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Stock market volatility and equity returns: Evidence from a two-state Markov-switching model with regressors

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ABSTRACT

This paper proposes a two-state Markov-switching model for stock market returns in which the state-dependent expected returns, their variance and associated regime-switching dynamics are allowed to respond to market information. More specifically, we apply this model to examine the explanatory and predictive power of price range and trading volume for return volatility. Our findings indicate that a negative relation between equity market returns and volatility prevails even after having controlled for the time-varying determinants of conditional volatility within each regime. We also find an asymmetry in the effect of price range on intra- and inter-regime return volatility. While price range has a stronger effect in the high volatility state, it appears to significantly affect only the transition probabilities when the stock market is in the low volatility state but not in the high volatility state. Finally, we provide evidence consistent with the 'rebound' model of asset returns proposed by Samuelson (1991), suggesting that long-horizon investors are expected to invest more in risky assets than short-horizon investors.

1. Introduction

Markov switching models have been extensively used in studies involving stock market returns, interest rates, foreign exchange rates, and return volatility forecasts. Researchers employ these regime switching techniques to account for specific features of macro-economic and financial time series such as the asymmetry of economic activity over the business cycle (Hamilton, 1989) or the fat tails, volatility clustering and mean reversion in stock prices (e.g., Cecchetti et al., 1990; Schaller and van Norden, 1997; Turner et al., 1989), interest rates (e.g., Ang and Bekaert, 2002; Gray, 1996; Hamilton, 1988), foreign exchange rates (e.g., Bollen et al., 2000; Dueker and Neely, 2007; Engel, 1994; Engel and Hamilton, 1990), and for improving volatility forecasts (e.g., Haas et al., 2004; Klaassen, 2002). The appeal of Markov switching models is that they give rise to parsimonious representations of state space models by letting the mean, variance as well as the dynamics of the series depend

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on the realization of a finite number of discrete states. They are particularly suited in situations where there are large changes in market volatility.[1]

In this paper we propose a two-state Markov switching model for stock market returns in which the state-dependent mean and return volatility as well as the transition probabilities of regime-switching are allowed to respond to changes in market information. We use this model to investigate the relationship between expected returns and market volatility accounting for shifts in investment opportunities linked to state-dependent changes in market volatility. In particular, we examine the explanatory and predictive power of price range and trading volume for return volatility. Our findings indicate that equity market returns are negatively correlated with volatility and that the effect of price range is asymmetric across the two regimes. However, we find no evidence that trading volume innovations are a significant determinant of state-dependent volatilities. Furthermore, although either the low expected return and high volatility state or the high expected return and low volatility state are persistent over short periods, our findings indicate a higher incidence of regime shifts between the two states over longer horizons. These results are consistent with the rebound model of mean-reverting asset returns proposed by Samuelson (1991), implying that long-horizon investors are expected to invest more in risky assets than short-horizon investors.

The time series of stock returns typically demonstrates several stylized facts characterizing the distributional and temporal properties of financial returns series; namely, leptokurtosis and asymmetry in the distribution, volatility clustering, structural breaks, and long memory. Markov switching models where the distribution of stock returns for a given period is a mixture of normals are capable of describing these stylized facts (Bulla and Bulla, 2006; Ryden et al., 1998). Turner et al. (1989) were the first to apply a Markov mixture of normal distributions to study the relation between the market risk premium and variance of stock returns using monthly excess returns on the S&P 500 index. They used a two-state Markov model characterized by two constant state-dependent expected returns and variances where the state dynamics are governed by a two-state first-order Markov chain with constant transition probabilities.

Extending the Turner et al. (1989) model, Schaller and van Norden (1997) include the price–dividend ratio as a determinant of both state-dependent expected returns and associated transition probabilities. Using monthly excess returns on the S&P 500 index, they provide robust evidence of Markov switching behavior for stock market returns. In particular, they report strong asymmetry in the effect of the price–dividend ratio on future expected returns; in the low-return state this effect is about four times larger than in the high-return state. On the other hand, they find that the price–dividend ratio has no significant effect on transition probabilities.

Our model for stock market returns is closest to the models considered by Schaller and van Norden (1997) and Turner et al. (1989), but with some important differences. We extend these models by incorporating regressors in the state-dependent volatilities through a link function, which allows us to directly assess sources of persistence on state-dependent volatilities. We use this model to produce new evidence on the relationship between market volatility and expected returns. In particular, we study the effect of two important volatility determinants; namely, price range and trading volume assessing their importance in terms of both explanatory power and predictability for return volatility.

The paper is organized as follows. Section 2 presents the proposed two-sate Markov switching model for stock market returns. Section 3 discusses the estimation method and volatility forecast performance measures. Section 4 describes the data, while Section 5 reports the empirical results. Finally, Section 6 presents conclusions.

2. The econometric model

Consider a stock portfolio. The return on the portfolio y_t for a period t has a mixed distribution of two normal densities with state-dependent expected returns and volatilities: (μ_{1t},σ_{1t}) if $S_t=1$ and (μ_{2t},σ_{2t}) if $S_t=2$ respectively, where the expected returns μ_{1t} and μ_{2t} may be affected by an $(m_1\times 1)$ vector of regressors, $x_{\mu,t}$, with two $(m_1\times 1)$ vectors of parameters, α_1 and α_2, through a linear link function $\mu_{jt}=\alpha'_j x_{\mu,t}, j=1,2$. The volatilities σ_{1t} and σ_{2t} may be affected by an $(m_2\times 1)$ vector of regressors, $x_{\sigma,t}$, with two $(m_2\times 1)$ vectors of parameters, β_1 and β_2, through an exponential link function $\sigma_{jt}=\exp(\beta'_j x_{\sigma,t}), j=1,2$. S_t is the state variable governed by a two-state first-order Markov chain with transition probabilities, $p_{ij}(t)$, associated with an $(m_3\times 1)$ vector of regressors, $x_{S,t}$, with two $(m_3\times 1)$ vectors of parameters, φ_1 and φ_2, through a logit link function

$$
\begin{aligned}
p_{11}(t) = P(S_t=1|S_{t-1}=1) = \mathrm{logit}\left(\varphi_1{}' x_{S,t-1}\right) p_{12}(t) = P(S_t=2|S_{t-1}=1) = 1-p_{11}(t) p_{22}(t) = P(S_t=2|S_{t-1}=2) \\
= \mathrm{logit}\left(\varphi_2{}' x_{S,t-1}\right) p_{21}(t) = P(S_t=1|S_{t-1}=2) = 1-p_{22}(t). \qquad (1)
\end{aligned}
$$

[1] More recent research has employed regime-switching vector autoregressions to capture the richer dynamics of bond and stock returns and their predictors (Guidolin and Timmermann, 2007; Henkel et al., 2011) and Markov-switching multifractals to model regime switching at low, intermediate and high frequencies (Calvet and Fisher, 2004, 2007). Markov-switching multifractal models compare favorably with standard volatility models both in- and out-of-sample yet their attractiveness stems from parsimony as they are able to capture the distributional nonlinearities of financial data with a very limited number of parameters. They permit estimation of switching models with an arbitrary large number of states thereby addressing some of the practical limitations encountered in the application of standard Markov-switching models (Calvet and Fisher, 2004). Most applications of multifractals involve univariate switching models as an extension to multivariate specifications would add considerable complexity to the parameter optimization problem.

