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Regime switches in the dependence structure of multidimensional financial data

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ABSTRACT

Misperceptions about extreme dependencies between different financial assets have been an important element of the recent financial crisis, which is why regulating entities do now require financial institutions to account for different behavior under market stress. Such sudden switches in dependence structures are studied using Markov switching regular vine copulas. These copulas allow for asymmetric dependencies and tail dependencies in high dimensional data. Methods for fast maximum likelihood as well as Bayesian inference are developed. The algorithms are validated in simulations and applied to financial data. The results show that regime switches are present in the dependence structure and that regime switching models provide tools for the accurate description of inhomogeneity during times of crisis.

1. Introduction

In the reverberations of the recent financial crisis, regulators turned their attention towards the fact that financial time series exhibit different behavior under market stress. This has led to new requirements for financial institutions addressing this issue. For European financial institutions, the European Banking Authority (EBA) has introduced the Stressed Value at Risk (SVaR) in addition to the standard VaR measure (European Banking Authority, 2012). Here, the distributions of underlying risk factors which are used to calculate VaR have to be calibrated using a period of significant stress for the banks' portfolios to appropriately reflect different behavior of time series during these times.

In the literature on financial time series different behavior during times of market stress has long been recognized (e.g. in the seminal paper of Engle (1982) which shows that variances are not constant over time). A class of models addressing this are Markov switching (MS), also called regime switching or hidden Markov models (Hamilton, 1989). There, the behavior of a financial time series exhibits two or more distinct regimes, which correspond to different states of the economy. Which regime is present at a particular point of time is governed by an underlying hidden Markov process. These different states of the Markov model fit nicely into the regulatory framework requiring stressed and non-stressed market conditions to be incorporated in risk management. While there is a huge range of univariate or low dimensional applications of Markov switching models (Pelletier, 2006; Garcia and Tsafack, 2011; Chauvet and Hamilton, 2006; Cerra and Saxena, 2005; Hamilton, 2005), their full potential for the modeling of a multidimensional set of underlyings and indices as it is required in the risk management of financial institutions has not yet been explored. In particular, inference methods which assess estimation uncertainty must be developed. Further, characteristics of financial data such as asymmetric dependence and tail dependence (Longin and Solnik, 2001; Ang and Chen, 2002) must be incorporated appropriately. These are outside the world of the non tail-dependent Gaussian and symmetric Student's t distributions. A class of multivariate dependence models which can describe such asymmetries (Joe et al., 2010; Nikoloulopoulos et al., 2012) are regular vine (R-vine) copulas.

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They are based on a series of linked trees called R-vine and have been introduced by Bedford and Cooke (2001, 2002) extending ideas of Joe (1996). R-vine distributions are built up hierarchically from bivariate copulas as building blocks and they have proven to be a tractable model in the multidimensional setting since also inference can be performed exploiting that hierarchical structure (Aas et al., 2009).

Our contribution is to introduce a general Markov switching R-vine (MS–RV) copula model and to develop efficient inference techniques. In particular, we go beyond the MS copula model of Chollete et al. (2009) in that we use the full flexibility of R-vine models. This superior flexibility will allow us to use truncation techniques (see Brechmann et al. (2012) and references therein), leading to a parsimonious parametrization of the model. Secondly, we develop an approximative Expectation–Maximization (EM) type procedure in the Maximum Likelihood (ML) framework which allows for fast parameter estimation and is scalable to high dimensional applications. The algorithm is based on the sequential estimation procedure developed by Aas et al. (2009), which has been shown to be asymptotically consistent by Hobæk Haff (2013). To address the issue of quantifying uncertainty, we further consider parameter inference for a prespecified MS–RV model in a Bayesian setup. For this, the algorithm of Min and Czado (2010), who consider Bayesian inference for a structural subclass of R-vine copulas, is generalized and extended to incorporate inference about the underlying Markov structure. In particular, we can also compute credible intervals for the probability of being in a given regime at a given point of time. While most existing models for time-varying dependence do not allow to quantify the uncertainty in the time variability of parameters, our Bayesian estimation procedure enables us to do so.

In order to demonstrate the applicability and performance of our procedures for parameter estimation, we perform a simulation study and investigate an application to exchange rate data. We also show how model selection can be performed for time-varying dependence structures by conducting a rolling window analysis and compare different models using the Bayesian deviance information criterion (DIC, Spiegelhalter et al., 2002). The remainder is structured as follows: Section 2 introduces the MS–RV model by first introducing R-vine distributions in Section 2.1 and then combining them with an underlying Markov structure in Section 2.2. The focus of Section 3 is parameter estimation, the stepwise procedure is discussed in Section 3.1 and Bayesian estimation in Section 3.2. In Section 4, we present the simulation results before turning to the empirical application in Section 5. Section 6 gives an outlook to directions of future research.

2. The Markov switching regular vine copula model

2.1. Regular vine distributions

R-vines as a graph theoretic tool for the construction of multivariate distributions have been introduced by Bedford and Cooke (2001, 2002). An R-vine $\mathcal{V}$ on d variables, which consists of a sequence of connected trees $T_1, \ldots, T_{d-1}$, with nodes N_i and edges E_i, $1 \le i \le d-1$, satisfies the following properties (Bedford and Cooke, 2001):

1. T_1 is a tree with nodes $N_1 = \{1, \ldots, d\}$ and edges E_1.
2. For $i \ge 2$, T_i is a tree with nodes $N_i = E_{i-1}$ and edges E_i.
3. If two nodes in T_{i+1} are joined by an edge, the corresponding edges in T_i must share a common node. (*proximity condition*)

There are two popular subclasses of R-vines. We call an R-vine *Canonical vine* (C-vine) if in each tree T_i there is one node which has edges with all $d-i$ other nodes. It is called *Drawable vine* (D-vine) if each node has at most two edges. Examples of regular vine tree structures can be found for example in Czado (2010) or in the Appendix. The notation we employ throughout our paper follows Czado (2010). Let $\mathbf{X} = (X_1, \ldots, X_d)$ be a random vector with marginal densities $f_1, \ldots, f_d$, respectively. To build up a statistical model using the R-vine, we associate to each edge $j(e), k(e)|D(e)$ in E_i, for $1 \le i \le d-1$, a bivariate copula density $c_{j(e),k(e)|D(e)}$. We call $j(e)$ and $k(e)$ the *conditioned set* while $D(e)$ is the *conditioning set*. Let $\mathbf{X}_{D(e)}$ denote the subvector of $\mathbf{X}$ determined by the set of indices $D(e)$. For the definition of the R-*vine distribution* we associate the bivariate copula densities $c_{j(e),k(e)|D(e)}$ with the conditional densities of $X_{j(e)}$ and $X_{k(e)}$ given $\mathbf{X}_{D(e)}$ equal to $c_{j(e),k(e)|D(e)}(F_{j(e)|D(e)}(x_{j(e)}|\mathbf{x}_{D(e)}), F_{k(e)|D(e)}(x_{k(e)}|\mathbf{x}_{D(e)}))f_{j(e)|D(e)}f_{k(e)|D(e)}$. In general, $c_{j(e),k(e)|D(e)}$ can depend on the values of variables which are conditioned on. To keep the number of parameters tractable, it is usually assumed that the conditional copula is constant, i.e. $c_{j(e),k(e)|D(e)}(.,.|\mathbf{x}_{D(e)}) = c_{j(e),k(e)|D(e)}(.,.)$, as discussed in Stöber et al. (2012), Haff et al. (2010) and Acar et al. (2012). The joint density of $\mathbf{X}$ is then uniquely determined by

$$f_{1,\ldots,d}(x_1, \ldots, x_d) = \prod_{i=1}^{d} f_i(x_i) \cdot \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}(F_{j(e)|D(e)}(x_{j(e)}|\mathbf{x}_{D(e)}), F_{k(e)|D(e)}(x_{k(e)}|\mathbf{x}_{D(e)})) \qquad (1)$$

as shown by Bedford and Cooke (2001). If the marginal densities are uniform on [0, 1], we call the distribution in (1) an R-vine copula. Given an R-vine $\mathcal{V}$, a set of corresponding parametric bivariate copulas $\mathbf{B}$ and their parameter vector $\boldsymbol{\theta}$, we denote the R-vine copula density by $c(.|\mathcal{V}, \mathbf{B}, \boldsymbol{\theta})$.

While also other iterative decompositions of a multivariate density into bivariate copulas and marginal densities are possible, R-vine distributions have the particularly appealing feature that the values for $F(x_{j(e)}|\mathbf{x}_{D(e)})$ and $F(x_{k(e)}|\mathbf{x}_{D(e)})$ appearing in Eq. (1) can be derived recursively without high dimensional integrations (see Dißmann et al., 2013 for details).

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2.2. Markov switching copula models

We want to develop a model for a multivariate financial time series $\{\mathbf{X}_t = (X_{1t}, \dots, X_{dt}), t = 1, \dots, T\}$ using R-vine copulas and the general MS approach introduced by Hamilton (1989). In this context the dependency among $\mathbf{X}_t$ depends on a hidden latent state variable S_t, which takes on only finitely many values $k = 1, \dots, p$. These are called regimes and represent the different states of the economy. As it is usual in the MS approach we assume that $S_t, t = 1, \dots, T$ is a homogeneous Markov chain (MC) in discrete time. For simplicity, we restrict the model to a first order MC, which is characterized by its transition matrix P with elements $P_{k,k'} := P(S_t = k'|S_{t-1} = k)$.

