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Question 1

a) Marginal probability of:

$$P(X=x_1) = \sum_{i=1}^{3} P(x=x_1, Y=i)$$

$$= 0,01 + 0,05 + 0,1$$
$$= 0,16$$

$$P(X=x_2) = \sum_{i=1}^{3} P(x=x_2, Y=i)$$

$$= 0,02 + 0,1 + 0,05$$
$$= 0,17$$

Using the same method we can calculate the rest.

$P(X=x_3) = 0,11$

$P(X=x_4) = 0,22$

$P(X=x_5) = 0,34 \Rightarrow$

$P(Y=y_1) = 0,26$

$P(Y=y_2) = 0,47$

$P(Y=y_3) = 0,27$

Marginal distribution

Y						0,26
						0,47
						0,27
	0,16	0,17	0,11	0,22	0,34	1
	X					

b) The conditional probability of:

$$P(X=x_1 \mid Y=y_1) = \frac{P(X \cap Y)}{P(Y)} = \frac{\cancel{0,01}}{}$$

$$= \frac{P(X=x_1 \cap Y=y_1)}{P(Y=y_1)} = \frac{0,01}{0,26}$$

$$= \frac{1}{26}$$

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Using the same method we calculate the rest:

$P(X_1 = x_2 \mid Y = y_1) = \frac{1}{13}$

$P(X = x_3 \mid Y = y_1) = 3/126$

$P(X = x_4 \mid Y = y_1) = 5/13$

$P(X = x_5 \mid Y = y_1) = 5/13$

$P(X = x_1 \mid Y = y_3) = 10/27$

$P(X = x_2 \mid Y = y_3) = 5/27$

$P(X = x_3 \mid Y = y_3) = 1/9$

$P(X = x_4 \mid Y = y_3) = 5/27$

$P(X = x_5 \mid Y = y_3) = 4/27$

Conditional distribution

Y \ X	x_1	x_2	x_3	x_4	x_5
y_1	1/126	1/13	3/126	5/13	5/13
y_3	10/27	5/27	1/9	5/27	4/27

Question 2:

We have the formula for the ~~expectation~~ conditional expectation:

$$E_X[x \mid y] = \sum_x x \cdot p(x \mid Y)$$

Therefore,

$$E_Y[E_X(x \mid y)] \cancel{= E_Y[E_X}$$

$$= E_Y\Big[\sum_x x \,.\, P(x \mid y)\Big]$$

$$= \sum_y p(Y) \,.\, \cancel{x} \,.\, p(x \mid y) \,.\, \sum_x x$$

Baye's rule state that $p(x,y) = p(x \mid y) \,.\, p(Y)$

so:

$$= \sum_y \cancel{x} \,.\, p(x,y) \sum_x x$$

The sum rules stated $\sum_y p(x,y) = p(x)$

So,

$\Rightarrow \sum_x x \cdot p(x)$ (1)

We have, $E_x[X] = \sum_x x \cdot p(x)$ (2)

Consider (1) and (2) we conclude

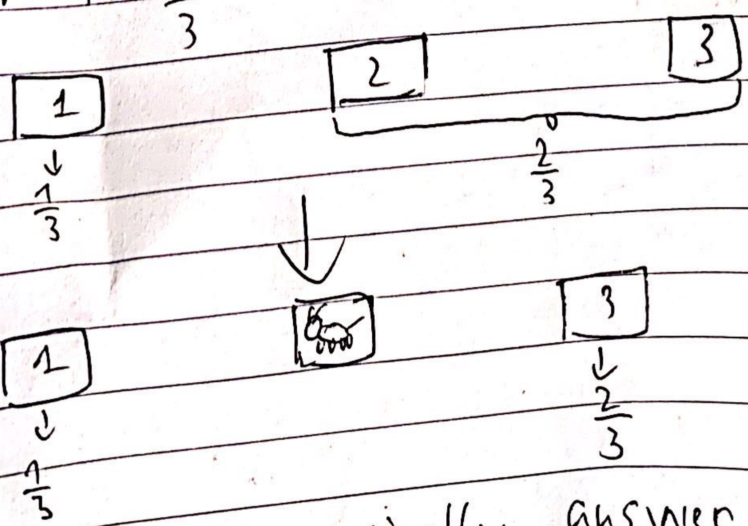
$$E_y[E_x(x|y)] = E_x[X]$$

Question 5:

At first, when we choose door no. 1 we have a $\frac{1}{3}$ chance of winning

which also means there is a $\frac{2}{3}$ chance the car is on ~~door no.2 or no.3~~ other doors.

However, Monty opened one of the door contains a goat. That means the chance for the car to be on the remaining door is $\frac{2}{3}$ now.



So, the mathematically answer is to switch

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Statistically speaking, let A be the event which the car is behind door no 1 and B the event Monty open door no. 2. Applying Baye's rules we have:

$$P(x,y) = P(X|y).P(Y)$$

$P(X|Y)$ means that if monty open door no. 2 what is the prob. the car is behind door no 1

$P(Y)$ is the probability Monty open door no. 2

Again we have,

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

$P(Y|X)$ is the probability that monty open door number 2 and the car is behind door no. 1.

Suppose that car is behind door no 1.

The probability that Monty open door no 2 is $\frac{1}{2}$ (Monty can only choose 2 or 3)

so $P(Y|X) = \frac{1}{2}$ and $P(X) = \frac{1}{3}$, $P(Y) = \frac{1}{2}$

So: $P(X|Y) = \dfrac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$

Therefore, when Monty open door no 2, the probability the car is behind door 1 is $\frac{1}{3}$.

On the other hand, we know door 1 or 3 has the car so let Z be the event the car

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is behind door 3, we can see that
$P(Z) = P(\overline{X})$ because the car can only
be in one of the door and we have:

$$P(X) + P(Z) = 1$$

$$P(Z) = \frac{2}{3}$$

So the probability the car is behind door 3
is $\frac{2}{3}$, twice as much as the probability
of being behind door 1. So to conclude
we should always SWITCH

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Question 3:

a) $P(X) = 0{,}207$, $P(Y) = 0{,}5$, $P(X|Y) = 0{,}365$

* $P(X,Y) = ?$

Using Bayes rules:

$$P(X,Y) = P(X|Y) \cdot P(Y)$$
$$= 0{,}365 \times 0{,}5$$
$$= 0{,}1825$$

b) $\cancel{P(\bar{X}|Y)} = ?$

$P(Y|\bar{X}) = ?$

Using Bayes rules:

$$P(Y|\bar{X}) = \frac{P(\bar{X}|Y) \cdot P(Y)}{P(\bar{X})}$$

$$= \frac{(1 - P(X|Y)) \cdot P(Y)}{1 - P(X)}$$

$$= \frac{(1 - 0{,}365) \cdot 0{,}5}{1 - 0{,}207} \approx 0{,}4$$

Question 4: