



ETF TIME SERIES ANALYSIS : UNIVARIATE AND MULTIVARIATE GARCH WITH VAR MODELING



2024-2025

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PART 1 : Presentation and descriptive analysis of the selected ETFs

In order to diversify the client's portfolio, three Amundi trackers were selected that replicate indices that appear to be completely opposite at first glance. All three are denominated in euros and are capitalization ETFs. The figures presented in this study are weekly.

The first is the Amundi MSCI World UCITS ETF listed in France on Euronext Paris, a broad market index that aims to replicate the MSCI World Index as closely as possible. The three main sectors that make it up are information technology (including multinationals such as Apple and Microsoft), the financial sector, which mainly features Berkshire Hathaway, the famous conglomerate led by Warren Buffet, and the healthcare sector, which includes Novo Nordisk, a pharmaceutical company based in Denmark. Geographically, just over 70% of the companies in the index are based in the United States, followed by Japan, the United Kingdom, and Canada, demonstrating strong exposure to the US market. This ETF is eligible for the PEA.

The second ETF is the Amundi EUR Corporate Bond Climate Paris Aligned UCITS ETF Acc, an index fund listed on Euronext Paris that invests in bonds issued by companies in a wide range of economic sectors (industry, financial services, etc.) and denominated in euros, while complying with the climate objectives of the Paris Agreement. This ETF seeks to replicate as closely as possible the performance of the Bloomberg MSCI Euro Corporate Paris Aligned Index, which selects issuers that meet strict criteria in terms of carbon footprint and alignment with a global warming trajectory of less than 2 degrees. This product therefore offers geographical diversification focused on the eurozone with moderate exposure to corporate credit risk, adjusted according to demanding environmental criteria. It is part of a responsible and sustainable investment strategy while providing a stable bond yield. This ETF is not eligible for the PEA (French equity savings plan) as it is not an equity vehicle but a bond vehicle.

The last one is the Amundi Pan Africa UCITS ETF Acc, an index fund listed on Xetra in Germany, which aims to replicate as closely as possible the performance of the S&P Pan Africa Index, a broad index comprising the main listed companies on the African continent. This fund offers unique exposure to African stock markets. The three main sectors represented are financial services (with major groups such as FirstRand), telecommunications (notably MTN Group, a key player in Africa) and energy, which includes Sasol, a company specializing in energy. Geographically, the index is heavily weighted towards South Africa, which accounts for more than 70% of the index's capitalization, followed by Morocco, Egypt, and Kenya. The ETF thus allows investors to gain exposure to emerging African growth in a structured and diversified framework, although this investment offers a higher level of risk due to the volatility of African markets and geopolitical risks. The product is not eligible for the PEA (French equity savings plan), as it invests in non-European securities.

