

An International Comparison of Implied, Realized, and GARCH Volatility Forecasts

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We compare the predictive ability and economic value of implied, realized, and GARCH volatility models for 13 equity indices from 10 countries. Model ranking is similar across countries, but varies with the forecast horizon. At the daily horizon, the Heterogeneous Autoregressive model offers the most accurate predictions, whereas an implied volatility model that corrects for the volatility risk premium is superior at the monthly horizon. Widely used GARCH models have inferior performance in almost all cases considered. All methods perform significantly worse over the 2008–09 crisis period. Finally, implied volatility offers significant improvements against historical methods for international portfolio diversification. © 2016 Wiley Periodicals, Inc. Jrl Fut Mark

1. INTRODUCTION

Volatility is a key concept in finance especially in portfolio selection, option pricing, and risk management. Despite a variety of shortcomings and alternatives, volatility still lies at the heart of modern finance. It is not surprising that a vast methodological and empirical literature exists around the development, assessment, and application of volatility forecasts.¹ Unfortunately, it is difficult to draw clear conclusions from the existing literature as research designs vary considerably across different studies in terms of countries, asset classes, time periods, forecasting techniques, forecast horizons, and forecast evaluation methods. Our study aims to overcome this difficulty by comparing some of the most popular volatility models within a common framework. Our analysis employs 13 equity indices from 10 countries, three forecast horizons, and different market conditions, that is, before, during, and after the 2008–09 crisis. Comparisons between models are performed on the basis of different statistical tests and loss functions. Most importantly, we assess the economic

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¹Characteristically, on November 24, 2015, 6,110 papers in Google Scholar and 1,548 research outputs in Scopus include the term “volatility forecasting.”

significance of the competing volatility forecasting models within an international portfolio choice framework.

The models we consider span the three most prominent families of volatility forecasting methods. First, we have the GJR-GARCH of Glosten, Jagannathan, and Runkle (1993) which descends from the ARCH family of historical volatility models. We select this model on the basis of its popularity among academics and practitioners and the fact that it is often found to have superior performance against alternative ARCH specifications (e.g., see Brownlees, Engle, & Kelly, 2012). Second, we use two model specifications based on raw and volatility risk-premium-adjusted implied volatility levels, respectively. For the adjustment, we use the technique of DeMiguel, Plyakha, Uppal, and Vilkov (2013) and Prokopczuk and Wese Simen (2014) which reduces the bias in the raw implied volatility. It is the first time a study assesses the importance of adjusting for the volatility risk premium in volatility forecasting in the context of international equity markets. Third, we employ realized volatility-based forecasts using plain lagged values and the Heterogeneous Autoregressive (HAR) model of Corsi (2009). The latter approach is becoming increasingly popular due to its simplicity and modeling accuracy (e.g., see Busch, Christensen, & Nielsen, 2011; Corsi, Pirino, & Reno, 2010; Giot & Laurent, 2007; Patton & Sheppard, 2015).

We adopt the standard practice of using the daily realized volatility as a proxy for the “actual” latent volatility. We then employ a battery of different techniques in order to compare the volatility models on the basis of various statistical and economic criteria. In order to accommodate the possibility of different forecasting scenarios and tasks, we use horizons of 1, 5, and 22 days, respectively. We apply univariate Mincer–Zarnowitz as well as encompassing regressions to assess the information content of each set of volatility forecasts in an in-sample setting. Out-of-sample analysis is based on the Diebold–Mariano test of predictive accuracy. Two statistical loss functions are utilized: the root mean-squared error and the quasi-likelihood. We perform several robustness checks by considering alternative models, configurations, and estimation periods along with the effect of volatility spillovers from the United States.

We find that model rankings remain roughly the same across countries but vary with the forecast horizon. Although the results for the weekly horizon are mixed, the HAR model is superior at the daily horizon and the implied volatility model yields the best forecasts at the monthly horizon. The superiority of the volatility risk premium-adjusted implied volatility forecasts at the monthly horizon is likely due to the fact that all model-free implied volatility indices have a fixed forecast horizon of 1 month by construction. Our results also reveal that volatility risk premium-adjusted implied volatility forecasts outperform raw implied volatility forecasts in most markets under consideration. This finding highlights the importance of accounting for the volatility risk premium when forecasting volatility using information from option prices. In line with the literature, comparing raw implied volatility forecasts to lagged realized volatility forecasts leads to mixed results (see Andersen, Frederiksen, & Staal, 2007b; Blair, Poon, & Taylor (2001); Jiang & Tian, 2005; Martens & Zein, 2004; Pong, Shackleton, Taylor, & Xu, 2004). Finally, we find that the historical GJR-GARCH model underperforms the realized and implied volatility alternatives in almost all cases (similar results are reported by Fleming, 1998; Jorion, 1995; Andersen, Bollerslev, Diebold, & Labys, 2003; Covrig & Low, 2003; Giot, 2003; Andersen, Frederiksen, & Staal, 2007b; and Charoenwong, Jenwittayaroje, & Low, 2009, among others).

Accurate volatility predictions are particularly important during periods of market turmoil, such as that of 2008–2009, when risks typically soar (see Schwert, 2011). Given that our dataset spans the period 2000–2012, we examine whether the performance of our forecasting methods changes between periods of market calmness and unrest. The ranking of our models is comparable in the periods before, during, and after the crisis. However, all

models perform significantly worse during the crisis period. This result suggests that researchers and practitioners should be more cautious when forecasting volatility during periods of market downturn.

Finally, we examine the practical value of the forecasting models considered for an international investor. This reveals that implied volatility-based forecasts lead to superior out-of-sample portfolio performance. In particular, only this forecasting method yields portfolios that manage to outperform our benchmark strategy of an equally weighted portfolio (1/N). We also find that correcting for the volatility risk premium further improves portfolio performance.

The remainder of the paper is organized as follows: Section 2 presents our data and the forecasting models under consideration. The information content and predictive ability of the models is evaluated in Sections 3 and 4, respectively. Section 4 also examines the value of the volatility forecasts before, during, and after the 2008–09 crisis period. Section 5 assesses the economic value of volatility forecasts. Section 6 summarizes our robustness checks while the last section concludes the paper.

2. METHODOLOGY

2.1. Data

Our analysis involves three types of data. First, we collect daily dividend-adjusted closing levels for 13 international equity indices from Bloomberg. We use these to compute percentage index returns. Second, we use a daily realized volatility series for each equity index, constructed from equally spaced 5-minute intraday returns. These series are collected from the Oxford-Man Institute Realized Library of the University of Oxford. Finally, we obtain daily data on the option implied volatility indices which correspond to the equity markets under consideration, from Bloomberg. All implied volatility indices used are constructed in a model-free manner. Our sample spans the period from January 3, 2000 to October 26, 2012, although for some markets the sample is shorter. Table I reports the equity indices considered, along with the sample period covered and the relevant implied volatility indices.

2.2. Volatility Forecasting Models

This section describes our proxy of the actual unobserved volatility and the volatility forecasting models we use in our analysis, respectively.

2.2.1. Realized volatility

As the true volatility is unobservable, we need to proxy it using an observable variable. The most popular and theoretically appealing proxy is the so-called realized variance. This is given by the sum of squared intraday returns sampled at equally spaced intervals (see Andersen & Bollerslev, 1998; Barndorff-Nielsen & Shephard, 2002b). It has been shown that in a frictionless market, realized variance is a consistent estimator of the actual unobserved variance (Barndorff-Nielsen & Shephard, 2002a). In particular, it follows from the quadratic variation theory that realized variance asymptotically converges to the actual unobserved variance as the sampling frequency increases to infinity.

Suppose that on a trading day t , we observe $M + 1$ prices at times: $t_0, t_1, t_2, \dots, t_M$. If p_{t_j} is the logarithmic price at time t_j , then the corresponding return, r_{t_j} , for the j^{th} intraday interval

TABLE I
List of Equity and Implied Volatility Indices

<i>Equity Index</i>	<i>Stock Exchange</i>	<i>Country</i>	<i>Implied Volatility Index</i>	<i>Sample Period</i>
AEX	Amsterdam Stock Exchange	the Netherlands	VAEX	03/01/2000–26/10/2012
CAC 40	Euronext Paris	France	VCAC	03/01/2000–26/10/2012
DAX	Frankfurt Stock Exchange	Germany	V1X	03/01/2000–26/10/2012
Dow Jones Industrial Average (DJIA)	New York Stock Exchange	United States	VXD	03/01/2000–26/10/2012
Euro Stoxx 50	Multiple Exchanges in Europe	Europe	V2X	03/01/2000–26/10/2012
FTSE 100	London Stock Exchange	United Kingdom	VFTSE	03/01/2000–26/10/2012
Hang Seng Index	Hong Kong Stock Exchange	China	VHSI	02/01/2001–26/10/2012
Korea Composite Stock Price Index (KOSPI)	Korea Exchange	South Korea	VKOSPI	02/01/2003–26/10/2012
Nikkei 225	Tokyo Stock Exchange	Japan	VXJ	04/01/2000–26/10/2012
NASDAQ 100	New York Stock Exchange (NYSE)	United States	VXN	05/02/2001–26/10/2012
Russell 2000	NASDAQ, NYSE	United States	RVX	02/01/2004–26/10/2012
Swiss Market Index (SMI)	SIX Swiss Exchange	Switzerland	V3X	04/01/2000–26/10/2012
S&P 500	NYSE, NASDAQ	United States	VIX	03/01/2000–26/10/2012

of day t is defined as $r_{t_j} = p_{t_j} - p_{t_{j-1}}$. The *realized volatility* estimator for day t is then defined as the square root of the sum of squared intraday returns:

$$RV_t = \sqrt{\sum_{j=1}^M r_{t_j}^2} \quad (1)$$

Realized volatility over horizons of k days is given by $RV_{t:t+k} = \sqrt{\sum_{i=1}^k RV_{t+i}^2}$, under the convention that $RV_t = RV_{t-1:t}$. In our analysis, we consider horizons of 1, 5, and 22 days, respectively.

2.2.2. Forecasts from intraday returns (LRE)

The first forecasting model we consider involves the lagged realized volatility (LRE). This model stems from the assumption that volatility is a Markov process and therefore last period's volatility is highly informative of future values. An alternative to LRE would be to use historical volatility constructed from daily data. Nevertheless, extensive empirical evidence has shown that historical volatility estimators obtained using daily data are inferior to high-frequency-based counterparts (e.g., Andersen & Bollerslev, 1998; Andersen, Bollerslev, & Diebold, 2007a; Blair et al., 2001).

