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Regime-switching in volatility and correlation structure using range-based models with Markov-switching

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ABSTRACT

This study examines latent shifts in the conditional volatility and correlation for the U.S. stock and T-bond data using the two-state Markov-switching range-based volatility and correlation models. This paper comes up with clear evidence of volatility regime-switching in stock indices and T-bond over the crisis period. As regards the process of correlation, we also find evidence of regime changes in correlations between stock indices and T-bond over several financial crises. We conclude that the phenomena of both volatility and correlation regime-switching are triggered by these financial crises. In addition, the range-based volatility and correlation model with regime-switching method could explicitly point out the true date of structure changes in the data generating process for volatility and correlation variables.

1. Introduction

For the recent decade, global financial markets have suffered several devastating shocks. The period surrounding the WorldCom scandal in 2002, the beginning of subprime crisis in the fall of 2007, the Lehman collapse on September, 2008 and the 2009 European sovereign-debt crisis etc. These financial crises not only destroy the asset values but also implicitly change the volatility structure of asset returns and the correlation structure between two assets. Using a more flexible volatility and correlation model to reassess the related processes is essential for a major overhaul of bank legislation and bank regulation. This study uses a two-state Markov-switching range-based volatility and dynamic conditional correlation (DCC) model to explore the different financial turmoil triggered regime switches in volatilities and corresponding correlation structures. The classical Markov-switching approach proposed by Hamilton (1989, 1990) is developed to delineate the uncertain regime shifts in the data generating process about economic and financial variables. Therefore, it is natural to introduce the framework of the Markov-switching into the range-based volatility and correlation models to discuss the impact of unusual events on the patterns of volatility and correlation processes.

There are lots of related literatures about volatility and correlation models with the Markov-switching method. The idea of regime switches in stock return volatility has been documented by Lamoureux and Lastrapes (1990), Hamilton and Susmel (1994), Dueker (1997), and more recently by Liu et al. (2012). The foregoing studies presented that considering the Markov-switching method in model specification for stock market data can capture the richer dynamics volatility process and obtain accurately in data fitting and forecasts. Furthermore, Cai (1994), Gray (1996), Edwards and Susmel (2003), and Sun (2005) demonstrate the phenomenon of regime shifts in interest rate volatility process, too. According to their empirical results, they explicitly point out that the distinct volatility regimes are highly related to macroeconomic shocks. In terms of literature on the issue of dynamical correlation pattern, Billio and Caporin (2005) and Haas (2010) illustrate the phenomenon of regime-switching in correlation processes between global stock market indices and evidence that the Markov-switching DCC model is superior to those multivariate GARCH models. Generally speaking, the previous studies provide empirical supports that the volatility and correlation models with the Markov-switching method outperform the single-regime models in data fittings and statistic prediction. In the case of estimating the dynamical volatility and correlation models, some recent literature consider that using the range data to replace the return data can obtain many advantages in parameter

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Table 1
Descriptive statistics for the daily ranges of the Nasdaq, S&P 500 and T-bond (2002.1.3–2011.12.31).

	Nasdaq	S&P 500	T-bond
Mean	1.656	1.509	2.091
Median	1.383	1.192	1.739
Maximum	11.129	10.904	20.104
Minimum	0.252	0.239	0.207
Std. Dev.	1.074	1.158	1.341
Skewness	2.614	3.082	2.616
Kurtosis	14.729	17.963	20.693
Bera–Jarque	17,306.80	27,488.85	35,429.05
Observations	2519	2519	2498

Notes: The Bera–Jarque is the statistic for normality test.

estimates of models and out-of-sample prediction for dependent variable.[1] For this reason, we take both the range-based volatility and correlation models as the basic frameworks for our analysis then introduce the Markov-switching method into these two models, respectively. We expect that the estimation results of our proposed models can capture how financial market volatility and correlation processes respond to the impacts of financial turmoil.

Our contributions to the related literature are twofold. Firstly, we confirm definitely that the volatility regime-switching in both two stock indices and T-bond are salient. In particular, there is a specific corresponding relationship between volatility regime and financial turmoil for these market data. Namely, during the financial crisis period, the volatility process stays in the high volatility regime; but in the tranquil period, the volatility process moves to the low volatility state. Besides, we find that the impacts of these financial crises on T-bond volatility are usually more persistent than that on stock indices' volatility. This finding could be attributed to the expectation of a falling interest rate environment. According to the switching frequency, it seems that the influences of shocks to S&P 500 volatility are more sensitive than that to Nasdaq and T-bond volatility. Secondly, from empirical results, the regime-switching in dynamical correlation processes between stock indices and T-bond is an obvious phenomenon. Moreover, using the range-based DCC model without structure change consideration for data fitting is liable to underestimate the short-run effect of correlation process over the tranquil period.[2] In addition, the impacts of the 2002 WorldCom scandal on dynamical correlations between stock indices and T-bond are more persistent than the dynamical correlation between the stock indices of Nasdaq and S&P 500. A series of financial crises from 2007 to 2009 triggered the structure changes in correlation processes between stock indices and T-bond in advance.

The remainder of this paper is organized as follows. Section 2 describes the setting of the two-state Markov-switching range-based volatility and correlation models. Section 3 reports the empirical results and makes constructively discussions for these findings. Section 4 summarizes the results and presents the concluding remarks.

2. Methodology

The main purpose for this section is to express volatility and correlation models with the Markov-switching mechanism.

[1] The range data can be defined as the difference of the highest and lowest asset prices during a fixed time interval. Also see Parkinson (1980), Alizadeh et al. (2002), Brandt and Jones (2006), Chou (2005), Chou et al. (2009) and Chou and Liu (2010).

