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Predicting volatility using the Markov-switching multifractal model: Evidence from S&P 100 index and equity options

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ABSTRACT

In this paper, we evaluate the performance of the ability of Markov-switching multifractal (MSM), implied, GARCH, and historical volatilities to predict realized volatility for both the S&P 100 index and equity options. Some important findings are as follows. First, we find that the ability of MSM and GARCH volatilities to predict realized volatility is better than that of implied and historical volatilities for both the index and equity options. Second, equity option volatility is more difficult to be forecast than index option volatility. Third, both index and equity option volatilities can be better forecast during non-global financial crisis periods than during global financial crisis periods. Fourth, equity option volatility exhibits distinct patterns conditional on various equity and option characteristics and its predictability by MSM and implied volatilities depends on these characteristics. And finally, we find that MSM volatility outperforms implied volatility in predicting equity option volatility conditional on various equity and option characteristics.

1. Introduction

Both academics and practitioners have been interested in predicting financial volatilities for the purpose of option pricing and financial asset and risk management. In the option market, option

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implied volatility is regarded as the predictor of future volatility of the underlying asset over the remaining life of the option. If the option market is efficient, implied volatility should contain the relevant information that is embedded in all other variables, including historical volatility, regarding future volatility.

There is a long-standing debate concerning whether implied volatility is a more efficient predictor of future volatility than GARCH and historical volatilities. On the one hand, many researchers find that implied volatility outperforms historical volatility in forecasting future volatility.[1] On the other hand, Lamoureux and Lastrapes (1993) and Canina and Figlewski (1993) provide evidence to the contrary. Specifically, Canina and Figlewski (1993) find that historical volatility is significantly related to future volatility, whereas implied volatility is unrelated to future volatility. They provide two explanations for their findings. First, the market is not enough efficient and the irrational investors usually make noises in the markets. Second, the trading cost is not considered in the Black-Scholes model and therefore the model is biased in this regard.

Prior studies that evaluate the performance of the ability of implied, GARCH, and historical volatilities to predict realized volatility tend to focus on index options, and the few exceptions are Gemmill (1986), Lamoureux and Lastrapes (1993), and Vasilellis and Meade (1996). For example, Lamoureux and Lastrapes (1993) analyze 10 equity options traded on the Chicago Board Options Exchange for the period 1982 through 1984 and find that implied volatility contains the incremental information relative to historical volatility in predicting realized volatility. These studies, however, do not attempt to investigate whether the option volatility with different equity and option characteristics will display different behavior and the degree of predictability. More recently, researchers have directed greater attention to this issue. For example, Ammann, Skovmand, and Verhofen (2009) find that implied volatility tends to overestimate realized volatility for underlying stocks with small capitalization, low beta, low market-to-book ratio, and no price momentum, and vice versa. Their study highlights the importance that stock characteristics should be taken into account in estimating the realized volatility of the equity options. However, their study does not take into account option characteristics such as the put-call ratio and option bid-ask spread.

In the literature, implied, GARCH, and historical volatilities are three widely used predictors for future volatility, even though no consensus is reached regarding which one has the best performance. However, these approaches cannot fully capture the outliers and long-memory feature in volatility. Calvet and Fisher (2004) propose a Markov-switching multifractal (MSM) model characterized by a small number of parameters but an arbitrarily large number of frequencies. The multifractal structure of this model can properly capture the outliers, moment scaling, and long memory exhibited in the financial volatility time series. In their empirical investigation of four daily exchange rate series, Calvet and Fisher (2004) find that the MSM model outperforms the GARCH, the fractionally integrated GARCH, and the Markov switching GARCH models both in- and out-of-sample. Their finding suggests that MSM volatility might be a good alternative approach to forecasting realized volatility that has significant outliers and long-memory.

Another important issue associated with volatility forecasting is the significant impact of extreme events on volatility. One well-known example is the market crash in October 1987 in U.S. stock markets. Similar observations can also be found in other stock markets. For example, Duan and Zhang (2001) have found that the 1997 Asian financial crisis had resulted in a substantial increase in market volatility in the Hong Kong stock market (see also Connolly, Stivers, & Sun, 2005; Poon & Granger, 2005; Fung, 2007). Poon and Granger (2005) point out that care must be taken in dealing with volatility estimation, modeling, and forecasting in the presence of extreme events.

In this paper, we evaluate the performance of the ability of MSM, implied, GARCH-type, and historical volatilities to predict future realized volatility for both the S&P 100 index and equity options. Motivated by the findings of Ammann et al. (2009), using the equity options allows us to conduct a cross-sectional analysis conditional on not only option characteristics but also equity characteristics.

[1] See, for example, among others, Latane and Rendleman (1976), Schmalensee and Trippi (1978), Chiras and Manaster (1978), Beckers (1981), Gemmill (1986), Vasilellis and Meade (1996), Christensen and Prabhala (1998), Szakmary, Ors, Kim, and Davidson (2003), and Yu, Lui, and Wang (2010).

Moreover, although Calvet and Fisher's (2004) finding implies that MSM volatility might be superior to implied volatility in forecasting realized volatility, whether this is the case is still an important, but unanswered, question. Our paper aims to address this important question. Since volatility forecasting is affected by extreme events, we also investigate the issue of whether the forecasting performance of our volatility measures is affected by the recent global financial crisis (hereafter, GFC) in 2007. Moreover, the effective risk management is especially important before and during any financial crisis periods and this will depend on how effectively to predict asset volatility. We also attempt to identify which volatility measures do the best job in forecasting future volatility during the GFC period.

Our results show that the ability of MSM and GARCH volatilities to predict the realized volatilities of the index and equity options is better than that of implied and historical volatilities. As for the degree of predictability of the volatilities of the index versus equity options, we find that it is more difficult to forecast equity option volatility than index option volatility. Comparing the degree of predictability of the index and equity options in non-global financial crisis (hereafter, non-GFC) periods with that in GFC periods, we find that the predictive power of our volatility measures significantly decreases during GFC periods.

Conditional on various characteristics, we find that the volatilities of the equity options increase with trading volume and the put-call ratio, decrease with firm size and the option bid-ask spread, are higher for options with large positive and negative momentum in underlying equity returns than for those with less momentum in underlying equity returns, and are larger for options with more positive and negative skewness than for those with less skewness. We also find that the predictive power of MSM and implied volatilities for the volatilities of the equity options increases with firm size and volume, decreases with the option bid-ask spread and put-call ratio, is better for options with less momentum in underlying equity returns than for those with large positive and negative momentum in underlying equity returns, and is better for options with less skewness than for those with more positive and negative skewness. Lastly, we find that MSM volatility outperforms implied volatility in predicting the volatilities of the equity options conditional on various characteristics.

The remainder of the paper is organized as follows. Section 2 describes the data and reports some descriptive statistics. Section 3 presents various volatility measures. Section 4 introduces our empirical frameworks. Section 5 presents and discusses the empirical results. Finally, we conclude the paper in Section 6.

2. Data description

Our sample consists of the S&P 100 index and equity options. The reason of using these two types of options is that the S&P 100 index option market is the most active index option market in the world (see also Harvey & Whaley, 1993; Christensen & Prabhala, 1998). The data of the S&P 100 index and equity options are obtained from the OptionMetrics Ivy DB database, which provides the daily option data on implied volatility, open interest, and closing bid-ask spread. In addition, the data on stock prices, trading volume, the number of shares outstanding, and industry classification are obtained from the Center for Research in Security Prices (CRSP), and the data on book value of equity are obtained from the Compustat. The sample period covers from 3 January 2000 to 31 October 2009. We exclude 8 equity options that miss some relevant data from our sample, leaving 92 equity options in the sample.[2]

MSM and GARCH volatilities are estimated using the daily returns of the index and equity options over the last one year. Consequently, our studying period is from the 2 January 2001 through 31 October 2009, which covers 2,221 trading days. To investigate the impact of the global financial turmoil caused by the subprime mortgage crisis in July 2007 on the degree of predictability of realized volatility, we conduct a sub-period analysis of all our empirical tests where the period from 2 January 2001 to 30 June 2007 is defined as the non-GFC period with the total transactions of 1,631 and that from 1 July

[2] We choose the sample period starting from 3 January 2000 as the best compromise between maximizing the sample period while minimizing the exclusion of the equity options that miss some relevant data, and the sample period ends on 31 October 2009 because it is the date of all relevant data available to us when we initiated our research project.

2007 to 31 October 2009 as the GFC period with the total transactions of 590.[3] Specifically, we follow Cecchetti (2009), Poulsen, Schenk-Hoppe, and Ewald (2009), and Diebold and Yilmaz (2012) to define the GFC period.

To see whether the predictive power of volatility measures for realized volatility is affected by equity and option characteristics, we consider four equity characteristics (firm size, momentum, volume, and industry classification) and three option characteristics (bid-ask spread, skewness, and put-call ratio) to form the groups. At the beginning of each year from 2001 to 2009, three groups are formed by ranking 92 equity options in our sample by various characteristics, except for industry classification. As for the industry group, we use the SIC classification of the CRSP to stratify equity options into three industries: financial industry (group 1), technology industry (group 4), and the other industries (group 5). Specifically, the other industries are composed of the equity options whose underlying stocks do not belong to financial and technology industries. Since the subprime mortgage crisis triggers the recent GFC, we further stratify the equity options in the financial industry into the banking sector (group 2) and the non-banking sector (group 3) to see whether there is a significant difference in the degree of the predictive power of our volatility measures for realized volatility between these two sectors.

