

EXTENDS *TLC*, *Integers*, *Sequences*

CONSTANTS *Key*, *Client*, *nil*

VARIABLES *server_state*, *wait_list*, *push_back_list*,
client_keys, *client_states*,
next_val, *server_pc*, *client_pc*, *locked*,
channels, *client_channel*, *client_queue*,
consume_channel, *outer_states*

vars \triangleq \langle *server_state*, *wait_list*, *push_back_list*,
client_keys, *client_states*,
next_val, *server_pc*, *client_pc*, *locked*,
channels, *client_channel*, *client_queue*,
consume_channel, *outer_states* \rangle

client_vars \triangleq \langle
client_keys, *client_states*, *client_pc*,
consume_channel, *outer_states* \rangle

min_val \triangleq 21

max_val \triangleq 23

Value \triangleq *min_val* .. *max_val*

NullValue \triangleq *Value* \cup {*nil*}

KevVal \triangleq [*Key* \rightarrow *NullValue*]

emptyKV \triangleq [*k* \in *Key* \mapsto *nil*]

Pair \triangleq *Key* \times *Value*

NullPair \triangleq *Pair* \cup {*nil*}

Channel \triangleq [*data* : *NullPair*, *status* : {“Empty”, “Ready”, “Consumed”}]

ClientState \triangleq {“Init”, “ClientCheckQueue”, “GetFromQueue”, “WaitOnChan”}

TypeOK \triangleq

- \wedge *server_state* \in *KevVal*
- \wedge *wait_list* \in [*Key* \rightarrow SUBSET *Client*]
- \wedge *push_back_list* \in [*Key* \rightarrow SUBSET *Client*]
- \wedge *next_val* \in (*min_val* - 1) .. *max_val*
- \wedge *client_keys* \in [*Client* \rightarrow SUBSET *Key*]
- \wedge *client_states* \in [*Client* \rightarrow *KevVal*]
- \wedge *server_pc* \in {“Init”, “CheckWaitList”, “SetBackWaitList”}
- \wedge *client_pc* \in [*Client* \rightarrow *ClientState*]

$$\begin{aligned}
& \wedge \text{locked} \in \text{BOOLEAN} \\
& \wedge \text{channels} \in \text{Seq}(\text{Channel}) \\
& \wedge \text{client_channel} \in [\text{Client} \rightarrow 1 \dots \text{Len}(\text{channels}) \cup \{\text{nil}\}] \\
& \wedge \text{client_queue} \in [\text{Client} \rightarrow \text{SUBSET Key}] \\
& \wedge \text{consume_channel} \in [\text{Client} \rightarrow 1 \dots \text{Len}(\text{channels}) \cup \{\text{nil}\}] \\
& \wedge \text{outer_states} \in [\text{Client} \rightarrow \text{KevVal}]
\end{aligned}$$

$$\begin{aligned}
\text{Init} & \triangleq \\
& \wedge \text{server_state} = \text{emptyKV} \\
& \wedge \text{wait_list} = [k \in \text{Key} \mapsto \{\}] \\
& \wedge \text{push_back_list} = [k \in \text{Key} \mapsto \{\}] \\
& \wedge \text{client_keys} \in [\text{Client} \rightarrow \text{SUBSET Key}] \\
& \wedge \forall c \in \text{Client} : \text{client_keys}[c] \neq \{\} \quad \text{client keys should not be empty} \\
& \wedge \text{client_states} = [c \in \text{Client} \mapsto \text{emptyKV}] \\
& \wedge \text{next_val} = \text{min_val} - 1 \\
& \wedge \text{server_pc} = \text{"Init"} \\
& \wedge \text{client_pc} = [c \in \text{Client} \mapsto \text{"Init"}] \\
& \wedge \text{locked} = \text{FALSE} \\
& \wedge \text{channels} = \langle \rangle \\
& \wedge \text{client_channel} = [c \in \text{Client} \mapsto \text{nil}] \\
& \wedge \text{client_queue} = [c \in \text{Client} \mapsto \text{client_keys}[c]] \\
& \wedge \text{consume_channel} = [c \in \text{Client} \mapsto \text{nil}] \\
& \wedge \text{outer_states} = [c \in \text{Client} \mapsto \text{emptyKV}]
\end{aligned}$$

