Black-Litterman Model with Factor: Practice to Theory

Week I: Algorithmic Trading & Mean-Variance Portfolio in Practice

Pasin Marupanthorn, Ph.D, CQF

25th Jaunary 2025

Lecturer Biography

Who am I?

Pasin Marupanthorn, Ph.D., CQF

Experience

- Quantitative Researcher/Trader
- Senior Advisor - Groundup Academy - Thailand
- Quantitative Researcher/Trader - ResilientML - Australia
- Quantitative Researcher - QRSLab - United Kingdom
- Graduate Researcher in Analysis & Probability Research Group - Maxwell Institute of Mathematical Science - United Kingdom
- Lecturer - RMUTSB

Lecturer Biography

Who am I?

Education

- Ph.D. Actuarial Mathematics & Statistics - Heriot-Watt University, UK
- Certificate in Quantitative Finance - CQF Institute
- M.Sc. Financial Engineering - WorldQuant University, US
- M.Sc. Mathematical Modelling - University of Birmingham, UK
- M.Sc. Mathematics - Thammasat University
- B.Sc. Applied Mathematics with Physics - Thammasat University

Areas of Expertise Financial Mathematics, Actuarial Mathematics, Stochastic Modelling & Statistical Machine Learning

Lecturer Biography

Collaboration

Industry Project Collaboration

Research Collaboration

Co-supervise Ph.D. students in Financial Mathematics and Actuarial Mathematics in collaboration with labs from the UK, US, France, Australia, or Japan.

https://quantfilab.github.io/pmarupanthorn/

A Guideline for Three Weeks

Lecture Series

Black-Litterman Model with Factor: Practice to Theory

- Week 1 *Algorithmic Trading & Mean-Variance Portfolio in Practice* - 25$^{\text{th}}$ January 2025 - Practice Focus
- Week 2 *Black-Litterman Model & Investor View with Factors in Practice* - 8$^{\text{th}}$ February 2025 - Practice Focus
- Week 3 *Advanced Mathematics behind Back- Litterman Model & other Bayesian Approach Portfolios* - 22$^{\text{nd}}$ March 2025 - Theory Focus

Markowitz Portfolio Theory $\equiv$ *Modern Portfolio Theory* $\equiv$ *Mean-Variance Portfolio Theory*

These terms are not exactly identical, but for the purpose of this lecture, I will assume they are equivalent.

- **What is Investment?** Allocating resources (money, time) into assets or projects to generate future returns.
- **Why Do We Invest?** To grow wealth, achieve financial goals, hedge against inflation, and manage risk.
- **Why Is Investment Important to Companies?** - Provides capital for growth and innovation. - Enhances market valuation and competitiveness.

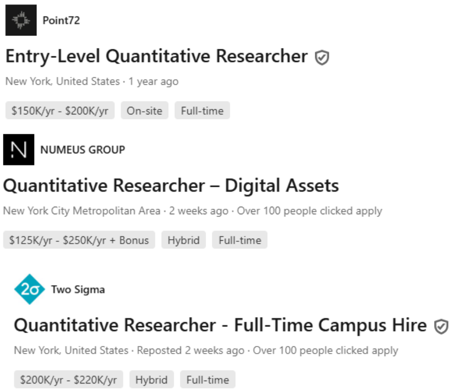
Investment in the Financial Market

- **Liquidity:**
  - Financial markets like stocks and bonds offer high liquidity, allowing investors to buy and sell assets quickly.
  - Other markets, such as real estate, often require more time to execute transactions.
- **Diversification:**
  - Wide range of assets, including stocks, bonds, ETFs, and derivatives, enables effective risk management.
  - Other markets may lack the variety needed for proper diversification.
- **Accessibility:**
  - Financial markets are accessible to both retail and institutional investors through online platforms.
  - Other markets, like private equity, often have higher entry barriers.

Why Should You Study Investment?

1 Motivation

- **Salary Insights:** Median salary for investment analysts: $90,000+ annually (varies by region and experience).
- **Global Opportunities:** The financial market operates worldwide, offering careers in major financial hubs.
- **Job Security:** Investment professionals are in demand across economic cycles, especially in wealth management and risk advisory.
- **Personal Finance Benefits:** Gain the knowledge to manage your own wealth effectively.



The Quant Era in Investment

1 Motivation

- The rise of data-driven and algorithmic investment strategies.
- Heavily relies on quantitative models, machine learning, and big data.
- Prominent players: Renaissance Technologies, Two Sigma, Citadel.

Some Quant Firms

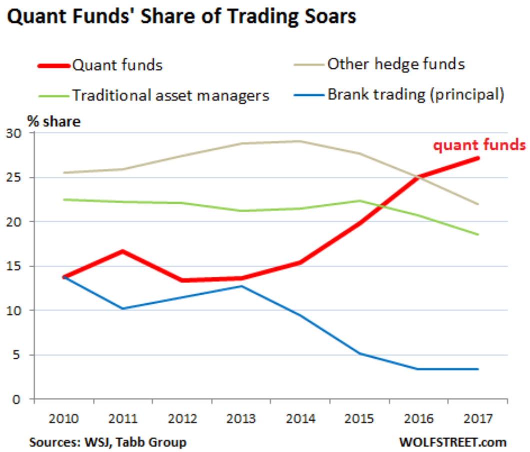


Figure: Quant Cap

QuantFI

Jim Simons: The King of Quant

- Founder of **Renaissance Technologies**, one of the most successful hedge funds in history.
- A former mathematician specializing in geometry and topology.
- Known as the ”Quant King” for pioneering quantitative investment strategies.
- *Medallion Fund*, a highly profitable fund averaging 40% annual returns after fees.
- Proved the power of mathematics and science in financial markets.

