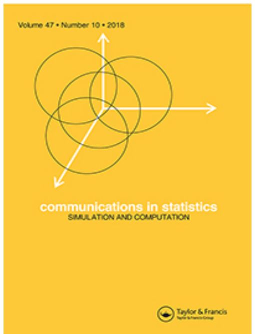

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Nonlinear high-frequency stock market time series: Modeling and combine forecast evaluations

Wen Cheong Chin[a] and Min Cherng Lee[b]

[a]Department of Mathematics, Xiamen University Malaysia, Sepang, Selangor Darul Ehsan, Malaysia; [b]School of Information Technology, Monash University, Subang Jaya, Selangor, Malaysia

ABSTRACT

This study intends to examine two major challenges in high-frequency (intraday) volatility analysis: *first*, the selection of a suitable realized volatility and *second*, the appropriate statistical model for the intraday volatility analysis. In order to address the first challenge, the multiple power variation volatility, nearest neighboring truncated volatility and range-based volatility have been included in the volatility selections. For the intraday volatility modeling, a skewed heterogeneous autoregressive (HAR) model with the features of leverage effect and structural break has been used to improve the standard HAR model specifications. Various volatility representation forecasts using the individual improved-HAR models as well as the combined models are evaluated under the combine forecast evaluations. The Dow Jones Industrial Average (DJIA) and MERVAL Argentina stock indexes have been selected for the empirical study. The empirical results showed that the mixture of best performing forecast models are relying on the type of volatility representation and also the selected proxy for the latent volatility.

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KEYWORDS
Realized volatility; Heterogeneous autoregressive model; Combine forecast methods

1. Introduction

In financial applications, the accuracy of volatility estimation and forecasting is crucial in providing a reliable tool for risk management, option trading, asset pricing (Juma et al. 2016; Juma et al. 2017) and etc. A vast amount of discussions on volatility modeling are available in the literature, as reviewed by Poon and Granger (2003). The increasing availability of high frequency data leads to the development of various volatility measures. Andersen and Bollerslev (1998) defined realized variance (RV) as the sum of squared intraday returns, which is proven to be an unbiased and less noisy estimator for the daily unobserved volatility compared to the squared daily returns proxy (McAleer and Medeiros 2008). Nonetheless, RV is ruled out from estimating the integrated variance in the presence of discontinuities or jumps. To deliver jump-robust volatility estimates, Barndorff-Nielsen and Shephard (2004) introduced realized bi-power variation (BV) and realized power variation (RPV). In BV, volatility is estimated using the sum of products of adjacent absolute returns, whereas an RPV of order p is a scaled sum of products of exponentiated absolute intraday returns (with power p). In the

CONTACT Min Cherng Lee brandonlmc24@gmail.com School of Information Technology, Monash University, Subang Jaya, Selangor, 47500, Malaysia.
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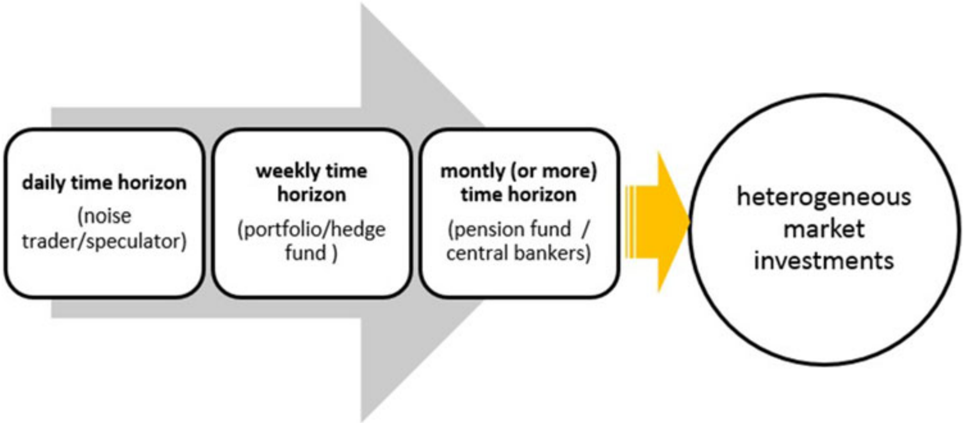

Figure 1. Various time horizon investments.

subsequent work, BV is generalized to tripower (TV) and higher order multipower variation (MPV) measures to overcome the finite sample jump distortion. However, TV and MPV are sensitive to the presence of very small returns arising from stale quotes, hence introducing another source of bias. To overcome the problems due to jumps and microstructure noise, Andersen, Dobrev, and Schaumburg (2012) proposed an improvement to these prevailing measures by using the nearest neighbor truncation, giving rise to minRV and MedRV estimators. Alternatively, Martens and van Dijk (2007) proposed a range-based (RRV) volatility estimator using the high-low price range that is believed to be more efficient as compared to RV under the condition of no market frictions. On the other hand, Dimitrios (2014) using three high frequency Parkinson estimators in an optimal sampling frequency analysis with the property of jump components of volatility for another type of range-based volatility estimator. Using stochastic approach, Asai, Chang, and McAleer (2017) had developed a realized stochastic volatility method to estimate the volatility and out-of-sample forecasts for three financial assets. The method is able to incorporates the general asymmetry and long memory of realized volatility. Recently, Ping and Li (2018) have proposed to use truncated two-scale realized volatility estimators in the SSEC index where they claimed that the empirical results are outperform the original heterogeneous autoregressive models in both statistical and economic aspects.

In the implementation of volatility modeling, RV has been a popular measure used with the autoregressive fractionally integrated moving average (ARFIMA) specification due to its long memory characteristics (Andersen et al. 2003; Giot and Laurent 2004; Martens, van Dijk, and de Pooter 2009). On the other hand, based on the heterogeneous market hypothesis, Corsi (2009) proposed the heterogeneous autoregressive realized volatility model (HAR). This model captures the long memory property of RV by taking volatility components of various time resolutions as regressors. The HAR model has dominated the high-frequency volatility modeling due to its simplicity (can be estimated using ordinary least-square method) and strong theoretical support from the HMH. Instead of homogeneous assumption by the traditional efficient market hypothesis, the HAR suggested heterogeneous market participants interpret new inflow market information based on their prior believe, capital constraint, risk profile, and others. In short, the HAR suggested that the heterogeneity among market participants is based on their investment time horizons which refer to daily, weekly and monthly duration. As illustrated in Figure 1, the market participants with dissimilar time horizon of investment observe, respond and create unique volatility components in each duration.

