Black-Litterman Model with Factor: Practice to Theory

Week II: Black-Litterman Model & Investor View with Factors

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Target of Mean-Variance Portfolio

- The mean-variance portfolio aims to optimize asset allocation by balancing expected return and risk.
- Investors seek to maximize returns for a given level of risk or minimize risk for a given level of return.
- The efficient frontier represents portfolios that offer the best risk-return tradeoff.
- The tangency portfolio, incorporating a risk-free asset, leads to the Capital Market Line (CML).

Mathematical Formulation of Mean-Variance Portfolio

Portfolio Expected Return:

$$\mathbb{E}[R_p] = \sum_{i=1}^{n} w_i \mathbb{E}[R_i] = \mathbf{w}'\boldsymbol{\mu} \tag{1}$$

Portfolio Variance:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \mathbf{w}'\Sigma\mathbf{w} \tag{2}$$

Optimization Problem:

$$\min_{\mathbf{w}} \mathbf{w}'\Sigma\mathbf{w}, \quad \text{subject to } \mathbf{w}'\mathbf{1} = 1 \tag{3}$$

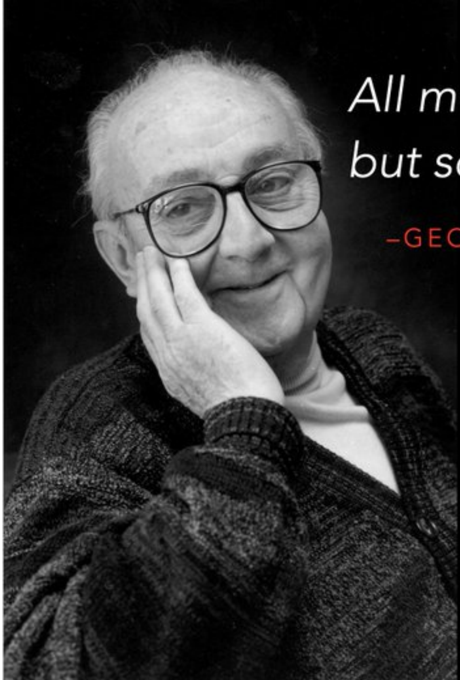
where $\mathbf{w}$ is the vector of portfolio weights, $\boldsymbol{\mu}$ is the vector of expected returns, and Σ is the covariance matrix of asset returns.

Where MVP Benefits

- **Long-Term Portfolio:** Designed for stability and consistent returns over time.
- **Institutional Portfolio:** Focuses on minimizing volatility to achieve sustainable long-term growth.
- **Leverage:** Used strategically to enhance returns while controlling overall risk exposure.
- **Order Management System (OMS):** A dynamic risk management tool such as stop loss and trailing stop loss.

Section

2 Why is Mean-Variance Portfolio Old School?



All models are wrong, but some are useful.

–GEORGE BOX, UW-MADISON

Section 2.1

Limitations of MVP

Sensitivity to Input Estimates of Expected Return

- Highly sensitive to input estimates, leading to extreme and unstable allocations.
- Often Produces concentrated portfolios that may not be well-diversified

Example

- **Objective:** Construct an optimal portfolio by minimizing variance for a given annual target return.
- **Retrieve Data:** Download historical stock prices for 5 assets using Yahoo Finance. The portfolio consists of AAPL, MSFT, GOOGL, AMZN and TSLA during 2023 to 2024.

E[R]	AAPL	MSFT	GOOGL	AMZN	TSLA
0.46	0.73	0.05	0.03	0.22	-0.03
2.80	-3.86	2.86	-1.23	-0.46	3.69
5.14	-8.45	5.68	-2.49	-1.14	7.41
7.47	-13.05	8.49	-3.76	-1.82	11.13
9.81	-17.64	11.31	-5.02	-2.50	14.85

Section

3 Introduction to Black-Litterman Portfolio

Section 3.1

Improvement from MVP

Benefits of Black-Litterman Portfolio

- **Stable and Diversified Allocations:** Overcomes the instability of Mean-Variance optimization.
- **Incorporates Investor Views:** Allows investors to adjust return estimates with confidence levels.
- **Uses Market Equilibrium Returns:** Starts with an equilibrium CAPM-based prior, reducing estimation risk.
- **More Realistic Portfolios:** Produces well-balanced portfolios that reflect both historical data and expert opinions.

Section 3.2

Pioneers

Fischer Black and His Achievements

- **Fischer Black (1938–1995):** Renowned economist, mathematician, and financial theorist.
- **Key Contributions:**
  - **Black-Scholes Model (1973):** Pioneered modern option pricing.
  - **Black-Litterman Model (1992):** Improved mean-variance optimization.
  - **Capital Market Theory:** Explored equilibrium models in finance.

Robert Litterman and His Achievements

- **Robert Litterman:** Economist, risk management expert, and former head of Quantitative Resources at Goldman Sachs.
- **Key Contributions:**
  - **Black-Litterman Model (1992):** Co-developed a Bayesian approach to portfolio optimization, improving on mean-variance models.
  - **Climate Risk and Sustainable Finance:** Advocates for integrating climate-related risks into financial decision-making.