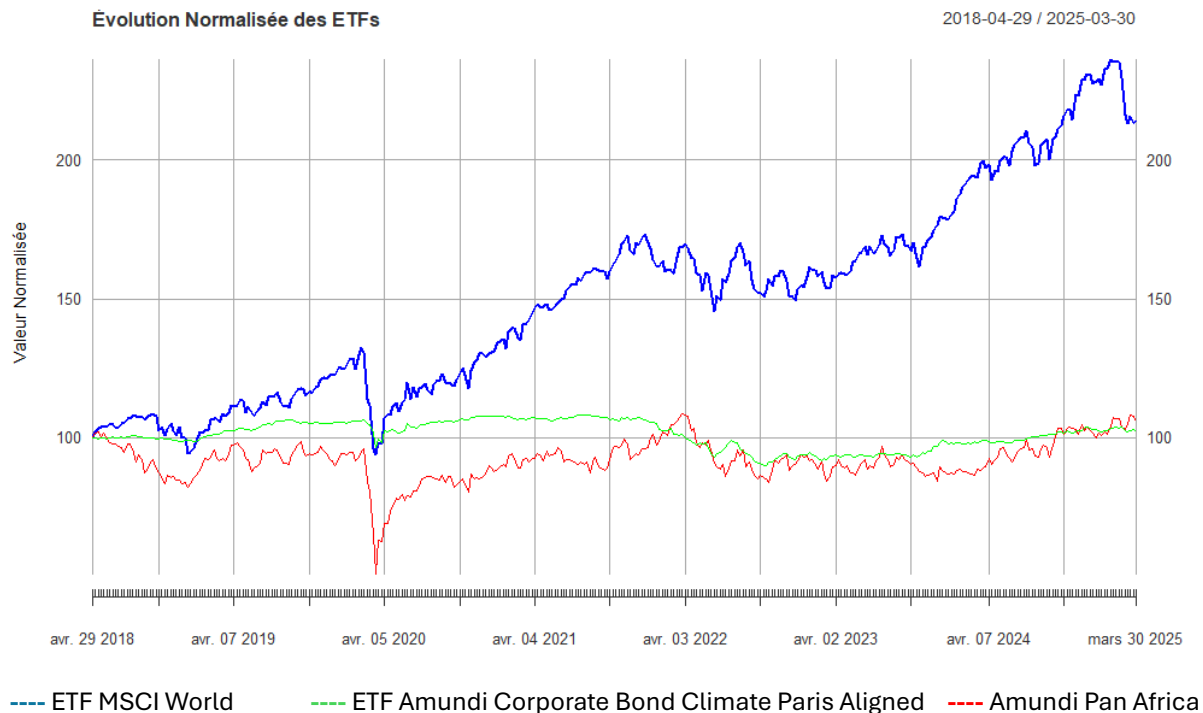


Chart 1 : Normalized performance of ETFs from April 29, 2018, to March 30, 2025

The advantage of normalizing the performance of the three ETFs to a base of 100 is comparative, as it allows all series to be brought back to a common starting point in order to identify which is outperforming or underperforming another, which is growing the fastest, etc.

The MSCI World ETF posted a net performance over the entire period, more than doubling in value over the selected period, despite a temporary decline in 2020 due to Covid-19. The ETF accurately reflects the growth of large global multinationals, particularly American ones, which make up a large part of the index.

The Amundi Corporate Bond Climate curve is stable and volatility is low, which is characteristic of corporate bonds. The yield is moderate but stable and reflects the ETF's defensive profile, i.e., low growth while taking less risk.

However, the Amundi Pan Africa ETF reflects the high instability of African markets, with high volatility, marked downturns, and fairly low growth. Returns are rather uncertain, but there is long-term growth potential for investors who are not averse to risk.

PART 2 : Statistical analysis of tracker returns

ETF	Mean	Median	Min	Max	StdDev	Skewness	Kurtosis_Excess	Annualized_Return_Percent
LGQM.DE	0.0006944761	0.001039584	-0.2205768	0.2392	0.03130143	-0.4638803	15.96787	3.611276
CW8.PA	0.002359303	0.003893703	-0.1421133	0.09390188	0.02158182	-1.373007	6.238738	12.26838
CRP.PA	9.926769e-05	0.0002603811	-0.05026089	0.04259672	0.006988905	-0.5595311	9.218143	0.516192

Table 1 : Descriptive statistics for ETFs

Amundi MSCI World (CW8.PA) :

We note that Amundi MSCI World (CW8.PA) has an annualized performance of 12.27%, making it the best-performing ETF among those studied. This return is due in particular to its broad exposure to large listed companies, especially US tech giants (such as Apple and Microsoft) that make up this index fund.

Volatility, measured by the standard deviation of returns, is 0.0215, reflecting moderate variability in returns: the ETF does not experience extreme fluctuations on a regular basis and remains stable.

The skewness coefficient is -1.37, indicating a marked negative asymmetry in the distribution of returns. Extreme losses are more prevalent than extreme gains. This means that investors may be exposed to significant declines even if the overall trend remains bullish.

The excess kurtosis is 6.24, indicating a leptokurtic distribution, meaning that the returns of this ETF experience more extreme events, whether positive or negative, than would be predicted by a normal distribution.

