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Forecasting Volatility

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ABSTRACT

In this paper, we investigate the time series properties of S&P 100 volatility and the forecasting performance of different volatility models. We consider several nonparametric and parametric volatility measures, such as implied, realized and model-based volatility, and show that these volatility processes exhibit an extremely slow mean-reverting behavior and possible long memory. For this reason, we explicitly model the near-unit root behavior of volatility and construct median unbiased forecasts by approximating the finite-sample forecast distribution using bootstrap methods. Furthermore, we produce prediction intervals for the next-period implied volatility that provide important information about the uncertainty surrounding the point forecasts. Finally, we apply intercept corrections to forecasts from misspecified models which dramatically improve the accuracy of the volatility forecasts.

KEY WORDS implied and realized volatility; model-based volatility; strong persistence; forecast evaluation

INTRODUCTION

Forecasting future volatility is of great practical importance for derivative pricing, portfolio optimization and value-at-risk analysis. The voluminous literature on forecasting financial volatility is surveyed in Poon and Granger (2003). Several recent studies on volatility forecasting have appeared in this journal: Brooks (1998, 2001) and Chong *et al*. (1999) assess the forecasting performance of various GARCH models; Brooks and Persand (2003), Dunis and Huang (2002), and Mittnik and Paolella (2000) implement volatility forecasting for value-at-risk and trading purposes; Lopez (2001) and Taylor (1999) discuss different approaches to evaluating the accuracy of volatility forecasts.

In this paper, we investigate the performance of different volatility models for forecasting S&P 100 volatility. Availability of data on implied volatility from option prices allows us to treat volatility as an observed rather than a latent process and use conditional mean methods for inference and forecasting. Our predictive regression results suggest that the implied volatility can serve as an unbiased forecast of future volatility. This finding is somewhat surprising given the large number of

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studies that document empirical rejection of the unbiasedness hypothesis but it may be attributed to differences in the definition of the volatility series and the inclusion of more recent data.

Given the empirical support that the implied volatility is an unbiased forecast of the future integrated volatility, we use conditional mean models to forecast next-period integrated volatility proxied either by the implied or the realized volatility. After a careful analysis of the time series properties of implied and realized volatility, we find that while the presence of unit roots is not supported by the data, the estimated largest autoregressive root in the volatility process appears to be very close to one. Since these findings suggest that the high persistence in volatility should be taken explicitly into account, we use the method proposed by Gospodinov (2002) to model volatility as a near-integrated process and construct median unbiased forecasts and prediction intervals. The forecasting results from this method are compared to the forecasts from a long-memory model fitted to implied volatility as well as EGARCH, FIEGARCH and stochastic volatility models that use information only from stock returns. We also investigate the properties of some combination and intercept-corrected forecasts.

Despite the vast literature on volatility forecasting, the paper makes contributions along several important dimensions. First, our study pays particular attention to the highly persistent nature of the volatility process for forecasting purposes. For instance, when the largest autoregressive root is near the nonstationarity boundary, the parameter estimates are downward biased and the forecasts constructed from these estimates tend to underpredict. For this reason, we construct median unbiased forecasts of implied volatility by approximating the finite-sample forecast distribution by bootstrap methods. Second, we produce confidence intervals for the implied volatility forecasts that provide important information about the uncertainty surrounding the point forecasts. Third, we apply intercept corrections to forecasts from misspecified models and find that these corrections dramatically improve the accuracy of the forecasts. Although these intercept corrections are ad hoc, they can serve as a useful practical tool for volatility forecast adjustment and may provide important insights about the direction of misspecification of volatility models and the deficiencies in volatility filtering.

The remainder of the paper is structured as follows. In the next section, we introduce the notion of implied volatility and forecast evaluation criteria. The third section discusses several conditional mean methods for modeling and forecasting implied volatility and provides a brief review of models for estimating and forecasting unobserved volatility from stock returns data. The fourth section presents the empirical results on the properties of the data and forecasting performance of different methods. The final section provides concluding remarks and outlines some directions for future research.

IMPLIED VOLATILITY AND FORECAST EVALUATION

When asset return volatility is time-varying, the pricing of contingent claims on this asset requires a forecast of future volatility. In the case of stochastic volatility with no leverage effect, the price of an option on an underlying asset with price S is given by (Hull and White, 1987)

$$Y(S, X, r, V, \tau) = E^{Q}[BS(S, X, R, \overline{V}, \tau)] \tag{1}$$

where X is the exercise price, $\tau = T - t$ is the option's time to maturity, $\overline{V} = \frac{1}{T-t}\int_t^T V(s)\mathrm{d}s$ is the average volatility over the life of the option, V is the instantaneous volatility of underlying asset

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returns, R is the risk-free rate, BS(•) is the Black–Scholes option pricing formula and E^Q denotes that the expectation is taken with respect to the risk-neutral measure.

Since the option price is observed in the market, we can back out the volatility by inverting the option price formula. In practice, there is a large cross-section of call and put options at each point of time that differ in terms of their maturities, exercise prices and degrees of moneyness. The options that are most informative about volatility[1] are near-the-money, close-to-maturity options. Since near-the-money, close-to-maturity options make the second derivative of the option price with respect to volatility close to zero, the Black–Scholes formula for these options is nearly linear in volatility and hence

$$E^Q\left[\frac{1}{T-t}\int_t^T V(t)\mathrm{d}s\right] \approx IV_{t,\tau} \tag{2}$$

where $\mathrm{IV}_{t,\tau} = \mathrm{BS}^{-1}(S, X, \tau, Y, R)$ is the Black–Scholes implied volatility at time t from a contract which matures in $(T - t)$ periods. This suggests that the implied volatility extracted from near-the-money, close-to-maturity options should be an unbiased forecast of future volatility under the risk-neutral measure although this may not be the case under the physical measure.[2] In the empirical analysis, we use realized volatility $\mathrm{RV}_{t,\tau} = \frac{1}{\tau}\sum_{i=t+1}^{T}[\ln(S_i) - \ln(S_{i-1})]^2$ as a proxy of the latent integrated volatility process[3] in (1) and (2).

