



# Forecasting realized volatility with changing average levels



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## ABSTRACT

We explore the abilities of regime switching with Markovian dynamics (MS) and of a smooth transition (ST) nonlinearity within the class of Multiplicative Error Models (MEMs) to capture the slow-moving long-run average in the realized volatility. We compare these models to some alternatives, including considering (quasi) long memory features (HAR class), the benefits of log transformations, and the presence of jumps. The analysis is applied to the realized kernel volatility series of the S&P500 index, adopting residual diagnostics as a guidance for model selection. The forecast performance is evaluated and tested via squared and absolute losses both in- and out-of-sample, as well as based on a robustness check on different sample choices. The results show very satisfactory performances of both MS and ST models, with the former also allowing for the dating and interpretation of regimes in terms of market events.

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## 1. Introduction

A consolidated body of literature in financial econometrics is devoted to the measurement of asset volatility, exploiting the information contained in asset price data collected at a very high frequency. The volatility estimators, known as realized volatility (RV) measures, have allowed for deeper insights into the dynamics of volatility, which are traditionally analyzed in a modeling and forecasting framework within the GARCH paradigm as conditional variances of returns (see [Bollerslev, 1986](#), [Engle, 1982](#), and further extensions; for a review, see [Teräsvirta, 2009](#); for nonlinear models, see [Teräsvirta, 2011](#)). Starting from the plain realized volatility, studied in detail by [Andersen, Bollerslev, Diebold, and Labys \(2000, 2003\)](#), various other measures have been introduced to take into consideration the presence of jumps and other market mi-

crostructure issues (for a review, see [Andersen, Bollerslev, & Diebold, 2010](#)). The most recent addition to the family of volatility estimators is the realized kernel volatility (developed by [Barndorff-Nielsen, Hansen, Lunde, & Shephard, 2008](#)), which is designed to possess robustness to market microstructure noise.

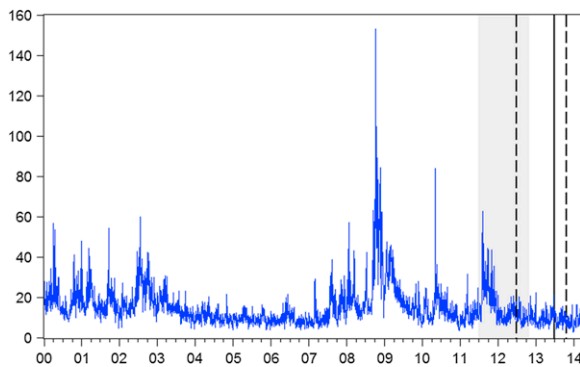
While volatility measurement from an end-of-day perspective has reached a mature stage, the modeling and forecasting of conditional volatility is open to refinements, building on existing dynamic models (cf. among others, [Brownlees & Gallo, 2010](#); [Cipollini, Engle, & Gallo, 2013](#); [Hansen, Huang, & Shek, 2012](#); [Shephard & Sheppard, 2010](#)). In Multiplicative Error Models (MEMs, developed by [Engle, 2002](#), and [Engle & Gallo, 2006](#)), the volatility series is specified as the product of a time-varying scale factor that evolves autoregressively and a random disturbance with a suitable distribution. As it is applied to non-negative values, a MEM captures dynamics without resorting to logs, thus producing forecasts of volatility (not log-volatility); adopting multiplicative (rather than additive) disturbances accommodates heteroskedasticity naturally (“vol-of-vol”).

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**Fig. 1.** Realized kernel volatility of the S&P500 index (Jan. 3, 2000 to Mar. 10, 2014); the gray shadowed area represents the out-of-sample period of the main analysis (between Jul. 5, 2011, and Oct. 26, 2012). For the robustness check, the vertical dashed lines identify the first out-of-sample period (Jul. 5, 2012 to Oct. 25, 2013) and the solid line is the beginning of the second out-of-sample period (Jul. 5, 2013).

The volatility profile over long periods typically exhibits fairly persistent local average volatilities (cf., as a leading example, the realized kernel volatility of the S&P500 in Fig. 1). Competing models must capture this empirical regularity with possibly alternative or complementary features (discrete changes in volatility, slow-moving long-run dynamics, other nonlinearities, or explicit long memory), and will be evaluated for both their in-sample fitting capabilities and their out-of-sample forecasting performances. The specific suggestions that we make involve extending MEMs to have regime-switching (Hamilton, 1989, 1990) or smooth-transition (Teräsvirta, 1994) representations, keeping variants of the heterogeneous autoregressive model (HAR, also known as a quasi-long memory model) of Corsi (2009) for RV and log-RV, for comparison purposes.

The consideration of time-varying local averages is not new to the volatility literature (see below for a review); our extended class of Markov Switching (MS) MEMs highlights regime-specific dynamics. Relative to other MEMs, our approach is distinct from both the mixture MEM (either that of De Luca & Gallo, 2004, and Lanne, 2006, which suffers from the shortcoming of time-independent regime indicator variables, cf. Bauwens, Hafner, & Laurent, 2012; or that of De Luca & Gallo, 2009, who make the mixing weights dependent on a lagged observable variable) and the P-Spline MEM used by Brownlees and Gallo (2010) for realized volatility with a time varying average level (which is a statistical fit to a supposed smooth underlying trend). A standard HAR may face serious challenges in fitting the observed pattern given its linear nature; the HAR- $j$  of Andersen, Bollerslev, and Diebold (2007) reacts to a separate measure of jumps as a possible accommodation of abrupt changes, but its out-of-sample performance depends crucially on jointly forecasting jumps as well. The issue of whether a log-transformation mitigates the importance of peaks in the series, flattening out the extent to which local averages may be present, seems to be mostly an empirical question. With many models at hand, we need a suitable model selection strategy; we adopt a nonparametric Bayesian procedure (Otranto & Gallo, 2002) for selecting the number of regimes, and take the view that the pres-

ence of autocorrelation in residuals and/or squared residuals may be seen as evidence of the presence of some form of model misspecification (in the dynamics and/or nonlinearity). In-sample fitting capabilities will not necessarily correspond to good forecasting performances, especially at different forecasting horizons; and finally, but possibly as a by-product, it is useful to evaluate the compatibility of the results with the interpretation of major events in financial markets.

The paper is organized as follows: in Section 2, we review the main literature on the nonlinear modeling of the realized volatility. In Section 3, we discuss the issues behind discrete and slow-moving local average volatilities, with a description of the Markov Switching and Smooth Transition extensions within the MEM class; we analyze the estimators' properties and provide some Monte Carlo evidence as to the small-sample properties of the estimators. In the same section, we also detail some of the alternative parameterizations that may capture different features of the data. An empirical application to the realized kernel volatility S&P500 series is the object of Section 4. The estimation of 13 models is a preliminary step to determining certain features of the data and allowing for a reduction in the number of models based on their in-sample properties and forecasting performances (one- and ten-step-ahead forecasts with the Diebold & Mariano, 1995 (DM), test and the Model Confidence Set (MCS) sequential test of Hansen, Lunde, & Nason, 2011, using absolute and squared losses). The inference on the regimes and the comparison with the smooth transition function provide some interesting insights on the classification of the periods and the calculation of the corresponding average levels. The robustness of the model performances is also checked on different time spans. Concluding remarks follow. Finally, some extra details on model estimation and comparison are provided in a web appendix.

## 2. High persistence in volatility

There is a major debate in the literature about the nature of the high level of persistence in realized volatility, and whether it may be the result of some nonlinearity in the process. The HAR model (Corsi, 2009), although not a formal long-memory model, can reproduce the observed hyperbolic-type decay of the autocorrelation function of (log-)volatility by specifying a sum of volatility components over different horizons. Andersen et al. (2007) insert a volatility jump component for capturing the abrupt changes that characterize the realized volatility, resulting in significant improvements in the forecasting performance. Baillie and Kapetanios' (2007) reasoning about the existence of both non-linear and long memory components in many economic and financial time series is developed by McAleer and Medeiros (2008), who introduce a multiple regime smooth transition extension of the HAR; their model is also able to capture the presence of sign and size asymmetries. Bordignon and Raggi (2012) propose an elegant solution for combining, in a single model, both non-linearity effects, through a Markov switching process, and high persistence, through fractionally integrated dynamics, that are capable of improving the accuracy of both in- and out-of-sample forecasts. Alternatively, concentrating on long memory explanations, Andersen et al. (2003)

suggest an autoregressive fractionally integrated moving average (ARFIMA) model for the log-realized volatility; while [Ohanissian, Russell, and Tsay \(2008\)](#) also find evidence of long memory in exchange rate dynamics. However, as was noted by [Lanne \(2006\)](#), the ARFIMA model may not be optimal for several reasons: (a) a simple short-memory ARMA model can be as good at forecasting the realized volatility of stock returns as a long-memory ARFIMA model ([Pong, Shackleton, Taylor, & Xu, 2004](#)); (b) the parameters of the FI and ARMA parts can capture similar characteristics ([Bos, Franses, & Ooms, 2002](#)); and (c) a feasible ARFIMA model must involve a truncation of the infinite-order lag polynomial in practical applications, and hence, it is only an approximation to the “true” model anyway. [Corsi, Mittnik, Pigorsch, and Pigorsch \(2008\)](#) underline how the empirical distributions of ARFIMA and HAR residuals, derived from the realized volatility series, tend to exhibit yet unmodeled volatility clustering. In this respect, the presence of regimes (mixture distribution) is also capable of capturing the slowly decaying autocorrelation function of the observed realized volatility series. [Maheu and McCurdy \(2002\)](#), but see also [2007, 2011](#)) find strong statistical evidence of regime changes in both the conditional mean and the conditional variance of the realized volatility, using a Markov Switching ARMAX representation, where the transition probabilities and the conditional mean of the volatility are both functions of the duration of the state.<sup>1</sup> [Scharth and Medeiros \(2009\)](#) extend a regression tree model to accommodate smooth splits in regimes controlled by past cumulated returns, which account for long-range dependence in volatility.

