Introductory Workshop

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1 Introduction

This document was created by the University of Southampton Quant Finance Society. This document covers the contents of the society's introductory workshop.

This documentation is intended to keep track of progress, and to provide information to anyone interested. We will try to keep this as simple as possible to ensure maximum understanding.

1.1 Document Focus

This document explains the thought process of finding new methods to model a financial market, and proceeds to give an example.

This is only a proof of concept, and is not a completely working model. The only aim of the document is to share the thought process of brainstorming and creating new ideas/models.

The example in this document uses a mathematical equation to model the S&P 500, in particular, over the last 24 years.

The code used to generate each equation will be added at the end of each explanation section.

1.2 Other Material

The code used to generate this, along with other material, can be found here on the society's GitHub or by visiting this link:

`https://github.com/QuantFinanceSocietySouthampton/QuantFinanceModel/tree/main`

2 Idea Formulation

The idea came from the fact that the S&P 500 follows an exponential trajectory, in the long term, and experiences regular fluctuations, in the short term.

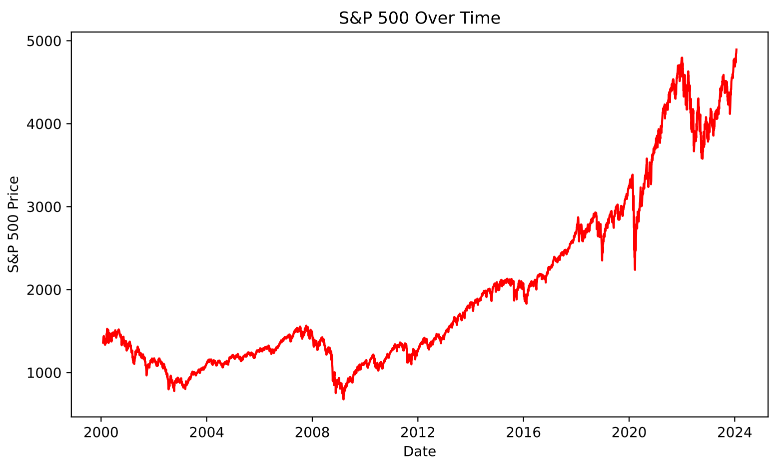

Figure 1: S&P 500, Years 2000-2024

As can be seen in Figure 1, market value does indeed follow a general exponential path, and fluctuates in the short term; as previously stated.

Note: This idea came from the perspective of a Mechatronic Engineering student, familiar with signal processing. The idea was inspired by looking at the market as if it were an electronic signal to be analysed using Fourier Transforms.

This society aims to gather people from various educational backgrounds in hopes of sharing novel perspectives on market modelling, and possibly creating something completely original.

3 Mathematical Implementation

Let the price of the S&P be x, and the time t. We will compare the model with the data as we go.

```
growthRate = 0.08
```

3.1 Step 1

We will begin with an exponential curve with growth rate α (= 0.8 in this model).

$$x = e^{\alpha t} \tag{1}$$

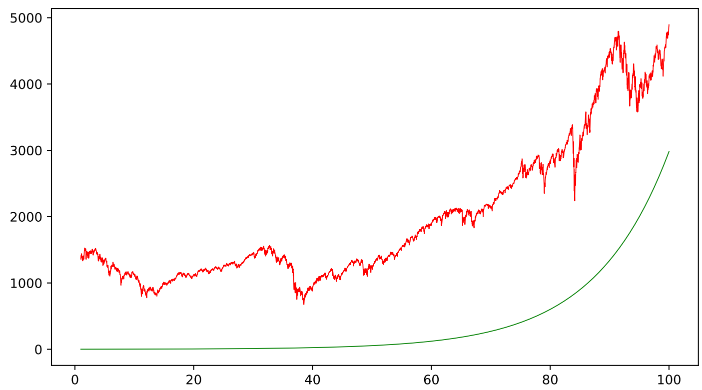

Figure 2: Step 1 Model vs. Market

Plotted here, in Figure (2), is an exponential to match the general movement of the market, the realisation of the first part of the idea from section (2).

```
x = np.exp(growthRate*t)
```

3.2 Step 2

For the second part of the idea, a fluctuating sinusoid is to be added to the exponential.

$$x = e^{\alpha t} + A_i \sin\left(2\pi f_i t + \phi_i\right) \tag{2}$$

Where A_i is an arbitrary amplitude, f_i is an arbitrary frequency and ϕ_i is an arbitrary phase shift.

As can be seen, in Figure (3), the sinusoid is barely visible as its amplitude is far less than the value of the exponential. It needs to be scaled with the exponential. $(A_i \ll e^{\alpha t})$

```
x = np.exp(growthRate*t) + 5*np.sin(2*pi*1*t)
```

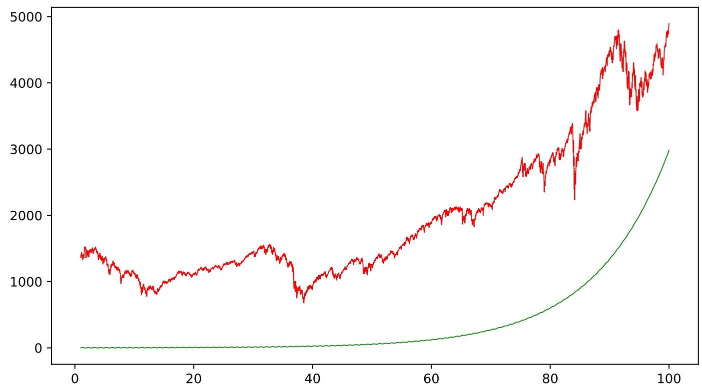

Figure 3: Step 2 Model vs. Market

3.3 Step 3

The sinusoid is therefore scaled to match the exponential.

$$x = e^{\alpha_1 t} + e^{\alpha_2 t} A_i \sin(2\pi f_i t + \phi_i)$$

$$x = e^{\alpha t}(1 + A_i \sin(2\pi f_i t + \phi_i)) \tag{3}$$

The growth rate $\alpha_1 = \alpha_2 = \alpha$ because both exponentials grow at the same rate. If one were to grow faster than the other, over time, the effect of the slower exponential would be insignificant.