We use a copula based approach to model the dependency of $\mathbf{X}_t$ in regime k ($S_t = k$). For this we assume that we know or can estimate the marginal distributions of X_{it} for $i = 1, \dots, d$. In particular we assume that we have (pseudo) copula data $\mathbf{u}_t = (u_{1t}, \dots, u_{dt}) \in [0, 1]^d$ for $t = 1, \dots, T$ available. The Markov switching R-vine (MS–RV) copula for $\mathbf{u}_t$ is now fully characterized by specifying conditional densities as follows

$$c(\mathbf{u}_t|(\mathcal{V}, \mathbf{B}, \boldsymbol{\theta})_{1,\dots,p}, S_t) = \sum_{k=1}^{p} 1_{\{S_t=k\}} \cdot c(\mathbf{u}_t|(\mathcal{V}, \mathbf{B}, \boldsymbol{\theta})_k). \tag{2}$$

The complete MS–RV copula model is thus specified in terms of p R-vine copula specifications and the transition matrix P which contains the parameters of the underlying Markov chain. For inference, we will always assume the R-vine structures $\mathcal{V}_k$ and corresponding sets of copulas $\mathbf{B}_k,\ k = 1, \dots, p$, to be given and thus suppress them in the following notation. The MS–RV copula is then completely described by its parameters

$$\boldsymbol{\eta}' = (\boldsymbol{\theta}'_{\text{cop}}, \boldsymbol{\theta}'_{\text{MC}}) = ((\boldsymbol{\theta}'_1, \dots, \boldsymbol{\theta}'_p), \boldsymbol{\theta}'_{\text{MC}}),$$

where the subscript "cop" stands for copula parameters and "MC" for parameters needed for the transition matrix P. In particular, $\boldsymbol{\theta}_k$ are the copula parameters corresponding to regime k. While this model does not include switching margins, the switching copula regimes induce serial dependence. The individual marginal time series $(u_{i,t})_{t=1,2,\dots}$, however, are i.i.d. uniform for $i = 1, \dots, d$.

3. Inference for Markov switching models

The first challenge in developing inference methods for MS models is that we face unobserved latent variables. In order to derive an expression for the full likelihood of $\tilde{\mathbf{u}}_T = (\mathbf{u}_1, \dots, \mathbf{u}_T)$, we consider a decomposition of their joint density $f(\tilde{\mathbf{u}}_T|\boldsymbol{\eta})$ into conditional densities:

$$\begin{aligned} f(\tilde{\mathbf{u}}_T|\boldsymbol{\eta}) &= f(\mathbf{u}_1|\boldsymbol{\eta}) \cdot \prod_{t=2}^{T} f(\mathbf{u}_t|\tilde{\mathbf{u}}_{t-1}, \boldsymbol{\eta}) = \left[\sum_{k=1}^{p} f(\mathbf{u}_1|S_1 = k, \boldsymbol{\theta}_k) P(S_1 = k|\boldsymbol{\theta}_{\text{MC}})\right] \\ &\quad \cdot \prod_{t=2}^{T}\left[\sum_{k=1}^{p} f(\mathbf{u}_t|S_t = k, \boldsymbol{\theta}_k) \cdot P(S_t = k|\tilde{\mathbf{u}}_{t-1}, \boldsymbol{\theta}_{\text{MC}})\right], \end{aligned} \tag{3}$$

where $\tilde{\mathbf{u}}_t := (\mathbf{u}_1, \dots, \mathbf{u}_t)$ and $f(\mathbf{u}_t|S_t = k, \boldsymbol{\theta}_k)$ is known from (2) for $t = 1, \dots, T$. We assume the unconditional probabilities $P(S_1 = k)$ to be given. To obtain the state prediction probabilities $\boldsymbol{\Omega}_{t|t-1} \in \Delta^p \subset \mathbb{R}^{p\times 1}$ on the p-simplex with elements

$$\left(\boldsymbol{\Omega}_{t|t-1}(\boldsymbol{\eta})\right)_k := P(S_t = k|\tilde{\mathbf{u}}_{t-1}, \boldsymbol{\eta}) \quad \text{for } k = 1, \dots, p$$

we can apply the filter of Hamilton (1989). Assuming $\boldsymbol{\Omega}_{t-1|t-1}$ to be given,

$$\begin{aligned} &\boldsymbol{\Omega}_{t|t-1}(\boldsymbol{\eta}) = P' \cdot \boldsymbol{\Omega}_{t-1|t-1}(\boldsymbol{\eta}) \quad \text{and} \\ &\boldsymbol{\Omega}_{t|t}(\boldsymbol{\eta}) = \frac{\boldsymbol{\Omega}_{t|t-1}(\boldsymbol{\eta}) \odot (f(\mathbf{u}_t|S_t = k, \tilde{\mathbf{u}}_{t-1}, \boldsymbol{\theta}_k))_{k=1,\dots,p}}{\sum_{k=1}^{p}\left(\boldsymbol{\Omega}_{t|t-1}(\boldsymbol{\eta})\right)_k \odot f(\mathbf{u}_t|S_t = k, \tilde{\mathbf{u}}_{t-1}, \boldsymbol{\theta}_k)}, \end{aligned}$$

and we obtain all probabilities which are required to evaluate the density (3) recursively. The operator $\odot$ denotes componentwise multiplication of two vectors. Similarly, the probability $\left(\boldsymbol{\Omega}_{t|T}(\boldsymbol{\eta})\right)_{s_t} := P(S_t = s_t|\tilde{\mathbf{u}}_T, \boldsymbol{\eta})$, to which we will refer as the "smoothed" probability of being in state s_t at time t, can be determined by applying the following backward iterations.

$$\left(\boldsymbol{\Omega}_{t|T}(\boldsymbol{\eta})\right)_{s_t} = \left(\left(P \cdot \frac{\boldsymbol{\Omega}_{t+1|T}(\boldsymbol{\eta})}{\boldsymbol{\Omega}_{t+1|t}(\boldsymbol{\eta})}\right) \odot \boldsymbol{\Omega}_{t|t}(\boldsymbol{\eta})\right)_{s_t}, \tag{4}$$

where also the division is to be understood componentwise. Because of the latent state variables and the resulting dependence between parameters, direct maximization of the likelihood for given $(\mathcal{V}_k, \mathbf{B}_k)_{k=1,\dots,p}$ is analytically not possible and numerically difficult. In the following, we discuss a frequentist and a Bayesian approach to make inference for this model tractable.

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3.1. EM algorithm for MS–RV models

Hamilton (1990) proposed to overcome the problems in maximum likelihood estimation for an MS model by using an EM type (Dempster et al., 1977) algorithm. This algorithm iteratively determines parameter estimates $\boldsymbol{\eta}^l$, $l = 1, 2, \ldots$, which converge to the ML estimate for $l \to \infty$. Let us consider the expected pseudo log likelihood function $Q(\boldsymbol{\eta}^{l+1}; \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l)$ for $\boldsymbol{\eta}^{l+1}$, given observations $\tilde{\mathbf{u}}_T$ and the current parameter estimate $\boldsymbol{\eta}^l$,

$$\begin{aligned} Q(\boldsymbol{\eta}^{l+1}; \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l) &:= \int_{\tilde{\mathbf{S}}_T} \log\left(f(\tilde{\mathbf{u}}_T, \tilde{\mathbf{S}}_T | \boldsymbol{\eta}^{l+1})\right) P(\tilde{\mathbf{S}}_T | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l) \\ &\propto \sum_{t=1}^{T} \int_{\tilde{\mathbf{S}}_T} \log\left(f(\mathbf{u}_t | S_t, \boldsymbol{\theta}_{\mathrm{cop}}^{l+1})\right) \cdot P(\tilde{\mathbf{S}}_T | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l) \\ &\quad + \int_{\tilde{\mathbf{S}}_T} \left[\sum_{t=2}^{T} \log\left(P(S_t | S_{t-1}, \boldsymbol{\theta}_{\mathrm{MC}}^{l+1})\right) + \log(P(S_1)^{l+1})\right] \cdot P(\tilde{\mathbf{S}}_T | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l), \end{aligned} \tag{5}$$

where we write $\tilde{\mathbf{S}}_t := (S_1, \ldots, S_t)$ and $\int_{\tilde{\mathbf{S}}_T} g(\tilde{\mathbf{S}}_T) := \sum_{s_1=1}^{n} \cdots \sum_{s_T=1}^{n} g(S_1 = s_1, \ldots, S_T = s_T)$ for an arbitrary function g of $\tilde{\mathbf{S}}_T$. The algorithm iterates the following steps:

1. Expectation: Obtain the smoothed probabilities $\boldsymbol{\Omega}_{t|T}(\boldsymbol{\eta}^l)$ of the latent states $\tilde{\mathbf{S}}_T = (S_1, \ldots, S_T)$ given the current parameter vector $\boldsymbol{\eta}^l$.
2. Maximization: Maximize $Q(\boldsymbol{\eta}^{l+1}; \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l)$ with respect to $\boldsymbol{\eta}^{l+1}$.

Using the Markov property of $\tilde{\mathbf{S}}_T$, Kim and Nelson (2006) show that the maximum of the pseudo likelihood is attained at

$$P_{i,j}^{l+1} = \frac{\sum_{t=1}^{T} P(S_t = j, S_{t-1} = i | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l)}{\sum_{t=1}^{T} P(S_{t-1} = i | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l)},$$

similarly $P(S_1 = k)^{l+1} = P(S_1 = k | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l),\ k = 1, \ldots, p.$

In contrast to the model originally considered by Hamilton where all maximization steps could be performed analytically, this is not possible for the maximization with respect to the copula parameters $\boldsymbol{\theta}_{\mathrm{cop}}^{l+1}$ in our case. This means that, while the transition probabilities can be obtained directly, the second part of the maximization step has to be performed using numerical optimization methods. Since a d-dimensional R-vine copula specification, in which each pair copula has k parameters, contains $d(d-1)/2 \cdot k$ parameters, this is computationally still very challenging. To circumvent this issue, we can exchange the joint maximization with respect to $\boldsymbol{\theta}_{\mathrm{cop}}^{l+1}$ with the stepwise maximization procedure of Aas et al. (2009) which is modified to weight each observation by $P(S_t = s_t | \tilde{\mathbf{u}}_T, \boldsymbol{\eta}^l)$.

