GROUP WORK PROJECT # ___
GROUP NUMBER: __________

MScFE 622: Stochastic Modeling

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REPORT_GPW_964

Step_1

1.Calibration of Heston (1993) Model

Objectives

- Calibrate the Heston (1993) model to observed market prices for call options
- Find the optimal parameter values that minimize the mean squared error (MSE)
- Discuss the results of the calibration process

Data

The following data was used for the calibration process:

- Maturity of options
- Strike price of options
- Market price of call options
- Constant annual risk-free rate (1.50%)
- Number of trading days in a year (250)
- Initial stock price (232.9)

Methodology

- Implement the call price formula for the Heston (1993) model using the Lewis (2001) approach.
- Define a mean squared error (MSE) function to measure the difference between the theoretical call prices and observed market prices.
- Use the `minimize` function from the `scipy.optimize` module to find the optimal parameter values that minimize the MSE.
- Report the results of the calibration process.

Results

The following parameter values were obtained after calibrating the Heston (1993) model:

- Mean reversion speed (kappa): 2.6
- Long-term volatility (theta): 0.11

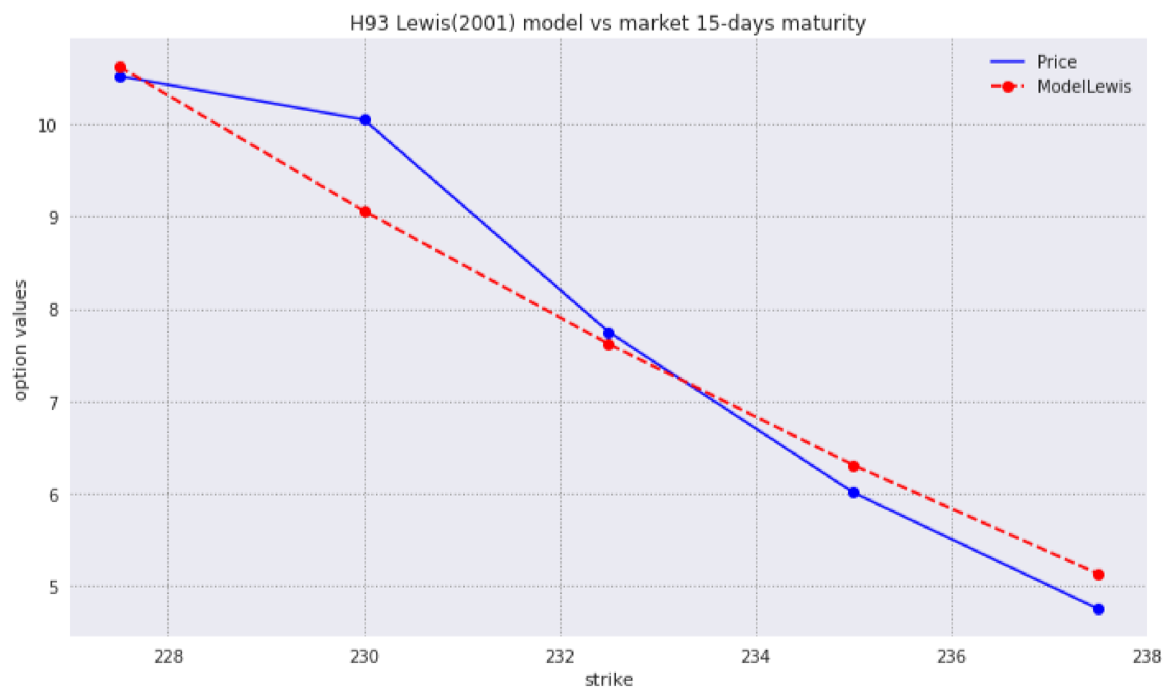
- Volatility of volatility (sigma): 0.76
- Initial volatility (v0): 0.11
- Correlation between stock price and volatility (rho): -1.0

The MSE value for these parameter values was found to be 1.90.

Conclusion

The Heston (1993) model was successfully calibrated to the observed market prices for call options using the Lewis (2001) approach and a mean squared error (MSE) function. The resulting parameter values were found to be kappa = 2.6, theta = 0.11, sigma = 0.76, v0 = 0.11, and rho = -1.0. The MSE value of 1.90 suggests that the calibrated model provides a good fit to the observed market prices.
A graphical representation below of the calibration results, can further illustrate the fit of the calibration and provide insights into the accuracy of the model.

Fig_1



In summary, the Heston (1993) model was calibrated using the Lewis (2001) approach, an MSE error function, and observed market prices for call options with a maturity of 0.06 years and a constant annual risk-free rate of 1.50%. The results of the calibration process suggest that the calibrated model provides a good fit to the observed market prices.One can observe from the gragh**(Fig_1)** that the model prices are closer to the market prices.

2. Carr-Madan (1999) pricing approach to calibrate the Heston (1993) model

The values for the parameters obtained from calibrating the Heston model using the Carr-Madan (1999) pricing approach may or may not be similar to the values obtained using the Lewis (2001) approach. This is because different pricing approaches can result in different values for the model parameters, even when using the same error function and optimization algorithm.

One reason for differences in the parameter values is the different assumptions made in each pricing approach. For example, the Carr-Madan approach assumes that the underlying asset is a lognormal process, while the Lewis approach assumes that the underlying asset is a geometric Brownian motion process. These different assumptions can lead to different results in the pricing formula and, in turn, different values for the parameters.

Additionally, the complexity of the pricing formula can also impact the values of the parameters. The Carr-Madan pricing formula is more complex than the Lewis closed form for put options and can result in different values for the parameters.