Firm size is defined as market capitalization at the end of the previous year. Following common practice in the literature, we define momentum as the cumulative returns for the 11-month period from 12 through 2 months prior to the group formation date. Volume is the daily average trading volume of the underlying equity over the previous one year. As for three option characteristics, the bid-ask spread is the average daily closed bid-ask spread of individual stock options over the previous one year. The skewness is measured as the third moment of underlying equity returns divided by their cube of standard deviation. The put-call ratio is the daily open interest contracts of put options divided by those of call options, and then the average put-call ratio is calculated over the previous one year.

3. Volatility measures

3.1. Realized and historical volatilities

Following Jorion (1995), we use 252 trading days to annualize the standard deviation of daily underlying stock returns to obtain realized volatility as below:

$$RV = \left(\frac{252}{T_M - 1} \sum_{t=1}^{T_M} (r_t - \bar{r})^2 \right)^{1/2}, \tag{1}$$

where $r_t = \ln(P_t/P_{t-1})$ is the daily return and P_t is the closing underlying index price of the index option or the closing underlying stock price of the equity options, $\bar{r}$ the sample mean of r_t, calculated as $\bar{r} = 1/T_M \sum_{t=1}^{T_M} r_t$, and T_M denotes the number of trading days left until the option expiration date. As for historical volatility, following Szakmary et al. (2003), we use the 30-day window to estimate it.[4]

3.2. Implied volatility

The measure of implied volatility of call and put options is obtained from the OptionMetrics Ivy DB database.[5] Implied volatility provided by the option database is estimated using the number of

[3] As expected, we also find that the realized volatilities of the S&P 100 index and equity options have a significant structure change due to the subprime mortgage crisis of the Fannie Mae and Freddie Mac occurred in July 2007.

[4] We also estimate historical volatility over the 60- and 90-day windows. We find that the ability of historical volatility estimated over the 30-day window to predict realized volatility is, on average, better than that estimated over the 60- and 90-day windows. We, therefore, choose to compare the predictive power of historical volatility estimated over the 30-day window with that of the other volatility measures.

[5] The implied volatilities yielded from the OptionMetrics Ivy DB database are calculated based on the Black and Scholes (1973) formula for the options that have an European-style feature, and are estimated using a binomial tree model of Cox et al. (1979) for the options with the American-style feature.

calendar days until the expiration date. Previous studies, however, have shown that the volatility tends to be lower on the non-trading days than on trading days in many markets.[6] Taking this into account, we follow Szakmary et al. (2003) to adjust implied volatility as follows:

$$IV_t = IV_t^{\text{OptionMetrics}} \times \sqrt{\frac{T_C}{T_M}}, \tag{2}$$

where T_C and T_M are the number of calendar days and that of trading days, respectively, over the remaining life of the option contract, $IV_t^{\text{OptionMetrics}}$ is the implied volatility of call and put options obtained from the OptionMetrics Ivy DB database at day t, and IV_t is the adjusted implied volatility of call and put options at day t. Following Yu et al. (2010), we use the equal-weighted implied volatility of call and put options in our following empirical tests.[7] Moreover, we adopt implied volatility from the nearest at-the-money options until ten trading days prior to option expiration and then switch to the next deferred option series. As noted by Szakmary et al. (2003), this procedure can help avoid distortion effects associated with estimating volatility with too short of an option maturity and reduce distortion effects due to the use of Black's model for too long of an option maturity, resulting in uniformity in volatility estimation across the contracts we examine.

3.3. GARCH model

Engle (1982) has been presented that the autoregressive conditional heteroskedasticity (ARCH) provides a good characterization for many financial time series. Under the ARCH model, the persistence of shocks to variance follows an autoregressive structure, which can well capture the non-normality and non-stability of financial asset return distributions (e.g., Lamoureux & Lastrapes, 1990). Afterward, Bollerslev (1986) extends Engle's (1982) ARCH model and proposes the generalized ARCH (GARCH) model. The GARCH model is specified as follows:

$$r_t = \mu + \varepsilon_t, \varepsilon_t \sim N(0, h_t), \tag{3}$$

$$h_t = \omega + \alpha(L)\varepsilon_{t-1}^2 + \beta(L)h_{t-1}, \tag{4}$$

where r_t is the index or equity return, μ is the mean of returns, h_t is the conditional variance, L is the lag operator, and ε_t is the random error.

Bollerslev (1987) proposes that the GARCH (1, 1) specification is an adequate representation for most economic and financial time series. As such, the GARCH (1, 1) specification is extensively used as a measure of volatility to predict realized volatility in many empirical studies.[8] Following this line of study, we also use the GARCH (1, 1) specification to measure volatility and consequently rewrite Eq. (4) as follows:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{5}$$

And following Engle and Bollerslev (1986), we obtain a daily s-step-ahead forecast of volatility as follows:

$$\hat{h}_{t+s} = \hat{\omega}\sum_{j=1}^{s-1}\left((\hat{\alpha}+\hat{\beta}\right)^j + (\hat{\alpha}+\hat{\beta})^{s-1}\hat{h}_{t+1}, \quad s = 1, 2, \ldots, T_M, \tag{6}$$

[6] See, for example, French (1984), Fleming, Ostdiek, and Whaley (1996), Jorion (1995), Davidson et al. (2001), and Szakmary et al. (2003).

[7] We also examine the relative performance of call option and put option implied volatilities in forecasting realized volatility and find that call option implied volatility performs better, on average, than put option implied volatility.

[8] See, for example, Day and Lewis (1992), Lamoureux and Lastrapes (1993), Jorion (1995), Vasilellis and Meade (1996), Brailsford and Faff (1996), Covrig and Low (2003), Szakmary et al. (2003), and Yu et al. (2010).

where $\hat{h}_{t+s}$ denotes the s-day ahead volatility predictor and the parameters of $\hat{\omega}$, $\hat{\alpha}$ and $\hat{\beta}$ are estimated using daily underlying stock returns over the previous year. Then following French (1984) and Jorion (1995), we generate the annualized GARCH volatility forecast (GV) as follows:

$$GV = \left(\frac{252}{T_M} \sum_{s=1}^{T_M} \hat{h}_{t+s} \right)^{1/2}, \tag{7}$$

where T_M is the number of trading days until the expiration of the option contract, as defined in Eq. (1).

An extension of the GARCH model is the asymmetric GARCH model such as GJR-GARCH and EGARCH models proposed by Glosten, Jagannathan, and Runkle (1993) and Nelson (1991), respectively. The asymmetric GARCH model allows the prior positive and negative volatility shocks to have asymmetric impact on the conditional variance. However, prior studies have shown that the GARCH model outperforms the GJR-GARCH and EGARCH models in forecasting future volatilities (e.g., Akgiray, 1989; Brooks, 1998; Day & Lewis, 1992; Doidge & Wei, 1998; Franses & Dijk, 1996). We also find that the asymmetric relation between conditional variance and prior volatility shocks is not significant in the index option and in most of the equity options. As a consequence, we choose to employ the GARCH model in our empirical tests.

3.4. MSM model

Mandelbrot, Fisher, and Calvet (1997) develop the Multifractal Model of Asset Returns (MMAR model) based on Mandelbrot (1972), Mandelbrot (1974), which are the forerunning studies in multifractal measures. This model takes into account the Noah effect and Joseph effect that are now well known in finance. The Noah effect denotes the long-tails of the return distribution as show in Mandelbrot (1963), while the Joseph effect indicates the long-dependence in volatility, which is the property of fractional Brownian motion presented by Mandelbrot and Van Ness (1968). However, in contrast to the models of Mandelbrot (1963) and Mandelbrot and Van Ness (1968), the MMAR model does not require that the variance be infinite and price increments be correlated. Mandelbrot et al. (1997) also suggest that the MMAR model is an alternative to ARCH-type representations based on the results of their examination on the foreign exchange market. However, one shortcoming of the MMAR model is that it lacks the applicable statistical measures.

Calvet and Fisher (2004) modify the MMAR model and propose the Markov-switching multifractal model (MSM model) that can overcome the shortcoming of the MMAR model. The MSM model is briefly described below.

Assuming the economic system is driven by a first-order Markov state vector at time t, the Markov state vector composed of different k stochastic multiplier is shown as follows:

$$M_t = (M_{1,t};\ M_{2,t};\ \ldots;\ M_{\bar{k},t} \in R_+^{\bar{k}}), \tag{8}$$

where each of stochastic multiplier of M_t has the same marginal distribution, but its frequency is heterogeneous at $r_1, r_2, \ldots, r_{\bar{k}}$. As for $k \in \{1, \ldots, \bar{k}\}$, the dynamics of $M_{k,t}$ can be presented that the $M_{k,t}$ drawn from distribution M with probability $r_{\bar{k}}$ and $M_{k,t} = M_{k,t-1}$ with the probability of $1 - r_k$. Here, the transferring events and new draws from M are assumed to be independent across k and t. The distribution M should be satisfied with the conditions, $M > 0$ and $E(M) = 1$. And the stochastic volatility process is built as follows:

$$\sigma(M_t) \equiv \bar{\sigma}(\Pi_{k=1}^{\bar{k}} M_{k,t})^{1/2}, \tag{9}$$

where $\bar{\sigma}$ is the standard deviation of non-conditional probability returns, $r_t = \sigma(M_t)\varepsilon_t$ denotes the return, and $\varepsilon_t \sim N(0, 1)$ is the random error. The switching probability $r_k \equiv (r_1, r_2, \ldots, r_k)$ presented as

$$r_k = 1 - (1 - r_1)^{(b^k - 1)}, \tag{10}$$

where $r_k \in (0, 1)$ and $b \in (1, \infty)$. The (r_k, b) is selected as the set of the transition probabilities. And this process is called Markov-Switching Multifractal (MSM). Additionally, we also manipulate the MSM volatility to be annualized by multiplying the square root of $252/T_M$.

