Statistics For Quants : Revision Notes

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July 26, 2025

1 Foundational Proportional Relationships

1.1 Central Limit Theorem as Proportional Convergence

Core Relationship:

$$\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0,1) \propto \frac{\text{Sample Mean Deviation}}{\text{Standard Error}}$$

What to Remember: - The CLT states that the distribution of sample means approaches normality regardless of the underlying distribution, provided $n > 30$ typically. - Key formula: Standardized mean $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$, where variance scales as $1/n$. - Proportional insight: Precision (inverse of SE) $\propto \sqrt{n}$; doubling sample size reduces SE by $\sqrt{2} \approx 1.41$. - Edge cases: For small n or heavy tails, use Berry-Esseen bounds for convergence rate $\propto 1/\sqrt{n}$. - Implementation tip: In Python (numpy), simulate with `np.mean(samples)` and standardize; in C++, use ¡random¿ for distributions.

When to Use: - In risk modeling for aggregating returns in portfolios: Use CLT to estimate annual volatility from daily data, e.g., $\sigma_{annual} \propto \sigma_{daily}\sqrt{252}$. - Backtesting interest rate models: Approximate confidence intervals for simulated zero rates to validate model calibration. - Interview context: Jane Street often asks about CLT limitations in fat-tailed markets; use for quick approximations in high-frequency trading simulations. - In quant workflows: Apply when backtesting credit spread changes over large datasets to infer population parameters without assuming normality.

Example 1 (Portfolio Return Aggregation)**.** *For a portfolio with daily returns $R_i \sim$ some distribution with $\mu = 0.0005$, $\sigma = 0.01$. For $n = 252$ trading days:*

$$\frac{\bar{R} - \mu}{\sigma/\sqrt{252}} \approx Z \implies \textit{Annualized SE} \propto \frac{0.01}{\sqrt{252}} = 0.00063$$

Proportional insight: Estimation precision improves by factor $\sqrt{n}$ as data accumulates.

Interview Question 1 (Jane Street Style)**.** ***Question:*** *In risk modeling for positions, you sample 100 daily equity returns with mean 0.001 and SD 0.02. What's the approximate 95% confidence interval for the true mean? How does it change if you use 400 samples? Adjust for fat tails using a proportionality approximation.*

Solution using Proportionality: *Using CLT: $CI \propto \bar{X} \pm z \cdot \frac{\sigma}{\sqrt{n}}$*

For $n = 100$: $CI \approx 0.001 \pm 1.96 \cdot \frac{0.02}{\sqrt{100}} = 0.001 \pm 0.00392$

For $n = 400$: Scaling factor $\sqrt{4} = 2$, so SE halves: $CI \approx 0.001 \pm 0.00196$

Fat-tail adjustment (e.g., t-distribution with $\nu = 5$): $\frac{t_{0.025,99}}{\Phi^{-1}(0.975)} \approx 1.05 \times 1.96 \implies$ CI widens by 5%.

1.2 Law of Large Numbers Proportionality

$$\frac{\bar{X}_n - \mu}{1/n} \to 0 \quad \Rightarrow \quad \text{Convergence Rate} \propto \frac{1}{n}$$

What to Remember: - Weak LLN: Sample mean converges in probability to population mean; strong LLN converges almost surely. - Proportional insight: Bias reduction $\propto 1/n$; for variance, combine with CLT for $\sqrt{1/n}$ rate. - Key assumption: Finite variance; fails for Cauchy distribution (infinite moments). - Implementation: In SQL, use AVG() over large tables; in C++, simulate with loops for convergence checks.

When to Use: - Backtesting FX spot models: Ensure long-run averages of simulated paths converge to historical means for calibration. - Risk factor modeling: Use in Monte Carlo simulations for positions to average out noise in large samples. - Interview tip: Citadel questions often probe LLN in infinite variance scenarios (e.g., stable distributions); approximate convergence with truncation. - In quant workflows: Apply to inflation zero rate backtesting, where large historical datasets ensure model estimates stabilize.

