Quanser Qube 2 – Report

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# Introduction

In the following chapters we’ll resume our work on the project of the Inverter Rotary Pendulum Quanser Cube 2 by explaining how did we approach to it, starting from the mechanical equations, to a state space model used in the simulations, and finally to the actual control of it. An important aspect was the parameter estimation, including Frictions, Disturbances and others.

Once a good model was found we could go through different control scheme, based on output feedback and state feedback.

# Equations of motion

In this section we will compute the equations of motion of the system, starting from the mechanical part and then adding the DC motor effects. We’ll use the Lagrange theory.

## Lagrange Equations

A simplified representation of the QUBE-Servo 2 is shown below, along with some necessary notation :

* Immagine che contiene schizzo, diagramma, design, disegno

  Descrizione generata automaticamente**Θ** : Motor angle (Positive clockwise)
* **Φ**: Pendulum angle (Positive counterclockwise and null for the pendulum pointing
* **Jr** : Rod moment of inertia with respect to its rotation axis
* **Lr** : Rod Lenght
* **Jp** : Pendulum moment of inertia with respect to its centre of mass
* **Lp** : Pendulum lenght
* **lp** : Distance of the pendulum centre of mass from the rotating axis
* **mp** : Pendulum mass

Fig 16 : Model

With “rod” we indicate the element of the QUBE-Servo rigidly connected to the motor shaft that holds the pendulum. The Position of the origin of the fixed reference frame and its axes orientation are shown in the drawing above.

According to Lagrange method, the equations of motion of the system can be written as (1.1.1):

(1.1.1)

Where represents our state space variables (1.1.2)

(1.1.2)

Since the damping energy can be neglected (1.1.3)

(1.1.3)

We can rewrite the Lagrange formula after simplifications by writing an equation for each state and considering L as the difference between kinetic and potential energy. (1.1.4)

(1.1.4)

We can rewrite the Lagrange equations as (1.1.5)

(1.1.5)

Where the total kinetic energy will be given by the sum of the kinetic energy of the pendulum and the kinetic energy of the rod (1.1.6)

(1.1.6)

where represent pendulum’s center of mass velocity, where its contributions on the 3 axes may be written as (1.1.7)

(1.1.7)

where each velocity component can be described based on the derivatives of the coordinates of the center of mass of the pendulum (1.1.8)

(1.1.8)

which can be explicitly calculated in Chapter 1.2 Reference System. Regarding the potential energy of the system, it will only include the contribution of the pendulum. (1.1.9)

(1.1.9)

Now considering that we will control a torque applied to a motor directly connected to the rod, with no external work applied to the pendulum, we can express (1.1.10)

(1.1.10)

By solving the system in equation (1.1.2), we can find the two equations that fully describe the dynamics of our system (1.1.11)

(1.1.11)

Which can be also expressed as equation (1.1.12)

(1.1.12)

With the following matrices (1.1.13)

(1.1.13)

The torque has to be substituted with the actual torque equation that can be found at chapter 1.3

## Reference System

The Reference System can be written as in the following system of equations (1.2.1)

(1.2.1)

We can see from the two images below (Fig 17 and Fig 18) how we derived the system of equations (1.2.1). We have denoted the position of the CoM on all three axes relative to the origin of the Cartesian coordinates as , and .

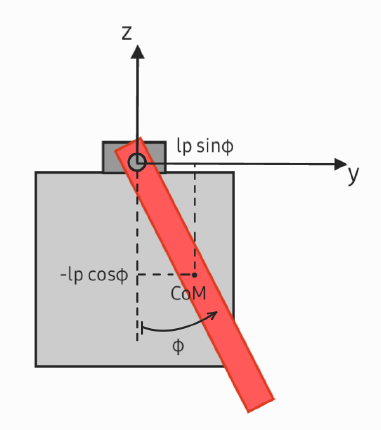
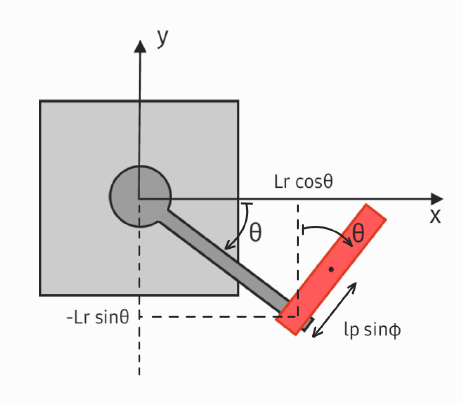
 

Fig 17 : ZY Plane Fig 18 : XY Plane

## DC Motor

Torque is generated by the DC permanent magnets motor according to the following electrical dynamic equation (1.3.1)

(1.3.1)

Following the necessary notation :

* : armature voltage
* : armature current
* : armature resistance
* : armature inductance
* E : back electromotive force
* : back-emf constant
* : torque constant (it holds )

As it will be clear in the next section where the parameter values are introduced, the armature inductance is small enough so that the term can be reasonably neglected (in other words, the motor electrical time constant is small).