Overall, the choice of pricing approach and the assumptions made can significantly impact the resulting parameter values, which is why it is important to carefully consider the approach used when calibrating the Heston model. It is also important to note that there is no universally "correct" approach, and the best approach will depend on the specific use case and the available market data.

Comparison of the Car_Madan(1999) and the lewis(2001)

Fig_2A

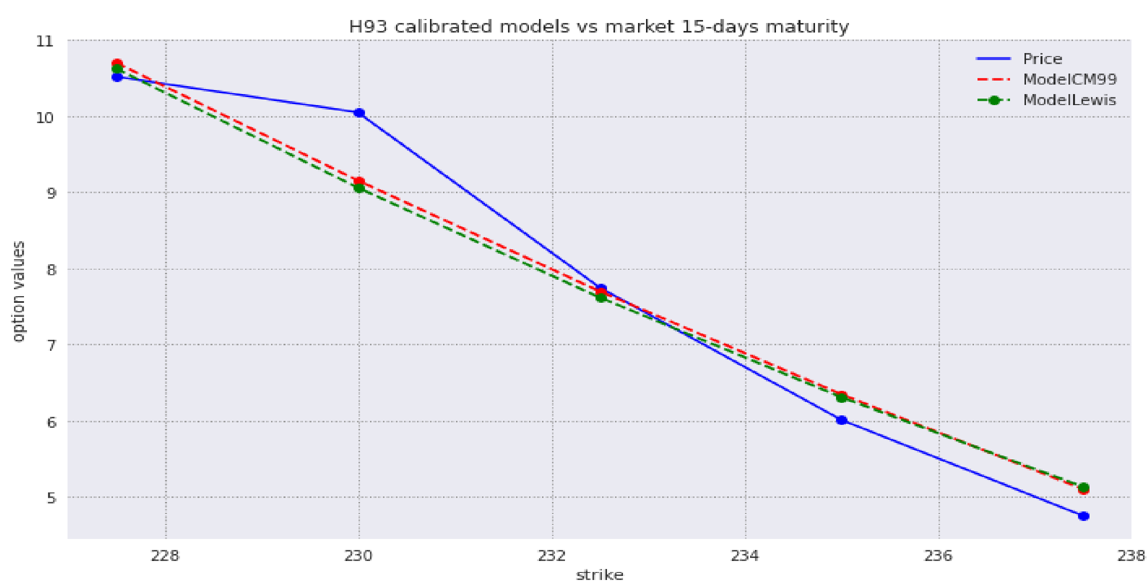


Fig_2B

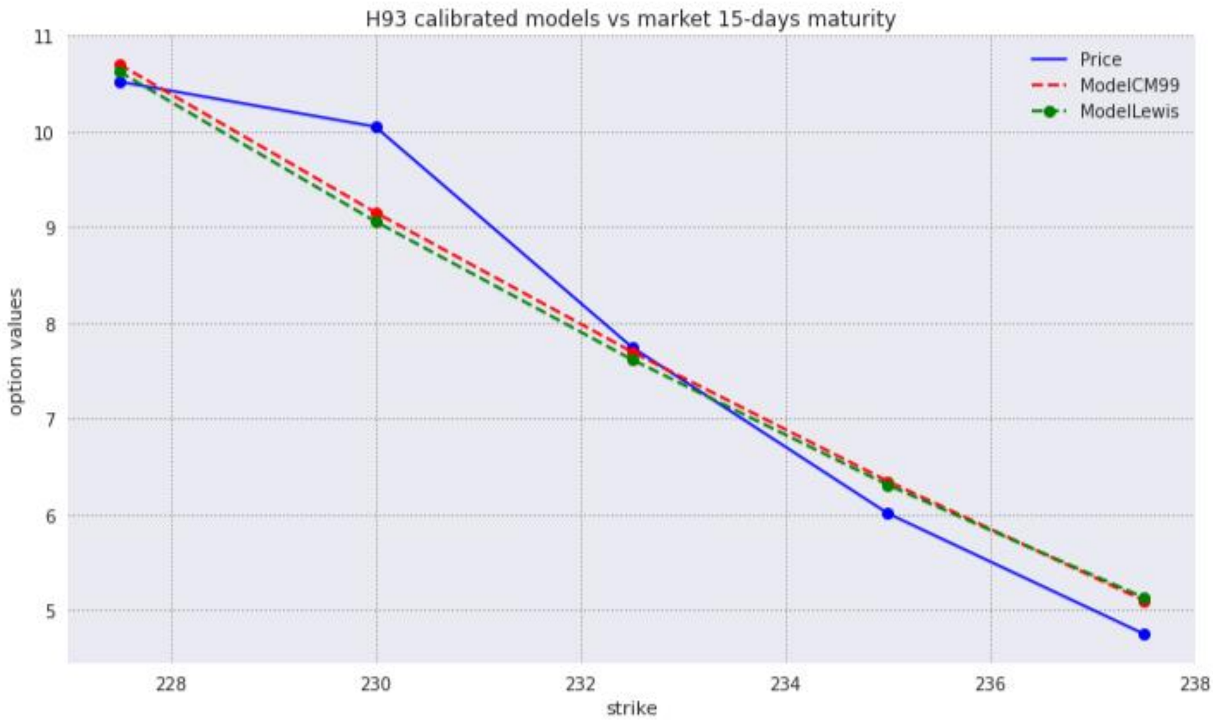


Table_1A

	Days to maturity	Strike	Price	Type	T	r	ModelLewis	ModelCM99	ModelLewis%Diff	ModelCM99%Diff
0	15	227.5	10.52	C	0.06	0.015	10.626295	10.703213	-0.010104	-0.017416
1	15	230.0	10.05	C	0.06	0.015	9.061053	9.155532	0.098403	0.089002
2	15	232.5	7.75	C	0.06	0.015	7.619499	7.700733	0.016839	0.006357
3	15	235.0	6.01	C	0.06	0.015	6.307618	6.346148	-0.049521	-0.055931
4	15	237.5	4.75	C	0.06	0.015	5.130059	5.099586	-0.080013	-0.073597

