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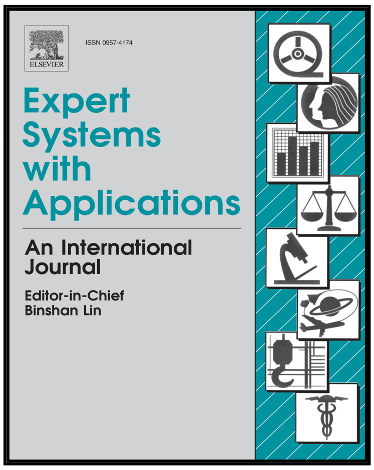

A hybrid volatility forecasting framework integrating GARCH, Artificial Neural network, Technical Analysis and Principal Components Analysis

Werner Kristjanpoller , Marcel C. Minutolo

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Highlights.

- A hybrid model is analyzed to predict the price volatility of Bitcoin
- The hybrid model used is an ANN-GARCH model with PCA pre processing.
- The technical analysis indexes are used as input data.
- The incorporation PCA preprocessing increases the accuracy of the hybrid model.
- An analysis of cryptocurrency.

A hybrid volatility forecasting framework integrating GARCH, Artificial Neural network, Technical Analysis and Principal Components Analysis.

Werner Kristjanpoller *
Departamento de Industrias,
Universidad Técnica Federico Santa María, Chile.
Av. España 1680, Valparaíso, Chile
Phone: (56 32) 2654571, Fax: (56 32) 2654815
Email: werner.kristjanpoller@usm.cl

Marcel C. Minutolo
Department Management,
Robert Morris University
324 Massey 6001 University Blvd Moon Township,
PA 15108, United State
Phone 412-397-5451, Fax
Email: minutolo@rmu.edu

*: corresponding author.

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Werner Kristjanpoller, Marcel C. Minutolo

Abstract.

Measurement, prediction, and modeling of currency price volatility constitutes an important area of research at both the national and corporate level. Countries attempt to understand currency volatility to set national economic policies and firms to best manage exchange rate risk and leverage assets. A relatively new technological invention that the corporate treasurer has to turn to as part of the overall financial strategy is cryptocurrency. One estimate values the total market capitalization of cryptocurrencies at $557 billion USD at the beginning of 2018. While the overall size of the market for cryptocurrency is significant, our understanding of the behavior of this instrument is only beginning. In this article, we propose a hybrid Artificial Neural Network-Generalized AutoRegressive Conditional Heteroskedasticity (ANN-GARCH) model with preprocessing to forecast the price volatility of bitcoin, the most traded and largest by market capitalization of the cryptocurrencies.

1. Introduction.

Cryptocurrency is rapidly becoming an integral part of the global economy. With the introduction of blockchain the potential of non-minted currency with a decentralized governance mechanism was realized. Since the introduction of bitcoin many more variations on the model have been introduced. One source, coinmarketcap.com, lists 1536 different types of digital currencies with a total market capitalization of $321 billion USD of which bitcoin accounts for $15.5 billion USD or approximately 42-percent of the total capitalization as of 16 April, 2018. This is interesting since 371 cryptocurrencies were added, the market capitalization went down $52.5 billion USD, and bitcoin's dominance increased slightly between the time of the initial draft of this paper and the revision (approximately a four month period). In this article, we have adopted the convention in the computer science literature to use Bitcoin (capital-B) to refer to the system and bitcoin (small-b) to refer to the unit of account (Böhme, Christin, Edelman, & Moore, 2015).

Many global firms are accepting bitcoin for payment of goods and services. With the high volatility in the value of bitcoin and its treatment as a property, firms who take large positions in it or hold large amounts put themselves at risk. Conversely, firms that integrate the use of bitcoin and other cryptocurrencies into their overall strategy have the potential to gain a competitive advantage. To this end, the current work is motivated by the desire to 1) build better forecasting models of the volatility of bitcoin; and, 2) understand how the currency behaves. In this work, we develop a hybrid volatility forecasting framework integrating GARCH, artificial neural network, technical analysis and principal components analysis.

Donaldson and Kamstra (1997) were one of the first to demonstrate that neural networks can capture the nonlinear effects of volatility that GARCH models and their derivatives are not capable of modeling. Since this work, there have been several studies of hybrid models such as Bahrammirzaee (2010), Bildirici and Ersin (2009), Guresen, Kayakutlu, and Daim (2011), Hajizadeh, Seifi, Zarandi, and Turksen (2012), Bildirici and Ersin (2013), Kristjanpoller, Fadic, and Minutolo (2014), Monfared and Enke (2014), Kristjanpoller and

Minutolo (2015), Kristjanpoller and Minutolo (2016), Lu, Que, and Cao (2016) and Kristjanpoller and Hernandez (2017). This study differs from the others for various reasons. First, it is the first hybrid application to predict the volatility of a cryptocurrency. Second, the bitcoin volatility series is highly volatile when compared to traditional currencies, which makes it a challenge to predict. Third, there is an innovation in the framework to include an analysis of major components such as preprocessing. Fourth, seven indicators of technical analysis are introduced as variables to improve the forecast. Finally, all the variables used as "exogenous" are derived from the bitcoin closing price series.

In the remainder of this paper, we first present a literature review of cryptocurrency and the development of the model. Following the literature review, we present the methodology and data used. We then present an analysis of the results and findings. In the final section, we present our conclusions to include a discussion of potential future research.

2. Literature Review.

2.1 Cryptocurrency

In this paper, we explore cryptocurrency in general by looking at bitcoin in particular. While there are other cryptocurrencies, we consider only bitcoin since it is the largest by market capitalization. As noted by Pichl and Kaizoji (2017), the amount of research about cryptocurrency is still “scarce” but increasing. We have found some coverage in the area of law which is not surprising as governments attempt to understand how to manage cryptocurrency (Böhme, Christin, Edelman, and Moore, 2015; Lim et al., 2016; Ju, Lu, and Tu, 2016); investments (Brière, Oosterlinck, and Szafarz, 2015; Callen-Naviglia and Alabdan, 2016; Wu and Pandey, 2014); markets (Bhattacharjee, 2016; Carrick, 2016); and, currency (Carrick, 2016; Davidson and Block, 2015; Li and Wang, 2017; McCallum, 2015; Polasik et al. 2016).

The creation of the algorithm and model that resulted in Bitcoin is attributed to a Japanese programmer who released the paper via the internet and goes by the pseudonym Satoshi Nakamoto. Despite the lack of a known person to attribute the work to, a decentralized, open-source community has developed around the concept and continues to promote it. Bitcoin promises to overcome many of the challenges that plague *traditional* currencies. For instance, the supply of bitcoin is fixed at 21 million coins and therefore not subject to the influences of regulatory bodies. Unlike minted currencies that may be forged or counterfeited, bitcoin is protected by an algorithm that is "uncrackable". The introduction of *new* coins is controlled by an algorithm through the process of *mining* which is the maintenance of the ledgers of all transactions without fee and available to the public (Lo and Wang, 2014). As time passes and coins are introduced the number of coins issued over a given period of time diminishes until the final coin is mined. Estimates of when the final coin will be mined place it sometime in the year 2140.