Jim Simons, Founder of Renaissance Technologies, 1938 - 2024

Thai's Quant Community: QuantCorner

https://www.facebook.com/quantcornerthailand/

Section

2 Algorithmic Trading

- Motivation
- **Algorithmic Trading**
- Brief Story of Modern Portfolio Theory
- Background on Financial Mathematics
- Mean-Variance Portfolio

Classification by Time Value

- **Spot Markets:** Involve immediate settlement and delivery of assets.
- **Derivatives Markets:** Include contracts based on future value, such as options, futures, and swaps.

Classification by Market Type

- **Equity:** Shares and stock-related securities.
- **Rates:** Bonds, interest rates, and fixed income products.
- **Commodities:** Physical goods like gold, oil, and agricultural products.
- **Carbon and Green Assets:** Emission trading credits and sustainable financial products.
- **Cryptocurrency:** Digital assets such as Bitcoin, Ethereum, and altcoins.

Steps in Demo and Paper Trading

1. **Choose a Platform:** Select a trading platform offering demo or paper trading.
2. **Set Up a Virtual Account:** Create an account and allocate virtual funds for practice.
3. **Learn Market Basics:** Understand trading instruments, market conditions, and strategies.
4. **Test Strategies:** Apply trading strategies and evaluate performance without financial risk.
5. **Analyze Performance:** Review trades, refine strategies, and measure metrics like profit, loss, and drawdown.

Interface for Algorithmic Trading

A platform or API used to automate trading strategies and allows traders to connect to financial markets programmatically.

Key Features:

- **Order Placement:** Send market, limit, or stop orders directly to the market.
- **Market Data Access:** Real-time and historical data for analysis and strategy development.
- **Strategy Execution:** Run automated trading algorithms based on predefined rules.
- **Risk Management:** Automate stop-loss, take-profit, and position sizing.

Type of Interfaces:

- **Direct APIs:** Interactive Brokers API, Alpaca, Binance API.
- **Platforms APIs:** MetaTrader, QuantConnect, TradingView, Interactive Brokers TWS API.

Free Data Sources

Daily Data

- Yahoo Finance
- Investing.com
- Provider; SETTrade,

Instaday Data usally from Specific Provider

- Crypto: Binance, Deripit,
- Equity: Globlex (Thai), IBKR (US)
- Exchange & Metral; All broker linked to MT4 and MT5

Free Data Sources

Daily Data: General Availability

- **Yahoo Finance:** Comprehensive financial data for global markets.
- **Investing.com:** Real-time data and market analysis tools.
- **SETTrade:** Thai stock market data provided by the Stock Exchange of Thailand (SET).

Intraday Data: Specific Providers

- **Cryptocurrency:**
  - Binance: Leading global crypto exchange with detailed intraday data.
  - Deribit: Specializing in cryptocurrency derivatives and options.
- **Equity Markets:**
  - Globlex: Thai brokerage offering real-time stock data.
  - IBKR (Interactive Brokers): U.S.-based broker providing real-time equity and options data.
- **Forex and Commodities:**
  - All brokers linked to MT4 and MT5: Includes forex, metals, and other financial instruments.

Section

3 Brief Story of Modern Portfolio Theory

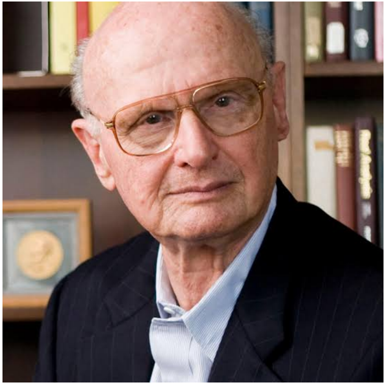
Investment Before Modern Portfolio Theory, 1930s

- **Rule of Thumb:** Investments were often based on intuition, experience, or simple heuristics rather than mathematical models.
- **Focus on Individual Assets:** Investors analyzed and selected assets based on their standalone performance without considering diversification.
- **Lack of Risk Quantification:** Risk was subjectively assessed, with no formal framework to measure or manage it systematically.
- **Popular Strategies:**
  - Investing in blue-chip stocks for stability.
  - Real estate and tangible assets were favored for long-term growth.
  - Dividend yields and fixed-income securities were prioritized for income generation.

Harry Markowitz: The Pioneer of Modern Portfolio Theory

Who is Harry Markowitz?

- An American economist and pioneer in quantitative finance.
- Introduced **Modern Portfolio Theory (MPT)** in 1952 through his paper *Portfolio Selection*.
- Awarded the **Nobel Prize in Economic Sciences** in 1990.



Harry Markowitz (1927–2023)

How Markowitz Revolutionized Investment

Key Innovations:

- **Diversification Principle:** Emphasized reducing risk by investing in multiple assets.
- **Efficient Frontier:** Proposed the set of portfolios that provide the highest return for a given risk level.
- **Risk-Return Optimization:** Introduced variance (or standard deviation) as a measure of portfolio risk.

Impact on Investment:

- Shifted from intuition-based investing to data-driven portfolio construction.
- Popularized diversification to reduce unsystematic risk.
- Laid the foundation for modern investment strategies used globally.

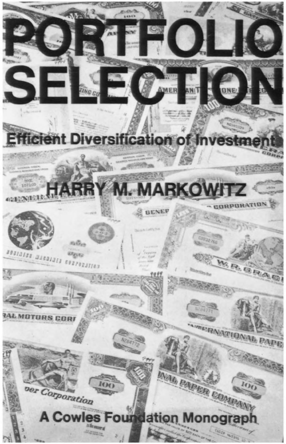
Don't Put All Your Eggs in One Basket.