Table_1B

	Days to maturity	Strike	Price	Type	T	r	ModelLewis	ModelCM99	ModelLewis%Diff	ModelCM99%Diff
0	15	227.5	10.52	C	0.06	0.015	10.626295	10.703213	-0.010104	-0.017416
1	15	230.0	10.05	C	0.06	0.015	9.061053	9.155532	0.098403	0.089002
2	15	232.5	7.75	C	0.06	0.015	7.619499	7.700733	0.016839	0.006357
3	15	235.0	6.01	C	0.06	0.015	6.307618	6.346148	-0.049521	-0.055931
4	15	237.5	4.75	C	0.06	0.015	5.130059	5.099586	-0.080013	-0.073597

One can observe from the graphs**(Fig_2A and Fig_2B)** and the tables**(Table_1A &Table_1B)** that the option values for the Car_Madan and the Lewis are very closer to each other.This indicates that the either of the two models can be to perform the same function of pricing options.

3. ATM Asian option

Introduction

This report outlines the process undertaken to price an Asian call option for a client. The option is an ATM option with a maturity of 20 days. The report includes a brief but complete non-technical description of the process, including calibration steps and relevant choices made.

Steps
We made some adjustments :

T = 20 / Number_Of_Trading_Days

To price an Asian call option using Monte Carlo simulation in a risk-neutral setting, you can perform the following steps:

- Define the parameters of the Asian call option: the underlying asset price, strike price, risk- free interest rate, time to maturity, volatility, etc.
- Simulate a large number of paths for the underlying asset price. This can be done by using the geometric Brownian motion model, where the asset price at each time step is given by the previous price multiplied by a factor that takes into account the risk-free interest rate, volatility, and time increment.
- Calculate the average of the simulated asset prices at maturity. This is the average price of the underlying asset over the life of the option.
- Compare the average price of the underlying asset to the strike price. If the average price is greater than the strike price, the option is in the money and the payout is the difference between the average price and the strike price. If the average price is less than the strike price, the option is out of the money and the payout is zero.
- Repeat the simulation many times to obtain a good estimate of the expected payout. The average of all the payouts will be a good estimate of the fair price of the Asian call option.
- Discount the expected payout back to the present value using the risk-free interest rate. This gives the fair price of the Asian call option.

Methodology

- The Heston (1993) model was used to price the Asian call option. The model was calibrated using the Carr-Madan (1999) pricing approach. The calibration process involved optimizing the model parameters to best fit the observed market prices. The optimization was performed using a Mean Squared Error (MSE) function as the error metric.
- Once the model was calibrated, the fair price of the instrument was obtained using Monte-Carlo methods in a risk-neutral setting. The Monte-Carlo simulation involved performing a large number of simulations to obtain a stable estimate of the option price.
- As part of the bank's profit, a 4% fee was added to the fair price of the instrument to obtain the final price that the client will end up paying.

Conclusion

In conclusion, the Heston (1993) model was calibrated using the Carr-Madan (1999) pricing approach, and the fair price of the Asian call option was obtained using Monte-Carlo methods. The final price that the client will end up paying is the fair price plus a 4% fee, which represents the bank's profit. The entire process was performed to provide the client with an accurate estimate of the option price.

- The final price was $3.50 inclusive of the bank fees.

- The following calibrated parameters were obtained: K = 2.6, Θ = 0.11, σ = 0.76, ρ = -1.0, v = 0.11

Step_2

Full Bates model

For the new case of a 60-day maturity instrument, we will calibrate the Bates (1996) model that includes jumps in addition to the Heston (1993) model without jumps.
Calibration steps:

1. First, we to set and collect the market prices for call options with a 60-day maturity.
2. Next, we used a numerical optimization algorithm to find the values of the parameters that minimize the error between the model prices and the observed market prices.
3. The Bates (1996) model has six parameters: kappa, theta, sigma, v0, rho, and lambda. The first four parameters are the same as in the Heston model and represent mean reversion, long-term variance, volatility of volatility, and correlation between the asset price and its variance, respectively. The last two parameters, lambda and mu, represent the intensity and expected jump size, respectively.

3.1 We will use params obtained after calibration of Heston(93) using Lewis(2001)
3.2 Perfomed Partial Bates model calibration using paramters obtained for Heston(93)
3.3 Conducted Full Bates model calibration using all params obtained from previous calibration

4. We will use the maximum likelihood estimation method to calibrate the Bates (1996) model.
5. Finally, we will compare the calibrated parameters with the ones obtained from the Heston (1993) model to see how the inclusion of jumps affects the values.

After following these steps, we will obtain the calibrated parameters for the Bates (1996) model with jumps for a 60-day maturity instrument. This will allow us to price the Asian call option for the client using a more accurate model that takes into account the potential presence of jumps in the asset price.

