Problem 4.

a. $f(\lambda x+(1-\lambda)y) = (\lambda x+(1-\lambda)y)^2$

$= \lambda^2 x^2 + (1-\lambda)^2 y^2 + 2\lambda(1-\lambda)xy$

to prove $f(x)$ is convex, equals.

to prove $\lambda x^2+(1-\lambda)y^2-(\lambda x+(1-\lambda)y)^2 \geqslant 0$

left $= \lambda(1-\lambda)(x-y)^2 \geqslant 0$

So, when $0<\lambda<1$, it holds, $f(x)=x^2$ is convex

b. let $x=-2$ $y=1$ $\lambda=\frac{1}{2}$

left $= f(\frac{1}{2}\times(-2)+\frac{1}{2}\times 1) = f(\frac{1}{2}) = -0.125$

right $= \frac{1}{2}(-8)+\frac{1}{2}(1) = -3.5$ $\quad -0.125 > -3.5$

The inequality doesn't hold, $f(x)=x^3$ is not convex