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MODULE *PendingKeys*

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EXTENDS *TLC*, *Naturals*

CONSTANTS *Key*, *Slave*, *Client*, *nil*

VARIABLES *info*, *pending*, *pc*,  
*slave\_map*,  
*client\_slave*, *client\_state*, *client\_keys*,  
*num\_action*

*aux\_vars*  $\triangleq$   $\langle \textit{client\_slave}, \textit{num\_action} \rangle$

*vars*  $\triangleq$   $\langle$   
*info*, *pending*, *pc*,  
*slave\_map*,  
*client\_state*, *client\_keys*,  
*aux\_vars*  
 $\rangle$

*max\_action*  $\triangleq$  7

*Seq*  $\triangleq$  0 .. 30

*NullSlave*  $\triangleq$  *Slave*  $\cup$  {*nil*}

*SlaveState*  $\triangleq$  [  
*running* : SUBSET *Key*,  
*latest\_seq* : *Seq*,  
*wait\_list* : SUBSET *Client*  
 $\rangle$

*Channel*  $\triangleq$  [*data* : SUBSET *Key*, *status* : {“Empty”, “Ready”, “Consumed”}]

*ClientState*  $\triangleq$  [  
*chan* : *Channel*,  
*consumed\_seq* : *Seq*  
 $\rangle$

*init\_slave\_state*  $\triangleq$  [  
*running*  $\mapsto$  {},  
*latest\_seq*  $\mapsto$  0,  
*wait\_list*  $\mapsto$  {}  
 $\rangle$

*init\_client\_state*  $\triangleq$  [  
*chan*  $\mapsto$  [*data*  $\mapsto$  {}], *status*  $\mapsto$  “Consumed”,  
 $\rangle$

$$\begin{aligned}
& consumed\_seq \mapsto 0 \\
& ] \\
TypeOK & \triangleq \\
& \wedge \text{ info} \in [Key \rightarrow NullSlave] \\
& \wedge \text{ pending} \subseteq Key \\
& \wedge \text{ pc} \in [Client \rightarrow \{ \text{"Init"}, \text{"GetRunningKeys"}, \text{"WaitOnChan"} \}] \\
& \wedge \text{ client\_slave} \in [Client \rightarrow Slave] \\
& \\
& \wedge \text{ slave\_map} \in [Slave \rightarrow SlaveState] \\
& \wedge \text{ client\_state} \in [Client \rightarrow ClientState] \\
& \wedge \text{ client\_keys} \in [Client \rightarrow \text{SUBSET } Key] \\
& \wedge \text{ num\_action} \in 0 \dots max\_action \\
& \\
Init & \triangleq \\
& \wedge \text{ info} = [k \in Key \mapsto nil] \\
& \wedge \text{ pending} = \{ \} \\
& \wedge \text{ pc} = [s \in Client \mapsto \text{"Init"}] \\
& \wedge \text{ client\_slave} \in [Client \rightarrow Slave] \\
& \\
& \wedge \text{ slave\_map} = [s \in Slave \mapsto \text{init\_slave\_state}] \\
& \wedge \text{ client\_state} = [c \in Client \mapsto \text{init\_client\_state}] \\
& \wedge \text{ client\_keys} = [c \in Client \mapsto \{ \}] \\
& \wedge \text{ num\_action} = 0 \\
& \\
allowAction & \triangleq \\
& \wedge \text{ num\_action} < max\_action \\
& \wedge \text{ num\_action}' = \text{ num\_action} + 1 \\
& \\
pushToClient(client\_set, old\_state) & \triangleq \\
LET \\
& \text{ get\_slave}(c) \triangleq \text{ slave\_map}'[\text{ client\_slave}[c]] \\
& \text{ curr\_seq}(c) \triangleq \text{ get\_slave}(c).latest\_seq \\
& \text{ can\_push}(c) \triangleq \\
& \quad \wedge c \in \text{ client\_set} \\
& \quad \wedge \text{ old\_state}[c].chan.status = \text{"Empty"} \\
& \quad \wedge \text{ old\_state}[c].consumed\_seq < \text{ curr\_seq}(c) \\
& \text{ new\_chan}(c) \triangleq \\
& \quad [data \mapsto \text{ get\_slave}(c).running, status \mapsto \text{"Ready"}] \\
& \text{ new\_state}(c) \triangleq \\
& \quad [\text{ chan} \mapsto \text{ new\_chan}(c), \text{ consumed\_seq} \mapsto \text{ curr\_seq}(c)]
\end{aligned}$$

