Live Coding Exercise: Time Series Model

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Introduction

We describe a coding exercise.

We'll ask a coding agent to reproduce all of the figures in this document.

In particular, we'll ask the agent to

- write a solution using Fortran
- write another solution using Python
- optimize both
- compare outcomes

Consider the one-dimensional dynamic system

$$x_{t+1} = g(x_t) \quad \text{where} \quad g(x) := 4x(1-x). \tag{1}$$

Defines **trajectories** $(x_t)_{t \geqslant 0}$ recursively by $x_{t+1} = g(x_t)$ and some fixed initial condition x_0

Another way to write this trajectory is

$$x_t = g^t(x_0) \quad \text{where} \quad g^t := g \circ \cdots \circ g$$

The product on the right is t compositions of g with itself.

The map g is called either the **logistic map** or the **quadratic map**

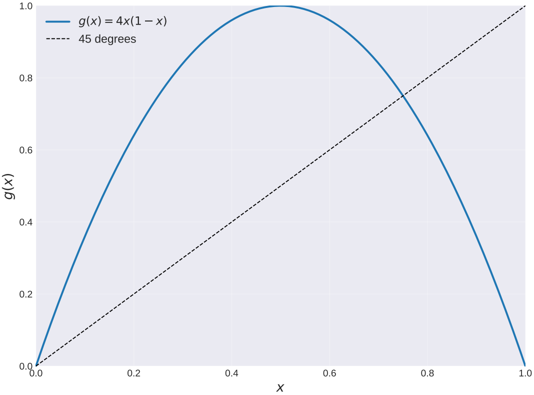

Figure: The quadratic map and 45 degree line

Cycles occur when we iterate with the quadratic map

This is easily confirmed by plotting time series

The next figure shows one such series when $x_0 = 0.3$

We see that the behavior of the state is not only cyclical but also highly erratic

It shows no sign of convergence to regular behaviour

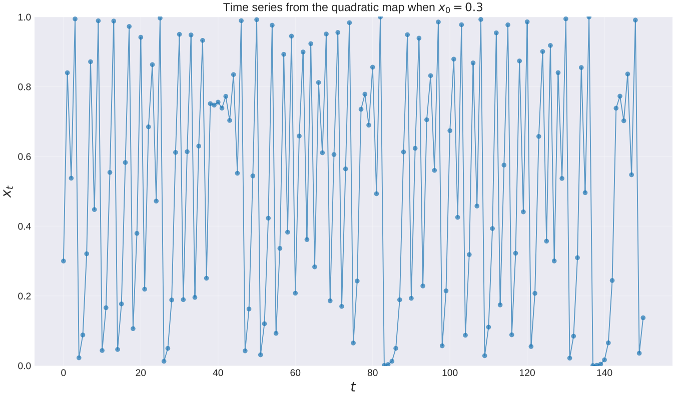

Figure: Time series from the quadratic map when $x_0 = 0.3$

At the same time, the model can generate strong "statistical" predictions over long time horizons

As one way to see this, consider the histogram shown in the next figure

This is a histogram of the sequence $(g^t(x_0))_{t=0}^n$ when $x_0 = 0.3$ and $n = 100,000$

Evidently there is some pattern to the apparent randomness, since the histogram is symmetric and U-shaped

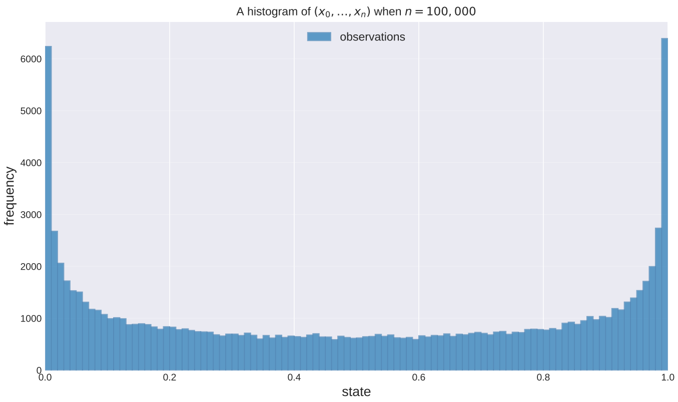

Figure: A histogram of $(x_0, \ldots, x_n)$ when $n = 100,000$