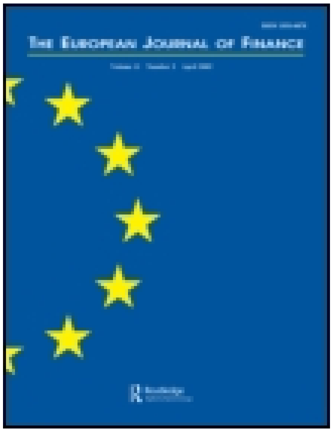

The European Journal of Finance
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rejf20

Option-based forecasts of volatility: an empirical study in the DAX-index options market

S. Muzzioli [a]

[a] Department of Economics and CEFIN , University of Modena and Reggio Emilia , Viale Berengario 51, 41100, Modena, Italy

Published online: 16 Apr 2010.

To cite this article: S. Muzzioli (2010) Option-based forecasts of volatility: an empirical study in the DAX-index options market, The European Journal of Finance, 16:6, 561-586, DOI: 10.1080/13518471003640134

To link to this article: http://dx.doi.org/10.1080/13518471003640134

The European Journal of Finance
Vol. 16, No. 6, September 2010, 561–586

Option-based forecasts of volatility: an empirical study in the DAX-index options market

S. Muzzioli*

Department of Economics and CEFIN, University of Modena and Reggio Emilia, Viale Berengario 51, 41100 Modena, Italy

Volatility estimation and forecasting are essential for both the pricing and the risk management of derivative securities. Volatility forecasting methods can be divided into option-based ones, which use prices of traded options in order to unlock volatility expectations, and time series volatility models, which use historical information in order to predict future volatility. Among option-based volatility forecasts, we distinguish between the 'model-dependent' Black–Scholes implied volatility and the 'model-free' implied volatility, proposed by Britten-Jones and Neuberger [Option prices, implied price processes and stochastic volatility. *Journal of Finance* 55: 839–66], that does not rely on a particular option pricing model. The aim of this paper is to investigate the unbiasedness and efficiency, with respect to past realised volatility, of the two option-based volatility forecasts. The comparison is pursued by using intra-daily data on the DAX-index options market. Our results suggest that Black–Scholes implied volatility subsumes all the information contained in past realised volatility and is a better predictor for future realised volatility than model-free implied volatility.

Keywords: Black–Scholes implied volatility; model-free implied volatility; volatility forecasting

JEL Classification: G13, G14

1. Introduction

Volatility estimation and forecasting are essential for both the pricing and the risk management of derivative securities. Various are the contributions aimed at assessing the best way to forecast volatility: we distinguish between option-based volatility forecasts, which use prices of traded options in order to unlock volatility expectations, and time series volatility models, which use historical information in order to predict future volatility.

Following Poon and Granger (2003), among time series volatility models, we find predictions based on past standard deviation, ARCH conditional volatility models and stochastic volatility models (for more details, see Section 9.2). Among option-based volatility forecasts, we have the Black–Scholes (B–S) implied volatility, which is a 'model-dependent' forecast since it relies on the Black and Scholes (1973) model, and the so-called 'model-free' implied volatility, proposed by Britten-Jones and Neuberger (2000), which does not rely on a particular option pricing model.

B–S implied volatility is usually extracted from a single option, by inverting the Black and Scholes formula, by means of a numerical method such as the bisection method. A drawback of using B–S implied volatility is clearly its dependence on the strike price of the option (the so-called smile effect), time to maturity of the option (term structure of volatility) and option

*Email: silvia.muzzioli@unimore.it

ISSN 1351-847X print/ISSN 1466-4364 online

DOI: 10.1080/13518471003640134
http://www.informaworld.com

type (call versus put). Nonetheless, at-the-money B–S implied volatility is usually considered as the market's expectation of future volatility between now and the expiration date of the option. Even if, from a theoretical point of view, there is no clear reason for that, since the Black and Scholes model postulates a constant volatility, from an empirical point of view, various papers have demonstrated the soundness of such a choice (see Section 2 for a literature review on the issue).

Model-free implied volatility is derived by using a cross section of option prices differing in strike price and option type. Therefore, theoretically, model-free implied volatility should be more informative than the implied volatility backed out from a single option (Carr and Wu 2006). Moreover, while the examination of the forecasting power of the B–S implied volatility is a joint test of model specification and market efficiency, model-free implied volatility, being independent of a particular option pricing model, provides a direct test of market efficiency.

However, some difficulties in the implementation of model-free implied volatility may undermine its theoretical advantages. In fact, the computation of model-free implied volatility requires a continuum of strike prices ranging from zero to infinity, condition that is not met in the reality of financial markets. Given the limited availability of strike prices, there is an ongoing debate (Jiang and Tian 2007) about the necessity of extending the strike price domain with respect to methods that 'cut the wings' (as it is done for the computation of the VDAX-new index).

Up to now, very few papers have dealt with the forecasting power of the different implementation techniques of model-free implied volatility. Moreover, the evidence in favour of the superiority of model-free implied volatility against B–S volatility is little and mixed (see Section 2 for a literature review on the issue).

The aim of this paper is twofold: to investigate the information content of B–S implied volatility and model-free implied volatility, and to compare the two option-based forecasts with historical volatility in order to see if they subsume all the information contained in the latter. As opposed to time series forecasts of volatility, which are based on historical information, the focus of the present paper is on the information content of a variable that is directly traded in the market: volatility that is bought and sold by dealers. The question we are going to answer is the following: is it better to use the prices of the most traded and thus most liquid at-the-money options, converted into a volatility measure via the B–S model, or is it more convenient to use the prices of the whole cross section of options of a given maturity, converted into a volatility measure via the 'model-free' variance definition that does not rely on any particular option pricing model? The comparison is performed by using intra-daily data on the DAX-index options market. The market is chosen for two main reasons. First, the options are European, and therefore the estimation of an early exercise premium is not needed and cannot influence the results; second, the DAX-index is a capital-weighted performance index composed of 30 major German stocks and is adjusted for dividends, stock splits and changes in capital: dividends are assumed to be reinvested in the shares and they do not affect the index value.