$$\begin{aligned}
& \text{new\_state\_or\_unchanged}(c) \triangleq \\
& \quad \text{IF } \text{can\_push}(c) \\
& \quad \quad \text{THEN } \text{new\_state}(c) \\
& \quad \quad \text{ELSE } \text{old\_state}[c] \text{ UNCHANGED} \\
& \text{IN} \\
& \quad [c \in \text{Client} \mapsto \text{new\_state\_or\_unchanged}(c)] \\
\\
& \text{AddKey}(k) \triangleq \\
& \quad \text{LET} \\
& \quad \quad \text{added} \triangleq \\
& \quad \quad \quad \text{IF } k \in \text{pending} \\
& \quad \quad \quad \quad \text{THEN } \{\} \\
& \quad \quad \quad \quad \text{ELSE } \{k\} \\
& \quad \quad \text{do\_add\_key}(s) \triangleq \\
& \quad \quad \quad \wedge \text{info}[k] = \text{nil} \\
& \quad \quad \quad \wedge \text{allowAction} \\
& \quad \quad \quad \wedge \text{info}' = [\text{info} \text{ EXCEPT } ![k] = s] \\
& \quad \quad \quad \wedge \text{slave\_map}' = [\text{slave\_map} \text{ EXCEPT} \\
& \quad \quad \quad \quad ![s].\text{latest\_seq} = @ + 1, \\
& \quad \quad \quad \quad ![s].\text{running} = @ \cup \text{added}] \\
& \quad \quad \quad \wedge \text{client\_state}' = \text{pushToClient}(\text{slave\_map}[s].\text{wait\_list}, \text{client\_state}) \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{pending} \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{pc} \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{client\_slave} \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{client\_keys} \\
& \quad \text{IN} \\
& \quad \quad \exists s \in \text{Slave} : \text{do\_add\_key}(s) \\
\\
& \text{RemoveKey}(k) \triangleq \\
& \quad \text{LET} \\
& \quad \quad \text{do\_remove\_key}(s) \triangleq \\
& \quad \quad \quad \wedge \text{info}[k] \neq \text{nil} \\
& \quad \quad \quad \wedge \text{info}[k] = s \\
& \quad \quad \quad \wedge \text{allowAction} \\
& \quad \quad \quad \wedge \text{info}' = [\text{info} \text{ EXCEPT } ![k] = \text{nil}] \\
& \quad \quad \quad \wedge \text{slave\_map}' = [\text{slave\_map} \text{ EXCEPT} \\
& \quad \quad \quad \quad ![s].\text{running} = @ \setminus \{k\}, \\
& \quad \quad \quad \quad ![s].\text{latest\_seq} = @ + 1] \\
& \quad \quad \quad \wedge \text{client\_state}' = \text{pushToClient}(\text{slave\_map}[s].\text{wait\_list}, \text{client\_state}) \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{pending} \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{pc} \\
& \quad \quad \quad \wedge \text{UNCHANGED } \text{client\_slave}
\end{aligned}$$