We attempt a comparison across this rich set of contributions, concentrating on a few issues: volatility vs. log-volatility, long memory, smooth transition, and Markov Switching. We keep in mind the discussion by [Diebold and Inoue \(2001\)](#) about the observational equivalence of long memory and nonlinear effects in volatility, and the common occurrence in nonlinear models that one model may significantly outperform others for specific series and sample periods. Thus, we concentrate on the class of MEMs either with regimes (considering both abrupt and smooth changes) or without, and the class of HARs (estimated on RV and log-RV, with and without jumps); we favor economic interpretability when identifying regimes and the models’ abilities to capture certain episodes that characterize the evolution of volatility, especially in times of distress. A convenient way to proceed is to evaluate our set of models in terms of residual diagnostics and in- and out-of-sample performances, verifying the robustness of the results to the sample period considered as well.

<sup>1</sup> In a GARCH framework, previous contributions addressed regime switching, cf. the SWARCH model ([Hamilton & Susmel, 1994](#)), the MS GARCH model ([Dueker, 1997](#); [Klaassen, 2002](#)), and the recent multivariate extensions ([Edwards & Susmel, 2003](#); [Gallo & Otranto, 2007, 2008](#); [Higgs & Worthington, 2004](#)). An alternative way to consider changes in regime is given by smooth transition GARCH models ([Teräsvirta, 2009](#)) and other nonlinear models ([Teräsvirta, 2011](#)). Several other authors have indicated the presence of level shifts in GARCH ([Perron & Qu, 2010](#)), or breaks at unknown points, also in GARCH ([He & Maheu, 2010](#)), as the cause of an apparent high persistence. The issue of a time-varying underlying level of volatility is also addressed by [Engle and Rangel \(2008\)](#), who adapt a spline function in GARCH to enable them to capture a low frequency component of the volatility (which they connect to macroeconomic factors).

### 3. Asymmetric MEMs with changing average volatility levels

The basic MEM idea was introduced by [Engle \(2002\)](#) and subsequently developed by [Engle and Gallo \(2006\)](#), with an extension to the multivariate case by [Cipollini et al. \(2013\)](#). The volatility  $x_t$  of a certain financial time series is modeled as the product of a time-varying scale factor  $\mu_t$  (the conditional mean of  $x_t$ ), which follows GARCH-type dynamics, and a positive valued error  $\varepsilon_t$ :

$$\begin{aligned} x_t &= \mu_t \varepsilon_t, \quad \varepsilon_t \sim \text{Gamma}(a, 1/a) \text{ for each } t, \\ \mu_t &= \omega + \alpha x_{t-1} + \beta \mu_{t-1} + \gamma D_{t-1} x_{t-1}, \end{aligned} \quad (3.1)$$

where the dummy variable  $D_t$  is equal to 1 if the return  $r_t$  is negative. This base specification takes into account the presence of asymmetric responses of the volatility to the sign of the previous returns ([Engle & Gallo, 2006](#)), paralleling the GJR-GARCH model ([Glosten, Jagannathan, & Runkle, 1993](#)); the coefficient  $\gamma$  captures a stronger reaction to past negative returns. Setting  $\gamma$  to zero in this *Asymmetric MEM* (AMEM), we get the simple MEM. Constraints can be imposed to ensure the positiveness of  $\mu_t$  (e.g.,  $\omega > 0$ ,  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\gamma \geq 0$ ) and the stationarity of the process (persistence  $\alpha + \beta + \gamma/2 < 1$  under a zero median of the returns). The Gamma distribution of the innovation depends only on a single parameter  $a$ , providing a unit mean and a variance equal to  $1/a$ . Larger values of  $a$  correspond to a symmetric density that is similar to the Normal distribution ([Engle & Gallo, 2006](#), provide a graphical summary of various cases), whereas smaller values provide asymmetry with a larger variance.<sup>2</sup> Correspondingly, conditional on the information  $\Psi_{t-1}$  available at time  $t - 1$ , the mean and variance of  $x_t$  are  $\mu_t$  and  $\mu_t^2/a$  respectively. Further lags can also be added to the specification of  $\mu_t$ .

For the specification provided in Eq. (3.1), the unconditional mean of the volatility across the entire period is given by:

$$\mu = \frac{\omega}{1 - \alpha - \beta - \gamma/2}. \quad (3.2)$$

#### 3.1. The AMEM with Markov Switching

In order to extend the capabilities of the model to capture extreme events that may have an impact on the market dynamics, we introduce switching parameters that follow a discrete Markov chain. We define the Markov-Switching AMEM (MS-AMEM) as:

$$\begin{aligned} x_t &= \mu_{t,s_t} \varepsilon_t, \quad \varepsilon_t | s_t \sim \text{Gamma}(a_{s_t}, 1/a_{s_t}) \text{ for each } t, \\ \mu_{t,s_t} &= \omega + \sum_{i=1}^n k_i I_{s_t} + \alpha_{s_t} (x_{t-1} - \mu_{t-1,s_{t-1}}) \\ &\quad + \beta_{s_t}^* \mu_{t-1,s_{t-1}} + \gamma_{s_t} D_{t-1} (x_{t-1} - \mu_{t-1,s_{t-1}}), \end{aligned} \quad (3.3)$$

where  $s_t$  is a discrete latent variable in the range  $[1, \dots, n]$ , representing the regime at time  $t$ .  $I_{s_t}$  is an indicator that

<sup>2</sup> Of course, less restrictive (but less parsimonious) specifications could also be adopted, such as the Generalized Gamma.

is equal to one when  $s_t \leq i$  and zero otherwise;  $k_i \geq 0$  and  $k_1 = 0$ . Thus, the constant in regime  $j$  is given by  $(\omega + \sum_{i=1}^j k_i)$ , and reflects non-decreasing levels of volatility passing to higher regimes. The changes in regime are driven by a Markov chain, such that:

$$\Pr(s_t = j | s_{t-1} = i, s_{t-2}, \dots) = \Pr(s_t = j | s_{t-1} = i) = p_{ij}. \quad (3.4)$$

The positiveness and stationary constraints given for Eq. (3.1) hold within each regime in Eq. (3.3) as well. The unconditional expected value within state  $j$ ,  $j = 1, \dots, n$ , is equal to:

$$\mu_j = \frac{\omega + \sum_{i=1}^j k_i}{1 - \alpha_j - \beta_j - \gamma_j/2}, \quad (3.5)$$

where  $\beta_j = \beta_j^* - \alpha_j - \gamma_j/2$ . In view of this reparameterization, the same unconditional expected value is shared by the model:

$$\mu_{t,s_t} = \omega + \sum_{i=1}^n k_i I_{s_t} + \alpha_{s_t} x_{t-1} + \beta_{s_t} \mu_{t-1,s_{t-1}} + \gamma_{s_t} D_{t-1} x_{t-1}. \quad (3.6)$$

Extensions of the model to make the transition probabilities in Eq. (3.4) dependent on past observable variables are possible (cf. Maheu & McCurdy, 2002, who make the transition probabilities dependent on the duration in the state); we will propose a particular specification in the next section.

We can estimate the model in Eq. (3.3) by adopting Hamilton's filter and smoother (Hamilton, 1994, Chapter 22). There is a path dependence issue which causes a well-known computational problem, due to the unobservability of  $\mu_{t,s_t}$  and the dependence on past values of  $s_t$ ; evaluating the likelihood function recursively, we would need to keep track of all possible paths taken by the regime between  $t = 1$  and  $t = T$ , making the model intractable. The solution adopted in this case is the one proposed by Kim (1994) for a state space MS model (see also the many examples provided by Kim & Nelson, 1999). After each step of the Hamilton filter, we collapse the  $n^2$  possible values of  $\mu_t$  at time  $t$  into  $n$  values, by an average over the probabilities at time  $t - 1$ :

$$\hat{\mu}_{t,s_t} = \frac{\sum_{i=1}^n \Pr[s_{t-1} = i, s_t = j | \Psi_t] \hat{\mu}_{t,s_{t-1},s_t}}{\Pr[s_t = j | \Psi_t]}, \quad (3.7)$$

where a hat indicates an estimate of the unknown variable and the probabilities in Eq. (3.7) are obtained using the Hamilton filter.<sup>3</sup>

<sup>3</sup> In principle, the approximation can be improved by considering  $n^3$  possible values of  $\mu_t$  (including  $t$ ,  $t - 1$  and  $t - 2$ ) and collapsing them into  $n^2$ , with a substantial increase in the size of the Markov chain and of the computational burden. An alternative is to avoid the path dependence by dropping the term  $\mu_{t-1,s_{t-1}}$  from the right part of Eq. (3.6) and possibly increasing the number of lagged  $x_t$  (see below).

**Table 1**

Average estimates and root mean squared errors of the parameters relative to 200 simulated series generated by the MS(3)-AMEM model with different lengths.

