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This study provides evidence of periodically collapsing bubbles in the British pound to US dollar exchange rate in the post-1973 period. We develop two- and three-state regime-switching (RS) models that relate the expected exchange rate return to the bubble size and to an additional explanatory variable. Specifically, we consider six alternative explanatory variables that have been proposed in the literature as early warning indicators of a currency crisis. Our findings suggest that the RS models are, in general, more accurate than the Random Walk model in terms of both statistical and especially economic evaluation criteria for exchange rate forecasts. Our three-state RS model outperforms the two-state models and among the variables considered in our analysis, the short-term interest rate is the optimal variable, closely followed by imports. Results are more promising for one-month predictions and are qualitatively robust over sample spans. However, various robustness checks based on other exchange rates show that the optimal bubble measures and optimal predictors critically depend on the exchange rate.

Keywords: bubbles; exchange rates; regime switching; forecasting

JEL Classification: F3; G1; C3

1. Introduction

Forecasting exchange rates is in the core of international economics. However, modeling nominal exchange rate behavior is one of the most challenging tasks imposed to economists since the seminal contribution of Meese and Rogoff (1983), who found that exchange rate models based on fundamentals fail to beat the Random Walk (RW) model for horizons up to one year.[1] Subsequent studies attempted to develop more sophisticated econometric techniques in order to overturn the findings of Meese and Rogoff but results did not turn out promising. To mention a few, Meese and Rose (1991) and Kilian and Taylor (2003) allowed for functional nonlinearities, while the system-based approach of MacDonald and Marsh (1997) yielded superior forecasts in certain circumstances.[2] Cheung, Chinn, and Garcia (2005) re-assessed the forecasting ability of a wide set of linear models that have been proposed in the last decade including interest rate parity, productivity-based models, and a composite specification and found that models that work well in one period do not necessarily work well in another period.

One promising route was opened up by Engel and Hamilton (1990) who modeled exchange rates as a two-state Markov-switching process. The authors showed that during the floating period their model outperformed the RW model both in-sample and out-of-sample at short horizons.

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Following this seminal contribution, Markov-switching models of exchange rates have been subsequently employed in the literature. Specifically, Evans and Lewis (1995) developed a model where regime switches are linked with rational traders' forecasts of the future exchange rate. Frommel, MacDonald, and Menkhoff (2005) provided evidence of a nonlinear relationship between exchange rates and fundamentals and found that the key determinant of regimes is the interest rate differential. More recently, Dewachter (2001) and Dueker and Neely (2007) merged the Markov-switching models with technical trading rules and found that such an approach can be fairly successful.[3] On the other hand, Engel (1994) found that a regime-switching (RS) model fits well in-sample for many exchange rates, but it is not able to generate forecasts superior to the RW model. Similarly, Klaassen (2005) enhanced Engel's model with a GARCH error structure, but also failed to find any nominal exchange rate predictability. However, more recently, Yuan (2011) employed a multi-state Markov-switching model with smoothing and showed that this model outperforms the RW model and is robust over sample spans. Moreover, Nikolsko-Rzhevskyy and Prodan (2012) modeled the constant of the term using the two-state Markov-switching stochastic segmented trend model and presented evidence of both short-run (one month) and long-run (up to one year) predictability for monthly exchange rates over the post-Bretton Woods period.

In this study, we also employ an RS specification linking the probabilities of regimes to fundamentals and deviations from them. More specifically, our starting point is the van Norden (1996) RS model which was developed as a test for exchange rate bubbles.[4] Van Norden linked speculative bubbles to a two-state RS model where both the future return and the probability of appreciation or depreciation are functions of the bubble and found mixed evidence with respect to the bubble hypothesis by employing alternative models of fundamental values for exchange rates (bubble measures). As the author states a bubble model may be equivalent to a non-bubble model when a different specification of fundamentals is employed. In this respect, the RS behavior induced by bubbles may just denote switching in fundamentals. Moreover, a number of factors can contribute to the apparent long swings in exchange rate data. These factors include the 'peso problem', the changing importance of chartists and fundamentalists in the foreign exchange market, monetary and fiscal policy changes between the relevant countries, changing business cycles, transactions costs, etc.

Following van Norden (1996), we also employ the simple two-state model that relates the future return of the exchange rate to the deviation from fundamentals (bubble size).[5] Bubbles are calculated in four alternative ways as proposed by the literature on exchange rate determination.[6] We then develop two extensions of this specification. The first one is a two-regime model enriched with an explanatory variable that enters in both the conditional mean and the probability equations. We consider six explanatory variables that have been proposed in the literature as early warning indicators of a currency crisis, namely exports, imports, international reserves, long- and short-term interest rates and the yield spread. Finally, following Brooks and Katsaris (2005) and Yuan (2011) along with the observation that exchange rates exhibit range-bound behavior for a sustained period of time, we extend our two-regime specification to a three-regime one by allowing for a third trendless regime in the dynamics of the exchange rate. The evaluation of our models is carried out in terms of both statistical and economic significance relative to a RW model. The statistical evaluation of the models is based on the mean-squared forecast error (MSFE) criterion, while the statistical significance is established through the forecast efficiency test and the methodology developed by Clark and West (2006, 2007) to allow for comparisons of nested models. More importantly, following the pioneering work of West, Edison, and Cho (1993) and recent contributions on the economic significance of exchange rate forecasts (Cheung and Valente 2009; Della Corte, Sarno, and Tsiakas 2009; Della Corte, Sarno, and Valente 2010) we also examine the forecasting power of our models in a stylized asset allocation framework, where a mean–variance investor maximizes expected utility. More specifically, a risk-averse investor will be willing to pay for switching from the portfolio constructed based on the forecasts of the simple RW model to a portfolio based on our proposed RS specifications. This performance fee, which forms the evaluation criterion, is the fraction of the wealth which when subtracted from the RS proposed portfolio returns equates the average utilities of the competing models. As a complement to the performance fee measure, we also employ the manipulation-proof performance measure proposed by Goetzmann et al. (2007), which can be interpreted as a portfolio's premium return after adjusting for risk. Finally, we develop two simple trading rules that use information from our estimated models to predict movements in the exchange rate market and lead to increased profits. The first trading rule is based on the estimated probabilities of a crash and a boom in the exchange rate market, and specifically states that the investor decides about the allocation of her wealth by comparing the probability of a crash with the probability of a boom. The second one compares the return of investing in the local currency (UK pounds in this study) to the expected return of investing in US dollars, which incorporates the forecast for the gross return in the exchange rate based on one of the estimated models, and selects the higher one.

To anticipate our key results, we find evidence of superior forecasting performance of our RS models for the UK pound to US dollar exchange rate relative to the RW benchmark during the post-1973 period. More in detail, a three-state RS model outperforms the two-state models and among the variables considered in our analysis, the short-term interest rate is the optimal variable, closely followed by imports, in both statistical and economic evaluation terms. Our findings suggest that a risk-averse investor would be willing to pay an annual performance fee of up to 519 bps to switch from the RW model to our three-regime specification. The risk-adjusted abnormal return is fully consistent (size and sign) with the results obtained from the performance fees. With respect to our trading rules, the first rule seems to work better than the second one, especially for our three-regime model augmented with either imports or interest rates (both long and short term). In general, the results are both model and bubble dependent. Our findings seem to be qualitatively robust to the settings of our out-of-sample experiment (i.e. number of out-of-sample observations and the starting and end dates of the estimation period). However, when we extend the analysis to other exchange rates, we find that the optimal bubble measures and optimal predictors critically depend on the exchange rate.

The layout of this paper is as follows: Section 2 motivates the paper by presenting the results of a recently introduced test for the detection of periodically collapsing bubbles in the UK pound to US dollar exchange rate. Section 3 presents various models of exchange rate determination used to calculate alternative bubble measures and presents our econometric methodology. Section 4 describes the data set and the main estimation results. Section 5 performs the statistical and economic evaluation of our exchange rate forecasts, while Section 6 employs our forecasts to develop two trading strategies aiming at increasing the profits of an investor. Robustness checks are presented in Section 7, and Section 8 summarizes the main findings of the paper.

2. Testing for the existence of a bubble

Several attempts have been made in the literature to develop econometric tools to test for the existence of bubbles in asset prices (Gurkaynak 2008 offers a stimulating review). Among the available procedures of bubble detection are the integration/cointegration-based tests (Diba and Grossman 1988). However, Evans (1991) criticizes this approach by showing that these tests cannot identify the existence of bubbles when they manifest periodically collapsing behavior in the sample under scrutiny.

In a recent study, Phillips, Wu, and Yu (2011, PWY hereafter) introduce a forward recursive right-tailed augmented Dickey–Fuller (ADF) test to identify the existence of bubbles. In the presence of a single bubble, this test is shown to be consistent. Moreover, Homm and Breitung (2012) show that the PWY procedure outperforms other tests, especially in the presence of periodically collapsing bubbles. However, if the sample contains more than one bubble, the PWY test is sometimes not capable of identifying all bubbles. In an attempt to improve the discriminatory power of the PWY test, Phillips, Shi, and Yu (2011, PSY hereafter) propose a generalized version of the PWY test that performs much better when we have multiple collapsing bubble episodes in our sample. Similar to the PWY test, the methodology introduced by PSY performs right-tailed ADF tests in a recursive manner and takes the supremum value. However, under the PSY approach the sample sequence is expanded by allowing the starting point to range within a feasible range (while the PWY test keeps the starting point fixed). Specifically, given a sample of T observations, the PSY methodology is based on the following statistic:

$$\mathrm{GSADF}(r_0) = \sup_{\substack{r_2 \in [r_0,1] \\ r_1 \in [0, r_2 - r_0]}} \{\mathrm{ADF}_{r_1}^{r_2}\},$$

where $r_0 = [Tr_0]$ is the size of the smallest sample window ($[\cdot]$ denotes the integer part of the argument), while r_1 and r_2 are the starting and ending points of the sample over which the ADF test is performed.

We first use the $\mathrm{GSADF}(r_0)$ statistic to test for the existence of a bubble in the UK pound to US dollar (£/$) exchange rate. We use monthly data of end-of-period market exchange rates ranging from January 1973 to January 2011. The smallest sample window contains 36 observations. The results, reported in Table 1, suggest that a bubble is present in the exchange rate (at a 5% confidence level). Table 1 also reports the PWY statistic, defined as $\mathrm{SADF} = \sup_{r_2 \in [r_0,1]}\{\mathrm{ADF}_0^{r_2}\}$, that leads to the same conclusion. All critical values are calculated by means of Monte Carlo simulations with a sample size equal to the number of available observations.

PSY also suggest a backward expanding sample sequence procedure to identify periods of bubble episodes. This analysis provides useful insights about the number, dating and duration of the bubble incidents. Figure 1 presents the sequence of the $\mathrm{GSADF}(r_0)$ statistic together with the simulated 95% critical values. Periods when the $\mathrm{GSADF}(r_0)$ statistic lies above the critical values indicate bubble episodes. The procedure reveals a number of bubble episodes mainly in the mid-1970s and the mid-1980s, while we also observe evidence of a bubble at the end of 2008 continuing into the beginning of 2009.

Table 1. Recursive right-tailed augmented Dickey–Fuller test of bubble detection (£/$).

		Finite sample critical values		
	Statistic	90%	95%	99%
SADF	2.682***	1.205	1.519	2.020
GSADF	2.682**	2.083	2.344	2.826

Note: Null hypothesis, 'No bubble in the exchange rate'.
**Rejection of the null hypothesis at a 5% confidence level.
***Rejection of the null hypothesis at a 1% confidence level.

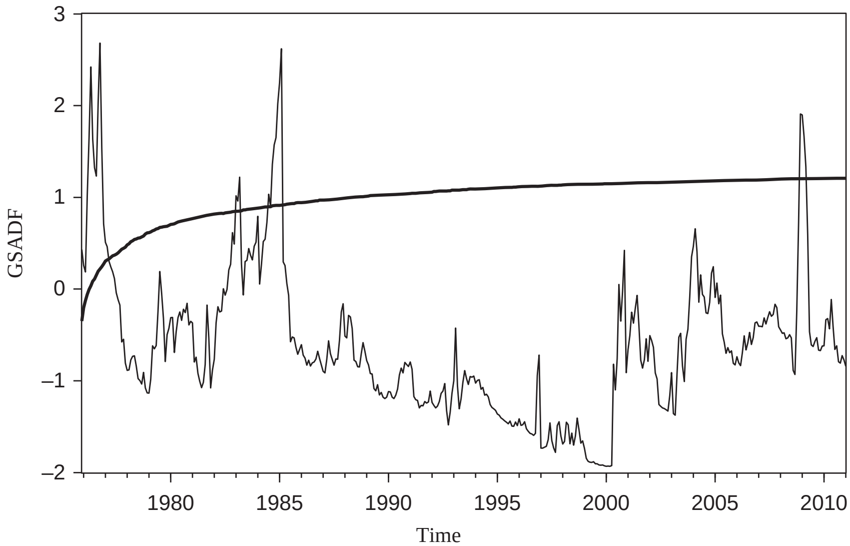

Figure 1. Date-stamping bubble periods in the £/$ exchange rate (GSADF test).

In summary, our findings provide clear evidence in favor of the existence of a periodically collapsing bubble in the British pound to US dollar exchange rate. In the following section, we apply various theories of exchange rate determination to calculate alternative bubble measures. Next, we use RS models, which explicitly account for the size of the bubble in the exchange rate, in an attempt to find an econometric model that generates reliable forecasts for the exchange rate.

3. Speculative bubble measures and RS models

3.1 *Speculative bubble measures*

Speculative bubbles in asset prices are systematic departures from the fundamental price of the asset. In this respect, any model of exchange rate determination can be employed to estimate a speculative bubble measure, which is defined as the deviation of the logarithm of the nominal spot exchange rate (e_t) from its fundamental value, denoted by f_t. In this mode, the size of the bubble, b_t, is defined as

$$b_t = e_t - f_t.$$

As already mentioned, a deviation from fundamentals may not always represent a bubble. However, this measure can help predict the future movement in the exchange rate as current deviations of the exchange rate from the equilibrium level determined by fundamentals induce future changes in the exchange rate so as to align to its long-run equilibrium. The predictive ability of such deviations/bubbles has been tested by Mark (1995), Abhyankar, Sarno, and Valente (2005), Della Corte, Sarno, and Tsiakas (2009) and Chen and Chou (2010) among others.

Drawing from the literature on exchange rate determination, we employ four measures of exchange rate deviations from fundamental price. The first bubble measure naturally arises in the context of Purchasing Power Parity (PPP) which posits that the nominal exchange rate moves one to one with the relative price differential rendering the real exchange rate stationary. Formally, the PPP fundamental price is defined as $f_t = p_t - p_t^*$, where p_t is the domestic price level, p_t^* the foreign price level and f_t is measured in units of domestic currency per unit of foreign currency. The validity of PPP is often tested in the context of a cointegrating relationship between the nominal exchange rate and relative prices (in logs). Specifically, the following equation is estimated:

$$e_t = \delta_0^A + \delta_1^A (p_t - p_t^*) + b_t^A, \tag{1}$$

where the deviation from fundamental prices, $b_t^A = e_t - f_t^A$, is given by the cointegrating residual (b_t^A) and f_t^A is calculated by the fitted values.

The next two measures of fundamental prices are two variants of the flexible monetary model. Starting with the following money demand functions of the two countries (domestic and foreign)

$$m_t - p_t = \alpha_1 y_t - \alpha_2 i_t,$$
$$m_t^* - p_t^* = \alpha_1 y_t^* - \alpha_2 i_t^*,$$

where m_t (m_t^*) is the log of the domestic (foreign) money supply, y_t (y_t^*) is the log domestic (foreign) income and i_t (i_t^*) is the domestic (foreign) nominal interest rate and assuming that PPP holds, i.e. $f_t = p_t - p_t^*$, we have the fundamental price as a function of relative money supplies, relative income and nominal interest rate differentials

$$f_t = (m_t - m_t^*) - \alpha_1 (y_t - y_t^*) + \alpha_2 (i_t - i_t^*). \tag{2}$$

Equation (2) represents the long-run equilibrium exchange rate and as such it can be viewed as a cointegrating relationship.[7] As previously, we estimate the following equation:

$$e_t = \delta_0^B + \delta_1^B (m_t - m_t^*) + \delta_2^B (y_t - y_t^*) + \delta_3^B (i_t - i_t^*) + b_t^B, \tag{3}$$

where the deviation from fundamental prices, $b_t^B = e_t - f_t^B$, is given by the cointegrating residual (b_t^B) and f_t^B is calculated by the fitted values.

If we further assume that inflation expectations by market agents are taken into account, we get the third model of fundamentals for exchange rates. This is model (2) enriched with the expectations about domestic and foreign inflation rates as these are depicted in the expected inflation rate differential ($\pi_t - \pi_t^*$) as follows:

$$f_t = \alpha_1 (m_t - m_t^*) + \alpha_2 (y_t - y_t^*) + \alpha_3 (i_t - i_t^*) + \alpha_4 (\pi_t - \pi_t^*). \tag{4}$$

Similar to the previous cases, our third bubble measure (b_t^C) is given by the residual of the following cointegrating equation:

$$e_t = \delta_0^C + \delta_1^C (m_t - m_t^*) + \delta_2^C (y_t - y_t^*) + \delta_3^C (i_t - i_t^*) + \delta_4^C (\pi_t - \pi_t^*) + b_t^C \tag{5}$$

and f_t^C is calculated by the fitted values.

The fourth method we employ stems from the real interest rate parity which requires that the real interest rate differentials correspond to expected changes in real exchange rates, denoted by re_t. If we further assume that the long-run real exchange rate is expected to converge to its long-run value ($\overline{\mathrm{re}}$), we have

$$E_t(\mathrm{re}_{t+k} - \mathrm{re}_t) = \mathrm{ri}_t - \mathrm{ri}_t^*$$
$$E_t(\mathrm{re}_{t+k}) \to \overline{\mathrm{re}}, \quad \text{as } k \to \infty,$$

where ri_t and ri_t^* are the k-period ahead domestic and foreign real interest rates, respectively. The implied bubble measure (b_t^D) is then given by the following equation:

$$b_t^D = e_t - (p_t - p_t^*) + (\mathrm{ri}_t - \mathrm{ri}_t^*) - \overline{\mathrm{re}}. \tag{6}$$

As van Norden (1996) argues, Equation (6) provides a measure of deviation from the fundamental values even in the presence of a constant risk premium since this would only shift the intercept term.

