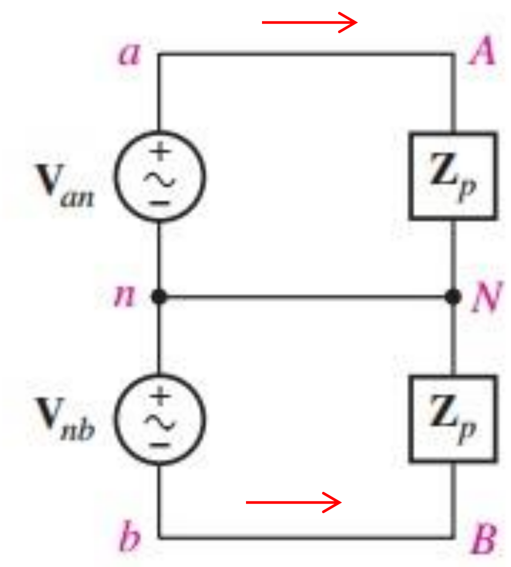
4.7 Single-Phase Three-Wire Systems

A single-phase three-wire source is defined as a source having three output terminals, such as a, n, and b as shown in Figure, at which the phasor voltages V_{an} and V_{nb} are equal



(a) A single-phase three-wire source.
(b) The representation of a single-phase three-wire source by two identical voltage sources.

4.7 Single-Phase Three-Wire Systems



A simple single-phase three-wire system. The two loads are identical, and the neutral current is zero.

$$\mathbf{V}_{an} = \mathbf{V}_{nb}$$

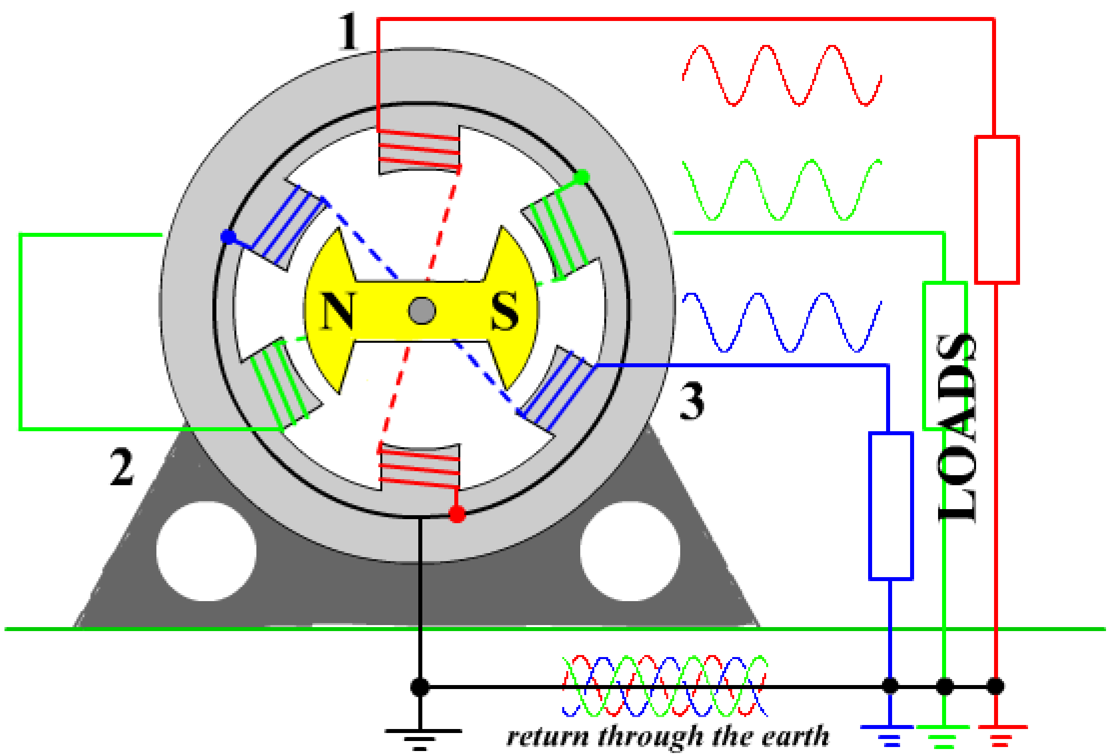
$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p} = \mathbf{I}_{Bb} = \frac{\mathbf{V}_{nb}}{\mathbf{Z}_p}$$

