XM531 Problem Set 4

Problem 1

The *gamma function* is denoted by $\Gamma(p)$ and is defined by the integral

$$\Gamma(p+1) = \int_0^\infty e^{-x} x^p dx.$$

a. Show that, for $p > 0, \Gamma(p+1) = p\Gamma(p)$.

b. Show that $\Gamma(1) = 1$.

c. If p is a positive integer n, show that $\Gamma(n+1) = n!$. Since $\Gamma(p)$ is also defined when p is not an integer, this function provides an extnsion of the factorial function to nonintegral vales of the independent variable. Note that it is also consistent to define $0! = 1$.

d. Show that, for $p > 0$,

$$p(p+1)(p+2)\ldots(p+n+1) = \frac{\Gamma(p+n)}{\Gamma(p)}.$$

e. Thus $\Gamma(p)$ can be determined for al positive values of p if $\gamma(p)$ is known in a single interval of unit length , say $0 < p \leq 1$. It is possible to show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. Find $\Gamma\left(\frac{3}{2}\right)$ and $\Gamma\left(\frac{11}{2}\right)$.

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Problem 2

Use the Laplace transform to solve the given intial value problem.

$$y'' + 3y' + 2y = 0; \qquad y(0) = 1, \quad y'(0) = 0.$$

Problem 3

Let α be a real number. Express the solution of the given initial value problm in terms of a convolution integral.

$$y'' + 2y' + 2y = \sin(\alpha t); \qquad y(0) = 0, \quad y'(0) = 0.$$

Problem 4

Let α be a real number. Find the Laplace transform of the following:

a. $f(t) = t^2 \sinh(\alpha t)$

b. $g(t) = \begin{cases} t, & 0 \leq t < 1, \\ 2 - t, & 1 \leq t < 2, \text{ and} \\ 0, & 2 \leq t < \infty \end{cases}$

Additional sheet for Problem ______