This paper makes at least three contributions to the ongoing debate on the performance of option-based volatility forecasts and their efficiency with respect to historical volatility. First, unlike previous studies, which use settlement prices, it uses a rich data set consisting of the more informative synchronous prices between the options and the underlying asset. This is important to stress, since our implied volatilities are real 'prices', as determined by synchronous no-arbitrage relations. Moreover, the choice of the DAX-index option market avoids the estimation of the early exercise premium and of the dividend yield and makes our data set less prone to measurement error.

Second, our methodology in order to compute option-based forecasts is more robust than the one used in previous studies and the results of the present paper are very important for an assessment of the best practice in order to compute model-free implied volatility. As for model-free implied volatility, we consider three different implementation techniques that vary in the extrapolation of the strike price domain. As for B–S implied volatility, we use a weighted average of implied volatilities backed out from different option classes.

Third, since the forecasting performance of model-free implied volatility has not been extensively tested and the few papers that address the issue provide mixed evidence on its superiority with respect to B–S implied volatility, our results can contribute to assess the relation among the various implementation techniques of both option-based forecasts and their relative forecasting performance.

The plan of the paper is as follows. Section 2 contains a brief literature review on the forecasting performance of option-based measures of volatility. Section 3 illustrates the theoretical concept of model-free implied volatility and highlights the main implementation issues. Section 4 presents the data set and the sampling procedure. Section 5 describes the computation of the volatility measures. Section 6 shows the methodology used in order to address the unbiasedness and efficiency of the different volatility forecasts. Section 7 reports the results of univariate and encompassing regressions. Sections 8 and 9 present some robustness tests on the computation of model-free implied volatility and on the methodology used in order to address the unbiasedness and efficiency of the volatility forecasts, respectively. The last Section concludes. The appendix discusses some implementation issues for model-free implied volatility.

2. Literature review on the information content of option-based measures of volatility

As recalled in the introduction, among option-based measures of volatility, we find the 'model-dependent' B–S implied volatility and the so-called 'model-free' implied volatility. While the information content of B–S implied volatility has been investigated in various papers (Poon and Granger (2003) in their survey examine 93 studies on the issue of volatility forecasting and conclude that predictions based on B–S implied volatility are on average superior to time series volatility models), the information content of model-free implied volatility has not been extensively tested yet.

From a theoretical point of view, Carr and Wu (2006) highlight that model-free implied volatility should be superior to B–S implied volatility. In fact, they show that at-the-money B–S implied volatility can be considered as a proxy for a volatility swap rate, while model-free implied volatility is a proxy for a variance swap rate. While the payoff on a volatility swap is difficult to replicate, the payoff of a variance swap rate is easily replicable by using a static position in a continuum of European options and a dynamic position in futures (for more details, see Carr and Wu (2006)). From an empirical point of view, the evidence in favour of the superiority of model-free implied volatility against B–S volatility is little and mixed.

As for B–S implied volatility, some early contributions find evidence that it is a biased and/or inefficient forecast of future realised volatility. Canina and Figlewski (1993) use a data set of daily closing prices of options on the S&P 100 from March 1983 to March 1987 and find a poor relationship between B–S implied and realised volatility. In the same market, Day and Lewis (1992) examine the predictive power of B–S implied volatility over a longer time period, from 1983 to 1989, and find that it is not better than standard time series models such as GARCH and EGARCH. Lamourex and Lastrapes (1993) examine the information content of B–S implied volatility extracted from options on ten stocks from 1982 to 1984 and find that implied volatility is biased and inefficient. Although the results of some of these studies (e.g. Day and Lewis 1992; Lamourex and Lastrapes 1993) are affected by overlapping samples, as recalled by Christensen, Hansen, and Prabhala (2001), or mismatching maturities between the option and the volatility forecast horizon, they constitute early evidence against the unbiasedness and information efficiency of implied volatility.

More recently, numerous papers analyse the empirical performance of B–S implied volatility in various option markets, ranging from indexes, futures, individual stocks or currencies and find that implied volatility is an unbiased and/or efficient forecast of future realised volatility. In the index options market, among others, Christensen and Prabhala (1998) examine the relation between B–S implied and realised volatility, using S&P100 options, over the 1983–1995 time period. They look for a possible regime shift around October 1987 and use non-overlapping samples and instrumental variables in order to account for possible errors in variables. They found that B–S implied volatility is a good predictor of future realised volatility. Christensen, Hansen, and Prabhala (2001) use options on the S&P 100 and non-overlapping samples and find evidence for the efficiency of B–S implied volatility as a predictor of future realised volatility. Yu, Liu, and Wang (2010) analyse the information content of implied volatility (both traded over the counter and on the official exchanges) of the Hang Seng index and the Nikkei-225 index from May 1998 to February 2005 and find that it is superior to either historical volatility or a GARCH-type volatility forecast.

In the futures options market, among others, Ederington and Guan (2002) analyse the S&P 500 futures options market and find that B–S implied volatility is an efficient forecast of future realised volatility. Szakmary et al. (2003) consider options on 35 different Futures contracts on a variety of asset class. They find that B–S implied volatility, while not a completely unbiased estimate of future realised volatility, is more informative than past realised volatility. Busch, Christensen, and Nielsen (2008) analyse futures contracts on three assets: \$/DM, S&P500 index, 30 year US T-bond. They show that implied volatility contains more information on the sample path and jump components of future volatility than high frequency return-based measures of volatility.

In the stock options market, among others, Godbey and Mahar (2007) analyse the information content of B–S implied call and put volatilities extracted from options on 460 stocks that compose the S&P500 index. They find that implied volatility contains some information on future realised volatility that is superior both to past realised volatility and to a GARCH(1,1) estimate. Moreover they highlight that the information content of implied volatility decreases as option volumes decrease.

In the currency markets, among others, Jorion (1995) uses data on options on various currency futures traded at the Chicago Mercantile Exchange and finds that B–S implied volatility is a biased but efficient predictor of future realised volatility. Pong et al. (2004) compare forecasts of realised volatility of the pound, mark, and yen exchange rates against the dollar and find that implied volatility is superior to historical forecasts only when the forecasting horizon is greater than one week.