We call this the *stepwise EM algorithm*. Since tree-wise estimation of copula parameters is asymptotically consistent (Hobæk Haff, 2013), this constitutes a close approximation (Hobæk Haff, 2012) to the "proper" EM algorithm. While there are theoretical results on the convergence of the EM algorithm (Wu, 1983), we loose these properties with our approximation. All limit theorems however do rely on proper maximization at each step of the algorithm. This is almost impossible to guarantee in our case where we are faced with high dimensional optimization problems and have to rely on numerical techniques. While all existing models for time-varying dependence structures in high dimensions suffer from the computational burden for numerical estimation, we do only need to maximize the likelihoods of bivariate copulas in this tree-wise procedure. This reduces computation time and avoids the curse of dimensionality. We denote the obtained estimate by

$$\left(\hat{\boldsymbol{\eta}}^{\mathrm{EM}}\right)' = \left(\left(\hat{\boldsymbol{\theta}}_{\mathrm{cop}}^{\mathrm{EM}}\right)' = \left(\left(\hat{\boldsymbol{\theta}}_1^{\mathrm{EM}}\right)', \ldots, \left(\hat{\boldsymbol{\theta}}_p^{\mathrm{EM}}\right)'\right), \left(\hat{\boldsymbol{\theta}}_{\mathrm{MC}}^{\mathrm{EM}}\right)'\right).$$

3.2. Gibbs sampling for the MS–RV model

Having derived an approximative ML procedure for our MS copula, we will now consider Bayesian estimation methods, which will enable us to quantify the uncertainty in parameter estimates. In particular, credible intervals (CIs) and posterior standard deviations are determined naturally while the uncertainty in ML parameter estimates is very hard to assess in this context. Building on ideas of Albert and Chib (1993), the Gibbs sampler which we develop consists of updates for the copula parameters, the Markov chain parameters and the latent state vector, respectively. Iterating through all three outlined update steps will yield a sample

$$\left(\left(\boldsymbol{\eta}^{r,\mathrm{MCMC}}\right)', \tilde{\mathbf{S}}_T^{r,\mathrm{MCMC}}\right) = \left(\left(\left(\left(\boldsymbol{\theta}_1^{r,\mathrm{MCMC}}\right)', \ldots, \left(\boldsymbol{\theta}_p^{r,\mathrm{MCMC}}\right)'\right), \left(\boldsymbol{\theta}_{\mathrm{MC}}^{r,\mathrm{MCMC}}\right)'\right), \tilde{\mathbf{S}}_T^{r,\mathrm{MCMC}}\right),$$

for $r = 1, \ldots, R$, where R is the number of realizations.

Update of copula parameters

In order to complete the model specification in a Bayesian framework, we first have to specify a prior distribution for each component of $\boldsymbol{\theta}_{\text{cop}}$. Following Min and Czado (2010), we assume independent uniform priors for all copula parameters in the model. For bivariate copula families where the parameter range is not compact, we restrict its support to some finite interval to avoid numerical instabilities for very small or large parameter values. If for all bivariate copulas in a given family there is a one-to-one correspondence between parameter values and Kendall's τ given in closed form, priors for τ can be considered as an alternative (Almeida and Czado, 2012). Furthermore, we can use a uniform prior for the correlation matrix of the model if all bivariate building blocks are Gaussian or Student's t copulas, cf. Lewandowski et al. (2009). Since the conditional distributions of the copula parameters given the remaining parameters are not available, we use a Metropolis–Hastings (MH) update here. There are several choices for proposal distributions available. Min and Czado (2010) use a modification of standard random walk proposals where the normal distribution is truncated to the parameter support, while proposal variances are tuned to achieve suitable acceptance rates. This leads to poor acceptance rates in some cases with strong dependencies and to high autocorrelations in general. To overcome these problems, we consider a two point mixture of a random walk proposal with an independent normal distribution at the mode of the likelihood function for each parameter. The modes are approximated by the stepwise estimation procedure for R-vines and the corresponding variances are determined from the inverse Hessian. Both distributions are assigned a weight of 0.5. Independence proposals centered around the mode have been proposed by Gamerman and Lopes (2006) and have been applied in a context similar to ours by Czado et al. (2010). While there are parameter constellations where pure random walk proposals are more favorable than independence proposals and vice versa, simulation studies showed that the chosen mixture distribution works well for all settings.

Update of Markov chain parameters

For this, we will assume independent Dirichlet distributions as prior distributions for the rows of the transition matrix P, i.e. $(P_{k,k'})_{k'=1,\dots,p} \sim \text{Dirichlet}\big((\alpha_{k,k'})_{k'=1,\dots,p}\big)$ for $k = 1, \dots, p$. The conditional posterior distribution of the transition probabilities in P, given the other parameters, depends only on the latent state vector $\tilde{\mathbf{S}}_T$. Here, the likelihood function

$$l(P|\tilde{\mathbf{S}}_T) = \prod_{k=1}^{p} \prod_{k'=1}^{p} p_{k,k'}^{n_{k,k'}},$$

where $n_{k,k'}$ denotes the number of transitions from state k to state k' in $\tilde{\mathbf{S}}_T$ is multinomial. Since the Dirichlet and the multinomial distribution are conjugate distributions (see Kotz et al., 2000), also the conditional posterior distributions are Dirichlet distributions with parameters $\alpha_{k,k'}^{posterior} = \alpha_{k,k'} + n_{k,k'}$. From these we can sample directly.

Update of the latent state vector

We follow the approach by Kim and Nelson (1998) to update $\tilde{\mathbf{S}}_T$ jointly assuming independent non informative priors and decompose

$$P(\tilde{\mathbf{S}}_T|\tilde{\mathbf{u}}_T, \boldsymbol{\theta}_{\text{cop}}, \boldsymbol{\theta}_{\text{MC}}) = P(\tilde{\mathbf{S}}_T|\tilde{\mathbf{u}}_T) = P(S_T|\tilde{\mathbf{u}}_T) \cdot \prod_{i=1}^{T-1} P(S_t|S_{t+1}, \tilde{\mathbf{u}}_T).$$

This allows to generate S_T from $P(S_T|\tilde{\mathbf{u}}_T)$ and S_t for $t = T-1, \dots, 1$ from

$$P(S_t|\tilde{\mathbf{u}}_t, S_{t+1}) \propto P(S_{t+1}|S_t)P(S_t|\tilde{\mathbf{u}}_t),$$

where $P(S_t|\tilde{\mathbf{u}}_t) = \boldsymbol{\Omega}_{t|t}(\boldsymbol{\eta})$ can again be determined using the Hamilton filter. Robustness studies show that the influence of the choice of the prior distribution of $P(S_1 = k)$ on the joint posterior is negligible.

4. Simulation study

This section presents the results of a simulation study which has been performed in order to demonstrate the ability of the developed Bayesian inference procedure to capture the true model in simulated data. We consider two regimes and five parameter setups, see Table 1. In all scenarios, we set the Markov parameters to $P(S_t = 1|S_{t-1} = 1) = 0.95$ and $P(S_t = 2|S_{t-1} = 2) = 0.9$, the corresponding prior distributions are chosen to be uniform. For each scenario, we simulate a time series with 800 four dimensional observations. Keeping the (true) R-vine structure and copula families we used for simulations, we obtain a posterior estimate for the parameters as follows:

1. Starting values for the EM algorithm: fit the copula parameters for each regime to the whole data set using the stepwise estimation procedure, and cluster the observations according to their likelihood values. Re-fit to the 400 observations which have the highest log likelihood.
2. Starting values for MCMC: obtained using the stepwise EM algorithm.
3. Obtain 1000 independent MCMC samples from the posterior distribution of the parameters. A burn-in period is discarded and the chain is sub-sampled according to the effective sample size.

Table 1
Simulation scenarios investigated and empirical coverage probabilities based on 120 data sets from each scenario.