Parameters	True	T = 1000		T = 2000	
		Average	RMSE	Average	RMSE
$\omega$	1.92	1.81	1.49	1.82	0.22
$k_2$	0.44	0.52	0.45	0.48	0.37
$k_3$	3.64	3.49	2.86	3.41	0.71
$\alpha_{13}$	0.14	0.14	0.11	0.14	0.02
$\alpha_2$	0.12	0.13	0.09	0.13	0.02
$\beta_{13}$	0.58	0.59	0.45	0.58	0.04
$\beta_2$	0.67	0.67	0.52	0.67	0.04
$\gamma_{13}$	0.13	0.13	0.10	0.13	0.01
$\gamma_2$	0.08	0.08	0.06	0.08	0.01
$p_{11}$	0.99	0.99	0.76	0.99	0.00
$p_{12}$	0.01	0.01	0.01	0.01	0.00
$p_{21}$	0.01	0.01	0.01	0.01	0.01
$p_{22}$	0.96	0.95	0.74	0.96	0.01
$p_{31}$	0.01	0.02	0.01	0.01	0.01
$p_{32}$	0.06	0.07	0.06	0.06	0.02
$a_1$	17.72	17.78	13.70	17.63	0.87
$a_2$	23.52	24.55	18.99	23.58	1.66
$a_3$	10.63	11.68	8.31	10.80	1.31

We rely on the work of Engle and Gallo (2006) and Kim (1994) for the Quasi Maximum Likelihood properties of the MS-AMEM estimator. The issue arises of the small-sample consequences of the approximation involved with the absence of analytical results. In the original contribution, Kim (1994) compares the exact maximum likelihood estimates of the generalized Hamilton model (Lam, 1990) with the results obtained by applying his procedure to real data. In our case, in order to provide some evidence of the properties of the estimator of the model in Eq. (3.6) in finite samples, we resort to a small simulation experiment. We generate data from the MS(3)-AMEM (Eq. (3.6)), adopting a reasonable set of values as “true” parameters (cf. the application below); the sample size is equal to  $T = 1000, 2000$ , and we generate 200 time series for each experiment. The results are shown in Table 1, where the average of the estimated coefficients is compared with the true parameters. The estimators of the dynamic parameters and the transition probabilities seem unbiased, whereas the constants and the parameters of the Gamma distributions show a small bias. The biases and overall root mean squared errors (RMSEs) decrease with the increase in  $T$ . The results point to the fact that, with more than 2000 observations, we can expect the approximated likelihood to provide satisfactory results.

Tests based on the likelihood function cannot be used to compare the AMEM to the corresponding MS models because of the presence of nuisance parameters that are present only under the alternative hypothesis; in this case, with the proper caution, a classical BIC or AIC could provide some information (see Psaradakis & Spagnolo, 2003, with the AIC giving better results when the parameter changes are not too small and the hidden Markov chain is fairly persistent).

An alternative which has behaved well in previous applications is the nonparametric Bayesian approach of Otranto and Gallo (2002). The procedure identifies the numbers of regimes in Markov switching models, based on the detection of the empirical posterior distribution of the number of regimes, via Gibbs sampling, using the nonpara-



metric Bayesian techniques derived from the Dirichlet process theory.

### 3.2. The AMEM with smooth transition

To explore the possibility that the abrupt shifts from one regime to another in the MS model should be replaced by a more gradual passage across average volatilities, we propose a smooth transition approach, which is similar to the smooth transition model of [Teräsvirta \(1994\)](#). We have the following specification (ST-AMEM):

$$\begin{aligned} x_t &= \mu_t \varepsilon_t, \quad \varepsilon_t \sim \text{Gamma}(a, 1/a) \text{ for each } t \\ \mu_t &= \omega + (\alpha_0 + \alpha_1 F(\xi_{t-1}))x_{t-1} \\ &\quad + (\beta_0 - \beta_1 F(\xi_{t-1}))\mu_{t-1} + \gamma D_{t-1}x_{t-1} \\ F(\xi_t) &= (1 + \exp(-g(\xi_t - c)))^{-1}. \end{aligned} \quad (3.8)$$

Here,  $F(\xi_t)$  is the smooth transition function that depends on a forcing variable  $\xi_t$ , which drives the smooth changes in the parameters of the model. Estimation is performed via maximum likelihood, which retains its QMLE interpretation under misspecification of the choice of the distribution for the innovations. In a financial framework, the changes could depend on market conditions represented by, for example, lagged values of the realized volatility or an independent measure of volatility. In the application illustrated in Section 4, the forcing variable adopted is the VIX (an average of implied volatilities on at-the-money options on the Standard and Poor 500 index).

### 3.3. Alternative specifications

Following the suggestions discussed in Section 2, we can explore three alternative approaches that may capture additional features, as follows.

1. To explore the possibility that the passage from one regime to another depends on market conditions, we apply a time-varying transition probability (TVTP) approach rather than a smooth transition function for the average volatility as in the ST-AMEM; here, the forcing variable enters the transition probabilities directly. The TVTP-MS(n)-AMEM is specified as

$$\begin{aligned} x_t &= \mu_{t,s_t} \varepsilon_t, \quad \varepsilon_t | s_t \sim \text{Gamma}(a_{s_t}, 1/a_{s_t}) \text{ for each } t \\ \mu_{t,s_t} &= \omega + \sum_{i=1}^n k_i I_{s_t} + \alpha_{s_t} x_{t-1} + \beta_{s_t} \mu_{t-1,s_{t-1}} \\ &\quad + \gamma_{s_t} D_{t-1} x_{t-1}, \\ p_{ij,t} &= \frac{\exp(\theta_{ij} + \phi_{i,j} \xi_{t-1})}{1 + \exp(\theta_{i,j} + \phi_{i,j} \xi_{t-1})}. \end{aligned} \quad (3.9)$$

2. To investigate the possibility that long-memory features may be able to capture the slow-moving underlying level of the volatility, we adopt a specification similar to [Corsi's \(2009\)](#) HAR model, where the conditional volatility is made dependent on past volatilities aggregated at different frequencies (HMEM; D(aily), 5 is for W(eekly), 22 is for M(onthly)):

$$\begin{aligned} x_t &= \mu_t \varepsilon_t, \quad \varepsilon_t \sim \text{Gamma}(a, 1/a) \text{ for each } t \\ \mu_t &= \omega + \alpha_D x_{t-1} + \alpha_W \bar{x}_{t-1}^{(5)} + \alpha_M \bar{x}_{t-1}^{(22)}, \end{aligned} \quad (3.10)$$

with the possible introduction of regimes (MS(n)-HMEM):<sup>4</sup>

$$x_t = \mu_{t,s_t} \varepsilon_t,$$

$$\varepsilon_t | s_t \sim \text{Gamma}(a_{s_t}, 1/a_{s_t}) \text{ for each } t$$

$$\begin{aligned} \mu_{t,s_t} &= \omega + \sum_{i=1}^n k_i I_{s_t} + \alpha_{D,s_t} x_{t-1} + \alpha_{W,s_t} \bar{x}_{t-1}^{(5)} \\ &\quad + \alpha_{M,s_t} \bar{x}_{t-1}^{(22)}. \end{aligned} \quad (3.11)$$

3. Finally, we adopt the HAR-J model proposed by [Anderesen et al. \(2007\)](#): abrupt changes may be related to the presence of jumps, so the HAR model is augmented by an additive variable  $J_t$ . Therefore,

$$\begin{aligned} x_t &= \omega + \alpha_D x_{t-1} + \alpha_W \bar{x}_{t-1}^{(5)} + \alpha_M \bar{x}_{t-1}^{(22)} \\ &\quad + \zeta J_{t-1} + \varepsilon_t, \end{aligned} \quad (3.12)$$

where  $J_t$  is obtained as:

$$J_t = \max(x_t - x_t^B, 0)$$

and  $x_t^B$  is the standardized realized bi-power variation of the financial time series analyzed.

We will also keep track of the performance of the original HAR, where the innovation term is added to the conditional expectation, rather than multiplied by it (as in HMEM). Moreover, we will consider HAR and HAR-J models for both RV and log-RV; in the latter case, we assume the log-normality of the disturbances in order to obtain the forecasts of RV from log-RV.

## 4. Nonlinear dynamics in the volatility of the S&P500 index

Turning now to the empirical analysis, we choose the realized kernel volatility of the S&P500 index as being representative of some typical features in the volatility that may be common to other stock indices. Our data are taken from the *Oxford-Man Institute's Realized Library* version 0.2 ([Heber, Lunde, Shephard, & Sheppard, 2009](#)), and are expressed as the percentage annualized volatility (i.e. the square root of the realized kernel variance multiplied by  $100\sqrt{252}$ ), as shown in [Fig. 1](#) for the period between Jan. 3, 2000, and Mar. 10, 2014.<sup>5</sup> The time series shows the high level of persistence discussed above. The presence of changing levels of the prevailing average volatility by sub-periods or other types of nonlinearity can be conjectured; in actual fact, the series shows alternating regimes which visually involve a shift in the average level but may also be accompanied by changes in the dynamics of the series. This is particularly clear in the sample at hand, with the turbulence leading to the burst of the tech bubble, the 2001 re-

<sup>4</sup> Note that the MS(n)-HMEM does not suffer from the path-dependence problem mentioned above.

<sup>5</sup> The Realized Library, which is updated continuously, provides a number of indicators, including the realized kernel variances for many financial indices. The estimator uses a weighted combination of squared returns and cross products of returns  $h$  lags apart ( $h = 1, \dots, H$ ) with a Parzen weight function and the bandwidth  $H$  chosen optimally as in Section 2.1 of [Barndorff-Nielsen, Hansen, Lunde, and Shephard \(2009\)](#), who also note the similarity of the realized kernel to a HAC-type estimator.