The objective of this paper is to study the forecast properties of the future average volatility process $\bar{V}_{t+1,\tau} = \frac{1}{\tau}\sum_{i=2}^{\tau+1} V_{t+i}$. The forecasts of $\bar{V}_{t+1,\tau}$ are obtained either from one of the conditional mean models fitted to the observed implied volatility or as $\frac{1}{\tau}\sum_{i=2}^{\tau+1}\hat{V}_{t+i|t}$, where $\hat{V}_{t+i|t}$ is the forecast of volatility made at time t from one of the GARCH or stochastic volatility models estimated from stock returns. The appropriate evaluation of the forecast accuracy should depend on the purpose of the forecasting exercise. For example, forecasts of the future average volatility $\bar{V}_{t+1,\tau}$ can be used to compute the next-period option price $Y_{t+1} = \mathrm{BS}(S_t, X, r. \bar{V}_{t+1,\tau}, \tau)$. Furthermore, one could use the next-period forecasts of call and put option prices to construct straddle trading strategies as in Noh *et al*. (1994). In this paper, we conduct the forecast evaluation not in terms of the actual option prices next period but in terms of the next-period implied volatility extracted from these prices. This is not problematic since most traders quote implied volatility and use these quotes to compute the option price. Moreover, a growing segment of the market trades pure volatility positions in terms of implied or realized volatility (Neftci, 2004) which makes our forecast comparison directly applicable. Similarly, if volatility forecasts are produced for value-at-risk analysis, the forecast accuracy of different models needs to be assessed in terms of realized volatility.

The choice of an appropriate criterion for volatility forecast evaluation is under active investigation (Andersen *et al*., 1999; Brailsford and Faff, 1996; Lopez, 2001; Taylor, 1999; among others).

[1] The options that contain most precise information about volatility are the options that maximize the 'vega' of the option $\frac{\partial(\mathrm{BS})}{\partial\sigma^2}$.

[2] See Chernov (2001) for further details. Jones (2003) also argues that the additional approximation error in (2) from introducing a leverage effect is likely to be small.

[3] For a review of various volatility measures and their properties, see Andersen *et al*. (2002).

Engle *et al*. (1993) and West *et al*. (1993) proposed profit-based and utility-based criteria for evaluating the accuracy of volatility forecasts. The directional accuracy of volatility forecasts is also of practical importance since the direction of predicted volatility change can be used for constructing trading strategies such as straddles (Engle *et al*., 1993). In this study, we consider only statistical evaluation criteria based on the following loss functions:

$$\text{MAE} = \frac{1}{s}\sum_{i=T-s+1}^{T}\left|\bar{V}_i - \hat{\bar{V}}_i\right|,$$

$$\text{MAPE} = \frac{1}{s}\sum_{i=T-s+1}^{T}\left|1 - \hat{\bar{V}}_i/\bar{V}_i\right|,$$

$$\text{MALE} = \frac{1}{s}\sum_{i=T-s+1}^{T}\left|\ln\left(\hat{\bar{V}}_i/\bar{V}_i\right)\right|,$$

$$\text{LINEX} = \frac{1}{s}\sum_{i=T-s+1}^{T} b\left\{\exp\left[a\left(\bar{V}_i - \hat{\bar{V}}_i\right)\right] - a\left(\bar{V}_i - \hat{\bar{V}}_i\right) - 1\right\}$$

where s is the number of out-of-sample forecasts, $\hat{\bar{V}}$ denotes the forecast of the future average volatility at maturity τ and $a \in (-\infty, 0) \cup (0, +\infty)$, $b \in (0, +\infty)$ are given parameters.

The mean absolute error is a commonly used measure for forecast evaluation but it imposes the same penalty on over- and under-predictions of volatility and is not invariant to scale transformations. The mean absolute percentage error (MAPE) accommodates possible heteroskedasticity in forecast errors but it can be unstable if volatility is very low. The logarithmic MAE (MALE) penalizes differently forecast errors in low volatility and high volatility periods. Finally, the linear-exponential (LINEX) criterion weighs differently over- and under-predictions.[4] The asymmetry in the loss function can have important practical implications for option pricing since an over-prediction of volatility would result in an upward bias of the call option price. Therefore, the buyer of an option might want to penalize more heavily positive forecast errors of volatility ($a > 0$) whereas the seller of an option would assign higher penalty to negative errors ($a < 0$). In the empirical section, the parameter b is set equal to 2 and $a = 0.5$ or -1.

MODELS FOR FORECASTING VOLATILITY

Conditional mean models

An advantage of using implied volatility is that we can treat it as an observed rather than a latent process and use conditional mean methods for modeling and forecasting. Since it is widely documented in the literature that volatility is strongly persistent, we model volatility as a long-memory and a slowly mean-reverting (near unit root) process. Below, we briefly review two models that account explicitly for the high persistence in volatility.

ARFIMA models

Suppose that the implied volatility IV_t follows an ARFIMA (p, d, q) process

[4] In this paper, we do not deal with optimal prediction under different loss functions. For optimal prediction under asymmetric loss, see Christoffersen and Diebold (1997).

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$$\Phi(L)(1-L)^d(IV_t-\mu)=\Theta(L)\varepsilon_t$$

where $\varepsilon_t \sim iid(0, \sigma^2)$, d is the fractional integration (differencing) parameter, $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ and $\Theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ are lag polynomials with roots that lie outside the unit circle and no common roots. The properties of the process depend on the value of the differencing parameter d. For example, the fractional integration process with $0 < d < 1$ provides a gradual transition from the integrated process of order 0 ($d = 0$) to the integrated process of order one ($d = 1$), or unit root process. The parameters of the model can be estimated by exact maximum likelihood (Sowell, 1992) and k-step ahead forecasts of IV_{T+k} can be computed from the AR representation of the process (see Brockwell and Davis, 1991). A similar approach to implied volatility forecasting has been used by Hwang and Satchell (1998).

Near-integrated AR model

An alternative way to ensure a smooth transition between the asymptotics for short-memory processes and the unit root limiting theory is the local-to-unity framework (Chan and Wei, 1987; Phillips, 1987). Suppose that the dynamics of implied volatility is described by the near-integrated model

$$\mathrm{IV}_t-\mu=\rho(\mathrm{IV}_{t-1}-\mu)+\varepsilon_t$$

where $\rho=\exp(c/T)\approx 1+\dfrac{c}{T}$, $c \le 0$ is a fixed constant and $\varepsilon_t \sim iid(0, \sigma^2)$. This model is observationally equivalent to the long-memory model discussed in the precious section.[5] The advantage of this model, however, is that the inference in this model is based entirely on the OLS estimator whose limiting theory is fully developed for the stationary, near-nonstationary and nonstationary cases.

