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# The role of trading volume in volatility forecasting<sup>☆</sup>

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#### ABSTRACT

Current models of volatility generally either use historical returns or option implied volatility to generate forecasts. Motivated by recent findings in the strand of literature focusing on volume-return (price) volatility relationships, this paper proposes the introduction of trading volume into various ARCH frameworks to improve forecasts. In particular, ex-ante evidence indicates that the incorporation of option implied volatility and trading volume into an EGARCH model leads to outperformance over other alternate forecast approaches. Noticeably, abnormal returns obtained from trading simulation underscores the improvement in forecast accuracy to be economically significant. These results remain robust to different measures of volatility and volume and offers scope for investors to more accurately predict volatility in the future.

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#### 1. Introduction

Forecasting volatility has been the subject of much investigation by academics and practitioners in recent years, due to the increasing recognition of its great practical importance in derivatives pricing, risk analysis, and portfolio management. Given that volatility serves as a critical input in most financial asset pricing models, the question of whether its dynamics can be forecasted falls within the vast literature on the predictability of asset prices and market efficiency.

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Despite the extensive literature in developing sophisticated models to forecast volatility dynamics [refer to Poon and Granger, 2003 for a review], little investigation exists on whether a synthesis between the research that focuses on option implied volatility and that of price-volume relationships can lead to an improved time series model for volatility forecasting. Our line of inquiry emerges from recent developments in two strands of studies independently existing in the literature which may give potential to improving forecasts. One investigates the dynamics of volatility driven by the market trading process to incorporate new information reflected in trading volume, while another seeks to capture the market expectation subsumed by option implied volatility. While independent forecasts can be estimated from single-factor models which are structured to extract the information content of each factor, anecdotal evidence gives higher merit to combining forecasts (Clemen, 1989; Becker, 2008). Besides, the appropriate incorporation of multiple factors may potentially offer an advanced forecasting model. It is documented in Poon and Granger's (2003) review that singular-factor time series models have some limitations which few empirical studies have attempted to overcome.

In this paper, the information content of option implied volatility and trading volume in forecasting the return volatility of individual stocks and the S&P 500 index in the US market will be tested for the period from 2 January 2003 to 30 June 2008. This study is necessary due to conflicting evidence surrounding whether additional information can be gleaned from option implied volatility. Work by authors such as Lamoureux and Lastrapes (1993), Mayhew and Stivers (2003) and Donaldson and Kamstra (2005) have shown mixed, if no improvement at all, in using option implied volatility over historical volatility in forecasting volatility.

In addition, the investigation into the information content of trading volume will provide further insight into three competing hypotheses currently upheld in the literature regarding the nature of the volume–volatility relation. The mixture of distribution hypothesis (MDH), proposed by Clark (1973), implies that the volume–volatility relation is simultaneous since they inherit a joint dependence on an underlying latent event and information flow variable. Hence, past volume does not contain any additional useful information on the future dynamics of volatility. Though the empirical confirmation of the hypothesis shows inconsistent results, its stance in the literature has firmly been established through the development of many influential theoretical models (Tauchen and Pitts, 1983; Andersen, 1996). In contrast, the sequential information arrival hypothesis (SIH) [see Copeland, 1976] and the noise trading hypothesis (Brock and LeBaron, 1996; Iori, 2002; Milton and Raviv, 1993) both suggest a lead-lag (causal) relation exists, and can be exploited for forecasting purposes. Given that we intend on incorporating volume into the forecasting model, results from our study can further elucidate the efficacy of the above models. Our particular approach not only can be intuitively appealing to determine the importance of any lag relationship that exists but also may prove to yield a beneficial improvement in the volatility forecast itself.

Our special interest in the information content of trading volume in volatility forecast rests on the scarcity of studies which link the volume–volatility relation with forecasting applications. This line of research has not yet been pursued vigorously in the past, either because of the conflicting evidence surrounding the nature of the volume–volatility relation (Lamoureux and Lastrapes, 1990a; Wagner and Marsh, 2005; Abu Hassan Shaari Mohd and Chin Wen, 2007) or due to discouraging results found in the earliest examinations of the role of volume in forecasting volatility (Brooks, 1998). Furthermore, it is important to note that the forecast performance of trading volume in those studies usually focused on statistical evaluations of the model, with little emphasis on examining trading strategies that may have led to a possible different conclusion. Some recent work by Donaldson and Kamstra (2005) did examine the role of trading volume in volatility forecasting by using it as a switching mechanism between the relative informativeness of ARCH and option implied volatility estimates. Although their results suggested forecasts did improve, volume only appeared as a dummy parameter switch and the results did not focus on possible economic viability of using the forecasts in trading strategies. In this paper we extend our investigation to evaluate whether the inclusion of trading volume does lead to tangible benefits from trading strategies, whilst modeling trading volume directly into our formulations.

Our study therefore intends to fill the gap in the literature by addressing the relative importance of stock and option trading volumes, combined with option implied volatility, in volatility forecasts to highlight the potential information captured from trading activities taken from both the stock and

**Table 1** List of equities and index.

Exchange ticker (CBOE)	Company name	Industry classification	Stock volume ('000) Total 2003-mid 2008	Option volume Total 2003-mid 2008
AIG	AMERICAN INTL.GP.INCO.	Financials	13,938,876.7	32,397,760
IBM	INTL.BUS.MCHS.CORP.	Technology	9,283,961.7	38,022,632
GM	GENERAL MOTORS	Industrials	14,774,256.1	48,046,712
GE	GENERAL ELECTRIC CO.	Industrials	38,434,988.3	84,263,186
WMT	WAL MART STORES INCO.	Consumer services	18,505,915.0	44,983,968
HPQ	HEWLETT-PACKARD CO.	Technology	18,295,916.0	32,934,816
TXN	TEXAS INSTS.INCO.	Technology	19,366,638.6	32,500,162
JNJ	JOHNSON & JOHNSON	Healthcare	12,351,670.3	19,059,264
XRX	XEROX CORP.	Technology	6,373,009.7	4,667,630
SPX	S&P500 INDEX	N/A	3,033,027,318.0	293,644,820

options markets. Unlike previous studies, our results suggest the information content on future stock volatility is shared between option implied volatility, actual option trading volume, and stock trading volume when we interact these factors in an augmented ARCH model. Our results show that, at the very least, stock and option trading volume can improve the forecast quality. This would provide support to research that has examined the role of trading activity in the option market in alleviating the information assimilation process in the underlying market (see Anthony, 1988; Easley et al., 1998; Chakravarty et al., 2004; Pan and Poteshman, 2006). Our findings suggest its nature is important to explain the varying pattern of the volume–volatility dynamics across different stocks and markets reported in previous studies.

The remaining of the paper will be structured as follows. The next section describes the nature of the data set and variable construction. It is followed by the methodology section which outlines the forecast evaluation procedure where multiple alternate assessment criteria (statistical functions of loss versus trading simulation) are appraised in direct comparison to other forecasting techniques. The information content of implied volatility and trading volume is then discussed in the empirical results, while the final section provides concluding remarks and outlines some directions for future research.

## 2. Data and variable computation

The primary data set employed in this study focuses on a period between 2 January 2003 and 30 June 2008. Of this sample, the first period up until 21 November 2007 is used for in-sample hypothesis testing and model construction, while the remaining data is utilized for out-of-sample evaluation. The time frame was limited to these dates due to what was available from the data vendors. It does, however, provide an interesting mix of bull and bear market trends, therefore incorporating an environment containing various market dynamics.

Apart from the S&P500 index, nine individual shares were chosen and are listed in Table 1. The selection was based on identifying stocks that had the most liquid, in terms of daily trading volume, options written on the underlying equity traded on the Chicago Board Options Exchange (CBOE). Given the primary objective of this research is to evaluate the forecast quality of trading volume, this sample set should provide the best chance of possibly identifying if this factor can contribute to volatility forecasts. If trading volume is not useful in this set of stocks, then it is unlikely to be in stocks with less liquid options. Daily option contract information, price and volume data for all active options on each of the underlying stocks were obtained from the data provider Stricknet Ltd. All remaining data relating to the underlying was collected from DataStream International. This included closing prices and daily trading volume for both the index and shares, dividend yield of the S&P500 index, the discrete cash dividends paid on the equities and the 30-day bank accepted bills rate which we utilize as the risk-free rate.

The option implied volatility is also required for our analysis and for the S&P500 index we utilize the CBOE's implied volatility index (VIX) also obtained from DataStream. Its construction is designed to represent the implied volatility of an at-the-money (ATM) option with 22 days to maturity, by taking the weighted average of four calls and four puts nearest to the money at the two nearest expiration dates. While it is intended to mitigate pricing bias and measurement error caused by staleness by using a wide band of the most liquid call and put options, this measurement inherits many other beneficial attributes, including consistency, efficiency and vega-maximizing. Due to these preferable features, the weighting scheme used to compute the VIX index of the CBOE is re-applied in this study to produce a single estimate of implied volatility of all traded options on each of the stocks we analyse. Consequently, on every trading day, eight nearest-to-the-money calls and puts from two nearby option expiration dates are chosen. Their implied volatilities are calculated from the observed trading prices using the binomial tree that explicitly account for early exercise and discrete dividends. We follow the procedure outlined in Harvey and Whaley (1992), conducted in Matlab. Despite the computational expense, its application ensures a precise estimation of implied volatilities. These eight estimates are then aggregated using the VIX weighting procedure, whose exact algorithm can be found in Fleming et al. (1995), and Corrado and Miller (2005).

Finally, this paper focuses on using realized volatility (RV) as the instrument that is to be forecasted. Daily stock returns are computed from closing prices, i.e.  $r_t = \ln(S_t) - \ln(S_{t-1})$ , and then utilized to calculate our proxy for the latent integrated volatility process, where  $RV_{t,\tau} = (1/\tau) \sum_{i=t+1}^T r_i^2$  with  $\tau$  being the frequency of observations. For the in-sample tests we only utilize daily data. However, for the out-of-sample test period we examine a number of alternate proxies including realized volatility from 5-min intra-day price data obtained from CQG Ltd.

#### 2.1. Descriptive statistics

Descriptive statistics for the return and daily volatility series reported in Table 2 suggest that all series do not conform to having a normal distribution. The Box-Pierce statistic also rejects the null hypothesis of independence for each series which shows a high auto-correlation up to the 10th lag. In particular, the pervasive evidence of volatility persistence in both the equity and options markets can be highlighted by the extremely high Qs(10) statistics found in the volatility series, consistent with previous studies (see Ding et al., 1993; Brailsford and Faff, 1996; also refer to Gospodinov et al., 2006 for a discussion of possible explanations for volatility persistence). All returns series are stationary, plus none of the volatility series show signs of significant persistence/long memory effects.<sup>2</sup>

#### 2.2. Methodology

In this paper we use the lowest order EGARCH (1,1) model which removes satisfactorily the residual autocorrelation, ARCH and sign-ARCH effects noted in Table 2 from the underlying data series. Hence, the specification of the augmented-ARCH model we employ to test the information content of option implied volatility and of trading volume is as follows:

$$r_t = \mu_1 + \mu_2 r_{t-1} + \varepsilon_t$$
  

$$\varepsilon_t = \sqrt{h_t z_t}$$
  

$$z_t \sim N(0, 1)$$

<sup>&</sup>lt;sup>1</sup> Both series tend to have a leptokurtic distribution, with the volatility series being considerably right-skewed. Departures from normality are further illustrated by the Jarque-Bera statistic strongly rejecting the null hypothesis of normality at the 5% level of significance in all cases.