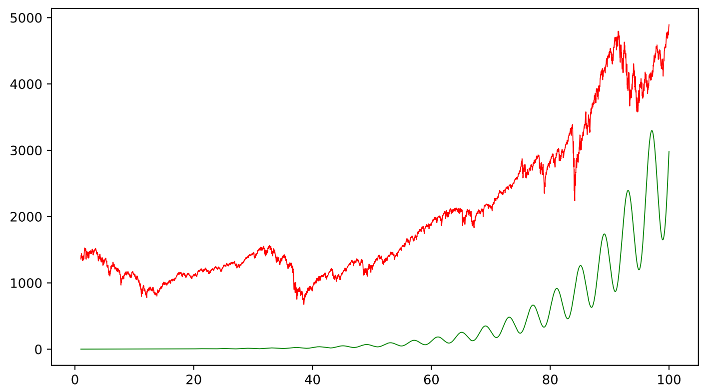

Figure 4: Step 3 Model vs. Market

As can be seen, in Figure (4), the model is already starting to take proper shape.

```
x = np.exp(growthRate*t) + 0.4*np.exp(growthRate*t)*(np.sin(2*pi*0.25*t))
```

3.4 Step 4

To better approximate the market, more sinusoid additions would be needed to follow the curves of market movement, and an offset to allow for one more degree of freedom in the model.

A more general equation is the following:

$$x = x_0 + e^{\alpha t}(1 + \sum_{i=0}^{N} A_i \sin(2\pi f_i t + \phi_i)) \tag{4}$$

where N is a finite number of sinusoids and x_0 is the offset.

In Figure (5), with the addition of more sinusoids and an offset, looks much closer to the actual market movement. The theory of the idea is that a fitted model with more sinusoids will better mimic market motion.

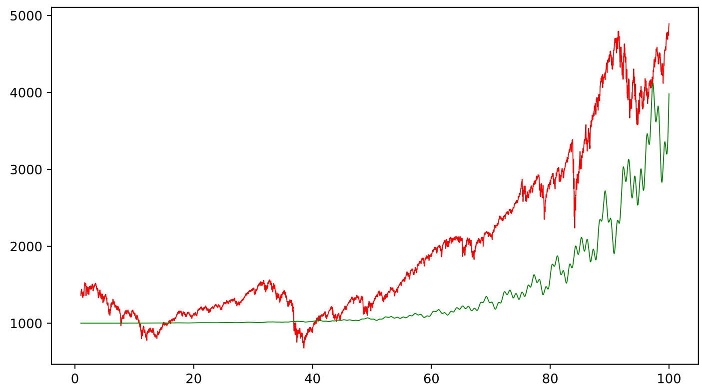

Figure 5: Step 4 Model vs. Market

```
x = 1000 + np.exp(growthRate*t)*(1 + 0.1*np.sin(2*pi*1*t) +
    0.2*np.sin(2*pi*0.25*t) + 0.08*np.sin(2*pi*0.35*t))
```

3.5 Parameter Conditions

For the model to behave properly, some conditions to take note of are the following:

- Ensure that $0 \leq A_i \leq 1$ to satisfy the condition $x \geq 0$.
- Ensure that $-\pi \leq \phi_i \leq \pi$ to limit the range of values of ϕ_i when fitting the model to the market data. Any function with values outside this range can be represented by values inside the range.
- Ensure that $A_i \propto \frac{1}{f_i}$ to satisfy the market property of "the quicker the change, the smaller it is".

4 Results

The end goal of making and fitting this model to market data, is to extract from it the market cycles ad when they happen.

When fitted, the useful information is the part containing the sinusoids in equation (4). The sinusoids contain the information on when, exactly, the market turns.

The key information is held in this equation:

$$\sum_{i=0}^{N} A_i \sin\left(2\pi f_i t + \phi_i\right) \tag{5}$$

The plot of this equation from the example is shown in Figure (6).

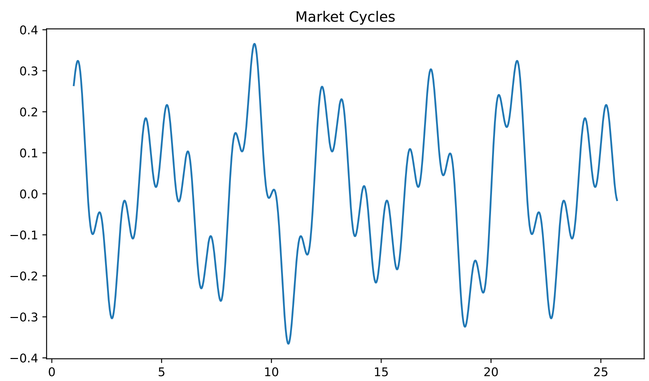

Figure 6: Market cycles extracted from demonstration example

```
cycles = 0.1*np.sin(2*pi*1*t) + 0.2*np.sin(2*pi*0.25*t) + 0.08*np.sin(2*pi*0.35*t)
```

Disclaimer

This document should not be used as financial advice, and should be used at your own personal risk. The Quant Finance Society takes no responsibility for any financial losses.