Institute for QC, University of Waterloo

(IQC)

Grand Unification of Quantum Algorithms

Isaac Chuang

NMR composite pulses 1983

Factoring (1994 Shor)

Simulation (1996) (Feynman)

Search (1996 Grover)

Arbitrarily Accurate Pulses 2004

Q. Phase Estim.

Q. Walks

Ampl. Amplif. 1997

Linear Systems HHL 2005

Lin. Comb. U (BCCKS 2015)

Adiabatic 2000

Signal Processing (LC 2016)

Fixed Point Adiabatic Amplification

Q. Sing. Value Transform

Gilyen, Su, Low, Wiebe

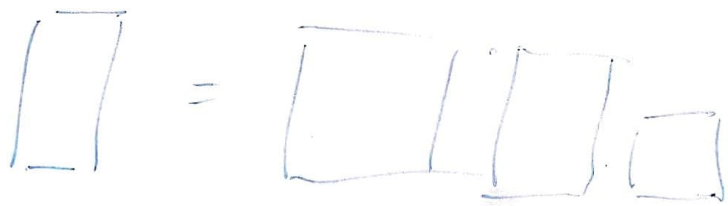
1) Search: Single qubit rotation

2) Simulation: Map H (non-unitary) to U (unitary)

3) Factoring: Focus on phase of eigenvalues

Canon: Sing. values

Recall... SVD for any $m \times n$ matrix A.
Sing. V. Decomposition



$A = W \cancel{\Sigma} V^{\dagger}$

unitary $m \times m$ | Diag. | Unitary $n \times n$ → Singular Vectors

Sig Values $\in [0,1]$

Core concept for
- Principal components (machine learning)
- Low rank approx. (collaborative filtering)
- Pseudoinverses
- Bipartite entanglement

SVD for large matrices hard to compute!

Singular Values for Quantum Algorithms

New: "unitary embedding of A"

based on

Theorem 1) Quantum Singular Value Transform

Each plane defined by a common pair of left & right singular vectors can be manipulated independently as if like separate SU(2) qubits w/o computing SVD.

Theorem 2) Quantum Signal Processing

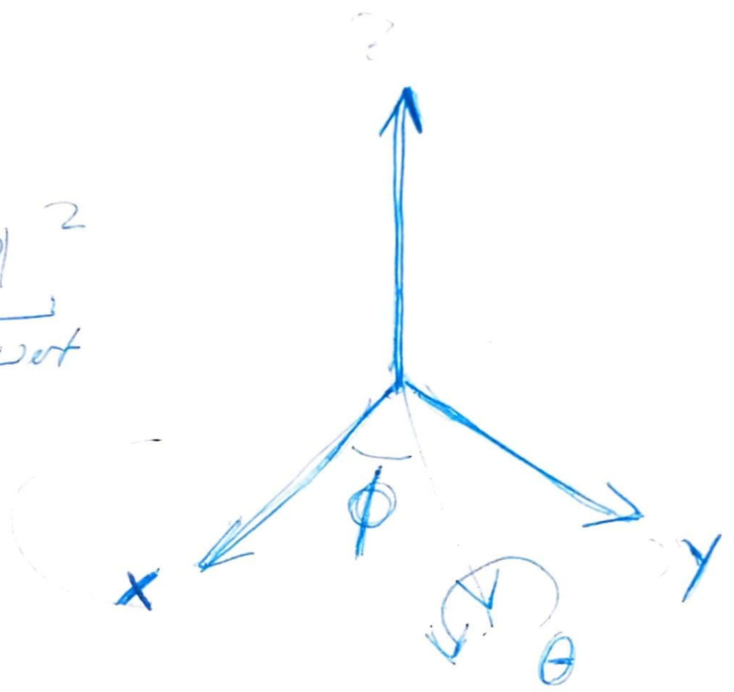
Each singular value can be transformed by any arbitrary polynomial.

