Quant Finance Portfolio Metrics

Return on Investment
How much money you made (or lost) compared to what you started with

Dividend Yield
How much a stock pays you each year in dividends, compared to its price.

Volatility
How much returns go up and down — a measure of *riskiness*.

Beta
How much your portfolio moves when the *market* moves.

Alpha
How much extra return you made beyond what the market explains.

Sharpe Ratio
How much *return* you get for each unit of *total risk*.

Value at Risk
The most you could *probably* lose in a given period, at a chosen confidence level.

Conditional Value at Risk
If things go *really bad* (beyond the VaR), how much you *actually* lose on average

R–squared
How much of your portfolio's movement is explained by the market (or benchmark)

Return Metrics

Total Return (Holding Period Return)

$$HPR = \frac{V_{end} - V_{begin} + CF}{V_{begin}}$$

$$V_{begin} = Initial\ Investment\ Value$$

$$V_{end} = Final\ Value$$

$$CF = Cash\ flows\ (dividens, coupons, distributions)$$

Some top dividend stocks grow slowly and have small capital gains. Focusing solely on capital gains ignores other ways the stock's value can increase, like price hikes. For example, an investor buys shares of Company B, and the share price increases 24.5% in one year. The investor gains 24.5% from the price change alone. Company B's dividends add a 4.1% yield, making the total return, including price changes, 28.6%.

Dividend Yield

$$Dividend\ Yield = \frac{Annual\ Dividends\ per\ Share}{Current\ Price}$$

A stock's dividend yield is a ratio showing how much a company pays out in dividends each year relative to its stock price. The reciprocal of the dividend yield is the dividend payout ratio.

Understanding Dividend Yield

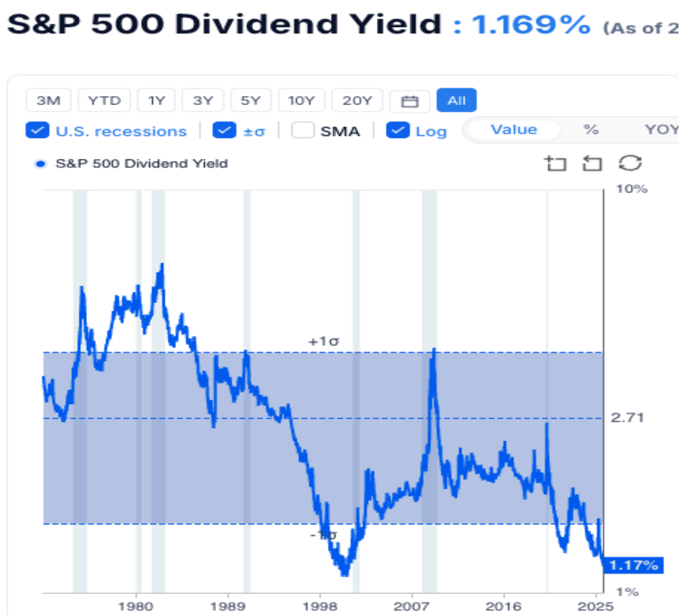
- *Utilities: yields > 4% (stable cash flows).*
- *Growth tech: < 1% (retain earnings).*
- *REITs/MLPs: very high yields due to pass-through structures.*

Dividends are payments made by a corporation to its shareholders, usually derived from the company's profits. These payments represent a portion of the company's earnings that is distributed to its investors as a reward for their ownership. The dividend yield is an estimate of the dividend only return of a stock investment. Assuming the dividend is not raised or lowered, the yield will rise when the stock price falls. Conversely, it will fall when the price of the stock rises. Because dividend yields change relative to the stock price; it can often look unusually high for stocks that are falling in value quickly. New companies that are relatively small, but still growing quickly, may pay a lower average dividend than mature companies in the same sectors. Generally, mature companies that aren't growing very quickly pay the highest dividend yields. Dividends can be issued in various forms, including cash payments, additional shares of stock, or other property. The most common form is cash dividends.