One of the many promises of Bitcoin is the reduction of transaction costs. Due to the reduction of the transaction costs, it has become a popular mechanism to transfer money across borders. All that the individual needs for the transaction to take place is the key in the ledger. Since the key is all that there is to identify the coin, bitcoin has become a popular form of 'anonymous' transactions. In fact, given the relative anonymity of the transaction, there have been allegations that bitcoin is associated with illegal business activities (Cheung, Roca, and Su, 2015).

Li and Wang (2017) suggest that, in the short-run, the price of bitcoin adjusts to changes in both economic fundamentals and market conditions. However, in the long-run, it is more volatile with respect to changes in economic fundamentals than to market conditions. Brière, Oosterlinck, and Szafarz (2015) looked at weekly return data for bitcoin from an investor's perspective. The authors built a diversified portfolio that included both traditional assets as well as alternative investments during the period 2010 to 2013. Their findings suggest that even a modest inclusion of bitcoin in a portfolio results in improvement to the risk-return trade-off. However, they do note that caution ought to be exercised since the results may be a factor of early-stage behavior that may not hold over the long-run.

Polasik, Piotrowska, Wisniewski, Kotkowski, and Lightfoot (2016), applying network externality theory, found that the trading price of bitcoin is dependent upon its overall popularity, media sentiment, and the total number of transactions handled. One particularly interesting article in this domain looked at both the correlation of the complexity of the network and flow with the volatility of bitcoin (Yang and Kim, 2015). They found that as the amount of bitcoin moving through the network increased, they were better able to predict the returns and volatility.

Carrick (2016) evaluated the use of bitcoin as a complement to emerging market currencies. The author begins the paper by asking the question of whether bitcoin is truly a currency addressing the exchange, account and value questions. In all three instances, Carrick concludes, bitcoin appears to resemble a currency. The author finds that those baskets that contain bitcoin seem to outperform those that do not suggesting that bitcoin is a good complement to foreign currency investment. Likewise, Dyhrberg (2016) applied a GARCH model to bitcoin time series and found that the currency behaved like gold and the dollar. Her results suggested that bitcoin may be appropriate for those investors who are risk adverse.

Van Alstyne (2014) in inquiring into why bitcoin has value stated that it has value for four reasons: 1) the technical aspect of bitcoin solves the “double spend” problem; 2) transactions that are near frictionless; 3) its ability to detect fraud due to the public ledger; and, 4) people value it. Yermack (2013) argued that currencies typically have three attributes: 1) they act as a mechanism of exchange; 2) are units of account; and, 3) are stores of value. Bhattacharjee (2016) argues that bitcoin demonstrates the first and is evident for the second in that merchants are accepting it as a form of payment but that it fails to meet the third criteria. Kristoufek (2015), using wavelet decomposition, found that bitcoin exhibits properties of both traditional financial assets and speculative ones. In the following section, we present a summary of the relevant literature as it relates to the forecasting models developed herein.

2.2 Artificial Neural Networks

The measurement, prediction, and modeling of the price volatility of different financial assets, for example commodities and exchange rates, constitute a widely-studied topic in financial research. Governments and central banks alike constantly track volatility in order to analyze fiscal budgets and determine which policies to implement in the country’s economy. It is for this reason that authors have tried to capture these time series characteristics with the purpose of reaching more accurate projections.

Even modest improvements to forecasting methods for market returns and volatility results in better understanding of markets for different financial assets which is important to those making investment decisions. Among those decision makers are government leaders, congressmen, pension fund managers, investment portfolio managers, and investors in general. As investors look for instruments to improve their returns, mitigate risk, and diversify their holdings, cryptocurrencies have gained a lot of interest. As a new technology the volatility has been very high, and for this reason it is a great challenge to forecast.

Traditional methods of volatility prediction of financial time series include AutoRegressive Conditional Heteroskedasticity (ARCH) models (Engle, 1982) and the generalized form (Bollerslev, 1986) as Generalized AutoRegressive Conditional Heteroskedasticity (GARCH). The ARCH and GARCH models incorporate the typical characteristic of heteroskedasticity in financial time series. From these the basic ARCH and GARCH models different extensions have been developed to include the EGARCH (Nelson, 1991), APGARCH (Ding et al., 1993), SWARCH (Hamilton & Susmel, 1994), FIAPGARCH (Tse, 1998), HYGARCH (Davidson, 2004), and more.

The artificial neural network method has proven to be potent for time-series forecasting. For example, White (1988) implemented a model based on the aforementioned method to forecast daily returns of IBM stock. At the stock market level, various studies have applied this technique to predict profitability and volatility: Kryzanowski et al. (1993), Gençay (1996), Qi (1999), Qi and Maddala (1999), Schittenkopf, Dorffner and Dockner (2000). Donaldson and Kamstra (1997) proposed the use of neural networks in combination with the GARCH model to capture the properties of volatility on stock returns, analyzing markets in London, New York, Tokyo, and Toronto. The comparisons within the sample show that the neural networks model captures properties of volatility that were not captured by the GARCH, EGARCH, or GJR models alone.

Roh (2007) proposed hybrid models in conjunction with neural networks and time series models to forecast the volatility of a price index from two points of view: the deviation and direction. The model was applied in South Korea's market and the results showed greater accuracy. Tseng et al. (2008) integrated a hybrid asymmetric approach of volatility forecasting with a neural network model to improve the predictive power of the price of financial derivatives in the Taiwanese market, concluding that the Grey-EGARCH models offer a better predictive power than other volatility forecasting models.

Bildirici and Ersin (2009) utilized a neural network model combined with diverse models derived from GARCH with the Istanbul Stock Exchange, identifying improvements in RMSE for the majority of the applied models. Hajizadeh et al. (2012) utilized two hybrid neural network models for the purpose of improving the predictive power of the GARCH family of models by utilizing explanatory variables to predict return volatility of the S&P 500, outperforming the GARCH method in results as measured through the comparison of realized volatility.

As illustrated above, the field of forecasting volatility has evolved rapidly. In the search for improvements in prediction, researchers have recently begun to explore the use of hybrid models. For instance, researchers have merged artificial intelligence with the traditional models to predict volatility of financial time-series (Kristjanpoller, Fadic, and Minutolo, 2014); neural network with GARCH (Hajizadeh et al., 2012); support vector machines and heterogeneous autoregressive (SVM-HAR) (Khan, 2011); evolving hybrid neural fuzzy network (eHFN) (Rosa et al., 2013), neural network with HAR (NNHAR) (Fernandes et al., 2014); and genetic algorithm and heterogeneous autoregressive (AHAR) (Qu and Ji, 2014); to name only a few.

As mentioned, different volatility studies have demonstrated that forecasting can be improved by including a neural network which corrects the forecasting of previous models such as in Bildirici and Ersin (2009), Kristjanpoller et al. (2014), and Monfared and Enke (2014). Hung (2011) proposed an adaptive GARCH model, in which through a genetic algorithm, the parameters are determined by the membership functions and the GARCH models with a fast convergence rate. The particular innovation of this research is the proposition of a new framework for volatility prediction composed of artificial intelligence and time series forecasting models.