4. Regression analysis and forecast error

4.1. Regression models

Following Canina and Figlewski (1993), Jorion (1995), Christensen and Prabhala (1998), Szakmary et al. (2003), and Yu et al. (2010), we examine the predictive ability of the volatility measures for realized volatility by the following regression model for each volatility measure:

$$RV_{i,t} = \alpha + \beta VYM_{ij,t} + \varepsilon_t, \qquad (11)$$

where RV and VYM are the realized volatility and volatility measure, respectively, the subscript i of RV denotes the S&P 100 index or equity options, and the subscript j of VYM denotes four volatility measures: implied volatility (IV), MSM volatility (MV), GARCH volatility (GV), and historical volatility (HV). As in typical tests of market efficiency, if the volatility measure is an unbiased estimator of realized volatility, we would expect the intercept to be zero and the slope coefficient to be unity in Eq. (11). In addition, it is expected to see that more information contained in the volatility measure will result in a higher adjusted-R^2 in Eq. (11).[9]

Since two series of RV and VYM are considered, it would be natural to use the bivariate methods such as the bivariate vector autoregression (BVAR) and bivariate GARCH (BGARCH) as alternatives to Eq. (11). We do not take the bivariate approaches for two reasons. First, the specification of the BVAR cannot provide a market efficiency test as in Eq. (11) since the right-hand-side variables in the BVAR should be lagged. Second, the BGARCH approach increases computational complexity substantially. To the best of our knowledge, we are unaware of any previous studies that employ the bivariate approaches by addressing the same issues as ours, which might be due to the same reasons.

It is well known that the financial market volatility generally increases sharply during any financial crisis periods, which might make realized volatility more difficult to be forecasted. This implies that the ability of our volatility measures to predict realized volatility might be different across GFC versus non-GFC periods. To address this issue, we divide our sample period into two sub-periods: the non-GFC from 1 January 2001 to 30 June 2007 and the GFC from 1 July 2007 to 31 October 31 2009. Then we also run the following regression model:

$$RV_{i,t} = \alpha_1 + \alpha_2 FC_t + \beta_1 VYM_{ij,t} + \beta_2 (FC_t \times VYM_{ij,t}) + \varepsilon_t, \qquad (12)$$

where the variables are defined as above and the dummy variable FC takes on a value of one during the GFC period and zero otherwise.

In Eq. (12), α_1 and β_1 measure the constant and slope coefficients, respectively, during the non-GFC period, whereas $\alpha_1 + \alpha_2$ and $\beta_1 + \beta_2$ measure the constant and slope coefficients, respectively, during the GFC period. In other words, α_2 and β_2 measure the increase or decrease in the constant and slope coefficients, respectively, in the GFC period relative to in the non-GFC period. If we find that the predictive power of volatility measures for future volatility decreases, as expected, during the GFC period, Eq. (12) allows us to know that the decline in the predictive power is due to α_2 and/or β_2. It should be noted that the error term in both Eqs. (11) and (12) should be serially uncorrelated. To reduce the bias of series correlation resulting from using the overlapping data procedure, we calculate the t-statistics in all our regression models using the Newey and West (1987) covariance matrix.

[9] We also test whether implied volatility reflects all information regarding future volatility. Following Szakmary et al. (2003) and Yu et al. (2010), we regress IV and each of MV, GV, and HV individually on RV. The results show that MV and GV can improve the predictive power of IV for RV for both the index and equity options in the full sample and two sub-sample periods. To conserve space, we do not report these results, but they are available upon request from the authors.

4.2. Measure of forecasting performance

In addition to the adjusted-R^2 of the regression models, we also use the mean absolute error (MAE), root mean squared error (RMSE), and heteroskedasticity adjusted root mean squared error (HRMSE) to evaluate the forecasting performance of volatility measures (see also Gemmill, 1986; Lamoureux & Lastrapes, 1993; Martens & Zein, 2004).[10] These three criteria are estimated by comparing the volatility measures with realized volatility and are described as follows:

$$\text{MAE} = \frac{1}{N}\sum_{n=1}^{N} \text{RV}_{i,n} - \text{VYM}_{ij,n} \Big| , \tag{13}$$

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum\nolimits_{n=1}^{N}(\text{RV}_{i,n} - \text{VYM}_{ij,n})^2}, \tag{14}$$

$$\text{HRMSE} = \sqrt{\frac{1}{N}\sum\nolimits_{n=1}^{N}\left(1 - \frac{\text{VYM}_{ij,n}}{\text{RV}_{i,n}}\right)^2}, \tag{15}$$

where the variables are defined as above and N is the number of observations. In our empirical tests, we find that the three forecast error measures lead to the consistent conclusion about the relative performance of the predictive power of our volatility measures for future volatility in most of cases.

5. Empirical results

5.1. Descriptive statistics

Table 1 reports the descriptive statistics on realized volatility (RV), implied volatility (IV), MSM volatility (MV), GARCH volatility (GV), and historical volatility (HV) for the full sample, non-GFC, and GFC periods. Specifically, Panels A and B of Table 1 report the descriptive statistics for the S&P 100 index and equity options, respectively. The first thing to notice from Table 1 is that the mean volatility and corresponding standard deviation increase substantially from the non-GFC period to the GFC period for all volatilities. For example, the mean realized volatility (corresponding standard deviation) increases from 16.59% (9.49%) in the non-GFC period to 30.36% (19.16%) in the GFC period. These findings indicate that the GFC does lead to a significant increase in the volatility of the options market.

Another thing deserves to notice is that the index option has lower mean volatility and corresponding standard deviation for all volatilities than the equity options in the full sample period as well as in two sub-sample periods. For example, the index option has 23.48%, 20.35%, and 32.11% (11.88%, 9.16%, and 14.07%) of the mean IV (corresponding standard deviation) in the full sample, non-GFC, and GFC periods, respectively, while the equity options have 37.96%, 34.27%, and 48.15% (17.27%, 12.94%, and 19.36%) of the mean IV (corresponding standard deviation) in these periods, respectively. These findings point out that the equity options tend to be more volatile than the index option.

5.2. Results of forecasting realized volatility of the index option

Table 2 reports the results of using four volatilities to forecast the realized volatility of the S&P 100 index option in three sample periods. Specifically, Panel A of Table 2 reports the estimation results and forecast errors of Eq. (11), and Panel B of Table 2 reports the estimation results of Eq. (12). Panel A of Table 2 shows that all slope coefficients of the four volatility measures are positive, less than 1, and significant at the 1% level in all three sample periods and that the constant coefficients are

[10] The squared root mean squared error (RMSE) is the mean squared error (MSE) that is also commonly used in measuring the forecasting performance. To make the measurement unit consistent with that of the MAE and HRMSE, we choose to use the RMSE in our analysis.

Table 1
Descriptive statistics.

	Full sample period					Non-GFC period					GFC period				
	RV	IV	MV	GV	HV	RV	IV	MV	GV	HV	RV	IV	MV	GV	HV
Panel A: S&P 100 index option															
No	2221	2221	2221	2221	2221	1631	1631	1631	1631	1631	590	590	590	590	590
Mean	0.2025	0.2348	0.2152	0.2173	0.1889	0.1659	0.2035	0.1873	0.1863	0.1563	0.3036	0.3211	0.2923	0.3030	0.2791
Stdev	0.1416	0.1188	0.1236	0.1274	0.1218	0.0949	0.0916	0.0853	0.0833	0.0815	0.1916	0.1407	0.1713	0.1789	0.1627
Median	0.1655	0.2192	0.1879	0.1904	0.1579	0.1377	0.1704	0.1668	0.1671	0.1266	0.2354	0.2691	0.2273	0.2363	0.2223
Min	0.0501	0.0510	0.0690	0.0682	0.0587	0.0501	0.0510	0.0690	0.0682	0.0587	0.0809	0.1329	0.1093	0.1059	0.1176
Max	1.2477	0.8914	1.0104	1.0512	0.7871	0.7089	0.5402	0.5949	0.5898	0.4539	1.2477	0.8914	1.0104	1.0512	0.7871
Skew	2.5526	1.5763	2.3960	2.4663	2.2607	1.6524	0.9946	1.1913	1.1064	1.3085	2.0099	1.5603	1.8936	1.8341	1.7346
Kurtosis	9.4322	3.5055	8.4949	8.9465	6.7026	3.1642	0.2414	1.7519	1.3660	1.2709	4.2757	1.9706	3.4542	3.2211	2.1985
Panel B: S&P100 equity options															
No	2221	2221	2221	2221	2221	1631	1631	1631	1631	1631	590	590	590	590	590
Mean	0.3416	0.3796	0.3483	0.3794	0.3205	0.2970	0.3427	0.3146	0.3509	0.2814	0.4647	0.4815	0.4416	0.4581	0.4284
Stdev	0.2235	0.1727	0.1969	0.2047	0.1911	0.1628	0.1294	0.1479	0.1576	0.1377	0.2699	0.1936	0.2321	0.2362	0.2260
Median	0.2761	0.3295	0.2898	0.3250	0.2625	0.2510	0.3000	0.2702	0.3101	0.2382	0.3759	0.4171	0.3683	0.3791	0.3483
Min	0.0850	0.1610	0.1099	0.1371	0.1056	0.0851	0.1610	0.1106	0.1375	0.1061	0.1480	0.2256	0.1494	0.1748	0.1540
Max	1.8920	1.3621	1.6406	1.7083	1.2159	1.2904	0.9230	1.1960	1.2173	0.8305	1.7381	1.2889	1.4220	1.4496	1.1212
Skew	2.3769	1.6955	2.0836	2.0281	2.0772	1.9129	1.2264	1.6145	1.2945	1.5646	1.7687	1.4186	1.4977	1.6072	1.4627
Kurtosis	8.8142	3.8755	7.9923	7.1156	5.7969	5.9153	1.5642	6.8097	3.5516	3.1764	3.8864	1.8058	2.5842	2.7885	1.7503