Gospodinov (2002) derived the asymptotic representations of the conditional predictive distribution of k-step ahead forecast $\widehat{IV}_{t+k|T}$ for two specifications of the forecast horizon: fixed k and k that increases linearly with the sample size. Unfortunately, the conditional limiting representations of the forecast $\widehat{IV}_{t+k|T}$ depend on the local-to-unity parameter c that is not consistently estimable, and the last observation IV_T which renders the asymptotic approach impractical. Gospodinov (2002) proposed a bootstrap-based method for constructing median unbiased forecasts of IV_{T+k} by bootstrapping the series backwards in time, conditional on the realized value of IV_T, on a grid of points for c. This approach can also be used for obtaining prediction intervals for IV_{T+k}. See Gospodinov (2002) for further details on the properties and the implementation of the bootstrap approximation to the conditional predictive distribution and the construction of forecasts.

Time-varying volatility models estimated from stock returns

EGARCH and FIEGARCH models

The generalized autoregressive conditional heteroskedasticity (GARCH) class of models (Engle, 1982; Bollerslev, 1986; Bollerslev *et al*., 1994) present a parametric alternative to estimating and

[5] Note that the ARFIMA (1, d, 0) model $(1 - \alpha L)(1 - L)^d \mathrm{IV}_t = \varepsilon_t$ can be rewritten as $\mathrm{IV}_t=(\alpha+d)\mathrm{IV}_{t-1}+\left[\dfrac{d(1-d)}{2}-\alpha d\right]$ $\mathrm{IV}_{t-2}+\ldots+\varepsilon_t$. Then, the AR representations of the near-integrated model with α close to one and d close to 0 and the long-memory model with d close to one and α close to 0 are observationally equivalent.

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forecasting the unobserved volatility process V_t from the asset returns $r_t = \ln(S_t) - \ln(S_{t-1})$. Assuming zero unconditional mean and lack of autocorrelation, the dynamics of returns is specified as $r_t = V_t^{1/2}\varepsilon_t$, where $\varepsilon_t \sim iid(0, 1)$. In order to account for a leverage effect whose presence appears to be a stylized fact for stock price data, we use the Gaussian EGARCH (1, 0) model of Nelson (1991):

$$(1-\beta L)(\ln(V_t)-\alpha) = g(\varepsilon_{t-1})$$

where $g(\varepsilon_t) = \gamma\varepsilon_t + \delta(|\varepsilon_t| - \sqrt{2/\pi})$ and $\varepsilon_t \sim N(0, 1)$. The model is estimated by maximum likelihood and the k-step ahead ($k > 1$) forecast of V_t is obtained by plugging the parameter estimates into the recursion $V_{T+k} = V_{T+k-1}^{\beta} e^{\varphi}[e^{(\gamma+\delta)^2/2}\Phi(\gamma+\delta) + e^{(\gamma-\delta)^2/2}\Phi(\gamma-\delta)]$, where $\varphi = \alpha(1-\beta) - \delta\sqrt{2/\pi}$, $\Phi(\cdot)$ is the CDF of a standard normal variable and $V_{T+1} = V_T^{\beta} e^{\alpha(1-\beta)} e^{g(\varepsilon_T)}$ (Cao and Tsay, 1992; Tsay, 2002).

Furthermore, to mimic the slow decay in the autocorrelation function of volatility, we follow Bollerslev and Mikkelsen (1996, 1999) and consider the fractionally integrated (FI)EGARCH(1, d, 0) model:

$$(1-\beta L)(1-L)^d(\ln(V_t)-\alpha) = g(\varepsilon_{t-1})$$

where d again is the fractional integration parameter. Using the expansion $(1 - L)^d = 1 - \Sigma_{i=1}^{\infty}\psi_i L^i$, where $\psi_1 = d$ and $\psi_i = \dfrac{i-1-d}{i}\psi_{i-1}$ for $i \geq 2$, we rewrite the FIEGARCH(1, d, 0) model in a more convenient form (Taylor, 2000) as

$$\ln(V_t) = \alpha + \sum_{i=1}^{m}\phi_i[(\ln(V_{t-i})-\alpha)] + g(\varepsilon_{t-1})$$

where m is a truncation parameter set equal to 1000, $\phi_1 = d + \beta$ and $\phi_i = \psi_i - \beta\psi_{i-1}$ for $i \geq 2$. As for the EGARCH model, the parameters are estimated by maximum likelihood and multi-step forecasts for the conditional volatility V_t are constructed under the assumption that ε_t is a standard normal random variable.

Stochastic volatility models

The stochastic volatility (SV) model has the form

$$r_t = \sigma\sqrt{V_t}\varepsilon_t$$
$$\ln(V_t) = \lambda\ln(V_{t-1}) + \sigma_v\xi_t$$

where ε_t and ξ_t are $iidN(0, 1)$ and mutually uncorrelated. The SV model is more flexible than the GARCH class of models because the persistence of volatility and the kurtosis of returns are modeled separately.[6]

[6] For a detailed discussion on the properties of the SV model and its relation to the GARCH class of models, see the survey articles by Chib (2002), Ghysels *et al*. (1996) and Shephard (1996), among others.

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Unfortunately, the estimation of SV models is particularly difficult due to the latent nature of the volatility process. In this paper, we use the MCMC method of Kim *et al*. (1998) for estimating the parameters (λ, σ_v) and filtering the unobserved volatility $\{V_t\}_{t=1}^T$.[7] Given the estimates of (λ, σ_v) and $\{V_t\}_{t=1}^T$, we construct multi-step forecasts of the conditional volatility by the recursion $\hat{V}_{T+k} = e^{\ln(\widehat{V}_{T+k})}e^{\sigma_v^2/2}$, where $\ln(\widehat{V}_{T+k}) = \hat{\lambda}^k \widehat{\ln(V_t)}$ (see also Mahieu and Schotman, 1998).

EMPIRICAL RESULTS

Data

For the conditional mean models, the implied volatility index VIX is constructed as a weighted average of eight close-to-maturity, near-the-money, nearby and second nearby puts and calls written on Standard and Poor's 100 (OEX) index with an average time to maturity of 30 calendar (22 trading) days (for more details see Fleming *et al*., 1995, and Whaley, 2000). The implied volatility index VIX was converted from annual into daily volatility.

The GARCH class of models and the stochastic volatility model use stock returns data computed from daily close prices of the S&P 100 index (not adjusted for dividends) as $r_t = \ln(S_t) - \ln(S_{t-1})$. Daily data for the S&P 100 index and the implied volatility index VIX for the period June 1, 1988 to May 17, 2002, were downloaded from http://table.finance.yahoo.com/k?s=^oex&g=d and http://www.cboe.com/MktData/vix.asp, respectively. The S&P 100 returns for this period exhibit very little autocorrelation, which is ignored in the subsequent empirical analysis.