$$\mathbf{I}_{nN} = \mathbf{I}_{Bb} + \mathbf{I}_{Aa} = \mathbf{I}_{Bb} - \mathbf{I}_{aA} = 0$$

Thus there is no current in the neutral wire, and it could be removed with-out changing any current or voltage in the system. This result is achieved through the equality of the two loads and of the two sources.

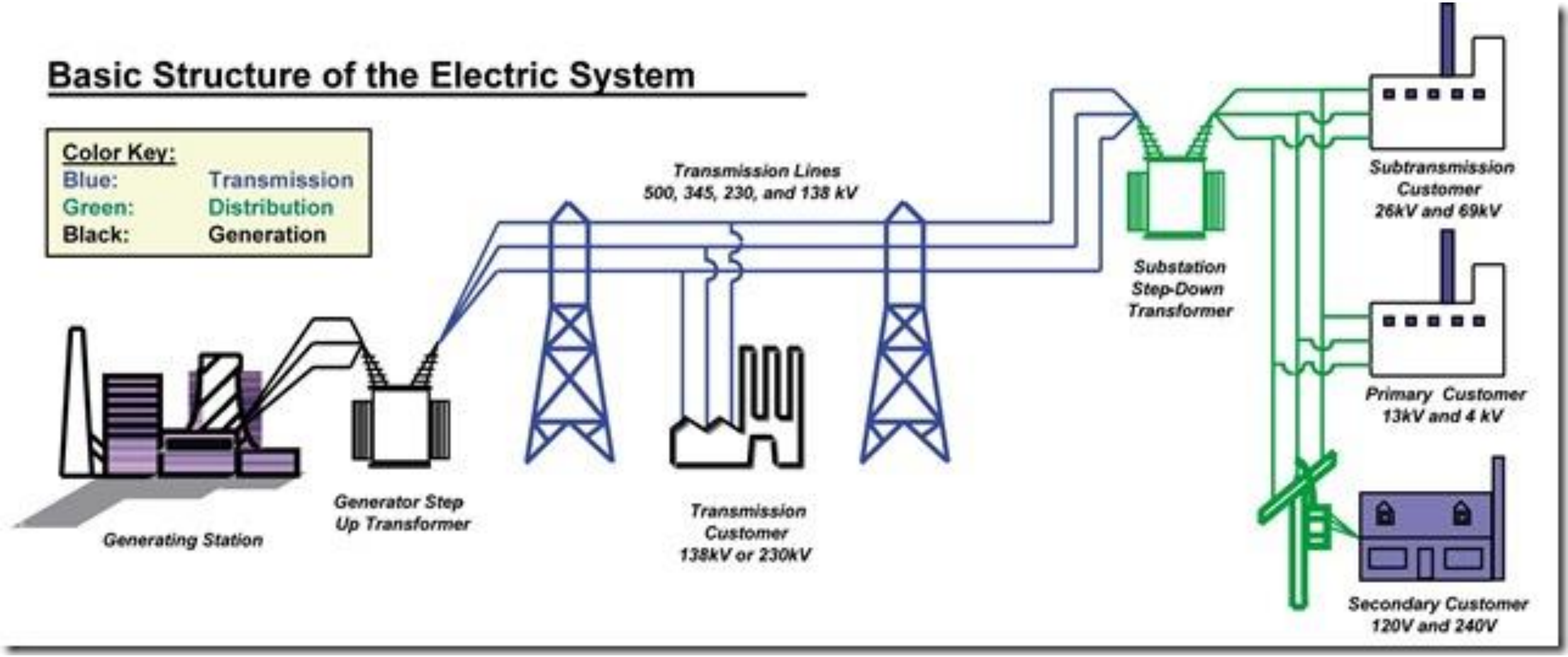
Balanced 3-phase systems

Generation of 3-phase voltage: Generator



Balanced 3-phase systems

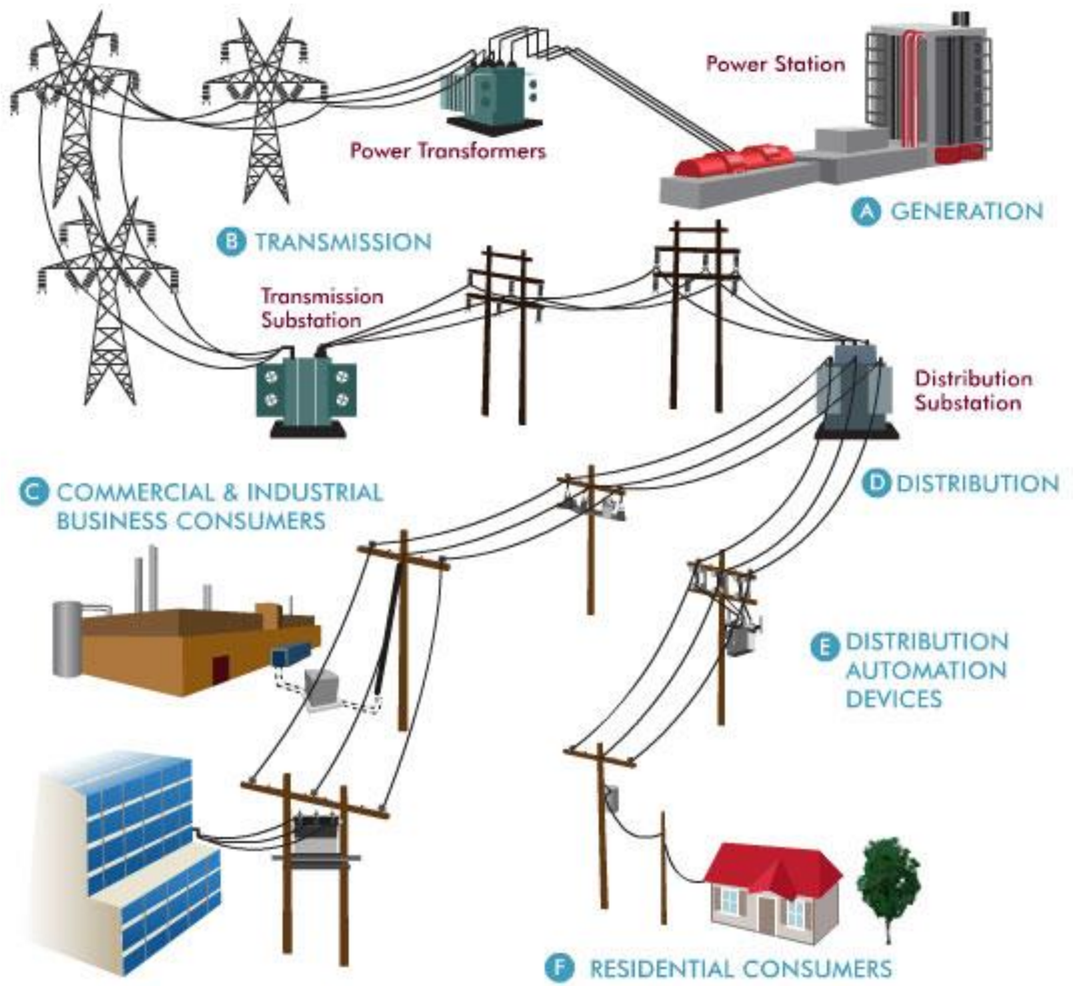
Generation, Transmission and Distribution