By doing so we are neglecting the DC motor electrical dynamics, and as a consequence the electrical torque can be expressed as in equation (1.3.2)

(1.3.2)

There is no need to introduce the motor mechanical dynamic equation as the motor is directly coupled to the rod without transmission gears and the motor moment of inertia is already accounted inside , the rod moment of inertia (which, as already stated, is considered with respect to the motor rotating axis).

According to matrices (1.1.13) we can conclude the notation by substituting equation (1.3.2) into torque vector, while all the others matrices remain the same (1.3.3)

(1.3.3)

# Model Identification

In this section the value of the various parameters appearing in the equations of motion is introduced, validating what is provided by the QUBE-Servo 2 datasheet with appropriate experiments. Also, to ensure a better correspondence between simulation and the real system, new phenomena like friction are introduced and modelled.

## Directly Measurable parameters

First, the parameters that could be directly measured were determined. Those are :

After some simple measurements, the values on the datasheet resulted accurate, thus we settled with :



## DC Motor Parameters

The DC motor dynamic equations, with no loads connected to its shaft, are (2.2.1) :

(2.2.1)

The various parameters, according to the datasheet, have the following values :



It is now clear why the motor electrical dynamics can be negletted, as we did in the inverted pendulum equations of motion, since the electrical time constant is small enough. (2.2.2)

(2.2.2)

Also, the motor inertia takes into account both the rotor inertia and the motor hub inertia ), being the motor hub the cylindrical component fixed to the motor shaft that allows the magnetic connection with the rod.

Now, a series of experiments were carried out in order to compute the motor parameters comparing the results with the datasheet values.

In order to to compute the armature resistance , constant imput voltages were applied (Fig 1) to the motor while keeping the rotor fixed, and the resulting current were measured (Fig 2).

Immagine che contiene testo, linea, Diagramma, diagramma

Descrizione generata automaticamente

Fig 1

Immagine che contiene linea, testo, diagramma, Diagramma

Descrizione generata automaticamente

Fig 2

The voltage current relation is then (2.2.3) :

(2.2.3)

As the current measurement showed possibly the presence of an offset, and to reduce the effect of other sources of measurement errors, the currents and the corresponding voltages are plotted and the resulting points are fitted with a linear function.

The angular coefficient of the obtained linear relationship will correspond to the estimated armature resistance. (Fig 3, Fig 4, Fig 5)

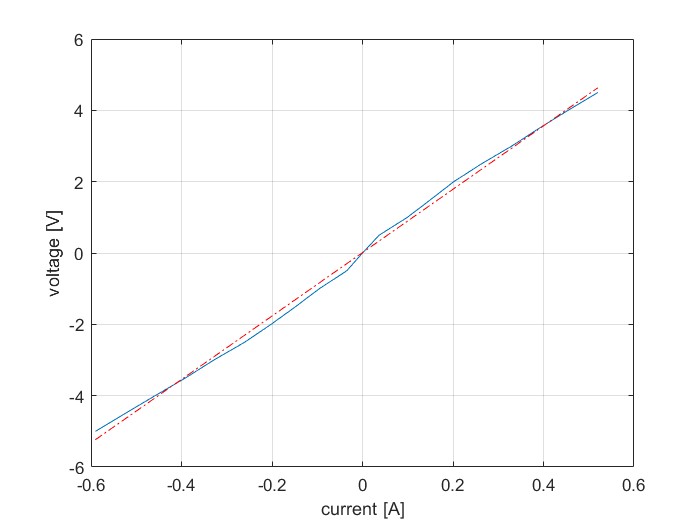


Fig 3 : Ω = 8.8

Immagine che contiene linea, Diagramma, testo, diagramma

Descrizione generata automaticamente

Fig 4 : Ω = 8.3

Immagine che contiene linea, testo, Diagramma, diagramma

Descrizione generata automaticamente

Fig 5 : Ω = 8.1

Depending whether the whole data points or only positive/negative values are considered, the resulting appears to be greater or smaller with respect to the datasheet value. This is probably due to non ideal behaviours of the current sensor and of the PWM voltage generator.

To avoid an arbitrary choice, we decided to keep the datasheet value which proves to be an acceptable compromise between the estimated values.