Figure: Diversification

Portfolio Selection: Efficient Diversification of Investments

By Harry Markowitz:

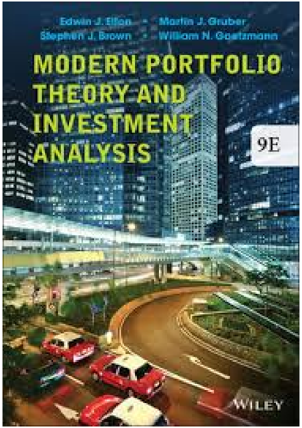
- The foundational text introducing **Modern Portfolio Theory (MPT)**.
- Focused on:
  - Risk-return tradeoff.
  - The principle of diversification.
  - Developing the efficient frontier.



Modern Portfolio Theory and Investment Analysis

By Edwin J. Elton, Martin J. Gruber, Stephen J. Brown, and William N. Goetzmann:

- A comprehensive guide covering:
  - Theory and practical applications of Modern Portfolio Theory.
  - Risk-adjusted returns and portfolio performance.
  - Real-world examples and case studies.
- Widely used in academic and professional finance courses.

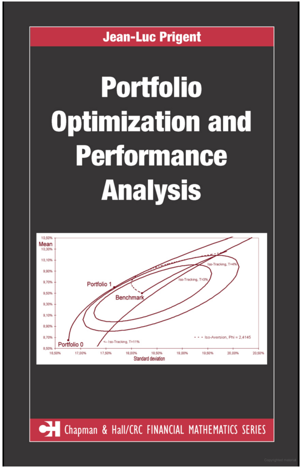


Portfolio Optimization and Performance Analysis

3 Brief Story of Modern Portfolio Theory

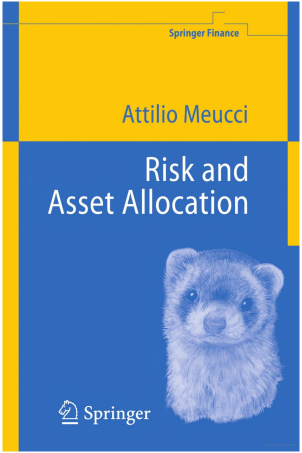
By Stephen G. Kellison:

- Covers foundational principles in portfolio optimization.
- Focuses on:
  - Analyzing portfolio performance.
  - Optimizing asset allocation for better returns.
  - Practical insights for risk management.
- Suitable for students and professionals.



By Bernhard Pfaff:

- Focuses on advanced mathematical models for portfolio construction.
- Key topics include:
  - Risk management frameworks.
  - The Black-Litterman Model for portfolio optimization.
  - Advanced quantitative techniques for asset allocation.
- Designed for advanced practitioners and researchers.



Section 4.1

Stationary of Time Series

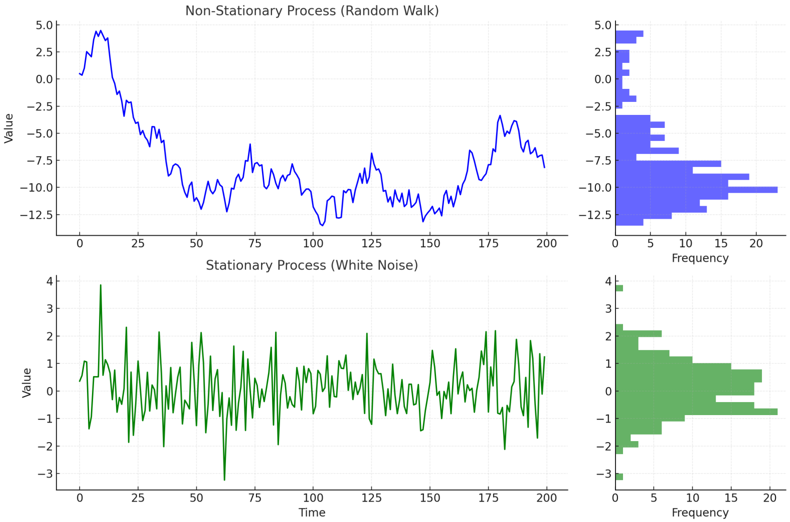
Stationary Time Series Process

A time series $\{X_t\}$ is **strictly stationary** if the joint distribution of $(X_{t_1}, X_{t_2}, \ldots, X_{t_k})$ is the same as that of $(X_{t_1+h}, X_{t_2+h}, \ldots, X_{t_k+h})$ for all $t_1, t_2, \ldots, t_k$, h, and k. A **weakly stationary** (or second-order stationary) time series satisfies the following conditions:

- **Constant Mean:**
$$\mathbb{E}[X_t] = \mu \quad \text{for all } t.$$
- **Constant Variance:**
$$\mathrm{Var}(X_t) = \mathbb{E}[(X_t - \mu)^2] = \sigma^2 \quad \text{for all } t.$$
- **Time-Invariant Autocovariance:**
$$\mathrm{Cov}(X_t, X_{t+h}) = \mathbb{E}[(X_t - \mu)(X_{t+h} - \mu)] = \gamma(h),$$
where $\gamma(h)$ depends only on the lag h, not on t.