Goldman Sachs

Founded: 1869
Headquarters: New York City, USA
CEO: David M. Solomon (as of 2024)
Industry: Investment Banking, Asset Management,
Trading, and Securities

Goldman Sachs is a leading global investment banking, securities, and investment management firm. It provides a wide range of financial services to corporations, financial institutions, governments, and individuals.

High Return University Endowment

- **Columbia University** 11.5%
- **Brown University Endowment** 11.3%
- **Harvard University** 9.6%
- **University of California, San Diego Foundation** 12.3%

Section

4 Implementation

Step 1: Define Market Equilibrium Returns

- Assume the market is in equilibrium, and expected returns follow the *Capital Asset Pricing Model (CAPM)*.
- Compute the *implied excess returns* based on the market capitalization weights:

Implied Excess Returns

$$\Pi = \lambda \Sigma w_m \tag{4}$$

where:

- Π = vector of implied excess returns,
- λ = risk aversion coefficient,
- Σ_m = covariance matrix of asset returns,
- w_m = market-capitalization weights.

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Real-World Example: Market-Capitalization Weights

- **What is Market-Capitalization Weight?**
  - It represents an asset' s share of the total market value of all assets in a given market.
  - Used in stock indices, ETFs, and portfolio construction.
- **Example: S&P 500 Market-Cap Weights**

Company	Market Cap (in Trillions)	Market Weight
Apple (AAPL)	2.8	$\frac{2.8}{40} = 7.0\%$
Microsoft (MSFT)	2.5	$\frac{2.5}{40} = 6.25\%$
Amazon (AMZN)	1.7	$\frac{1.7}{40} = 4.25\%$
Tesla (TSLA)	0.9	$\frac{0.9}{40} = 2.25\%$
Total (S&P 500)	**40.0**	**100%**

Capital Asset Pricing Model

- **Definition:** The CAPM describes the relationship between expected return and systematic risk.
- **Formula:**

$$\mathbb{E}[R_i] = R_f + \beta_i(\mathbb{E}[R_m] - R_f) \tag{5}$$

where:

- $\mathbb{E}[R_i]$ = Expected return of asset i,
- R_f = Risk-free rate,
- β_i = Asset' s sensitivity to market risk,
- $\mathbb{E}[R_m]$ = Expected market return.

Derivation of Market Equilibrium Returns from Mean-Variance Optimization

- **Objective:** Investors maximize expected return while minimizing risk.
- **Mean-Variance Optimization Problem:**

$$\max_{w} \quad \mathbb{E}[R_p] - \frac{\lambda}{2}\mathrm{Var}(R_p) \tag{6}$$

where:

- w = portfolio weights,
- $\mathbb{E}[R_p] = w'\mathbb{E}[R]$ = expected portfolio return,
- $\mathrm{Var}(R_p) = w'\Sigma w$ = portfolio variance,
- λ = risk aversion coefficient.

- **First-Order Condition:** Differentiate w.r.t. w:

$$\frac{\partial}{\partial w}\left(w'\mathbb{E}[R] - \frac{\lambda}{2}w'\Sigma w\right) = 0 \tag{7}$$

- *Taking the derivative:*

$$\mathbb{E}[R] - \lambda\Sigma w = 0 \tag{8}$$

- *Solving for* w

$$w^* = \frac{1}{\lambda}\Sigma^{-1}\mathbb{E}[R] \tag{9}$$

- **Market Equilibrium Assumption:**
  - In equilibrium, investors collectively hold the market portfolio.
  - The *market-clearing condition* means the equilibrium portfolio is given by market capitalization weights w_m.
- **Setting $w = w_m$ in equilibrium:**

$$\mathbb{E}[R] = \lambda \Sigma w_m \tag{10}$$

- *Defining Market-Implied Excess Returns:*

$$\Pi = \lambda \Sigma w_m \tag{11}$$

Why Define Market Equilibrium Returns?

- *Problem in Mean-Variance Optimization (MVO):*
  - Expected returns (μ) are difficult to estimate accurately.
  - Small errors in μ lead to extreme and unrealistic portfolio allocations.
- *Black-Litterman Solution:*
  - Instead of estimating μ directly, start with a *stable market equilibrium* assumption.
  - Use *Capital Asset Pricing Model (CAPM)* to derive *implied excess returns* Π.
- **Key Idea:** The market portfolio represents an optimal balance of risk and return, providing a reasonable starting point for expected returns.

Step 2: Incorporate Investor Views

- Investors introduce their own *views* on expected asset returns.
- Views are expressed in the form:

Expected Returns based on Investor Beliefs

$$Q = P\Pi + \epsilon \tag{12}$$

where:

- Q = vector of expected returns based on investor beliefs,
- P = matrix defining which assets are affected by views,
- ϵ = error term capturing uncertainty in views.

- Each view has an associated confidence level, influencing portfolio weights.

Why Incorporate Investor Views?

- *Problem with Market Equilibrium Returns:*
  - $\Pi = \lambda\Sigma w_m$ assumes investors accept market-implied returns.
  - In reality, investors have subjective opinions on expected returns.
- *Solution:*
  - The Black-Litterman model allows investors to express *views* on assets.
  - These views are integrated into the market equilibrium returns.
  - Helps construct *more realistic* and *customized* portfolios.
- *Key Idea:*
  - We modify market equilibrium returns using investor inputs while maintaining *stability*.