2.2.3. Model-free implied volatility forecasts (MFIV)

The second forecasting model we adopt is based on volatility measures implied from the prices of liquid traded options. Option implied volatility can be constructed using the Black–Scholes (BS) option pricing model. However, this approach fails to accommodate empirically documented facts, such as stochastic volatility, non-Gaussian returns, etc. To address these issues, recent research advocates the derivation of implied volatility in a model-free manner that employs mid-quote prices of out-of-the money call and put options with different strike prices and maturities (e.g., Britten-Jones & Neuberger, 2000). The implied volatility indices we adopt in this work are constructed this way.

Implied volatility indices represent the market expectation of future volatility of the underlying equity index over the next 30 calendar days. In order to predict volatility at various horizons, we re-scale implied volatility so that it forecasts the k -day volatility (see, e.g., Blair et al., 2001). This is done by setting $MFIV_{t:t+k} = \sqrt{\frac{k}{252}} IV_t$, where IV_t is the daily value of the implied volatility index. Then, $MFIV_{t:t+k}$ corresponds to the k -day option implied volatility forecast.

2.2.4. Model-free implied volatility adjusted for the volatility risk premium (C-MFIV)

It is well known that implied volatility is a biased estimator of future volatility (Andersen et al., 2007b; Jiang & Tian, 2005). One of the main reasons for this bias is the existence of an economically significant variance risk premium (see Chernov, 2007). To correct for this premium, DeMiguel, Plyakha, Uppal, and Vilkov (2013) and Prokopczuk and Wese Simen

(2014) implement a simple non-parametric adjustment to the implied volatility. They show that accounting for the volatility risk premium leads to superior volatility forecasts, compared to time series models, Black–Scholes implied volatility, and the non-adjusted MFIV.

Motivated by this evidence, we assess the value of the aforementioned adjustment in the context of international equity markets. In particular, we consider a relative variance risk premium, defined as

$$\text{VRP}_t = E_t^Q(\text{Var}_{t:t+k})/E_t^P(\text{Var}_{t:t+k}), \quad (2)$$

where P and Q are the physical and risk-neutral probability measures, respectively, and $\text{Var}_{t:t+k}$ is the variance from day t to day $t+k$. Empirically, the risk neutral expectation of variance between t and $t+k$ can be estimated using $\text{MFIV}_{t:t+k}$. To estimate the ex-ante expectation of variance under the physical measure, we employ the realized variance computed from 5-minute intraday returns. Then, following DeMiguel, Plyakha, Uppal, and Vilkov (2013), the average estimated relative variance risk premium over a period of 252- k days is

$$\bar{\text{VRP}}_t = \frac{1}{252-k} \sum_{i=t-251}^{t-k} \frac{\text{MFIV}_{i:i+k}^2}{\text{RV}_{i:i+k}^2}, \quad (3)$$

where k is the forecast horizon ($k = 1, 5$, or 22). Finally, the *corrected model-free implied volatility* (C-MFIV) forecast for a horizon of k days is obtained as follows:

$$C - \text{MFIV}_{t:t+k} = \frac{\text{MFIV}_{t:t+k}}{\sqrt{\bar{\text{VRP}}_t}}. \quad (4)$$

Other approaches in the literature that correct for the variance risk premium include Chernov (2007) and Kang, Kim, and Yoon (2010). Chernov relies on at-the-money option contracts and models the volatility risk premium as an affine function of the underlying spot volatility. Kang et al. take a more structural approach that employs investor risk preferences and higher order risk neutral moments. While both of these alternatives successfully correct for the variance risk premium, we adopt the technique of DeMiguel, Plyakha, Uppal, and Vilkov and Prokopczuk and Wese Simen (2014), as it is simpler to apply in our framework without requiring additional estimations from options data. Moreover, as argued by Prokopczuk and Wese Simen, dividing raw implied volatility by the squared root of the variance risk premium, instead of using a volatility spread as in Chernov, can improve volatility forecasting in time-periods that include regime switches. This makes the method of Prokopczuk and Wese Simen more suitable for our empirical analysis, as our datasets cover the 2008–09 financial crisis. A potential downside of Equation (4) is that the choice of the averaging period is not theoretically justified. To deal with this issue, we consider alternative estimation periods, namely 18 and 24 months, and we do not observe any significant change in our results.

2.2.5. GJR-GARCH model forecasts (GJR)

The fourth forecasting model in our analysis is the so-called GJR-GARCH model of Glosten, Jaganathan, and Runkle (1993). This captures serial dependencies in an asymmetrical manner in that negative shocks have a greater impact on volatility than positive shocks of the same magnitude. Moreover, it offers superior forecasting performance against other ARCH specifications (see, e.g., Brownlees, Engle, & Kelly, 2012). The GJR-GARCH model is

specified as follows:

$$h_t = \omega + \alpha e_{t-1}^2 + \gamma I_{\{e_{t-1} < 0\}} e_{t-1}^2 + \beta h_{t-1}, \quad (5)$$

where e_t are the residuals from the mean equation $r_t = \mu + e_t$, μ is the unconditional mean of the return series, h_t is the conditional variance of the residuals, and $I_{\{e_{t-1} < 0\}}$ is an indicator function that equals 1 if the previous period innovation is negative and zero otherwise. To produce out-of-sample volatility forecasts, we estimate the above model on a rolling basis using a window of 800 observations and construct k -step ahead forecasts in a recursive manner.²

2.2.6. Heterogeneous autoregressive model forecasts

The literature highlights the importance of long memory for modeling and forecasting volatility (Andersen et al., 2003; Areal & Taylor, 2002; Comte & Renault, 1998; Deo, Hurvich, & Lu, 2006). Alas, many of the conventional long memory models, such as the Fractional Integrated GARCH (FIGARCH), present significant estimation and computational difficulties. To overcome such issues, we adopt the HAR model introduced by Corsi (2009). This is relatively straightforward to estimate and can capture known stylized facts of financial data, such as long memory and fat tails.

In the HAR setting, realized volatility over the next k days is modeled as an affine function of past realized volatilities computed over three different horizons:

$$RV_{t:t+k} = \omega + \beta_d RV_t + \beta_w RV_{t-5:t} + \beta_m RV_{t-22:t} + e_{t+k} \quad (6)$$

To be consistent with the application of the GJR-GARCH model, we estimate the above model using a rolling sample of the 800 most recent observations. We then use the parameter estimates to forecast the volatility of equity returns over horizons of 1, 5, and 22 business days, respectively.

3. INFORMATION CONTENT OF VOLATILITY FORECASTS

3.1. Univariate Regressions

To assess the information content of individual volatility forecasts, we estimate Mincer–Zarnowitz regressions (Mincer & Zarnowitz, 1969). These involve regressing the k -day realized volatility for each equity index on forecasts from the different volatility models:

$$RV_{t:t+k} = \alpha + \beta \hat{F}_{t:t+k} + e_t, \quad (7)$$

where $\hat{F}_{t:t+k}$ is the k -day ahead forecast from a particular volatility forecasting model. A forecast is informative about future volatility if it has a coefficient in regression Equation (7) that is statistically different from zero.

The results from estimating the above regressions are reported in Tables II and III for the daily and monthly horizon, respectively (to preserve space, results for the weekly horizon can be found in the Appendix SA). The tables give the coefficient estimates (β) for each forecasting model along with the associated t -statistics in parentheses. The bottom section of each table presents the average values of the coefficients for each forecasting model across the thirteen indices. We observe that all slope coefficient estimates are statistically

²In Section 6, we also consider sample lengths of 1,000 and 1,200 observations, respectively.

TABLE II
Univariate Regressions for Volatility Forecasts: 1-Day Forecast Horizon

	<i>LRE</i>	<i>MFIV</i>	<i>C-MFIV</i>	<i>GJR</i>	<i>HAR</i>
AEX					
α	0.022 (7.42)	-0.022 (-5.20)	0.005 (1.49)	0.021 (5.99)	0.003 (0.88)
β	0.855 (38.48)	0.726 (36.06)	1.062 (34.10)	0.652 (32.60)	0.977 (40.83)
\bar{R}^2	73.90%	71.29%	69.69%	74.29%	76.45%
CAC 40					
α	0.028 (6.68)	-0.044 (-7.40)	-0.006 (-1.29)	0.011 (2.51)	0.005 (1.01)
β	0.829 (29.46)	0.889 (30.94)	1.150 (29.61)	0.732 (29.52)	0.970 (31.44)
\bar{R}^2	68.72%	70.04%	66.93%	71.56%	71.78%
DAX					
α	0.028 (7.00)	-0.046 (-6.81)	-0.010 (-1.73)	-0.002 (-0.37)	0.004 (0.93)
β	0.837 (32.27)	0.910 (28.98)	1.167 (28.53)	0.848 (28.15)	0.968 (31.22)
\bar{R}^2	70.36%	70.34%	65.80%	71.30%	72.77%
DJIA					
α	0.030 (8.15)	-0.043 (-6.30)	-0.015 (-2.51)	0.001 (0.17)	0.006 (1.29)
β	0.794 (27.41)	0.986 (24.45)	1.278 (23.36)	0.893 (22.54)	0.948 (27.45)
\bar{R}^2	63.01%	68.01%	66.71%	70.39%	69.07%
Euro Stoxx 50					
α	0.037 (6.60)	-0.042 (-5.75)	-0.015 (-2.24)	0.010 (1.80)	0.007 (1.10)
β	0.788 (22.80)	0.881 (26.14)	1.220 (25.04)	0.789 (25.03)	0.958 (23.33)
\bar{R}^2	62.09%	64.80%	62.05%	67.94%	65.55%
FTSE 100					
α	0.020 (6.83)	-0.022 (-5.68)	-0.002 (-0.50)	0.015 (3.89)	0.001 (0.28)
β	0.840 (31.82)	0.718 (32.37)	1.110 (31.09)	0.632 (24.39)	0.984 (34.28)
\bar{R}^2	70.53%	72.51%	71.31%	70.54%	73.64%
Hang Seng					
α	0.036 (6.77)	0.000 (-0.02)	-0.014 (-1.67)	0.073 (15.00)	0.003 (0.40)
β	0.731 (16.83)	0.564 (15.68)	1.202 (15.99)	0.256 (10.90)	0.968 (15.95)
\bar{R}^2	53.46%	59.02%	60.33%	45.27%	60.19%
KOSPI					
α	0.025 (5.24)	-0.031 (-3.39)	-0.019 (-2.18)	-0.001 (-0.17)	0.006 (0.90)
β	0.841 (25.07)	0.737 (18.54)	1.225 (18.33)	0.711 (20.95)	0.954 (21.40)
\bar{R}^2	70.75%	65.05%	64.29%	65.45%	72.86%
Nikkei 225					
α	0.028 (7.54)	-0.004 (-0.59)	0.002 (0.30)	0.022 (3.88)	0.004 (0.79)
β	0.803 (27.48)	0.570 (19.37)	1.074 (19.98)	0.528 (19.09)	0.960 (26.16)
\bar{R}^2	64.43%	57.91%	61.16%	59.77%	67.59%
NASDAQ					
α	0.024 (6.67)	-0.046 (-8.12)	-0.011 (-2.34)	-0.002 (-0.31)	0.003 (0.80)
β	0.832 (29.93)	0.787 (29.83)	1.218 (28.29)	0.690 (24.72)	0.970 (28.75)
\bar{R}^2	69.29%	71.51%	69.57%	68.24%	72.70%
Russell 2000					
α	0.040 (7.27)	-0.065 (-7.61)	0.000 (0.05)	0.015 (2.15)	0.006 (0.95)
β	0.793 (24.97)	0.818 (26.72)	1.136 (25.17)	0.631 (21.83)	0.969 (26.35)
\bar{R}^2	62.80%	63.57%	61.45%	62.31%	66.95%
SMI					
α	0.019 (6.74)	-0.015 (-3.12)	0.005 (1.13)	0.013 (3.23)	0.003 (0.89)
β	0.850 (33.80)	0.748 (25.77)	1.026 (23.90)	0.705 (25.32)	0.971 (34.58)
\bar{R}^2	72.54%	70.43%	68.58%	71.05%	75.12%

continued

TABLE II
(Continued)