Stationary Time Series Process

4 Background on Financial Mathematics

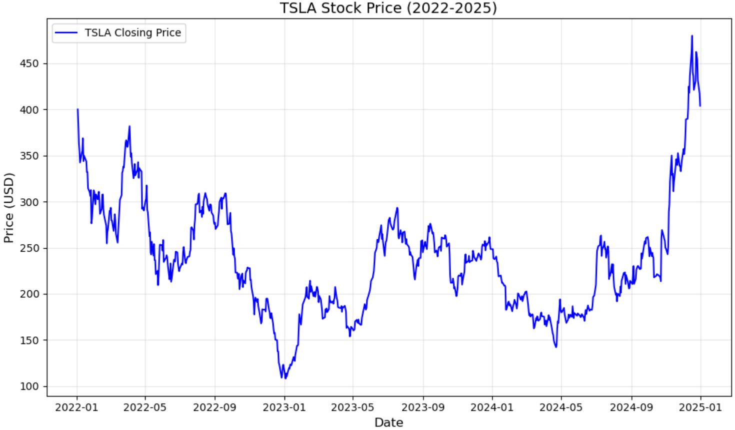


Section 4.2

Price and Return Processes

Price Processes in Finance: Non-Stationary Nature

Common in finance, as asset prices evolve unpredictably and depend on external factors like market trends and macroeconomic conditions.



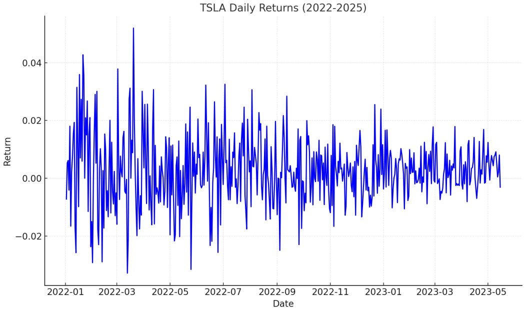
Return Process and Near Stationarity

4 Background on Financial Mathematics

- Let P_t denote the price of an asset at time t.
- The **logarithmic return** is given by:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln(P_{t-1}),$$

where R_t: Return at time t and P_t: Asset price at time t.

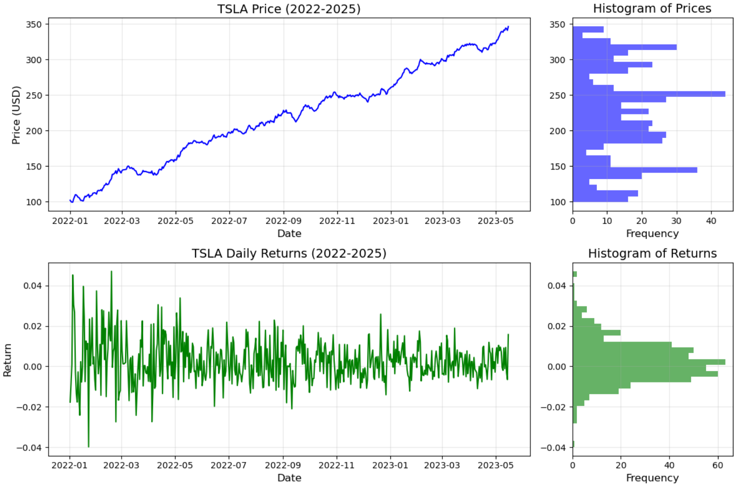


Section 4.3

Financial Data & Stylized Fact

Stylized Facts of Financial Data

- **Heavy Tails:** The distribution of returns often has fatter tails than the normal distribution, meaning extreme events (outliers) occur more frequently.
- **Volatility Clustering:** High volatility tends to be followed by high volatility, and low volatility follows low volatility, indicating periods of market calm and turbulence.
- **Leverage Effect:** Negative returns tend to increase volatility more than positive returns of the same magnitude.
- **Autocorrelation:** Returns often exhibit short-term autocorrelation, meaning past returns may influence future returns, although this is weak over long periods.
- **Non-Stationarity:** Asset prices are often non-stationary, meaning their statistical properties, such as mean and variance, change over time.


TSLA Price (2022-2025)
Price (USD)
Date
Histogram of Prices
Frequency
TSLA Daily Returns (2022-2025)
Return
Date
Histogram of Returns
Frequency

Section

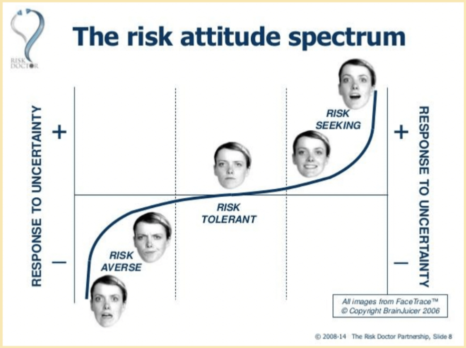
5 Mean-Variance Portfolio

Section 5.1

Mean-Variance Portfolio

Risk-Averse vs. Risk-Seeking vs. Risk-Neutral Investors

- Risk-Averse Investors prefer lower risk and are willing to sacrifice some return to avoid uncertainty.
- Risk-Seeking Investors prefer higher risk, even if it means a lower expected return.
- Risk-Neutral Investors are indifferent to risk and base decisions solely on expected return.



Risk

Definition of a Portfolio in Mathematical Notation

What is a Portfolio?

- A portfolio is a collection of financial assets, such as stocks, bonds, or other securities, represented by their weights.
- The weights represent the proportion of total investment allocated to each asset.