[2] The short-run effect of correlation process means that the estimated immediate impacts of shocks on dynamic correlation process from the DCC model, also see the empirical results in this study later.

Table 2
Range-based volatility model fitting for the Nasdaq, S&P 500 and T-bond (2002.1.3–2011.12.31).

$R_t = \lambda_t \varepsilon_t \quad R_t | I_{t-1} \sim \exp(1, \cdot)$
$\lambda_t = \omega + \alpha R_{t-1} + \beta \lambda_{t-1}$

	Nasdaq	S&P 500	T-bond
$\hat{\omega}$	0.003 (0.001)	0.003 (0.001)	0.001 (<0.001)
$\hat{\alpha}$	0.128 (0.020)	0.107 (0.015)	0.044 (0.006)
$\hat{\beta}$	0.838 (0.024)	0.857 (0.019)	0.948 (0.007)
$Q^2(10)$	12.068 [0.281]	9.636 [0.473]	8.058 [0.623]
LLF	−382.970	−398.718	−1355.670

Notes: λ_t and R_t are the range-based conditional volatility and range, respectively. *LLF* is the log likelihood function, p-values are in brackets and the numbers in parentheses are robust standard errors proposed by Bollerslev and Wooldridge (1992). $Q^2(10)$ is the statistics for serial correlation up to the 10th order in the squared standardized residuals.

2.1. Two-state Markov-switching range-based volatility model

Considering the two-state nonlinear structure in dynamical volatility process, we construct the two-state Markov-switching range-based volatility model as

$$R_t = \lambda_{s_t,t}\varepsilon_t, \quad \varepsilon_t | I_{t-1} \sim exp(1, .) \tag{1.1}$$

$$p_{ij} = Pr(s_t = j | s_{t-1} = i), \quad i, j = 1, 2 \tag{1.2}$$

$$\lambda_{s_t,t} = \omega_{s_t} + \alpha_{s_t} R_{t-1} + \beta_{s_t} \lambda_{s_t,t-1} \tag{1.3}$$

where R_t is the observed range in logarithm type during the time interval t, ε_t is assumed to follow the exponential distribution with a unit mean, and S_t follows a Markov chain with two-state space $\mathbf{S} = \{1,2\}$. The transition probability is presented in Eq. (1.2). According to the probability axiom, the sum of probabilities has to satisfy $\sum_{j=1}^{2} p_{ij} = 1$ for $i = 1$, 2, and

Table 3
Markov-switching range-based volatility model for the Nasdaq, S&P 500 and T-bond (2002.1.3–2011.12.31).

$R_t = \lambda_{s_t,t}\varepsilon_t \varepsilon_t | I_{t-1} \sim exp1, (\cdot)$
$\lambda_{s_t,t} = \omega_{s_t} + \alpha_{s_t} R_{t-1} + \beta_{s_t} \lambda_{s_t,t-1}$
$p_{ij} = \Pr(s_t = j | s_{t-1} = i) i, j = 1, 2.$

	Nasdaq	S&P 500	T-bond
Low volatility regime			
$\hat{\omega}_1$	0.001 (<0.001)	0.001 (0.001)	<0.001 (<0.001)
$\hat{\alpha}_1$	0.032 (0.015)	0.028 (0.020)	0.008 (0.005)
$\hat{\beta}_1$	0.949 (0.019)	0.949 (0.028)	0.985 (0.007)
$\hat{p}_{11}$	0.998 (0.120)	0.996 (0.176)	0.999 (0.117)
$\hat{\pi}^1_\infty$	0.732	0.556	0.574
High volatility regime			
$\hat{\omega}_2$	0.020 (0.006)	0.005 (0.003)	0.004 (0.002)
$\hat{\alpha}_2$	0.305 (0.140)	0.171 (0.065)	0.098 (0.027)
$\hat{\beta}_2$	0.596 (0.126)	0.790 (0.074)	0.883 (0.030)
$\hat{p}_{22}$	0.993 (0.117)	0.995 (0.245)	0.998 (0.158)
$\hat{\pi}^2_\infty$	0.268	0.444	0.426
LLF	−369.880	−392.205	−1340.835
LR-test statistic	26.180 [<0.001]	13.026 [0.011]	29.670 [<0.001]

Notes: λ_t and R_t are the range-based conditional volatility and range, respectively. *LLF* is the log likelihood function, p-values are in brackets and the numbers in parentheses are robust standard errors proposed by Bollerslev and Wooldridge (1992). The probability of staying in the low volatility state is p_{11}, and that of staying in the high volatility state is p_{22}. The stationary regime probabilities, π^1_∞ and π^2_∞, are computed by the expression: $\pi^1_\infty = (1-p_{22})/(2-p_{11}-p_{22})$ and $\pi^2_\infty = (1-p_{11})/(2-p_{11}-p_{22})$, respectively. The LR-test statistic is equal to twice the difference in the *LLF* of the Markov-switching and single-regime model. The null hypothesis is that single-regime specification against the alternative of two-regime case. The critical values are 13.277 ($\chi^2(4) = 1\%$), 9.488 ($\chi^2(4) = 5\%$) and 7.779 ($\chi^2(4) = 10\%$).

