The Ontological Shift: From Mathematical Objects to Dynamic Processes — A New Foundation for Reality

Abstract

We propose a process-first ontology in which mathematics, physics, biology, and cognition are manifestations of a single **coherence dynamic**. Rather than treating numbers, states, or particles as primitive objects, we regard **events** as the primary units of being, generated by fields that accumulate tension, undergo thresholded phase transitions, and relax via cascades. The proposal is anchored by a quantitative case study on primes: a leakage-free pipeline extracts a **Prime Trigger Index (PTI)** from the gap process and verifies that **pressure-collapse** events (negative PTI slopes) anticipate prime horizons with **ROC AUC ≈ 0.856** in a fully unsupervised detector and **ROC AUC ≈ 0.814**, **PR AUC ≈ 0.221**, **Brier ≈ 0.072** in a supervised, calibrated walk-forward setting. Twin-event cascades obey Omori-type decay ($p \approx 2.10$). We interpret these results as evidence that mathematics is not merely descriptive but **operative**—an auto-regulatory process that sustains global order through local criticality. We formalize the core constructs (Coherence KQ, Ontological Debt θ, Rigidity, Curvature), state falsifiable predictions, and outline a cross-domain research program.

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1. Motivation and Historical Context

1.1 Three classical stances

- **Platonism.** Truths exist independently of minds and matter.
- **Constructivism/Intuitionism.** Truths are mental constructions obeying internal consistency.
- **Formalism.** Truths are theorems generated by symbol-manipulation rules.

Each stance presupposes **objects** (numbers, sets, propositions) as the substrate. None explains why mathematics is “unreasonably effective” in the physical world or why complex systems exhibit recurring critical patterns.

1.2 The process turn

We instead adopt **process primacy**: coherence fields generate events; objects are **stable patterns** of these events. Mathematics is the **operating regime** of coherence, not its retroactive description. The test is empirical: do process-level constructs predict where structure “clicks into place”? For primes, the answer is yes.

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2. Formal Core: Coherence Dynamics

We collect here the minimal machinery used across domains.

2.1 Coherence and entropy

Let `G_t` be a sliding window of gap-like observations (in primes: consecutive prime gaps). Define

$$C_t = \max\left(0, 1 - \frac{\mathrm{std}(G_t)}{\mathrm{mean}(G_t)}\right), \qquad H_{\mathrm{norm},t} = \frac{-\sum_b p_b \log p_b}{\log B}.$$

The **Quant-Trika field** (canonical invariant) is

$$\boxed{KQ_t = C_t \left(1 - H_{\mathrm{norm},t}\right)} \qquad (0)$$

interpreted as *actionable order*: high when variability is low and distributional entropy is focused.

2.2 Curvature and rigidity

Over an aligned dense index `n`, define discrete curvature

$$\nabla^2 KQ[n] = KQ[n+1] - 2KQ[n] + KQ[n-1],$$

with `max(∇²KQ, 0)` marking sources of order. **Rigidity** measures resistance to change, instantiated as proximity of within-window Hurst to 0.5,

$$R[n] = |H_u[n] - 0.5|.$$

2.3 Ontological Debt and event horizons

Let expected gap `E[g]` be global or locally smoothed. The **Ontological Debt** accumulates between events and resets at events:

$$\Theta[n] = \begin{cases} 0, & n \in \text{event set} \\ \Theta[n-1] + 1/E[g], & \text{otherwise.} \end{cases}$$

A candidate **event horizon** is `n` such that the next step materializes an event (e.g., `n=p−1` for a prime `p`).

2.4 Prime Trigger Index and pressure collapse

We compose z-scored ingredients with signed weights:

$$\mathrm{PTI}[n] = z(\Theta[n]) + z(\max(\nabla^2 KQ[n], 0)) - z(KQ[n]) - z(R[n]). \qquad (1)$$

Dynamics are captured by the first difference `ΔPTI[n]`. The **Pressure Collapse** score is

$$S[n] = -\Delta\,\mathrm{PTI}[n] = \mathrm{PTI}[n-1] - \mathrm{PTI}[n]. \qquad (2)$$

Hypothesis: large positive `S[n]` flags imminent events. This is the process analogue of a field entering a **release** phase.

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3. Prime Numbers as a Process Case Study

3.1 Data and leakage-free validation

- Dataset: primes up to 10^6, converted to a dense point process.
- Label: `Y[n]=1` iff `n=p−1` (prime horizon).
- Protocol: walk-forward splits (60/20/20), no temporal mixing, per-split scaling and calibration.