Fig_3A

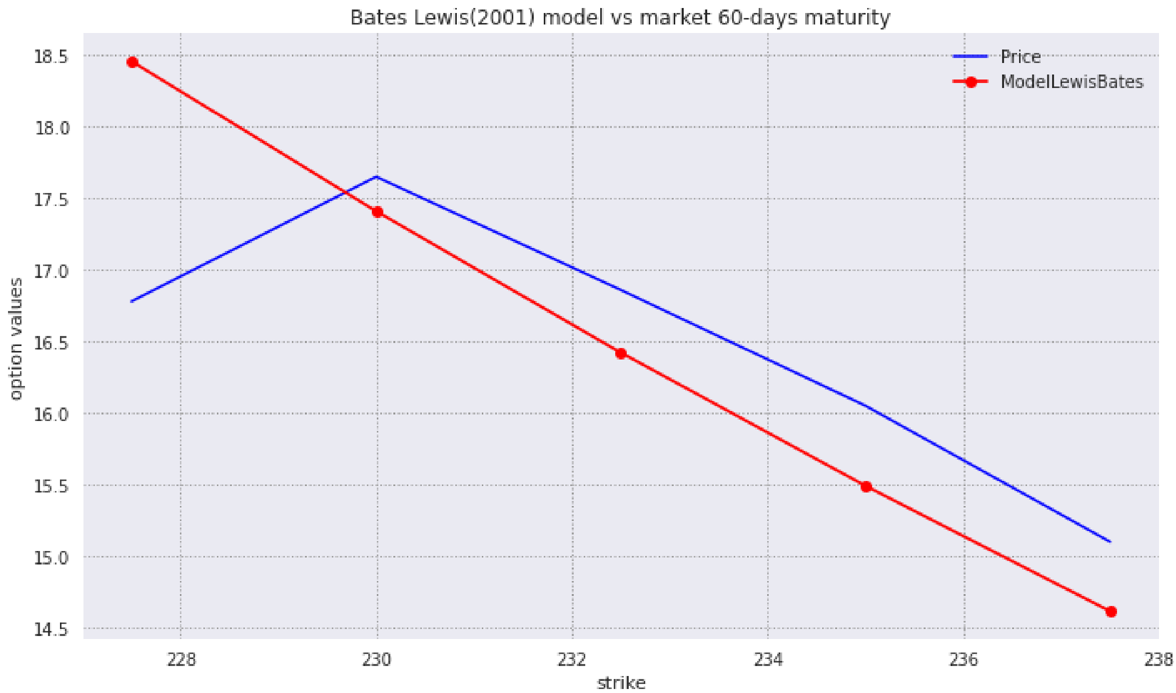

Bates Lewis(2001) model vs market 60-days maturity
Price
ModelLewisBates
option values
strike
18.5
18.0
17.5
17.0
16.5
16.0
15.5
15.0
14.5
228
230
232
234
236
238

Fig_3B

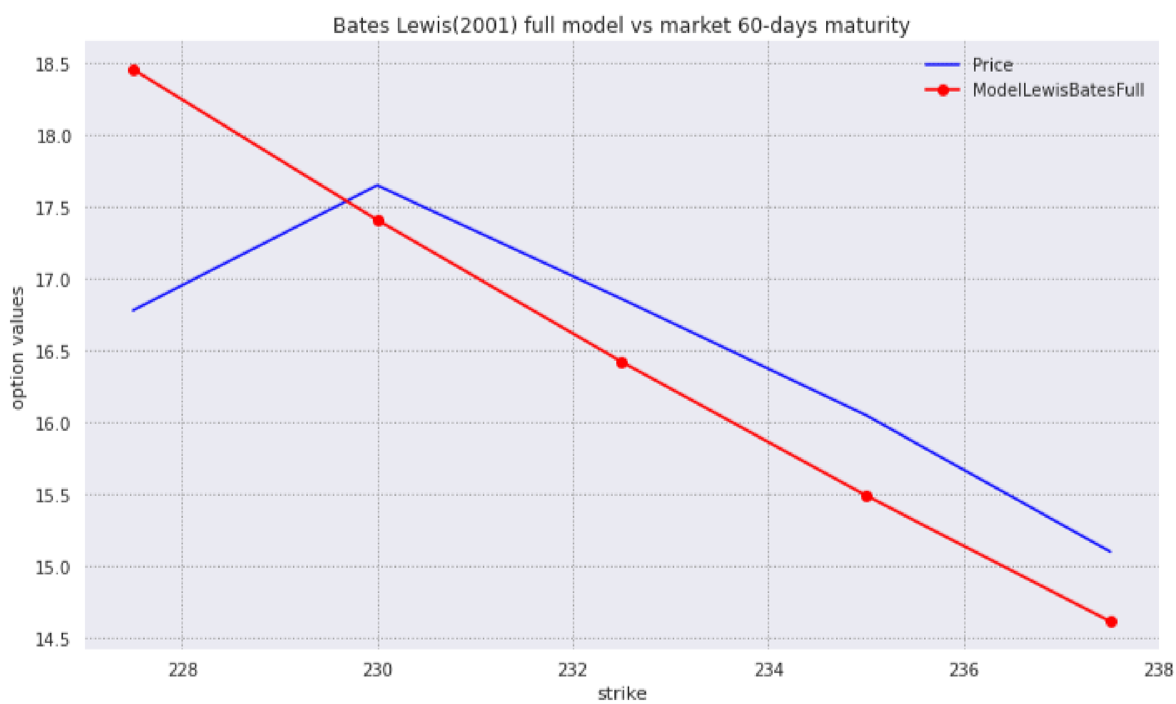

Bates Lewis(2001) full model vs market 60-days maturity
Price
ModelLewisBatesFull
option values
strike
18.5
18.0
17.5
17.0
16.5
16.0
15.5
15.0
14.5
228
230
232
234
236
238