Definition of a Portfolio in Mathematical Notation

Mathematical Representation:

- Let n be the number of assets in the portfolio.
- The portfolio is represented as a vector of weights:

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}^T,$$

  where w_i is the proportion of total capital allocated to asset i.
- The total weight must satisfy the following constraints:

$$\sum_{i=1}^{n} w_i = 1, \quad \text{(fully invested portfolio).}$$

Portfolio Return and Variance

Expected Return of the Portfolio:

- If μ is the vector of expected returns of the assets, the portfolio's expected return is:

$$\mu_p = \mathbf{w}^T \mu = \sum_{i=1}^{n} w_i \mu_i.$$

Portfolio Variance:

- If Σ is the covariance matrix of asset returns, the portfolio variance is:

$$\sigma_p^2 = \mathbf{w}^T \Sigma \mathbf{w}.$$

Introduction to Mean-Variance Portfolio

- The goal is to construct an optimal portfolio by balancing:
  - **Expected Return (μ):** Defined as:

    $$\mu_p = \mathbf{w}^T \mu,$$

    where:
    - μ_p: Portfolio expected return.
    - $\mathbf{w}$: Vector of portfolio weights $(w_1, w_2, \ldots, w_n)$.
    - μ: Vector of individual asset expected returns.
  - **Volatility (σ):** Defined as the square root of portfolio variance:

    $$\sigma_p = \sqrt{\mathbf{w}^T \Sigma \mathbf{w}},$$

    where Σ is the covariance matrix of asset returns.

Risk-Return Trade-Off: Maximizing Return

- Investors aim to achieve the highest return (μ_p) for a specific level of risk (σ_{target}).
- Optimization Problem:

$$\max_{\mathbf{w}} \mathbf{w}^T \mu,$$

subject to:

$$\mathbf{w}^T \Sigma \mathbf{w} \leq \sigma_{\text{target}}^2, \quad \sum_{i=1}^{n} w_i = 1.$$

- Constraints:
  - $\mathbf{w}^T \Sigma \mathbf{w}$: Ensures portfolio risk is below the target.
  - $\sum_{i=1}^{n} w_i = 1$: Fully invested portfolio.
- Investors with a low-risk tolerance choose a lower σ_{target}.
- This approach finds portfolios on the efficient frontier corresponding to a fixed risk level.

Maximizing Return for a Given Risk

Analytical Solution:

$$\mathbf{w} = \frac{\Sigma^{-1}(\mu - \lambda \mathbf{1})}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}},$$

where:

- $\mathbf{w}$: Vector of portfolio weights.
- μ: Vector of expected returns.
- Σ: Covariance matrix of asset returns.
- λ: Determined by the risk constraint σ^2_{target}.

Risk-Return Trade-Off: Minimizing Risk

- Investors aim to achieve the lowest possible risk (σ_p^2) for a desired return (μ_{target}).
- Optimization Problem:

$$\min_{\mathbf{w}} \mathbf{w}^T \Sigma \mathbf{w},$$

subject to:

$$\mathbf{w}^T \mu = \mu_{\text{target}}, \quad \sum_{i=1}^{n} w_i = 1.$$

- Constraints:
  - $\mathbf{w}^T \mu$: Ensures the portfolio meets the desired return.
  - $\sum_{i=1}^{n} w_i = 1$: Fully invested portfolio.
- This approach identifies the minimum-risk portfolio for a specific return level on the efficient frontier.

Minimizing Risk for a Desired Return

Analytical Solution:

$$\mathbf{w} = \Sigma^{-1}(\mathbf{A}\mu + \mathbf{B}\mathbf{1}),$$

where:

$$\mathbf{A} = \frac{\mu_{\text{target}}(\mathbf{1}^T\Sigma^{-1}\mathbf{1}) - (\mathbf{1}^T\Sigma^{-1}\mu)}{D},$$

$$\mathbf{B} = \frac{(\mu^T\Sigma^{-1}\mu) - \mu_{\text{target}}(\mathbf{1}^T\Sigma^{-1}\mu)}{D},$$

and:

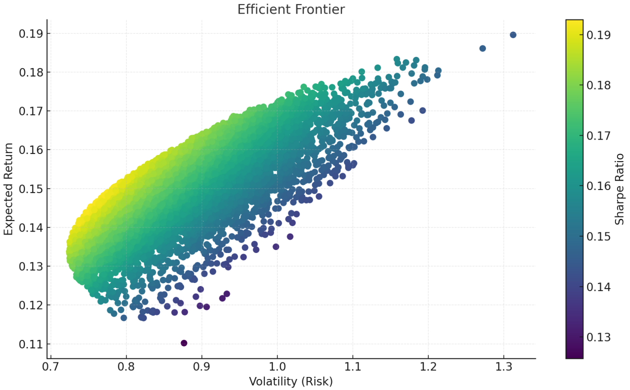
$$D = (\mu^T\Sigma^{-1}\mu)(\mathbf{1}^T\Sigma^{-1}\mathbf{1}) - (\mathbf{1}^T\Sigma^{-1}\mu)^2.$$

Efficient Frontier

5 Mean-Variance Portfolio

What is the Efficient Frontier?

- A graphical representation of optimal portfolios that maximize return for a given risk or minimize risk for a given return.
- Portfolios on the frontier are efficient, while portfolios below it are suboptimal.



Efficient Frontier

- Each portfolio on the frontier satisfies:

$$\sigma_p^2 = \mathbf{w}^T \Sigma \mathbf{w}, \quad \mu_p = \mathbf{w}^T \mu.$$

- The shape of the frontier is a parabola in risk-return space due to the quadratic nature of variance.
- The frontier's upper portion represents portfolios that dominate in both return and risk.

Minimum Variance Portfolio

Objective:

$$\min_{\mathbf{w}} \mathbf{w}^T \Sigma \mathbf{w},$$

subject to:

$$\sum_{i=1}^{n} w_i = 1.$$

Analytical Solution:

- The weights for the minimum variance portfolio are:

$$\mathbf{w} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}},$$

  where Σ is the covariance matrix of asset returns, $\mathbf{1}$ is the vector of ones of size n (number of assets).