Example 2 (Backtesting Interest Rate Models)**.** *Simulating 1,000 paths for zero rates with true mean 0.03. Sample mean* $\bar{r} = 0.031$*. Error* $\propto 1/1000 = 0.001$*, explaining the deviation.*

2 Distribution Proportionalities

2.1 Normal Distribution Proportional Structure

$$\phi(x) \propto e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{normalization} \propto \frac{1}{\sqrt{2\pi\sigma^2}}$$

What to Remember: - PDF is symmetric; 68-95-99.7 rule for $\pm 1, 2, 3\sigma$. - Proportional properties: Linear combinations remain normal; conditional normal is also normal. - Moments: Skew=0, Kurtosis=3; tail decay $\propto e^{-x^2/2}$, faster than power-law. - In code: Python's scipy.stats.norm for fitting; C++ Boost.Math for quantiles.

When to Use: - Modeling small equity returns or interest rate changes assuming no jumps. - In VaR calculations for derivatives: Approximate tails when data is symmetric and light-tailed. - Interview: Two Sigma may ask about normality tests (e.g., Jarque-Bera $\propto$ skew^2+ (kurt-3)^2); use for baseline before switching to t-dist. - In quant workflows: Fit normal to bond z-spread changes for initial risk assessment, then test for deviations.

Example 3 (Credit Spread Modeling)**.** *Credit spreads* $S \sim N(0.5\%, 0.2\%^2)$*. Probability of spike* $> 1\%$*:* $P(S > 1) \propto 1 - \Phi(2.5) \approx 0.0062$*.*

2.2 Student's t-Distribution Proportionalities

$$f(t) \propto \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \quad \text{heavier tails as } \nu \downarrow$$

What to Remember: - Degrees of freedom ν control tails; as $\nu \to \infty$, approaches normal. - Variance $\propto \nu/(\nu-2)$ for $\nu > 2$; no variance if $\nu \leq 2$. - Proportional: Tail prob $\propto t^{-\nu}$ vs. normal's exponential. - Code: Use statsmodels in Python for robust regression; custom implementations in C++ for efficiency.

When to Use: - Fat-tailed returns in positions: Model when kurtosis ¿3, e.g., during market crashes. - Robust estimation in credit spread backtesting: Use t-errors to handle outliers. - Interview: Hudson asks for t vs. normal in risk models; approximate VaR scaling as above. - In quant workflows: Apply to equity factor models with limited data ($n < 30$) for wider confidence intervals.

Example 4 (Fat-Tail Risk in Positions)**.** *For returns with* $\nu = 4$*, VaR scaling:* $\frac{VaR_t}{VaR_N} \propto \sqrt{\frac{\nu-2}{\nu}} \cdot t_{\nu,0.99}/2.33 \approx 1.5$.

2.3 Log-Normal Distribution Proportionalities

$$f(x) \propto \frac{1}{x} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}} \quad E[X] = e^{\mu+\sigma^2/2}$$

What to Remember: - For positive variables; log(X) is normal. - Proportional: Mean/median = $e^{\sigma^2/2}$; variance $\propto (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$. - Black-Scholes assumes log-normal prices. - Code: SQL for log-transform queries; Python for simulations.

When to Use: - FX spot modeling: Prices can't be negative, use for volatility scaling. - Inflation zero rates: Model compounded rates. - Interview: IMC probes log-normal in options; approximate moments for quick calcs. - In quant workflows: Fit to basis spreads for positive deviations.

Example 5 (FX Spot Volatility Modeling)**.** *FX rates often log-normal; volatility* $\sigma \propto \sqrt{Var(\ln X)}$*. For* $\sigma = 0.1$*, mean/median ratio* $\propto e^{0.005} \approx 1.005$.

3 Inferential Statistics Proportionalities

3.1 Hypothesis Testing Proportional Framework

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \propto \frac{\text{Signal}}{\text{Noise}}$$

p-value $\propto P(|T| > t|H_0)$.

What to Remember: - t-test for means; z-test if σ known. - Power = 1 - Type II error $\propto \sqrt{n}\cdot$ effect size. - Multiple testing: Bonferroni correction α/m. - Code: Python hypothesis with scipy.stats.ttest.

When to Use: - Testing zero mean in FX changes for arbitrage models. - Backtesting: Validate if new risk factor adds value. - Interview: Citadel on power in small samples. - In quant workflows: Test significance of credit spread predictors.