4.7 Single-Phase Three-Wire Systems

Balanced 3-phase systems

Generation, Transmission and Distribution



4.8 Three-Phase Y-Y Connection

Balanced 3-phase systems

Y and Δ connections

Balanced 3-phase systems can be considered as 3 equal single phase voltage sources connected either as Y or Delta (Δ) to 3 single three loads connected as either Y or Δ

SOURCE CONNECTIONS | **LOAD CONNECTIONS**

Y connected source ⟷ **Y connected load**

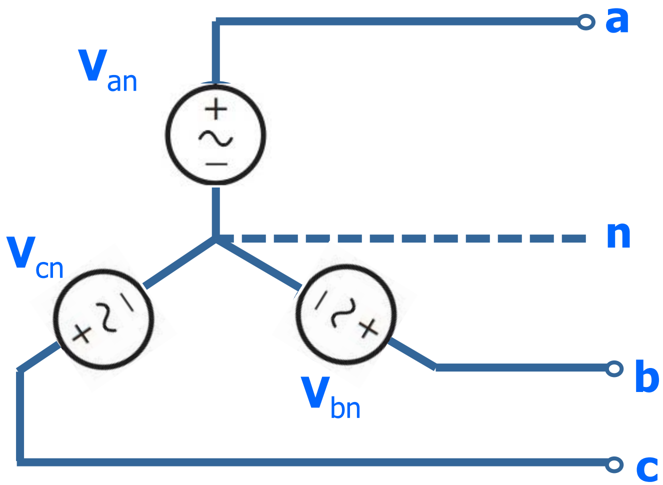
Δ connected source ⟷ **Δ connected load**

