IMO51 Homework Chapter 7

Exercise 7.1 The vectors $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ span a plane through the origin in $\mathbb{R}^3$.

(a) Compute an orthogonal basis for this plane.

(b) Compute the orthogonal projection of the point $\mathbf{x} = \begin{bmatrix} 10 \\ 10 \\ -20 \end{bmatrix}$ onto the plane.

(c) Compute the distance from $\mathbf{x}$ to the plane.

(d) Compute the distance from $\mathbf{y} = \begin{bmatrix} 6 \\ -11 \\ 1 \end{bmatrix}$ to the plane. What can you conclude about $\mathbf{y}$ from your answer?

Exercise 7.2. A plane $\mathcal{P}$ in $\mathbf{R^3}$ is defined by $-x + 2y - 3y = 0$. It has $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ as basis vectors.

(a) Find 3-vectors $\mathbf{v}'$ and $\mathbf{w}'$ so that $\{\mathbf{v}, \mathbf{v}'\}$ and $\{\mathbf{w}, \mathbf{w}'\}$ are orthogonal bases for $\mathcal{P}$ (there are many possible answers).

(b) For $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, compute $\mathbf{Proj}_{\mathcal{P}}(\mathbf{u})$ in terms of the bases $\{\mathbf{v}, \mathbf{v}'\}$ and $\{\mathbf{w}, \mathbf{w}'\}$ that you found in part (a); i.e., find scalars a, b, and c, d for which $\mathbf{Proj}_{\mathcal{P}}(\mathbf{u}) = a\mathbf{v} + b\mathbf{v}'$ and $\mathbf{Proj}_{\mathcal{P}}(\mathbf{u}) = c\mathbf{w} + d\mathbf{w}'$.

(c) Consider the two forms for $\mathbf{Proj}_{\mathcal{P}}(\mathbf{u})$ that you derived in part (b). Show that they describe the same vector.

Exercise 7.3. Find the distance between the point (-1, 4, 3) and the plane $2x - 3y + 6z = 0$.

Exercise 7.4 The vectors $\mathbf{v} = (1, -1, 2, 4, -3)$ and $\mathbf{w} = (5, -3, 4, 3, -1)$ span a plane through the origin in $\mathbb{R}^5$.

(a) Compute an orthogonal basis for this plane.

(b) Let $\mathbf{x} = (1, 6, -1, -6, 0)$. Compute $\mathbf{y} = \mathbf{x} - \mathbf{Proj}_{\mathrm{span}(\mathbf{v},\mathbf{w})}(\mathbf{x})$, and verify that adding $\mathbf{y}$ to the orthogonal basis from part (a) forms an orthogonal basis of a 3-dimensional subspace of $\mathbb{R}^5$.

Exercise 7.5 Planes P_1 and P_2 are parallel to each other. P_1 contains the point (0, 0, 0) and P_2 contains the point (0, 0, 1). The vector $\mathbf{n} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is normal to P_1. Find the distance between the two planes.

Exercise 7.6 Compute the shortest distance between the lines $L_1(t) = \begin{bmatrix} 4-t \\ 4+4t \\ -7-2t \end{bmatrix}$ and $L_2(t) = \begin{bmatrix} -t \\ 2+t \\ -6+5t \end{bmatrix}$ in $\mathbb{R}^3$.

In the problems below, use the methods from our class with projections, showing all your work. Do not just plug into formulas for the coefficients that you may have learned in another class-the latter will not earn any points. However, you may use technology to calculate dot products and do other computations.

Exercise 7.7 Find the line of best fit (in the least squares sense) through the points $(2, -1), (1, 1), (4, 1)$. What is the r^2 value?

Exercise 7.8 Find the line of best fit (in the least squares sense) through the points $(0, -2), (2, 1), (4, 1), (6, 3)$. What is the r^2 value?

Exercise 7.9 How long is one year in a cat's life? Sometimes people say "one year in a cat's (or dog's) life is like seven years in a person's life". This implies a linear relationship; that is, $y = 7x$ where x is the cat's age in years and y is the corresponding human age. But that relationship is not terribly accurate. Here is a set of ordered pairs where the x coordinate is a cat age and the y coordinate approximates the corresponding human age.

$$(1,7),(2,13),(3,20),(4,26),(5,33),(6,40),(7,44),(8,48),(9,52),(10,56),(11,60),(12,64),$$

$$(13,68),(14,72),(15,76),(16,80),(17,84),(18,88),(19,92),(20,96),(21,100),(22,104).$$

Important! If you don't have access to technology that makes it easy to work with vectors that have so many entries, just use the following points: $(1,7),(6,40),(11,60),(17,84),(22,104)$.

(a) Find the line of best fit (in the least squares sense) through these points.

(b) Find the r^2 value. Is it reasonable to say this is a linear relationship?