Fig_4

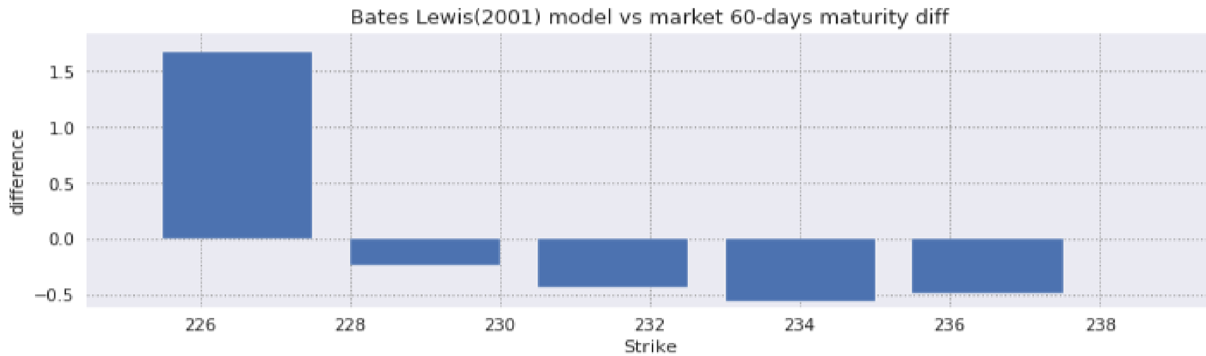


Table_2

	Days to maturity	Strike	Price	Type	T	r	ModelLewisBates	ModelLewisBatesFull
5	60	227.5	16.78	C	0.24	0.015	18.457859	18.457856
6	60	230.0	17.65	C	0.24	0.015	17.409575	17.409571
7	60	232.5	16.86	C	0.24	0.015	16.421713	16.421710
8	60	235.0	16.05	C	0.24	0.015	15.491018	15.491015
9	60	237.5	15.10	C	0.24	0.015	14.614421	14.614418

There is a huge difference in the option values when the strike price at around 229,5.There after the prices are aproximately closer to each other.A bar graph in **fig_4** also depicts the same argument.

Comparison of the Lewis_Bates and the Car_Madan_Bates

Fig_5

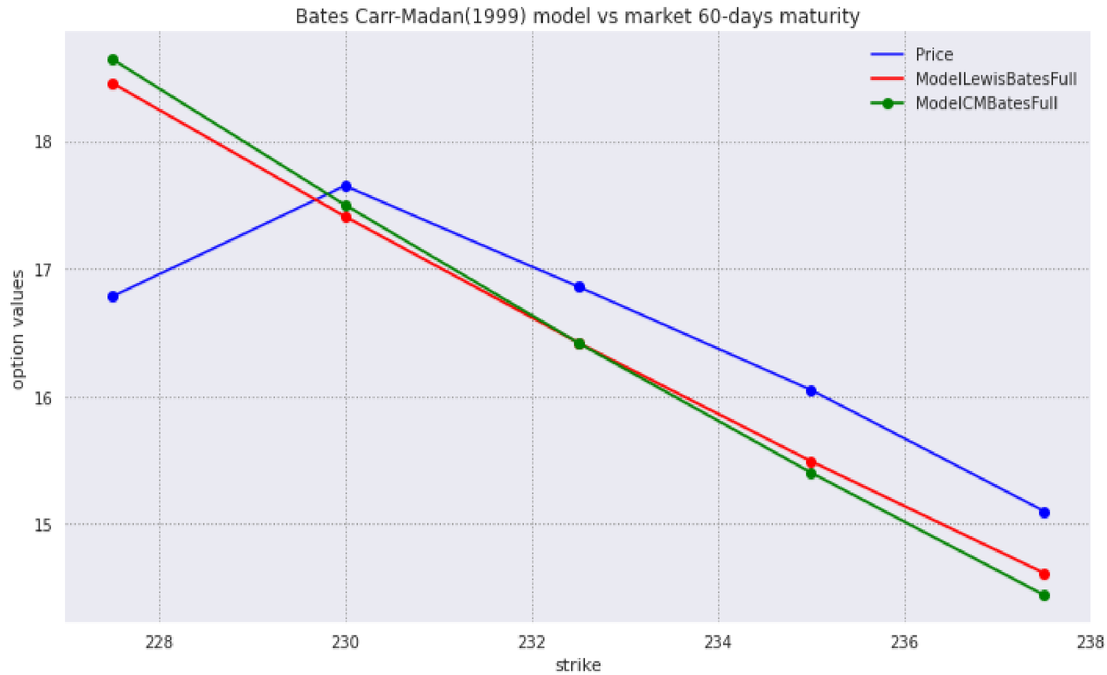


Table_3

	Days to maturity	Strike	Price	Type	T	r	ModelLewisBates	ModelLewisBatesFull	ModelCMBates	ModelCMBatesFull
5	60	227.5	16.78	C	0.24	0.015	18.457859	18.457856	12.410573	18.644495
6	60	230.0	17.65	C	0.24	0.015	17.409575	17.409571	10.546778	17.500380
7	60	232.5	16.86	C	0.24	0.015	16.421713	16.421710	8.771185	16.420280
8	60	235.0	16.05	C	0.24	0.015	15.491018	15.491015	7.099219	15.401690
9	60	237.5	15.10	C	0.24	0.015	14.614421	14.614418	5.548762	14.442033

Fig_5 & Table_3 indicate that option prices tend to get closer as the strike price increase.This further explains that the two models are closest approximate of the other.