Y-Y **Y-Δ** **Δ-Y** **Δ-Δ**

Balanced 3-phase systems

SOURCE CONNECTIONS

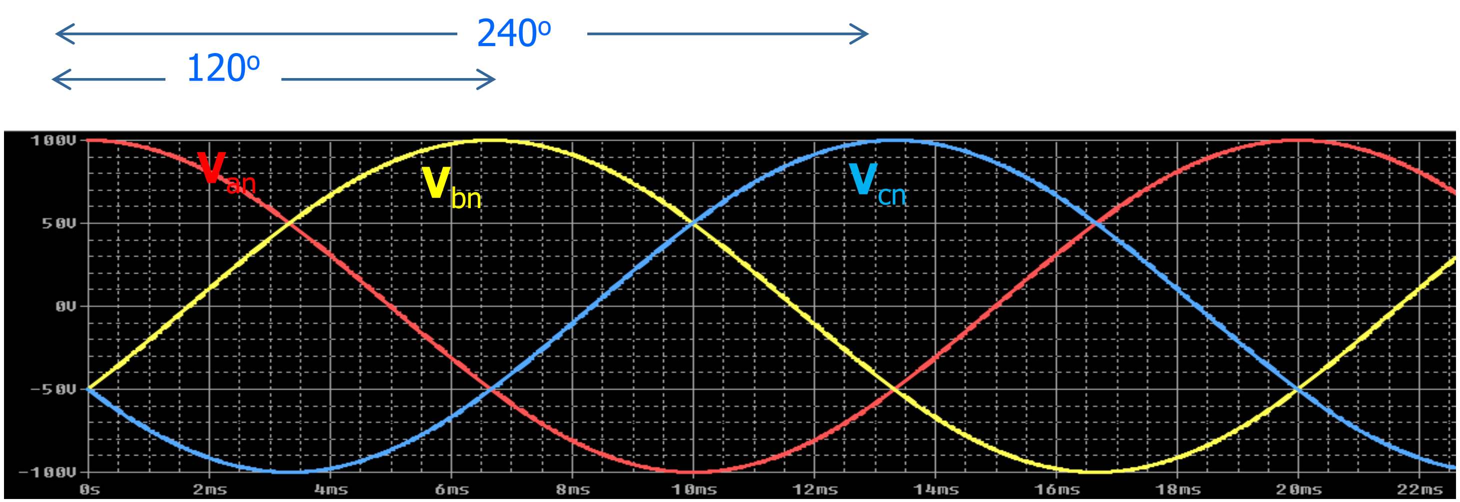
Source : Y connection



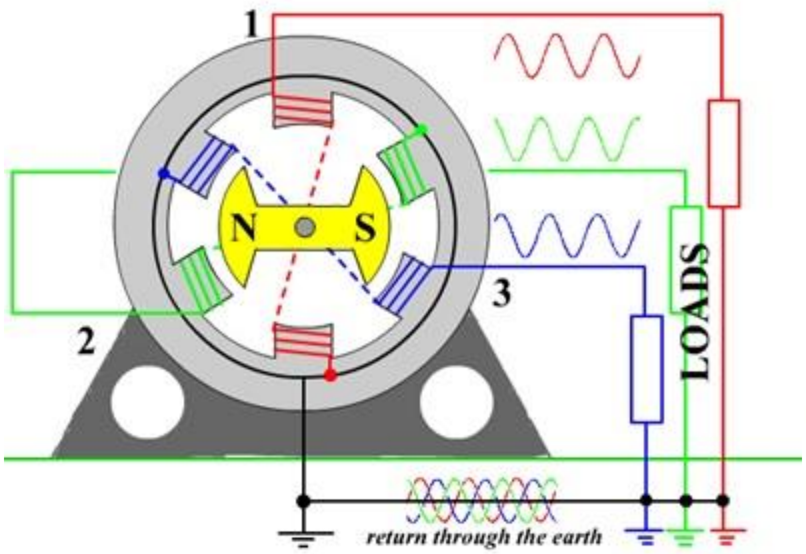
$$v_{an}(t) = \sqrt{2}V_p\cos(\omega t) \Rightarrow \mathbf{V}_{an} = V_p\angle 0^o$$