As for model-free implied volatility, its empirical performance has not been extensively tested and the few studies provide mixed evidence. Some papers find that it is an unbiased and an efficient forecast of future realised volatility. Lynch and Panigirtzoglou (2003) analyse the predictive power of model-free implied volatility on four different markets: S&P500, FTSE100, Eurodollar and sterling futures and find that model-free implied volatility is a biased though efficient estimate of future volatility. Jiang and Tian (2005) investigate the predictive power of model-free implied volatility in the S&P500 options. They find that model-free implied volatility is an efficient and unbiased forecast (after a constant adjustment) of future realised volatility and subsumes all the information contained in B–S implied volatility. Bollerslev, Tauchen, and Zhou (2009) show that model-free implied volatility, proxied by the Chicago Board Options Exchange volatility index (VIX), is better than at-the-money B–S implied volatility in predicting future stock returns, when the variance risk premium is accounted for.

On the other hand, some papers find opposite results. Andersen and Bondarenko (2007) investigate the forecasting performance of model-free implied volatility in the S&P 500 futures market and find that it does not perform better than the simple B–S implied volatility. Taylor, Zhang, and Yadav (2006) analyse the information content of volatility using stock options for 149 US individual firms and find that overall there is less information in model-free implied volatility than in at-the money B–S implied volatility. Becker, Clements, and White (2007) examine whether the VIX index has any additional information relevant for forecasting volatility beyond that available in time series volatility forecasts and conclude that the VIX index does not contain any such information. Tsiaras (2009) examines the information content of alternative implied volatility measures for the 30 components of the DJIA index and concludes that it is impossible to distinguish between the information content of B–S and model-free implied volatilities.

3. Model-free implied volatility

Under mild conditions, Britten-Jones and Neuberger (2000) show how to derive the variance of the asset returns from a set of option prices. Suppose that the underlying asset S follows a diffusion process with time varying volatility, does not pay dividends and that the risk-free rate is zero. Suppose that a continuum of option prices $C(T, K)$ in strikes and maturities is available. The risk neutral expected sum of squared returns between two dates T_1 and T_2 is completely defined by a set of option prices expiring on the two dates:

$$E^Q\left[\int_{T_1}^{T_2}\left(\frac{\mathrm{d}S_t}{S_t}\right)^2\right] = 2\int_0^{\infty}\frac{C(T_2, K) - C(T_1, K)}{K^2}\,\mathrm{d}K, \tag{1}$$

where the expectation is taken under the risk neutral measure Q, S_t is the underlying asset and $C(T, K)$ is a call option with strike K that expires at time T. As the methodology does not rely on any particular assumption on the underlying stochastic process, Equation (1) is called a 'model-free' measure of variance.

The squared root of variance is the so-called model-free implied volatility σ:

$$\sigma = \sqrt{2\int_0^{\infty}\frac{C(T_2, K) - C(T_1, K)}{K^2}\,\mathrm{d}K}$$

Note that (Britten-Jones and Neuberger 2000) this introduces an upward bias in the volatility since

$$E^Q\left[\sqrt{\int_{T_1}^{T_2}\left(\frac{\mathrm{d}S_t}{S_t}\right)^2}\right] \leqslant \sqrt{2\int_0^{\infty}\frac{C(T_2, K) - C(T_1, K)}{K^2}\,\mathrm{d}K}$$

Jiang and Tian (2005) introduce several theoretical and practical modifications in order to compute model-free implied volatility. From a theoretical point of view, they relax the assumptions of no dividends and zero risk-free rate. By taking into account dividends and non-zero interest rates, Equation (1) becomes (see Jiang and Tian (2005) for the complete derivation)

$$E^Q\left[\int_{T_1}^{T_2}\left(\frac{\mathrm{d}S_t}{S_t}\right)^2\right] = 2\int_0^{\infty}\frac{C(T_2, Ke^{rT_2}) - C(T_1, Ke^{rT_1})}{K^2}\,\mathrm{d}K, \qquad (2)$$

where S_t is considered as the observed underlying price minus the expected value of the dividends.

Taking $T_1 = 0$ and $T_2 = T$, Equation (2) simplifies to

$$E^Q\left[\int_0^{T}\left(\frac{\mathrm{d}S_t}{S_t}\right)^2\right] = 2\int_0^{\infty}\frac{C(T, Ke^{rT}) - \max(S_0 - K, 0)}{K^2}\,\mathrm{d}K. \qquad (3)$$

Note that in this case, in order to compute model-free variance, only one set of options maturing at time T is needed.

As in the market, only options with a limited number of strike prices are traded, both truncation and discretisation errors occur. Truncation errors are faced since a limited range of strike prices $K \in [K_{\min}, K_{\max}]$ is used, instead of $K \in [0, \infty]$. Discretisation errors are due to the fact that only a finite number (instead of a continuum) of strike prices is available. In order to cope with these limits, Jiang and Tian (2005) propose the following approximation to Equation (3):

$$2\int_0^{\infty}\frac{C(T, Ke^{rT}) - \max(S_0 - K, 0)}{K^2}\,\mathrm{d}K \approx \sum_{i=1}^{m}\left[g(T, K_i) + g(T, K_{i-1})\right]\Delta K, \qquad (4)$$

where $\Delta K = (K_{\max} - K_{\min})/m$, m is the number of abscissas, $K_i = K_{\min} + i\Delta K$, $0 \leqslant i \leqslant m$, $g(T, K_i) = [C(T, K_i e^{rT}) - \max(S_0 - K_i, 0)]/K_i^2$ and the trapezoidal rule for numerical integration has been used.

In order to mitigate both truncation and discretisation errors, Jiang and Tian (2005) propose to extend the strike price domain and apply a curve-fitting method to interpolate implied volatilities between strike prices. The methodology develops into four steps. First, call and put prices are translated into implied volatilities by using the Black and Scholes formula (only out-of-the-money call and put prices are used). Second, a curve fitting method (cubic splines) is used in order to interpolate implied volatilities. Third, in order to extend the strike prices domain, constant volatility is supposed outside the available range of strike prices: for strikes below (above) the minimum (maximum) value, implied volatility is constant and equal to the volatility that corresponds to $K_{\min}$ ($K_{\max}$). Note that the latter assumption introduces a third source of approximation error that is different from both truncation and discretisation. Last, Black and Scholes formula is used in order to convert implied volatilities into call prices at the desired strike price frequency. Remark that the Black and Scholes formula is merely used in order to translate options prices into implied volatilities and vice versa; therefore it does not affect the 'model-free' attribute of volatility.