3. Data.

The data analyzed in this article are the series of bitcoin expressed as US dollar per bitcoin (BTC) for the period from September 13th, 2011 to August 26th, 2017. The return or price variation, r_t, is calculated as the first difference of the log (P_t), where P_t is the BTC closing price at time *t*. The historical volatility (*HV*) of bitcoin is represented as the variance of the price return. In particular, the *HV* is analyzed at 22 days, which is the volatility of a month with daily data. We also analyze a short-term *HV* of 10 days (2 weeks) and a longer term *HV* to 44 days (2 months).

For the period analyzed, the average price of the BTC was 479 USD per BTC, a value higher than the median of 329 USD per BTC, which is due to the fact that during the first years of the analysis the price was very low compared to last 6 months. The standard deviation is higher than the average shown by the high dispersion of the BTC price, while the difference between the minimum and maximum values is just under 2,000 times. The daily variation of the price is 0.34% and its standard deviation of 0.049, evidencing the high volatility of this currency. Neither the BTC price series nor the BTC returns are normal, and the return series is stationary. While the bitcoin price series is not stationary, the series of returns is. Table 1 presents the series descriptive statistics.

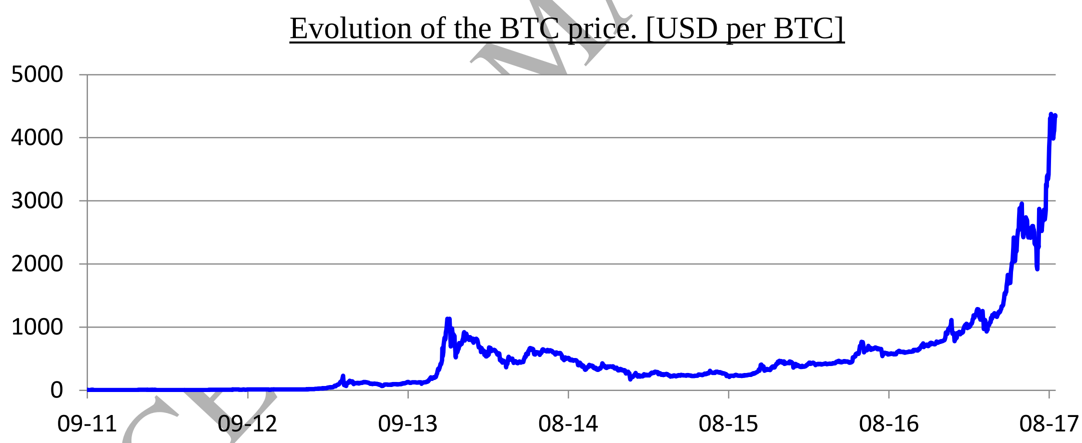
Table 1: Descriptive Statistics of price and price return of bitcoin.

Statistics	BTC	Return
Mean	479.063	0.0034
Median	328.975	0.0023
Standard deviation	631.004	0.04929
Minimum	2.250	-0.66394
Maximum	4,378.840	0.33748
Skewness	3.040	-1.503
Kurtosis	14.446	27.679
Normality	14.768 ***	54,338 ***
ADF	3.926	-46.977 ***

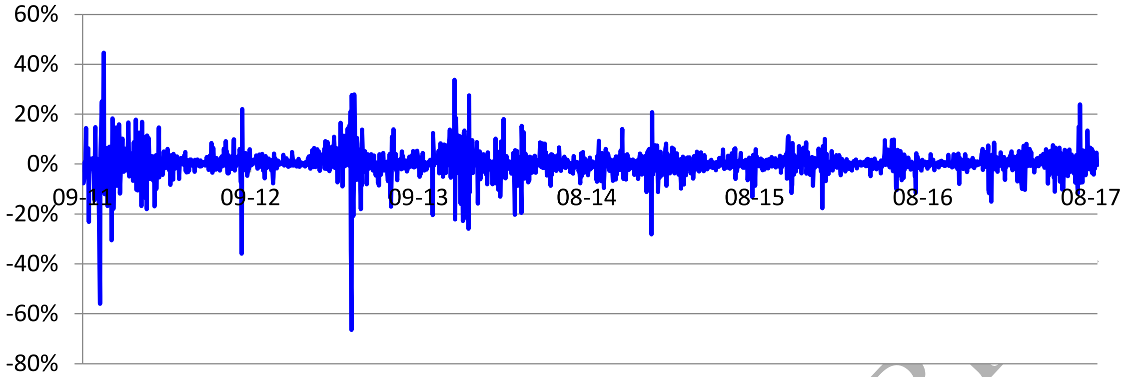
The normality test used is the Jarque-Bera test, which has $\chi^2(q)$ distribution with 2 degrees of freedom, and its null hypothesis indicates that errors are normally distributed. The Stationarity test is the ADF. *** denotes statistically significance at 1%.

In Figure 1, the evolution of the price, return and historical volatility of BTC are shown. It can be observed that in Sept 2011 the price was around 3 USD per BTC and remained relatively static for 2 years; then the first sharp rise between the end of 2013 and the beginning of 2014 where it exceeded 1,000 USD per BTC. The exchange rate descended slowly between 2014 and mid-2015 until reaching 363 USD per BTC; and, then started an upward trend that leads in August 2017 to surpass the 4,000 USD per BTC barrier. From the point of view of the return it can be seen that the end of 2011 and beginning of 2012 was a period of positive and negative returns with a high dispersion, which is also reflected in the graph of historical volatility for all three terms analyzed. Towards the end of 2013 and early 2014, there is an erratic behavior of ups and downs in returns, but it does not have the same correspondence in historical volatility. The upward trend since mid-2015 has been with much more moderate returns with the exception of the last months of the analysis period. From the figures of return and variances, it can be observed that there is variance and that this is not constant over time, thus showing that heterocedasticity exists. To confirm the heteroscedasticity of the series, the Ljung Box test was applied, Box and Pierce (1970), Ljung and Box (1978) and Harvey (1993), rejecting the null hypothesis for all the lags between 1 and 40 days. From the point of view of volatility, another cluster of high volatility can be seen at the beginning of 2013. To confirm the non-linearity of the historical volatility series, the Terasvirta test was applied (Teraesvirta et al., 1993), rejecting the null hypothesis in each case with a statistical significance of 5%. Thus, when detecting non-linearity it is appropriate to apply ANN.