REITs, MLPs, and BDCs: In some cases, the dividend yield may not provide much information about the kind of dividend, the company pays. For example, the average dividend yield in the market can be very high among real estate investment trusts (REITs). However, those are the yields from ordinary dividends, which differ from qualified dividends in that the former is taxed as regular income while the latter is taxed as capital gains. Along with REITs, master limited partnerships (MLPs) and business development companies (BDCs) typically also have very high dividend yields. The structure of these companies is such that the U.S. Treasury requires them to pass on the majority of their income to their shareholders. This is referred to as a "passthrough" process, and it means that the company doesn't have to pay income taxes on profits that it distributes as dividends. U.S. Securities and Exchange Commission. "Investor Bulletin: Real Estate Investment Trusts (REITs)."

https://fred.stlouisfed.org/graph/?g=3fbu

https://www.gurufocus.com/economic_indicators/150/sp-500-dividend-yield



Volatility

$$\sigma = \sqrt{\left\{\frac{1}{N-1}\sum_{t=1}^{n}(r_t - \bar{r})^2\right\}}, \qquad \sigma_{ann} = \sigma\sqrt{k}$$

$$\sigma = Standard\ Deviation\ of\ Returns$$

$$r_t = return\ at\ time\ t$$

$$\bar{r} = mean\ return$$

$$N = number\ of\ periods$$

$$k = number\ of\ periods\ per\ year$$

$$\sigma_{ann} = annualized\ volatility$$

where k is the number of periods in the time horizon; specifically, k = 252 for daily returns and k = 12 for monthly.

Why It Matters

Volatility is a measurement of how varied the returns of a given security or market index are over time. It is often measured from either the standard deviation or variance between those returns. In most cases, the higher the volatility, the riskier the security.

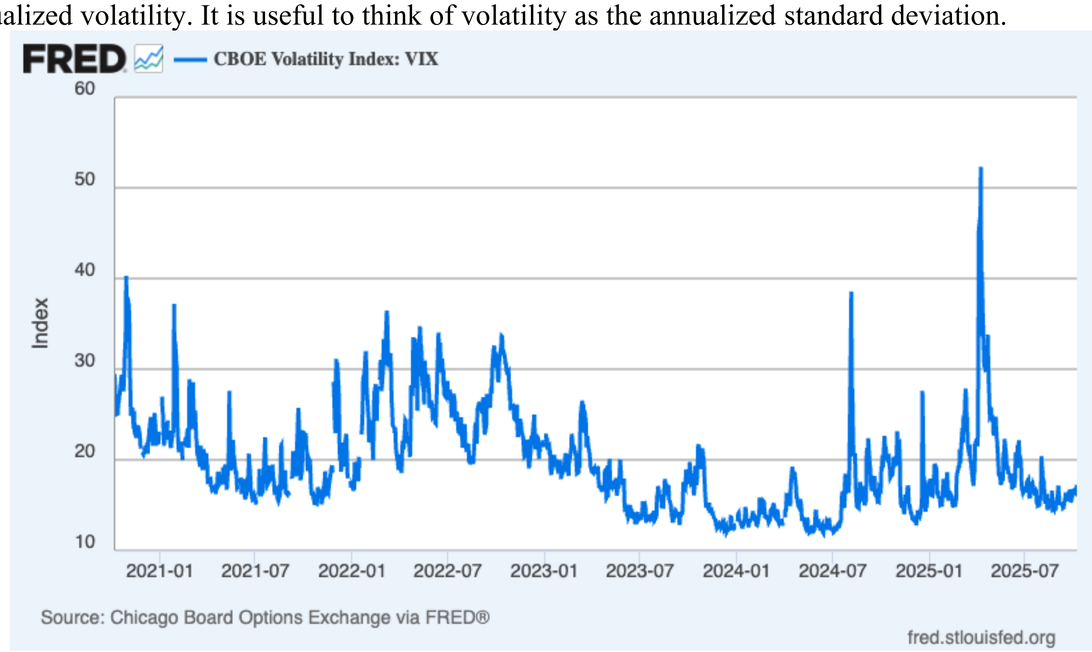
In the securities markets, volatility is often associated with big price swings, either up or down. For example, when the stock market rises and falls more than 1% over a sustained period of time, it is called a volatile market.