Next, we turn to the description of three alternative RS models that are designed to capture the observed dynamics in the £/$ exchange rate under the assumption of the existence of a periodically collapsing bubble in the exchange rate.

3.2 *RS speculative behavior models*

The development of rational speculative bubbles has been extensively researched in the literature (Blanchard 1979; Blanchard and Watson 1982; Diba and Grossman 1988; West 1988). The Blanchard and Watson (1982) model assumes that the collapsing state is induced by a positive bubble burst which does not regenerate. Recently, van Norden and Schaller (1993), van Norden (1996) and Schaller and van Norden (1999) propose a model where both positive and negative bubbles are permitted and the probability of collapse depends on the bubble size. As Brooks and Katsaris (2005) state, the van Norden and Schaller model focuses only on the explosive state of the bubble and as such the asset price either grows with explosive expectations ('Survives') or reverses to fundamental values ('Collapses'). To this end, the authors propose an extension to accommodate a third state ('Dormant') where the bubble grows at the required rate of return without explosive expectations. Both the Van Norden–Schaller and the Brooks–Katsaris specifications give rise to RS models, two-regime and three-regime models, respectively. In this paper, we also consider RS models which we modify and extend in various directions. These extensions and modifications are detailed below.

We begin our analysis with the van Norden and Schaller model (Model 1) which serves as a natural benchmark in our analysis. Assuming that the asset market of interest (in our case, the exchange rate market for a specific currency) can be in either the 'Survival' (S) or the 'Collapse' (C) state, the exchange rate return follows a two-regime model. In the 'Survival' state, the bubble continues to exist (and grows), while in the latter case the bubble collapses (partly). The gross return of the exchange rate, R_t, which is a function of the bubble (b_t), can be in two different states (regimes) with different means, slopes and variances.[8] As shown in the following set of equations, the probability of collapse (q_t) depends on the size of the bubble and is bounded between 0 and 1 given that we adopt the same approach as in Probit models where Φ is the cumulative density function of the standard normal distribution.[9]

Model 1:

$$
\begin{aligned}
R_{c,t+1} &= \beta_{c0} + \beta_{c1} b_t + \varepsilon_{c,t+1}, \quad \text{where } \varepsilon_{c,t+1} \sim N(0, \sigma_c^2),\\
R_{s,t+1} &= \beta_{s0} + \beta_{s1} b_t + \varepsilon_{s,t+1}, \quad \text{where } \varepsilon_{s,t+1} \sim N(0, \sigma_s^2),\\
\Pr(\text{State}_{t+1} = C) &= q_t = \Phi(\beta_{q_0} + \beta_{q_1} b_t).
\end{aligned}
\tag{7}
$$

This switching regression nests a general normal mixture model where $\beta_{c1} = \beta_{s1} = \beta_{q_1} = 0$ and the linear regression model where $\beta_{c0} = \beta_{s0}$, $\beta_{c1} = \beta_{s1}$, $\beta_{q_1} = 0$.[10] Model 1 is typically estimated by maximizing the following likelihood function:

$$
\prod_t \left[q_t \varphi\left(\frac{R_{t+1} - \beta_{c0} - \beta_{c1} b_t}{\sigma_c}\right) \sigma_c^{-1} + (1 - q_t) \varphi\left(\frac{R_{t+1} - \beta_{s0} - \beta_{s1} b_t}{\sigma_s}\right) \sigma_s^{-1} \right], \tag{8}
$$

where φ is the standard normal probability density function (pdf), while σ_s and σ_c are the standard deviations of $\varepsilon_{s,t+1}$ and $\varepsilon_{c,t+1}$, respectively. The probability of being in regime i at time $t + 1$ is given by the formula $\Phi(1(i)(\beta_{q_0} + \beta_{q_1} b_t))$, where $1(i) = 1$ in the 'Collapse' state and -1 in the 'Survival' state.

The basic assumption of the speculative bubble models is that the arrival of news may fuel a bubble collapse. This collapse is often viewed as a random occasion that causes investors to liquidate their position at a certain point in time. Although investors observe the built-up of the bubble and expect the bubble to collapse, they cannot precisely estimate the time of the collapse. We assume that they monitor a set of observable variables that help them to find the optimal time to exit from the market. Given that our market of interest is the exchange rate market, we assume that a speculative attack exists in the form of extreme pressure in the foreign exchange market which results in a devaluation (or revaluation) of the currency. This assumption is directly related to the literature on the Early Warning Systems (EWS) which are set up to identify an impending crisis before it occurs. The first EWS was proposed by Kaminsky, Lizondo, and Reinhart (1998) who employ a large database of 15 indicator variables covering the external position, the financial sector, the real sector, the institutional structure and the fiscal policy of the country. This line of research was developed to account for any type of financial crises (Bordo et al. 2001). Lestano and Jacobs (2007) compare currency crisis dating methods adopting various definitions of currency pressure indexes and provide a review for the latest developments on the issue. More recently, Candelon, Dumitrescu, and Hurlin (2012) propose a new statistical framework for evaluating EWSs and find that the introduction of forward-looking variables improves the forecasting properties of the EWS.

Based on the above, we conjecture that any of the early warning indicators can act as a signal of changing market expectations about the evolution of the speculative bubble. Specifically, we employ six variables that have been proposed in the literature as early warning indicators. Three of the variables, namely the annual growth rate of exports, imports and international reserves, capture the external sector position of the country. The long-term and short-term interest rate differential proxy for the financial sector health relative to the foreign country. Attempting to gauge the self-fulfilling origins of a currency crisis, we also add the term spread (long-term government bond minus short-term interest rate) in our indicator variable list. Such a variable captures the feeling of the market with respect to future inflationary pressures and output growth (Estrella and Hardouvelis 1991). Under this setting, we model the probability of collapse as a function of both the bubble size and one of the indicators (z_t). This signal of the collapse of the bubble will induce abrupt changes in the exchange rate and in this respect the expected return in the collapse regime is a function of the candidate indicator as well.

Model 2:

$$\begin{aligned} R_{c,t+1} &= \beta_{c0} + \beta_{c1} b_t + \beta_{c2} z_t + \varepsilon_{c,t+1}, \quad \text{where } \varepsilon_{c,t+1} \sim N(0, \sigma_c^2), \\ R_{s,t+1} &= \beta_{s0} + \beta_{s1} b_t + \varepsilon_{s,t+1}, \quad \text{where } \varepsilon_{s,t+1} \sim N(0, \sigma_s^2), \\ \Pr(\text{State}_{t+1} = C) &= q_t = \Phi(\beta_{q_0} + \beta_{q_1} b_t + \beta_{q_2} z_t). \end{aligned} \tag{9}$$

The next extension we propose (Model 3) draws from the Brooks and Katsaris model (2005). Following their work, we propose a three-state RS model by adding a third state ('Dormant' state) of the bubble to model (9). More in detail, we assume that when the bubble size is small, probably market participants believe that the bubble will continue to grow at a steady rate and as such the bubble size does not enter the mean equation of the dormant state.[11] Furthermore, we follow Brooks and Katsaris by modeling the probability of being in the dormant state (η_t) as a function of the bubble and the absolute value of the average six-month actual returns minus the absolute value of the average six-month returns of the estimated fundamental values (denoted as spread, sp_t) implied by the four models presented in the previous section. The intuition behind this specification is quite clear. When investors observe large spreads, i.e. larger average returns than average fundamental returns, they tend to believe that the bubble has entered the explosive state and the probability of being in the dormant state falls.[12] The third model we consider is given by the following equations:

Model 3:

$$\begin{aligned} R_{d,t+1} &= \beta_{d0} + \varepsilon_{d,t+1}, \quad \text{where } \varepsilon_{d,t+1} \sim N(0, \sigma_d^2), \\ R_{c,t+1} &= \beta_{c0} + \beta_{c1} b_t + \beta_{c2} z_t + \varepsilon_{c,t+1}, \quad \text{where } \varepsilon_{c,t+1} \sim N(0, \sigma_c^2), \\ R_{s,t+1} &= \beta_{s0} + \beta_{s1} b_t + \varepsilon_{s,t+1}, \quad \text{where } \varepsilon_{s,t+1} \sim N(0, \sigma_s^2), \\ \Pr(\text{State}_{t+1} = D) &= \eta_t = \Phi(\beta_{\eta_0} + \beta_{\eta_1} b_t + \beta_{\eta_2} \text{sp}_t), \\ \Pr(\text{State}_{t+1} = C) &= q_t = \Phi(\beta_{q_0} + \beta_{q_1} b_t + \beta_{q_2} z_t). \end{aligned} \tag{10}$$

Model 3 is estimated by maximizing the following log-likelihood function:[13]

$$\ln \prod_t \left[\eta_t \varphi\left(\frac{R_{t+1} - \beta_{d0}}{\sigma_d}\right) \sigma_d^{-1} + (1-\eta_t) q_t \varphi\left(\frac{R_{t+1} - \beta_{c0} - \beta_{c1} b_t - \beta_{c2} z_t}{\sigma_c}\right) \sigma_c^{-1} \right. \\ \left. + (1-\eta_t)(1-q_t) \varphi\left(\frac{R_{t+1} - \beta_{s0} - \beta_{s1} b_t}{\sigma_s}\right) \sigma_s^{-1} \right]. \tag{11}$$

Obviously, the simple two-regime model (Model 1) and the extended two-regime model (Model 2) are both nested in the three-regime model (Model 3).

For each one of the three aforementioned models, we can calculate the related ex ante probability of R_{t+1} being in each regime by (i) conditioning on b_t for Model 1, (ii) conditioning on b_t and z_t for Model 2 and (iii) conditioning on b_t, z_t and sp_t for Model 3. For example, given a set of estimates for Model 3, the ex ante probabilities can be easily calculated by η_t, $(1-\eta_t)q_t$ and $(1-\eta_t)(1-q_t)$, as given above, for the dormant, collapse and surviving state, respectively. Furthermore, we can calculate the ex post probabilities of each state in a similar manner but this time we also condition on the realized return R_{t+1}. For example, the ex post probability of collapse for Model 3 is given by the following equation:

$$P_t^{x,c} = \frac{(1-\eta_t)q_t}{\sigma_c} \varphi\left(\frac{R_{t+1} - \beta_{c0} - \beta_{c1} b_t - \beta_{c2} z_t}{\sigma_c}\right) \left[\frac{\eta_t}{\sigma_d} \varphi\left(\frac{R_{t+1} - \beta_{d0}}{\sigma_d}\right) \right. \\ + \frac{(1-\eta_t)q_t}{\sigma_c} \varphi\left(\frac{R_{t+1} - \beta_{c0} - \beta_{c1} b_t - \beta_{c2} z_t}{\sigma_c}\right) \\ \left. + \frac{(1-\eta_t)(1-q_t)}{\sigma_s} \varphi\left(\frac{R_{t+1} - \beta_{s0} - \beta_{s1} b_t}{\sigma_s}\right) \right]^{-1}. \tag{12}$$

Similar expressions can be obtained for the ex post probabilities of being in the survival and dormant states.[14]

We can also calculate the probability of an unusually low or high return in the next period, denoted as the probability of a crash and a boom, respectively. This is a crude measure of the ability of our models to determine optimal policy (i.e. optimal entry and exit times) in the context of a trading strategy. Both probabilities are linked to the probability of observing large movements in the exchange rate (i.e. large negative or positive returns) that would generate profits (losses) to the investors given that they hold the correct (incorrect) position in the market. More in detail, the probability of a crash (i.e. a return more than two standard deviations, σ_{R_t}, below the mean return) can be calculated as follows:

$$\Pr(R_{t+1} < x) = \eta_t \Phi\left(\frac{x - \beta_{d0}}{\sigma_d}\right) + (1 - \eta_t) q_t \Phi\left(\frac{x - \beta_{c0} - \beta_{c1} b_t - \beta_{c2} z_t}{\sigma_c}\right) + (1 - \eta_t)(1 - q_t) \Phi\left(\frac{x - \beta_{s0} - \beta_{s1} b_t}{\sigma_s}\right), \tag{13}$$

where $x = \bar{R}_t - 2\sigma_{R_t}$. Similarly, the probability of a boom (i.e. a return more than two standard deviations above the mean return) is given by

$$\Pr(R_{t+1} > x) = \eta_t \Phi\left(\frac{-x + \beta_{d0}}{\sigma_d}\right) + (1 - \eta_t) q_t \Phi\left(\frac{-x + \beta_{c0} + \beta_{c1} b_t + \beta_{c2} z_t}{\sigma_c}\right) + (1 - \eta_t)(1 - q_t) \Phi\left(\frac{-x + \beta_{s0} + \beta_{s1} b_t}{\sigma_s}\right), \tag{14}$$

where $x = \bar{R}_t + 2\sigma_{R_t}$. The above probabilities can be calculated for the two-regime models (Models 1 and 2) in a similar manner.

4. Estimation results

4.1 *Data and bubble estimates*

The empirical work presented below focuses on the UK pound to US dollar (£/\$) exchange rate in the post-Bretton Woods floating exchange rate period. We use monthly data of end-of-period market exchange rates ranging from January 1973 to January 2011. Price levels are proxied by the Consumer Price Index and inflation rates are calculated from the y-o-y growth rates of prices. We employ the industrial production index and the M3 monetary aggregate for the income and money supply levels. Short- and long-term interest rates are the overnight interbank rates and the long-term government yields, respectively. International reserves are expressed in US dollars, while exports and imports refer to the volume of the respective variables for the UK. The source of our data set is mainly the IMF-IFS database.[15] The first step in our analysis is the calculation of the bubble measures ($b_t^i, i = A, B, C, D$). The first three bubble measures are calculated from the residuals of cointegration equations (1), (3) and (5), whereas the fourth is calculated from Equation (6). Table 2 reports the estimates of the parameters of models (1), (3) and (5).

Figure 2, which plots the four calculated bubble measures together with the nominal exchange rate, shows that the bubble measures exhibit a similar behavior revealing periods of positive and negative deviations of the exchange rate from the fundamental values.

4.2 *Models' estimates – in-sample statistics*

We first estimate Models 1–3 for each one of the four bubble measures and for each one of the six explanatory variables considered in this study.[16] Table 3 reports the estimates for the three-state RS model for the first bubble measure.[17] The majority of coefficients appear to be statistically significant. At this point, we should note that even when the coefficient of an explanatory variable is not statistically different from zero, this does not necessarily mean that the variable has no predictive power for the exchange rate. It is often the case that a variable that is insignificant

Table 2. Estimates of the exchange rate determination models.

	Equation (1)		Equation (3)		Equation (5)
δ_0^A	−**0.485** (0.007)	δ_0^B	−**0.333** (0.033)	δ_0^C	−**0.413** (0.037)
δ_1^A	**0.595** (0.036)	δ_1^B	**0.107** (0.015)	δ_1^C	**0.060** (0.018)
		δ_2^B	−0.063 (0.154)	δ_2^C	−0.085 (0.151)
		δ_3^B	**0.012** (0.002)	δ_3^C	**0.013** (0.002)
				δ_4^C	−**1.045** (0.223)

Notes: Standard errors are reported in parentheses. Entries in bold indicate a coefficient that is statistically different from zero (at a 10% confidence level).

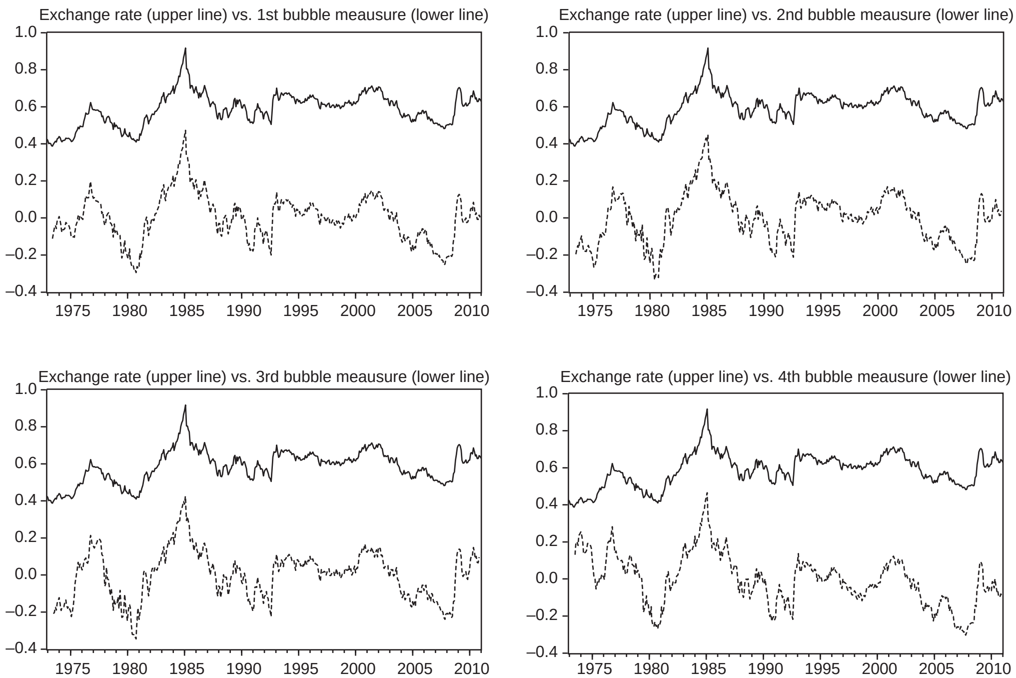

Figure 2. The bubble measures and the £/$ exchange rate.

in-sample has good predictive power out-of-sample and vice versa. Furthermore, we also observe some differences in the point estimates depending on the explanatory variable included in the model. For example, the mean return for the dormant state appears to be negative, as indicated by the estimated value of β_{d0} which is lower than unity, in two cases (that is, when we use either the long- or short-term interest rate as a predictor) and positive in all other four cases. Another example is the estimated β_{η_2} coefficient which captures the effect of the spread, sp_t, on the probability of being in the dormant state η_t. The estimates are usually negative with two exceptions that correspond to models with either the international reserves or the yield spread as a predictor.