Maximum Sharpe Ratio Portfolio

Objective:

$$\max_{\mathbf{w}} \frac{\mathbf{w}^T \mu - r_f}{\sqrt{\mathbf{w}^T \Sigma \mathbf{w}}},$$

subject to:

$$\sum_{i=1}^{n} w_i = 1.$$

Analytical Solution:

- The weights for the maximum Sharpe ratio portfolio are:

$$\mathbf{w} = \frac{\Sigma^{-1}(\mu - r_f \mathbf{1})}{\mathbf{1}^T \Sigma^{-1}(\mu - r_f \mathbf{1})},$$

where μ: is the vector of expected returns and r_f is the risk-free

Efficient Portfolio: Convex Combination of Portfolios

Theory: Efficient Portfolio Combination

- Any portfolio on the **efficient frontier** is a linear combination of:
  - The **Minimum Variance Portfolio** ($\mathbf{w}_{\text{min}}$), which minimizes risk.
  - The **Maximum Sharpe Ratio Portfolio** ($\mathbf{w}_{\text{max}}$), which maximizes return per unit of risk.

Mathematical Representation:

- The weights of an efficient portfolio are given by:

$$\mathbf{w}_{\text{efficient}} = \alpha \mathbf{w}_{\text{min}} + (1 - \alpha)\mathbf{w}_{\text{max}},$$

  where:
  - α: The proportion of the portfolio allocated to the minimum variance portfolio.
  - $1 - \alpha$: The proportion allocated to the maximum Sharpe ratio portfolio.
- The value of α depends on the investor's risk preference or return target.

Efficient Portfolio Weights

Given the target expected return, α convex combination parameter, calculated as:

$$\alpha = \frac{\mu_{\text{max}} - \mu_{\text{target}}}{\mu_{\text{max}} - \mu_{\text{min}}},$$

where:

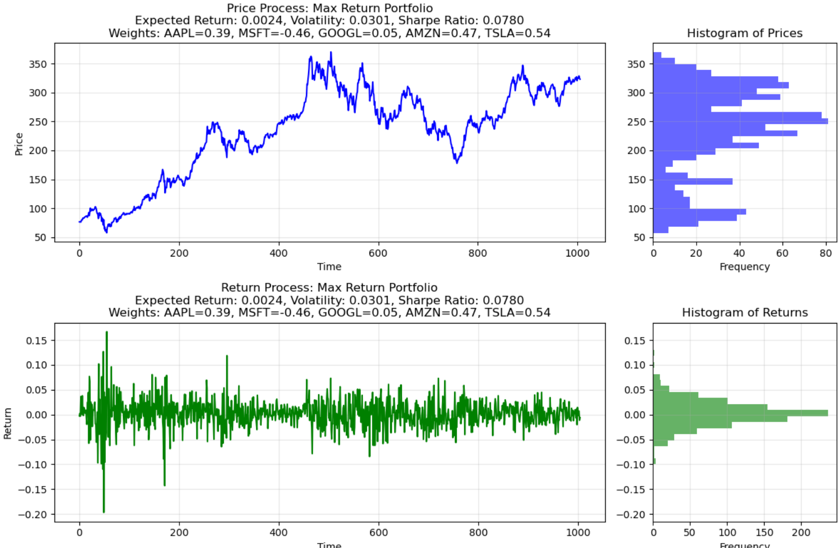
- μ_{max}: Expected return of the maximum return portfolio.
- μ_{min}: Expected return of the minimum variance portfolio.
- μ_{target}: Target expected return.

Assumptions of the Mean-Variance Portfolio Model

- **Asset Returns are Normally Distributed:**
  - The returns of assets are assumed to follow a normal distribution.
  - This ensures that the mean and variance fully describe the distribution of returns.
- **Investors are Rational and Risk-Averse:**
  - Investors prefer higher returns for a given level of risk.
  - They aim to maximize utility, which is a function of return and risk.
- **Stationarity of Returns:**
  - The statistical properties (mean, variance, and covariance) of returns are assumed to remain constant over time.

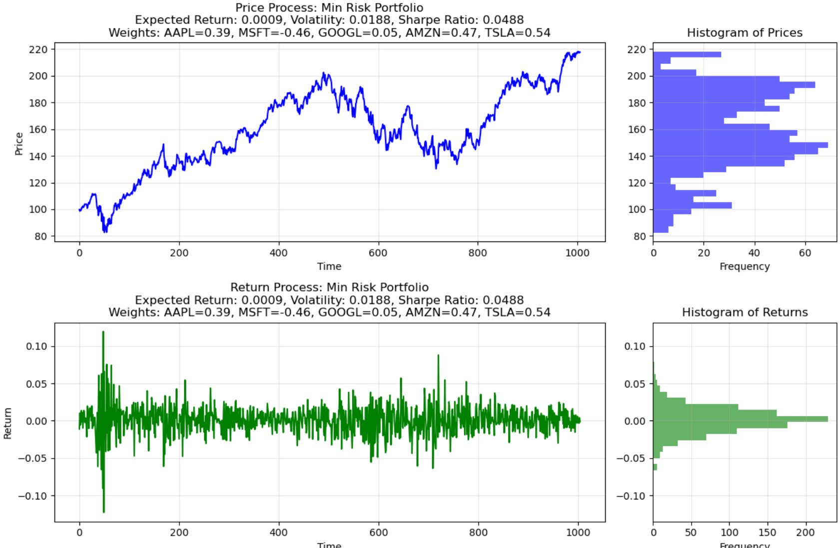
Example: Max Sharpe

5 Mean-Variance Portfolio



Example: Min Var

5 Mean-Variance Portfolio



Section 5.2

Case Study I: Small US Equity Portfolio

Portfolio Analysis of 5 Stocks (Part 1)

Selected Assets:

- The portfolio consists of 5 leading US technology and automotive stocks:
  - **AAPL**: Apple Inc.
  - **MSFT**: Microsoft Corporation
  - **GOOGL**: Alphabet Inc. (Google)
  - **AMZN**: Amazon.com Inc.
  - **TSLA**: Tesla, Inc.

