Group #332. Submission #1
WorldQuant University, USA
September 4, 2022

OPTIONS – PUT-CALL PARITY & GREEKS

For following option questions, we assume S=100, K=100, r=5%, sigma=20%, and time=3 months.

1. ***Does put-call parity apply for European options? Why or why not?***

Here below is put-call parity equation:

$$c + Ke^{-rt} = p + S_0$$

Yes. Put-call parity applies to European options. We demonstrate its validity below with prices obtained for put and call using Black Scholes model.

Put-call parity holds as the portfolio of instruments with same risk and payoff profile should provide same expected gain or loss at the maturity of the instrument for no arbitrage opportunity to exist. Put-call parity states that payoff from portfolio of stock and right to sell that stock (put option) is equivalent to the payoff of portfolio of a riskless zero-coupon bond (maturity as expiration of the option and face value same as strike) and right to buy (call option) the same stock. As the payoff of these two portfolio at the expiration of the two portfolios is the same so the value of these portfolio should be same at times prior to the expiration as well.

2. ***Rewrite put-call parity to solve for the call in terms of everything else.***

Equation for put-call parity to solve for call option in terms of everything else is as below:

$$c = S_0 + p - Ke^{-rt}$$

3. ***Rewrite put-call parity to solve for the put in terms of everything else.***

Equation for put-call parity to solve for put option in terms of everything else is as below:

$$p = c + Ke^{-rt} - S_0$$

4. ***Does put-call parity apply for American options? Why or why not?***

No. Put call parity does not apply to American options due to the feature of early exercise. American options provide extra optionality to holder of the option. Due to the extra optionality, as American options can be exercised during the life of the option before its expiry, the two sides of the put-call parity might show divergence in the option values.

5. ***Price an ATM European call and put using Black-Scholes.***

For values given above, using Black-Scholes model, price of ATM European call option is \$4.61 and that of put option is \$3.37.

6. ***Compute the Greek Delta for the European call and European put. How do they compare?***

For values given above, Delta for European call option is 0.5695 and that of put option is -0.4305. As expected, we observe delta of call option to be positive and that of put option to be negative.

7. ***Compute the Greek Gamma for the European call and European put. How do they compare?***

   For values given above, Gamma for European call and put value is 0.0393. As expected, we observe Gamma for both call and put options to be the same.

8. ***Price an ATM American call and put using a binomial tree.***

   For values given above, using Black-Scholes model, price of ATM American call option is \$4.61 and that of put option is \$3.48.

9. ***Compute the Greek delta for the American call and American put. How do they compare?***

   For values given above, Delta for European call option is 0.5694 and that of put option is -0.4495. As expected, we observe delta of call option to be positive and that of put option to be negative.

10. ***Compute the Greek gamma for the American call and American put. How do they compare?***

    For values given above, Gamma for European call is 0.0393 and put value is 0.0423. As expected, we observe Gamma for both call and put options to be positive.

11. ***If the team answered Question 1 as "Yes", then show that the European call and put satisfy put-call parity.***

    Here below is put-call parity equation:

$$c + Ke^{-r} \ = p + \ S_0$$

    For given values above, we calculate both left and right-hand side portfolio values to be \$103.37. Thus, we conclude that put-call parity holds for European options.

12. ***If the team answered Question 4 as "Yes", then show that the American call and put satisfy put-call parity.***

    Here below is put-call parity equation:

$$c + Ke^{-rt} = p + \ S_0$$

    For given values above, we calculate both left-hand side portfolio value to be \$103.37 and right-hand side portfolio value to be \$103.48. This, we conclude that put-call parity does not hold for American options.

13. ***Confirm that the European call is less than or equal to the American call. Show the difference if any.***

    For given values above, European and American call options value is \$4.61. We observe value for both types of options to be very close and not to show any meaningful divergence. This makes sense and is congruence with what's observed in practice - it does not make sense for a call options on a non-dividend paying underlying to be exercised prior to expiration.