Table 3. Estimates of the three-state RS model (first bubble measure).

	Exports	Imports	International reserves	Long-term interest rate	Short-term interest rate	Yield spread
β_{d0}	**1.000**	**1.001**	**1.032**	**0.999**	**0.999**	**1.007**
	(0.002)	(0.002)	(0.009)	(0.002)	(0.002)	(0.004)
β_{s0}	**1.018**	**1.013**	**1.015**	**1.031**	**1.019**	**0.997**
	(0.008)	(0.006)	(0.008)	(0.014)	(0.006)	(0.002)
β_{s1}	**−0.258**	**−0.266**	**−0.306**	**−0.180**	**−0.250**	0.024
	(0.071)	(0.076)	(0.098)	(0.084)	(0.062)	(0.015)
β_{c0}	**0.933**	**0.934**	**0.997**	**0.940**	**0.975**	**1.025**
	(0.004)	(0.014)	(0.003)	(0.013)	(0.008)	(0.012)
β_{c1}	**−0.197**	**−0.276**	**−0.001**	**−0.103**	**0.144**	**−0.330**
	(0.026)	(0.073)	(0.016)	(0.057)	(0.040)	(0.097)
β_{c2}	−0.047	0.032	**−0.004**	0.369	**−0.530**	−0.175
	(0.032)	(0.050)	(0.002)	(0.374)	(0.161)	(0.568)
σ_d^2	**0.022**	**0.022**	**0.018**	**0.023**	**0.021**	**0.033**
	(0.001)	(0.002)	(0.008)	(0.001)	(0.002)	(0.003)
σ_s^2	**0.034**	**0.034**	**0.034**	**0.033**	**0.033**	**0.022**
	(0.005)	(0.004)	(0.005)	(0.007)	(0.004)	(0.002)
σ_c^2	**0.005**	**0.013**	**0.022**	0.011	**0.018**	**0.033**
	(0.003)	(0.004)	(0.002)	(0.007)	(0.005)	(0.008)
$\beta_{\eta 0}$	**1.302**	**0.995**	**−1.926**	**1.650**	**0.980**	−2.352
	(0.321)	(0.370)	(0.782)	(0.377)	(0.306)	(1.572)
$\beta_{\eta 1}$	1.107	**2.362**	1.665	0.676	**−2.830**	−3.576
	(0.956)	(1.094)	(1.775)	(1.385)	(1.339)	(3.120)
$\beta_{\eta 2}$	**−25.576**	**−23.909**	**24.631**	**−29.050**	**−29.762**	**85.072**
	(9.817)	(10.950)	(11.071)	(9.743)	(9.857)	(51.468)
β_{q0}	**−1.300**	**−1.832**	**1.028**	**−1.152**	−0.556	**−1.168**
	(0.426)	(0.582)	(0.247)	(0.535)	(0.342)	(0.251)
β_{q1}	**−5.528**	**−12.388**	−0.165	4.298	**3.471**	0.709
	(3.107)	(3.650)	(1.003)	(4.572)	(1.378)	(1.124)
β_{q2}	−4.310	**5.360**	1.124	19.451	2.238	−6.738
	(3.668)	(3.136)	(0.867)	(19.659)	(5.386)	(11.606)

Notes: Standard errors are reported in parentheses. Entries in bold indicate a coefficient that is statistically different from zero (at a 10% confidence level). The estimated parameters correspond to the three-state RS model (Equation (10)).

We then apply likelihood ratio (LR) tests to choose among Models 1–3 for each combination of bubble measure and explanatory variable.[18] We have a total of 24 (= 4 bubble measures ×6 explanatory variables) combinations and the results are reported in Table 4. The general picture that emerges from the LR tests indicates that Model 3 is preferable to the other two models in almost all cases. We can identify only six cases (using a 10% confidence level) where the LR test selects either Model 1 (three cases) or Model 2 (three cases) instead of the general Model 3. When we focus on the selection between Models 1 and 2, the results of the LR test generate a mixed picture (i.e. Model 1 is selected over Model 2 in half of the cases) that depends on both the bubble measure and the explanatory variable included in Model 2.

In an attempt to evaluate the ability of an RS model to fit our data, we implement the Regime Classification Measure (RCM) introduced by Ang and Bekaert (2002). According to Ang and Bekaert (2002), a good RS model should be able to classify regimes sharply, i.e. the smoothed ex

Table 4. LR test statistic for model selection.

	Model 1 vs. Model 2		Model 1 vs. Model 3		Model 2 vs. Model 3	
	LR	*p*-Value	LR	*p*-Value	LR	*p*-Value
b_t^A						
Exports	0.837	0.658	18.654	0.009	17.818	0.003
Imports	4.729	0.094	18.644	0.009	13.916	0.016
International reserves	5.475	0.065	12.316	0.091	6.841	0.233
Long-term interest rate	0.112	0.946	11.735	0.110	11.623	0.040
Short-term interest rate	14.411	0.001	23.338	0.001	8.927	0.112
Yield spread	0.371	0.831	12.689	0.080	12.318	0.031
b_t^B						
Exports	1.365	0.505	19.901	0.006	18.537	0.002
Imports	3.501	0.174	19.882	0.006	16.380	0.006
International reserves	5.062	0.080	20.447	0.005	15.385	0.009
Long-term interest rate	0.952	0.621	13.700	0.057	12.748	0.026
Short-term interest rate	14.631	0.001	27.158	0.000	12.527	0.028
Yield spread	0.651	0.722	13.031	0.071	12.380	0.030
b_t^C						
Exports	1.358	0.507	21.733	0.003	20.374	0.001
Imports	4.667	0.097	19.732	0.006	15.065	0.010
International reserves	4.897	0.086	32.354	0.000	27.457	0.000
Long-term interest rate	0.890	0.641	10.215	0.177	9.325	0.097
Short-term interest rate	14.414	0.001	31.748	0.000	17.334	0.004
Yield spread	0.927	0.629	9.537	0.216	8.610	0.126
b_t^D						
Exports	2.348	0.309	15.073	0.035	12.726	0.026
Imports	12.450	0.002	21.855	0.003	9.405	0.094
International reserves	5.188	0.075	17.395	0.015	12.207	0.032
Long-term interest rate	13.004	0.002	17.330	0.015	4.326	0.504
Short-term interest rate	14.805	0.001	29.005	0.000	14.200	0.014
Yield spread	0.972	0.615	33.246	0.000	32.274	0.000

Notes: LR $= -2(\log\ l_r - \log\ l_g)$, where $\log\ l_r$ and $\log\ l_g$ stand for the maximized values of the log-likelihood function of the restricted and general (unrestricted) models, respectively.

post probabilities $P_t^{x,i}$ for each regime i should be close to either zero or one. For a three-regime model, RCM is given by

$$\text{RCM} = \frac{100K^2}{T} \sum_{t=1}^{T} \prod_{i=d,c,s} P_t^{x,i}, \quad K = 3.$$

Lower RCM values denote better regime classification, since a perfect RS model will generate an RCM close to zero, while a model that cannot distinguish between regimes will produce an RCM close to 100. Our results, reported in Table 5, provide strong support for Model 3 since the majority of RCM values are small and close to zero (they usually are below 5). On the other hand, Models 1 and 2 are associated with RCMs that range from 11.96 to 38.79.

Figure 3 plots the ex ante probability (often called the 'filtered probability') of R_{t+1} being in each regime, together with the spot exchange rate. It is obvious that the variable spends most of

Table 5. The RCM.

	b_t^A	b_t^B	b_t^C	b_t^D
Model 1	37.488	26.904	29.361	24.748
Model 2				
Exports	33.189	24.444	28.156	23.532
Imports	37.002	28.550	29.531	23.424
International reserves	36.956	27.647	29.441	24.693
Long-term interest rate	38.793	26.713	31.437	26.146
Short-term interest rate	12.811	14.206	13.983	11.959
Yield spread	35.086	25.793	28.382	31.949
Model 3				
Exports	0.244	0.451	0.023	0.569
Imports	0.909	0.699	0.466	3.426
International reserves	3.915	9.167	0.022	4.022
Long-term interest rate	0.197	0.484	0.309	0.776
Short-term interest rate	6.044	6.773	0.007	1.237
Yield spread	3.465	0.359	0.155	0.165

its time in the dormant regime. However, it switches periodically to the survival regime, which is usually followed by a transition to the collapse state.

As noted in the previous section, a measure that provides useful insights about the ability of our models to predict large movements of the exchange rate is the probability of a crash or a boom given by Equations (13) and (14), respectively. If these probabilities have some predictive power for the dynamics in the exchange rate market, we might be able to use them to develop proper strategies that can generate positive returns to the investor. Figure 4 illustrates the calculated probability of a crash for Model 3 for each one of the explanatory variables, together with the first bubble measure. The figure highlights both similarities and differences among the six alternative explanatory variables. For example, in all cases we observe a significant increase in the probability of a crash around 1985, just before the collapse of the bubble. A similar picture emerges around 2009 but in this case the bubble collapse is not captured by the models that include either international reserves or the yield spread as an explanatory variable. However, we should note that the probabilities of a crash plotted in Figure 4 have been calculated based on the full-sample estimates of Model 3. If we want to test whether the estimated probability of a crash (or a boom) has any predictive power for large movements in the market, we should estimate it in a recursive way using only the available to the investor information in each period. We come back to this point in Section 6 where we develop simple strategies based on the estimated probability of a crash or boom that lead to positive returns for the investor.

5. Out-of-sample forecasts

So far, the results indicate that the three-state RS models provide a satisfactory in-sample description of the data. We now move to the main focus of our study, which is the ability of the RS models to generate reliable out-of-sample forecasts for the exchange rate, starting with a forecast horizon of one period (month). To do so, we set an out-of-sample forecast exercise and afterwards we perform both a statistical and economic evaluation of the forecasts.

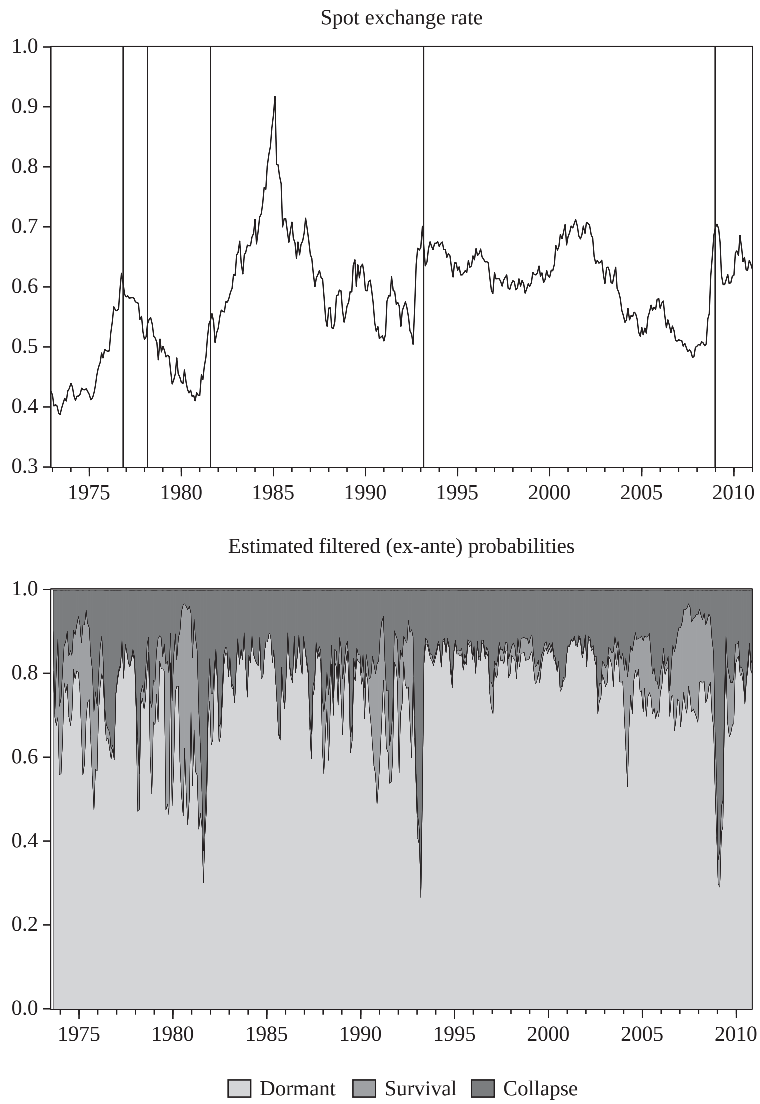

Figure 3. Estimated filtered probabilities and spot exchange rate.

5.1 *Forecast efficiency and accuracy*

Before presenting our findings, it is useful to briefly discuss specific issues that are relevant to forecasting and the evaluation of forecasts in economics (West 2006 and Clark and McCracken 2013 provide detailed reviews on recent developments in forecasting). In all forecasting exercises, the researcher must (i) decide on the optimal way to generate the predictions and (ii) choose the proper statistical procedure to evaluate the predictions. Both decisions are crucial and entail various choices that eventually affect both the forecasting accuracy and the evaluation of the estimated models. For example, given a total sample of T observations, the researcher must determine the way to split the sample into the estimation part (say R observations) and the out-of-sample part

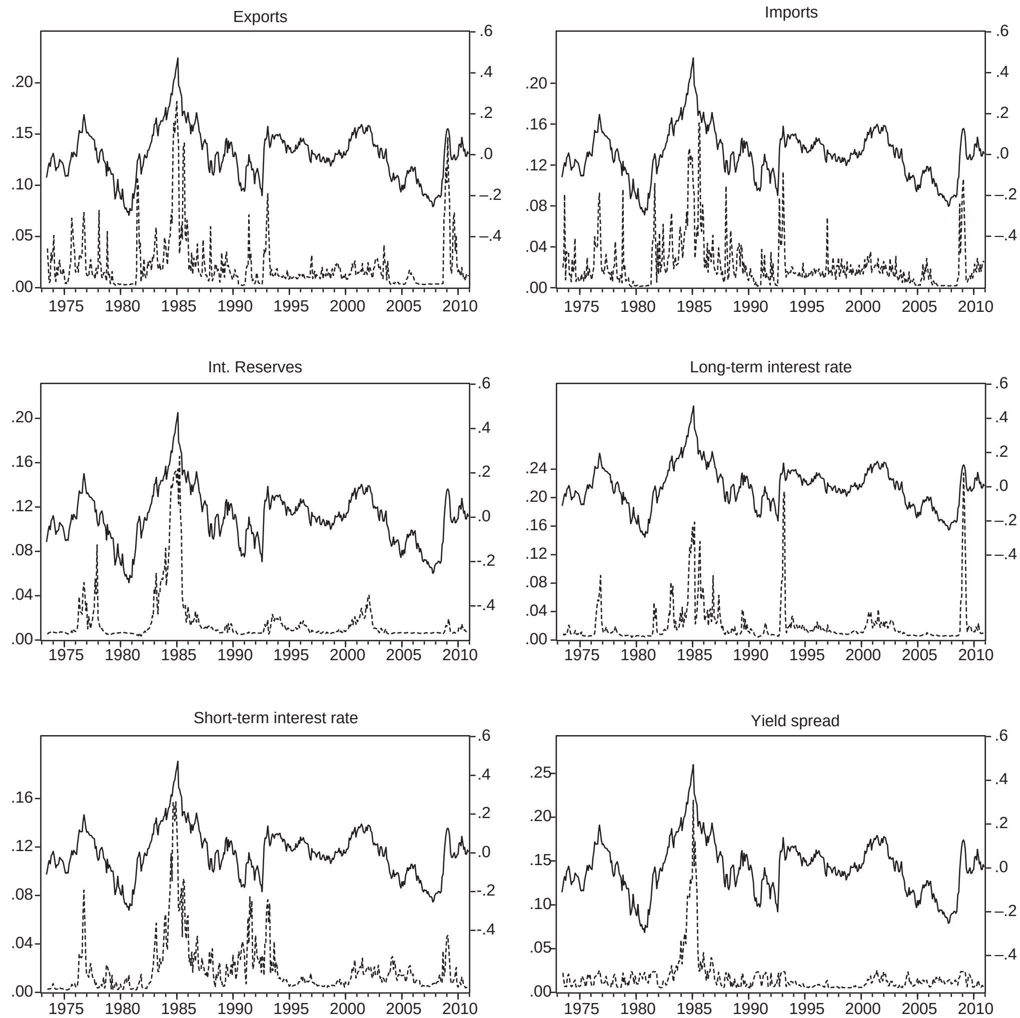

Figure 4. Probabilities of a crash (Model 3) and the first bubble measure.

(say $P := T - R$ observations). Obviously, there is a trade-off, since a large R improves the quality of the estimated parameters of the model but, at the same time, leaves few observations for the out-of-sample forecast exercise making the evaluation of the predictive ability of the model difficult. In our analysis, we keep about one-third of the available sample for out-of-sample forecasting. This choice gives us a sufficient number of forecasts to evaluate the estimated models, while keeping enough observations to obtain reliable estimates for the parameters of our RS models. Moreover, in the context of a forecast exercise the researcher can choose between three alternative schemes to generate the predictions, namely the recursive, rolling and fixed scheme. Under the recursive scheme, the initial estimation sample uses the first R observations; the second estimation sample goes up to $R + 1$ and so on. Thus, the estimation sample increases by one observation each time we re-estimate our model to generate the next forecast. Under the rolling scheme, the size of the estimation sample remains fixed (equal to R), since for every observation we add at the end of the estimation sample we remove one from the beginning of our sample. Finally, under the fixed scheme, the model is estimated only once using the first R observations. In our study, we choose to use the recursive scheme that is more efficient than the other two, since it uses all available information for the estimation of the model. On the other hand, numerous studies in the literature show that the utilization of the recursive scheme greatly complicates the asymptotics for various tests of predictive ability, especially when multi-step forecasts are considered.