Data Source:

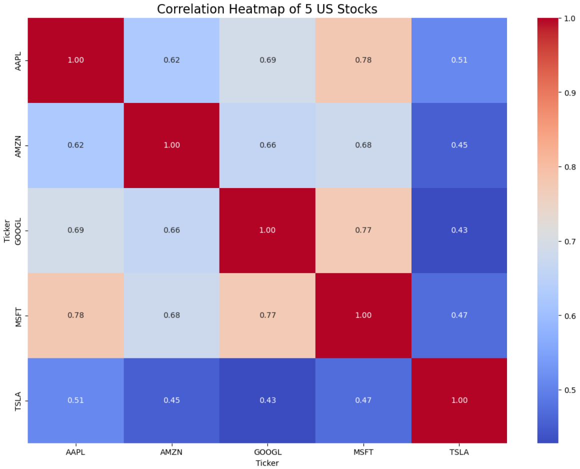
- Historical stock data retrieved using Yahoo Finance API.
- Time period: **January 1, 2020 – December 31, 2023**.

Portfolio Construction:

- Daily adjusted closing prices are used to calculate returns.
- The covariance matrix of returns is computed for risk assessment.
- Portfolio weights are optimized to achieve the target of the daily expected return of 1% with minimum volatility.

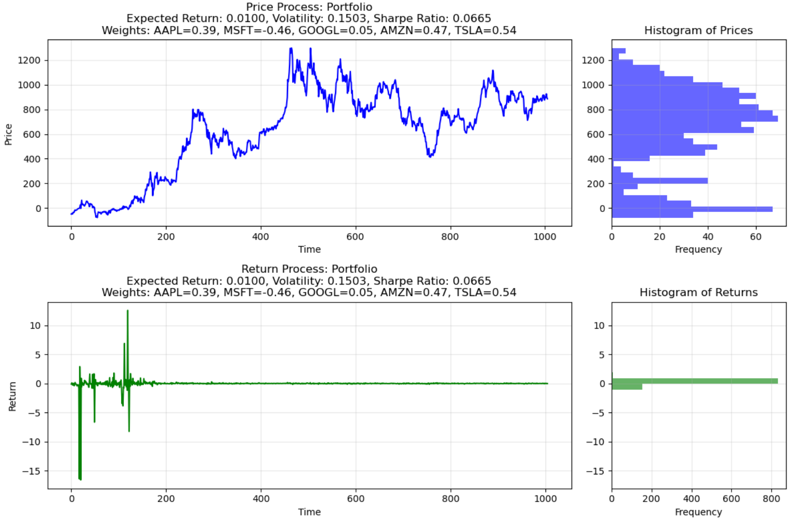
Validating Covariance

5 Mean-Variance Portfolio



Results

5 Mean-Variance Portfolio



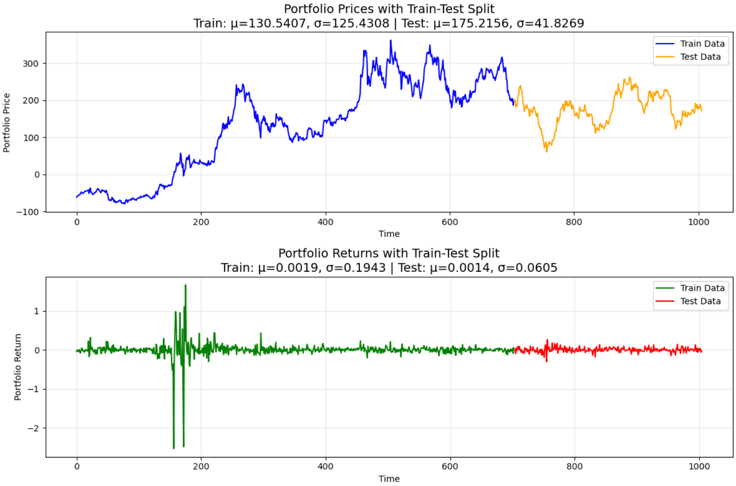
Warning

If this were valid, everyone would be a billionaire.

- You cannot determine the optimal weights until empirical data is obtained - it is data-dependent and not currency-dependent.
- If investing in the stock market, odd lots and fractional shares may affect your results, especially with a small budget.

Frame Title

5 Mean-Variance Portfolio



Methods for Non-Stationary Covariance Matrix

1. **Rebalancing:**
   - Regularly update portfolio weights based on the latest covariance estimates.
   - Choose a rebalancing frequency (e.g., daily, monthly) based on market dynamics.
2. **Predicted Covariance Structure:**
   - Use machine learning models (e.g., GARCH, LSTM) to forecast the covariance matrix.
   - Incorporate macroeconomic factors or market indicators into predictions.
3. **Rolling Windows:**
   - Estimate the covariance matrix using a rolling window of historical data.
   - Adjust the window size to balance stability and responsiveness.