Next there are the motor constant and , which have the same value and therefore can both be estimated with a single experiment. Theoretically, by providing a constant voltage to the motor, at steady state the current and speed can be measured and can be computed as described in equation (2.2.4).

(2.2.4)

However, due to the aforementioned non-idealities of the measuring setup, this formula does not lead to acceptable result. So, for the moment, we will keep the datasheet value of these constants.

Next, at steady state the motor does not reach exactly the expected speeds, and a current different from zero flows through its windings. This suggests the presence of a friction torque acting on the motor. To model this phenomenon, we are going to use the classical Coulomb and viscous friction model (Fig 6).

Immagine che contiene linea, diagramma, testo, Diagramma

Descrizione generata automaticamente

Fig 6

To characterize the friction curve, a “ladder-like” input voltage is set as the motor input, from -7V up to 7V increasing of 0.1V each second. The resulting steady state current and speed are measured, which allows to compute the friction torque with the formula (2.2.5)

(2.2.5)

And to plot the speed-torque curve (Fig 7, Fig 8, Fig 9, Fig 10) :

Immagine che contiene linea, testo, Diagramma, diagramma

Descrizione generata automaticamente

Fig 7

Immagine che contiene linea, testo, Diagramma, diagramma

Descrizione generata automaticamente

Fig 8

Immagine che contiene testo, linea, schermata, Diagramma

Descrizione generata automaticamente

Fig 9

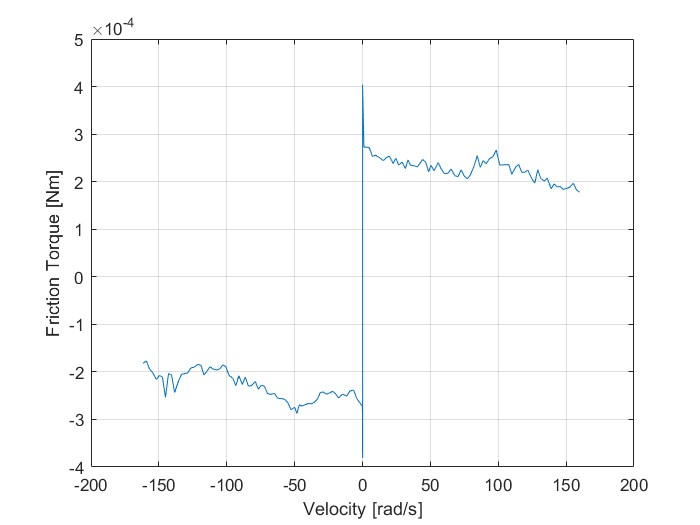


Fig 10

The resulting friction torque characteristic, after having removed the offset due to the current sensor, mostly matches the theoretical one, though no viscous friction effect is apparently visible. Thus, our friction model will only consider static friction, and it will be implemented in the Simulink version of the model using a lookup table block, which allows a very simple implementation and avoids introducing the discontinuity at zero speed. Notice that the Coulomb friction value we selected is 2.5 [N\*m].

It's worth mentioning that during these experiments we noticed a different behavior of the motor depending on the direction of rotation, meaning that for equal but opposite in sign constant input voltages the motor reached different steady state velocities. More specifically, with negative input voltages the motor reaches lower velocities with respect to positive input voltages. This suggests either a slightly different behaviour of the PWM voltage generator depending on the voltage sign, or, more likely, a non symmetric torque characteristic, maybe caused by wear of the DC motor brushes-collector system. This should be visible from the friction experiment, but the current sensor offset, which proved to be quite unpredictable, hid such phenomenon.

Now that friction has been modelled, by comparing the simulated motor with the real system in terms of steady state velocities, it can be noticed a good match at low speeds, yet a gap becomes more and more evident as the velocity increases.

As the friction torque characteristic suggested, the system is not affected by any noticeable viscous friction, thus it’s very likely that the cause of this mismatch is related to an incorrect value of and . But, since the available measuring devices do not allow a precise estimation of those parameters, we will keep and introduce an “equivalent damping effect”, meaning we add a fictitious viscous damping to the system so that the DC motor model matches the real measured steady state velocities. To tune this damping coefficient, we used the Simulink “Parameter Estimator”, which allows, given a Simulink dynamical model, a signal of such model, and a set of measured data, to estimate the chosen model parameters that minimize the distance between the measured data and the selected signal (the mathematical procedure is based on least squared method).