	Conditional Kendall's τ		Coverage probability			
	Gumbel D-vine regime	Gaussian C-vine regime	90% CI		95% CI	
			Sym. (%)	HPD (%)	Sym. (%)	HPD (%)
(1)	$\tau_{43\|21} = 0.4$ $\tau_{42\|1} = 0.6$, $\tau_{32\|1} = 0.6$ $\tau_{41} = 0.8$, $\tau_{31} = 0.8$, $\tau_{21} = 0.8$	$\tau_{41\|23} = 0.4$ $\tau_{42\|3} = 0.6$, $\tau_{31\|2} = 0.6$ $\tau_{43} = 0.8$, $\tau_{32} = 0.8$, $\tau_{21} = 0.8$	92	92	94	94
(2)	$\tau_{43\|21} = 0.4$ l $\tau_{42\|1} = 0.6$, $\tau_{32\|1} = 0.6$ $\tau_{41} = 0.8$, $\tau_{31} = 0.8$, $\tau_{21} = 0.8$	$\tau_{41\|23} = 0.1$ $\tau_{42\|3} = 0.2$, $\tau_{31\|2} = 0.2$ $\tau_{43} = 0.3$, $\tau_{32} = 0.3$, $\tau_{21} = 0.3$	89	89	92	92
(3)	$\tau_{43\|21} = 0.1$ $\tau_{42\|1} = 0.2$, $\tau_{32\|1} = 0.2$ $\tau_{41} = 0.3$, $\tau_{31} = 0.3$, $\tau_{21} = 0.3$	$\tau_{41\|23} = 0.4$ $\tau_{42\|3} = 0.6$, $\tau_{31\|2} = 0.6$ $\tau_{43} = 0.8$, $\tau_{32} = 0.8$, $\tau_{21} = 0.8$	85	84	92	93
(4)	$\tau_{43\|21} = 0.1$ $\tau_{42\|1} = 0.2$, $\tau_{32\|1} = 0.2$ $\tau_{41} = 0.3$, $\tau_{31} = 0.3$, $\tau_{21} = 0.3$	$\tau_{41\|23} = 0.1$ $\tau_{42\|3} = 0.2$, $\tau_{31\|2} = 0.2$ $\tau_{43} = 0.3$, $\tau_{32} = 0.3$, $\tau_{21} = 0.3$	75	75	82	93
(5)	$\tau_{43\|21} = 0.3$ $\tau_{42\|1} = 0.5$, $\tau_{32\|1} = 0.3$ $\tau_{41} = 0.7$, $\tau_{31} = 0.5$ $\tau_{21} = 0.3$	$\tau_{41\|23} = 0.3$ $\tau_{42\|3} = 0.5$, $\tau_{31\|2} = 0.3$ $\tau_{43} = 0.7$, $\tau_{32} = 0.5$, $\tau_{21} = 0.3$	85	85	92.8	93

Table 2
R-vine models considered for the exchange rate data with tree structures $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V}_3$, copula families and parameters given in the Appendix.

Model	Defined in section	Regime 1 (no crisis)	Regime 2 (crisis)	Copulas regime 1		Copulas regime 2	Parameters
(1)	5.2	$\mathcal{V}_{11} = \mathcal{V}_1$	$\mathcal{V}_{12} = \mathcal{V}_1$	Mixed	$=$	Mixed	Table A.4
(2a) (2a*) (2b) (2c)	5.4	$\mathcal{V}_{21} = \mathcal{V}_1$ $\mathcal{V}_{21} = \mathcal{V}_1$ $\mathcal{V}_{21} = \mathcal{V}_1$ $\mathcal{V}_{21} = \mathcal{V}_1$	$\mathcal{V}_{22} = \mathcal{V}_2$ $\mathcal{V}_{22} = \mathcal{V}_2$ $\mathcal{V}_{22} = \mathcal{V}_2$ $\mathcal{V}_{22} = \mathcal{V}_2$	N N N N		SG SG, N G Student's t	Tables A.5 and A.6
(3)	5.4	$\mathcal{V}_{31} = \mathcal{V}_1$	$\mathcal{V}_{32} = \mathcal{V}_3$	Mixed	$\neq$	Mixed	Table A.7

From the obtained samples, we estimate 90% and 95% symmetric and highest posterior density (HPD) CIs for the copula parameters and check whether all true copula parameters lie within these intervals. The procedure was repeated 120 times for each scenario with results reported in Table 1. Relative bias and mean squared error (MSE) for all parameter estimates are available upon request. For all parameter setups, except Scenario 4, we observe about 90% (95%) frequentist coverage. Scenario 4 corresponds to low dependence in both regimes, thus regimes are less distinguishable. Therefore, with clearly distinguishable regimes the outlined procedure is able to identify the true model.

5. Application: US exchange rates

In this section, we apply the estimation procedures of Section 3 to analyze US exchange rate data using the MS–RV model. Since the focus is on modeling dependence structures of multivariate data, we apply a two step estimation approach as suggested by Joe and Xu (1996). In the first step, appropriate parametric models for the marginal time series are fitted separately and used to transform the standardized residuals to approximately uniform margins. To this transformed data, we apply the copula model in the second step. While joint estimation of marginal and copula models is more efficient and allows to take marginal estimation errors into consideration, it is also computationally more challenging. Given the size of our data set we choose the two step procedure.

Before Bayesian or frequentist parameter inference for the MS–RV model can be conducted, appropriate R-vine structures and sets of bivariate copulas for each regime need to be selected in a preanalysis. To do so, we apply the heuristic model selection techniques as outlined in Dißmann et al. (2013) (see Czado et al., 2013 for a comparison of different model selection strategies) which select an R-vine tree structure sequentially to capture the most important dependencies on the first trees. In all our applications we assume the presence of two regimes. Unless mentioned otherwise, the copula families we will consider are the Gaussian (N) copula and the Gumbel (G) copula. Since the Gumbel copula is not invariant with respect to rotations, we consider its standard form and rotations by 90° (G90), 180° (survival Gumbel, SG) and 270° (G270), respectively. For all models studied, we run the MCMC for 20 000 iterations discarding the first 1000 as burn-in, and keep every fifth observation to reduce autocorrelations. For estimating quantiles of the posterior distribution, we further thin the output according to what Kass et al. (1998) call "effective sample size" (cf. Carlin and Louis, 2009). After this, we end up with ca. 1000 approximately i.i.d. samples. On a standard PC with Intel® Core™ 2 Duo CPU and 4 GB RAM, these computations take about one day, the EM procedure converges in less than a minute.

This section is structured as follows: Section 5.1 introduces the exchange rate data we analyze, and an initial MS–RV model in which only the copula parameters are switching is fitted in Section 5.2. To gain additional information about

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Table 3
DIC values for the different (MS) *R*-vine models that have been considered. Lower values indicate a better fit of the model to the data.

Model	(1)	(2a)	(2a*)	(2b)	(2c)	(3)	No MS
DIC	−4398	−4280	−4312	−4199	−4346	−4430	−4146

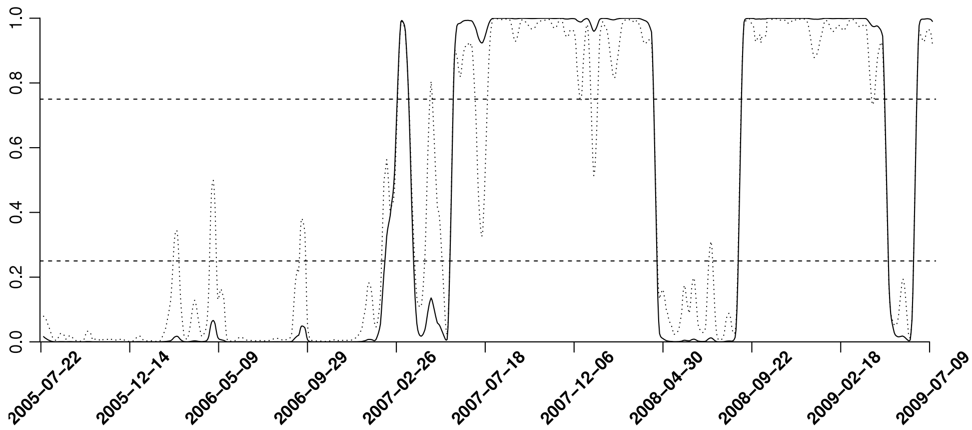

Fig. 1. Estimated probabilities of being in state 2 for Model (1) (solid: EM estimates smoothed by an MA(7) filter, dotted: Bayesian estimates).

possible regime switches also in the copula families we conduct a rolling window analysis in Section 5.3. Subsequently, *R*-vine copula structures for the "crisis" regime are selected in Section 5.4. Section 5.5 presents findings of our analysis, and the in-sample fit of all analyzed MS–RV models is compared in Section 5.6.

5.1. Data set

The data set consists of 9 exchange rates to the US dollar, namely for the Euro, British pound, Canadian dollar, Australian dollar, Brazilian real, Japanese yen, Chinese yuan, Swiss franc and Indian rupee. The observed time period is from July 22, 2005 to July 17, 2009, resulting in 1007 daily observations. The modeling of the one dimensional margins with appropriate ARMA–GARCH models and the transformation to copula data has been performed by Czado et al. (2012). In total, we consider 6 models which will be defined as we proceed, their defining tree structures and the allowed copula families are listed in Table 2.

5.2. R-Vine copula with switching parameters

As a first model, we consider a common *R*-vine copula structure for the two regimes but with different parameters. To do so, we fit an *R*-vine with corresponding bivariate copulas to the data using the procedure of Dißmann et al. (2013). Since the estimated pair-copula parameters corresponding to higher trees indicate conditional independencies, we truncate the *R*-vine copula after the second tree, i.e. we associate all edges on higher trees with independence copulas. The *R*-vine copula structure $\mathcal{V}_1$ resulting from this procedure is given in the Appendix. We call this model as Model (1).

Fig. 1 shows the probability $P(S_t = 2|\tilde{\mathbf{u}}_T, \hat{\boldsymbol{\eta}}^{\text{EM}})$ that the hidden state variable S_t indicates the presence of regime 2 plotted against time. While regime 1 is predominant until around February 2007, regime 2 becomes more important during the later times of the financial crisis.