$$\begin{aligned}
\text{waitListEmpty} & \triangleq \\
& \forall k \in \text{Key} : \text{wait_list}[k] = \{\}
\end{aligned}$$

$$\begin{aligned}
\text{SetServerState}(k) & \triangleq \\
& \wedge \text{next_val} < \text{max_val} \\
& \wedge \neg \text{locked} \\
& \wedge \text{server_pc} = \text{"Init"} \\
& \wedge \text{IF } \text{waitListEmpty} \\
& \quad \text{THEN} \\
& \quad \quad \wedge \text{UNCHANGED } \text{locked} \\
& \quad \quad \wedge \text{UNCHANGED } \text{server_pc} \\
& \quad \text{ELSE} \\
& \quad \quad \wedge \text{locked}' = \text{TRUE} \\
& \quad \quad \wedge \text{server_pc}' = \text{"CheckWaitList"} \\
& \wedge \text{next_val}' = \text{next_val} + 1 \\
& \wedge \text{server_state}' = [\text{server_state} \text{ EXCEPT } ![k] = \text{next_val}'] \\
& \wedge \text{UNCHANGED } \langle \text{wait_list}, \text{push_back_list} \rangle \\
& \wedge \text{UNCHANGED } \text{channels}
\end{aligned}$$

\wedge UNCHANGED $client_channel$
 \wedge UNCHANGED $client_queue$
 \wedge UNCHANGED $client_vars$

$setOutputChan(c, k) \triangleq$
 LET
 $index \triangleq client_channel[c]$
 $oldState \triangleq channels[index]$
 $val \triangleq server_state[k]$
 $newState \triangleq [oldState \text{ EXCEPT } !.data = \langle k, val \rangle, !.status = \text{"Ready"}]$
 IN
 $\wedge channels' = [channels \text{ EXCEPT } ![index] = newState]$
 $\wedge client_channel' = [client_channel \text{ EXCEPT } ![c] = nil]$
 $\wedge client_states' = [client_states \text{ EXCEPT } ![c][k] = server_state[k]]$

$waitListEmptyNew \triangleq$
 $\forall k \in Key : wait_list'[k] = \{\}$

$handleWaitEntryNoChange(c, k) \triangleq$
 $\wedge push_back_list' = [push_back_list \text{ EXCEPT } ![k] = @ \cup \{c\}]$
 \wedge UNCHANGED $channels$
 \wedge UNCHANGED $client_states$
 \wedge UNCHANGED $client_queue$
 \wedge UNCHANGED $client_channel$

$handleWaitEntryChanged(c, k) \triangleq$
 IF $client_channel[c] \neq nil$
 THEN
 $\wedge setOutputChan(c, k)$
 $\wedge client_queue' = [client_queue \text{ EXCEPT } ![c] = @ \cup \{k\}]$
 \wedge UNCHANGED $push_back_list$
 ELSE
 $\wedge client_queue' = [client_queue \text{ EXCEPT } ![c] = @ \cup \{k\}]$
 \wedge UNCHANGED $channels$
 \wedge UNCHANGED $client_channel$
 \wedge UNCHANGED $client_states$
 \wedge UNCHANGED $push_back_list$

$ServerCheckWaitList(k, c) \triangleq$
 $\wedge server_pc = \text{"CheckWaitList"}$
 $\wedge c \in wait_list[k]$
 $\wedge wait_list' = [wait_list \text{ EXCEPT } ![k] = @ \setminus \{c\}]$
 \wedge IF $client_states[c][k] = server_state[k]$
 THEN $handleWaitEntryNoChange(c, k)$

