

*Basic Research as Open Source Project between Science and Industry:*

# Computing the Condition Numbers of the Quantum Algebraic Attack on chosen Cryptosystems

*Identifying Security Levels of globally used Encryption*



# Global Industrial Context



Massive global funding for the development of **Quantum Technologies** bring

- **hope** for solving health care, environmental and other global problems but also
- **threats** to global data security on the other side





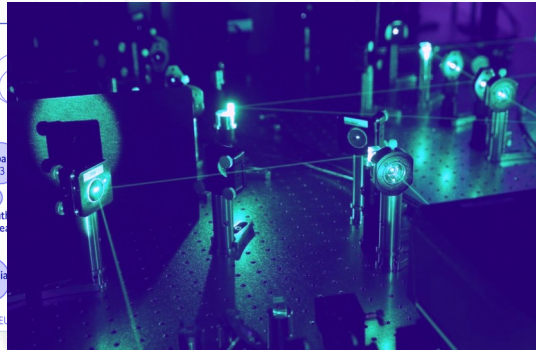
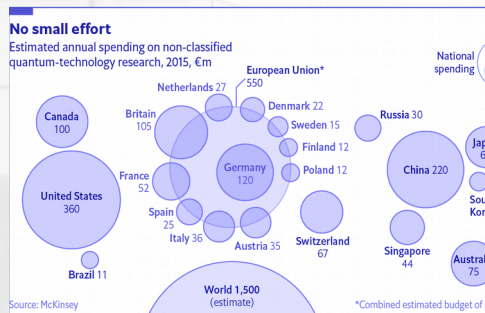
# Post-Quantum Threat Intelligence



## Threat

Confidentiality, Integrity and Authenticity of data in transport and rest are in **danger**. [TLS and VPN (Web, Mobile), SSH, PGP (Email), Databases, etc.]

The international competition in the development of Quantum Technologies it is often called a **war race**.



## Intelligence

In parallel, the development of Quantum Secure Technologies is massively funded as well.

# Our Mission



## Basic Research for Post-Quantum Threat Intelligence by

- Identifying **Quantum Secure Encryption** among globally algorithms wrt. recently published *Quantum Algebraic Attack* by Chen-Gao
- Refining requirements for the development of new **Quantum Secure Encryption Algorithms**, resisting the quantum algebraic attack

### Remark

The *Quantum Algebraic Attack* is not in the scope of the NIST Post-Quantum standardization process which started in 2017! (only for asymmetric systems)

We investigate the security levels of symmetric crypto systems!

$$\begin{aligned} & (y f(2x) + d_0(x)^2) y_1 + c_2(x) y_2 + c_3(x) y_3 \\ & (x+1) = \left( \frac{x(x-2)}{2} \right) 1 + (x(x-1)) 0 + \left( \frac{x(x-1)}{2} \right) \\ & )^2 \\ & = \left( \frac{(x-1)(x-2)}{2} \right) 1 + (x(x-1)) 0 + \left( \frac{x(x-1)}{2} \right) \\ & f_p(x, y) \\ & )^2 (y + 6x + 2)^4 - (y + 3)^4 + 8x^2 (y + 9x + 6)^4 (y + 1 \\ & 1) (x + 6)^4 (x + 9)^4 \quad x(x + 1) (x + 2)^4 \\ & - 9b + \sqrt{3} \sqrt[4]{4a^3 + 27b^2} y^3 - 6x^2 (y + 10x + 8) x + 1 \\ & \frac{2^{1/3} 3^{2/3}}{x(x+6)^2} \quad (y+9x+ \\ & \frac{(y+8x)^2}{(1-i\sqrt{3})(-9b+\sqrt{3}\sqrt[4]{4a^3+27b^2})^{1/3}} \quad (y+8x+ \\ & 1/3 + \frac{2^{1/3} 3^{2/3} x + 9}{(y+8x)^2 (y+7x+4)^4 (y+ \end{aligned}$$

