Lateral control

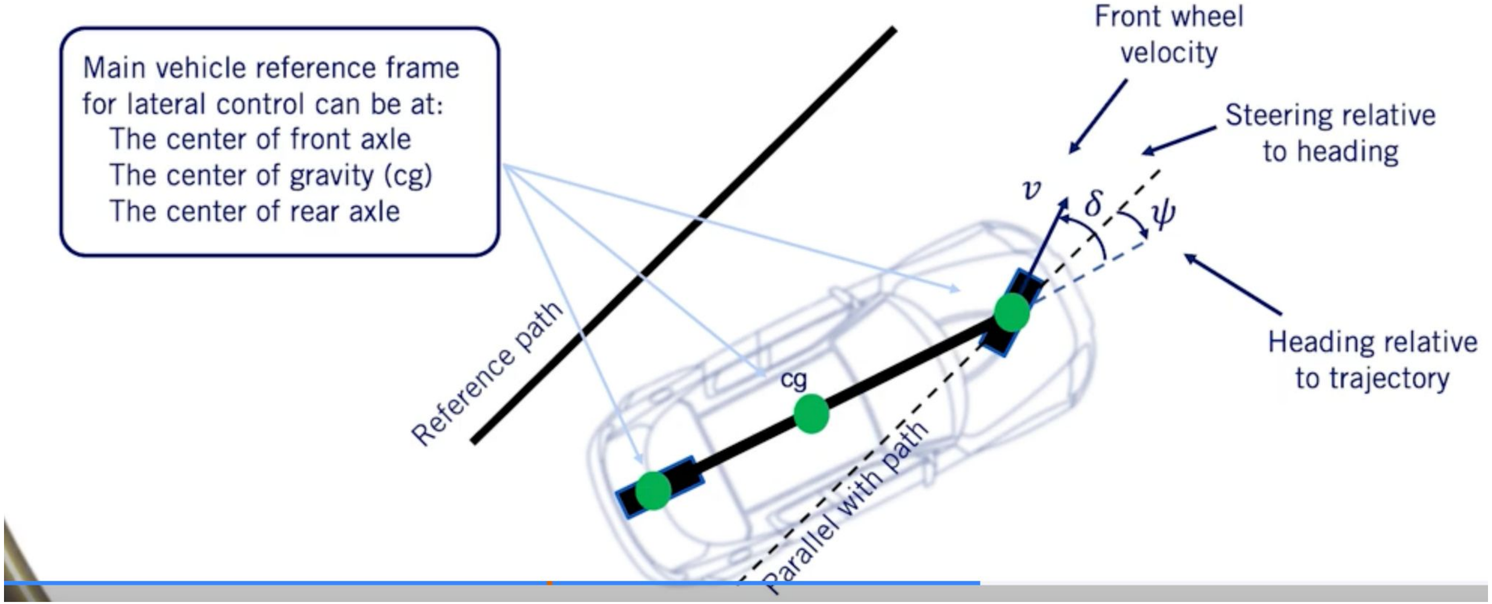
- Define the geometry of the lateral control problem, including heading and cross track errors
- Design a geometric steering controller to track a straight line segment
- Identify the limits of geometric controllers as wheel slip increases
- Explore options for dynamic control, including model predictive control