5.1.1 Forecast efficiency

We organize the out-of-sample forecast exercise as follows. We recursively estimate all three models considered in this study adding one observation at a time and generate a one-period ahead forecast for each model. The forecast exercise is organized in real-time terms. In other words, we obtain a forecast for period $t+1$ using all available information in period t (i.e. all bubble measures and models are re-estimated recursively to include all the available observations). The first estimation sample ends in December 1997, leaving the last 13 years of our sample for predictions. Before analyzing the relative forecasting accuracy of our competing models, we first evaluate the forecasts of each model from a different perspective. To be more specific, we conduct a forecast efficiency test based on the following standard efficiency regression put forward by Mincer and Zarnowitz (1969) and employed by various researchers:

$$R_{t+h} = \gamma_0 + \gamma_1 \hat{R}_{t+h} + w_t,$$

where $\hat{R}_{t+h}$ stands for the h-step ahead forecast of the actual gross return R_{t+h} taken from one of our estimated models. We can claim that the forecasts are efficient if we cannot reject the null hypothesis $H_0 : \gamma_0 = 0 \cap \gamma_1 = 1$. This forecast efficiency regression is estimated by ordinary least squares but we correct the standard errors of the estimated parameters for heteroskedasticity and serial correlation by means of the HAC estimator introduced by Newey and West (1987). For brevity, Table 6 reports only the asymptotic p-values of the standard Wald test for the aforementioned null hypothesis based on the $X^2(2)$ distribution. In general, the third bubble measure seems to perform better than the other three bubble measures, while the fourth bubble measure is clearly the measure that generates the less efficient forecasts. If we focus on the one-month horizon, all our RS models generate efficient forecasts. There are only two exceptions corresponding to Model 2 with exports (for the third and fourth bubble measure). The results are mixed for the three- and six-month horizon. Of the 52 tests of H_0 for each horizon, we observe (at the 5% significance level) 32 and 34 rejections for the three- and six-month horizon, respectively. Finally, none of our models generates efficient forecasts for a 12-month forecast horizon. The only two exceptions correspond to the third bubble measure and specifically to our two- and three-state models with the short-term interest rate as a predictor.

5.1.2 Forecast accuracy

The forecast accuracy of our three RS models is evaluated relative to the following RW model[19] that often serves as a benchmark in empirical studies of exchange rates:

$$R_{t+1} = c + \varepsilon_{t+1}, \quad \text{where } \varepsilon_{t+1} \sim N(0, \sigma^2).$$

The evaluation of the models is based on the MSFE criterion. Table 7 (Panel A) reports the ratio of the MSFE of each one of the RS models over the MSFE of the RW model. A ratio below unity (highlighted in bold) indicates that the RS model outperforms the RW model, while ratios over unity suggest that the regime model fails to generate more accurate forecasts than the RW

Table 6. Forecast efficiency measure (p-values).

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. h = 1 month				
Model 1	**0.528**	**0.461**	**0.642**	**0.412**
Model 2				
Exports	**0.052**	**0.072**	0.050	0.001
Imports	**0.610**	**0.619**	**0.782**	**0.692**
International reserves	**0.363**	**0.493**	**0.580**	**0.273**
Long-term interest rate	**0.404**	**0.568**	**0.643**	**0.490**
Short-term interest rate	**0.176**	**0.069**	**0.391**	**0.291**
Yield spread	**0.580**	**0.461**	**0.630**	**0.209**
Model 3				
Exports	**0.398**	**0.766**	**0.498**	**0.074**
Imports	**0.515**	**0.319**	**0.849**	**0.582**
International reserves	**0.364**	**0.521**	**0.805**	**0.358**
Long-term interest rate	**0.585**	**0.127**	**0.102**	**0.161**
Short-term interest rate	**0.787**	**0.912**	**0.887**	**0.122**
Yield spread	**0.129**	**0.168**	**0.161**	**0.480**
Panel B. h = 3 months				
Model 1	0.023	**0.207**	**0.232**	0.000
Model 2				
Exports	0.000	0.000	0.000	0.000
Imports	**0.149**	**0.454**	**0.230**	0.002
International reserves	0.008	**0.374**	**0.416**	0.000
Long-term interest rate	0.001	0.001	0.001	0.000
Short-term interest rate	0.012	0.001	0.001	0.032
Yield spread	**0.440**	**0.228**	0.004	0.000
Model 3				
Exports	0.000	0.000	0.000	0.000
Imports	**0.129**	**0.346**	**0.363**	0.005
International reserves	0.015	**0.147**	**0.252**	0.000
Long-term interest rate	0.003	0.006	0.002	0.001
Short-term interest rate	**0.308**	0.000	**0.124**	**0.583**
Yield spread	**0.312**	**0.274**	**0.671**	0.000
Panel C. h = 6 months				
Model 1	0.015	0.003	**0.243**	0.001
Model 2				
Exports	0.000	0.000	0.001	0.000
Imports	**0.164**	**0.216**	**0.267**	0.000
International reserves	0.013	0.018	**0.073**	0.000
Long-term interest rate	0.000	0.000	0.000	0.000
Short-term interest rate	**0.073**	**0.427**	**0.182**	0.000
Yield spread	**0.071**	0.045	**0.424**	0.006
Model 3				
Exports	0.000	0.000	0.000	0.000
Imports	**0.262**	**0.487**	**0.594**	0.001
International reserves	**0.060**	0.045	**0.347**	0.002
Long-term interest rate	0.001	0.000	0.001	0.000
Short-term interest rate	0.004	**0.075**	**0.778**	0.000
Yield spread	0.016	0.000	**0.552**	0.000

(*Continued*)

Table 6. Continued.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel D. h=12 months				
Model 1	0.000	0.001	0.010	0.000
Model 2				
Exports	0.000	0.000	0.000	0.000
Imports	0.002	0.004	0.019	0.000
International reserves	0.000	0.000	0.002	0.000
Long-term interest rate	0.002	0.001	0.002	0.000
Short-term interest rate	0.042	0.047	**0.271**	0.000
Yield spread	0.003	0.002	0.026	0.000
Model 3				
Exports	0.000	0.001	0.000	0.000
Imports	0.001	0.011	0.000	0.000
International reserves	0.000	0.010	0.012	0.000
Long-term interest rate	0.000	0.001	0.004	0.000
Short-term interest rate	0.017	0.015	**0.226**	0.000
Yield spread	0.001	0.002	0.032	0.000

Notes: The entries correspond to the *p*-values of the Wald test for forecast efficiency. Entries greater than 0.05 are highlighted in bold indicating forecast efficiency of the corresponding model.

model. Starting with the most parsimonious RS model considered in this study, that is Model 1, the MSFE criterion suggests that Model 1 outperforms the RW model for three out of four bubble measures but the ratios are very close to unity. Even when we consider Model 2 that enriches the specification of Model 1 by including one of the six explanatory variables under scrutiny, we often fail to generate lower MSFEs compared with the RW model. Specifically, when Model 2 includes either exports, international reserves or long-term interest rates, it generates higher MSFEs relative to the RW model for all bubble measures. The remaining explanatory variables seem to improve the forecasting performance of Model 2 leading to lower MSFEs compared with the benchmark in most cases. Specifically, Model 2 that includes either imports or short-term interest rates outperforms the RW model, while the same holds for the yield spread for all the bubble measures with the exception of the fourth one. For example, in the case of the short-term interest rate, the ratios range from 0.970 (b_t^A) to 0.974 (b_t^C).

The predictive performance of Model 3 crucially depends on the explanatory variable considered and to a lesser extend the bubble measure employed. To be more specific, when Model 3 includes either imports or short-term interest rates, it always produces more accurate forecasts than the RW model (in terms of the MSFE criterion). For example, when we consider Model 3 with imports (short-term interest rate) the ratios range from 0.962 (b_t^D) to 0.984 (b_t^C) (0.967 (b_t^D) to 0.989 (b_t^C)). Similarly, Model 3 with exports outperforms the RW model for three out of four bubble measures. On the other hand, when Model 3 includes international reserves or the yield spread, it is outperformed by the RW model in almost all cases (three out of four). Finally, the results are mixed for the long-term interest rate. As far as the bubble measures are concerned, we have to note that in this case the most accurate bubble measure is the fourth one that attains the lowest MSFEs for four out of six specifications, followed by the first and third one. The second bubble measure ranks last as it improves forecasts in three out of six specifications. In summary, the findings, so far, suggest that (i) for all three alternative RS models we can identify parameterizations that improve the forecasting performance compared with the RW model, (ii) the behavior of Models 2 and 3 crucially depends on the explanatory variable we choose to include in the specification, (iii) Model 3 has the best forecasting performance among the three RS models given that we employ specific explanatory variables, (iv) out of the six explanatory variables considered in our analysis, imports and short-term interest rates appear to improve the forecasting accuracy of the RS models, while the results are mixed for the other variables and (v) the fourth bubble measure proves more accurate.

We now extend our analysis to multi-step forecasts. In general, there are two ways to obtain multiperiod forecasts for the variable of interest: the 'direct' and the 'iterated' one. The former approach is based on the estimation of a horizon-specific model. Specifically, the researcher estimates a multiperiod model regressing a multiperiod-ahead value of the variable of interest (R_{t+h} in our case) on current and past values of the predictors (b_t and z_t in our case). In this way, the h-step ahead forecast is directly obtain from the estimated model. On the other hand, the 'iterated' approach simply iterates forward (for the desired number of periods) the one-step ahead forecast. In other words, in order to obtain the two-step ahead forecast, the researcher uses the one-step ahead forecast. Then, the three-step ahead forecast is based on the two-step ahead forecast and so on. In theory, if the estimated models are correctly specified, iterated forecasts are more efficient, but direct forecasts are more robust to model misspecification. In practice, evidence in the literature suggests that we cannot a priori know which method is optimal.[20] We choose to generate direct multi-period forecasts which are more robust to misspecification. To do so, we re-estimate Models 1–3, where R_{t+1} is replaced by R_{t+h}, where h is the forecast horizon which we set equal to 3, 6 and 12 periods (months). Table 7 (Panels B–D) reports the MSFE ratios of the RS models relative to the RW model for horizons of 3, 6 and 12 months ahead. The main findings can be summarized as follows:

- $h = 3$: Model 1 outperforms the RW model for two out of four bubble measures (the MSFE ratio is around 0.97 in these two cases). The forecasting accuracy of Model 2 is, in general, poor. The only exception arises when Model 2 includes the short-term interest rate. In this particular case, Model 2 is superior to the RW model for all bubble measures with a ratio that is below 0.9. Finally, the predictive performance of Model 3 is also poor beating the RW model in only one-third of the cases. Even in these cases, the MSFE ratio is usually close to unity.
- $h = 6$: Model 1 produces lower MSFEs than the RW model for all bubble measures yielding an MSFE ratio that is usually around 0.97. Model 2 is, in most cases, beaten by the RW model unless we choose to use the yield spread as an explanatory variable. Similarly, Model 3 generally fails to improve upon the RW model. However, when we use either imports or international reserves, Model 3 does better than the RW model for three out of four bubble measures.
- $h = 12$: The forecasting accuracy of the RS models (relative to the RW model) seems to improve. Model 1 outperforms the RW model for three bubble measures generating an MSFE ratio that ranges from 0.905 to 0.933 in these cases. Models 2 and 3 provide more accurate forecasts than the RW model when the last three explanatory variables are used (these are the long- and short-term interest rates and the yield spread). Once again, models with the short-term interest rate appear optimal.

In summary, our findings show that the predictive performance of our RS models relative to the RW model deteriorates when we consider multi-step forecasts. Similar to the one-month forecast horizon, the short-term interest rate emerges as the optimal variable (among the six considered in this study) for inclusion in Models 2 and 3. However, we should also note that the yield spread

Table 7. Ratios of MSFEs (out-of-sample period: 1998–2010).

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. h = 1 month				
Model 1	**0.997**	**0.992**	**0.991**	1.002
Model 2				
Exports	1.033	1.028	1.033	1.086
Imports	**0.997**	**0.991**	**0.994**	**0.981**
International reserves	1.005	1.003	1.000	1.012
Long-term interest rate	1.005	1.002	1.000	1.004
Short-term interest rate	**0.970**	**0.971**	**0.974**	**0.973**
Yield spread	**0.997**	**0.993**	**0.996**	1.015
Model 3				
Exports	**0.995**	**0.987**	**0.999**	1.027
Imports	**0.979**	**0.978**	**0.984**	**0.962**
International reserves	1.004	1.001	**0.976**	1.007
Long-term interest rate	**0.987**	1.022	1.025	**0.977**
Short-term interest rate	**0.988**	**0.989**	**0.981**	**0.967**
Yield spread	1.021	1.017	1.019	**0.997**
Panel B. h = 3 months				
Model 1	1.016	**0.974**	**0.970**	1.114
Model 2				
Exports	1.098	1.091	1.138	1.306
Imports	1.010	**0.989**	1.001	1.064
International reserves	1.039	**0.978**	**0.991**	1.176
Long-term interest rate	1.072	1.064	1.074	1.096
Short-term interest rate	**0.895**	**0.879**	**0.891**	**0.889**
Yield spread	**0.973**	**0.992**	1.025	1.291
Model 3				
Exports	1.283	1.162	1.163	1.364
Imports	1.006	**0.987**	**0.987**	1.056
International reserves	1.028	**0.989**	**0.995**	1.170
Long-term interest rate	1.062	1.053	1.069	1.079
Short-term interest rate	**0.956**	1.107	1.012	**0.957**
Yield spread	**0.970**	**0.990**	**0.963**	1.163
Panel C. h = 6 months				
Model 1	**0.971**	**0.967**	**0.894**	**0.999**
Model 2				
Exports	1.081	1.131	1.042	1.205
Imports	1.000	**0.991**	**0.994**	1.082
International reserves	1.022	1.022	1.000	1.089
Long-term interest rate	1.156	1.140	1.124	1.148
Short-term interest rate	1.014	**0.990**	1.001	1.081
Yield spread	**0.946**	**0.995**	**0.925**	1.030
Model 3				
Exports	1.101	1.071	1.116	1.339
Imports	**0.973**	**0.949**	**0.960**	1.065
International reserves	**0.973**	**0.961**	**0.941**	1.058
Long-term interest rate	1.068	1.082	1.075	1.119
Short-term interest rate	1.020	**0.923**	**0.870**	1.133
Yield spread	**0.980**	1.043	**0.901**	1.322

(*Continued*)

Table 7. Continued.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel D. h = 12 *months*				
Model 1	**0.909**	**0.933**	**0.905**	1.006
Model 2				
Exports	1.193	1.213	1.208	1.360
Imports	**0.929**	**0.937**	**0.972**	1.060
International reserves	1.012	1.033	1.005	1.116
Long-term interest rate	**0.958**	**0.989**	**0.977**	1.013
Short-term interest rate	**0.877**	**0.896**	**0.881**	**0.965**
Yield spread	**0.897**	**0.921**	**0.901**	**0.997**
Model 3				
Exports	1.174	**0.942**	1.024	1.360
Imports	**0.962**	**0.924**	1.073	1.105
International reserves	**0.974**	**0.922**	**0.967**	1.067
Long-term interest rate	**0.977**	**0.974**	**0.952**	1.047
Short-term interest rate	**0.918**	**0.899**	**0.886**	**0.988**
Yield spread	**0.872**	**0.904**	**0.937**	1.122

Notes: The table reports the ratio of the MSFE of the RS model over the MSFE of the RW model. Bold indicates superiority of the RS model.

seems to help in terms of forecasting accuracy as the forecast horizon increases. Finally, the first three bubble measures seem to outperform the fourth one for $h > 1$.

5.2 *Statistical evaluation*

Up to this point, we examined the forecasting accuracy of the RS models based on the point estimates of the MSFE which is sample dependent. We now use formal statistical testing to evaluate the forecasting performance of the three RS models examined in our study relative to the RW model. Given that we are interested in comparing nested models, we apply the methodology developed by Clark and West (2006, 2007). Before presenting our findings, it is essential to provide a brief description of the approach that Clark and West (2006, 2007) use for the forecast evaluation of a parsimonious model A relative to a larger model B.[21] The authors show that under the null hypothesis, model B should generate larger MSFE than model A. The intuition behind this argument is that since under the null hypothesis the additional parameters of model B do not help predictions, in finite samples model B loses efficiency due to the estimation of these parameters that introduces noise into the forecasts. This inflates the MSFE of model B. Therefore, even if model A generates smaller MSFE than model B, we should not consider this as prima facie evidence of superiority of model A over B. In this respect, Clark and West (2006, 2007) introduce a testing procedure that corrects for the inflation in the MSFE of the larger model before evaluating the relative forecasting accuracy of the two models. Let $\hat{R}_{A,t}$ and $\hat{R}_{B,t}$ denote the forecasts for R_t obtained from models A and B, respectively (in our case, model A corresponds to the RW model while model B corresponds to one of the RS models). Given a sequence of P forecasts, we first calculate

$$f_t = (R_t - \hat{R}_{A,t})^2 - (R_t - \hat{R}_{B,t})^2 + (\hat{R}_{A,t} - \hat{R}_{B,t})^2, \quad t = 1, 2, \ldots, P.$$

The test statistic of Clark and West, denoted as CW-t, is given by the standard *t-statistic* of the regression of f_t on a constant. We should note that under the alternative hypothesis of the test, model B has lower MSFE than model A. Thus, this is a one-sided test. Clark and West (2006, 2007) recommend using 1.282 and 1.645 for a 0.10 and 0.05 test, respectively. Extensive simulations performed by them that consider a variety of different processes and settings show that the aforementioned critical values provide reliable results.

The CW-*t* statistics for one-step ahead forecasts are reported in Table 8 (Panel A). Our findings can be summarized as follows:

(1) Model 1 fails to produce statistically lower MSFEs compared with the RW model for all four bubble measures.
(2) Model 2 outperforms the RW model for all bubble measures, when we use the short-term interest rate as an explanatory variable. For the remaining variables, the difference in the MSFE of Model 2 relative to that of the RW model is statistically significant in just one case, that is when we use imports as an explanatory variable and consider the fourth bubble measure.
(3) The picture changes when we consider Model 3. Specifically, when Model 3 includes imports, it yields statistically lower MSFE than the RW model for all bubble measures. The same holds for the short-term interest rate with the exception of the last bubble measure where the decrease in the MSFE of Model 3 relative to the RW model is not statistically significant. Depending on the explanatory variable and the bubble measure, we can still identify few other cases where the RW model is beaten by Model 3.
(4) When comparing the performance of the bubble measures, our fourth bubble measure ranks first as it provides statistically significant forecasts in two and four cases for Models 2 and 3, respectively.