An asset's volatility is a key factor when pricing options contracts.

Understanding Volatility

Volatility often refers to the amount of uncertainty or risk related to the size of changes in a security's value. A higher volatility means that a security's value can potentially be spread out over a larger range of values. This means that the price of the security can move dramatically over a short time period in either direction. A lower volatility means that a security's value does not fluctuate dramatically and tends to be steadier. One way to measure an asset's variation is to quantify the daily returns (percent move on a daily basis) of the asset. Historical volatility is based on historical prices and represents the degree of variability in the returns of an asset. This number is without a unit and is expressed as a percentage.

While variance captures the dispersion of returns around the mean of an asset in general, volatility is a measure of that variance bounded by a specific time period. Thus, we can report daily, weekly, monthly, or annualized volatility. It is useful to think of volatility as the annualized standard deviation.



Beta

$$\beta = \frac{\mathrm{Cov}(r_p, r_m)}{\mathrm{Var}(r_m)}$$

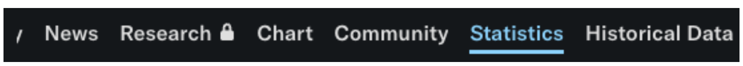
$$r_p = \ portfolio\ (\ or\ asset)\ return$$

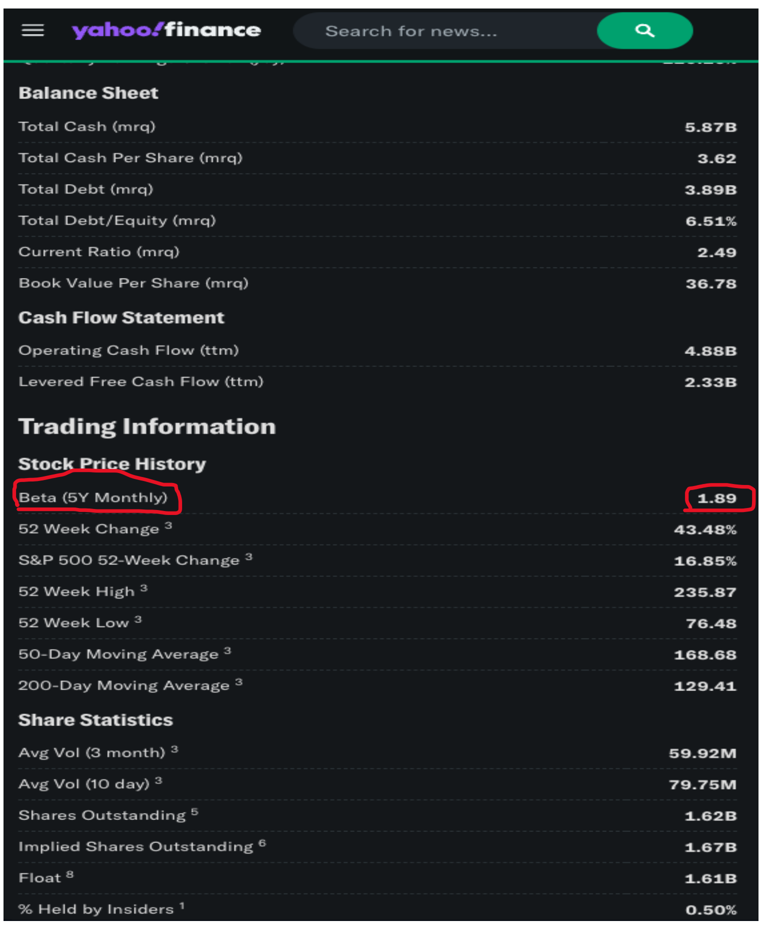
$$r_m = market\ return$$

- β = 1: *moves with market*
- β > 1: *more volatile (e.g. tech)*
- β < 1: *more stable (e.g. utilities)*
- β < 0: *inverse to market (e.g. gold, inverse ETFs)*

Beta is an indicator of the price volatility of a stock or other asset in comparison with the broader market. It suggests the level of risk that an investor takes on in buying the stock. The higher the beta number, the higher the risk. The market that the stock trades in, such as the S&P 500, always has a beta of 1.0. A number above one indicates that the stock's price swings more, up or down, then the market in general. A number below one indicates its price is more stable.