Effect of Rounding Estimate Weight

5 Mean-Variance Portfolio

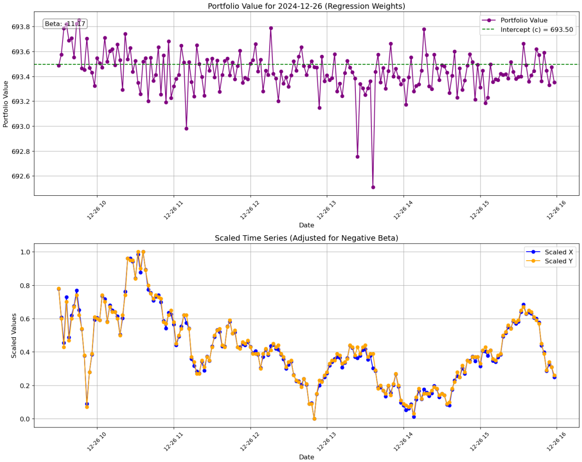


Figure: Estimate Weight

Effect of Rounding Estimate Weight

5 Mean-Variance Portfolio

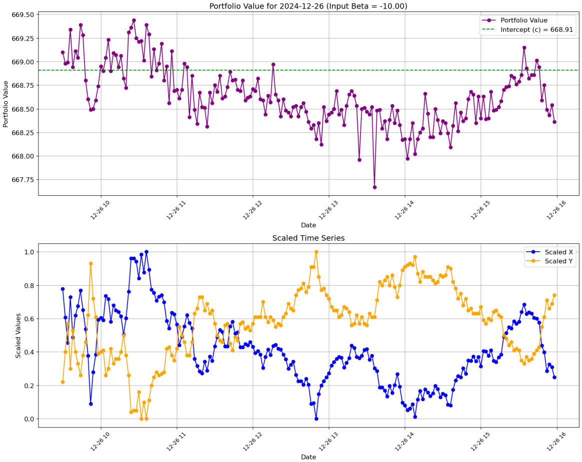


Figure: Rounded Weight

Section 5.3

Case Study II: Medium-Size US Equity Portfolio

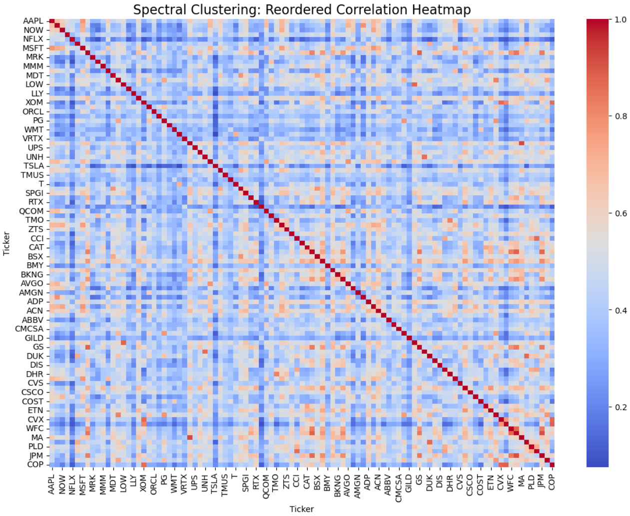
Portfolio of Assets

The portfolio consists of 100 US stocks from diverse sectors, including:

- Technology: AAPL, MSFT, AMZN, GOOGL, META
- Consumer Goods: KO, PEP, PG, WMT
- Healthcare: JNJ, PFE, ABBV, MRK, LLY
- Finance: JPM, BAC, GS, MA, MS
- Energy: XOM, CVX, COP, NEE

Correlation Matrix fo Asset Return

5 Mean-Variance Portfolio



Problem: Condition Number of Covariance Matrix

- A **high condition number** implies that Σ is nearly singular or ill-conditioned.
- This means that the matrix has eigenvalues with large disparities.
- It can lead to numerical instability when performing operations such as inversion.

Analysis for Σ:

- **Computed Condition Number:** 887.30.
- **Implication:**
  - The covariance matrix is moderately ill-conditioned.
  - Some assets in the portfolio have highly correlated returns, resulting in a lack of diversification.
  - Numerical methods (e.g., portfolio optimization) may require regularization to ensure stability.

Methods to Address Singularity of Covariance Matrix (Part 1)

Regularization:

- Add a small constant λI to the diagonal of the covariance matrix:

$$\Sigma_{\text{regularized}} = \Sigma + \lambda I$$

- Ensures numerical stability and invertibility.

Dimensionality Reduction:

- Use Principal Component Analysis (PCA) to reduce the dimensionality of the dataset.
- Retain the principal components that explain the majority of the variance.

Methods to Address Singularity of Covariance Matrix (Part 2)

Shrinkage Estimators:

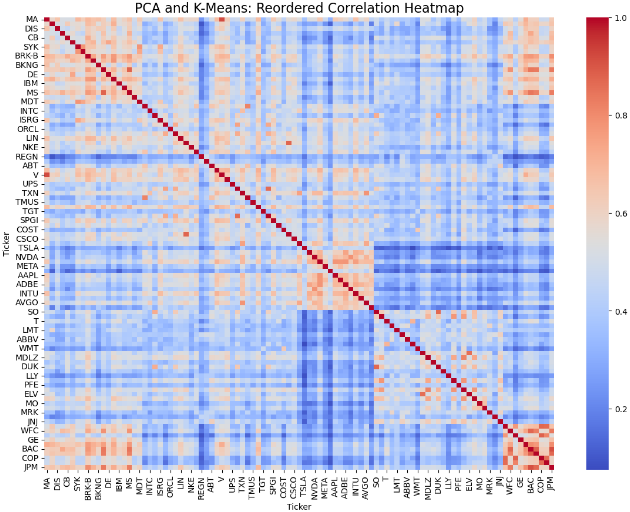
- Use methods like Ledoit-Wolf shrinkage to estimate a more robust covariance matrix.
- Combines the sample covariance matrix with a structured estimator (e.g., identity matrix).

Asset Selection:

- Reduce the number of assets in the portfolio.
- Remove assets with near-perfect correlations or redundant information.

Asset Selection by Clustering

5 Mean-Variance Portfolio



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Q&A

Thank you for your attention