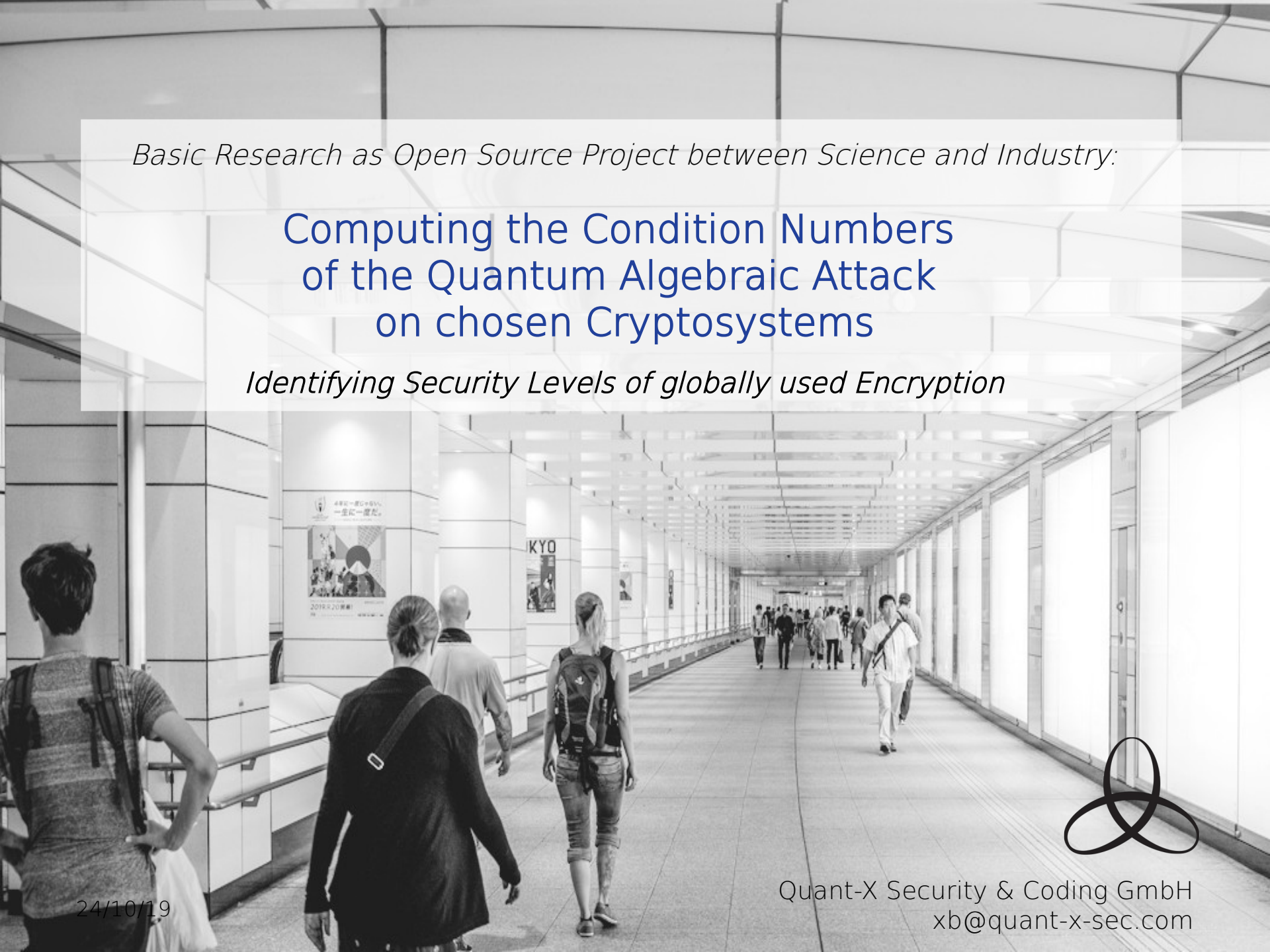

Basic Research as Open Source Project between Science and Industry:

Computing the Condition Numbers of the Quantum Algebraic Attack on chosen Cryptosystems