$$v_{bn}(t) = \sqrt{2}V_p\cos(\omega t - 120^o) \Rightarrow \mathbf{V}_{bn} = V_p\angle -120^o$$

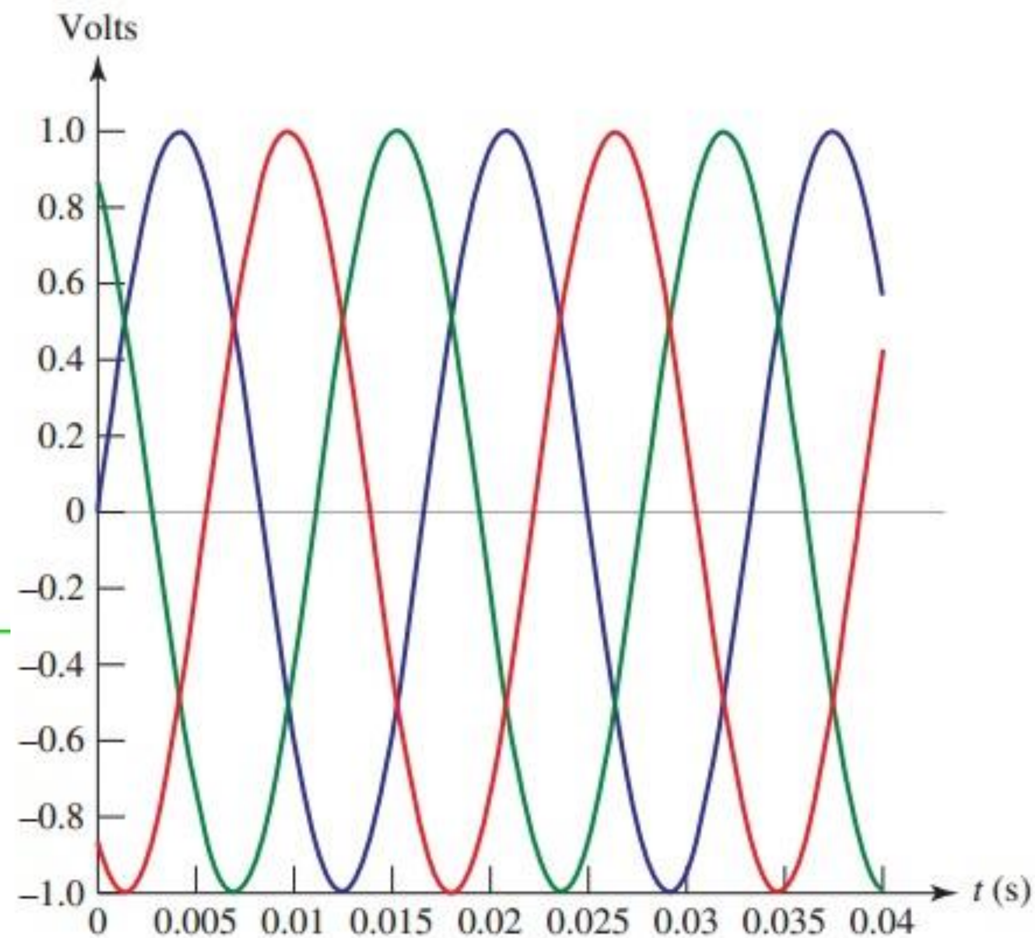
$$v_{cn}(t) = \sqrt{2}V_p\cos(\omega t + 120^o) \Rightarrow \mathbf{V}_{cn} = V_p\angle 120^o$$