A beta coefficient shows the volatility of an individual stock compared to the systematic risk of the entire market. Beta represents the slope of the line through a regression of data points. In finance, each point represents an individual stock's returns against the market. Beta effectively describes the activity of a security's returns as it responds to swings in the market. It is used in the capital asset pricing model (CAPM), which describes the relationship between systematic risk and expected return for assets. CAPM is used to price risky securities and to estimate the expected returns of assets, considering the risk of those assets and the cost of capital.
https://smartasset.com/investing/how-to-calculate-the -beta-of-a-portfolio





Balance Sheet	
Total Cash (mrq)	5.87B
Total Cash Per Share (mrq)	3.62
Total Debt (mrq)	3.89B
Total Debt/Equity (mrq)	6.51%
Current Ratio (mrq)	2.49
Book Value Per Share (mrq)	36.78
Cash Flow Statement	
Operating Cash Flow (ttm)	4.88B
Levered Free Cash Flow (ttm)	2.33B
Trading Information	
Stock Price History	
Beta (5Y Monthly)	1.89
52 Week Change [3]	43.48%
S&P 500 52-Week Change [3]	16.85%
52 Week High [3]	235.87
52 Week Low [3]	76.48
50-Day Moving Average [3]	168.68
200-Day Moving Average [3]	129.41
Share Statistics	
Avg Vol (3 month) [3]	59.92M
Avg Vol (10 day) [3]	79.75M
Shares Outstanding [5]	1.62B
Implied Shares Outstanding [6]	1.67B
Float [8]	1.61B
% Held by Insiders [1]	0.50%

How Investors Use Beta

An investor uses beta to gauge how much risk a stock adds to a portfolio. While a stock that deviates very little from the market doesn't add a lot of risk to a portfolio, it also doesn't increase the potential for greater returns. Investors must ensure that a specific stock is compared to the right benchmark and review the R-squared value relative to the benchmark. R-squared is statistical measure that compares the security's historical price movements to the benchmark index. A security with a high R-squared value indicates a relevant benchmark. A gold exchange-traded fund (ETF), such as the SPDR Gold Shares (GLD), is tied to the performance of gold bullion. Consequently, a gold ETF would have a low beta and R-squared relationship with the S&P 500.

Value at Risk (VaR)

$$\mathrm{VaR}_q = \mu - z_q$$

$$\mu = mean\ return$$

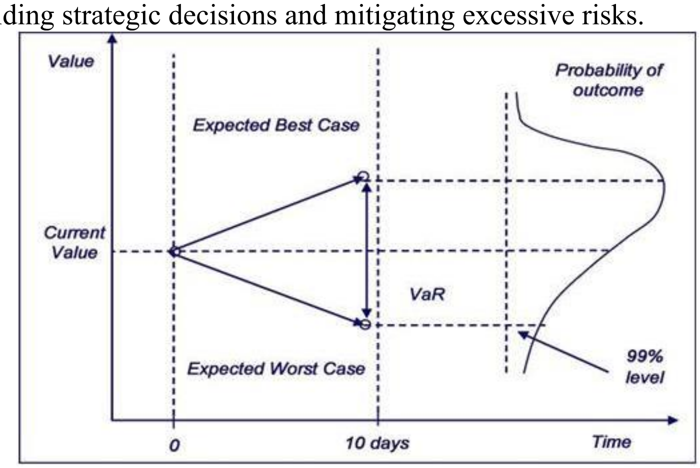
$$z_q = z - score\ for\ confidence\ level$$