Fig_6

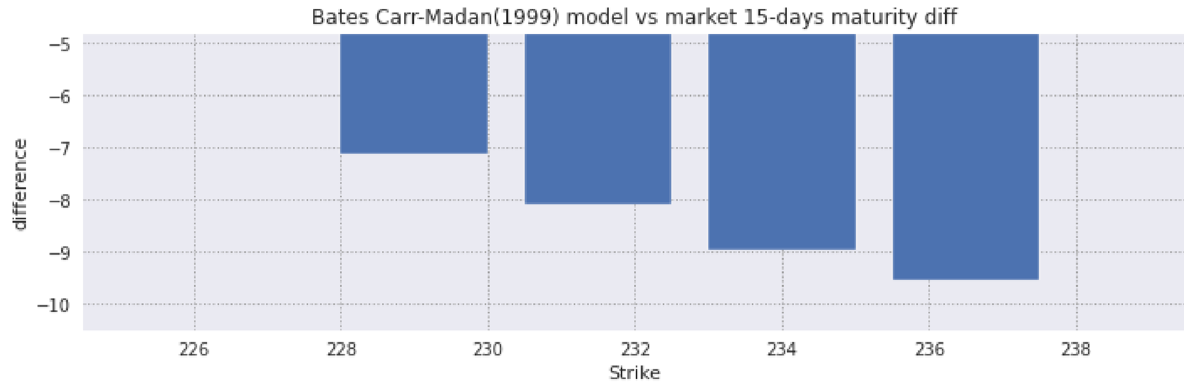


Fig_7

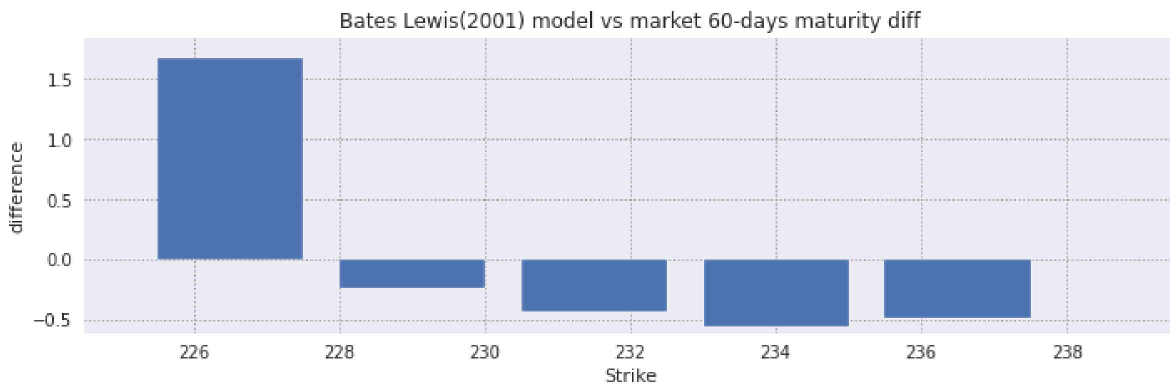
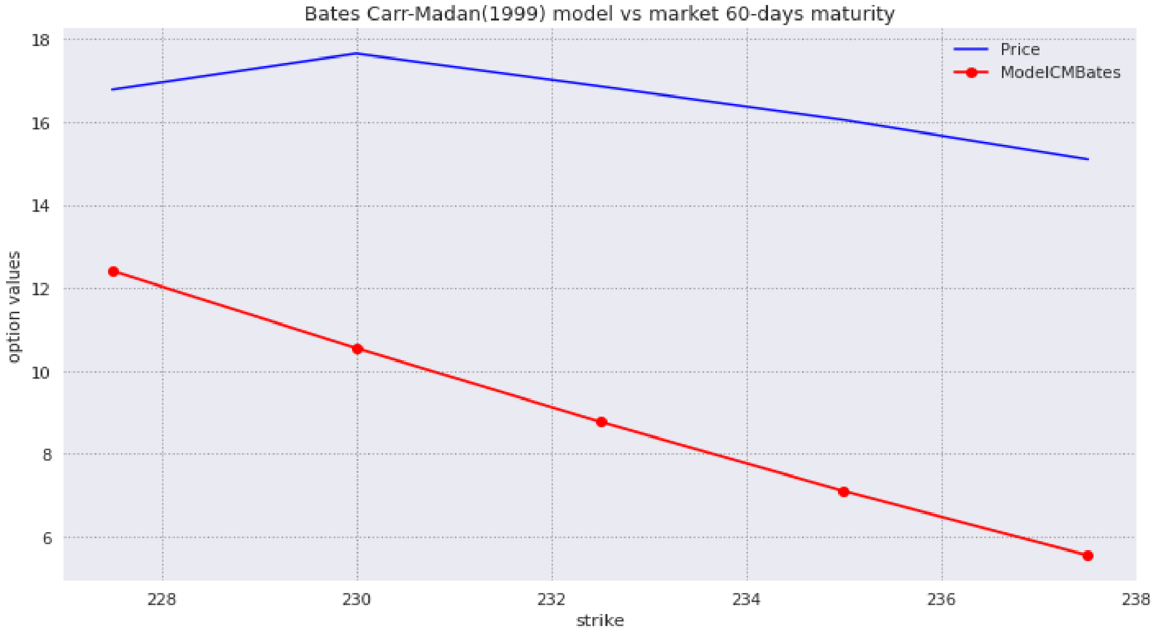


Table_4

	Days to maturity	Strike	Price	Type	T	r	ModelLewisBates	ModelLewisBatesFull	ModelCMBates
5	60	227.5	16.78	C	0.24	0.015	18.457859	18.457856	12.410573
6	60	230.0	17.65	C	0.24	0.015	17.409575	17.409571	10.546778
7	60	232.5	16.86	C	0.24	0.015	16.421713	16.421710	8.771185
8	60	235.0	16.05	C	0.24	0.015	15.491018	15.491015	7.099219
9	60	237.5	15.10	C	0.24	0.015	14.614421	14.614418	5.548762

