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Volatility Threshold Dynamic Conditional Correlations: An International Analysis

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ABSTRACT
This article proposes a modeling framework for the study of changes in cross-market comovement conditional on volatility regimes. Methodologically, we extend the Dynamic Conditional Correlation multivariate GARCH model to allow the dynamics of correlations to depend on asset variances through a threshold structure. The empirical application of our model to a sample of international stock markets in 1994–2011 indicates that the periods of market turbulence are associated with an increase in cross-market comovement. The modeling framework proposed in the article represents a useful tool for the study of market contagion. (*JEL*: C50, F37, G11, G15)

KEYWORDS: comovement, contagion, dynamic correlations, volatility thresholds

Understanding the relationship between correlations and volatility is crucial for risk management, optimal portfolio allocation strategies, and analysis of

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market interdependence. Correlations that increase in volatile periods reduce the power of portfolio diversification when it is needed most. The present article suggests a modeling framework for the study of changes in cross-market comovement conditional on volatility regimes. Methodologically, the paper extends Engle's (2002) multivariate Dynamic Conditional Correlation (DCC) model and its generalization by Cappiello, Engle, and Sheppard (2006) with the purpose to investigate the dynamic relationship between correlations and volatilities of the underlying assets. We apply the proposed specification to an analysis of return comovement of a selection of international stock markets in the period 1994–2011, which is characterized by a number of episodes of market turbulence.

The analysis of market comovement around the periods of crisis is the central theme of the literature on market contagion. Early studies on the relationship between correlations and volatility in international markets often relied on an analysis of these risk measures computed over sub-periods of the data sample. In particular, a range of studies focused on a comparison of the correlation coefficients measured in stable and volatile market periods (e.g., Bertero and Mayer (1990), King and Wadhwani (1990), Lee and Kim (1993), Erb et al. (1994), Calvo and Reinhart (1996), among others). One of the critical issues faced by this literature are the biases related to the heteroskedasticity of financial return series when measuring changes in correlations over periods of turbulence and tranquility. In particular, the studies by Stambaugh (1995), Boyer et al. (1999), and Forbes and Rigobon (2002) argue that since correlations are a positive function of volatility, the evidence of an increase in correlations in volatile periods (such as crisis periods) does not necessarily imply a change in the level of interdependence, or contagion, between the markets. In this context, the current literature makes a distinction between interdependence and contagion, with contagion reflecting a change in the level of market interdependence, or more generally a change in the data generating process in times of crisis.[1]

A range of studies addresses the issue of testing for market contagion, leading to a set of alternative approaches. We cite, among others, the adjusted correlation tests of Forbes and Rigobon (2002), the outlier tests of Favero and Giavazzi (2002), the threshold tests of Pesaran and Pick (2007), the co-exceedance approach of Bae et al. (2003), the probability model of Eichengreen et al. (1995, 1996), the extreme value theory approach of Longin and Solnik (2001), the latent factor approach of Dungey and Martin (2007), and the coskewness analysis of Fry et al. (2010).[2] For an overview of empirical literature on contagion, we refer to Dornbusch et al. (2000), Pericoli and Sbracia (2003), and Dungey et al. (2005).

Several studies which analyze changes in comovement across periods of crisis and tranquility employ GARCH-type models to account for heteroskedasticity of financial return series. As an example, in Dungey and Martin (2007) asset returns

[1]In our study we adopt this general definition of market contagion, consistent with Forbes and Rigobon (2002) and Pericoli and Sbracia (2003).

[2]Further recent studies on market contagion include Beirne et al. (2009) and Markwat et al. (2009).

are driven by a set of latent factors, with the factor dynamics specified as an AR(1) processes with GARCH residuals. The analysis of contagion is implemented on the basis of the interaction between the residuals of the factor model. Dungey et al. (2010) propose a structural multivariate GARCH model, where the mean dynamic follows a VAR(1) process, additionally including the 3-month U.S. Treasury Bill rate to control for the impact of global financial conditions.[3] The effect of crises on cross-market linkages is studied by allowing the parameters of the mean specification to vary in tranquil and crisis periods.

Longin and Solnik (1995), Pelletier (2006) and Silvennoinen and Teraesvirta (2005, 2009) analyze changes in correlations between different market regimes by extending the constant conditional correlation (CCC) GARCH model of Bollerslev (1990). Longin and Solnik (1995) allow the estimated correlation value for the turbulent market periods to differ from the constant correlation coefficient for the rest of the sample by introducing a threshold conditional on the contemporaneous value of volatility. Pelletier (2006) proposes a regime-switching correlation structure driven by an unobservable state variable that follows a first-order Markov chain. In the smooth transition conditional correlation (STCC) GARCH models of Silvennoinen and Teraesvirta (2005, 2009), the variation in conditional correlations is governed by observed transition variables, one of considered variables being the volatility index VIX. All three studies allow changes in correlations between the states, but imply constant correlation within individual states.

The modeling framework we suggest in the present study relates to the previous literature on time variation in cross-market comovement in a number of ways. Our mean specification is defined as a VAR process with a number of predictive economic variables to control for common shocks to international market returns, which is similar in spirit to the mean specifications in, for instance, Forbes and Rigobon (2002) and Dungey et al. (2010). The second moments are modeled by means of a multivariate GARCH in order to control for heteroskedasticity of returns when analyzing changes in correlations across volatility regimes. In this regard, our specification generalizes the approach of the aforementioned studies which employ CCCs. In particular, we model correlations in a dynamic way by employing the DCC model of Engle (2002) and its generalization by Cappiello et al. (2006). Taking advantage of the dynamic behavior of correlations and volatilities, we explicitly include in the correlation model a threshold structure based on volatility levels. We term the proposed specification the Volatility Threshold Generalized DCC (VT-GDCC) model. As we show in the article, the model represents a useful tool for the analysis of variation in the level of market comovement by providing a framework to study the impact of changes in volatility states on cross-market correlations.

We apply the proposed model to a set of international stock markets in the period from 1994 to 2011. We consider two alternative specifications of the Volatility Threshold component. In the first specification, we use the VT component to

[3]Forbes and Rigobon (2002) employ a similar specification of the mean dynamic.

capture the impact of high volatility states in the underlying markets on their contemporaneous cross-correlation. In the second specification, we use the VT component for the analysis of contagion associated with extreme U.S. volatility. For both specifications, we find evidence of an increase in cross-market comovement in turbulent periods, consistent with contagion between stock markets.

We proceed as follows. Section 1 introduces our modeling strategy. Section 2 deals with the model estimation, testing, and parameter interpretation. Section 3 contains a description of the data. Section 4 presents the empirical results. Finally, Section 5 concludes.

1 VOLATILITY THRESHOLD DCC

In this section, we introduce the Volatility Threshold Generalized DCC model which extends the DCC specifications proposed by Engle (2002) and Cappiello et al. (2006). In the VT-GDCC model, the correlation dynamic depends on variance values through a threshold structure.

Consider an n-variate conditional process ε_t with zero mean and covariance matrix H_t, which is identically distributed following an unspecified density $D(.)$:

$$\varepsilon_t | F_{t-1} \sim D(0, H_t), \tag{1}$$

where F_{t-1} denotes the conditioning information set, including the information up to time $t-1$. The vector ε_t may represent either a vector of zero mean returns or the vector of residuals obtained from a return mean model. The VT-GDCC model has the following representation. The conditional covariance matrix is decomposed into volatility and correlation components:

$$H_t = D_t R_t D_t, \tag{2}$$

where D_t is a diagonal matrix of conditional volatilities:

$$D_t = \mathrm{diag}\left\{\sqrt{h_{1,t}}, \sqrt{h_{2,t}}, \ldots \sqrt{h_{n,t}}\right\}. \tag{3}$$

$h_{i,t}$ is the time t conditional variance of asset i, and R_t represents the time-varying conditional correlation matrix. Furthermore, we denote by η_t the variance standardized residuals

$$\eta_t = D_t^{-1} \varepsilon_t, \tag{4}$$

which are correlated and have a unit variance.

Following Bollerslev (1990), Engle (2002), and further studies generalizing their models, the conditional variance $h_{i,t}$ could follow any univariate GARCH model. In

Cappiello et al.'s (2006) GDCC model, the following equations drive the correlation dynamic[4]:

$$R_t = (\mathrm{diag}(Q_t))^{-1/2} Q_t (\mathrm{diag}(Q_t))^{-1/2} \tag{5}$$

$$Q_t = (\bar{Q} - A\bar{Q}A' - B\bar{Q}B') + A(\eta_{t-1}\eta_{t-1}^{'})A' + BQ_{t-1}B', \tag{6}$$

where $\bar{Q}$ is a correlation matrix of dimension n and A and B are $n \times n$ parameter matrices. Cappiello et al. (2006) propose to restrict the parameter matrices to be diagonal. As a result, the individual elements of Q_t are characterized by the following dynamic:

$$q_{ij,t} = (1 - \alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1}, \tag{7}$$

where α_i and β_i, $i,j = 1,2,...n$, are the elements of the parameter matrices A and B. The GDCC model addresses a drawback of the original DCC model by Engle (2002), which assumes a common dynamic for all correlations. It includes, as a special case, Engle's (2002) DCC model under the restrictions $A = \alpha^{1/2} I_n$ and $B = \beta^{1/2} I_n$ (where I_n represents an identity matrix of dimension n).

Building on the GDCC specification in (7), the Volatility Threshold GDCC model provides a framework for analyzing the impact of varying volatility states on the correlation dynamic. In particular, taking advantage of the dynamic behavior of correlations and volatilities, we explicitly include in the correlation model a threshold structure based on volatility levels as follows:

$$q_{ij,t} = (1 - \alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j\left(v_{ij,t} - \bar{v}_{ij}\right), \tag{8}$$

where $v_{ij,t}$ are the elements of a dummy variable matrix V_t, which indicate threshold violations, $\bar{V} = E[V_t]$, where $E[\cdot]$ is the unconditional expectation operator, and γ_i are the elements of the diagonal parameter matrix Γ. Since the inputs of the conditional correlation process in (8) are the variance standardized residuals, η_t, we are able to explicitly control for return interdependence (in a mean model) and heteroskedasticity (using GARCH specifications) when studying the impact of volatility states on cross-market comovement. In this regard, our general approach is similar to that of Forbes and Rigobon (2002), who propose variance-adjusted correlation tests with the purpose of distinguishing between the effects of interdependence and contagion.

The general specification of the volatility threshold dummies $v_{ij,t}$ depends on the hypothesis to be tested. In this study, we consider two alternative designs of $v_{ij,t}$ which will be employed in the empirical application in Section 4. In the first case, the introduction of the volatility threshold component serves to evaluate the

[4] The Asymmetric Generalized DCC in Cappiello et al. (2006) includes an additional component accounting for the asymmetric impact of the past negative shocks on the correlation process.

potential changes in cross-market comovement when both markets in the pair are experiencing periods of turbulence:

$$v_{ij,t} = \begin{cases} 1 \text{ if } h_{i,t} > \chi_i \text{ and } h_{j,t} > \chi_j \\ 0 \qquad\quad \text{otherwise} \end{cases}, \tag{9}$$

where χ_i and χ_j are the volatility thresholds for assets i and j, respectively. Note that the threshold violations are defined with respect to the conditional variances at time t, being functions of the information set at time $t-1$. Consequently, the threshold violations, and hence the values of the dummy variables $v_{ij,t}$, are known at time $t-1$.

Many of the existing studies of contagion focus on cross-market transmission of shocks with the shocks originating in a specific market or region. One could adapt the volatility threshold specification to provide the related tests. For instance, one could analyze the impact of an increase in the risk of a specific market on the cross-market correlations—or, in other words, analyze the contagion effects associated with shocks originating in this market. Let the index $i=1$ denote this market of interest. The V_t dummy is now a scalar which assumes a unit value when the volatility of market 1 is above a threshold:

$$v_{ij,t} = v_{1,t} = \begin{cases} 1 & \text{if } h_{1,t} > \chi_1 \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

An important issue to be addressed is the definition of the thresholds χ_i in (9) and (10), taking into account that the conditional variances, $h_{i,t}$, are not directly observable. The multi-stage approach usually employed in the estimation of DCC models (which we will present in detail in the following section) may be helpful in simplifying this task. In particular, conditional variances are typically obtained in the first stage of the estimation by focusing only on marginal densities of mean residuals. Then, in the second stage, the correlation parameters are estimated conditionally on the first-stage outputs. Thus, in the second stage the conditional variance series are available (as they are functions of the parameters estimated in the first stage) and, therefore, we can compute quantities depending on them, such as the V_t matrix. We suggest to define the volatility threshold as $\chi_i = \mathbb{Q}_i(\delta_i)$, where δ_i is the quantile level (a parameter assuming values between 0 and 1) and $\mathbb{Q}_i(\delta_i)$ denotes the empirical δ_i quantile of the conditional variance process, $h_{i,t}$. Given the first-stage estimates and $\mathbb{Q}_i(\delta_i)$, the path of V_t is known in the second stage, and its expectation $\bar{V}$ can be computed with the sample estimator. The quantile levels δ_i could be either fixed a priori or estimated as an additional model parameter. We follow the second approach. The next section of the article discusses the inferential and computational issues associated with our approach.