4. The data set and the sampling procedure

The data set[1] consists of intra-daily data on DAX-index options, recorded from 1 January 2001 to 31 December 2005. Each record reports the strike price, expiration month, transaction price, contract size, hour, minute, second and centisecond. As for the underlying asset, intra-daily prices of the DAX-index recorded in the same time period are used. As for the risk-free rate, the one-month Euribor rate is recorded in the same time period at a daily frequency. The Euribor rate used is the one that corresponds to the day in which the option is traded and it is assumed to remain constant over the lifetime of the option.

DAX-options started trading on the German Options and Futures Exchange (EUREX) in August 1991. They are European options on the DAX-index, which is a capital-weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks splits and changes in capital. DAX-index options are quoted in index points, carried out one decimal place. The contract value is EUR 5 per DAX-index point. The tick size is 0.1 of a point representing a value of EUR 0.50. Expiration months are the three near calendar months within the cycle March, June, September and December as well as the two following months of the cycle June and December. Dividends are assumed to be reinvested in the shares and they do not affect the index value. Moreover, the fact that the options are European avoids the estimation of the early exercise premium. This feature is important since our data set is by construction less prone to estimation error if compared with the majority of previous studies that use American-style options.

Several filters are applied to the option data set. First, we eliminate option prices that are less than one Euro, since the closeness to the tick size may affect the true option value. Second, in order not to use stale quotes, we eliminate options with trading volumes of less than one contract. Third, following Ait-Sahalia and Lo (1998), only at-the-money and out-of-the-money options are retained (call options with moneyness $(X/S) > 0.97$ and put options with moneyness $(X/S) < 1.03$). Fourth, option prices violating the standard no-arbitrage constraints are eliminated. Finally, in order to reduce computational burden, we only retain options that are traded between 3.00 and 4.00 p.m. (the choice is motivated by the level of trading activity in this interval).

As for the sampling procedure, in order to avoid the telescoping problem described in Christensen, Hansen, and Prabhala (2001), we use monthly non-overlapping samples. In particular, we collect the prices recorded on the Wednesday following the expiry of the option (third Saturday of the expiry month) because the week immediately following the expiry date is one of the most active. These options have a fixed maturity of almost one month (from 17 to 22 days to expiry). If the Wednesday is not a trading day, we move to the trading day immediately after.

5. The volatility measures

We compute four volatility measures: realised volatility (σ_{r}), historical volatility (σ_{h}), B–S implied volatility (σ_{BS}) and model-free implied volatility (σ_{mf}). Realised volatility is computed, in annual terms, as the squared root of the sum of 5 min frequency squared index returns over the lifetime of the option:

$$\sigma_{\mathrm{r}} = \sqrt{\sum_{t=1}^{n}\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right]^2} * 12,$$

where n is the number of index prices spaced by 5 min in the 1-month period. The choice of using 5 min frequency data is made following Andersen and Bollerslev (1998) and Andersen et al. (2001) who showed the importance of using high frequency returns in order to measure realised volatility and point out that returns at a frequency higher than 5 min are affected by serial correlation. Following previous studies (see e.g. Canina and Figlewski 1993; Christensen and Prabhala 1998), as a proxy for historical volatility (σ_{h}) we use lagged (one month before) realised volatility. The robustness of our results with respect to the use of other proxies for historical volatility is shown in Section 9.2.

As for the model-free implied volatility (σ_{mf}) computation, the following procedure, based on Jiang and Tian (2005), has been used. We start from the cleaned data set of option prices that is composed of at-the-money and out-of-the-money call and put prices recorded from 3.00 to

4.00 p.m. As a limited number of strike prices is available, we need to interpolate option prices in order to generate the missing prices. Due to the nonlinear relation between option prices and strike prices, we follow Shimko (1993) and Ait-Sahalia and Lo (1998) and perform a curve-fitting method to interpolate implied volatilities between strike prices, rather than option prices. We compute call and put implied volatilities by using synchronous prices, matched in a 1 min interval, by inverting the Black and Scholes formula. As we are using option prices that are traded in a 1 h interval, we obtain different implied volatilities for the same strike price, depending on the time of the trade. Therefore, in order to have a one-to-one mapping between strikes and implied volatilities, we compute the average of implied volatilities that correspond to the same strike price. With the aim of having a smooth function, following Bates (1991) and Campa, Chang, and Reider (1998), cubic splines are used to interpolate implied volatilities.

In order to extend the domain of strike prices, following Jiang and Tian (2005), we suppose constant volatility outside the available range of strikes: for strikes below (above) the minimum (maximum) value, implied volatility is equal to the volatility that corresponds to $K_{\min}$ ($K_{\max}$). The domain of strike prices is extended by using a factor u such that $S/(1+u) \leqslant K \leqslant S(1+u)$; for the current implementation, u has been chosen to be equal to 0.5. In order to have a sufficient discretisation of the integration domain, we compute strikes spaced by an interval $\Delta K = 10$. Finally, we use the Black and Scholes formula in order to convert implied volatilities into call prices. As we need a single value for the underlying asset, we take the average value of the underlying in the hour of trades.

In the appendix, we discuss the truncation (choice of u) and discretisation (choice of ΔK) errors of model-free implied volatility, and we show that the parameters u and ΔK chosen for the current implementation are likely to reduce both the truncation and the discretisation errors to a negligible value. In Section 8, we address the extrapolation error given by the artificial extension of the strike price domain outside the existing range, by deriving model-free implied volatility in two different ways: without extending the strike price domain and by using the methodology proposed in Jiang and Tian (2007).