$$\sigma = standard\ deviation\ of\ returns$$

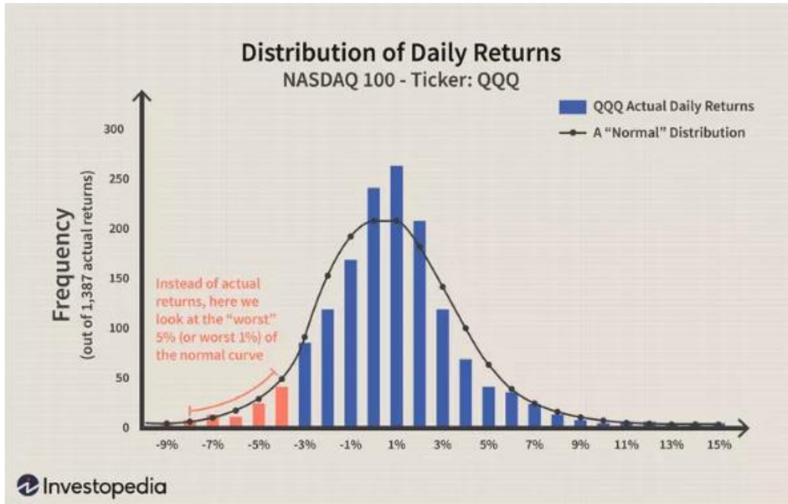
$$\mathrm{VaR}_\alpha = \mu + \sigma\Phi^{-1}(\alpha)$$

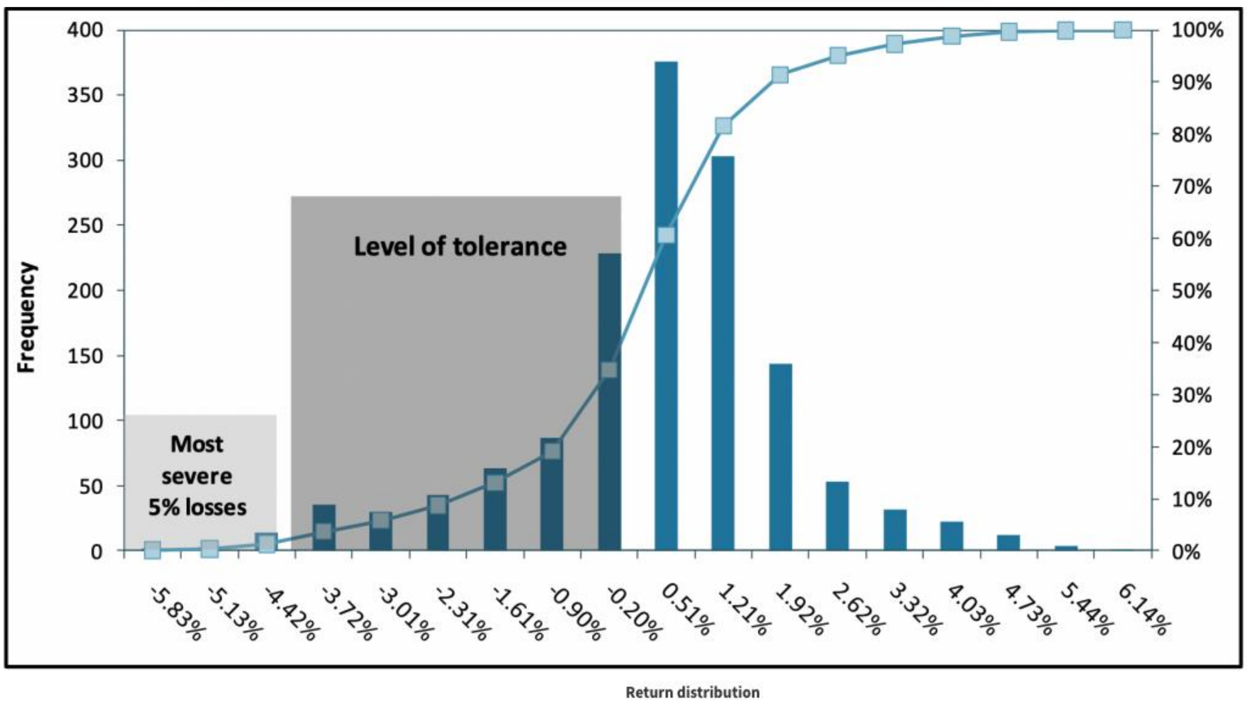
What is VaR

Value at Risk (VaR) is an essential tool for investment and commercial banks to measure potential financial losses over a set time period. VaR calculations help risk managers understand the probabilities and extents of potential losses in portfolios, specific positions, or an entire firm. This insight allows institutions to assess their risk exposure and determine the adequacy of their capital reserves, ultimately guiding strategic decisions and mitigating excessive risks.



