PID

As classic Control there is the PID and we proceed by using two way to find a proper combination of the gains: one way is by using the matlab function “pidtuneOptions” which provides you the gains after you put inside as input the desired Phase Margin and cut-off Frequency (wc) and the other way is by trial and error. In the second way we start by setting all the gain equal to 0, then changing the Proportionl Gain and se how would be the Step Response of the Syst, and then changing also the Integral one and after the Derivative one. At the end we tried to fix the combination as the one we like the most in terms of Settling Time and Overshoot for both θ (rod angle) and ϕ (pendulum bar angle).

Since we couldn’t find a proper combination of Gain for the PID to control the bar pendulum at the unstable position, we use to implement the PID Control only for the Stable position, bar at downside position.

From the State Space equation we can derive two Transfer Funtions, one from the Input Voltage to the ϕ angle and the other one is from the Input Voltage to the ϕ angle. Combining these two transfer funtions we can derive a new one which provides the relation from the θ angle to the ϕ angle.

“Equations”

Immagine che contiene diagramma, schermata, linea, Carattere

Descrizione generata automaticamenteIn this way we can design a Control for the Gθ and one for Gϕ separately, the first one will control the θ angle and the second one will act to mantain the ϕ angle at the desired position, in our case to be at the downside (0°).

“Add text to specify the Gθ, Gϕ”

The Control for θ is at the inner loop, we must be sure that it is faster than the outer loop, the one controlling the ϕ angle when designing each control. (“I don’t kknow if in this case is should be or they can have dufferent time”)



Immagine che contiene testo, schermata, diagramma, linea

Descrizione generata automaticamenteImmagine che contiene testo, schermata, diagramma, Carattere

Descrizione generata automaticamente

Looking at the Step Response of the θ angle it seems to be very fast, setting time of about 0.15s

Immagine che contiene testo, diagramma, Diagramma, schermata

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Immagine che contiene testo, diagramma, linea, Diagramma

Descrizione generata automaticamenteThe settling time for ϕ due to a step seems to be around 1s

Immagine che contiene testo, linea, diagramma, Diagramma

Descrizione generata automaticamenteImmagine che contiene testo, schermata, diagramma, linea

Descrizione generata automaticamente“gains for theta”

Immagine che contiene testo, linea, diagramma, schermata

Descrizione generata automaticamente“gains for phi”

Immagine che contiene testo, schermata, diagramma, Diagramma

Descrizione generata automaticamente

The settling time seems to be around 1.2s

The final block scheme (the one used in simulink) is the following:

Immagine che contiene schermata, linea, diagramma, testo

Descrizione generata automaticamente

“add text to specify that the phi and theta are in the feedback”

To see how fast it takes to the bar pendulum to stabilize at downside position and to be steady after a perturbation on it, we hit the bar with a finger and this could be modeled in the simulink block as an impulse along the phi feedback.

Immagine che contiene diagramma, schermata, linea

Descrizione generata automaticamente

“add text to indicate the dist on phi”

Now we can observe how is the response of the system due to a perturbation on ϕ of 15° which last 0.05s

Immagine che contiene schermata, testo, Diagramma, linea

Descrizione generata automaticamenteWe can observe that the PID designed using the 1st way is faster but the variation on phi is higher, instead with the second one the ϕ angle is oscillating more but small variation.

The measurment of the ϕ angle from the real model is confermating the behavior of the simulated model:

* Ts = 1s for the 1st one
* Ts = 1.5s for the 2nd one

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamenteThe particlar difference between the two version of PID is that in the 1st one the action of the control variable is more agressive, that’s why we have a quick oscillation of ϕ after the perturbation. With the 2nd one, instead we don’t see any of these small oscillations.

Immagine che contiene schermata, testo

Descrizione generata automaticamenteComparison between the simulation behavior and the real behavior of the bar pendulum using the 1st design procedure.

Comparison between the simulation behavior and the real behavior of the bar pendulum using the 2nd Immagine che contiene schermata, testo, Diagramma, Software per la grafica

Descrizione generata automaticamentedesign procedure.

The quick small oscillation of the ϕ with the 1st way PID design procedure can be observe also by looking at the Voltage that should enter the DC Motor

Immagine che contiene schermata, testo

Descrizione generata automaticamenteAfter the perturbation at time 5 the Input Voltage starts to oscillate very fast between the limit we add fo the voltage that enters the DC Motor. This is not observe with the 2nd way PID design procedure.

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamente

We could already predict these beahvior by looking at the simulation. The voltage variation due to the perturbation is higher, quick and more socillations for the 1st strategy.

Mind also that the Voltage shows up here is the ideal one limited by a saturation, we are assuming that the Voltage providing from the Power Supply is fast enough to follow the Voltage we request. We might requesting something faster than how the Power supply is actually fast.

Immagine che contiene schermata, Software per la grafica, Software multimediale

Descrizione generata automaticamente

These oscillation are due to the voltage oscillation and so an oscillation also for the θ angle, infact the quick small oscillation causes a slow movement for the rod

Robustness test

What about if some of the parameters are wrong or we didn’t consider them:

* Ir = formula
* α = 0
* β = 0

basically we ignore the presence of the encoder on the rod so the Inertia is due only to the rod and we completely ignore the presence of damping for the bar pendulum and for the DC motor.

Immagine che contiene schermata, testo

Descrizione generata automaticamenteBy changing the parameter only for the 2nd design prosedure we observe as result more higher oscillation and so longer settling time, but at the end the bar pendulum reach a steady downside position.

Trajectory tracking control

One thing to try is also to put a reference on the θ angle, like a sinusoidal signal of 45°with a Period of 20s (f=0.05Hz), and see if the system is able to follow the reference and at the same time keeping the bar at a steady position at 0°.

As we observed is that the Control system is able to track the trajectory while keeping the bar at the downside position, but the amplitude of the the sinusoidal beahavior of the real system is lower than the 45° of the reference.

Immagine che contiene schermata, testo, Diagramma

Descrizione generata automaticamenteImmagine che contiene schermata, Diagramma, linea, diagramma

Descrizione generata automaticamente

Immagine che contiene schermata

Descrizione generata automaticamenteIncreasing the frequency like at the cut-off frequency: f = 0.8HZ

Immagine che contiene schermata, testo

Descrizione generata automaticamente

The Control System is still able to follow the trajectory for θ, but the amplitude of the variation is lower, less than 10°, the bar pendulum now is also oscillating since we are at a frequency where the attenuation is enough to see this beahavior.

Immagine che contiene schermata, Policromia

Descrizione generata automaticamenteIncreasing more the frequency, beyond the cut-off frequency, at f = 8Hz:

Immagine che contiene schermata, testo, Software per la grafica

Descrizione generata automaticamente

Both rod and bar pendulum are quickly oscillating, the rod even with a lower amplitude