The two-state Markov-switching model (hereafter MSM) with regressors as described above leads to the specification of the state-dependent portfolio return in the form

$$y_t \Big| S_t = \mu_{S_t t} + \sigma_{S_t t}\varepsilon_t, \varepsilon_t \tilde{} N(0,1) \tag{2}$$

where the errors ε_t are assumed to be independently distributed with respect to all past and future realizations of the state variable. As S_t is assumed to be the unobserved state variable in our model, conditional on the regressors $x_t = \{x_{\mu,t}; x_{\sigma,t}; x_{S,t}\}$, the portfolio return for period t has a two-component mixed normal distribution with expectation and volatility given by

$$E(y_t|x_t) = \mu_t(x_t) = \Pr(S_t = 1)\mu_{1t} + \Pr(S_t = 2)\mu_{2t} \tag{3}$$

$$\sqrt{Var(y_t|x_t)} = \sigma_t(x_t) = \sqrt{\Pr(S_t = 1)(\sigma_{1t}^2 + \mu_{1t}^2) + \Pr(S_t = 2)(\sigma_{2t}^2 + \mu_{2t}^2) - \mu_t^2}. \tag{4}$$

Note that $\Pr(S_t = 1) + \Pr(S_t = 2) = 1$, and $\Pr(S_t = 1)$ are either 0 or 1 for period t when the states are known. In the two-state MSM with regressors, stock market returns are allowed to switch between two states of market returns with time varying state-dependent volatilities. The state in each period determines which of the two normal distribution models with regressors is used to generate the portfolio return for that period. The two states are characterized by both the expectation and associated volatility function for the portfolio return. Moreover, the stochastic process of the state is governed by a two-state first-order Markov chain where the transition probabilities between the two states are dependent on regressors.

The likelihood function for this model is

$$L(\theta) = \left(p_1 f_{1,1}(y_1) + p_2 f_{2,1}(y_1)\right) \prod_{t=2}^{T} \left[\sum_{u=1}^{2}\sum_{v=1}^{2} p_{uv}(t) f_{v,t}(y_t)\right] \tag{5}$$

where $\theta = (\alpha_1{}' \alpha_2{}' \ \beta_1{}' \ \beta_2{}' \ \varphi_1{}' \ \varphi_2{}')'$ is a vector of all the model parameters, and $f_{i,t}(\cdot)$ is the normal density function with mean μ_{it} and standard deviation σ_{it}. Note that $p_1 = \Pr(S_1 = 1)$ and $p_2 = \Pr(S_1 = 2)$ are the unconditional probabilities of the initial state in the Markov chain and they are assumed to be known parameters in our model. The simulation study by Liu (2008) shows that the choice of these two probability values has little impact on the parameter estimates when the sample size is large.

Turner et al. (1989) consider a special case of the two-state MSM with no regressors and use it to model the portfolio excess return. They assume that there are two states in the market characterized by low and high variances of stock returns; and market agents know the structure of the Markov process generating the states, but are unsure of the prevailing state in the past, present and future. Hence market agents base their buying and selling decisions in period t on a prior distribution of the state in that period; i.e., $\Pr(S_t = i|\psi_{t-1}), i = 1, 2$, where ψ_{t-1} is the information set through to period $t-1$ which is available at the beginning of period t. Moreover, market agents evaluate both the expected values of the portfolio return and variance for period t conditional on the information set ψ_{t-1} as follows[2]:

$$E(y_t|\psi_{t-1}) = \Pr(S_t = 1|\psi_{t-1})\mu_1 + \Pr(S_t = 2|\psi_{t-1})\mu_2, \tag{6}$$

$$E(Var(y_t|\psi_{t-1})) = \Pr(S_t = 1|\psi_{t-1})\sigma_1^2 + \Pr(S_t = 2|\psi_{t-1})\sigma_2^2. \tag{7}$$

And they only use past returns as the information set ψ_{t-1} when they form their prior distribution of the state.

To extend the model considered by Turner et al. (1989), we assume that market agents observe not only past realizations of stock returns but also other relevant information. Agents use this information to form their prior distribution of the state and evaluate the expected portfolio return and volatility for each period. Specifically, market agents determine the prior distribution of the state for period t, $\Pr(S_t = i|\psi_{t-1})$, using relevant information including explanatory variables or regressors ($x_{S,t}$) in such a way that these variables are associated with the transition probabilities by a logit link function in a Markov setting with $\Pr(S_t = i|\psi_{t-1})$ determined by

$$\Pr(S_t = i|\psi_{t-1}) = \sum_{j=1}^{2} \Pr(S_t = i|S_{t-1} = j)\Pr(S_{t-1} = j|\psi_{t-1}) = \sum_{j=1}^{2} p_{ji}(t)\Pr(S_{t-1} = j|\psi_{t-1}). \tag{8}$$

Note that $\Pr(S_{t-1} = j|\psi_{t-1})$ is the posterior distribution of the state for period $t-1$, and is available information at the beginning of period t. Further extending the model considered by Turner et al. (1989), we allow the two state-dependent expected portfolio returns and volatilities to be associated with explanatory variables (or regressors), $x_{\mu,t}$ and $x_{\sigma,t}$ respectively, via a link function. Market agents use the information on these variables to evaluate the expected return and volatility for period t, and they make their buying and selling decisions in period t according to standard mean-variance theory and the prior

[2] Alternatively the expected variance can be estimated by

$$E(Var(y_t|\psi_{t-1})) = \Pr(S_t = 1|\psi_{t-1})\sigma_1^2 + \Pr(S_t = 2|\psi_{t-1})\sigma_2^2 + \{\Pr(S_t = 1|\psi_{t-1})\mu_1^2 + \Pr(S_t = 2|\psi_{t-1})\mu_2^2 - [\Pr(S_t = 1|\psi_{t-1})\mu_1 + \Pr(S_t = 2|\psi_{t-1})\mu_2]^2\}.$$

distribution of the state for that period. Adopting a deterministic link function, we can evaluate the effects of these variables on expected returns and volatility in a way similar to standard regression analysis. To the best of our knowledge, this is the first study that examines the explanatory power and predictability of regressors for return volatility in the class of two-state MSMs.

3. Estimation and forecasting

The maximum likelihood estimates of the parameters in the proposed model can be obtained by applying the Expectation Maximization (EM) algorithm of Dempster et al. (1977) as outlined by Hamilton (1990).[3]

To forecast the expected return and volatility of portfolio return for period $T+1$, we use all the information up to period T, and estimate the prior distribution of the state for period $T+1$ as

$$\Pr(S_{T+1}=i|\psi_T)=\sum_{j=1}^{2}\Pr(S_{T+1}=i|S_T=j)\Pr(S_T=j|\psi_T)=\sum_{j=1}^{2}p_{ji}(T+1)\Pr(S_T=j|\psi_T). \tag{9}$$

Note that the posterior distribution of the state for period T, $\Pr(S_T=j|\psi_T)$, is estimated by fitting the model to a sample of T periods. At the beginning of period $T+1$, we forecast the expected return and volatility as follows:

$$\tilde{\mu}_{T+1}=\Pr(S_{T+1}=1|\psi_T)\mu_{1T}+\Pr(S_{T+1}=2|\psi_T)\mu_{2T} \tag{10}$$

$$\tilde{\sigma}_{T+1}=\sqrt{\Pr(S_{T+1}=1|\psi_T)(\sigma_{1T}^2+\mu_{1T}^2)+\Pr(S_{T+1}=2|\psi_T)(\sigma_{2T}^2+\mu_{2T}^2)-\tilde{\mu}_{T+1}^2}. \tag{11}$$

After we observe the values of the regressors in the model for period $T+1$, we can update the expected return and volatility forecast as

$$\hat{\mu}_{T+1}=\Pr(S_{T+1}=1|\psi_{T+1})\mu_{1T+1}+\Pr(S_{T+1}=2|\psi_{T+1})\mu_{2T+1} \tag{12}$$

$$\hat{\sigma}_{T+1}=\sqrt{\Pr(S_{T+1}=1|\psi_{T+1})(\sigma_{1T+1}^2+\mu_{1T+1}^2)+\Pr(S_{T+1}=2|\psi_{T+1})(\sigma_{2T+1}^2+\mu_{2T+1}^2)-\hat{\mu}_{T+1}^2} \tag{13}$$

where μ_{jT+1} and σ_{jT+1} are evaluated with the observed values of the regressors for period $T+1$ and the parameter estimates from the in-sample estimation. The posterior distribution of the state is obtained from Bayes' theorem as

$$\Pr(S_{T+1}=i|\psi_{T+1})=\frac{\Pr(S_{T+1}=i|\psi_T)f_{i,T+1}(y_{T+1})}{\sum_{j=1}^{2}\Pr(S_{T+1}=j|\psi_T)f_{j,T+1}(y_{T+1})},i=1,2. \tag{14}$$

Note that $f_{j,T+1}(\cdot)$ is the normal density function with mean μ_{jT+1} and standard deviation σ_{jT+1}.