 $<sup>^2</sup>$  The differencing long memory parameter d of the volatility process is estimated from the Geweke and Hudak (1983) (GPH) regression with a bandwidth parameter equaling  $T^{0.6}$ , similar to Seungmook and Wohar (1992). Also to differentiate between long memory dependence and the presence of structural break, we repeat it for different temporal aggregates of data [using daily, weekly, biweekly and monthly frequencies as proposed by Gospodinov et al. (2006) in the spirit of Andersen et al. (2001). Also refer to Lamoureux and Lastrapes (1990b) and Diebold and Inoue (2001) for a discussion. The results show unstable estimates of d. We therefore test for the consistency of GPH estimates using the methodology proposed by Ohanissian et al. (2008), which rejects the null hypothesis of a true long memory for all stocks.

**Table 2**Descriptive statistics of the daily return series (%) and the daily volatility series (%) for the period from 2 January 2003 to 30 June 2008.

	AIG	IBM	GM	GE	WMT	HPQ	TXN	JNJ	XRX	SPX
Panel A: daily return	n series (%)									
Mean	-0.057	0.031	-0.084	0.007	0.008	0.068	0.045	0.013	0.038	0.026
SD (%)	1.725	1.204	2.381	1.223	1.184	1.859	2.065	0.939	1.770	0.888
Skewness	-0.633	-0.268	0.193	-0.788	0.226	-0.659	0.335	0.236	0.787	-0.104
Excess kurtosis	7.115	3.789	4.725	12.643	1.594	10.155	2.655	2.288	7.124	1.987
Minimum	-12.466	-8.662	-15.045	-13.684	-5.224	-16.790	-8.651	-3.922	-8.922	-3.569
Maximum	9.279	5.246	16.647	5.756	5.938	12.367	12.330	4.102	16.034	4.132
J-B	721	363	517	1499	95	1193	200	169	624	143
ARCH (10)	13.429	5.912	4.876	1.406	3.941	0.427	2.296	5.699	2.674	17.465
Q(10)	9.190***	9.680***	11.750***	9.490***	7.940***	12.470***	10.420***	10.960***	23.820***	21.350
Qs(10)	242.190***	92.220***	43.130***	16.050***	46.460***	4.770***	28.790***	94.510***	29.540***	308.530
ADF test	$-35.790^{***}$	$-37.500^{***}$	$-35.050^{***}$	$-38.370^{***}$	-38.000***	$-38.430^{***}$	$-37.240^{***}$	$-39.280^{***}$	$-41.830^{***}$	-41.450
Panel B: daily volati	lity series (%)									
Mean	0.030	0.031	-0.084	0.007	0.008	0.068	0.045	0.013	0.038	0.026
SD	0.090	0.035	0.147	0.057	0.027	0.120	0.092	0.018	0.095	0.016
Skewness	8.363	9.663	9.169	25.360	5.137	14.894	6.734	4.448	16.004	4.529
Excess kurtosis	99.057	159.540	130.370	803.030	40.300	285.550	70.915	24.587	380.760	27.141
Minimum	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	1.554	0.750	2.771	1.873	0.353	2.819	1.520	0.168	2.571	0.171
J-B	23825	16948	21390	327470	7227	102170	11406	9512	73645	8356
Q(10)	242.190***	92.220***	43.130***	16.050***	46.460***	4.770***	28.790***	94.510***	29.540***	308.530
d(m=1)	0.064	0.390	0.370	0.434	0.463	0.348	0.272	0.440	0.561	0.635
d(m=5)	0.029	0.430	0.283	0.234	0.378	0.521	0.241	0.248	0.299	0.752
d(m=10)	0.051	0.303	0.349	0.398	0.364	0.302	0.421	0.098	0.500	0.413
d(m=22)	0.039	0.084	0.533	0.180	0.705	0.280	0.180	0.045	0.156	0.518
W_stat	0.279	8.104**	4.306	13.627***	9.371**	8.780**	6.371	18.592***	27.097***	13.739

Notes: JB statistic measures the difference of the skewness and kurtosis of the series with those of the normal distribution. Under the null hypothesis of a normal distribution, the JB statistic follows a Chi<sup>2</sup> distribution with  $2^{\circ}$  of freedom. The F-stat of the test of autoregressive conditional heteroskedasticity (ARCH) in the residuals for up to 10th-order serial correlation is also reported. The Q(10) and Qs(10) are the Box-Pierce test statistics for the return (volatility) series and the squared return series for up to the 10th-order serial correlation, respectively. Under the null hypothesis of independence, the test statistic is distributed asymptotically as a Chi-square distribution with  $10^{\circ}$  of freedom.

The *t*-stat of the ADF test is reported. Null hypothesis is the time series contains a unit root I (1) process.

The fractional integration parameter d of the volatility process has been estimated from the log-periodogram regression of Geweke and Hudak (1983) for different temporal aggregates, including daily, weekly, biweekly and monthly. W.stat refers to the test statistic proposed by Ohanissian et al. (2008) to test the null hypothesis of a true long memory process, i.e. d(m=1)=d(m=5)=d(m=10)=d(m=22)=d where 0 < d < 0.5. The test statistic is distributed asymptotically as a Chi-square distribution with  $3^{\circ}$  of freedom.

<sup>\*\*</sup> Rejection at the 5% significance level.

<sup>\*\*\*</sup> Rejection at the 1% significance level.

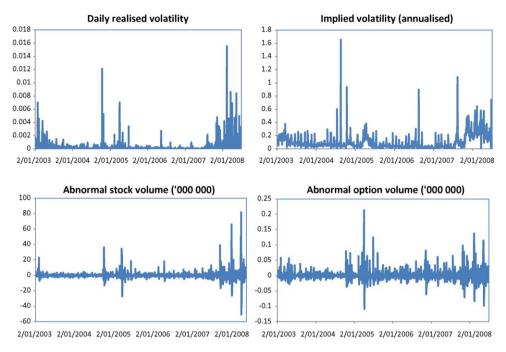


Fig. 1. The realized volatility series, the implied volatility series, the abnormal stock volume series and the abnormal option volume series of the stock AIG being plotted for the period from 2 January 2003 to 30 June 2008.

$$\ln(h_t) = \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 \ln h_{t-1} + \beta_{IVI} IVI_{t-1} + \beta_{AOV} AOV_{t-1} + \beta_{ASV} ASV_{t-1}$$
(1)

where  $r_t$  is the daily arithmetic stock return from close on day t-1 to close on day t;  $h_t$  is the conditional variance generated from the model based on past information;  $\varepsilon_t$  is the residual from the mean equation representing news about volatility;  $z_{t-1}$  is  $\varepsilon_{t-1}$  standardized on past volatility, which represents the leverage effect;  $IVI_{t-1}$  is the option implied volatility at the close on day t-1;  $AOV_{t-1}$  and  $ASV_{t-1}$  are the lagged option and stock abnormal volume amounts, respectively. We define abnormal trading volume as the unexpected above-average trading volume after filtering for stochastic trends, similar to Wagner and Marsh (2005). In order to remove a stochastic trend in volume series a standard moving average filtering method is applied. These procedures yield series of normalised volume, conditional on available information at any point in time,  $V_t$ . Abnormal volume is then computed as the de-trended volume series by taking the difference between the actual and normalised volume series.

For the purpose of this study,  $IVI_{t-1}$ ,  $AOV_{t-1}$  and  $ASV_{t-1}$  are designed to assess whether option implied volatility and trading volume contain any useful information of the future dynamics of stock return volatility, beyond what has been incorporated in past movements of the stock return. A graphical illustration of the intuitive relation between these factors and the return volatility can be found in Fig. 1 when comparing plots of these different series for the whole sample period.

In order to determine the performance of the model from adding option implied volatility and trading volume, we compare results in the empirical section when just one of the two additional

<sup>&</sup>lt;sup>3</sup> Both volume series have been rescaled to be out of 1,000,000 and of 1,000,000,000 for normal stocks and the index, respectively.

<sup>&</sup>lt;sup>4</sup> A non-centred 5 day-moving average is utilised in this paper, though in-sample analysis shows no significant qualitative difference would have occurred if the 20-day or 50-day moving average was used instead. As an alternative, we also considered the filtering method of Hodrick and Prescott (1997), but found the results did not change significantly. They also had a very high correlation with the moving average technique.

variables are added, a combined forecast from both, plus when both are included at the same time in the model.<sup>5</sup> Also, as a benchmark for an alternative to ARCH-type forecasting models, we check performance relative to three alternate specifications, that being an autoregressive integrated moving average model (ARMA(1,1)), a non-parametric principle component model (PCA) and a stochastic volatility model (SVM). The PCA (see Anderson and Vahid, 2007; Konstantinidi et al., 2008 for examples of this approach) specifies the volatility dynamics as a composition of a number of common factors

$$\Delta V_t = c_1 + \sum_{i=1}^{7} \phi_i PC_{t-1,i} + \varepsilon_t \quad \text{with } \varepsilon_t \sim N(0,1)$$
 (2)

where  $\Delta V_t$  is the change in volatility;  $PC_i$  with i=1,...,7 are the common factors; and  $\phi_{i,i}=1,...,7$  are coefficients to be estimated. Based on in-sample results of the statistical significance of parameter estimates and the explanatory power of the model measured by the adj- $R^2$ , only the first seven PCs are retained. For the SVM, we specify it as:

$$r_t = \sigma \sqrt{V_t} \varepsilon_t$$
 with  $\varepsilon_t \sim N(0, 1)$   $\ln(V_t) = \lambda \ln(V_{t-1}) + \sigma_v \xi_t$  with  $\xi_t \sim N(0, 1)$  (3)

where  $r_t$  is the daily stock return as similar to Eq. (1),  $V_t$  the "unobservable" stochastic volatility. The deterministic terms  $\lambda$  and  $\sigma_v$  measure the volatility persistence and the volatility of volatility, respectively.<sup>6</sup>

Forecast accuracy can be assessed with reference to statistical loss functions, namely mean absolute error (MAE), mean absolute logarithmic error (MALE), mean absolute percent error (MAPE), and a linear exponential function (LINEX). The major challenge is the choice of appropriate criterion because these measurements of error entail different implications on the forecast inaccuracy. The complication of this task is mainly due to the non-normality of the distribution of volatility which can either lead to the possibility of finding a significant difference between alternate forecast methods or to conflicting verdicts derived from different evaluation measures (Poon and Granger, 2003). Despite this matter, these statistics have often been used in the literature due to their simplicity for calculation and interpretation purposes.

We also employ the mean correct prediction (MCP) of the direction of change in volatility which is defined as the percentage of observations for which the model predicts a change of the same sign as the realized change in volatility. Its appraisal can be found in many papers (see GonÃÇalves and Guidolin, 2006; Konstantinidi et al., 2008) and is based on the perception that correct direction forecasts provide investors with profit opportunities through option trading. The key theme employed in our study however will be the orthogonality test which has proven to be a more powerful technique in comparing between forecasts, with its application being found in Day and Lewis (1992), Day (1993) and Donaldson and Kamstra (2005). Specifically, the verification of forecasting power is based on the regression of the "actual" volatility on multiple forecasts generated from different models (Mincer-Zarnowitz regression), i.e.  $\sigma_t^2 = \alpha + \sum_{i=1}^n \beta_i \hat{\sigma}_{ti}^2 + e_t$ , where  $\sigma_t^2$  is the ex post volatility, and  $\hat{\sigma}_{ti}^2$  is the forecasted volatility. The criteria of assessment include the t-statistic and the  $R^2$  (adj- $R^2$ ) of the regression.