	<i>LRE</i>	<i>MFIV</i>	<i>C-MFIV</i>	<i>GJR</i>	<i>HAR</i>
S&P 500					
α	0.027 (7.29)	-0.042 (-6.98)	-0.015 (-2.82)	0.008 (1.68)	0.005 (1.17)
β	0.820 (29.50)	0.912 (27.72)	1.268 (26.52)	0.805 (25.18)	0.956 (29.90)
\bar{R}^2	67.21%	70.27%	68.69%	72.46%	72.19%
Aggregate results					
Average α	0.028	-0.032	-0.007	0.014	0.004
Average β	0.816	0.788	1.164	0.682	0.966
Average \bar{R}^2	66.85%	67.29%	65.89%	66.97%	70.53%

This table presents results from estimating regressions of realized volatility on individual volatility forecasts. The forecast horizon is 1 trading day. Each panel corresponds to a different market. Columns show estimation results for a particular forecasting model. α and β denote the intercept and slope of the regression, while the row " \bar{R}^2 " shows the adjusted R^2 coefficient of the regression. Numbers in parentheses denote t -statistics. The bottom panel of the table contains the average values of the intercepts and slopes across markets for each forecasting model. All regressions are estimated using Newey–West (1987) heteroskedasticity and autocorrelation consistent standard errors. Forecasts are based on the lagged realized volatility (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model. Significant coefficients at the 5% level are highlighted in bold. HAR and GJR model forecasts are produced using a rolling sample of 800 daily observations.

significant, indicating that all forecasts bear information about future volatility. Looking at the results across markets, we observe that according to the adjusted R^2 (\bar{R}^2) values, the HAR model has the greatest explanatory power at the daily horizon for 9 out of 13 indices. However, at the monthly horizon, the C-MFIV model exhibits the highest \bar{R}^2 values in 12 out of 13 markets. At the weekly forecast horizon, the results are more mixed with the GJR, HAR, and MFIV producing the most informative forecasts in the majority of markets. Notably, the GJR model is superior for the S&P 500 index for all three forecast horizons considered.

The regression of Equation (7) can also be employed to assess the biasness of volatility forecasts. In particular, a volatility forecast is an unbiased forecast of future realized volatility, if $\alpha = 0$ and $\beta = 1$. We test the validity of this joint restriction through a standard Wald-type test. As we use overlapping samples, the errors from Equation (7) are serially correlated for forecasting horizons that exceed 1 day. For this reason, we employ the heteroscedasticity and autocorrelation consistent standard errors of Newey and West (1987) with k lags. We draw three conclusions from these results which are available upon request. First, for most markets and forecast horizons, the regression intercept α is statistically significant while β is different from 1. This result means that most volatility forecasts are biased and is in line with Jiang and Tian (2005) and Andersen, Frederiksen, and Staal (2007b). The only notable exception is the HAR model at the 1-day forecast horizon where the restriction cannot be rejected for 6 out of the 13 equity markets at the 5% significance level. Second, the observed bias is significantly smaller for the HAR forecasts and to a lesser extent for the C-MFIV forecasts, respectively. Third, our results indicate that the volatility risk premium adjustment significantly reduces the bias in the model-free implied volatility. The reduction is more pronounced at the monthly forecast horizon.

3.2. Encompassing Regressions

We further assess the incremental information content of the volatility forecasts by estimating encompassing regressions for each of the 13 equity markets. These regressions

TABLE III
Univariate Regressions for Volatility Forecasts: 22-Day Forecast Horizon

	<i>LRE</i>	<i>MFIV</i>	<i>C-MFIV</i>	<i>GJR</i>	<i>HAR</i>
AEX					
α	0.037 (4.94)	0.011 (1.37)	0.032 (4.51)	0.037 (5.82)	0.010 (1.11)
β	0.743 (13.38)	0.602 (15.08)	0.819 (14.92)	0.580 (17.23)	0.916 (14.49)
\bar{R}^2	60.29%	63.05%	67.40%	60.46%	60.32%
CAC 40					
α	0.039 (4.83)	−0.004 (−0.49)	0.026 (3.25)	0.029 (4.02)	0.015 (1.79)
β	0.758 (13.56)	0.743 (15.93)	0.883 (14.55)	0.660 (16.95)	0.898 (15.82)
\bar{R}^2	59.10%	62.16%	65.36%	58.30%	59.32%
DAX					
α	0.046 (4.91)	−0.001 (−0.11)	0.028 (2.96)	0.020 (1.79)	0.018 (1.85)
β	0.731 (12.58)	0.745 (14.78)	0.874 (13.64)	0.758 (12.44)	0.877 (14.56)
\bar{R}^2	57.07%	63.04%	63.06%	57.04%	55.84%
DJIA					
α	0.032 (2.79)	−0.010 (−0.66)	0.015 (1.11)	0.012 (0.87)	0.024 (2.05)
β	0.783 (8.41)	0.853 (8.95)	0.964 (8.49)	0.856 (8.42)	0.810 (9.01)
\bar{R}^2	61.25%	63.75%	66.18%	65.81%	60.23%
Euro Stoxx 50					
α	0.052 (5.41)	0.003 (0.26)	0.026 (2.38)	0.034 (3.65)	0.018 (1.78)
β	0.706 (11.63)	0.724 (12.61)	0.902 (11.69)	0.694 (13.54)	0.896 (14.49)
\bar{R}^2	51.97%	59.20%	60.61%	57.86%	51.66%
FTSE 100					
α	0.025 (3.85)	0.000 (0.05)	0.021 (3.17)	0.024 (3.29)	0.007 (1.02)
β	0.802 (13.26)	0.628 (15.86)	0.863 (13.49)	0.593 (13.01)	0.913 (14.46)
\bar{R}^2	64.42%	68.52%	71.44%	62.02%	63.13%
Hang Seng					
α	0.041 (3.20)	0.026 (3.21)	0.020 (2.42)	0.094 (11.21)	0.022 (1.89)
β	0.701 (6.50)	0.476 (10.59)	0.877 (10.89)	0.180 (5.89)	0.806 (8.35)
\bar{R}^2	48.81%	56.42%	57.94%	28.44%	49.71%
KOSPI					
α	0.049 (3.30)	0.006 (0.30)	0.020 (1.15)	0.002 (0.09)	0.025 (1.27)
β	0.696 (6.29)	0.618 (6.81)	0.916 (6.99)	0.742 (6.39)	0.830 (6.27)
\bar{R}^2	48.02%	57.96%	58.81%	46.78%	48.50%
Nikkei 225					
α	0.047 (3.44)	0.031 (2.01)	0.033 (2.42)	0.035 (2.47)	0.019 (1.09)
β	0.677 (6.43)	0.456 (6.87)	0.810 (7.58)	0.491 (7.36)	0.841 (6.84)
\bar{R}^2	45.67%	49.89%	57.50%	47.43%	46.16%
NASDAQ					
α	0.033 (3.29)	−0.011 (−0.87)	0.019 (1.74)	0.010 (0.85)	0.015 (1.34)
β	0.770 (9.55)	0.662 (10.91)	0.927 (10.35)	0.655 (10.12)	0.889 (10.43)
\bar{R}^2	59.04%	63.08%	65.95%	62.84%	60.29%
Russell 2000					
α	0.051 (3.32)	−0.001 (−0.05)	0.040 (2.52)	0.048 (2.37)	0.022 (1.20)
β	0.745 (9.30)	0.645 (9.65)	0.845 (9.78)	0.550 (7.30)	0.899 (9.30)
\bar{R}^2	54.83%	53.49%	59.98%	51.22%	54.63%
SMI					
α	0.031 (4.77)	0.008 (1.00)	0.026 (3.20)	0.024 (4.00)	0.008 (1.18)
β	0.759 (13.06)	0.645 (13.58)	0.824 (11.20)	0.644 (16.07)	0.910 (15.71)
\bar{R}^2	60.21%	64.09%	66.71%	56.39%	62.19%

continued

TABLE III
(Continued)

	<i>LRE</i>	<i>MFIV</i>	<i>C-MFIV</i>	<i>GJR</i>	<i>HAR</i>
S&P 500					
α	0.032 (2.95)	−0.009 (−0.64)	0.015 (1.21)	0.019 (1.51)	0.017 (1.46)
β	0.790 (9.36)	0.786 (10.07)	0.966 (9.44)	0.774 (9.18)	0.869 (10.25)
\bar{R}^2	62.31%	64.06%	66.20%	67.45%	62.36%
Aggregate results					
Average α	0.040	0.004	0.025	0.030	0.017
Average β	0.743	0.660	0.882	0.629	0.873
Average \bar{R}^2	56.38%	60.67%	63.63%	55.54%	56.49%

This table presents results from estimating regressions of realized volatility on individual volatility forecasts. The forecast horizon is 22 trading days. Each panel corresponds to a different market. Columns show estimation results for a particular forecasting model. α and β denote the intercept and slope of the regression, while the row " \bar{R}^2 " shows the adjusted R^2 coefficient of the regression. Numbers in parentheses denote t -statistics. The bottom panel of the table contains the average values of the intercepts and slopes across markets for each forecasting model. All regressions are estimated using Newey–West (1987) heteroskedasticity and autocorrelation consistent standard errors. Forecasts are based on the lagged realized volatility (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model. Significant coefficients at the 5% level are highlighted in bold. HAR and GJR model forecasts are produced using a rolling sample of 800 daily observations.

replace the individual forecasts in the right hand-side of Equation (7) with a pair of forecasts from competing volatility models. Apart from the relative importance of two competing volatility forecasts, encompassing regressions can detect whether a volatility forecast subsumes all the information included in another forecast.³

Table IV presents aggregate results for daily and weekly encompassing regressions, namely, the average intercept and slope as well as the average \bar{R}^2 coefficient across the thirteen indices. Extensive results for daily and weekly forecasts are not reported, as they indicate that all forecasting models contain information about future volatility with statistically significant coefficients at the 5% level. We find that all models explain a large fraction of the variation in realized volatility, since the average \bar{R}^2 values range between 70.7% (74.2%) and 72.7% (75%) at the daily (weekly) forecast horizon.