To sum up, our findings for the one-period forecast horizon and for Models 1 and 2 reflect the well-known difficulty of finding a model that beats the RW model in terms of forecasting accuracy for the exchange rate. The only exception seems to be Model 2 that includes the short-term interest rate. Moreover, when we consider a three-state RS model, we obtain lower MSFEs than the RW model that are statistically significant given that we choose to include either imports or short-term interest rates in the specification. Thus, the short-term interest rate appears to be the optimal explanatory variable (in terms of the MSFE of the forecasts for the exchange rate) among the ones considered in our analysis, closely followed by imports.

As noted in a previous section, the statistical evaluation of multi-step forecasts of nested models is not straightforward, since the limiting distribution depends, in general, on data-specific parameters. However, Clark and West (2007) mention that the CW-*t* statistic performs relatively well for multi-step forecasts given a medium or large sample size.[22] We, therefore, choose to apply the CW-*t* statistic for our multi-period forecasts but results should be read with caution. Panels B–D of Table 8 report our results that apply the CW-*t* statistic for forecast horizons 3, 6 and 12 months. As suggested by Clark and McCracken (2013), an autocorrelation consistent standard error should be employed in order to improve the accuracy of inference in evaluating recursive multi-step predictions. As is evident, our RS models rarely generate statistically more accurate forecasts than the RW model. Specifically, for $h = 3$, only six specifications yield statistically significant MSFE reductions, while for the six-month horizon, only three specifications generate statistically significant lower MSFEs. Quite interestingly, the 12-month horizon is associated with no significant predictive ability irrespective of the specification employed. We now move to the economic evaluation of the forecasts of our models.

Table 8. The Clark–West t-statistic.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. $h = 1$ month				
Model 1	0.983	1.057	1.114	0.554
Model 2				
Exports	−0.182	−0.499	−0.491	−1.366
Imports	0.957	1.115	0.889	**1.582**
International reserves	0.536	0.303	0.453	−0.031
Long-term interest rate	0.593	0.458	0.604	0.428
Short-term interest rate	**1.662**	**1.636**	**1.480**	**1.412**
Yield spread	0.936	0.997	0.891	0.123
Model 3				
Exports	1.251	**1.519**	1.052	0.307
Imports	**1.582**	**1.689**	**1.356**	**2.333**
International reserves	0.716	0.577	**1.843**	0.223
Long-term interest rate	**1.584**	0.283	−0.075	**2.374**
Short-term interest rate	**1.667**	**1.474**	**1.655**	**1.251**
Yield spread	−0.139	0.470	0.043	**1.292**
Panel B. $h = 3$ months				
Model 1	1.072	0.891	0.896	0.117
Model 2				
Exports	0.905	0.516	−0.239	−1.181
Imports	0.381	0.499	0.446	−0.270
International reserves	0.671	0.704	0.633	0.185
Long-term interest rate	−0.651	−0.501	−0.509	−1.404
Short-term interest rate	1.247	**1.388**	**1.376**	1.096
Yield spread	**1.606**	0.946	1.872	0.319
Model 3				
Exports	−0.263	0.238	−0.229	−1.144
Imports	0.584	0.706	0.720	0.096
International reserves	0.751	0.858	0.705	0.094
Long-term interest rate	0.163	0.137	−0.071	−0.284
Short-term interest rate	**1.311**	0.494	0.275	1.057
Yield spread	1.039	**1.301**	**3.048**	0.051
Panel C. $h = 6$ months				
Model 1	1.033	**1.433**	1.233	1.000
Model 2				
Exports	0.202	0.093	0.744	−0.595
Imports	0.575	0.518	0.495	−0.589
International reserves	0.607	0.545	0.666	0.024
Long-term interest rate	−0.027	−0.246	−0.053	−0.673
Short-term interest rate	0.507	0.719	0.461	0.301
Yield spread	0.776	0.385	1.208	0.612
Model 3				
Exports	0.462	0.571	0.170	−0.600
Imports	0.782	0.713	0.845	0.019
International reserves	0.709	0.779	0.743	0.329
Long-term interest rate	0.318	0.211	0.349	−0.390
Short-term interest rate	1.251	**1.351**	**1.520**	0.789
Yield spread	1.216	0.882	1.280	0.697

(*Continued*)

Table 8. Continued.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel D. h = 12 months				
Model 1	1.092	1.086	0.998	0.828
Model 2				
Exports	0.370	0.277	0.197	0.058
Imports	1.081	0.956	0.891	0.586
International reserves	0.635	0.482	0.552	0.492
Long-term interest rate	1.028	0.762	0.961	0.757
Short-term interest rate	1.062	0.892	1.003	0.902
Yield spread	1.148	0.979	1.025	0.944
Model 3				
Exports	0.721	0.789	1.190	0.152
Imports	1.155	1.160	0.890	0.552
International reserves	0.754	0.786	0.671	0.527
Long-term interest rate	0.996	0.896	1.081	0.778
Short-term interest rate	1.140	0.991	0.984	0.981
Yield spread	1.238	1.145	1.031	1.081

Note: Bold indicate cases where RS forecasts are statistically superior to the RW model forecasts.

5.3 *Economic evaluation*

In this section, we examine the forecasting power of our models in a stylized asset allocation framework, where a mean-variance investor maximizes expected utility. This utility-based approach initiated by West, Edison, and Cho (1993) has been extensively employed in the literature as a measure for ranking the performance of competing models (Fleming, Kirby, and Ostdiek 2001; Marquering and Verbeek 2004; Cheung and Valente 2009; Della Corte, Sarno, and Tsiakas 2009; Della Corte, Sarno, and Valente 2010).

Consider a UK investor who has a one-month horizon and constructs a dynamically rebalanced portfolio. Her portfolio choice problem is how to allocate wealth between the safe domestic (UK) bond and the risky foreign (US) bond. The only source of risk with respect to the foreign bond stems from the uncertainty over the future path of the exchange rate. Since only one risky asset is involved, this approach could be thought of as a standard exercise of market timing in the FX market. Let i_t and i_t^* denote the domestic and foreign risk-free rate.[23] In a mean–variance framework, the solution to the maximization problem of the investor yields the following weight (w_t) on the risky (foreign) asset

$$w_t = \frac{E_t(i_t^* + r_{t+1} - i_t)}{\gamma\ \mathrm{Var}_t(r_{t+1})} = \frac{i_t^* - i_t + \hat{r}_{t+1}}{\gamma\ \mathrm{Var}(\hat{r}_{t+1})}, \tag{15}$$

where E_t and Var_t denote the conditional expectation and variance operators, r_{t+1} is the return of the exchange rate, $\hat{r}_{t+1}$ is the predicted return of the exchange rate and γ is the Relative Risk Aversion (RRA) coefficient that controls the investor's appetite for risk (Campbell and Viceira 2002; Campbell and Thomson 2008; Rapach, Strauss, and Zhou 2010). The conditional variance of the portfolio is approximated by the sample variance of the exchange rate return and is estimated recursively over the out-of-sample period.[24] Under this setting the optimally constructed portfolio return over the out-of-sample period is equal to

$$r_{p,t+1} = w_t(i_t^* + r_{t+1}) + (1 - w_t)i_t.$$

Over the forecast evaluation period the investor with initial wealth of $W_o = 1$£ realizes an average utility of

$$\bar{U} = \frac{1}{P}\sum_{t=0}^{P-1}\left(r_{p,t+1} - \frac{\gamma}{2}(r_{t+1-}\bar{r}_{t+1})^2\right), \tag{16}$$

where $\bar{r}_{t+1}$ denotes the average exchange rate return over the evaluation period. At any point in time, the investor prefers the model for conditional returns that yields the highest average realized utility. More specifically, a risk-averse investor will be willing to pay for switching from the portfolio constructed based on the forecasts of the simple RW model to a portfolio based on our proposed RS specifications if the RS model has good forecasting ability. Let $r^{\mathrm{RS}}_{p,t+1}, r^{\mathrm{RW}}_{p,t+1}$ denote the portfolios generated by employing the RS and the RW models, respectively. The performance fee, denoted by Ω, is the fraction of the wealth which when subtracted from the RS proposed portfolio returns equates the average utilities of the competing model. In our setup, the performance fee is calculated as the difference between the realized utilities as follows:

$$\frac{1}{P}\sum_{t=0}^{P-1}(r^{\mathrm{RS}}_{p,t+1} - \Omega - \frac{\gamma}{2}(r_{t+1-}\bar{r}_{t+1})^2) = \frac{1}{P}\sum_{t=0}^{P-1}(r^{\mathrm{RW}}_{p,t+1} - \frac{\gamma}{2}(r_{t+1-}\bar{r}_{t+1})^2) \Longrightarrow$$

$$\Omega = \bar{U}^{\mathrm{RS}} - \bar{U}^{\mathrm{RW}}. \tag{17}$$

If our proposed model does not contain any economic value, the performance fee is negative ($\Omega \leq 0$), while positive values of the performance fee suggest superior predictive ability against the RW benchmark.

As a complement to the performance fee measure, we also employ the manipulation-proof performance measure proposed by Goetzmann et al. (2007). This measure can be interpreted as a portfolio's premium return after adjusting for risk and it remedies potential caveats associated with the popular Sharpe ratio such as the effect of non-normality (Jondeau and Rockinger 2006), the underestimation of the performance of dynamic strategies (Marquering and Verbeek 2004; Han 2006) and the choice of utility function (Della Corte, Sarno, and Sestieri 2012). This measure is defined as

$$M(R_p) = \frac{1}{1-\gamma}\ln\left\{\frac{1}{P}\sum_{t=0}^{P-1}\left(\frac{1+r_{p,t+1}}{1+i_t}\right)^{1-\gamma}\right\}.$$

Let Θ denote the difference between the $M(R_p)$s of competing models which is defined as follows and is employed to assess the most valuable model:

$$\Theta = M(R_p)^{\mathrm{RS}} - M(R_p)^{\mathrm{RW}} \tag{18}$$

Both Ω and Θ are reported in annualized basis points.

As previously, we recursively estimate all three models considered in this study adding one observation at a time and generate a one-period ahead forecast for each model. Utilizing this forecast, we calculate the weight (Equation (15)) the candidate model attaches on the risky asset. Following Abhyankar, Sarno, and Valente (2005), we restrict the degree of short-selling and leveraging by setting $-1 < w_t < 2$ assuming that the investor can borrow no more than 100% of his wealth. We also assume a coefficient of RRA equal to 3 and in this way we construct the portfolio return implied by each of the competing models. This procedure is repeated for the whole out-of-sample period of 13 years. As already mentioned, our candidate models are evaluated against the benchmark RW model. To this end, we calculate the ex post realized utility

(Equation (16)) of the RW model and of each one of our proposed RS specifications. The respective performance fee (Ω) and the manipulation-proof performance measure difference (Θ) are reported in Table 9 (Panel A).

The overall picture emerging from Table 9 points to significant utility gains for the investor who is willing to switch from the benchmark RW model to one of the RS models. More in detail, the investor is willing to pay annual performance fees that range from 41 (fourth bubble measure) to 142 (third bubble measure) basis points when employing the strategy implied by Model 1. Turning to Model 2, we observe that the economic significance of our models depends on both the bubble measure and the indicator variable employed. Depending on the specification, the performance fee reaches 341 bps for the fourth bubble measure with imports. Negative fees are mainly associated with exports, while the best performing models are the ones employing either imports or the short-term interest rate. More importantly, our results suggest that the fees can increase substantially when employing our full model (Model 3). Specifically, an investor would pay up to 519 bps annually for employing forecasts of the exchange rate generated by a three-regime model built on imports and the fourth bubble measure. Out of 24 possible specifications, only 4 generate negative fees. Irrespective of the bubble measure, the models employing exports, imports or the short-term interest rate are the ones generating utility gains (positive performance fees) for the investor. Finally, the risk-adjusted abnormal return Θ is fully consistent (size and sign) with the results obtained from the performance fees. Whenever an investor is willing to pay a high performance fee for utilizing a strategy based on the RS model, this strategy is associated with a substantial premium relative to the benchmark one and vice versa.

To test the sensitivity of our findings to the value of the coefficient of RRA, we repeat the calculation of Ω and Θ setting the coefficient of RRA equal to either 2 or 5. The respective findings are reported in Panels B and C of Table 9, respectively. Overall, our results are qualitatively similar, since we rarely observe any changes in the sign of Ω and Θ. As expected, a low risk aversion of 2 leads to increased performance fees. Specifically, an investor would be willing to pay up to 243, 457 and 612 bps to utilize our models 1, 2 and 3, respectively. On the other hand, both Ω and Θ decrease as the coefficient of RRA increases. For $\gamma = 5$, the maximum performance fee is 86, 208 and 355 bps for models 1, 2 and 3, respectively. As previously, the risk-adjusted abnormal return Θ is fully consistent (size and sign) with the results obtained from the performance fees.

For completeness, we also extend the economic evaluation of our proposed models relative to the RW benchmark to longer horizons, namely 3, 6 and 12 months. The economic significance of our proposed models could provide new insights for the quality of our models. Panels A, B and C of Table 10 report our results for the 3-, 6- and 12-month horizon, respectively.

With respect to the three-month horizon, the simple two-regime model (Model 1) generates positive performance fees that range from 59 to 240 bps. Turning to the extended two-regime specification (Model 2), positive fees are generated by the models that include either the short-term interest rate, the international reserves or the yield spread, while the opposite picture emerges when the long-term interest rate is employed. Positive performance fees are depicted for the remaining variables for the first three bubble measures. As for the three-regime model, our results vary with the choice of the bubble measure; generating positive performance fees for our first two bubble measures and mixed for the remaining ones. Once again the performance-proof measure points to the same direction.

The performance of the simple two-regime specification at the six-month horizon is similar to the three-month one generating positive fees up to 238 bps. Furthermore, utility gains are depicted by either the two-regime models augmented by the yield spread or the three-regime model augmented by imports, the short- and long-term interest rates. The remaining specifications produce

Table 9. Economic evaluation vs. the random walk model (one-step ahead forecasts).

	b_t^A		b_t^B		b_t^C		b_t^D	
	Ω	Θ	Ω	Θ	Ω	Θ	Ω x	Θ
Panel A. Risk aversion coefficient $\gamma = 3$								
Model 1	133	135	135	138	142	144	41	44
Model 2								
Exports	15	16	−47	−43	−64	−62	−213	−212
Imports	133	138	150	156	100	109	341	355
International reserves	31	37	1	7	32	36	−88	−84
Long-term interest rate	104	106	59	61	69	69	14	15
Short-term interest rate	254	266	261	274	195	207	191	204
Yield spread	112	114	134	141	103	108	−62	−59
Model 3								
Exports	153	166	252	266	134	147	10	20
Imports	268	282	289	303	191	205	519	532
International reserves	75	82	48	55	334	347	−41	−34
Long-term interest rate	214	221	113	117	−148	−146	328	332
Short-term interest rate	219	223	187	187	297	309	167	180
Yield spread	−38	−37	103	113	−135	−128	198	200
Panel B. Risk aversion coefficient $\gamma = 2$								
Model 1	243	247	232	238	228	231	112	117
Model 2								
Exports	99	102	1	7	−41	−37	−202	−199
Imports	223	229	248	254	141	147	457	464
International reserves	125	132	20	27	62	69	−95	−90
Long-term interest rate	214	217	161	164	170	171	93	95
Short-term interest rate	323	330	338	344	238	244	224	231
Yield spread	219	223	240	247	189	196	−7	−2
Model 3								
Exports	252	259	315	322	229	236	115	124
Imports	337	344	333	340	237	244	612	617
International reserves	180	188	97	104	394	401	−65	−58
Long-term interest rate	253	262	252	259	−151	−148	439	444
Short-term interest rate	250	256	216	218	373	379	204	211
Yield spread	93	96	218	226	−133	−126	260	262
Panel C. Risk aversion coefficient $\gamma = 5$								
Model 1	76	77	81	83	85	86	25	26
Model 2								
Exports	−59	−60	−81	−79	−92	−92	−196	−196
Imports	82	85	91	95	66	71	208	225
International reserves	12	16	0	4	19	22	−53	−51
Long-term interest rate	25	25	19	19	32	31	4	4
Short-term interest rate	174	205	166	196	138	168	134	165
Yield spread	69	70	80	85	62	65	−60	−59
Model 3								
Exports	94	105	170	180	68	76	−64	−60
Imports	180	206	195	209	136	155	355	372
International reserves	34	38	27	31	221	231	−25	−21
Long-term interest rate	133	140	8	9	−108	−107	204	206
Short-term interest rate	178	180	157	158	190	198	112	142
Yield spread	−66	−66	33	39	−88	−83	121	123

Notes: The performance fee, Ω, is the fraction of the wealth which when subtracted from the RS proposed portfolio returns equates the average utilities of the competing model (i.e. the RS and the RW models). Θ is the difference between the manipulation-proof performance measure of competing models (RS and RW).