The accuracy of the generated out of sample forecasts are also evaluated in an economic setting, using trading simulation to determine if profit arises from volatility forecasts. It is noted this evaluation approach is currently under active investigation in the literature (see, for example, Engle et al., 1993; West et al., 1993; Konstantinidi et al., 2008). While volatility is not a direct tradable asset for stocks (and most indices), volatility trading can still be achieved by establishing synthetic positions in options and their underlying assets. Similar to Guo (2000), "dynamic" trading on ATM straddles is simulated

<sup>&</sup>lt;sup>5</sup> It is worth highlighting that the authors tried several other specifications that allowed, for example, volume in the conditional mean equation, but none of the results led to a significant improvement in fit above and beyond the model illustrated in the paper.

<sup>&</sup>lt;sup>6</sup> Similar to Gospodinov et al. (2006), we apply the MCMC method to estimate the model parameters and generate forecasts recursively.

<sup>&</sup>lt;sup>7</sup> Refer to Gospodinov et al. (2006) for the algorithm and interpretation of these metrics.

in this study by simultaneously going long (short) on a call and a put of nearest-to-the-money and nearest-time-to-maturity available when the return volatility of the underlying asset is expected to increase (decrease). Dynamic trading requires closing the position on the next trading day. Whilst also taking into account transaction costs, the economic significance of these trading strategies over the long run will be evaluated using traditional measurements of portfolio performance, namely the Sharpe ratio and the Leland's (1999) modified alpha to account for non-normality, along with their 95% bootstrapped confidence interval.

# 3. Empirical results

## 3.1. In sample testing

To assess whether lagged implied volatility and trading volume help to improve forecasting stock return volatility, we examine the regression results of the augmented-EGARCH (1,1) model specified in Eq. (1). Table 3 shows joint significance for both factors has been found in six of the stocks. In particular, it is observed that the estimate of volatility persistence ( $\alpha_3$ ) drops remarkably after the introduction of option implied volatility and trading volume into the conditional variance equation. An intuitive explanation, similar to Wagner and Marsh's (2005) suggestion, would be that the benchmark EGARCH (1,1) model is likely to be restrictive in explaining volatility persistence. Moving across the table,  $IVI_{t-1}$  takes the expected sign for all stocks and the index, and indicates a positive relationship between the market expectation of future stock volatility reflected in option price and the realized volatility. Except for one stock (GE), a positive relationship between lagged trading volume and future stock volatility is also evident in cases where volume is statistically significant at the conventional 5% level of significance. This provides early indicative support to the sequential information hypothesis which predicts that future stock volatility is positively correlated to the abnormal volume generated from trading activities revealing news into the market.

Considering the construction of two volume variables is meant to capture the informed trading component in each market, the testing results would address whether each proxy of the arrival of news has any additional information content beyond what has been captured by the other (refer to Chordia, 2009 for a similar approach in the context of testing different proxies of liquidity). Our finding, being option trading volume stays significant at the presence of stock trading volume in the same regression, suggests that the incorporation of information would have been initiated through the speculative trading activities of informed traders in the option market. Our conjecture builds on Black's (1975) argument that informed traders may prefer to trade in the options market due to the high financial leverage, reduced transactions costs and wider trading opportunities (e.g. speculation on volatility). Indirect evidence of informed trading in the options market has been highlighted through numerous testing methods, including the causality test (Easley et al., 1998; Anthony, 1988), and the "information share" approach (Chakravarty et al., 2004). Intriguingly, the insignificance of the abnormal stock trading volume suggests that the rejection of the sequential information hypothesis in earlier studies into the stock market (Brooks, 1998; Wagner and Marsh, 2005) may arise from the omission of option trading activities.

In addition, the statistical significance of  $IVI_{t-1}$  and  $AOV_{t-1}$  suggests each factor possesses a separate information set of the future dynamics of the return volatility of the underlying asset. The caveat of the above analysis is that the implied volatility derived from the prevailing option price would not capture the impact of trading activities attributable to the heterogeneity of traders at the arrival of new information. This aspect is re-examined in later analyses (viz., Table 4 onwards). The forecasting quality of both factors is further confirmed by the enhanced model fit, as illustrated by an improvement in the AIC, the Schwartz criteria and the log likelihood statistic of the augmented-model relative to the benchmark EGARCH (1,1) across the sample. At the same time, results of the ARCH and Ljung-Box

<sup>&</sup>lt;sup>8</sup> Refer to Leland (1999) for the argument of why these metrics are more appropriate to measure the performance of portfolios containing options or assets with non-linear payoffs.

**Table 3**Results of in-sample hypothesis testing.

	AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
$\mu_1$	-0.0001	0.0003	-0.0001	0.0010	0.0004	0.0005	0.0002	-0.0001	0.0005	0.0002
	(0.0003)	(0.0003)	(0.0005)	(0.0005)**	(0.0003)	(0.0005)	(0.0002)	(0.0003)	(0.0004)	(0.0002)
$\mu_2$	0.050	-0.009	0.077	-0.032	-0.019	0.016	-0.038	-0.012	-0.138	-0.055
	(0.031)**	(0.032)	(0.030)***	(0.030)	(0.029)	(0.026)	(0.032)	(0.032)	(0.032)***	(0.026)**
$\alpha_0$	-2.18	-0.89	-5.76	-2.97	-2.44	-1.26	-0.21	-6.57	-1.95	-3.74
-	(0.98)**	(0.30)***	(1.78)***	(0.70)***	(0.68)***	(0.39)***	(0.05)***	(2.94)**	(0.57)***	(0.81)***
$\alpha_1$	0.10	0.08	0.06	0.11	0.06	-0.03	-0.02	0.17	0.19	-0.13
-	(0.045)**	(0.039)**	(0.079)	(0.066)**	(0.051)	(0.036)	(0.015)*	(0.076)**	(0.068)***	(0.080)**
$\alpha_2$	-0.07	-0.03	0.07	-0.03	-0.10	-0.02	-0.02	0.04	-0.05	-0.24
2	(0.045)*	(0.024)	(0.062)	(0.062)	(0.043)***	(0.026)	(0.014)*	(0.046)	(0.040)	(0.039)***
$\alpha_3$	0.81	0.93	0.41	0.70	0.78	0.87	0.98	0.33	0.80	0.67
-	(0.088)***	(0.027)***	(0.188)**	(0.074)***	(0.062)***	(0.041)***	(0.005)***	(0.321)	(0.060)***	(0.072)***
$\beta$ _IVI	1.77	0.65	2.76	1.33	1.60	0.67	0.24	1.81	0.57	0.04
	(0.837)**	(0.229)***	(0.801)***	(0.358)***	(0.493)***	(0.188)***	(0.067)***	(0.632)***	(0.202)***	(0.009)***
$\beta$ _AOV	11.52	1.61	-2.30	7.27	3.60	7.69	1.49	0.26	-6.40	-0.62
	(4.906)***	(0.576)***	(1.020)**	(4.797)*	(1.821)**	(3.573)**	(5.094)	(2.312)	(6.965)	(0.617)
$\beta$ -ASV	-0.02	0.00	0.01	-0.02	0.03	-0.01	0.02	0.00	0.03	0.00
	(0.030)	(0.005)	(0.012)	(0.013)*	(0.025)*	(0.008)	(0.016)*	(0.012)	(0.016)**	(0.145)
LLH	3602.1	3736.2	2926.1	3146.8	3717.9	2979.2	3900.1	3616.5	3204.9	4149.1
AIC	-6.12	-6.34	-4.97	-5.34	-6.31	-5.06	-6.62	-6.15	-5.44	-7.05
Schwarz	-6.08	-6.31	-4.93	-5.30	-6.27	-5.02	-6.58	-6.12	-5.40	-7.01
Skewness	-0.49	0.18	0.36	-0.23	-0.03	0.25	0.20	0.09	0.10	-0.43
Kurtosis	9.69	3.82	8.38	8.71	4.52	4.18	5.07	4.20	4.98	3.90
LB (10)	3.92	4.37	5.44	15.20	7.43	10.43	3.40	14.63	9.63	10.46
LB_2 (10)	3.39	6.67	1.04	2.16	10.07	8.22	9.29	6.49	6.92	10.96
ARCH (10)	3.40	6.56	0.95	2.11	10.14	8.42	9.34	6.59	6.67	10.27

The specification of the model underlying the results reported is as follows:

$$\begin{split} r_t &= \mu_1 + \mu_2 r_{t-1} + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t z_t} \\ z_t \sim N(0, 1) \\ \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 \ln h_{t-1} + \beta_{IVI} IVI_{t-1} + \beta_{AOV} AOV_{t-1} + \beta_{ASV} ASV_{t-1} \end{split}$$

Within the model construction,  $r_t$  is the daily return from close on day t-1 to close on day t, -t is the disturbance term,  $z_t$  is the standardized residual of stock return,  $z_t$  is the stock return variance conditional on the information set at time  $z_t$ ,  $z_t$  is one lag weighted implied volatility generated from eight options nearest to the money and nearest time to maturity using settlement price at close on day  $z_t$  are measurements of the abnormal trading volume on day  $z_t$  relative to its one-week lagged moving average of stocks and options, respectively.

Notes: Standard errors are in the parentheses below corresponding to the parameter estimates. LLH is the value of maximised Gaussian log likelihood. AIC (Akaike information criteria) and Schwarz are two information criteria which indicate the model fit. The LB(10) and LB.2(10) are the Ljung-Box test statistics at the 10th lag of the standardized residuals and standardized squared residuals of the estimated model. Under the null hypothesis of no autocorrelation, the test statistic is distributed asymptotically as Chi-square distribution with 10° of freedom. The statistics of the Engle (1982)'s LM ARCH test is also reported. It also follows a Chi-square distribution under the null hypothesis of no ARCH effects. The model has been estimated for the period from 2 January 2003 to 21 November 2007. The distribution of the standardized residuals has also been reported in terms of skewness and kurtosis.

- \* Rejection at the 10% significance level.
- \*\* Rejection at the 5% significance level.
- \*\*\* Rejection at the 1% significance level.

 Table 4

 In sample testing for different specifications of the conditional variance equation.

