XM531 Problem Set 2

Problem 1

If $W(f,g)$ is the Wronskian of f and g, and if $u = 2f - g$, $v = f + 2g$, find the Wronskian of $W(u,v)$ of u and v in terms of $W(f,g)$

Problem 2

Assume that y_1 and y_2 are a fundamental aset of solutions of $y'' + p(t)y' + q(t)y = 0$ and let $y_3 = a_1y_1 + a_2y_2$ and $y_4 = b_1y_1 + b_2y_2$, where a_1, a_2, b_1, and b_2 are any constants. Show that

$$W(y_3, y_4) = (a_1b_2 - a_2b_1)W(y_1, y_2).$$

Are y_3 and y_4 also a fundamental set of solutions? Why or why not?

Problem 3

Find the fundamental set of solutions specified by Theorem 3.2.5 for the given differential equation and initial point.

$$y'' + y' - 2y = 0; \qquad t_0 = 0$$

Problem 4

Find the characteristic polynomial, and then write the general solution to the ODEs.

(a) $y'' - 4y' - 12y = 0$.

(b) $-\frac{1}{2}y'' = 13y + 5y'$.

(c) $y'' + 9y = 6y'$.

Problem 5

A 3-foot spring measures 9 feet long after a mass weighing 12 pounds is attached to it. The medium through which the mass moves offers a damping force numerically equal to $\sqrt{3}$ times the instantaneous velocity. Find the equation of motion if the mass is initially released from the equilibrium position with a downward velocity of 5 ft/s. Hint: Use $g = 32$ ft/s^2 for the acceleration due to gravity.

Additional sheet for Problem _____