So, by applying this to the DC motor Simulink model fed by a “ladder-like” imput voltage, choosing as a signal the motor velocity and as dataset the measured velocities from the real system, the resulting damping coefficient is (2.2.6)

(2.2.6)

Immagine che contiene testo, Diagramma, linea, diagramma

Descrizione generata automaticamente

Fig 11 : No Damping

Immagine che contiene testo, linea, Diagramma, diagramma

Descrizione generata automaticamente

Fig 12 : With Damping

Finally, as the transient of the DC motor model across the previous experiments matched the transient of the real system fairly well, the motor inertia provided by the datasheet proved to be accurate.

## Rod Inertia Identification

The rod without the pendulum attached is connected to the motor hub. The equations of motion of the motor do not change, besides of an increased inertia that now is equal to (2.3.1)

(2.3.1)

Being  the total rod inertia, the one used in the equations of the complete inverted pendulum model, the motor inertia as described in the previous section and the inertia of the rod component only.

After having implemented the Simulink model to this setup, comprising of Coulomb and equivalent viscous friction, various constant voltages are applied to the real system and the resulting velocity response is measured. Then, using Simulink’s Parameter Estimator the value of the rod inertia is estimated to best follow the measured response, obtaining (2.3.2) and showed in Fig 13.

(2.3.2)

Immagine che contiene testo, linea, Diagramma, diagramma

Descrizione generata automaticamente

Fig 13

## Pendulum Parameters

The main parameter associated to the pendulum that has not yet been determined is its moment of inertia. This, given the pendulum mass and its length, can be computed as (2.4.1)

(2.4.1)

To better represent the real pendulum motion, viscous friction is added to model the damped oscillations. The dynamic equation of the pendulum, so when the rod is kept fixed and only the pendulum is allowed to move, is described in equation (2.4.2)

(2.4.2)

Where is the pendulum damping coefficient. To approximately estimate this parameter, first the free motion of the pendulum for different initial positions was measured. Then, the peaks of the measured damped oscillations were extracted from the dataset, and were fitted with a negative exponential function. The exponent of the resulting function, averaged over the multiple experiments, can be considered as the pendulum viscous friction coefficient (2.4.3)

[N\*m\*s/rad] (2.4.3)

Immagine che contiene linea, tipografia

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Fig 14 : Real vs No Damping

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Descrizione generata automaticamente

Fig 15 : Real vs Damping

As it can be seen from (Fig 14, Fig 15) the simulated pendulum motion compared to the real system, the oscillations frequency is still slightly different, but the oscillations damping is represented well enough.

## Encoder Cable Effect

The rotary encoder that measures the pendulum rotation angle is fixed to the rod and is connected to the body of the QUBE-Servo via cable. Such cable affects the motion of the system, acting as a sort of rotational spring around the motor angle , as it tends to bring the rod back to its initial position.

This means that the cable effect can be introduced into the model by adding to the first equation of motion a term proportional to the rod angle , whose coefficient would represent an equivalent elastic coefficient.

However, setting up an experiment to characterize such a parameter proved to be tricky as the effect associated to the cable tends to be inconsistent and unpredictable.

For this reason, in order to still include this phenomenon into the model without arbitrarily choosing an elastic coefficient, it will be added to the Simulink version of the dynamic model as a direct “feedback” over of the angle .

Further details will be given as soon as the Simulink model is introduced in the following chapters.

# Complete Model

In this chapter, we can summarize everything discussed in the previous chapters. Starting with a complete nonlinear model written in the Lagrange form (1.1.13) and after identifying all the parameters, we aim to obtain a linearized model. This model will be used both for the controls to be discussed in the next chapter and to have a complete model for conducting analyses and simulations that are as realistic and reliable as possible compared to the real system.

## Non-Linear Model

Immagine che contiene testo, schermata, Carattere, diagramma

Descrizione generata automaticamenteBefore starting with the actual linearization, it is important to have a complete nonlinear model as seen in the previous chapters and transcribe it into Simulink block form, so that we can simulate the nonlinear model (i.e., the real one) when building controllers that will act on the real system.

The linearized model will primarily be used to design controllers and observers that are clearly based on a linear system described by matrices A, B, C, and D.

