

# Factor Model

## Factor Investing for Retail Investors

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## Mathematical Tool

Factor models play a crucial role in the analysis of financial markets and portfolio management. They aim to decompose the returns on securities into contributions from various underlying factors.

The general expression of a factor model is given by:

$$R_i = \alpha_i + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{in}F_n + \epsilon_i$$

where:

- ▶  $R_i$  is the return on asset  $i$ ,
- ▶  $\alpha_i$  represents the asset's expected return not explained by the factors,
- ▶  $\beta_{ij}$  measures the sensitivity of the return on asset  $i$  to factor  $j$ ,
- ▶  $F_j$  is the value of factor  $j$ ,
- ▶  $\epsilon_i$  is the idiosyncratic return component for asset  $i$ .

# Python Code

```
import numpy as np
model = LinearRegression()
model.fit(X, y)
print(Slope:, model.coef[0])
print(Intercept:, model.intercept)
"""
```

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Generating Alpha

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## Introduction

The Fama-French Five-Factor model is an extension of the original Three-Factor model, designed to explain stock returns through five distinct factors. These factors capture various aspects of stocks' risk and return profiles.

1. **Market Risk** (Market): Excess return of the market over the risk-free rate.
2. **Size** (SMB, Small Minus Big): Stocks with smaller market capitalizations have higher returns than those with larger market caps.
3. **Value** (HML, High Minus Low): Stocks with high book-to-market ratios (value stocks) outperform those with low book-to-market ratios (growth stocks).

# Fama-French Five-Factor Model: Additional Factors

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4. **Profitability** (RMW, Robust Minus Weak): Companies with high operating profitability outperform those with low profitability, independent of their size.
5. **Investment** (CMA, Conservative Minus Aggressive): Firms with lower investment rates have higher returns than firms with higher investment rates.

## Selection of Stock Generating Alpha

It is the constant after regression in the equation of the factor model

- ▶ A positive  $\alpha$  indicates the portfolio has earned excess returns to that predicted by its risk (beta).
- ▶ A negative  $\alpha$  indicates underperformance relative to expectations.



## Index-Tracking Optimal Portfolio Selection

# Mathematical Model for Index-Tracking Optimal Portfolio Selection

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- ▶ **Objective:** Minimize the tracking error between the portfolio's returns and the index's returns.
- ▶ **Tracking Error Minimization:**

$$\min_w \sqrt{\sum_{t=1}^T (r_{p,t} - r_{i,t})^2}$$

where  $r_{p,t} = w^\top r_t$  and  $r_{i,t}$  is the return of the index at time  $t$ .

- ▶ **Constraints:**

- ▶ Budget Constraint:

$$\sum_{j=1}^n w_j = 1$$

- ▶ Non-negativity (if no short selling allowed):

$$w_j \geq 0, \quad \forall j$$