Value at Risk





Exploring VaR Methodologies: Historical, Variance–Covariance, and Monte Carlo

There are three main ways of computing VaR: the historical method, the variance–covariance method, and the Monte Carlo method.

Historical Method. The historical method looks at one's prior returns history and orders them from worst losses to greatest gains—following from the premise that past returns experience will inform future outcomes.

Variance–Covariance Method. Rather than assuming that the past will inform the future, the variance–covariance method, also called the parametric method, instead assumes that gains and losses are normally distributed. This method frames potential losses as standard deviation events from the mean. The variance–covariance method works best for risk measurement in which the distributions are known and reliably estimated. It is less reliable if the sample size is very small.

Monte Carlo Method. A third approach to VaR is to conduct a Monte Carlo simulation. This technique uses computational models to simulate projected returns over hundreds or thousands of possible iterations. Then, it takes the chances that a loss will occur—say, 5% of the time— and reveals the impact. The Monte Carlo method applies to various risk measurement problems, assuming the probability distribution for risk factors is known.

$$\mathrm{CVaR}_q = \mu - \sigma \frac{\phi(z_q)}{q}$$

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

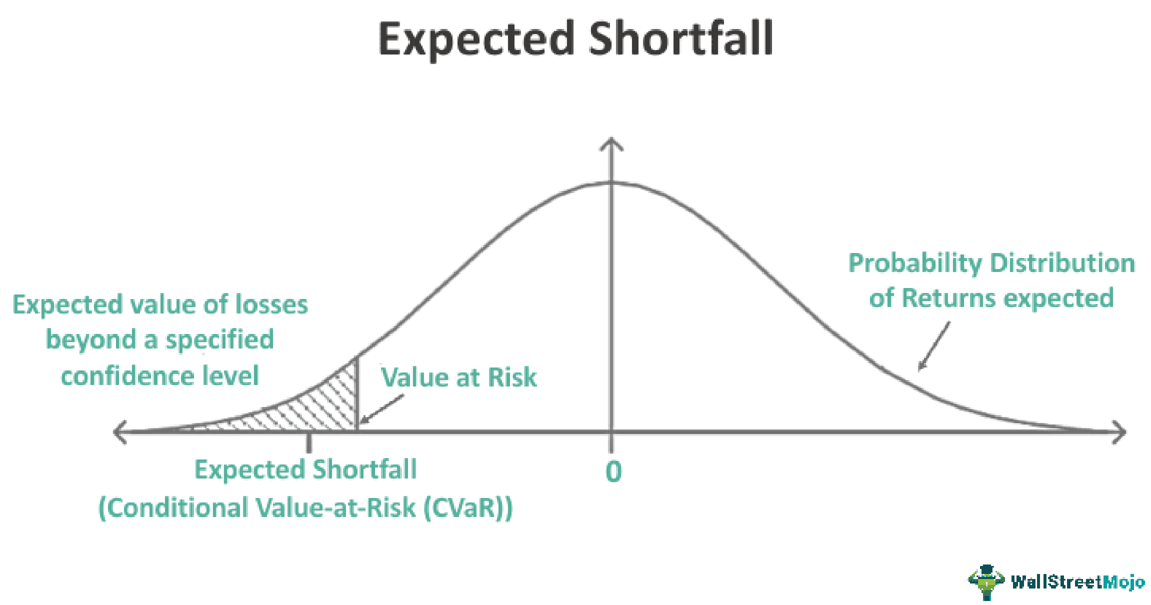
$$\phi(z_q) = standard\ normal\ probability\ density\ function$$

$$q = confidence\ level$$

The PDF tells us how likely it is for a normally distributed variable to take a value near a given point.