Amundi EUR Corporate Bond Climate Paris Aligned (CRP.PA) :

The Amundi Corporate Bond Climate Paris Aligned ETF has a very low annualized performance of 0.51% despite a slight jump towards the end of the period (see Chart 1). The ETF has significantly underperformed.

Its volatility is low, at 0.00699. At first glance, this may suggest that the ETF is low-risk, stable, or safe and would suit a cautious investor profile. However, other statistical indicators reveal another side to the ETF: the skewness coefficient and excess kurtosis.

The former is negative, estimated at -0.56, indicating a negative asymmetry in the distribution of returns, suggesting that returns tend to experience significant losses more often than extreme gains. The latter is high, estimated at 9.21, so the distribution is leptokurtic. There are therefore more peaks of extreme returns than in a normal distribution.

The real risk is therefore masked by the low dispersion of returns, and the ETF may suffer significant one-off losses, despite the impression of “stability” it gives.

Amundi Pan Africa (LGQM.DE) :

The African ETF has an annualized performance of 3.61%, which may seem reasonable but remains modest given the high risk involved. It is the most volatile of the three ETFs, with a standard deviation of 0.031, indicating frequent and significant movements and therefore uncertainty regarding future returns.

The negative skewness of -0.46 reveals, as before, that extreme declines are more frequent than rises, while the very high kurtosis of 15.97 highlights the probability of extreme returns. Although the product offers exposure to African growth, it remains unstable and impacted by external shocks, which does not offer a favorable risk/return ratio.

	CW8.PA	CRP.PA	LGQM.DE
CW8.PA	1	0.509	0.603
CRP.PA	0.509	1	0.501
LGQM.DE	0.603	0.501	1

Table 2 : Correlation matrix

Calculating the correlation matrix between ETFs is fundamental in finance because it allows us to measure how assets move together in order to build an optimized portfolio with the best possible diversification.

It is constructed by relating the covariance of the returns of X and Y to the standard deviation of the returns of X multiplied by that of Y.

The highest correlation in the matrix is between the MSCI World (CW8.PA) and the Pan Africa ETF (LGQM.DE), suggesting that the performance of the global ETF and the African ETF evolve in a similar manner. This correlation between the two can be explained by the fact that African markets are influenced by developed markets (e.g., the cost of raw materials).

The CW8.PA & CRP.PA correlation is estimated at 0.509, indicating a positive but moderate relationship, as bond markets can react differently from equity markets, and vice versa: for example, in times of stress, bonds can serve as a safe haven and an alternative to equity products.

Finally, the correlation between CRP.PA and LGQM.DE, at 0.501, is also moderate, while being the weakest in the matrix. This may reflect the difference between a European bond ETF, where the focus is on stable and fixed income, with a profile that can be described as cautious and defensive, and an African ETF, where a riskier approach is favored because it is more volatile. However, the two are not completely uncorrelated, as the estimate suggests.

