GARCH and EGARCH Models for Estimating Volatility: AAPL

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1. Problem Statement

Volatility is a central concept in financial markets, influencing risk management, derivatives pricing, and strategic asset allocation. Accurately estimating volatility enables financial professionals to better assess risk, optimize portfolios, and refine investment strategies.

This case study investigates how to estimate and forecast the volatility of Apple Inc. (AAPL) using historical closing price data. Volatility is typically modeled using time series techniques, and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is widely used for its ability to capture time-varying volatility.

The key objectives of this analysis are:

- **Problem Definition:** Estimate AAPL's volatility from historical price data to support decision-making in risk management and portfolio optimization.
- **Data Collection and Preprocessing:** Gather historical closing prices for AAPL, compute daily returns, and address any missing data points.
- **Model Selection:** Fit a GARCH(1,1) model to the return series, a common model for capturing volatility clustering and time-varying variance.
- **Model Fitting:** Estimate the parameters of the GARCH model and assess its adequacy through residual diagnostics.
- **Volatility Forecasting:** Forecast future volatility over a 30-day horizon and compare model performance.

This paper demonstrates how the GARCH model is applied to time series data, the importance of model validation, and the value of volatility forecasting for financial decision-making.

2. Methodology

The methodology follows a clear sequence from data collection to model estimation, validation, and forecasting:

1. **Data Collection and Preprocessing:** Historical closing prices for AAPL were downloaded from Yahoo Finance. Log returns were calculated as:

$$r_t = 100 \times \log\left(\frac{P_t}{P_{t-1}}\right)$$

   Missing values were removed using the ‘dropna()‘ function, and the time series was aligned to ensure consistency.

2. **Initial Modeling with GARCH(1,1):** A constant mean model combined with a GARCH(1,1) volatility process was fitted. This model assumes returns are normally distributed with time-varying variance:

$$
\begin{aligned}
r_t &= \mu + \epsilon_t \\
\epsilon_t &= z_t \sqrt{h_t} \\
h_t &= \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}
\end{aligned}
$$

3. **Model Diagnostics:** Residuals and autocorrelations were examined to assess model adequacy. Based on these diagnostics, the model was refined to better capture the volatility dynamics.

4. **Improved Modeling with EGARCH(1,1):** An EGARCH(1,1) model was implemented to address shortcomings in the GARCH model residuals. The EGARCH model accounts for asymmetries in volatility and allows for leverage effects, which are commonly observed in financial time series.

5. **Forecasting:** Both the GARCH(1,1) and EGARCH(1,1) models were used to forecast volatility over a 30-day horizon. Forecasts were compared to assess model performance and illustrate the advantage of model refinement.

3. Initial Model Attempt: GARCH(1,1)

The GARCH(1,1) model was fitted to the return series, assuming a constant mean. The results suggested that both the ARCH and GARCH terms were statistically significant, capturing some degree of volatility clustering.

- **Mean Equation:** $\mu = 0.1408$ (significant at 1%)
- **Volatility Equation:** $\omega = 0.1338$, $\alpha = 0.0961$, $\beta = 0.8719$
- **Log-likelihood:** -2747.48, **AIC:** 5502.96

Although the model was statistically valid, residual diagnostics revealed autocorrelation in squared standardized residuals, indicating a potential misfit.

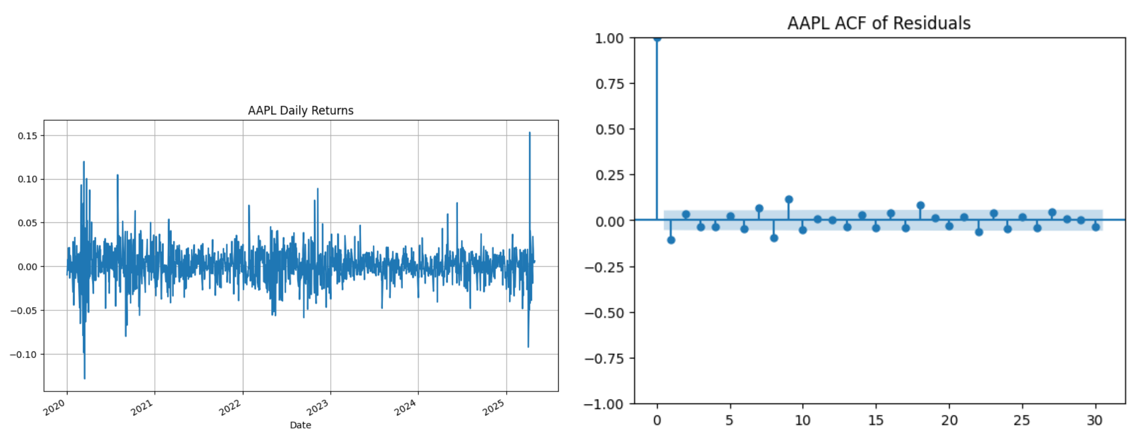

Figure 1: Daily Returns and ACF of Standardized Residuals (GARCH)

Forecasting from the GARCH(1,1) model provided a baseline estimate of future volatility:

Despite these initial results, the diagnostics prompted further exploration, leading to the adoption of a more sophisticated model.