	AIG	GM	GE	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
Panel A	A: Akaike int	formation c	riterion							
(1)	-6.082	-6.327	-4.957	-5.286	-6.244	-5.000	-6.602	-6.142	-5.422	-6.988
(2)	-6.102	-6.339	$-4.966^{a}$	-5.334	-6.306	-5.047	-6.620	-6.144	-5.439	-7.047
(3)	-6.100	-6.331	-4.960	-5.307	-6.278	-4.969	-6.605	-6.142	-5.423	-6.990
(4)	-6.093	-6.329	-4.943	-5.293	-6.281	-5.002	-6.606	-6.140	-5.436	-6.986
(5)	-6.098	-6.330	-4.960	-5.334	-6.283	-5.025	-6.605	-6.140	-5.431	-6.988
(6)	$-6.117^{a}$	$-6.346^{a}$	$-4.966^{a}$	-5.340	-6.312	$-5.057^{a}$	-6.621	-6.142	-5.438	$-7.049^{a}$
(7)	-6.103	-6.341	-4.965	-5.333	-6.307	-5.045	-6.620	$-6.146^{a}$	-5.437	-7.047
(8)	-6.116	-6.344	-4.965	$-5.341^{a}$	$-6.313^{a}$	-5.056	$-6.623^{a}$	-6.145	$-5.440^{a}$	-7.047
Panel E	3: Schwarz i	nformation	criterion							
(1)	-6.056	-6.301	-4.931	-5.260	-6.218	-4.974	-6.576	-6.116	-5.397	-6.962
(2)	-6.072	-6.309	$-4.936^{a}$	-5.303	-6.276	-5.017	$-6.590^{a}$	-6.114	$-5.409^{a}$	$-7.017^{a}$
(3)	-6.069	-6.301	-4.929	-5.277	-6.247	-4.939	-6.575	-6.112	-5.393	-6.959
(4)	-6.063	-6.299	-4.913	-5.263	-6.251	-4.971	-6.575	-6.110	-5.405	-6.956
(5)	-6.064	-6.296	-4.926	-5.299	-6.249	-4.990	-6.571	-6.106	-5.397	-6.953
(6)	$-6.082^{a}$	$-6.311^{a}$	-4.932	$-5.305^{a}$	$-6.278^{a}$	$-5.022^{a}$	-6.587	-6.108	-5.403	-7.014
(7)	-6.068	-6.307	-4.930	-5.298	-6.273	-5.011	-6.585	-6.111	-5.403	-7.012
(8)	-6.077	-6.305	-4.926	-5.302	-6.274	-5.017	-6.584	-6.117 <sup>a</sup>	-5.401	-7.008

The specification of models underlying the results reported is as follows:

```
\begin{split} r_t &= \mu_1 + \mu_2 r_{t-1} + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t z_t} \\ z_t \sim N(0,1) \\ (1) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} \\ (2) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{IVI} IVI_{t-1} \\ (3) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{AOV} AOV_{t-1} \\ (4) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{AOV} AOV_{t-1} \\ (5) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{AOV} AOV_{t-1} + \beta_{ASV} ASV_{t-1} \\ (6) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{IVI} IVI_{t-1} + \beta_{AOV} AOV_{t-1} \\ (7) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{IVI} IVI_{t-1} + \beta_{AOV} ASV_{t-1} \\ (8) \ln(h_t) &= \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 & \ln h_{t-1} + \beta_{IVI} IVI_{t-1} + \beta_{AOV} AOV_{t-1} + \beta_{ASV} ASV_{t-1} \\ \end{split}
```

where  $IVI_{t-1}$  is one lag weighted implied volatility generated from eight options nearest to the money and nearest time to maturity using settlement price at close on day t-1.  $AOV_{t-1}$  and  $ASV_{t-1}$  are measurements of the abnormal trading volume on day t-1 relative to its 1-week lagged moving average of stocks and options, respectively.

Notes: The Akaike information criterion is defined as the obtained maximum log-likelihood, penalised for the number of coefficients in the model AIC = -2l/T + 2k/T where l is log-likelihood, k is the number of parameters and T is the number of observations. The Schwarz criterion is an alternative to AIC which imposes a larger penalty for additional coefficient (s). The results are reported for the period from 2 January 2003 to 21 November 2007.

tests on the standardized residuals and squared residuals listed at the bottom of the table indicate that volatility clustering in the residuals is not significant.

In our next step, a progression of different model specifications has been examined empirically in order (1) to understand better the information content of option implied volatility and trading volume and (2) to provide justification on competing hypotheses regarding the volume–volatility relation. We report in Table 4 statistics of the empirical fit which would help to address the question of how to best model the conditional variance of stock return. While the AIC criterion in panel A provides an overwhelming evidence supporting a nested model of the implied volatility and trading volume (being specified as either an integrated IVI-AOV-EGARCH (1,1) model in row 6 or an integrated IVI-AOV-ASV-EGARCH (1,1) model in row 8), a greater penalty on additional coefficients imposed by the Schwarz criterion means that the former seems to provide the most suitable specification overall. This verdict is broadly consistent with the insignificant evidence of any additional information content of stock trading volume found previously.

To provide further insight into the impact of incorporating both factors into the ARCH framework, Table 5 sets out four common statistics of the in-sample performance, namely the MAE, MAPE, MALE and LINEX. Considering the insignificance of stock trading volume in the evidence at this stage, the

<sup>&</sup>lt;sup>a</sup> Highlights the model of best fit.

**Table 5**Statistics of the in-sample performance with daily squared return being used as the realized volatility.

		AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
Panel A: EGARCH (1,1)	MAE MAPE MALE LINEX	0.0468 120.444 1.8504 0.0008	0.0417 58.082 1.7942 0.0014	0.1220 112.293 1.8156 0.0022	0.0834 45.383 1.6597 0.0014	0.0464 194.833 1.9711 0.0023	0.1048 43.158 1.6589 0.0036	0.0468 202.897 2.2289 0.0035	0.0474 120.069 1.8381 0.0027	0.0637 12.577 1.5199 0.0002	0.0230 7547.419 2.2615 0.0003
Panel B: IVI-EGARCH (1,1)	MAE MAPE MALE LINEX	0.0493 116.024 1.8388 0.0014	0.0288 <sup>^</sup> 46.470 <sup>^</sup> 1.6542 <sup>^</sup> 0.0003 <sup>^</sup>	0.1395 121.198 1.9494 0.0067	0.0943 69.110 1.8653 0.0026	0.0327 <sup>^</sup> 131.045 <sup>^</sup> 1.8067 <sup>^</sup> 0.0003 <sup>^</sup>	0.1118 57.789 1.7589 0.0045	0.0235 <sup>^</sup> 93.852 <sup>^</sup> 1.8644 <sup>^</sup> 0.0002 <sup>^</sup>	0.0342 <sup>^</sup> 79.521 <sup>^</sup> 1.7935 <sup>^</sup> 0.0003 <sup>^</sup>	0.0749 29.999 1.7134 0.0015	0.0159 <sup>^</sup> 4409.740 <sup>^</sup> 1.8540 <sup>^</sup> 0.0001 <sup>^</sup>
Panel C: AOV-EGARCH (1,1)	MAE MAPE MALE LINEX	0.0522 210.828 1.8292 0.0036	0.0296 57.788 1.6519 0.0004	0.1396 121.149 1.9486 0.0067	0.0938 70.845 1.8598 0.0025	0.0327 133.868 1.8019 0.0003	0.1094 59.206 1.7479 0.0037	0.0235 <sup>^</sup> 93.287 <sup>^</sup> 1.8602 <sup>^</sup> 0.0002 <sup>^</sup>	0.0342 <sup>^</sup> 79.905 <sup>^</sup> 1.7935 <sup>^</sup> 0.0003 <sup>^</sup>	0.0750 29.895 1.7142 0.0015	0.0160 <sup>^</sup> 4655.127 <sup>^</sup> 1.8526 <sup>^</sup> 0.0001 <sup>^</sup>
Panel D: IVI-AOV-EGARCH (1,1)	MAE MAPE MALE LINEX	0.0516 173.312 1.8306 <sup>^*</sup> 0.0027	0.0294 <sup>2</sup> 55.011 <sup>2</sup> 1.6522 <sup>2</sup> 0.0003 <sup>2</sup>	0.1399 121.106* 1.9468* 0.0069	0.0931* 70.548 1.8534* 0.0024*	0.0329 <sup>^</sup> 134.262 <sup>^</sup> 1.8024 <sup>^,*</sup> 0.0003 <sup>^</sup>	0.1092* 59.840 1.7482* 0.0037*	0.0237 94.636 1.8609 <sup>*</sup> 0.0002	0.0342 <sup>^</sup> 79.586 <sup>^</sup> 1.7937 <sup>^</sup> 0.0003 <sup>^</sup>	0.0762 30.381 1.7154 0.0018	0.0160 <sup>^</sup> 4655.039 <sup>^</sup> 1.8526 <sup>^*</sup> 0.0001 <sup>^*</sup>
Panel E: COMB(IVI- EGARCH(1,1),AOV-EGARCH(1,1))	MAE MAPE MALE LINEX	0.0482* 120.491 1.9562 0.0007^,*	0.0289 <sup>^</sup> 45.481 <sup>^*</sup> 1.6407 <sup>^*</sup> 0.0003 <sup>^</sup>	0.1360* 114.395* 1.9724 0.0047*	0.0939* 58.147* 1.8278* 0.0028	0.0324 <sup>^*</sup> 130.303 <sup>^*</sup> 1.8052 <sup>^*</sup> 0.0003 <sup>^*</sup>	0.1103* 61.274 1.7655 0.0033^.*	0.0234 <sup>°</sup> * 92.017 <sup>°</sup> * 1.8639 <sup>°</sup> * 0.0002 <sup>°</sup> *	0.0341 <sup>*</sup> 84.277 <sup>*</sup> 1.7826 <sup>*</sup> 0.0003 <sup>*</sup>	0.0754 30.673 1.7249 0.0015	0.0155 <sup>*</sup> 4706.712 <sup>°</sup> 1.8840 <sup>°</sup> 0.0001 <sup>°</sup>

Table 5 (Continued)

		AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
Panel F: COMB (AOV-EGARCH (1,1), IVI)	MAE	0.0495	0.0300 <sup>^</sup>	0.1358*	0.0943*	0.0327 <sup>^</sup>	0.1109*	0.0235 <sup>^</sup>	0.0340^*	0.0771	0.0157 <sup>°</sup>
	MAPE	131.506	54.103 <sup>^</sup>	121.214	55.791*	128.420 <sup>^*</sup>	59.630	86.285 <sup>^*</sup>	85.881^	34.387	5290.427 <sup>°</sup>
	MALE	1.9298	1.6949 <sup>^</sup>	1.9503	1.8195*	1.7795 <sup>^*</sup>	1.7580*	1.8756 <sup>^</sup>	1.7813^*	1.7720	1.8745 <sup>°</sup>
	LINEX	0.0007 <sup>^*</sup>	0.0002 <sup>^,*</sup>	0.0048*	0.0028	0.0003 <sup>^</sup>	0.0034^.*	0.0001 <sup>^*</sup>	0.0003^	0.0013*	0.0001 <sup>°</sup>

The specification of models underlying the results reported is as follows:

```
\begin{split} r_t &= \mu_1 + \mu_2 r_{t-1} + \varepsilon_t \\ \varepsilon_t &= \sqrt{h_t z_t} \\ z_t &= iid \, N(0,1) \\ (1) &\ln(h_t) = \alpha_0 + \alpha_1 \, |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 \, \ln \, h_{t-1} \\ (2) &\ln(h_t) = \alpha_0 + \alpha_1 \, |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 \, \ln \, h_{t-1} + \beta_{NI} IVI_{t-1} + \beta_{AOV} AOV_{t-1} + \beta_{ASV} ASV_{t-1} \\ (3) &\ln(h_t) = \alpha_0 + \alpha_1 \, |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 \, \ln \, h_{t-1} + \beta_{IVI} IVI_{t-1} \\ (4) &\ln(h_t) = \alpha_0 + \alpha_1 \, |z_{t-1}| + \alpha_2 z_{t-1} + \alpha_3 \, \ln \, h_{t-1} + \beta_{AOV} AOV_{t-1} \\ (5) &\hat{\sigma}_t^2 = \alpha_0 + \mu_1 (\hat{h}_{IVI-egarch})_t + \mu_2 (\hat{h}_{AOV-egarch})_t \\ (6) &\hat{\sigma}_t^2 = \alpha_0 + \mu_1 (IV\hat{I})_t + \mu_2 (\hat{h}_{AOV-egarch})_t \end{split}
```

where  $IVI_{t-1}$  is one lag weighted implied volatility generated from eight options nearest to the money and nearest time to maturity using settlement price at close on day t-1.  $AOV_{t-1}$  and  $ASV_{t-1}$  are measurements of the abnormal trading volume on day t-1 relative to its 1-week lagged moving average of stocks and options, respectively.