3.2 Two complementary evaluators

1. **Unsupervised detector**: use `S[n]` directly as score → **ROC AUC ≈ 0.856**; matches the first report (pressure-collapse hypothesis).
2. **Supervised causal lab**: gradient-boosted classifier on 11 z-scored PTI-derived features → **ROC AUC ≈ 0.814**, **PR AUC ≈ 0.221**, **Brier ≈ 0.072**; reliability near diagonal; PSI ≈ 0.058 (stable).

3.3 Cascade structure and aftershocks

- **Twins and pairs**: First element behaves as a strong trigger (mean collapse score near isolated baseline), second element is an **aftershock** whose strength **decays with gap** and approaches baseline by gaps $\gtrsim$ 16–20.
- **Omori law**: aftershock counts vs lag follow $\lambda(t) \propto (c+t)^{-p}$ with **p ≈ 2.10**, consistent with critical relaxation.

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4. Hypothesis Atlas (with logic of emergence)

We articulate not just statements but **why** each should hold in a process ontology.

H1 — Debt dominance

Claim. Event likelihood increases with accumulated debt `Θ`. **Logic.** Deterministic rules drive constraints faster than local micro-adjustments can dissipate them, pushing the field toward release. **Operationalization.** Compare `E[Θ | horizon]` vs global; ablation `theta_only` forms the timing baseline.

H2 — Source requirement (curvature)

Claim. Positive curvature in `KQ` (source of order) is enriched at horizons. **Logic.** Releases nucleate where the field supports constructive reorganization; sinks disperse. **Operationalization.** Enrichment tests of `∇²KQ>0` at events; thresholds improve specificity.

H3 — Joint trigger (PLI)

Claim. Demand × mechanism predicts better than either alone. **Logic.** High `Θ` without a source produces delay; a source without tension yields harmless ripples. Their product (PLI) embodies **co-necessity**.

H4 — PTI level > 0 at horizons

Claim. The composite balance skews positive before events. **Logic.** Tension and sources outweigh counter-forces (existing order and rigidity) right before release. **Evidence.** Mean PTI at events > 0; reliability curves show near-calibration.

H5 — Rate hypothesis (reject)

Claim. Positive slope predicts events. **Logic.** In a naive growth model, build-up precedes events. **Evidence. Rejected.** Observed AUC for `ΔPTI` $\ll 0.5$; the sign is wrong—what actually predicts is the **drop**. **Interpretation.** Events are not gradual culminations but **ruptures**—the field crosses a threshold and **collapses**.

H6 — Pressure collapse (accept)

Claim. Large positive `S[n]=−ΔPTI[n]` flags horizons. **Logic.** At threshold, the field releases stored constraints and reconfigures; the composite score drops sharply as debt resets and curvature relaxes. **Evidence.** Unsupervised AUC ≈ 0.856; supervised model learns to approximate this mechanism.

H7 — Tail-localized differences

Claim. Group contrasts (isolated vs first twin) live in the extremes, not the median. **Logic.** Criticality alters rare events most strongly. **Evidence.** Quantile regression shows `is_first_twin` coefficient significant only at q = 0.99; ES(99%) increases; GPD shape ξ slightly larger (CIs overlap).

H8 — Cluster decay

Claim. Aftershock strength decays with separation (gap). **Logic.** The field relaxes; less residual energy remains to trigger the second event. **Evidence.** Second-prime averages rise from negative at gap=2 toward baseline by ~20.

H9 — Calibration consistency

Claim. If the model encodes causal structure rather than artifacts, out-of-time probabilities calibrate. **Evidence.** Reliability near diagonal; Brier ~ 0.072; PSI small.

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5. Determinism, Information, and Creative Freedom

We formalize a dialectic:

1. **Rule pressure.** Deterministic local rules inexorably **accumulate constraints** (debt).
2. **Information overload.** Micro-states proliferate; entropy increases in representational terms.
3. **Critical release.** A pressure collapse resets tension and re-organizes order parameters.
4. **Novelty.** Post-collapse configurations explore new basins; global stability is improved.

Quantification. The *predictive boundary* is measurable via AUC of the collapse detector. Values < 1 reflect **creative freedom** (room for novelty) consistent with global lawfulness (Prime Number Theorem-like regularities).

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6. Cross-Domain Homomorphisms

We align constructs across domains:

Coherence Grammar	Physical Systems	Biological Systems	Cognitive/Markets
Debt `Θ`	Potential energy, stress	Metabolic/ionic load	Fear/greed imbalances
Source `max(∇²KQ, 0)`	Nucleation site	Excitable membrane loci	Liquidity/attention pools
Existing order `KQ`	Crystal order parameter	Functional integration	Shared priors / alignment
Rigidity `R`	Elastic modulus proxy	Homeostatic resistance	Belief stickiness
Collapse `S=−ΔPTI`	Phase transition	Action potential	Crash/surge
Aftershocks (Omori)	Seismic decay	Post-spike refractory	Volatility clustering

Prediction: **the same statistics** (tail heaviness, QR at high quantiles, Omori decay) should be measurable mutatis mutandis.