In Fig. 19, the previously described block is shown, and as we can see, it has an input that represents the voltage input to the motor, and four outputs that describe the arbitrarily chosen states of the system, see chapter 3.2 for more, while in Fig. 20 we can see the internal structure of what is shown in Fig. 19

Immagine che contiene testo, diagramma, Piano, Disegno tecnico

Descrizione generata automaticamente

Fig 20 : Exploited Fig 19

## Linearized Model

Using the Taylor series expansion, it is possible to linearize a nonlinear system around the equilibrium point of our system. Clearly, in the case of a pendulum, there are two equilibrium points:

* , Asymptotically stable equilibrium like in the Fig 16
* [Rad] , Instable equilibrium, Pendulum upwards

As mentioned in Chapter 3.1, we have chosen the position and velocity of both angles and as state variables. For convenience, we describe them within a vector as shown in (3.2.1)

(3.2.1)

The two equilibrium points can be described as well as (3.2.2)

(3.2.2)

Let’s now use Taylor series and for this purpose we need to specify a generic function as follow (3.2.3)

(3.2.3)

As seen in the two equations (1.1.11), in our case, we need to linearize both equations around both equilibrium points, thereby obtaining two linearized systems: one for the asymptotically stable equilibrium and one for the unstable equilibrium. Thus, using the well-known Taylor series expansion (3.2.4)

Using this equation and performing some tedious mathematical steps, we can then derive the two linearized equations at both equilibrium points. At the stable equilibrium position (for example), we have a new system of equations described as: (3.2.4)

(3.2.4)

M =

which, after various simplifications, can finally be described as (3.2.5)

(3.2.5)

We can now derive the transfer function G(s), which is described as (3.2.6)

(3.2.6)

Now we can perform all the analyses on the linearized model, including Bode and Nyquist plots, and particularly obtain the eigenvalues to verify if the previously mentioned stability of the two equilibrium points is consistent. In the results (3.2.7), we can see on the left the eigenvalues of the system linearized at the stable point, and on the right, those linearized at the unstable point.

(3.2.7)

# Controllers and state estimation

In this chapter, we will introduce the concepts purely related to the control of our system, after having previously discussed the nonlinear equations that describe the real system and the parameter estimation that will be used in the latter. Specifically, we will first introduce the model that we will use for the simulations and its relative linearization. Then, we will move on to the actual control systems, both based on output feedback and state feedback, and always comparing what we expected from the simulation and what we obtain from the real system.

The goal is to control the pendulum’s rod at the upside position (180°) and bring it from the stable position to the unstable one, using two different type of controllers, one called Swing-up and the other to Stabilize once it reach the upright position.

## Output Feedback Controller (PI)

As classic Control there is the PID and we proceed by using two way to find a proper combination of the gains: one way is by using the matlab function “pidtuneOptions” which provides you the gains after you put inside as input the desired Phase Margin and cut-off Frequency (wc) and the other way is by trial and error. In the second way we start by setting all the gain equal to 0, then changing the Proportionl Gain and se how would be the Step Response of the Syst, and then changing also the Integral one and after the Derivative one. At the end we tried to fix the combination as the one we like the most in terms of Settling Time and Overshoot for both θ (rod angle) and ϕ (pendulum bar angle).

Since we couldn’t find a proper combination of Gain for the PID to control the bar pendulum at the unstable position, we use to implement the PID Control only for the Stable position, bar at downside position.

From the State Space equation we can derive two Transfer Funtions, one from the Input Voltage to the ϕ angle and the other one is from the Input Voltage to the ϕ angle. Combining these two transfer functions we can derive a new one which provides the relation from the θ angle to the ϕ angle.

“Equations”

Immagine che contiene diagramma, schermata, linea, Carattere

Descrizione generata automaticamenteIn this way we can design a Control for the Gθ and one for Gϕ separately, the first one will control the θ angle and the second one will act to mantain the ϕ angle at the desired position, in our case to be at the downside (0°).

“Add text to specify the Gθ, Gϕ”

The Control for θ is at the inner loop, we must be sure that it is faster than the outer loop, the one controlling the ϕ angle when designing each control. (“I don’t kknow if in this case is should be or they can have dufferent time”)



Immagine che contiene testo, schermata, diagramma, linea

Descrizione generata automaticamenteImmagine che contiene testo, schermata, diagramma, Carattere

Descrizione generata automaticamente

Looking at the Step Response of the θ angle it seems to be very fast, setting time of about 0.15s

Immagine che contiene testo, diagramma, Diagramma, schermata

Descrizione generata automaticamente

Immagine che contiene testo, diagramma, linea, Diagramma

Descrizione generata automaticamenteThe settling time for ϕ due to a step seems to be around 1s

Immagine che contiene testo, linea, diagramma, Diagramma

Descrizione generata automaticamenteImmagine che contiene testo, schermata, diagramma, linea

Descrizione generata automaticamente“gains for theta”

Immagine che contiene testo, linea, diagramma, schermata

Descrizione generata automaticamente“gains for phi”

Immagine che contiene testo, schermata, diagramma, Diagramma

Descrizione generata automaticamente

The settling time seems to be around 1.2s

The final block scheme (the one used in simulink) is the following:

Immagine che contiene schermata, linea, diagramma, testo

Descrizione generata automaticamente

“add text to specify that the phi and theta are in the feedback”

To see how fast it takes to the bar pendulum to stabilize at downside position and to be steady after a perturbation on it, we hit the bar with a finger and this could be modeled in the simulink block as an impulse along the phi feedback.