Example 6 (Backtesting FX Spot Models)**.** *Test* $H_0 : \mu_{FX} = 0$ *vs.* $H_1 : \mu > 0$ *with* $\bar{X} = 0.0002$*,* $s = 0.01$*,* $n = 100$*:* $t \propto 0.0002/(0.01/10) = 0.2$*, p-value high, fail to reject.*

Interview Question 2 (Citadel Style)**.** ***Question:*** *In derivative risk modeling, you have 50 observations of bond z-spread changes with sample mean 0.1 bps and SD 2 bps. Test if true mean is zero at 5% level. Approximate power if true mean is 0.5 bps.*

Solution using Proportionality: $t = \frac{0.1}{\sqrt{50}}/2 \approx 0.035$*,* $p > 0.05$*, fail to reject.*

Power $\propto 1 - \beta$*, where* $\beta = P(T < t_{0.05} - \delta\sqrt{n}/s)$*. For* $\delta = 0.5$*: shift* $\propto 0.5\sqrt{50}/2 \approx 1.77$*, power* $\approx 70\%$*.*

3.2 Bayesian Inference Proportionalities

$$P(\theta|D) \propto P(D|\theta)\cdot P(\theta) \quad \text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

What to Remember: - Conjugate priors (e.g., normal-normal) yield closed-form posteriors. - Credible interval from posterior quantiles. - Bayes factor for model comparison $\propto$ marginal likelihood ratio. - Code: PyMC3 for MCMC; C++ for custom samplers.

When to Use: - Updating priors in inflation models with new data. - Risk modeling with uncertainty: Bayesian VaR for positions. - Interview: Two Sigma on prior sensitivity. - In quant workflows: Bayesian regression for equity factors with informative priors from historical data.

Example 7 (Prior Updating in Inflation Models)**.** *Prior on inflation rate* $\theta \sim N(2\%, 0.5\%^2)$*. Data likelihood* $N(2.2\%, 0.3\%^2)$*. Posterior mean* $\propto \frac{2/0.5^2 + 2.2/0.3^2}{1/0.5^2 + 1/0.3^2} \approx 2.15\%$*.*

Interview Question 3 (Two Sigma Style)**.** ***Question:*** *In equity risk modeling, prior on beta is* $N(1, 0.2^2)$*. You observe 100 returns with sample beta 1.1 and SE 0.1. Compute posterior beta and approximate 95% credible interval using proportionality.*

Solution: *Precision ratio: prior* $1/0.04$*, data* $1/0.01 = 100$*. Posterior mean* $\propto (1\cdot 25 + 1.1\cdot 100)/125 = 1.08$*. Posterior SD* $\propto 1/\sqrt{125} \approx 0.089$*. CI* $\approx 1.08 \pm 1.96\cdot 0.089$*.*

4 Regression Proportionalities

4.1 Linear Regression Structure

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad R^2 \propto \frac{\text{Explained Variance}}{\text{Total Variance}}$$

What to Remember: - OLS minimizes RSS; assumptions: linearity, independence, homoscedasticity. - Standard error $\propto \sqrt{(1 - R^2)/(n - p)}$. - Proportional: Beta interpretation as change per unit X. - Code: scikit-learn LinearRegression; C++ for matrix ops.

When to Use: - Equity factor models: Regress returns on market, size. - Backtesting: Predict credit spreads from macro variables. - Interview: Jane Street on collinearity effects. - In quant workflows: Model bond z-spreads on interest rates.

Example 8 (Equity Risk Factor Model)**.** *Regress stock returns on market:* $\beta \propto Cov(R_s, R_m)/Var(R_m)$*.*

4.2 Multiple Regression Proportional Adjustments

$$VIF_j \propto \frac{1}{1-R_j^2} \quad \text{collinearity inflation}$$

What to Remember: - VIF ¿5 indicates multicollinearity; ridge regression shrinks betas $\propto 1/(\lambda+$ eigenvalue). - Adjusted R^2 penalizes for p: $\propto 1-(1-R^2)(n-1)/(n-p-1)$. - Code: Statsmodels for VIF; SQL for data prep.

When to Use: - Multi-factor models for risks: Handle correlated predictors like rates and spreads. - Inflation modeling: Adjust for collinear economic indicators. - Interview: Citadel on VIF in high-dim data. - In quant workflows: Use in basis risk models with multiple currencies.

Example 9 (Inflation Zero Rates Modeling)**.** *Regress rates on factors; if VIF=5, SE inflates by* $\sqrt{5} \approx 2.24$.

5 Time Series Proportionalities

5.1 AR(1) Model

$$X_t = \phi X_{t-1} + \epsilon_t \quad \text{Stationarity: } |\phi| < 1$$

ACF $\rho_k \propto \phi^k$.