The specification in (8) allows cross-sectional heterogeneity both in the standard parameters of the GDCC model as well as in the conditional correlations' response to volatility states. In some cases, the restrictions imposing common dynamics on the elements in Q_t could be well justifiable (integrated markets may have correlations characterized by the same dynamic evolution), leading to a more parsimonious specification and/or making the model estimation feasible also in large cross-sectional dimensions. For instance, the dynamic evolution of the conditional correlations might be restricted to be the same for groups of assets, similar to Billio et al. (2006), Billio and Caporin (2010), and Engle and Kelly (2009). In particular, we could impose block structures on the parameter matrices (on all of them or on some only), reducing therefore the parameter space dimension. If we cluster the n assets into $m<n$ nonoverlapping groups, the dynamics of the elements of Q_t follows:

$$q_{ij,t}=(1-\alpha_l\alpha_k-\beta_l\beta_k)\bar{q}_{ij}+\alpha_l\alpha_k\eta_{i,t-1}\eta_{j,t-1}+\beta_l\beta_k q_{ij,t-1}+\gamma_l\gamma_k\left(v_{ij,t}-\bar{v}_{ij}\right) \tag{11}$$
$$i\in l,\ j\in k,\ l,k=1,2,\ldots m,\ l\neq k,$$
$$q_{ij,t}=(1-\alpha_l^2-\beta_l^2)\bar{q}_{ij}+\alpha_l^2\eta_{i,t-1}\eta_{j,t-1}+\beta_l^2 q_{ij,t-1}+\gamma_l^2\left(v_{ij,t}-\bar{v}_{ij}\right), \tag{12}$$
$$i,j\in l,\ l=1,2,\ldots m,$$

where l and k are the groups including assets i and/or j. We refer to the model in (11)–(12) as Block-VT-GDCC. Finally, the correlation dynamics could be restricted to be the same across all the markets, resulting in the scalar VT-GDCC specification, or simply VT-DCC:

$$q_{ij,t}=(1-\alpha-\beta)\bar{q}_{ij}+\alpha\eta_{i,t-1}\eta_{j,t-1}+\beta q_{ij,t-1}+\gamma\left(v_{ij,t}-\bar{v}_{ij}\right),\ i,j=1,2,\ldots n. \tag{13}$$

Finally, we would like to note that given the quadratic structure in (5), R_t is guaranteed to be positive definite if Q_t is positive definite. Building on the results in Cappiello et al. (2006), for the VT-GDCC (as well as the Block-VT-GDCC) specification, Q_t is positive definite if the following conditions are satisfied: (i) the initial Q_0 is positive definite and (ii) the matrix $\bar{Q}-A\bar{Q}A'-B\bar{Q}B'-\Gamma\bar{V}\Gamma'$ is positive semi-definite. For the VT-DCC specification, the sufficient condition for positive definiteness is $1-\alpha-\beta-\delta\gamma<0$, where δ is the maximum eigenvalue of $\bar{Q}^{-1/2}\bar{V}\bar{Q}^{-1/2}$.[5]

2 MODEL ESTIMATION

DCC multivariate GARCH models generally allow for a two-stage estimation. Specifically, under a Quasi Maximum Likelihood (QML) approach we can write the normal likelihood function of the DCC models as a sum of a volatility part and

[5]For the VT-GDCC and Block-VT-GDCC with diagonal parameter matrices, the model intercept must be constrained to be positive definite within the model estimation process.

a correlation part. We first express the quasi-normal likelihood of the VT-GDCC model as follows:

$$
\begin{aligned}
L(\theta) &= \sum_{t=1}^{T} L_t = K - \frac{1}{2}\sum_{t=1}^{T}\left(\ln|H_t| + \varepsilon_t H_t^{-1}\varepsilon_t'\right) \\
&= K - \frac{1}{2}\sum_{t=1}^{T}\left(\ln|D_t R_t D_t| + \varepsilon_t D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t'\right),
\end{aligned}
\tag{14}
$$

where θ denotes the entire parameter set (including both the variance and the correlation parameters). We may rewrite (14) using the following decomposition of the time t log-likelihood:

$$
\begin{aligned}
&\ln|D_t R_t D_t| + \varepsilon_t D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t' = 2\ln|D_t| + \ln|R_t| + \varepsilon_t D_t^{-1} R_t^{-1} D_t^{-1}\varepsilon_t' \\
&= \left(2\ln|D_t| + \varepsilon_t D_t^{-2}\varepsilon_t'\right) + \left(\ln|R_t| + \eta_t R_t^{-1}\eta_t' - \varepsilon_t D_t^{-2}\varepsilon_t'\right) \\
&= L_{1,t} + L_{2,t}
\end{aligned}
\tag{15}
$$

and obtain the final representation of the quasi log-likelihood:

$$
L(\theta) = \sum_{t=1}^{T} L_t = K - \frac{1}{2}\sum_{t=1}^{T}\left(L_{1,t} + L_{2,t}\right) = L_1(\theta_1) + L_2(\theta_1, \theta_2, \delta),
\tag{16}
$$

where θ_1 is the subset of θ that includes only the parameters entering the variance equations, θ_2 contains the parameters driving the conditional correlation dynamic, and $\delta = \{\delta_1, \delta_2, \ldots \delta_n\}$is the vector of the variance quantile levels. Following Engle (2002), we can determine the estimates of the volatility parameters by maximizing L_1 in (16). Note that the first-stage log-likelihood is simply the sum of the individual series volatility log-likelihoods. Conditionally on the parameter estimates and the variance sequences obtained in the first stage, we can compute the variance standardized residuals and then estimate θ_2 and δ by maximizing the second-stage log-likelihood L_2 in (16). We stress that the second stage also includes the evaluation of the thresholds $\mathbb{Q}_i(\delta_i)$, $i = 1, 2, \ldots n$, the construction of the dummy matrix V_t, and the computation of its expected value $\bar{V}$.

A limitation of the two-step estimation approach is that it affects the efficiency of the estimated parameters. To overcome this limitation, we suggest using the Maximization-by-Parts (MBP) approach introduced in Fan, Pastorello, and Renault (2007). For simplicity, assume for the moment that the parameters δ are known (we will further address this issue below). The first-order conditions for the full likelihood are as follows:

$$\begin{aligned}\frac{\partial L(\theta)}{\partial \theta_1'} &= \frac{\partial L_1(\theta_1)}{\partial \theta_1'} + \frac{\partial L_2(\theta_1,\theta_2,\delta)}{\partial \theta_1'} = 0\\ \frac{\partial L(\theta)}{\partial \theta_2'} &= \frac{\partial L_2(\theta_1,\theta_2,\delta)}{\partial \theta_2'} = 0.\end{aligned} \tag{17}$$

Solving the second equation alone is equivalent to the maximization of the second-stage likelihood in Engle (2002). However, if we follow the two-stage estimation approach of Engle (2002), the first-stage likelihood does not take into account the derivative of the second-stage likelihood with respect to the variance dynamic parameters. The MBP method suggests iterating across the two following equations:

$$\begin{aligned}\frac{\partial L_1(\theta_1)}{\partial \theta_1'} + \frac{\partial L_2\left(\hat{\theta}_{1,j-1},\hat{\theta}_{2,j-1},\delta\right)}{\partial \theta_1'} &= 0\\ \frac{\partial L_2\left(\hat{\theta}_{1,j},\theta_2,\delta\right)}{\partial \theta_2'} &= 0,\end{aligned} \tag{18}$$

where hats denote estimated values and the subscripts j denote the iteration steps. At step j, we maximize the first-stage likelihood also taking into account the gradient of the second-stage likelihood with respect to volatility parameters evaluated at step $j-1$. Next, given step j estimates of the variance parameters, we maximize the second-stage likelihood. The initialization of the procedure is given by the output of the standard two-stage estimation approach of Engle (2002). As suggested by Fan et al. (2007), a few iterations are generally sufficient to achieve convergence. In the VT-GDCC model, the evaluation of the gradient $\frac{\partial L_2\left(\hat{\theta}_{1,j-1},\hat{\theta}_{2,j-1},\delta\right)}{\partial \theta_1}$ must be done with numerical techniques given the complexity of the analytical gradient (see Hafner and Herwatrz, 2008, and Caporin, 2011, for analytical expressions of the DCC gradient).

We emphasize that in the VT-GDCC model the correlation dynamic and, therefore, the second-stage likelihood depend on the first-stage parameters via the variance standardized residuals (as in the standard DCC model) and the estimated first-stage conditional variances. The latter enter the equation driving the correlation dynamic through the dummy variable matrix V_t. Since the parameters of the volatility models are determined exclusively in the first stage of estimation (see equations (17) and (18)), and the second-stage estimation is performed conditionally on the first-stage output, the fitted volatility series could be considered as given in the second stage.

As explained in the previous section, we suggest estimating the conditional variance thresholds using the first-stage output. These thresholds are functions of the estimated conditional variances (available from the first stage) and of an unknown set of parameters (the vector containing quantile levels). One might consider two alternative approaches to determining the quantile levels defining volatility thresholds: either fixing them a priori or estimating them within the second-stage likelihood. When the parameters δ_i, $i=1,2,...n$, are calibrated, the estimation of the Volatility Threshold models could be performed using the MBP method imposing the first-order conditions in (18). It must be emphasized that if we do not calibrate the quantile levels included in δ, the joint estimation of the parameter set (δ,θ_2) may create computational problems due to the highly nonlinear impact of quantile levels on the likelihood function. To mitigate this problem, we suggest a specific-to-general estimation approach. In the following, we consider the case when the V_t dummy matrix is defined in (9):

(1) estimate the model using MBP with calibrated $\delta_1=\delta_2=...=\delta_n=\delta$ parameters (conditional variance quantiles levels are set to be equal for all series) and determine over a 1% grid the optimal δ value and call it δ_0; we consider δ values from 5% to 99%; the optimal value is associated with the maximum level of the full likelihood function;

(2) consider a grid search in the boundaries of the $\boldsymbol{\delta}_k$, the vector of quantile levels which is initialized using the vector $\delta_0\mathbf{1}$, where $\mathbf{1}$ denotes a n-dimensional vector of ones; within the grid search we use a 1% step, a maximum deviation of 2% from $\boldsymbol{\delta}_k$, and determine the new optimal vector $\boldsymbol{\delta}_{k+1}$; we note that, as in the previous step, at each point in the grid the model parameters are estimated by MPB and the optimal value is associated with the highest likelihood function; furthermore, the grid search evaluates the possible changes of each quantile level, therefore the total number of likelihood maximizations in step (2) is equal to 5^{k} given that each quantile level could remain unchanged, decrease by 1% or 2%, or increase by 1% or 2%;

(3) iterate step (2) many times until we reach a convergence of $\boldsymbol{\delta}_k$ using a tolerance criteria over the likelihood function.

This iterative approach, in our experience, guarantees convergence with a limited number of iterations of step (2), but has the disadvantage of being time consuming given the repeated use of the MBP approach.[6]

In contrast to the standard DCC models, we do not impose correlation targeting constraints by fixing the $\bar{Q}$ matrix at the sample correlation of the standardized residuals η_t. The simultaneous estimation of all parameters in (8), including $\bar{Q}$, allows to overcome the inconsistency problems discussed in Aielli (2008).

In the VT-GDCC-type specification, the impact of the standard DCC and the additional volatility threshold elements on the correlation dynamic is measured by functions of the model parameters. For example, the product of the volatility

[6] As a result, by using MBP we reduce the efficiency loss associated with the separate estimation of variance and correlation parameters, but we are not able to also overcome the loss of efficiency due to the estimation of the variance quantiles δ.

threshold coefficients $\gamma_i \gamma_j$ captures the impact of high volatility on the correlation of the asset pair (i, j). To evaluate the significance of this impact, the standard errors of functions of the coefficients are estimated using the delta method.

2.1 Monte Carlo Simulation

The previous section has presented an estimation approach that combines a grid search for the volatility quantiles and MBP for the remaining model parameters of the VT-GDCC. The model is characterized by highly nonlinear patters associated with the presence of the VT component in the correlation dynamic, and with the standardization that provides correlation matrices from the dynamic matrices Q_t. Furthermore, the estimation approach is not standard and does not provide the joint estimation of all parameters. These elements open a number of questions. First, full efficiency is lost. Second, the consistency of estimators could also be influenced by the fact that—due to the introduction of the VT component—the correlation estimates are functions of the GARCH parameters and the conditional variance sequences obtained in the first stage of the estimation. To verify if our iterative estimation procedure produces consistent estimators, we perform Monte Carlo simulations.

The simulation of the VT-GDCC requires several steps given that volatility quantiles are determined using the volatility density, which, in turn, is not known. Conditional on the choice of parameters θ_1, θ_2, and δ we simulate a VT-GDCC model with GARCH(1,1) variances in three steps:

(1) simulate a GDCC model with GARCH(1,1) variances and use the simulated conditional variances to initialize the variance thresholds $\mathbb{Q}_i(\delta_i)$, $i=1,2,...n$;
(2) simulate a VT-GDCC with GARCH(1,1) variances conditional on the variance thresholds determined in (1), and update their value in order to reduce the dependence on step (1);
(3) simulate again a VT-GDCC with GARCH(1,1) variances conditional on the variance thresholds determined in (2).