Generation of 3-phase voltage: Generator



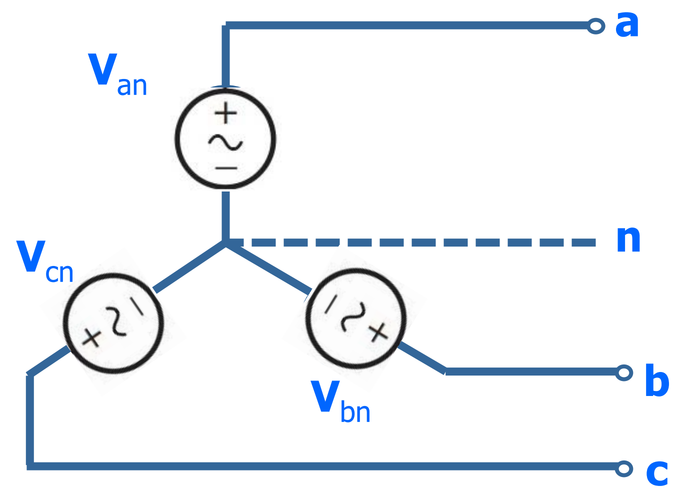
An example set of three voltages



An example set of three voltages, each of which is 120° out of phase with the other two. As can be seen, only one of the voltages is zero at any particular instant.