In order to evaluate the in-sample and out-of-sample fit in terms of volatility estimates and forecasts, we need a benchmark for true volatility. Most of the volatility forecast comparisons in the literature rely on some variant of the squared-return based method. As pointed out by Andersen and Bollerslev (1998), among others, for time series reflecting the return of closing price to next closing price, the squared return provides an unbiased measure of the true volatility. Andersen et al. (2003) propose an integrated volatility measure for the true intraday volatility by first discretely sampling prices at regular time intervals throughout a day, and then calculating the integrated variance based on the squared returns over each time interval. Since we consider weekly portfolio returns in this study, we use the weekly integrated volatility as the benchmark for the true weekly volatility defined by

$$\overline{\sigma}_t=\sqrt{\sum_{\tau=1}^{5}y_{t,\tau}^2} \tag{15}$$

where $y_{t,\tau}$ is the daily return of day τ in week t.

The in-sample fit of the volatility estimate is assessed using standard mean square error (MSE) and mean absolute deviation (MAD) criteria:

$$MSE_1(\text{in}-\text{sample})=T^{-1}\sum_{t=1}^{T}(\overline{\sigma}_t-\hat{\sigma}_t)^2 \tag{16}$$

$$MAD_1(\text{in}-\text{sample})=T^{-1}\sum_{t=1}^{T}|\overline{\sigma}_t-\hat{\sigma}_t| \tag{17}$$

[3] The estimation method and the issues related to the implementation of the EM algorithm for the two-state MSM with regressors are discussed in detail in Liu (2008).

where T is the in-sample size and $\hat{\sigma}_t$ is the estimated volatility for period t. For the out-of-sample forecast of volatility, we use two measures similar to MSE_1 and MAD_1 as:

$$MSE_2(\text{out}-\text{of}-\text{sample}) = T^{*-1}\sum_{t=T+1}^{T+T^*}(\overline{\sigma}_t-\tilde{\sigma}_t)^2 \tag{18}$$

$$MAD_2(\text{out}-\text{of}-\text{sample}) = T^{*-1}\sum_{t=T+1}^{T+T^*}|\overline{\sigma}_t-\tilde{\sigma}_t| \tag{19}$$

where T^* is the out-of-sample size and $\tilde{\sigma}_t$ is the one-step-ahead forecast of volatility.

Following the referee's comments, we consider the following in-sample and out-of-sample *MSE* and *MAD* measures for the performance of expected return forecasting:

$$MSE_1(\text{mean}) = T^{-1}\sum_{t=1}^{T}(y_t-\hat{\mu}_t)^2, \tag{20}$$

$$MAD_1(\text{mean}) = T^{-1}\sum_{t=1}^{T}|y_t-\hat{\mu}_t|, \tag{21}$$

$$MSE_2(\text{mean}) = T^{*-1}\sum_{t=T+1}^{T+T^*}(y_t-\tilde{\mu}_t)^2, \tag{22}$$

$$MAD_2(\text{mean}) = T^{*-1}\sum_{t=T+1}^{T+T^*}|y_t-\tilde{\mu}_t|. \tag{23}$$

4. Data and regressors

The raw data for our study includes the daily close, high and low prices of the S&P 500 stock index as well as the daily trading volume. We define a week as the period from the stock market close time on Wednesday to the stock market close time on the following Wednesday, and we calculate the weekly returns (y_t) on the S&P 500 stock index as 100 times the difference in the natural logarithm of the index using two consecutive Wednesday close prices for the sample period from 19 January, 1983 to 21 November, 2007.[4] The total number of observations in our sample is 1297. We split the sample into two parts: (1) an in-sample part used for parameter estimation corresponding to the period from 19 January, 1983 to 26 December, 2001 with 989 observations; and (2) an out-of-sample part used for volatility forecasting corresponding to the period from 2 January, 2002 to 21 November, 2007 with 308 observations.

To investigate variations in the volatility of portfolio returns, we relate volatility fluctuations to changes in two indicators of market conditions; namely, price range and trading volume. Stock price range is defined as the difference between the highest and lowest stock prices over a fixed sampling interval and is closely related to volatility of returns as price range is regarded as an alternative measure to return volatility. Parkinson (1980) derives a closed form expression for the relation between security price range and return volatility under the assumption that the price of security follows a normal diffusion process. As highlighted by Alizadeh et al. (2002), the price range is a highly efficient return volatility proxy, distilling volatility information from the entire price path, in contrast to volatility proxies based on the return, which use only the opening and closing prices. Andersen and Bollerslev (1998), Chou (2005), and Brandt and Diebold (2006) also conclude that price range is an efficient proxy of return volatility. In addition, the price range is commonly used as a technical indicator in stock investment (Edwards and Magee, 1997).

In our empirical analysis we define the price range as 100 times the difference in the natural logarithm of the index between the highest and lowest prices over a week. We use the natural logarithm of the lagged price range, $x_{1,t}=\ln(R_{t-1})$, where R_{t-1} is the price range for week $(t-1)$, as a regressor in the conditional volatility model. The augmented Dickey–Fuller unit root test for the price range series is highly significant with a test statistic of -6.81 (p-value $=0.000$),[5] suggesting that the series is stationary.

Since market agents can easily access past data on price range, we assume that market agents may also use this information to form their prior and posterior distributions of the state of stock market returns. Consequently, we use the average of the price ranges for week $(t-26)$ through week $(t-1)$ as a regressor in the transition probabilities, denoted as $x_{3,t}=\frac{1}{26}\sum_{k=1}^{26}R_{t-k}$. The augmented Dickey–Fuller unit root test for the series is highly significant with a test statistic of -3.50 (p-value $=0.008$), suggesting that the series is stationary.

[4] We do not use any data before July, 1982 because the compilation method of the high and low prices for the S&P 500 was amended around April, 1982.

[5] MacKinnon (1996) one-sided p-value.

Table 1
Descriptive statistics.

	y_t	$\bar{\sigma}_t$	$x_{1,t}$	$x_{2,t}$	$x_{3,t}$
Whole sample (19 January, 1983–21 November, 2007)					
Mean	0.175	1.892	0.953	0.002	2.965
Median	0.301	1.636	0.925	0.001	2.718
Std	2.126	1.264	0.504	0.144	1.082
Minimum	−16.663	0.217	−0.367	−1.064	1.418
Maximum	10.182	25.708	3.537	0.946	6.641
Skewness	−0.634	6.474	0.344	−0.036	1.141
Kurtosis	7.595	103.207	3.436	8.317	3.980
Number of observations = 1297					
In-sample (19 January,1983–26 December, 2001)					
Mean	0.208	1.890	0.971	0.001	2.995
Median	0.380	1.648	0.943	−0.001	2.798
Std	2.151	1.302	0.498	0.149	1.007
Minimum	−16.663	0.217	−0.367	−1.064	1.418
Maximum	7.505	25.708	3.537	0.946	6.565
Skewness	−0.888	7.365	0.302	−0.042	1.034
Kurtosis	7.968	118.982	3.536	8.600	3.947
Number of observations = 989					
Out-of-sample (2 January, 2002–21 November, 2007)					
Mean	0.068	1.898	0.894	0.004	2.870
Median	0.107	1.577	0.837	0.007	2.219
Std	2.043	1.139	0.518	0.130	1.292
Minimum	−7.161	0.345	−0.273	−0.577	1.499
Maximum	10.182	8.519	2.755	0.540	6.641
Skewness	0.304	2.035	0.499	0.005	1.345
kurtosis	6.367	9.238	3.259	6.145	3.809
Number of observations = 308					

Trading volume is another indicator of return volatility. Andersen (1996) studies the contemporaneous relation between return volatility and trading volume and concludes that it should be advantageous to utilize trading volume in conjunction with returns when constructing measures of return volatility. The rationale for this is that the information flow represents a stochastic volatility process that drives both returns and volume, so trading volume series will provide information regarding the state of the unobserved return volatility process.