- VaR: maximum loss not exceeded with q confidence
- CVaR: expected loss if VaR is breached (tail severity)

Conditional Value at Risk (CVaR), also known as expected shortfall, provides a deeper insight into the tail risk of investments than traditional value at risk (VaR). By calculating CVaR, investors can understand the extent of potential extreme losses beyond the VaR cutoff and apply this knowledge in portfolio optimization for better risk management



Expectation Form

The CVaR at confidence level α is the expected loss given that the loss L exceeds the Value at Risk (VaR) at the same level:

$$\mathrm{CVaR}_\alpha(L) = E[\, L \mid L \geq \mathrm{VaR}_\alpha(L)\,]$$

Quantile Function
CVaR can be written as the average of the quantiles above α:

$$\text{CVaR}_{\alpha}(L) = \frac{1}{1-\alpha}\int_{\alpha}^{1} \text{VaR}_{u}(L)\; du$$

PDF Form
If the loss distribution L is continuous with PDF fL(x), then:

$$\text{CVaR}_{\alpha}(L) = \frac{1}{1-\alpha}\int_{\text{VaR}_{\alpha}(L)}^{\infty} x\, f_L(x)\, dx$$

L : Random variable representing portfolio losses
$E\,[\cdot]$: Expected value operator (average)
$VaR_u(L) = Value\ at\ Risk\ at\ confidence\ level\ \alpha$
α : Confidence level (e.g., 0.95 for 95 % VaR)
u : Quantile variable for tail probability integration
$f_L(x)$: Probability density function (PDF) of loss distribution
x : Possible loss value beyond the VaR threshold
1 − α : Tail probability (e.g., 5 % for 95 % confidence)

Sharpe Ratio

$$\text{Sharpe} = \frac{R_p - R_f}{\sigma_p}$$

$$R_p = Portfolio\ return$$

$$R_f = Risk\ free\ rate$$

$$\sigma = Portfolio\ Volatility$$

The Sharpe ratio is one of the most widely used methods for measuring risk-adjusted relative returns. It compares a fund's historical or projected returns relative to an investment benchmark with the historical or expected variability of such returns.

Sortino Ratio

$$\text{Sortino} = \frac{R_p - MAR}{\sigma_{down}}$$

$$MAR = Minimum\ Acceptable\ Return$$

$$\sigma_{down} = Downside\ volatility$$

Treynor Ratio

$$\text{Treynor} = \frac{R_p - R_f}{\beta_p}$$

Sharpe Alternatives:

The Sortino and the Treynor. The standard deviation in the Sharperatio’s formula assumes that price movements in either direction are equally risky. The risk of an abnormally low return is very different from the possibility of an abnormally high one for most investors and analysts. A variation of the Sharpe called the Sortino ratio ignores the above-average returns to focus solely on downside deviation as a better proxy for the risk of a fund or portfolio. The standard deviation in the denominator of a Sortino ratio measures the variance of negative returns or those below a chosen benchmark relative to the average of such returns. Another variation of the Sharpe is the Treynor ratio, which divides excess return over a risk-free rate or benchmark by the beta of a security, fund, or portfolio as a measure of its systematic risk exposure. Beta measures the degree to which the volatility of a stock or fund correlates with that of the market as a whole. The goal of the Treynor ratio is to determine whether an investor is being compensated for extra risk above that posed by the market.

Example of How to Use the Sharpe Ratio

The Sharpe ratio is sometimes used to assess how adding an investment might affect the portfolio’s risk-adjusted returns. For example, an investor is considering adding a hedge fund allocation to a portfolio that has returned 18% over the last year. The current risk-free rate is 3%, and the annualized standard deviation of the portfolio’s monthly returns was 12%, which gives it a one year Sharpe ratio of 1.25, or (18% − 3%)/12%. The investor believes that adding the hedge fund to the portfolio will lower the expected return to 15% for the coming year, but also expects the portfolio’s volatility to drop to 8% as a result. The risk-free rate is expected to remain the same over the coming year. Using the same formula with the estimated future numbers, the investor finds the portfolio would have a projected Sharpe ratio of 1.5, or (15% − 3%)/8%. In this case, while the hedge fund investment is expected to reduce the absolute return of the portfolio, based on its projected lower volatility, it would improve the portfolio’s performance on a risk-adjusted basis. If the new investment lowered the Sharpe ratio, it would be assumed to be detrimental to risk-adjusted returns, based on forecasts. This example assumes that the Sharpe ratio based on the portfolio’s historical performance can be fairly compared to that using the investor’s return and volatility assumptions.