In order to compute the B–S implied volatility (σ_{BS}), we use the following procedure. First we compute call and put implied volatilities, with the Black and Scholes formula, for the options closest to being at-the-money, i.e. with strikes one below and one above the underlying price, by using synchronous prices, matched in a 1 min interval. As we are using option prices that are traded in a 1 h interval, we compute the average implied volatility for each strike price. B–S implied volatility is defined as the weighted average of the two implied volatilities that correspond to the two strikes that are closest to being at-the-money, with weights inversely proportional to the distance to the moneyness (e.g. if the DAX-index is 5355 and the closest strikes are 5400 and 5350, the implied volatility of the 5400 strike will be weighted 5/50 against the implied volatility 5350 strike which is weighted 45/50). As we need a single value for the underlying asset, we take the average value of the underlying in the hour of trades.

We report descriptive statistics for volatility and log volatility series in Table 1. Figure 1 plots the three log-volatility series in our sample period. On average, realised volatility is lower and less volatile than both implied volatility estimates. Model-free implied volatility is, on average, higher and more volatile than B–S implied volatility. The volatility series are highly skewed (long right tail) and leptokurtic and the hypothesis of a normal distribution is strongly rejected. Option-based measures of volatility are on average higher than realised volatility, since they carry a substantial risk premium. The difference can also be attributed to hedging costs that are incurred in the option replication by trading in the underlying asset.

Table 1. Descriptive statistics.

Statistic	σ_{mf}	σ_{BS}	σ_r	$\ln(\sigma_{mf})$	$\ln(\sigma_{BS})$	$\ln(\sigma_r)$
Mean	0.290	0.265	0.236	−1.338	−1.431	−1.569
Std. dev.	0.139	0.132	0.125	0.441	0.451	0.497
Skewness	1.094	1.311	1.090	0.396	0.416	0.298
Kurtosis	3.201	4.189	3.305	2.327	2.522	2.206
Jarque Bera	11.660	20.040	11.710	2.613	2.224	2.379
p-Value	0.003	0.000	0.002	0.270	0.329	0.304

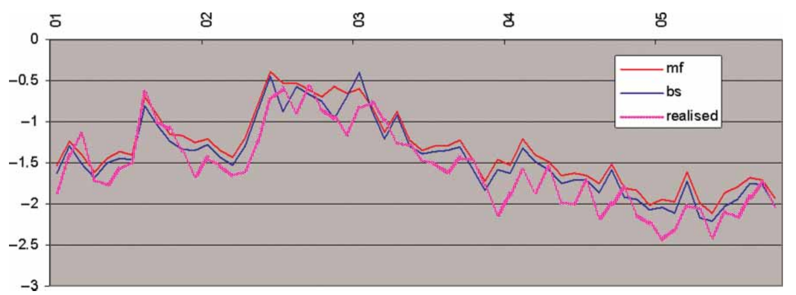

Figure 1. Time series plot of realised, B–S and model-free implied volatilities (in logs).

Option-based volatilities are more volatile than realised volatility; this is at odds with the interpretation of implied volatility as a smoothed expectation of realised volatility, in which case it should be less volatile than realised volatility. The puzzle can be explained by the more noisy observations in option-based volatilities. In fact, option-based volatilities are computed from market prices of options with almost one month to expiration: in order to have a continuous series, the records are taken from different option contracts, whereas realised volatility is taken as the variation in the returns of the underlying asset.

Model-free implied volatility is, on average, higher than B–S implied volatility: it contains the information content of out-of-the-money put options that convey a higher volatility than the one of at-the-money options. In fact, it is well known that the prices of out-of-the-money options are higher than the corresponding at-the-money ones, since they provide portfolio insurance. Moreover, model-free implied volatility is more volatile than B–S implied volatility since it embeds the extreme observations of option prices in the tails of the strike price domain.

As for the log-volatility series, we note that they have a slightly positive skewness (long right tail) and they are a little platykurtic; therefore the hypothesis of a normal distribution is not rejected. In the following empirical analysis, in line with the literature (Jiang and Tian 2005), we use the natural logarithms of the volatility series, since they conform more to normality.

6. The methodology

The information content of implied volatility is examined both in univariate and in encompassing regressions. Even if, from a theoretical point of view, it is good practice to start from the general encompassing regression and analyse in turn the nested regressions, in order to keep the outline of the analysis consistent with the related literature, we examine first the univariate regressions and second the more general encompassing regressions. In univariate regressions, realised volatility is regressed against one of the three volatility forecasts – B–S implied volatility (σ_{BS}), model-free implied volatility (σ_{mf}) or historical volatility (σ_{h}) – in order to examine the forecasting ability of each volatility forecast. The univariate regressions are the following:

$$\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_i) + \varepsilon, \tag{5}$$

where σ_{r} is the realised volatility and σ_{i} the volatility forecast, $i = h$, BS, mf.

In encompassing regressions, realised volatility is regressed against two or more volatility forecasts in order to distinguish which one has the highest explanatory power and to address whether a single volatility forecast subsumes all the information contained in the others. Therefore, we first compare pairwise one option-based forecast with historical volatility in order to address the efficiency of the forecasts. Second, we compare B–S implied volatility and model-free implied volatility, in order to see which one has the highest information content on future realised volatility. Third, realised volatility is regressed on the three volatility forecasts in order to see the relative importance of each forecast.

The encompassing regressions used are the following:

$$\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_{\mathrm{h}}) + \varepsilon, \tag{6}$$

where σ_{r} is the realised volatility, σ_i the implied volatility, $i =$ BS, mf and σ_{h} the historical volatility.

$$\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf}}) + \varepsilon, \tag{7}$$

where σ_{r} is the realised volatility, σ_{BS} the B–S implied volatility and σ_{mf} the model-free implied volatility.

$$\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf}}) + \delta \ln(\sigma_{\mathrm{h}}) + \varepsilon, \tag{8}$$

where σ_{r} is the realised volatility, σ_{BS} the B–S implied volatility and σ_{mf} the model-free implied volatility and σ_{h} the historical volatility.