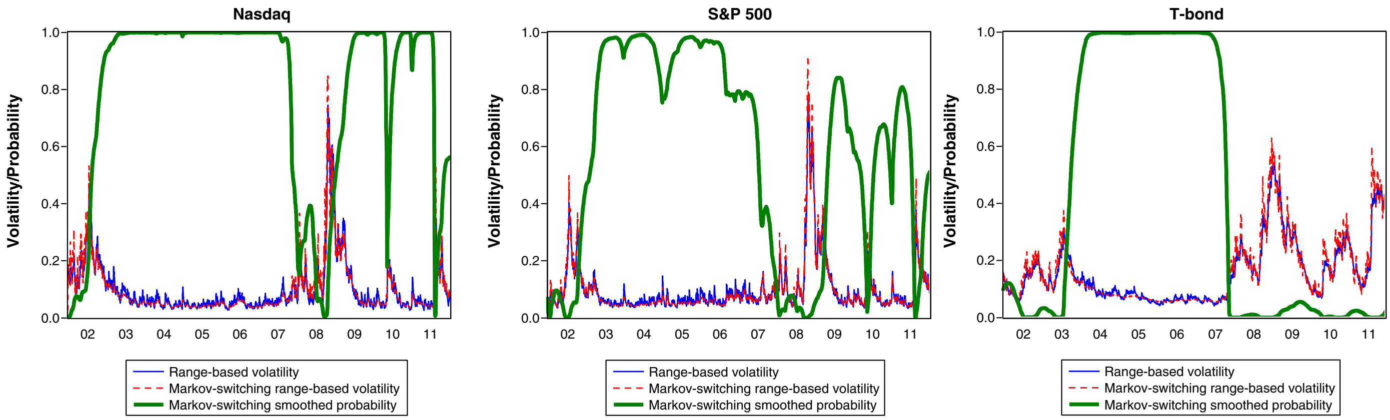

Fig. 1. The estimated range-based, Markov-switching range-based volatility pattern and smoothed probability for low volatility regime for the Nasdaq, S&P 500 and T-bond (2002.1.3–2011.12.31). This figure plots the estimated range-based volatility (thin solid line) pattern, Markov-switching range-based volatility (dashed line) pattern and smoothed probability (thick solid line) at a daily frequency.

$0 \le p_{ij} \le 1$ for $i,j = 1,2$. If we skip both Eq. (1.2) and the Markov chain process, then the two-state Markov-switching range-based volatility model can revert to the original range-based volatility model proposed by Chou (2005). By the time varying transition probabilities, it is easy to infer the stationary distribution of the Markov chain as $\pi_\infty^{s_t}$. The regime variance, $\lambda_{s_t,t}$, follows the basic range-based volatility framework shown in Eq. (1.3). The coefficients $(\omega_{s_t}, \alpha_{s_t}, \beta_{s_t})$ in the conditional range equation are all positive to ensure non-negative constraint for $\lambda_{s_t,t}$. Furthermore, Eq. (1.3) can be rewritten as $\lambda_{s_t,t} = \omega_{s_t}\left(1-\beta_{s_t}\right)^{-1} + \alpha_{s_t}\sum_{i=1}^{\infty}\beta_{s_t}^{i-1}R_{t-i}$, for $s_t = 1,2$. By this specification, there are some economic insights from this expression, the conditional range with regime-change, $\lambda_{s_t,t}$, depends only on the within-regime parameters. In addition, the parameters $\omega_{s_t}\left(1-\beta_{s_t}\right)^{-1}$, α_{s_t}, and β_{s_t} can be regarded as the total impact effect, the short-run impact effect, and the long-run effect of shocks to regime s_t conditional range respectively.

The log-likelihood function for the two-state Markov-switching range-based volatility model is derived carefully in Appendix A and can be written as,

$$\log f(R_1, R_2, \dots, R_T | R_0; \theta) = \sum_{t=1}^{T} \log f(R_t | I_{t-1}; \theta). \tag{2}$$

The unknown vector parameters (θ) can be obtained by maximizing Eq. (2).

2.2. Two-state Markov-switching range-based DCC model

To introduce the regime-switching structure to the range-based dynamic conditional correlation process, the two-state Markov-switching range-based DCC model can be shown as:

$$R_{k,t} = \lambda_{k,t}\varepsilon_{k,t} \quad \varepsilon_{k,t}\left|I^{t-1} \sim exp(1,\cdot),\right. \quad k = 1,2. \tag{3.1}$$

$$\lambda_{k,t} = \omega_k + \alpha_k R_{k,t-1} + \beta_k \lambda_{k,t-1} \tag{3.2}$$

$$\eta_{k,t} = r_{k,t}/\lambda_{k,t}^*, \quad where \quad \lambda_{k,t}^* = adj_k \times \lambda_{k,t}, \quad adj_k = \bar{\sigma}_k/\hat{\lambda}_k \tag{3.3}$$

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i), \quad i,j = 1,2. \tag{3.4}$$

$$Q_t^{ij} = \left(1-A_j-B_j\right)\bar{Q}_j + A_j\eta_{t-1}\eta'_{t-1} + B_j Q_{t-1}^i \tag{3.5}$$

$$\Gamma_t^{ij} = diag\left\{Q_t^{ij}\right\}^{-1/2} Q_t^{ij} diag\left\{Q_t^{ij}\right\}^{-1/2}, \tag{3.6}$$