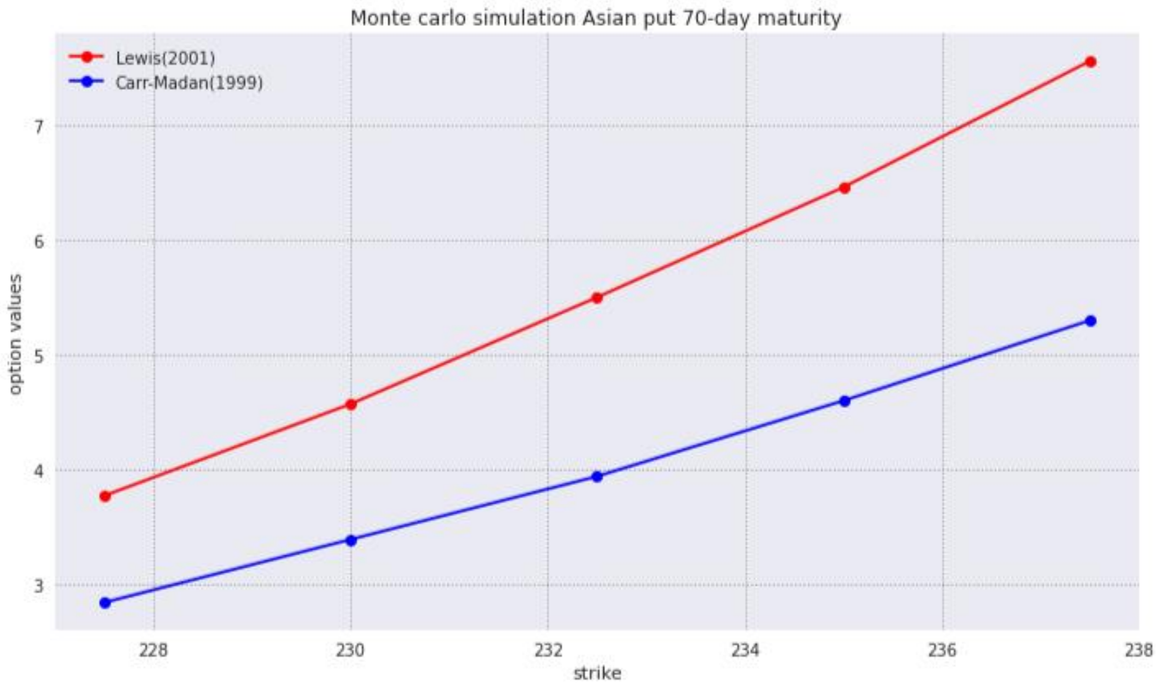
Fig_8



The difference in option prices for the 15_day and 60_day maturity increases as the strike price increases.This indicates a linear possitive relationship btween the differences and the strike prices.This is clearly indicated by both **Fig_6** and **Fig_8** and **Table_4** and partially **Fig_7.**

Monte carlo simulation for the Asian put option

Fig_9



- We would use the Heston model with jumps (Bates, 1996) to determine the option price. This would involve simulating the evolution of the underlying asset price over time, taking into account volatility and any jumps in the price. The parameters calibrated would be used in this simulation process.
- Finally, the fair price of the instrument would be determined using Monte Carlo methods in a risk-neutral setting.
- The results of this pricing process would provide an estimate of the fair price of the 70-day maturity, 95%-moneyness put option for the client. It is important to note that this estimate is based on the assumptions and inputs used in the calibration process and the simulation, and may not reflect the true market value of the option.