REFERENCES

Control Theory: https://www.youtube.com/watch?v=lBC1nEq0_nk

Vehicle model

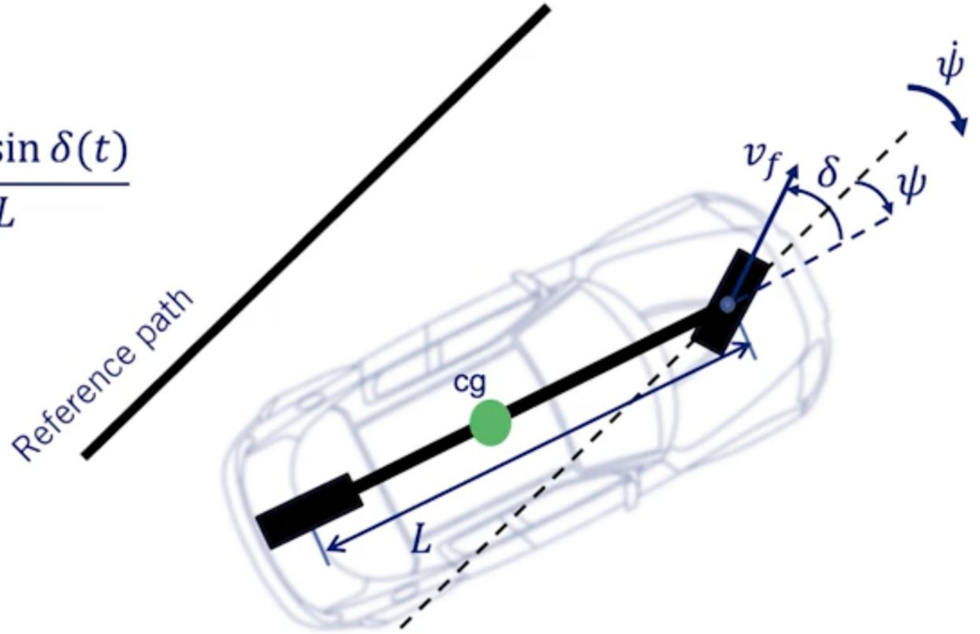
- Vehicle (bicycle) model & parameters
  - All states variables and inputs defined relative to the centre of front axle



Controller error terms

- Heading error
  - Component of velocity perpendicular to trajectory divided by ICR radius

$$\dot{\psi}_{des}(t) - \dot{\psi}(t) = \frac{v_f(t)\sin\delta(t)}{L}$$



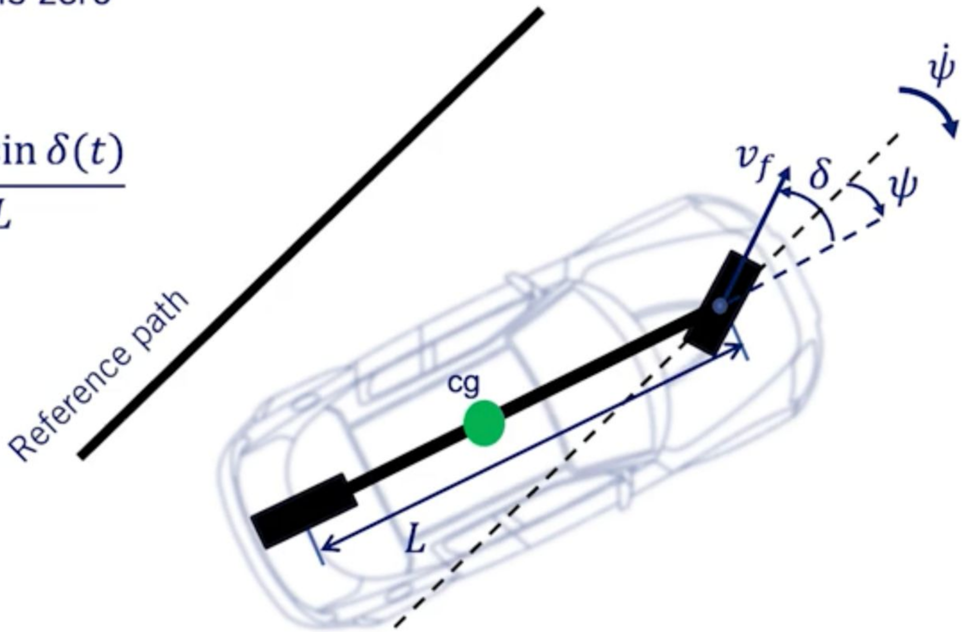
Heading error

- Heading error
  - Component of velocity perpendicular to trajectory divided by ICR radius
  - Desired heading is zero