In steps (1) and (2) we simulate sequences of length $M=5000$ observations, while in step (3) we simulate series of $M=1000+T$ observations where T is the length of the series used for model estimation. In the Monte Carlo simulation, we focus on a single cross-sectional dimension $n=3$ in order to reduce the time required for running a complete experiment counting 1000 replications. To analyze the impact of the sample length on the simulation results, we consider three values of T: 500, 1000, and 2000. We report the results for two parameter sets: (i) all parameters in $\bar{Q}$ are equal to 0.8 (diagonal elements are all equal to 1) and parameters $\{\alpha_i, \beta_i, \gamma_i\}_{i=1}^n = \{0.2, 0.9, 0.2\}_{i=1}^n$; (ii) all parameters in $\bar{Q}$ are equal to 0.8 (diagonal elements are all equal to 1) and parameters $\alpha=\{0.10, 0.15, 0.20\}$, $\beta=\{0.88, 0.9, 0.92\}$,

Table 1 Simulation results for the first parameter set

	$N=500$			$N=1000$			$N=2000$		
	Bias	St. dev.	RMSE	Bias	St. dev.	RMSE	Bias	St. dev.	RMSE
ρ_{12}	0.001	0.020	0.020	0.001	0.015	0.015	0.000	0.011	0.011
ρ_{13}	0.001	0.020	0.020	0.000	0.015	0.015	0.000	0.010	0.010
ρ_{23}	0.001	0.021	0.021	0.001	0.015	0.015	0.000	0.011	0.011
α_1	0.046	0.061	0.077	0.014	0.042	0.045	0.005	0.031	0.032
α_2	0.039	0.056	0.068	0.014	0.043	0.045	0.006	0.030	0.031
α_3	0.041	0.060	0.073	0.015	0.042	0.045	0.005	0.028	0.029
β_1	−0.075	0.137	0.156	−0.022	0.073	0.077	−0.009	0.040	0.041
β_2	−0.067	0.131	0.147	−0.022	0.068	0.072	−0.010	0.042	0.043
β_3	−0.069	0.139	0.155	−0.024	0.069	0.073	−0.009	0.038	0.039
γ_1	−0.005	0.105	0.105	−0.005	0.071	0.071	−0.003	0.051	0.051
γ_2	−0.001	0.105	0.105	−0.009	0.073	0.073	−0.005	0.049	0.049
γ_3	−0.018	0.102	0.104	−0.010	0.072	0.073	−0.003	0.049	0.049
δ	−0.034	0.063	0.072	−0.025	0.060	0.065	−0.015	0.053	0.055
Iter.	3.371	0.996		3.103	0.789		2.954	0.695	

This table reports the Monte Carlo bias, standard deviation, and root mean squared error of the MBP estimators obtained from the multi-step estimation procedure for different series lengths (reported in the first row). All experiments are based on 1000 replications. The data generating process is a trivariate VT-GDCC model with true parameter values equal to $\rho_{12}=\rho_{13}=\rho_{23}=0.8, \alpha_1=\alpha_2=\alpha_3=0.2, \beta_1=\beta_2=\beta_3=0.9, \gamma_1=\gamma_2=\gamma_3=0.2, \delta=0.8$. The last row reports the average and standard deviation of the number of MBP iterations.

and $\gamma=\{0.20, 0.15, 0.10\}$. Hence, while in the first case the parameters are set to be equal across the series, in the second case we allow for small differences between them. For both cases and for all series we specify GARCH(1,1) parameters as follows: variance intercept 0.01, ARCH coefficient 0.05, GARCH coefficient 0.9. In both experiments, we generate data by using a single quantile level set equal to $\delta=0.8$ for all the three series.

To mimic the grid search procedure outlined in the previous section, and at the same time to avoid the estimation of a very large number of models and limit the total time required to complete the Monte Carlo experiment, estimations are performed under the restriction of having the same quantile δ for all three series.[7] With this restriction, for each set of the simulated series (associated with the two parameter sets defined above) we estimate 21 models by varying the quantile from $\delta=0.7$ to $\delta=0.9$ with a step of 0.01. We choose the specification with the highest log-likelihood among the 21 cases.

In Tables 1 and 2 we report the Monte Carlo bias, standard error, and mean squared error for the two VT-GDCC parameter sets defined above.[8] In both cases,

[7]This restriction is relaxed in the empirical application of the model.

[8]The estimates of GARCH coefficients are not reported for brevity. The Monte Carlo results highlight that GARCH estimates are consistent.

Table 2 Simulation results for the second parameter set

	$N=500$			$N=1000$			$N=2000$		
	Bias	St. dev.	RMSE	Bias	St. dev.	RMSE	Bias	St. dev.	RMSE
ρ_{12}	0.002	0.017	0.017	0.001	0.012	0.012	0.000	0.013	0.013
ρ_{13}	0.001	0.016	0.016	0.001	0.012	0.012	0.001	0.012	0.012
ρ_{23}	0.001	0.019	0.019	0.000	0.013	0.013	−0.001	0.013	0.013
α_1	0.066	0.070	0.096	0.034	0.052	0.062	0.032	0.048	0.058
α_2	0.061	0.069	0.092	0.031	0.050	0.059	0.028	0.051	0.058
α_3	0.040	0.072	0.083	0.021	0.055	0.059	0.020	0.055	0.058
β_1	−0.124	0.203	0.238	−0.072	0.148	0.165	−0.067	0.145	0.159
β_2	−0.105	0.181	0.210	−0.065	0.142	0.156	−0.058	0.136	0.148
β_3	−0.088	0.164	0.186	−0.052	0.131	0.141	−0.050	0.128	0.137
γ_1	0.006	0.110	0.110	0.005	0.086	0.086	0.001	0.082	0.082
γ_2	0.022	0.105	0.107	0.013	0.078	0.079	0.007	0.077	0.077
γ_3	0.033	0.112	0.116	0.023	0.084	0.087	0.019	0.084	0.086
δ	−0.022	0.068	0.071	−0.017	0.065	0.068	−0.018	0.065	0.067
Iter.	3.375	1.152		3.534	1.094		3.505	1.176	

This table reports the Monte Carlo bias, standard deviation, and root mean squared error of the MBP estimators obtained from our multi-step estimation procedure for different series lengths (reported in the first row). All experiments are based on 1000 replications. The data generating process is a trivariate VT-GDCC model with true parameter values equal to $\rho_{12}=\rho_{13}=\rho_{23}=0.8$, $\alpha_1=0.10$, $\alpha_2=0.15$, $\alpha_3=0.2$, $\beta_1=0.88$, $\beta_2=0.9$, $\beta_3=0.92$, $\gamma_1=0.20$, $\gamma_2=0.15$, $\gamma_3=0.1$, $\delta=0.8$. The last row reports the average and standard deviation of the number of MBP iterations.

these quantities all converge to zero by increasing the sample dimension. The results present evidence that our estimation approach provides consistent estimates of the parameters. Furthermore, we observe that the MBP estimation requires a limited number of iterations, consistent with Fan et al. (2007).

We note that the simulations performed here do not consider the case of the scalar VT component defined in (10). However, since the specification with a scalar VT is a special case of the model adopted in the presented Monte Carlo experiment, our conclusions on the appropriateness of the estimation approach also extend to this alternative specification.

2.2 Testing and Interpreting the Volatility Threshold Effects

Before turning to the empirical application of our model, in this section we provide a discussion of some further aspects related to (i) the tests of the VT specification and (ii) the interpretation of the VT parameters in the conditional correlation process. While, for simplicity, the following discussion is based on the scalar VT-DCC specification in (13), the argumentation can be easily generalized to the VT-GDCC.

When evaluating the VT-DCC specification, the main interest is on testing the significance of the γ parameter. However, under the null hypothesis of $\gamma=0$

the volatility quantiles (or, in general, the parameters entering the functions $\mathbb{Q}_i(\delta_i)$, $i=1,2,...n$) are not identified. To overcome this problem, we resort to the simulation-based approach of Hansen (1996) and consider the likelihood ratio (LR) test between the unrestricted VT-DCC model and the restricted DCC specification. The critical values for the LR test are determined through simulations. We consider the following steps:

(1) simulate under the null of DCC using the parameters α, β, and $\bar{Q}$ obtained from the estimated VT-DCC specification; thus, we are simulating under the null hypothesis we would like to test, but with parameter values corresponding to those obtained from the estimated VT specification; the simulation procedure has been described in Section 2.1;

(2) use the simulated series to estimate both the DCC and the VT-DCC models (for the latter following the grid search procedure previously described); evaluate and store the LR test statistic obtained from the estimated models;

(3) repeat (1)–(2) 2500 times and determine critical values for the test as quantiles of the empirical density of the LR statistic determined on the basis of the tests computed in step (2).

Given the critical values, we evaluate the null hypothesis of absence of the VT effect on correlations by comparing these critical values to the LR test statistics obtained on the basis of the likelihoods of the VT-DCC and DCC models fitted to the original data.

With respect to the interpretation of the γ parameter in the conditional correlation process, one might be tempted to interpret it as the magnitude of an increase in correlations due to the occurrence of a volatility threshold violation. However, the interpretation is not as direct as what it may seem for a number of reasons. First of all, the dynamic matrices Q_t are not correlations but covariance matrices of the standardized residuals η_t and therefore the impact of the γ on correlations is not linear; in fact, correlations are given as $\rho_{ij,t}=q_{ij,t}q_{ii,t}^{-1/2}q_{jj,t}^{-1/2}$ and the impact of instances of high volatility on correlations also depends on the level of the diagonal elements of Q_t; in turn, these can reduce the impact on correlations if they are larger than 1, while they might increase the impact if smaller than 1 (note that the diagonal elements of Q_t are not constrained to be equal to 1, but they assume values that oscillate around 1). As a result, a statistically significant γ or, more generally, the improvement in the likelihood associated with the introduction of the VT component signals the existence of an impact of high volatility states on the correlations. However, the magnitude of this impact is not given by the γ coefficient. Furthermore, the occurrences of high volatility, and the associated violations of the thresholds are typically not spotted around, but they generally appear in clusters with violations over consecutive observations. The impact is therefore persistent due to the presence of *periods* of high volatility and due to the autoregressive dynamic of Q_t.

To analyze the impact of introduction of the VT component on correlation dynamic, one could compare the correlation sequences obtained from two models, with and without the VT component. The difference between the dynamic correlations can be interpreted as the VT impact. To evaluate the relevance of the VT component for correlations, the simple graphical analysis of the dynamic evolution of $\gamma_i\gamma_j\left(v_{ij,t}-\bar{v}_{ij}\right)$ in (10) would not be appropriate. In fact, $\gamma_i\gamma_j\left(v_{ij,t}-\bar{v}_{ij}\right)$ does not directly reflect the change in correlations for two reasons: (i) the standardization of Q_t; (ii) the autoregressive dynamic of the model (note that the introduction of the VT component has an impact on all the parameters of the correlation process).

3 DATA DESCRIPTION

The empirical part of the article focuses on an analysis of the time-varying correlations between some of the largest American, European, and Asia-Pacific stock markets, including the United States, Canada, France, Germany, UK, Australia, Hong Kong, and Japan. The analysis is based on the Financial Times Stock Exchange Index (FTSE) total return indices denominated in U.S. Dollars.[9] The data are collected weekly. The sample spans the period from January 1994 to January 2011 and consists of 890 return observations. The use of weekly data addresses the issue of nonsynchronicity of the trading hours of the analyzed markets. Furthermore, all the data are expressed in a common currency to include the effects coming through the exchange rate channel. In fact, by using a common currency, the return for a given stock market equals the sum of the local currency stock market return and of the exchange rate return with respect to the common currency. An analysis of the information transmission mechanism that allows separation of the effects of the exchange rate channel from that of the pure stock market channel is beyond the scope of the present article and is left for future research.

Table 3 presents descriptive statistics for the returns of the eight stock market indices considered in our analysis. All series show the typical nonnormality of financial time series. They are negatively skewed and display excess kurtosis. At the 5% level of significance, the Ljung-Box statistics indicate serial autocorrelation in the returns of all market indices with exception of Canada and Japan. The squared returns of all series are highly auto correlated, which may be interpreted as an evidence of ARCH effects in the analyzed data.

Table 4 shows unconditional correlations of the returns. The highest correlations are between the three developed European markets, France, Germany, and the UK (ranging from 0.80 to 0.89). These are followed by the correlations between the United States and Canada (0.75) and between the United States and European markets (ranging from 0.70 to 0.72). The three Asia-Pacific markets, and especially

[9]The focus on the FTSE indices has a number of advantages: a long common sample period for the total return indices (the data on the FTSE market indices are available from January 1994); a common methodology to index construction; a common conversion rule from local currencies to the U.S. Dollars.

Table 3 Descriptive statistics

	Australia	Canada	France	Germany	Hong Kong	Japan	UK	United States
Mean	0.002	0.002	0.002	0.002	0.001	1.24E-04	0.001	0.002
(t-stat)	(1.976)	(2.114)	(1.496)	(1.370)	(0.866)	(0.121)	(1.514)	(1.887)
Max	0.215	0.206	0.209	0.152	0.186	0.118	0.199	0.123
Min	−0.223	−0.211	−0.166	−0.176	−0.290	−0.153	−0.159	−0.157
St. dev.	0.033	0.031	0.032	0.034	0.040	0.031	0.027	0.025
Skewness	−0.620	−0.531	−0.263	−0.584	−0.468	0.067	−0.311	−0.601
Kurtosis	10.048	8.460	7.344	5.740	8.405	4.584	8.846	7.404
JB	1901.534	1148.476	710.8665	329.3876	1116.904	93.87541	1282.966	773.765
LB(6)	16.057	9.599	27.674	15.633	15.870	7.309	22.677	21.629
LBS(6)	379.22	406.63	381.59	272.76	173.91	77.708	429.49	158.51

This table reports descriptive statistics for the weekly returns on the FTSE stock market indices from 1/4/1994 to 2/1/2011 (890 observations). The figures in parentheses are t-statistics for the null hypothesis of the mean to be equal to zero. JB is the Jarque-Bera test statistic, distributed as a χ^2_2; LB(6) and LBS(6) are Ljung-Box test statistics with 6 lags for return levels and return squares, respectively, distributed as a χ^2_6. The upper 1 and 5 percentile points of the χ^2_2 distribution are 9.21 and 5.99, respectively. The upper 1 and 5 percentile points of the χ^2_6 distribution are 16.81 and 12.59, respectively.

Table 4 Correlations between market returns

	United States	Canada	France	Germany	UK	Australia	HK	Japan
United States	1							
Canada	0.751	1						
France	0.702	0.726	1					
Germany	0.723	0.71	0.887	1				
UK	0.699	0.724	0.848	0.795	1			
Australia	0.537	0.679	0.652	0.607	0.684	1		
HK	0.453	0.531	0.508	0.509	0.541	0.643	1	
Japan	0.385	0.447	0.496	0.454	0.452	0.544	0.497	1

the Japanese market, show in tendency the lowest correlations with the rest of the markets.