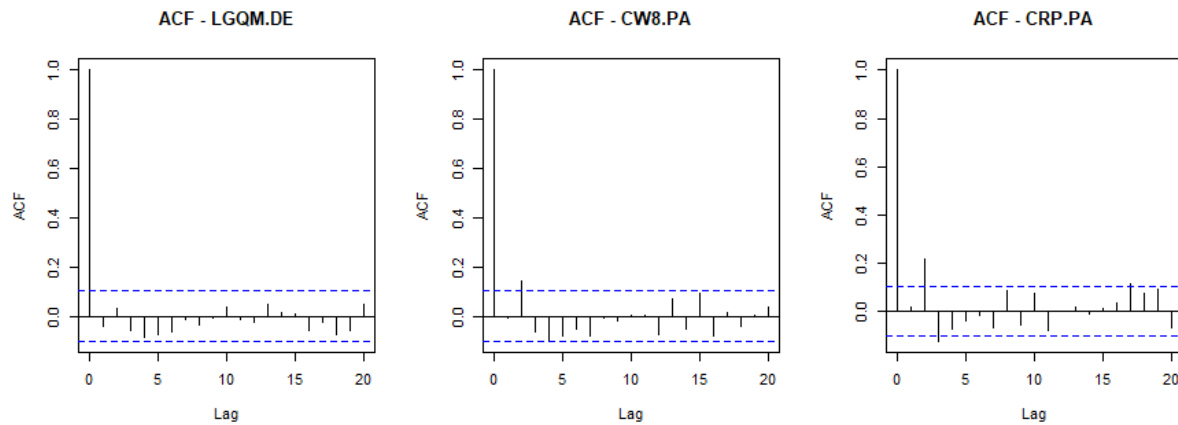


Chart 2: Autocorrelation functions (ACF) of ETFs returns

In finance, the autocorrelation function is used to test the independence of returns and thus detect any dependency structure that may exist. The blue dotted lines represent the 95% confidence intervals for testing the significance of autocorrelations under the following hypothesis: H_0 : there is no temporal dependency.

There is short-term dependence in the autocorrelation function of the LGQM.DE ETF, as all the bars remain within the confidence bands.

For the global ETF CW8.PA, none of the bars exceed the confidence limits, except for the fourth lag, which proves that there is a non-zero linear dependence between today's return and that of four periods ago, here in weeks because our returns are weekly. The hypothesis of independence at this lag, at the 5% threshold, is rejected.

Finally, we note that the ACF function of the CRP.PA bond ETF shows, at approximately the third lag, a non-zero linear dependence between today's return and that of three weeks ago. The hypothesis of independence at the third lag is therefore rejected.

PART 3 : Modeling and evaluating return volatility

GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models aim to model and predict the conditional volatility of time series. Unlike ARCH models with constant variance, GARCH models allow volatility to vary over time. GARCH models capture non-constant volatility and its persistence. Here, μ is the conditional mean of the return, ω is the variance constant, α_1 is the effect of shocks, and β_1 is the persistence of past volatility. An estimated parameter is considered significant when its p-value is < 0.05 , meaning it will have a real effect in the model.

The model applied to the ETFs studied is a GARCH(1,1) model, which means that we include 1 past variance and 1 past shock.

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*               GARCH Model Fit               *
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Conditional variance dynamics
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GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution      : norm

Optimal Parameters
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	Estimate	Std. Error	t value	Pr(> t)
mu	0.000529	0.001214	0.43575	0.663021
omega	0.000255	0.000062	4.12817	0.000037
alpha1	0.243699	0.068683	3.54816	0.000388
beta1	0.390390	0.112291	3.47659	0.000508

Table 3: GARCH(1,1) model applied to the LGQM.DE ETF

We note that the volatility of the LGQM ETF is not constant over time but varies according to past shocks and variance. In the results presented, the α_1 coefficient means that previous shocks, regardless of their evolution, have an impact on 24.37% of today's volatility. The β_1 coefficient, meanwhile, indicates that 39% of today's variance is significantly influenced by past dynamics. The model shows that the ETF is exposed to reactive volatility, structural uncertainty, and political instability, which are characteristics of the African market.

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*-----*
*          GARCH Model Fit          *
*-----*

Conditional Variance Dynamics
-----
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution      : norm

Optimal Parameters
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```

	Estimate	Std. Error	t value	Pr(> t)
mu	0.003244	0.000906	3.5803	0.000343
omega	0.000060	0.000019	3.0943	0.001973
alpha1	0.252759	0.070657	3.5773	0.000347
beta1	0.635994	0.075620	8.4104	0.000000

Table 4: GARCH(1,1) model applied to the CW8.PA ETF

Applying the GARCH model to the global ETF, we see that the alpha1 and beta1 coefficients are significant, and therefore play a major role in this model.

The alpha1 coefficient indicates that approximately 25.28% of current volatility is explained by the previous day's shock. As the coefficient is significant, the model accurately captures the market's immediate reactions to recent shocks.

The beta1 coefficient reveals that 63.60% of current volatility depends on the previous day's volatility, which is highly persistent given its p-value close to 0.

In summary, the GARCH estimate for this product shows that the volatility of the MSCI World ETF is both sensitive to recent shocks and influenced by its own "memory."