Table 10. Economic evaluation vs. the RW model (multi-step ahead forecasts).

	b_t^A		b_t^B		b_t^C		b_t^D	
	Ω	Θ	Ω	Θ	Ω	Θ	Ω	Θ
Panel A. 3-month forecast horizon								
Model 1	240	259	236	274	200	212	59	68
Model 2								
Exports	14	18	21	28	45	52	−265	−264
Imports	100	154	161	228	116	201	−181	−116
International reserves	252	293	147	222	82	97	120	184
Long-term interest rate	−85	−86	−69	−70	−48	−48	−216	−219
Short-term interest rate	302	400	284	383	321	419	248	348
Yield spread	298	302	164	172	335	342	20	23
Model 3								
Exports	39	61	102	144	−74	−55	−376	−377
Imports	134	214	156	247	187	278	−153	−73
International reserves	190	247	137	238	109	140	−88	−72
Long-term interest rate	67	69	12	15	32	33	−26	−27
Short-term interest rate	197	290	0	4	−57	−62	120	214
Yield spread	184	195	196	208	377	375	−2	13
Panel B. 6-month forecast horizon								
Model 1	104	371	238	501	118	386	103	374
Model 2								
Exports	135	170	54	115	70	80	−94	−82
Imports	181	182	152	156	62	58	−52	−53
International reserves	46	64	−19	2	22	35	−91	−81
Long-term interest rate	−19	−10	−37	−33	−10	−6	−57	−64
Short-term interest rate	7	−16	118	118	−95	−180	158	162
Yield spread	250	286	82	93	160	288	143	190
Model 3								
Exports	278	333	160	274	150	171	−48	−39
Imports	335	337	305	324	252	252	98	101
International reserves	177	216	−56	221	135	159	19	31
Long-term interest rate	201	205	100	103	107	110	9	5
Short-term interest rate	350	453	42	310	129	393	343	415
Yield spread	92	359	−87	195	226	371	−30	241
Panel C. 12-month forecast horizon								
Model 1	−52	214	−119	155	−8	179	−153	99
Model 2								
Exports	−339	−43	−403	−107	−364	−74	−467	−160
Imports	−37	219	−73	161	86	181	−129	28
International reserves	−99	31	−123	−46	−76	−38	−208	−88
Long-term interest rate	101	278	73	209	77	212	89	203
Short-term interest rate	146	342	120	268	194	282	67	286
Yield spread	−2	258	−90	178	36	197	−122	146
Model 3								
Exports	−154	132	−109	167	−191	105	−427	−119
Imports	106	357	49	309	68	93	−69	122
International reserves	−71	131	−28	57	−178	−121	−107	21
Long-term interest rate	103	354	3	267	70	270	106	271
Short-term interest rate	131	351	−2	264	181	264	72	323
Yield spread	46	297	−61	208	−160	25	15	259

Note: See notes below Table 9. Portfolios are restructured every 3 (Panel A), 6 (Panel B) or 12 (Panel C) months. The risk aversion (γ) coefficient is equal to 3.

mixed evidence. It is worth noting that in this horizon we find a few inconsistencies between the performance fees and the manipulation-proof measure. For example, while the three-regime model that contains the yield spread as the indicator variable generates negative performance fees for the second bubble measure, the risk-adjusted return is estimated at 195 bps annually.

Contrary to shorter horizons, the 12-month horizon does not favor the employment of Model 1 based on the performance fee measure. However, the premium return adjusted for risk is positive

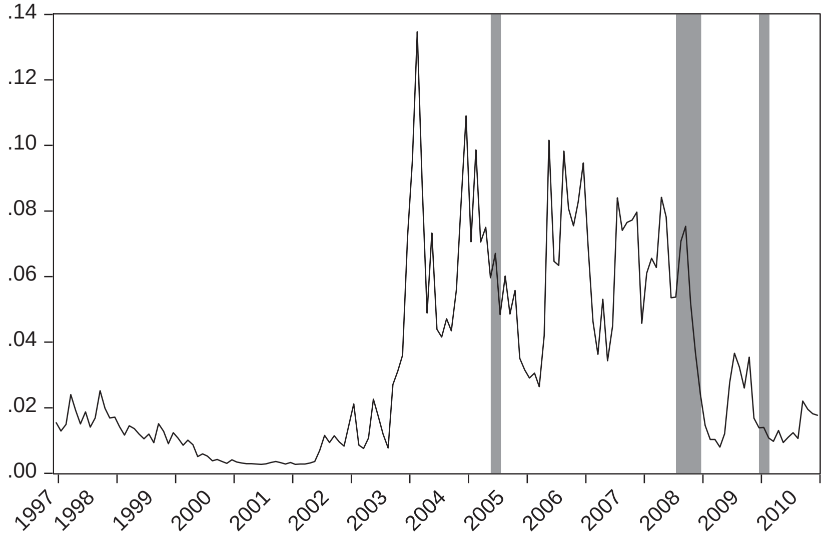

Figure 5. Recursive probability of a boom (Model 3 with exports, first bubble measure).

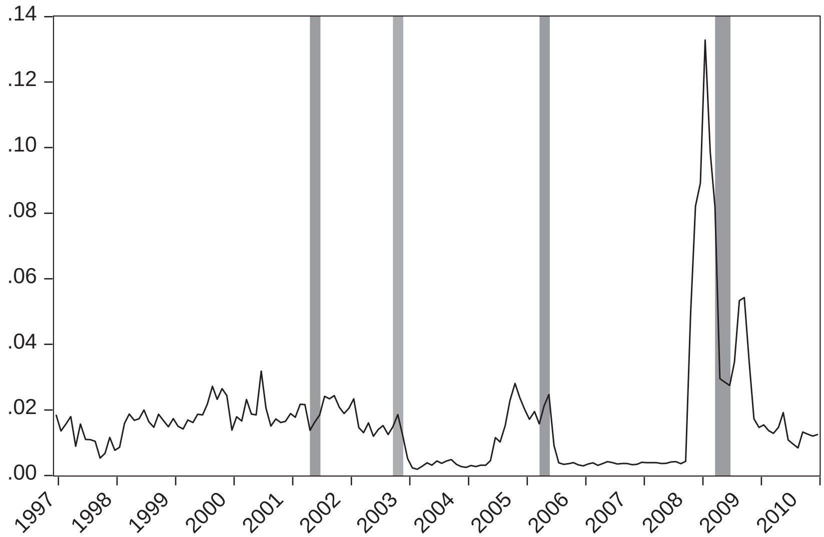

Figure 6. Recursive probability of a crash (Model 3 with exports, first bubble measure).

Table 11. Trading strategies – excess end-of-period wealth of an initial investment of £1 over the benchmark strategy.

	Transaction cost = 0.5%				Transaction cost = 1%				Transaction cost = 0.1%			
	b_t^A	b_t^B	b_t^C	b_t^D	b_t^A	b_t^B	b_t^C	b_t^D	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. Strategy 1												
Model 1	14.2	9.9	−2.5	10.7	4.6	−6.6	−11.2	5.0	22.2	24.3	4.9	15.4
Model 2												
Exports	−9.1	−0.1	4.1	2.6	−10.8	−0.1	−10.2	−11.7	−7.7	−0.1	16.4	14.8
Imports	−3.1	−0.1	−0.1	0.0	−6.7	−0.1	−0.1	0.0	−0.2	−0.1	−0.1	0.0
International reserves	0.0	−0.1	−0.1	0.0	0.0	−0.1	−0.1	0.0	0.0	−0.1	−0.1	0.0
Long-term interest rate	0.0	−0.1	−0.1	−8.2	0.0	−0.1	−0.1	−19.9	0.0	−0.1	−0.1	1.9
Short-term interest rate	13.9	10.4	−24.1	−23.9	4.3	0.9	−28.7	−27.0	21.9	18.2	−20.2	−21.3
Yield spread	0.0	−0.1	−0.1	0.0	0.0	−0.1	−0.1	0.0	0.0	−0.1	−0.1	0.0
Model 3												
Exports	−4.9	22.9	0.7	0.6	−16.9	5.2	−13.4	−20.1	5.3	38.2	12.8	19.0
Imports	21.6	9.1	14.0	15.6	7.8	−5.7	4.4	2.2	33.3	21.7	22.0	27.0
International reserves	14.2	−18.4	9.2	−21.1	4.6	−29.5	−2.0	−25.9	22.2	−9.0	18.6	−17.2
Long-term interest rate	−5.0	6.1	9.6	9.1	−23.5	−11.9	−6.9	1.6	11.2	21.8	23.9	15.3
Short-term interest rate	33.1	25.2	8.3	32.9	20.5	9.2	0.9	24.5	43.7	38.9	14.5	39.9
Yield spread	−29.9	6.0	−19.5	−12.1	−38.8	−8.5	−33.5	−17.2	−22.4	18.5	−7.4	−8.0
Panel B. Strategy 2												
Model 1	23.9	−3.6	17.9	0.2	6.1	−10.6	6.2	−8.7	39.3	2.2	27.7	7.6
Model 2												
Exports	15.6	−14.1	−41.2	−35.8	0.3	−34.6	−53.3	−51.1	28.6	4.3	−30.7	−22.4
Imports	21.3	0.6	−11.0	−10.1	7.5	−11.8	−28.8	−34.1	33.0	11.1	4.7	11.8
International reserves	−9.3	−24.4	−18.3	−11.9	−17.7	−29.0	−26.3	−20.2	−2.2	−20.5	−11.6	−4.9
Long-term interest rate	32.3	−6.9	−7.5	−7.8	15.8	−22.0	−21.0	−18.0	46.5	6.1	4.0	0.8
Short-term interest rate	4.8	9.3	−9.0	16.1	−6.1	−10.7	−20.7	−1.0	14.0	26.9	0.9	30.8
Yield spread	25.0	16.3	2.4	−2.5	9.0	8.5	−13.6	−9.6	38.7	22.8	16.1	3.3
Model 3												
Exports	17.7	6.5	−0.5	−0.2	2.3	−16.6	−19.5	−9.1	30.9	27.1	16.2	7.2
Imports	−5.9	−0.1	−33.6	26.3	−27.4	−17.5	−50.5	−8.1	13.4	15.0	−18.7	58.6
International reserves	1.1	−12.7	−2.0	−21.9	−7.9	−24.2	−20.8	−31.3	8.5	−3.0	14.6	−14.0
Long-term interest rate	−24.8	2.8	−22.7	25.3	−45.4	−11.4	−33.5	7.4	−6.2	15.0	−13.5	40.8
Short-term interest rate	−21.1	−9.3	−7.6	−6.1	−40.8	−24.2	−27.4	−22.9	−3.5	3.5	10.0	8.5
Yield spread	6.2	13.0	−27.5	10.0	−1.2	−2.1	−38.0	−6.6	12.4	25.9	−18.6	24.3

Note: The benchmark strategy involves an investor who avoids exchange rate risk by investing all her money in local currency (i.e. British pounds) in the UK risk-free rate. Assuming an initial wealth of one British pound, the benchmark strategy generates an end-of-period amount of 1.813 pounds. Positive entries indicate that the trading strategy based on the RS model outperforms the benchmark strategy.

Table 12. Ratios of MSFEs (out-of-sample period: 2001–2010).

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. $h = 1$ month				
Model 1	**0.998**	**0.998**	**0.997**	1.004
Model 2				
Exports	1.039	1.037	1.045	1.096
Imports	**0.998**	**0.996**	**0.999**	**0.979**
International reserves	1.006	1.007	1.005	1.012
Long-term interest rate	1.008	1.008	1.007	1.004
Short-term interest rate	**0.964**	**0.969**	**0.972**	**0.969**
Yield spread	**0.997**	**0.999**	1.002	1.018
Model 3				
Exports	**0.996**	**0.989**	1.004	1.016
Imports	**0.977**	**0.980**	**0.987**	**0.953**
International reserves	1.004	1.003	0.979	1.006
Long-term interest rate	**0.984**	1.031	1.035	0.974
Short-term interest rate	**0.985**	**0.991**	**0.984**	**0.960**
Yield spread	1.026	1.027	1.029	0.995
Panel B. $h = 3$ months				
Model 1	1.011	**0.968**	**0.967**	1.116
Model 2				
Exports	1.071	1.072	1.125	1.273
Imports	1.011	**0.988**	1.000	1.067
International reserves	1.037	**0.968**	**0.987**	1.145
Long-term interest rate	1.068	1.058	1.066	1.091
Short-term interest rate	**0.887**	**0.868**	**0.881**	**0.882**
Yield spread	**0.961**	**0.984**	1.010	1.293
Model 3				
Exports	1.258	1.128	1.128	1.325
Imports	1.001	**0.977**	**0.979**	1.056
International reserves	1.020	**0.971**	**0.981**	1.166
Long-term interest rate	1.051	1.038	1.056	1.068
Short-term interest rate	**0.948**	1.089	1.006	**0.947**
Yield spread	**0.963**	**0.977**	**0.940**	1.165
Panel C. $h = 6$ months				
Model 1	**0.960**	**0.952**	**0.877**	**0.995**
Model 2				
Exports	1.041	1.094	**0.987**	1.160
Imports	1.001	**0.988**	**0.991**	1.086
International reserves	**0.985**	**0.979**	**0.962**	1.049
Long-term interest rate	1.131	1.116	1.101	1.130
Short-term interest rate	1.010	**0.983**	**0.996**	1.079
Yield spread	**0.944**	**0.993**	**0.905**	1.029
Model 3				
Exports	1.061	1.025	1.081	1.288
Imports	**0.968**	**0.938**	**0.951**	1.067
International reserves	**0.943**	**0.920**	**0.911**	1.020
Long-term interest rate	1.051	1.059	1.053	1.100
Short-term interest rate	**0.995**	**0.889**	**0.828**	1.123
Yield spread	**0.960**	1.027	**0.878**	1.326

(*Continued*)

Table 12. Continued.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel D. $h = 12$ months				
Model 1	**0.907**	**0.900**	**0.897**	1.007
Model 2				
Exports	1.138	1.145	1.144	1.291
Imports	**0.919**	**0.920**	**0.954**	1.063
International reserves	**0.984**	**0.989**	**0.973**	1.086
Long-term interest rate	**0.926**	**0.944**	**0.931**	**0.991**
Short-term interest rate	**0.861**	**0.875**	**0.856**	**0.956**
Yield spread	**0.889**	**0.908**	**0.887**	1.000
Model 3				
Exports	1.124	**0.922**	**0.959**	1.318
Imports	**0.941**	**0.899**	1.054	1.086
International reserves	**0.940**	**0.865**	**0.930**	1.035
Long-term interest rate	**0.941**	**0.926**	**0.904**	1.025
Short-term interest rate	**0.883**	**0.855**	**0.864**	**0.977**
Yield spread	**0.841**	**0.855**	**0.911**	1.117

Notes: The table reports the ratio of the MSFE of the RS model over the MSFE of the RW model. Bold indicates superiority of the RS model.

for all bubble measures. Turning to Models 2 and 3, the best performance is achieved by the two interest rates, while results for the remaining variables vary with the choice of the bubble measure.

6. Trading strategies

In the last part of our analysis, we develop simple trading strategies that are based on the estimated RS models. Each trading rule uses information obtained by our models to predict the timing of large movements in the exchange rate market in order to take advantage of them and increase profits. We assume an investor located in the UK who has two choices: she can invest all available funds in Eurodeposits nominated in either local (i.e. British pound) or foreign (i.e. US dollar) currency. The investor starts with an initial wealth of one pound and decides about the allocation of the available funds on a monthly basis. The period we use for the evaluation of the trading strategies coincides with the period of the out-of-sample forecast exercise performed in the previous section. Specifically, we assume that the investor takes her first investment decision at the end of December 1997 concerning how to invest her money in January 1998. The exercise is organized on a real-time basis, that is, we use all available information up to time T to decide how to invest the money from T to $T+1$. Thus, all estimates, probabilities and forecasts are calculated recursively using only the available information.

We consider two alternative trading rules. The first one simply states that the investor decides about the allocation of money by comparing the probability of a crash with the probability of a boom. If the former is higher than the latter, the investors chooses to invest in British pounds earning interest equal to the monthly UK rate, say i_t. In the opposite case, she invests in US dollars and earns interest equal to the monthly US rate, say i_t^*. The second trading rule is also very simple since it compares the return of investing in the local currency, $(1+i_t)$, to the expected return of investing in US dollars, $(1+i_t^*)\hat{R}_{t+1}$ (where $\hat{R}_{t+1}$ is the forecast for the gross return in the exchange rate based on one of the estimated models), and chooses the highest one. Once again, the same transaction fee applies whenever the investor switches from one currency to the other.

Table 13. Ratios of MSFEs (out-of-sample period: 2004–2010).

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. $h = 1$ month				
Model 1	1.001	**0.996**	**0.998**	1.003
Model 2				
Exports	1.051	1.039	1.058	1.112
Imports	1.004	**0.996**	1.001	**0.964**
International reserves	1.000	**0.996**	**0.997**	1.003
Long-term interest rate	1.013	1.011	1.010	1.006
Short-term interest rate	**0.965**	**0.973**	**0.977**	**0.968**
Yield spread	**0.999**	**0.998**	1.002	1.019
Model 3				
Exports	**0.995**	**0.986**	1.006	1.002
Imports	**0.968**	**0.970**	**0.978**	**0.943**
International reserves	**0.998**	**0.992**	**0.977**	**0.999**
Long-term interest rate	**0.988**	1.046	1.048	**0.972**
Short-term interest rate	**0.994**	**0.999**	**0.986**	**0.958**
Yield spread	1.030	1.035	1.038	**0.998**
Panel B. $h = 3$ months				
Model 1	1.030	**0.971**	**0.958**	1.149
Model 2				
Exports	1.055	1.058	1.125	1.282
Imports	1.026	**0.999**	1.007	1.082
International reserves	1.050	**0.957**	**0.977**	1.166
Long-term interest rate	1.085	1.073	1.079	1.108
Short-term interest rate	**0.876**	**0.852**	**0.869**	**0.862**
Yield spread	**0.971**	**0.991**	1.036	1.356
Model 3				
Exports	1.288	1.143	1.111	1.331
Imports	1.019	**0.989**	**0.989**	1.074
International reserves	1.031	**0.970**	**0.967**	1.207
Long-term interest rate	1.076	1.057	1.077	1.090
Short-term interest rate	**0.952**	1.137	1.035	**0.956**
Yield spread	**0.961**	**0.991**	**0.956**	1.206
Panel C. $h = 6$ months				
Model 1	**0.982**	**0.982**	**0.885**	1.016
Model 2				
Exports	1.013	1.052	**0.932**	1.102
Imports	1.029	**0.996**	1.010	1.115
International reserves	**0.936**	**0.921**	**0.905**	**0.993**
Long-term interest rate	1.151	1.132	1.118	1.153
Short-term interest rate	1.059	1.006	1.031	1.124
Yield spread	**0.939**	**0.964**	**0.913**	1.020
Model 3				
Exports	1.057	1.010	1.059	1.253
Imports	**0.992**	**0.959**	**0.966**	1.088
International reserves	**0.918**	**0.890**	**0.877**	**0.982**
Long-term interest rate	1.084	1.092	1.084	1.129
Short-term interest rate	1.059	**0.925**	**0.865**	1.190
Yield spread	**0.985**	1.018	**0.888**	1.393

(*Continued*)

Table 13. Continued.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel D. h = 12 months				
Model 1	**0.910**	**0.923**	**0.884**	**0.976**
Model 2				
Exports	1.026	**0.991**	**0.987**	1.163
Imports	**0.935**	**0.899**	**0.979**	1.075
International reserves	**0.917**	**0.900**	**0.889**	**0.992**
Long-term interest rate	**0.996**	**0.999**	**0.991**	1.018
Short-term interest rate	**0.918**	**0.903**	**0.899**	1.008
Yield spread	**0.902**	**0.893**	**0.879**	**0.990**
Model 3				
Exports	1.077	**0.924**	**0.902**	1.213
Imports	**0.996**	**0.945**	1.166	1.150
International reserves	**0.892**	**0.865**	**0.898**	**0.982**
Long-term interest rate	1.012	**0.986**	**0.966**	1.068
Short-term interest rate	**0.952**	**0.884**	**0.900**	1.029
Yield spread	**0.876**	**0.888**	**0.957**	1.183

Notes: The table reports the ratio of the MSFE of the RS model over the MSFE of the RW model. Bold indicates superiority of the RS model.