$$\cancel{\dot{\psi}_{des}(t)} - \dot{\psi}(t) = \frac{v_f(t)\sin\delta(t)}{L}$$

$$\Downarrow$$

$$\dot{\psi}(t) = \frac{-v_f(t)\sin\delta(t)}{L}$$

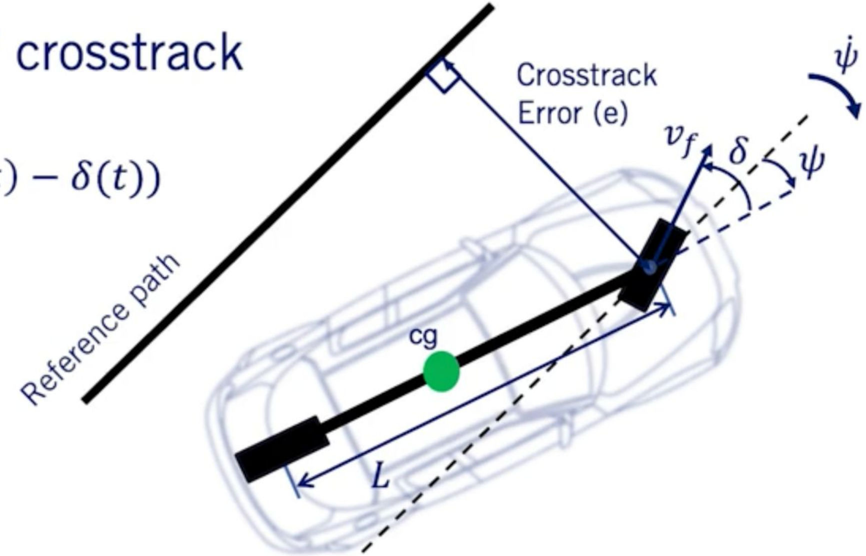


Crosstrack error

- Crosstrack error (e) :
  - Distance from center of front axle to the closest point on path

- Rate of change of crosstrack error ($\dot{e}$)

$$\dot{e}(t) = v_f(t)\sin(\psi(t) - \delta(t))$$



Supplementary Reading: Introduction to Lateral Vehicle Control

To learn more about the lateral control of autonomous vehicles, read the article below:

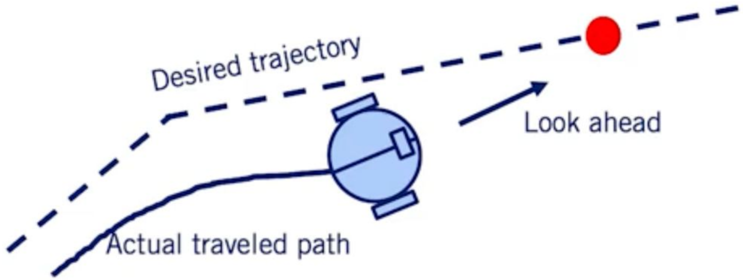
J. Jiang and A. Astolfi, "Lateral Control of an Autonomous Vehicle," in IEEE Transactions on Intelligent Vehicles, vol. 3, no. 2, pp. 228-237, June 2018. URL: http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=8286943&isnumber=8363076

To compute the minimum distance to a curved path defined by a spline:

Wang, H., Kearney, J., & Atkinson, K. (2002, June). Robust and efficient computation of the closest point on a spline curve. In Proceedings of the 5th International Conference on Curves and Surfaces (pp. 397-406). URL : http://homepage.divms.uiowa.edu/~kearney/pubs/CurvesAndSurfaces_ClosestPoint.pdf

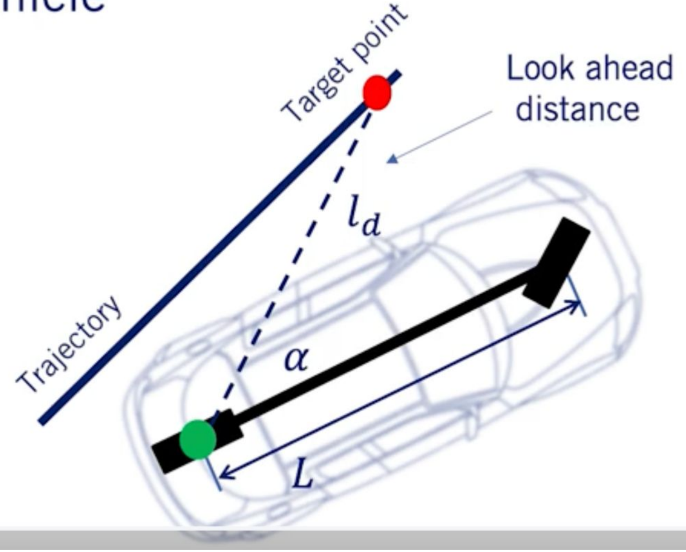
Geometric path tracking controllers

- One of the most popular classes of path tracking in robotics and autonomous vehicle
  - Exploits geometric relationship between the vehicle and the path resulting in compact control law solutions to the path tracking problem
  - Use of reference point on path to measure error of the vehicle, can be ahead of the vehicle