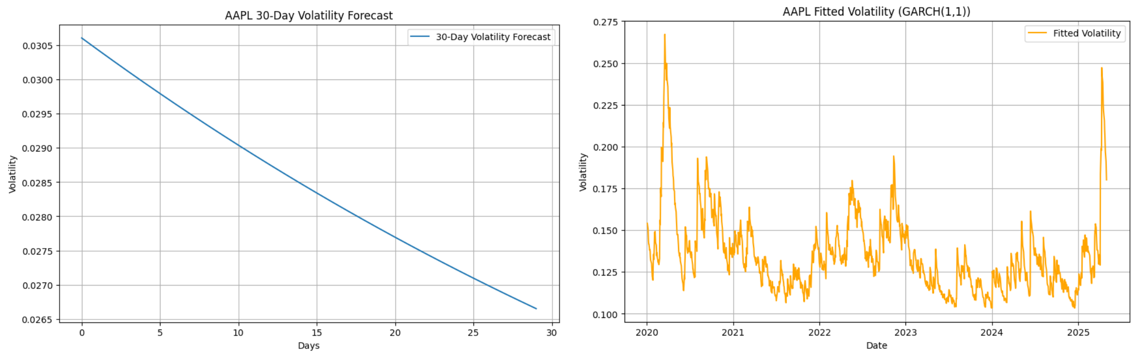

Figure 2: 30-Day Volatility Forecast and Fitted Volatility (GARCH(1,1))

4. Model Validation and Refinement

Residual diagnostics from the GARCH model revealed patterns inconsistent with the white noise assumptions. The standardized residuals and their distribution were analyzed to check for normality and independence:

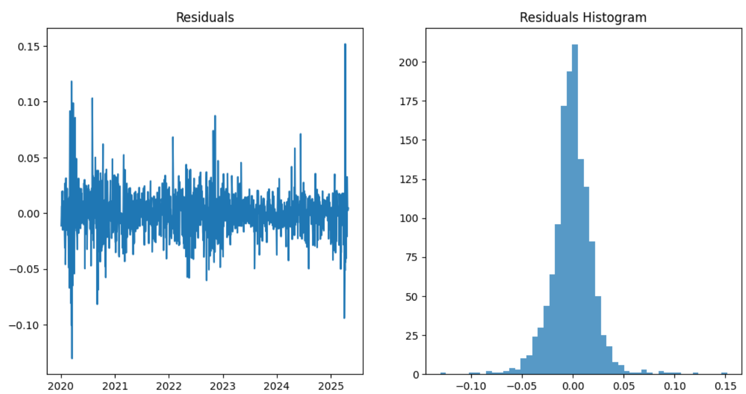

Figure 3: Standardized Residuals and Histogram (GARCH(1,1))

These plots revealed fat tails and residual structure unaccounted for by the GARCH specification. To address this, an EGARCH(1,1) model was implemented. This model captures asymmetric volatility responses and better accounts for leverage effects in financial data.

The EGARCH(1,1) model estimates were:

- **Mean Equation:** $\mu = 0.1323$ (significant at 1%)
- **Volatility Equation:** $\omega = 0.0525$, $\alpha = 0.2053$, $\beta = 0.9667$
- **Log-likelihood:** -2748.05, **AIC:** 5504.10

Post-estimation diagnostics showed notable improvements in residual behavior:

The EGARCH residuals exhibited closer alignment with white noise, showing reduced kurtosis and autocorrelation. This validated the EGARCH model as a better fit compared to the GARCH model.

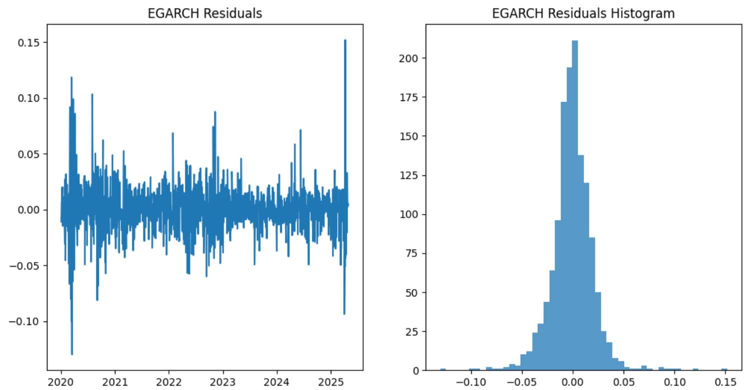

Figure 4: Standardized Residuals and Histogram (EGARCH(1,1))