Notes: This table represents the in-sample fitting performance in terms of different statistical loss functions, including the mean absolute error (MAE), the mean absolute percentage error

(MAPE), the mean absolute logarithmic error (MALE) and the linear exponential function (LINEX), using annualised measurement of volatility and its forecasts. Details of the calculation and interpretation of these metrics can be referred to in Gospodinov et al. (2006). The results are reported for the period from 2 January 2003 to 21 November 2007.

^indicates better performance relative to the benchmark EGARCH (1,1) model.

<sup>\*</sup> indicates better performance relative to the integrated IVI-EGARCH (1,1) model.

set of models of our interest encompasses different forms of integrating the option implied volatility (IVI) and the abnormal option volume (AOV) into the EGARCH (1,1) specification in panel B through to D, and two combined forecasts in panels E and F.

While the integrated models are central in validating the information content of option implied volatility and trading volume in our previous discussion, the practice of combining forecasts introduces a novel way to account for the separate effects of those factors. This paper proposes two alternate models out of many different specifications which are capable of optimally utilizing different information sets possessed by option implied volatility and trading volume to improve forecasts. One employs forecasts generated by each of the augmented IVI-EGARCH (1,1) model and the augmented AOV-EGARCH (1,1) model conditional on past information, while another makes use of naïve forecasts from option implied volatility combined with forecasts of the AOV-EGARCH (1,1) model. Both forms of combined forecasts can be obtained in a joint linear regression defined as follows:

$$\hat{\sigma}_t^2 = \alpha_0 + \mu_1 (\hat{h}_{\text{IVI-egarch}})_t + \mu_2 (\hat{h}_{\text{AOV-egarch}})_t \tag{4}$$

$$\hat{\sigma}_t^2 = \alpha_0 + \mu_1(IV\hat{I})_t + \mu_2(\hat{h}_{AOV-egarch})_t \tag{5}$$

where  $(\hat{h}_{\text{IVI-egarch}})_t$ ,  $(\hat{h}_{\text{AOV-egarch}})_t$  are forecasts generated from integrating lagged option implied volatility and lagged abnormal option trading volume respectively into an EGARCH (1,1) model;  $IV\hat{l}_t$  is the naïve forecast derived from the historical implied volatility, which means  $IV\hat{l}_t = IVI_{t-1}$  where IVI is the implied volatility prevailing in the market at time (t-1). Though the parameter estimates of the combining regressions for the in-sample period are not presented here, it is noticed that the statistical significance of  $\mu_1$  and  $\mu_2$  suggests that implied volatility and trading volume each possesses different information sets with regards to the future dynamics of volatility. Hence, one's contribution to improve volatility forecast is independent of another.

With daily squared return being used to measure the realized volatility in Table 5, it is clearly depicted that the introduction of either option implied volatility or volume or both into the EGARCH (1,1) variance equation in panels B through F helps to reduce the gap between the model estimate of volatility and the actual volatility. This is illustrated by an improvement across the majority of the sample. While the MAE results show no clear distinction between the relative performances between models on a cross-sectional basis, models of combining forecasts in the last two panels outperform the EGARCH (1,1) model under the LINEX criterion. Of particular interest, while MAPE seems to favour the IVI-EGARCH (1,1) model in panel B, the AOV-EGARCH (1,1) model in panel C is more preferred under the MALE statistic cross-sectionally. Consequently, the integration of both factors into the EGARCH (1,1) model shown in panel D leads to its outperformance over each of the IVI-EGARCH (1,1) model and the AOV-EGARCH (1,1) model under MALE and MAPE, respectively. Another interesting observation is that two combined forecasts perform relatively better than the IVI-GARCH (1,1) model for the majority of stocks in the sample. The model which combines forecasts of the IVI-EGARCH (1,1) model and of the AOV-EGARCH (1,1) model in particular provides the best performance, as one observes much lower MAPE across the board relative to the rest of the models.

It is worth noting that the MAPE is relatively high across the table. One possibility is bad proxy being used while another points to a poor forecast performance due to model misspecifications. We re-examine these aspects by (1) comparing the characteristics of different proxies of realized volatility and (2) by re-assessing the performance of these models in a different context in later analyses (viz., Table 6 where the sum of intra-day squared returns at 5-min intervals provides a more appropriate proxy of daily realized volatility, see Poon and Granger, 2003). In sum, the proper inference seems to be that daily squared returns misrepresent the true latent volatility process<sup>9</sup> [refer to Andersen and Bollerslev, 1998; Andersen et al., 1999 for a formal scrutiny]. To avoid misleading results inherent from this problem, we therefore use the 22-day-average daily squared returns as an alternative to proxy for daily realized volatility, similar to Gospodinov et al. (2006). Results, not being reproduced

<sup>&</sup>lt;sup>9</sup> The possibility of model specification is also unlikely since we found the performance of all models worsened after applying the ad-hoc intercept-correction proposed by Gospodinov et al. (2006).

**Table 6**Statistics of the out-of-sample performance with intra-day squared return being used as the realized volatility.

	•					-					
		AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
Panel A: our models											
Panel A1: IVI-AOV-EGARCH (1,1)	MAE MAPE MALE LINEX	0.2673 1.8976* 0.9362 0.0401	0.0983 1.9368 0.7512 0.0113	0.2461* 1.2868* 0.7876* 0.0307*	0.0732 <sup>^</sup> 0.5874 <sup>^</sup> 0.5889 <sup>^</sup> 0.0014 <sup>^</sup>	0.0671 0.7742* 0.6608 0.0015	0.0969 <sup>*</sup> 0.7304 <sup>*</sup> 0.6797 <sup>*</sup> 0.0019 <sup>°</sup>	0.0148 <sup>*</sup> 0.8436 <sup>*</sup> 0.6836 <sup>*</sup> 0.0001 <sup>*</sup>	0.0434 <sup>^</sup> 0.6278 <sup>^*</sup> 0.6840 <sup>^*</sup> 0.0003 <sup>*</sup>	0.0656 <sup>^*</sup> 0.9584 <sup>*</sup> 0.6131 <sup>^*</sup> 0.0021	0.0291 0.9362 0.6446 0.0006
Panel A2: COMB(IVI- EGARCH(1,1),AOV- EGARCH(1,1))	MAE MAPE MALE LINEX	0.1792 <sup>*</sup> 0.8075 <sup>*</sup> 0.7380 <sup>*</sup> 0.0064 <sup>*</sup>	0.0759 1.3401 0.7730 0.0014	0.2204* 1.0308^* 0.7255* 0.0194*	0.0728 <sup>°</sup> * 0.6311 <sup>°</sup> 0.5925 0.0016 <sup>°</sup>	0.0611* 0.6158* 0.6030* 0.0010*	0.0961 <sup>°</sup> * 0.7331 <sup>°</sup> * 0.6714 <sup>°</sup> * 0.0018 <sup>°</sup>	0.0154 <sup>^</sup> 0.9271 0.7022 <sup>^*</sup> 0.0001	0.0437 <sup>^</sup> 0.6299 <sup>^*</sup> 0.6900 <sup>^</sup> 0.0003 <sup>^*</sup>	0.0679 0.9515* 0.6454 0.0020*	0.0269 0.8224 0.6068 0.0005
Panel A3: COMB (AOV-EGARCH (1,1), IVI)	MAE MAPE MALE LINEX	0.1785 <sup>*</sup> 0.9379 <sup>*</sup> 0.7191 <sup>*</sup> 0.0072 <sup>*</sup>	0.0729 0.9358^* 0.6810^* 0.0028	0.2227* 1.0471^;* 0.7319* 0.0189*	0.0744 0.6066 0.6320 0.0014	0.0652* 0.7889 0.6405* 0.0014	0.0988 0.7445^* 0.6979^ 0.0018^	0.0147 <sup>*</sup> 0.8333 <sup>*</sup> 0.6691 <sup>*</sup> 0.0001 <sup>*</sup>	0.0425 <sup>*</sup> 0.6544 <sup>°</sup> 0.6619 <sup>*</sup> 0.0004	0.0643 <sup>^*</sup> 0.8357 <sup>^*</sup> 0.5950 <sup>^*</sup> 0.0015 <sup>^*</sup>	0.0279 0.9068 0.6308 0.0005
Panel B: Benchmark mode		0.4040	0.000	0.0404	0.0704	0.0500	0.0005	0.0454	0.0450	0.0000	0.0000
Panel B1: EGARCH (1,1)	MAE MAPE MALE LINEX	0.1849 0.9716 0.7504 0.0097	0.0697 1.0623 0.7366 0.0008	0.2101 1.0769 0.7049 0.0173	0.0734 0.7348 0.5914 0.0020	0.0596 0.5089 0.5996 0.0007	0.0985 0.8798 0.7009 0.0021	0.0154 0.9222 0.7074 0.0001	0.0450 0.6884 0.7210 0.0003	0.0676 0.9405 0.6418 0.0020	0.0266 0.6541 0.5946 0.0003
Panel B2: IVI-EGARCH (1,1)	MAE MAPE MALE LINEX	0.2557 1.9638 0.9095 0.0379	0.0685 <sup>^</sup> 1.0292 <sup>^</sup> 0.7109 <sup>^</sup> 0.0009	0.2551 1.3350 0.8102 0.0317	0.0730 <sup>^</sup> 0.5708 <sup>^</sup> 0.5844 <sup>^</sup> 0.0012 <sup>^</sup>	0.0656 0.7864 0.6531 0.0014	0.0973 <sup>^</sup> 0.7543 <sup>^</sup> 0.6885 <sup>^</sup> 0.0017 <sup>^</sup>	0.0153 <sup>^</sup> 0.9257 0.7100 0.0001 <sup>^</sup>	0.0434 <sup>2</sup> 0.6342 <sup>2</sup> 0.6870 <sup>2</sup> 0.0003	0.0657 <sup>^</sup> 0.9603 0.6198 <sup>^</sup> 0.0020	0.0285 0.9156 0.6301 0.0006
Panel B3: AOV-EGARCH (1,1)	MAE MAPE MALE LINEX	0.2198 1.2744 0.7975 0.0232	0.0992 1.9383 0.7727 0.0111	0.1975 <sup>^</sup> 0.8135 <sup>^</sup> 0.6506 <sup>^</sup> 0.0129 <sup>^</sup>	0.0793 0.7628 0.6687 0.0018	0.0634 0.5849 0.6610 0.0010	0.0984 0.8889 0.7023 0.0024	0.0151 <sup>^</sup> 0.8653 <sup>^</sup> 0.6873 <sup>^</sup> 0.0001	0.0444 <sup>^</sup> 0.6092 <sup>^</sup> 0.7092 <sup>^</sup> 0.0003 <sup>^</sup>	0.0661 <sup>^</sup> 0.8731 <sup>^</sup> 0.6298 <sup>^</sup> 0.0019 <sup>^</sup>	0.0274 0.6789 0.6153 0.0003