Despite its simplicity, HAR shows good performance in volatility estimation and forecasting. It has then become a popular choice in volatility forecasting (Forsberg and Ghysels 2007; Martens, van Dijk, and de Pooter 2009) and Value-at-risk analysis (Clements, Galvão, and Kim 2008). To improve the accuracy of volatility forecast, the asymmetric volatility phenomenon has to be taken into consideration. Such motivation was contributed by Louzis, Xanthopoulos-Sisinis, and Refenes (2012), in which a logarithmic HAR with asymmetries is modeled in the form of lagged standardized returns and absolute standardized returns occurring at distinct time horizons, taking RPV as a regressor. Other extension of HAR models are such as HAR with jumps (Corsi, Audrino, and Reno 2012; Huang et al. 2013), HAR with logistic smooth transition (Qu et al. 2016), multifractal HAR (Tao et al. 2018), vector HAR (Cubadda, Guardabascio, and Hecq 2017), among others. To capture long range dependence in the volatility of realized volatility, the authors included fractionally integrated GARCH (FIGARCH) implementation for the conditional heteroscedasticity of the residuals, giving rise to asymmetric HAR-FIGARCH model (AHAR-FIGARCH). Nonetheless, it is noted that the order in the RPV is arbitrarily determined, and hence, a more definite jump-robust volatility measure is desired. For this particular study, we are focusing on the HAR-family models only. We would like to look into the forecast performance of the proposed HAR and other HAR-family models. Other nonlinear models such as principal components combining and neural networks are not considered in this specific study. For financial realized volatility forecasting, the HAR-family models seem to be outperformed other nonlinear models. Empirically, this has been proven by Dimitrios (2017) by using spot equity, spot foreign exchange rates, exchange traded funds, equity index futures, US Treasury bonds futures, energy futures, and commodities options for U.S. financial markets. Arneri, Poklepovi, and Teai (2018) has also commented that the HAR models describe the realized volatility very well. However, the accuracy of forecasting can be improved by combining the neural network and HAR-family models. In future research, we would like to include more nonlinear models and the combination of them to improve the accuracies in realized volatility forecasting.

Combining individual forecast has been proven to be improving the accuracy of forecasts, with simple combination methods generally perform better than complex methods (Clemen 1989; Chin and Lee 2017). The methods being considered in this study includes simple-average (SA), simple-median (SM), least squares (LS), mean square error (MSE) and MSE ranks approaches. To evaluate the performances of our forecast model, four measurements namely the root mean squared error (RMSE), mean absolute error (MAE), mean absolute percentage Error (MAPE) and the Theil inequality coefficient (TIC) have been selected in the forecast evaluations. These will help to decide which forecasting models (individual or combine) perform better in our empirical studies.

On the other hand, as stock markets are susceptible to external shocks such as financial crises, US global recession and European debt crisis, sizable structural changes in the unconditional variance of stock market returns are evident. This motivates us to extend the study of Louzis, Xanthopoulos-Sisinis, and Refenes (2012) by including the structural break component to AHAR-FIGARCH model. Besides, we also consider the stylized fact that stock market volatility is fat-tailed. With these considerations, we have proposed heavy-tailed structural AHAR-FIGARCH model. The regressor of the proposed model is not limited to RPV only, but the model will be examined with other jump-robust volatility measures as well, namely TV, minRV and MedRV.

2. Methodology

2.1. Types of volatility proxy

One of the most widely used proxy in estimating the integrated variance used in financial time series is the realized volatility (RV) (Andersen and Bollerslev 1998). Due to economic events such as finance crisis, politically instability and etc, the RV encounters inconsistency issue contributed by the presence of sudden break in the data. In order to tackle this problem, Barndorff-Nielsen and Shephard (2002) have recommended a more general realized multipower variation proxy which employs the products of power of several adjacent absolute returns. The tripower variation (TV) mentioned by Andersen, Dobrev, and Schaumburg (2012) is theoretically the most efficient estimators among others. Although the TV is able to smoothen the abrupt jump, it is somehow sensitive to the presence of small returns arising from the discretization prices, which subsequently lead to inaccurate estimation of latent volatility. Other methods such as the abrupt jump volatility proxies are constructed using the minimum and median nearest neighbor truncation approaches (Andersen, Dobrev, and Schaumburg 2012). These estimators, namely the minRV and medRV, are asymptotically consistent estimators for the underlying integrated variance. An alternative realized range-based (RangeRV) volatility estimator is also considered in this study. The high-low price range proposed by Martens and van Dijk (2007) to create the RangeRV which is believed to be more efficient as compared to RV under the condition of no market frictions. In order to formulized the volatility proxies, lets define the continuously compounded intraday returns of day t with sampling frequency N as $r_{t,j} = 100(\ln P_{t,j} - \ln P_{t-1,j})$ with $j = 1, \ldots, n$ and $t = 1, \ldots, T$. The five types of proxies are as follows:

Type I: Realized volatility

$$RV_t = MPV_t(n = 1, p = 2) = \mu_2^{-1} \sum_{j=1}^{t} |r_j|^2 \tag{1}$$

Type II: Tripower variation volatility

$$TV_t = MPV_t(n = 3, p = 2) = \mu_{2/3}^{-3} \left(\frac{t}{t-2} \right) \sum_{j=1}^{t-2} |r_j|^{2/3} |r_{j+1}|^{2/3} |r_{j+2}|^{2/3} \tag{2}$$

Type III: MinRV

$$medRV_t = \frac{\pi}{\pi - 2} \left(\frac{t}{t-1} \right) \sum_{j=2}^{t-1} \left[min\left(|r_{t,j}|, |r_{t,j+1}| \right) \right]^2 \tag{3}$$

Type IV: MedRV

$$medRV_t = \frac{\pi}{6 - 4\sqrt{3} + \pi}\left(\frac{t}{t-2}\right)\sum_{j=2}^{t-1}\left[med\left(|r_{t,j-1}|, |r_{t,j}|, |r_{t,j+1}|\right)\right]^2 \tag{4}$$

where $\mu_{p/i} = 2^{\frac{p}{2i}}\Gamma\left[\frac{p/i+1}{2}\right]\Gamma\left[\frac{1}{2}\right]$.

Type V: RRV

$$RRV_t = \frac{1}{4ln2}\sum_{j=1}^{t}\left[100 \times \left[ln(H_{j,t}) - ln(L_{j,t}) -\right]\right]^2 \tag{5}$$

where H and L represent the highest and lowest asset prices observed during the fixed intraday interval.

2.2. Volatility model

This study adapted the heterogeneous autoregressive (HAR) model for the realized volatility modeling. The extended HAR model is expected to account several important empirical stylized facts in the financial stock markets. These include the clustering realized volatility, long memory volatility of realized volatility, risk premium, leverage effect, structural break impact and skewed and heavy tails distributed error. For comparison, we have included other HAR models as follows:

2.3. Model 1: HAR model

This is the original HAR model proposed by Corsi (2009). The heterogeneous volatility components are composed by three different time investment horizon volatilities namely the short-, medium- and long-term horizons as follows:

$$\begin{aligned} ln\left(RV_t^d\right) &= \theta_0 + \theta_{d1} ln(RV_{t-1}) + \theta_{d2} ln(RV_{t-2}) \\ &+ \theta_w ln\left(RV_{t-1}^w\right) + \theta_m ln\left(RV_{t-1}^m\right) + a_t \end{aligned} \tag{6}$$

where a_t are the errors that assumed to be normally distributed with $N(0, \sigma^2)$ and the aggregated $ln(RV)$ follows $ln(RV_t^k) = \frac{1}{k}[ln(RV_t) + \dots + ln(RV_{t-k+1})]$ with k equivalent to 1, 5 and 22 for daily, weekly and monthly duration respectively.

