XM531 Problem Set 1

Your Name

Problem 1

For each of the following give an example. You do not need to solve your example.

(a) Give an example of a second-order ODE that is non-linear.

(b) Give an example of a first-order ODE that is linear but non-separable.

(c) Give an example of a first-order ODE that is separable but non-linear.

(d) Give an example of a first-order ODE that is non-separable and non-linear or explain why such a differential equation cannot exist.

(e) Give an example of a first-order ODE that is separable and linear or explain why such a differential equation cannot exist.

Problem 2

Sometimes it is possible to solve a nonlinear equation by making a change of the dependent variable that converts it into a linear equation. The most important equation has the form

$$y' + p(t)y = q(t)y^n,$$

and is called a Bernoulli equation after Jakob Bernoulli.

(a) Solve the Bernoulli Equation when $n = 0$; when $n = 1$.

(b) Show that if $n \neq 0, 1$, then the substitution $v = y^{1-n}$ reduces Bernoulli's equation to a linear equation. This method of solution was found by Leibniz in 1696.

Problem 3

Find a substitution of v that reduces the problem

$$y' = f(ax + by + c),$$

into a separable differential equation.

Problem 4

Using an appropriate substitution, find the general solution to the ODEs.

(a) $y' = (2x + 3y)^2$.

(b) $y^2y' + \dfrac{y^3}{t} = \dfrac{2}{t^2}$.

(c) $(x^2 - y^2)dx + xydy = 0$.

Problem 5

Verify that the DE is not exact. Find an integration factor $\mu(x)$, such that multiplying by $\mu(x)$ makes the DE exact. You must clearly state how $\mu(x)$ was found. Finally, solve the DE.

$$(\sin(y) - 2ye^{-x}\sin(x))dx + (\cos(y) + 2e^{-x}\cos(x))dy = 0.$$

Additional sheet for Problem _____