Identifying Security Levels of globally used Encryption

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24/10/19

Global Industrial Context

Massive global funding for the development of **Quantum Technologies** brings

- hope for solving health care, environmental and other global problems

but also

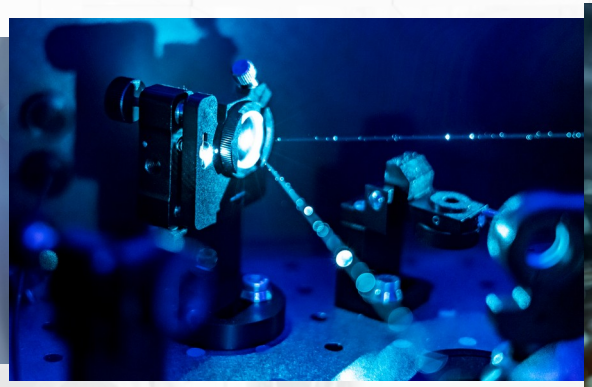
- threats to global data security on the other side

Post-Quantum Threat Intelligence

Threat

Confidentiality, Integrity and Authenticity of data in transport and in rest are in danger. [TLS and VPN (Web, Mobile), SSH, PGP (Email), Databases, etc.]

The international competition in the development of Quantum Technologies it is often called a war race.


Intelligence

In parallel, the development of Quantum Secure Technologies is massively funded as well.

Our Mission

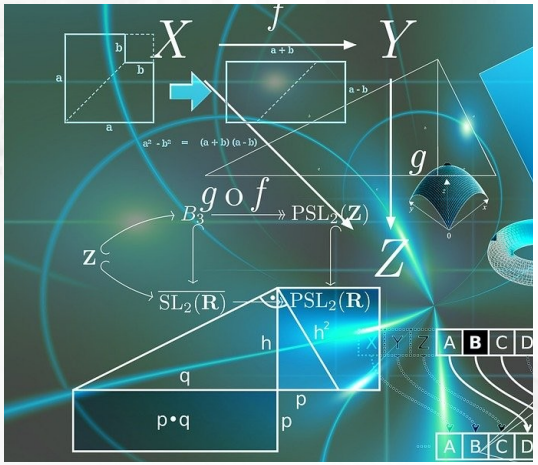
Basic Research for Post-Quantum Threat Intelligence by

- Identifying Quantum Secure Encryption among globally algorithms wrt. recently published *Quantum Algebraic Attack* by Chen-Gao
- Refining requirements for the development of new Quantum Secure Encryption Algorithms, resisting the quantum algebraic attack

Remark

The *Quantum Algebraic Attack* is not in the scope of the NIST Post-Quantum standardization process which started in 2017! (only for asymmetric systems)

We investigate the security levels of symmetric crypto systems!