Analyzing the parameter estimates $\hat{\boldsymbol{\theta}}_1^{\text{EM}}$ and $\hat{\boldsymbol{\theta}}_2^{\text{EM}}$ (Table A.4) for the two regimes, we find that regime 1 has stronger dependencies on the first tree, whereas regime 2 has stronger dependencies on the second tree. In particular, regime 2 exhibits stronger conditional negative dependencies reflected by rotated Gumbel copulas, thus creating a more asymmetric dependence structure. In order to apply our Bayesian estimation procedure, we need to distinguish both regimes to avoid model identification problems. For a detailed consideration of this issue we refer to Frühwirth-Schnatter (2001). Using our observations with regard to the strength of dependence in the two regimes identified by the EM algorithm, we define regime 1 to correspond to weaker conditional dependence on the second tree and regime 2 to correspond to stronger dependence on the second tree, compared by the sum of absolute values of Kendall's τ corresponding to parameters $\boldsymbol{\theta}_1^{r,\text{MCMC}}$ and $\boldsymbol{\theta}_2^{r,\text{MCMC}}$. The resulting posterior probability estimates for the state variable, i.e.

$$\hat{P}(S_t = 1|\tilde{\mathbf{u}}_T) := \frac{1}{R}\sum_{r=1}^{R} S_t^{r,\text{MCMC}},$$

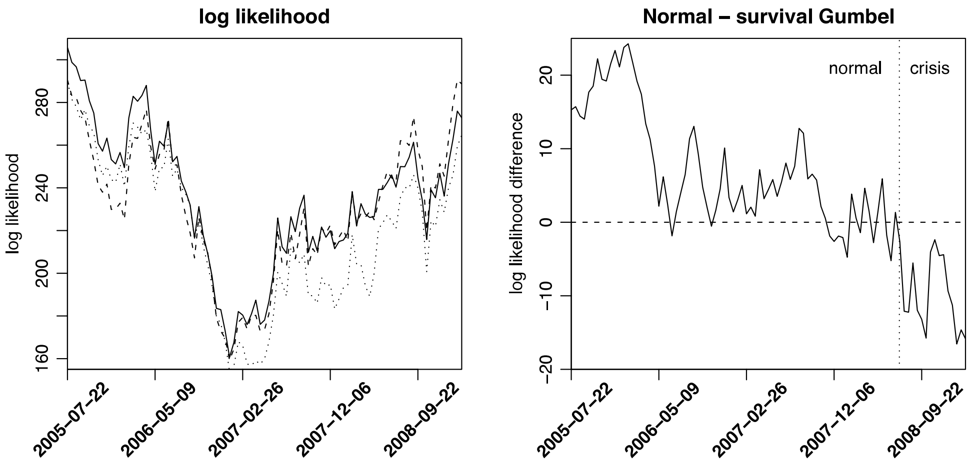

Fig. 2. Left-panel: log likelihood values resulting from fitting *R*-vines with normal (solid), survival Gumbel (dashed) and Gumbel (dotted) copulas. Right panel: difference between the values for the normal and survival Gumbel model, we indicate the period to which the structure for Model (3) is fitted. The dates are the starting observations of the rolling windows.

for *R* independent MCMC samples, are plotted as dotted points in Fig. 1. These Bayesian estimates follow those obtained from the EM algorithm closely while showing a slightly higher degree of variability.

5.3. Rolling window analysis

Having identified parameter switches in an *R*-vine copula model for our data set, we will now try to identify switches in the overall dependence structure. Since there is empirical evidence that dependence structures can change in times of crisis (cf. Longin and Solnik, 1995, Ang and Bekaert, 2002 or Garcia and Tsafack, 2011) and since tail dependencies become more important in times of extremal returns, we want to select two different *R*-vine copula structures. To do so, we start with a rolling window analysis, selecting and fitting *R*-vine models to a rolling window of 100 data points. To reduce model complexity, we decide to work again with truncated *R*-vines, resulting in a sufficiently flexible and parsimonious model. The copulas on the first tree were chosen to be either all Gaussian, Gumbel or survival Gumbel. The copulas on the second tree were set to Gaussian and the *R*-vines were truncated after this second tree. The resulting rolling log likelihoods are given in the left panel of Fig. 2. For an AIC (BIC) comparison this is sufficient since the number of parameters remains the same in all models considered.

We see that, while the range of overall likelihood estimates is similar, the Gaussian model tends to give the best fit, i.e. the highest log likelihood, (left panel of Fig. 2) over the whole data set. However, the survival Gumbel model starts to outperform the Gaussian model towards the end of the observation period (right panel of Fig. 2). Furthermore, the survival Gumbel model, in which the exchange rates taken into consideration are assumed to be lower tail dependent, tends to outperform the model with standard Gumbel copulas, corresponding to upper tail dependence. This is in accordance with the observation that the financial crisis during the observation period originated in the US dollar area, quickly spreading to the world economy but with different severity e.g. to other developed countries and developing countries. Because of this, cash flows out of the US dollar area, resulting in higher Foreign Currency/US exchange rates, tend to be less extremely correlated than the cash flows into the dollar area to settle liabilities denominating in US dollar, which results in more lower than upper tail dependence.

5.4. Identifying crisis regimes

Based on the observations from the rolling window analysis we will now define Models (2a)–(2c) and (3). For Models (2a)–(2c) we incorporate our knowledge about the evolution of the financial crisis into the model selection. They will be used to investigate the influence of different tail dependence structures in the modeling of the exchange rate data. Since restricting to a particular kind of tail dependence will make the MS–RV model look less favorable as compared to classical models in terms of goodness of fit, we do further consider Model (3). Here, we select tree structures in a semi-automatic way based on the rolling window analysis and allow for all bivariate copula families.

For Models (2a)–(2c), we decide to select *R*-vine copulas as follows: For regime 1, the tree structure ($\mathcal{V}_1$) is again fitted to the whole data set, but we use only Gaussian copulas as bivariate building blocks. Since parameter estimates on the higher trees indicate weak dependence, we truncate the *R*-vine copula after the second tree. To determine a second structure ($\mathcal{V}_2$) we apply the procedure of Dißmann et al. (2013) to the time frame from July 10, 2008 to December 3, 2008 (first 100 days of the "crisis" period indicated in Fig. 2). Doing so, we capture many high-impact events of the financial crisis. For the copulas on the first tree associated to $\mathcal{V}_2$ we consider survival Gumbel copulas (strong dependence for negative returns, Model (2a)), Gumbel copulas to capture dependencies in the upper tail (Model (2b)) and Student's *t* copulas to cover symmetric tail

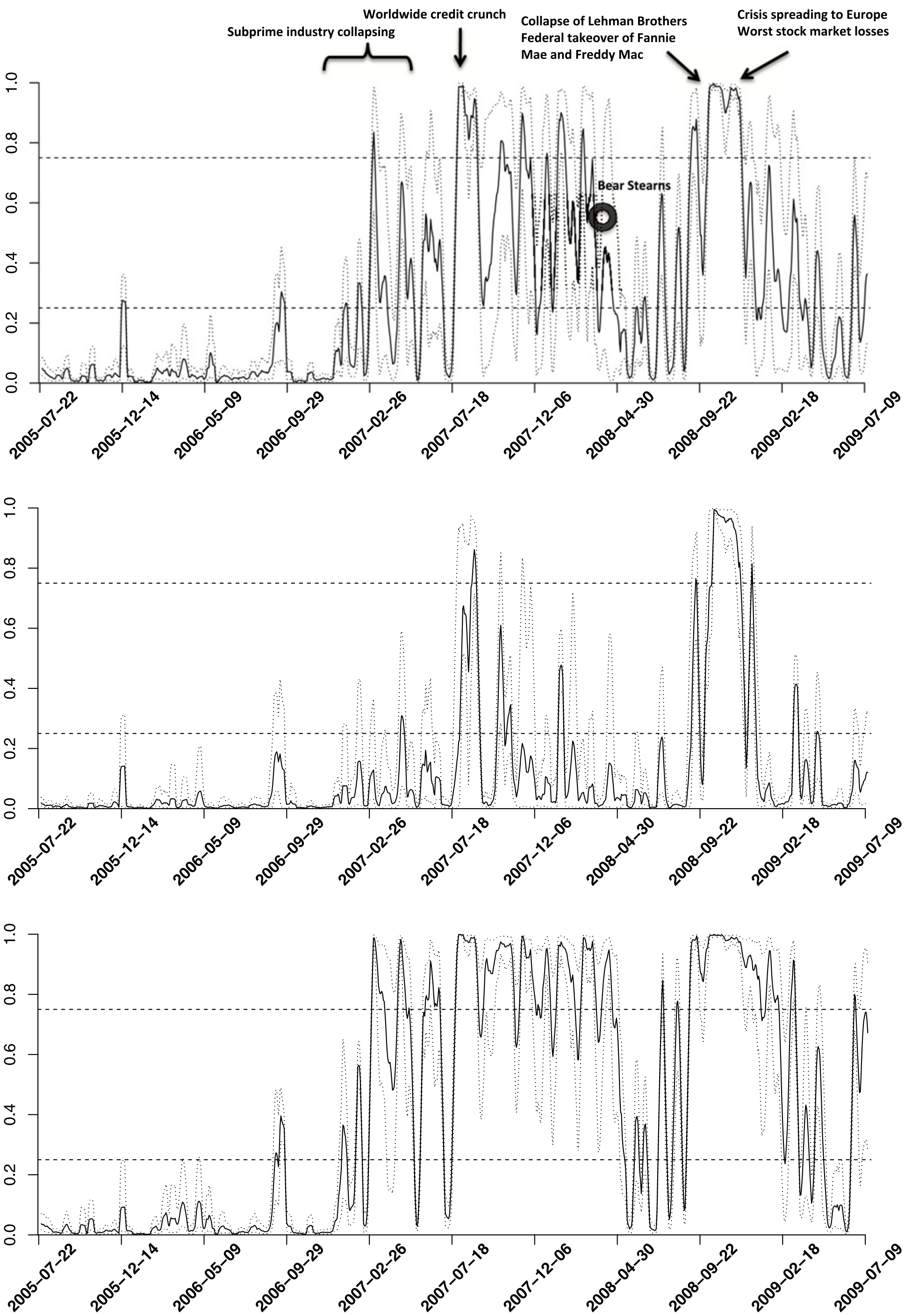

Fig. 3. Smoothed probabilities that the hidden state variable indicates the non-Gaussian regime. Models are from top to bottom: (2a), (2b), (2c). The solid lines correspond to Bayesian MCMC estimates, the dotted lines to 90% CIs. High impact crisis events are annotated in the upmost graph.

dependencies (Model (2c)). The copulas corresponding to edges on the second tree are again chosen to be Gaussian and we truncate after tree 2. While the survival Gumbel model is preferred in the rolling window analysis, we include Models (2b) and (2c) to investigate the impact of different tail dependencies. The resulting smoothed probabilities for being in the non-Gaussian regime using Models (2a)–(2c) are given in Fig. 3.