2.4. Model 2: HAR-FIGARCH(1,d,1)-skewed-t model with leverage and break

The original HAR model proposed by Corsi (2009) consists of several heterogeneous volatility components by three different time investment horizon volatility namely the short-, medium- and long-term horizons volatilities. Later, Corsi et al. (2008) suggested the inclusion of GARCH component in the HAR which allows volatility of realized volatility. The heteroscedastic residuals are expected to remain and the GARCH model is used to account for the conditional heteroscedasticity of the original HAR residuals. This study extended the HAR model to account several important empirical stylized facts in the financial stock markets. These include the clustering realized volatility, long memory volatility of realized volatility, risk premium, leverage effect, structural break impact and skewed heavy tail distributed error.

$$\begin{aligned}
ln\left(RV_t^d\right) &= \theta_0 D_t + \theta_{d1} D_t ln\left(RV_{t-1}^d\right) + +\theta_{d2} D_t ln\left(RV_{t-2}^d\right) + \theta_w D_t ln\left(RV_{t-1}^w\right) \\
&\quad + \theta_m D_t ln\left(RV_{t-1}^m\right) \\
&\quad + \mu_{d1} sr_{t-1}^d + \mu_{d2} sr_{t-2}^d + \mu_w sr_{t-1}^w + \mu_m sr_{t-1}^m \\
&\quad + \pi_{d1}|sr_{t-1}^d| + \pi_{d2}|sr_{t-2}^d| + \pi_w|sr_{t-1}^w| + \pi_m|sr_{t-1}^m| \\
a_t &= \sigma_t \epsilon_t, \quad \epsilon_t|\Omega_{t-1} \sim \text{Skewed}-t(\nu, \gamma) \\
\sigma_t^2 &= \alpha_0 + \left[1 - \beta_1 L - \left(1 - \phi_1(L)\right)(1-L)^d\right] a_t^2 + \beta_1 \sigma_{t-1}^2
\end{aligned} \tag{7}$$

To capture the structural break impact, we use the dummy variable with $D_t = 1$ if ($t > DB$) and zero otherwise. For leverage effects, the standardized returns of daily, weekly and monthly are defined as $sr_t^d = \frac{r_t}{\sqrt{RV_t}}$, $sr_t^w = \frac{\sum_{i=1}^{5} r_{t-i+1}}{\sqrt{\sum_{i=1}^{5} RV_{t-i+1}}}$ and $sr_t^m = \frac{\sum_{i=1}^{22} r_{t-i+1}}{\sqrt{\sum_{i=1}^{22} RV_{t-i+1}}}$ respectively. The asymmetry (positive and negative) response function with respect to the logarithmic realized volatility is defined as

$$\frac{\partial ln\left(RV_t^d\right)}{\partial sr_{t-1}^{(\cdot)}} = \begin{cases} \mu_{(\cdot)} + \pi_{(\cdot)} & \text{if} \quad sr_{t-1}^{(\cdot)} > 0 \\ \mu_{(\cdot)} - \pi_{(\cdot)} & \text{if} \quad sr_{t-1}^{(\cdot)} < 0 \end{cases}$$

In other words, the good news ($if \;\; sr_{t-1}^{(\cdot)} > 0$) and bad news ($if \;\; sr_{t-1}^{(\cdot)} < 0$) have different impact on the realized volatility. The leverage effect occurs if bad news yield greater impact on future realized volatility. The ARCH-type specification are represented by the polynomial $\phi_1(L) = [1 - \alpha(L) - \beta(L)](1-L)^{-1}$ with $\alpha(L) = \alpha_1 L, \beta(L) = \beta_1 L$ and $(1-L)^d$ is the fractional differencing operator with an infinite order with the specification $\sum_{k=0}^{\infty} \Gamma(k-d)\Gamma(k-1)^{-1}\Gamma(-d)^{-1}L^k$. Using the Baillie, Bollerslev, and Mikkelsen (1996) specification, the summation is truncated at 1000 lags. In order to accommodate the fat tail and asymmetric features, a skewed-t (Lambert and Laurent 2001; Fernndez and Steel 1998) distribution for $\epsilon_t|\Omega_{t-1} \sim \text{Skewed}-t(0.1, \gamma)$ with the density function is used as follows:

$$f(\epsilon_t;\nu;\gamma) = \begin{cases} \dfrac{\Gamma\left[\frac{\nu+1}{2}\right]}{\Gamma\left[\frac{\nu}{2}\right]\sqrt{\pi(\nu-2)}}\left(\dfrac{2s}{\gamma+\gamma^{-1}}\right)\left(1+\dfrac{s\epsilon_{i,t}+m}{\nu-2}\gamma\right)^{-\frac{\nu+1}{2}}, & \text{if} \quad \epsilon_t < -ms^{-1} \\ \dfrac{\Gamma\left[\frac{\nu+1}{2}\right]}{\Gamma\left[\frac{\nu}{2}\right]\sqrt{\pi(\nu-2)}}\left(\dfrac{2s}{\gamma+\gamma k^{-1}}\right)\left(1+\dfrac{s\epsilon_{i,t}+m}{\nu-2}\gamma\right)^{-\frac{\nu+1}{2}}, & \text{if} \quad \epsilon_t \geq -ms^{-1} \end{cases}$$

with ν and γ are the tail and asymmetry parameters respectively where $s = \sqrt{\gamma^2 + \gamma^{-2} - m^2 - 1}$ and $m = \frac{\gamma - \gamma^{-1}}{\Gamma(\left[\frac{\nu-1}{2}\right]\sqrt{\nu-2}\Gamma\left[\frac{\nu}{2}\right]\sqrt{\pi})}$. Under the specific conditions, this model can reduce to original Heterogeneous Autoregressive, HAR-GARCH Corsi (2009) and HAR-FIGARCH Chin, Lee, and Yap (2016) models respectively. Hence, we have included the aforementioned three models in the estimation and forecast evaluations.

2.5. Combining forecast methods

Forecast averaging's main objective is to combine several competitive forecasts into a single forecast, which normally outperformed the individual benchmark models. A constant or time-varying weight averaging schemes are typically used (Timmermann 2006). Here, we have considered a few commonly used averaging weighting schemes namely the simple-average (SA), simple-median (SM), least squares (LS), mean square error (MSE) and MSE ranks approaches. Mathematically, assume that the h-step-ahead forecasts, Y^A_{t+h} is composed by a weighted average of the S individual-model forecasts Y^r_{t+h} for $r = 1, ..., S$ as follows:

$$Y^A_{t+h} = W^1 Y^1_{t+h} + W^2 Y^2_{t+h} + \cdots + W^S Y^S_{t+h} \tag{8}$$

The weights for each scheme are described as follows:

1. SA: $W^r = \frac{1}{S}$, where every forecast is given the similar weight Clements, Galvão, and Kim (2008).
2. SM: The median of each of the forecasts and the weights are time-varying since every individual method can be the median in the forecast sample.
3. LS: The weights are the estimated coefficients of ordinary a least-squares regression: $Y_{t+h} = W^0 + W^1 Y^1_{t+h} + W^2 Y^2_{t+h} + \cdots + W^S Y^S_{t+h} + \epsilon_{t+h}$, where W^0 is the intercept coefficient Granger and Ramanathan (1984).
4. MSE: $W^r = \frac{1/MSE_{r,t+h}}{\sum_{l=1}^{S} /MSE_{l,t+h}}$, which is the ratio of individual forecasts MSE to the total of all MSEs Stock and Watson (2001).
5. MSE-ranks: $W^r = \frac{1/Rank_{r,t+h}}{\sum_{l=1}^{S} /Rank_{l,t+h}}$, where the smallest MSE will has the rank 1 (Aiolfi and Timmermann (2006)).