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*-----*
*           GARCH Model Fit           *
*-----*

Conditional variance Dynamics
-----
GARCH Model       : sGARCH(1,1)
Mean Model        : ARFIMA(0,0,0)
Distribution       : norm

Optimal Parameters
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```

	Estimate	Std. Error	t value	Pr(> t)
mu	0.000452	0.000209	2.15968	0.030798
omega	0.000002	0.000002	0.92301	0.356001
alpha1	0.312224	0.056207	5.55490	0.000000
beta1	0.686776	0.041173	16.68008	0.000000

Table 5 : GARCH(1,1) model applied to the CRP.PA ETF

This GARCH(1,1) model applied to the bond ETF indicates that 68.68% of conditional volatility depends heavily on past volatility (beta1 significant with a p-value close to 0) and recent shocks (alpha1 being significant and also having a p-value close to 0). The model reveals the importance for cautious investors not to underestimate the risk of corporate bonds in times of stress, whether related to monetary policy, credit markets, or other factors.

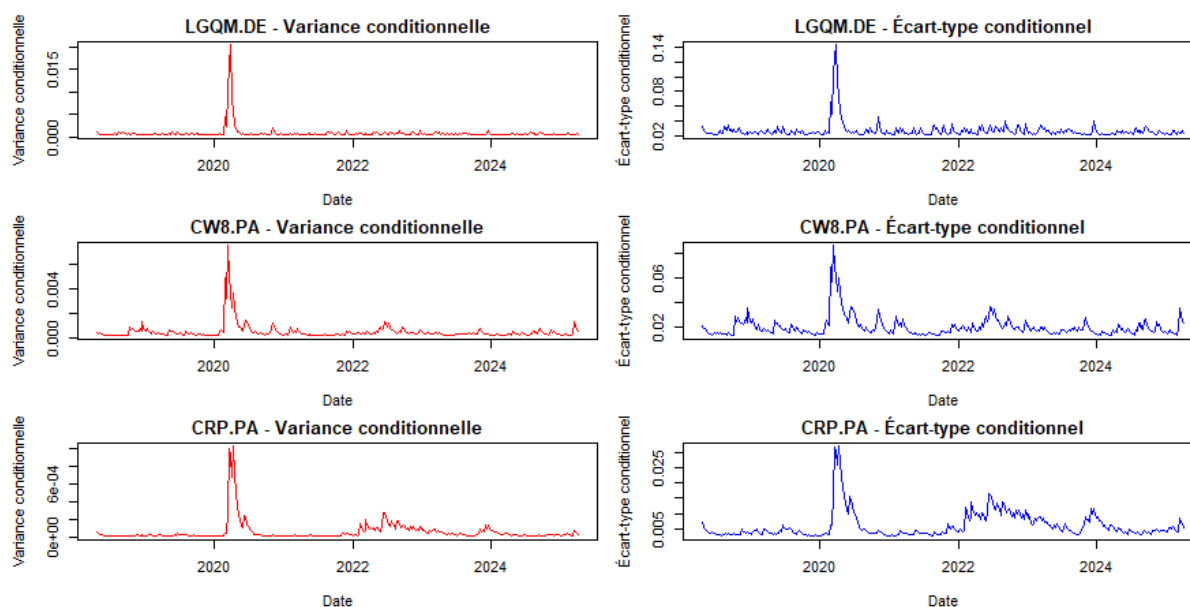


Chart 3 : Standard deviations (blue) and conditional variances (red) of ETFs

The conditional variance and conditional standard deviation charts from the previously estimated GARCH(1,1) models reveal a volatility structure marked by a global shock in March 2020 linked to the Covid-19 pandemic. Although ETFs react differently to different shocks given their different asset classes and geographical areas, they all reacted to the systemic shock in the same way, which may call into question the effectiveness of geographical or sector diversification in the event of a global crisis.