Pure pursuit controllers

- Pure pursuit method consists of geometrically calculating the trajectory curvature
- Connect the centre of rear axle location to a target point on the path ahead of the vehicle



Pure pursuit controllers

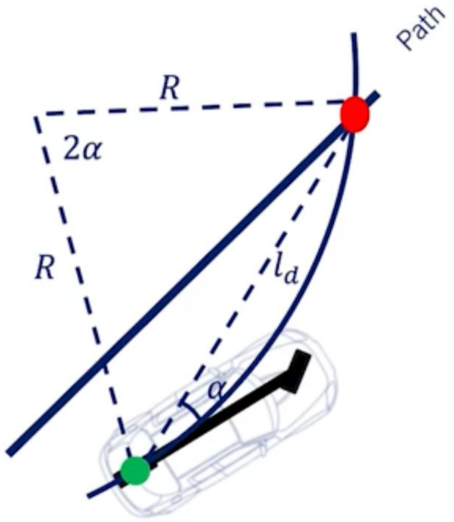
- Steering angle determined by target point location and angle between the vehicle's heading direction and lookahead direction.
- From the *law of sines*:

$$\frac{l_d}{\sin 2\alpha} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{l_d}{2\sin\alpha\cos\alpha} = \frac{R}{\cos(\alpha)}$$

$$\frac{l_d}{\sin\alpha} = 2R$$

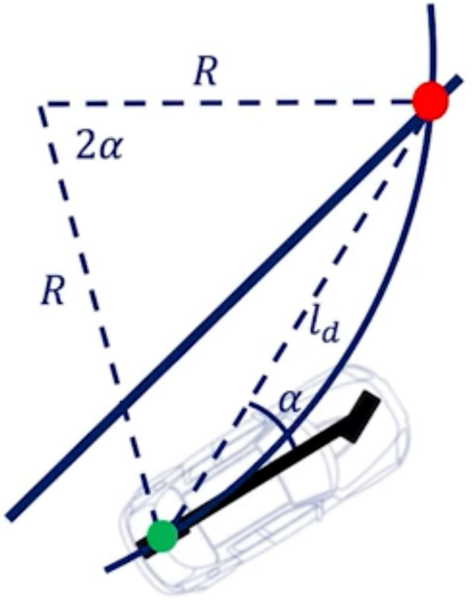
$$\kappa = \frac{1}{R} = \frac{2\sin\alpha}{l_d}$$ Path curvature



Steering angle to follow the arc

$$\kappa = \frac{2 \sin \alpha}{l_d} \qquad \delta = \tan^{-1} \kappa L$$

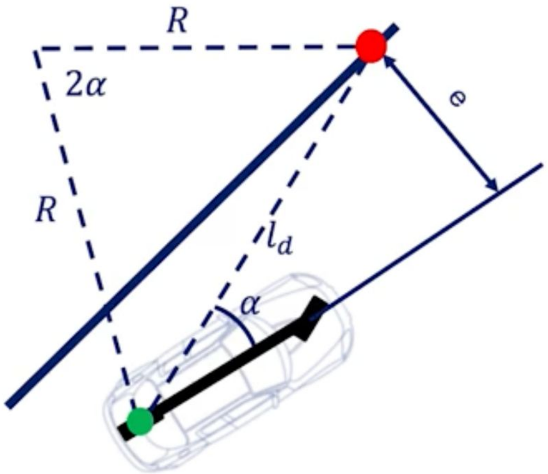
$$\delta = \tan^{-1}\left(\frac{2L \sin \alpha}{l_d}\right)$$



Curvature K is proportional to cross-track error {e}

Crosstrack error (e) is defined here as the lateral distance between the heading vector and the target point so:

$$\left.\begin{aligned} \sin\alpha &= \frac{e}{l_d} \\ \kappa &= \frac{2\sin\alpha}{l_d} \end{aligned}\right\} \quad \kappa = \frac{2}{l_d^2}e$$



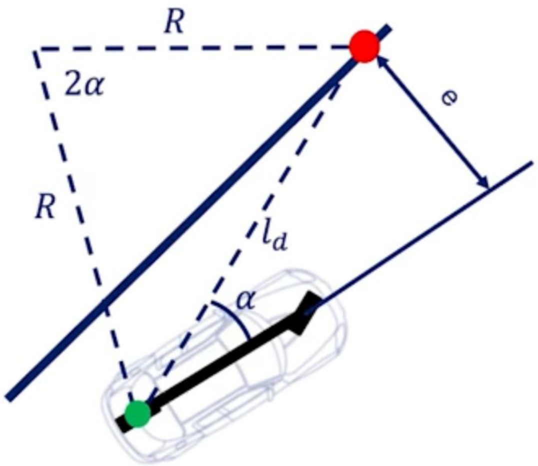
Steering angle delta for pure pursuit controller