2.6. Combining forecast evaluations

Four measurements have been selected in order to obtain an objective forecast accuracy evaluations: root mean squared error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and the Theil inequality coefficient (TIC) in the forecast evaluations. In order to decide whether to conduct an individual forecast or averaging forecast, a regression test can be performed under the following specifications:

$$Y_{t+h} - \hat{Y}_{t+h,r} = \phi_0 + \sum_{l=1}^{S} \phi_l \hat{Y}_{t+h,l} \tag{9}$$

where Y_{t+h} and $\hat{Y}_{t+h,r}$ represent the actual values and the forecast values under model r. The null hypothesis assumes all $\phi_l = 0$, which implies that individual forecast model r contains all the information of other forecasts by a total of S models. In other words, the non-rejection of this hypothesis leads to the conclusion of forecast r can be used individually whereas the rejection suggested that a combination of various models should be performed for the forecast.

Table 1. Descriptive statistics for various realized volatility representations.

DJIA	LOGRV	LOGTV	LOGMINRV	LOGMEDRV	LOGRRV
Mean	−9.898	−10.360	−10.405	−10.393	−10.280
Std. Dev	1.016	0.945	0.955	0.950	1.039
Skewness	0.464	0.475	0.451	0.454	1.739
Kurtosis	3.295	3.519	3.527	3.391	14.164
Jarque-Bera	67.897*	83.994*	78.251*	70.022*	9787.300*
MERVAL	LOGRV	LOGTV	LOGMINRV	LOGMEDRV	LOGRRV
Mean	−8.807	−9.111	−9.096	−9.103	−9.507
Std. Dev	0.712	0.687	0.718	0.708	0.668
Skewness	0.417	0.248	0.157	0.151	0.303
Kurtosis	4.242	3.581	3.254	3.360	3.687
Jarque-Bera	159.129*	41.481*	11.562*	15.722*	59.774*

Note: *Indicates significant at 5% level.

3. Empirical studies

Two stock market indexes have been included in this study: Dow Jones Industrial Average (DJIA) and Argentina MERVAL indexes. The former index consists of thirty large publicly owned companies based in the United States under the S&P Dow Jones Indices; whereas the latter index is one of the most important indexes of the Buenos Aires Stock Exchange. Both the indexes are selected due to their representative of financial health status in each respective country. The dataset consists of 5-minute interval prices started from January 2009 and ended at December 2015 with 1677 and 1757 observations for MERVAL and DJIA respectively. For in-sample estimation, the data are used up to 31st December 2014 whereas the rest of the data are utilized for out-of-sample forecast evaluations.

3.1. Descriptive statistics

The descriptive statistics in Table 1 reports the well-known statistical features for all the realized volatility representations which included the logarithmic of RV, TV, minRV, medRV and RangeRV for DJIA and MERVAL indexes respectively. For mean and standard deviation, all the volatility representations indicate similar magnitudes for both DJIA and MERVAL indexes. For DJIA only, except for the RangeRV, all the volatility representations have comparable positive skewness (approximately 0.45) and slightly excess kurtosis (approximately 0.40) as compared to a standard normal distribution. The RRV for DJIA shows a strong positive skewed and heavily excess kurtosis. Overall, all the Jarque-Bera tests rejected all the series at 5% significance level and conclude that they are not normally distributed with the strongest rejection by the RRV series for DJIA and RV for MARVAL index respectively.

In Figure 3, the quantile-quantile plots for DJIA are conducted for all the series and it is found that all the series portrait concave downward patterns which indicate positive skewness. The quantile-quantile plots for MERVAL (Figure 2) show better fitting against normal distribution for most of the center observations. However, both the tails for all the volatility representations are heavier that the normal distributions.

Figure 4 illustrates all the autocorrelation function (ACF) of volatility representations for DJIA and MARVEL. For both the ACFs for DJIA and MARVEL are found to be

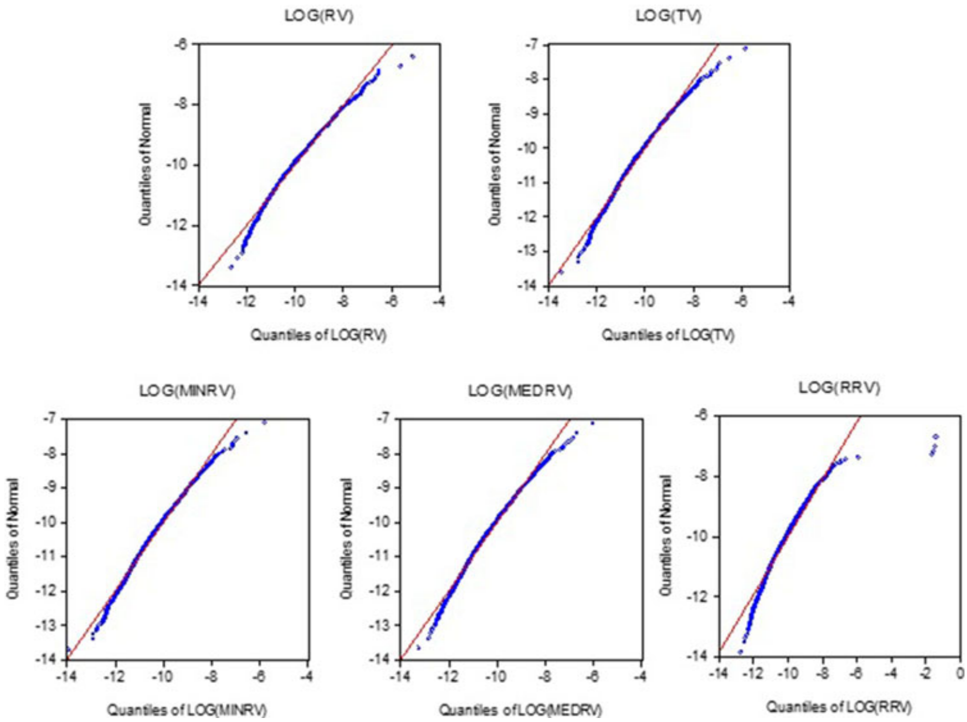

Figure 2. Quantile-quantile plots for MERVAL.