For Model (3), the R-vine structure $\mathcal{V}_3$ together with the corresponding copulas for the "crisis" regime is selected by applying the stepwise selection procedure of Dißmann et al. (2013) to the data points where the rotated Gumbel copula is outperforming the normal copula in the rolling window analysis (July 10, 2008–July 17, 2009, annotated with "crisis" in Fig. 2). The R-vine structure for the "normal" regime with corresponding copulas as identified from the remaining data

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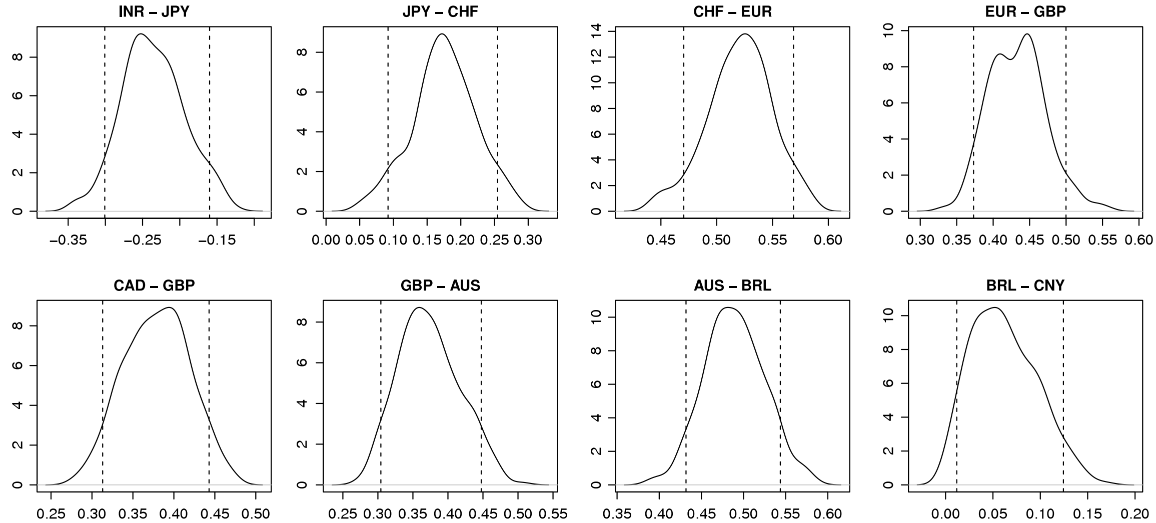

Fig. 4. Estimated marginal posterior densities in the crisis regime of Model (2a⋆) with 90% CIs. The plotted densities correspond to the (unconditional) copulas associated to Tree 1 of the vine $\mathcal{V}_2$.

points (July 22, 2005–July 9, 2008) coincides with the structure for the "normal" regime in Models (2a)–(2c), $\mathcal{V}_1$. While the pair copulas corresponding to this tree structure were all chosen to be Gaussian before, we allow for all bivariate copula families here to make use of the full flexibility the MS–RV model provides.

We employ the EM procedure for an initial fit of MS–RV models with the selected regimes. Subsequently, a Bayesian analysis using the Gibbs sampler and prior assumptions of Section 3.2 is performed.

5.5. Empirical findings

While the overall strength of dependence modeled in the two regimes (judging by the fitted values of Kendall's τ, Tables A.5 and A.6) is similar for all models, the results for Model (2c) with Student's t copulas are close to the results of Model (1), whereas the other two differ significantly. This was expected, since the model with Student's t copulas is close to a Gaussian copula model with regime switching parameters. Analyzing the estimated Kendall's τs further, we find that in Model (2c) the τ between the Japan–US and the India–US exchange rate indicates negative dependence. Since the Gumbel and survival Gumbel family model positive dependence, this cannot be captured in Model (2a) or (2b), respectively. Replacing the copula for this bivariate margin by a Gaussian copula (Model (2a⋆)) so that it captures the negative dependence, does however not significantly change the posterior estimates of the hidden state variable. This means that the observed difference in the behavior of Models (2a) and (2b) as compared to Model (2c) cannot be explained by the lack of Gumbel and Gumbel survival copulas to allow for negative dependence. Instead, these models tend to be preferred during specific times of high impact events of the financial crisis, where the bivariate dependence structures are closer to a Gumbel copula, as indicated in the top panel of Fig. 3 where important events are annotated. The probability of being in a given state at a given time is a function of the observations from the multivariate time series $\tilde{\mathbf{u}}_T$ and the model parameters $(\boldsymbol{\theta}_{\text{cop}}, \boldsymbol{\theta}_{\text{MC}})$, i.e.

$$p_{t,i} = P(S_t = i|\tilde{\mathbf{u}}_T, \boldsymbol{\theta}_{\text{cop}}, \boldsymbol{\theta}_{\text{MC}}),$$

is determined by (4) for given $(\boldsymbol{\theta}_{\text{cop}}, \boldsymbol{\theta}_{\text{MC}})$. This means that we also obtain the posterior distribution of the state probability $p_{t,i}$, from which we can compute CIs (see Fig. 3). The obtained pointwise 90% credible intervals (CIs) are quite narrow for Models (2a)–(2c) which shows that the time variations which are detected are in fact characteristics of the data.

Fig. 4 shows several marginal posterior density estimates for the copula parameters in the crisis regime of Model (2a⋆). As we can see, the parameter value of $\tau_{\text{INR–JPY}} = 0$ which would correspond to independence is nowhere near a 90% or 95% CI, the dependence is significantly negative. For the copula between Brazil–US and China–US in contrast, the parameter values in our posterior sample are all close to 0, which means that the two time series are only weekly dependent or maybe independent.

5.6. Model comparison

Having discussed the stylized features of the investigated MS–RV models, we want to compare them in terms of their fit. For this, we rely on in-sample methods, and use our Bayesian Gibbs sampling procedure to calculate the deviance information criterion (DIC) which has been proposed by Spiegelhalter et al. (2002). Table 3 shows DIC values for all models under investigation. For comparison, we also include an *R*-vine model without regime switches, but where the vine tree structure has not been truncated after tree 2. The first two trees of this structure correspond to Structure $\mathcal{V}_1$, it has been selected by applying the method proposed by Dißmann et al. (2013) to the full data set.

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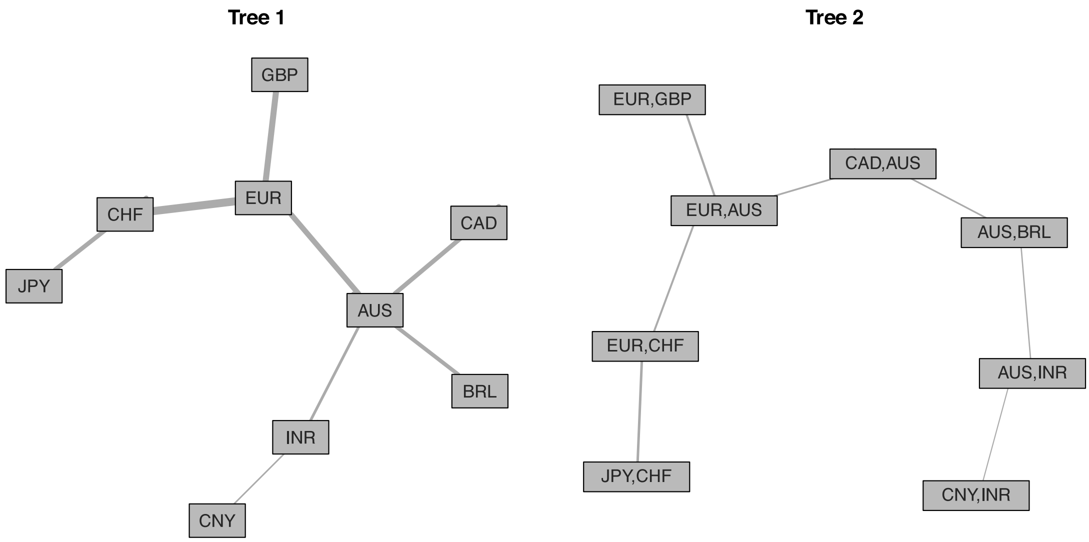

Fig. A.5. First and second tree of the tree structure $\mathcal{V}_1$ of Model (1). This structure represents also the non-crisis regime in Models (2a)–(2c) and (3).

Although the full *R*-vine model has 36 parameters and the MS–RV models where we use truncated vines and one parametric pair copulas only have 32, even the worst MS–RV copula outperforms the model without Markov structure, which clearly supports the use of models with time-varying dependence. The DIC values further show that in terms of in-sample fit, the model with standard Gumbel copulas in the crisis regime is outperformed by the other models, which was to be expected from the rolling window analysis. Since the copulas in Model (1) were chosen maximizing pairwise AIC, it outperforms the models where we restricted the choice of copulas. The best-performing model however is Model (3), where the copula families were chosen using pair-wise AIC but the *R*-vine structure differs between the regimes. This shows that MS–RV models with switching vine structure and parameters are more suitable for this kind of data than models where only the copula parameters are switching.