- Lookahead l_d is assigned as a linear function of vehicle speed: $l_d = Kv_f$

$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{l_d}\right) \qquad \kappa = \frac{2}{l_d^2}e$$

$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{K_{dd}v_f}\right)$$

Forward velocity



Stanley controller

- Stanley method is the path tracking approach used by Stanford University's Darpa Grand Challenge team
    - Uses the center of the front axle as a reference point
    - Look at both the error in heading and the error in position relative to the closest point on the path
    - Define an intuitive steering law to
        - Correct heading error
        - Correct position error
        - Obey max steering angle bounds

Stanley heading control

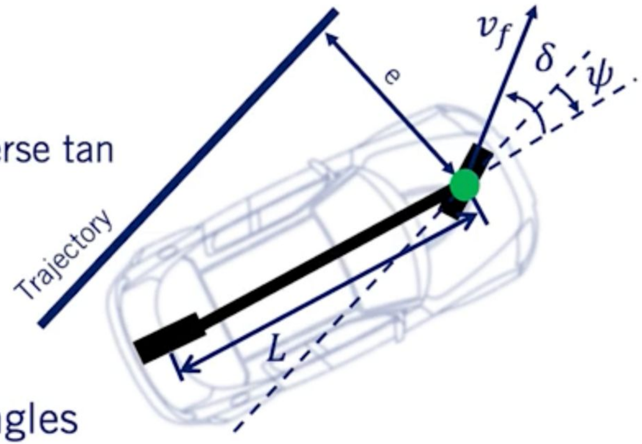
Combine three requirements:

- Steer to align heading with desired heading (proportional to heading error)

$$\delta(t) = \psi(t)$$

- Steer to eliminate crosstrack error
  - Essentially proportional to error
  - Inversely proportional to speed
  - Limit effect for large errors with inverse tan
  - Gain k determined experimentally

$$\delta(t) = \tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right)$$



- Maximum and minimum steering angles

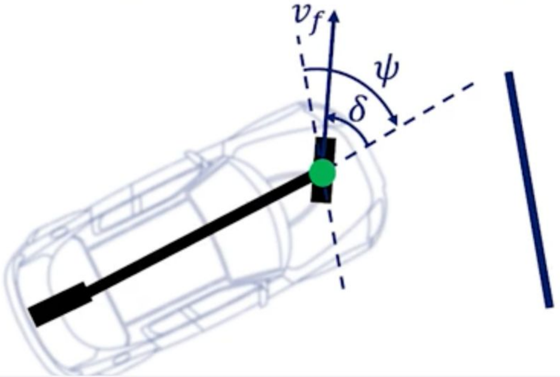
$$\delta(t) \in [\delta_{min}, \delta_{max}]$$

Stanley control law

- Stanley Control Law

$$\delta(t) = \psi(t) + \tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right), \quad \delta(t) \in [\delta_{min}, \delta_{max}]$$

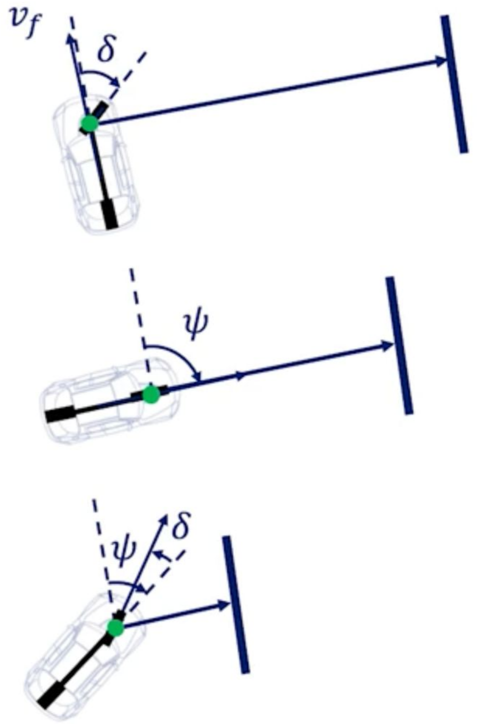
- For large heading error, steer in opposite direction
  - The larger the heading error, the larger the steering correction
  - Fixed at limit beyond maximum steering angle, assuming no crosstrack error