To construct the trading volume regressor for the two-state MSM, we first calculate the natural logarithm of the average trading volume of the S&P 500 stock index over a week. Visually, this weekly trading volume series appears to be non-stationary with an upward trend. The augmented Dickey–Fuller unit root test for the series is insignificant with the test statistic of 0.254 (p-value = 0.976), confirming that the weekly trading volume series is not stationary. We then evaluate the first difference of the series and check it for stationarity. The augmented Dickey–Fuller unit root test for the first difference series is highly significant with a test statistic of −19.12 and (p-value = 0.000). Hence we define a trading volume regressor for conditional volatility as $x_{2,t} = \ln(v_{t-1}) - \ln(v_{t-2})$ where $v_t$is the weekly average trading volume for week t.

Table 1 reports the summary statistics of weekly returns, integrated weekly volatility, and MSM regressors. The sample distribution of the weekly returns appears asymmetric and leptokurtic with thicker tails than those of the normal distribution.

5. Empirical results

We first examine whether there is evidence for two distinct regimes in stock market returns. We start by comparing the performance of a constant mean and volatility model with that of a two-state MSM with no regressors. We then repeat the same exercise but include regressors in the volatility equation of the single regime model as well as in the volatility equation and transition probabilities of the two-state MSM. We also assess the explanatory and predictive power of trading volume and price range for return volatility; and we do this by examining in-sample fit and out-of-sample forecasts of return volatility for different specifications of single and dual regime models including the two-state MSM with regressors. Finally we interpret our findings.

5.1. Models with no regressors

The basic hypothesis of the two-state MSM with no regressors is that there exist two regimes in the stock market where the weekly stock return is characterized by a two-component mixed normal distribution with two constant regime-dependent expected returns and volatilities. To evaluate this hypothesis empirically, we fit the weekly returns series to both a simple linear model with constant mean and volatility (M) and a two-state MSM with no regressors (M1). Table 2 reports parameter estimates, log-likelihood values, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for the two

Table 2
Parameter estimates and standard errors (in brackets) of the normal distribution of returns with constant expected return and volatility model (M), and the two-state MSM with no regressor (M1).

Model	Sate j	Expected return	Return volatility	Transition probability	Log-likelihood	AIC	BIC
		μ_j	σ_j	p_{jj}			
M	1	0.208	2.151		−2160.2	4324.4	4334.2
		(0.068)	(0.048)				
M1	1	0.301	1.564	0.988	−2091.4	4194.7	4224.1
		(0.064)	(0.060)	(0.007)			
	2	0.035	2.937	0.980			
		(0.166)	(0.144)	(0.015)			

models. Both AIC and BIC are in favor of the two-state MSM with no regressors, supporting the presence of two distinct states for US stock market returns. Furthermore, the likelihood ratio test statistic of Hansen (1992) for the comparison between models M and M1 is 137.6. This value is well in excess of the 5% and 1% critical values of 13.52 and 17.67, respectively, as tabulated in Garcia (1998). Clearly we can reject the simple linear model with constant mean and volatility in favor of the two-state MSM with no regressors (M1).

According to model M1, when the US stock market is at state 1, the expected weekly market portfolio return is 0.301 (or 15.65% per year) with volatility 1.564 (or 11.23% per year); and when the US stock market is at state 2, the expected weekly market portfolio return drops to 0.035 (or 1.82% per year) with higher volatility of 2.937 (or 21.18% per year). These results are consistent with the findings of Schaller and van Norden (1997) who observe a negative correlation between state-dependent variances and returns. As suggested by Black (1976) and Christie (1982) changing financial leverage may cause higher variances to be associated with lower average returns.[6] High return-low volatility and low return-high volatility states are typically associated with the presence of bull and bear markets, respectively (see Ang and Bekaert, 2002; Chen, 2007; Maheu and McCurdy, 2000).

The estimated transition probabilities are $\hat{p}_{11}=0.988$ and $\hat{p}_{22}=0.980$, meaning that once the stock market is in the low volatility state (State 1) for one week then on average 98.8% of the time it will remain in that state the following week. Hence according to the results of model M1 there is only 1.2% probability that the market will switch to state 2 next week. Similarly, there is only a 2% probability that it will switch out of the high volatility state (State 2) in the ensuing week. This implies that either of the two states is very persistent. As in Turner et al. (1989), we can use these transition probabilities to estimate the duration during which the stock market remains in a given state by computing the maximum number of consecutive periods, denoted as n, satisfying

$$\Pr\left(S_{t+n}=i, S_{t+n-1}=i, \ldots, S_{t+1}=i \middle| S_t=i\right) > 0.5. \tag{24}$$

Hence, the duration is about 57 weeks for remaining in state 1 and 34 weeks in state 2, respectively.

Fig. 1 displays the S&P 500 stock index during the whole sample period (top panel), time plots of the estimated posterior probabilities of being in state 1 (middle panel) as well as the smoothed volatility estimates (bottom panel) based on model M1. A visual inspection of these graphs shows that the probability of being in state 1 is either very close to 1 or very close to 0 for most of the in-sample period. Periods where the probability of being in state 1 is very close to 0 (or equivalently, the probability of being in state 2 is close to 1) correspond to well-known market crashes, such as October 1987, the 1997 Asian financial crisis, and the 'dot-com' collapse in early 2000. The parameter estimates of Table 2 and the time plots of Fig. 1 provide complementary pictures of the nature of the two states, where state 1 may be referred as a tranquil state of the stock market and state 2 as a turbulent state. Note that we indentify the stock market to be in state 1 if the estimated probability of being state 1 exceeds 0.5 and in state 2, otherwise. The shaded areas in the middle and bottom panels of Fig. 1 represent the stock market to be in state 2.

5.2. Models with regressors

To examine the predictive power of price range and trading volume for return volatility, we consider two single-regime models with a volatility regressor (M01, M02) and three different specifications of the two-state MSMs with regressors (M2, M3, M4). The first two models, M01 and M02, assume that the weekly stock returns are normally distributed with constant expected return and return volatility associated with either the price range regressor ($x_{1,t}$) or the trading volume regressor ($x_{2,t}$), respectively. M2 includes $x_{1,t}$ as a regressor in state-dependent volatility but assumes constant transition probabilities, whereas M3 includes price range regressors, $x_{1,t}$ in the conditional volatility and $x_{3,t}$ in the transition probabilities. M4 includes the trading volume innovation series ($x_{2,t}$) as a regressor in both conditional volatility and transition probabilities. Table 3 reports in-sample parameter estimates for these models whereas Table 4 presents in-sample and out-of-sample model performance measures.

[6] Another explanation for the negative relationship between stock returns and their variances is volatility feedback (see Bae et al., 2007; Mayfield, 2004; Turner et al., 1989). And negative correlations may also arise from simple 'buy' and 'sell' trading rules (Brock et al., 1992).