6. Discussion and outlook

This paper provides a detailed investigation of estimation methods for Markov switching regular vine models. We believe that this constitutes a significant contribution towards making time-varying copula models a standard tool for the description of multivariate time series. Allowing flexible pair copula constructions, which can account for asymmetric and tail-dependence as observed in the dependence among financial time series, to vary over time, we have created a model which is limited mainly by its need for efficient computational treatment.

The quick EM algorithm, based on the step-wise estimator for *R*-vine copulas, which we have introduced, allows to perform inference in almost arbitrary dimensions. For a more thorough study including the uncertainty of parameter estimates, we propose a Bayesian inference procedure which has been shown to correctly capture the true model in simulations.

In particular, we provide a tool to identify different dependence behavior during times of crisis periods as required in today's regulatory framework. The application illustrates that regime switches are present in exchange rates and are related to periods of market stress. Similar experience has been made investigating stock returns where the present regime will strongly influence portfolio Value at Risk forecasts.

While we believe that the methods for parameter estimation presented here will satisfy most statisticians and practitioners needs, improvements in model selection are still desirable. Given the regulatory requirements, further research will also address the joint estimation and detection of regime switches in marginal time series models and the dependence structure.

Acknowledgments

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Appendix. Selected *R*-vine structures

Selected R-vine structure for the US exchange rates

Model (1). Here, only the copula parameters are switching (parameter estimates are given in Table A.4) while tree structure (Fig. A.5) and copula families are common to both regimes. The tree structure is selected by applying the algorithm of Dißmann et al. (2013) to the whole data set (July 22, 2005–July 17, 2009).

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Table A.4
Estimated Kendall's τ for the first and second tree of Model (1).

Tree 1	GBP, EUR	EUR, CHF	CHF, JPY	AUS, EUR	AUS, BRL	INR, AUS	CAD, AUS	CNY, INR
cop. fam.	SG	SG	SG	N	G	N	N	G
Regime 1								
$\hat{\tau}_1^{EM}$	0.55	0.78	0.46	0.46	0.19	0.14	0.29	0.11
$\hat{\tau}_1^{MCMC}$	0.56	0.79	0.47	0.46	0.18	0.14	0.28	0.13
5% quant.	0.53	0.77	0.43	0.43	0.14	0.10	0.24	0.09
95% quant.	0.60	0.81	0.50	0.49	0.23	0.20	0.32	0.17
Regime 2								
$\hat{\tau}_2^{EM}$	0.44	0.58	0.24	0.41	0.45	0.26	0.44	0.07
$\hat{\tau}_2^{MCMC}$	0.44	0.58	0.22	0.40	0.43	0.25	0.44	0.05
5% quant.	0.40	0.55	0.17	0.36	0.39	0.20	0.40	0.02
95% quant.	0.47	0.60	0.27	0.43	0.47	0.29	0.50	0.10

Tree 2	GBP, AUS| EUR	CAD, EUR| AUS	CAD, BRL| AUS	BRL, INR| AUS	CNY, AUS| INR	JPY, EUR| CHF	CHF, AUS| EUR
cop. fam.	G	G	SG	0	SG	G270	G270
Regime 1							
$\hat{\tau}_1^{EM}$	0.15	0.11	0.07	0.02	0.01	−0.06	−0.03
$\hat{\tau}_1^{MCMC}$	0.15	0.11	0.07	0.01	0.02	−0.06	−0.03
5% quant.	0.10	0.07	0.03	−0.04	0.00	−0.10	−0.07
95% quant.	0.20	0.15	0.13	0.06	0.05	−0.02	−0.00
Regime 2							
$\hat{\tau}_2^{EM}$	0.15	0.11	0.11	0.11	0.10	−0.31	−0.24
$\hat{\tau}_2^{MCMC}$	0.16	0.13	0.11	0.11	0.11	−0.31	−0.24
5% quant.	0.10	0.08	0.04	0.06	0.07	−0.36	−0.28
95% quant.	0.21	0.17	0.17	0.16	0.16	−0.26	−0.19

Table A.5
Estimated Kendall's τ values corresponding to the first tree of Models (2a)–(2c), respectively. For the Student's t copula used in Model (2c), the first parameter is Kendall's τ, the second gives the degrees of freedom (with $\nu_{\max} = 30$).

"normal", $\mathcal{V}_1$	GBP, EUR	EUR, CHF	CHF, JPY	AUS, EUR	AUS, BRL	INR, AUS	CAD, AUS	CNY, INR
(2a) $\hat{\tau}_1^{EM}$	0.53	0.75	0.45	0.46	0.28	0.21	0.35	0.12
(2a) $\hat{\tau}_1^{MCMC}$	0.56	0.78	0.45	0.48	0.24	0.21	0.33	0.15
5% quant.	0.52	0.75	0.41	0.45	0.19	0.17	0.29	0.10
95% quant.	0.60	0.80	0.49	0.51	0.29	0.25	0.36	0.20
(2b) $\hat{\tau}_1^{EM}$	0.54	0.75	0.44	0.47	0.29	0.22	0.35	0.12
(2b) $\hat{\tau}_1^{MCMC}$	0.52	0.74	0.43	0.44	0.29	0.21	0.34	0.11
5% quant.	0.49	0.72	0.40	0.41	0.26	0.17	0.31	0.07
95% quant.	0.55	0.76	0.46	0.48	0.32	0.24	0.37	0.14
(2c) $\hat{\tau}_1^{EM}$	0.60	0.81	0.47	0.48	0.21	0.19	0.32	0.16
(2c) $\hat{\tau}_1^{MCMC}$	0.60	0.80	0.47	0.49	0.21	0.21	0.31	0.17
5% quant.	0.57	0.79	0.44	0.46	0.16	0.17	0.27	0.13
95% quant.	0.62	0.82	0.51	0.52	0.26	0.22	0.35	0.22
"crisis", $\mathcal{V}_2$	GBP, EUR	EUR, CHF	CHF, JPY	JPY, INR	AUS, GBP	BRL, AUS	BRL, CNY	CAD, GBP
(2a) $\hat{\tau}_2^{EM}$	0.44	0.45	0.11	0.00	0.41	0.49	0.11	0.41
(2a) $\hat{\tau}_2^{MCMC}$	0.42	0.52	0.22	0.01	0.37	0.47	0.07	0.37
5% quant.	0.36	0.47	0.11	0.00	0.30	0.41	0.01	0.29
95% quant.	0.49	0.56	0.30	0.02	0.44	0.53	0.13	0.43
(2b) $\hat{\tau}_2^{EM}$	0.37	0.37	0.10	0.00	0.32	0.41	0.08	0.36
(2b) $\hat{\tau}_2^{MCMC}$	0.45	0.37	0.05	0.01	0.35	0.44	0.13	0.38
5% quant.	0.34	0.27	0.00	0.00	0.25	0.36	0.03	0.30
95% quant.	0.55	0.45	0.15	0.04	0.44	0.53	0.24	0.46
(2c) $\hat{\tau}_2^{EM}$	0.43, 10.8	0.58, 8.6	0.27, 7.9	−0.13, 30	0.37, 10.7	0.44, 5.9	0.06, 30	0.33, 30
(2c) $\hat{\tau}_2^{MCMC}$	0.42, 14.2	0.56, 9.7	0.25, 9.8	−0.15, 21.4	0.35, 15.4	0.45, 9.3	0.05, 20.8	0.34, 21.8
5% quant.	0.38, 7.0	0.53, 5.6	0.21, 5.2	−0.21, 11.6	0.31, 7.0	0.45, 4.8	−0.01, 10.7	0.29, 11.0
95% quant.	0.46, 25.7	0.60, 16.3	0.29, 18.3	−0.11, 29.2	0.40, 28.1	0.49, 19.0	0.11, 29.2	0.39, 29.3

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Table A.6
Estimated Kendall's τ, corresponding to the tree 2 of (2a)–(2c).

"normal", $\mathcal{V}_1$	JPY, EUR| CHF	AUS, CHF| EUR	AUS, GBP| EUR	CAD, EUR| AUS	CAD, BRL| AUS	INR, BRL| AUS	CNY, AUS| INR
(2a) $\hat{\tau}_1^{EM}$	−0.14	−0.13	0.14	0.10	0.12	0.07	0.01
(2a) $\hat{\tau}_1^{MCMC}$	−0.10	−0.08	0.13	0.10	0.11	0.06	0.00
5% quantile	−0.16	−0.14	0.09	0.06	0.07	0.01	−0.05
95% quantile	−0.02	0.00	0.18	0.14	0.15	0.10	0.04
(2b) $\hat{\tau}_1^{EM}$	−0.16	−0.14	0.15	0.10	0.12	0.07	0.01
(2b) $\hat{\tau}_1^{MCMC}$	−0.17	−0.17	0.16	0.10	0.11	0.07	0.02
5% quantile	−0.21	−0.20	0.13	0.06	0.07	0.03	−0.02
95% quantile	−0.13	−0.13	0.20	0.13	0.14	0.11	0.06
(2c) $\hat{\tau}_1^{EM}$	−0.01	0.02	0.15	0.10	0.12	0.03	0.00
(2c) $\hat{\tau}_1^{MCMC}$	−0.03	0.00	0.14	0.10	0.11	0.04	0.00
5% quantile	−0.08	−0.06	0.09	0.05	0.07	−0.01	−0.05
95% quantile	0.03	0.05	0.18	0.15	0.16	0.09	0.04
"crisis", $\mathcal{V}_2$	CNY, AUS| BRL	GBP, BRL| AUS	AUS, CAD| GBP	CAD, EUR| GBP	GBP, CHF| EUR	EUR, JPY| CHF	CHF, INR| JPY
(2a) $\hat{\tau}_2^{EM}$	0.21	0.04	0.24	0.14	−0.22	−0.42	0.03
(2a) $\hat{\tau}_2^{MCMC}$	0.17	0.04	0.28	0.15	−0.19	−0.36	0.07
5% quantile	0.09	−0.03	0.19	0.07	−0.26	−0.45	0.01
95% quantile	0.25	0.12	0.35	0.21	−0.13	−0.31	0.15
(2b) $\hat{\tau}_2^{EM}$	0.19	0.11	0.26	0.17	−0.16	−0.36	−0.02
(2b) $\hat{\tau}_2^{MCMC}$	0.22	0.14	0.39	0.28	−0.14	−0.42	−0.02
5% quantile	0.11	0.02	0.21	0.14	−0.23	−0.52	−0.10
95% quantile	0.34	0.28	0.56	0.42	−0.04	−0.31	0.07
(2c) $\hat{\tau}_2^{EM}$	0.10	0.03	0.30	0.18	−0.15	−0.35	0.12
(2c) $\hat{\tau}_2^{MCMC}$	0.11	0.04	0.30	0.18	−0.16	−0.36	0.11
5% quantile	0.06	−0.01	0.25	0.12	−0.20	−0.40	0.05
95% quantile	0.16	0.10	0.35	0.22	−0.11	−0.31	0.16