**Table 2**

Descriptive statistics for the S&P500 realized kernel volatility (in annualized percentage terms).

	Full	In-sample	Out-of-sample
Mean	14.90	15.60	15.00
Median	12.54	13.10	12.95
Min	2.43	3.49	4.50
Max	153.19	153.19	62.43
St. dev.	9.78	10.16	8.76
Skewness	3.31	3.34	2.06
Kurtosis	21.45	21.43	5.73
$\rho(1)$	0.82	0.82	0.74
$\rho(5)$	0.71	0.71	0.57
$\rho(22)$	0.55	0.55	0.41

Notes: Full span: January 3, 2000 to March 10, 2014; main out-of-sample period: July 5, 2011 to October 26, 2012.  $\rho(i)$  indicates the autocorrelation at lag  $i$ .

cession, the low level of volatility mid-decade, and then the explosion of uncertainty following the subprime mortgage crisis.

The figure may be complemented by the descriptive statistics reported in Table 2 for the overall period: we also report a split analysis between the in-sample and out-of-sample periods, with July 1, 2011, as the end of the estimation period (when the shaded area starts, corresponding to the out-of-sample period). Vertical bars are added to identify the additional periods chosen for the robustness check, see below. It is clear that the first period has more turbulent features than the second, with a very large range and a thick right tail (high kurtosis), determined by the bursts of volatility after September 2008 in particular, and reaching a maximum at over 150% (relative to an overall mean of about 13%). The slow decay in the time dependence is reflected by the autocorrelation function, reported at lags 1 (one day), 5 (one week) and 22 (one month), confirming the features found in the literature. The more recent period is definitely less turbulent, with more moderate peaks and with the skewness and kurtosis reflecting a substantially less thick right tail.

#### 4.1. Estimation results

The MEM and AMEM are estimated as benchmarks, with the estimation results reported in Table 3.<sup>6</sup> The asymmetry coefficient  $\gamma$  is statistically significant, in line with previous evidence; this favors the AMEM, as do the information criteria. The estimated persistence is very high (around 0.93 for both models). The model diagnostics show the presence of serially correlated residuals (cf. the  $p$ -values of the Ljung–Box test statistics at lags 1, 5 and 10), which is not corrected by increasing the order of the AMEM model (results not shown here).

When exploring the possibility of the presence of regimes, we start by applying the nonparametric Bayesian procedure suggested by Otranto and Gallo (2002). Table 4 reports the results for two choices of the hyperparameter

$A$  that regulates the prior probabilities on the number of regimes (a lower  $A$  favors a smaller number of regimes); the presence of autocorrelation in the raw data suggests the choice of a low value of  $A$ , as was indicated by Otranto and Gallo (2002). There is strong evidence (an absence of regimes has a zero posterior probability) in favor of three regimes, with little support for a fourth regime.

As with other MS models, it is crucial to avoid overfitting; we adopt a MS(3)-AMEM using the most general specification and then proceed to some coefficient testing in order to select a more parsimonious model and improve the precision of the parameter estimates. We can calculate the Wald test statistic for the joint hypothesis in the MS-AMEM:

$$\alpha_i = \alpha_j, \quad \beta_i = \beta_j, \quad \gamma_i = \gamma_j$$

for each  $i, j = 1, 2, 3$  and  $i \neq j$ . The corresponding  $p$ -values are 0.00 for  $(i, j) = (1, 2)$ , 0.97 for  $(i, j) = (1, 3)$ , and 0.28 for  $(i, j) = (2, 3)$ , due to the large standard errors of the coefficients in regime 3; a restricted model having the same AMEM dynamics in regimes 1 and 3 confirms the significant difference in the dynamic coefficients in state 2.<sup>7</sup>

The shapes of the Gamma densities based on the estimated parameters  $a_{it}$  in the MS(3)-AMEM (cf. Table 5) give an idea of the interactions between conditional means and the innovations in each regime (Fig. 2): we see an approximate symmetry in the medium volatility state (23.52, the largest value) and a fairly pronounced asymmetry for the high volatility state (smallest  $a$  equal to 10.63), as one would expect. The larger variance of the regime representing the highest volatility is evident, having peaks of different magnitudes belonging to this state. The low volatility of regime 1 has a larger variance than regime 2, which could be considered a sort of *normal* volatility state. Our findings seem to be in line with the well-documented finding that the volatility of volatility is high if the level of volatility is also high, due to a heteroskedastic measurement error (Barndorff-Nielsen & Shephard, 2005; Corsi et al., 2008).

We can assign each day to a regime based on the mode of the smoothed probabilities  $\Pr(s_t | \Psi_T)$  of being in one of the states for each  $t$  from the MS(3)-AMEM (fortunately, these probabilities are generally near either zero or one, favoring a clear-cut association). These results are shown in Fig. 3, where the regime classifications are reported (1, 2 and 3, from bottom to top), keeping the realized volatility series in the background, together with the S&P500 index (log-scale, to make local trends comparable). Higher regimes are associated with market downturns, and there are many oscillations between states 2 and 3 until the second half of 2003, a feature which is consistent with the uncertainty that followed the burst of the dot-com bubble (which had its peak in March 2000), the 2001 recession, 9/11, and the starts of the Afghan war in October 2001 and the Iraqi war in March 2003. There is a long quiet period between October 2003 and July 2007, characterized by a long-lasting bullish rally in the stock market, with a sudden transition to the high volatility regime on July 19, 2007, which lasts for about a month, marking the beginning of the sub-prime mortgage crisis and the first measures undertaken by the FED to contain the credit crunch. The second part of 2007 and the first part of 2008 are character-

<sup>6</sup> The standard errors are derived from the so-called 'sandwich matrix', which provides estimators that are robust to possible misspecifications in the distributional hypotheses (see for example Bollerslev & Wooldridge, 1992).

<sup>7</sup> The restricted model with the same dynamics in states 2 and 3 does not get similar empirical support and induces residual autocorrelation.

**Table 3**  
S&P500 realized kernel volatilities (in annualized percentage terms).

	$\omega$	$\alpha$	$\beta$	$\gamma$	$a$	AIC	BIC	LB(1)	LB(5)	LB(10)
MEM	1.05 (0.07)	0.40 (0.03)	0.53 (0.03)		13.44 (0.48)	5.47	5.48	0.02	0.01	0.00
AMEM	0.94 (0.07)	0.27 (0.01)	0.61 (0.01)	0.11 (0.01)	14.21 (0.51)	5.42	5.43	0.02	0.03	0.00

Notes: The table shows coefficient estimates for the two benchmark models, MEM and AMEM (with standard errors in parentheses), together with information criteria, and  $p$ -values for the Ljung–Box test statistics. Sample: Jan. 3, 2000 to Jul. 1, 2011.

**Table 4**  
Nonparametric Bayesian approach for detecting the number of regimes: empirical posterior distribution of the number of regimes as a function of the hyperparameter  $A$ .

$A$	Number of regimes			
	1	2	3	4
0.1	0.00	0.00	0.93	0.07
0.3	0.00	0.00	0.91	0.09

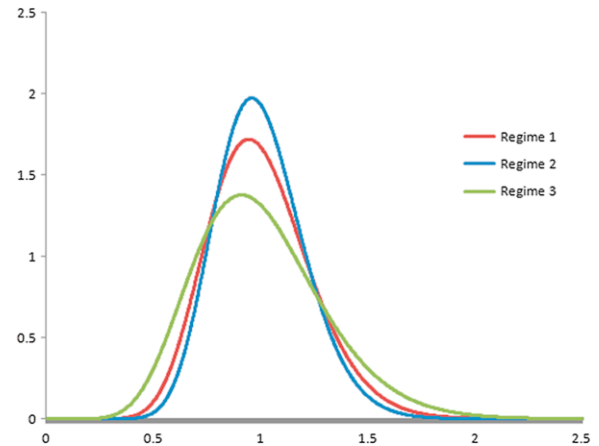
The nonparametric Bayesian approach of Otranto and Gallo (2002) is a procedure for identifying the number of regimes in Markov switching models, based on the detection of the empirical posterior distribution of the number of regimes, via Gibbs sampling, using the nonparametric Bayesian techniques derived from the Dirichlet processes theory. The hyperparameter  $A$  regulates the prior probabilities on the number of regimes. The other prior distributions adopted are the same as were used by Otranto and Gallo (2002).

**Table 5**  
Coefficient estimates for Markov Switching AMEM specifications with three regimes and Gamma innovations (with standard errors in parentheses). Sample: Jan. 3, 2000 to Jul. 1, 2011.