To allow for predictability and to control for common shocks to international market returns, we consider a mean model defined as a VAR specification with a number of predictive variables. The choice of the predictive variables is based on the evidence in Ait-Sahalia and Brandt (2001) and Pesaran and Timmerman (1995, 2000). In particular, we consider (i) short-term interest rates for the European market, measured by the German 3-month money rate for the period from January 1994 through December 1998, and the Euro Interbank Offered Rate (EURIBOR) for the rest of the sample (the Euro was introduced on January 1, 1999); (ii) short-term U.S. interest rates, measured by the 3-month U.S. Treasury Bill rate; (iii) long-term

Table 5 Correlations between variance standardized residuals

	United States	Canada	France	Germany	UK	Australia	HK	Japan
United States	1							
Canada	0.721	1						
France	0.644	0.671	1					
Germany	0.655	0.654	0.849	1				
UK	0.643	0.648	0.782	0.751	1			
Australia	0.525	0.635	0.599	0.581	0.627	1		
HK	0.488	0.525	0.532	0.537	0.552	0.604	1	
Japan	0.375	0.432	0.482	0.454	0.435	0.521	0.484	1

European interest rates, measured by the German 10-year government bond yield; (iv) long-term U.S. interest rates measured by the U.S. 10-year government bond yield; and finally (v) the Organization of the Petroleum Exporting Countries (OPEC) oil price. We consider the weekly difference for the interest rates and log-returns for the OPEC oil price.

4 EMPIRICAL RESULTS

The mean dynamic is specified as a VARX(1)[10] model over the stock market indices and a set of explanatory variables:

$$X_t = \mu + \phi X_{t-1} + \theta Z_{t-1} + \varepsilon_t, \tag{19}$$

where X_t is the set of stock market returns and Z_t is the set of additional economic variables described in the previous section.

The residuals from the mean specification[11] are used to model the volatility of the stock markets. For all eight residual series, we fit the standard GARCH(1,1) specification of Bollerslev (1986) and the asymmetric generalization of Glosten et al. (1993), the GJR(1,1) model. The choice between the volatility models is based on the Schwarz Information Criterion (SIC). For all series the model preferred by SIC is GJR(1,1).[12]

Table 5 presents the cross-country correlations of variance standardized residuals η_t as defined in (4), while Table 6 presents the correlations of the fitted volatility series. For the sample on average, the correlations of the standardized residuals tend to be somewhat lower than the correlations of the return series.

[10]The lag length is selected according to standard information criteria.

[11]The parameter estimates of the mean model are not reported but are available from the authors upon request. Ljung-Box tests indicate that mean residuals are uncorrelated.

[12]The parameters of the estimated volatility models are not reported but are available from the authors upon request.

Table 6 Correlations between fitted GJR-GARCH volatilities

	United States	Canada	France	Germany	UK	Australia	HK	Japan
United States	1							
Canada	0.815	1						
France	0.884	0.844	1					
Germany	0.853	0.707	0.886	1				
UK	0.869	0.905	0.955	0.803	1			
Australia	0.613	0.812	0.758	0.638	0.803	1		
HK	0.265	0.360	0.287	0.301	0.291	0.525	1	
Japan	0.442	0.501	0.454	0.410	0.473	0.538	0.551	1

Table 7 DCC model estimates

	Coeff		*t*-statistic
α	0.018		11.630
β	0.971		325.055
LL		−7411.702	

This table reports the estimated coefficients, *t*-statistics, and log-likelihood (LL) of the scalar DCC model.

With respect to volatility, we note that the cross-correlations of volatility of the American and European markets are higher than the correlations of their return series. The evidence for the Asia-Pacific markets is mixed, with Hong Kong and Japan showing lower correlations of volatility series relative to return series in a number of cases.

4.1 Volatility Threshold DCC Estimates

We start with the estimation of the benchmark DCC models specified in (7). We consider three different specifications imposing alternative restrictions on the diagonal parameter matrices, A and B: (i) the scalar DCC specification, which imposes a common correlation dynamic for all market pairs in our sample; (ii) the Block-GDCC specification, which restricts the elements of the A and B matrices to be the same for the markets from each of the three geographical regions considered; and (iii) the unrestricted GDCC specification, with no restrictions on the parameter matrices.

Tables 7–9 summarize the estimates of the three specifications. For the Block-GDCC and GDCC specifications, we report the cross-products of the elements of the A and B matrices, $\alpha_i\alpha_j$ and $\beta_i\beta_j$, and their *t*-statistics.[13] The results of the LR

[13] The tables do not report the elements in $\bar{Q}$ since they are almost identical to the unconditional correlations reported in Table 5.

Table 8 Block-GDCC model estimates

	Coeff	t-stat		Coeff	t-stat
$\alpha_{\text{us}}\alpha_{\text{can}}$	0.034	7.107	$\beta_{\text{us}}\beta_{\text{can}}$	0.931	99.445
$\alpha_{\text{us}}\alpha_{\text{fr}}$ $\alpha_{\text{us}}\alpha_{\text{ger}}$ $\alpha_{\text{us}}\alpha_{\text{uk}}$ $\alpha_{\text{can}}\alpha_{\text{fr}}$ $\alpha_{\text{can}}\alpha_{\text{ger}}$ $\alpha_{\text{can}}\alpha_{\text{uk}}$	0.026	10.900	$\beta_{\text{us}}\beta_{\text{fr}}$ $\beta_{\text{us}}\beta_{\text{ger}}$ $\beta_{\text{us}}\beta_{\text{uk}}$ $\beta_{\text{can}}\beta_{\text{fr}}$ $\beta_{\text{can}}\beta_{\text{ger}}$ $\beta_{\text{can}}\beta_{\text{uk}}$	0.953	183.692
$\alpha_{\text{us}}\alpha_{\text{aus}}$ $\alpha_{\text{us}}\alpha_{\text{hk}}$ $\alpha_{\text{us}}\alpha_{\text{jap}}$ $\alpha_{\text{can}}\alpha_{\text{aus}}$ $\alpha_{\text{can}}\alpha_{\text{hk}}$ $\alpha_{\text{can}}\alpha_{\text{jap}}$	0.017	7.052	$\beta_{\text{us}}\beta_{\text{aus}}$ $\beta_{\text{us}}\beta_{\text{hk}}$ $\beta_{\text{us}}\beta_{\text{jap}}$ $\beta_{\text{can}}\beta_{\text{aus}}$ $\beta_{can}\beta_{\text{hk}}$ $\beta_{\text{can}}\beta_{\text{jap}}$	0.956	171.085
$\alpha_{\text{fr}}\alpha_{\text{ger}}$ $\alpha_{\text{fr}}\alpha_{\text{uk}}$ $\alpha_{\text{ger}}\alpha_{\text{uk}}$	0.020	10.147	$\beta_{\text{fr}}\beta_{\text{ger}}$ $\beta_{\text{fr}}\beta_{\text{uk}}$ $\beta_{\text{ger}}\beta_{\text{uk}}$	0.976	353.751
$\alpha_{\text{fr}}\alpha_{\text{aus}}$ $\alpha_{\text{fr}}\alpha_{\text{hk}}$ $\alpha_{\text{fr}}\alpha_{\text{jap}}$ $\alpha_{\text{ger}}\alpha_{\text{aus}}$ $\alpha_{\text{ger}}\alpha_{\text{hk}}$ $\alpha_{\text{ger}}\alpha_{\text{jap}}$ $\alpha_{\text{uk}}\alpha_{\text{aus}}$ $\alpha_{\text{uk}}\alpha_{\text{hk}}$ $\alpha_{\text{uk}}\alpha_{\text{jap}}$	0.013	7.550	$\beta_{\text{fr}}\beta_{\text{aus}}$ $\beta_{\text{fr}}\beta_{\text{hk}}$ $\beta_{\text{fr}}\beta_{\text{jap}}$ $\beta_{\text{ger}}\beta_{\text{aus}}$ $\beta_{\text{ger}}\beta_{\text{hk}}$ $\beta_{\text{ger}}\beta_{\text{jap}}$ $\beta_{\text{uk}}\beta_{\text{aus}}$ $\beta_{\text{uk}}\beta_{\text{hk}}$ $\beta_{\text{uk}}\beta_{\text{jap}}$	0.978	284.738
$\alpha_{\text{aus}}\alpha_{\text{hk}}$ $\alpha_{\text{aus}}\alpha_{\text{jap}}$ $\alpha_{\text{hk}}\alpha_{\text{jap}}$	0.008	4.214	$\beta_{\text{aus}}\beta_{\text{hk}}$ $\beta_{\text{aus}}\beta_{\text{jap}}$ $\beta_{\text{hk}}\beta_{\text{jap}}$	0.981	158.434
		LL	−7384.98		

$$q_{ij,t} = (1-\alpha_l\alpha_k-\beta_l\beta_k)\bar{q}_{ij} + \alpha_l\alpha_k\eta_{i,t-1}\eta_{j,t-1} + \beta_l\beta_k q_{ij,t-1}$$

$$i\in l,\ j\in k,\ l,k=1,2,\ldots m,\ l\neq k,$$

$$q_{ij,t} = (1-\alpha_l^2-\beta_l^2)\bar{q}_{ij} + \alpha_l^2\eta_{i,t-1}\eta_{j,t-1} + \beta_l^2 q_{ij,t-1},$$

$$i,j\in l,\ l=1,2,\ldots m,$$

This table reports the estimated coefficients, t-statistics, and log-likelihood (LL) of the Block-GDCC model above; t-statistics of the parameter functions are calculated using the delta method.

Table 9 GDCC model estimates

	Coeff	t-stat		Coeff	t-stat
$\alpha_{us}\alpha_{can}$	0.032	6.432	$\beta_{us}\beta_{can}$	0.938	104.520
$\alpha_{us}\alpha_{fr}$	0.023	7.587	$\beta_{us}\beta_{fr}$	0.964	158.050
$\alpha_{us}\alpha_{ger}$	0.024	7.034	$\beta_{us}\beta_{ger}$	0.960	144.527
$\alpha_{us}\alpha_{uk}$	0.022	6.747	$\beta_{us}\beta_{uk}$	0.961	151.011
$\alpha_{can}\alpha_{fr}$	0.026	8.902	$\beta_{can}\beta_{fr}$	0.954	144.098
$\alpha_{can}\alpha_{ger}$	0.028	7.733	$\beta_{can}\beta_{ger}$	0.951	134.073
$\alpha_{can}\alpha_{uk}$	0.026	7.251	$\beta_{can}\beta_{uk}$	0.951	135.948
$\alpha_{us}\alpha_{aus}$	0.016	4.681	$\beta_{us}\beta_{aus}$	0.966	137.830
$\alpha_{us}\alpha_{hk}$	0.016	4.476	$\beta_{us}\beta_{hk}$	0.961	131.200
$\alpha_{us}\alpha_{jap}$	0.011	3.550	$\beta_{us}\beta_{jap}$	0.968	132.856
$\alpha_{can}\alpha_{aus}$	0.018	4.590	$\beta_{can}\beta_{aus}$	0.957	118.970
$\alpha_{can}\alpha_{hk}$	0.018	4.415	$\beta_{can}\beta_{hk}$	0.951	123.365
$\alpha_{can}\alpha_{jap}$	0.013	3.492	$\beta_{can}\beta_{jap}$	0.958	123.537
$\alpha_{fr}\alpha_{ger}$	0.020	7.983	$\beta_{fr}\beta_{ger}$	0.976	276.925
$\alpha_{fr}\alpha_{uk}$	0.019	8.251	$\beta_{fr}\beta_{uk}$	0.977	310.776
$\alpha_{fr}\alpha_{aus}$	0.013	5.335	$\beta_{fr}\beta_{aus}$	0.983	246.083
$\alpha_{fr}\alpha_{hk}$	0.013	4.600	$\beta_{fr}\beta_{hk}$	0.977	170.745
$\alpha_{fr}\alpha_{jap}$	0.009	3.557	$\beta_{fr}\beta_{jap}$	0.984	199.634
$\alpha_{ger}\alpha_{uk}$	0.020	7.449	$\beta_{ger}\beta_{uk}$	0.973	250.036
$\alpha_{ger}\alpha_{aus}$	0.014	5.133	$\beta_{ger}\beta_{aus}$	0.979	212.465
$\alpha_{ger}\alpha_{hk}$	0.014	4.426	$\beta_{ger}\beta_{hk}$	0.974	153.130
$\alpha_{ger}\alpha_{jap}$	0.010	3.476	$\beta_{ger}\beta_{jap}$	0.981	177.643
$\alpha_{uk}\alpha_{aus}$	0.013	4.906	$\beta_{uk}\beta_{aus}$	0.979	195.139
$\alpha_{uk}\alpha_{hk}$	0.013	4.381	$\beta_{uk}\beta_{hk}$	0.974	163.336
$\alpha_{uk}\alpha_{jap}$	0.009	3.414	$\beta_{uk}\beta_{jap}$	0.981	187.070
$\alpha_{aus}\alpha_{hk}$	0.009	3.583	$\beta_{aus}\beta_{hk}$	0.980	137.217
$\alpha_{aus}\alpha_{jap}$	0.006	3.110	$\beta_{aus}\beta_{jap}$	0.987	166.934
$\alpha_{hk}\alpha_{jap}$	0.006	2.851	$\beta_{hk}\beta_{jap}$	0.981	128.162
		LL	−7379.523		

$$q_{ij,t} = (1-\alpha_i\alpha_j-\beta_i\beta_j)\bar{q}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1}$$

This table reports the estimated coefficients, t-statistics, and log-likelihood (LL) of the GDCC model above; t-statistics of the parameter functions are calculated using the delta method.

tests which compare the three specifications suggest that both the Block-GDCC and unrestricted GDCC are preferred to the scalar model. The more parsimonious Block-GDCC specification is, however, preferred to the unrestricted GDCC.[14] The

[14] In particular, the results of the Likelihood Ratio tests are as follows:
LR(DCC,block-GDCC) = 53.444 (the upper 1 and 5 percentile points of the χ^2_4 distribution are 9.49 and 13.3);
LR(DCC,GDCC) = 64.358 (the upper 1 and 5 percentile points of the χ^2_{14} distribution are 23.7 and 29.1);

Block-GDCC will serve as a benchmark model for the evaluation of the models with the Volatility Threshold component.