where Eqs. (3.1)–(3.3) present the range-based volatility specification, $R_{k,t}$ is the observed range in the logarithm of the k^{th} asset during the time interval t, $\lambda_{k,t}$ is the conditional mean of the range, $\varepsilon_{k,t}$ is the disturbance term, which follows the exponential distribution with a unit mean, $\eta_{k,t} = r_{k,t}/\lambda_{k,t}^*$ is the standardized residual, and the scaled expected range $(\lambda_{k,t}^*)$ is substituted for the conditional standard deviation. The unconditional standard deviation of the return series k and the sampling mean of the estimated conditional range of the series k are represented as $\bar{\sigma}_k$ and $\hat{\lambda}_k$, respectively. The two-state Markov chain transition probability, which has the constraints $\sum_{j=1}^{S} p_{ij} = 1$ for $i = 1,2$, and $0 \le p_{ij} \le 1$ for $i,j = 1,2$, is shown in Eq. (3.4). In Eq. (3.5), A_j and B_j are the parameter matrices. If we omit both Eq. (3.4) and the Markov chain process, then the two-state Markov-switching range-based DCC model can revert to the original range-based DCC model proposed by Chou et al. (2009). It is natural to define the stationary distribution of the Markov chain as $\pi_\infty^{s_t}$ by the time-varying transition probabilities. The dynamic conditional covariance and correlation processes with the Markov-switching property are reported in Eqs. (3.5) and (3.6). The unconditional correlations can be presented as $\bar{Q}_j = \frac{1}{T}\sum_{i=1}^{T}\eta_t\eta'_t$, a time-varying correlation matrix is denoted by Γ_t^{ij}, and Q_t^{ij} denotes a time-varying covariance matrix. In Eqs. (3.5) and (3.6), the superscript symbol represents the regime-shift from i to j. In short, the Markov-switching range-based DCC model is composed of

Table 4
The estimation of regime-switching date in volatility process.

	Date to low regime	Date to high regime
Nasdaq	August 21, 2002	November 16, 2007
	December 31, 2008	April 30, 2010
	May 26, 2010	July 27, 2011
	October 27, 2011	
S&P 500	February 7, 2003	July 13, 2007
	April 22, 2009	March 10, 2010
	July 7, 2010	December 10, 2010
	January 4, 2011	July 5, 2011
	December 7, 2011	
T-bond	October 9, 2003	October 30, 2007

Notes: According to Hamilton (1989), the regime-switching date is calculated by the boundary of $\Pr(s_t = j | I^T) = 0.5$ for $j = 1,2$.

Table 5
The estimation of range-based dynamic conditional correlation model of daily data of the Nasdaq, S&P 500 and T-bond (2002.1.3–2011.12.31).

$$R_{k,t} = \lambda_{k,t}\varepsilon_{k,t} \quad \varepsilon_{k,t}|I^{t-1}\sim \exp(1,\cdot), \quad k = asset1, asset2$$
$$\lambda_{k,t} = \omega_k + \alpha_k R_{k,t-1} + \beta_k \lambda_{k,t-1}, \eta_{k,t} = r_{k,t}/\lambda^*_{k,t}, \quad where \quad \lambda^*_{k,t} = adj_k \times \lambda_{k,t}, \quad adj_k = \bar{\sigma}_k/\hat{\lambda}_k,$$
$$Q_t = (1-A-B)\bar{Q} + A\eta_{t-1}\eta'_{t-1} + BQ_{t-1},$$
$$\Gamma_t = diag\{Q_t\}^{-1/2}Q_t diag\{Q_t\}^{-1/2}$$

	Nasdaq & SP 500	T-bond & Nasdaq	T-bond & SP 500
$\hat{A}$	0.037 (0.008)	0.038 (0.007)	0.058 (0.008)
$\hat{B}$	0.948 (0.012)	0.952 (0.010)	0.935 (0.009)
LLF	2566.756	198.839	227.781

Notes: λ_t and R_t are the range-based conditional volatility and range, respectively. The time-varying correlation matrix is denoted by Γ^{ij}_t. The number in parentheses is robust standard error proposed by Bollerslev and Wooldridge (1992). *LLF* is the log likelihood function.

the range-based volatility model for the conditional variance process and the Markov-switching method for the conditional covariance and correlation case.

The log-likelihood function for the two-state Markov-switching DCC model is reported in this section. According to Engle (2002a) and Chou et al. (2009), the two-step quasi-maximum likelihood approach is suitable for estimating the models in the DCC family. It is intuitive to execute the first step quasi-maximum likelihood estimation (QMLE) by Eq. (4), since our specification allows only the conditional correlation to change in different regimes. The likelihood function of the volatility component, $L_V(\kappa)$, can be expressed as

$$L_V(\kappa) = -\frac{1}{2}\sum_t \sum_{k=1}^{2} \left(\log(2\pi) + \log\left(\lambda^*_{k,t}\right) + \frac{r^2_{k,t}}{\lambda^{*2}_{k,t}} \right). \tag{4}$$

By maximizing the QMLE of Eq. (4), we can estimate and obtain the parameters $\kappa = \{\omega_k, \alpha_k, \beta_k\}$. Using the above parameter estimates to the second step of the estimation, the related estimation procedure is discussed in Appendix B.

3. Empirical results

Many crisis events are directly or indirectly related to the U.S. market. We, therefore, select two main stock indices and an important interest rate data from the United States for the empirical analysis, namely, the Nasdaq index, the S&P 500 index, and the T-bond interest rate. These data can be collected from the Yahoo Finance (http://finance.yahoo.com/) for the sample period starting from January 3, 2002 to December 31, 2011. The daily high-low range data are collected to fit the range-based volatility and correlation models. Table 1 provides descriptive statistics for daily range data. All the variables of ranges are apparently not following a normal distribution by Bera–Jarque criterion.

We estimate the range-based volatility model for these target markets. Table 2 reports the parameter estimates of these models. It is appropriate to adopt the quasi-maximum likelihood estimation (QMLE) to estimate the range-based volatility model.[3] According to the significance of parameter estimates and Ljung-Box Q^2 statistics, we can infer that the estimation results conform to the stationarity and non-negative conditions of range-based volatility model.