$$\begin{aligned}
& \wedge \text{UNCHANGED } client\_keys \\
& \text{IN} \\
& \quad \exists s \in Slave : do\_remove\_key(s) \\
\\
addPendingKeyUpdateSlaveMap(k) & \triangleq \\
& \text{LET} \\
& \quad s \triangleq info[k] \\
& \text{IN} \\
& \quad \wedge slave\_map' = [slave\_map \text{ EXCEPT} \\
& \quad \quad ![s].running = @ \setminus \{k\}, \\
& \quad \quad ![s].latest\_seq = @ + 1 \\
& \quad ] \\
& \quad \wedge client\_state' = pushToClient(slave\_map[s].wait\_list, client\_state) \\
\\
AddPendingKey(k) & \triangleq \\
& \wedge k \notin pending \\
& \wedge allowAction \\
& \wedge pending' = pending \cup \{k\} \\
& \wedge \text{IF } info[k] \neq nil \\
& \quad \text{THEN } addPendingKeyUpdateSlaveMap(k) \\
& \quad \text{ELSE} \\
& \quad \quad \wedge \text{UNCHANGED } slave\_map \\
& \quad \quad \wedge \text{UNCHANGED } client\_state \\
& \wedge \text{UNCHANGED } info \\
& \wedge \text{UNCHANGED } pc \\
& \wedge \text{UNCHANGED } client\_slave \\
& \wedge \text{UNCHANGED } client\_keys \\
\\
removePendingKeyUpdateSlaveMap(k) & \triangleq \\
& \text{LET} \\
& \quad s \triangleq info[k] \\
& \text{IN} \\
& \quad \wedge slave\_map' = [slave\_map \text{ EXCEPT} \\
& \quad \quad ![s].running = @ \cup \{k\}, \\
& \quad \quad ![s].latest\_seq = @ + 1 \\
& \quad ] \\
& \quad \wedge client\_state' = pushToClient(slave\_map[s].wait\_list, client\_state) \\
\\
RemovePendingKey(k) & \triangleq \\
& \wedge k \in pending \\
& \wedge allowAction \\
& \wedge pending' = pending \setminus \{k\} \\
& \wedge \text{IF } info[k] \neq nil
\end{aligned}$$

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    THEN removePendingKeyUpdateSlaveMap(k)
    ELSE
      ∧ UNCHANGED slave_map
      ∧ UNCHANGED client_state

    ∧ UNCHANGED info
    ∧ UNCHANGED pc
    ∧ UNCHANGED client_slave
    ∧ UNCHANGED client_keys

InitClient(c)  $\triangleq$ 
  LET
    s  $\triangleq$  client_slave[c]
  IN
    ∧ pc[c] = "Init"
    ∧ pc' = [pc EXCEPT ![c] = "GetRunningKeys"]
    ∧ slave_map' = [slave_map EXCEPT ![s].wait_list = @ ∪ {c}]
    ∧ UNCHANGED client_state
    ∧ UNCHANGED client_keys
    ∧ UNCHANGED info
    ∧ UNCHANGED pending
    ∧ UNCHANGED aux_vars

init_channel  $\triangleq$  [data  $\mapsto$  {}, status  $\mapsto$  "Empty"]

GetRunningKeys(c)  $\triangleq$ 
  LET
    new channel and assign to client_state
    updated_chan  $\triangleq$  [client_state EXCEPT ![c].chan = init_channel]
  IN
    ∧ pc[c] = "GetRunningKeys"
    ∧ pc' = [pc EXCEPT ![c] = "WaitOnChan"]

    ∧ UNCHANGED slave_map
    ∧ client_state' = pushToClient({c}, updated_chan)

    ∧ UNCHANGED client_keys
    ∧ UNCHANGED info
    ∧ UNCHANGED pending
    ∧ UNCHANGED aux_vars

ConsumeChan(c)  $\triangleq$ 
  ∧ pc[c] = "WaitOnChan"
  ∧ client_state[c].chan.status = "Ready"
  ∧ pc' = [pc EXCEPT ![c] = "GetRunningKeys"]