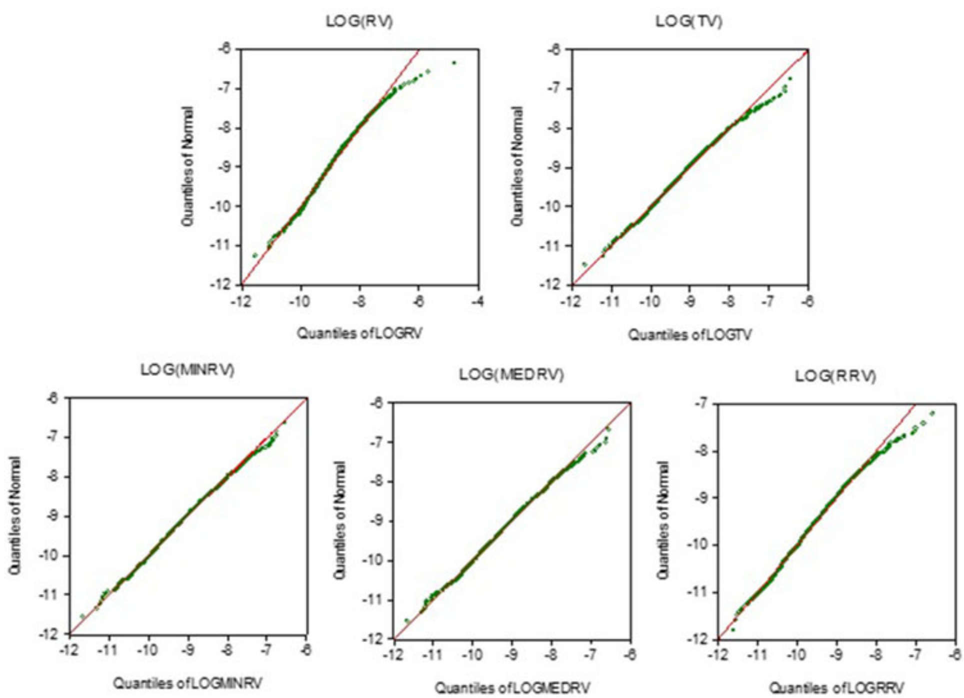

Figure 3. Quantile-quantile plots for DJIA.

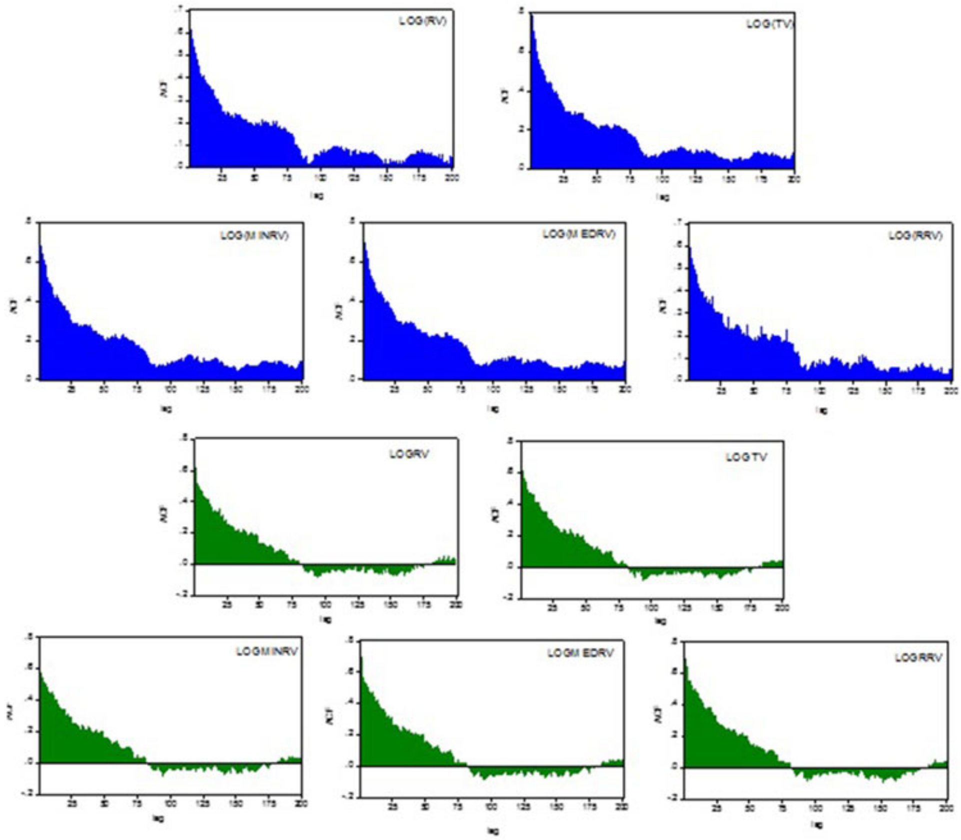

Figure 4. Autocorrelation function for MERVAL and DJIA. *Note*: the horizontal line showing the upper bound of $\frac{1.96}{\sqrt{T}}$.

shown slow hyperbolic decay rate and suggested that the presence of long-range dependence property.

3.2. Estimation results

The estimation results for both the DJIA and MARVEL are presented in Tables 2 and 3 respectively. Overall, both DJIA and MERVAL are undergone the model estimations namely the HAR model, HAR-GARCH-Normal model, HAR-FIGARCH-skewed-*t* model and leverage-break-HAR-FIGARCH-skewed-t model respectively which incorporate with five volatility representations with RV, TV, minRV, medRV and RRV. Under the specific conditions, this model can reduce to original Heterogeneous Autoregressive Corsi (2009) -Type I, HAR-GARCH Corsi et al. (2008) – Type II and HAR-FIGARCH Chin, Lee, and Yap (2016) – Type III models and our proposed model respectively. We will consider our proposed model estimation for the empirical studies.

Besides the ordinary least square estimation for HAR, the rest of the models are conducted using the simulated annealing maximum likelihood (MaxSA) estimation by G@RCH packages. As expected the HAR models for DJIA and MERVAL indicated issue of volatility of realized volatility whereas the HAR-GARCH corrected this dispute with

Table 2. HAR-FIGARCH-skewed-*t* with leverage and break for DJIA.