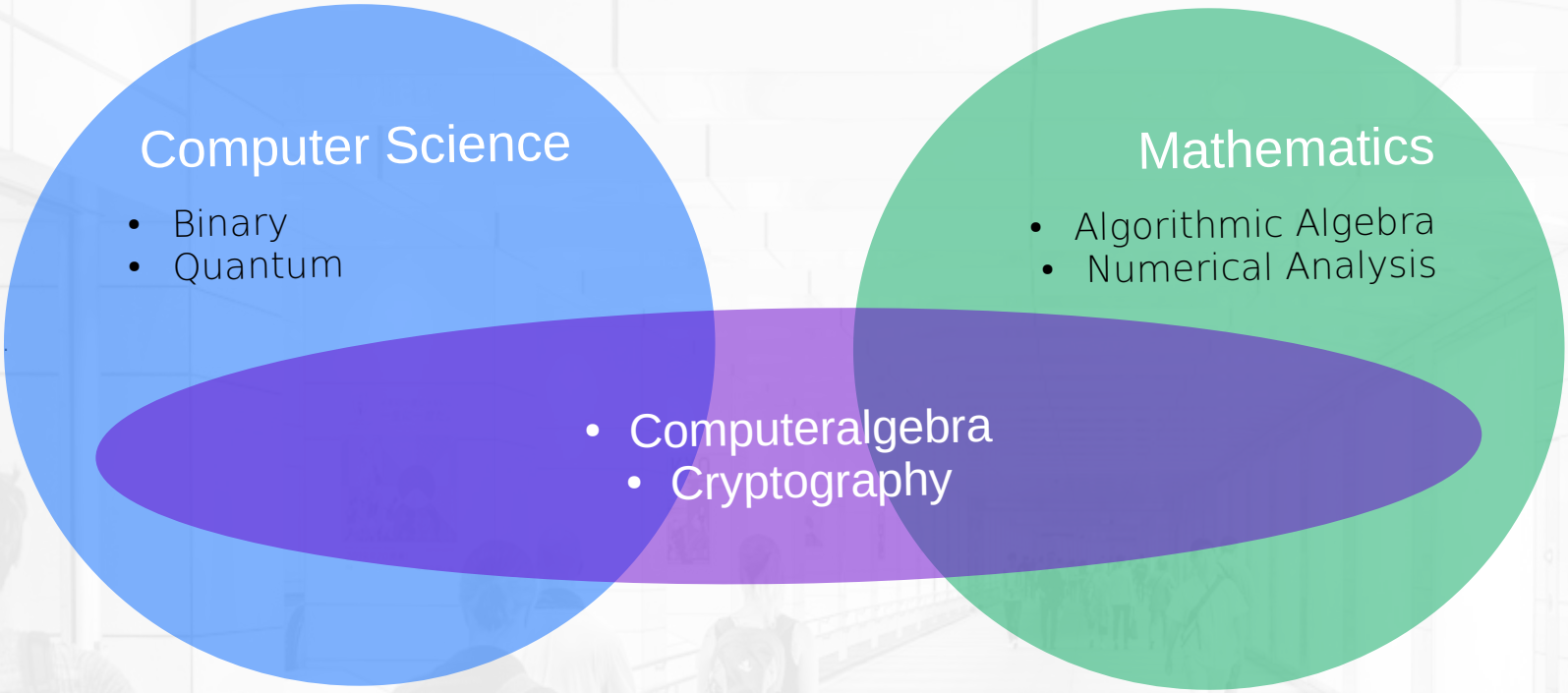
Scope of our Mission

Post-Quantum Security for Binary and Quantum Technologies

- Our results cover both, the **binary and quantum computing aspects**!
- Therefore our results will bring benefit to the development of **quantum resistant binary and quantum technologies**.

Complexity of the Topic

Scientific and Technological Context



Security Level Computation

The condition number κ in the complexity of the Quantum Algebraic Attack by Chen-Gao is the condition number of the Macaulay matrix defined in their paper.

The computation of κ is not trivial due to the size of the matrix, which cannot be handled by classical methods on classical computing systems anymore.

AES	N_k	N_r	#Vars	#Eqs	T-Sparseness	Complexity
AES-128	4	4	1792	4400	101376	$2^{68.61}c\kappa^2$
AES-128	4	6	2624	6472	151680	$2^{70.68}c\kappa^2$
AES-128	4	8	3456	8544	201984	$2^{72.16}c\kappa^2$
AES-128	4	10	4288	10616	252288	$2^{73.30}c\kappa^2$
AES-192	6	12	7488	18096	421248	$2^{76.59}c\kappa^2$
AES-256	8	14	11904	29520	696384	$2^{78.53}c\kappa^2$

Security Level Estimations

The joint work of Jianqiang Li, Jintai Ding, Vlad Gheorghiu, Andŕas Gilyen, Sean Hallgren,

Limitations of the Macaulay matrix approach for using the HHL algorithm to solve multivariate polynomial systems,

offers security level estimations for affected cryptosystems: Chen/Gao's algorithm is exponential in the Hamming weight (number of one's) of the solution (the secret key + the expanded key + derived sub keys).

A randomly generated symmetric secret key should have about equally many zero's as one's. Under this assumption, the Hamming weight * of the solution is about half the size of the solution vector. The structure of the AES Macaulay matrix implies a far lower Hamming weight. It is our next goal to compute the Hamming weight of this matrix.

** The Hamming weight of a string is the number of symbols that are different from the zero-symbol of the alphabet used.*

https://indico.physik.uni-muenchen.de/event/84/attachments/248/553/S2C.Li.slides.pdf

Remark

Thanks to the participants of the Industrial Computeralgebra Conference 2021 for this hint!

Research Results and Next Steps

Results are published on Github, where we invited collaborators and reviewers:

Project:
https://github.com/Quant-X-Security-Coding-GmbH/QAA_Condition_Number

Computeralgebra Magazine Publication:
https://fachgruppe-computeralgebra.de/data/CA-Rundbrief/car67.pdf

Scientific Paper:
https://github.com/Quant-X-Security-Coding-GmbH/QAA_Condition_Number/blob/main/official_paper/QAA_on_AES_paper.pdf

Next Steps:

1) Compute the Hamming weight of the AES-Macaulay matrix
2) Compute the condition number of the AES-Macualay matrix

Core Team

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- Christoph Stockhammer, MathWorks