Stanley control law (2)

- For larger positive crosstrack error

$$\tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right) \approx \frac{\pi}{2} \;\rightarrow\; \delta(t) \approx \psi(t) + \frac{\pi}{2}$$



  - As heading changes due to steering angle, the heading correction counteracts the crosstrack correction, and drives the steering angle back to zero
  - The vehicle approaches the path, crosstrack error drops, and steering command starts to correct heading alignment.

Stanley controller error dynamics

- The error dynamics when not at maximum steering angle are:

$$\dot{e}(t) = -v_f(t)\sin(\psi(t) - \delta(t)) = -v_f(t)\sin\left(\tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right)\right)$$

$$= \frac{-ke(t)}{\sqrt{1 + \left(\frac{ke(t)}{v_f}\right)^2}}$$

- For small crosstrack errors, leads to exponential decay characteristics

$$\dot{e}(t) \approx -ke(t)$$

Stanley controller adjustment

- Low speed operation
  - Inverse speed can cause numerical instability
  - Add softening constant to controller

$$\delta(t) = \psi(t) + \tan^{-1}\left(\frac{ke(t)}{k_s + v_f(t)}\right)$$

- Extra damping on heading
  - Becomes an issue at higher speeds in real vehicle

- Steer into constant radius curves
  - Improves tracking on curves by adding a feedforward term on heading

Supplementary Reading: Geometric Lateral Control - Stanley

To learn more about the Stanley Control, check out the PDF listed below:

Snider, J. M., "Automatic Steering Methods for Autonomous Automobile Path Tracking", Robotics Institute, Carnegie Mellon University, Pittsburg (February 2009). https://www.ri.cmu.edu/pub_files/2009/2/Automatic_Steering_Methods_for_Autonomous_Automobile_Path_Tracking.pdf. Automatic_Steering_Methods_for_Autonomous_Automobile_Path_Tracking.pdf

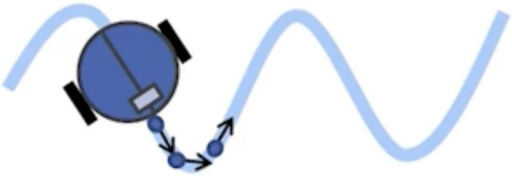
PDF File

You can also read the original paper by Der. Hoffmann et al. in the PDF below:

Hoffmann, G. et al., "Autonomous Automobile Trajectory Tracking for Off-Road Driving: Controller Design, Experimental Validation and Racing", Stanford University, (2007). http://ai.stanford.edu/~gabeh/papers/hoffmann_stanley_control07.pdf

MPC

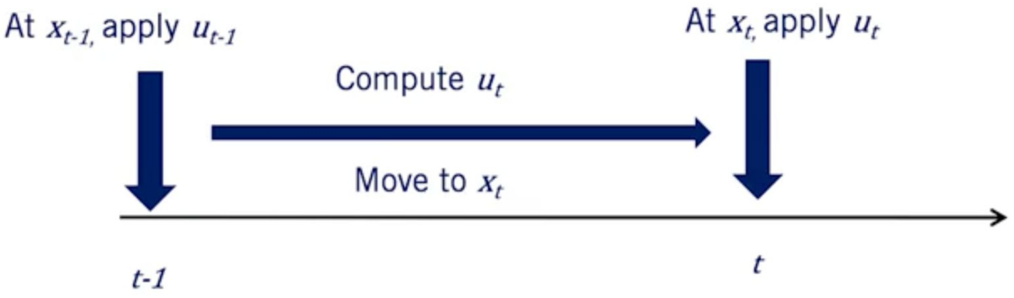
- Model predictive control (MPC)
  - Numerically solving an optimization problem at each time step
  - Receding horizon approach
- Advantages of MPC
  - Straightforward formulation
  - Explicitly handles constraints
  - Applicable to linear or nonlinear models
- Disadvantages of MPC
  - Computationally expensive