	DJIA:HAR-FIGARCH-skewed(t)				
Estimation	RV	TV	minRV	medRV	RRV
HAR: θ_0	−0.981**	−0.718**	−0.872*	−0.695**	−0.773**
	(0.234)	(0.180)	(0.190)	(0.167)	(0.182)
$\theta_{day,t-1}$	0.126**	0.360**	0.312**	0.365**	0.227**
	(0.047)	(0.040)	(0.041)	(0.041)	(0.040)
$\theta_{day,t-2}$	0.165**	0.016	0.002	0.016	0.166**
	(0.047)	(0.043)	(0.042)	(0.042)	(0.040)
$\theta_{week,t-1}$	0.284**	0.331**	0.364**	0.295**	0.280**
	(0.082)	(0.072)	(0.074)	(0.073)	(0.071)
$\theta_{month,t-1}$	0.344**	0.280**	0.257**	0.273**	0.263**
	(0.069)	(0.046)	(0.048)	(0.048)	(0.050)
Break					
RV859 (M)	−2.501**	−2.930**	−2.968**	−2.560**	−2.655**
	(0.624)	(0.438)	(0.579)	(0.548)	(0.576)
	Brv(749)	Btv903	Bmin903	Bmed869	Brrv898
bRV1 (M)	–	−0.012	0.004	−0.028	0.110*
		(0.069)	(0.061)	(0.060)	(0.058)
bRV2 (M)	−0.060	0.145**	0.142**	0.118*	−0.030
	(0.062)	(0.066)	(0.065)	(0.064)	(0.060)
bw1RV (M)	−0.089	−0.208*	−0.220*	−0.135	−0.109
	(0.107)	(0.116)	(0.113)	(0.109)	(0.102)
bm1RV (M)	−0.209**	−0.191**	−0.194**	−0.187**	−0.211**
	(0.093)	(0.076)	(0.083)	(0.078)	(0.081)
Leverage:					
Radj day	−0.124**	−0.081**	−0.079**	−0.077**	−0.100**
	(0.021)	(0.015)	(0.016)	(0.015)	(0.014)
Abs(radj day)	0.075113**	0.086420**	0.109992**	0.094109**	0.062635**
	(0.033348)	(0.021499)	(0.022587)	(0.021537)	(0.021490)
Radj week	−0.092**	−0.035*	−0.048**	−0.046**	−0.045**
	(0.028)	(0.019)	(0.020)	(0.019)	(0.018)
Abs(Radj week)	0.095**	0.043*	0.037	0.048*	0.056**
	(0.035)	(0.023)	(0.025)	(0.024)	(0.022)
Radj month	−0.0145	−0.000	0.001	0.010	−0.008
	(0.031)	(0.019)	(0.020)	(0.018)	(0.019)
Abs(Radj month)	**0.054**	**0.037**	**0.044***	**0.029**	**0.037**
	(0.040)	(0.027)	(0.027)	(0.026)	(0.026)
Cst(ν)	0.040	0.023	0.035*	0.021	0.000
	(0.043)	(0.014)	(0.020)	(0.014)	(0.000)
d-Figarch	0.186*	0.291**	0.234**	0.279**	1.123**
	(0.901)	(0.091)	(0.080)	(0.096)	(0.060)
ARCH(ϕ_1)	0.664	0.318**	0.366**	0.421**	0.014
	(0.350)	(0.091	(0.102)	(0.085)	(0.065)
GARCH(β_1)	0.721*	0.561**	0.545**	0.648**	0.994**
	(0.305)	(0.115)	(0.113)	(0.094)	(0.005)
Asymmetry	0.253**	0.153**	0.128**	0.156**	0.242**
	(0.045)	(0.042)	(0.040)	(0.041)	(0.045)
Tail	16.483**	14.284**	13.412**	13.513**	12.064**
	(6.359)	(4.406)	(4.034)	(4.091)	(3.767)
Model selection					
L	−1478.322	−1122.975	−1202.294	−1148.593	−1123.697
AIC	2.018	1.540	1.647	1.574	1.541
SIC	2.096	1.618	1.725	1.659	1.619
HIC	2.047	1.569	1.676	1.604	1.570
Diagnostic					
$\tilde{a}_t$, Q(10)	17.580	5.779	6.504	5.278	10.041
$\tilde{a}_t^2$, Q(10)	8.607	4.769	5.19592	10.498	4.904
ARCH (10)	0.805	0.508	0.511	1.028	0.488

1. $\tilde{a}_t$ represents the standardized residual. Ljung Box Serial Correlation Test.
(Q-statistics) on $\tilde{a}_t$ and $\tilde{a}_t^2$: Null hypothesis - No serial correlation.
2. For estimation, the parentheses values represent standard error.
3. For diagnostic, the parentheses values represent *p*-value.
4. * and ** denote 5% and 10% level of significance respectively.

Table 3. HAR-FIGARCH-skewed-*t* with leverage and break for MERVAL.

	DJIA:HAR-FIGARCH-skewed(t)				
Estimation	RV	TV	minRV	medRV	RRV
HAR: θ_0	−2.298**	−1.541**	−2.114**	−1.544**	−2.268**
	(0.239)	(0.242)	(0.565)	(0.262)	(0.39792)
$\theta_{day,t-1}$	0.140**	0.225**	0.218**	0.234**	0.210**
	(0.033)	(0.0359)	(0.0341)	(0.033)	(0.039582)
$\theta_{day,t-2}$	0.0773**	0.091**	0.073**	0.074**	0.082
	(0.033)	(0.036)	(0.033)	(0.031)	(0.051)
$\theta_{week,t-1}$	0.246**	0.183**	0.164**	0.149**	0.125
	(0.058)	(0.069)	(0.066)	(0.066)	(0.100)
$\theta_{month,t-1}$	0.324**	0.365**	0.353**	0.407**	0.394**
	(0.050)	(0.055)	(0.070)	(0.056)	(0.090)
Break					
RV859 (M)	−2.430**	−3.082**	−2.940**	−3.663**	−2.585
	(0.880)	(0.736)	(0.916)	(0.755)	(97.541)
	Brv1123	Btv1123	Bmin1146	Bmed1148	Brrv1119
bRV1 (M)	0.0709	0.079	0.032	−0.000	−0.064
	(0.069)	(0.071)	(0.073)	(0.072)	(1.107)
bRV2 (M)	−0.056	−0.078	−0.014	–	−0.087
	(0.067)	(0.071)	(0.069)		(0.803)
bw1RV (M)	0.057	0.085	0.031	0.078	0.312**
	(0.120)	(0.115)	(0.105)	(0.097)	90.114
bm1RV (M)	−0.365**	−0.439**	−0.393**	−0.497**	−0.435
	(0.103)	(0.098)	(0.105)	(0.098)	(8.063)
Leverage:					
Radj day	−0.019	−0.031**	−0.035**	−0.027**	−0.004
	(0.014)	(0.010)	(0.011)	(0.010)	(0.045)
Abs(radj day)	0.109**	0.073**	0.084**	0.079**	0.057
	(0.020)	(0.016)	(0.018)	(0.016)	(0.069)
Radj week	0.016	0.015	0.018	0.009	0.012
	(0.015)	(0.010)	(0.019)	(0.010)	(0.018)
Abs(Radj week)	0.069**	0.054**	0.060**	0.042**	0.035
	(0.020)	(0.016)	(0.018)	(0.015)	(0.116)
Radj month	−0.005	−0.001	0.004	0.006	0.001
	(0.015)	(0.010)	(0.011)	(0.010)	(0.154)
Abs(Radj month)	0.049**	0.021	0.033*	0.025*	0.017
	(0.020)	(0.015)	(0.017)	(0.015)	(0.105)
GARCH					
Cst(ν)	0.124**	0.084*	0.108	0.064	0.007
	(0.058)	(0.047)	(0.086)	(0.056)	(0.026)
d-Figarch	0.139**	−0.011	0.011	0.045	0.089
	(0.053)	(0.083)	(0.108)	(0.094)	(0.082)
ARCH(ϕ_1)	0.273	0.717**	0.682**	0.697**	0.963**
	(0.238)	(0.098)	(0.190)	(0.222)	(0.104)
GARCH(β_1)	0.288	0.640**	0.622**	0.656**	0.951**
	(0.235)	(0.097)	(0.185)	(0.225)	(0.179)
Asymmetry	0.460**	0.308**	0.375**	0.312**	0.370**
	(0.048)	(0.045)	(0.049)	(0.043)	(0.043)
Tail	7.285**	12.504**	9.898**	10.226**	7.374
	(1.294)	(3.397)	(2.189)	(2.323)	(5.089)
Model Selection					
L	−1344.128	−1040.170	−1226.640	−1074.886	−1230.130
AIC	1.932	1.502	1.766	1.550	1.771
SIC	2.014	1.584	1.848	1.628	1.853
HIC	1.963	1.533	1.797	1.579	1.802
Diagnostic					
$\tilde{a}_t$, Q(10)	17.514	7.146	7.101	3.791	6.028
$\tilde{a}_t^2$, Q(10)	4.856	7.004	3.632	5.429	7.742
ARCH (10)	0.504	0.691	0.380	0.554	0.775

1. $\tilde{a}_t$ represents the standardized residual. Ljung Box Serial Correlation Test. (Q-statistics) on $\tilde{a}_t$ and $\tilde{a}_t^2$: Null hypothesis – No serial correlation.
2. For estimation, the parentheses values represent standard error.
3. For diagnostic, the parentheses values represent *p*-value.
4. * and ** denote 5% and 10% level of significance respectively.

the GARCH specification. However the model can be further improved by including the long memory asymmetry and also heavy tail property using the HAR-FIGARCH-skewed-t models. However, only the DJIA indicated stationary and invertible differencing parameter (d) for all the volatility representations except for RRV. For MERVAL, only the RV is significantly different from zero at 5% significance level. Lastly, the leverage and break effects are comprised in the previous model. Since the information criteria (AIC, SIC and HIC) indicate that the fourth model are best (lowest) among others, the discussion of the estimation will based on the leverage-break HAR-FIGARCH-skewed-t models for the DJIA and MERVAL indexes.