We use the returns on S&P 100 to obtain measures of realized and historical volatility as proxies of the latent integrated volatility process. In particular, we construct a rolling sample for realized volatility from overlapping data using the expression $\mathrm{RV}_{t,\tau} = \frac{1}{\tau}\sum_{i=t+1}^{T} r_i^2$ with $\tau = 22$. For example, the first sample point of realized volatility is computed as a sample average of the squared S&P 100 returns from observation 2 to observation 24; the second sample point of realized volatility uses data from observation 3 to observation 25 etc. In fact, this measure is closely related to the structure of the implied volatility, which allows direct comparisons between these volatility measurements. The historical volatility is constructed in a similar fashion using the expression $\mathrm{HV}_{t-\tau,\tau} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} r_{t-i}^2$.

Summary statistics of IV and RV and predictive regressions

Figure 1 plots the implied and realized volatility and shows clearly volatility clustering over time. The first part of the sample, 1989–1991, is characterized by relatively high volatility which coincides with the recession period in the US economy. This is followed by a long tranquil period of low volatility. Since the second half of 1997, the volatility (and the volatility of volatility) increases significantly and remains at high levels until the end of the sample period.

As a next step, we take a preliminary look at the summary statistics of different proxies of S&P 100 volatility. In addition to the implied and realized volatility processes described above, we consider some transformations of the original series that have also been used in empirical work. In

[7] In an earlier version, we also used the efficient method of moments by Gallant and Tauchen (1996) for estimating the parameters and the reprojection algorithm for filtering the latent volatility process. The forecasting results from this method are inferior to the results obtained by the MCMC method and are not reported but are available from the authors upon request.

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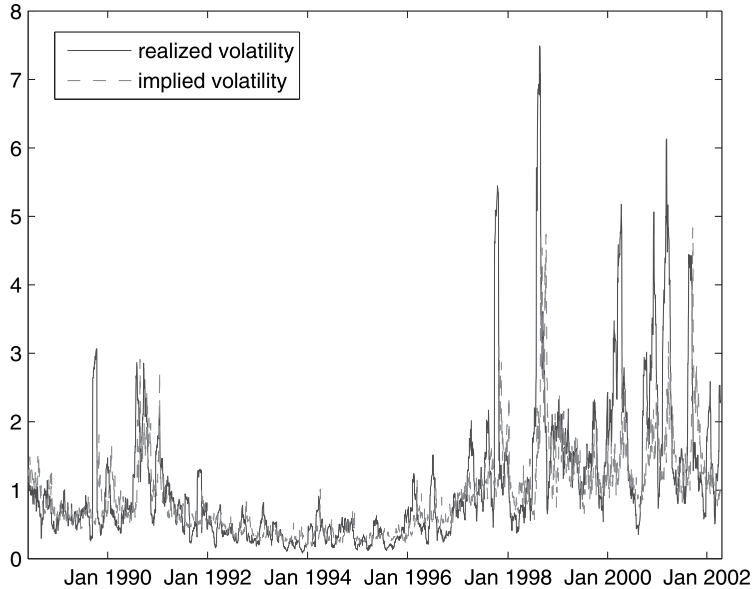

Figure 1. Realized and implied volatility of Standard and Poor's 100 (OEX) index

Table I. Summary statistics for implied (VIX) and realized volatility

	Mean	SD	Skewness	Kurtosis
$\overline{V}_t$				
Implied	0.900	0.591	1.867	8.943
Realized	1.064	1.032	2.537	11.120
$\overline{V}_t^{1/2}$				
Implied	0.905	0.284	0.798	3.851
Realized	0.945	0.413	1.279	4.920
$\ln(\overline{V}_t^{1/2})$				
Implied	−0.148	0.309	0.061	2.393
Realized	−0.142	0.411	0.172	2.775

particular, we consider separately $\overline{V}_t$, $\overline{V}_t^{1/2}$ and $\ln(\overline{V}_t^{1/2})$, where $\overline{V}_t$ denotes either implied or realized volatility at time t. Table I reports the summary statistics for the different series and reveals some interesting properties of the data. The volatility series $\overline{V}_t$ is characterized by positive skewness and high kurtosis. The square root transformation reduces the skewness and the leptokurtosis of the distribution. Interestingly, the distribution of log-volatility appears to be very close to a normal distribution. This feature of the log-volatility process has also been documented by Andersen *et al.* (2001) for the realized volatility constructed from intradaily exchange rate data.

Using realized volatility as a proxy for the unobserved integrated volatility, we test empirically the validity of relationship (2) in the predictive regression (Chernov, 2001):

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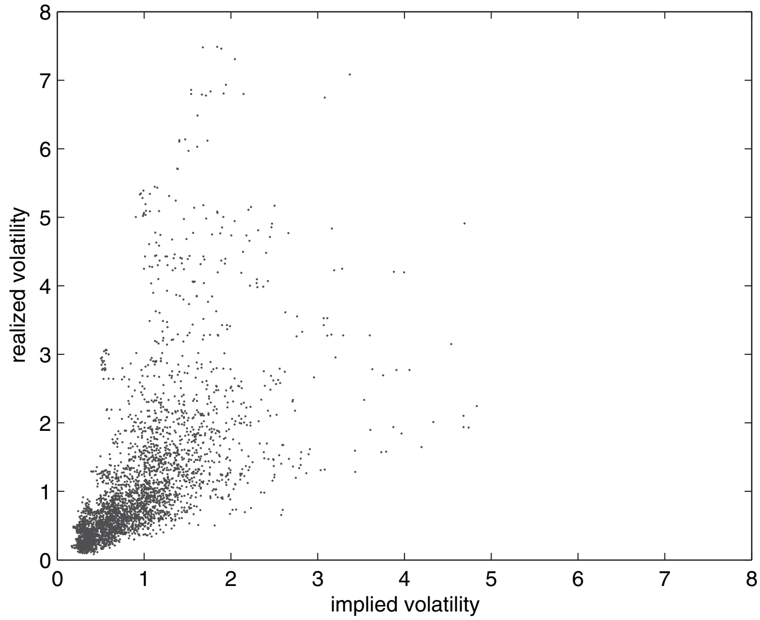

Figure 2. Scatter plot of realized versus implied volatility

$$\mathrm{RV}_{t,\tau} = a_1 + a_2\mathrm{IV}_{t,\tau} + a_3\mathrm{HV}_{t-\tau,\tau} + \varepsilon_{t+\tau} \tag{4}$$

The unbiasedness of implied volatility as a forecast of future volatility implies that $a_1 = 0$ and $a_2 = 1$. The historical volatility is included in the regression to verify that implied volatility is an efficient forecast of future volatility in a sense that economic variables belonging to the information set up to time t do not have additional predictive power, i.e. $a_3 = 0$. For more detailed discussion of the predictive regression (4), see Bandi and Perron (2001), Canina and Figlewski (1993), Chernov (2001), Christensen and Prabhala (1998), Day and Lewis (1992), Fleming (1998), Lamoureux and Lastrapes (1993), Poteshman (2000), among others.