14. ***Confirm that the European put is less than or equal to the American put. Show the difference if any.***

    For given values above, European put option value is \$3.37 and that of American put option is \$3.48. We observe a difference of \$0.11 and American put options to be prices higher than similar European options. This difference can be explained due to presence of extra optionality of early exercise for American put options.

15. ***Use a strike level so that calls are ITM and puts are OTM. Repeat Questions 8 through 10.***

    For following option questions, we assume S=100, K=90, r=5%, sigma=20%, and time=3 months.

    a. ***Price an American call and put using a binomial tree.***

       For values given above, we calculate price of American ITM call option to be \$11.67 and that of OTM put option to be \$0.56 using binomial tree.

    b. ***Compute the Greek delta for the American call and American put. How do they compare?***

       For values given above, we calculate deltas of American ITM call option to be 0.8904 and that of OTM put option to be -0.1122 using binomial tree. We observe deltas for call to approach 1 and put options to approach 0.

    c. ***Compute the Greek gamma for the American call and American put. How do they compare?***

       For values given above, we calculate gammas for both American call to be 0.0188 and put to be 0.0193, which are both positive and very close to each other.

16. ***Use another strike level so that calls are deep ITM and puts are deep OTM.***

    For following option questions, we assume S=100, K=80, r=5%, sigma=20%, and time=3 months.

    a. ***Price an American call and put using a binomial tree.***

       For values given above, we calculate price of American deep-ITM call option to be \$21.02 and that of deep-OTM put option to be \$0.03 using binomial tree.

    b. ***Compute the Greek delta for the American call and American put. How do they compare?***

       For values given above, we calculate deltas of American deep-ITM call option to be 0.992 and that of deep-OTM put option to be -0.0081 using binomial tree. We observe deltas for call to be almost 1 and put options to be almost 0.

    c. ***Compute the Greek gamma for the American call and American put. How do they compare?***

       For values given above, we calculate gammas for both deep-ITM American call and deep-OTM put options to be 0.0022, which is positive and same for both.

17. ***Use a strike level so that calls are OTM and puts are ITM.***

    For following option questions, we assume S=100, K=110, r=5%, sigma=20%, and time=3 months.

    a. ***Price an American call and put using a binomial tree.***

       For values given above, we calculate price of American OTM call option to be $1.19 and that of ITM put option to be $10.33 using binomial tree.

    b. ***Compute the Greek delta for the American call and American put. How do they compare?***

       For values given above, we calculate deltas of American OTM call option to be 0.2181 and that of ITM put option to be -0.8485 using binomial tree. We observe deltas for call to be approaching 0 and put options to approach -1.

    c. ***Compute the Greek gamma for the American call and American put. How do they compare?***

       For values given above, we calculate gammas for both American call to be 0.0295 and put to be 0.0369, which are both positive and close to each other.

18. ***Use another strike level so that calls are deep OTM and puts are deep OTM.***

    For following option questions, we assume S=100, K_{call}=120, K_{put}= 80, r=5%, sigma=20%, and time=3 months.

    a. ***Price an American call and put using a binomial tree.***

       For values given above, we calculate price of American OTM call option to be $0.2 and that of OTM put option to be $0.03 using binomial tree.

    b. ***Compute the Greek delta for the American call and American put. How do they compare?***

       For values given above, we calculate deltas of American OTM call option to be 0.0496 and that of OTM put option to be -0.0081 using binomial tree. We observe delta for deep OTM call option to be near zero but small positive. The delta for deep OTM put option is also close to 0 and small negative.

    c. ***Compute the Greek gamma for the American call and American put. How do they compare?***

       For values given above, we calculate gammas for both American call to be 0.0102 and put to be 0.0022, which are both positive and close to each other.