PART 4 : Multivariate modeling of conditional volatility

Multivariate GARCH models aim to model the joint dynamics of volatility and correlations between several financial assets over time. They allow interactions between time series to be taken into account. Here, the model used is the GARCH-BEKK model, which required extensive research to find a BEKK package that could be downloaded from Rstudio.

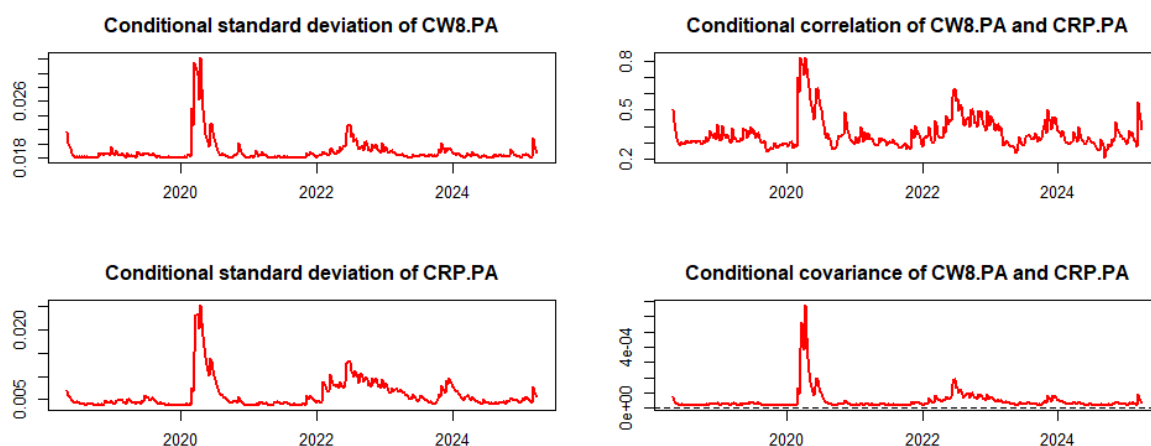


Chart 4 : GARCH-BEKK model for CW8.PA and CRP.PA

The BEKK model applied to global ETFs and bond ETFs reveals that, during crises, their behavior becomes synchronized, which limits diversification effects. However, during normal periods, their behavior differs, suggesting that the two are complementary in an asset allocation strategy.

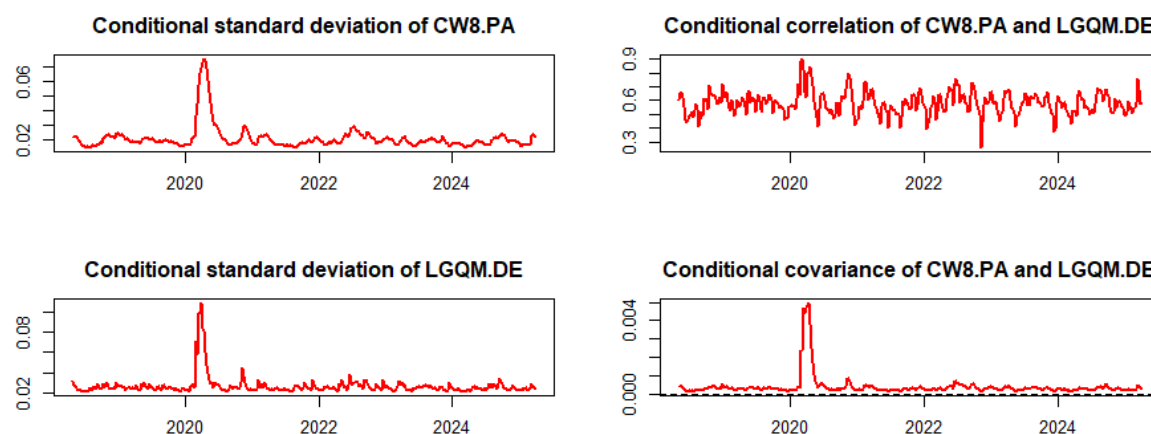


Chart 5 : GARCH-BEKK model for CW8.PA and LGQM.DE

The GARCH-BEKK model applied here to the global ETF and the African ETF highlights very different risk or volatility profiles between these two assets. On the one hand, the MSCI World ETF, representative of the global market, shows moderate volatility with a justified increase in 2020 during the pandemic. On the other hand, the Pan Africa ETF shows significantly higher volatility and is more sensitive to shocks, reflecting the structural characteristics of African markets, which are considered emerging: low liquidity, high risk, and instability.