Following Christensen and Prabhala (1998), in univariate regressions (5), there are three hypotheses to be tested. First, we test $H_0 : \beta = 0$: if the volatility forecast contains some information about future realised volatility, then the slope coefficient should be different from zero. Second, we test $H_0 : \alpha = 0$ and $\beta = 1$ in order to assess the unbiasedness of the volatility forecast. In case this hypothesis is rejected, we see if at least the slope coefficient is equal to one ($H_0 : \beta = 1$) and, if accepted, we interpret the volatility forecast as unbiased after a constant adjustment. Finally, if implied volatility is efficient, then the error term should be white noise and uncorrelated with the information set.

In encompassing regressions (6), there are two hypotheses to be tested. The first one is $H_0 : \gamma = 0$, i.e. whether one of the two option-based forecasts subsumes all the information contained in historical volatility. The second is a joint test of information content and efficiency: we test in Equation (6) if the slope coefficients of historical volatility and of the option-based forecast are equal to zero and one respectively ($H_0 : \gamma = 0$ and $\beta = 1$). Following Jiang and Tian (2005), we ignore the intercept in the null hypothesis, and if our null hypothesis is not rejected, we interpret the volatility forecast as unbiased after a constant adjustment.

In encompassing regression (7), we test if $\gamma = 0$ and $\beta = 1$, in order to see if B–S implied volatility subsumes all the information contained in model-free implied volatility. Finally in encompassing regression (8) we test if B–S implied volatility subsumes all the information contained in both model-free implied volatility and historical volatility, i.e. the joint null hypothesis $H_0 : \delta = 0$, $\gamma = 0$ and $\beta = 1$.

Christensen and Prabhala (1998) compared the information content of B–S implied volatility with historical volatility in the S&P100 index options market. They run both ordinary least squares (OLS) regressions and errors-in-variables (EIV) regressions in order to correct for potential errors in variables due to the early exercise feature of the options and the dividend yield estimation and found different results in OLS and EIV regressions. As our dataset consists of prices of options on the DAX-index that are European-style and are written on a non-dividend paying index, we avoid measurement errors that may arise in the estimation of the dividend yield and the early exercise premium. Moreover, we carefully cleaned the dataset by applying rigorous filtering constraints detailed in Section 4 and we use synchronous prices for the index and the option that are matched in a 1 min window. Therefore, we expect our data set to be less prone to measurement errors than that of Christensen and Prabhala (1998).[2]

The regressions run in Christensen and Prabhala (1998) and used in the present paper have been criticised by Christensen and Nielsen (2006) and Bandi and Perron (2006) for being fractional cointegrating relations. In that setting, the regressions cannot be interpreted as tests of option market efficiency, since cointegration is associated with long-run comovements. Moreover, OLS is generally inconsistent for beta and more involved methods such as the narrow band least squares should be used (see Christensen and Nielsen (2006) for the necessary asymptotic distribution theory for inference on the beta coefficient).

7. The results

The results of the OLS univariate and encompassing regressions are reported in Table 2 (standard errors in parentheses). In all the regressions, the residuals are not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch–Godfrey LM test confirms non-autocorrelation up to lag 12[3]), although they are not normal.[4] Moreover, as there is no significant evidence of heteroscedasticity in all the regressions' residuals, the OLS estimators have the minimum variance among the class of unbiased estimators.

Some comments are in order. First, in all the three univariate regressions, the beta coefficients are significantly different from zero: this means that all the volatility forecasts (B–S, model-free and historical volatility) contain some information about future realised volatility. However, the null hypothesis that any of the three volatility forecasts is an unbiased estimate of future realised volatility is strongly rejected in all cases. In particular, in our sample, realised volatility is, on average, lower than the two option-based volatility forecasts, suggesting that option-based forecasts over predict realised volatility. This is in line with the results in Jiang and Tian (2005) and Lynch and Panigirtzoglou (2003), who document a positive risk premium for stochastic volatility.

The average risk premium (defined in this setting, as the difference between implied and subsequent realised volatility, using the logs of the two series) is 0.23 (with a standard deviation of 0.21) for model-free implied volatility and 0.14 (with a standard deviation of 0.20) for B–S implied volatility. Therefore, both implied volatility measures overwhelmingly overpredict future realised volatility, the error being much higher and a little more volatile for model-free implied volatility. The positive risk premium can be explained by the fact that agents are risk averse (Jiang and Tian 2005). Also, in Bollerslev, Tauchen, and Zhou (2009), it has been argued that the difference between the model-free risk neutral expected variance and the actual variance is a proxy for the aggregate degree of risk aversion of the market.

Moreover, even if there is no evidence of time trend in the sample, since volatility at the beginning and at the end of the sample is almost at the same level, from mid-2004 onwards we can see that realised volatility remains substantially low, consistently with the findings of

Table 2. OLS regressions.

Dependent variable: log realised volatility									
Independent variables									
Intercept	$\ln(\sigma_{\mathrm{BS}})$	$\ln(\sigma_{\mathrm{mf}})$	$\ln(\sigma_{\mathrm{h}})$	Adj. R^2	DW	χ^{2a}	χ^{2b}	χ^{2c}	Hausman test
−0.13 (0.07)	1.01^{+++} (0.05)			0.83	1.89	26.91 (0.00)			0.572
−0.20 (0.07)		1.02^{+++} (0.05)		0.82	1.97	69.04 (0.00)			0.121
−0.24 (0.10)			0.85^{+++***} (0.06)	0.69	1.96	6.71 (0.04)			
−0.13 (0.07)	1.01^{+++} (0.12)		−0.01 (0.10)	0.83	1.89		0.02 (0.99)		0.085
−0.21 (0.07)		1.05^{+++} (0.16)	−0.03 (0.13)	0.82	1.96		0.17 (0.92)		0.002
−0.1 (0.07)	0.83^{+++} (0.30)	0.2 (0.30)		0.83	1.91		0.40 (0.82)		0.047
−0.14 (0.07)	0.83^{+++} (0.31)	0.21 (0.37)	−0.03 (0.12)	0.83	1.90			0.38 (0.94)	0.062