Examples of Investor Views in Matrix Form

- *Example 1: An investor believes Asset 1 will have a return of 6%.*

$$P = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.06 \end{bmatrix} \tag{13}$$

- *Example 2: An investor expects Asset 2 to outperform Asset 3 by 2%.*

$$P = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.02 \end{bmatrix} \tag{14}$$

- *Example 3: Multiple Views*

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.06 \\ 0.02 \end{bmatrix} \tag{15}$$

- **Key Benefit:** Views modify only specific assets without disturbing overall equilibrium.

Incorporating View Confidence Levels

- Investor views have different levels of certainty.
- Confidence levels are represented by a covariance matrix:

$$\Omega = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \tag{16}$$

  where:
  - σ_i^2 represents the uncertainty (variance) in each investor view.
  - Higher σ_i^2 means lower confidence in that view.
- *Why Use Ω?*
  - Ensures that views with low confidence do not strongly influence portfolio weights.
  - Maintains stability while integrating subjective insights.

Why This Approach Works

- *Avoids Extreme Portfolio Weights:*
  - Investor views alone could lead to unrealistic allocations.
  - Ω prevents overreliance on any single view.
- *Combines Market Data with Expert Judgment:*
  - Bayesian inference integrates both efficiently.
  - Ensures a more balanced final return estimate.
- *Provides a Structured Framework:*
  - Unlike ad-hoc modifications, this method ensures mathematical consistency.
- **Key Insight:** Investor views are incorporated smoothly into expected returns without disrupting overall market stability.

Step 3: Compute Posterior Returns Using Bayesian Updating

- Update the market-implied returns Π using Bayesian inference:

Adjusted Expected Returns Incorporating Investor Views

$$\mu^* = \left((\tau\Sigma)^{-1} + P'\Omega^{-1}P\right)^{-1} \left((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right) \tag{17}$$

where:

- μ^* = adjusted expected returns incorporating investor views,
- τ = scalar controlling uncertainty in equilibrium returns,
- Ω = diagonal matrix representing uncertainty in views.

- This process refines expected returns by combining *market data* and *investor opinions*.

Why Use Bayesian Updating?

- *Problem with Market Returns and Investor Views:*
  - Market equilibrium returns (Π) alone may not reflect real-world expectations.
  - Investor views (Q) can be biased or uncertain.
- *Solution: Bayesian Updating:*
  - Bayesian inference provides a *weighted combination* of market equilibrium and investor beliefs.
  - The influence of each is determined by *confidence levels* in the views.
- *Key Idea:*
  - Instead of replacing Π with Q, we create a *blended return estimate* μ^* that accounts for both.

Understanding the Role of Each Term

- **Market Confidence (Left Term):**

$$(\tau\Sigma)^{-1}\Pi \tag{18}$$

  - Reflects equilibrium returns weighted by covariance Σ.
  - τ controls how much we trust equilibrium estimates.

- **Investor Confidence (Right Term):**

$$P'\Omega^{-1}Q \tag{19}$$

  - Represents investor views weighted by confidence levels in Ω.

- **Blending Both:**

$$\left((\tau\Sigma)^{-1} + P'\Omega^{-1}P\right)^{-1} \tag{20}$$

  - Determines the final weight given to market vs. investor views.

Example –Impact of Confidence Levels

- *Case 1: High Confidence in Market Equilibrium*
  - τ is large $\Rightarrow$ Market returns Π dominate.
  - Portfolio remains close to market-cap weights.
- *Case 2: High Confidence in Investor Views*
  - Ω is small $\Rightarrow$ Investor views Q dominate.
  - Portfolio shifts strongly toward investor beliefs.
- *Case 3: Balanced Confidence Levels*
  - Both terms contribute equally.
  - Portfolio adjusts in a smooth and stable manner.
- **Key Takeaway:** Bayesian updating *avoids extreme changes* while incorporating investor insights.

Why This Approach Works

- *Mathematically Well-Structured:*
  - Uses Bayesian statistics for a robust and stable update.
  - Ensures smooth integration of multiple data sources.
- *Prevents Extreme Portfolio Weights:*
  - Unlike MVO, does not lead to highly concentrated portfolios.
- *Balances Market Data and Investor Views:*
  - Avoids over-reliance on either one.
- *Creates a Realistic Portfolio Allocation:*
  - Better reflects both historical data and expert judgment.

Step 4: Optimize the Portfolio

- Use *Mean-Variance Optimization* to compute optimal portfolio weights:

Optimal Portfolio Weights Incorporating Investor Views

$$w^* = \frac{1}{\lambda}\Sigma^{-1}\mu^* \tag{21}$$

where:

- w^* = optimal portfolio weights incorporating investor views,
- μ^* = adjusted expected returns,
- Σ = covariance matrix of asset returns,
- λ = risk aversion coefficient.

- This results in a *well-diversified, stable, and realistic* asset allocation.

Summary of Black-Litterman Process

1. *Define Market Equilibrium Returns* using CAPM-based implied excess returns.
2. *Incorporate Investor View* by specifying return expectations and confidence levels.
3. *Compute Posterior Expected Returns* using Bayesian updating.
4. *Optimize the Portfolio* using Mean-Variance Optimization.