ELSE *handleWaitEntryChanged*(*c*, *k*)
 ∧ IF *waitListEmptyNew*
 THEN
 ∧ *server_pc*' = "SetBackWaitList"
 ∧ UNCHANGED *locked*
 ELSE
 ∧ UNCHANGED *server_pc*
 ∧ UNCHANGED *locked*

 ∧ UNCHANGED *server_state*
 ∧ UNCHANGED ⟨*client_keys*, *client_pc*, *consume_channel*, *outer_states*⟩
 ∧ UNCHANGED *next_val*

ServerSetBackWaitList \triangleq
 ∧ *server_pc* = "SetBackWaitList"
 ∧ *server_pc*' = "Init"
 ∧ *locked*' = FALSE
 ∧ *wait_list*' = *push_back_list*
 ∧ *push_back_list*' = [*k* ∈ *Key* ↦ {}]
 ∧ UNCHANGED *server_state*
 ∧ UNCHANGED *channels*
 ∧ UNCHANGED ⟨*client_channel*, *client_queue*⟩
 ∧ UNCHANGED *client_vars*
 ∧ UNCHANGED *next_val*

clientGoto(*c*, *state*) \triangleq *client_pc*' = [*client_pc* EXCEPT ![*c*] = *state*]

newChannel \triangleq
 ∧ *channels*' = *Append*(*channels*, [*data* ↦ *nil*, *status* ↦ "Empty"])

newChannelIndex \triangleq *Len*(*channels*')

GetState(*c*) \triangleq
 ∧ *client_pc*[*c*] = "Init"
 ∧ ¬*locked*
 ∧ *locked*' = TRUE
 ∧ *newChannel*
 ∧ *client_channel*' = [*client_channel* EXCEPT ![*c*] = *newChannelIndex*]
 ∧ *clientGoto*(*c*, "ClientCheckQueue")
 ∧ UNCHANGED ⟨*client_keys*, *client_states*, *client_queue*⟩
 ∧ UNCHANGED *next_val*
 ∧ UNCHANGED ⟨*server_pc*, *server_state*⟩
 ∧ UNCHANGED ⟨*consume_channel*, *outer_states*⟩
 ∧ UNCHANGED ⟨*wait_list*, *push_back_list*⟩

$$\begin{aligned}
& ClientCheckQueue(c) \triangleq \\
& \quad \wedge client_pc[c] = \text{"ClientCheckQueue"} \\
& \quad \wedge \text{IF } client_queue[c] = \{\} \\
& \quad \quad \text{THEN} \\
& \quad \quad \quad \wedge clientGoto(c, \text{"WaitOnChan"}) \\
& \quad \quad \quad \wedge consume_channel' = [consume_channel \text{ EXCEPT } ![c] = client_channel[c]] \\
& \quad \quad \quad \wedge locked' = \text{FALSE} \\
& \quad \quad \quad \wedge \text{UNCHANGED } client_channel \\
& \quad \quad \text{ELSE} \\
& \quad \quad \quad \wedge clientGoto(c, \text{"GetFromQueue"}) \\
& \quad \quad \quad \wedge \text{UNCHANGED } locked \\
& \quad \quad \quad \wedge \text{UNCHANGED } client_channel \\
& \quad \quad \quad \wedge \text{UNCHANGED } consume_channel \\
& \quad \wedge \text{UNCHANGED } channels \\
& \quad \wedge \text{UNCHANGED } \langle client_queue, client_states \rangle \\
& \quad \wedge \text{UNCHANGED } \langle client_keys \rangle \\
& \quad \wedge \text{UNCHANGED } \langle server_pc, server_state \rangle \\
& \quad \wedge \text{UNCHANGED } next_val \\
& \quad \wedge \text{UNCHANGED } \langle outer_states \rangle \\
& \quad \wedge \text{UNCHANGED } \langle wait_list, push_back_list \rangle
\end{aligned}$$