This table presents the descriptive statistics on realized volatility (RV), implied volatility (IV), MSM volatility (MV), GARCH volatility (GV), and historical volatility (HV) for both S&P 100 index and equity options. In addition to the index option, the sample consists of 92 equity options that have all relevant data during the sample period. No is the number of trading days in the sample. The whole sample period has 2,221 trading days (from 2 January 2001 to 31 October 2009), the non-GFC period has 1,631 trading days (from 2 January 2001 to 30 June 2007), and the GFC period has 590 trading days (1 July 2007 to 31 October 2009). Mean and Median are the average and median volatilities, respectively, during the sample period; Stdev denotes the standard deviation; Min and Max are the minimum and maximum, respectively; Skew and Kurtosis indicate the third and fourth moments of index or equity returns, respectively. Here we delete the options without the full sample period and include 92 individual stock options in our study.

Table 2
The regression results and forecasting performance on the S&P 100 index option.

	Constant	Slope	Adjusted-R^2	MAE	RMSE	HRMSE
Panel A: The estimation results of Eq. (11)						
Full sample period						
IV	−0.011	0.909***	0.582	0.069	0.098	0.522
	(−0.849)	(13.681)				
MV	0.006	0.913***	0.635	0.056	0.087	0.3878
	(0.409)	(11.365)				
GV	0.009	0.889***	0.640	0.057	0.088	0.387
	(0.632)	(11.178)				
HV	0.046***	0.827***	0.506	0.064	0.103	0.386
	(4.010)	(11.046)				
Non-GFC period						
IV	0.003	0.801***	0.598	0.058	0.073	0.537
	(0.346)	(15.608)				
MV	0.003	0.868***	0.608	0.047	0.064	0.403
	(0.374)	(14.533)				
GV	−0.001	0.895***	0.617	0.047	0.063	0.396
	(−0.102)	(15.809)				
HV	0.045***	0.775***	0.443	0.050	0.074	0.374
	(5.087)	(11.348)				
GFC period						
IV	0.014	0.902***	0.438	0.102	0.145	0.480
	(0.420)	(7.791)				
MV	0.060***	0.833***	0.555	0.084	0.131	0.342
	(2.122)	(7.298)				
GV	0.063***	0.794***	0.548	0.086	0.134	0.364
	(2.272)	(7.311)				
HV	0.097***	0.739***	0.393	0.103	0.157	0.421
	(3.928)	(7.172)				

	α_1	α_2	β_1	β_2	Adjusted-R^2
Panel B: The estimation results of Eq. (12)					
IV	0.003	0.011	0.801***	0.102	0.594
		(0.346)	(0.292)	(15.604)	(0.738)
MV	0.003	0.057*	0.868***	−0.034	0.655
		(0.374)	(1.745)	(14.529)	(−0.245)
GV	−0.001	0.064**	0.895***	−0.101	0.654
		(−0.102)	(2.011)	(15.804)	(−0.762)
HV	0.054***	0.042**	0.713***	0.075	0.522
		(5.684)	(2.470)	(10.211)	(0.503)

Panel A of the table reports the estimation results and forecast errors of Eq. (11) over the full sample, Non-GFC, and GFC periods as follows:

$$RV_{i,t} = \alpha + \beta VYM_{ij,t} + \varepsilon_t, \tag{11}$$

where RV and VYM are the realized volatility and volatility measure, respectively, the subscript i denotes the S&P 100 index option, and the subscript j denotes the four volatility measures: implied volatility (IV), MSM volatility (MV), GARCH volatility (GV), and historical volatility (HV). Panel B of the table reports the estimation results and forecast errors of Eq. (12) over the full sample period as follows:

$$RV_{i,t} = \alpha_1 + \alpha_2 FC_t + \beta_1 VYM_{ij,t} + \beta_2 (FC_t \times VYM_{ij,t}) + \varepsilon_t, \tag{12}$$

where the variables are defined as above, the dummy variable FC takes on a value of one during the GFC period and zero otherwise. MAE, RMSE, and HRMSE are the mean absolute error, root mean squared error, and heteroskedasticity adjusted root mean squared error, respectively. The t-statistics are corrected for heteroskedasticity and autocorrelation using the Newey and West (1987) covariance matrix and are reported in parentheses beneath the estimated coefficients.
Note: ***, **, * denote significant level at the 1%, 5%, and 10% level, respectively.

significantly positive in 5 out of the 12 cases. These results are inconsistent with the notion of market efficiency. They suggest that the four volatility measures are biased forecasts of realized volatility, which is consistent with the findings of Jorion (1995) and Szakmary et al. (2003).

As for forecasting performance, Panel A of Table 2 shows that MV and GV have higher adjusted-R^2 than do IV and HV in all three sample periods and that MV and GV have lower forecast errors than do IV and HV in all three sample periods, except for the case of HRMSE in non-GFC periods. These findings suggest that MV and GV are, on average, more useful in predicting future volatility on the S&P 100 index option than IV and HV. Except for HRMSE of MV and GV, the three measures of forecast errors are higher in GFC periods for all volatility measures than those in non-GFC periods. Consistent with intuition, these findings indicate that future volatility is more difficult to be forecasted during GFC periods than during non-GFC periods. Moreover, we find that MV has better forecasting performance in GFC periods than the other three volatility measures.

Panel B of Table 2 shows that the α_2 coefficient, a constant estimate in GFC periods, is significantly positive at conventional significance levels in the cases of MV, GV, and HV and that the β_2 coefficient, a slope estimate in GFC periods, is not significant at conventional significance in all cases of volatility measures. Consistent with the results reported in Panel A of Table 2, these findings suggest that the predictive power of volatility measures for the future volatility of the S&P 100 index option falls during GFC periods. Furthermore, the significant α_2 coefficient suggests that the increase in the forecasting errors of MV, GV, and HV for future volatility during GFC periods emanates primarily from the biased estimates of the constant coefficient.

5.3. *Results of forecasting realized volatility of the equity options*

Table 3 presents the results of using four volatilities to forecast the realized volatility of the S&P 100 equity options in three sample periods. Specifically, Panel A of Table 3 presents the mean constant and slope coefficients, mean adjusted-R^2 of Eq. (11) and the three corresponding mean forecast errors. It shows that the estimated mean constants are significantly positive in all cases, except for the case of IV in the full sample period. It also shows that the estimated mean slopes are less than one and significantly positive in all cases. Similar to what we find in Table 2, these findings indicate that the four types of volatility measures are biased predictors for the future volatility of the equity options. Looking at the mean adjusted-R^2 to gauge forecasting performance among the four volatility measures, we find that MV and GV have better forecasting performances than IV and HV in three sample periods. We also find that MV and GV have, on average, smaller mean forecast errors than IV and HV in three sample periods. These findings suggest that future volatility on the equity options, on average, tends to be better forecasted based on MV and GV than based on IV and HV, which is similar to what we find in Panel A of Table 2 for the case of the index option.

Before we move to Panel B of Table 3, we compare the forecasting performances of volatility measures in non-GFC versus GFC periods, as we do in Table 2. Panel A of Table 3 shows that MV and GV have higher mean adjusted-R^2 in non-GFC periods than in GFC periods, while IV and HV have higher the adjusted-R^2 in GFC periods than in non-GFC periods. Directing our attention to the three forecast errors, we find that, except for HRMSE of GV and HV, the three forecast errors of each volatility measure are higher in GFC periods than those in non-GFC periods. Consistent with what we observe in Panel A of Table 2, these findings indicate that it tends to be easier to forecast the future volatility of the equity options during non-GFC periods than during GFC periods. Moreover, we find that GV has the best forecasting performance during GFC periods.