Fig_10

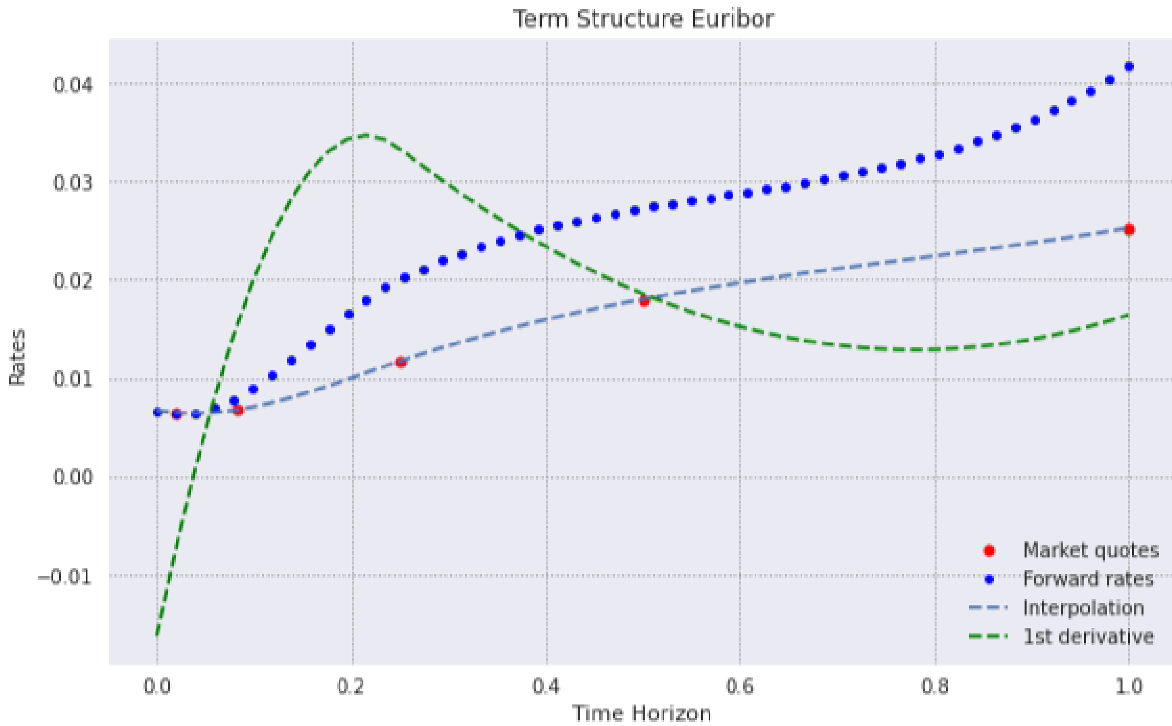

Term Structure Euribor
Rates
Time Horizon
0.04
0.03
0.02
0.01
0.00
−0.01
0.0
0.2
0.4
0.6
0.8
1.0
Market quotes
Forward rates
Interpolation
1st derivative

Fig_11

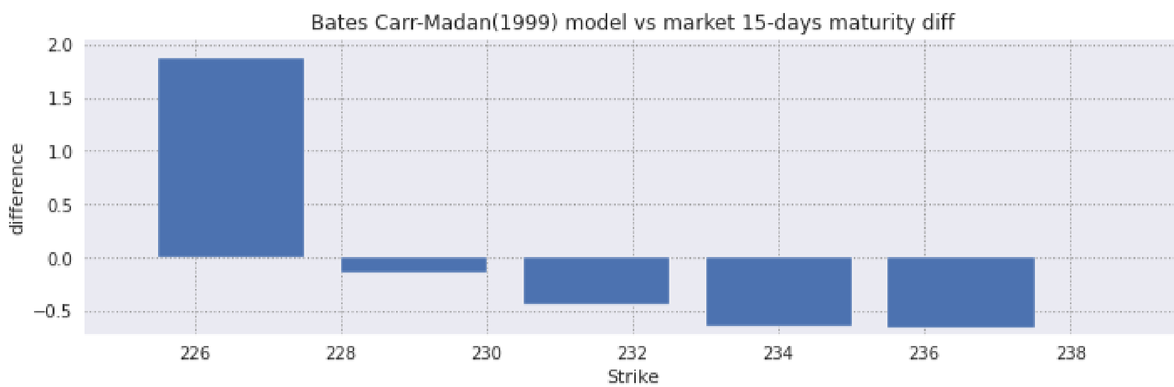
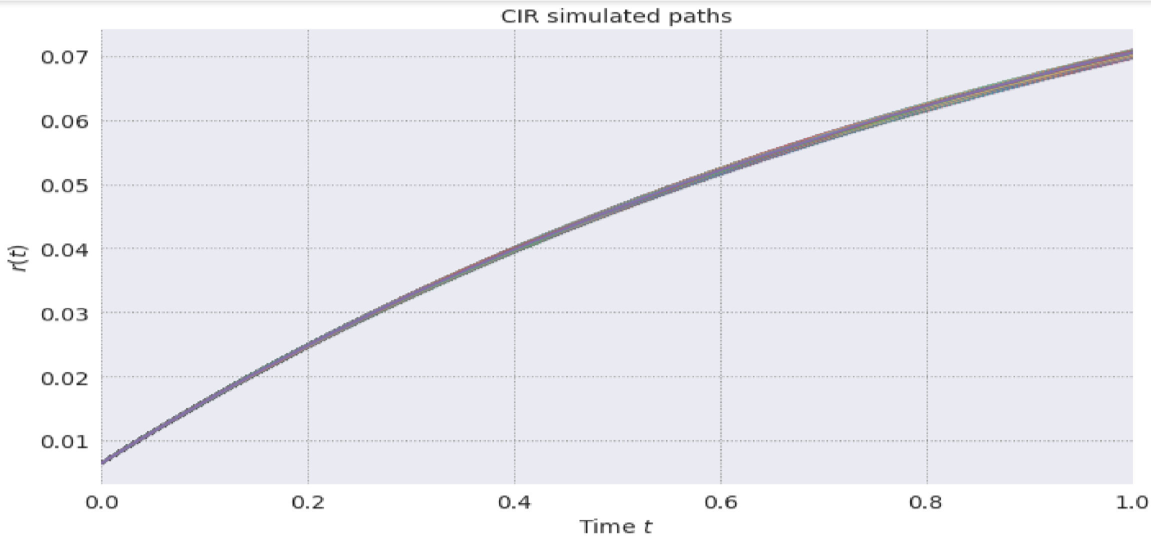

Bates Carr-Madan(1999) model vs market 15-days maturity diff
difference
Strike
2.0
1.5
1.0
0.5
0.0
−0.5
226
228
230
232
234
236
238

Fig_12



The following are our calibrated parametrs Kappa = 0.999, θ = 0.107, σ =0.001.

- High values of kappa indicate that the rate reverts back to its mean quickly, while low values of kappa indicate that the rate reverts back slowly.Our Kappa value is low which means the rate reverts back slowly.
- High values of theta indicate that the long-term mean of the rate is high, while low values of theta indicate that the long-term mean of the rate is low.In this case the value of theta is low as well which mean that the long term mean of the rate is low.
- High values of sigma indicate that the rate is highly volatile, while low values of sigma indicate that the rate is less volatile.Again the value of sigma is very low which indicates that the rate is less volatile