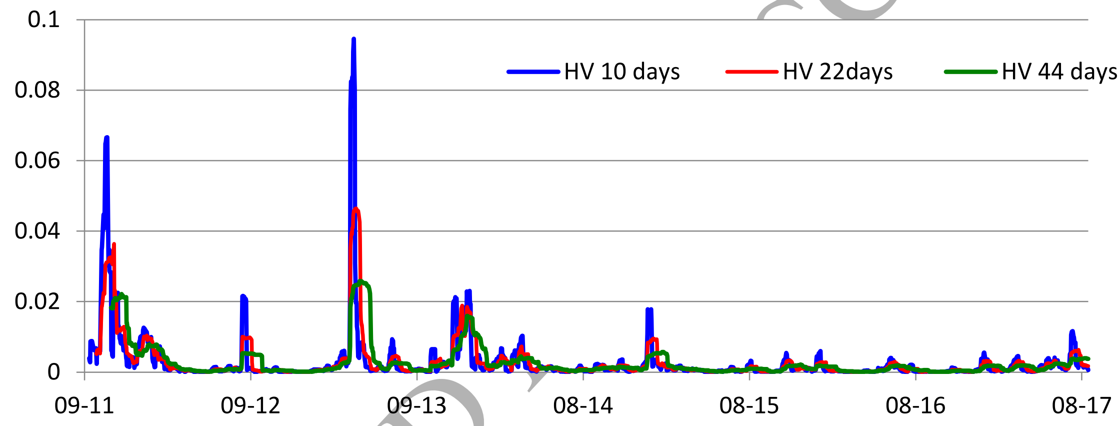
Figure 1. Time series for daily close price, daily return and the historical volatility of BTC.



Evolution of BTC price return.



Evolution of BTC Historical Volatility.



In addition to BTC we develop 7 technical indexes (TA) related with the selected time series. These are: Momentum 12 days (M12); Moving Average Convergence Divergence (MACD); MACD line (MACD-l); Relative Strength Index (RSI); Stochastic RSI (S-RSI); Bollinger Upper Bound (BUB); and, Williams Percentage 14 days (WP14). These TA represent different characteristics of the time series, information that is processed, and expected to enhance the results of the proposed methodology. Specifically, these 7 TA where chosen according to the relevance of information that they contain, and are the most common TA used in the literature. To the original algorithms, a percentage variation transformation was performed to include more information (DTA), expecting that the Multi-Layer Perceptron (MLP) explained in section 4.2 could perform better and improve forecasting.

Also, the PCA algorithm (Whittle, 1952) was conducted over the TA and its transformations. From the PCA, we take every component (fourteen of them) and use them as new features to feed the MLP. The idea is that the new factors are independent from each other and will facilitate the computation of the MLP. Henceforth, the transformed TA factor is referred to as the FTA and FDTA for the transformed DTA.

4. Methodology.

In this Section, the data studied and the characteristics of the models proposed for the study are explained in detail.

4.1. GARCH models.

Given the characteristic of heteroskedasticity of economic and financial time series, the AutoRegressive Conditional Heteroskedasticity (ARCH) models and their generalization (GARCH) are established for modeling by Engle (1982) and Bollerslev (1986). The GARCH (*p*,*q*) model is described in Equations 1a, 1b, and 1c.

$$r_t = \alpha_0 + \sum_{i=1}^{k} \beta_i r_{t-i} + e_t \qquad \text{(Equation 1a)}$$

$$e_t | \Psi_{t-1} \sim N(0, h_t) \qquad \text{(Equation 1b)}$$

$$h_t = c + \sum_{i=1}^{p} \theta_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \qquad \text{(Equation 1c)}$$

where k is the autoregressive order of the mean equation, p is the order of the lagged squared error and q the order for the variance lagged.

In this study the asymmetric power GARCH model (APGARCH) is used, since this model is the generalization of different models. The APGARCH (Eq. 2a, 2b, and 2c) model incorporates all of the modifications and derivatives of the original GARCH model. We include the power term (*d*) to the GARCH model, which governs the variance equation (2c). The inclusion of the power term allows for a linear relationship of standard deviation, *d*=1, establishing itself as a model of stochastic volatility, Taylor (1986).

$$r_t = \alpha_0 + \sum_{i=1}^{k} \beta_i r_{t-i} + e_t \qquad \text{(Equation 2a)}$$

$$e_t \sim N(0, h_t^{d/2}) \qquad \text{(Equation 2b)}$$

$$h_t^{d/2} = c + \sum_{j=1}^{q} \beta_i h_{t-j}^{d/2} + \sum_{i=1}^{p} \theta_i (|e_{t-i}| + \omega\, e_{t-i})^d \qquad \text{(Equation 2c)}$$

The exponential GARCH (EGARCH) is also used since different studies have demonstrated its ability to predict volatility of financial series. The EGARCH model is described in equations 3a, 3b and 3c.

$$r_t = \alpha_0 + \sum_{i=1}^{k} \beta_i r_{t-i} + e_t \qquad \text{(Equation 3a)}$$

$$e_t \sim N(0, h_t\ ) \qquad \text{(Equation 3b)}$$

$$\log(h_t\ ) = c + \sum_{i=1}^{p} \theta_i \frac{e_{t-i}}{h_{t-i}} + \sum_{j=1}^{q} \beta_j \log(h_{t-j}) + \gamma \left|\frac{e_{t-i}}{h_{t-i}}\right| \qquad \text{(Equation 3c)}$$

4.2. Multi-Layer Perceptron

Artificial intelligence has been widely used in financial applications, especially in quantitative ones. The most common architecture is the Multi-Layer Perceptron (MLP), and can be defined as follows: consider *L* as the total number of layers; *N* as the total number of units per layer; and, I as the total number of inputs of the networks. Thus, the MLP is represented as:

$$S_j^1 = \sum_{k=1}^{I} w_{j,k}^1 x_k \qquad \forall j \in N \qquad (4)$$
$$A_j^1 = \theta_j^1(S_j^1)$$

$$S_j^l = \sum_{i=1}^{N} w_{j,i}^l A_i^{l-1} \qquad \forall j \in N, \; \forall l \in \{2, \dots, L-1\} \qquad (5)$$
$$A_j^l = \theta_j^l(S_j^l + b_l)$$

$$S_1^L = \sum_{i=1}^{N} w_{1,i}^{L-1} A_i^{L-1} \qquad (6)$$
$$A_1^L = \psi_1^L(S_1^L)$$

where θ_j^l is the activation function for layer l and unit j, ψ_1^L is a linear activation function of the l as layer L and $w_{j,i}^l$ is the weight from unit i of the previous layer to unit j in layer l. Notice that equation (4) corresponds to the first layer, equation (5) corresponds to the intermediate layers, and equation (6) corresponds to the last layer. This algorithm is trained and learns its parameters by the backpropagation algorithm (Rumelhart et al. 1986).

4.3. Proposed models

There are twelve models evaluated considering different combinations of models and inputs. Each of the models is explained in detail in the following descriptions. The models are conducted via the MLP, and their variations are the input features that are used to build the forecasts.

Model 1 (BTC.L + GARCH) uses l lagged values of the original bitcoin time series and the forecast of the best GARCH model to develop the predictions. The parameter l was set to 11, which resembles two weeks of financial time. Thus, the model has 12 input features to develop the prediction and represents a hybrid approach since the GARCH forecast is considered, making it an econometric-AI model.

