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1.1 The individual's problem

$$\max_{C_t^y, s_t, C_{t+1}^o, b_{t+1}} \frac{(C_t^y)^{1-\eta}-1}{1-\eta} + \beta\left[\frac{(C_{t+1}^o)^{1-\eta}-1}{1-\eta} + \chi\frac{[(1-\zeta)b_{t+1}]^{1-\eta}-1}{1-\eta}\right]$$

s.t. $C_t^y + s_t = w_t l_t + (1-\zeta)b_t + g_t$

$C_{t+1}^o + b_{t+1} = (1+r_{t+1})s_t$.

$\boxed{\text{Problem 1:}}$

Set $f(C_t^y, s_t, C_{t+1}^o, b_{t+1}, \lambda_1, \lambda_2) = \frac{(C_t^y)^{1-\eta}-1}{1-\eta} + \beta\left[\frac{(C_{t+1}^o)^{1-\eta}-1}{1-\eta} + \chi\frac{[(1-\zeta)b_{t+1}]^{1-\eta}-1}{1-\eta}\right]$

$+\lambda_1(C_t^y + s_t - w_t l_t - (1-\zeta)b_t - g_t)$

$+\lambda_2[C_{t+1}^o + b_{t+1} - (1+r_{t+1})s_t]$.

$$\begin{cases} \frac{\partial f}{\partial C_t^y} = (C_t^y)^{-\eta} + \lambda_1 = 0 \\ \frac{\partial f}{\partial s_t} = \lambda_1 - (1+r_{t+1})\lambda_2 = 0 \\ \frac{\partial f}{\partial C_{t+1}^o} = \beta(C_{t+1}^o)^{-\eta} + \lambda_2 = 0 \\ \frac{\partial f}{\partial b_{t+1}} = \beta\chi(1-\zeta)^{1-\eta}(b_{t+1})^{-\eta} + \lambda_2 = 0 \\ C_t^y + s_t = w_t l_t + (1-\zeta)b_t + g_t \\ C_{t+1}^o + b_{t+1} = (1+r_{t+1})s_t \end{cases} \Rightarrow \begin{cases} C_t^y = \frac{(1+r_{t+1})[w_t l_t + (1-\zeta)b_t + g_t]}{A_{t+1} + B_{t+1} + 1 + r_{t+1}} \\ b_{t+1} = A_{t+1} C_t^y \\ C_{t+1}^o = B_{t+1} C_t^y \\ s_t = \frac{A_{t+1} + B_{t+1}}{1+r_{t+1}} C_t^y \end{cases}$$

where $A_{t+1} = \left[\frac{1}{\beta\chi(1+r_{t+1})(1-\zeta)^{1-\eta}}\right]^{\frac{1}{-\eta}}$

$B_{t+1} = \left[\frac{1}{\beta(1+r_{t+1})}\right]^{-\frac{1}{\eta}}$

Problem 2:

$$S_t = \frac{A_{t+1}+B_{t+1}}{1+r_{t+1}} \cdot \frac{1+r_{t+1}}{A_{t+1}+B_{t+1}+1+r_{t+1}} \cdot [w_t l_t + (1-\xi) b_t + g_t]$$

$$= \frac{A_{t+1}+B_{t+1}}{A_{t+1}+B_{t+1}+1+r_{t+1}} [w_t l_t + (1-\xi) b_t + g_t]$$

$$S_{t-1} = \frac{A_t+B_t}{A_t+B_t+1+r_t} [w_{t-1} l_{t-1} + (1-\xi) b_{t-1} + g_{t-1}]$$

$$b_t = A_t C_{t+1}^y = \frac{A_t}{A_t+B_t+1+r_t}(1+r_t)[w_{t-1} l_{t-1} + (1-\xi) b_{t-1} + g_{t-1}]$$

$$= \frac{A_t}{A_t+B_t}(1+r_t) S_{t-1}$$

$$\therefore S_t = \frac{A_{t+1}+B_{t+1}}{A_{t+1}+B_{t+1}+1+r_{t+1}} \left[w_t l_t + (1-\xi) \frac{A_t(1+r_t)}{A_t+B_t} S_{t-1} + g_t\right]$$

1.2 The firm's problem

$$\max_{K_t, L_t} \{ A K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t+\delta) K_t \}$$

Problem 3:

Set $f(K_t, L_t) = A K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t+\delta) K_t$

$$\begin{cases} \frac{\partial f}{\partial K_t} = A\alpha K_t^{\alpha-1} L_t^{1-\alpha} - (r_t+\delta) = 0 \\ \frac{\partial f}{\partial L_t} = A(1-\alpha) K_t^{\alpha} L_t^{-\alpha} - w_t = 0 \end{cases} \Rightarrow \begin{cases} r_t = A\alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta \\ w_t = A(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} \end{cases}$$