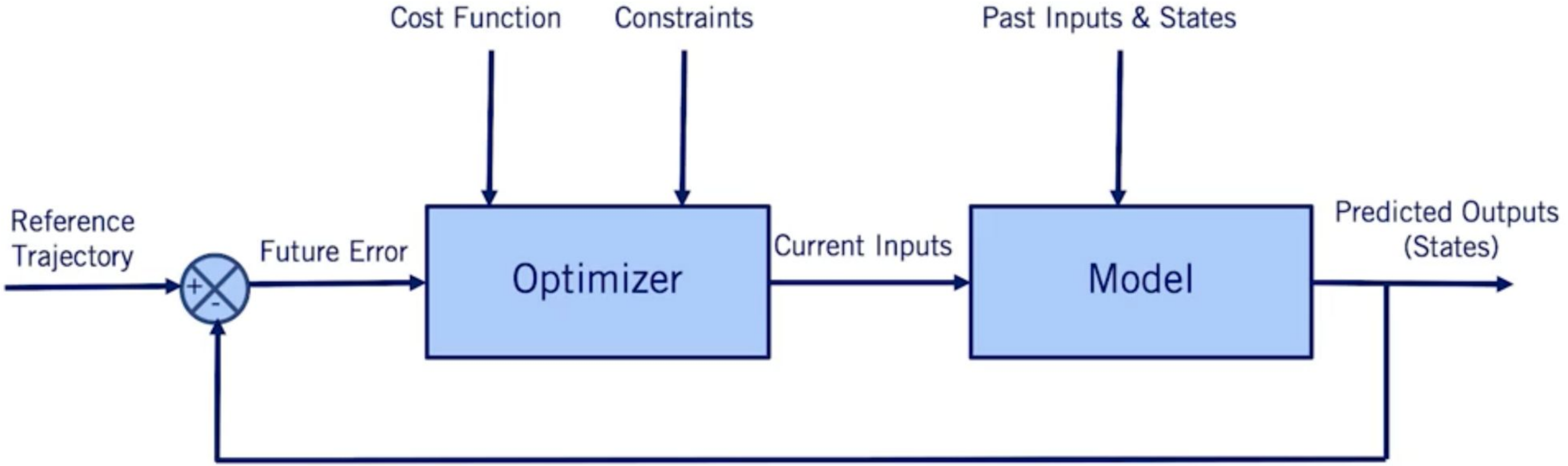
MPC

Receding Horizon Control Algorithm

- Pick receding horizon length (T)
- For each time step, t
- Set initial state to predicted state, x_t
  - Perform optimization over finite horizon t to T while traveling from x_{t-1} to x_t
  - Apply first control command, u_t, from optimization at time t



MPC structure



Linear MPC formulation

- Linear time-invariant discrete time model:

$$x_{t+1} = Ax_t + Bu_t$$

- MPC seeks to find control policy U

$$U = \{u_{t|t}, u_{t+1|t}, u_{t+2|t}, \dots\}$$

- Objective function - regulation:

$$J(x(t), U) = \sum_{j=t}^{t+T-1} x_{j|t}^T Q x_{j|t} + u_{j|t}^T R u_{j|t}$$

- Objective function - tracking:

$$\delta x_{j|t} = x_{j|t,des} - x_{j|t} \qquad J(x(t), U) = \sum_{j=t}^{t+T-1} \delta x_{j|t}^T Q \delta x_{j|t} + u_{j|t}^T R u_{j|t}$$

Linear MPC Solution

- Unconstrained, finite horizon, discrete time problem formulation:

$$\min_{U \triangleq \{u_{t|t}, u_{t+1|t}, \dots\}} J(x(t), U) = x_{t+T|t}^{\mathrm{T}} \, Q_f \, x_{t+T|t} + \sum_{j=t}^{t+T-1} x_{j|t}^{\mathrm{T}} Q x_{j|t} + u_{j|t}^{\mathrm{T}} R u_{j|t}$$

$$s.t. \qquad x_{j+1|t} = A x_{t|t} + B u_{t|t} \, , \qquad t \leq j \leq t + T - 1$$

- Linear quadratic regulator, provides a closed form solution
  - Full state feedback: $u_t = -K x_t$
  - Control gain K is a matrix
  - Refer to supplemental materials