5. Results and Forecast Comparison

The final comparison focused on the volatility forecasts from both models. A combined forecast plot illustrates how the two models behave over a 30-day horizon:

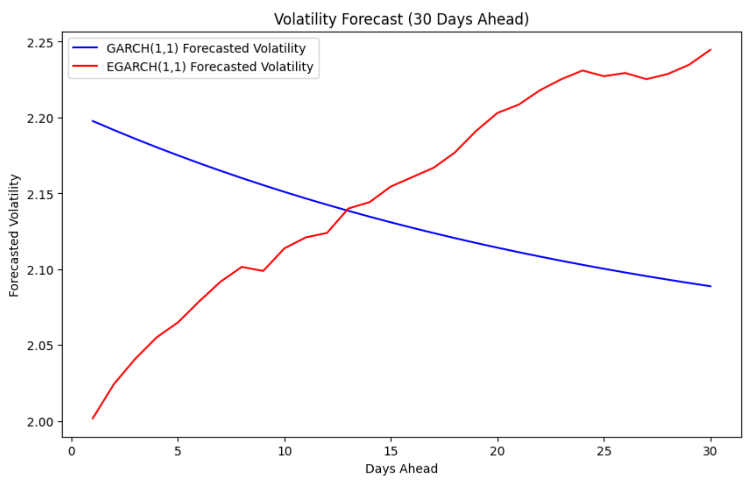

Figure 5: 30-Day Ahead Forecasted Volatility: GARCH(1,1) vs EGARCH(1,1)

To benchmark the forecasts, they were compared against a 30-day rolling historical volatility:

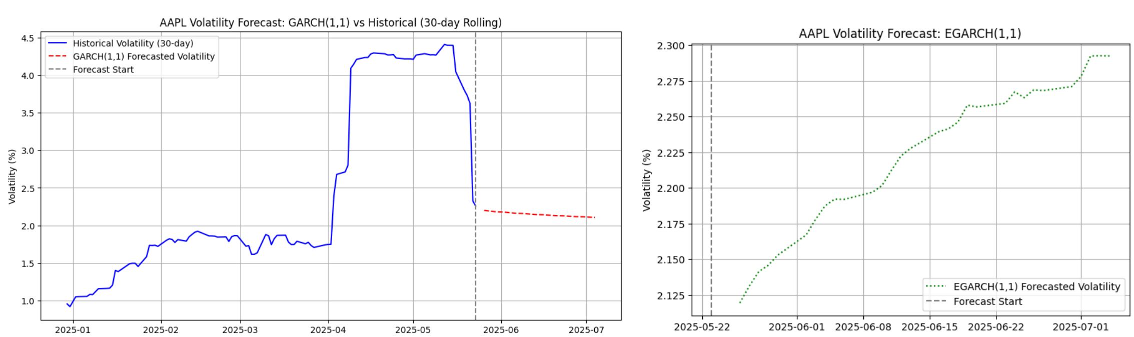

Figure 6: Forecasted Volatility vs Historical Rolling: GARCH(1,1) and EGARCH(1,1)

These results reinforce the importance of model validation in volatility forecasting. While the GARCH(1,1) model captures general trends, the EGARCH(1,1) model offers superior flexibility, producing forecasts that align more closely with observed market behavior. This highlights the value of refining models based on diagnostic checks for more accurate and reliable predictions.

6. Conclusion (STAR Framework)

- **Situation:** Accurately modeling volatility is essential in quantitative finance, particularly in applications like risk management, derivatives pricing, and portfolio construction. However, asset returns often display volatility clustering, asymmetry, and non-normality—patterns that simple models may overlook.
- **Task:** The goal was to estimate and forecast the volatility of Apple Inc. (AAPL) using historical return data, while ensuring the model provided both statistical validity and practical forecasting utility.
- **Action:** A GARCH(1,1) model was initially fitted to the return series. Diagnostic analysis revealed residual autocorrelation and fat tails, suggesting a misfit. An EGARCH(1,1) model was then implemented to address these issues, capturing asymmetry and improving residual behavior. Each model was rigorously evaluated through standardized residuals, histograms, and 30-day ahead forecast plots.
- **Result:** The EGARCH(1,1) model outperformed the GARCH(1,1), delivering more realistic and interpretable volatility forecasts. Its improved handling of market asymmetries and tail behavior led to better alignment with observed volatility. This demonstrates how model validation and refinement directly enhance forecasting accuracy—an insight with clear applications in trading, hedging, and portfolio risk management.