Figure 2 is a scatter plot of realized versus implied volatility. The scales of the two volatility measures in Figure 2 are set to be the same in order to emphasize that realized volatility appears to be more noisy than implied volatility, which is also evident from their unconditional standard deviations in Table I. The OLS estimates of the predictive regression (4) with Newey–West heteroskedasticity and autocorrelation consistent (HAC) standard errors are reported in Table II. Contrary to the previous findings in the literature, we can not reject the unbiasedness hypothesis for all the specifications we consider. This result is surprising given the large number of studies that document empirically the rejection of the unbiasedness hypothesis and offer possible explanations for the bias. Our results appear to suggest that the implied volatility is an optimal forecast of future volatility. As we argue below, the relationship between realized and implied volatility is highly unstable and differences in the results may arise from the particular choice of a sample period as well as differences in the construction of the volatility series.

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Table II. Estimation of the volatility forecasting regression

	$\overline{V}_t$	$\overline{V}_t^{1/2}$	$\ln(\overline{V}_t^{1/2})$
$\hat{a}_1$	0.104	0.000	0.010
	(0.081)	(0.049)	(0.014)
$\hat{a}_2$	1.107	1.084	1.029
	(0.175)	(0.107)	(0.074)
$\hat{a}_3$	−0.031	−0.037	−0.004
	(0.078)	(0.069)	(0.056)

Note: Newey–West HAC standard errors in parentheses.

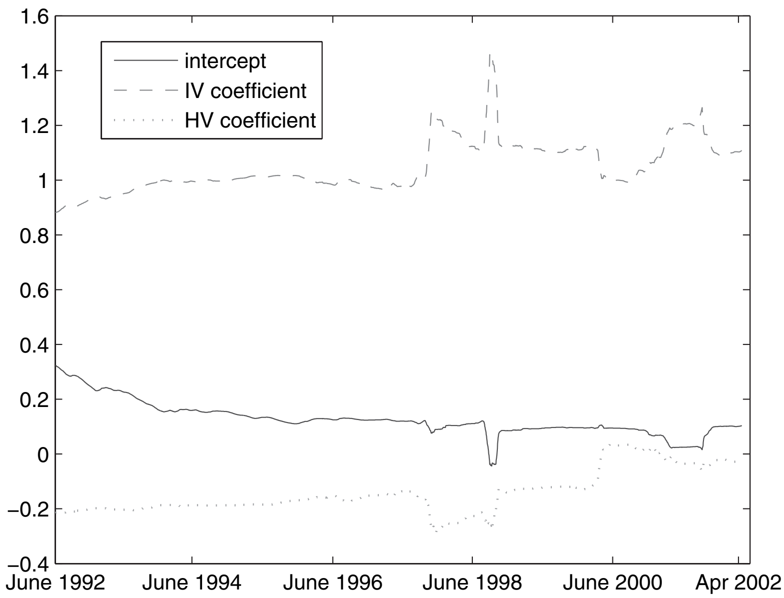

Figure 3. Recursive sample estimates from predictive regression (4)

To investigate the robustness of the results, we examine recursive sample estimates[8] of the parameters in the predictive regression. The initial sample size for the recursive estimates is set to 1000 observations. The results are plotted in Figure 3 and look quite interesting. The parameter estimates appear to be unstable with large variations over time. In fact, the test statistics for parameter instability in models with strongly persistent regressors proposed by Hansen (1992) have values $L = 27.8$, $\text{Ave}F_T = 222.7$ and $\sup F_T = 617.2$ that show unambiguously that the relationship between these variables is characterized by high instability.

[8] The results from rolling sample estimation with a rolling window of size 1000 are qualitatively similar and are available from the authors upon request.

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The recursive sample results reveal that, for most of the sample period until mid 1997, $\hat{a}_1 > 0$, $\hat{a}_2 < 1$ and $\hat{a}_3 < 0$. This is consistent with the previous results found in the literature and indicates that the implied volatility measure under-predicts low volatility and over-predicts high volatility. Since model misspecification and measurement error are expected to have second-order effects on the bias, the prime suspects for the rejection of the unbiasedness hypothesis with pre-1997 data appear to be the omission of volatility risk premium (Chernov, 2001; Poteshman, 2000) and the persistence properties of the data (Bandi and Perron, 2001). The significant parameter instability that is evident from the structural break tests and Figure 3 casts some doubts on the use of predictive regressions for testing the unbiasedness hypothesis and calls for further research.

Persistence in volatility

Unit root tests

It is a stylized fact that financial volatility is highly persistent. Possible explanations of the persistency in the volatility process include nonuniform information flows and aggregation over heterogeneous information processes (Andersen and Bollerslev, 1998), structural breaks (Lamoureux and Lastrapes, 1990a), omission of important factors such as trading volume (Lamoureux and Lastrapes, 1990b) etc. The persistence of implied and realized volatility is even further amplified by the averaging (smoothing) that is involved in the construction of these volatility measures. Although it is theoretically unjustified to believe that a shock to volatility has a permanent effect on the level of the series, the empirical evidence suggests that the speed at which the shock dissipates over time is very slow. Therefore, it is empirically possible that the presence of unit root in volatility cannot be rejected for a particular sample. In this case, imposing a unit root may be beneficial for short-term forecasting of volatility since it reduces the parameter uncertainty and hence the forecast error.

To test the null hypothesis of a unit root against the alternative of stationarity, we consider the augmented Dickey–Fuller (ADF) test for $\rho = 1$ from the regression

$$\tilde{V}_t = \rho \tilde{V}_{t-1} + \sum_{j=1}^{p} \psi_j \Delta \tilde{V}_{t-j} + \varepsilon_t \tag{5}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$ and $\tilde{V}_t$ is the demeaned volatility series.[9] To increase the power of the test, we follow Elliott *et al*. (1996) and demean the series using GLS demeaning where the quasi-differences are constructed with autoregressive coefficient $\bar{\rho} = 1 - 7/T$.

Psaradakis and Tzavalis (1999) and Wright (1999) test for a unit root in the latent volatility process using ARMA representations of the observed squared returns. As argued in Andersen and Bollerslev (1998), the squared returns provide a noisy measure of the true volatility with a number of large outliers. In this paper, we use the implied and realized volatility to test if the persistence in the volatility is driven by the presence of a unit root.