We now turn to the estimation of the Volatility Threshold specifications. In the empirical implementation below, we follow the threshold calibration approach discussed in Section 2. We start with the estimation of the correlation model in (8) employing the VT component defined in (9). We perform a grid search over conditional variance quantiles from 0.5 to 0.99, in each case setting the conditional variance quantile levels equal across the series. We consider three specifications of the correlation dynamic. In all three specifications, we impose block-restrictions on the A and B parameter matrices, as described above. In the first specification, the diagonal matrix Γ is restricted to be a scalar. In the second specification, we impose block restrictions on Γ similar to the restrictions on the matrices A and B. Finally, in the third specification, the elements of the Γ matrix are unrestricted. In all three cases, the log likelihood reaches its global maximum for the specification with the threshold set at the 72% quantile level. Having identified the optimal common threshold, we perform LR tests to evaluate the restrictions on the parameters of Γ. The tests indicate that the specification with block restrictions is preferred to the two alternatives considered.[15] Next, for the Block-VT-GDCC specification we consider a grid search in the boundaries of the common 72% threshold, as we detailed in Section 2, to determine the series-specific thresholds. The identified conditional volatility quantile levels together with the associated volatility values are summarized in Table 10. The highest volatility threshold value is observed for the Hong Kong market and the lowest for the U.S. market.[16]

Table 11 reports the estimates of the block VT-GDCC specification in (8) and (9) with series-specific thresholds. The parameter products $\gamma_i \gamma_j$ capture the effect of high volatility in the underlying markets on their correlation levels. To compare the performance of the VT specification and the benchmark block-GDCC in Table 8, we perform a LR test. Given that under the null of GDCC model the volatility quantiles are not identified, the critical values of the LR test have been estimated following the simulation-based approach of Hansen (1996), as described in Section 2.2. The LR test statistic based on the log-likelihoods of the VT and the benchmark GDCC specifications equals 46.3, while critical values are equal to 38.935, 24.372, and 19.466 at the 1%, 5%, and 10% confidence levels, respectively[17]. Hence, the VT model outperforms the benchmark model.

LR(block-GDCC,GDCC) = 10.914 (the upper 1 and 5 percentile points of the χ^2_{10} distribution are 18.3 and 23.2).

[15] The LR tests are available upon request.

[16] Given the identified series specific thresholds, we have also estimated two additional VT-GDCC specifications, the first with a scalar VT and the second with an unrestricted VT parameter matrix. Again, the LR tests have indicated that the specification with block restrictions is preferred to the two alternatives considered.

[17] Critical values have been determined following Hansen (1996): (i) simulate under the null of GDCC with parameter estimates of the VT-GDCC specification (thus with the VT parameters set to zero); (ii) use the simulated series to estimate both the GDCC and the VT-GDCC models (in case of the latter following

Table 10 Volatility thresholds

	72% variance quantile	Series specific variance quantile	
	Value	Quantile (%)	Value
United States	0.026	73	0.026
Canada	0.030	71	0.030
France	0.031	76	0.034
Germany	0.035	71	0.034
UK	0.027	72	0.027
Australia	0.031	72	0.031
Hong Kong	0.040	74	0.041
Japan	0.033	77	0.034

This table reports the values of the identified conditional standard deviation series quantiles used in the empirical application.

The results indicate a highly significant VT effect on correlations of all market pairs in our sample. Hence, there is evidence that the periods of turbulence in the markets are associated with an increase in their comovement. We find that the magnitude and significance of the VT effect are the strongest for the pair of the U.S. and Canadian markets and for the pairs which include the combinations of the American and European markets.

Next, we employ the VT-GDCC framework to present an example of analysis of the impact of high volatility states in a specific market on cross-market correlations. This could be viewed as an analysis of contagion effects associated with the shocks (potentially originating) in a specific market. Given the widely perceived prominent role of the U.S. market among world markets and, in particular, the influence of U.S. stock market fluctuations on developments in other stock markets, in our example we focus on contagion effects associated with high volatility states in the U.S. market.

The conditional correlation process is now specified as follows:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} + \alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \psi_i\psi_j\left(v_{\mathrm{us},t} - \bar{v}_{\mathrm{us}}\right), \tag{20}$$

where the VT dummy is a scalar, which assumes the value of 1 when the U.S. volatility exceeds a threshold and zero otherwise.

As before, we impose block restrictions on the diagonal A, B, and Γ parameter matrices. The model is estimated using the grid search procedure described in Section 2 varying the U.S. conditional variance quantile from 0.5 to 0.99. We find that the log-likelihood of the estimated specification reaches its maximum at the 97% quantile of the U.S. volatility series. However, our analysis indicates that the

the grid search procedure described in Section 2); (iii) store the LR statistic computed on the basis of the models estimated in (ii); (iv) repeat (i)–(iii) 2500 times and determine critical values over the set of estimated LR statistics.

Table 11 Block-VT-GDCC model estimates with the VT dummy defined in (9)

	Coeff	t-stat		Coeff	t-stat		Coeff	t-stat
$\alpha_{us}\alpha_{can}$	0.030	6.148	$\beta_{us}\beta_{can}$	0.926	100.330	$\gamma_{us}\gamma_{can}$	0.039	5.199
$\alpha_{us}\alpha_{fr}$ $\alpha_{us}\alpha_{ger}$ $\alpha_{us}\alpha_{uk}$ $\alpha_{can}\alpha_{fr}$ $\alpha_{can}\alpha_{ger}$ $\alpha_{can}\alpha_{uk}$	0.027	9.924	$\beta_{us}\beta_{fr}$ $\beta_{us}\beta_{ger}$ $\beta_{us}\beta_{uk}$ $\beta_{can}\beta_{fr}$ $\beta_{can}\beta_{ger}$ $\beta_{can}\beta_{uk}$	0.945	169.280	$\gamma_{us}\gamma_{fr}$ $\gamma_{us}\gamma_{ger}$ $\gamma_{us}\gamma_{uk}$ $\gamma_{can}\gamma_{fr}$ $\gamma_{can}\gamma_{ger}$ $\gamma_{can}\gamma_{uk}$	0.026	5.564
$\alpha_{us}\alpha_{aus}$ $\alpha_{us}\alpha_{hk}$ $\alpha_{us}\alpha_{jap}$ $\alpha_{can}\alpha_{aus}$ $\alpha_{can}\alpha_{hk}$ $\alpha_{can}\alpha_{jap}$	0.018	6.685	$\beta_{us}\beta_{aus}$ $\beta_{us}\beta_{hk}$ $\beta_{us}\beta_{jap}$ $\beta_{can}\beta_{aus}$ $\beta_{can}\beta_{hk}$ $\beta_{can}\beta_{jap}$	0.946	140.793	$\gamma_{us}\gamma_{aus}$ $\gamma_{us}\gamma_{hk}$ $\gamma_{us}\gamma_{jap}$ $\gamma_{can}\gamma_{aus}$ $\gamma_{can}\gamma_{hk}$ $\gamma_{can}\gamma_{jap}$	0.020	4.165
$\alpha_{fr}\alpha_{ger}$ $\alpha_{fr}\alpha_{uk}$ $\alpha_{ger}\alpha_{uk}$	0.024	9.535	$\beta_{fr}\beta_{ger}$ $\beta_{fr}\beta_{uk}$ $\beta_{ger}\beta_{uk}$	0.964	244.019	$\gamma_{fr}\gamma_{ger}$ $\gamma_{fr}\gamma_{ger}$ $\gamma_{ger}\gamma_{uk}$	0.017	4.393
$\alpha_{fr}\alpha_{aus}$ $\alpha_{fr}\alpha_{hk}$ $\alpha_{fr}\alpha_{jap}$ $\alpha_{ger}\alpha_{aus}$ $\alpha_{ger}\alpha_{hk}$ $\alpha_{ger}\alpha_{jap}$ $\alpha_{uk}\alpha_{aus}$ $\alpha_{uk}\alpha_{hk}$ $\alpha_{uk}\alpha_{jap}$	0.016	7.117	$\beta_{fr}\beta_{aus}$ $\beta_{fr}\beta_{hk}$ $\beta_{fr}\beta_{jap}$ $\beta_{ger}\beta_{aus}$ $\beta_{ger}\beta_{hk}$ $\beta_{ger}\beta_{jap}$ $\beta_{uk}\beta_{aus}$ $\beta_{uk}\beta_{hk}$ $\beta_{uk}\beta_{jap}$	0.965	178.205	$\gamma_{fr}\gamma_{aus}$ $\gamma_{fr}\gamma_{hk}$ $\gamma_{fr}\gamma_{jap}$ $\gamma_{ger}\gamma_{aus}$ $\gamma_{ger}\gamma_{hk}$ $\gamma_{ger}\gamma_{jap}$ $\gamma_{uk}\gamma_{aus}$ $\gamma_{uk}\gamma_{hk}$ $\gamma_{uk}\gamma_{jap}$	0.013	3.927
$\alpha_{aus}\alpha_{hk}$ $\alpha_{aus}\alpha_{jap}$ $\alpha_{hk}\alpha_{jap}$	0.010	4.005	$\beta_{aus}\beta_{hk}$ $\beta_{aus}\beta_{jap}$ $\beta_{hk}\beta_{jap}$	0.966	99.470	$\beta_{aus}\beta_{hk}$ $\gamma_{aus}\gamma_{jap}$ $\gamma_{hk}\gamma_{jap}$	0.010	2.401
				LL	−7361.83			

$$q_{ij,t} = (1-\alpha_l\alpha_k - \beta_l\beta_k)\bar{q}_{ij} + \alpha_l\alpha_k\eta_{i,t-1}\eta_{j,t-1} + \beta_l\beta_k q_{ij,t-1} + \gamma_l\gamma_k\left(v_{ij,t} - \bar{v}_{ij}\right)$$
$$i \in l,\ j \in k,\ l,k = 1,2,\ldots m,\ l \neq k,$$
$$q_{ij,t} = (1-\alpha_l^2 - \beta_l^2)\bar{q}_{ij} + \alpha_l^2\eta_{i,t-1}\eta_{j,t-1} + \beta_l^2 q_{ij,t-1} + \gamma_l^2\left(v_{ij,t} - \bar{v}_{ij}\right),$$
$$i,j \in l,\ l = 1,2,\ldots m.$$

This table reports the estimated coefficients, t-statistics, and log-likelihood (LL) of the Block-VT-GDCC model above; t-statistics of the parameter functions are calculated using the delta method. The VT dummy is defined in (9). The volatility thresholds are the series specific thresholds reported in Table 10.

U.S. volatility threshold effect has a statistically significant impact on all cross-market correlations (both including and not including the U.S. market) in our sample starting from the 81% volatility quantile. In other words, there is evidence of contagion effects associated with the U.S. volatility exceeding its 80% quantile. We acknowledge that we do not present *direct* evidence on the original source of the shocks generating contagion effects. What we do document is that the events of high volatility in the U.S. market are associated with an increase in cross-market correlations. This evidence confirms the significant role of the U.S. stock market in the international context. The periods of turbulence in the United States are associated with a change in return dynamics of other major stock markets of the world, as reflected in the change of their correlations.

Table 12 presents the estimates of the block VT-GDCC specification with the VT dummy equal to 1 when the U.S. volatility exceeds its 97% quantile. The results indicate a highly significant effect of the extreme U.S. volatility on correlations of all market pairs in our sample. To compare the US-VT specification with the benchmark block-GDCC in Table 8, we perform a LR test. Again, given that under the null of GDCC model the volatility quantiles are not identified, the critical values of the LR test have been estimated following the simulation-based approach of Hansen (1996). The LR test statistic based on the log-likelihoods of the VT and the benchmark GDCC specifications equals 32.0, while critical values are equal to 12.293, 8.738, and 6.595 at the 1%, 5%, and 10% confidence levels, respectively. Hence, the US-VT model outperforms the benchmark model.

We have presented the estimates of the VT-GDCC specification for two alternative specifications of the VT component. To examine if the two VT effects have a distinct impact on conditional correlations of the analyzed markets, we consider a specification which includes both VT dummies:

$$\begin{aligned} q_{ij,t} = {} & (1-\alpha_i\alpha_j-\beta_i\beta_j)\bar{q}_{ij}+\alpha_i\alpha_j\eta_{i,t-1}\eta_{j,t-1}+\beta_i\beta_j q_{ij,t-1}+\gamma_i\gamma_j\left(v_{ij,t}-\bar{v}_{ij}\right) \\ & +\psi_i\psi_j\left(v_{\mathrm{us},t}-\bar{v}_{\mathrm{us}}\right). \end{aligned} \tag{21}$$

The estimates reported in Table 13 show evidence of an increase in cross-market correlations associated with both VT effects. This increase is significant at the 1% level for correlations between all markets with exception of those between the Asia-Pacific markets. The rise in correlations among the latter markets is significant at the 10% level only.