Panel B of Table 3 presents the estimation results of Eq. (12). Specifically, the α_2 and β_2 coefficients in Eq. (12) are the estimated constant and slope, respectively, during GFC periods. It shows that the α_2 and β_2 coefficients are significantly positive at conventional significance levels in the cases of MV, GV, and HV and in the cases of IV, GV, and HV, respectively. Consistent with what we observe in Panel A of Table 2 for the index option, these findings also suggest that the predictive power of volatility measures for the future volatility of the equity options significantly declines during GFC periods. Moreover, comparing the results in Panel A of Table 2 with those in Panel A of Table 3, we find that the volatility measures tend to have higher adjusted-R^2 and lower forecast errors in the case of the index option than in the case of the equity options, suggesting that the volatility measures help better forecast the future volatility of the index option than that of the equity options. This makes sense since

Table 3
The regression results and forecasting performance on the S&P 100 equity options.

	Constant	Slope	Adjusted-R^2	MAE	RMSE	HRMSE
Panel A: The estimation results of Eq. (11)						
Full sample period						
IV	−0.002	0.895***	0.486	0.112	0.161	0.526
	(−0.533)	(8.099)				
MV	0.061***	0.797***	0.501	0.098	0.158	0.388
	(20.548)	(19.187)				
GV	0.039***	0.801***	0.507	0.110	0.168	0.479
	(8.492)	(16.119)				
HV	0.105***	0.723***	0.390	0.111	0.177	0.432
	(26.672)	(21.400)				
Non-GFC period						
IV	0.026***	0.776***	0.403	0.099	0.120	0.466
	(4.741)	(13.226)				
MV	0.060***	0.743***	0.468	0.081	0.123	0.384
	(17.671)	(23.096)				
GV	0.045***	0.724***	0.458	0.101	0.143	0.509
	(9.412)	(19.436)				
HV	0.119***	0.615***	0.290	0.095	0.144	0.431
	(24.267)	(22.775)				
GFC period						
IV	0.031***	0.888***	0.405	0.147	0.209	0.473
	(4.661)	(7.464)				
MV	0.129***	0.700***	0.427	0.145	0.216	0.393
	(20.014)	(17.276)				
GV	0.104***	0.785***	0.455	0.134	0.203	0.372
	(16.228)	(24.809)				
HV	0.174***	0.657***	0.308	0.157	0.234	0.429
	(26.035)	(27.482)				

	α_1	α_2	β_1	β_2	Adjusted-R^2
Panel B: The estimation results of Eq. (12)					
IV	0.026***	0.005	0.776***	0.111***	0.500
		(4.471)	(0.556)	(13.226)	(4.714)
MV	0.060***	0.076***	0.743***	−0.010	0.521
		(17.671)	(11.973)	(23.096)	(−0.794)
GV	0.045***	0.059***	0.724***	0.061***	0.539
		(9.412)	(7.114)	(19.436)	(3.486)
HV	0.119***	0.055***	0.615***	0.041**	0.410
		(24.267)	(7.692)	(22.775)	(2.132)

See the note to Table 2 for the detailed description of Table 3.
Note: ***, **, * denote significant level at the 1%, 5%, and 10% level, respectively.

the underlying stocks of the equity options tend to contain more firm-specific information, making the future volatility of the equity options more difficult to be forecast.

5.4. Predictive power conditional on stock and option characteristics

5.4.1. Volatility conditional on stock and option characteristics

To see whether the predictive power of volatility measures for the future volatility of the equity options is affected by the equity and option characteristics, we begin by estimating realized volatility (RV), implied volatility (IV), and MSM volatility (MV) conditional on these characteristics, and the results are presented in Table 4.[11] It is noted that, except for the industry group, the group 1 denotes the smallest size, bid-ask spread, skewness, and the ratio of the open interest of put to call options

[11] We also estimate GARCH volatility (GV) and historical volatility (HV) conditional on various characteristics, as we do in Table 4. We find that the pattern of GV and HV is similar to that of IV and MV in all characteristics groups. Moreover, we still find that the forecasting performance of MV and GV for realized volatility is, on average, better than that of IV and HV conditional

Table 4
Volatility conditional on the equity and option characteristics.

Group	RV 1	RV 2	RV 3	RV 4	RV 5	IV 1	IV 2	IV 3	IV 4	IV 5	MV 1	MV 2	MV 3	MV 4	MV 5
Volume															
FULL	0.294	0.339	0.392			0.337	0.375	0.427			0.300	0.345	0.399		
Non-GFC	0.258	0.294	0.338			0.306	0.339	0.382			0.272	0.312	0.359		
GFC	0.394	0.461	0.540			0.422	0.473	0.551			0.380	0.438	0.508		
DIFF	0.136	0.167	0.202			0.116	0.134	0.169			0.108	0.126	0.149		
Put-call ratio															
FULL	0.335	0.346	0.367			0.365	0.397	0.461			0.338	0.371	0.421		
Non-GFC	0.296	0.306	0.291			0.333	0.365	0.406			0.309	0.331	0.350		
GFC	0.443	0.458	0.574			0.453	0.484	0.611			0.419	0.483	0.615		
DIFF	0.147	0.152	0.283			0.120	0.119	0.205			0.110	0.152	0.265		
Bid-ask spread															
FULL	0.405	0.328	0.297			0.446	0.365	0.330			0.415	0.333	0.298		
Non-GFC	0.362	0.281	0.254			0.412	0.324	0.296			0.387	0.295	0.265		
GFC	0.524	0.460	0.417			0.541	0.475	0.425			0.492	0.438	0.390		
DIFF	0.162	0.179	0.163			0.129	0.151	0.129			0.105	0.143	0.125		
Size															
FULL	0.363	0.338	0.315			0.409	0.378	0.349			0.378	0.348	0.324		
Non-GFC	0.310	0.299	0.273			0.365	0.345	0.315			0.339	0.321	0.290		
GFC	0.507	0.446	0.430			0.531	0.469	0.445			0.486	0.424	0.419		
DIFF	0.197	0.147	0.157			0.166	0.124	0.130			0.147	0.103	0.129		
Momentum															
FULL	0.425	0.303	0.348			0.488	0.333	0.402			0.441	0.301	0.376		
Non-GFC	0.347	0.269	0.320			0.410	0.304	0.376			0.384	0.272	0.351		
GFC	0.639	0.399	0.427			0.704	0.414	0.472			0.600	0.381	0.448		
DIFF	0.292	0.130	0.107			0.294	0.110	0.096			0.216	0.109	0.097		
Skewness															
FULL	0.360	0.321	0.347			0.415	0.348	0.395			0.381	0.323	0.367		
Non-GFC	0.312	0.279	0.303			0.375	0.312	0.361			0.350	0.289	0.338		
GFC	0.494	0.439	0.468			0.523	0.448	0.488			0.467	0.416	0.448		
DIFF	0.182	0.160	0.165			0.148	0.136	0.127			0.117	0.127	0.110		
Industry															
FULL	0.400	0.387	0.412	0.395	0.320	0.419	0.402	0.436	0.441	0.359	0.398	0.375	0.421	0.410	0.326
Non-GFC	0.262	0.224	0.299	0.379	0.286	0.307	0.272	0.342	0.431	0.330	0.279	0.237	0.322	0.407	0.301
GFC	0.781	0.837	0.724	0.437	0.413	0.729	0.763	0.695	0.468	0.439	0.725	0.756	0.694	0.419	0.395
DIFF	0.519	0.613	0.425	0.058	0.127	0.422	0.491	0.353	0.037	0.109	0.446	0.519	0.372	0.012	0.094

This table presents the realized volatility (RV), implied volatility (IV), and MSM volatility (MV) conditional on the equity and option characteristics during the full sample period (FULL), the non-GFC period, and the GFC period. DIFF is the difference in RV, IV, and MV between non-GFC and GFC periods. Volume is the daily average trading volume of the underlying equity over the previous one year. The put-call ratio is the daily open interest contracts of put options divided by those of call options, and then the average put-call ratio is calculated over the previous one year. The bid-ask spread is the average daily closed bid-ask spread of individual stock options over the previous one year. Size is defined as market capitalization at the end of the previous year. Momentum is defined as the cumulative returns for the 11-month period from 12 through 2 months prior to the group formation date. The skewness is measured as the third moment of underlying equity returns divided by their cube of standard deviation. The group 1 denotes the smallest size, bid-ask spread, skewness, and the ratio of the open interest of put to call options groups, the lowest volume group, and the momentum group with the worse performance of the underlying equity, while the group 3 denotes the largest size, bid-ask spread, skewness, and the ratio of the open interest of put to call options groups, the highest volume group, and the momentum group with the best performance of the underlying equity. As for the industry group, group 1 is the financial industry, group 2 the banking sector of the financial industry, group 3 the non-banking sector of the financial industry, group 4 the technology industry, and group 5 the other industries.

groups, the lowest volume group, and the momentum group with the worse performance of the underlying equity, while the group 3 denotes the largest size, bid-ask spread, skewness, and the ratio of the open interest of put to call options groups, the highest volume group, and the momentum group

on various characteristics. We also find that the predictive power of GV and HV for realized volatility across the groups within each characteristic is similar to that of IV and MV reported in Table 5. To conserve space, we do not report the results associated with GV and HV, but they are available upon request from the authors.

with the best performance of the underlying equity. As for the industry groups, it is recalled that the group 1 denotes the financial industry, the group 2 the banking sector of the financial industry, the group 3 the non-banking sector of the financial industry, the group 4 the technology industry, and the group 5 the other industries.