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1.3 The gavern ment

$$g_t = \zeta \int b_t \, di$$

1.4 The general equilibrium

$$L_t = \int l_t \, di = 1$$

$$K_{t+1} = \int s_t \, di$$

Problem 4:

$$K_{t+1} = \int s_t \, di = \frac{A_{t+1} + B_{t+1}}{A_{t+1} + B_{t+1} + 1 + r_{t+1}} \left[w_t + (1-\zeta) \frac{A_t(1+r_t)}{A_t + B_t} K_t + g_t \right]$$

$$\because g_t = \zeta \int b_t \, di = \zeta \int \left[\frac{A_t}{A_t + B_t} (1 + r_t) s_{t-1} \right] di = \zeta \frac{A_t(1+r_t)}{A_t + B_t} K_t$$

$$\therefore K_{t+1} = \frac{A_{t+1} + B_{t+1}}{A_{t+1} + B_{t+1} + 1 + r_{t+1}} \left[w_t + \frac{A_t(1+r_t)}{A_t + B_t} K_t \right]$$

$$\because \begin{cases} r_t = A\alpha (K_t)^{\alpha - 1} - \delta \\ w_t = A(1-\alpha)(K_t)^{\alpha} \end{cases}$$

$$\therefore K_{t+1} = \frac{A_{t+1} + B_{t+1}}{A_{t+1} + B_{t+1} + 1 + r_{t+1}} \left[\left(1 - \alpha + \frac{A_t}{A_t + B_t} \alpha\right) A (K_t)^{\alpha} + \frac{A_t}{A_t + B_t} (1-\delta) K_t \right]$$

1.5 The wealth distribution

Problem 5:

$$a_{t+1} = S_t = \frac{\bar{A}+\bar{B}}{\bar{A}+\bar{B}+1+\bar{r}}\left[\bar{w}\, l_t + (1-\xi)\frac{\bar{A}(1+\bar{r})}{\bar{A}+\bar{B}}\, a_t + \xi \frac{\bar{A}(1+\bar{r})}{\bar{A}+\bar{B}}\bar{K}\right]$$

$$\therefore \quad a_{t+1} = c_3 l_t + c_4 a_t + c_5$$

$$\begin{cases} c_3 = \dfrac{\bar{A}+\bar{B}}{\bar{A}+\bar{B}+1+\bar{r}} \cdot \bar{w} \\[2ex] c_4 = \dfrac{\bar{A}+\bar{B}}{\bar{A}+\bar{B}+1+\bar{r}} \cdot (1-\xi)\dfrac{\bar{A}(1+\bar{r})}{\bar{A}+\bar{B}} = \dfrac{\bar{A}(1+\bar{r})(1-\xi)}{\bar{A}+\bar{B}+1+\bar{r}} \\[2ex] c_5 = \dfrac{\bar{A}(1+\bar{r})\xi}{\bar{A}+\bar{B}+1+\bar{r}} \cdot \bar{K} \end{cases}$$

where $\bar{r}, \bar{w}, \bar{K}$ is the steady-state

$$\bar{A} = \left[\frac{1}{\beta\chi(1+\bar{r})(1-\xi)^{1-\eta}}\right]^{\frac{1}{\eta}}, \quad \bar{B} = \left[\frac{1}{\beta(1+\bar{r})}\right]^{-\frac{1}{\eta}}$$

1.6 Estate taxes and wealth inequality.

Problem 6:

$$l_{t+1} = \bar{l} + \nu(l_t - \bar{l}) + \varepsilon_{t+1}$$

$$a_{t+1} = c_3 l_t + c_4 a_t + c_5$$

$$\therefore \quad a_\infty = \frac{c_5}{1-c_4} + c_3 \sum_{s=1}^{+\infty} c_4^{s-1} l_s$$

$$E(a_\infty) = \frac{c_5}{1-c_4} + c_3 \sum_{s=1}^{\infty} c_4^{s-1} \cdot E(l_s) = \frac{c_3 + c_5}{1-c_4}$$

$$Var(a_\infty) = C_3^2 \frac{1}{(1-C_4^2)(1-v^2)} \frac{1+C_4 v}{1-C_4 v} \sigma^2$$

$$Var(a_\infty^{nt}) = C_3^2 \frac{1}{(1-(C_4^{nt})^2)(1-v^2)} \cdot \left(\frac{1+C_4^{nt} v}{1-C_4^{nt} v}\right) \sigma^2$$

$\because C_4 = \frac{C_4^{nt}}{1+t} < C_4^{nt}$, $\therefore Var(a_\infty) < Var(a_\infty^{nt})$

$\because E(a_\infty) = E(a_\infty^{nt})$ $\therefore CV(a_\infty) < CV(a_\infty^{nt})$.

So the estate taxes reduce wealth inequality in the long run.