Model 2 (BTC.L + GARCH.L) takes *l* lagged values of the original bitcoin time series and the *v* lagged values of the best GARCH model to make the predictions. Both *l* and *v* were set to 11, as in the previous model. Thus, this model takes 22 input features to develop forecasts and is also considered a hybrid approach because of the lagged values of the GARCH model feeding the MLP.

Model 3 (GARCH + GARCH.L) takes the best GARCH and its *v* lagged values to make the predictions. The parameter *v* was set to 11. This too is a hybrid model but does not rely on the characteristics of the original series, because it does not consider the lagged values of the bitcoin, but includes it indirectly through the forecast of the GARCH model. The MLP is feed by 12 inputs features.

Model 4 (BTC.L + TA) uses *l* lagged values of the original bitcoin series and the TA factors described in 3.1.2. to make the prediction. The parameter *l* was set to 11. The model is not hybrid because it does not take as features the GARCH forecast and is not autoregressive. Instead, the forecast is enhanced by the additional information that the TA has about the original time series. The total number of features fed into this model is 18.

Model 5 (BTC.L + TA + DTA) is like the model 4, but aggregates the percentage variation transformed factors to bring more information to the MLP. With *l* set to 11 (two weeks), the model has 25 inputs features to make the prediction. The inclusion of additional features allow the MLP to develop more complex relationships and this could lead to better performance regarding forecast accuracy.

Model 6 (BTC.L + GARCH + TA + DTA) follows the same description as the previous models but this model also includes the best GARCH forecast to make the predictions. Thus, the model has 26 input features to make predictions and is a hybrid approach due to the consideration of the GARCH forecast. The inclusion of even more features is to see if the MLP can extract more relationships in the data and make more accurate predictions than previous models.

Model 7 (BTC.L + GARCH + GARCH.L + TA) considers the lagged values of the original bitcoin time series, the best GARCH forecast, the autoregressive of the best GARCH, and the TA factors to make predictions. This model has 30 inputs features to make predictions. There are fewer features compared to the last model, but this includes the forecast information provided by the GARCH and is autoregressive which can help to achieve a better prediction.

Model 8 (BTC.L + GARCH + GARCH.L + TA + DTA) follows the same descriptions of the previous model but also includes the transformed factors, DTA, which adds an additional 7 features totaling 37 inputs to make predictions. This model has the most input features and it is expected that the MLP could identify deeper relationships enhancing even further the forecasting ability.

Model 9 (GARCH + FTA) considers the best GARCH forecast and the PCA transformed TA factors as inputs features to make predictions. This model is hybrid due to the consideration of the GARCH forecast and does not include directly the information of the original time series. This approach has 8 inputs features to make forecasts.

Model 10 (GARCH + FTA + FDTA) follows the same description as the previous model and includes the transformed factors, DTA, passed through the PCA algorithm. Model 10 has 15 input features to make predictions. The inclusion of all the PCA factors is expected to enhance the prediction capability of the MLP, having more data to establish useful relationships and forecasts.

Model 11 (GARCH + GARCH.L + FTA) is similar to Model 9, but it also considers the lagged values of the best GARCH to make predictions. The model has 19 inputs features to feed the MLP. The idea behind this model is to evaluate the predicting capability of the autoregressive components of the best GARCH, with the expectation that the performance of the model will be better than the Model 9.

Model 12 (GARCH + GARCH.L + FTA + FDTA) takes the same inputs as the previous model but also includes the transformed TA passed through PCA. Model 12 has 26 inputs features to make predictions. This model is the second highest with respect to features to feed the MLP and is directly related to Model 10 and Model 11. With respect to the Model 10, the idea of this model is to evaluate the predicted capability of the autoregressive components of the best GARCH and with respect to the second Model 10 the idea is to test the predicted capability of the FDTA factors.

The period studied has 2,110 days of which the last 1,293 days were used for prediction by all models, due to the data required to predict GARCH models and hybrids. These 12 proposed models plus all the optimization to find the best forecast of the econometric models can be combined into a framework to predict the volatility of bitcoin. The aforementioned framework aims to determine the best forecast for the volatility of bitcoin and is executed day-by-day with a sliding window which means that for each of the 1,293 predictions of each of the three volatilities all models were run with all their possible combinations of variables and sizes.

4.4. Loss function and Confidence Test.

To evaluate the accuracy of the forecasting for the different models, the Mean Square Error (MSE) is utilized as a loss function, because of its robustness (Fuertes et al. (2009)). To test the performance, the Model Confidence Set (MCS) is applied (Hansen et al., 2003; Hansen et al., 2011). The MCS is defined as the set of surviving models and provides *p*-values of the test for each model.

5. Analysis of Results.

5.1 Econometric Models.

The first analysis is to predict each of the three volatilities with the GARCH models. As described in the methodology, the GARCH based model, the EGARCH model and the APGARCH model are used. The orders k, p and q are set with a maximum of 2; while sliding window sizes are 63, 126, 189, 252 and 378 days. This implies that for each of the volatility series there are 120 models of the GARCH family. In Table 2, the best models according to the MSE for each of the series are provided. It can be observed that for all terms of volatility the best model is the EGARCH model. The concentration of the best models focuses mainly on the two largest sizes, in particular the best model of each volatility is with a window of 252 days. It can also be seen that the longer the term the lower the MSE, which means that it is easier to estimate, which is partly explained because the longer term volatility is more stable.

Table 2: GARCH Models MSE for different volatility terms.

Volatility	Model	length	*r*	*p*	*q*	MSE
10 days						
	EGARCH	252	1	1	1	4.768E-05
	EGARCH	378	1	1	1	4.802E-05
	EGARCH	252	2	1	1	4.816E-05
22 days						
	EGARCH	252	2	1	1	2.839E-05
	EGARCH	252	1	1	2	2.873E-05
	EGARCH	378	1	1	1	2.889E-05
44 days						
	EGARCH	252	1	1	1	1.834E-05
	EGARCH	252	1	1	2	1.915E-05
	EGARCH	252	2	1	2	1.915E-05

Length is the size of the windows used to estimate the parameters of each model. r, p, and q are the orders of the GARCH, EGARCH and APGARCH model.

To make the econometric model more demanding, the best models are analyzed by changing conditional distribution of errors. For this, we analyze the t-student distribution and Generalized Error Distribution (GED). In Table 3, it can be seen that the best models are the ones with only GED and not with t-student. It is also interesting to note that only for the 10-day volatility is the MSE improved with the EGARCH-GED model, while for the other two cases the models with Normal distribution are better.

Table 3: MSE for EGARCH Models with different conditional error distribution assumption for different volatility terms.

Volatility	Distribution	length	*r*	*p*	*q*	MSE
10 days						
	GED	252	1	1	1	4.746E-05
	GED	378	2	1	2	4.747E-05
	GED	252	2	1	1	4.775E-05
22 days						
	GED	378	2	1	2	2.844E-05
	GED	378	1	1	1	2.850E-05
	GED	378	2	1	1	2.867E-05
44 days						
	GED	378	2	1	1	1.938E-05
	GED	252	1	1	1	1.939E-05
	GED	378	1	1	2	1.940E-05

Finally, to further improve the econometric forecasts, the technical analysis indicators were included as technical indicators variables in the variance equation. Although there are 7 technical indicators that were defined, in order not to generate multicollinearity, a correlation analysis was done to leave only those that are not highly correlated, since it would imply possible problems in the determination of the models. To discriminate, a correlation threshold of 0.4 was used, as can be seen in Table 4, both MACD and MACD-1 have a high correlation with M12, therefore, these two indicators will not be taken into account. The RSI also has a high correlation with M12, whereas WP14 has it with M12 and S-RSI. Therefore, only M12, S-RSI, and BUB were used as indicators.