4.2 Evaluating the Relevance of the VT Component

To demonstrate the impact of introduction of the Volatility Threshold component on the dynamic correlations, we present some illustrative figures. Figure 1 shows the evolution of the U.S. volatility during our sample period, 1994–2011. We note that the high volatility states are associated with the well-known periods of market

Table 12 Block-VT-GDCC model estimates with the VT dummy defined in (10)

	Coeff	t-stat		Coeff	t-stat		Coeff	t-stat
$\alpha_{us}\alpha_{can}$	0.025	6.412	$\beta_{us}\beta_{can}$	0.924	121.038	$\psi_{us}\psi_{can}$	0.156	3.089
$\alpha_{us}\alpha_{fr}$ $\alpha_{us}\alpha_{ger}$ $\alpha_{us}\alpha_{uk}$ $\alpha_{can}\alpha_{fr}$ $\alpha_{can}\alpha_{ger}$ $\alpha_{can}\alpha_{uk}$	0.023	10.374	$\beta_{us}\beta_{fr}$ $\beta_{us}\beta_{ger}$ $\beta_{us}\beta_{uk}$ $\beta_{can}\beta_{fr}$ $\beta_{can}\beta_{ger}$ $\beta_{can}\beta_{uk}$	0.939	214.190	$\psi_{us}\psi_{fr}$ $\beta_{us}\beta_{ger}$ $\psi_{us}\psi_{uk}$ $\psi_{can}\psi_{fr}$ $\psi_{can}\psi_{ger}$ $\psi_{can}\psi_{uk}$	0.101	4.503
$\alpha_{us}\alpha_{aus}$ $\alpha_{us}\alpha_{hk}$ $\alpha_{us}\alpha_{jap}$ $\alpha_{can}\alpha_{aus}$ $\alpha_{can}\alpha_{hk}$ $\alpha_{can}\alpha_{jap}$	0.014	6.479	$\beta_{us}\beta_{aus}$ $\beta_{us}\beta_{hk}$ $\beta_{us}\beta_{jap}$ $\beta_{can}\beta_{aus}$ $\beta_{can}\beta_{aus}$ $\beta_{can}\beta_{jap}$	0.940	180.963	$\psi_{us}\psi_{aus}$ $\psi_{us}\psi_{hk}$ $\psi_{us}\psi_{jap}$ $\psi_{can}\psi_{aus}$ $\psi_{can}\psi_{hk}$ $\psi_{can}\psi_{jap}$	0.073	3.648
$\alpha_{fr}\alpha_{ger}$ $\alpha_{fr}\alpha_{uk}$ $\alpha_{ger}\alpha_{uk}$	0.020	10.524	$\beta_{fr}\beta_{ger}$ $\beta_{fr}\beta_{uk}$ $\beta_{ger}\beta_{uk}$	0.954	347.566	$\psi_{fr}\psi_{ger}$ $\psi_{fr}\psi_{uk}$ $\psi_{ger}\psi_{uk}$	0.066	4.388
$\alpha_{fr}\alpha_{aus}$ $\alpha_{fr}\alpha_{hk}$ $\alpha_{fr}\alpha_{jap}$ $\alpha_{ger}\alpha_{aus}$ $\alpha_{ger}\alpha_{hk}$ $\alpha_{ger}\alpha_{jap}$ $\alpha_{uk}\alpha_{aus}$ $\alpha_{uk}\alpha_{hk}$ $\alpha_{uk}\alpha_{jap}$	0.013	7.099	$\beta_{fr}\beta_{aus}$ $\beta_{fr}\beta_{hk}$ $\beta_{fr}\beta_{jap}$ $\beta_{ger}\beta_{aus}$ $\beta_{ger}\beta_{hk}$ $\beta_{ger}\beta_{jap}$ $\beta_{uk}\beta_{aus}$ $\beta_{uk}\beta_{hk}$ $\beta_{uk}\beta_{jap}$	0.955	249.454	$\psi_{fr}\psi_{aus}$ $\psi_{fr}\psi_{hk}$ $\psi_{fr}\psi_{jap}$ $\psi_{ger}\psi_{aus}$ $\psi_{ger}\psi_{hk}$ $\psi_{ger}\psi_{jap}$ $\psi_{uk}\psi_{aus}$ $\psi_{uk}\psi_{hk}$ $\psi_{uk}\psi_{jap}$	0.048	4.089
$\alpha_{aus}\alpha_{hk}$ $\alpha_{aus}\alpha_{jap}$ $\alpha_{hk}\alpha_{jap}$	0.008	3.841	$\beta_{aus}\beta_{hk}$ $\beta_{aus}\beta_{jap}$ $\beta_{hk}\beta_{jap}$	0.956	134.451	$\psi_{aus}\psi_{hk}$ $\psi_{aus}\psi_{jap}$ $\psi_{hk}\psi_{jap}$	0.035	2.267
				LL	−7368.98			

$$q_{ij,t} = (1-\alpha_l\alpha_k-\beta_l\beta_k)\bar{q}_{ij}+\alpha_l\alpha_k\eta_{i,t-1}\eta_{j,t-1}+\beta_l\beta_k q_{ij,t-1}+\psi_l\psi_k\left(v_{us,t}-\bar{v}_{ij}\right)$$

$$i\in l,\ j\in k,\ l,k=1,2,\ldots m,\ l\neq k,$$

$$q_{ij,t} = (1-\alpha_l^2-\beta_l^2)\bar{q}_{ij}+\alpha_l^2\eta_{i,t-1}\eta_{j,t-1}+\beta_l^2 q_{ij,t-1}+\psi_l^2\left(v_{us,t}-\bar{v}_{ij}\right),$$

$$i,j\in l,\ l=1,2,\ldots m.$$

This table reports the estimated coefficients, t-statistics, and log-likelihood (LL) of the Block-VT-GDCC model above; t-statistics of the parameter functions are calculated using the delta method. The VT dummy is defined in (10). It assumes the value of 1 when the U.S. volatility exceeds the 97% quantile.

Table 13 Block-VT-GDCC model estimates with two VT dummies defined in (9) and (10)

	Coeff	t-stat		Coeff	t-stat		Coeff	t-stat		Coeff	t-stat
$\alpha_{us}\alpha_{can}$	0.025	5.654	$\beta_{us}\beta_{can}$	0.916	112.057	$\gamma_{us}\gamma_{can}$	0.031	4.536	$\psi_{us}\psi_{can}$	0.115	2.212
$\alpha_{us}\alpha_{fr}$			$\beta_{us}\beta_{fr}$			$\gamma_{us}\gamma_{fr}$			$\psi_{us}\psi_{fr}$		
$\alpha_{us}\alpha_{ger}$			$\beta_{us}\beta_{ger}$			$\gamma_{us}\gamma_{ger}$			$\psi_{us}\psi_{ger}$		
$\alpha_{us}\alpha_{uk}$	0.024	9.531	$\beta_{us}\beta_{uk}$	0.932	190.693	$\gamma_{us}\gamma_{uk}$	0.019	4.814	$\psi_{us}\psi_{uk}$	0.074	2.964
$\alpha_{can}\alpha_{fr}$			$\beta_{can}\beta_{fr}$			$\gamma_{can}\gamma_{fr}$			$\psi_{can}\psi_{fr}$		
$\alpha_{can}\alpha_{ger}$			$\beta_{can}\beta_{ger}$			$\gamma_{can}\gamma_{ger}$			$\psi_{can}\psi_{ger}$		
$\alpha_{can}\alpha_{uk}$			$\beta_{can}\beta_{uk}$			$\gamma_{can}\gamma_{uk}$			$\psi_{can}\psi_{uk}$		
$\alpha_{us}\alpha_{aus}$			$\beta_{us}\beta_{aus}$			$\gamma_{us}\gamma_{aus}$			$\psi_{us}\psi_{aus}$		
$\alpha_{us}\alpha_{hk}$			$\beta_{us}\beta_{hk}$			$\gamma_{us}\gamma_{hk}$			$\psi_{us}\psi_{hk}$		
$\alpha_{us}\alpha_{jap}$	0.015	6.354	$\beta_{us}\beta_{jap}$	0.932	155.486	$\gamma_{us}\gamma_{jap}$	0.013	3.118	$\psi_{us}\psi_{jap}$	0.064	2.867
$\alpha_{can}\alpha_{aus}$			$\beta_{can}\beta_{aus}$			$\gamma_{can}\gamma_{aus}$			$\psi_{can}\psi_{aus}$		
$\alpha_{can}\alpha_{hk}$			$\beta_{can}\beta_{hk}$			$\gamma_{can}\gamma_{hk}$			$\psi_{can}\psi_{hk}$		
$\alpha_{can}\alpha_{jap}$			$\beta_{can}\beta_{jap}$			$\gamma_{can}\gamma_{jap}$			$\psi_{can}\psi_{jap}$		
$\alpha_{fr}\alpha_{ger}$			$\beta_{fr}\beta_{ger}$			$\gamma_{fr}\gamma_{ger}$			$\psi_{fr}\psi_{ger}$		
$\alpha_{fr}\alpha_{uk}$	0.023	9.693	$\beta_{fr}\beta_{uk}$	0.947	273.876	$\gamma_{fr}\gamma_{uk}$	0.012	3.134	$\psi_{fr}\psi_{uk}$	0.047	2.683
$\alpha_{ger}\alpha_{uk}$			$\beta_{ger}\beta_{uk}$			$\gamma_{ger}\gamma_{uk}$			$\psi_{ger}\psi_{uk}$		

(*continued*)

Table 13 Continued

	Coeff	t-stat		Coeff	t-stat		Coeff	t-stat		Coeff	t-stat
$\alpha_{\mathrm{fr}}\alpha_{\mathrm{aus}}$			$\beta_{\mathrm{fr}}\beta_{\mathrm{aus}}$			$\gamma_{\mathrm{fr}}\gamma_{\mathrm{aus}}$			$\psi_{\mathrm{fr}}\psi_{\mathrm{aus}}$		
$\alpha_{\mathrm{fr}}\alpha_{\mathrm{hk}}$			$\beta_{\mathrm{fr}}\beta_{\mathrm{hk}}$			$\gamma_{\mathrm{fr}}\gamma_{\mathrm{hk}}$			$\psi_{\mathrm{fr}}\psi_{\mathrm{hk}}$		
$\alpha_{\mathrm{fr}}\alpha_{\mathrm{jap}}$			$\beta_{\mathrm{fr}}\beta_{\mathrm{jap}}$			$\gamma_{\mathrm{fr}}\gamma_{\mathrm{jap}}$			$\psi_{\mathrm{fr}}\psi_{\mathrm{jap}}$		
$\alpha_{\mathrm{ger}}\alpha_{\mathrm{aus}}$			$\beta_{\mathrm{ger}}\beta_{\mathrm{aus}}$			$\gamma_{\mathrm{ger}}\gamma_{\mathrm{aus}}$			$\psi_{\mathrm{ger}}\psi_{\mathrm{aus}}$		
$\alpha_{\mathrm{ger}}\alpha_{\mathrm{hk}}$	0.015	6.968	$\beta_{\mathrm{ger}}\beta_{\mathrm{hk}}$	0.948	195.236	$\gamma_{\mathrm{ger}}\gamma_{\mathrm{hk}}$	0.008	2.866	$\psi_{\mathrm{ger}}\psi_{\mathrm{hk}}$	0.041	3.095
$\alpha_{\mathrm{ger}}\alpha_{\mathrm{jap}}$			$\beta_{\mathrm{ger}}\beta_{\mathrm{jap}}$			$\gamma_{\mathrm{ger}}\gamma_{\mathrm{jap}}$			$\psi_{\mathrm{ger}}\psi_{\mathrm{jap}}$		
$\alpha_{\mathrm{uk}}\alpha_{\mathrm{aus}}$			$\beta_{\mathrm{uk}}\beta_{\mathrm{aus}}$			$\gamma_{\mathrm{uk}}\gamma_{\mathrm{aus}}$			$\psi_{\mathrm{uk}}\psi_{\mathrm{aus}}$		
$\alpha_{\mathrm{uk}}\alpha_{\mathrm{hk}}$			$\beta_{\mathrm{uk}}\beta_{\mathrm{hk}}$			$\gamma_{\mathrm{uk}}\gamma_{\mathrm{hk}}$			$\psi_{\mathrm{uk}}\psi_{\mathrm{hk}}$		
$\alpha_{\mathrm{uk}}\alpha_{\mathrm{jap}}$			$\beta_{\mathrm{uk}}\beta_{\mathrm{jap}}$			$\gamma_{\mathrm{uk}}\gamma_{\mathrm{jap}}$			$\psi_{\mathrm{uk}}\psi_{\mathrm{jap}}$		
$\alpha_{\mathrm{aus}}\alpha_{\mathrm{hk}}$			$\beta_{\mathrm{aus}}\beta_{\mathrm{hk}}$			$\gamma_{\mathrm{aus}}\gamma_{\mathrm{hk}}$			$\psi_{\mathrm{aus}}\psi_{\mathrm{hk}}$		
$\alpha_{\mathrm{aus}}\alpha_{\mathrm{jap}}$	0.009	3.833	$\beta_{\mathrm{aus}}\beta_{\mathrm{jap}}$	0.948	106.183	$\gamma_{\mathrm{aus}}\gamma_{\mathrm{jap}}$	0.006	1.744	$\psi_{\mathrm{aus}}\psi_{\mathrm{jap}}$	0.035	1.807
$\alpha_{\mathrm{hk}}\alpha_{\mathrm{jap}}$			$\beta_{\mathrm{hk}}\beta_{\mathrm{jap}}$			$\gamma_{\mathrm{hk}}\gamma_{\mathrm{jap}}$			$\psi_{\mathrm{hk}}\psi_{\mathrm{jap}}$		
					LL	−7354.99					

$$q_{ij,t} = (1-\alpha_l\alpha_k-\beta_l\beta_k)\bar{q}_{ij}+\alpha_l\alpha_k\eta_{i,t-1}\eta_{j,t-1}+\beta_l\beta_k q_{ij,t-1}+\gamma_i\gamma_j\left(v_{ij,t}-\bar{v}_{ij}\right)+\psi_l\psi_k\left(v_{\mathrm{us},t}-\bar{v}_{ij}\right)$$

$$i\in l,\ j\in k,\ l,k=1,2,\ldots m,\ l\neq k,$$

$$q_{ij,t} = (1-\alpha_l^2-\beta_l^2)\bar{q}_{ij}+\alpha_l^2\eta_{i,t-1}\eta_{j,t-1}+\beta_l^2 q_{ij,t-1}+\gamma_l^2\left(v_{ij,t}-\bar{v}_{ij}\right)+\psi_l^2\left(v_{\mathrm{us},t}-\bar{v}_{ij}\right),$$

$$i,j\in l,\ l=1,2,\ldots m.$$

This table reports the estimated coefficients, t-statistics, and log-likelihood (LL) of the Block-VT-GDCC model above; t-statistics of the parameter functions are calculated using the delta method. For the definition of the VT dummies see Tables 11 and 12.