$$\begin{aligned}
& GetFromQueue(c, k) \triangleq \\
& \quad \wedge client_pc[c] = \text{"GetFromQueue"} \\
& \quad \wedge k \in client_queue[c] \\
& \quad \wedge \text{IF } client_states[c][k] = server_state[k] \\
& \quad \quad \text{THEN} \\
& \quad \quad \quad \wedge client_queue' = [client_queue \text{ EXCEPT } ![c] = @ \setminus \{k\}] \\
& \quad \quad \quad \wedge clientGoto(c, \text{"ClientCheckQueue"}) \\
& \quad \quad \quad \wedge wait_list' = [wait_list \text{ EXCEPT } ![k] = @ \cup \{c\}] \\
& \quad \quad \quad \wedge \text{UNCHANGED } channels \\
& \quad \quad \quad \wedge \text{UNCHANGED } client_channel \\
& \quad \quad \quad \wedge \text{UNCHANGED } client_states \\
& \quad \quad \quad \wedge \text{UNCHANGED } locked \\
& \quad \quad \quad \wedge \text{UNCHANGED } consume_channel \\
& \quad \quad \quad \wedge \text{UNCHANGED } push_back_list \\
& \quad \quad \text{ELSE} \\
& \quad \quad \quad \wedge clientGoto(c, \text{"WaitOnChan"}) \\
& \quad \quad \quad \wedge locked' = \text{FALSE} \\
& \quad \quad \quad \wedge setOutputChan(c, k) \\
& \quad \quad \quad \wedge consume_channel' = [consume_channel \text{ EXCEPT } ![c] = client_channel[c]] \\
& \quad \quad \quad \wedge \text{UNCHANGED } client_queue \\
& \quad \quad \quad \wedge \text{UNCHANGED } wait_list \\
& \quad \quad \quad \wedge \text{UNCHANGED } push_back_list \\
& \quad \wedge \text{UNCHANGED } \langle server_pc, server_state \rangle
\end{aligned}$$

\wedge UNCHANGED $client_keys$
 \wedge UNCHANGED $next_val$
 \wedge UNCHANGED $outer_states$

$ConsumeFromChan(c) \triangleq$

LET

$index \triangleq consume_channel[c]$
 $old_state \triangleq channels[index]$
 $k \triangleq old_state.data[1]$
 $val \triangleq old_state.data[2]$
 $new_state \triangleq [old_state \text{ EXCEPT } !.data = nil, !.status = \text{"Consumed"}]$

IN

$\wedge client_pc[c] = \text{"WaitOnChan"}$
 $\wedge channels[index].status = \text{"Ready"}$
 $\wedge clientGoto(c, \text{"Init"})$
 $\wedge channels' = [channels \text{ EXCEPT } ![index] = new_state]$
 $\wedge outer_states' = [outer_states \text{ EXCEPT } ![c][k] = val]$
 \wedge UNCHANGED $\langle client_keys, client_states, client_queue, client_channel \rangle$
 \wedge UNCHANGED $consume_channel$
 \wedge UNCHANGED $locked$
 \wedge UNCHANGED $\langle server_pc, server_state, next_val, wait_list \rangle$
 \wedge UNCHANGED $push_back_list$

$TerminateCond \triangleq$

$\wedge server_pc = \text{"Init"}$
 $\wedge \forall c \in Client :$
 $\quad \wedge client_pc[c] = \text{"WaitOnChan"}$
 $\quad \wedge channels[consume_channel[c]].status = \text{"Empty"}$
 $\wedge next_val = max_val$