Outcome: A robust portfolio that integrates market equilibrium and investor insights for better diversification and stability.

Section

5 Case Study I: BL Maximum Sharpe Ratio Portfolio with Personal Views

Experiment Overview

- **Objective:** Compare the *Maximum Sharpe Ratio Portfolio* for the *Mean-Variance (MV) model* and Black-Litterman (BL) model.
- **Methodology:**
  1. Download historical data for five assets and S&P 500 as the market portfolio.
  2. Compute market-implied equilibrium returns (Π) using the *CAPM model*.
  3. Incorporate three *investor views* into the Black-Litterman model.
  4. Compute *Maximum Sharpe Ratio Portfolio* analytically using the formula:

$$w^* = \frac{\Sigma^{-1}(\mu_{ML} - r_f 1)}{1'\Sigma^{-1}(\mu_{ML} - r_f 1)} \tag{22}$$

  5. Compare *portfolio weights, expected return, volatility, and Sharpe ratio* for both models.

Market Portfolio and Assets Considered

- **Market Portfolio: S&P 500**
  - The *S&P 500 Index* represents the overall US equity market.
  - Used as the *market-capitalization-weighted portfolio* in the Black-Litterman framework.
  - The *expected return of the market portfolio* is estimated using the *historical average return* of the S&P 500, $\Pi = \Sigma_m w_m \approx \frac{1}{N}\mathbf{1}$, i.e. equal-weight benchmark (Just in Case Unknown Weight). The market-cap weight for S&P500 index can be found on https://www.slickcharts.com/sp500.
- **Assets Included in the Portfolio**: AAPL, MSFT,GOOGL, AMZN, and TSLA

Mathematical Model: Maximum Sharpe Ratio Portfolio

- **Objective:** Maximize the Sharpe Ratio

$$\max_{w} \frac{\mathbb{E}[R_p] - R_f}{\sigma_p} \tag{23}$$

- **Analytical Solution:**

$$w^* = \frac{\Sigma^{-1}(\mu_{BL} - r_f 1)}{1' \Sigma^{-1}(\mu_{BL} - r_f 1)} \tag{24}$$

Investor Views in Black-Litterman Model

- **Investor Views in This Experiment**

View	**Assets Involved**	**Expected Excess Return**
1	AAPL will outperform GOOGL	+2.00%
2	MSFT will outperform TSLA	+1.50%
3	AMZN will outperform TSLA	+1.00%

- **View Matrix (P), Expected Returns (Q), and Uncertainty (Ω),** $\tau = 1$

$$P = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.02 \\ 0.015 \\ 0.01 \end{bmatrix} \quad \Omega = \begin{bmatrix} 0.005 & 0 & 0 \\ 0 & 0.005 & 0 \\ 0 & 0 & 0.005 \end{bmatrix} \tag{25}$$

- $$\mu_{BL} = \left((\tau\Sigma)^{-1} + P'\Omega^{-1}P\right)^{-1} \left((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right) \tag{26}$$

Portfolio Metric Comparison

- The table below compares the key metrics for the Maximum Sharpe Ratio Portfolio:

Metric	**Max Sharpe Ratio (MV)**	**Max Sharpe Ratio (BL)**
Expected Return	0.5674	0.1347
Volatility	0.2130	0.2166
Sharpe Ratio	2.5699	0.5295

- **Observations:**
  - The Mean-Variance portfolio has a *higher expected return (0.5674 vs. 0.1347)*.
  - The Black-Litterman portfolio has a slightly *higher volatility (0.2166 vs. 0.2130)*.
  - The Sharpe Ratio is significantly *higher for the MV Portfolio (2.5699 vs. 0.5295)*.

Portfolio Weights Comparison

- The table below shows the *optimal asset allocation* for both portfolios:

Ticker	Max Sharpe Weights (MV)	Max Sharpe Weights (BL)
AAPL	0.5223	0.2841
MSFT	0.1908	0.3766
GOOGL	-0.0227	0.0290
AMZN	0.1758	0.2700
TSLA	0.1338	0.0403

- **Observations:**
  - The *Mean-Variance Portfolio* heavily invests in *AAPL (52.23%)*.
  - The *Black-Litterman Portfolio* shifts more weight to *MSFT and AMZN* due to investor views.
  - The MV Portfolio includes *short positions in GOOGL (-2.27%)*, while BL avoids shorts.
  - *TSLA receives a significantly lower allocation in the BL model* compared to MV.

Key Takeaways

- **Mean-Variance vs. Black-Litterman:**
  - The *Mean-Variance Portfolio (MV)* achieves a *higher Sharpe Ratio* but is highly sensitive to expected return estimates.
  - The *Black-Litterman Portfolio (BL)* integrates market equilibrium and investor views, resulting in a more stable allocation.
- **Portfolio Performance:**
  - *MV Portfolio achieves higher returns but has extreme allocations.*
  - *BL Portfolio smooths allocations but sacrifices return potential.*
  - *BL avoids short positions* and reallocates weights towards more diversified holdings.

Using LLM for Sentiment and Scoring the View Matrices

As you are Financial New Sentiment Expert, please read the financial news and stock price and give me the comparison view in Black-Litterman Model in matrix form of P, Q and Omega. Please consider portfolio consists of TSLA, AAPL, GOOGL, AMZN and MSFT.