The two alternative strategies exploit diverse aspects of information coming from the estimated RS models; the first one is based on the estimated probabilities of a crash and a boom, while the second one uses the forecasts for the exchange rate return. Before presenting our results, let us investigate whether the estimated probabilities of a crash or boom used in the first trading strategy have any predictive power for significant movements in the exchange rate market. Figure 5 plots the probability of a boom estimated recursively from Model 3 using the first explanatory variable (that is, exports) and the first bubble measure. Shaded areas highlight the periods with the highest (positive) three-month cumulative return in the exchange rate. We observe a significant increase in the estimated probability of a boom before the occurrence of a large positive return in the exchange rate market. Similarly, Figure 6 shows the estimated probability of a crash for the same model, together with the periods that have the lowest (negative) three-month cumulative return in the exchange rate. Once again, our model appears to have some predictive power to warn us for large negative movements in the exchange rate market, although results seem more promising for positive exchange rate returns.

We evaluate each strategy in terms of its ability to lead to higher profits than a benchmark strategy (B) that avoids exchange rate risk. Under the benchmark strategy the investor always invests in local currency, earning interest i_t and therefore she never pays any transaction fees. In our case, the benchmark strategy leads to an end-of-period wealth equal to 1.813. Each time the investor exchanges funds from one currency to the other, we assume that she pays a transaction fee. Table 11 presents the results for three levels of transaction fees (0.5%, 1% and 0.1%) and for both strategies in Panels A and B, respectively. We report our findings in terms of excess end-of-period wealth over the benchmark strategy (in percentage terms). As such, positive values are associated with superior performance. The general picture that emerges suggests that the results are both model and bubble dependent. Model 2 displays the worst performance for both strategies, since it rarely outperforms the benchmark strategy. The findings for Model 1 suggest that it beats the benchmark strategy for three out of four bubble measures for both strategies. Finally, the results are

Table 14. Ratios of MSFEs (in-sample period: 1976–2000).

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. h = 1 *month*				
Model 1	**0.993**	**0.998**	**0.996**	1.001
Model 2				
Exports	1.018	**0.990**	1.024	1.100
Imports	**0.997**	**0.999**	1.000	**0.973**
International reserves	**0.998**	1.003	1.001	1.009
Long-term interest rate	**0.996**	1.005	1.005	**0.997**
Short-term interest rate	**0.972**	**0.967**	**0.973**	**0.974**
Yield spread	1.000	1.012	1.007	1.012
Model 3				
Exports	**0.995**	**0.976**	1.012	1.013
Imports	1.005	**0.985**	1.001	**0.965**
International reserves	**0.972**	1.000	**0.993**	1.004
Long-term interest rate	**0.997**	1.020	**0.998**	**0.965**
Short-term interest rate	**0.985**	**0.987**	1.001	**0.981**
Yield spread	1.031	**0.998**	1.011	1.000
Panel B. h = 3 *months*				
Model 1	**0.967**	**0.990**	**0.965**	**0.992**
Model 2				
Exports	1.089	1.139	1.048	1.185
Imports	**0.993**	1.005	**0.975**	**0.952**
International reserves	**0.987**	**0.983**	**0.977**	1.009
Long-term interest rate	1.018	1.031	1.026	1.017
Short-term interest rate	**0.987**	1.000	1.061	1.013
Yield spread	**0.994**	1.043	0.999	1.004
Model 3				
Exports	1.238	1.084	1.123	1.262
Imports	1.038	1.089	0.927	1.102
International reserves	**0.979**	**0.992**	**0.988**	1.021
Long-term interest rate	**0.948**	1.039	1.179	1.060
Short-term interest rate	**0.962**	**0.964**	**0.943**	**0.999**
Yield spread	**0.992**	1.024	**0.994**	1.138
Panel C. h = 6 *months*				
Model 1	**0.928**	**0.931**	**0.925**	**0.974**
Model 2				
Exports	1.050	1.100	1.038	1.228
Imports	**0.971**	**0.956**	**0.959**	**0.920**
International reserves	**0.953**	**0.951**	**0.958**	**0.997**
Long-term interest rate	1.043	1.014	1.026	1.023
Short-term interest rate	1.028	1.034	1.004	**0.979**
Yield spread	**0.969**	**0.931**	**0.953**	1.023
Model 3				
Exports	**0.928**	1.054	1.025	1.063
Imports	**0.946**	**0.947**	**0.990**	1.177
International reserves	**0.920**	**0.957**	**0.906**	1.002
Long-term interest rate	1.013	1.010	1.015	1.063
Short-term interest rate	1.005	**0.998**	**0.900**	1.105
Yield spread	**0.924**	**0.927**	**0.930**	1.014

(*Continued*)

Table 14. Continued.

	b_t^A	b_t^B	b_t^C	b_t^D
Panel D. h = 12 months				
Model 1	**0.881**	**0.976**	**0.947**	**0.957**
Model 2				
Exports	1.030	**0.945**	1.059	1.146
Imports	**0.946**	**0.969**	**0.985**	**0.968**
International reserves	**0.935**	**0.979**	**0.979**	1.035
Long-term interest rate	**0.920**	**0.885**	**0.909**	**0.997**
Short-term interest rate	**0.945**	1.074	**0.968**	**0.802**
Yield spread	**0.944**	**0.974**	**0.972**	1.058
Model 3				
Exports	**0.979**	**0.964**	**0.973**	**0.919**
Imports	**0.882**	**0.896**	**0.883**	1.011
International reserves	**0.908**	**0.943**	**0.937**	1.012
Long-term interest rate	**0.989**	**0.947**	1.018	1.009
Short-term interest rate	**0.979**	1.032	**0.880**	**0.682**
Yield spread	**0.930**	**0.960**	**0.997**	1.021

Notes: The table reports the ratio of the MSFE of the RS model over the MSFE of the RW model. Bold indicates superiority of the RS model.

more promising for Model 3, especially for the first trading rule. Specifically, the implementation of the first trading strategy based on Model 3 that contains either imports or interest rates (both long- and short-term) generates higher return relative to the benchmark rule in almost all cases, reaching the highest value of 33.1 percentage points for the short-term rate and the first bubble measure (for a 0.5% transaction fee). As expected, a higher transaction cost of 1% leads to fewer profitable strategies. Specifically, Model 3 can generate an excess return of up to 24.5 percentage points over the benchmark strategy. On the other hand, lower transaction costs (i.e. 0.1%) render the majority of Model 3 specifications more profitable than the benchmark strategy.

7. Robustness checks

We now perform a series of checks to test the robustness of our main findings to various factors that may have an impact on our results. We start by investigating the sensitivity of our findings to the date we choose to start the out-of-sample forecast exercise. We, therefore, repeat the forecast exercise twice by setting the starting date to January 2001 and January 2004. In other words, we move forward the starting date of our forecast exercise by three and six years (that is, 36 and 72 observations, respectively). Table 12 reports the ratio of the MSFE of each one of our RS models over the MSFE of the RW model for the first case (i.e. when we set the staring date to January 2001). Our main findings remain quantitatively similar. Specifically, the first three bubble measures seem to outperform the fourth one for all forecast horizons, while short-term interest rates and imports are still the optimal predictors in the context of our RS models. It is also interesting to note the significant improvement in the predictive power of the second and third bubble measures for long-term forecasts ($h = 12$). Once again, the yield spread improves the forecasting accuracy of our models only when $h > 1$. Table 13 presents the MSFE ratios for the second case (i.e. when we set the staring date to January 2004). The findings are, in general, similar to the previous case.

Table 15. Ratios of MSFEs (one-step ahead forecasts).

	b_t^A	b_t^B	b_t^C	b_t^D
Panel A. ¥/$				
Model 1	1.001	1.048	1.042	**0.993**
Model 2				
Exports	1.035	1.062	1.042	**0.992**
Imports	1.001	1.041	1.037	**0.995**
International reserves	1.001	1.061	1.068	**0.993**
Long-term interest rate	1.005	1.046	1.068	**0.996**
Short-term interest rate	1.002	1.062	1.038	**0.994**
Yield spread	1.006	1.057	1.046	1.009
Model 3				
Exports	1.016	1.049	1.066	**0.998**
Imports	1.012	1.013	1.055	1.004
International reserves	1.025	1.067	1.098	**0.992**
Long-term interest rate	1.029	1.039	1.023	1.030
Short-term interest rate	1.014	1.009	1.011	**0.997**
Yield spread	1.016	1.016	1.081	1.009
Panel B. SEK/$				
Model 1	**0.995**	1.003	1.005	**0.994**
Model 2				
Exports	1.000	1.046	1.043	1.001
Imports	1.004	1.065	1.075	1.008
International reserves	**0.995**	1.005	1.005	**0.994**
Long-term interest rate	1.004	1.015	1.013	1.003
Short-term interest rate	1.003	**0.983**	**0.991**	1.003
Yield spread	**0.993**	**0.984**	**0.986**	**0.992**
Model 3				
Exports	**0.999**	**0.997**	1.010	**0.996**
Imports	**0.999**	1.007	1.028	1.002
International reserves	**0.989**	1.002	1.000	**0.992**
Long-term interest rate	**0.993**	1.010	1.005	**0.996**
Short-term interest rate	1.000	1.027	1.003	**0.990**
Yield spread	**0.997**	1.016	**0.999**	**0.992**
Panel C. CAD/$				
Model 1	1.006	1.040	**0.999**	1.005
Model 2				
Exports	1.005	1.022	**0.999**	1.006
Imports	1.023	1.413	**0.999**	1.017
International reserves	1.011	1.040	1.027	1.006
Long-term interest rate	**0.999**	1.040	**0.992**	1.000
Short-term interest rate	**0.998**	1.037	**0.996**	**0.997**
Yield spread	1.020	1.092	1.026	1.018
Model 3				
Exports	1.002	1.026	1.022	**0.999**
Imports	1.008	1.036	1.009	**0.998**
International reserves	1.020	1.007	1.021	1.008
Long-term interest rate	1.011	1.012	1.016	1.005
Short-term interest rate	1.003	1.028	1.041	1.009
Yield spread	1.019	1.036	1.033	1.027

Notes: The table reports the ratio of the MSFE of the RS model over the MSFE of the RW model. Bold indicates superiority of the RS model.

We further test the robustness of our findings by changing the starting date of the estimation sample. To do so, we repeat the analysis by moving the in-sample period three years (that is, 36 observations) forward. The results are reported in Table 14 and reveal that, in general, the main findings of our forecast exercise remain unchanged.

Finally, we extend our analysis to three other currency pairs. Specifically, we repeat the analysis, covering the same period, for the US dollar against the Japanese Yen, the UK pound against the Swedish Krona and the US dollar against the Canadian dollar. For brevity, we only report, in Table 15, the MSFE ratios for the one-month forecast horizon. In all three cases, we can identify specifications of our RS models that outperform the RW model. However, we observe significant differences among exchange rates. Specifically, in the case of the US dollar against the Japanese Yen (Panel A of Table 15) the only bubble measure that has good predictive power for the exchange rate is the fourth one. All other bubble measures fail to generate more accurate forecasts compared with the RW model. This finding contradicts the evidence from the UK pound to US dollar exchange rate where the fourth bubble measure was the one with the poorest forecasting ability. Turning our attention to the UK pound against the Swedish Krona exchange rate, the results, reported in Panel B of Table 15, suggest that the first and fourth bubble measures outperform the other two. Interestingly, the yield spread seems to be the optimal predictor. Finally, our RS models rarely beat the RW model when we consider the US dollar against the Canadian dollar exchange rate. In this case (Panel C of Table 15), the short-term interest rate appears to improve the forecasting accuracy of the two-state RS model but this is not true for the three-state model.

In summary, the robustness analysis reveals that the findings of our study are, in general, robust to the number of out-of-sample observations and the starting and end dates of the estimation period. When we extend the analysis to alternative exchange rates, we can still identify RS that generate reliable forecasts. However, we also observe some important differences among exchange rates. It seems that the optimal bubble measures and optimal predictors critically depend on the exchange rate.

8. Conclusions

Forecasting exchange rates is of great interest to all participants in international financial markets. In recent years, the exchange rate markets exhibit increased variability and we often witness extreme market movements. This has attracted researchers to develop various models in an effort to obtain reliable forecasts for the movements in exchange rates. However, numerous studies in the literature reveal that, in most cases, sophisticated econometric models fail to generate more accurate predictions than a naive RW model.

The motivation of this study stems from the results obtained from the implementation of a new methodology for bubble identification, introduced by Phillips, Shi, et al. (2011), on the UK pound to US dollar exchange rate during the post-1973 period. The test provides strong evidence of a periodically collapsing bubble in the exchange rate. We, therefore, use various models of exchange rate determination to calculate four different bubble measures. In all cases, we observe systematic divergence of actual prices from fundamental values in the exchange rate market. At some point however, a sharp reversal of market exchange rates to fundamental values starts. This kind of behavior can be explained by the theory of speculative bubbles and self-fulfilling expectations. Among others, van Norden and Schaller (1993) and Brooks and Katsaris (2005) link speculative bubbles to RS models. In line with the literature, we implement three alternative RS models to examine whether speculative bubbles drive the dynamics in the exchange rate under examination. Our first model is a simple two-state model, typically used in the literature, which relates the future return of the exchange rate to the bubble size. Model 2 enriches the specification of the first model by adding an explanatory variable that enters in both the conditional mean and the probability equations. We consider six explanatory variables that have been proposed in the literature as early warning indicators of a currency crisis, namely exports, imports, international reserves, long- and short-term interest rates and the yield spread. Finally, our third model extends Model 2 by allowing for a third state in the dynamics of the exchange rate. The evaluation of our models is carried out in terms of both statistical and economic significance relative to an RW model. The analysis is repeated for all four alternative bubble measures to reveal the effect of the bubble measure on the behavior of our models.

An out-of sample forecast exercise reveals that for one-period (i.e. one-month) forecasts all three RS models considered in this paper display predictive power for the exchange rate under scrutiny. Model 3 appears to be the optimal one (in terms of the MSFE criterion) given that either imports or short-term interest rates are included in the specification of the model. Under this specification, the superiority of Model 3 over the RW model is statistically significant and the same result holds for Model 2 with short-term interest rate as an explanatory variable. For multi-period forecasts, the predictive performance of our RS models relative to the RW model seems to deteriorate but we can still identify parameterizations that improve the forecasting performance of Models 2 and 3 compared with the RW model. Specifically, our findings suggest that the short-term interest rate remains the optimal variable (among the six considered in this study) for inclusion in Models 2 and 3, leading to lower MSFEs relative to the RW model.

Our three RS models do even better when we go beyond statistical criteria and apply economic evaluation criteria. Specifically, we use both a performance fee measure (calculated in a stylized asset allocation framework, where a mean–variance investor maximizes expected utility) and the manipulation-proof performance measure (proposed by Goetzmann et al. 2007) to compare our models relative to the RW model. The first measure can be interpreted as the fee that a risk-averse investor is willing to pay for switching from a portfolio allocation strategy based on the RW model to a portfolio that uses an RS model to forecast the exchange rate movements, while the second measure can be interpreted as a portfolio's premium return after adjusting for risk. Both measures of economic evaluation provide similar results and suggest significant utility gains for the investor from using our RS models for predicting changes in the exchange rate. Once again, Model 3 outperforms the other two RS models generating the higher utility gains (that reach up to 519 bps for a one-period horizon), especially when imports or short-term interest rates are included in its specification.

Finally, we develop two simple trading rules that use information from our estimated models to predict movements in the exchange rate market and lead to increased profits. The first trading rule is based on the estimated probabilities of a crash and a boom, while the second one uses the point forecasts of each RS model for the exchange rate return. In general, the results are both model and bubble dependent. The first trading strategy seems to work better than the second one, especially for Model 3 that contains either imports or interest rates (both long- and short-term).

In summary, our analysis for the British pound to US dollar exchange rate provides evidence that (i) the forecasting performance of our RS models is, in general, better than that of an RW model, especially in terms of economic evaluation criteria, (ii) a three-state RS model outperforms the two-state models and (iii) among the variables considered in our analysis, the short-term interest rate is the optimal variable, closely followed by imports, in both statistical and economic evaluation terms. A series of checks support the robustness of our main findings to the settings of our out-of-sample exercise. However, the extension of our analysis to other currency pairs reveals that the predictive power of the bubble measures and predictors examined in this study critically depends on the exchange rate. This raises the question as to whether there are alternative explanatory variables, such as microstructure variables (Gradojevic and Yang 2006) and liquidity measures (Adrian, Etula, and Shin 2011) that can be used to improve the accuracy of exchange rate forecasts. Moreover, it is possible that alternative models, such as Taylor rule-based models (Molodtsova and Papell 2012), neural networks (Qi and Wu 2003; Pacelli 2012) or time-varying parameter models (e.g. the RW coefficient model by Stock and Watson 1998 and the Bayesian time-varying parameter model by Canova 1993) could contribute to the quest for the optimal model for predicting exchange rates. Rossi (2013) provides a stimulating review of the methods, variables and models employed to predict exchange rates.