$Terminated \triangleq$

$\wedge TerminateCond$
 \wedge UNCHANGED $vars$

$Next \triangleq$

$\vee \exists k \in Key :$
 $\quad \vee SetServerState(k)$
 $\vee \exists c \in Client :$
 $\quad \vee GetState(c)$
 $\quad \vee ClientCheckQueue(c)$
 $\vee \exists k \in Key :$
 $\quad \vee GetFromQueue(c, k)$
 $\quad \vee ServerCheckWaitList(k, c)$
 $\vee ConsumeFromChan(c)$

$$\begin{aligned} & \vee \text{ServerSetBackWaitList} \\ & \vee \text{Terminated} \end{aligned}$$

$$\text{Spec} \triangleq \text{Init} \wedge \Box[\text{Next}]_{\text{vars}}$$

$$\text{FairSpec} \triangleq \text{Spec} \wedge \text{WF}_{\text{vars}}(\text{Next})$$

$$\begin{aligned} \text{Inv} & \triangleq \\ & \text{TerminateCond} \Rightarrow \\ & \quad \forall c \in \text{Client} : \forall k \in \text{client_keys}[c] : \\ & \quad \quad \wedge \text{client_states}[c][k] = \text{server_state}[k] \\ & \quad \quad \wedge \text{outer_states}[c][k] = \text{server_state}[k] \end{aligned}$$

$$\text{AlwaysTerminate} \triangleq \Diamond \text{TerminateCond}$$

$$\begin{aligned} \text{ChannelInv} & \triangleq \\ & \forall \text{index} \in 1 \dots \text{Len}(\text{channels}) : \\ & \quad \text{LET} \\ & \quad \quad \text{ch} \triangleq \text{channels}[\text{index}] \\ & \quad \text{IN} \\ & \quad \quad \vee \text{ch.data} = \text{nil} \wedge \text{ch.status} = \text{"Empty"} \\ & \quad \quad \vee \text{ch.data} = \text{nil} \wedge \text{ch.status} = \text{"Consumed"} \\ & \quad \quad \vee \text{ch.data} \neq \text{nil} \wedge \text{ch.status} = \text{"Ready"} \end{aligned}$$

$$\begin{aligned} \text{LockedCorrectly} & \triangleq \\ & (\text{server_pc} = \text{"Init"} \wedge \forall c \in \text{Client} : \text{client_pc}[c] = \text{"Init"}) \Rightarrow \neg \text{locked} \end{aligned}$$

$$\begin{aligned} \text{allChannelConsumedExceptWaiting} & \triangleq \\ & \forall i \in \text{DOMAIN channels} : \\ & \quad (\forall c \in \text{Client} : \text{consume_channel}[c] \neq i) \Rightarrow \text{channels}[i].\text{status} = \text{"Consumed"} \end{aligned}$$

$$\begin{aligned} \text{AllChannelConsumed} & \triangleq \\ & \text{TerminateCond} \Rightarrow \text{allChannelConsumedExceptWaiting} \end{aligned}$$

$$\begin{aligned} \text{channelPushOrRecv} & \triangleq \\ & \forall \text{index} \in 1 \dots \text{Len}(\text{channels}) : \\ & \quad \text{LET} \\ & \quad \quad \text{before} \triangleq \text{channels}[\text{index}] \\ & \quad \quad \text{after} \triangleq \text{channels}'[\text{index}] \\ & \quad \text{IN} \\ & \quad \quad \vee \wedge \text{before.status} = \text{"Empty"} \\ & \quad \quad \quad \wedge \text{after.status} = \text{"Ready"} \\ & \quad \quad \vee \wedge \text{before.status} = \text{"Ready"} \\ & \quad \quad \quad \wedge \text{after.status} = \text{"Consumed"} \end{aligned}$$

$$\vee \text{ before} = \text{after}$$

$$\begin{aligned} \text{channelPushRecvOrAppend} &\triangleq \\ &\vee \text{ channelPushOrRecv} \\ &\vee \text{Len}(\text{channels}') = \text{Len}(\text{channels}) + 1 \end{aligned}$$

$$\begin{aligned} \text{ChannelPushInv} &\triangleq \\ &\Box[\text{channelPushRecvOrAppend}]_{\text{channels}} \end{aligned}$$

$$\text{Symm} \triangleq \text{Permutations}(\text{Key}) \cup \text{Permutations}(\text{Client})$$