Table 4: Correlation matrix of technical indicators.

	M12	MACD	MACD-l	RSI	S-RSI	BUB	WP14
M12	1.0000	**0.7668**	**0.7465**	**0.5081**	0.1945	0.2826	**0.4321**
MACD		1.0000	0.2853	**0.4491**	0.2865	-0.1607	0.3900
MACD-l			1.0000	0.3574	-0.0948	**0.5960**	0.2557
RSI				1.0000	**0.4836**	-0.0264	**0.7577**
S-RSI					1.0000	-0.0894	**0.5142**
BUB						1.0000	-0.0074
WP14							1.0000

Bold values indicate correlations higher than 0.4.

The percentage variation of all technical indicators, as well as the quadratic values of both series, and their variations were incorporated as possible variables. For each period of historical volatility, the best models were adjusted by incorporating the technical indicators' variables into the equation of variance with different groups of variables.

For the 22-day and 44-day volatility, the MSE of the best GARCH model improves by including technical indicators' variables in the variance; while for the 10-day volatility the inclusion of technical indicators does not improve the MSE. With this, the best econometric model is obtained to predict each volatility. The best results for each volatility can be observed in Table 5, as well as the technical indicators variables with which the best MSE is achieved.

Table 5: MSE for GARCH Models with technical indicators variables for different volatility terms.

Volatility	Distribution	length	*r*	*p*	*q*	Vars.	MSE
10 days							
	GED	252	1	1	1	Yes	4.846E-05
						No	4.746E-05
22 days							
	Normal	252	2	1	1	Yes	2.723E-05
						No	2.839E-05
44 days							
	Normal	252	1	1	1	Yes	1.726E-05
						No	1.834E-05

Vars. indicates if the model has or not technical indicators variables in the variance equation. For the 10-days volatility the variables added in the variance for the best model are: M12; S-RSI; BUB; $M12^2$; $S\text{-}RSI^2$; BUB^2. For the 22-days and 44-days volatility the variables added in the variance for the best model are: M12; S-RSI; BUB; $M12^2$; $S\text{-}RSI^2$; BUB^2; MACD; MACD-l; S-RSI; WP14; $MACD^2$; $MACD\text{-}l^2$; $S\text{-}RSI^2$; $WP14^2$. The superscript ^{2} indicates the squared value and no superscript indicates percentile variation.

5.2 Proposed models.

After determining the best econometric forecast for each of the time periods of the historical volatility of the bitcoin price return, we analyzed the 12 proposed hybrid models. For this the artificial neural network was fed with the different combinations described above for different configurations of the network. The configurations used for the network were 1, 2, 3 and 4 hidden layers and 5, 10, 15 and 20 neurons with 252 days. First, the MLP hybrid models were analyzed with only econometric data, that is, models 1, 2 and 3. In Table 6 it can be seen that for short-term volatility Model 1 has the best MSE, with a configuration of 1 hidden layer and 10 neurons, decreasing the MSE by 3.32% with respect to the best GARCH model. For volatility at 22 and 44 days the best model is Model 2 with a configuration of 2 hidden layers for 22 days and 3 for 44 days. In both cases the number of neurons of the best 5 models is 5 neurons. For its part, the best hybrid lowers the MSE by 2.16% for 22-day volatility and by 4.53% for 44-day. It is interesting to note that in all cases the hybrid models improve the MSE. Model 1 only has the best results for short term while the best 5 models of 22 and 44 days are Model 2 and Model 3.

Table 6: MSE for Hybrid MLP and Econometrics models.

Volatility	Model	Hidden layers	Neurons	MSE	% Var.
10-days					
	Model 1	1	10	5.221E-06	-3.32
	Model 1	2	5	5.225E-06	-3.25
	Model 2	2	5	5.236E-06	-3.04
	Best GARCH			5.401E-06	
22-days					
	Model 2	2	5	3.337E-06	-2.16
	Model 2	3	5	3.343E-06	-2.01
	Model 3	4	5	3.346E-06	-1.92
	Best GARCH			3.411E-06	
44-days					
	Model 2	3	5	2.297E-06	-4.53
	Model 3	2	5	2.320E-06	-3.55
	Model 2	2	5	2.321E-06	-3.53
	Best GARCH			2.406E-06	

% Var. means the percentage of improvement of the MSE related to the MSE of the best GARCH model. The MSE of the best GARCH models change because the sample for the hybrids is less given the data used as input for the ANN.

The following analysis focuses on the hybrid models that incorporate the MLP with the econometric models and with the technical indicators, that is, Model 4 to Model 8. In Table 7, it can be observed that for the 10-day volatility the best model is 4; it reduces the MSE of the best GARCH by 4.5%. The configuration of the ANN is 1 layer and 10 neurons; while for 22-day volatility it is the same model but with only 5 neurons and reduces the MSE by 4.99%. For the case of the 44-day volatility the best model is 5 with a configuration of 2 layers and 5 neurons. In general, the best hybrid models have few layers and few neurons in this group.

Table 7: MSE for Hybrid MLP Econometric and Technical indicators models.

Volatility	Model	Hidden layers	Neurons	MSE	% Var.
10-days					
	Model 4	1	10	5.157E-06	-4.50
	Model 8	1	10	5.174E-06	-4.20
	Model 5	2	5	5.183E-06	-4.03
	Best GARCH			5.401E-06	
22-days					
	Model 4	1	5	3.241E-06	-4.99
	Model 5	2	5	3.283E-06	-3.75
	Model 6	1	5	3.321E-06	-2.62
	Best GARCH			3.411E-06	
44-days					
	Model 5	2	5	2.257E-06	-6.18
	Model 7	2	5	2.286E-06	-4.98
	Model 4	3	5	2.297E-06	-4.53
	Best GARCH			2.406E-06	

% Var. means the percentage of improvement of the MSE related to the MSE of the best GARCH model. The MSE of the best GARCH models change because the sample for the hybrids is less given the data used as input for the ANN.

Finally, Models 9 to 12 are analyzed, which incorporate preprocessed technical analysis indicators for each window using Principal Component Analysis. This is an innovation in hybrid models, which is justified since obtaining the factors derived from the PCA analysis of the indicators ensures the independence of the input data being fed to the ANN. Thus, it should improve the results of the ANN adjustments and, therefore, are expected to improve the forecasts.