Panel B4: ARMA (1,1)	MAE	0.1959*	0.0773	0.2105*	0.0783	0.0618*	0.1017	0.0146 <sup>*</sup>	0.0447	0.0692	0.0328
	MAPE	1.0551*	1.2911	0.8820^*	0.6886	0.5496*	0.8786	0.8676 <sup>*</sup>	0.7078	0.6896 <sup>°,*</sup>	1.1065
	MALE	0.8043*	0.7899	0.7100*	0.6527	0.6420*	0.7432	0.6903 <sup>*</sup>	0.7126	0.6838	0.7411
	LINEX	0.0106*	0.0015	0.0107^*	0.0012 <sup>^*</sup>	0.0007^,*	0.0017	0.0001 <sup>*</sup>	0.0004	0.0009 <sup>°,*</sup>	0.0005*
Panel B5: PCA	MAE	0.2891	0.1216	0.2852	0.1229	0.1053	0.1575	0.0226	0.0694	0.1255	0.0504
	MAPE	1.8362*	2.1816	1.1185*	1.4757	1.3277	1.6464	1.2352	1.2807	1.5290	1.5860
	MALE	2.3629	1.2373	1.7228	2.7692	2.0239	2.3940	2.6267	1.3865	2.0297	1.5212
	LINEX	0.0358*	0.0178	0.0438	0.0082	0.0053	0.0108	0.0002	0.0026	0.0092	0.0016
Panel B6: SVM	MAE	0.2269*	0.0788	0.6318	0.8441	0.1684	0.1182	0.4321	0.4085	0.4422	0.0361
	MAPE	1.5391*	1.4672	3.3144	15.5947	3.2130	1.3974	35.4781	13.8973	8.6155	1.3625
	MALE	0.8951*	0.8020	1.2628	2.1168	1.0333	0.8462	2.1115	1.7629	1.6781	0.7896
	LINEX	0.0219*	0.0021	0.2266	0.3006	0.0353	0.0046	0.1804	0.1188	0.1219	0.0007

Panel A comprises of different model specifications which incorporate the information sets of both option implied volatility and trading volume within the EGARCH (1,1) framework, including the integrated EGARCH (1,1) model (panel A1), the combined forecast of the IVI-EGARCH (1,1) and the AOV-EGARCH (1,1) (panel A2), as well as the combined forecast of the AOV-EGARCH (1,1) and option implied volatility (panel A3). Their performance will be compared to the benchmark EGARCH (1,1) model (panel B1), the integrated IVI-EGARCH (1,1) model (panel B2), the integrated AOV-EGARCH (1,1) model (panel B3), the ARMA (1,1) model (panel B4), the principal component analysis model (panel B5) and the stochastic volatility model (panel B6). Details of models in panels A1 through to B3 are analogous to those presented in the in-sample test in Table 5, while the specifications of the last three models can be found in Eqs. (3)–(5) in the methodology section.

Notes: This table represents the out-of-sample forecast performance in terms of different statistical loss functions, including the mean absolute error (MAE), the mean absolute percentage error (MAPE), the mean absolute logarithmic error (MALE) and the linear exponential function (LINEX), using annualised measurement of volatility and its forecasts. Details of the calculation and interpretation of these metrics can be referred to in Gospodinov et al. (2006). The results are reported for daily forecasts over the period from 24 November 2007 to 30 June 2008.

<sup>^</sup>indicates better performance relative to the benchmark EGARCH (1,1) model.

<sup>\*</sup> indicates better performance relative to the integrated IVI-EGARCH (1.1) model.

here, yield a very similar observation of forecast improvement while the MAPE statistic is maintained at a reasonable level of less than 1 at this instance.

Overall, the in-sample evidence thus far demonstrates that the combination of both factors either by way of integrating them in an EGARCH (1,1) variance equation or through the exercise of combining forecasts leads to a better performance than the use of option implied volatility alone.

#### 3.2. Out-of-sample forecast performance

In this section, the out-of-sample forecast performance of six models previously considered is assessed based on the statistical criteria and the economic significance of forecast accuracy obtained through trading simulation, in direct comparison to three other techniques specified in the methodology section. The forecast exercise is performed for the period from 24th October 2007 to 30th June 2008 by repetitively rolling the whole set of data input following the in-sample period. In the other words, the oldest data is dropped whenever a new observation is added.

For our out-of-sample tests we examined a host of alternate proxies to measure the latent realized volatility. We settle on producing in this paper the outcome from using intra-day squared returns, simply because it is a widely practiced method and the results differ little when applying other measures, such as the rolling daily averages used for the in-sample results. Intra-day data does, however, have the advantage of utilizing the most recent information sets available to determine the forecast horizon.

# 3.3. Statistics of forecast accuracy

Table 6 reports the out-of-sample forecast performance expressing in term of different statistical loss functions analogous to those presented in the in-sample test in Table 5. Panel A comprises of three alternate models which we have developed in an attempt to employ a synthesis of option implied volatility and trading volume to derive forecasts, namely the integrated IVI-AOV-EGARCH (1,1) model specified in Eq. (1) and two combined forecasting models specified in Eqs. (4) and (5). Results of forecasts generated from many different benchmarks, either being drawn from the ARCH approach or other techniques, are summarized in panel B, following the same criteria. These panels address a number of questions of interest, including (1) whether the synthesis of option implied volatility and trading volume (a) helps to improve forecasts generated from the ARCH approach and (b) performs better than each component standing alone within the ARCH framework, and (2) how it performs relative to other forecasting techniques prominent in literature.

To briefly summarize the results presented in Table 6, we find some evidence of forecast improvement consistently spanning across different error measures and model specifications considered in panel A even though the cross-sectional performance of our models does not fare as well as in the sample. Of particular interest, a combination of naïve forecasts of option implied volatility and forecasts generated from the integration of volume into an EGARCH (1,1) model in subpanel A3 clearly dominates all the benchmark models considered in panel B. Another interesting observation is a significant reduction in the MAPE statistic relative to results reported in the in-sample test, as one would expect. This confirms the use of squared returns to proxy the actual volatility itself leads to the overestimation of MAPE. Diagnostics presented in subpanels A1 to B3 indicate that all models of the EGARCH family perform relatively well. A cross-check on other competing models highlights they uniformly perform better than the PCA and the SVM in subpanels B5 and B6 while two combined forecasts slightly dominate the ARMA (1,1) model in subpanel B4. The robustness of these findings has also been confirmed in other forecast exercises we performed with longer horizon (such as weekly and monthly), for which we found a similar, if not greater, improvement in performance from the addition of trading volume as the forecast horizon was extended. Our results are similar to Donaldson and Kamstra's (2005) finding that the monthly data horizon is more apt in highlighting the switching role trading volume can serve

The role of trading volume can be highlighted by a consistent improvement of the direction forecast performance presented in Table 7 when different measures of realized volatility are considered. It is found in panel A that trading volume helps to increase the percentage of correct direction of forecasts

**Table 7**The direction forecast performance for the out of sample period with different alternate measures of realized volatility.

		AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
anel A: our models											
Panel A1: IVI-AOV-EGARCH	Daily	45.26 <sup>,*</sup>	32.85 <sup>,*</sup>	50.36 <sup>^*</sup>	37.23 <sup>*</sup>	35.77 <sup>^,*</sup>	55.47	27.74	37.23 <sup>ˆ,*</sup>	34.31 <sup>^*</sup>	52.55
	22-Day-average	48.91	43.80,*	49.64	51.82 <sup>*</sup>	51.82 <sup>^,*</sup>	50.36	39.42	43.80,*	45.99 <sup>*</sup>	60.58
1,1)	Intra-day	56.93	51.09 <sup>^*</sup>	47.45	49.64*	45.99	53.28,*	46.72	43.80	48.18	46.72
Panel A2:	Daily	36.50	27.74	49.64	42.34*	29.93	57.66 <sup>^</sup>	21.17	35.77	32.12,*	45.99
OMB(IVI-EGARCH(1,1),AOV-	22-Day-average	51.82 <sup>*</sup>	38.69	53.28 <sup>*</sup>	51.09 <sup>*</sup>	45.99 <sup>^</sup>	58.39 <sup>*</sup>	41.61	42.34,*	50.36 <sup>^*</sup>	59.12
GARCH(1,1))	Intra-day	48.18	44.53	49.64	51.82*	43.07	56.93,*	45.99	42.34	51.09 <sup>*</sup>	47.45
Panel A3: COMB	Daily	46.72	35.77 <sup>^,*</sup>	52.55 <sup>^*</sup>	42.34*	32.85,*	48.18	29.20,*	40.15,*	41.61,*	50.36
OV-EGARCH (1,1), IVI)	22-Day-average	53.28 <sup>*</sup>	48.18 <sup>^*</sup>	48.91	51.09 <sup>*</sup>	47.45 <sup>*</sup>	47.45	39.42	51.09 <sup>^</sup> ,*	43.07	58.39
OV-EGARCH (1,1), IVI)	Intra-day	55.47	49.64	49.64	44.53	41.61	47.45	48.18,*	45.26 <sup>^</sup>	51.82 <sup>^</sup> ,*	45.99
nel B: benchmark models											
Panel B1: EGARCH (1,1)	Daily	37.96	31.39	43.07	46.72	29.93	51.82	21.90	34.31	29.20	37.23
	22-Day-average	53.28	40.88	58.39	55.47	43.07	59.85	42.34	37.23	45.99	52.55
	Intra-day	46.72	49.64	47.45	51.82	45.99	51.09	46.72	44.53	51.09	50.36
Panel B2: IVI-EGARCH (1,1)	Daily	40.15	29.93	49.64	35.04	31.39	57.66 <sup>^</sup>	27.74	36.50	29.20	54.74
	22-Day-average	48.18	42.34	50.36	48.18	45.99	56.93	43.80	41.61	44.53	62.77
	Intra-day	57.66	46.72	54.01	47.45	45.99	52.55	46.72	45.99	48.91	47.45
Panel B3: AOV-EGARCH	Daily	40.88	35.77	46.72	49.64	29.93	45.26	25.55	37.23	31.39	37.96
1)	22-Day-average	45.99	42.34	50.36	49.64	38.69	50.36	41.61	48.18 <sup>^</sup>	45.26	54.74
	Intra-day	49.64	51.09	45.26	50.36	41.61	48.91	44.53	48.18	47.45	48.18
Panel B4: ARMA (1,1)	Daily	25.55	29.20	27.01	42.34*	33.58 <sup>^,*</sup>	52.55	44.53,*	32.12	34.31 <sup>^</sup> ,*	24.09
	22-Day-average	45.26	41.61 <sup>^</sup>	54.01 <sup>*</sup>	48.18	42.34	40.15	43.07	36.50	45.99 <sup>*</sup>	43.07
	Intra-day	45.99	51.82 <sup>ˆ,*</sup>	45.99	45.99	49.64 <sup>^,*</sup>	60.58 <sup>^*</sup>	51.82 <sup>ˆ,*</sup>	51.09 <sup>^</sup> ,*	45.26	43.07
Panel B5: PCA	Daily	31.39	32.85,*	31.39	33.58	35.04 <sup>^,*</sup>	34.31	37.96 <sup>^,*</sup>	39.42,*	31.39 ,*	33.58
	22-Day-average	46.72	41.61 <sup>^</sup>	56.93 <sup>*</sup>	40.15	39.42	40.15	39.42	39.42	43.80	43.07
	Intra-day	45.99	46.72	45.99	44.53	45.99	43.80	44.53	43.80	43.80	46.72
Panel B6: SVM	Daily	52.55 <sup>^,*</sup>	45.99 <sup>^,*</sup>	42.34	27.01	48.91 <sup>^,*</sup>	45.99	36.50 <sup>^,*</sup>	34.31	45.26 <sup>^,*</sup>	38.69
	22-Day-average	50.36 <sup>*</sup>	46.72 <sup>,*</sup>	40.15	31.39	59.12 <sup>^*</sup>	51.09	39.42	38.69	46.72 <sup>,*</sup>	48.18
	Intra-day	49.64	45.26	41.61	30.66	54.74 <sup>^*</sup>	37.96	32.12	38.69	44.53	43.07