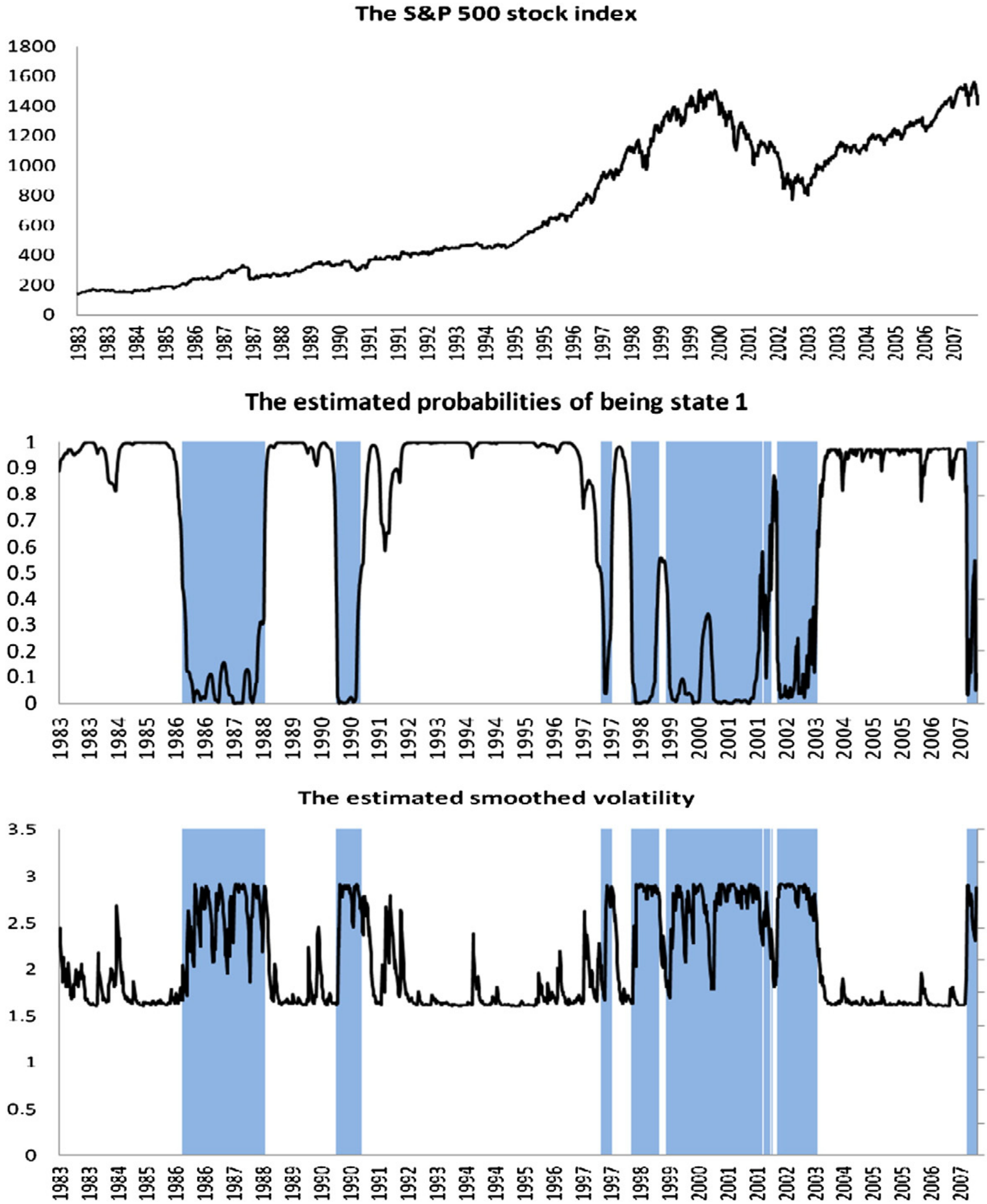

Fig. 1. Time plots of the S&P 500 stock index, the estimated probabilities of being in state 1 and smoothed volatility based on model (M1).

The results in Tables 3 and 4 show that both AIC and BIC favor the two-state MSMs (M1, M2, M3 and M4) over the single-regime models with volatility regressors (M01 and M02). This provides strong evidence for the presence of two distinct regimes in the US stock market characterized by different conditional expected returns and time-varying volatilities, corroborating the findings reported in Section 5.1.

Based on the likelihood ratio test, M3 is preferred to M1 with a likelihood ratio test statistic of 60.2 and p-value of 0.000; and M3 is preferred to M2 with a likelihood ratio statistic of 16.4 and p-value of 0.000. On the other hand, M1 is preferred to M4 as the associated likelihood ratio test statistic is 0.8 with p-value of 0.938. Furthermore, in terms of the in-sample and out-sample MSEs and MADs, M3 performs best among the four two-state MSMs, whereas M4 is not better than M1. This provides evidence that price range regressors have both explanatory and forecasting power for return volatility, but the trading volume regressor does not.

Table 3
Parameter estimates (Est) and standard errors (Std) of five models: two single-regime models where returns are assumed to be normally distributed with constant expected return and volatility on price range regressor $x_{1,t}$ (M01) and trading volume regressor $x_{2,t}$ (M02), respectively; and three specifications of the two-state MSM with regressors (M2, M3, M4).

	Regressor	M01	M02	M2	M3	M4
		Est (Std)	Est (Std)	Est (Std)	Est (Std)	Est (Std)
State 1						
Mean	Constant	0.161 (0.062)***	0.212 (0.068)***	0.439 (0.102)***	0.430 (0.093)***	0.299 (0.065)***
Volatility	Constant	0.266 (0.048)***	0.764 (0.022)***	0.047 (0.083)	0.091 (0.110)	0.448 (0.038)***
	$x_{1,t}$	0.454 (0.044)***		0.490 (0.054)***	0.184 (0.113)	
	$x_{2,t}$		0.123 (0.110)			0.101 (0.218)
Transition	Constant			1.883 (1.222)	4.971 (1.902)***	4.484 (0.664)***
Probabilities	$x_{3,t}$				−1.795 (0.870)**	
	$x_{2,t}$					−0.484 (5.843)
State 2						
Mean	Constant			−1.899 (1.731)	−0.195 (0.182)	0.048 (0.166)
Volatility	Constant			0.332 (0.385)	0.591 (0.121)***	1.077 (0.049)***
	$x_{1,t}$			0.745 (0.317)**	0.381 (0.083)***	
	$x_{2,t}$					0.101 (0.166)
Transition	Constant			−11.417 (99.322)	−4.723 (2.865)*	4.014 (0.871)***
Probabilities	$x_{3,t}$				1.317 (0.826)	
	$x_{2,t}$					−2.084 (5.242)

* Significant at 10%.
** Significant at 5%.
*** Significant at 1%.

In Table 3, the coefficient estimates of $x_{1,t}$ in the conditional volatility equations of M2 and M3 are all positive as expected, suggesting that an increase (decrease) in the price range this week signals an increase (decrease) in return volatility next week irrespective of the state of stock market returns. However, this effect is much stronger in state 2 as is evident from the relative magnitude of estimated coefficients. The effect of price range on volatility is broadly consistent with the results of the conditional autoregressive range model of Chou (2005) for the weekly S&P 500 index. We also find evidence that the price range has an effect on the regime switching dynamics. The coefficient estimate of $x_{3,t}$ in M3 is negative and significant for $p_{11}(t)$, indicating that an increase (decrease) in the average price range over the past 26 weeks lowers (raises) the probability of remaining in the low volatility state. Alternatively, an increase in the average price range raises the probability of switching from the low volatility to the high volatility regime. On the other hand, the point estimate of the coefficient of $x_{3,t}$ in $p_{22}(t)$ is positive but at a much smaller magnitude and is also statistically insignificant, suggesting that the likelihood of shifts from the high to the low volatility state does not decrease (increase) in response to decreases (increases) in the average price range.

According to the results of model M3, the two states of the stock market are characterized by high expected return (0.430) and a low expected return (−0.195), respectively; and the high expected return state (State 1) corresponds to low return volatility, and the low expected return state (State 2) to high return volatility. In Fig. 2, the top panel displays the estimated state-dependent volatilities for the two states of high and low expected returns and the bottom panel the estimated transition probabilities of remaining in state 1 and state 2, respectively. Note that both the volatility level and the difference between the two volatility regimes are much larger during market turmoil periods like the 1987 stock market crash and the dot-com bubble in the early 2000s. Note also that the estimated smoothed volatility based on model M3 is plotted in Fig. 3.

Table 4
In-sample and out-of-sample performance measures for the seven models: the three single regime models (M, M01 and M02) and the four two-state MSMs (M1–M4).