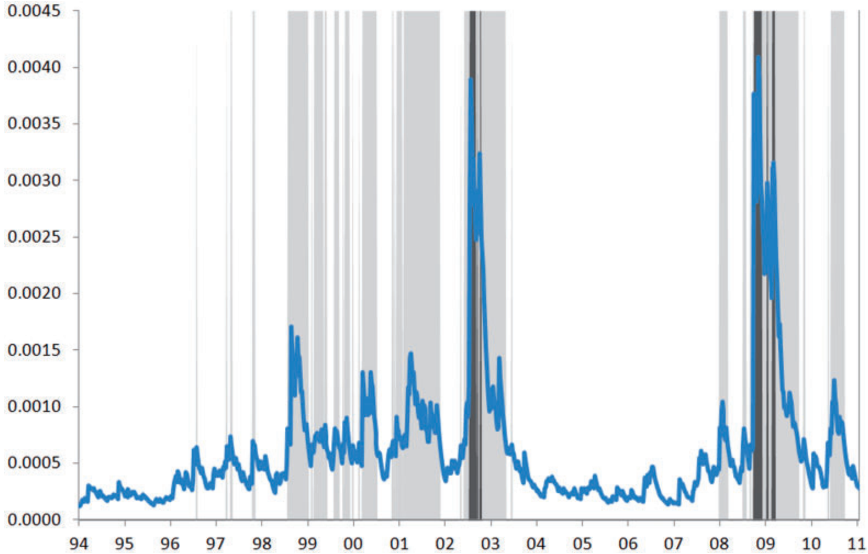

Figure 1 The conditional volatility and the volatility threshold violations of the U.S. market. This figure presents the development of the U.S. conditional volatility over the sample period. The highlighted areas indicate the violations of the 73% quantile (light areas) and the 97% quantile (dark areas) of distribution of the volatility.

crises, for example the period around the 1998 Russian default, the September 11, 2001, the burst of the dot-com bubble in 2002, and the subprime crisis in 2008–2009. We observe that spikes in volatility are followed by slow reversion to its long-term mean, consistent with the persistence of volatility.

The VT-GDCC specification with the VT component defined in (9) detects the events of volatility co-exceedances (or co-violations) and their impact on cross-market comovement. To get an idea about the distribution of threshold violations over time, we construct an index of violations by summing-up the diagonal elements of the V_t matrix in (9). The index measures the number of markets with an above-the-threshold volatility[18] at each point in time over the analyzed sample period. Figure 2 presents the evolution of this index. The index might be interpreted as a measure of diffusion of market turbulence. It is similar to the co-exceedance index of Bae et al. (2003) who analyze coincident extreme returns in the international markets (see Section 4.3). We observe that the cases of the most diffused market turbulence are associated with the 1998, 2002, and 2008–2009 events. Hence, the VT-GDCC specification identifies the periods of turbulence in the markets and then uses this information to evaluate the impact on cross-market correlations.

To illustrate the differences between conditional correlations implied by specifications without and with the VT component defined in (9)—specifically,

[18]The volatility thresholds used in this example are those reported in Table 10.

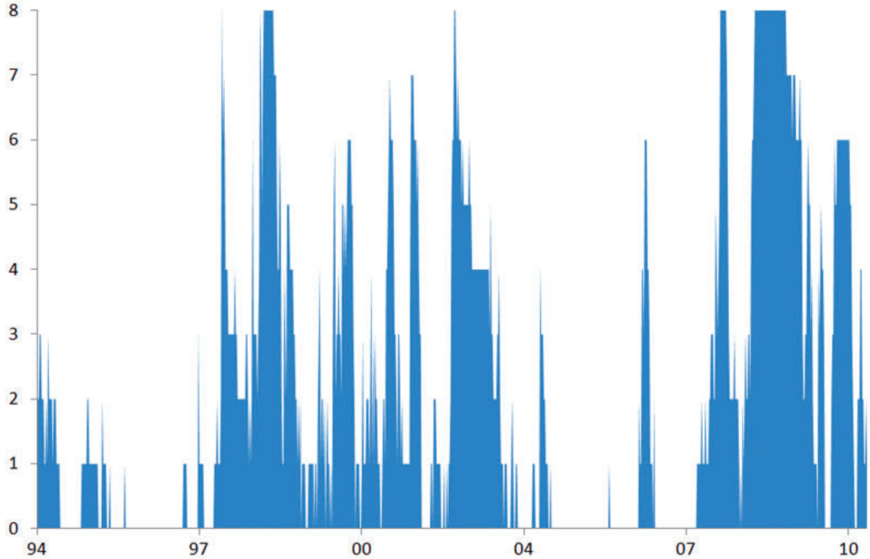

Figure 2 The diffusion of the volatility threshold violations across the markets. This figure presents the development of the index of volatility threshold violations. The index measures the number of markets with the above-the-threshold volatility at each point in time over the sample period. The thresholds are the series specific thresholds reported in Table 10.

between the block GDCC and VT-GDCC—in Figure 3 we report the deviations between the conditional correlations resulting from these two models, averaged across all market pairs. We document several peaks in the average differences between correlations obtained with and without introduction of the VT component. The most pronounced differences are observed around the crises in 1998 and 2008–2009 (clearly these differences will be even larger for some market pairs and smaller for others). This is consistent with the pattern observed for the co-exceedance index in Figure 2. In particular, the peaks in 1998 and 2008–2009 coincide with the periods when all eight markets in our sample experience "excess" (above-the-threshold) volatility. The presence of negative deviations between the estimated correlations should not be surprising for the following reasons. First, the VT-GDCC and GDCC specifications have different parameter estimates that induce differences in time variation of the correlations. Second, in the presented example we consider the average cross-market correlations; thus, the volatility states defining the value of the VT dummy would differ across the market pairs and the potential VT impact for some pairs might not show up in the market average.

Finally, in Figure 4, we present the differences between conditional correlations implied by specifications without and with the VT component, where the VT dummy is defined as in (10) and assumes a unit value when the volatility of the U.S. market exceeds its 97% quantile. Again, we consider differences in correlations

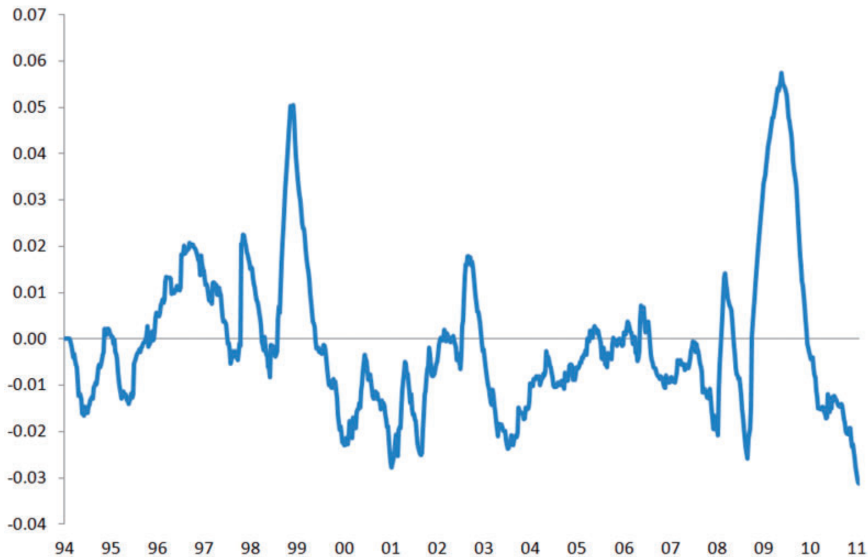

Figure 3 The volatility threshold impact on conditional correlations with the VT component Defined in (9). This figure presents the differences between conditional correlations implied by Block VT-GDCC and Block-GDCC specifications averaged across all market pairs. The VT dummy is defined in (9). The volatility thresholds are reported in Table 10.

averaged across all market pairs in the sample. We observe that the two events of extreme U.S. volatility in 2002 and 2008–2009 are associated with differences in correlations of almost 12%.

The illustrative evidence presented in this section is consistent with state-contingent comovement patterns in the international stock markets and with the existence of cross-market contagion.

4.3 A Comparison with the Exogenous Thresholds in Bae et al. (2003)

The index in Figure 2 is similar to the co-exceedance index of Bae et al. (2003) who analyze joint occurrences of extreme returns in the international stock markets. In their study, an extreme return (exceedance) is arbitrarily defined as one that lies in the 5% tails of the return distribution.[19] In this section, we compare our

[19]We would like to emphasize that although both Bae et al. (2003) and our study employ the events of coincident extreme observations for the analysis of contagion, there are significant differences between the two approaches, which makes their direct comparison not trivial. Our study focuses on the impact of coincident high volatility on changes in the level of cross-market correlations estimated using variance standardized residuals [specification (8)–(9)]. Bae et al. identify the cases of coincident extreme returns in the markets and interpret these cases as evidence of contagion. In particular, they acknowledge that

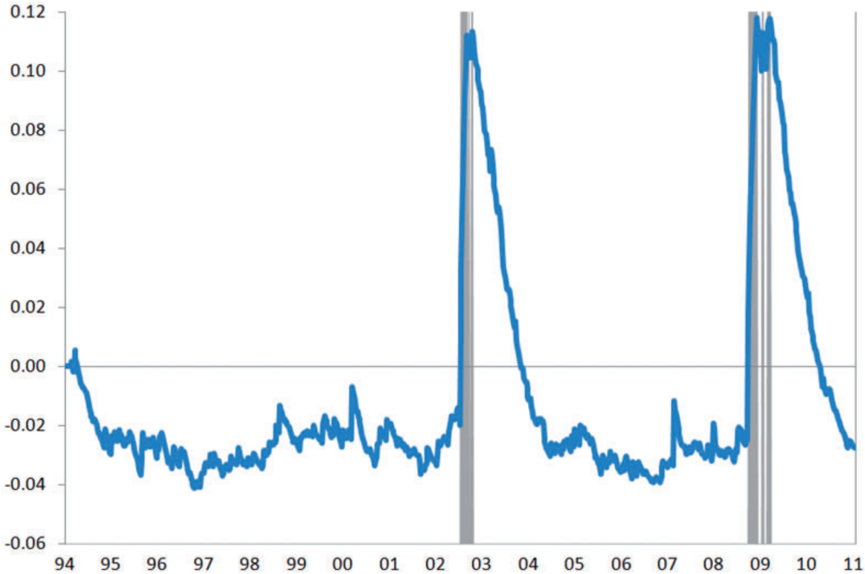

Figure 4 The volatility threshold impact on conditional correlations with the VT component defined in (10). This figure presents the differences between conditional correlations implied by Block VT-GDCC and Block-GDCC specifications averaged across all market pairs. The VT dummy is defined in (10). It assumes the value of 1 when the U.S. volatility exceeds the 97% quantile.

approach to the definition of thresholds to the approach adopted by Bae et al. (2003). In addition to the potential differences related to the arbitrary setting versus endogenous estimation of the thresholds, we may expect differences resulting from the fact that our thresholds are based on volatility, which, in contrast to returns, is persistent. In the following, we provide a comparison of the co-exceedance indices resulting from the two alternative definitions of thresholds. We also provide a comparison of the performance of the VT-GDCC models estimated using these alternative thresholds.

Figure 5 presents the co-exceedance index based on the thresholds as defined in Bae et al. (2003), it means using the 5% tails of the return distribution. In particular, at each point in time, the co-exceedence index shows the number of markets with returns in either the lower or upper 5% tails of distribution. A comparison of Figures 2 and 5 indicates that the co-exceedances of volatility in Figure 2 are more persistent and more concentrated than the co-exceedances of returns in Figure 5.

in the presence of a common factor and constant correlations during the sample period the coincident extreme (positive/negative) returns may be expected and, therefore, do not necessarily signal contagion. To demonstrate that the coincident extreme returns imply contagion, Bae et al. perform simulations assuming a constant correlation matrix during the sample period. The simulated series do not deliver the same count of the co-exceedance events as the empirical data. On the basis of this result, Bae et al. interpret the co-exceedances as evidence of contagion.

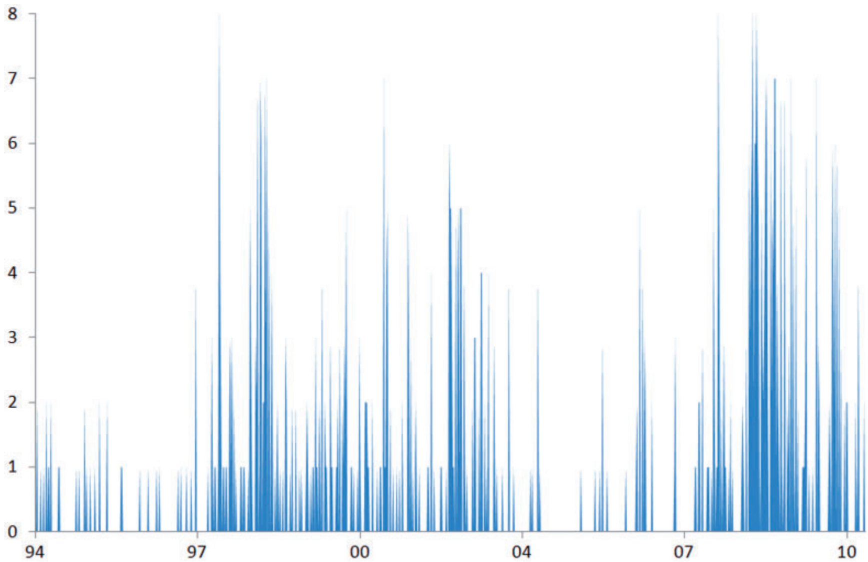

Figure 5 The co-exceedance index of extreme returns. This figure presents the co-exceedence index of extreme returns, which measures the number of markets with the returns in either the lower or upper 5% tails distribution at each point in time during the sample period.

The observed differences are likely to be attributed to (i) the persistence of the conditional variances which after a shock revert slowly to their long-term mean, and (ii) the fact that the endogenously estimated volatility thresholds are not as extreme as the arbitrarily chosen 5% tails of return distribution.