More interesting results emerge from the monthly encompassing regressions presented in Table V. First, focusing on the fourth column of the table (labeled "LRE + C-MFIV") reveals that the corrected model-free implied volatility subsumes all information contained in lagged realized volatility. The coefficients of C-MFIV are all highly significant and lie between 0.70 and 0.99. In contrast, the raw MFIV is not significantly superior to lagged realized volatility. This result confirms the importance of accounting for the volatility risk premium for volatility forecasting. Second, C-MFIV subsumes the information in HAR forecasts in all markets except for the S&P 500. The raw MFIV also outperforms HAR forecasts in 7 out of 13 markets. Third, C-MFIV yields superior forecasts to GARCH in 6 of the 13 markets (see column labeled "GJR + C-MFIV"), while for the remaining markets both are informative, but C-MFIV has a more pronounced impact on realized volatility as indicated by the greater regression coefficients. The latter result extends to all regressions that involve raw or adjusted

³We do not present combinations of in-sample forecasts for CMFIV + MFIV and LRE + HAR in Tables IV and V. The main reason is that the MFIV is essentially nested within the C-MFIV and LRE is nested within HAR. Moreover, from an econometric perspective, inclusion of MFIV (LRE) and the C-MFIV (HAR) in the same regression would make it much more susceptible to multicollinearity.

TABLE IV
Aggregate Results from Encompassing Regressions for the 1- and 5-Day Forecast Horizons

	LRE + GJR	LRE + MFIV	LRE + C-MFIV	HAR + GJR	HAR + MFIV	HAR + C-MFIV	GJR + MFIV	GJR + C-MFIV
Panel A: 1-day forecast horizon								
Average α	0.010	-0.016	-0.002	0.000	-0.017	-0.006	-0.017	-0.006
Average β_1	0.429	0.432	0.461	0.613	0.623	0.670	0.375	0.414
Average β_2	0.388	0.439	0.613	0.297	0.316	0.407	0.399	0.555
Average \bar{R}^2	72.60%	72.70%	72.36%	72.46%	72.20%	71.92%	70.69%	70.78%
Panel B: 5-day forecast horizon								
Average α	0.013	-0.008	0.006	0.004	-0.012	-0.001	-0.010	0.001
Average β_1	0.436	0.431	0.443	0.530	0.517	0.535	0.372	0.384
Average β_2	0.379	0.411	0.551	0.350	0.371	0.492	0.386	0.542
Average \bar{R}^2	74.57%	74.68%	74.66%	74.67%	74.23%	74.17%	74.55%	75.03%

This table presents average coefficient estimates as well as average adjusted R^2 coefficients (\bar{R}^2) from regressions of realized volatility on competing volatility forecasts at the 1- and 5-day forecast horizons, respectively. Each column reports results for a different pair of forecasting models. For example, the column headed "HAR + GJR" contains the aggregate results from estimating a two-variable regression of realized volatility on the forecasts from the HAR and the GJR model, respectively. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model. All regressions are estimated using Newey–West (1987) heteroskedasticity and autocorrelation corrected standard errors.

TABLE V
Encompassing Regressions for Volatility Forecasts: 22-Day Forecast Horizon

	LRE + GJR	LRE + MFIV	LRE + C-MFIV	HAR + GJR	HAR + MFIV	HAR + C-MFIV	GJR + MFIV	GJR + C-MFIV
AEX								
α	0.030 (4.12)	0.016 (1.86)	0.032 (4.76)	0.019 (2.49)	0.009 (1.04)	0.032 (4.09)	0.016 (2.37)	0.030 (4.00)
β_1	0.392 (3.94)	0.266 (4.03)	0.035 (0.29)	0.470 (2.63)	0.241 (1.70)	-0.002 (-0.01)	0.239 (2.12)	0.134 (1.58)
β_2	0.312 (3.21)	0.405 (5.92)	0.785 (4.94)	0.306 (2.27)	0.453 (4.39)	0.820 (4.51)	0.381 (4.23)	0.656 (7.14)
\bar{R}^2	64.30%	64.10%	67.40%	62.80%	63.40%	67.40%	64.80%	67.90%
CAC 40								
α	0.025 (3.06)	0.005 (0.50)	0.026 (3.17)	0.016 (2.07)	-0.002 (-0.25)	0.024 (3.20)	0.000 (0.01)	0.021 (2.71)
β_1	0.424 (5.57)	0.268 (4.29)	0.114 (1.11)	0.515 (3.85)	0.268 (2.05)	0.088 (0.47)	0.239 (2.83)	0.183 (2.65)
β_2	0.338 (4.21)	0.506 (6.22)	0.765 (5.10)	0.308 (2.58)	0.538 (4.23)	0.805 (3.94)	0.508 (6.56)	0.677 (7.20)
\bar{R}^2	62.90%	63.20%	65.50%	61.20%	62.70%	65.40%	63.50%	66.30%
DAX								
α	0.022 (1.99)	0.003 (0.22)	0.028 (2.91)	0.012 (1.17)	-0.001 (-0.11)	0.030 (3.11)	-0.002 (-0.18)	0.020 (1.92)
β_1	0.394 (6.42)	0.101 (1.20)	0.089 (0.86)	0.409 (2.99)	-0.040 (-0.19)	-0.051 (-0.20)	0.193 (1.96)	0.218 (2.27)
β_2	0.407 (4.60)	0.654 (6.17)	0.779 (5.28)	0.440 (2.82)	0.776 (4.02)	0.919 (3.47)	0.581 (7.32)	0.659 (6.68)
\bar{R}^2	61.40%	63.20%	63.20%	59.10%	63.00%	63.10%	63.70%	64.00%
DJIA								
α	0.014 (0.92)	0.001 (0.04)	0.016 (1.16)	0.012 (0.88)	-0.004 (-0.29)	0.014 (1.06)	0.000 (0.00)	0.009 (0.64)
β_1	0.216 (1.73)	0.281 (2.57)	0.112 (1.10)	0.079 (0.36)	0.267 (3.63)	0.145 (1.50)	0.552 (2.10)	0.424 (2.42)
β_2	0.644 (3.16)	0.572 (6.46)	0.838 (4.86)	0.780 (2.62)	0.600 (6.59)	0.811 (4.54)	0.326 (1.79)	0.518 (4.31)
\bar{R}^2	66.40%	64.70%	66.30%	65.90%	64.70%	66.40%	66.90%	68.10%
Euro Stoxx 50								
α	0.031 (3.37)	0.007 (0.51)	0.026 (2.32)	0.027 (3.53)	0.004 (0.33)	0.030 (3.12)	0.010 (1.07)	0.022 (2.23)
β_1	0.249 (3.23)	0.105 (1.38)	0.015 (0.12)	0.161 (0.81)	-0.071 (-0.30)	-0.098 (-0.35)	0.312 (2.91)	0.281 (3.58)
β_2	0.495 (4.80)	0.632 (5.86)	0.886 (4.56)	0.586 (3.31)	0.774 (3.71)	0.988 (3.31)	0.427 (5.43)	0.578 (6.03)
\bar{R}^2	59.50%	59.40%	60.60%	58.10%	59.20%	60.70%	60.90%	62.30%
FTSE 100								
α	0.017 (2.44)	0.004 (0.53)	0.020 (3.32)	0.010 (1.54)	-0.001 (-0.08)	0.023 (3.99)	0.002 (0.36)	0.020 (2.83)
β_1	0.481 (5.60)	0.234 (3.63)	0.077 (0.72)	0.520 (3.85)	0.149 (1.58)	-0.053 (-0.36)	0.137 (1.22)	0.063 (0.78)
β_2	0.281 (3.19)	0.462 (6.62)	0.789 (5.06)	0.285 (2.52)	0.535 (6.68)	0.907 (5.24)	0.501 (5.59)	0.785 (7.58)
\bar{R}^2	68.00%	69.20%	71.50%	65.70%	68.70%	71.40%	69.00%	71.50%

continued

TABLE V
(Continued)

	LRE + GJR	LRE + MFIV	LRE + C-MFIV	HAR + GJR	HAR + MFIV	HAR + C-MFIV	GJR + MFIV	GJR + C-MFIV
Hang Seng								
α	0.041 (3.10)	0.026 (3.01)	0.020 (2.40)	0.016 (1.14)	0.024 (2.55)	0.019 (2.13)	0.019 (1.81)	0.015 (1.47)
β_1	0.719 (5.59)	0.087 (0.67)	0.026 (0.24)	0.933 (6.40)	0.102 (0.71)	0.053 (0.39)	− 0.069 (−2.80)	− 0.048 (−2.25)
β_2	−0.008 (−0.32)	0.427 (5.02)	0.850 (5.83)	−0.046 (−1.54)	0.424 (4.73)	0.828 (5.28)	0.580 (7.72)	1.006 (7.88)
\bar{R}^2	48.80%	56.50%	57.90%	50.30%	56.50%	57.90%	57.80%	58.70%
KOSPI								
α	0.008 (0.30)	0.005 (0.22)	0.019 (1.07)	−0.002 (−0.10)	0.005 (0.24)	0.017 (1.11)	−0.004 (−0.18)	0.010 (0.38)
β_1	0.412 (3.81)	−0.031 (−0.15)	−0.071 (−0.35)	0.494 (3.41)	0.102 (0.66)	0.082 (0.55)	0.173 (1.28)	0.141 (0.92)
β_2	0.405 (2.09)	0.641 (2.79)	0.993 (3.00)	0.382 (1.98)	0.556 (3.35)	0.843 (3.65)	0.510 (7.39)	0.785 (8.31)
\bar{R}^2	54.00%	57.90%	58.90%	52.90%	58.10%	58.90%	58.70%	59.30%
Nikkei 225								
α	0.030 (2.05)	0.030 (1.96)	0.034 (2.69)	0.019 (1.38)	0.020 (1.28)	0.037 (3.20)	0.024 (1.68)	0.027 (1.88)
β_1	0.336 (3.11)	0.241 (2.12)	−0.120 (−0.81)	0.407 (2.42)	0.306 (2.33)	−0.107 (−0.63)	0.218 (1.87)	0.106 (1.21)
β_2	0.293 (2.55)	0.319 (3.28)	0.927 (4.20)	0.286 (2.03)	0.314 (3.36)	0.894 (4.15)	0.284 (4.17)	0.673 (6.12)
\bar{R}^2	50.90%	51.20%	57.70%	50.00%	51.20%	57.60%	52.20%	58.10%
NASDAQ								
α	0.013 (0.89)	−0.003 (−0.24)	0.019 (1.72)	0.008 (0.65)	−0.007 (−0.52)	0.017 (1.70)	−0.006 (−0.60)	0.010 (0.77)
β_1	0.279 (1.79)	0.196 (1.59)	0.047 (0.30)	0.315 (1.31)	0.245 (1.71)	0.107 (0.62)	0.331 (1.85)	0.256 (2.20)
β_2	0.447 (2.63)	0.509 (4.14)	0.877 (3.88)	0.442 (2.04)	0.492 (3.92)	0.827 (3.75)	0.352 (2.68)	0.605 (5.59)
\bar{R}^2	64.20%	63.50%	66.00%	63.70%	63.50%	66.00%	65.20%	67.60%
Russell 2000								
α	0.042 (2.29)	0.022 (1.18)	0.038 (2.43)	0.025 (1.39)	0.008 (0.40)	0.034 (2.12)	0.010 (0.56)	0.036 (2.14)
β_1	0.499 (4.92)	0.443 (3.60)	0.147 (0.99)	0.707 (3.51)	0.552 (2.98)	0.134 (0.62)	0.202 (1.52)	0.089 (0.90)
β_2	0.212 (2.08)	0.286 (3.03)	0.697 (3.79)	0.129 (0.77)	0.265 (1.99)	0.733 (3.29)	0.429 (3.84)	0.732 (5.62)
\bar{R}^2	56.40%	56.40%	60.20%	54.90%	55.50%	60.10%	54.40%	60.20%
SMI								
α	0.019 (2.76)	0.011 (1.30)	0.026 (3.58)	0.007 (1.14)	0.005 (0.69)	0.021 (3.23)	0.007 (0.88)	0.023 (2.84)
β_1	0.481 (6.54)	0.202 (2.31)	0.089 (0.65)	0.680 (6.56)	0.326 (2.32)	0.188 (1.04)	0.159 (1.60)	0.109 (1.38)
β_2	0.291 (3.59)	0.489 (4.91)	0.738 (3.77)	0.191 (1.93)	0.428 (3.78)	0.669 (3.36)	0.513 (6.33)	0.710 (6.40)
\bar{R}^2	63.60%	64.60%	66.80%	63.10%	64.80%	67.00%	64.80%	67.10%