As before, we consider three different transformations of our volatility series: $\bar{V}_t$, $\bar{V}_t^{1/2}$ and $\ln(\bar{V}_t^{1/2})$. We also construct median unbiased estimates and 68% confidence intervals for the persistence parameter ρ by inverting the bootstrap version of the ADF statistic (Stock, 1991). The results are reported in Table III and show that the null hypothesis of a unit root is rejected for all variables. Even though the median unbiased estimate of the largest autoregressive root indicates a very high

[9] The lag order p is selected by the modified AIC (Ng and Perron, 2001).

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Table III. Unit root tests and interval estimates of the largest AR root

	ADF-GLS	MUE of ρ	68% CI of ρ
Implied volatility			
$\bar{V}_t$	−4.173	0.974	[0.970, 0.978]
$\bar{V}_t^{1/2}$	−3.286	0.984	[0.981, 0.988]
$\ln(\bar{V}_t^{1/2})$	−3.079	0.990	[0.987, 0.992]
Realized volatility			
$\bar{V}_t$	−4.577	0.984	[0.981, 0.986]
$\bar{V}_t^{1/2}$	−3.849	0.987	[0.984, 0.989]
$\ln(\bar{V}_t^{1/2})$	−3.255	0.988	[0.985, 0.990]

Note: The critical value for the ADF-GLS unit root test at 5% significance level is −1.95. The median unbiased estimates (MUE) and the 68% confidence interval for the parameter ρ from equation (5) are constructed by inverting the bootstrap version of the ADF statistic.

degree of persistence in the series, none of the 68% confidence intervals contain the value of one. Therefore, modeling volatility as a near-integrated (local-to-unity) or a fractionally integrated (long-memory) process may provide a better representation of the data.

Long memory in volatility

The differencing (long-memory) parameter d of the volatility process is estimated from the log-periodogram regression of Geweke and Porter-Hudak (1983) (GPH) with a bandwidth parameter equal to $T^{0.6}$. The results for the different volatility series are reported in the first column of Table IV. The estimates of the fractional integration parameter clearly show the existence of long memory in volatility. Most of the estimates of d lie in the non-stationary region $0.5 \leq \hat{d} < 1$.[10] As we pointed out above, the high estimates of the fractional differencing parameter may be partially due to the smoothing involved in the construction of the volatility series. Implied volatility appears to have a stronger long-memory component than realized volatility and the square root transformation has the largest values of $\hat{d}$ across the different specifications of volatility.[11]

Lamoureux and Lastrapes (1990a) argue that the presence of structural breaks may produce spurious high persistence in volatility. Diebold and Inoue (2001) provide theoretical justification and simulation evidence that structural breaks in the short-memory component may generate sample behavior which resembles long memory dependence. One informal way to distinguish between these two processes is based on the observation that the long-memory processes are self-similar and, therefore, the fractional integration is preserved under temporal aggregation (Andersen *et al*., 2001). For this reason, we aggregated data over 5 and 10 days (corresponding to weekly and bi-weekly frequencies) and then compared the estimates of the fractional integration parameter for the different temporal aggregates. The results are reported in the second and third columns of Table IV. Although we observe some differences in the point estimates of d as we go from daily to weekly frequency, these differences do not appear to be statistically significant and tend to vanish as we further increase

[10] The consistency of the GPH estimator of $d \in [0.5, 1)$ has been established by Velasco (1999).

[11] See Dittmann and Granger (2002) for a discussion on how different estimates of the fractional integration parameter can arise from different nonlinear transformations of the data.

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Table IV. GPH estimates of d for impled and realized volatility aggregated over m periods

	$m = 1$	$m = 5$	$m = 10$
Implied volatility			
$\bar{V}_t$	0.814	0.651	0.678
	(0.058)	(0.102)	(0.140)
$\bar{V}_t^{1/2}$	0.859	0.832	0.887
	(0.061)	(0.128)	(0.173)
$\ln(\bar{V}_t^{1/2})$	0.837	0.804	0.774
	(0.062)	(0.104)	(0.119)
Realized volatility			
$\bar{V}_t$	0.723	0.460	0.496
	(0.067)	(0.111)	(0.160)
$\bar{V}_t^{1/2}$	0.771	0.608	0.694
	(0.067)	(0.125)	(0.182)
$\ln(\bar{V}_t^{1/2})$	0.725	0.535	0.559
	(0.056)	(0.087)	(0.126)

Note: The numbers in parentheses are the regression standard errors.

the length of the aggregation period. This provides some informal evidence that the volatility series are characterized by long range dependence.

Properties of model-based volatility

Table V summarizes the estimation results from the EGARCH, FIEGARCH and stochastic volatility models.[12] The results confirm the strongly persistent behavior of volatility that was detected in the previous sections. The estimates from the EGARCH and FIEGARCH models indicate the presence of a leverage effect. Positive news has almost no impact on volatility, whereas negative shocks lead to a large increase in volatility ($\gamma - \delta = -0.19$ for EGARCH and -0.29 for FIEGARCH). The fractional integration parameter in the FIEGARCH model is estimated to lie near but slightly below the boundary of the nonstationary region. The filtered volatility series from the EGARCH (FIEGARCH) and stochastic volatility models (not shown here to preserve space) exhibit similar patterns although there are some noticeable differences that might produce differences in the forecasts.

Forecasting results

The forecast evaluation of the different methods is performed for one-step ahead forecasts[13] of implied and realized volatility using the statistical evaluation criteria defined above ('implied volatility and forecast evaluation'). In addition, we explicitly recognize the possibility of model misspecification and consider an intercept correction of the forecasts of the form $\tilde{V}_{T+i} = \hat{V}_{T+i} + (V_{T+i-1} - \hat{V}_{T+i-1})$, where $\hat{V}_{T+i}$ is the model forecast and V_{T+i} is the true value (for a discussion see Clements

[12] The stochastic volatility model is estimated using Mahieu and Schotman's (1998) GAUSS code available from the *Journal of Applied Econometrics Data Archive* (http://qed.econ.queensu.ca/jae/). The estimation of the ARFIMA model is performed using the ARFIMA package in Ox. All the other computations in the paper were coded by the authors in GAUSS and are available upon request.

[13] Results for longer forecast horizons are available from the authors upon request.