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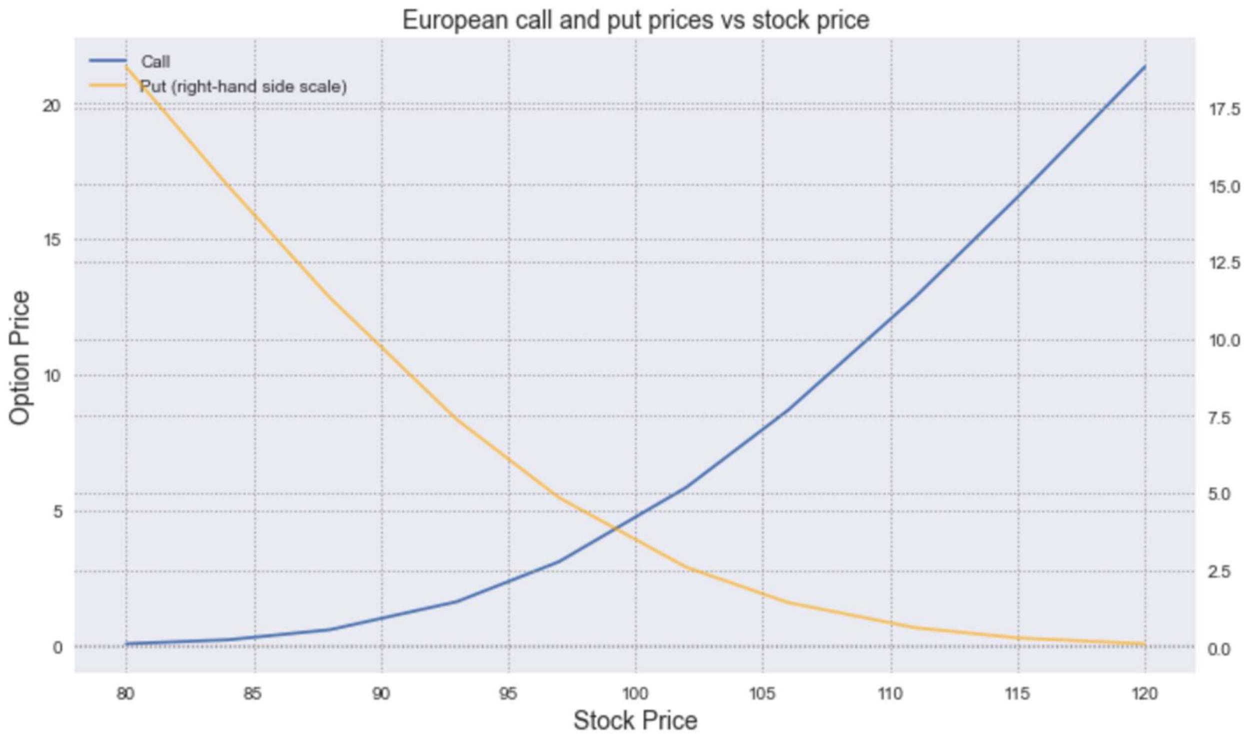
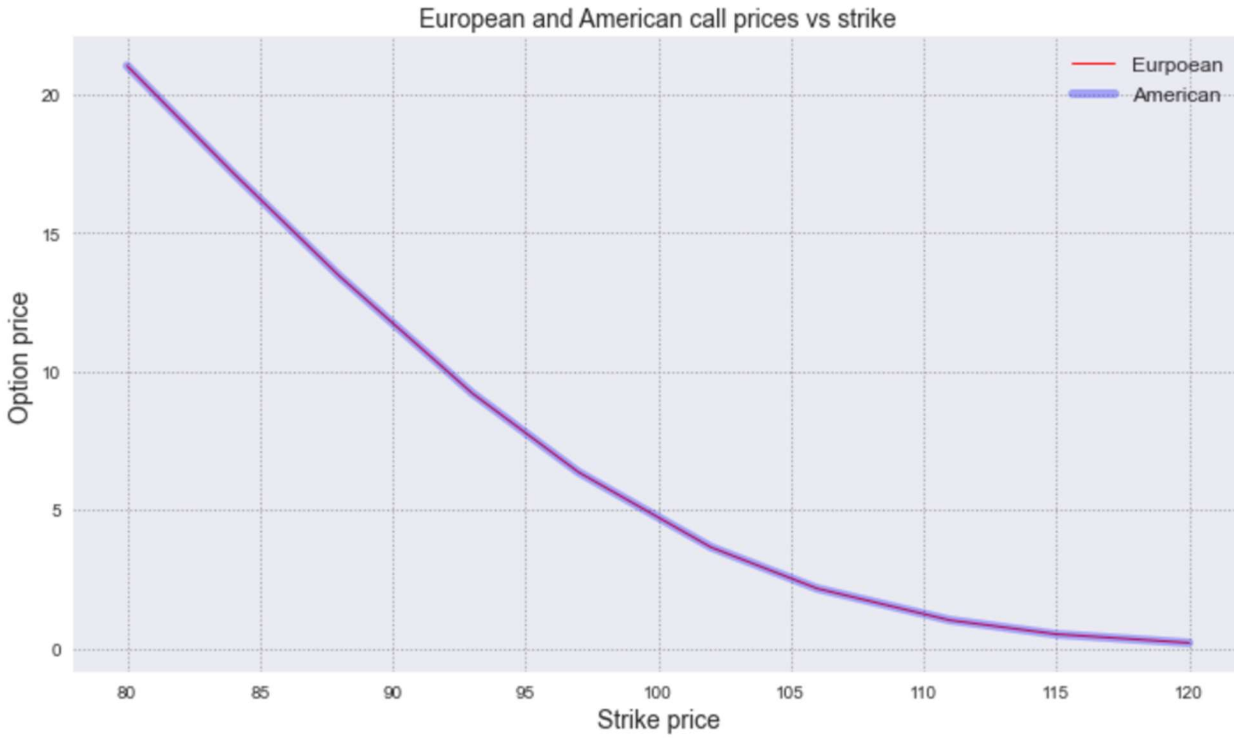
23. Graph #1. Graph European call prices and put prices versus stock price.