# Scope of our Mission



## Post-Quantum Security for Binary and Quantum Technologies

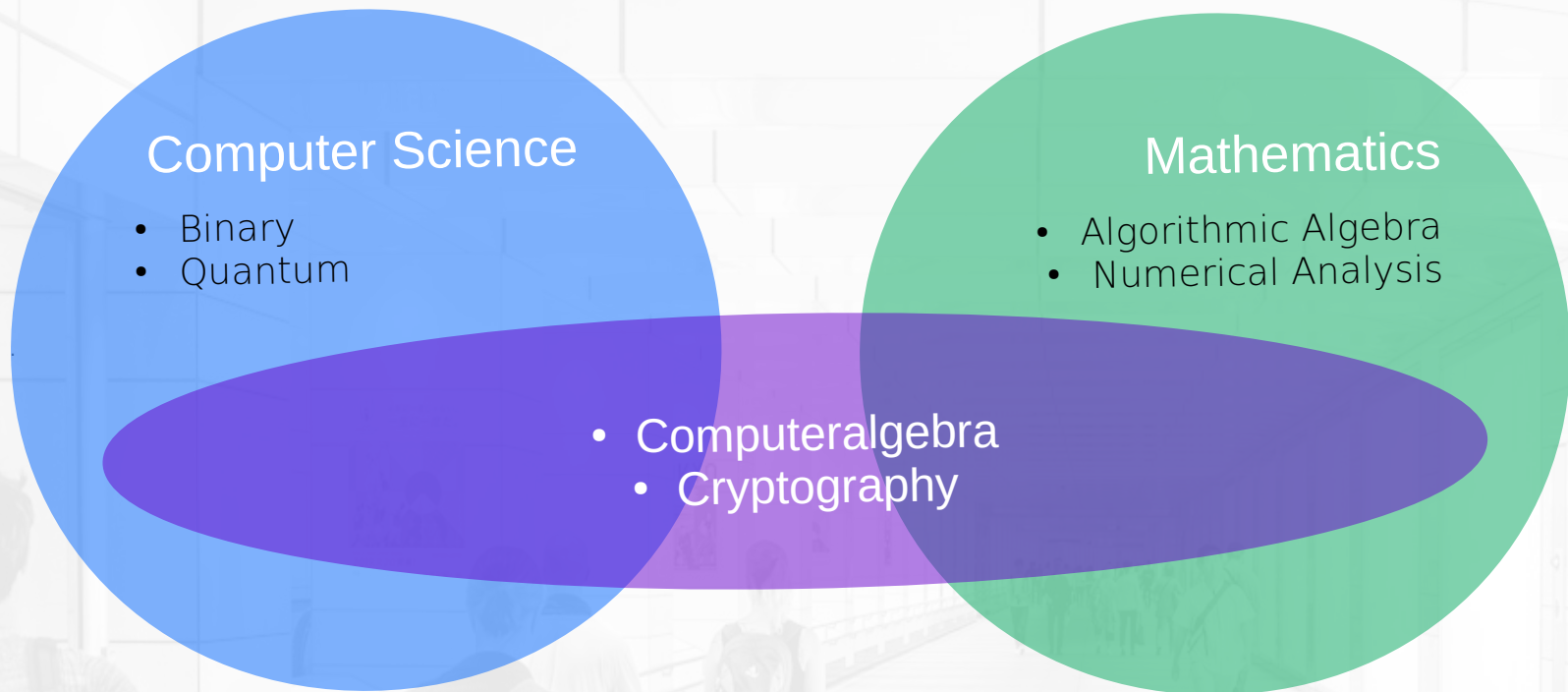
- Our results cover both, the **binary and quantum computing aspects!**
- Therefore our results will bring benefit to the development of **quantum resistant binary and quantum technologies.**





# Complexity of the Topic

## Scientific and Technological Context



# Ongoing Research in Need for Funding



*Basic Research as Open Source Project between Science and Industry*

Results are published on Github, where we invite collaborators and reviewers:

Overview:

[https://github.com/XeniaGabriela/QAA\\_Condition\\_Nr](https://github.com/XeniaGabriela/QAA_Condition_Nr)

Paper with achieved Results:

[https://github.com/XeniaGabriela/QAA\\_Condition\\_Nr/blob/master/official\\_paper/QAA\\_on\\_AES\\_paper.pdf](https://github.com/XeniaGabriela/QAA_Condition_Nr/blob/master/official_paper/QAA_on_AES_paper.pdf)

Considerations for ongoing Research:

[https://github.com/XeniaGabriela/QAA\\_Condition\\_Nr/blob/master/results\\_nonnenmann\\_rump/The%20case%20of%20nonzero%20s%20.pdf](https://github.com/XeniaGabriela/QAA_Condition_Nr/blob/master/results_nonnenmann_rump/The%20case%20of%20nonzero%20s%20.pdf)

# Prototype: Security Level of AES



- 1) Compute the value of the condition number  $\kappa$  in the complexity of the Quantum Algebraic Attack by Chen-Gao on AES.

AES	$N_k$	$N_r$	#Vars	#Eqs	T-Sparseness	Complexity
AES-128	4	4	1792	4400	101376	$2^{68.61} c\kappa^2$
AES-128	4	6	2624	6472	151680	$2^{70.68} c\kappa^2$
AES-128	4	8	3456	8544	201984	$2^{72.16} c\kappa^2$
AES-128	4	10	4288	10616	252288	$2^{73.30} c\kappa^2$
AES-192	6	12	7488	18096	421248	$2^{76.59} c\kappa^2$
AES-256	8	14	11904	29520	696384	$2^{78.53} c\kappa^2$

- 2) Compute the value of the condition number of crypto systems in the Chen-Gao paper (Trivium, Keccak, MPKC)
- 3) Investigate more affected crypto systems (crypto systems which can be reduced to a Boolean Multivariate Quadratic Equation System).



# Steps for Crypto Systems in Chen-Gao Paper



## 1) Compute the condition number of the Macaulay Matrix of the S-Box for AES

- a) For the classical notion of a condition number  $\kappa$  including zero singular values,  $\kappa(\text{AES}) = \text{infinity}$  (see "QAA\_on\_AES\_paper.pdf" on Github)

Done

*Cooperation with Prof. Dr. S. Rump, Institute for reliable computing, TU Hamburg*

- b) For the modified notion of a condition number excluding zero singular values,  $\kappa(\text{AES})$  has to be explicitly computed (see "the case of nonzero s.pdf" on Github)

Ongoing

*Cooperation with Mathworks*

## 2) With identified mechanisms of step 1b), compute the condition numbers of Trivium, Keccak and MPKC.

Open

Polynomials are already identified by Chen-Gao!

*Cooperation with Mathworks*

# Core Team



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- CEO Quant-X Security & Coding
- Information Security Specialist
- Fachreferentin Industrie  
Fachgruppe Computeralgebra



## Dr. Peter Nonnenmann

- Independent Scientific  
Researcher
- Quantum Theory
- Computer Science



Fachgruppe Computeralgebra



# Feedback and Collaborators



Fachgruppe Computeralgebra

In August 2020 we will have a meeting with the Computeralgebra Professionals Group Germany's Speaker to identify potential further cooperation on the topic.

