Econophysics

How Does Physics Explain the Financial Market?
Panel Session

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Motivation

Financial Returns

$$(Today\ Price - Yesterday\ Price)/Yesterday\ Price$$

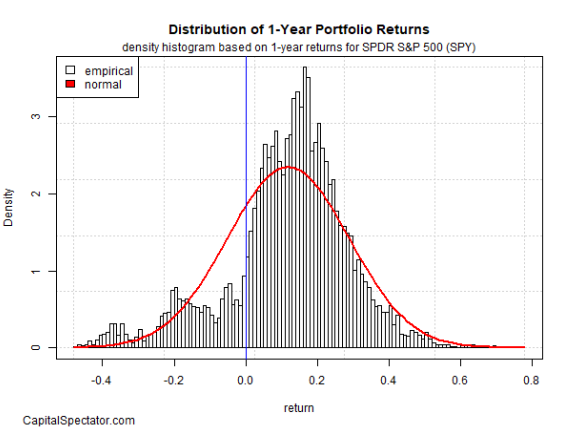


Figure 1: Returns over Time

Stylized Facts in Financial Data

- *Fat Tails*: Financial returns often show 'heavier' tails than a normal distribution.
- *Asymmetry*: Returns can exhibit skewness, not always symmetric around the mean.
- *Aggregate Normality*: As the frequency of return lengthen, aggregate returns tend to normality.
- *Absence of Serial Correlation*: Return series are generally not autocorrelated, except at high frequency.
- *Volatility Clustering*: Volatility of the returns is serially correlated.
- *Time-Varying Cross-Correlation*: Correlations between assets return tends to increase during high volatility period.

Quantitative Finance VS Econophysics?

- Econophysics aims to explain the causes of the results.
    - Why does it happen?
- Quantitative Finance focuses on Management.
    - How does we achieve a goal?

Physics behind Market Micro-Structure: Modeling Market Interactions

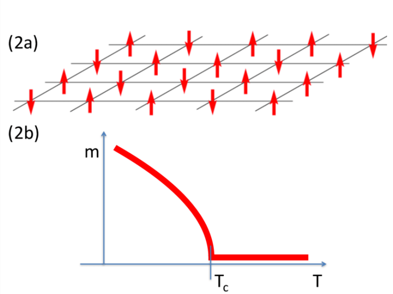
Herding Behavior: Cont and Bouchaud (2000)

- **Model Setup:**
  - A market with N agents trading a stock with price $p(t)$.
  - Each agent chooses to buy or sell one unit of stock ($i(t) = \pm 1$) with probability a, or stays idle with probability $1 - 2a$.
- **Price Change:**

$$p(t+1) = p(t) + \frac{1}{\lambda} \sum_{i=1}^{N} i(t)$$

  - $D(t) = \sum_{i=1}^{N} i(t)$ is the excess demand.
  - λ measures the liquidity or market depth.
- **Stock Return Distribution:**
  - If the demand i is IID with finite variance, then the distribution of $\Delta p(t)$ converges to a Gaussian distribution as N goes to infinity, according to the Central Limit Theorem.
  - The distribution of stock returns $\Delta p(t) = p(t+1) - p(t)$ depends on the joint distribution of individual demands $(i(t))_{1 \leq i \leq N}$.

Ising Model VS Traders Interaction



Ising Model: Synchronization of Coupled Oscillators—Phase Transitions and Entropy Production

Complex Network: Dynamics of Information Diffusion on Online Social Networks

Lux and Marchesi (2000) Model

Lux and Marchesi proposed a model in line with agent-based models in behavioral finance. The model considers a market with N agents, comprising two groups:

- n_f 'fundamentalists' with a perceived fundamental price p_f.
- n_c 'chartists' or trend followers.

The total number of agents is constant: $n_f + n_c = N$.

Demand:

- Each fundamentalist trades a volume V_f proportional to $(p_f - p)$.
- Each chartist trades a constant volume V_c.
- Excess demand by chartists: $(n^+ - n^-)V_c$.
- Demand from noise trader ϵ

Global Excess Demand:

$$ED = (n^+ - n^-)V_c + n_f\gamma(p_f - p) + \epsilon$$

Physics behind Market Micro-Structure: Order-Driven Market Modeling

Limit Order Book

- A Limit Order Book (LOB) is a record of unexecuted limit orders in a financial market.
- It is organized by security and price level, often displayed electronically.

Key Components:

- *Bid*: The price and quantity that buyers are willing to purchase.
- *Ask*: The price and quantity that sellers are willing to sell.
- *Spread*: The difference between the highest bid and the lowest ask.

Functions:

- Price Discovery
- Order Matching
- Trade Execution

Note:

- Orders are generally executed on a "first-come, first-served" basis.
- Limit Order Books are dynamic and can change rapidly.