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Table V. Estimation results for volatility models

Parameters	Estimate	SE
EGARCH(1, 0)		
β (persistence)	0.979	0.007
γ (leverage effect)	–0.082	0.019
δ (size effect)	0.106	0.022
FIEGARCH(1, d, 0)		
d (fractional integration)	0.443	0.046
β (persistence)	0.555	0.117
γ (leverage effect)	–0.125	0.034
δ (size effect)	0.166	0.032
Stochastic volatility		
λ (persistence)	0.986	0.003
σ_v (volatility of volatility)	0.138	0.011

and Hendry, 1998). Note that this is a GLS-type correction to the original forecast in the case when the forecast errors are serially correlated and follow a random walk process. Finally, we consider two combination forecasts that are sample averages of the forecasts from all models with and without the stochastic volatility model, denoted by COMB1 and COMB2, respectively.

We leave the last 1000 observations for out-of-sample forecasting which covers the high volatility (and volatility-of-volatility) period starting in April 1998. In this case, the first forecast from the conditional mean models fitted to implied volatility is constructed by estimating the parameters from the initial 2493 observations and then using the estimated parameters to compute a one-step-ahead forecast. The next forecast is computed from the estimated parameters based on the first 2494 observations etc. until the end of the sample.

Similarly, for the first forecast from the EGARCH, FIEGARCH and stochastic volatility models, we estimate the parameters and filter the volatility process using the initial 2493 observations. Next, we produce a sequence of 1-, 2-, ... , 22-step-ahead forecasts from the estimated parameters and filtered volatility. The forecast of average future volatility is finally obtained by taking a sample average of the 1-step, 2-step, ... , 22-step-ahead forecasts. Then, we continue with the next-period forecast by re-estimating the model using the first 2494 observations etc.

Table VI reports the results from forecasting next-period implied volatility measured by VIX. The estimated fractional integration parameter in the ARFIMA model used for out-of-sample forecasting varied between 0.7 and 0.73. The bootstrap procedure of Gospodinov (2002) for the near-integrated model approximates the whole predictive distribution of volatility, but we use only the 50th percentile of this distribution and construct median unbiased forecasts (MUF). We also employ this method to construct 68% prediction intervals for next-period VIX in order to assess the uncertainty surrounding the MUF. The lower and upper limits of the 68% prediction intervals are plotted in Figure 4. The empirical coverage rate of the prediction intervals for these 1000 observations is 64%.[14]

[14] The difference between the empirical coverage and nominal level may be due to the small number of observations used for out-of-sample forecasting. When we increase the number of observations for out-of-sample forecasting from 1000 to 1500, the empirical coverage rates of the 68% prediction intervals is 67%.

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Table VI. Forecast evaluation of volatility models for 1-step-ahead forecasts of implied volatility

Loss	EGARCH	FIEGARCH	SV-Bayes	ARFIMA	MUF	COMB1	COMB2
MAE	0.3353 (5)	0.3717 (6)	0.6399 (7)	0.1511 (2)	0.1508 (1)	0.2605 (4)	0.1952 (3)
MAPE	0.2203 (5)	0.2559 (6)	0.4726 (7)	0.0969 (2)	0.0957 (1)	0.1828 (4)	0.1298 (3)
MALE	0.2210 (5)	0.2474 (6)	0.3600 (7)	0.0968 (1)	0.0973 (2)	0.1681 (4)	0.1280 (3)
LINEX1	0.0545 (5)	0.0837 (6)	0.1500 (7)	0.0150 (1)	0.0152 (2)	0.0310 (4)	0.0201 (3)
LINEX2	0.2362 (6)	0.2203 (5)	1.4821 (7)	0.0516 (1)	0.0531 (2)	0.1308 (4)	0.0692 (3)

Note: COMB1 denotes a linear combination of the forecasts from all models including SV model; COMB2 denotes a linear combination of the forecasts from all models excluding SV model; MAPE is the mean absolute percentage error; MALE is the logarithmic MAE; LINEX1 is the linear exponential loss function with parameter $a = 0.5$; LINEX2 is the linear exponential loss function with $a = -1$; MUF is the median unbiased forecast of VIX as described above ('Near-integrated AR model'). The numbers in parentheses are the relative ranking of each method for the corresponding loss function.

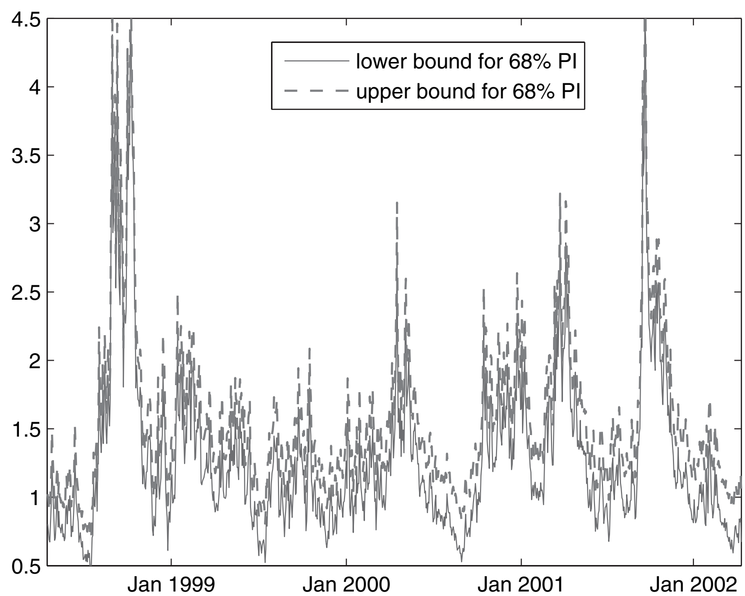

Figure 4. 68% Prediction intervals of next day implied volatility for April 20, 1998 to April 17, 2002

Although the 68% prediction intervals appear relatively tight, they may still be economically significant and deliver a large variation in the option prices once these implied volatility forecasts are plugged into the option price formula.

The results from Table VI for the one-step-ahead forecasts show that the conditional mean models fitted to implied volatility produce the best forecasts for all evaluation criteria. The combination forecasts perform consistently well across the different loss functions and appear to be using efficiently the information from the model forecasts. The stochastic volatility model produces the largest forecast errors for all loss functions.

A closer examination of the autocorrelation structure of the forecast errors from the EGARCH, FIEGARCH and SV models suggests that these models are severely misspecified for forecasting

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implied volatility. This is not surprising because implied volatility incorporates richer information content which involves adjustments for risk, market imperfections etc. For this reason, we construct intercept-corrected forecasts, which are presented in Table VII. Since the forecast errors from the ARFIMA and near-integrated models do not reveal any presence of misspecification, the intercept correction applied to the forecasts from these models tends to deteriorate their performance, and so we leave them uncorrected. The intercept correction improves substantially the forecasting performance of the conditional volatility models. The gains in forecast precision from intercept correction are particularly impressive for the FIEGARCH models, whose forecast errors are up to 25% lower compared to the uncorrected forecasts from the conditional mean models fitted to implied volatility.