continued

TABLE V
(Continued)

	LRE + GJR	LRE + MFIV	LRE + C-MFIV	HAR + GJR	HAR + MFIV	HAR + C-MFIV	GJR + MFIV	GJR + C-MFIV
S&P 500								
α	0.019 (1.46)	0.003 (0.26)	0.016 (1.31)	0.019 (1.83)	-0.003 (-0.26)	0.012 (1.01)	0.008 (1.03)	0.013 (1.07)
β_1	0.184 (1.25)	0.319 (2.76)	0.177 (1.55)	0.044 (0.17)	0.342 (3.20)	0.230 (2.01)	0.574 (2.13)	0.454 (2.51)
β_2	0.612 (3.09)	0.492 (6.20)	0.767 (4.62)	0.738 (2.55)	0.497 (6.60)	0.732 (4.43)	0.220 (1.04)	0.430 (2.85)
\bar{R}^2	67.90%	65.20%	66.50%	67.40%	65.10%	66.70%	68.00%	69.10%
Aggregate results								
Average α	0.024	0.010	0.025	0.014	0.004	0.024	0.006	0.020
Average β_1	0.390	0.209	0.057	0.441	0.215	0.055	0.251	0.185
Average β_2	0.364	0.492	0.822	0.371	0.512	0.829	0.432	0.678
Average \bar{R}^2	60.65%	61.47%	63.73%	59.63%	61.26%	63.74%	62.30%	64.62%

This table presents results from estimating regressions of realized volatility on competing volatility forecasts for the 22-day forecast horizon. Each column reports the results for a different specification. For example, the column headed "HAR + GJR" contains the results from estimating a two-variable regression of the 22-day realized volatility on the forecasts from the HAR and GJR-GARCH(1,1) model, respectively. Rows report regression coefficients and their associated *t*-statistics in parentheses. The row labeled " \bar{R}^2 " contains the adjusted \bar{R}^2 coefficient of each regression. α is the intercept, while β_1 (β_2) is the slope coefficient of the left (right) hand-side model in a column. The last three rows of the table contain the mean values of the intercepts and slopes across markets for each forecasting model. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model. All regressions are estimated using Newey–West (1987) heteroskedasticity and autocorrelation corrected standard errors with 22 lags. Significant coefficients at the 5% level are highlighted in bold.

implied volatility, indicating that these models have a stronger effect on future volatility compared to the remaining models. This finding is also clear from the bottom panel of Table V, which contains the cross-sectional average of the regression coefficients.

Overall, our results indicate the superiority of implied volatility forecasts in the monthly horizon. The superior predictive ability of the C-MFIV at the monthly horizon is likely a consequence of its construction from options with maturity of 1 month. This is in line with Taylor, Yadav, and Zhang (2010) who show that option implied volatility forecasts are more informative than historical volatility when the forecast horizon matches the maturity date of the underlying options.

In summary, the results from univariate and encompassing regressions suggest that all forecasts are informative about future volatility across all markets, even though they generally produce biased forecasts. The forecast horizon determines the most informative model with the HAR and the C-MFIV being clearly superior at the daily and monthly horizons, respectively.

4. OUT-OF-SAMPLE FORECAST EVALUATION

So far, our analysis has been devoted to regressions that explore the information content of volatility forecasts in-sample. In this section, we assess the out-of-sample forecasting accuracy of the competing models. To this end, we consider two of the most frequently used loss functions: the root mean squared error (RMSE) and the quasi-likelihood (QLIKE). These are defined as

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{t=1}^N \left(RV_{t:t+k} - \hat{F}_{t:t+k} \right)^2} \quad (8)$$

$$\text{QLIKE} = \frac{1}{N} \sum_{t=1}^N \left[\log(\hat{F}_{t:t+k}) + \frac{RV_{t:t+k}}{\hat{F}_{t:t+k}} \right], \quad (9)$$

where N is the number of out-of-sample volatility forecasts. RMSE is a popular choice in empirical applications, while QLIKE is known to be robust to noise in the volatility proxy (Patton, 2011).

To formally investigate whether the differences between the forecast errors of competing volatility models are statistically significant, we employ the Diebold–Mariano (DM) predictive accuracy test (Diebold & Mariano, 1995). In implementing the DM test, we account for possible autocorrelation in overlapping multi-period forecasts using the covariance estimator of Newey–West (1987).

Tables VI and VII report our out-of-sample results (results for the weekly forecast horizon are reported in Table SAI of the appendix to save space). Rows correspond to markets and columns correspond to forecasting models. Panel A (Panel B) reports the results for the RMSE (QLIKE) loss function. The model with the lowest forecast errors for each loss function is highlighted in bold. Models with significantly higher forecast errors than the best model according to the DM test at the 5% (10%) are marked with two (one) asterisks. The results generally indicate model rankings that are similar to those obtained previously by the in-sample univariate and encompassing regressions. Starting with Table VI, we observe that HAR is clearly the best performing model at the daily horizon. All models are inferior to the HAR model, according to both performance criteria. The model with the worst overall performance is the MFIV, which yields the highest forecast error in almost all markets,

TABLE VI
Out-of-Sample Forecasting Performance: 1-Day Horizon

	Panel A: RMSE					Panel B: QLIKE				
	LRE	MFIV	C-MFIV	GJR	HAR	LRE	MFIV	C-MFIV	GJR	HAR
AEX	0.301*	0.648*	0.325*	0.492*	0.275	-3.762*	-3.667*	-3.758*	-3.734*	-3.771
CAC 40	0.355*	0.552*	0.368*	0.465*	0.322	-3.675*	-3.614*	-3.676*	-3.655*	-3.685
DAX	0.375*	0.560*	0.407*	0.423*	0.345	-3.608*	-3.557*	-3.608*	-3.600*	-3.618
DJIA	0.431*	0.475*	0.424*	0.386*	0.375	-3.787*	-3.766*	-3.799*	-3.805*	-3.808
Euro Stoxx 50	0.451*	0.616*	0.455*	0.472*	0.407	-3.594*	-3.541*	-3.594*	-3.590*	-3.605
FTSE 100	0.273*	0.583*	0.270*	0.462*	0.248	-3.960*	-3.849*	-3.963*	-3.917*	-3.968
Hang Seng	0.356*	0.779*	0.319	1.212*	0.306	-3.825*	-3.704*	-3.835*	-3.723*	-3.840
KOSPI	0.348*	0.738*	0.389*	0.586*	0.323	-3.700*	-3.590*	-3.694*	-3.638*	-3.707
Nikkei 225	0.311*	0.838*	0.318*	0.709*	0.282	-3.775*	-3.629*	-3.773*	-3.685*	-3.784
NASDAQ	0.319*	0.685*	0.331*	0.555*	0.288	-3.808*	-3.685*	-3.806*	-3.743*	-3.818
Russell 2000	0.479*	0.901*	0.489*	0.796*	0.428	-3.504*	-3.387*	-3.499*	-3.446*	-3.517
SMI	0.262*	0.500*	0.275*	0.382*	0.240	-3.898*	-3.823*	-3.899*	-3.870*	-3.904
S&P 500	0.408*	0.533*	0.417*	0.416*	0.360	-3.780*	-3.733*	-3.786*	-3.787*	-3.798

This table presents out-of-sample forecasting errors for the indices under consideration. Columns correspond to forecasting models. We report the results for the root mean squared error (RMSE) and the quasi-likelihood (QLIKE). The forecast horizon is 1 trading day. Out-of-sample forecasts for GJR and HAR are obtained using a rolling window of 800 observations. In order to facilitate the presentation of our results, we multiply RMSE by 100. The model with the lowest forecast errors is highlighted in bold. One (two) asterisk shows that the indicated model is inferior to the best model (i.e., has significantly higher forecast errors) using a Diebold–Mariano test at the 10% (5%) significance level. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model.