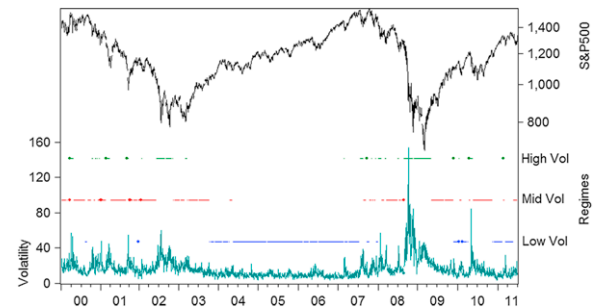
$\omega$	$k_2$	$k_3$	$a_1$	$a_2$	$a_3$
1.92 (0.13)	0.44 (0.19)	3.64 (0.36)	17.72 (0.70)	23.52 (1.25)	10.63 (0.91)
$\alpha_{13}$	$\alpha_2$	$\beta_{13}$	$\beta_2$	$\gamma_{13}$	$\gamma_2$
0.14 (0.01)	0.12 (0.01)	0.58 (0.03)	0.67 (0.03)	0.13 (0.01)	0.08 (0.01)
$p_{11}$	$p_{12}$	$p_{21}$	$p_{22}$	$p_{31}$	$p_{32}$
0.99 (0.00)	0.01 (0.00)	0.01 (0.00)	0.96 (0.01)	0.01 (0.00)	0.06 (0.01)

ized basically by mid-volatility, with occasional passages to the higher regime, until the beginning of September 2008, when the switch to regime 3 became more persistent, gaining momentum with the collapse of the Lehman Brothers (with an outburst in the series on Oct. 20, 2008, of which more later), and lasting until May 2009. The second half of 2009 is spent mostly in regime 2, with a passage to regime 1 at the end of the year. The following months see changes between regimes 1 and 2, although the month of May 2010 is marked by a sudden switch to regime 3 (following the flash crash of May 6).

The characterization of the regime switching is confirmed by the duration  $\frac{1}{1-p_{ii}}$  in a certain regime  $i$ , estimated on the basis of the transition probabilities reported in the lower part of Table 5. On average, we have an 87-day permanence in the state of low volatility, which decreases to 28 days for the intermediate volatility state, and to 13 days for the high volatility state. This result is in line with the empirical evidence that periods of turmoil have shorter durations than quieter spells.



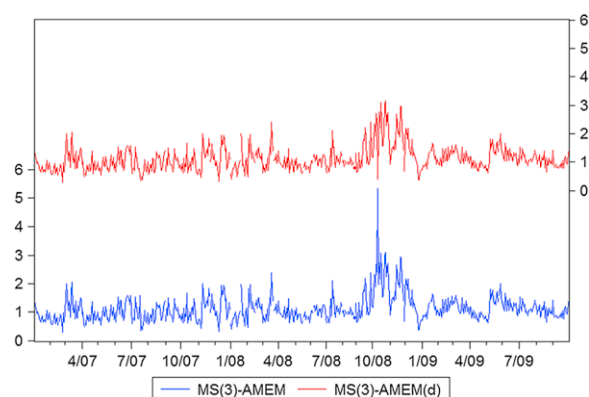
**Fig. 2.** Gamma densities by regime for the MS(3)-AMEM.



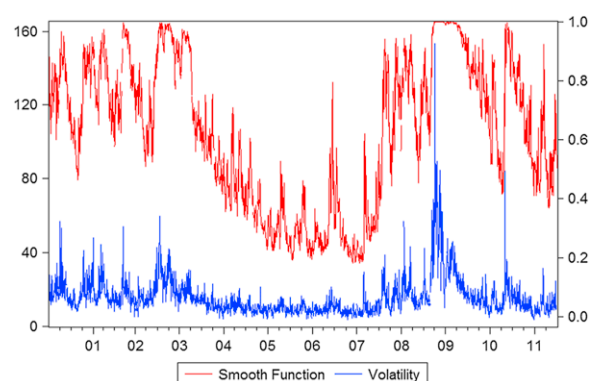
**Fig. 3.** Realized volatility and inference on the regimes (horizontal dots), obtained as the mode of the smoothed regime probabilities using the MS(3)-AMEM. For reference, the S&P500 index is reported at the top (log-scale).

The off-diagonal elements of the transition probability matrix point to a strong interaction between regimes 2 and 3, while the period of low volatility is a sort of stand-alone regime. As a matter of fact, once in regime 1 there is a very low probability of switching to either of the other regimes. When in the regime of intermediate volatility, there is a higher probability of moving to the high volatility regime than of reverting to a low volatility regime. By the same token, note that downward transitions from the high volatility state preferably occur through a move to the intermediate state.

The ability of the Markov Switching AMEM to remove the underlying slow-moving trend in the evolution of volatility can be seen by dividing the original data by the regime-specific average volatilities, estimated at 9.21, 14.39 and 28.80, respectively (cf. Eq. (3.5)). This is done



**Fig. 4.** S&P500 realized kernel volatility series divided by the regime-specific average volatilities obtained from the MS(3)-AMEM and MS(3)-AMEM(d).



**Fig. 5.** Smooth transition function obtained from ST-AMEM (right axis) and the S&P500 realized kernel volatility (left axis).

in the bottom panel of Fig. 4, and results in a fitted series that oscillates around 1, with a single peak being present, corresponding to Oct. 20, 2008. Rather than fitting an extra regime,<sup>8</sup> given the exceptional nature of the market reaction to the demise of Lehman Brothers, we insert a dummy variable in a modified MS(3)-AMEM(d) (d stands for dummy); the estimation results are substantially the same with a value of the dummy variable coefficient of 71.75 and a slightly lower value of  $k_3$ . The top panel of Fig. 4 shows the better outcome graphically.

The alternative modeling of the series sees a smooth evolution across volatility levels being achieved by estimating the ST-AMEM (details are available in the web appendix). A synthetic representation of its properties is shown in Fig. 5, where, as in Fig. 3, we report how the smooth transition function follows the dynamics of the underlying realized volatility series. The relationship between the classifications by regimes and by smooth transitions across states can be summarized ex post by regressing the values of the smooth transition function on the smoothed probabilities of being in one of the three

states from the MS(3)-AMEM(d). The results are interesting, with a high fit ( $R^2 = 0.97$ ) and estimated coefficients equal to 0.37, 0.75 and 0.93, respectively. The issue of which model delivers the best outcomes in terms of forecasting performances then becomes an empirical question.

#### 4.2. Model comparison

As a reference for practical applications, we suggest a general model selection procedure, in view of the non-nested nature of the models involved and the problem of nuisance parameters being present only under the alternative for the MS and ST models. The sequence proposed is:

1. Identify the number of regimes  $n$  in the series; for this purpose, we adopt the nonparametric Bayesian procedure of Otranto and Gallo (2002), which can be performed before the estimation step.
2. Estimate alternative models with this number of regimes and alternative linear and nonlinear specifications; select the models that are able to capture both the residual autocorrelation and the squared residual autocorrelation, using the Ljung–Box statistics.<sup>9</sup> In particular, the presence of autocorrelation in the squared residuals is seen as a symptom of nonlinearities not being modeled adequately.
3. If more models (at least partially, say at lag 1) satisfy point 2, compare them in terms of in-sample and out-of-sample performances; to this end, we use the MSE, the MAE, the DM statistic, and the MCS approach of Hansen et al. (2011).

For the series at hand, we consider a large set of models in the MS class, even evaluating the performance of the MS(4)-AMEM. Thus, the summary diagnostics results for 12 models (MEM, AMEM, MS(3)-AMEM(d), TVTP-MS-AMEM, MS(4)-AMEM, ST-AMEM, HMEM, MS(3)-HMEM, HAR, log-HAR, HAR-J and log-HAR-J) are presented in Table 6. We highlight in boldface values that do not lead to a rejection of the null hypothesis of no autocorrelation at a 5% significance level. The table shows that the only model that does not present any serial correlation is the MS(3)-AMEM(d) (even without the dummy variable, not reported). Rather than stopping here as per the procedure, we retain the models in the MEM class (MS(3)-AMEM(d), ST-AMEM, HMEM, MS(3)-HMEM) that absorb at least lag-1 autocorrelation in both residuals and squared residuals, together with the base model AMEM. For comparison purposes, we also keep the other models in the HAR class (HAR, log-HAR, HAR-J and log-HAR-J), in spite of them completely missing point 2 above.

In Table 7, we report the results in terms of in- and out-of-sample (1 and 10 steps ahead) loss function mean squared errors (MSE) and mean absolute errors (MAE). For the out-of-sample case, we use a period between Jul. 5, 2011, and Oct. 26, 2012 (334 observations), but we do not

<sup>8</sup> Estimation results for the MS(4)-AMEM are reported in the web appendix; its properties and performance are discussed below.

<sup>9</sup> For the MS( $n$ ) models, we have used the generalized residuals, introduced by Gouriéroux, Monfort, and Trognon (1987) for latent variable models, defined as  $E(\hat{\varepsilon}_t | \Psi_{t-1}) = \sum_{i=1}^n \hat{\varepsilon}_{s_t, t} \Pr(s_t = i | \Psi_{t-1})$ , where  $\hat{\varepsilon}_{s_t, t}$  are the residuals at time  $t$  derived from the parameters of the model in state  $s_t$ .



**Table 6**Residual ( $\hat{\varepsilon}$ ) and squared residual ( $\hat{\varepsilon}^2$ ) autocorrelation of several models: Ljung–Box Q statistics.