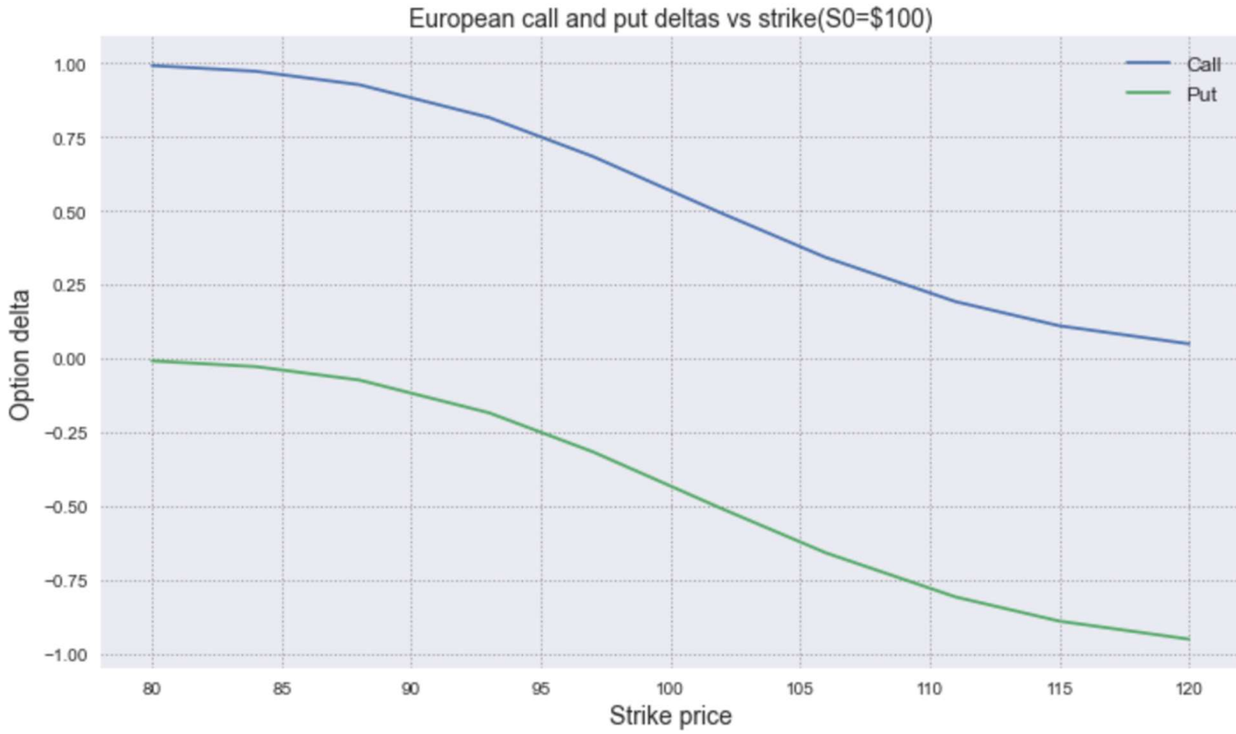
24. Graph #2. Graph American call prices and put prices versus strike.