Reaction–Diffusion Systems

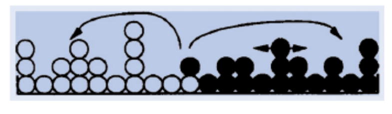
Reaction–diffusion systems are mathematical models that correspond to various physical phenomena.
https://demonstrations.wolfram.com/DiffusionOfGasesInATube/ **Equation:**

$$\frac{\partial u}{\partial t} = D\nabla^2 u + R(u)$$

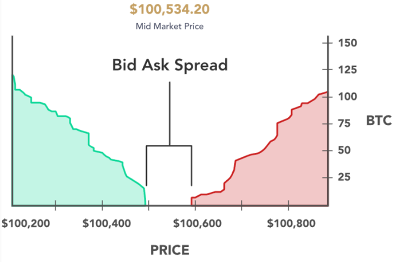
Two Key Components:

- *Local Chemical Reactions ($R(u)$)*: Substances are transformed into each other.
- *Diffusion ($D\nabla^2 u$)*: Causes substances to spread out over a surface in space.

Order Book vs Particles Collision



Particles Collision



Order Books

Table: Analogy between Physics and Bak et al. (1997)

Table 1: Analogy between the $A + B \to \emptyset$ reaction model and the order book of Bak et al. (1997)

Physics	Bak et al. (1997)
Particles	Orders
Finite pipe	Order book
Collision	Transaction

The Order Book as a Reaction–Diffusion Model

Bak et al. (1997) presented a simple model based on Physics:

- Market with N noise traders.
- Each trader can exchange one share at a time.
- Price $p(t)$ is an integer with an upper bound Δp.

Initial Conditions:

$$p_j^b(0) \in \{0, \Delta p/2\}, \quad j = 1, \ldots, N/2,$$
$$p_j^s(0) \in \{\Delta p/2, \Delta p\}, \quad j = 1, \ldots, N/2.$$

Price Update:

$$p_j^s(t+1) = p_j^s(t) \pm 1, \quad p_j^b(t+1) = p_j^b(t) \pm 1$$

Transaction: A transaction occurs if $p_i^b(t+1) = p_j^s(t+1)$, and the orders are removed.

Note: The number of orders remains constant.

Simulation Process and Analogy to Physics

The simulation process is akin to the reaction–diffusion model $A + B \to \emptyset$ in Physics.

Physics Analogy:

- Two types of particles are inserted on each side of a pipe of length Δp.
- Particles move randomly with steps of size one.
- Upon collision, particles are annihilated and replaced.

Price Variation:

$$\Delta p(t) \approx t^{1/4} \ln \left(\frac{t}{t_0} \right)^{1/2}$$

Hurst Exponent:

- Simulated price increments exhibit $H = 1/4$.
- Contrasts with the random walk expectation $H = 1/2$.
- Indicates sub-diffusive behavior, a step in the wrong direction for $H \geq 0.7$.

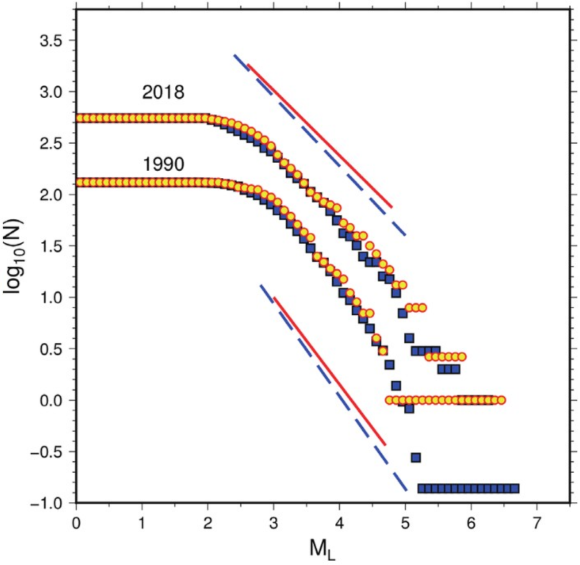
The Black–Scholes PDE

- The Black–Scholes PDE describes the price dynamics of a European option.
- The equation is given by:

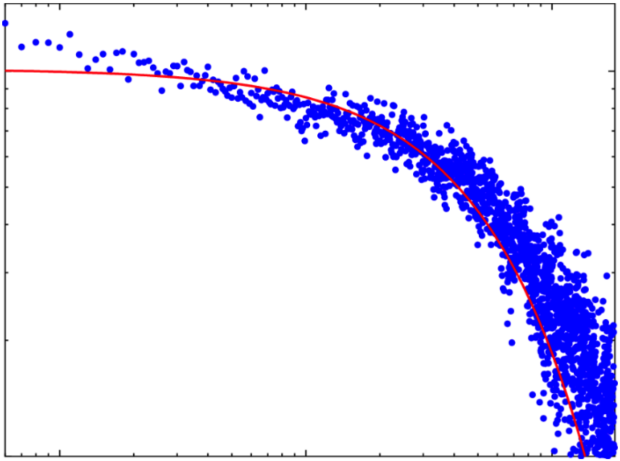
$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

- Where:
    - V is the option price
    - t is time
    - S is the price of the underlying asset
    - σ is the volatility
    - r is the risk-free rate

Econophysics Model in Practice



Frequency-magnitude distributions of foreshock (circles) and aftershock (squares) sequences for the 1990 and 2018 Hualien earthquakes, with the total number normalized between each earthquake's foreshocks and aftershocks. The line segments indicate the range of M L for inverting b-values. Note that the circles are distributed to the right of the squares, suggesting a genuinely depressed b-value for foreshock sequences.



Autocorrelation of the process Y (t) in log-log scale. The solid line is the autocorrelation of y = ln(σ/m) for the