Immagine che contiene diagramma, schermata, linea

Descrizione generata automaticamente

“add text to indicate the dist on phi”

Now we can observe how is the response of the system due to a perturbation on ϕ of 15° which last 0.05s

Immagine che contiene schermata, testo, Diagramma, linea

Descrizione generata automaticamenteWe can observe that the PID designed using the 1st way is faster but the variation on phi is higher, instead with the second one the ϕ angle is oscillating more but small variation.

The measurment of the ϕ angle from the real model is confermating the behavior of the simulated model:

* Ts = 1s for the 1st one
* Ts = 1.5s for the 2nd one

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamenteThe particlar difference between the two version of PID is that in the 1st one the action of the control variable is more agressive, that’s why we have a quick oscillation of ϕ after the perturbation. With the 2nd one, instead we don’t see any of these small oscillations.

Immagine che contiene schermata, testo

Descrizione generata automaticamenteComparison between the simulation behavior and the real behavior of the bar pendulum using the 1st design procedure.

Comparison between the simulation behavior and the real behavior of the bar pendulum using the 2nd Immagine che contiene schermata, testo, Diagramma, Software per la grafica

Descrizione generata automaticamentedesign procedure.

The quick small oscillation of the ϕ with the 1st way PID design procedure can be observe also by looking at the Voltage that should enter the DC Motor

Immagine che contiene schermata, testo

Descrizione generata automaticamenteAfter the perturbation at time 5 the Input Voltage starts to oscillate very fast between the limit we add fo the voltage that enters the DC Motor. This is not observe with the 2nd way PID design procedure.

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamente

We could already predict these beahvior by looking at the simulation. The voltage variation due to the perturbation is higher, quick and more socillations for the 1st strategy.

Mind also that the Voltage shows up here is the ideal one limited by a saturation, we are assuming that the Voltage providing from the Power Supply is fast enough to follow the Voltage we request. We might requesting something faster than how the Power supply is actually fast.Immagine che contiene schermata, Software per la grafica, Software multimediale

Descrizione generata automaticamente

These oscillation are due to the voltage oscillation and so an oscillation also for the θ angle, infact the quick small oscillation causes a slow movement for the rod

Robustness test

What about if some of the parameters are wrong or we didn’t consider them:

* Ir = formula
* α = 0
* β = 0

basically we ignore the presence of the encoder on the rod so the Inertia is due only to the rod and we completely ignore the presence of damping for the bar pendulum and for the DC motor.

Immagine che contiene schermata, testo

Descrizione generata automaticamenteBy changing the parameter only for the 2nd design prosedure we observe as result more higher oscillation and so longer settling time, but at the end the bar pendulum reach a steady downside position.

Trajectory tracking control

One thing to try is also to put a reference on the θ angle, like a sinusoidal signal of 45°with a Period of 20s (f=0.05Hz), and see if the system is able to follow the reference and at the same time keeping the bar at a steady position at 0°.

As we observed is that the Control system is able to track the trajectory while keeping the bar at the downside position, but the amplitude of the the sinusoidal beahavior of the real system is lower than the 45° of the reference.

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamenteImmagine che contiene schermata, Diagramma, linea, diagramma

Descrizione generata automaticamente

Immagine che contiene schermata

Descrizione generata automaticamenteIncreasing the frequency like at the cut-off frequency: f = 0.8HZ

Immagine che contiene schermata, testo

Descrizione generata automaticamente

The Control System is still able to follow the trajectory for θ, but the amplitude of the variation is lower, less than 10°, the bar pendulum now is also oscillating since we are at a frequency where the attenuation is enough to see this beahavior.

Immagine che contiene schermata, Policromia

Descrizione generata automaticamenteIncreasing more the frequency, beyond the cut-off frequency, at f = 8Hz:

Immagine che contiene schermata, testo, Software per la grafica

Descrizione generata automaticamente

Both rod and bar pendulum are quickly oscillating, the rod even with a lower amplitude

## State Feedback Controllers (PP-LQR)

LQR

“some theory”

For the matrices Q and R to set for the LQR implementation, we’ve chosen the parameter such that the states would reach 0 as soon as possible and the Control system wouldn’t use too much control variable: the q >> r

Control at the Unstable Position

After some trials, changing the parameters q and r, with the real model, we’ve found the proper matrices:

[qθ=10 qθ\_dot=1 qϕ=10 qϕ\_dot=10] and [r=1] “fix this visualization lol”

The qs related to the states of the bar should be higher if we want them to reach the equilibrium quickly (ϕ=180° and ϕ\_dot=0), instead the qs related to the states of the rod can be lower like q=1 and tis means they can move due to a perturbation and return to the equilibrium point slower than the bar, the important thing is that the bar is kept upside. What we notice when fixing the parameters is that the rod is not reaching the equilibrium point (at 0° position), maybe because while trying it the bar is not that steady, so the rod tries to reach new static position everytime and as a result the rod is not steady. To solve this solution we set the value for θ the same as for ϕ the .