Section

6 Case Study II: Thai BL Minimum Variance - Target Return Portfolio

Experiment Overview

6 Case Study II: Thai BL Minimum Variance - Target Return Portfolio

Objective: Construct and compare the Black-Litterman Portfolio for Thai stocks with different investor views.

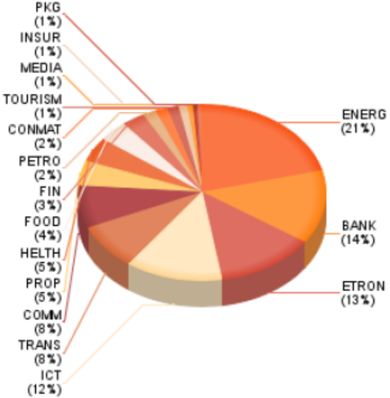
- *Market Considered:* Thai Stock Market with SET100 index. https://www.set.or.th/th/market/statistics/market-statistics/monthly-report/set100
- *Stocks Used:* DELTA.BK, PTT.BK, ADVANC.BK, AOT.BK, GULF.BK during 2023 and 2024

SET100 : Top 10 constituents

Symbol	Sector	Market cap. (mil. THB)	Index weight (%)
DELTA	ETRON	1,571,700.83	11.92
PTT	ENERG	899,734.38	6.83
ADVANC	ICT	844,675.57	6.41
AOT	TRANS	789,284.93	5.99
GULF	ENERG	677,589.41	5.14
PTTEP	ENERG	506,173.14	3.84
CPALL	COMM	467,121.27	3.54
SCB	BANK	420,888.41	3.19
TRUE	ICT	404,259.58	3.07
KBANK	BANK	380,277.08	2.88

Index weight data as of: January 2025

Sector breakdown chart



Market Equilibrium and CAPM-Based Returns

Market capitalization weights:

$$w_m = \frac{\text{Market Cap}_i}{\sum \text{Market Cap}_i} \tag{27}$$

Equilibrium excess returns:

$$\Pi = \lambda \Sigma w_m \tag{28}$$

where:

- λ = Risk aversion coefficient (set to 10)
- Σ = Covariance matrix of asset returns
- w_m = Market capitalization weights

Investor Views and Confidence Levels

Three Investor Views Considered:

View	Assets Involved	Expected Excess Return
1	DELTA > PTT, PTT > AOT	+2.00%, +1.50%
2	PTT > GULF, DELTA > AOT	+1.00%, +1.80%
3	ADVANC > GULF, DELTA > ADVANC	+1.50%, +1.20%

Black-Litterman Model: View Matrix and Parameters

View Matrix (P) - Defines Assets Affected by Each View:

$$P_{\text{View 1}} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}, \quad P_{\text{View 2}} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$P_{\text{View 3}} = \begin{bmatrix} 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

Expected Returns Vector (Q) - Investor' s Expected Excess Returns:

$$Q_{\text{View 1}} = \begin{bmatrix} 0.02 \\ 0.015 \end{bmatrix}, \quad Q_{\text{View 2}} = \begin{bmatrix} 0.01 \\ 0.018 \end{bmatrix}, \quad Q_{\text{View 3}} = \begin{bmatrix} 0.015 \\ 0.012 \end{bmatrix}$$

Confidence Matrix (Ω) - Variance of View Errors:

$$\Omega = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix}$$

Other Key Parameters:

- $\lambda = 10$ (Risk Aversion Coefficient)
- $\tau = 0.05$ (Market Uncertainty Factor)

$R_{target} = 0.1$ (Target Expected Return)

Black-Litterman Model

Updating Market Equilibrium Returns with Views:

$$\mu_{BL} = \left((\tau\Sigma)^{-1} + P'\Omega^{-1}P\right)^{-1}\left((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right) \tag{29}$$

Objective Function (Minimize Portfolio Variance):

$$\min_{\mathbf{w}} \mathbf{w}'\Sigma\mathbf{w}$$

2. Constraints:

- *Budget Constraint (Total Weights Sum to 1):*

$$\sum_{i=1}^{n} w_i = 1$$

- *Expected Return Constraint:*

$$\mathbf{w}'\mu_{BL} = R_{\text{target}}$$

Portfolio Performance Comparison

Portfolio	Expected Return	Volatility	Sharpe Ratio
BL View 1	0.0597	0.2463	0.2424
BL View 2	0.0703	0.2465	0.2853
BL View 3	-0.1570	0.5149	-0.3049

Portfolio Weights Comparison

	DELTA.BK	PTT.BK	ADVANC.BK	AOT.BK	GULF.BK
BL View 1	0.4127	0.0898	0.3778	0.1826	-0.0629
BL View 2	0.4517	0.0763	0.3782	0.1673	-0.0735
BL View 3	0.7737	-0.8570	0.3053	1.8925	-1.1144

Conclusion

- The Black-Litterman Model provides a *more flexible framework* by incorporating investor views into portfolio construction.
- *Risk-Return Tradeoff:* Investors should balance views with portfolio stability.

Section

7 Case Study III: BL Portfolio with Environmental Consideration VS Environmental Factor Portfolio

Section 7.1

Objectives

Objectives

1. To compare the Black-Litterman (BL) portfolio with environmental considerations to the factor portfolio.
2. To compare the BL portfolio across investors with different levels of risk aversion.