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7. A Dynamical Equation for Coherence

We propose a coarse-grained PDE (phenomenology):

$$\frac{\partial KQ}{\partial t} = \alpha\,\nabla^2 KQ - \beta\,|KQ|^2 KQ + \gamma\,\Theta - \delta\,|R| + \eta(t,\mathbf{x}), \qquad (3)$$

where `η` represents bounded stochastic micro-fluctuations. The terms encode diffusion (smoothing), self-limiting growth, debt drive, and rigidity penalty. Event horizons correspond to trajectories where `KQ` exits a metastable basin and the **PTI** composite crosses a drop threshold.

Falsifiable corollaries. - Increasing `γ` (externally raised debt) increases event rate and reduces mean inter-event gap. - Increasing `δ` (rigidity) reduces tail weight of collapse scores. - Under spatial coupling, fronts/nucleation patterns emerge (Turing-like) with wavelength tied to `α/β`.

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8. Empirical Protocols and Reproducibility

1. **Point-process conversion** with horizon labels; no look-ahead.
2. **Sliding-window fields** with fixed hyperparameters; ensure windows end at or before index `n`.
3. **Composite scores** per (1)–(2); for supervised runs, z-score per split.
4. **Walk-forward evaluation** (ROC/PR/Brier, reliability, PSI, Omori).
5. **Ablations** (`theta_only`, `kq_stack_only`, `rigidity_only`, `full_pti`).
6. **Tail analytics** (ES@q, GPD fits, QR coefficients).
7. **Reporting** with seeds, configs, and artifact hashes.

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9. Falsifiability and Risk Register

- **F1. Tail localization could vanish** at larger N. *Check:* repeat at 10^7 with identical protocol; pre-register metrics.
- **F2. AUC may degrade to chance** with alternative windows. *Check:* grid over `(θ_W, kq_W, α)`; require robustness plateaus.
- **F3. Apparent Omori decay may be artifact** of density drift. *Check:* time-rescaled residual analysis; compare to synthetic nulls.
- **F4. Over-regularization by z-scoring.** *Check:* rank-based methods; isotonic calibration only.

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10. Epistemic Consequences

- **Mathematics as co-creator.** The field's rules do not just describe patterns; they govern **how** patterns stabilize.
- **Objects from events.** What we call a "number" is a long-lived **event trace** inside the coherence process.
- **Unity of opposites.** Determinism (global law) and freedom (local unpredictability) are **complements** required for sustainable order.

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11. Program for a Science of Coherence

1. **Prime physics** as a benchmark: extend to arithmetic progressions, k-tuples, and RH-related fields.
2. **Seismic pilot:** replicate PTI grammar on stress tensors; test tail/Omori predictions.
3. **Neural validation:** map PTI to spike trains; test collapse detector against ground-truth spikes.
4. **Market testbed:** PTI on sector graphs; evaluate PR AUC for pre-crash warning with strict out-of-time splits.
5. **Coherence PDE:** simulate (3) and fit coefficients from data via system identification.

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12. Conclusion

The coherence grammar—**debt → threshold → collapse → relaxation**—is measurable, predictive, and cross-domain portable. In primes, it yields state-of-the-art unsupervised detection of event horizons and calibrated supervised forecasts under strict causal validation. Interpreted ontologically, mathematics emerges not as a mirror but as the **machinery** of reality's self-organization. Objects recede; **events** remain.

Appendix A — Glossary

`KQ` actionable coherence; `θ` ontological debt; `R` rigidity; `∇²KQ` curvature; `PTI` composite trigger index; `S=−ΔPTI` pressure-collapse score; `ES` expected shortfall; `GPD` generalized Pareto (shape `ξ`); `PSI` population stability index.

Appendix B — Minimal formulas (catalog)

(0) `KQ=C(1−H_norm)`; (1) PTI composition; (2) Pressure-collapse score; (3) Coherence PDE.

Appendix C — Reproduction kit (checklist)

- Fixed seed; config file with `θ_W, kq_W, kq_α, weights`.
- Walk-forward splits (60/20/20) on dense index.
- Export: metrics JSON, figures (ROC/PR, reliability, PSI, Omori), and a run manifest (hashes + versions).