For the Control Variable we initially set it to a very low value, r=0.01, since we wanted the qs much higher as we wanted the state to reach the equilibrium faster rather than penalizing the control variable. This, tho, causes the Control variable to be agressive (fast and high variation of the Voltage input), the response to a disturbance was very good tho, very fast to keep the bar at the vertical position. The problem was that the rod wasn’t steady, it was keeping to move either one side o the otherside depending on the perturbation on the bar. So as final value we choose for the Control variable is r=1.

In this phase of finding the proper values for the matrices we observed that:

* higher value for the qs causes a strong action, to solve this a higher value for r is needed
* the couple qs for each angle should not be that so differenti, even intuitively
* if you want the rod to not randomly walk and also hit the end stop, you should choose the same value as for the bar pendulum

As result we obtain the following verctor containing the gains: [-3.1623 -1.9351 -53.2574 -5.6413]

“image of how the scheme should be, the statefeedback one”

Disturb on ϕ

POLE PLACEMENT

Pole Placement Control is still a feedback Control logic implementation, by using the states as feedback we generate the control variable to enter the syst…

The Control variable is proportional to the States through a proper gain K and this is calculated through … by setting the poles we desired . Mind that the Poles influenced a lot the Response of the syst in terms of Settling time, overshoot and possible oscillations.

Immagine che contiene diagramma, linea, schermata, Piano

Descrizione generata automaticamente

Immagine che contiene testo, Carattere, schermata, design

Descrizione generata automaticamenteBy looking at the Linearized System Poles at the downside position we decided to remove the pole at the origin and also sustitute the Complex Coniugate poles with real poles. In addition, we took as a starting poles the poles we’ve found with LQR at downside position and tried to add a small variation in some poles which are influencing the system response. …

LQR poles: [-1 -3.47+i8.45 -3.47-i8.45 -153.4]

[-1 -3.5 -4 -150]: as a starting point, we observed that the response is a little bit noisy, the rod was not that steady and so also the bar pendulum, the reaction was still good tho, fast to bring the bar at the downside position after a disturbance on it.

[-2 -5.5 -7 -150]: with these poles the response was more steady, not noisy, and the rod have a better reaction againts the disturance on the bar pendulum.

Increasing more the poles will just make the response very noisy, infact a guide lines to choose the poles is to not choose them very far away from the Openloop poles, otherwise this will lead a high control effort, and not very negative thinking a very fast reacting system is alwasy better, in frequency domain this means having a very large Band width which leads to noise amplification.

The gain obtained: 2.9151 2.3769 55.7466 2.3375

Immagine che contiene testo, diagramma, linea, Diagramma

Descrizione generata automaticamenteImmagine che contiene testo, linea, schermata, Carattere

Descrizione generata automaticamente

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamenteFrom the matlab code simulation the Settling time seems to be around 1.5s and the variation of ϕ to be small.

Looking at the Mathematical model simulation the response seems to assest more or less after 1.5s due to a perturbation on the ϕ angle of 15° for 0.05s.

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamenteComparing the behavior of the real system the Assestment time is the same, but the rod, after the perturbation, is not so steady causing some small oscillation at the bar pendulum.

Immagine che contiene schermata, testo, Diagramma, diagramma

Descrizione generata automaticamenteThe rod is not steady, but at least is fast to recover after the perturbation

Immagine che contiene testo, schermata

Descrizione generata automaticamente

Immagine che contiene schermata, testo, Software per la grafica, Software multimediale

Descrizione generata automaticamenteAs usual we can assume the oscillation by looking at the Voltage reference behavior, in simulation the variation is high and very fast and in the real system is even more worst.

Robustness Test

Now let’s see if the Control System is robust due to a variation in the parameters:

* Ir = formula
* α = 0
* Immagine che contiene schermata, testo

  Descrizione generata automaticamenteβ = 0

With respect to the nominal parameters the behavior seems more or less the same, maybe there are just more oscillations.

We can observe the difference in the Voltage variation and the rod variation as usual.