Additionally, we list the parameter estimates for the two-state Markov-switching range-based volatility model in Table 3. Conventionally, one can define the smaller coefficient of (ω_{s_t}) as the low volatility regime, and the larger coefficient of (ω_{s_t}) as the high volatility regime. The estimated persistent rate ($\hat{\alpha}_{s_t} + \hat{\beta}_{s_t}$) of low volatility regime is higher than that of high volatility regime for all these markets' data. This finding indicates that the effect of a shock to the expectation of the volatility process is more persistent in low volatility regime than in high volatility regime. It is of interest to observe that the parameter estimates for the Markov-switching range-based volatility model are significantly different from zero in statistics for all these market data. Thus, it is reasonable to infer that the volatility structure for all market data has remarkable regime changes during these sample periods. Moreover, we can get the expectation of transition period by the implied transition probability. The expectation of transition period from the low level to high level volatility ($1/(1-\hat{p}_{11})$) is approximately two years for the Nasdaq, one year for the S&P 500 and four years for the T-bond market. However, the expectation of transition period from the high level to low level volatility ($1/(1-\hat{p}_{22})$) is approximately 0.57 years for the Nasdaq, 0.8 years for the S&P 500 and two years for the T-bond market. These findings indicate that the expectation of transition period from the low level to high level volatility is longer than that of reverse. This study also reports the stationary regime probability ($\pi^{s_t}_\infty$) to describe how the frequency of volatility will move to a specific regime in the next period. The stationary regime probability shows that the volatility will move to a low (high) level regime in the next period is 0.732 (0.268) for the Nasdaq, 0.556 (0.444) for the S&P

Table 6
The estimation of Markov-switching range-based dynamic conditional correlation model for daily data of the Nasdaq, S&P 500 and T-bond (2002.1.3–2011.12.31).

$$R_{k,t} = \lambda_{k,t}\varepsilon_{k,t}\varepsilon_{k,t}|I^{t-1}\sim \exp(1,\cdot), k = asset1, asset2$$
$$\lambda_{k,t} = \omega_k + \alpha_k R_{k,t-1} + \beta_k \lambda_{k,t-1},$$
$$\eta_{k,t} = r_{k,t}/\lambda^*_{k,t}, \quad where \quad \lambda^*_{k,t} = adj_k \times \lambda_{k,t}, \quad adj_k = \bar{\sigma}_k/\hat{\lambda}_k,$$
$$p_{ij} = Pr(s_t = j|s_t = i),$$
$$Q^{ij}_t = \left(1-A_j-B_j\right)\bar{Q}_j + A_j\eta_{t-1}\eta'_{t-1} + B_jQ^i_{t-1},$$
$$\Gamma^{ij}_t = diag\left\{Q^{ij}_t\right\}^{-1/2}Q^{ij}_t diag\left\{Q^{ij}_t\right\}^{-1/2}, \quad i,j = 1,2.$$

	Nasdaq & SP 500	T-bond & Nasdaq	T-bond & SP 500
High correlation regime			
$\hat{A}_1$	−0.024 (0.001)	0.099 (0.008)	0.079 (0.012)
$\hat{B}_1$	0.538 (0.061)	0.860 (0.010)	0.910 (0.015)
$\hat{p}_{11}$	0.999 (0.210)	0.999 (0.835)	0.998 (0.815)
$\hat{\pi}^1_\infty$	0.000	0.000	0.002
Low correlation regime			
$\hat{A}_2$	0.037 (0.007)	0.020 (0.004)	0.039 (<0.001)
$\hat{B}_2$	0.960 (0.009)	0.979 (0.005)	0.960 (0.002)
$\hat{p}_{22}$	1.000 (130.97)	1.000 (0.351)	0.999 (0.459)
$\hat{\pi}^2_\infty$	1.000	1.000	0.998
LLF	2576.776	209.185	234.308
LR-test statistic	20.040 [<0.001]	20.692 [<0.001]	13.053 [0.005]

Notes: λ_t and R_t are the range-based conditional volatility and range, respectively. The time-varying correlation matrix is denoted by Γ^{ij}_t. *LLF* is the log likelihood function, p-values are in brackets and the numbers in parentheses are robust standard errors proposed by Bollerslev and Wooldridge (1992). The probability of staying in the low correlation state is p_{11}, and that of staying in the high correlation state is p_{22}. The stationary regime probabilities, π^1_∞ and π^2_∞, are computed by the expression: $\pi^1_\infty = (1-p_{22})/(2-p_{11}-p_{22})$ and $\pi^2_\infty = (1-p_{11})/(2-p_{11}-p_{22})$, respectively. The LR-test statistic is equal to twice the difference in the *LLF* of the Markov-switching and single-regime range-based DCC model. The null hypothesis is that single-regime specification against the alternative of two-regime case. The critical values are 11.345 ($\chi^2(3) = 1\%$), 7.815 ($\chi^2(3) = 5\%$) and 6.251 ($\chi^2(3) = 10\%$).

[3] Engle and Russell (1998), Engle (2002b) and Chou (2005) adopt QMLE to estimate the parameters of the range-based volatility models which can obtain a consistent estimation of the parameters.