What to Remember: - Mean reversion speed $\propto 1-\phi$; half-life $-\ln(2)/\ln(\phi)$. - Forecast: $E[X_{t+h}] \propto \phi^h X_t$. - Dickey-Fuller test for unit root. - Code: ARIMA in statsmodels; custom in C++.

When to Use: - Mean-reverting spreads: Model basis convergence. - FX spot: Forecast short-term moves. - Interview: Hudson on stationarity tests. - In quant workflows: AR for interest rate paths in backtesting.

Example 10 (Basis Risk Modeling)**.** *Basis spreads with* $\phi = 0.9$*: Long-run mean reversion time* $\propto 1/(1-0.9) = 10$ *periods.*

5.2 GARCH(1,1) Proportionalities

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad \text{Persistence: } \alpha+\beta < 1$$

What to Remember: - Unconditional variance $\omega/(1-\alpha-\beta)$. - Proportional: Volatility clustering if $\alpha > 0$. - Extensions: EGARCH for asymmetry. - Code: arch library in Python; C++ for optimization.

When to Use: - Volatility modeling: Capture clustering in equity returns. - Risk backtesting: Simulate conditional VaR. - Interview: IMC on GARCH vs. stochastic vol. - In quant workflows: Apply to credit spread volatility for stress testing.

Example 11 (Volatility Clustering in Positions)**.** *With* $\alpha = 0.1$*,* $\beta = 0.8$*: Unconditional var* $\propto \omega/(1-0.9) = 10\omega$.

6 Advanced Concepts: Multivariate and Nonparametric

6.1 PCA Proportional Decomposition

$$\text{Eigenvalue}\lambda_i \propto \text{Variance Explained by PC}_i\text{Variance Explained by PC}_i{}_{\text{Variance Explained by PC}_i{}_{\text{Variance Explained by PC}_i}}$$

Total explained $\propto \sum \lambda_i/\text{trace}(\Sigma)$.

What to Remember: - PCs are orthogonal; select by cumulative variance ¿80%. - Proportional: Factor loading = eigenvector. - Scree plot for elbow. - Code: sklearn.decomposition.PCA; SQL for cov matrix.

When to Use: - Dimensionality reduction in multi-factor models. - Correlated risks: Decompose equity and rates. - Interview: Two Sigma on PCA in covariance estimation. - In quant workflows: PCA on inflation and rate factors for compression.

Example 12 (Risk Factor Reduction)**.** *For correlated rates, first PC explains 80%: Dimensionality reduction factor* $\propto 1/5$ *for 5 factors.*

6.2 Kernel Density Estimation

$$\hat{f}(x) \propto \frac{1}{nh} \sum K\left(\frac{x - X_i}{h}\right)$$

Bandwidth $h \propto n^{-1/5}$.

What to Remember: - Gaussian kernel common; optimal h by cross-validation. - Proportional: Smoother with larger h, but bias increases. - For multivariate: Product kernels. - Code: scipy.stats.gaussian$_kde$.

When to Use: - Nonparametric density for credit spreads in backtesting. - Empirical distributions for FX without assumptions. - Interview: Jane Street on bandwidth selection. - In quant workflows: KDE for empirical VaR in positions.

Example 13 (Nonparametric Credit Spread Density)**.** *For spreads data, optimal $h \propto$ Silverman's rule:* $1.06\hat{\sigma}n^{-1/5}$.

6.3 Copula Proportional Dependencies

$$C(u,v) \propto \text{Joint CDF Transformation} \quad \text{Tail Dependence} \propto \lim_{q \to 1} \frac{1 - 2q + C(q,q)}{1 - q}$$

What to Remember: - Sklar's theorem: Any joint dist = copula of margins. - Gaussian: No tail dep; Clayton: Lower tail. - Fit via max likelihood. - Code: copulae library in Python.

When to Use: - Joint modeling of equity drops and credit widenings. - Stress testing correlated risks. - Interview: Hudson on tail dep in crises. - In quant workflows: Copulas for basis and FX joint risks.