Section 7.2

Factor Model with Environmental Factor

Fama-French Five-Factor Model

$$R_i - R_f = \alpha + \beta_m(R_m - R_f) + \beta_s \mathrm{SMB} + \beta_v \mathrm{HML} + \beta_p \mathrm{RMW} + \beta_i \mathrm{CMA} + \epsilon_i$$

where:

- R_i = Return of asset i
- R_f = Risk-free rate
- R_m = Market return
- SMB = Small Minus Big (size factor)
- HML = High Minus Low (value factor)
- RMW = Robust Minus Weak (profitability factor)
- CMA = Conservative Minus Aggressive (investment factor)
- $\beta_m, \beta_s, \beta_v, \beta_p, \beta_i$ = Factor loadings
- ϵ_i = Idiosyncratic risk

Introduction to Environmental Factor

- The Environmental Risk Factor represents a sustainability-based factor in portfolio analysis.
- It is constructed as a Best Minus Worst (BMW) factor, similar to other factor investing approaches.
- The factor captures excess returns of firms with high environmental standards over those with poor environmental practices.

Construction of Environmental Factor

- Firms are ranked based on their environmental risk scores, sourced from Sustainalytics.
- The top one-third of firms with the best environmental scores are long positions.
- The bottom one-third of firms with the worst environmental scores are short positions.
- The EAll factor is computed as:

$$E = R_{\text{best}} - R_{\text{worst}}$$

- Five Factor model plus Environment,

$$R_i - R_f = \alpha + \beta_m (R_m - R_f) + \beta_s \text{SMB} + \beta_v \text{HML} + \beta_p \text{RMW} + \beta_i \text{CMA} + \beta_e \text{E} \epsilon_i$$

Performance Metrics

Factor	Exp. Return	Volatility	Cumulative Return	Sharpe	Sortino
Mkt	0.01	1.29	2.68	0.00	0.01
SMB	-0.00	0.74	-0.36	-0.00	-0.00
HML	0.10	1.18	48.43	0.08	0.12
RMW	0.05	0.80	27.54	0.07	0.10
CMA	0.07	0.70	35.39	0.10	0.15
EAll	-0.02	0.43	-10.10	-0.05	-0.06
SP500	0.01	1.23	6.07	0.01	0.01

Table: Performance Metrics for Selected Factors (2021-2023)

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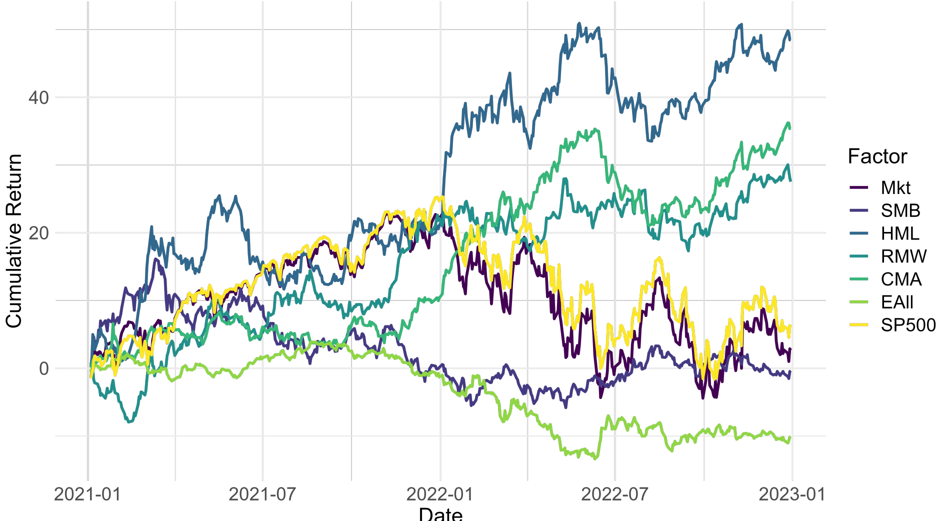
Cumulative Returns of Selected Factors



QuantFI

Market and Economic Context

- From 2021 to 2023, value stocks outperformed due to economic uncertainty and rising interest rates.
- The underperformance of EAll was driven by:
  - Surge in energy prices favoring high-carbon firms.
  - Short-term market focus on financial returns over sustainability.
  - High volatility and regulatory uncertainty in ESG investing.
- Future improvements may involve dynamic reweighting of environmental risk exposure.

Section 7.3

Asset Selection

Introduction

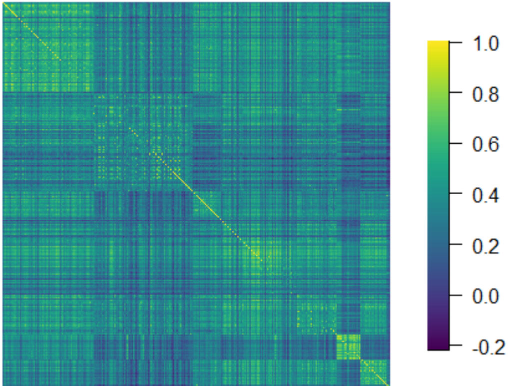
- Asset selection is a crucial step in constructing an optimal portfolio with risk-adjusted returns.
- The goal is to achieve diversification while minimizing tracking error relative to a benchmark (S&P 500).
- A two-step process is used:
    1. **Spectral Clustering** for dimension reduction.
    2. **Minimum Tracking Error** portfolio optimization.