Notes: The numbers in brackets are the standard errors. The χ^{2a} reports the statistic of a χ^2 test for the joint null hypothesis $\alpha = 0$ and $\beta = 1$ (p-values in parentheses) in the following univariate regressions: $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_i) + \varepsilon$, where σ_{r} is the realised volatility and σ_i the volatility forecast, $i = h$, BS, mf. The χ^{2b} reports the statistic of a χ^2 test for the joint null hypothesis $\gamma = 0$ and $\beta = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_{\mathrm{h}}) + \varepsilon$, where σ_{r} is the realised volatility, σ_i the implied volatility, $i =$ BS, mf and σ_{h} the historical volatility and $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf}}) + \varepsilon$, where σ_{r} is the realised volatility, σ_{BS} the B–S implied volatility and σ_{mf} the model-free implied volatility. The χ^{2c} reports the statistic of a χ^2 test for the joint null hypothesis $\delta = 0$, $\gamma = 0$ and $\beta = 1$ (p-values in parentheses) in the following encompassing regression: $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf}}) + \delta \ln(\sigma_{\mathrm{h}}) + \varepsilon$, where σ_{r} is the realised volatility, σ_{BS} the B–S implied volatility and σ_{mf} the model-free implied volatility and σ_{h} the historical volatility. The last column reports the Hausman (1978) specification test statistic (one degree of freedom) 5% critical level = 3.841.

*Slope coefficient is significantly different from 1 at the 10%.

**Slope coefficient is significantly different from 1 at the 5%.

***Slope coefficient is significantly different from 1 at the 1%.

+Slope coefficient is significantly different from 0 at the 10%.

++Slope coefficient is significantly different from 0 at the 5%.

+++Slope coefficient is significantly different from 0 at the 1%.

Panetta et al. (2006). As they state, the decrease in realised volatility can be explained by the phase of expansion and low inflation experienced in those years. Also firm specific components, such as the increase in firm's profitability and the reduction in leverage and uncertainty determined the reduction in volatility. For what concerns implied volatility, some financial factors such as the improvement in market liquidity and the increase in the supply of options from institutional investors (hedge funds, investment banks and pension funds), a consequence of the reduction in risk aversion documented in that period (Panetta et al. 2006), can have pushed down option prices and in turn implied volatilities. However, the reduction caused by financial factors was lower than that caused by real factors, since implied volatility remains higher than realised volatility also in that period. This can also explain the imperfect fit in the regressions.

As neither one of the forecasts is unbiased, we test if β is significantly different from one. The hypothesis cannot be rejected at the 10% critical level for the two option-based forecasts, while it is strongly rejected for historical volatility. We can therefore consider both option-based forecasts as unbiased after a constant adjustment given by the intercept of the regression. As for the adjusted R^2, B–S implied volatility is ranked first in explaining future realised volatility, strictly followed by model-free implied volatility, whereas historical volatility has the lowest forecasting power.

Let us turn to the analysis of the encompassing regressions. First of all, we can observe that both option-based forecasts subsume all the information contained in historical volatility. This is evident by comparing the adjusted R^2 of univariate and encompassing regressions: the inclusion of historical volatility does not improve the goodness of fit. Moreover, the coefficient of historical volatility is not significant in both regressions. Second, both option-based volatility forecasts are efficient and unbiased after a constant adjustment given by the intercept of the regression. In fact, the slope coefficients of both option-based volatility forecasts are not significantly different from one, at the 10% level, and the joint test of information content and efficiency $\gamma = 0$ and $\beta = 1$ does not reject the null hypothesis.

In order to see if B–S implied volatility subsumes all the information contained in model-free implied volatility, we test in encompassing regression (7) if $\gamma = 0$ and $\beta = 1$. First of all, we observe that only the slope coefficient of B–S implied volatility is significantly different from zero, while the slope coefficient of model-free implied volatility is not. Moreover, the joint test $\gamma = 0$ and $\beta = 1$ does not reject the null hypothesis, providing evidence for the superiority of B–S implied volatility with respect to model-free implied volatility. As an additional test, we compare the predictive accuracy of B–S implied volatility with respect to model-free implied volatility, by computing the Diebold and Mariano test statistic (for more details, see Diebold and Mariano (1995)). The loss function chosen is the mean squared error. The Diebold and Mariano test statistic is 5.93: under the null of equal predictive accuracy it is distributed as a $N(0,1)$; therefore we can strongly reject the null hypothesis.

For completeness, let us analyse the results of encompassing regression (8) in which we compare all the three volatility forecasts. First of all, the inclusion of both model-free implied volatility and historical volatility does not improve the goodness of fit given by the adjusted R^2. In fact, both the coefficients of historical volatility and model-free implied volatility are not significantly different from zero. Moreover, also in this case, B–S implied volatility is both efficient and unbiased after a constant adjustment, as it is evident by looking at the χ^{2c} column that jointly tests if $\delta = 0$, $\gamma = 0$ and $\beta = 1$ and does not reject the null hypothesis. The results are robust with respect to an EIV procedure.[5]

The comparison between the two option-based volatility forecasts has highlighted that B–S implied volatility subsumes all the information contained in model-free implied volatility. Model-free implied volatility, as a risk neutral expectation of future realised volatility, carries several advantages: first, it is independent of any option pricing model, and therefore it is not subject to model specification error; second, it extracts and aggregates information across all option prices instead of using a single implied volatility or an ad hoc weighting scheme to combine implied volatilities of options with different strikes. However, as the model-free implied volatility definition hinges on the existence of a continuum of strike prices, condition that is not met in the reality of financial markets, its practical implementation is questioned. In particular, the lack of liquid options in the tails of the risk neutral distribution and the extrapolation procedure that is necessary in order to extend the strike price domain are the main causes of the slightly worse performance of model-free implied volatility. In particular, the better performance of B–S implied volatility, with respect to previous papers, can be attributed to the fact that it has been computed by using synchronous prices, from a vast set of options in a 1 h window.

Moreover, in our sample, both option-based measures carry a substantial risk premium, that is more pronounced in model-free implied volatility than in B–S implied volatility. As the market volatility risk premium is likely to fluctuate over time, the better performance of B–S implied volatility can also be attributed to the fact that it is less sensitive to the latter. A similar result was found in Andersen and Bondarenko (2007), who analyse the forecasting performance of different corridor implied volatility measures spanning from the narrow B–S implied volatility to the wide model-free implied volatility. They find that model-free implied volatility perform worse than less premium sensitive measures, such as B–S implied volatility and other narrower corridor implied volatility measures that rely on different truncations of the strike price domain.