In general, our results and the findings in the existing literature imply that we are still unable to identify a model or predictor that systematically provides reliable forecasts for all exchange rates, time periods, data frequencies and forecast evaluation criteria. This clearly implies that central bankers and policymakers, who rely on exchange rate forecasts when taking crucial policy decisions, should be cautious on selecting the appropriate model/predictor. The same holds for international investors and corporations with foreign currency exposure. In general, FX market participants ought to be careful when generating exchange rate forecasts and take into account the aforementioned sensitivity of the forecasts. This raises the challenge of finding new ways to overcome the observed instabilities in the predictability of exchange rates.

Our analysis opens routes for future research where, for example, alternative explanatory variables related to exchange rates are considered as possible predictors of future movements in exchange rate markets. Another interesting extension is to investigate whether combination of forecasts from various exchange rate models can lead to accuracy gains for exchange rate forecasts. Finally, it would be very useful to extend the analysis to a more general investment environment by considering a more diversified portfolio where equities (both local and foreign) are included.

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Notes

1. For a literature review on the monetary approach, see Neely and Sarno (2002).
2. Panel regression techniques in conjunction with long-run relationships have shown some potential usefulness (Mark and Sul 2001).
3. See also Bollen, Gray, and Whaley (2000) and Chen and Lee (2006).
4. See also van Norden and Schaller (1993), Schaller and van Norden (1997, 1999) and van Norden and Vigfusson (1998).
5. The term ‘bubble’ is employed in the text to denote the deviation from fundamentals.
6. Details on bubble calculations and fundamental-based models are given in Section 3.
7. For a more detailed description of the model, see Rapach and Wohar (2002) and references therein.
8. For notational simplicity, we drop the superscript $i = A, B, C, D$ for the four bubble measures b_t^i.
9. van Norden (1996) includes an additional term, b_t^2, in the probability equation, while Brooks and Katsaris (2005) employ the absolute value of the bubble, $|b_t|$.
10. The subscripts c_0, c_1 refer to the constant and slope coefficient of the collapse state. Similarly, s_0, s_1 refer to the survival state and q_0, q_1 refer to the coefficients associated with the probability of collapse, q_t.
11. See also Evans (1991).
12. For an elaborate discussion on this issue, see Brooks and Katsaris (2005).
13. The derivation of the likelihood function is given in Appendix 1.
14. The calculation of the smoothed (ex post) probabilities is given in Appendix 2.

15. The following IFS codes are employed: nominal exchange rate: xx..AE.ZF…; price levels: xx64…ZF…; output levels (y-o-y growth rate): xx66…ZF…; short-term interest rate: xx60B..ZF…; long-term interest rate: xx60C…ZF…; international reserves: xx.1D.DZF…; exports: xx72..CZF…and imports: xx73..CZF…, xx refers to the country code, which is 112 for UK and 111 for US. The M3 monetary aggregate is sourced from the OECD Main Economic Indicators.
16. The explanatory variables do not enter Model 1, so Model 1 is estimated just once for each bubble measure.
17. The results for the other bubble measures and models are not reported for brevity but are available from the authors upon request.
18. Instead of using the asymptotic chi-squared distribution, we can apply Monte Carlo simulations (following the approach of Cheung and Erlandsson 2005) to obtain the empirical distribution of the LR statistic. We do not follow this approach because our goal is to examine the forecasting performance of all three RS models and thus model selection is not important in our analysis.
19. The (full-sample) estimated value of c is about -1.18×10^{-6}, while the estimate of σ is about 0.04.
20. See Marcellino, Stock, and Watson (2006) for additional details.
21. Appendix 3 provides a detailed description of the Clark and West methodology.
22. This is based on unreported results from Monte Carlo simulations (Clark and West 2007).
23. We employ the three-month Eurosterling and Eurodollar deposit rate for the domestic and foreign interest rate, respectively. The series are available from the Bank of England database.
24. Details on the forecasting scheme are given in Section 5.1.

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Appendix 1. Log-likelihood function of the three-state RS model

Let us assume that the variable of interest, R_{t+1}, can be in three different regimes (states) denoted by d (Dormant), c (Collapse) and s (Survival). We define the unobserved state variable S_t that determines the regime of the process in time t ($S_t = d, c$ or s). We can write the joint density function of R_{t+1} and S_{t+1} given information up to time t (we denote this information set by ξ_t) as the product of the marginal and conditional functions, that is,

$$f(R_{t+1}, S_{t+1}|\xi_t) = f(S_{t+1}|\xi_t)f(R_{t+1}|S_{t+1}, \xi_t).$$

If we sum over all possible values of S_{t+1}, we obtain the marginal density of the variable of interest:

$$\begin{aligned} f(R_{t+1}|\xi_t) &= \sum_{S_{t+1}=d,c,s} f(R_{t+1}, S_{t+1}|\xi_t) \\ &= \sum_{S_{t+1}=d,c,s} f(S_{t+1}|\xi_t)f(R_{t+1}|S_{t+1}, \xi_t). \end{aligned}$$

Given a sample of T observations, the log-likelihood function is given by

$$\begin{aligned} \ln L &= \sum_{t=1}^{T} \ln\left[\sum_{S_{t+1}=d,c,s} f(S_{t+1}|\xi_t)f(R_{t+1}|S_{t+1}, \xi_t)\right] \\ &= \sum_{t=1}^{T} \ln[f(S_{t+1} = d|\xi_t)f(R_{t+1}|S_{t+1} = d, \xi_t) \\ &\quad + f(S_{t+1} = c|\xi_t)f(R_{t+1}|S_{t+1} = c, \xi_t) \\ &\quad + f(S_{t+1} = s|\xi_t)f(R_{t+1}|S_{t+1} = s, \xi_t)]. \end{aligned}$$

In the context of our three-state RS model, we allow the conditional density function $f(R_{t+1}|S_{t+1}, \xi_t)$ to depend on various variables, such as the bubble measure (b_t) and the predictor (z_t). (For notational simplicity we drop the superscript $i = A, B, C, D$ for the four bubble measures b_t^i.) To be more specific, under regime d we have

$$R_{t+1} := R_{d,t+1} = \beta_{d0} + \varepsilon_{d,t+1}.$$

Under regime c, R_{t+1} is given by

$$R_{t+1} := R_{c,t+1} = \beta_{c0} + \beta_{c1}b_t + \beta_{c2}z_t + \varepsilon_{c,t+1},$$

while under regime s we have

$$R_{t+1} := R_{s,t+1} = \beta_{s0} + \beta_{s1}b_t + \varepsilon_{c,t+1}.$$

Therefore, the pdf of an observation under the assumption of being generated by a specific regime is given by

$$\begin{aligned} f_d(\varepsilon_{d,t+1}) &= f_d(R_{t+1} - \beta_{d0}), \\ f_c(\varepsilon_{c,t+1}) &= f_c(R_{t+1} - \beta_{c0} + \beta_{c1}b_t + \beta_{c2}z_t), \\ f_s(\varepsilon_{s,t+1}) &= f_s(R_{t+1} - \beta_{s0} + \beta_{s1}b_t), \end{aligned}$$

respectively. We now need to determine the probability law governing the switches from one regime to another. In the context of our three-state model, the probability of being in regime d (i.e. $R_{t+1} = R_{d,t+1}$) given the information set ξ_t, denoted by η_t, is given by $\Phi(\beta_{\eta_0} + \beta_{\eta_1}b_t + \beta_{\eta_2}\mathrm{sp}_t)$, where Φ is the cumulative density function of the standard normal distribution, b_t is the bubble and sp_t is the spread. This specification ensures that η_t lies between 0 and 1. Moreover, the probability of being in regime c (i.e. $R_{t+1} = R_{c,t+1}$) given the information set ξ_t equals $(1 - \eta_t)q_t$, where $q_t = \Phi(\beta_{q_0} + \beta_{q_1}b_t + \beta_{q_2}z_t)$. Finally, the probability of being in regime s (i.e. $R_{t+1} = R_{s,t+1}$) given the information set ξ_t is $(1 - \eta_t)(1 - q_t)$. In this way, the three probabilities sum up to unity. Under this specification, the likelihood function for each observation is

$$\eta_t \frac{f_d(\varepsilon_{d,t+1})}{\sigma_d} + (1-\eta_t)q_t \frac{f_c(\varepsilon_{c,t+1})}{\sigma_c} + (1-\eta_t)(1-q_t)\frac{f_s(\varepsilon_{s,t+1})}{\sigma_s}$$
$$= \eta_t \frac{f_d(R_{t+1}-\beta_{d0})}{\sigma_d} + (1-\eta_t)q_t \frac{f_c(R_{t+1}-\beta_{c0}+\beta_{c1}b_t+\beta_{c2}z_t)}{\sigma_c}$$
$$+ (1-\eta_t)(1-q_t)\frac{f_s(R_{t+1}-\beta_{s0}+\beta_{s1}b_t)}{\sigma_s}.$$

Let us further assume that $\varepsilon_{d,t+1}$, $\varepsilon_{c,t+1}$ and $\varepsilon_{s,t+1}$ follow i.i.d. normal distribution with zero mean and variances σ_d^2, σ_c^2 and σ_s^2, respectively. In this way, the log-likelihood function of our three-state RS model can be written as follows:

$$\ln L = \sum_{t=1}^{T} \ln\left[\eta_t \varphi\left(\frac{R_{t+1}-\beta_{d0}}{\sigma_d}\right)\sigma_d^{-1} + (1-\eta_t)q_t\varphi\left(\frac{R_{t+1}-\beta_{c0}-\beta_{c1}b_t-\beta_{c2}z_t}{\sigma_c}\right)\sigma_c^{-1}\right.$$
$$\left. +(1-\eta_t)(1-q_t)\varphi\left(\frac{R_{t+1}-\beta_{s0}-\beta_{s1}b_t}{\sigma_s}\right)\sigma_s^{-1}\right]$$
$$= \ln\prod_{t=1}^{T}\left[\eta_t \varphi\left(\frac{R_{t+1}-\beta_{d0}}{\sigma_d}\right)\sigma_d^{-1} + (1-\eta_t)q_t\varphi\left(\frac{R_{t+1}-\beta_{c0}-\beta_{c1}b_t-\beta_{c2}z_t}{\sigma_c}\right)\sigma_c^{-1}\right.$$
$$\left. +(1-\eta_t)(1-q_t)\varphi\left(\frac{R_{t+1}-\beta_{s0}-\beta_{s1}b_t}{\sigma_s}\right)\sigma_s^{-1}\right],$$

where $\varphi(\cdot)$ stands for the density function of the standard normal distribution.

Appendix 2. Calculation of the smoothed (ex post) probabilities of the three-state RS model

Once again, we define the unobserved state variable S_t that determines the regime of the process in time t ($S_t = d, c$ or s). The ex post (smoothed) probabilities, that is, $\Pr(S_{t+1} = i|\xi_t, R_{t+1})$ for $i = d, c, s$, of our three-state model are calculated as follows:

$$\Pr(S_{t+1}=i|\xi_t,R_{t+1}) = \frac{f(S_{t+1}=i,R_{t+1}|\xi_t)}{f(R_{t+1}|\xi_t)}$$
$$= \frac{f(R_{t+1}|S_{t+1}=i,\xi_t)\Pr(S_{t+1}=i|\xi_t)}{\sum_{i=d,c,s}(R_{t+1}|S_{t+1}=i,\xi_t)\Pr(S_{t+1}=i|\xi_t)}.$$

Thus, in the context of our specification, we obtain the following ex post probabilities:

$$\Pr(S_{t+1}=d|\xi_t,R_{t+1}) := P_t^{x,d} = \frac{\eta_t}{\sigma_d}\varphi\left(\frac{R_{t+1}-\beta_{d0}}{\sigma_d}\right)\left\{\frac{\eta_t}{\sigma_d}\varphi\left(\frac{R_{t+1}-\beta_{d0}}{\sigma_d}\right)\right.$$
$$+\frac{(1-\eta_t)q_t}{\sigma_c}\varphi\left(\frac{R_{t+1}-\beta_{c0}-\beta_{c1}b_t-\beta_{c2}z_t}{\sigma_c}\right)$$
$$\left.+\frac{(1-\eta_t)(1-q_t)}{\sigma_s}\varphi\left(\frac{R_{t+1}-\beta_{s0}-\beta_{s1}b_t}{\sigma_s}\right)\right\}^{-1},$$
$$\Pr(S_{t+1}=c|\xi_t,R_{t+1}) := P_t^{x,c} = \frac{(1-\eta_t)q_t}{\sigma_c}\varphi\left(\frac{R_{t+1}-\beta_{c0}-\beta_{c1}b_t-\beta_{c2}z}{\sigma_c}\right)\left\{\frac{\eta_t}{\sigma_d}\varphi\left(\frac{R_{t+1}-\beta_{d0}}{\sigma_d}\right)\right.$$
$$+\frac{(1-\eta_t)q_t}{\sigma_c}\varphi\left(\frac{R_{t+1}-\beta_{c0}-\beta_{c1}b_t-\beta_{c2}z_t}{\sigma_c}\right)$$
$$\left.+\frac{(1-\eta_t)(1-q_t)}{\sigma_s}\varphi\left(\frac{R_{t+1}-\beta_{s0}-\beta_{s1}b_t}{\sigma_s}\right)\right\}^{-1},$$

$$\Pr(S_{t+1}=s|\xi_t,R_{t+1}) := P_t^{x,s} = \frac{(1-\eta_t)(1-q_t)}{\sigma_s}\varphi\left(\frac{R_{t+1}-\beta_{s0}-\beta_{s1}b_t}{\sigma_s}\right)\left\{\frac{\eta_t}{\sigma_d}\varphi\left(\frac{R_{t+1}-\beta_{d0}}{\sigma_d}\right)\right.$$
$$+\frac{(1-\eta_t)q_t}{\sigma_c}\varphi\left(\frac{R_{t+1}-\beta_{c0}-\beta_{c1}b_t-\beta_{c2}z_t}{\sigma_c}\right)$$
$$\left.+\frac{(1-\eta_t)(1-q_t)}{\sigma_s}\varphi\left(\frac{R_{t+1}-\beta_{s0}-\beta_{s1}b_t}{\sigma_s}\right)\right\}^{-1}.$$

Appendix 3. Brief description of the Clark–West test (CW-t) for equal predictive accuracy

Let us assume that we are interested in comparing the forecast accuracy of a parsimonious model A relative to that of a larger model B (i.e. we have two nested models) based on the MSFE criterion. Let $\hat{R}_{A,t}$ and $\hat{R}_{B,t}$ denote the forecasts for R_t obtained from models A and B, respectively (in our case, model A corresponds to the RW model, while model B corresponds to one of the RS models), while P stands for the number of predictions obtained from each model. The forecast errors are then given by $(R_t-\hat{R}_{A,t})$ and $(R_t-\hat{R}_{B,t})$ for models A and B, respectively. The sample MSFEs, denoted by σ_A^2 and σ_B^2 for models A and B, respectively, are then easily calculated taking the sample averages of $(R_t-\hat{R}_{A,t})^2$ and $(R_t-\hat{R}_{B,t})^2$, respectively. That is,

$$\sigma_A^2=\frac{1}{P}\sum(R_t-\hat{R}_{A,t})^2,$$
$$\sigma_B^2=\frac{1}{P}\sum(R_t-\hat{R}_{B,t})^2.$$

Under the null hypothesis that the parsimonious model generates the data, the additional parameters of model B do not help predictions. Thus, in finite samples, model B loses efficiency due to the estimation of these parameters that introduces noise into the forecasts. This inflates the MSFE of model B. In summary, under the null we expect σ_A^2 to be smaller than σ_B^2. In this framework, the distribution used by Diebold and Mariano (1995), which assumes a zero mean for the difference $\sigma_A^2-\sigma_B^2$, provides a poor finite sample approximation since $\sigma_A^2-\sigma_B^2$ is expected to be negative in small samples. Clark and West (2007) properly adjust their statistic to have, under the null, approximate mean zero. Specifically, instead of using $\sigma_A^2-\sigma_B^2$ to construct a statistic to test the null hypothesis, they propose using the following ‘adjusted’ difference:

$$\sigma_A^2-\sigma_{B,\text{adjusted}}^2,$$

where

$$\sigma_{B,\text{adjusted}}^2=\frac{1}{P}\sum(R_t-\hat{R}_{B,t})^2-\frac{1}{P}\sum(\hat{R}_{A,t}-\hat{R}_{B,t})^2.$$

In this way, they properly adjust the statistic for the inflated MSFE of model B due to the estimation of parameters that are zero under the null. To be more specific, Clark and West introduce the following test statistic, denoted as CW-t:

$$\text{CW-}t=\sqrt{P}\frac{\bar{f}_t}{\sqrt{\text{var}(f_t-\bar{f}_t)}},$$

where

$$f_t=(R_t-\hat{R}_{A,t})^2-(R_t-\hat{R}_{B,t})^2+(\hat{R}_{A,t}-\hat{R}_{B,t})^2,\quad t=1,2,\ldots,P,$$

while $\bar{f}_t$ denotes the sample mean of f_t (i.e. $\bar{f}_t=(1/P)\sum f_t$) and $\text{var}(f_t-\bar{f}_t)$ is the sample variance of the difference $(f_t-\bar{f}_t)$. A straightforward way to obtain the CW-t statistic is by regressing f_t on a constant. CW-t is then given by the standard *t-statistic* of the constant term. Note that this is a one-sided test since under the alternative hypothesis, model B has lower MSFE than model A. Clark and West (2006, 2007) recommend using 1.282 and 1.645 for a 0.10 and 0.05 test, respectively. Their findings of extensive simulations suggest that these critical values provide reliable results.