TABLE VII
Out-of-Sample Forecasting Performance: 22-Day Horizon

	Panel A: RMSE					Panel B: QLIKE				
	LRE	MFIV	C-MFIV	GJR	HAR	LRE	MFIV	C-MFIV	GJR	HAR
AEX	1.612*	3.042*	1.365	2.411**	1.492	-2.152*	-2.084*	-2.161	-2.127**	-2.155
CAC 40	1.735*	2.495*	1.475	2.283**	1.632	-2.068*	-2.028**	-2.079	-2.049**	-2.071*
DAX	1.913	2.518*	1.611	2.076**	1.810	-1.996*	-1.969**	-2.008	-1.991**	-2.000*
DJIA	1.862*	2.018*	1.613	1.727	1.860*	-2.172*	-2.161**	-2.184	-2.179	-2.176*
Euro Stoxx 50	2.117*	2.698*	1.731	2.251**	1.963	-1.974*	-1.948**	-1.988	-1.974**	-1.979*
FTSE 100	1.283*	2.663*	1.088	2.165**	1.255*	-2.354*	-2.268**	-2.364	-2.311**	-2.352*
Hang Seng	1.527	3.558*	1.250	6.058*	1.447*	-2.236*	-2.127**	-2.241	-2.107**	-2.235**
KOSPI	2.016	3.368*	1.617	2.547**	1.889	-2.076	-2.002**	-2.086	-2.033**	-2.080
Nikkei 225	1.612*	3.878*	1.311	3.187**	1.501	-2.158*	-2.048**	-2.170	-2.083**	-2.166
NASDAQ	1.577**	3.151*	1.336	2.490**	1.480	-2.196*	-2.103**	-2.205	-2.147**	-2.202
Russell 2000	2.149*	4.052*	1.896	3.582**	2.031	-1.862*	-1.784**	-1.872	-1.820**	-1.869
SWI	1.385*	2.317*	1.192	1.928**	1.272	-2.298*	-2.245**	-2.308	-2.269**	-2.300*
S&P 500	1.875	2.369*	1.649	1.889	1.810	-2.163**	-2.134**	-2.174	-2.166	-2.167

This table presents out-of-sample forecasting errors for the indices under consideration. Columns correspond to forecasting models. We report the results for the root mean squared error (RMSE) and the quasi-likelihood (QLIKE). The forecast horizon is 22 trading days. Out-of-sample forecasts for GJR and HAR are obtained using a rolling window of 800 observations. In order to facilitate the presentation of our results, we multiply RMSE by 100. The model with the lowest forecast errors is highlighted in bold. One (two) asterisk shows that the indicated model is inferior to the best model (i.e., has significantly higher forecast errors) using a Diebold–Mariano test at the 10% (5%) significance level. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1, 1) model (GJR), and the Heterogeneous Autoregressive (HAR) model.

followed by the GJR-GARCH. For instance, in the case of the NASDAQ index, HAR has a RMSE value of 0.29, while at the same time the RMSE of MFIV is more than double as high and equal to 0.69.

Table VII shows that at the monthly horizon, C-MFIV exhibits the lowest forecast errors among almost all competing models for all markets and loss functions. The loss differences of most models against the C-MFIV are statistically significant at the 5% level. The only exception is the HAR model. Nevertheless, HAR still has larger forecast errors than C-MFIV for the majority of the markets. HAR also appears to outperform the raw implied volatility forecasts in all markets according to both criteria. The GJR-GARCH model underperforms most of the models except for the MFIV. The latter results shows that correcting implied volatility for the volatility risk premium is of high importance when forecasting volatility.

The results for the weekly horizon suggest that C-MFIV produces the lowest forecast errors for the majority of the markets (Table SAII in appendix). In particular, according to the RMSE (QLIKE), C-MFIV yields the best performance in 10 (8) out of 13 markets. Nevertheless, the differences in forecast errors between C-MFIV and HAR are insignificant in all but 2 cases. Again, MFIV and GJR-GARCH result in the worst performance. For example, we find that the RMSE value for C-MFIV is 0.46 compared to a 0.98 for GJR-GARCH in the FTSE 100 index.

We further investigate whether the accuracy of volatility forecasting changes in periods of market turmoil. In this context, we perform a separate out-of-sample forecast evaluation over three different sub-periods: (i) the pre-crisis period (January 2003–July 2008), (ii) the crisis period (August 2008–December 2009), and (iii) the after-crisis period (January 2010–August 2012). We define the crisis period to be the shortly after the Lehman collapse until the end of 2009, similar to Genre, Kenny, Meyler, and Timmermann (2013).⁴ Tables VIII and IX present forecast losses for each market and forecasting model over the three sub-periods considered (weekly results are in Table SAIII of the appendix to conserve space).

This analysis leads to three major conclusions. First, the forecasting ability of all models significantly deteriorates over the crisis period according to both loss functions. Also for most markets, the forecast errors in the post-crisis period are higher than the errors in the pre-crisis period. One of the possible explanations for this is the anomalous situation around European sovereign debt in the post-crisis period. These results indicate that academics and practitioners should be more cautious when forecasting volatility in periods of turmoil. Second, as in the full sample, HAR and C-MFIV are superior to the remaining models during the crisis period, at the 1- and 22-day horizons, respectively. Although model rankings do not change for these horizons, the statistical differences among models are less significant. Third, the ranking of the models at the weekly horizon substantially changes in the crisis period and the evidence regarding the best model is fairly mixed. However, the loss differences between the best models are almost never statistically significant.

5. ECONOMIC VALUE OF VOLATILITY FORECASTS

We now assess the economic value of the five volatility forecasting methodologies under study from the perspective of an international portfolio investor. We consider a market of 11 assets, which correspond to 11 out of the 13 international indices we consider. We exclude KOSPI and Russell 2000 from this analysis due to the relatively smaller period that their data span. We also do not account for currency risk, as this goes beyond the aims of this study. We

⁴We have experimented with different periods such as 2007–2009 and obtained almost identical results. These results are available upon request.

TABLE VIII
Forecast Losses for 1-Day Horizon: Calm Versus Turmoil Periods

Panel A: RMSE						Panel B: QLIKE				
	LRE	MFIV	C-MFIV	GJR	HAR	LRE	MFIV	C-MFIV	GJR	HAR
I. Pre-crisis period										
AEX	0.245*	0.578**	0.280**	0.382**	0.225	-3.928**	-3.822**	-3.925**	-3.904**	-3.939
CAC 40	0.253*	0.478**	0.271**	0.342**	0.232	-3.897**	-3.822**	-3.898**	-3.877**	-3.907
DAX	0.306**	0.485**	0.320**	0.343**	0.278	-3.759**	-3.704**	-3.761**	-3.753**	-3.770
DJIA	0.281**	0.332**	0.259**	0.245	0.242	-3.957**	-3.938**	-3.970**	-3.972**	-3.975
Euro Stoxx 50	0.327**	0.495**	0.346**	0.340**	0.297	-3.788**	-3.736**	-3.790**	-3.791**	-3.802
FTSE 100	0.223*	0.468**	0.232*	0.320**	0.205	-4.151**	-4.035**	-4.153**	-4.113**	-4.159
Hang Seng	0.327**	0.706**	0.289	0.569**	0.277	-3.864**	-3.761**	-3.876**	-3.817**	-3.881
KOSPI	0.278**	0.642**	0.297**	0.508**	0.250	-3.714**	-3.615**	-3.707**	-3.662**	-3.721
Nikkei 225	0.257**	0.632**	0.255**	0.535**	0.226	-3.800**	-3.691**	-3.798**	-3.734**	-3.809
NASDAQ	0.227**	0.563**	0.244**	0.418**	0.201	-3.927**	-3.813**	-3.929**	-3.865**	-3.939
Russell 2000	0.379*	0.642**	0.461**	0.431**	0.349	-3.529**	-3.457**	-3.511**	-3.513**	-3.541
SMI	0.200*	0.405**	0.209**	0.324**	0.184	-4.016**	-3.944**	-4.017**	-3.985**	-4.022
S&P 500	0.265**	0.377**	0.254**	0.247**	0.231	-3.962**	-3.922**	-3.974**	-3.971**	-3.979
II. Crisis period										
AEX	0.495**	0.984**	0.490	0.842**	0.451	-3.221**	-3.134**	-3.220**	-3.199**	-3.228
CAC 40	0.611*	0.764**	0.580	0.677**	0.548	-3.189**	-3.146**	-3.185**	-3.178**	-3.195
DAX	0.597	0.800**	0.648*	0.606	0.563	-3.133**	-3.090**	-3.123**	-3.126**	-3.137
DJIA	0.718*	0.770**	0.764**	0.651	0.645	-3.200**	-3.178**	-3.204**	-3.214**	-3.220
Euro Stoxx 50	0.836*	0.946**	0.783	0.770	0.745	-3.145**	-3.091**	-3.139**	-3.136**	-3.151
FTSE 100	0.449*	0.884**	0.423	0.814**	0.408	-3.390**	-3.303**	-3.395**	-3.359**	-3.400
Hang Seng	0.649	1.212**	0.613	3.072**	0.584	-3.354**	-3.221**	-3.362	-3.144**	-3.366
KOSPI	0.513	0.990**	0.602*	0.660**	0.482	-3.312**	-3.221**	-3.304**	-3.282**	-3.318
Nikkei 225	0.487*	1.308**	0.516**	1.084**	0.450	-3.367**	-3.210**	-3.365**	-3.293**	-3.374
NASDAQ	0.528**	0.947**	0.504	0.817**	0.470	-3.335**	-3.226**	-3.331**	-3.287**	-3.342
Russell 2000	0.665**	1.115**	0.599	1.079**	0.584	-3.140**	-3.047**	-3.147**	-3.091**	-3.153
SMI	0.423*	0.788**	0.436*	0.591**	0.388	-3.398**	-3.330**	-3.395**	-3.384**	-3.403
S&P 500	0.681*	0.838**	0.731**	0.739**	0.618	-3.188**	-3.135**	-3.181**	-3.181**	-3.200

continued

TABLE VIII
(Continued)

	Panel A: RMSE					Panel B: QLIKE				
	LRE	MFIV	C-MFIV	GJR	HAR	LRE	MFIV	C-MFIV	GJR	HAR
III. Post-crisis period										
AEX	0.268**	0.557**	0.301**	0.434**	0.245	-3.717**	-3.639**	-3.712**	-3.682**	-3.725
CAC 40	0.343**	0.553**	0.387**	0.530**	0.316	-3.503**	-3.457**	-3.503**	-3.477**	-3.513
DAX	0.352**	0.549**	0.400**	0.451**	0.317	-3.557**	-3.508**	-3.557**	-3.542**	-3.568
DJIA	0.472**	0.509**	0.437**	0.421	0.397	-3.760**	-3.736**	-3.775**	-3.786	-3.790
Euro Stoxx 50	0.382**	0.619**	0.417**	0.495**	0.352	-3.445**	-3.391**	-3.444**	-3.431**	-3.454
FTSE 100	0.239**	0.585**	0.235*	0.442**	0.214	-3.889**	-3.776**	-3.893**	-3.830**	-3.898
Hang Seng	0.261**	0.724**	0.220	0.975**	0.215	-3.903**	-3.756**	-3.914	-3.742**	-3.918
KOSPI	0.291	0.658**	0.309*	0.604**	0.270	-3.887**	-3.758**	-3.880*	-3.798**	-3.894
Nikkei 225	0.289	0.880**	0.295	0.758*	0.269	-3.928**	-3.718**	-3.928**	-3.786**	-3.939
NASDAQ	0.297	0.695**	0.333**	0.574**	0.279	-3.866**	-3.722**	-3.861**	-3.789**	-3.876
Russell 2000	0.406**	0.890**	0.439*	0.766**	0.366	-3.674**	-3.522**	-3.669**	-3.590**	-3.687
SMI	0.262*	0.476**	0.281**	0.349*	0.238	-3.923**	-3.841**	-3.926**	-3.895**	-3.931
S&P 500	0.446**	0.583**	0.446**	0.445**	0.381	-3.733**	-3.676**	-3.737**	-3.743**	-3.758

This table presents out-of-sample forecast losses for the indices considered over three sub-samples. The first sub-sample covers the period from the beginning of our out-of-sample period until July 31, 2008. The second sub-sample covers the crisis period from August 1, 2008 to December 31, 2009 and finally the last sub-sample covers the after crisis period from January 1, 2010 to October 26, 2012. Columns correspond to forecasting models. We report the results for the root mean squared error (RMSE) and the quasi-likelihood (QLIKE). The forecast horizon is 1 trading day. Out-of-sample forecasts for GJR and HAR models are obtained using a rolling window of 800 observations. In order to facilitate the presentation of our results, we multiply RMSE by 100. The model with the lowest forecast errors is highlighted in bold. One (two) asterisk shows that the indicated model is inferior to the best model (i.e., has significantly higher forecast errors) using a Diebold–Mariano test at the 10% (5%) significance level. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model.