(Non) Linear MPC formulation

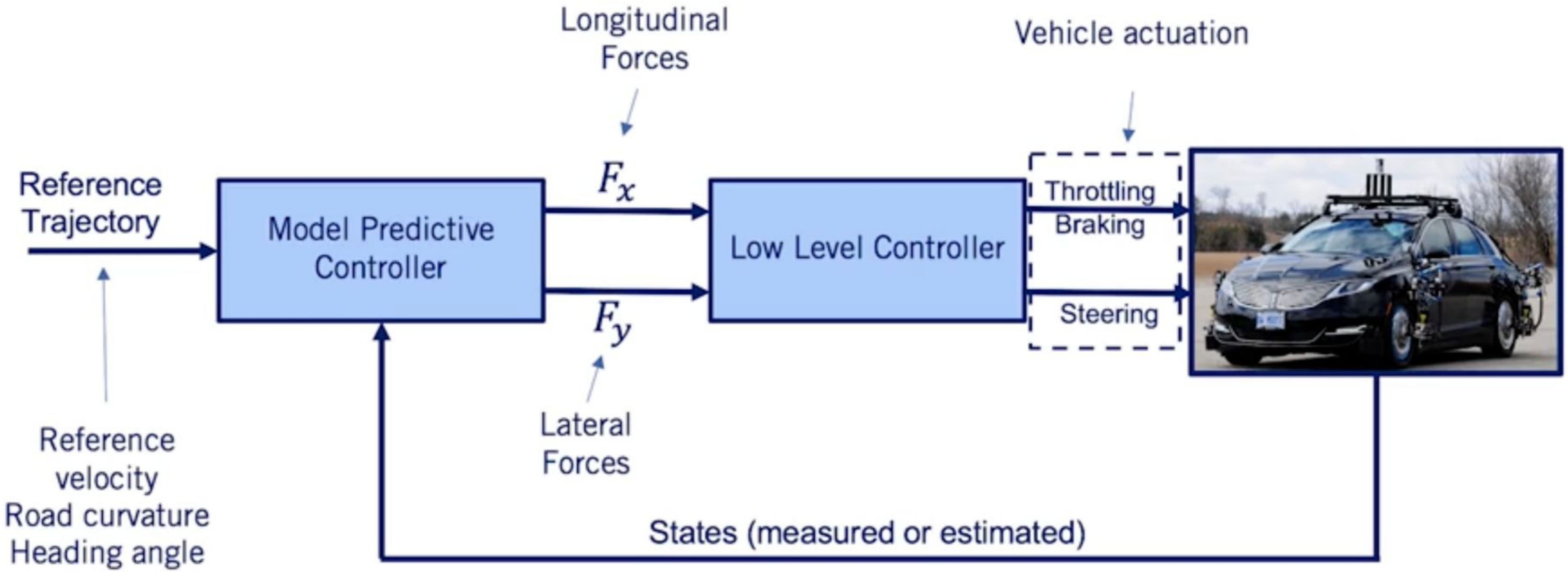
- Constrained (non)linear finite horizon discrete time case

$$\min_{U \triangleq \{u_{t|t}, u_{t+1|t}, \dots\}} J(x(t), U) = \sum_{j=t}^{t+T} C(x_{j|t}, u_{j|t})$$

$$\begin{aligned}
s.t. \quad & x_{j+1|t} = f\left(x_{j|t}, u_{j|t}\right), && t \le j \le t+T-1 \\
& x_{min} \le x_{j+1|t} \le x_{max}, && t \le j \le t+T-1 \\
& u_{min} \le u_{j|t} \le u_{max}, && t \le j \le t+T-1 \\
& g\left(x_{j|t}, u_{j|t}\right) \le 0, && t \le j \le t+T-1 \\
& h\left(x_{j|t}, u_{j|t}\right) = 0, && t \le j \le t+T-1
\end{aligned}$$

- No closed form solution, must be solved numerically

MPC for lateral control



MPC for lateral control

- Cost Function - Minimize
  - Deviation from desired trajectory
  - Minimization of control command magnitude
- Constraints - Subject to
  - Longitudinal and lateral dynamic models
  - Tire force limits
- Can incorporate low level controller, adding constraints for:
  - Engine map
  - Full dynamic vehicle model
  - Actuator models
  - Tire force models

Supplementary Reading: Advanced Steering Control - MPC

To learn more about Model Predictive Control (MPC) for vehicle control, read the ar below:

Falcone, P. et al., "Predictive Active Steering Control for Autonomous Vehicle Systems", IEEE (2007).
https://ieeexplore.ieee.org/document/4162483