Model	M	M01	M02	M1	M2	M3	M4
In-sample							
Log likelihood	−2160.2	−2102.7	−2159.6	−2091.4	−2069.5	−2061.3	−2091.0
AIC	4324.4	4211.3	4325.1	4194.7	4155.1	4142.6	4202.1
BIC	4334.2	4226.0	4339.8	4224.1	4194.2	4191.6	4251.1
MSE_1	1.76	1.41	1.76	1.36	1.14	1.14	1.36
MAD_1	0.87	0.72	0.87	0.69	0.64	0.60	0.69
MSE_1 (mean)	4.62	2.20	2.42	2.05	1.69	1.64	2.05
MAD_1 (mean)	1.59	1.17	1.25	1.12	1.03	0.99	1.12
Out-sample							
MSE_2	1.36	0.77	1.35	0.82	0.74	0.64	0.82
MAD_2	0.90	0.64	0.90	0.65	0.63	0.58	0.65
MSE_2 (mean)	4.18	1.80	2.43	1.89	1.78	1.65	1.89
MAD_2 (mean)	1.48	1.08	1.29	1.07	1.07	1.00	1.07

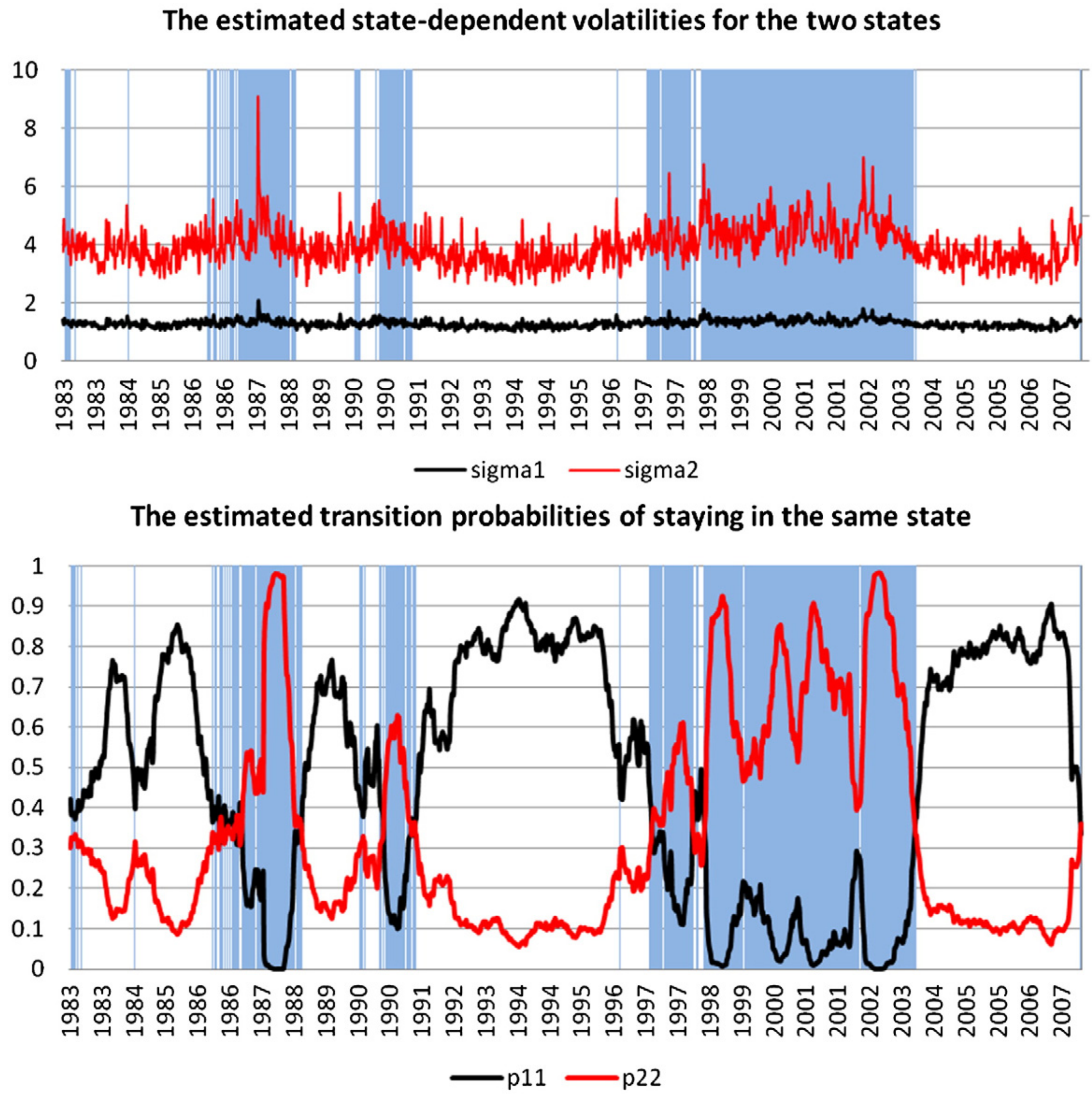

Fig. 2. Plots of estimated state-dependent volatilities and transition probabilities of remaining in the same state based on the parameter estimates of M3.

Therefore, our results show that after having controlled for the time-varying intra- and inter-regime determinants of volatility, there still exists evidence of a negative relationship between equity market return and volatility; volatility is high in the low expected return state and low in the high expected return state.[7] While evidence of asymmetry in the relationship between equity market returns and volatility is not new, previous results have been based on Markov-switching volatility models in which conditional volatility was constant within each regime (e.g., Schaller and van Norden, 1997; Turner et al., 1989). According to the new evidence we provide the negative relation between equity market return and volatility holds even when the conditional intra- and inter-regime volatility is time-varying.

Furthermore, the bottom panel of Fig. 2 shows that the two estimated transition probabilities for remaining in the same state vary over time and at most one of them exceeds 0.5 at any point of time. Based on this we can define three different types of time periods: Type I during which the stock market is more likely dominated by state 1 with low volatility and high expected return during, Type II during which the stock market is more likely dominated by state 2 with high volatility and low expected return,

[7] As pointed out by an anonymous referee, we have not controlled for leverage or volatility feedback in the specification of our model. While this is true, we note that to the extent that negative (positive) changes in returns are associated with increases (decreases) in price range, our evidence is consistent with the presence of volatility feedback as reported in the literature (see Bae et al., 2007). If this is the case, then our finding of a positive correlation between price range and subsequent volatility regime is consistent with a negative correlation between return innovations and the subsequent volatility regime (see Bae et al., 2007: p. 48).

and Type III during which neither of these two states are dominant.[8] For the whole sample period, Type I periods account for 52.0% of the time, Type II periods for 25.5% of the time, and Type III periods for 22.5% of the time.[9] The bottom panel of Fig. 2 also shows that the high volatility state with lower expected return is more likely to be persistent during market turmoil periods like the 1987 stock market crash and the dot-com bubble in the early 2000s.

Note that the shaded areas in Fig. 2 tend to coincide with the stock market being in the high volatility regime as the estimated probability of being in state 1 is less than 0.5. It is worth mentioning that the classification of volatility regimes for the two-state volatility MSMs is based on the Bayesian probabilities of being at either of the two states using information on observed returns. When the magnitude of a shock to returns is similar to the volatility level of a given state, it is more likely that the volatility regime will be identified with that state. However, the inference problem becomes much more difficult when volatilities are time-varying within each state where even relatively small volatility changes may trigger a regime change.[10]

To assess further the importance of price range and trading volume for return volatility, we compare the forecasting performance of MSM with three different types of GARCH volatility models with regressors; namely, GARCH(1,1), T-GARCH (1,1), and EGARCH(1,1), each with two alternative specifications including either the price range regressor ($x_{1,t}$) or the trading volume regressor ($x_{2,t}$) in the conditional variance equation. We also fit the data to the Markov-switching (MS)-GARCH(1,1) model with no regressors proposed by Klaassen (2002).[11] The parameter estimates and standard deviations of these models are presented in Table 5 and the associated performance measures for these models are shown in Table 6. Table 5 shows that the price range regressor $x_{1,t}$ is highly significant in the three GARCH models, whereas the trading volume regressor $x_{2,t}$ is not, confirming the findings of the two-state MSMs reported earlier.