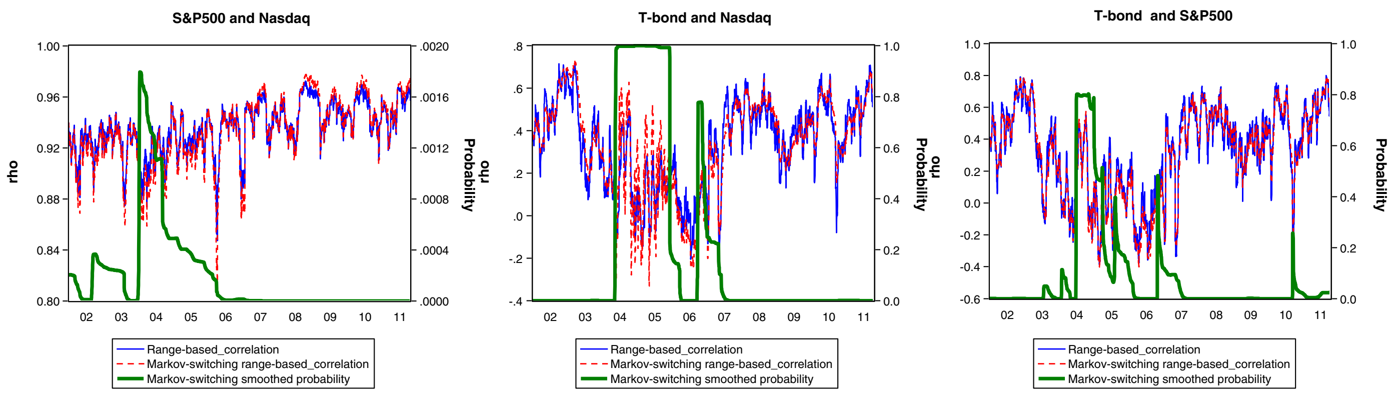

Fig. 2. The estimated range-based, Markov-switching range-based correlations and smoothed probability for high correlation regime (2002.1.3–2011.12.31). This figure plots the estimated range-based correlation (thin solid line) pattern, Markov-switching range-based correlation (dashed line) pattern and smoothed probability (thick solid line) at a daily frequency.

500 and 0.574 (0.426) for the T-bond, respectively. These indicate that the volatility process probably remains at the low level volatility regime for all markets. According to Ang and Bekaert's (2002) suggestion, we can use the conventional likelihood ratio test (LR-test) to examine whether there are any change in regimes.[4] The LR-test statistics support that the two-state Markov-switching structure for all market volatilities is statistically significant at the 5% level. In addition, comparing the figures of parameter estimates in Tables 2 and 3, we find that the range-based volatility model without Markov-switching method will understate (overstate) the persistence of shocks to volatility during the tranquil (turbulent) period. It means that considering the Markov-switching structure certainly improves the stability of estimating volatility process.

Fig. 1 shows the patterns of the estimated range-based volatilities, the Markov-switching range-based volatilities and the smoothed probabilities for all these market data. For the Nasdaq and S&P 500 indices, the patterns of the estimated Markov-switching range-based volatility depict sharply increasing trend than the range-based case during the financial market turmoil of 2007–2008. In terms of the smoothed probabilities for both stock indices, we find that they display some similar patterns. Furthermore, their timing for switching processes are related to some major financial crises, e.g. the 2002 WorldCom scandal, the 2007 subprime housing crisis, the 2008 credit crisis and the 2009 European sovereign-debt crisis. As to the graphs for the T-bond in Fig. 1, these graphs partly differ from that of the stock indices. The main finding for the T-bond shows that those financial crises indeed cause the volatility structure to shift to the more volatile state. Empirical results demonstrate that the influences of those financial crises on volatility process for the T-bond market are very strong and continual for a long time; nevertheless, that for stock markets are huge but intermittent during our sample period. We infer that this finding observed from T-bond volatility could be attributed to the expectation of a falling interest rate environment. When the interest rates are expected to remain at a sustained low level, the investors will prefer the T-bond to short-term bond. This interest rate expectation will cause not only the T-bond volatility regime to switch from the low level to high level but also the persistent influences of financial crises on T-bond volatility.

According to the estimation results for the time of regime-switching date in Table 4, the impacts of the 2002 WorldCom scandal on T-bond volatility are more persistent than that on stock indices' case.[5] Starting from the last half of 2007, a succession of financial disturbances of 2007–2009 triggered structure changes in volatility processes for all market data. Besides, the reactions of shocks to the S&P 500 index volatility are faster than Nasdaq and T-bond volatility in light of the switching frequency. We also calculate the truly average transition period from the low level to high level volatility regime to be approximately 2.6 years for the Nasdaq, 1.57 years for the S&P 500 and 4.05 years for the T-bond.[6] However, the truly average transition period from the high level to low level volatility regime is approximately 0.52 years for the Nasdaq, 0.75 years for the S&P 500 and 1.77 years for the T-bond. These findings are in line with the result of Table 3. All in all, the estimation results of the two-state Markov-switching range-based volatility model display an interesting phenomenon that the global financial crisis of 2007–2009 led to a succession of regime changes in volatility processes for stock indices' data. However, it is still difficult to ascertain that the real date of volatility regime switches is resulted from the 2008 Lehman collapse or 2009 European sovereign debt crisis. This finding is in line with the opinion of Kalbaska and Gatkowski (forthcoming). They consider that the global financial crisis of 2007–2009 triggered the European sovereign debt crisis; therefore the identification of regime-switching date by a specific event is not easy during this turbulent period.

Next, we also discuss whether the financial crises will change the dynamic correlation process from one regime to another. Estimating the range-based DCC model for all market data and their estimation results are shown in Table 5. The estimates of the two range-based DCC parameters are statistically significant. Furthermore, the parameter estimates ($\hat{A}+\hat{B}<1$) demonstrate that this model contains the property of mean reversion.