25. Graph #3. Graph European and American call prices versus strike.

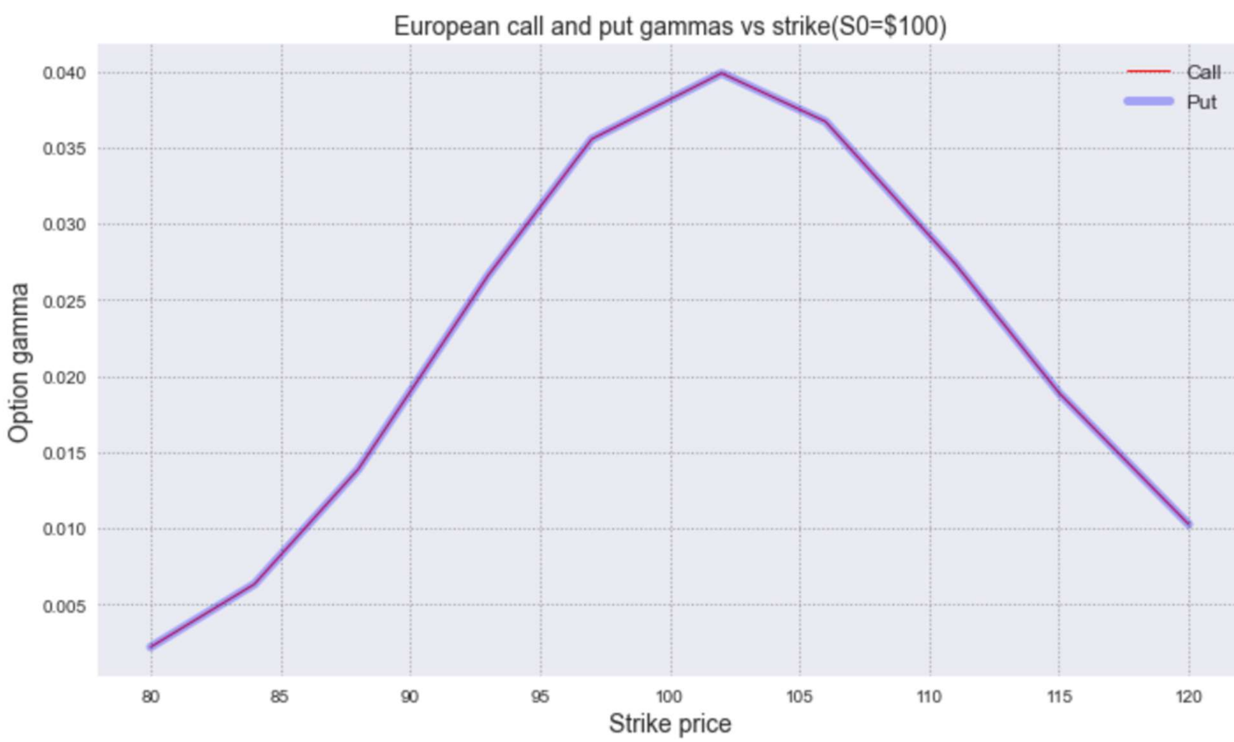


26. Graph #4. Graph European and American put prices versus strike.

27. Graph #5. Graph European deltas versus strike.



28. Graph #6. Graph European gammas versus strike.



29. Pick a non-FANG stock for which you can get option data.

For this exercise, we obtained options chain data for TSLA stock from Yahoo finance.