The conditional correlation between these two assets varies sharply, reinforcing the idea that the African ETF is a risky asset, making it attractive for diversification during calm periods but much riskier in times of global crisis.

PART 5 : Modeling dynamic relationships between ETFs using the VAR model

VAR (Vector Autoregressive) models are used to analyze the relationships between several time series. They enable us to study the interdependence between several assets and are a useful tool because we can understand the influence of one asset's returns on others and study the impact of a shock on another variable using impulse response functions.

In a VAR model, we seek to explain each variable by its past values and those of other variables. The residual correlation matrix allows us to measure the correlation between unexplained shocks in the different VAR equations at a given point in time.

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Correlation matrix of residuals:
      CW8  LGQM  CRP
CW8  1.0000  0.6173  0.5163
LGQM  0.6173  1.0000  0.4884
CRP   0.5163  0.4884  1.0000
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Table 6 : Residual correlation matrix

The correlation matrix of residuals obtained from the VAR model shows that the three ETFs exhibit significant correlations.

The high correlation between CW8 and LGQM suggests that African markets react strongly and quickly to shocks affecting developed markets, reflecting Africa's dependence on the rest of the world and continued influence from major stock exchanges, even though the Pan Africa ETF is geographically distant.

The moderate correlation between CW8 and CRP shows that bond markets are not entirely disconnected from equity markets. In normal times, their behavior differs, but in the event of a global crisis, they react in the same way as equities, which is undoubtedly justified by increases in risk premiums or changes in interest rates.

The weakest correlation is between LGQM and CRP. However, it is still reasonable (almost 0.5). They do not react systematically to shocks but probably share the same sensitivity to global risk factors.

IRFs can be used to measure the effect of an impulse on a given variable on the other variables drawn from the VAR model. Specifically, they show: the direction of the impact, its effect, its propagation duration, and the uncertainty surrounding the effect thanks to confidence intervals.

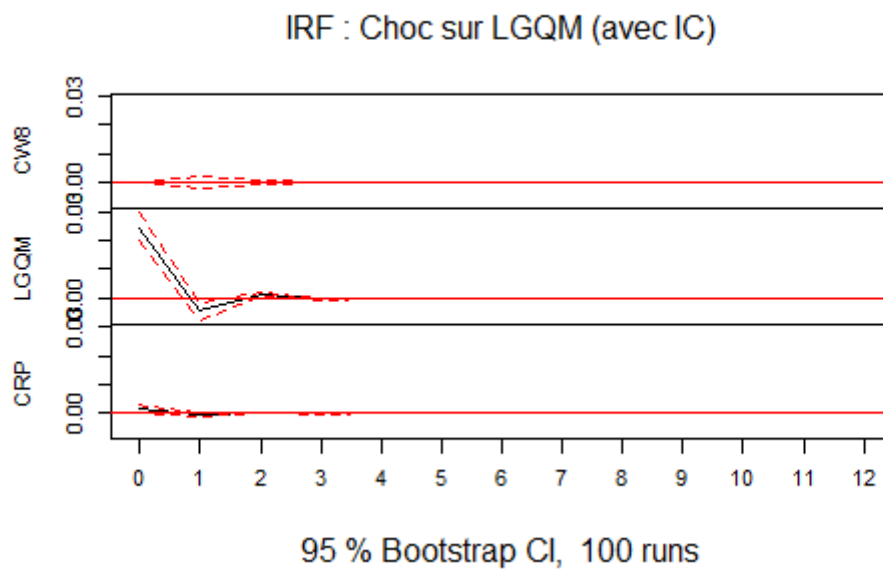


Chart 6 : LGQM impulse response function

This impulse response function shows that a shock to the Pan Africa ETF has an immediate effect on itself but no contagion effect on the global market, represented by the MSCI World ETF (CW8.PA), or on the bond market, represented by the CRP.PA ETF. This suggests that African markets do not have a significant influence on other markets.