Algor.	Sing. Vect.	Sing. Val.	Embedding	Transform
Search find t	$\vert s\rangle, \vert t\rangle$ start, target	$c = \langle s \vert t\rangle$	$\begin{bmatrix} c & \cdots \\ \vdots & \ddots \end{bmatrix}$	$c \to 1$
Simulation e^{-iHt} approx.	$\vert E\rangle$ Energy eigenstates	E Eigenenergies	$\begin{bmatrix} H & \cdots \\ \vdots & \ddots \end{bmatrix}$	$H \to \cos(H)$
Factoring eigenphase λ of U	Eigenvectors of $(1+U^\dagger)\cdot(1+U)$	$\cos(\lambda)$ ↑ eigenphases	$\begin{bmatrix} \frac{1+U}{\sqrt{2}} & \cdots \\ \vdots & \ddots \end{bmatrix}$	$\lambda \to \begin{cases} 0 & \text{if } \lambda < \frac{1}{2} \\ 1 & \text{otherwise} \end{cases}$

Composite Pulses

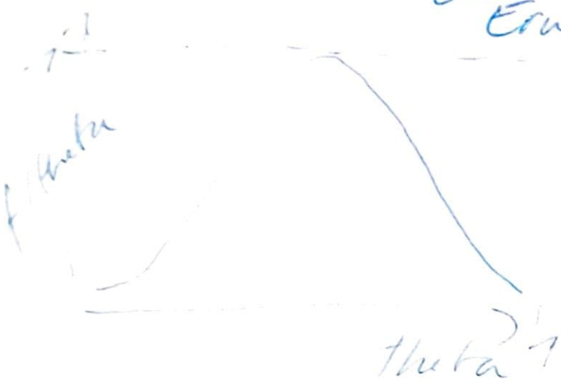
- Single qubit rotation around axis in x-y plane:

$$R_{\phi}(\theta) = e^{-i\frac{\theta}{2}[X\cos\phi + Y\sin\phi]}$$



- one pulse:

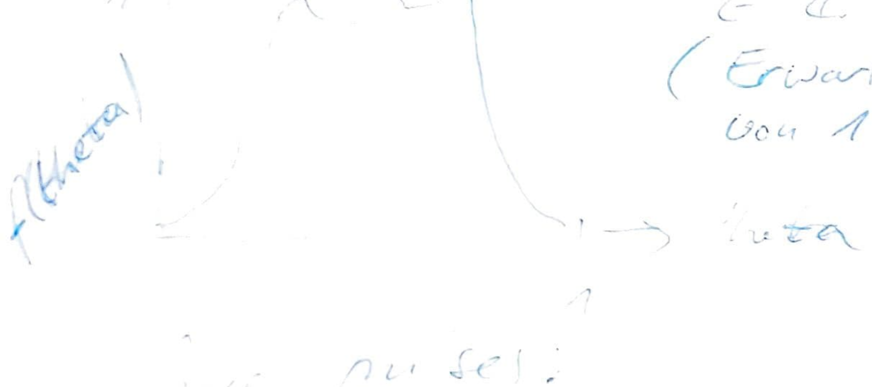
$$f(\theta) = |\underbrace{\langle 1 | R_0(\theta) | 0 \rangle}_{\text{Erwartungswert}}|^2$$



- three pulses:

$$f(\theta) = |\underbrace{\langle 1 | R_0(\theta) \cdot R_{1.5}(\theta) \cdot R_0(\theta) | 0 \rangle}_{\in \mathbb{C}}|^2$$

(Erwartungswert von 1 im Zustand 0)



- five pulses:

Composite gate operations

- gate sequence:
$\vec{R_\phi} = R_{\phi_{k-1}}(\theta) \cdots R_{\phi_0}(\theta) = \begin{bmatrix} P(\cos\theta) & \cdot \\ \cdot & \cdot \end{bmatrix}$

Q. Signal Processing Theorem:

~~[illegible]~~ P well-behaved $\rightarrow$
$\Rightarrow$ There exists $\vec{\phi}$ such that

Skalarprodukt $\langle 0 | R_{\vec{\phi}} | 0 \rangle = P(\cos\theta)$
$\in \mathbb{C}$

Zeilenvektor "bra" — Spaltenvektor "ket"
(aus Dualraum)

Quantum Search Problem

- $a \in [0, 2^n - 1]$ and $U|a\rangle = \begin{cases} -|a\rangle & \text{if } a = a_0 \\ |a\rangle & \text{otherwise} \end{cases}$