Basically there are *five* portions in this model, *first* the heterogeneous volatility components represented by three different time horizon daily, weekly and monthly components. For both DJIA and MERVAL, all the components are statistically different from zero at 5% level across the five volatility representations which supported the heterogeneous market hypothesis. In terms of the impact of each horizon volatility component, the DJIA seems to be quite even distributed among the lag one daily, weekly and monthly components whereas the MERVAL shows a tendency of stronger impact by the lag one monthly component as compared to other horizons components. *Second* is the feature of volatility of realized volatility. In this specific study, the fractionally integrated GARCH specification has been used in order to capture the long memory volatility of the various realized volatility representations. However, only the DJIA indicated the long memory property across the various volatility representations whereas only the RV representation shows this property in the MERVAL index. In short, it is necessary to include the long memory volatility in some of the volatility representation in order to obtain better in-sample estimation.

Third is the leptokurtic and skewed densities error series for the selected volatility representations. As indicated in Tables 2 and 3, the skewed-t distribution indicates slight improvement over the Gaussian across the various volatility representations according to the AIC, SIC and HIC criteria for both the DJIA and MERVAL indexes. Therefore, the additional two skewness and heavy-tail parameters are necessary to provide a better estimation result for both DJIA and MERVAL. *Fourth*, the leverage effect of asymmetric volatility that influence the past daily, weekly and monthly standardized returns. From Table, except for MERVAL (RV and RRV representations), the leverage coefficients $\mu_{(\cdot)}$ are statistically significant at a 5% significance level for past daily and weekly standardized return only. The negative weighting of $\mu_{(\cdot)}$ suggested that the past negative shocks produce a stronger impact on future volatility as compared to the past positive one. In terms of the impact, the past daily standardized returns have observed approximate double the intensity of past weekly standardized return. By including the past weekly standardized return in leverage effect analysis, one is able to preview the bad news impact of different time horizon instead of only the past daily component.

Fifth, the Bai-Perron sequential procedure has been used in order to detect the possible breakpoints in the long-run level of various volatility representations. The identification is based on the ordinary least squared standard HAR model. It is found that for DJIA, the only breakpoint falls at approximately July 2012 whereas September 2013 for MARVEL across the various volatility representations. Besides the long-run level shift, the impact of break is also included in the heterogeneous components for the past daily, weekly and monthly volatilities. For DJIA, only the TV, minRV and medRV indicate

Table 4. Combine forecast evaluation for MERVAL.

Actual: RV	Forecast	RMSE	MAE	MAPE	Theil
	$RV01_F$	0.660	0.495	6.297	0.039
	$TV\ 02_F$	0.831	0.635	8.134	0.048
	$MIN03_F$	0.810	0.613	7.864	0.047
	$MED04_F$	0.841	0.645	8.274	0.046
	$RRV\ 05_F$	1.518	1.390	17.331	0.084
	Simple mean	0.900	0.705	9.017	0.052
	Simple median	0.828	0.630	8.086	0.048
	Least-squares	0.660	0.495	6.297	0.039
	Mean square error	1.229	1.071	13.487	0.0690
	MSE ranks	0.791	0.594	7.633	0.046
Actual: TV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$RV01_F$	0.534	0.410	4.770	0.031
	$TV\ 02_F$	0.529	0.375	4.523	0.030
	$MIN03_F$	0.530	0.379	4.560	0.030
	$MED04_F$	0.542	0.386	4.661	0.031
	$RRV\ 05_F$	1.117	0.997	11.824	0.060
	Simple mean	0.570	0.407	4.939	0.032
	Simple median	0.535	0.380	4.587	0.030
	Least-squares	0.529	0.375	4.523	0.030
	Mean square error	0.818	0.660	7.945	0.045
	MSE ranks	0.535	0.379	4.581	0.0302
Actual: minRV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$RV01_F$	**0.586**	0.450	5.298	**0.034**
	$TV\ 02_F$	0.604	**0.425**	**5.191**	0.034
	$MIN03_F$	0.600	0.429	5.217	0.034
	$MED04_F$	0.617	0.439	5.368	0.035
	$RRV\ 05_F$	1.182	1.037	12.46	0.064
	Simple mean	0.645	0.458	5.635	0.036
	Simple median	0.608	0.432	5.269	0.034
	Least-squares	0.600	0.429	5.217	0.034
	Mean square error	0.917	0.742	9.031	0.050
	MSE ranks	0.609	0.431	5.274	0.034
Actual: medRV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$RV01_F$	0.551	0.423	4.908	0.032
	$TV\ 02_F$	0.546	0.386	4.649	0.031
	$MIN03_F$	**0.544**	**0.386**	**4.633**	**0.031**
	$MED04_F$	0.558	0.396	4.773	0.032
	$RRV\ 05_F$	1.119	0.991	11.767	0.060
	Simple mean	0.583	0.416	5.044	0.033
	Simple median	0.550	0.390	4.690	0.031
	Least-squares	0.558	0.396	4.773	0.031
	Mean square error	0.833	0.671	8.080	0.046
	MSE ranks	0.551	0.391	4.711	0.031
Actual: RRV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$RV01_F$	1.093	1.010	10.422	0.060
	$TV\ 02_F$	0.826	0.739	7.653	0.0446
	$MIN03_F$	0.865	0.774	7.997	0.0468
	$MED04_F$	0.826	0.735	7.602	0.0446
	$RRV\ 05_F$	**0.560**	**0.384**	4.241	**0.0290**
	Simple mean	0.755	0.667	6.913	0.041
	Simple median	0.840	0.749	7.748	0.045
	Least-squares	0.560	0.384	**4.241**	0.029
	Mean square error	0.925	0.839	8.659	0.050
	MSE ranks	0.652	0.561	5.865	0.035

that the long-run level, lagged daily, weekly and monthly heterogeneous components are influenced by the break. For instance the TV series, the long-run level encounters additional negative magnitude of 2.30392 after the break. For the heterogeneous

Table 5. Combine forecast evaluation for DJIA.