Lag	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$
	MEM		AMEM		MS(3)-AMEM(d)		TVTP-MS(3)-AMEM	
1	5.44	<b>3.18</b>	5.12	<b>2.79</b>	<b>1.17</b>	<b>1.03</b>	52.05	22.69
5	14.39	<b>8.43</b>	12.31	<b>6.86</b>	<b>3.36</b>	<b>3.58</b>	177.49	78.76
10	35.71	25.85	30.56	22.90	<b>5.17</b>	<b>5.96</b>	332.39	166.84
20	108.26	64.46	93.77	66.52	<b>18.30</b>	<b>21.79</b>	565.59	303.97
	MS(4)-AMEM		ST-AMEM		HMEM		MS(3)-HMEM	
1	15.93	<b>0.16</b>	<b>0.26</b>	<b>0.36</b>	<b>1.59</b>	<b>0.19</b>	<b>3.23</b>	<b>0.08</b>
5	31.52	<b>3.83</b>	<b>7.56</b>	<b>5.24</b>	46.24	24.10	98.49	21.90
10	67.34	33.44	20.70	19.48	74.82	45.78	148.65	29.84
20	145.47	88.14	40.99	37.45	145.23	81.49	212.98	48.15
	HAR		log-HAR		HAR-J		log-HAR-J	
1	6.12	129.28	11.79	509.10	9.37	155.61	14.79	475.90
5	66.01	226.57	54.24	1190.19	76.95	269.48	47.51	1071.17
10	88.92	382.42	58.16	1192.03	100.76	435.53	51.43	1073.72
20	101.32	474.71	82.64	1226.20	125.59	541.02	73.29	1111.93

Note: bold numbers indicate that the Q statistic leads to a non-rejection at the 5% significance level. Sample: Jan. 3, 2000 to Jul. 1, 2011.

**Table 7**

In-sample (Jan. 3, 2000 to Jul. 1, 2011) and out-of-sample (Jul. 5, 2011 to Oct. 26, 2012; 1 and 10 steps ahead) performances in terms of the mean square errors and mean absolute errors of the models estimated.

	In-sample		Out-of-sample			
	MSE	MAE	1 step ahead		10 steps ahead	
			MSE	MAE	MSE	MAE
AMEM	27.06	3.16	27.65	3.62	361.22	11.66
MS(3)-AMEM(d)	<b>23.01</b>	<b>2.89</b>	26.96	<b>3.61</b>	381.93	11.70
ST-AMEM	26.09	3.13	<b>26.92</b>	<b>3.61</b>	<b>270.67</b>	<b>9.38</b>
HMEM	29.38	3.28	29.91	3.76	374.39	11.69
MS(3)-HMEM	23.37	2.95	29.26	3.75	312.25	10.86
HAR	28.72	3.28	29.70	3.79	<i>310.68</i>	<i>9.60</i>
log-HAR	29.62	3.24	30.04	3.72	376.21	10.58
HAR-J	27.31	3.16	27.27	3.68	503.48	18.05
log-HAR-J	29.02	3.22	29.62	3.85	344.87	13.77

Note: the best result by column is reported in bold, and the second-best is in italics.

re-estimate the coefficient values (cf. a robustness check below). From a descriptive point of view, the table points to the better performances for both losses of the MS(3)-AMEM(d) in-sample (and the MS(3)-HMEM second) and of the ST-AMEM<sup>10</sup> for 1 and 10 steps ahead (with the HAR second). Overall, the presence of a time-varying average level of volatility seems to be beneficial for the quality of the forecasts. The Markov Switching models seem to perform well for the one-step-ahead forecasts (the MS(3)-AMEM(d) ties or is a close second), while their performances deteriorate at longer horizons; interestingly, the HAR has the second best performance, with the MS(3)-HMEM being very close to it, pointing to the benefits of temporal aggregation for longer run forecasting.

From an inferential point of view, the popular Diebold and Mariano (1995) test can be used to verify the null hypothesis of no difference in the accuracy levels of two competing forecasts: it boils down to verifying whether the means of the differences of the squared and absolute forecast errors of each pair of models are zero. Traditionally,

DM tests (and more generally, any predictive ability test) are applied out-of-sample and do not take into account parameter uncertainty; Inoue and Kilian (2005) show, both analytically and via Monte Carlo simulations, that out-of-sample tests have lower power than their in-sample equivalents for linear models with the same number of parameters (in our nonlinear context, the reference to Inoue and Kilian's results should be meant as a heuristic extension). Diebold (in press) stresses that the DM test is intended for comparing out-of-sample forecasts, not for model selection, while the in-sample test can give some comparative indications of model properties in terms of their fitting capabilities. Hansen and Timmerman (in press) advocate a joint analysis of in- and out-of-sample performances, referring also to a result of Hansen (2010), who shows that there is a strongly negative relationship between in- and out-of-sample performances. Since one expects an over-parameterized model to have a tendency to perform worse than a simpler model out-of-sample, if that does not happen, it could be interpreted as indicating support for the more complex model.

Let us consider Table 8, where we show synthetically the results of the DM statistics, with the correction proposed by Harvey, Leybourne, and Newbold (1997); we use symbols to indicate that the model in the column performs significantly better (at a 5% significance level) than the model in the row in terms of in-sample (presence of a star) or out-of-sample (1 step ahead with a diamond and 10 steps ahead with a black circle) fits. We will limit our comments to the two main models suggested here, namely the MS(3)-AMEM(d) and the ST-AMEM. For the former, we see that

- *in-sample*: it is significantly better than HMEM, HAR, log-HAR and log-HAR-J for squared errors and than all other models (except MS(3)-HMEM) for absolute errors; it is not worse than any in either case.
- *out-of-sample*: for one step ahead and for squared errors, it is better than the same models as in-sample, while for absolute errors, it is only better than HMEM, HAR and log-HAR. It is no worse than the others. For 10

<sup>10</sup> Recall that the model uses values of the VIX that are predicted out-of-sample with a simple AR(4); more sophisticated models could be used.

**Table 8**

Significance of the Diebold–Mariano statistics for in-sample predictions (Jan. 3, 2000 to Jul. 1, 2011) and out-of-sample (Jul. 5, 2011 to Oct. 26, 2012) 1- and 10-step-ahead forecasts.

	AMEM	MS(3)-AMEM(d)	ST-AMEM	HMEM	MS(3)-HMEM	HAR	log-HAR	HAR-J	log-HAR-J
Diebold–Mariano results									
<i>Squared errors</i>									
AMEM			★ ●		★			●	
MS(3)-AMEM(d)			●						
ST-AMEM					★				
HMEM	★ ◇	★ ◇	★ ◇ ●		★			●	★ ◇
MS(3)-HMEM			◇ ●						
HAR	◇	★ ◇	★ ◇		★		★ ◇		
log-HAR	★ ◇	★ ◇	★ ◇ ●	★	★ ●		★	★	
HAR-J		●	●	●	★ ●				●
log-HAR-J	★	★ ◇	★ ◇ ●		★			★ ◇	
<i>Absolute errors</i>									
AMEM		★	●		★		●	●	
MS(3)-AMEM(d)									
ST-AMEM		★			★				
HMEM	★ ◇	★ ◇	★ ◇ ●		★		●	●	★
MS(3)-HMEM			◇ ●				●	●	
HAR	★ ◇	★ ◇	★ ◇		★	★	★	★	
log-HAR	★	★	★		★			★	★
HAR-J		●	●	●	★ ●		●	●	●
log-HAR-J	★ ◇ ●	★ ◇ ●	★ ◇ ●	◇ ●	★ ◇ ●	◇ ●	◇ ●	★ ◇	

The model in the column is significantly better in forecasting performance than the model in the row (at the 5% significance level) if a symbol is present: in-sample (★), 1 step ahead (◇) and 10 steps ahead (●) out-of-sample.

steps ahead, HAR-J is added to the previous list for both losses, while log-HAR-J is added for the absolute loss. At the same horizon, it performs worse than ST-AMEM for the squared loss.

The ST-AMEM has a somewhat better performance (at least out-of-sample):

- *in-sample*: it is significantly better than HMEM, HAR, log-HAR and log-HAR-J for both losses, and than AMEM for the squared loss. It is worse than MS(3)-HMEM for both losses and than MS(3)-AMEM(d) for the absolute one (as has been seen).
- *out-of-sample*: for 1 step ahead, it is better than HMEM, MS(3)-HMEM, HAR and log-HAR-J for both losses and than log-HAR for the absolute loss. For 10 steps ahead, it is best of all (except for HAR) for the squared loss, and better than all except MS(3)-AMEM(d), HAR, and log-HAR for the absolute loss. It is not significantly worse than any other model at either 1 or 10 steps ahead.

We perform a simultaneous comparison of the models and show the results of the model confidence set (MCS) approach of Hansen et al. (2011);<sup>11</sup> Table 9 reports the results of the sequential procedure at a 95% confidence level, indicating, on the basis of the *p*-values, whether or not the model in the row is in the 'winning' set (highlighted in bold). We have six such sets (in-sample, and 1 step and 10 steps out-of-sample for each of the two losses). By and large, the models with the time-varying average volatility perform better, although with some qualifications. The

Markov Switching models perform well in-sample and one step ahead. For forecasting, ST-AMEM is the only model that is always present in the best MCS, with an interesting good performance by HAR over the longer forecast horizon (though this is limited to the absolute loss).

#### 4.3. Robustness check

In order to check the robustness of our results to the choice of the estimation/forecast samples, we perform two exercises. The first analysis starts from the same in-sample period as before (Jan. 3, 2000 to July 1, 2011) and projects the estimated model to produce out-of-sample forecasts. After 66 observations, the model is re-estimated including the new observations (but keeping the same starting period), and new forecasts are produced; we then proceed by calculating a new set of estimates every 66 observations, with the corresponding forecasts. We limit ourselves to the four models that emerged as the most interesting from the previous discussion, namely the MS(3)-AMEM(d) and the ST-AMEM, with the HAR and the AMEM as benchmarks. The results can be summarized as follows:

- The performances in terms of MSE and MAE (Table 10) show that ST-AMEM is the best model one step ahead, with MS(3)-AMEM(d) as a close second; ST-AMEM confirms its first position in MSE for 10 steps ahead, followed by HAR, while the positions are inverted for MAE.
- The DM pairwise comparison (Table 11) indicates that HAR performs significantly worse than MS(3)-AMEM(d), than ST-AMEM for squared errors, and even than AMEM for absolute errors. For 10 steps ahead, HAR is only better than AMEM, and only for absolute errors.
- Finally, the MCS analysis (Table 12) shows that HAR is excluded from the best one step ahead model set at a 95% confidence level; it also disappears from the best

<sup>11</sup> This by-passes a limitation of the Diebold and Mariano test which involves two models at a time with possible inconsistencies. To perform the MCS approach, we have used the quasi-likelihood loss function with semi-quadratic statistics; see Clements, Doonan, Hurn, and Becker (2009) for details.