The stylized fact of the positive relation between volatility and volume is well documented in prior studies (e.g., Karpoff, 1987; Lamoureux & Lastrapes, 1990).[12] Consistent with this stylized fact, Table 4 shows that realized volatility increases with volume in all cases. Prior studies document that the put-call ratio is a useful measure of market sentiment, with higher levels reflecting more bearish sentiment (e.g., Dennis & Mayhew, 2002; Simon & Wiggins, 2001). We find that realized volatility increases with the put-call ratio in the whole sample period and the GFC period (see also Wang, Keswani, & Taylor, 2006).

Table 4 also shows that realized volatility decreases with the bid-ask spread in all cases. This makes sense in that the smaller bid-ask spread leads to lower transactions costs and higher liquidity and trading volume, which in turn leads to higher volatility.[13] Realized volatility also decreases with size in all cases, which is consistent with the finding in prior studies that small firms tend to have higher volatility (e.g., Campbell, Lettau, Malkiel, & Xu, 2001). Compared with large stocks, small stocks also tend to have low volume and institutional holding (e.g., Badrinath, Kale, & Noe, 1995; Chordia & Swaminathan, 2000), high information asymmetry and illiquidity (e.g., Amihud, 2002; Brennan & Hughes, 1991), and large bid-ask spreads (e.g., Stoll & Whaley, 1983). All these factors lead to one consensus result that small stocks incorporate information more sluggishly than large stocks, which consequently makes the equity value of small stocks is more uncertain and volatile.

Some researchers argue that the momentum effect and higher volatility in stock prices are generated by behavior biases such as biased self-attribution and overconfidence (e.g., Daniel, Hirshleifer, & Subrahmanyam, 1998). Consistent with this argument, we find that the momentum groups with the worse and best performances of the underlying equities (i.e., groups 1 and 3) have higher realized volatility in all cases. Moreover, we find that the momentum group 1 has higher realized volatility than the momentum group 3, which is also consistent with the leverage effect in volatility documented in the finance literature that a negative return shock increases volatility more than does a positive return shock (e.g., Black, 1976; Christie, 1982).[14] For the skewness groups, as the skewness value is near to zero (i.e., group 2), the equity return distribution is less skewed. Because of this, realized volatility is larger in groups 1 and 3, while it is smaller in group 2.

The relation between realized volatility and the industry group is more complicated. Here, we focus on the changes in the relation between non-GFC and GFC periods, and some interesting observations emerge. Realized volatility is the smallest in the banking sector of the financial industry during non-GFC periods, whereas they become the largest during GFC periods. Similarly, the significant difference in realized volatility between non-GFC and GFC periods can also be found in the financial industry and the non-banking sector of the financial industry. The technology and the other industries have higher realized volatility in GFC periods than in non-GFC periods. Compared with the realized volatility of the financial industry, including the banking and non-banking sectors, the realized volatility of the technology and that of the other industries are relatively more stable across non-GFC and GFC periods. Consistent with our intuition, this implies that the asset prices of the financial industry are more affected by the recent global financial turmoil than those of the other industries.

Lastly, two remarkable observations deserve notice from Table 4. First, implied and MSM volatilities have the similar patterns within each characteristics group as those observed in the case of realized

[12] From theoretical perspectives, the positive relation between volume and volatility can be explained on the ground of the mixture-of-distributions hypothesis (Clark, 1973; Epps & Epps, 1976; Tauchen & Pitts, 1983), the differences-of-opinion models (Harris & Raviv, 1993; Shalen, 1993; Kandel & Pearson, 1995), and the heterogeneous-asymmetric information models (Campbell et al., 1993; Wang, 1994).

[13] The bid-ask spread can be decomposed into three cost components: order-processing costs (Tinic, 1972), inventory holding costs (Amihud & Mendelson, 1980; Ho & Stoll, 1981), and adverse information costs (Copeland & Galai, 1983; Glosten & Milgrom, 1985) and is used as a measure of liquidity (e.g., Amihud & Mendelson, 1986; Eleswarapu & Reinganum, 1993).

[14] We also form the momentum groups based on past 3-, 6-, and 9-month stock returns and find the similar results as reported in Tables 4 and 5.

Table 5
The forecasting performance on the S&P 100 equity options conditional on the equity and option characteristics.

	Full sample period					Non-GFC period					GFC period				
Group	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
The mean adjusted-R^2 of Eq. (11)															
Volume															
IV	0.481	0.557	0.676			0.511	0.565	0.692			0.413	0.461	0.579		
MV	0.606	0.668	0.722			0.646	0.709	0.792			0.514	0.555	0.591		
Put-call ratio															
IV	0.579	0.552	0.529			0.601	0.540	0.387			0.496	0.439	0.409		
MV	0.682	0.641	0.583			0.777	0.617	0.394			0.551	0.537	0.485		
Bid-ask spread															
IV	0.659	0.549	0.481			0.692	0.561	0.498			0.560	0.517	0.417		
MV	0.724	0.683	0.612			0.809	0.711	0.669			0.600	0.565	0.461		
Size															
IV	0.476	0.528	0.609			0.502	0.585	0.616			0.420	0.484	0.589		
MV	0.605	0.637	0.709			0.618	0.679	0.763			0.511	0.553	0.619		
Momentum															
IV	0.482	0.529	0.398			0.527	0.588	0.414			0.402	0.420	0.360		
MV	0.564	0.659	0.494			0.676	0.737	0.612			0.459	0.508	0.370		
Skewness															
IV	0.488	0.568	0.529			0.516	0.605	0.571			0.420	0.475	0.449		
MV	0.628	0.700	0.652			0.657	0.760	0.711			0.464	0.559	0.536		
Industry															
IV	0.611	0.629	0.603	0.553	0.510	0.632	0.643	0.611	0.667	0.554	0.548	0.568	0.520	0.450	0.465
MV	0.624	0.635	0.620	0.613	0.567	0.647	0.662	0.636	0.713	0.601	0.570	0.582	0.532	0.477	0.480

This table reports the mean adjusted-R^2 of Eq. (11). See the note to Table 2 for the detailed description of Eq. (11) and the note to Table 4 for the definition of the equity and option characteristic variables.

volatility. Second, consistent with what we observe in Table 1, realized, implied, and MSM volatilities are higher in GFC periods than in non-GFC periods.

5.4.2. Results of forecasting realized volatility conditional on equity and option characteristics

Implied volatility is a forward-looking measure of likely future volatility conditions, while MSM, GARCH, and historical volatilities are a backward-looking measure of recent volatility conditions. To gauge the forecasting performance of MSM volatility, our examination here focuses on the comparison of the predictive power of implied volatility (IV) versus MSM volatility (MV) for realized volatility. Table 5 reports the results of the mean adjusted-R^2 of Eq. (11) for each characteristics group.

Liquidity plays an important role in evaluating the option price (e.g., Brenner, Eldor, & Hauser, 2001; Dennis & Mayhew, 2002; Figlewski, 1989). High liquidity makes the underlying assets incorporate information in a timely manner (e.g., Chordia, Roll, & Subrahmanyam, 2008), which leads to less measurement errors and increases the predictability of realized volatility. Since high volume and small bid-ask spreads imply high liquidity (e.g., George & Longstaff, 1993), it can be seen from Table 5 that the mean adjusted-R^2 increases with volume but decreases with bid-ask spreads over full sample and two sub-sample periods for both IV and MV. Prior studies have also found that converting an option price into an implied volatility incurs errors due to bid-ask spreads (e.g., Christensen & Prabhala, 1998; Fleming, 1998; Jorion, 1995). Specifically, more measurement errors associated with larger bid-ask spreads. This may be another reason for the decrease in the predictive power of IV and MV for realized volatility due to larger bid-ask spreads.

When market participants are bearish, they buy put options for purposes of either hedging or just speculating market movements. This will increase the put option prices and lead to more negative risk-neutral skew and, as a result, the difference between implied and realized volatilities increases (see also Bollen & Whaley, 2004; Han, 2008). This implies that the ability of implied volatility to predict realized volatility decreases with the put-call ratio. Consistent with this prediction, Table 5 shows that the mean adjusted-R^2 decreases with the put-call ratio in all cases for both IV and MV.

Dennis and Mayhew (2002) find that in addition to market risk, firm size and trading volume also help explain cross-sectional variation in the skewness of the equity options traded on the Chicago Board Options Exchange. Dennis, Mayhew, and Stivers (2006) examine the bias of implied volatility for future realized volatility on 50 equity options and find a significant difference in the slope coefficients between the largest 10 firms and smallest 10 firms. Moreover, large firms tend to have less information asymmetry than small firms, implying that large firms could incorporate information into prices more quickly than small firms (e.g., Brennan & Hughes, 1991). This further implies that the estimation risk of the equity value decreases with firm size. As shown in Table 5, the predictive power of IV and MV for realized volatility increases with firm size in all cases. In unreported results, we find that the estimated slope coefficients of Eq. (11) of IV and MV are more close to 1 for large firms than small firms in all cases, which is consistent with the finding of Dennis et al. (2006).