TABLE IX
Losses for 22-Day Forecasts: Calm Versus Turmoil Periods

Panel A: RMSE						Panel B: QLIKE				
	LRE	MFIV	C-MFIV	GJR	HAR	LRE	MFIV	C-MFIV	GJR	HAR
I. Pre-crisis period										
AEX	1.303**	2.664**	1.056	2.013**	1.178**	-2.333*	-2.251**	-2.341	-2.305**	-2.333**
CAC 40	1.186**	2.111**	0.954	1.667**	1.161**	-2.305**	-2.249**	-2.314	-2.280**	-2.302**
DAX	1.429**	2.084**	1.097	1.646**	1.299**	-2.161**	-2.129**	-2.174	-2.157**	-2.164**
DJIA	1.019**	1.293**	0.795	1.011**	1.046**	-2.371**	-2.356**	-2.382	-2.370**	-2.368**
Euro Stoxx 50	1.498**	2.097**	1.154	1.583**	1.355**	-2.180**	-2.152**	-2.194	-2.182**	-2.184**
FTSE 100	0.961**	2.021**	0.768	1.454**	1.028**	-2.557**	-2.465**	-2.566	-2.514**	-2.548**
Hang Seng	1.132	3.042**	0.960	2.465**	1.139*	-2.289	-2.196*	-2.292	-2.239**	-2.288
KOSPI	1.103	2.819**	1.104	1.884**	1.017	-2.117	-2.041**	-2.116	-2.078**	-2.120
Nikkei 225	0.934	2.787**	0.871	2.382**	0.936	-2.207	-2.120**	-2.211	-2.144**	-2.208
NASDAQ	0.951**	2.492**	0.796	1.761**	0.900**	-2.347**	-2.249**	-2.353	-2.295**	-2.348*
Russell 2000	1.930*	2.611**	1.544	1.492	1.693	-1.877	-1.859**	-1.898	-1.899	-1.885
SMI	0.961**	1.810**	0.746	1.644**	0.919**	-2.438**	-2.381**	-2.446	-2.399**	-2.436**
S&P 500	1.063**	1.541**	0.814	1.071**	1.069**	-2.374**	-2.343**	-2.385	-2.371**	-2.371**
II. Crisis period										
AEX	2.632	4.643**	2.291	3.642*	2.552	-1.590	-1.534**	-1.602	-1.585	-1.585
CAC 40	2.917	3.522**	2.525	3.140	2.836	-1.557	-1.538	-1.566	-1.559	-1.551
DAX	3.269	3.712	2.830	2.962	3.292	-1.496	-1.484	-1.503	-1.497	-1.483
DJIA	3.664	3.620	3.283	3.155	3.783	-1.555	-1.541	-1.558	-1.568	-1.549
Euro Stoxx 50	3.870	4.158	3.181	3.508	3.755	-1.491	-1.475	-1.506	-1.504	-1.478
FTSE 100	2.229	4.146**	1.970	3.630	2.123	-1.765	-1.703**	-1.776	-1.751	-1.764
Hang Seng	3.731	6.041**	2.874	15.104**	3.308	-1.677	-1.593	-1.701	-1.487**	-1.686
KOSPI	3.565	4.668*	2.627	2.750*	3.306	-1.664*	-1.621*	-1.685	-1.676	-1.665
Nikkei 225	3.016	6.201**	2.370	4.209*	2.754	-1.727*	-1.618**	-1.749	-1.707	-1.730
NASDAQ	2.763*	4.486**	2.255	3.479	2.616	-1.682*	-1.616**	-1.697	-1.673	-1.684
Russell 2000	2.897	5.057**	2.560	4.594	2.873	-1.503	-1.440**	-1.512	-1.478	-1.501
SMI	2.194	3.670*	2.065	2.722	2.060	-1.766	-1.727**	-1.779	-1.769*	-1.768
S&P 500	3.639	4.056	3.266	3.430	3.578	-1.523	-1.503**	-1.533	-1.542	-1.523

continued

TABLE IX
(Continued)

	Panel A: RMSE					Panel B: QLIKE				
	LRE	MFIV	C-MFIV	GJR	HAR	LRE	MFIV	C-MFIV	GJR	HAR
III. Post-crisis period										
AEX	1.450**	2.667**	1.250	2.316**	1.282	-2.098	-2.046**	-2.106	-2.068**	-2.109
CAC 40	1.796**	2.528**	1.553	2.718**	1.549	-1.888*	-1.867**	-1.901	-1.867**	-1.903
DAX	1.794**	2.533**	1.593	2.257**	1.586	-1.935	-1.910**	-1.946	-1.926**	-1.948
DJIA	1.728**	2.001**	1.470	1.768**	1.558	-2.110**	-2.110**	-2.131	-2.129	-2.133
Euro Stoxx 50	1.884**	2.788**	1.622	2.510**	1.623	-1.825*	-1.800**	-1.838	-1.816**	-1.840
FTSE 100	1.143**	2.746**	0.958	2.268**	1.028	-2.277**	-2.189**	-2.288	-2.219**	-2.286
Hang Seng	0.701	3.312**	0.746	5.340**	0.822	-2.322	-2.177**	-2.322	-2.078**	-2.318
KOSPI	1.383	2.952**	1.240	2.872**	1.353	-2.251	-2.166**	-2.265	-2.178**	-2.260
Nikkei 225	1.602	4.107**	1.249	3.830**	1.469	-2.279	-2.127**	-2.302	-2.155**	-2.302
NASDAQ	1.497**	3.208**	1.352	2.794**	1.376	-2.231	-2.131**	-2.240	-2.167**	-2.246
Russell 2000	1.774	4.027**	1.634	3.679**	1.625	-2.033	-1.921**	-2.039	-1.955**	-2.044
SMI	1.524*	2.289**	1.272	1.941**	1.324	-2.302**	-2.247**	-2.315	-2.276**	-2.313
S&P 500	1.752	2.485**	1.598	1.985**	1.569	-2.092	-2.063**	-2.103	-2.100**	-2.113

This table presents out-of-sample forecast losses for the indices considered over three sub-samples. The first sub-sample covers the period from the beginning of our out-of-sample period until July 31, 2008. The second sub-sample covers the period of the crisis from August 1, 2008 to December 31, 2009 and finally the last sub-sample covers the after crisis period from January 1, 2010 to October 26, 2012. Columns correspond to forecasting models. We report the results for the root mean squared error (RMSE) and the quasi-likelihood (QLIKE). The forecast horizon is 22 trading days. Out-of-sample forecasts for GJR and HAR models are obtained using a rolling window of 800 observations. In order to facilitate the presentation of our results, we multiply RMSE by 100. The model with the lowest forecast errors is highlighted in bold. One (two) asterisk shows that the indicated model is inferior to the best model (i.e., has significantly higher forecast errors) using a Diebold–Mariano test at the 10% (5%) significance level. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1, 1) model (GJR), and the Heterogeneous Autoregressive (HAR) model.

assume weekly and monthly rebalancing instead of daily to avoid any significant bias caused from the non-synchronous trading times of the markets considered. So, let r_t be the vector of asset returns in period t , where t could be week or month. In each period, the investor adopts a “volatility-timing” strategy with portfolio weights $w_t = (w_{1t}, w_{2t}, \dots, w_{Nt})'$ defined as

$$w_{it} = \frac{(1/\sigma_{it}^2)}{\sum_{i=1}^N (1/\sigma_{it}^2)}, \quad i = 1, \dots, N, \quad (10)$$

where σ_{it}^2 is the conditional variance of the return on the i index at time $t + 1$, given the information available at time t . This portfolio strategy is proposed by Kirby and Ostdiek (2012) and is equivalent to the portfolio of minimum variance under the assumption of zero covariance between assets. We adopt this strategy as the weights are only a function of the variances and are not subject to other moments, such as means and covariances. This allows us to evaluate the different volatility forecasting models in a portfolio choice setting, protecting our conclusions from biases due to errors in estimates of means and covariances. Kirby and Ostdiek also show that the above volatility-timing strategy offers several practical advantages. For instance, it requires no optimization, does not yield short positions, leads to low transaction costs, and appears to perform at least as good as several alternative strategies that involve the estimation of means and/or covariances.

In practice, σ_{it}^2 's are unknown and need to be estimated. More accurate estimates lead to smaller levels of portfolio risk (for the effects of estimation errors in the portfolio choice process, see Michaud, 1989). By using each of the five volatility models under study to estimate σ_{it}^2 's in Equation (10), we can test how efficient a model is in improving portfolio performance. Through this process, we end up with five portfolios with each one corresponding to a different forecasting model. We evaluate the out-of-sample performance of each portfolio by following a standard rolling window approach from the portfolio choice literature (e.g., see DeMiguel, Garlappi, & Uppal, 2009). In particular, we first estimate the portfolio weights \hat{w}_t^s for each period t and forecasting model s . We then use the weights for each strategy to compute the respective portfolio return at time $t + 1$:

$$r_{t+1}^s = (\hat{w}_t^s)' r_{t+1} \quad (11)$$

The outcome of this procedure is a time-series of out-of-sample portfolio returns for each forecasting methodology. Using these series, we compute the following statistics that allow us to assess the economic significance of each forecasting method:

- Out-of-sample mean: $\hat{\mu}^s = \frac{1}{M} \sum_{t=1}^M r_t^s$
- Out-of-sample variance: $(\hat{\sigma}^s)^2 = \frac{1}{M-1} \sum_{t=1}^M (r_t^s - \hat{\mu}^s)^2$
- Out-of-sample mean-to-standard deviation ratio: $\hat{\theta}^s = \frac{\hat{\mu}^s}{\hat{\sigma}^s}$
- Average portfolio turnover: $\hat{\tau}^s = \frac{1}{M-1} \sum_{t=1}^{M-1} \|\hat{w}_{t+1}^s - \hat{w}_t^s\|_1$

In the above, M denotes the total number of out-of-sample returns, \hat{w}_{t+}^S is the vector of portfolio weights at time $t + 1$ before rebalancing to \hat{w}_{t+1}^S , and $\|\cdot\|_1$ is the 1-norm. The average portfolio turnover is important as a measure of the stability of the portfolio and of relevant transaction costs (see Kourtis, 2014).