Example 14 (Joint Tail Risk in Credit and Equity)**.** *Clayton copula: Tail dependence $\propto 2^{-1/\theta}$. For $\theta = 2$, lower tail $\propto 0.5$.*

Interview Question 4 (Hudson River Trading Style)**.** ***Question:*** *Model joint defaults in credit spreads and equity drops using Gaussian copula with $\rho = 0.6$. Approximate conditional default probability if equity drops below 1% quantile.*

Solution: *Conditional prob $\propto \Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\Phi^{-1}(0.01)}{\sqrt{1-\rho}}\right)$. For PD=0.05: $\approx \Phi((\Phi^{-1}(0.05) - 0.774 \cdot -2.33)/0.8) \approx 0.12$.*

6.4 Bootstrapping Proportionalities

$$\text{Bootstrap SE} \propto \sqrt{\frac{1}{B-1} \sum (\hat{\theta}_b^* - \bar{\theta}^*)^2}$$

Convergence rate $\propto 1/\sqrt{n}$.

What to Remember: - Resample with replacement; B¿1000 for accuracy. - Percentile CI: [2.5%, 97.5%] of bootstrap dist. - Bias-corrected accelerated (BCa) for skewness. - Code: bootstrap in resample (Python); parallel in C++.

When to Use: - Nonparametric CI for VaR with small samples. - Backtesting model parameters without dist assumptions. - Interview: IMC on bootstrap vs. parametric. - In quant workflows: Bootstrap SE for interest rate model params.

Example 15 (Bootstrapping VaR for Positions)**.** *Resample 1,000 times from 200 spread changes; 99% VaR CI width $\propto \sqrt{Var(V\hat{a}R_b)/1000}$.*

6.5 Monte Carlo Simulation Proportionalities

$$\text{MC Error} \propto \frac{\sigma}{\sqrt{m}} \quad \text{Convergence: } m \propto 1/\epsilon^2$$

What to Remember: - Variance reduction: Antithetic (halve var), control variates. - Quasi-MC for faster convergence $\propto 1/m$. - Seed for reproducibility. - Code: numpy.random; C++ for high-perf sims.

When to Use: - Pricing complex derivatives with no closed form. - Simulating paths for credit spread models. - Interview: Jane Street on variance reduction. - In quant workflows: MC for inflation scenario analysis.

Example 16 (Simulating Interest Rate Paths)**.** *For Vasicek model, path accuracy $\propto 1/\sqrt{m}$; use $m = 10,000$ for 1% error on mean rate.*

Interview Question 5 (IMC Style)**.** ***Question:*** *Use Monte Carlo to estimate option price with variance $\sigma^2 = 4$. How many simulations for SE ¡ 0.01? Approximate with antithetic variates (50% variance reduction).*

Solution: *$m \propto \sigma^2/\epsilon^2 = 4/0.0001 = 40,000$. With reduction: $m \propto 0.5 \times 40,000 = 20,000$.*

7 Machine Learning Statistical Foundations

7.1 Overfitting Proportionalities

$$\text{Generalization Error} \propto \text{Training Error} + \frac{k}{n} \quad \text{(AIC-style penalty)}$$

What to Remember: - AIC = -2 log-likelihood + 2k; BIC + k ln n. - Cross-validation to estimate. - Proportional: Complexity penalty grows with k/n. - Code: sklearn.model$_{s}electionforCV$.

When to Use: - ML models for FX prediction: Penalize to avoid overfitting historical data. - Risk factor selection in equity models. - Interview: Two Sigma on bias-variance tradeoff. - In quant workflows: AIC for selecting vars in credit spread ML models.

Example 17 (ML Model for FX Prediction)**.** *With 10 features, 500 samples: Penalty* $\propto 10/500 = 0.02$*, adjust validation error accordingly.*

7.2 Regularization Proportionalities

$$L_1 \text{ penalty} \propto \lambda \sum |\beta_j| \quad \text{Sparsity: } \beta_j = 0 \text{ if } |\text{corr}| < \lambda$$

What to Remember: - Lasso (L1) for selection; Ridge (L2) for shrinkage. - Optimal λ via CV. - Proportional: Stronger λ increases sparsity. - Code: sklearn.linear$_{m}odel.Lasso$.

When to Use: - High-dim risk models: Select key factors from many predictors. - Inflation modeling with correlated vars. - Interview: Jane Street on lambda tuning. - In quant workflows: Lasso for basis risk feature selection.

Example 18 (Lasso for Risk Factor Selection)**.** *In equity model,* $\lambda = 0.1$ *shrinks small betas to zero, reducing factors by* $\propto 30\%$.