Tables 4 and 6 show that while both AIC and BIC favor an EGARCH(1,1) model with exogenous price range regressor, $x_{1,t}$, for in-sample model selection, model M3 is superior to any of the alternative models for volatility estimation in terms of MSE_1 and MAD_1 values. More importantly, M3 is superior to any of these alternative models in terms of out-of-sample volatility forecasts as it yields the lowest MSE_2 and MAD_2 values. In addition, M3 has the smallest MSE and MAD values for mean forecasting for both in-sample and out-of-sample.

Following the suggestion of the referee, we have also examined the usefulness of historical realized volatility for return volatility forecasts. More specifically, we compare the two-state Markov model with the heterogeneous autoregressive model of realized volatility (HAR-RV) proposed by Corsi (2009) in terms of both in-sample and out-of-sample volatility forecast performances. We utilize weekly and monthly observed realized volatilities, $RV_t^{(w)}$ and $RV_t^{(m)}$, respectively, to generate weekly return volatility forecasts. For the two-state MSM, we include as regressors $\ln(RV_{t-1}^{(w)})$ and $\ln(RV_{t-1}^{(m)})$ in the conditional volatility, and $RV_{t-1}^{(w)}$ and $RV_{t-1}^{(m)}$ in the transition probabilities. Based on the in-sample estimates of this model,[12] we calculate in-sample and out-of-sample one-step-ahead volatility forecasts. The forecast performance measures are $MSE_1=1.12$ and $MAD_1=0.62$ for in-sample volatility forecasts, and $MSE_2=0.69$ and $MAD_2=0.62$ for out-of-sample.[13] In comparison the corresponding values we obtain using the fitted in-sample HAR-RV model[14] are $MSE_1=1.35$ and $MAD_1=0.66$ for in-sample, and $MSE_2=0.74$ and $MAD_2=0.60$ for out-of-sample volatility forecasts. Therefore the two-state MSM with realized volatility regressors performs better than the HAR-RV model in terms of in-sample volatility forecasts, and also exhibits better out-of-sample performance on the basis of MSE. Furthermore, we note that model M3 (see Table 4) is superior to both the HAR-RV model and the two-state MSM with realized volatility regressors in terms of both in-sample and out-of-sample forecast performances.

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[8] The average value of the probability of remaining in the current state over the whole sample period is less than 0.5 (0.483 for p_{11} and 0.335 for p_{22}), suggesting a higher incidence of regime shifts over longer horizons. This finding is broadly consistent with Samuelson's (1991) rebound model for mean reverting stock returns. Using a two-state Markov model, Samuelson defines the rebound case as a Markov process with frequent reversals where there is a higher probability of switching between states than remaining in the current state (i.e., p_{11}, $p_{22}<0.5$). He shows that risk-averse long-horizon investors with strictly concave long-term utility functions will tolerate in the rebound case a higher fraction of risky assets than in the random walk case (i.e., $p_{11}=p_{22}=0.5$). The reverse will be true in the case of a momentum process (i.e.,p_{11}, $p_{22}>0.5$); in this case long-horizon investors will tolerate a lower fraction of risky assets.

[9] Clearly once we allow for intra- and inter-regime volatility dynamics, we find much less persistence in either of the two states in comparison with the findings of Section 5.1. These differences may reflect the extra layer of uncertainty characterizing models with regressor dependent volatilities. In the two-state MSM (M1) with constant state-dependent volatilities there is no uncertainty over the magnitude of future volatility changes although there is uncertainty over the timing at which the volatility level will change. Mayfield (2004) reports a similar result; namely, a big drop in the probability that the market will remain in the high volatility state following a structural shift in the volatility process.

[10] As pointed out by an anonymous referee, the classification problem may be addressed by resorting to a model with nested regimes as in Geweke and Keane (2007).

[11] We used the code provided by Klaassen for the estimation of his model which does not include regressors in the conditional variance equation.

[12] The estimated two-state MSM (standard errors in brackets) is given by:

$$\underset{(0.110)}{\mu_1 = 0.493},\ \sigma_{1,t} = \exp\Big(\underset{(0.138)}{-0.334}\underset{(0.086)}{-0.050}\ln\Big(RV_{t-1}^{(w)}\Big) + \underset{(0.115)}{0.680}\ln\Big(RV_{t-1}^{(m)}\Big)\Big), p_{11}(t) = \text{logit}\Big(\underset{(1.169)}{1.873}\underset{(0.397)}{-0.079}RV_{t-1}^{(w)} + \underset{(0.268)}{0.132}RV_{t-1}^{(m)}\Big),$$

$$\underset{(1.141)}{\mu_2 = -2.183},\ \sigma_{2,t} = \exp\Big(\underset{(0.603)}{-0.681} + \underset{(0.337)}{0.078}\ln\Big(RV_{t-1}^{(w)}\Big) + \underset{(0.490)}{1.193}\ln\Big(RV_{t-1}^{(m)}\Big)\Big), p_{22}(t) = \text{logit}\Big(\underset{(2.242)}{-1.597} + \underset{(2.354)}{1.957}RV_{t-1}^{(w)}\underset{(1.201)}{-0.959}RV_{t-1}^{(m)}\Big).$$

[13] Consistent with previous findings (e.g. Corsi, 2009; Ghysels et al., 2006) our results provide further evidence that cascade models utilizing observed realized volatilities sampled at different frequencies are useful for volatility forecasting. The MSE and MAD values we obtain from the two-state MSM with realized volatility regressors are smaller than those of model M1 (see Table 4) both in-sample and out-of-sample.

[14] The fitted HAR-RV (standard errors in brackets) is given by:

$$RV_t^{(w)} = \underset{(0.076)}{0.852} + \underset{(0.043)}{0.029}RV_{t-1}^{(w)} + \underset{(0.025)}{0.246}RV_{t-1}^{(m)} + \omega_t.$$

Table 5
Parameter estimates and standard errors (in brackets) of GARCH(1,1), T-GARCH(1,1), EGARCH(1,1) and MS-GARCH(1,1) models.

GARCH(1,1) with the price range regressor ($x_{1,t}$):

$$y_t = \underset{(0.065)}{0.221} + \varepsilon_{t-1}, \varepsilon_t \tilde{} N(0,\sigma_t),$$
$$\sigma_t^2 = \underset{(0.026)}{-0.023} + \underset{(0.013)}{0.035}\varepsilon_{t-1}^2 + \underset{(0.028)}{0.886}\sigma_{t-1}^2 + \underset{(0.133)}{0.390}x_{1,t}$$

GARCH(1,1) with the trading volume regressor ($x_{2,t}$):

$$y_t = \underset{(0.063)}{0.235} + \varepsilon_{t-1}, \varepsilon_t \tilde{} N(0,\sigma_t),$$
$$\sigma_t^2 = \underset{(0.025)}{0.071} + \underset{(0.014)}{0.071}\varepsilon_{t-1}^2 + \underset{(0.017)}{0.917}\sigma_{t-1}^2 \underset{(0.934)}{-0.031}x_{2,t}$$

T-GARCH(1,1) with the price range regressor ($x_{1,t}$):

$$y_t = \underset{(0.062)}{0.181} + \varepsilon_t, \varepsilon_t \tilde{} N(0,\sigma_t), I_t = 1 \text{ if } \varepsilon_t < 0, \text{and } 0 \text{ otherwise},$$
$$\sigma_t^2 = \underset{(0.040)}{-0.004} \underset{(0.021)}{-0.085}\varepsilon_{t-1}^2 + \underset{(0.023)}{0.179}\varepsilon_{t-1}^2 I_{t-1} + \underset{(0.030)}{0.859}\sigma_{t-1}^2 + \underset{(0.129)}{0.585}x_{1,t}$$

T-GARCH(1,1) with the trading volume regressor ($x_{2,t}$):

$$y_t = \underset{(0.062)}{0.180} + \varepsilon_t, \varepsilon_t \tilde{} N(0,\sigma_t), I_t = 1 \text{ if } \varepsilon_t < 0, \text{and } 0 \text{ otherwise},$$
$$\sigma_t^2 = \underset{(0.062)}{0.278} \underset{(0.023)}{-0.006}\varepsilon_{t-1}^2 + \underset{(0.027)}{0.202}\varepsilon_{t-1}^2 I_{t-1} + \underset{(0.024)}{0.842}\sigma_{t-1}^2 \underset{(1.028)}{-0.398}x_{2,t}$$