Details of models in panel A1 through to B6 are analogous to those presented in Table 6, including the integrated IVI-AOV-EGARCH (1,1) model (panel A1), the combined forecast of the IVI-EGARCH (1,1) and the AOV-EGARCH (1,1) (panel A2), the combined forecast of the AOV-EGARCH (1,1) and option implied volatility (panel A3), the benchmark EGARCH (1,1) model (panel B1), the integrated IVI-EGARCH (1,1) model (panel B2), the integrated AOV-EGARCH (1,1) model (panel B3), the ARMA (1,1) model (panel B4), the principal component analysis model (panel B5) and the stochastic volatility model (panel B6). Three alternative measures have been used to proxy the latent realized volatility, including the daily squared returns, the 22-day-average squared returns and the sum of intra-day squared returns.

*Notes*: This table represents the out-of-sample forecast performance in terms of correct forecast of directional changes in volatility. The mean correct prediction statistic presented above can be defined as the percentage of observations for which the model predicts a change of the same sign as the realized change in volatility. The results are reported for daily forecasts over the period from 24 November 2007 to 30 June 2008.

^indicates better performance relative to the benchmark EGARCH (1,1) model.

<sup>\*</sup> indicates better performance relative to the integrated IVI-EGARCH (1,1) model.

**Table 8**The orthogonality test of the volatility forecasts of the S&P500 index returns with intra-day squared return being used as the realized volatility.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
α	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0001)	0.0000 (0.0001)	0.0001 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)***	0.0002 (0.0000)***	0.0002 (0.0000)***
β	0.7486 (0.2257)***	0.8568 (0.2673)***	1.1605 (0.4116)***	1.1768 (0.4036)***	0.6354 (0.2263)***	1.4697 (0.4356)***	0.0262 (0.0680)	0.0096 (0.0358)	0.1131 (0.1999)
R_2 Adj <i>R</i> _2	0.0753 0.0685	0.0707 0.0638	0.0556 0.0486	0.0592 0.0523	0.0552 0.0482	0.0778 0.0709	0.0011 -0.0063	0.0005 -0.0069	0.0024 -0.0050
Log likelihood	960.25	959.91	958.81	959.07	958.78	960.43	954.96	954.92	955.05

Details of models (1)–(9) are analogous to those presented in Tables 6 and 7, including (1) the integrated IVI-AOV-EGARCH (1,1) model, (2) the combined forecast of the IVI-EGARCH (1,1) and the AOV-EGARCH (1,1), (3) the combined forecast of the AOV-EGARCH (1,1) and option implied volatility, (4) the benchmark EGARCH (1,1) model, (5) the integrated IVI-EGARCH (1,1) model, (6) the integrated AOV-EGARCH (1,1) model, (7) the ARMA (1,1) model, (8) the principal component analysis model and (9) the stochastic volatility model. The results are reported for daily forecasts over the period from 24 November 2007 to 30 lune 2008.

*Notes*: This table represents the statistics of the Mincer–Zarnowitz regression in which the realized volatility is regressed against a constant and the forecasted volatility as follows:

$$\sigma_t^2 = \alpha + \beta \hat{\sigma}_t^2 + e_t$$

of (i) the EGARCH model, (ii) the option implied volatility and (iii) both, no matter whether it is integrated into the EGARCH model or whether the exercise of combining forecasts is applied. The evidence becomes overwhelming in case daily squared returns are used to proxy the realized volatility. Moving across the table, the ARMA (1,1) and the PCA are clearly the worst performers. The SVM, while performing comparably to the benchmark EGARCH (1,1) model in subpanel B1, is surpassed by our proposed models in subpanels A1 and A3. This particular finding further anchors the forecast quality of trading volume. Overall, the combined forecast in subpanel A3 again provides the best performance in a horse race among all models, as one would expect.

To get additional perspective on the forecast performance of a synthesis of option implied volatility and trading volume, we perform the orthogonality test by regressing the realized volatility on one or multiple forecasts of all models being covered thus far. For brevity, we only reported in Table 8 statistics of the Mincer–Zarnowitz regression for each model's forecasts of the S&P500 index return volatility. At the first glance, it clearly shows that all models of the EGARCH family fare better than other forecasting techniques when considering the statistical significance and the magnitude of  $\beta$  estimates obtained in columns (1)–(6) in contrast to the rest. The R squared (or the adj-R squared) and the maximum likelihood at the bottom of the table, however, stimulates that daily variations of volatility are best captured by forecasts generated from the integrated IVI-AOV-EGARCH (1,1) model or from the integrated AOV-EGARCH (1,1) model. Furthermore, all those statistics agrees that a combination of forecasts generated from the integrated IVI-EGARCH (1,1) and the integrated AOV-EGARCH (1,1) model.

#### 3.4. Economic significance

The economic significance of the volatility forecast exercise we have preceded thus far is the next focus of our analysis. Albeit the forecast exercise has been conducted for different forecast horizons, we only report in Table 9 simulated results of the ATM-straddle trading strategy based on daily volatility forecasts, being expressed in term of the Sharpe ratio (SR), the Leland's alpha (Ap) and their bootstrapped 95% confidence intervals. While the SR is arguably the most commonly employed criterion of evaluating portfolio performance in the existing literature, the Ap provides a better reference of the profitability of non-linear payoffs by taking into account higher-order moments of the return distributions to accommodate deviations from normality. This problem is especially acute in the context of option trading being investigated in this paper.

<sup>\*\*\*</sup> Rejection at the 1% significance level.

**Table 9**Out of sample trading simulation using daily volatility forecasts.

		AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
Panel A: our models											
Panel A1: IVI-AOV-EGARCH (1,1)	Sharpe ratio 95% CI Ap 95% CI	0.261** (0.154) (0.367) 0.117** (0.059) (0.183)	-0.104 -(0.228) (0.012) -0.072 -(0.162) (0.012)	0.010 -(0.134) (0.137) 0.005 -(0.067) (0.069)	-0.017 -(0.137) (0.126) -0.009 -(0.062) (0.061)	0.061 -(0.077) (0.211) 0.030 -(0.039) (0.109)	0.019 -(0.138) (0.160) 0.012 -(0.053) (0.062)	0.151** (0.014) (0.272) 0.056** (0.003) (0.104)	-0.111 -(0.237) (0.009) -0.050 -(0.125) (0.011)	0.113 -(0.026) (0.262) 0.052 -(0.015) (0.124)	-0.050 -(0.19 (0.100 -0.018 -(0.07 (0.036
Panel A2: COMB(IVI- EGARCH(1,1),AOV- EGARCH(1,1))	Sharpe ratio 95% CI Ap 95% CI	-0.036 -(0.199) (0.104) -0.018 -(0.092) (0.050)	-0.011 -(0.152) (0.127) -0.004 -(0.104) (0.091)	-0.002 -(0.127) (0.148) -0.001 -(0.064) (0.079)	-0.002 -(0.138) (0.159) -0.002 -(0.058) (0.073)	0.017 -(0.116) (0.146) 0.009 -(0.053) (0.070)	0.028 -(0.129) (0.173) 0.016 -(0.051) (0.068)	0.071 -(0.074) (0.213) 0.031 -(0.025) (0.074)	-0.110 -(0.225) (0.034) -0.049 -(0.113) (0.019)	0.100 -(0.031) (0.269) 0.046 -(0.012) (0.117)	-0.079 -(0.22 (0.055 -0.028 -(0.08
Panel A3: COMB (AOV-EGARCH (1,1), IVI)	Sharpe ratio 95% CI Ap 95% CI	-0.052 -(0.207) (0.080) -0.025 -(0.093) (0.036)	-0.071 -(0.219) (0.067) -0.047 -(0.145) (0.047)	0.005 -(0.148) (0.127) 0.002 -(0.073) (0.065)	0.006 -(0.153) (0.148) 0.003 -(0.064) (0.068)	0.084 -(0.060) (0.219) 0.041 -(0.029) (0.109)	0.001 -(0.137) (0.121) 0.006 -(0.055) (0.050)	0.054 -(0.084) (0.209) 0.024 -(0.033) (0.078)	-0.136** -(0.247) -(0.006) -0.062 -(0.127) (0.001)	0.098 -(0.041) (0.237) 0.045 -(0.019) (0.108)	-0.073 -(0.21 (0.060) -0.027 -(0.08 (0.023)
Panel B: benchmark models		(0.030)	(0.017)	(0.005)	(0.000)	(0.103)	(0.030)	(0.070)	(0.001)	(0.100)	(0.023
Panel B1: EGARCH (1,1)	Sharpe ratio 95% CI Ap 95% CI	-0.001 -(0.160) (0.171) -0.001 -(0.073) (0.080)	-0.079 -(0.231) (0.050) -0.053 -(0.166) (0.037)	0.066 -(0.066) (0.191) 0.030 -(0.033) (0.098)	0.100 -(0.030) (0.225) 0.044 -(0.016) (0.103)	0.010 -(0.143) (0.155) 0.005 -(0.068) (0.078)	0.064 -(0.074) (0.214) 0.031 -(0.030) (0.086)	0.063 -(0.060) (0.200) 0.028 -(0.022) (0.073)	-0.111 -(0.252) (0.041) -0.050 -(0.130) (0.020)	0.080 -(0.065) (0.233) 0.036 -(0.031) (0.108)	-0.09 -(0.24 (0.038 -0.03 -(0.09 (0.016
Panel B2: IVI-EGARCH (1,1)	Sharpe ratio 95% CI Ap 95% CI	0.158** (0.029) (0.289) 0.071** (0.011) (0.139)	-0.078 -(0.217) (0.066) -0.053 -(0.156) (0.051)	0.008 -(0.152) (0.164) 0.004 -(0.075) (0.086)	-0.022 -(0.162) (0.117) -0.012 -(0.069) (0.053)	0.013 -(0.111) (0.145) 0.006 -(0.052) (0.072)	0.030 -(0.139) (0.174) 0.018 -(0.054) (0.068)	0.155** (0.019) (0.294) 0.057** (0.003) (0.113)	-0.121 -(0.230) (0.012) -0.055 -(0.116) (0.010)	0.122 -(0.010) (0.248) 0.056 -(0.006) (0.116)	-0.04 -(0.16 (0.099 -0.01 -(0.06 (0.037
Panel B3: AOV-EGARCH (1,1)	Sharpe ratio 95% CI Ap 95% CI	0.014 -(0.127) (0.171) 0.004 -(0.059) (0.083)	-0.115** -(0.261) -(0.002) -0.079 -(0.189) (0.003)	-0.092 -(0.227) (0.058) -0.046 -(0.115) (0.031)	0.148** (0.007) -0.274 0.066** (0.000) (0.127)	-0.027 -(0.167) (0.126) -0.013 -(0.079) (0.063)	0.077 -(0.079) (0.227) 0.036 -(0.030) (0.089)	0.060 -(0.065) (0.194) 0.027 -(0.023) (0.070)	-0.123 -(0.238) (0.028) -0.055 -(0.121) (0.017)	0.051 -(0.098) (0.179) 0.023 -(0.045) (0.082)	-0.07 -(0.2 (0.083 -0.02 -(0.08 (0.031