Actual: RV	Forecast	RMSE	MAE	MAPE	Theil
	$DJRV_{F01}$	0.754	0.580	6.114	0.038
	$DJTV_{F01}$	0.970	0.764	8.331	0.047
	$DJMIN_{F01}$	1.009	0.799	8.720	0.049
	$DJMED_{F01}$	0.975	0.770	8.388	0.048
	$DJRRV_{F01}$	1.090	0.812	8.663	0.054
	Simple mean	0.891	0.701	7.604	0.044
	Simple median	0.959	0.754	8.222	0.047
	Least-squares	0.754	0.580	6.114	0.038
	Mean square error	0.965	0.762	8.304	0.047
	MSE ranks	0.853	0.668	7.201	0.042
Actual: TV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$DJRV_{F01}$	0.743	0.585	5.558	0.036
	$DJTV_{F01}$	0.600	0.456	4.581	0.029
	$DJMIN_{F01}$	0.620	0.470	4.744	0.029
	$DJMED_{F01}$	0.599	0.456	4.586	0.028
	$DJRRV_{F01}$	0.951	0.635	6.198	0.046
	Simple mean	0.609	0.475	4.679	0.029
	Simple median	**0.595**	**0.453**	**4.543**	**0.028**
	Least-squares	0.600	0.456	4.581	0.029
	Mean square error	0.702	0.548	5.249	0.034
	MSE ranks	0.601	0.464	4.631	0.029
Actual: minRV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$DJRV_{F01}$	0.788	0.617	5.828	0.038
	$DJTV_{F01}$	0.612	0.470	4.672	0.029
	$DJMIN_{F01}$	0.628	0.480	4.799	0.030
	$DJMED_{F01}$	0.610	**0.469**	4.664	0.029
	$DJRRV_{F01}$	0.967	0.654	6.328	0.047
	Simple mean	0.631	0.495	4.836	0.030
	Simple median	**0.609**	0.469	**4.655**	**0.029**
	Least-squares	0.628	0.480	4.799	0.030
	Mean square error	0.750	0.581	5.506	0.036
	MSE ranks	0.616	0.483	4.764	0.029
Actual: medRV	**Forecast**	**RMSE**	**MAE**	**MAPE**	**Theil**
	$DJRV_{F01}$	0.783	0.617	5.801	0.038
	$DJTV_{F01}$	0.607	0.474	4.689	0.029
	$DJMIN_{F01}$	0.622	0.488	4.852	0.030
	$DJMED_{F01}$	0.604	0.474	4.694	0.029
	$DJRRV_{F01}$	0.971	0.651	6.277	0.047
	Simple mean	0.628	0.492	4.784	0.030
	Simple median	**0.603**	**0.472**	**4.661**	**0.029**
	Least-squares	0.604	0.474	4.694	0.029
	Mean square error	0.745	0.580	5.477	0.04
	MSE ranks	0.611	0.483	4.749	0.029
Actual: RRV	Forecast	RMSE	MAE	MAPE	Theil
	$DJRV_{F01}$	1.290	0.693	14.476	0.063
	$DJTV_{F01}$	1.289	0.589	14.166	0.062
	$DJMIN_{F01}$	1.307	0.606	14.416	0.0624
	$DJMED_{F01}$	1.296	0.591	14.256	0.0620
	$DJRRV_{F01}$	1.472	0.755	15.624	0.0715
	Simple mean	1.279	0.591	14.045	0.062
	Simple median	1.287	0.586	14.145	0.0617
	Least-squares	1.472	0.755	15.624	0.0715
	Mean square error	**1.263**	**0.574**	**13.830**	**0.061**
	MSE ranks	1.300	0.614	14.332	0.0627

components, the lagged two daily volatility coefficient shows an extra positive 0.145121 shift and approximately negative 0.20000 for the weekly and monthly volatility components. In other words, it is important to capture these additional impacts from the presence of break in the volatility series. On the other hand, the MERVAL only indicates the shifts in long-run level and monthly volatility components.

3.3. Forecast evaluations

Overall, there are five individual models based on various volatility representations (RV, TV, minRV, medRV and RRV) and five combine models (SA, SM, LS, MSE and MSE ranks) in the forecast evaluations. The forecast results are shown in Tables 4 and 5 for MERVAL and DJIA respectively. Based on the in-sample estimation, the HAR-FIGARCH with leverage and break provides the best information criteria (lowest values) over its counterparts. Therefore, the evaluation will base on the specification of this model. The out-of-sample data or the test data is set typically about 20% of the total sample (Hyndman and Athanasopoulos 2013), however it is subjected on how long the sample is and how far ahead we would like to forecast. With the sample size of 1677 and 1757 for MERVAL and DJIA, we have selected the annual one-day-ahead forecasts with 242 and 249 with the respective markets. For the DJIA there are 249 one-day-ahead forecast and 242 for MERVAL for data started from January until December of 2015. Each volatility representations is selected once to act as the latent volatility in order to obtain the four loss function namely the RMSE, MAE MAPE and Theil index. This procedure can avoid the biasness issue and also improve the objectiveness of the forecast evaluation results.

We first focus on the DJIA evaluations. Overall, the forecast evaluations are in favor of combine forecast models regardless the type of selected latent volatility. The combine forecasts using the simple median (SM) method indicate superior results when the latent volatility is represented by TV, minRV and also medRV whereas the least-squares (LS) method and mean-square-error (MSE) approach outperform other when RV and RRV are selected as the latent volatility.

Next, the MERVAL indicates slightly different outcomes from the similar forecast evaluation procedures. The individual model seems to be more favorable than the combine forecast methods except when the RV is selected as the latent volatility. When TV acts as the actual volatility, the HAR(TV)-FIGARCH with leverage and break shows the best forecast evaluation under the RMSE, MAE, MAPE and Theil index. While move to minRV as the actual volatility, the mixture of HAR(RV) and HAR(TV) perform the best than other models and when shift to medRV, the HAR(minRV) outperforms others. Lastly, the HAR(RRV) shows the best forecast evaluations when RRV acts as the latent volatility. Only when RV represent the actual volatility, the least squares combine forecast indicates the best forecast results.

To summarize, after selecting the best model, namely the HAR-FIGARCH with leverage and break model, there is no single model, either individual or combine forecast model has absolute outperform the forecast evaluations for the DJIA and MERVAL indexes. This is because the type of volatility representation and also the selection of latent volatility have played a very important role in the forecast evaluations.

4. Conclusion

In this study, the high-frequency volatility of stock markets has been examined using the heterogeneous autoregressive model with the leverage effect and structural break features. Various realized volatility proxies have been selected in order to obtain a more thorough analysis. For forecast evaluations, the forecasts are based on five individual models with various volatility representations and another five with the combination of the individual forecasts. Using the combine forecast evaluation procedures based on four loss functions, the best forecast model is identified. In order to obtain a more objective examination, each volatility representation is alternately selected as the latent volatility in the forecast evaluations. Following this practice, the forecast evaluations show a mixture of the best individual and combine forecasts. For DJIA, the forecast evaluations are in favor of combine forecast methods such as simple median, least-square and mean-square-error while the MERVAL indicates individual HAR TV, and minRV are outperformed others when the selected actual forecasts move across from RV to TV, minRV, medRV and lastly RRV. In short, the DJIA enjoys the benefit of combining the individual forecasts and it is necessary to obtain a better forecast results whereas for MERVAL market, individual forecasts are sufficient. As a conclusion, in this specific study the forecast evaluations are depended on the selected latent volatility, type of volatility representation and the individual/combined forecast method.

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ORCID

Min Cherng Lee http://orcid.org/0000-0002-7904-4537

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