30. Select 5 strike levels for a call option: 2 ITM, 1 ATM, and 2 OTM.

As of the time of writing this report, the TSLA stock (underlying) is priced at $270.21. From the option chain data we obtained before, we dynamically select 5 strikes - $260, $266.67, $273.33, $280, $283.33 - for further examination.

31. Are these options European or American?

TSLA options are American options listed on Chicago Board Options Exchange (CBOE).

32. Using these strike levels, apply your code to price that option. Be sure to use the correct values of stock price, strike level, time to maturity, risk-free rate, and volatility (use the options implied volatility as the volatility value).

We use binomial tree to price TSLA options and observe the prices to be reasonably close to those observed in the market.

33. How well did Black-Scholes match the prices?

Black-Scholes model will not apply to the American options we chose to price. Please see response to #34.

34. How well did binomial trees match the prices?

Options prices obtained using binomial tree match very close (to not allow economically feasible arbitrage) to the market prices. This is not very surprising though as we used implied volatility to price these options and TSLA does not pay any dividends.

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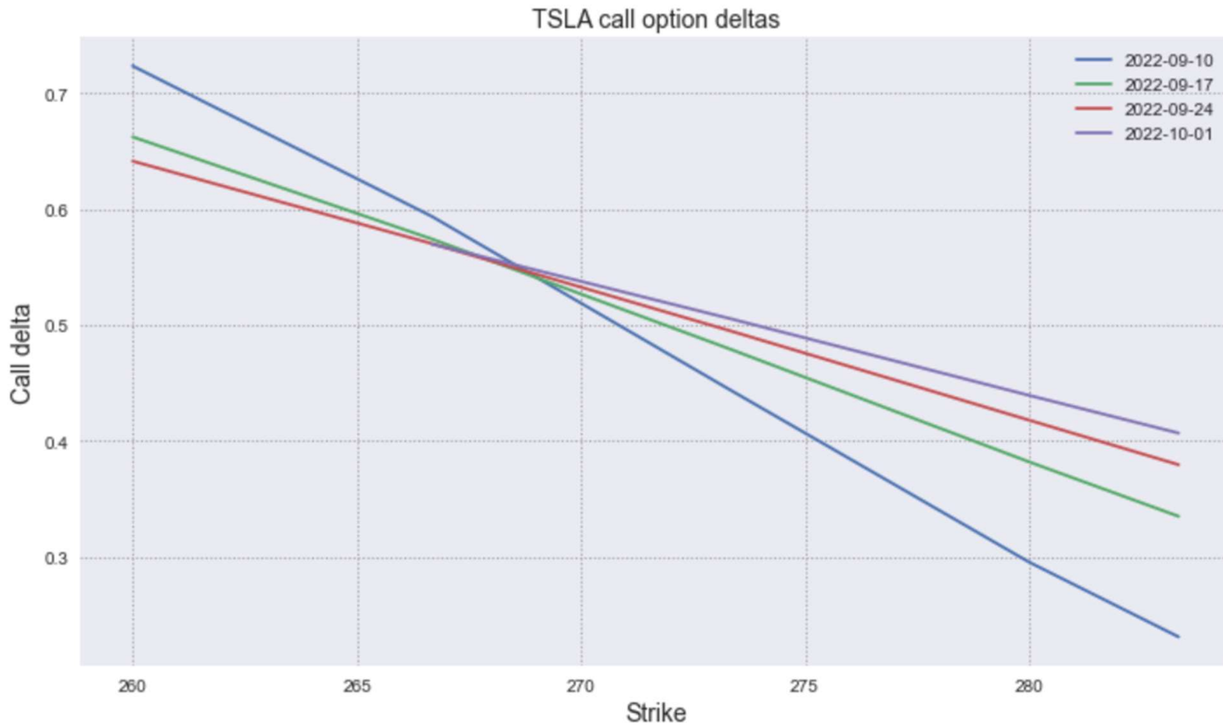
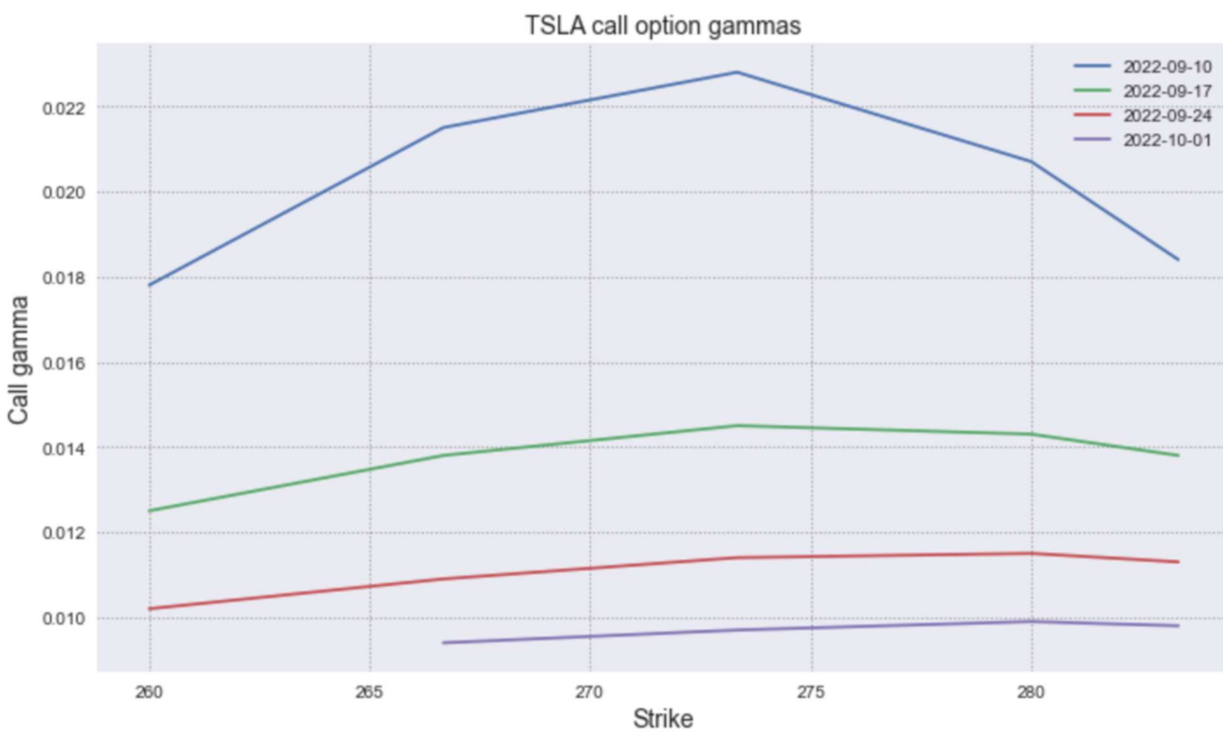
35. Graph the deltas and gammas.





36. Conclusions.

At the start of this exercise, we examined put-call parity for American and European options. We observed that put-call parity for the European options priced using BSM model holds, however, does not hold for American options. The feature of early exercise available to holders of American options allow for the divergence of values for these type of put and call options.

We also do price comparison between American and European options. We find not much difference in pricing of American and European call options as it is not economically feasible for these options to be exercised prior to expiration for a non-dividend paying stock. We found American put options to be more than or at least as valuable as European put options.

In later part of this study, we priced European options a different strike levels with both call and put options being OTM, deep-OTM, ATM, ITM and deep-ITM. We observed the progression of various greeks for call and put options. Graphs for European and American call option greeks were created for visual inspection and analysis.