Interview Question 6 (Jane Street Style)**.** ***Question:*** *In ML-based credit spread prediction, you have Lasso with* $\lambda = 0.05$ *on 20 features. Approximate number of non-zero coefficients if correlations average 0.1.*

Solution: *Non-zero if* $|\hat{\beta}_{OLS}| > \lambda/2 \propto 0.025$. *With corr=0.1,* $\hat{\beta} \propto 0.1\sigma_y/\sigma_x \approx 0.1$*; 50% features survive.*

8 Interview Questions with Proportional Solutions

Interview Question 7 (Two Sigma Style)**.** ***Question:*** *In backtesting a credit spread model, you have 200 daily changes with sample kurtosis 5 (vs. normal 3). Approximate the impact on 99% VaR compared to normal assumption. Use proportionality for fat tails.*

Solution: *Kurtosis excess = 2 implies heavier tails. Cornish-Fisher expansion: VaR adjustment* $\propto \frac{1}{6}(z^3 - 3z)\kappa/4 \approx 0.1z$ *for z=2.33, so VaR increases by 23%.*

Interview Question 8 (IMC Style)**.** ***Question:*** *For positions, regress returns on 3 factors with* $R^2 = 0.6$*. If adding a 4th factor increases* R^2 *to 0.65, test significance at 5%. Approximate F-statistic proportionality.*

Solution: $F = \frac{(R_2^2 - R_1^2)/(p_2 - p_1)}{(1-R_2^2)/(n-p_2)} \propto \frac{0.05/1}{0.35/(n-5)}$. *For n=100: F14.3 ¿* $F_{1,95}(0.05) = 3.94, significant.$

Interview Question 9 (Hudson River Trading Style)**.** ***Question:*** *In FX spot modeling, you have AR(1) with* $\phi = 0.95$ *over 252 days. Approximate the half-life of shocks and proportional impact on forecast error for 10-day horizon.*

Solution: *Half-life* $\propto -\ln(2)/\ln(\phi) \approx 13.5$ *days.*

Forecast variance $\propto \sigma^2(1-\phi^{2h})/(1-\phi^2)$*. For h=10: 0.99* $\sigma^2/(1-0.9025)10.1\sigma^2$.

9 Practical Risk Modeling Applications

9.1 VaR and ES Proportionalities

$$\text{ES}_\alpha \propto \frac{\text{VaR}_\alpha}{1-\alpha} \int_\alpha^1 q_u du$$

For normal: ES VaR + 0.5 σ.

What to Remember: - VaR = quantile; ES = conditional tail expectation. - Backtesting: Christoffersen test for clustering. - Proportional: ES ¿ VaR by factor depending on tail shape. - Code: QuantLib for VaR; Python for historical sim.

When to Use: - Portfolio risk: Compute ES for regulatory reporting. - Stress testing credit spreads. - Interview: Citadel on coherent measures. - In quant workflows: ES for tail risks.

Example 19 (Portfolio VaR Backtesting)**.** *If exceptions ¿ expected, proportionality: Kupiec test statistic* $\propto 2\ln[(x/n)^x(1-x/n)^{n-x}/p^x(1-p)^{n-x}]$.

10 Approximation Techniques for Quants

10.1 Taylor Expansion in Statistics

$$\hat{\theta} \approx \theta_0 + \frac{\partial l}{\partial \theta}(\theta - \theta_0) \quad \text{delta method var} \propto (\frac{\partial g}{\partial \theta})^2 \text{Var}(\hat{\theta})$$

What to Remember: - First-order for means; second for vars. - Delta method for transformed params (e.g., log-var). - Code: SymPy for derivatives.

When to Use: - Approximating SE of ratios in risk models. - Quick calcs in backtesting. - Interview: Two Sigma on approximations. - In quant workflows: Delta for Sharpe in equity models.

Example 20 (Approximating Sharpe Ratio SE)**.** *SR* $= \mu/\sigma$, *Var(SR)* $\propto (1 + 0.5SR^2)/n$.

11 Memory Aids and Interview Tips

11.1 Key Proportional Relationships

1. Confidence width $\propto 1/\sqrt{n}$
2. Power $\propto \sqrt{n}\delta/\sigma$
3. $R^2 \propto 1 - \text{RSS/TSS}$
4. Bayes factor $\propto P(D|H_1)/P(D|H_0)$
5. BIC penalty $\propto k \ln n$
6. MC simulations $\propto 1/\epsilon^2$
7. Lasso shrinkage $\propto \lambda$

11.2 Problem-Solving Strategy

1. Identify proportional scaling. 2. Check limiting cases. 3. Use approximations for quick estimates. 4. Relate to financial contexts like risk factors.