Balanced 3-phase systems

Source : Y connection

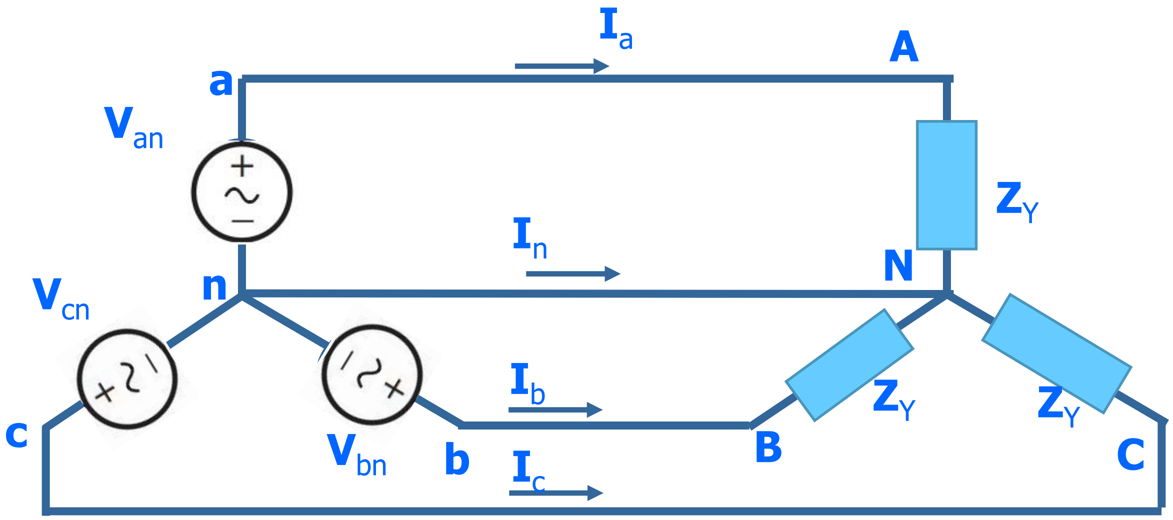


$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow \mathbf{V}_{an} = V_p\angle 0^o$$

$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t - 120^o) \Rightarrow \mathbf{V}_{bn} = V_p\angle -120^o$$

$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t + 120^o) \Rightarrow \mathbf{V}_{cn} = V_p\angle 120^o$$

4.8 Three-Phase Y-Y Connection



$$V_{an} = V_p\angle 0^o$$
$$V_{bn} = V_p\angle -120^o$$
$$V_{cn} = V_p\angle 120^o$$

Phase voltages

$$I_a = \frac{V_p\angle 0^o}{Z_Y}$$
$$I_b = \frac{V_p\angle -120^o}{Z_Y}$$
$$I_c = \frac{V_p\angle 120^o}{Z_Y}$$

line currents

$$\therefore I_a + I_b + I_c = I_n = 0$$

The wire connecting n and N can be removed !

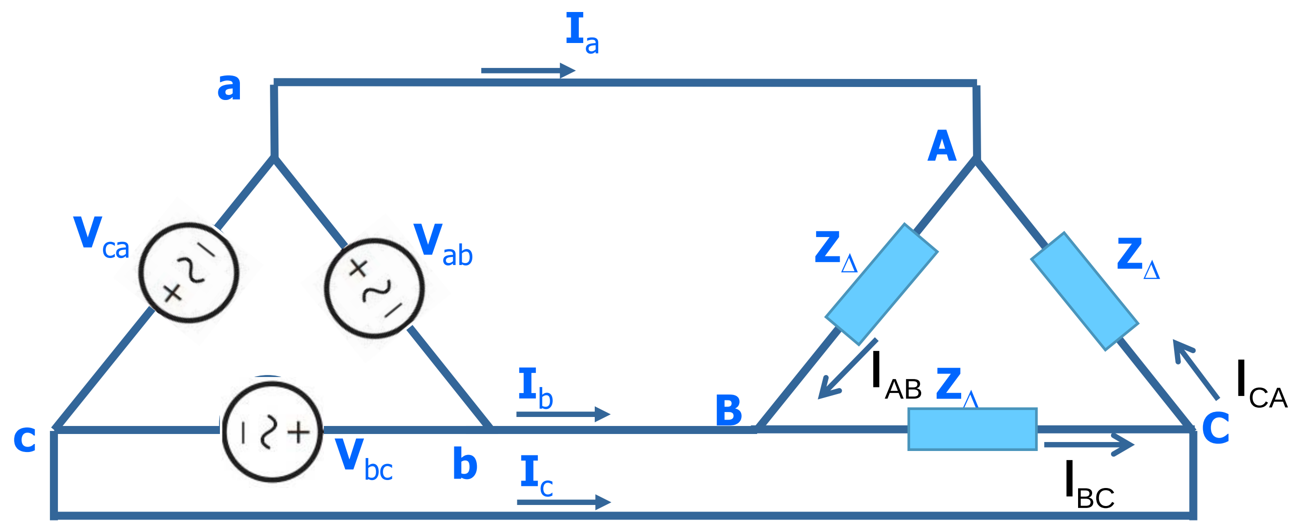
$$\begin{aligned} V_{ab} &= V_{an} + V_{nb} \\ &= V_p\angle 0^o + V_p\angle 60^o \\ &= \sqrt{3}V_p\angle 30^o \end{aligned}$$