EGARCH(1,1) with the price range regressor ($x_{1,t}$):

$$y_t = \underset{(0.062)}{0.187} + \varepsilon_t, \varepsilon_t \tilde{} N(0,\sigma_t),$$
$$\log(\sigma_t^2) = \underset{(0.038)}{0.064} \underset{(0.050)}{-0.086}\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| \underset{(0.022)}{-0.173}\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \underset{(0.041)}{0.787}\log(\sigma_{t-1}^2) + \underset{(0.060)}{0.300}x_{1,t}$$

EGARCH(1,1) with the trading volume regressor ($x_{2,t}$):

$$y_t = \underset{(0.062)}{0.151} + \varepsilon_t, \varepsilon_t \tilde{} N(0,\sigma_t),$$
$$\log(\sigma_t^2) = \underset{(0.031)}{-0.033} + \underset{(0.034)}{0.186}\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| \underset{(0.018)}{-0.153}\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \underset{(0.017)}{0.922}\log(\sigma_{t-1}^2) + \underset{(0.295)}{0.065}x_{2,t}$$

MS-GARCH(1,1):

$$\mu_1 = \underset{(0.054)}{0.33}, \; \sigma_{1,t}^2 = \underset{(0.052)}{0.14} + \underset{(0.020)}{0.034}e_{t-1}^2 + \underset{(0.026)}{0.834}\sigma_{t-1}^2, p_{11} = \underset{(0.008)}{0.960}$$

$$\mu_2 = \underset{(0.301)}{-3.69}, \sigma_{2,t}^2 = \underset{(1.040)}{0.00} + \underset{(0.020)}{0.406}e_{t-1}^2 + \underset{(0.411)}{0.594}\sigma_{t-1}^2, p_{22} = \underset{(0.008)}{0.054}$$

6. Conclusion

In this paper we have proposed a two-state Markov-switching model for stock market returns where the state-dependent expected returns and volatilities depend on regressors and the dynamics of the state for stock market returns are governed by a two-state first-order Markov chain with regressor-dependent transition probabilities. We have used different specifications of this model to investigate the explanatory and predictive power of two volatility determinants, namely price range and trading volume. We find that price range has significant explanatory and predictive power for return volatility. However, we find no evidence that trading volume shocks have a significant effect on intra- and inter-regime volatility. Our findings indicate that even if we account for shifts in investment opportunities (state-dependent changes in volatility) via price range, the level of risk measured by state-dependent return volatility remains negatively correlated with the state-dependent expected return. We also find state-dependent differences in the effect of price range on volatility; the intra-regime effect is stronger in the high volatility state whereas the inter-regime effect appears to be stronger in the low volatility state. These findings provide new evidence of asymmetry in the relationship between time-varying conditional volatility and equity market returns.

In addition, our analysis indicates that by using information on price range, market agents may be able to obtain better volatility forecasts. An increase (decrease) in price range over a week indicates an increase (decrease) in the market volatility, whereas an increase (decrease) in the average price range over the last six months signals an increase (decrease) in the likelihood that the stock market is in the high volatility regime.

Overall, our findings demonstrate strong evidence of switching behavior in the US stock market with equities switching between two states: a low expected return and high volatility state, and a high expected return and low volatility state, respectively. While either of the two states is persistent over short periods, the average probability of remaining in the current

Table 6
In-sample and out-of-sample performance measures for GARCH(1,1), T-GARCH(1,1), EGARCH(1,1) and MS-GARCH(1,1) models.

Model	GARCH(1,1)		T-GARCH(1,1)		EGARCH(1,1)		MS-GARCH(1,1)
	$x_{1,t}$	$x_{2,t}$	$x_{1,t}$	$x_{2,t}$	$x_{1,t}$	$x_{2,t}$	
In-sample							
Log likelihood	−2090.8	−2099.0	−2073.5	−2087.9	−2066.4	−2085.6	−2063.4
AIC	4191.6	4207.9	4159.0	4187.7	4144.8	4183.1	4146.8
BIC	4223.0	4232.4	4188.4	4217.1	4174.2	4212.5	4195.8
MSE_1	1.37	1.42	1.31	1.40	1.26	1.35	1.46
MAD_1	0.69	0.71	0.67	0.71	0.65	0.69	0.70
MSE_1 (mean)	4.62	4.62	4.62	4.62	4.62	4.62	2.25
MAD_1 (mean)	1.58	1.58	1.59	1.59	1.59	1.59	1.12
Out-sample							
MSE_2	0.88	1.67	0.85	1.33	0.66	1.33	0.75
MAD_2	0.68	1.09	0.64	0.88	0.59	0.88	0.60
MSE_2 (mean)	4.18	4.19	4.17	4.17	4.17	4.17	1.73
MAD_2 (mean)	1.48	1.48	1.48	1.48	1.48	1.48	0.98

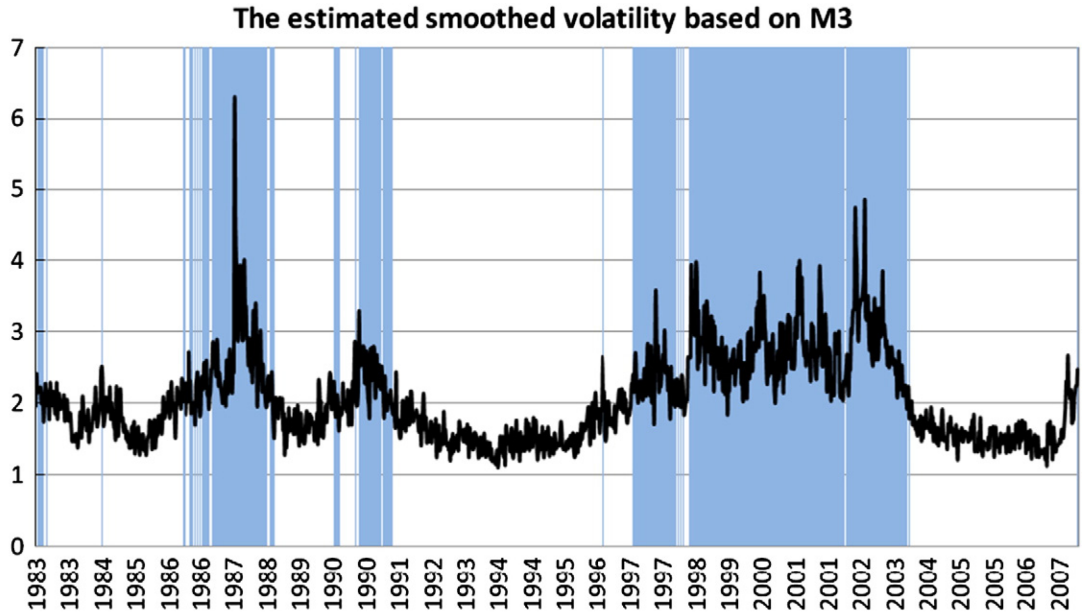

Fig. 3. Plot of estimated smoothed volatility based on M3.

state over the whole sample period is 0.483 for the high expected return state and 0.335 for the low expected return state, respectively. This suggests a higher incidence of regime shifts over longer horizons, which is broadly consistent with the rebound process for asset returns proposed by Samuelson (1991). Samuelson shows that in the presence of rebound or mean-reverting process for security returns risk-averse investors with strictly concave long-term utility functions will tolerate a higher fraction of risky assets in their portfolio than they would with serially independent returns; and they are more likely to increase their exposure to the risky asset as the investment horizon increases. Thus our findings have potentially important implications for financial practice. With mean-reverting security returns, long-horizon investors are expected to invest more in risky assets than short-horizon investors if there is a higher probability to switch between regimes rather than stay in the current state. These findings contrast earlier results suggesting that investors with longer horizons would favor fewer risky assets when there are recurrent but infrequent regime shifts.

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