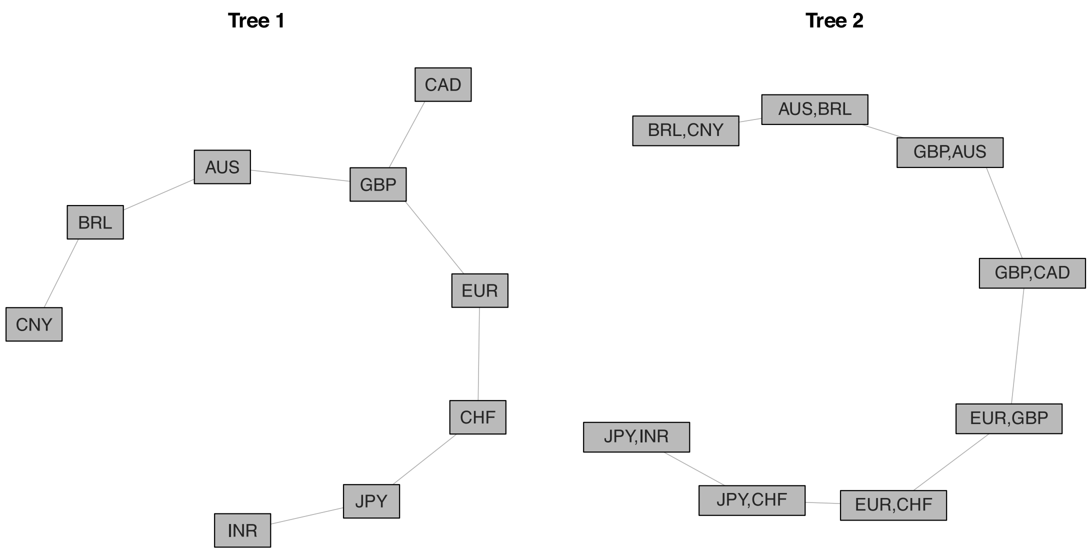

Fig. A.6. First and second tree of the "crisis" R-vine structure $\mathcal{V}_2$ which we have chosen for Model (2).

Model (2). For Models (2a)–(2c), two R-vine tree structures have been selected. The R-vine $\mathcal{V}_1$, corresponding to normal times, has again the structure displayed in Fig. A.5, the second R-vine ($\mathcal{V}_2$), is given in Fig. A.6. The corresponding parameter estimates are given in Tables A.5 and A.6. $\mathcal{V}_2$ was obtained by applying the model selection heuristic of Dißmann et al. (2013) to the period from July 10, 2008 to December 3, 2008.

Model (3). The structure for the first regime is $\mathcal{V}_1$ with copulas selected by AIC, the tree structure for the second regime is $\mathcal{V}_3$ (Fig. A.7) and the estimated parameters are given in Table A.7.

Here, the structures $\mathcal{V}_1$ and $\mathcal{V}_3$ and the copula families were selected by applying the Dißmann et al. (2013) algorithm to the periods from July 22, 2005 to July 9, 2008 and July 10, 2008 to July 17, 2009, respectively.

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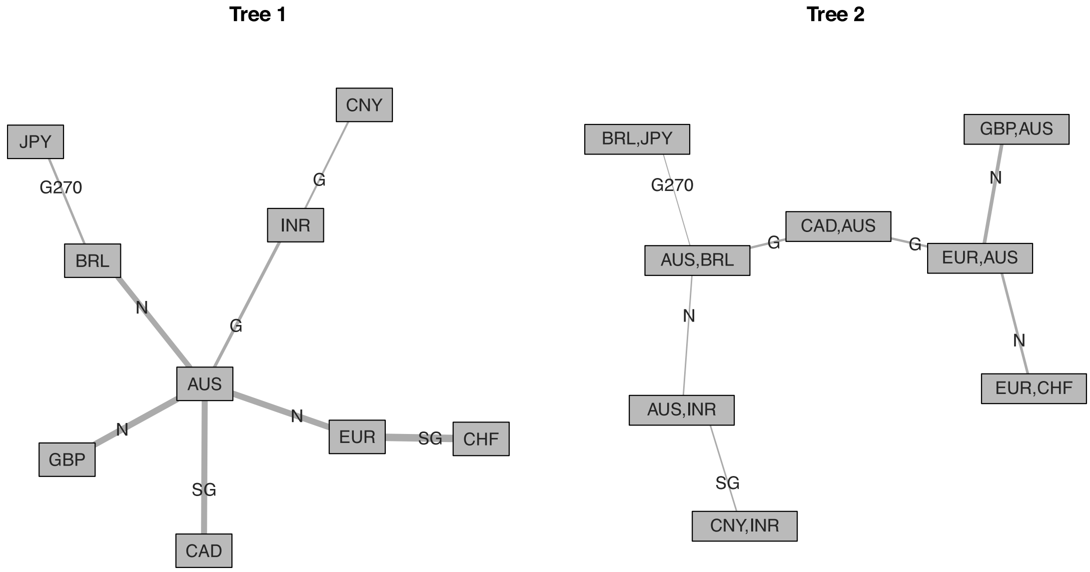

Fig. A.7. First and second tree of the *R*-vine structure representing the "crisis" regime of Model (3). We refer to this structure as $\mathcal{V}_3$.

Table A.7
Estimated Kendall's τ, corresponding to the first and second tree of the normal regime as well as the crisis regime in Model (3).

"normal", $\mathcal{V}_1$	GBP, EUR	EUR, CHF	CHF, JPY	AUS, EUR	AUS, BRL	INR, AUS	CAD, AUS	CNY, INR
cop. fam.	SG	N	N	N	G	N	N	G
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{EM}}$	0.51	0.76	0.49	0.45	0.22	0.16	0.30	0.11
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{MCMC}}$	0.52	0.76	0.49	0.45	0.22	0.16	0.30	0.11
5% quantile	0.49	0.75	0.46	0.42	0.17	0.12	0.26	0.07
95% quantile	0.55	0.78	0.52	0.48	0.26	0.20	0.34	0.16

"normal", $\mathcal{V}_1$	JPY, EUR| CHF	AUS, CHF| EUR	AUS, GBP| EUR	CAD, EUR| AUS	CAD, BRL| AUS	INR, BRL| AUS	CNY, AUS| INR
cop. fam.	G270	G 270	G	G	N	N	G
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{EM}}$	−0.09	−0.05	0.14	0.10	0.08	0.05	0.03
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{MCMC}}$	−0.09	−0.05	0.14	0.11	0.08	0.04	0.04
5% quant.	−0.14	−0.10	0.10	0.07	0.04	−0.01	0.01
95% quant.	−0.04	−0.02	0.18	0.15	0.13	0.09	0.07

"crisis", $\mathcal{V}_3$	CHF, EUR	EUR, AUS	GBP, AUS	AUS, CAD	AUS, BRL	BRL, JPY	INR, AUS	CNY, INR
cop. fam.	SG	N	N	SG	N	G270	G	G
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{EM}}$	0.54	0.42	0.42	0.50	0.52	−0.34	0.23	0.06
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{MCMC}}$	0.55	0.40	0.41	0.49	0.52	−0.35	0.23	0.06
5% quant.	0.50	0.35	0.35	0.45	0.48	−0.40	0.16	0.00
95% quant.	0.58	0.44	0.46	0.53	0.56	−0.30	0.29	0.12

"crisis", $\mathcal{V}_3$	CNY, AUS| INR	INR, BRL| AUS	AUS, JPY| BRL	CAD, BRL| AUS	CAD, EUR| AUS	GBP, EUR| AUS	AUS, CHF| EUR
cop. fam.	SG	N	G270	G	G	N	N
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{EM}}$	0.10	0.11	−0.14	0.10	0.15	0.34	−0.32
(2a) $\hat{\boldsymbol{\tau}}_1^{\text{MCMC}}$	0.11	0.11	−0.17	0.13	0.16	0.34	−0.32
5% quant.	0.05	0.05	−0.24	0.05	0.10	0.28	−0.37
95% quant.	0.19	0.17	−0.10	0.19	0.22	0.41	−0.27

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