Although the range-based DCC model seems appropriate for fitting the dynamic correlation process, this model might not directly point out the implicit correlation structure change. Fortunately, the Markov-switching range-based DCC model can improve this unambiguous problem. Table 6 presents the results of parameter estimates for the Markov-switching range-based DCC model. In terms of the regime specification, we define the smaller coefficient of $(1-A_j-B_j)$ as the low correlation regime, and the larger coefficient of $(1-A_j-B_j)$ as the high correlation regime. One of the main findings about correlation process indicates the estimated transition probability of staying in the same regime, $\hat{p}_{11}$ and $\hat{p}_{22}$, both exceeding 0.9 for any market. In

[4] Ang and Bekaert (2002) suggest approximating the critical values of the conventional LR-test statistic by the quantiles of the chi-square distribution with a degree of freedom equal to the difference in the number of parameters between alternative and null hypotheses. For instance, there are 4 different parameters for volatility model testing but 3 different parameters for correlation model testing.

[5] According to Hamilton's (1989) research, we can calculate the regime-switching date by the boundary of $\Pr(s_t=j|I^T)=0.5$ for $j=1,2$.

[6] We calculate the truly average transition period from the estimation results for the time of regime-switching date. For instance, the Nasdaq index truly average transition period from the low level to high level volatility regime can be computed by $((1320+334+295)/3)/250=2.6$ years.

Table 7
The estimation of regime-switching date in correlation process.

	Date to high regime	Date to low regime
Nasdaq & SP 500	N/A	N/A
T-bond & Nasdaq	June 8, 2004	January 11, 2006
	November 1, 2006	January 16, 2007
T-bond & SP 500	July 20, 2004	February 15, 2005
	April 22, 2005	April 29, 2005

Notes: According to Hamilton (1989), the regime-switching date is calculated by the boundary of $\Pr(s_t = j|I^T) = 0.5$ for $j = 1,2$.

addition, we can find the transition probability from the low level to high level correlation is lower than that of the reverse by the estimated transition probability figures. Table 6 also presents the stationary regime probability ($\pi_\infty^{s_t}$). The stationary regime probability displays that the correlation will move to a high (low) regime in the next period to be 0.000 (1.000) for correlation between the Nasdaq and S&P 500 index, 0.000 (1.000) for correlation between the T-bond and Nasdaq index, and 0.002 (0.998) for correlation between the T-bond and S&P 500 index.[7] This finding indicates that the probability of the expected correlation between any two markets moving to a low regime is over 0.99. The LR-test statistics support that the two-state Markov-switching structure for all market correlations is statistically significant at the 1% level. In addition, comparing the figures of parameter estimates in Tables 5 and 6, we find that the range-based DCC model without Markov-switching method will overstate (understate) the persistence of shocks to correlation during the tranquil (turbulent) period.

Fig. 2 plots the estimated range-based, Markov-switching range-based correlations and the corresponding smoothed probabilities for all markets' data. The correlation between the Nasdaq and S&P 500 index for Markov-switching range-based DCC model characterizes a more sharp pattern than that for range-based DCC case. The correlation between these two stock indices is highly positive and rambles in the level of 0.8 to 0.98. We infer that the weak regime-switching structure for correlation between these two stock indices is attributed to the similar market properties. In brief, the influences of financial crises on different stock indices are similar; hence the correlation between stock indices has a relatively stable structure than the correlation structures between stock indices and T-bond. The correlations between stock indices and T-bond are generally positive except for the period from 2003 to 2007 and the third quarter of 2010, when they become negative. We infer that the declines in correlations between stock indices and T-bond could be attributed to the variability of inflation and output growth. Because of the lower variability of inflation and stable output growth could lead to lower correlations between stock indices and T-bond. As to the smoothed probabilities for correlations between each stock index and T-bond, we can observe that they have partly similar patterns. The main finding is that the 2007 subprime housing crisis led to mainly regime switches in correlation processes from high correlation regime to low case. Generally speaking, the estimates of smoothed probability still support regime switches in correlation processes. Accordingly, we consider that using the Markov-switching range-based DCC model to fit these time series data for correlation is essential and appropriate.

According to the estimation results for the time of regime-switching date in Table 7, the impacts of the 2002 WorldCom scandal on correlations between stock indices and T-bond ended until the half of 2004.[8] A sequence of financial crises from 2007 to 2009 triggered the structure changes in correlation processes between S&P 500 index and T-bond on April, 2005. In addition, these disturbing events of 2007–2009 led to structure changes in correlation process between Nasdaq index and T-bond on January, 2007. We also calculate the truly average transition period from low to high regime to be approximately 1.6 years for the correlation between T-bond and Nasdaq index, and 1.34 years for the correlation between T-bond and S&P 500 index. However, the truly average transition period from high to low regime is approximately 0.88 years for the correlation between T-bond and Nasdaq, and 0.29 years for the correlation between T-bond and S&P 500. These findings conform with the estimation results of Table 6.

4. Conclusion

In this study, we examine whether the unexpected financial crises will change the volatility and correlation regime process for the US stock and bond markets. This study employs the Markov-switching range-based volatility model and the Markov-switching range-based DCC model to capture the dynamic volatility and correlation structures, respectively.

From the empirical results, we find statistically significant volatility regime-switching structures in these markets by the parameter estimates for the Markov-switching range-based volatility model. Consequently, we diagnose various financial crises bring regime switches for volatility processes. In addition, we also find that the influences of financial crises on volatility for the T-bond are more persistent than that for both stock indices.