$\rightarrow$ find a_0

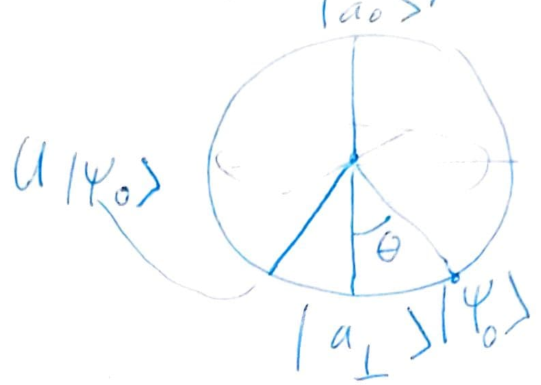
- Cost = # of uses of U
  - $\rightarrow$ Classical: $\sim N = 2^n$
  - $\rightarrow$ Quantum: $\sim \sqrt{N} = 2^{\frac{n}{2}}$ Grover

Hidden qubit: Start with equal superposition state

- $|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{a=0}^{N-1} |a\rangle$
- Oracle Operator: $U|\Psi_0\rangle = \overbrace{-\frac{1}{\sqrt{N}}|a_0\rangle}^{-\sin\theta|a_0\rangle} + \overbrace{\underbrace{\frac{1}{\sqrt{N-1}} \sum_{a \neq a_0} |a\rangle}_{\text{unchanged rest of states}}}^{\cos\theta|a_\perp\rangle}$

$|a_\perp\rangle$ composite state

Bloch Sphere.



- Initial state:

$$|\psi_0\rangle = \sin\theta\,|a_0\rangle + \cos\theta\,|a_\perp\rangle$$

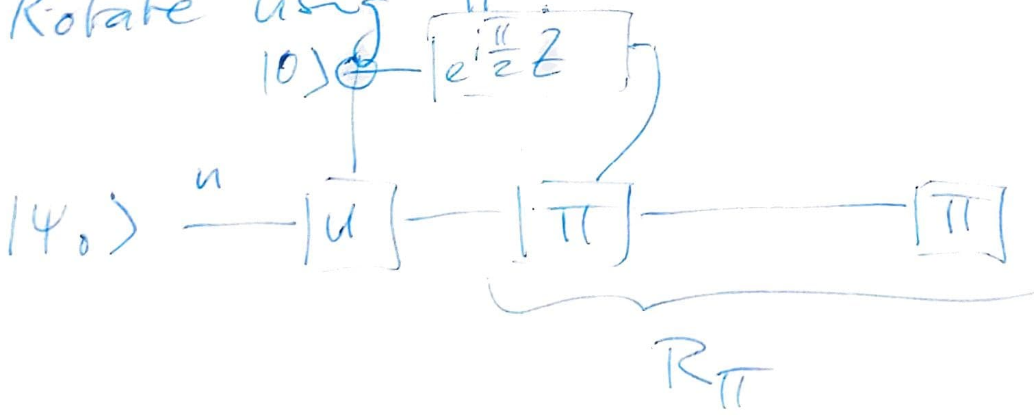
- Oracle:

$$U|\psi_0\rangle = -\sin\theta\,|a_0\rangle + \cos\theta\,|a_\perp\rangle$$

θ is unknown!

- Projector: $\Pi = 2\underbrace{|\psi_0\rangle}_{\text{Spalten}\cancel{\text{Zeilen}}\text{vektor}}\overbrace{\langle\psi_0|}^{\text{Zeilenvektor}} - I$

- Rotate using Π:



Quantum Search as Signal Processing

=> Fixed point search as alternative to Grover!

(Fixed point adiabatic amplification)

Question would it work with D-Wave ?

QUANTUM SINGULAR VALUE TRANSF.

- Quantum Simulation by Singular Value Transform

Performance is optimal for Qubitization and Quantum Signal Processing

Considering

1. Simulation time t
2. Simulation error ϵ
3. Hamiltonian sparsity d

- Factoring by Singular Value Transform

Binary search on Q. Sing. Value Transform Output.

– HHL by Q. Singular Value Transform

Linear Systems solved

New Algorithms found yet with this approach?

⟹ Quantum Channel Discrimination Techniques

Next Steps in Lab?

→ Block encoding (which can be realized?)

→ . . .