Tables VIII and IX report the forecasting results when the realized volatility is used as a proxy for the true integrated volatility. In this case, the different methods produce rather similar results and none of the methods appears to dominate the others uniformly across the different loss functions. The conditional mean models of implied volatility are the best performers under the heteroskedasticity-adjusted (percentage) MAE but they do not fare well under LINEX. As in the implied volatility forecasting, the stochastic volatility model produces the worst forecasts except for the LINEX loss function with parameter $a = 0.5$.

Again, some interesting findings emerge from the intercept-corrected forecasts. More specifically, the intercept correction leads to a substantial (by orders of magnitude) reduction in the forecast errors for all models. Except for the LINEX loss with $\alpha = -1$, the FIEGARCH intercept-corrected forecasts appear to be the best-performing individual forecasting method and is only slightly dominated by the combination forecasts constructed by excluding the SV model.

Table VII. Forecast evaluation of volatility models for 1-step-ahead intercept-corrected forecasts of implied volatility

Loss	EGARCH	FIEGARCH	SV-Bayes	ARFIMA	MUF	COMB1	COMB2
MAE	0.1190 (2)	0.1147 (1)	0.2634 (7)	0.1511 (6)	0.1508 (5)	0.1362 (4)	0.1217 (3)
MAPE	0.0794 (3)	0.0759 (1)	0.1805 (7)	0.0969 (6)	0.0957 (5)	0.0891 (4)	0.0789 (2)
MALE	0.0784 (3)	0.0749 (1)	0.1823 (7)	0.0968 (5)	0.0973 (6)	0.0878 (4)	0.0781 (2)
LINEX1	0.0080 (2)	0.0079 (1)	0.0375 (7)	0.0150 (5)	0.0152 (6)	0.0110 (4)	0.0092 (3)
LINEX2	0.0370 (2)	0.0343 (1)	0.2787 (7)	0.0516 (5)	0.0531 (6)	0.0500 (4)	0.0373 (3)

Note: See note to Table VI.

Table VIII. Forecast evaluation of volatility models for 1-step-ahead forecasts of realized volatility

Loss	EGARCH	FIEGARCH	SV-Bayes	ARFIMA	MUF	COMB1	COMB2
MAE	0.8575 (3)	0.8756 (6)	0.9802 (7)	0.8644 (3)	0.8655 (4)	0.8418 (2)	0.8412 (1)
MAPE	0.4231 (4)	0.4467 (6)	0.6205 (7)	0.4100 (3)	0.4046 (1)	0.4331 (5)	0.4057 (2)
MALE	0.4782 (6)	0.4753 (5)	0.5000 (7)	0.4557 (3)	0.4586 (4)	0.4462 (1)	0.4493 (2)
LINEX1	0.8364 (3)	0.9669 (7)	0.7696 (1)	0.8469 (5)	0.8520 (6)	0.8079 (2)	0.8465 (4)
LINEX2	0.9746 (4)	0.9222 (2)	2.5105 (7)	1.1130 (6)	1.0978 (5)	0.9733 (3)	0.9142 (1)

Note: See note to Table VI.

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Table IX. Forecast evaluation of volatility models for 1-step-ahead intercept-corrected forecasts of realized volatility

Loss	EGARCH	FIEGARCH	SV-Bayes	ARFIMA	MUF	COMB1	COMB2
MAE	0.2082 (5)	0.1823 (2)	0.3178 (7)	0.2021 (4)	0.2173 (6)	0.1925 (3)	0.1805 (1)
MAPE	0.1183 (5)	0.1045 (2)	0.2011 (7)	0.1158 (4)	0.1249 (6)	0.1118 (3)	0.1026 (1)
MALE	0.1160 (4)	0.1025 (1)	0.2045 (7)	0.1201 (5)	0.1289 (6)	0.1108 (3)	0.1025 (1)
LINEX1	0.0306 (6)	0.0243 (2)	0.0522 (7)	0.0264 (4)	0.0297 (5)	0.0251 (3)	0.0236 (1)
LINEX2	0.3665 (6)	0.1590 (5)	0.3967 (7)	0.1290 (1)	0.1456 (3)	0.1495 (4)	0.1430 (2)

Note: See note to Table VI.

CONCLUDING REMARKS

In this paper, we consider several volatility measures and investigate their time series properties and forecasting performance. The volatility process implied by the option prices on the S&P 100 index permits us to treat volatility as observed and use conditional mean models that take explicitly into account the high persistence of the series. We approximate the whole predictive distribution of the future implied volatility by bootstrap methods and use this approximation to construct median unbiased forecasts and prediction intervals for next period implied volatility. For comparison purposes, we also include volatility forecasts obtained from ARFIMA, EGARCH, FIEGARCH and SV models.

Our findings can be summarized as follows. First of all, our forecasting results show that the implied volatility provides valuable information about the future movements of volatility. The forecast improvements from models fitted to implied volatility may come from several sources. First, it is the richer information content of option prices compared to stock returns. Also, the observability of implied volatility allows us to use more efficient methods for modeling and forecasting that explicitly account for the high persistence of the volatility process. Although we do not attempt to disentangle the individual contributions of these factors here, this might prove to be a fruitful direction for future research.

Second, the forecasting fragility and possible misspecification of volatility models is best indicated by the nontrivial improvements that result from intercept corrections of the model forecasts. From a practical point of view, even though the suggested intercept correction appears ad hoc, this is an easy-to-use method for forecast improvement that should receive more attention in empirical work. On the other hand, models that exhibit greater forecast insensitivity to intercept corrections would be of great importance for policy analysis, regulation and investment decision making. Furthermore, we find that combining information from different volatility models tends to improve the performance of volatility forecasts, especially at long forecasting horizons (not reported in this paper). Therefore, intercept correction and constructing combination forecasts appears to be a promising avenue of future research on volatility forecasting.

Finally, this paper considers only forecasts from univariate models. Using simultaneously information from stock returns and option prices by including the implied volatility in a GARCH-type model as in Blair *et al*. (2001), or adding exogenous regressors that contain some incremental information about volatility such as trade volume (Brooks, 1998; Lamoureux and Lastrapes, 1990b), may lead to an increased forecast accuracy and deserves further attention.

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