Step 1: Reduction of Portfolio Dimensions

- Spectral clustering is used to identify groups of assets with similar behavior.
- The correlation matrix of 782 equities is analyzed using Singular Value Decomposition (SVD).
- The top 6 eigenvectors (explaining 50% variance) are selected for clustering.
- The **k-means algorithm** is applied to determine asset clusters.
- The optimal number of clusters (k) is determined using the "elbow method".
- Eight clusters are selected based on the between-cluster sum of squares.

Clustered Assets and Selection Criteria

After clustering, a heatmap of asset correlations reveals distinct blocks.



From each cluster, the top 10 assets with the highest **Sharpe ratios** are selected.

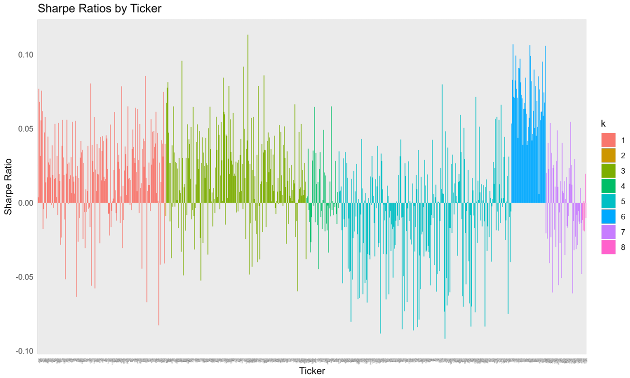


Figure: Sharpe Ration of 782 Equities in eight clusters

Step 2: Minimizing Tracking Error Portfolio

- Tracking error (TE) measures the deviation of portfolio returns from the benchmark.
- Defined as:

$$TE = \sqrt{E[(R_p - R_b)^2]}$$

- The objective is to minimize TE by selecting optimal asset weights.

Optimization Problem

- The tracking error minimization problem is formulated as:

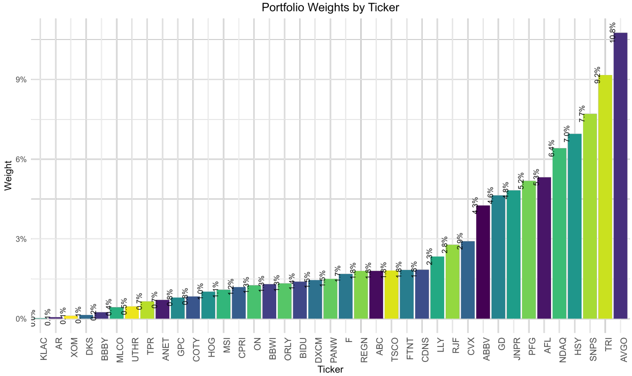
$$\min_{w} \sqrt{w^\top \Sigma w - 2w^\top \Sigma b + b^\top \Sigma b}$$

  where:
  - w = portfolio weights,
  - Σ = covariance matrix of asset returns,
  - b = benchmark index weights.
- A non-negative linear regression approach is applied to enforce long-only constraints.

Results: Selected Portfolio

7 Case Study III: BL Portfolio with Environmental Consideration VS Environmental Factor Portfolio

- After optimization, the portfolio is reduced to 39 assets.
- The selected portfolio achieves:
  - **Sharpe Ratio:** 0.077 (compared to 0.01 for S&P 500).
  - **Portfolio-Benchmark Correlation:** 0.97.



Section 7.4

Black-Litterman Models with Long-Only Constraint

Black-Litterman Model: Mean-Variance Optimization (Long-Only)

Objective: Minimize risk while maximizing expected returns based on the BL framework.

- The optimization problem is formulated as:

$$\min_{w} \frac{1}{2} w^\top \Sigma w - \lambda w^\top \mu_{BL}$$

- Subject to:

$$1^\top w = 1, \quad w \geq 0$$

- This ensures a fully invested, long-only portfolio.
- Suitable for investors in markets where short-selling is restricted.

Black-Litterman Model: Maximum Sharpe Ratio (Long-Only)

Objective: Maximize the Sharpe ratio under a long-only constraint.

- The optimization problem is formulated as:

$$\max_{w} \frac{w^\top \mu_{BL}}{\sqrt{w^\top \Sigma w}}$$

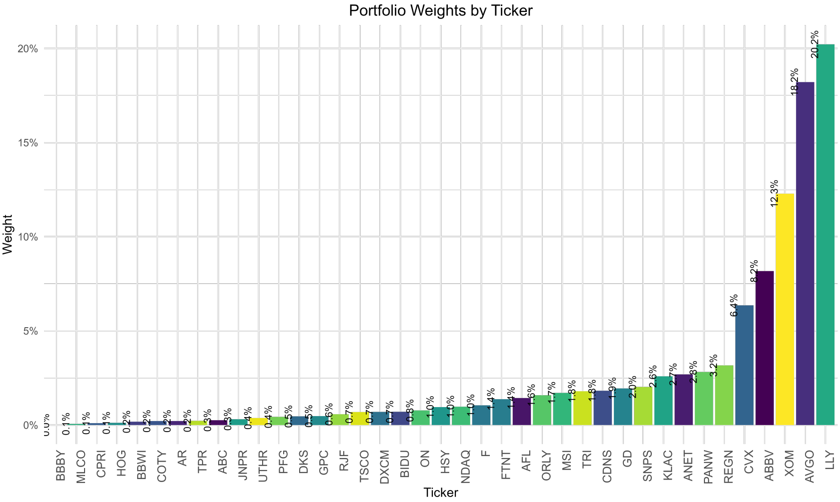
- Subject to:

$$1^\top w = 1, \quad w \geq 0$$

- This model finds the portfolio with the highest risk-adjusted return.
- Compared to mean-variance optimization, this approach prioritizes maximizing return per unit of risk.
- Suitable for investors seeking efficient risk-return trade-offs.

Market-Cap Weight

7 Case Study III: BL Portfolio with Environmental Consideration VS Environmental Factor Portfolio



Types of Investors Based on Risk Aversion

- **Risk-Averse Investors (High λ):**
  - Prioritize capital preservation and low volatility.
  - Prefer conservative portfolios with low-risk assets.
  - Example: *Trustees and institutional investors managing endowments or pensions.*
- **Moderate-Risk Investors (Medium λ):**
  - Balance risk and return by diversifying assets.
  - Seek portfolios that provide stable returns with controlled risk.
  - Example: *Market portfolio investors and balanced fund managers.*
- **Risk-Seeking Investors (Low λ):**
  - Accept higher volatility in pursuit of higher returns.
  - Allocate significant portions to high-growth or speculative assets.
  - Example: *Kelly criterion investors, hedge funds, and venture capitalists.*

Incorporating Environmental Views in Portfolio Construction

7 Case Study III: BL Portfolio with Environmental Consideration VS Environmental Factor Portfolio

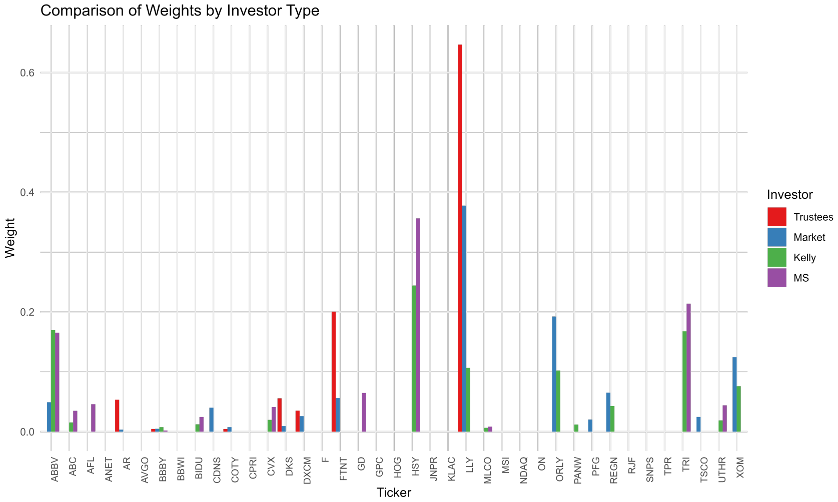
- Firms are ranked based on environmental risk scores (e.g., Sustainalytics ESG data).
- Investor views:
  - Top 10 firms with the lowest environmental risk are expected to outperform; AR, BBBY, TRI, ORLY, REGN, ANET, AFL, PFG, PANW, and ABBV.
  - Bottom 10 firms with the highest environmental risk are expected to underperform; DXCM, KLAC, GD, AVGO, F, COTY, HSY, ON, XOM, and CVX.
- This view is incorporated into the BL model through the investor view matrix P:

$$P = \begin{bmatrix} 1 & 1 & \dots & -1 & -1 & \dots \end{bmatrix}$$

- The expected return adjustment is $Q = 0.02 \times \mathbb{I}$

Optimal Weight Obtained by BL models

7 Case Study III: BL Portfolio with Environmental Consideration VS Environmental Factor Portfolio



Performance Metrics for Selected Factors and BL Portfolios

Portfolio	Expected Return	Volatility	Cumulative Return	Sharpe	Sortino
Trustees	0.18	1.63	88.40	0.11	0.17
Market	0.15	1.20	75.54	0.13	0.20
Kelly	0.11	0.87	55.26	0.13	0.19
MS	0.09	0.85	45.75	0.11	0.16
Mkt	0.01	1.29	2.68	0.00	0.01
SMB	-0.00	0.74	-0.36	-0.00	-0.00
HML	0.10	1.18	48.43	0.08	0.12
RMW	0.05	0.80	27.54	0.07	0.10
CMA	0.07	0.70	35.39	0.10	0.15
EAll	-0.02	0.43	-10.10	-0.05	-0.06

Conclusion

- BL portfolios outperform traditional factor-based and market-cap-weighted portfolios in risk-adjusted returns.
- The Environmental Risk Factor underperformed during the study period, suggesting challenges in aligning financial and sustainability objectives.
- Different investor risk aversion levels impact portfolio allocations and performance.

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Q&A

Thank you for your attention