$$\begin{aligned} V_{bc} &= V_{bn} + V_{nc} \\ &= \sqrt{3}V_p\angle -90^o \end{aligned}$$

$$\begin{aligned} V_{ca} &= V_{cn} + V_{na} \\ &= \sqrt{3}V_p\angle 150^o \end{aligned}$$

line-line voltages
OR
Line voltages

4.8 Three-Phase Δ- Δ Connection



$$V_{ab} = V_p\angle 0^o$$

$$V_{bc} = V_p\angle -120^o$$

$$V_{cn} = V_p\angle 120^o$$

$$V_{ab} = V_{AB} \qquad I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$V_{bc} = V_{BC} \qquad I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$V_{ca} = V_{CA} \qquad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Phase currents

Using KCL,

$$I_a = I_{AB} - I_{CA} = I_{AB}(1 - 1\angle 120^o) = I_{AB}\sqrt{3}\angle -30^o$$

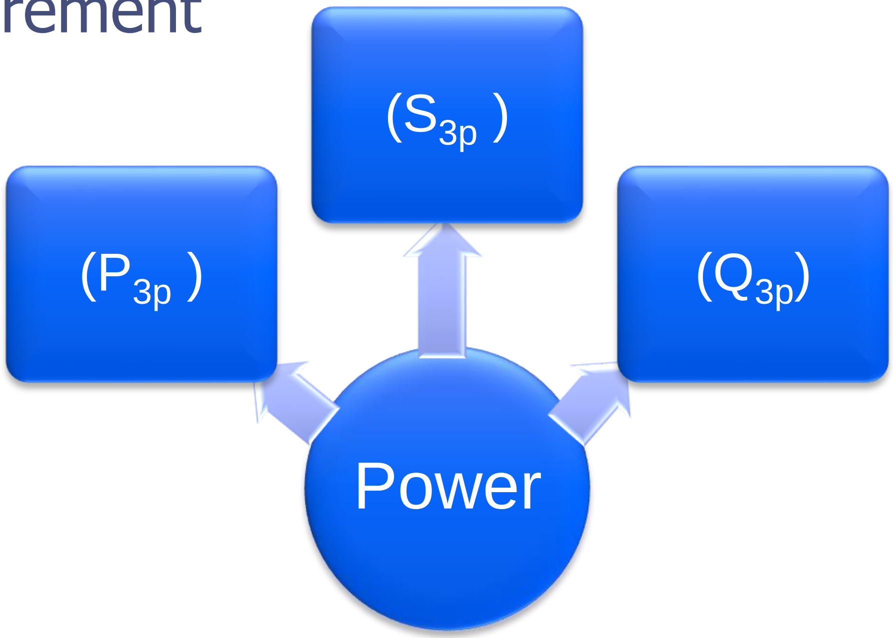
$$I_b = I_{BC} - I_{AB} = I_{BC}(1 - 1\angle 120^o) = I_{BC}\sqrt{3}\angle -30^o$$

$$I_c = I_{CA}\sqrt{3}\angle -30^o$$

line currents

4.9 Power Measurement

Power Measurement



$$P_{3p}=U_A\, I_A \cos\varphi_A + U_B\, I_B \cos\varphi_B + U_C\, I_C \cos\varphi_C$$

$$Q_{3p} = U_A\, I_A \sin\varphi_A + U_B\, I_B \sin\varphi_B + U_C\, I_C \sin\varphi_C$$

$$S_{3p} = \sqrt{P^2{}_{3p} + Q_{3p}{}^2}$$

THANK YOU