**Table 9**Sequential  $p$ -values from the model confidence set approach.

In-sample				Out-of-sample							
				1 step ahead				10 steps ahead			
$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $		$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $		$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $	
HMEM	0.00	HMEM	0.00	HMEM	0.00	log-HAR-J	0.00	HAR-J	0.00	HAR-J	0.00
log-HAR	0.00	HAR	0.00	log-HAR	0.00	HAR	0.00	MS(3)-AMEM(d)	0.00	log-HAR-J	0.00
HAR	0.00	log-HAR	0.00	HAR	0.00	<b>HMEM</b>	0.08	HMEM	0.00	MS(3)-AMEM(d)	0.00
log-HAR-J	0.00	log-HAR-J	0.00	log-HAR-J	0.02	<b>MS(3)-HMEM</b>	0.28	log-HAR	0.00	HMEM	0.00
HAR-J	0.00	HAR-J	0.00	<b>MS(3)-HMEM</b>	0.35	<b>log-HAR</b>	0.69	AMEM	0.00	AMEM	0.00
AMEM	0.01	AMEM	0.00	<b>AMEM</b>	0.94	<b>HAR-J</b>	0.96	log-HAR-J	0.00	log-HAR	0.00
<b>ST-AMEM</b>	0.05	ST-AMEM	0.00	<b>HAR-J</b>	0.93	<b>ST-AMEM</b>	0.99	HAR	0.01	MS(3)-HMEM	0.00
<b>MS(3)-AMEM(d)</b>	0.90	<b>MS(3)-AMEM(d)</b>	0.30	<b>MS(3)-AMEM(d)</b>	0.86	<b>AMEM</b>	0.90	MS(3)-HMEM	0.01	<b>HAR</b>	0.47
<b>MS(3)-HMEM</b>		<b>MS(3)-HMEM</b>		<b>ST-AMEM</b>		<b>MS(3)-AMEM(d)</b>		<b>ST-AMEM</b>		<b>ST-AMEM</b>	

For each forecast horizon and each criterion, the first row corresponds to the first model removed, down to the best performing model in the last row. We highlight the models belonging to the best MCS at a 95% confidence level in boldface.

**Table 10**

Out-of-sample (Jul. 5, 2011 to Oct. 22, 2012; 1 and 10 steps ahead) performances, in terms of the mean square errors and mean absolute errors of the models estimated by updating the estimates after 66 observations.

	Out-of-sample			
	1 step ahead		10 steps ahead	
	MSE	MAE	MSE	MAE
AMEM	27.84	3.63	361.12	11.68
MS(3)-AMEM(d)	27.14	<b>3.62</b>	388.06	11.98
ST-AMEM	<b>26.81</b>	<b>3.62</b>	<b>272.01</b>	9.42
HAR	29.97	3.80	307.47	<b>9.31</b>

Note: The best result in each column is reported in bold, with the second-best in italics.

**Table 11**

Significance of the Diebold–Mariano statistics for the comparison of out-of-sample (Jul. 5, 2011 to Oct. 22, 2012) 1- and 10-step-ahead forecasts of the models estimated by updating the estimates after 66 observations.

	AMEM	MS(3)-AMEM(d)	ST-AMEM	HAR
Diebold–Mariano results				
Squared errors				
AMEM				
MS(3)-AMEM(d)				
ST-AMEM				
HAR	◇		◇	
Absolute errors				
AMEM				•
MS(3)-AMEM(d)				
ST-AMEM				
HAR	◇	◇	◇	

The model in the column has a significantly better forecasting performance than that of the model in the row (at the 5% significance level) if a symbol is present: 1 step ahead (◇) and 10 steps ahead (•) out-of-sample.

set for 10 steps ahead for the absolute errors (where ST-AMEM is still the best model), and is included in it (next to the ST-AMEM) for the squared errors.

By and large, the performance parallels that evidenced in the previous section, with ST-AMEM confirming its good properties for longer horizons and the coherence of the MS(3)-AMEM(d) for one step ahead being confirmed likewise.

The second analysis consists of estimation and forecasting over two other samples, one year apart, of approximately the same length as the original one. The first starts

on Jan. 2, 2001, and ends on July 3, 2012 (forecasts from Jul. 5, 2012 to Oct. 25, 2013), and the second is from Jan. 7, 2002 to July 3, 2013 (forecasts from Jul. 5, 2013 to Mar. 10, 2014). In this case, we check for

- *autocorrelation in the residuals* in Table 13. As before, HAR is not satisfactory, while the AMEM shows substantial improvements in the first alternative sample, and more so in the second (as happens with the ST-AMEM). MS(3)-AMEM(d) is the only one that does not present any problem with these model diagnostics.
- *MSE and MAE in- and out-of-sample results* in Table 14. MS(3)-AMEM(d) has the best in-sample performance. While the ST-AMEM has the best out-of-sample performance in both losses, we notice that the MS(3)-AMEM(d) improves substantially in the longer out-of-sample evaluation. Generally speaking, the levels of MSE drop in both cases, from about 27 (cf. Table 7) to about eight for one step ahead and from 300 to 40 for 10 steps ahead, while the levels of MAE drop roughly from three to two and from ten to five, respectively. Recall that 2011 (and therefore the debt-ceiling crisis of mid-summer) is considered to be out-of-sample in the main analysis, while it is in the estimation period for both of the robustness check samples (cf. Fig. 1). While this does not affect ST-AMEM, which passes the robustness check very well, the improvement in performance of the MS(3)-AMEM(d) and the worsening of the HAR, especially at longer horizons, indicates that the quasi-long memory features of the HAR are more likely to be useful in more turbulent periods (which obviously will not be known in advance).
- *the DM test analysis* in Table 15. By and large, it confirms the same patterns, with generally better performances by the MS(3)-AMEM(d) and the ST-AMEM than the other two, especially in the second sample.
- *the MCS analysis* in Table 16. In-sample, the AMEM class performs well again, particularly the MS(3)-AMEM(d), while ST-AMEM is always present in the best set out-of-sample, accompanied by the MS(3)-AMEM(d) for longer horizons. For one step ahead, satisfactory results are offered by more than one model.

Summing up, as one would expect, the estimation period is relevant for the specific performances of the models, both in- and out-of-sample. However, the results are quite

**Table 12**

Sequential  $p$ -values from the model confidence set approach for a comparison of the out-of-sample (Jul. 5, 2011 to Oct. 22, 2012) 1- and 10-step-ahead forecasts of the models estimated by updating the estimates after 66 observations.

Out-of-sample							
1 step ahead				10 steps ahead			
	$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $		$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $
HAR	0.05	HAR	0.03	AMEM	0.00	MS(3)-AMEM(d)	0.00
<b>AMEM</b>	0.72	<b>AMEM</b>	0.99	MS(3)-AMEM(d)	0.00	AMEM	0.00
<b>MS(3)-AMEM(d)</b>	0.87	<b>MS(3)-AMEM(d)</b>	0.93	<b>ST-AMEM</b>	0.94	HAR	0.02
<b>ST-AMEM</b>		<b>ST-AMEM</b>		<b>HAR</b>		<b>ST-AMEM</b>	

For each forecast horizon and criterion, the first row corresponds to the first model removed, down to the best performing model in the last row. We highlight the models belonging to the best MCS at a 95% confidence level in bold.

**Table 13**

Residual ( $\hat{\varepsilon}$ ) and squared residual ( $\hat{\varepsilon}^2$ ) autocorrelations of several models: Ljung–Box  $Q$  statistics.

Lag	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$	$\hat{\varepsilon}$	$\hat{\varepsilon}^2$
First sample: Jan. 2, 2001 to Jul., 3, 2012								
	AMEM		MS(3)-AMEM(d)		ST-AMEM		HAR	
1	<b>1.37</b>	<b>0.48</b>	<b>0.18</b>	<b>2.17</b>	<b>1.41</b>	<b>1.09</b>	8.42	136.59
5	13.85	<b>7.63</b>	<b>6.45</b>	<b>4.61</b>	12.54	<b>9.04</b>	82.63	239.76
10	36.92	26.05	<b>11.40</b>	<b>9.01</b>	29.15	27.65	102.40	397.64
20	100.66	61.76	<b>25.89</b>	<b>25.85</b>	33.69	42.11	111.82	488.16
Second sample: Jan. 7, 2002 to Jul., 3, 2013								
	AMEM		MS(3)-AMEM(d)		ST-AMEM		HAR	
1	<b>2.34</b>	<b>1.08</b>	<b>0.01</b>	<b>1.33</b>	<b>0.60</b>	<b>0.54</b>	7.57	133.48
5	<b>5.41</b>	<b>4.07</b>	<b>1.84</b>	<b>4.13</b>	<b>3.96</b>	<b>4.03</b>	75.38	234.01
10	<b>13.30</b>	<b>13.06</b>	<b>5.46</b>	<b>6.47</b>	<b>15.16</b>	<b>16.31</b>	94.68	399.69
20	52.28	45.25	<b>19.61</b>	<b>25.56</b>	32.50	32.24	106.28	498.70

Note: bold values indicate that the  $Q$  statistic is not significant at the 5% significance level.