Amin, Coval, and Seyhun (2004) find that the momentum of the underlying asset has a significant effect on the pricing of the index options through investors' expectations about future asset returns. Positive momentum in equity returns makes investors place more bet on equities, call options, and futures contracts. However, it is more expensive for investors to buy equities and futures contracts than to buy call options. As a consequence, the demand pressure on call options will drive up call options prices (i.e., implied volatility of call options). On the contrary, negative momentum in equity returns will induce investors to buy more put options and therefore increase the bias of implied volatility. Hence, it is reasonable to conjecture that the ability of implied volatility to predict realized volatility will decrease when the underlying equities exhibit stronger momentum in returns. Consistent with this conjecture, Table 5 shows that the predictive power of IV and MV for realized volatility is lower in groups 1 and 3 with the worse and best performance of the underlying equities, respectively, than that in group 2 with the medium performance of the underlying equities in all cases.

As noted by Lamoureux and Lastrapes (1993), the skewness of the equity return distribution will generate the Black-Scholes option pricing bias. This implies that the predictability of realized volatility will be lower when the skewness is larger. Consistent with this prediction, the results of Table 5 show that larger skewness (i.e., skewness groups 1 and 3) results in the lower predictability of realized volatility for both IV and MV than does smaller skewness in all cases.

Merton (1976) argues that under the assumption of continuous trading in time the Black-Sholes (1973) model do not capture the abnormal variations in equity prices due to the non-marginal impact of industry-wide (and/or firm-specific) information, consequently leading to the bias on the equity options pricing. This implies that the predictability of realized volatility may be affected by, for example, the speed at which industry-wide information is incorporated into equity prices. Since this speed may be different among industries, it is expected to see that the predictability of realized volatility will be higher in some industries and lower in the others. Table 5 shows that the ability of IV and MV to predict realized volatility in the financial industry, including the banking and non-banking sectors, is on average better than that in the technology and the other industries in full sample and GFC periods. In non-GFC periods, IV and MV perform best in forecasting the realized volatility of the technology industry than that of all other industries in our sample.

After discussing each characteristics group, we first compare the relative predictive power of IV versus MV across the characteristics groups and sample periods. We find that the mean adjusted-R^2 associated with MV is higher than that associated with IV in all cases. This finding indicates that using MV to predict realized volatility is better than using IV. Then, we compare the predictive power of IV and MV for realized volatility across the characteristic groups in non-GFC versus GFC periods. Consistent with what we observe in Tables 2 and 3, we still find that the performance of using IV and MV to predict realized volatility across characteristic groups in non-GFC periods is better than in GFC periods in most cases. And more importantly, we find that MV outperforms IV (and GV and HV) in forecasting the realize volatility of the equity options in all cases during GFC periods. This is similar to what we find in the case of the index option, but we do not find that this is the case for the unconditional results of the equity options in Table 3. This highlights the importance of taking equity and option characteristics into proper account when evaluating the predictability of the equity options.

We also calculate the mean forecast errors of MAE, RMSE, and HRMSE for MV and IV within each characteristics group. To conserve space, we do not report these results but they are available upon request from the authors. In unreported results, we find that the mean forecast errors of MV is lower than those of IV in all characteristics groups and sample periods, regardless of using MAE, RMSE, or HRMSE. This finding provides additional evidence on better forecasting performance of MV for realized volatility than that of IV. We also find that three forecast error measures are higher in GFC periods than in non-GFC periods across all characteristics groups. This finding provides supplementary evidence that realized volatility tends to be less predictable in GFC periods than in non-GFC periods.

6. Concluding remarks

Predicting volatility is an important task for option pricing and financial asset and risk management. Implied, GARCH, and historical volatilities are three widely used predictors for future volatility in the literature. Using the Markov-switching multifractal (MSM) model to predict volatility has received far less attention. The MSM model has the advantage over implied, GARCH, and historical volatilities in predicting future volatility in that it can effectively capture the outliers, moment scaling, and long memory exhibited in the time series of financial volatility. Another issue that is also not so extensively explored in the literature is whether the predictability of realized volatility is affected by the equity and option characteristics.

In this paper, we evaluate the performance of the ability of MSM volatility as well as that of implied, GARCH, and historical volatilities to predict realized volatility for both the S&P 100 index and equity options. To show whether MSM volatility has superior forecasting performance in the presence of extreme events, we divide our sample period into two sub-sample periods: GFC and non-GFC periods. We expect to see that MSM volatility outperforms the other volatility measures in predicting future volatility, especially in GFC periods. We also conduct a cross-sectional analysis of the predictability of the equity options conditional on not only option characteristics but also equity characteristics to see whether their predictability differs in different characteristics.

We find that the ability of MSM and GARCH volatilities to predict the realized volatilities of the index and equity options is better than that of implied and historical volatilities in both GFC and non-GFC periods. As for the degree of predictability of equity option versus index option volatilities, we find that equity option volatility tends to be more difficult to be forecast than index option volatility, which may be due to more idiosyncratic risks embedded in the former than in the latter.

We also find that both index and equity option volatilities tend to more difficult to be forecast during GFC periods than during non-GFC periods. Specifically, during GFC periods, we find that MSM volatility outperforms the other three volatility measures for the index option and for the equity options conditional on the equity and option characteristics. These results provide evidence that the shock to volatility due to extreme events such as the recent global financial turmoil can be better captured by MSM volatility than by GARCH, implied, and historical volatilities. These findings offer an important practical insight that practitioners can use the MSM model to predict volatility to form their risk management strategy, especially during financial turmoil periods.

We find that equity option volatility exhibits distinct patterns conditional on various equity and option characteristics. For example, it increases with trading volume and the put-call ratio but decreases with firm size and the option bid-ask spread. And more importantly, its predictability by MSM and implied volatilities depends on these characteristics. For example, the predictive power of MSM and implied volatilities for equity option volatility increases with volume and firm size and decreases with the put-call ratio and option bid-ask spread. Finally, we find that MSM volatility outperforms implied volatility in predicting equity option volatility conditional on various equity and option characteristics during both GFC and non-GFC periods.

Taken together, our results in this paper document the superior performance of MSM volatility in predicting index and equity option volatilities over implied, GARCH, and historical volatilities. It would be an interesting issue to see whether MSM volatility helps develop better hedge and risk management strategies. We leave these research topics for future studies.

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References

Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time series effects. *Journal of Financial Markets*, *5*, 31–56.
Amihud, Y., & Mendelson, H. (1980). Dealership market: Market-making with inventory. *Journal of Financial Economics*, *8*, 31–53.
Amihud, Y., & Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics*, *17*, 223–249.
Amin, K., Coval, J. D., & Seyhun, H. N. (2004). Index option prices and stock market momentum. *Journal of Business*, *77*, 835–873.
Ammann, M., Skovmand, D., & Verhofen, M. (2009). Implied and realized volatility in the cross-section of equity options. *International Journal of Theoretical and Applied Finance*, *12*, 745–765.
Akgiray, V. (1989). Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts. *Journal of Business*, *62*, 55–80.
Badrinath, S. G., Kale, J. R., & Noe, T. H. (1995). Of shepherds, sheep and the cross-autocorrelations in equity returns. *Review of Financial Studies*, *8*, 401–430.
Beckers, S. (1981). Standard deviations implied in option prices as predictors of future stock price variability. *Journal of Banking and Finance*, *5*, 363–381.
Black, F. (1976). Studies of stock market volatility changes. In *Proceedings of the American statistical association, business and economic statistics section* (pp. 177–181).
Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, *81*, 637–654.
Bollen, N. P., & Whaley, R. E. (2004). Does net buying pressure affect the shape of implied volatility functions? *Journal of Finance*, *59*, 711–753.
Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*, 307–327.
Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economics and Statistics*, *69*, 542–547.
Brailsford, T. J., & Faff, R. W. (1996). An evaluation of volatility forecasting techniques. *Journal of Banking and Finance*, *20*, 419–438.
Brennan, M., & Hughes, P. (1991). Stock prices and the supply of information. *Journal of Finance*, *46*, 1665–1691.
Brenner, M., Eldor, R., & Hauser, S. (2001). The price of options illiquidity. *Journal of Finance*, *56*, 789–805.
Brooks, C. (1998). Predicting stock index volatility: Can market volume help? *Journal of Forecasting*, *17*, 59–80.
Calvet, L. E., & Fisher, A. J. (2004). How to forecast long-run volatility: Regime-switching and the estimation of multifractal processes. *Journal of Financial Econometrics*, *2*, 49–83.
Campbell, J. Y., Grossman, S. J., & Wang, J. (1993). Trading volume and serial correlation in stock returns. *Quarterly Journal of Economics*, *108*, 905–939.
Campbell, J. Y., Lettau, M., Malkiel, B. G., & Xu, Y. (2001). Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *Journal of Finance*, *56*, 1–43.
Canina, L., & Figlewski, S. (1993). The informational content of implied volatility. *Review of Financial Studies*, *6*, 659–681.
Cecchetti, S. G. (2009). Crisis and responses: The federal reserve in the early stages of the financial crisis. *Journal of Economic Perspectives*, *23*, 51–75.
Chiras, D. P., & Manaster, S. (1978). The information content of option prices and a test of market efficiency. *Journal of Financial Economics*, *6*, 213–234.
Chordia, T., Roll, R., & Subrahmanyam, A. (2008). Liquidity and market efficiency. *Journal of Financial Economics*, *87*, 249–268.
Chordia, T., & Swaminathan, B. (2000). Trading volume and cross-autocorrelations in stock returns. *Journal of Finance*, *55*, 913–935.
Christensen, B. J., & Prabhala, N. R. (1998). The relation between implied and realized volatility. *Journal of Financial Economics*, *50*, 125–150.
Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of Financial Economics*, *10*, 407–432.
Clark, P. K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica*, *41*, 135–155.
Connolly, R., Stivers, C., & Sun, L. (2005). Stock market uncertainty and the stock-bond return relation. *Journal of Financial and Quantitative Analysis*, *40*, 161–194.
Copeland, T., & Galai, D. (1983). Information effects on the bid-ask spread. *Journal of Finance*, *38*, 1457–1469.
Covrig, V., & Low, B. S. (2003). The quality of volatility traded on the over-the-counter currency market: A multiple horizons study. *Journal of Futures Markets*, *23*, 261–285.
Cox, J., Ross, S., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, *7*, 229–263.
Daniel, K., Hirshleifer, D., & Subrahmanyam, A. (1998). Investor psychology and security market under- and overreactions. *Journal of Finance*, *53*, 1839–1885.
Davidson, W. N., Kim, J. K., Ors, E., & Szakmary, A. (2001). Using implied volatility on options to measure the relation between asset returns and variability. *Journal of Banking and Finance*, *25*, 1245–1269.
Day, T. E., & Lewis, C. M. (1992). Stock market volatility and the information content of stock index options. *Journal of Econometrics*, *52*, 267–287.
Dennis, P., & Mayhew, S. (2002). Risk-neutral skewness: Evidence from stock options. *Journal of Financial and Quantitative Analysis*, *37*, 471–493.