Table 9 (Continued)

		AIG	GE	GM	HPQ	IBM	TXN	JNJ	WMT	XRX	SPX
Panel B4: ARMA (1,1)	Sharpe ratio 95% CI Ap 95% CI	-0.024 -(0.179) (0.106) -0.012 -(0.083) (0.050)	-0.091 -(0.245) (0.041) -0.063 -(0.174) (0.033)	0.054 -(0.070) (0.196) 0.026 -(0.036) (0.103)	0.030 -(0.103) (0.131) 0.012 -(0.045) (0.061)	-0.051 -(0.181) (0.099) -0.023 -(0.083) (0.046)	0.044 -(0.107) (0.187) 0.023 -(0.043) (0.074)	0.157** (0.038) (0.279) 0.059** (0.012) (0.103)	-0.124 -(0.256) (0.016) -0.056 -(0.128) (0.012)	-0.066 -(0.205) (0.081) -0.031 -(0.095) (0.039)	-0.160** -(0.292) -(0.017) -0.057** -(0.114) -(0.003)
Panel B5: PCA	Sharpe ratio 95% CI Ap 95% CI	-0.134 -(0.262) (0.008) -0.064 -(0.126) (0.006)	0.098 -(0.048) (0.227) 0.064 -(0.036) (0.165)	0.022 -(0.104) (0.157) 0.012 -(0.054) (0.081)	-0.070 -(0.182) (0.066) -0.026 -(0.085) (0.030)	-0.013 -(0.172) (0.123) -0.010 -(0.078) (0.061)	-0.107 -(0.235) (0.022) -0.042 -(0.095) (0.010)	-0.138** -(0.268) -(0.006) -0.049** -(0.101) -(0.001)	-0.075 -(0.216) (0.061) -0.036 -(0.105) (0.030)	-0.069 -(0.207) (0.060) -0.031 -(0.096) (0.027)	-0.017 -(0.157) (0.119) -0.005 -(0.059) (0.044)
Panel B6: SVM	Sharpe ratio 95% CI Ap 95% CI	-0.035 -(0.172) (0.091) -0.015 -(0.080) (0.044)	0.023 -(0.097) (0.157) 0.011 -(0.067) (0.110)	0.205** (0.053) (0.346) 0.102** (0.021) (0.182)	0.156** (0.019) (0.325) 0.072** (0.010) (0.137)	0.205** (0.079) (0.326) 0.091** (0.027) (0.160)	-0.003 -(0.163) (0.136) 0.003 -(0.063) (0.053)	0.129 -(0.011) (0.276) 0.045 -(0.006) (0.103)	0.144** (0.023) (0.284) 0.068** (0.008) (0.139)	0.106 -(0.012) (0.236) 0.054 -(0.008) (0.111)	-0.134 -(0.267) (0.011) -0.050 -(0.103) (0.006)
Panel B7: Long position	Sharpe ratio 95% CI Ap 95% CI	0.310** (0.210) (0.418) 0.138** (0.080) (0.203)	0.267** (0.161) (0.378) 0.182** (0.084) (0.278)	0.303** (0.185) (0.406) 0.147** (0.075) (0.210)	0.342** (0.245) (0.442) 0.141** (0.085) (0.210)	0.319** (0.210) (0.424) 0.143** (0.084) (0.203)	0.150** (0.007) (0.271) 0.057** (0.001) (0.109)	0.267** (0.153) (0.375) 0.092** (0.046) (0.144)	0.106 -(0.043) (0.249) 0.051 -(0.023) (0.124)	0.138** (0.019) (0.277) 0.067** (0.007) (0.128)	0.155** (0.021) (0.314) 0.052** (0.005) (0.117)

Option straddles have been employed to trade on volatility forecasts generated from alternative forecast models. This trading strategy involves undertaking simultaneously long (short) positions on ATM option contracts with the same exercise and time to maturity when future volatility is expected to increase (decrease). Daily volatility forecasts are generated by rolling the sample forward and re-estimating the model's parameters on a daily basis over the period from 24 November 2007 to 30 June 2008.

The strategy is based on daily forecasts obtained from the integrated IVI-AOV-EGARCH (1,1) model (panel A1), the combined forecast of the IVI-EGARCH (1,1) and the AOV-EGARCH (1,1) (panel A2), the combined forecast of the AOV-EGARCH (1,1) and option implied volatility (Panel A3), the benchmark EGARCH (1,1) model (panel B1), the integrated IVI-EGARCH (1,1) model (panel B2), the integrated AOV-EGARCH (1,1) model (panel B3), the ARMA (1,1) model (panel B4), the principal component analysis model (panel B5) and the stochastic volatility model (panel B6). In addition, profit is determined for a strategy of purely going long on ATM straddles in panel B7.

Notes: The forecast performance can be evaluated based on the Sharpe ratio and Leland's alpha (Ap) and their respective bootstrapped 95% confidence interval (CI).

<sup>\*\*</sup> The rejection of the null hypothesis of a zero Sharpe ratio (Ap) at a 5% level of significance.

Without transaction costs being considered, the performance of models of the EGARCH family covered in subpanels A1 to B3 is very encouraging with positive abnormal return being obtained for a majority of the sample, even when the non-normality of the empirical distribution of trading profits are taken into account. In some occasions, the magnitude of abnormal returns well exceeds a transaction cost of around 1–2% reasonably expected for option trading, which any investor would see as a dramatic increase in wealth on a daily basis. These results, while hardly being seen in the equity market, are reasonably expected for option trading due to high leverage and stay consistent with previous findings in the literature (Guo, 2000).

However, what of more interest is whether these models significantly yield abnormal returns. This being found for the integrated IVI-AOV-EGARCH (1,1) model in subpanel A1, the IVI-EGARCH (1,1) model and the AOV-EGARCH (1,1) model in subpanels B2 and B3, respectively, clearly demonstrates their outperformance over the EGARCH (1,1) model. These statistically significant returns are also associated with significantly large values of the Sharpe ratio, suggesting that their performance is truly abnormal. The dominance of the integrated IVI-AOV-EGARCH (1,1) model is of particular interest, since the results thus far show only slight differences in performance of models, which combine the information sets of both option implied volatility and trading volume, proposed in panel A. This model also compares favourably with other two benchmarks, namely the ARMA (1,1) model and the PCA model. on a cross-sectional basis.

Nevertheless, the performance of the SVM in subpanel B6 is striking, as illustrated by findings of a statistically significant positive Leland's alpha (Shape ratio) in four cases. This result is so contrary that it induces us to re-examine the forecast quality of the SVM to provide justification(s) of the source of abnormal returns. Given the substantial value of MAPEs being found in subpanel B6 of Table 6, a proper inference seems to be that abnormal returns actually stem from a persistent overestimation of forecasts for those particular stocks. Considering the fact that the SVM leads to the decision to buy for 75–91% of the times, this line of reasoning is further confirmed in subpanel B7 in Table 9 showing that abnormal returns would have been obtained in most cases (except for WMT stock) by simply going long on ATM straddles for the entire out-of-sample period.

Hence, an overall conclusion from this analysis is that the integration of option implied volatility and trading volume into the EGARCH (1,1) specification leads to outperformance relative to other competing forecasting techniques. Essentially, unreported results show that these findings are robust to different simulation set-ups, <sup>10,11</sup> or proxies of realized volatility. <sup>12</sup> As one would expect, the economic gain would be reduced if consideration is made of transaction costs. While the magnitude of this impact largely depends on the size of these costs, it imposes an impediment on real trade executions to realize profit from volatility forecasts.

#### 4. Conclusion

The implication of this study is considered not only in terms of its contribution into the current body of literature in volatility–volume relation but also of future research into volatility modeling and forecasting. While finding evidence in compliance with the sequential information hypothesis saying that trading volume contains useful information of future stock volatility, this study suggests its importance in volatility forecasting is shared between stock and option trading volume, with the latter possessing a better forecast quality. The information share would explain the inconsistency of results

<sup>&</sup>lt;sup>10</sup> Volatility trading is also simulated in this study [using the VIX futures traded on the CBOE, similar to Konstantinidi et al. (2008)] and tells the same tale. The integration of either implied volatility or volume or both leads to a better performance relative to the EGARCH (1,1) model, even though a comparison to other forecasting techniques shows no clear evidence of dominance.

<sup>&</sup>lt;sup>11</sup> In addition, we re-performed straddle trading simulation to assess the robustness of our findings across three different sub-out-of-sample periods, following the recursive pseudo scheme suggested by Konstantinidi et al. (2008). Main findings (statistical and economic significance) are not sensitive to the period under consideration.

<sup>12</sup> The same trading simulation set-up was re-applied where 22-day-average squared returns are used to proxy historical volatility. This is appropriate for the purpose of option trading since its construction has been designed to match closer with the volatility input required for option pricing. Results, not being reproduced here, yield a very similar observation to the above. We omitted the use of daily squared returns for being a bad proxy in our previous discussion.

in previous studies, such as (Brooks, 1998; Donaldson and Kamstra, 2005) among other, since they fail to adequately account for all the inherent implications of options trading activities when investigating into the volume–volatility relation. Also in this paper we provide several simple model specifications where the information content of trading volume can be utilized to improve the predictive power of the ARCH approach, or a combination of ARCH and option implied volatility. Both in and out-of-sample tests reveal the value of adding volume. In particular, we find that the accuracy of volatility forecasts can be improved by combining information from option implied volatility and volume accordingly, most markedly by integrating both factors into the variance equation of the EGARCH (1,1) model.

It is important to note that our study does not consider how further improvements could be made by accounting for possible bias within implied volatility estimations either from using standard option pricing models, or considering the efficiency of the options market as a whole [refer to Mayhew, 1995 for a review]. In the latter case, it has previously been shown that variations of the informational quality of the implied volatility across moneynesses and stocks may be attributable to the liquidity of the options market and the trading costs involved. In particular, Mayhew and Stivers (2003) found that the information content of option implied volatility, with respect to modeling and forecasting the volatility of stock returns, deteriorates for stocks with lower options volume. For this reason it may be worthwhile as a future research endeavor to investigate how useful trading volume is as a tool for forecasting volatility when the stocks being examined have options that are far less liquid. In particularly, it would be interesting to examine how quickly information content contained in trading volume potentially declines.

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