Meanwhile, we find that the correlations between stock indices and T-bond have remarkable regime-switching processes. But the correlation between two stock indices presents the ambiguous regime-switching structure. This phenomenon can be attributed to the similar market properties. Furthermore, this study observes that the correlations between stock indices and T-bond are usually positive except for the period from 2003 to 2007 and the third quarter of 2010, as they become negative. These declines in correlations between stock indices and T-bond could be attributed to the lower variability of inflation and output growth. Furthermore, we demonstrate that the unexpected financial crises make regime switches for correlation processes.

Appendix A. Construction of log-likelihood function for the two-state Markov-switching range-based volatility model

According to Hamilton (2008), we can make an inference of s_t through the observed range R_t. By the case of two probabilities, we infer that

$$\xi_{jt} = \Pr(s_t = j|I_t;\theta), j = 1,2, \tag{A.1}$$

where the information set is $I_t = \{R_t, R_{t-1}, \ldots, R_1, R_0\}$, the unknown vector parameters are $\theta = \left(\lambda_{s_t}, \omega_{s_t}, \alpha_{s_t}, \beta_{s_t}, p_{11}, p_{22}\right)'$, and the sum for two-state probabilities is unity. We can obtain the state probabilities by an iterative method. Eq. (A.2) can be regarded as the input element for generating Eq. (A.1).

$$\xi_{i,t-1} = Pr(s_{t-1} = i|I_{t-1};\theta), \quad i = 1,2. \tag{A.2}$$

The distribution for R_t follows the exponential distribution with unit mean, since the range data are all positive values. The density function for two regimes is

$$\eta_{jt} = f(R_t|s_t = j, I_{t-1};\theta) = \lambda_{s_t} \exp\left[-\lambda_{s_t} R_t\right], \quad j = 1,2. \tag{A.3}$$

[7] As the stationary regime probability is equal to 1, it means that the moving chance in the next time is close to zero.

[8] According to Hamilton's (1989) research, we can calculate the regime-switching date by the boundary of $\Pr(s_t = j|I^T) = 0.5$ for $j = 1,2$.

By introducing Eq. (A.2), we can estimate the conditional density as

$$f(R_t|I_{t-1};\theta) = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{ij}\xi_{i,t-1}\eta_{jt}. \tag{A.4}$$

The renewed joint probabilities can be expressed as

$$\xi_{jt} = \frac{\sum_{i=1}^{2} p_{ij}\xi_{i,t-1}\eta_{jt}}{f(R_t|I_{t-1};\theta).} \tag{A.5}$$

After we execute the iteration, the log-likelihood function can be written as,

$$\log f(R_1, R_2, \ldots, R_T|R_0;\theta) = \sum_{t=1}^{T} \log f(R_t|I_{t-1};\theta). \tag{A.6}$$

Appendix B. The estimation procedure for the two-state Markov-switching range-based DCC model

The estimation procedure for the two-state Markov-switching range-based DCC model can be finished as:

1. given the filtered probabilities as inputs, determine the joint probabilities as:

$$Pr\left(s_t = j, s_{t-1} = i\middle|I^{t-1}\right) = Pr(s_t = j, s_{t-1} = i) \times Pr\left(s_{t-1} = i\middle|\mathbf{I}^{t-1}\right) i, j = 1, 2 \tag{B.1}$$

2. estimate the regime-dependent log-likelihood as:

$$L\left(R_t|\boldsymbol{\lambda}_t, s_t = j, s_{t-1} = i, I^{t-1}\right) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log\left(\Gamma_t^{ij}\right) + \eta'_t\left(\Gamma_t^{ij}\right)^{-1}\eta_t\right) \tag{B.2}$$

3. estimate the log-likelihood of observation t as:

$$L\left(R_t|\lambda_t, I^{t-1}\right) = \sum_{j=1}^{S}\sum_{i=1}^{S} L\left(R_t|\boldsymbol{\lambda}_t, s_t = j, s_{t-1} = i, \mathbf{I}^{t-1}\right) \times Pr\left(s_t = j, s_{t-1} = i|\mathbf{I}^{t-1}\right) \tag{B.3}$$

$$L(R_t, \ldots, R_1) = L(R_{t-1}, \ldots, R_1) + L\left(R_t|\boldsymbol{\lambda}_t, I^{t-1}\right) \tag{B.4}$$

4. update the joint probabilities as:

$$Pr\left(s_t = j, s_{t-1} = i|I^{t-1}\right) = \frac{L\left(R_t|\boldsymbol{\lambda}_t, s_t = j, s_{t-1} = i, I^{t-1}\right) \times Pr\left(s_t = j, s_{t-1} = i|I^{t-1}\right)}{L\left(R_t|\boldsymbol{\lambda}_t, I^{t-1}\right)} \tag{B.5}$$

5. calculate the filtered probabilities as:

$$Pr\left(s_t = j|\mathbf{I}^t\right) = \sum_{i=1}^{S} \Pr\left(s_t = j, s_{t-1} = i|\mathbf{I}^t\right) \quad j = 1, 2 \tag{B.6}$$

6. update the dynamic conditional correlation matrix by the following approximation:

$$Q_t^j = \frac{\sum_{i=1}^{S} Pr\left(s_t = j, s_{t-1} = i|I^t\right) \times Q_t^{ij}}{Pr\left(s_t = j|I^t\right)} \tag{B.7}$$

7. iterate 1 to 6 until the end of the sample. The unknown parameters of the two-state Markov-switching range-based DCC model can be obtained with these estimation procedures.

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