Table X presents the results of this analysis for the cases of weekly and monthly rebalancing. For weekly (monthly) rebalancing the covariance matrix is estimated using the respective 5-day (22-day) ahead volatility forecasts. We compare the performance of the five estimated portfolios with that of the equally weighted portfolio (1/N). This is a typical benchmark in the portfolio choice literature because of its superiority over many sample-based portfolios (DeMiguel, Garlappi, & Uppal, 2009; Kourtis, 2015). The parentheses next to the portfolio variance give the P -values from testing the hypothesis of no-difference between the variance of the portfolio and that of the 1/N strategy, respectively. To derive the P -values, we use the non-parametric circular bootstrap method by Ledoit and Wolf (2011), assuming an average block size of 10 and 10,000 trials.

In the case of weekly rebalancing (Panel A), we find that only the forecasts based on implied volatility offer significant reduction in portfolio risk over 1/N. The portfolios computed using the MFIV and C-MFIV methodologies offer the lowest out-of-sample variance, that is, 0.0255. The variance for 1/N is 0.0268 with the difference to the implied-volatility-based portfolio being significant at the 1% level. The remaining data-driven portfolios also result to lower variance than 1/N; however, the difference is not statistically

TABLE X
Portfolio Performance

	$\hat{\mu}^s$	$(\hat{\sigma}^s)^2$	(P -Value)	$\hat{\theta}^s$	$\hat{\tau}^s$
1/N	0.0316	0.0268	–	0.1934	0.0095
Panel A: Weekly rebalancing					
LRE	0.0062	0.0260	(0.15)	0.0386	0.3489
MFIV	0.0293	0.0255***	(0.00)	0.1834	0.1994
C-MFIV	0.0265	0.0255***	(0.00)	0.1659	0.2519
GJR	0.0137	0.0259	(0.22)	0.0852	0.3218
HAR	0.0124	0.0259	(0.16)	0.0770	0.2917
Panel B: Monthly rebalancing					
LRE	0.0367	0.0248	(0.86)	0.2332	0.1967
MFIV	0.0356	0.0248	(0.71)	0.2260	0.1326
C-MFIV	0.0349	0.0237**	(0.04)	0.2266	0.1448
GJR	0.0355	0.0238	(0.29)	0.2297	0.2006
HAR	0.0380	0.0243	(0.70)	0.2437	0.1705

This table summarizes the performance of the minimum variance portfolios constructed using volatility estimates from the five forecasting methodologies under study. Portfolios are computed assuming a diagonal covariance matrix while short sales are not allowed. For Panel A, 5-day ahead volatility forecasts are used and portfolio rebalancing is performed weekly. For Panel B, 22-day ahead volatility forecasts are used and portfolio rebalancing is performed monthly. The first row reports the performance for the equally weighted portfolio (1/N). $\hat{\mu}^s$, $\hat{\sigma}^s$, and $\hat{\theta}^s$ denote the annualized out-of-sample portfolio mean return, variance, and mean-to-standard deviation ratio, respectively. $\hat{\tau}^s$ is the average turnover. The P -value next to the variance corresponds to the test of the hypothesis that the portfolio variance is equal to the variance of the 1/N portfolio. ** (***) indicate rejection of the above hypothesis at the 5% (1%) significance level. For this test, the non-parametric methodology of Ledoit and Wolf (2011) is adopted. Forecasting is based on the lagged realized volatility from 5-minute returns (LRE), the model-free implied volatility (MFIV), the MFIV adjusted for the volatility risk premium (C-MFIV), the GJR-GARCH(1,1) model (GJR), and the Heterogeneous Autoregressive (HAR) model.

significant. The two portfolios that use forward-looking information also outperform the history-based portfolios in terms of mean return. As a result, their mean-to-standard deviation ratio is higher. This finding is consistent with the results of Kempf, Korn, and Saßning (2014) who show that the use of implied moments is of significant merit for portfolio optimization. Between the two portfolios, MFIV offers slightly higher mean. Finally, both portfolios offer relatively low levels of turnover making them attractive under transaction costs.

When rebalancing is performed monthly, only the C-MFIV-based portfolio offers significantly lower risk than $1/N$, namely, 0.0237 versus 0.0268, respectively. The corrected model-free implied volatility model offers the lowest variance among all forecasting models, including the raw implied volatility (MFIV). A similar result is obtained by DeMiguel, Plyakha, Uppal, and Vilkov (2013) who find that correcting the implied volatility forecast for the risk-premium can enhance out-of-sample portfolio performance. As in the case of weekly rebalancing, portfolios that utilize implied volatility are more stable in terms of portfolio turnover. However, the HAR-based portfolio leads to a higher average return in this case.

6. ROBUSTNESS CHECKS

We conduct several additional tests to investigate the robustness of our findings. We begin by analyzing whether our main findings persist if we predict logarithmic volatility rather than the level of volatility. We then analyze our results with respect to other intraday volatility estimators, such as the realized kernel of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008). Next, we show that our findings are robust to the choice of the rolling sample used to obtain forecasts from the HAR and GARCH models. Finally, we check whether our core findings persist when volatility spillovers from the U.S. market are taken into consideration. The corresponding results are included in the appendix of this work. We find that for all alternatives our main conclusions generally remain the same.

6.1. Forecasting Logarithmic Volatility

Our analysis has focused so far on forecasting raw realized volatility. This is because volatility is the key input in option pricing and modern portfolio theories. Nonetheless, one may argue that volatility is heavily skewed and, therefore, the residuals from univariate and encompassing regressions violate the normal distribution assumption of the ordinary least squares (OLS) estimation. As a robustness check, we repeat our in-sample and out-of-sample model evaluation, replacing volatility by its natural logarithm. Tables SBI–BIX of the appendix present these results. Consistent with our main findings, we see that the results allow similar conclusions to those that assume raw realized volatility.

6.2. Robustness to Microstructure Noise

According to the theory of quadratic variation, realized variance is a consistent estimator of the true unobserved variance (i.e., “integrated variance”) in the absence of microstructure noise. However, numerous studies have shown that realized variance is susceptible to microstructure effects (Bandi & Russell, 2006; Hansen & Lunde, 2006; Lee & Mykland, 2012; Zhang, Mykland, & Ait-Sahalia, 2005). More specifically, in the presence of microstructure noise, realized variance fails to converge to the actual unobserved variance due to accumulation of error. To explore the robustness of our findings to microstructure

noise, we repeat our analysis using the realized kernel as a more robust proxy for the latent volatility in the presence of market frictions (for practical details on realized kernels the reader can refer to Barndorff-Nielsen, Hansen, Lunde, & Shephard, 2009). Our results are practically unaffected by this choice (see Tables SCI–CIX of the appendix).

6.3. Alternative Estimation Periods for the Forecasting Models

Our out-of-sample analysis rests on a rolling window of 800 observations. As this choice may seem arbitrary, we also consider windows of 1,000 and 1,200 observations. The results reported in Tables SDI–DIX of the appendix show that changing the width of the rolling window has virtually no impact on our main conclusions.

6.4. Controlling for Spillovers from the U.S. Market

Several papers document volatility spillovers from the United States to other international equity markets (e.g., Theodossiou & Lee, 1993; Ng, 2000; Baele, 2005). This means that the ability of the various models to predict future realized volatility may be influenced or even vanish once the effect from the U.S. market volatility is considered. To investigate this possibility, we re-estimate all univariate and encompassing regressions using the lagged realized volatility of the S&P 500 as an additional regressor for the indices of all countries except the United States. To avoid issues associated with asynchronous trading times across global stock markets (see Hamao, Masulis, & Ng, 1990), we limit our analysis to the weekly and monthly forecast horizon.

The results from Mincer–Zarnowitz regressions in Tables SEI and SEII of the appendix allow two conclusions. First, the coefficient estimates of the various volatility forecasts remain strongly significant in all markets. Second, most coefficients of the lagged U.S. volatility are significant showing that lagged U.S. volatility provides additional information to that contained in the volatility forecasts of the domestic equity market. However, as shown in the row labeled “ $\Delta \bar{R}^2$ ” the increase in the adjusted- R^2 is relatively low in most cases. For example, at the monthly forecast horizon, the \bar{R}^2 improvement ranges between 0.1% and 10%.

Turning to the results from the encompassing regressions presented in Tables SEIII and SEIV, we see that both at the weekly and monthly forecast horizons, most forecasts are still informative, while lagged U.S. volatility enters again with a highly significant coefficient. Similar to the two-variable regressions above, the improvement in the explanatory power of the regressions augmented with lagged realized U.S. volatility is relatively modest in most cases. Collectively, the above results highlight the importance of spillovers for volatility forecasting; however, they do not alter our core findings about the forecasting ability of the different models under study.

6.5. Alternative Estimation Periods for the Variance Risk Premium

One could possibly argue that the choice of a period of just under 1 year for the estimation of the VPR is not theoretically justified. Our choice is driven by the desire to strike a balance between estimation accuracy by using a sufficiently large sample for the VPR calculation and the use of an as recent as possible information set. To assess the robustness of our results to this choice, we experimented with alternative durations of 18 months (378 trading days) and 24 months (504 trading days) and found that our evidence is literally unaffected. These results are presented in Tables SFI–FVI of the appendix.

7. CONCLUSIONS

We study how the performance of several popular forecasting models for equity index volatility is affected by a series of factors, that is, country, forecast horizon, market conditions, statistical accuracy, and economic significance. The HAR model is the most accurate for deriving 1-day ahead forecasts. An implied volatility model that incorporates the volatility risk premium is more suitable for monthly forecasts. Forecasts based on GJR-GARCH models are inferior to realized and implied volatility forecasts in almost all markets considered. The accuracy of forecasts based on implied volatility can be improved by making an adjustment for the volatility risk premium. Forecasting volatility in periods of market turmoil should be undertaken with caution as forecasting performance is likely to deteriorate significantly. Finally, implied volatility forecasts can significantly enhance international portfolio choice over historical methods. Future research could seek to improve international volatility forecasting by combining individual models or incorporating spillover effects between markets.

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