Immagine che contiene schermata, testo, Software multimediale, Software per la grafica

Descrizione generata automaticamenteImmagine che contiene schermata, Diagramma

Descrizione generata automaticamente

Trajectory Tracking Control

Immagine che contiene schermata, testo

Descrizione generata automaticamenteLet’s test the behavior of the Control System by adding a Trajectory reference for the θ:

Immagine che contiene schermata, testo, Diagramma, linea

Descrizione generata automaticamente

As we observed is that the Control System was able to keep the bar pendulum at the downside position meanwhile following the reference for θ.

The amplitude of the sinusoidal behavior of the real system is not exactly the same of the reference, like it has a smaller amplitude but the period is the same, this is due probably to the fact that the Control system give priority to the control of ϕ rather than moving the rod.

Immagine che contiene schermata, testo

Descrizione generata automaticamenteWith f = 0.26, at the cut-off frequency:

Immagine che contiene schermata, testo

Descrizione generata automaticamenteAs predicted the amplitude of the variation is lower, but the bar is still kept at the downside position, with a small oscillation.

With f = 2.6Hz, beyond the cut-off frequency:

Immagine che contiene schermata, Software multimediale, Software per la grafica

Descrizione generata automaticamenteImmagine che contiene schermata, testo, Software per la grafica, Diagramma

Descrizione generata automaticamente

The rod tried to follow the reference, but since it’s too fast, it’s not moving, just small fast oscillation.

Control at the Unstable position

Immagine che contiene testo, Carattere, schermata, design

Descrizione generata automaticamente In the Unstable (upside) position the these are the Open loop poles fo the system, there is only on eunstable poles. For Pole Placement it’s better to remove it and also to remove the pole at the origin (as observed in the real syst this pole won’t be able to keep the rod at steady position, but it will move randomly untili t hit the end stop).

Taking also into account of the LQR poles at the upside position, we choose the poles accordingly and slightly changed them:

LQR poles: [-3.09 -4.64+i3.67 -4.64-i367 -115.67]

Starting poles: [-0.8 -0.9 15 115]

With these poles the response was noisy, the rod is not that steady and so the bar pendulum, even tho the bar is kept at the unstable position, the variation of the rod angle can be high enough to hit the end stop, due to a strong pertutbation, and causing the bar to fall down.

[-5 -5.2 -15 -115]: increasing the first two poles the response was still noisy, but the rod was very responsive againts a perturbation, the control will immediatly act due to a perturbation on the bar so the rod won’t move too much.

[-2 -2.2 -15 -115]: final poles, less noisy to the previous ones, still the rod is very responsive.

Immagine che contiene testo, linea, diagramma, Diagramma

Descrizione generata automaticamenteThe gains obtained: -1.9156 -2.0188 -74.6922 -5.9023

Immagine che contiene testo, linea, schermata, diagramma

Descrizione generata automaticamenteTesting the Step response with the matlab code it seems that the Settling time it would be around 2.5s, but the variation is very small so we can consider the bar to be more or less stable already before that time.

Againts a perturbation on the bar pendulum angle (ϕ): impulse of 15° for about 0.05s

Immagine che contiene schermata, testo, Diagramma, linea

Descrizione generata automaticamenteAs predicted the bar pendulum is able to reach the unstable position after 1s already with the mathematical model, instead with the real system the response is a bit noisy, there is a random variation of about 2° on ϕ. The real system is not that steady, but is fast to recover from the perturbation.

Immagine che contiene schermata, Diagramma, linea, testo

Descrizione generata automaticamenteThe rod is not stable and as usual this unstability can be observe on the Voltage referance which variating very fast and reaching the limit of the Voltage input.

Immagine che contiene schermata, Software multimediale, Software per la grafica, Modifica

Descrizione generata automaticamenteImmagine che contiene schermata, testo

Descrizione generata automaticamente

Robustness Test

Now let’s see if the Control System is robust due to a variation in the parameters:

* Ir = formula
* α = 0
* Immagine che contiene schermata, testo, Diagramma, Software per la grafica

  Descrizione generata automaticamenteβ = 0

The behavior is more or less the same.

Trajectory Tracking Control

Immagine che contiene schermata, Diagramma, testo, linea

Descrizione generata automaticamenteThe Control is able to track the sinusoidal signal of amplitude 45° and period T=20s and also to keep the bar at the vertical position.

Immagine che contiene schermata, testo

Descrizione generata automaticamenteThe bar is not that steady, buti t is kept at the upside position

## State Observer (KF-PP)

## Swing-up Controller (PP)

## Swing-up Controller (Energy Based)

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