Next, we estimate the model in (8) with the elements of the matrix V_t specified as

$$v_{ij,t}=\begin{cases} 1 \text{ if } \left(r_{i,t-1}<\mathbb{Q}_{R,i}^{0.05} \text{ or } r_{i,t-1}>\mathbb{Q}_{R,i}^{0.95}\right) \text{ and } \left(r_{j,t-1}<\mathbb{Q}_{R,j}^{0.05} \text{ or } r_{j,t-1}>\mathbb{Q}_{R,j}^{0.95}\right) \\ 0 \qquad \text{otherwise} \end{cases} \tag{22}$$

and alternatively as

$$v_{ij,t}=\begin{cases} 1 \text{ if } r_{i,t-1}<\mathbb{Q}_{R,i}^{0.05} \text{ and } r_{j,t-1}<\mathbb{Q}_{R,j}^{0.05} \\ 0 \qquad \text{otherwise} \end{cases}, \tag{23}$$

where $\mathbb{Q}_{R,i}^{\alpha}$ is the empirical α-quantile of the returns of market i. Note that the specification in (22) identifies the cases when the markets in the pair are both in the lower 5% quantile, or both in the upper 5% quantile, or in the opposite 5% quantiles (the latter case is not observed in our sample of international market returns). In specification (23), we consider the case when both markets in the pair

Table 14 Comparison of Block-VT-GDCC Model with VT dummies in (22) and (23) and the Benchmark Model with VT Dummy in (9)

	LL	ΔBIC
$v_{ij,t}=\begin{cases} 1 \text{ if } \left(r_{i,t-1}<\mathbb{Q}_{R,i}^{0.05} \text{ or } r_{i,t-1}>\mathbb{Q}_{R,i}^{0.95}\right) \text{ and } \left(r_{j,t-1}<\mathbb{Q}_{R,j}^{0.05} \text{ or } r_{j,t-1}>\mathbb{Q}_{R,j}^{0.95}\right) \\ 0 \quad \text{otherwise} \end{cases}$	−7380.703	37.7
$v_{ij,t}=\begin{cases} 1 \text{ if } r_{i,t-1}<\mathbb{Q}_{R,i}^{0.05} \text{ and } r_{j,t-1}<\mathbb{Q}_{R,j}^{0.05} \\ 0 \quad \text{otherwise} \end{cases},$	−7384.639	45.6
Benchmark model (Table 11)	−7361.830	

This table reports the log-likelihoods (LLs) of the Block-VT-GDCC with the thresholds defined in (22), (23) and (9), and the difference between the BIC information criterion (ΔBIC) of the specification with the threshold in (9) and the specifications with the thresholds in (22) and (23). The specification with the threshold in (9) is the benchmark model with the estimates reported in Table 11.

experience extreme negative returns. We also note that the returns are lagged in order to perform a fair comparison in terms of the conditioning information set (in our model the conditional volatility is contemporaneous but it is based on the time $t-1$ information set, thus excluding the contemporaneous returns).[20]

Table 14 reports the likelihoods of the estimated models. We note that the likelihoods of both specifications (with the thresholds defined in (22) and (23)) are lower than the likelihood of our benchmark VT-GDCC specification reported in the last row of the table (the estimates of the benchmark specifications are reported in Table 11). To compare two VT-GDCC models differing only in the specification of the Volatility Threshold component, we resort to a comparison of the likelihood values through the BIC information criterion using the taxonomy of Kass and Raftery (1995). The last column of Table 14 reports the difference between the BIC of the benchmark VT-GDCC model and the BIC of the specifications with the thresholds defined in (22) and (23). Following Kass and Raftery (1995), our model with endogenous volatility thresholds provides a strong improvement over the models with exogenous thresholds based on extreme returns.[21]

5 CONCLUSION

In this article, we propose a generalized DCC model which extends the representation by Cappiello et al. (2006) and makes the correlation dynamic dependent on variance values through a threshold structure. This model allows

[20] In this context, we emphasize that the economic intuition underlying the thresholds defined with respect to returns and conditional variances is not identical.

[21] In unreported results we also considered setting the thresholds in (22) and (23) at the 10% and 15% tails of the return distribution, thus covering 20% and 30% of the extreme return observations, respectively (note that these thresholds are closer to our endogenous volatility thresholds). Again, the results indicated that our specification provides a strong improvement over these alternatives.

an analysis of the dynamic behavior of correlations between assets conditional on volatility regimes, and, therefore, represents a useful tool in areas such as optimal portfolio allocation, hedging, and contagion analysis.

The proposed modeling framework is employed to study the behavior of correlations of a selection of the largest stock markets of the world in the period from 1994 to 2011. The analyzed sample period is characterized by a number of events of financial market turbulence. We consider two alternative specifications of the volatility threshold effect. In the first case, we study the impact of high volatility states in the underlying markets on their cross-correlations; in the second case, we analyze contagion effects associated with extreme U.S. volatility. The empirical evidence indicates that turbulent periods coincide with an increase in cross-market comovement. Our findings support the existence of contagion in the international stock markets.

From a general viewpoint, the volatility threshold effects may be integrated in the different dynamic correlation specifications proposed in the literature. The specifications to be considered in this context in the future are, for example, those proposed by Tse and Tsui (2002) and Pelletier (2006). In particular, the joint use of the volatility threshold structure and Markov switching dynamics may provide further useful tools for contagion analysis.

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REFERENCES

Aielli, G. P. 2008. “Consistent Estimation of Large Scale Dynamic Conditional Correlatins”, Department of Economics, Statistics, Mathematics and Sociology, University of Messina, Working paper collection “Quaderni di Economia, Statistica e Analisi del Territorio”.

Ait-Sahalia, Y., and M. W. Brandt. 2001. Variable Selection for Portfolio Choice. *Journal of Finance* 54: 1297–1351.

Bae, K. H., A. Karolyi, and R. Stulz. 2003. A New Approach to Measuring Financial Market Contagion. *Review of Financial Studies* 16: 717–764.

Beirne, J., G. M. Caporale, and M. Schulze-Ghattas. 2009. “Volatility Spillovers and Contagion from Mature to Emerging Stock Markets.” IMF Working Paper 08-286 and ECB Working Paper 1113.

Bertero, E., and C. Mayer. 1990. Structure and Performance: Global Interdependence of Stock Markets Around the Crash of 1987. *European Economic Review* 34: 1155–1180.

Billio, M., and M. Caporin, 2010. Market Linkages, Variance Spillovers, and Correlation Stability: Empirical Evidence of Financial Contagion. *Computational Statistics and Data Analysis* 54: 2443–2458.

Billio, M., M. Caporin, and M. Gobbo. 2006. Flexible Dynamic Conditional Correlation Multivariate GARCH for Asset Allocation. *Applied Financial Economics Letters* 2: 123–130.

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31: 307–327.

Bollerslev, T. 1990. Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Model. *Review of Economics and Statistics* 72: 498–505.

Boyer, B. H., M. S. Gibson, and M. Loretan. 1999. "Pitfalls in Tests for Changes in Correlations." IFS Discussion Paper No. 597R, Federal Reserve Board.

Calvo, S., and C. M. Reinhart. 1996. "Capital Flows to Latin America: Is there Evidence of Contagion Effects?" In G. Calvo, M. Goldstein and E. Hochreiter (eds.), *Private Capital Flows to Emerging Markets After the Mexican Crisis*. Washington: Institute for International Economics.

Cappiello, L., R. F. Engle, and K. Sheppard. 2006. Asymmetric Dynamics in the Correlations of Global Equity and Bond Markets. *Journal of Financial Econometrics* 4: 537–572.

Corsetti G, M. Pericoli, and M. Sbracia. 2005. Some Contagion, Some Interdependence: More Pitfalls in Tests of Financial Contagion. *Journal of International Money and Finance* 24: 1177–1199.

Dornbusch, R., Y. C. Park, and S. Claessens. 2000. Contagion: Understanding How it Spreads. *World Bank Research Observer* 15: 177–197.

Dungey, M., R. Fry, B. González-Hermosillo, and V. L. Martin. 2005. Empirical Modelling of Contagion: A Review of Methodologies. *Quantitative Finance* 5: 9–24.

Dungey, M., and V. L Martin. 2007. Unravelling Financial Market Linkages During Crises. *Journal of Applied Econometrics* 22: 89–119.

Dungey, M., Milunovich, G., and S. Thorp. 2010 Unobservable Shocks as Carriers of Contagion *Journal of Banking and Finance* 34: 1008–1021.

Eichengreen, B., A. K. Rose, and C. Wyplosz. 1995. Exchange Market Mayhem: The Antecedents and Aftermath of Speculative Attacks. *Economic Policy* 21: 249–312.

Eichengreen, B. J., A. Rose, and C. Wyplosz. 1996. Contagious Currency Crises: First Tests. *Scandinavian Journal of Economics* 98: 463–494.

Engle, R. F. 2002. Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models. *Journal of Business and Economic Statistics* 20: 339–350.

Engle, R. F., and B. Kelly. 2012. Dynamic Equicorrelation. *Journal of Business and Economic Statistics* 30: 212–228.

Engle, R. F., and V. Ng. 1993. Measuring and Testing the Impact of News on Volatility. *Journal of Finance* 48: 1749–1778.

Engle, R. F., and K. Sheppard. 2001. "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." Working Paper 2001-15, University of California at San Diego.

Erb, C. B., C. E. Harvey, and T. E. Viskanta. 1994. Forecasting International Correlations. *Financial Analyst Journal* 50: 322–345.

Fan, Y., S. Pastorello, and E. Renault. 2007. "Maximization by Parts in Extremum Estimation". Working Paper, University of North Carolina in Chapel Hill.

Favero, C. A., and F. Giavazzi. 2002. Is the International Propagation of Financial Shocks Non-linear? Evidence from the ERM. *Journal of International Economics* 57: 231–246.

Forbes, K., and R. Rigobon. 2002. No Contagion, Only Interdependence: Measuring Stock Market Comovements. *Journal of Finance* 57: 2223–2261.

Franses, P. H., and C. Hafner. 2009. A Generalized Dynamic Conditional Correlation Model for Many Asset Returns. *Econometric Reviews* 28: 612–631.

R. Fry, V. L. Martin, and C. Tang. 2010. A New Class of Tests of Contagion with Applications. *Journal of Business and Economic Statistics* 28: 423–437.

Garcia R., and G. Tsafack. 2006. "Dependence Structure and Extreme Comovements in International Equity and Bond Markets." Working Paper, Université de Montréal.

Glosten, L., R. Jagannathan, and D. Runkle. 1993. On the Relationship between the Expected Value and the Volatility of the Nominal Excess Returns on Stocks. *Journal of Finance* 48: 1779–1801.

Hansen, B. E. 1996. Inference When a Nuisance Parameter is Not Identified under the Null Hypothesis. *Econometrica* 64: 413–430.

Kass, R. E., and A. E. Raftery. 1995. Bayes Factor. *Journal of the American Statistical Association* 90: 773–795.

King, M., and S. Wadhwani. 1990. Transmission of Volatility between Stock Markets. *Review of Financial Studies* 3: 5–33.

Kodres, L. and M. Pritsker. 2002. A Rational Expectations Model of Financial Contagion. *Journal of Finance* 57: 769–799.

Lee, S. B., and K. J. Kim. 1993. Does the October 1987 Crash Strengthen the Co-Movements Among National Stock Markets. *Review of Financial Economics* 3: 89–102.

Longin, F., and B. Solnik. 1995. Is the Correlation in International Equity Returns Constant: 1960–1990? *Journal of International Money and Finance* 14: 3–26.

Longin, F., and B. Solnik. 2001. Extreme Correlations of International Equity Markets. *Journal of Finance* 56: 649–676.

Markwat, T., Kole, E., and D. van Dijk. 2009. Contagion as a Domino Effect in Global Stock Markets. *Journal of Banking and Finance* 33: 1996–2012.

Pelletier, D. 2006. Regime Switching for Dynamic Correlations. *Journal of Econometrics* 127: 445–473.

Pericoli, M., and M. Sbracia. 2003. A Primer on Financial Contagion. *Journal of Economic Surveys* 17: 571–608.

Pesaran, H., and Pick, A. 2007. Econometric Issues in the Analysis of Contagion. *Journal of Economic Dynamics and Control* 31: 1245–1277.

Pesaran, M. H., and A. Timmerman. 1995. Predictability of Stock Returns: Robustness and Economic Significance. *Journal of Finance* 50: 1201–1228.

Pesaran, M. H., and A. Timmerman. 2000. A Recursive Modeling Approach to Predict UK Stock Returns. *Economic Journal* 110: 159–191.

Rabemananjara, R., and J. M. Zakoian. 1993. Threshold ARCH Models and Asymmetries in Volatility. *Journal of Applied Econometrics* 8: 31–49.

Schinasi, G. J., and R. T. Smith. 2001. “Portfolio Diversification, Leverage and Financial Contagion.” In S. Claessens and K. J. Forbes (eds.), *International Financial Contagion*. Boston, MA: Kluwer Academic Press.

Silvennoinen, A., and T. Terasvirta 2005 “Multivariate Autoregressive Conditional Heteroskedasticity with Smooth Transitions in Conditional Correlations” SSE/EFI Working Paper Series in Economics and Finance No. 577.

Silvennoinen, A., and T. Terasvirta. 2009 Modelling Multivariate Autoregressive Conditional Heteroskedasticity with the Double Smooth Transition Conditional Correlation GARCH Model *Journal of Financial Econometrics* 7: 373–411.

Stambaugh, R. 1995. “Unpublished Discussion of Karolyi and Stulz (1996).” National Bureau of Economic Research Conference on Risk Management.

Tong, H. 1983. *Threshold Models in Non-Linear Time Series Analysis*. New York: Springer.

Tse, Y., and A. Tsui. 2002. A Multivariate GARCH Model with Time-Varying Correlations. *Journal of Business and Economic Statistics* 20: 351–362.

Zakoian, M. 1994. Threshold Heteroskedastic Models. *Journal of Economic Dynamics and Control* 18: 931–955.