**Table 14**

In-sample and out-of-sample (1 and 10 steps ahead) performances in terms of the mean square errors and mean absolute errors of the models estimated.

	In-sample		Out-of-sample			
	MSE	MAE	1 step ahead		10 steps ahead	
			MSE	MAE	MSE	MAE
First sample						
AMEM	27.46	3.18	9.11	2.36	63.68	6.82
MS(3)-AMEM(d)	<b>23.06</b>	<b>2.91</b>	9.84	2.36	49.99	5.37
ST-AMEM	26.05	3.12	<b>7.97</b>	<b>2.18</b>	<b>37.51</b>	<b>4.89</b>
HAR	29.11	3.30	17.41	3.16	55.09	6.09
Second sample						
AMEM	25.51	3.06	7.54	2.25	63.73	7.26
MS(3)-AMEM(d)	<b>21.47</b>	<b>2.92</b>	7.43	2.25	37.98	5.32
ST-AMEM	24.25	3.00	<b>7.01</b>	<b>2.21</b>	<b>36.76</b>	<b>5.19</b>
HAR	27.29	3.18	7.95	2.22	77.45	7.72

First in-sample period: Jan. 2, 2001 to Jul. 3, 2012; first out-of-sample period: Jul. 5, 2012 to Oct. 25, 2013. Second in-sample period: Jan. 7, 2002 to Jul., 3, 2013; second out-of-sample period: Jul. 5, 2013 to Mar. 10, 2014. We report the best result for each column in bold, with the second-best in italics.

remarkably stable in pointing out that a good in-sample performance tends to be accompanied by a good out-of-sample performance for the AMEM class of models when some time-varying average volatility is modeled explicitly.

## 5. Concluding remarks

In this paper, we have focused on the presence of a changing long-run volatility level around which some

short run dynamics can be identified. We devote particular attention to suggesting that such a slow-moving underlying trend may be attributable either to regime-switching behaviors or to some nonlinearities, where the transition is modeled as a smooth autoregression. Such a feature is shared by most volatility series, at least as measured from the year 2000 until more recent data. To this end, we have suggested several extensions of the univariate Asymmetric Multiplicative Error Model (AMEM) to accommodate sudden or smooth changes in the underlying average level of volatility. This phenomenon cannot be accommodated satisfactorily by a standard AMEM, given that, in practice, some substantial autocorrelation is left in the residuals. In general, since several models are available for dealing with the phenomenon, we resort to a nonparametric Bayesian procedure for identifying the number of regimes, and rely on residual diagnostics to indicate having captured relevant nonlinearities or to identify possible overparameterization. The comparison across models is completed with out-of-sample forecasting diagnostics, with the inference derived on squared and absolute forecast errors. Being aware of the possibility of the results being sample-specific, we also suggest the need for a robustness check across the estimation and out-of-sample periods.

We illustrate the performances of our models in reference to the series of realized kernel volatility of the S&P500. Our results (model MS(3)-AMEM) point to the presence of three regimes, with the highest volatility regime being attributed to the most severe bursts of volatility; we find



**Table 15**

Significance of the Diebold–Mariano statistics for comparisons of the in-sample and out-of-sample 1- and 10-step-ahead forecasts of the models estimated.

	AMEM	MS(3)-AMEM(d)	ST-AMEM	HAR
Diebold–Mariano results				
First sample				
<i>Squared errors</i>				
AMEM		★	★ ◇ ●	
MS(3)-AMEM(d)			◇	
ST-AMEM				
HAR	◇	★ ◇	★ ◇	
<i>Absolute errors</i>				
AMEM		★ ●	★ ◇ ●	
MS(3)-AMEM(d)			◇	
ST-AMEM		★		
HAR	★ ◇	★ ◇	★ ◇	
Second sample				
<i>Squared errors</i>				
AMEM		★ ●	★ ◇ ●	
MS(3)-AMEM(d)				
ST-AMEM				
HAR	★	★ ●	★ ◇ ●	
<i>Absolute errors</i>				
AMEM		★ ●	★ ●	
MS(3)-AMEM(d)				
ST-AMEM		★		
HAR	★	★ ●	★ ●	

The model in the column has a significantly better forecasting performance than the model in the row (at the 5% significance level) if a symbol is present: in-sample (★), and 1 step ahead (◇) and 10 steps ahead (●) out-of-sample. First in-sample period: Jan. 2, 2001 to Jul., 3, 2012; first out-of-sample period: Jul. 5, 2012 to Oct. 25, 2013. Second in-sample period: Jan. 7, 2002 to Jul., 3, 2013; second out-of-sample period: Jul. 5, 2013 to Mar. 10, 2014.

**Table 16**

Robustness check on two other samples.

				1 step ahead				10 steps ahead			
$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $		$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $		$\hat{\varepsilon}^2$		$ \hat{\varepsilon} $	
First sample											
In-sample: Jan. 2, 2001 to Jul., 3, 2012				Out-of-sample: Jul. 5, 2012 to Oct. 25, 2013							
HAR	0.04	HAR	0.00	HAR	0.00	HAR	0.00	AMEM	0.00	AMEM	0.00
AMEM	0.09	AMEM	0.00	MS(3)-AMEM(d)	0.00	MS(3)-AMEM(d)	0.00	HAR	0.03	HAR	0.01
ST-AMEM	0.30	ST-AMEM	0.00	AMEM	0.00	AMEM	0.00	MS(3)-AMEM(d)	0.03	MS(3)-AMEM(d)	0.20
MS(3)-AMEM(d)		MS(3)-AMEM(d)		ST-AMEM		ST-AMEM		ST-AMEM		ST-AMEM	
Second sample											
In-sample: Jan. 7, 2002 to Jul., 3, 2013				Out-of-sample: Jul. 5, 2013 to Mar. 10, 2014							
HAR	0.01	HAR	0.00	HAR	0.03	MS(3)-AMEM(d)	0.97	HAR	0.00	HAR	0.00
AMEM	0.55	AMEM	0.00	AMEM	0.05	AMEM	0.95	AMEM	0.00	AMEM	0.00
ST-AMEM	0.20	ST-AMEM	0.00	MS(3)-AMEM(d)	0.19	HAR	0.86	ST-AMEM	0.73	ST-AMEM	0.70
MS(3)-AMEM(d)		MS(3)-AMEM(d)		ST-AMEM		ST-AMEM		MS(3)-AMEM(d)		MS(3)-AMEM(d)	

Notes: The table shows sequential  $p$ -values from the model confidence set approach for a comparison of the in-sample and out-of-sample 1- and 10-step-ahead forecasts of the models estimated. For each forecast horizon and criterion, the first row corresponds to the first model removed, down to the best performing model in the last row. We highlight the models belonging to the best MCS at the 95% confidence level in bold.

that switches between regimes are most likely between the medium and high volatilities, while the long period of volatility moderation in the mid-2000s makes the low volatility regime almost an absorbing state. The short-lived spike in October 2008 may suggest the advisability of considering an extra regime; however, in doing so, we find that the four-state version suffers from numerical difficulties in convergence. While it classifies the October episode as ex-

trema, it nevertheless fails to capture the size of the jump, and ends up signaling other periods with different characteristics as extreme too, making the overall inference on the regimes less stable. We manage to improve the fit and general properties further simply by including a dummy variable to change the value of the constant in the conditional expectation expression just for the day of the jump (model MS(3)-AMEM(d)).

A direct regime-oriented interpretation of the Smooth Transition version of the AMEM is not possible, as it is not possible to estimate average levels of volatilities by regime in the latter. Nevertheless, we find a striking match between the transition function and the behavior of the estimated smoothed transition probabilities in the MS model, with a very close fit in the regression of the former to the three latter series.

We analyze the performances of the AMEM class of models against a relatively large number of competing models, including the four-regime MS-AMEM, a MS-AMEM where the transition probabilities are made time-varying (TVTP-MS(3)-AMEM), and versions of the MEM where we make use of the temporal aggregation that is typical of heterogeneous autoregressive (HAR) models with and without MS effects, in addition to four HAR models themselves, with and without jumps and using levels as opposed to logs. Most of the models exhibit some degree of autocorrelation in both the residuals and the squared residuals, with the only exception being the MS(3)-AMEM (with or without the dummy). We synthesize the in- and out-of-sample (1 and 10 steps ahead) performances relative to squared and absolute losses both descriptively (MSE and MAE) and inferentially (Diebold–Mariano tests and model confidence set analysis). We can say that the performances of our two main models (MS(3)-AMEM(d) and ST-AMEM) are never beaten by any other model (except in-sample for the MS(3)-HMEM versus ST-AMEM); there are other models that perform as well in general, but not in a systematic way. The log-HAR and log-HAR-J models are usually dominated, and the HAR in levels seems to be better only on longer horizons. The robustness check with a moving window of one year shows that moving the period of turbulence resulting from the debt-ceiling crisis of mid-summer 2011 from the forecast period to the estimation period has a very stabilizing effect on the forecasting performance, reducing the values of the loss functions and delivering a greater support for our main models.

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## Appendix A. Supplementary data

Supplementary material related to this article can be found online at <http://dx.doi.org/10.1016/j.ijforecast.2014.09.005>.

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