Dennis, P., Mayhew, S., & Stivers, C. (2006). Stock returns, implied volatility innovations, and the asymmetric volatility phenomenon. *Journal of Financial and Quantitative Analysis*, *41*, 381–406.
Diebold, F. X., & Yilmaz, K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting*, *28*, 57–66.
Doidge, C., & Wei, J. Z. (1998). Volatility forecasting and the efficiency of the Toronto 35 index options market. *Canadian Journal of Administrative Sciences*, *5*, 1–13.
Duan, J. C., & Zhang, H. (2001). Hang Seng index options around the Asian financial crisis—A GARCH approach. *Journal of Banking and Finance*, *25*, 1989–2014.
Eleswarapu, V., & Reinganum, M. (1993). The seasonal behavior of the liquidity premium in asset pricing. *Journal of Financial Economics*, *34*, 373–386.
Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, *50*, 987–1007.
Engle, R. F., & Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews*, *5*, 1–50.
Epps, T. W., & Epps, M. L. (1976). The stochastic dependence of security price changes and transaction volumes: Implications for the mixture-of-distributions hypothesis. *Econometrica*, *44*, 305–321.
Figlewski, S. (1989). Options arbitrage in imperfect markets. *Journal of Finance*, *44*, 1289–1311.
Fleming, J., Ostdiek, B., & Whaley, R. E. (1996). Trading costs and the relative rates of price discovery in stock, futures, and option markets. *Journal of Futures Markets*, *16*, 353–387.
Fleming, J. (1998). The quality of market volatility forecasts implied by S&P 100 index option prices. *Journal of Empirical Finance*, *5*, 317–345.
Franses, P. H., & Dijk, D. V. (1996). Forecasting stock market volatility using (non-linear) GARCH models. *Journal of Forecasting*, *15*, 229–235.
French, D. W. (1984). The weekend effect on the distribution of stock prices: Implications for option pricing. *Journal of Financial Economics*, *13*, 547–559.
Fung, J. K. W. (2007). Order imbalance and the pricing of index futures. *Journal of Futures Markets*, *27*, 697–717.
Gemmill, G. (1986). The forecasting performance of stock options on the London traded options market. *Journal of Business Finance & Accounting*, *13*, 535–546.
George, T. J., & Longstaff, F. A. (1993). Bid-ask spreads and trading activity in the S&P 100 index options market. *Journal of Financial and Quantitative Analysis*, *28*, 381–397.
Glosten, L. R., Jagannathan, R., & Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, *48*, 1779–1801.
Glosten, L. R., & Milgrom, P. R. (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, *14*, 71–100.
Han, B. (2008). Investor sentiment and option prices. *Review of Financial Studies*, *21*, 387–414.
Harris, M., & Raviv, A. (1993). Differences of opinion make a horse race. *Review of Financial Studies*, *6*, 473–506.
Harvey, C. R., & Whaley, R. E. (1993). Market volatility prediction and the efficiency of the S&P 100 index option market. *Journal of Financial Economics*, *31*, 43–73.
Ho, T., & Stoll, H. R. (1981). Optimal dealer pricing under transactions and return uncertainty. *Journal of Financial Economics*, *9*, 47–73.
Jorion, P. (1995). Predicting volatility in the foreign exchange market. *Journal of Finance*, *50*, 507–528.
Kandel, E., & Pearson, N. D. (1995). Differential interpretation of public signals and trade in speculative markets. *Journal of Political Economy*, *103*, 831–872.
Karpoff, J. (1987). The relation between price changes and trading volume: A survey. *Journal of Financial and Quantitative Analysis*, *22*, 109–126.
Lamoureux, C. G., & Lastrapes, W. D. (1990). Heteroskedasticity in stock return data: Volume versus GARCH effects. *Journal of Finance*, *45*, 221–229.
Lamoureux, C. G., & Lastrapes, W. D. (1993). Forecasting stock-return variance: Toward an understanding of stochastic implied volatilities. *Review of Financial Studies*, *6*, 293–326.
Latane, H. A., & Rendleman, R. J., Jr. (1976). Standard deviations of stock price ratios implied in option prices. *Journal of Finance*, *31*, 369–381.
Mandelbrot, B. B. (1963). The variation of certain speculative prices. *Journal of Business*, *36*, 394–419.
Mandelbrot, B. B., & Van Ness, J. W. (1968). Fractional Brownian motions, fractional noises and applications. *SIAM Review*, *10*, 422–437.
Mandelbrot, B. B. (1972). Possible refinement of the lognormal hypothesis concerning the distribution of energy dissipation in intermittent turbulence. *Statistical Models and Turbulence*, *12*, 333–351.
Mandelbrot, B. B. (1974). Intermittent turbulence in self-similar cascades: Divergence of high moments and dimension of the carrier. *Journal of Fluid Mechanics*, *62*, 331–358.
Mandelbrot, B. B., Fisher, A., Calvet, L. (1997). *A multifractal model of asset returns*. Working Paper, Cowles Foundation Discussion Paper No. 1164.
Martens, M., & Zein, J. (2004). Predicting financial volatility: High-frequency time-series forecasts vis-à-vis implied volatility. *Journal of Futures Markets*, *24*, 1005–1028.
Merton, R. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, *3*, 125–144.
Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, *59*, 347–370.
Newey, W. K., & West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, *55*, 703–708.
Poon, S. H., & Granger, C. W. J. (2005). Practical issues in forecasting volatility. *Financial Analysts Journal*, *61*, 45–56.
Poulsen, R., Schenk-Hoppe, K. R., & Ewald, C. O. (2009). Risk minimization in stochastic volatility models: Model risk and empirical performance. *Quantitative Finance*, *9*, 693–704.

Schmalensee, R., & Trippi, R. R. (1978). Common stock volatility expectations implied by option premia. *Journal of Finance*, *33*, 129–147.
Shalen, C. T. (1993). Volume, volatility, and the dispersion of beliefs. *Review of Financial Studies*, *6*, 405–434.
Simon, D. P., & Wiggins, R. A. (2001). S&P futures returns and contrary sentiment indicators. *Journal of Futures Markets*, *21*, 447–462.
Stoll, H. R., & Whaley, R. E. (1983). Transaction costs and the small firm effect. *Journal of Financial Economics*, *12*, 57–79.
Szakmary, A., Ors, E., Kim, J. K., & Davidson, W. N., III. (2003). The predictive power of implied volatility: Evidence from 35 futures markets. *Journal of Banking and Finance*, *27*, 2151–2175.
Tauchen, G. E., & Pitts, M. (1983). The price variability–volume relationship on speculative markets. *Econometrica*, *51*, 485–505.
Tinic, S. M. (1972). The economics of liquidity services. *Quarterly Journal of Economics*, *86*, 79–93.
Vasilellis, G. A., & Meade, N. (1996). Forecasting volatility for portfolio selection. *Journal of Business Finance & Accounting*, *23*, 125–143.
Wang, J. (1994). A model of competitive stock trading volume. *Journal of Political Economy*, *102*, 127–168.
Wang, Y.-H., Keswani, A., & Taylor, S. J. (2006). The relationships between sentiment, returns and volatility. *International Journal of Forecasting*, *22*, 109–123.
Yu, W. W., Lui, E. C., & Wang, J. W. (2010). The predictive power of the implied volatility of options traded OTC and on exchanges. *Journal of Banking and Finance*, *34*, 1–11.