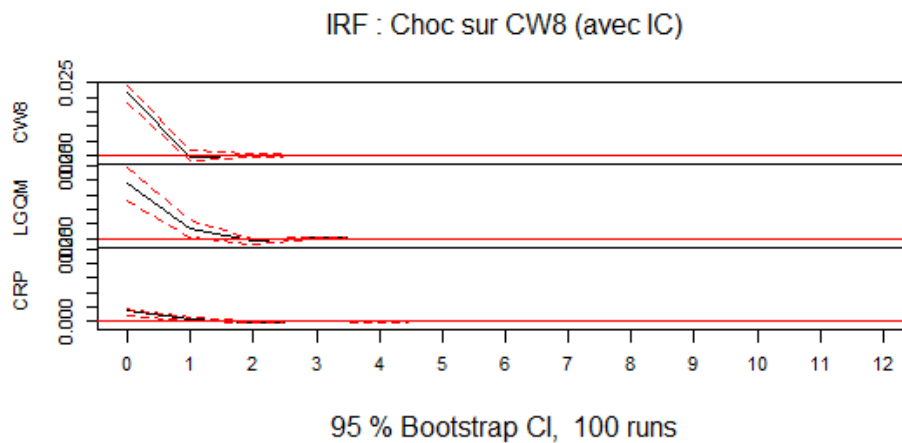


Chart 7 : CW8 impulse response function

This graph shows that shocks to the MSCI World Index have immediate effects on African markets, whereas, as seen previously, the reverse is not true. However, the shock does not have a very significant impact on bond markets. This may highlight the role of the CW8 ETF as a “dominant variable” in the financial system, with its movements influencing other financial assets.

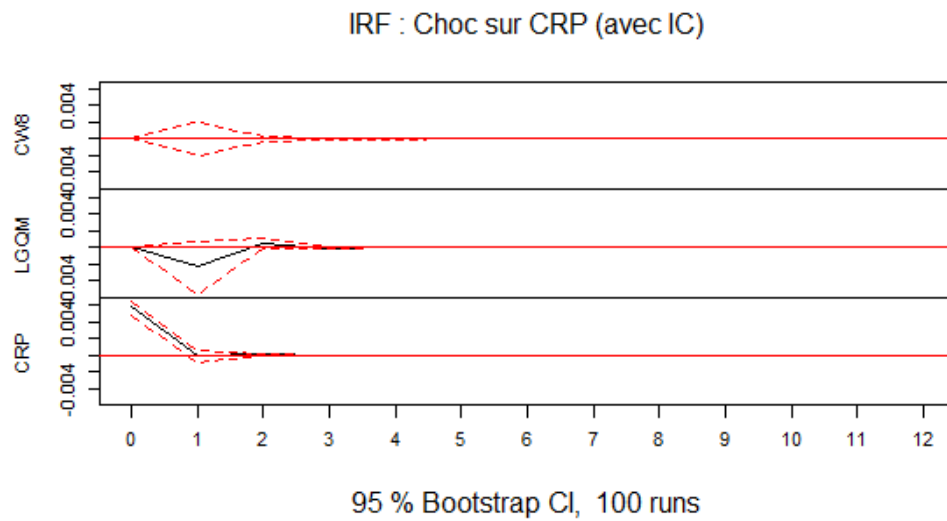


Chart 8 : CRP impulse response function

The shock to the CRP bond ETF has a limited impact on other assets. When a positive shock affects CRP (rise in rates, rise in risk premiums, reorientation of capital flows towards climate-related corporate bonds, etc.), the African ETF reacts negatively in the following periods. We see that a shock to CRP has no noticeable effect on developed markets, and therefore CW8. This may suggest that “sustainable” corporate bonds can be seen as autonomous assets that stabilize a portfolio.