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$$\begin{aligned}
& \wedge \text{client\_state}' = [\text{client\_state} \text{ EXCEPT } ![c].\text{chan.status} = \text{"Consumed"}] \\
& \wedge \text{client\_keys}' = [\text{client\_keys} \text{ EXCEPT } ![c] = \text{client\_state}[c].\text{chan.data}] \\
& \wedge \text{UNCHANGED } \text{slave\_map} \\
& \wedge \text{UNCHANGED } \text{info} \\
& \wedge \text{UNCHANGED } \text{pending} \\
& \wedge \text{UNCHANGED } \text{aux\_vars}
\end{aligned}$$

$$\begin{aligned}
\text{clientWaitOnChan}(c) & \triangleq \\
& \wedge \text{pc}[c] = \text{"WaitOnChan"} \\
& \wedge \text{client\_state}[c].\text{chan.status} = \text{"Empty"}
\end{aligned}$$

$$\begin{aligned}
\text{TerminateCond} & \triangleq \\
& \wedge \forall c \in \text{Client} : \text{clientWaitOnChan}(c) \\
& \wedge \text{num\_action} = \text{max\_action}
\end{aligned}$$

$$\begin{aligned}
\text{Terminated} & \triangleq \\
& \wedge \text{TerminateCond} \\
& \wedge \text{UNCHANGED } \text{vars}
\end{aligned}$$

$$\begin{aligned}
\text{Next} & \triangleq \\
& \vee \exists k \in \text{Key} : \\
& \quad \vee \text{AddKey}(k) \\
& \quad \vee \text{RemoveKey}(k) \\
& \quad \vee \text{AddPendingKey}(k) \\
& \quad \vee \text{RemovePendingKey}(k) \\
& \vee \exists c \in \text{Client} : \\
& \quad \vee \text{InitClient}(c) \\
& \quad \vee \text{GetRunningKeys}(c) \\
& \quad \vee \text{ConsumeChan}(c) \\
& \vee \text{Terminated}
\end{aligned}$$

$$\text{Spec} \triangleq \text{Init} \wedge \Box[\text{Next}]_{\text{vars}}$$

$$\text{FairSpec} \triangleq \text{Spec} \wedge \text{WF}_{\text{vars}}(\text{Next})$$

$$\text{AlwaysTerminate} \triangleq \Diamond \text{TerminateCond}$$

$$\text{running\_keys}(s) \triangleq \{k \in \text{Key} : \text{info}[k] = s\} \setminus \text{pending}$$

$$\begin{aligned}
\text{ClientKeysMatchSharedState} & \triangleq \\
& \forall c \in \text{Client} : \\
& \quad \text{clientWaitOnChan}(c) \Rightarrow \\
& \quad \wedge \text{client\_keys}[c] = \text{running\_keys}(\text{client\_slave}[c])
\end{aligned}$$

$$\text{SlaveMapRunningMatchSharedState} \triangleq$$

$$\forall s \in Slave : \\ slave\_map[s].running = running\_keys(s)$$

$$channelAction(c) \triangleq \\ \begin{aligned} & \vee \wedge client\_state[c].chan.status = \text{"Consumed"} \\ & \quad \wedge client\_state'[c].chan.status = \text{"Empty"} \\ & \vee \wedge client\_state[c].chan.status = \text{"Consumed"} \\ & \quad \wedge client\_state'[c].chan.status = \text{"Ready"} \\ & \vee \wedge client\_state[c].chan.status = \text{"Empty"} \\ & \quad \wedge client\_state'[c].chan.status = \text{"Ready"} \\ & \vee \wedge client\_state[c].chan.status = \text{"Ready"} \\ & \quad \wedge client\_state'[c].chan.status = \text{"Consumed"} \\ & \vee client\_state'[c].chan = client\_state[c].chan \end{aligned}$$

$$allChannelAction \triangleq \\ \forall c \in Client : channelAction(c)$$

$$ChannelSpec \triangleq \\ \Box[allChannelAction]_{client\_state}$$

$$ReadyAlwaysConsumed \triangleq \\ \begin{aligned} & \forall c \in Client : \\ & \quad client\_state[c].chan.status = \text{"Ready"} \\ & \quad \leadsto client\_state[c].chan.status = \text{"Consumed"} \end{aligned}$$

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