Capital Asset Pricing Model (CAPM) a.k.a. Alpha

$$\alpha = R_p - \left[R_f + \beta_p\left(R_m - R_f\right)\right]$$

$$\alpha = Excess\ return\ beyond\ systematic\ risk$$

$$R_m = Market\ return$$

$$\beta_p = Beta\ of\ the\ portfolio$$

$$R_f = Risk\ free\ rate$$

$$R_p = Portfolio\ return$$

The capital asset pricing model (CAPM) describes the relationship between systematic risk, or the general perils of investing, and expected return for assets, particularly stocks. It is a finance model that establishes a linear relationship between the required return on an investment and risk. CAPM is based on the relationship between an asset’s beta, the risk-free rate (typically the Treasury bill rate), and the equity risk premium, or the expected return on the market minus the risk-free rate.

Positive α = outperformance beyond systematic risk.

CAPM vs Beta

Beta compares a security or portfolio's volatility or systematic risk to the market. If a stock is riskier than the market, it will have a beta greater than one. If a stock has a beta of less than one, the formula assumes it will reduce the risk of a portfolio. A stock's beta is then multiplied by the market risk premium, which is the return expected from the market above the risk-free rate. The risk-free rate is then added to the product of the stock's beta and the market risk premium. The result should give an investor the required return or discount rate that they can use to find the value of an asset.

CAPM Example

Imagine an investor is contemplating a stock valued at $100 per share today that pays a 3% annual dividend. Say this stock has a beta compared with the market of 1.3, which means it is more volatile than a broad market portfolio (i.e., the S&P 500 Index). Also, assume that the risk-free rate is 3% and this investor expects the market to rise in value by 8% per year. The expected return of the stock based on the CAPM formula is 9.5%:

9.5% = 3% + 1.3 °ø (8% − 3%).

The expected return of the CAPM formula is used to discount the expected dividends and capital appreciation of the stock over the expected holding period. If the discounted value of those future cash flows is equal to $100, then the CAPM formula indicates the stock is fairly valued

$\boldsymbol{R^2}$

$$R^2 = \rho\left(r_p, r_b\right)^2$$

ρ (rho): The Correlation Coefficient measures the strength and direction of the linear relationship between two random variables, for example, portfolio returns and benchmark returns.

It's a standardized version of covariance.

$$\rho(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

ρ Value	Relationship	Interpretation
ρ = 1	Perfect positive correlation	Move together exactly
0 < **ρ** < 1	Positive correlation	Usually move together
ρ = 0	No linear correlation	Independent movements
-1 < **ρ** < 0	Negative correlation	Move oppositely
ρ = -1	Perfect negative correlation	Inverse relationship

Measures proportion of variance explained by benchmark.

• R2 ≈ 1: returns benchmark-driven
• Low R2: portfolio diverges (active bets)

How to Use R-Squared

In investing, R-squared is generally interpreted as the percentage of a funds or security's movements that can be explained by movements in a benchmark index. For example, an R-squared for a fixed income security vs. a bond index identifies the security's proportion of price movement that is predictable based on a price movement of the index. The same can be applied to a stock vs. the S&P 500 Index or any other relevant index. It may also be known as the coefficient of determination.

R-squared values range from 0 to 1 and are commonly stated as percentages from 0% to 100%.

An R-squared of 100% means that all of the movements of a security (or another dependent variable) are completely explained by movements in the index (or whatever independent variable you are interested in). In investing, a high R-squared, from 85% to 100%, indicates that the stock's or fund's performance moves relatively in line with the index. A fund with a low R-squared, at 70% or less, indicates that the fund does not generally follow the movements of the index. Higher R-squared value will indicate a more useful beta figure. For example, if a stock or fund has an R-squared value of close to 100%, but has a beta below 1, it is most likely offering higher risk-adjusted returns