In Table 8, it can be observed that for the 10-day volatility the best model is Model 10, which in a configuration of 4 hidden layers and 5 neurons. Model 10 improves the MSE by 5.24% with respect to the best econometric model, and of all the hybrid models it has the best overall performance. For 22-day volatility the best model is 9 with 1 hidden layer and five neurons, reducing the MSE by 7.71% with respect to the benchmark and is better than the other hybrid models. Finally, in the case of 44-days volatility the best MSE is achieved by Model 11 with 4 hidden layers and 5 neurons, decreasing the MSE of the benchmark by 6.51%. It can then be concluded that by incorporating the variables of the technical analysis with a preprocessing of PCA to the MLP the accuracy of the predictions of the volatility of bitcoin improves.

To confirm the best models, all hybrid models were run with 22 lagged values instead of 11 and also configurations with 5 and 6 hidden layers. In none of the cases was there an improvement to the MSE by the best hybrid model discussed above for each of the volatilities.

Table 8: MSE for Hybrid MLP Econometrics and Technical indicators processed by PCA models.

Volatility	Model	Hidden layers	Neurons	MSE	% Var.
10-days					
	Model 10	4	5	5.118E-06	-5.24
	Model 10	1	10	5.145E-06	-4.74
	Model 10	2	5	5.158E-06	-4.49
	Best GARCH			5.401E-06	
22-days					
	Model 9	1	5	3.148E-06	-7.71
	Model 12	1	10	3.152E-06	-7.59
	Model 11	1	5	3.205E-06	-6.03
	Best GARCH			3.411E-06	
44-days					
	Model 11	4	5	2.249E-06	-6.51
	Model 10	4	5	2.262E-06	-5.94
	Model 12	2	5	2.290E-06	-4.80
	Best GARCH			2.406E-06	

% Var. means the percentage of improvement of the MSE related to the MSE of the best GARCH model. The MSE of the best GARCH models change because the sample for the hybrids is less given the data used as input for the ANN.

5.3. Loss function test.

To determine if the models that deliver the lowest MSE are statistically significant we used the Model Confidence Set (MCS) (Hansen et al., 2003; Hansen et al. 2011). The MCS procedure yields *p*-values for each of the models tested. For a given model, the MCS p-value is the threshold at which the model belongs to the group of the best models ($\widehat{\mathcal{M}}_{1-\alpha}$). Thus, an object with a small MCS p-value makes it unlikely that it is one of the best alternatives in $\widehat{\mathcal{M}}_{1-\alpha}$. In this study, we use $\alpha = 0.05$ to evaluate the goodness of the models. This test applies to the best 50 models of each volatility and also includes the best GARCH model.

For the 10-day volatility it can be observed that the best model according to the lowest MSE is supported by the MCS test as the best of all (Table 9), but there are other models as good as it with a statistical significance of 5%. The best GARCH model is clearly discarded. There are 13 of 51 models $\widehat{\mathcal{M}}_{95}$ that are similar to the best, of these 7 (54%) are models that include preprocessing through PCA and technical indicators and 4 (31%) are hybrids with technical indicators. Although there is no statistically significant difference, most models are preprocessed by PCA.

For the 22-day window, it too is statistically confirmed as the best model with the lowest MSE, that is, Model 9 with 1 hidden layer and 5 neurons the best, but there are 20 models with similar significance. 16 models out of 20 are with the PCA preprocessing and technical indicators; the remaining 4 are hybrid models with technical indicators. No hybrid model without technical indicators and GARCH models are included in the best models according the MCS p-value.

Finally, for the 44-day volatility window, the MSE is again used to find the model(s) with the lowest MCS. Only 8 models have the same accuracy as the best mode. 5 of the 8 models including the PCA preprocessing and technical indicators, while 2 are only hybrid with technical indicators and the other one is a hybrid.

With this test, it can be ensured that the models that obtained the lowest MSE in each period of volatility are the statistically best models and the best model includes the PCA preprocessing and technical indicators.

Table 9: The best models according to MCS test.

Volatility	Model	Hidden layers	Neurons	MSE	MCS *p*-value
10-days					
	Model 10	4	5	5.118E-06	1.0000
	Model 10	1	10	5.145E-06	0.9430
	Model 10	2	5	5.158E-06	0.9430
	Model 4	1	10	5.157E-06	0.9430
	Model 8	1	10	5.174E-06	0.9430
	Model 5	2	5	5.183E-06	0.8420
	Model 12	4	5	5.183E-06	0.8420
	Model 5	4	5	5.188E-06	0.8140
	Model 11	1	5	5.213E-06	0.8140
	Model 10	2	10	5.207E-06	0.8140
	Best GARCH			5.401E-06	0.0000
22-days					
	Model 9	1	5	3.148E-06	1.0000
	Model 12	1	10	3.152E-06	0.9420
	Model 11	1	5	3.205E-06	0.9190
	Model 10	2	5	3.243E-06	0.9190
	Model 10	1	10	3.223E-06	0.9190
	Model 4	1	5	3.241E-06	0.9190
	Model 12	1	5	3.247E-06	0.9130
	Model 5	2	5	3.283E-06	0.9130
	Model 11	1	10	3.246E-06	0.9130
	Model 12	1	15	3.253E-06	0.9130
	Best GARCH			3.411E-06	0.0000
44-days					
	Model 11	4	5	2.249E-06	1.0000
	Model 5	2	5	2.257E-06	0.9370
	Model 10	4	5	2.263E-06	0.9370
	Model 7	2	5	2.286E-06	0.7520
	Model 12	2	5	2.290E-06	0.7070
	Model 9	3	5	2.293E-06	0.7070
	Model 11	2	10	2.296E-06	0.7070
	Model 2	3	5	2.297E-06	0.6910
	Model 4	3	5	2.297E-06	0.0000
	Model 11	3	5	2.296E-06	0.0000
	Best GARCH			2.406E-06	0.0000

MCS *p*-value is the p-value of the Model Confidence Set, Hansen et al. (2003) and Hansen et al. (2011).

In order to demonstrate the efficiency of the framework, and at the same time how difficult it is to predict the volatility of bitcoin, the proposed framework was applied to the EUR-USD exchange rate. In the first instance, the GARCH models were applied with normal error distribution, GED and T; with and without the variables of the technical indicators. All the configurations that were used for bitcoin were run for the 3 prediction horizons. The results of the best forecast models according to the MSE for each of the prediction horizons can be seen in Table 10. It can be seen that the longer the period, the lower the MSE. The APGARCH model is the most effective for predicting 10-day volatility; while at 22 and 44 days the EGARCH model performs the best. However, the best models use normal distribution and when including the variables of the technical indicators, the performance of the forecasts was not improved.

Table 10: MSE of the best GARCH models to predict EUR-USD volatility.

