Recent Trends in Scientific Computing

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November 2023

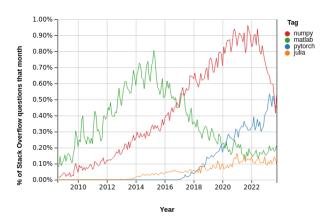
Topics

- Traditional compiled languages
- Modern JIT compilers
- Al-driven scientific computing
- Where are we heading?
- Economic applications

Sides, code:

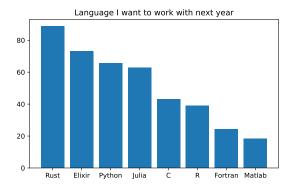
https://github.com/QuantEcon/treasury_2023

Some trends:



Source: Stackoverflow Trends

Stack Overflow 2023 Developer Survey (50 languages)



— https://survey.stackoverflow.co/2023/

A review of some scientific computing environments

General purpose scientific computing environments:

- 1. Fortran / C / Rust
- 2. MATLAB (\approx Python + NumPy)
- 3. Julia (\approx Python + Numba)
- 4. Python + Google JAX (\approx Python + PyTorch)

Fortran / C / Rust — static types and AOT compilers

Example. Suppose we want to compute the sequence

$$k_{t+1} = sk_t^\alpha + (1-\delta)k_t$$

from some given k_0

Let's write a function in C that

- 1. implements the loop
- 2. returns the last k_t

```
#include <stdio.h>
#include <math.h>
int main() {
    double k = 0.2:
    double alpha = 0.4;
    double s = 0.3;
    double delta = 0.1;
    int i:
    int n = 1000;
    for (i = 0; i < n; i++) {
        k = s * pow(k, alpha) + (1 - delta) * k;
    printf("k = %f \setminus n", k);
```

```
\varphi john on gz-precision .../treasury_2023 on \beta main )> gcc solow.c -o out -lm
```

 φ john on gz-precision .../treasury_2023 on β main $\rangle\rangle$./out

x = 6.240251

Pros

fast

Cons

- time consuming to write
- lack of portability
- hard to debug
- hard to parallelize
- low interactivity

For comparison, the same operation in Python:

```
α = 0.4
s = 0.3
δ = 0.1
n = 1_000
k = 0.2

for i in range(n-1):
    k = s * k**α + (1 - δ) * k

print(k)
```

Pros

- easy to write
- high portability
- easy to debug
- high interactivity

Cons

slow

So how can we get

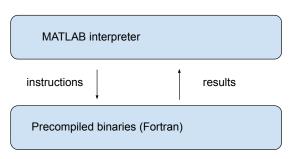
good execution speeds and high productivity / interactivity?

MATLAB

$$A = [2.0, -1.0 \\ 5.0, -0.5];$$

$$b = [0.5, 1.0]';$$

$$x = inv(A) * b$$



Python + NumPy

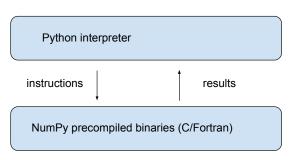
import numpy

$$A = ((2.0, -1.0), (5.0, -0.5))$$

$$b = (0.5, 1.0)$$

$$A, b = np.array(A), np.array(b)$$

$$x = np.inv(A) @ b$$



Julia — rise of the JIT compilers

Can do MATLAB / NumPy style vectorized operations

$$A = \begin{bmatrix} 2.0 & -1.0 \\ 5.0 & -0.5 \end{bmatrix}$$

$$b = [0.5 \ 1.0]$$

$$x = inv(A) * b$$

But also has fast loops via an efficient JIT compiler

Example. Suppose, again, that we want to compute

$$k_{t+1} = s k_t^\alpha + (1-\delta) k_t$$

from some given k_0

Iterative, not easily vectorized

```
function solow(k0, \alpha=0.4, \delta=0.1, n=1_000) 
 k = k0 
 for i in 1:(n-1) 
 k = s * k^\alpha + (1 - \delta) * k 
 end 
 return k 
end 
solow(0.2)
```

Julia accelerates solow at runtime via a JIT compiler

Python + Numba copy Julia

```
from numba import jit
@jit(nopython=True)
def solow(k0, \alpha=0.4, \delta=0.1, n=1_000):
    k = k0
    for i in range(n-1):
         k = s * k**\alpha + (1 - \delta) * k
    return k
solow(0.2)
```

Runs at same speed as Julia / C / Fortran

Al-driven scientific computing

Key players

- OpenAI (ChatGPT, Whisper), Microsoft
- Google Research, Google DeepMind
- Meta
- Apple?

Platforms / libraries

- PyTorch (Meta, OpenAI, Stable Diffusion)
- Google JAX (Google, Apple Ajax)
- Tensorflow, Mojo?

Lightening introduction to deep learning

Supervised deep learning: find a good approximation to an unknown functional relationship

$$y = f(x)$$

x is the input and y is the output

Examples.

- x = unfinished sentence, y = next word
- x = weather sensor data, y = max temp tomorrow

Relevant to economics?

"ECMWF's weather forecasting model is considered the gold standard for medium-term weather forecasting...Google DeepMind claims in an non-peer-reviewed paper to have beat it 90% of the time..."

"Traditional forecasting models are big, complex computer algorithms based on atmospheric physics and take hours to run. Al models can create forecasts in just seconds."

Source: MIT Technology Review July 2023

Training

Nonlinear regression: Take data set $(x_i, y_i)_{i=1}^n$ and solve

$$\min_{\theta} \ell(\theta) = \sum_{i=1}^{n} (y_i - f_{\theta}(x_i))^2 \quad \text{ s.t. } \quad \theta \in \Theta$$

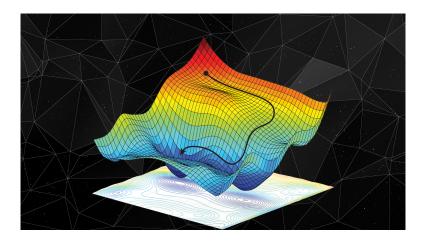
In the case of ANNs, we consider all f_{θ} having the form

$$f_\theta = \sigma \circ A_1 \circ \cdots \circ \sigma \circ A_k$$

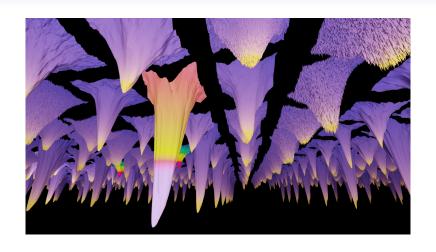
where

- $A_i x = W_i x + b_i$ is an affine map
- σ is a nonlinear "activation" function

Minimizing a smooth loss functions



Source: https://danielkhv.com/



Source: https://losslandscape.com/gallery/

Core elements

- automatic differentiation
- parallelization (CPUs / GPUs / TPUs)
- Compilers / JIT-compilers

```
import jax.numpy as inp
from jax import grad, jit
def predict(params, x):
  for W, b in params:
    y = inp.dot(W, x) + b
    x = inp.tanh(y)
  return y
def loss(params, x, targets):
  preds = predict(params, x)
  return jnp.sum((preds - targets)**2)
grad loss = jit(grad(loss))
```

Sample code

https://github.com/QuantEcon/treasury_2023/notebooks