Volatility	Model	length	k	p	q	dist	Vars.	MSE
10 days								
	APGARCH	Normal	no	252	2	2	2	5.85E-10
	EGARCH	Normal	no	252	2	1	1	5.87E-10
	EGARCH	GED	no	126	1	1	1	5.93E-10
22 days								
	EGARCH	Normal	no	252	2	1	1	3.11E-10
	EGARCH	Normal	no	252	1	1	1	3.23E-10
	EGARCH	GED	no	126	1	1	1	3.33E-10
44 days								
	EGARCH	Normal	no	252	2	1	1	2.20E-10
	EGARCH	Normal	no	252	1	1	1	2.44E-10
	EGARCH	Normal	no	126	1	1	1	2.60E-10

Length is the size of the windows used to estimate the parameters of each model. *r*, *p*, and *q* are the orders of the GARCH, EGARCH and APGARCH model. Vars. indicates if the model has or not technical indicators variables in the variance equation. For the 10-days volatility the variables added in the variance for the best model are: M12; S-RSI; BUB; $M12^2$; $S\text{-}RSI^2$; BUB^2. For the 22-days and 44-days volatility the variables added in the variance for the best model are: M12; S-RSI; BUB; $M12^2$; $S\text{-}RSI^2$; BUB^2; MACD; MACD-l; S-RSI; WP14; $MACD^2$; $MACD\text{-}l^2$; $S\text{-}RSI^2$; $WP14^2$. The superscript ^{2} indicates the squared value and no superscript indicates percentile variation.

In order to compare the results of the predictions of the GARCH models for the volatility of bitcoin with that of the volatility of the EUR-USD, it is necessary to use the Mean Absolute Percentage Error (MAPE) as a complement to the MSE. Analyzing the MAPEs (Table 11), it can be seen that the percentage error that is incurred when forecasting bitcoin is much higher than EUR-USD confirming that it is more difficult to predict the volatility of bitcoin.

Table 11: MSE and MAPE of the best model per horizon to forecast the EUR-USD volatility and BTC-USD volatility.

	EUR-USD			BTC-USD		
Horizont	Model	MSE	MAPE	Model	MSE	MAPE
10 days	APGARCH	5.85E-10	1.22	EGARCH	4.75E-05	2.80
22 days	EGARCH	3.11E-10	0.70	EGARCH	2.72E-05	2.39
44 days	EGARCH	2.20E-10	0.56	EGARCH	1.73E-05	1.64

Once the best forecast model of the GARCH family has been defined, forecasts of the volatility of the EUR-USD are made using the proposed framework composed of the 12 models. The results obtained are shown in Table 12. It can be seen that for the 10-day horizon, the best model is Model 10 with 3 hidden layers and 20 neurons, which has as input the GARCH forecast with the technical indicators and its transformations including a preprocessing PCA. In this case, the MSE is decreased by only 2.01% with respect to the best forecast obtained with the GARCH models. In the 22-day volatility period, the best model is Model 9 with only one hidden layer and 20 neurons; the ANN having the GARCH forecast and the technical indicators including a PCA preprocessing. The decrease in MSE is important, since it decreases by 9.05%. Finally, for the volatility of the EUR-USD at 44 days, the best model is Model 11 with 3 layers and 20 neurons. This model has as input the GARCH forecast and its lagged forecasts, the technical indicators and their transformations including a PCA preprocessing. In this horizon, the MSE is decreased by 8.34%, which is a relevant increase in accuracy. Finally, to check the significance of the superiority of the forecasts, the MCS test was applied. It can be seen that with a significance of 5% there is no superiority of the best hybrid models with respect to the best GARCH model, which indicates the predictive capacity of the GARCH model in the short term for the EUR-USD exchange rate.

Table 12: MSE for hybrid MLP econometrics and technical indicators processed by PCA to forecast the EUR-USD volatility.

Volatility	Model	Hidden layers	Neurons	MSE	% Var.	MCS *p*-value
10-days						
	Model 10	3	20	5.732E-10	-2.01%	1.000
	Model 9	3	20	5.802E-10	-0.83%	0.978
	Model 10	1	15	5.814E-10	-0.61%	0.978
	Best GARCH			5.850E-10		0.978
22-days						
	Model 9	1	20	2.828E-10	-9.05%	1.000
	Model 10	1	20	2.897E-10	-6.85%	0.648
	Model 9	1	15	2.916E-10	-6.23%	0.648
	Best GARCH			3.110E-10		0.648
44-days						
	Model 11	3	20	2.016E-10	-8.34%	1.000
	Model 11	3	15	2.028E-10	-7.83%	0.894
	Model 12	4	15	2.029E-10	-7.77%	0.894
	Best GARCH			2.200E-10		0.791

% Var. means the percentage of improvement of the MSE related to the MSE of the best GARCH model. The MSE of the best GARCH models change because the sample for the hybrids is less given the data used as input for the ANN. MCS *p*-value is the p-value of the Model Confidence Set, Hansen et al. (2003) and Hansen et al. (2011).

6. Conclusions.

In this article we accomplished several important items. First, we provided a brief review of both cryptocurrency in general and bitcoin in particular. Second, we provided a review of the application and development of artificial neural networks, machine learning, and

GARCH models. We illustrated the application of a hybrid Artificial Neural Network-Generalized AutoRegressive Conditional Heteroskedasticity (ANN-GARCH) model to forecast the price volatility of bitcoin, the most traded and largest by market capitalization of the cryptocurrencies.

As we stated at the beginning of this work, the measurement, prediction, and modeling of currency price volatility in general constitutes an important area of research at both the national and corporate level. Countries attempt to understand currency volatility to set national economic policies and firms to best manage exchange rate risk and leverage assets. Cryptocurrency is a relatively new technological invention that the corporate treasurer has to turn to as part of the overall financial strategy. The estimates of the total market capitalization for all cryptocurrencies is quite high and prior to this study little effort has been made to study the nature of the volatility of the currency. Like the introduction of any new technological invention, the early period of adoption can be very volatile and it is during this period that investors are exposed to the greatest risk. Hence, models that can capture this volatility in order to mitigate the exposure to risk are very important.

While the results for this particular study are applicable for bitcoin, there are many more cryptocurrencies. For instance, Ethereum currently is trading more than $1.9-billion USD daily with a total market capitalization of more than $83-billion USD. Ripple, has more than $2.2-billion USD trading daily. There are more than 134 cryptocurrencies that have market capitalizations of more than $100-million USD. The model developed herein may not apply broadly to the cryptocurrency markets. Future research is needed to determine how and if other cryptocurrencies behave in similar manners. One approach may be to develop similar ANN-GARCH models for a variety of cryptocurrencies and compare model performance across currencies.

Similar to the previous comment, the technical elements chosen for this study may not be broadly applicable to other currencies. In short, future work may be done that looks at the inclusion, or lack, of the technical analysis indicators. It is the case that the seven technical indicators selected for this particular analysis worked well with the models developed, it may not be the case for some of the other cryptocurrencies. To this end, additional models need to be developed and tested against other cryptocurrencies.

Finally, legislation is likely to change in the coming years as governments attempt to understand how cryptocurrencies fit into the national and global economic systems. As stated earlier, bitcoin is decentralized and not regulated by any one country; no country can create or control the release of bitcoin. Hence, federal regulators are not able to manipulate the currency in order to promote national agenda. Therefore, it is likely that most countries will continue to work on methods for centralizing a decentralized asset. As the political and legal environment evolves, the results of this study will need to be tested to see if the findings continue to hold.

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