8. Robustness tests on model-free implied volatility computation

The extrapolation method of Jiang and Tian (2005) is based on the extension of the strike price domain, by supposing that for strikes below (above) the minimum (maximum) value, implied volatility is constant and equal to the volatility of $K_{\min}$ ($K_{\max}$). By examining different option maturities and different strike price ranges of availability, Jiang and Tian (2005) show that in general their methodology is better than truncating the strike price domain to the available one (even if, for 1-month maturity options, the results are mixed and seem in favour of the truncation). Moreover, as the flat extrapolation scheme may underestimate volatility in the tails, Jiang and Tian (2007) suggest extrapolating volatilities outside the listed strike price range by using a linear function that matches the slope of the smile function at $K_{\min}$ and $K_{\max}$. This methodology has the advantage that the smile function remains smooth at $K_{\min}$ and $K_{\max}$.

Therefore, in this section, we compute model-free implied volatility in two different ways: first, without extending the domain of strike prices (σ_{mf2}); second, by using the methodology proposed in Jiang and Tian (2007) (σ_{mf3}). As this latter methodology of extending the strike price domain by a segment that matches the slope of the smile function at $K_{\min}$ and $K_{\max}$ may generate implied volatilities that are artificially too high (in case the slope is positive) or too low (in case the slope is negative), we have imposed both a lower and an upper bound to implied volatilities equal to 0.001 and 0.999, respectively. Descriptive statistics are reported in Table 3. We can see that on average σ_{mf2} (σ_{mf3}) is greater than σ_{BS} and lower (greater) than σ_{mf}; therefore, it contains a lower (higher) volatility risk premium than σ_{mf}. Given that the natural logarithm of the series conforms more to normality, we use the latter in the regressions.

We run both univariate regression (5) and encompassing regressions (6) and (7) in order to investigate the performance of the two different model-free implied volatility estimates. The results are reported in Table 4. We can see that both the alternative model-free implied volatility computation methodologies obtain a worse performance than model-free implied volatility

Table 3. Descriptive statistics for model-free volatility computed either without extending the strike price domain (σ_{mf2}) or with the methodology proposed in Jiang and Tian (2007) (σ_{mf3}).

Statistic	σ_{mf2}	σ_{mf3}	$\ln(\sigma_{mf2})$	$\ln(\sigma_{mf3})$
Mean	0.279	0.296	−1.370	−1.313
Std. dev.	0.127	0.141	0.427	0.432
Skewness	1.075	1.188	0.309	0.473
Kurtosis	3.444	3.479	2.345	2.44
Jarque Bera	11.640	14.20	1.960	2.923
p-Value	0.002	0.000	0.375	0.231

computed by extending the strike price domain as suggested by Jiang and Tian (2005). Although both model-free implied volatilities remain still efficient and unbiased forecasts of future realised volatility, they obtain an adjusted R^2 that is inferior to the one of the original model-free implied volatility estimate. Moreover, also in this case, B–S implied volatility subsumes all the information contained in both the alternative model-free implied volatility estimates.

Therefore, we conclude that, in this data set, the Jiang and Tian (2005) extrapolation method, which assumes a flat smile outside the range of traded strike prices, is superior both to a methodology that uses only the existing strike price domain and to the methodology of extending the strike price domain by using a function that matches the slope of the smile at the bounds of the existing strike prices.

9. Further tests for robustness

In the following, we discuss some additional checks aimed at assessing the robustness of our findings across different regression specifications; different proxies used for historical volatility and different performance evaluation measures. The possibility of a relation between the forecasting ability of option-based measures and the level (high or low) of realised volatility is also explored.

9.1 *Forecasting ability under different regression specifications*

The regressions discussed so far have all being run in the logarithmic form. This is standard in the literature and brings several advantages. First, the logarithmic specification is less likely to be affected by outliers. Second, log-series conform more to the normality assumption, making the inference based on asymptotic more reliable. Moreover, using the logs lifts the difference between volatility or variance since $\ln(\sigma^2) = 2\ln(\sigma)$.

However, there is a concavity issue that arises since if $E[\ln(\sigma_r)] = E[\ln(\sigma_{BS})]$, then this does not imply that $E[(\sigma_r)] = E[(\sigma_{BS})]$ or that $E[(\sigma_r^2)] = E[(\sigma_{BS}^2)]$; therefore it is very likely that the unbiasedness results may not agree with each other, depending on the regression specification. To this end, we have replicated the analysis by using two different regression specifications: volatility and variance. In order to save space, we do not include any additional table, and the results are available on request.

The main findings are independent of the regression specification. First, both in the volatility and in the variance specifications, B–S implied volatility remains the best forecast, according to the adjusted R^2 measure (which is a little lower if compared with the one in the logarithmic specification), closely followed by model-free implied volatility (σ_{mf}) computed following

Table 4. OLS regressions for σ_{mf2} and σ_{mf3}.

Dependent variable: log realised volatility									
Independent variables									
Intercept	$\ln(\sigma_{\mathrm{BS}})$	$\ln(\sigma_{\mathrm{mf2}})$	$\ln(\sigma_{\mathrm{mf3}})$	$\ln(\sigma_{\mathrm{h}})$	Adj. R^2	DW	χ^{2a}	χ^{2b}	χ^{2c}
−0.15 (0.10)		1.03^{+++} (0.07)			0.79	2.09	46.63 (0.00)		
−0.22 (0.09)			1.03^{+++} (0.07)		0.79	1.91	77.11 (0.00)		
−0.12 (0.07)	0.94^{+++} (0.16)	0.08 (0.16)			0.83	1.896		0.25 (0.88)	
−0.13 (0.07)	0.99^{+++} (0.25)		0.01 (0.25)		0.83	1.89		0.02 (0.98)	
−0.15 (0.10)		0.95^{+++} (0.17)		0.08 (0.15)	0.78	2.1		0.54 (0.76)	
−0.21 (0.08)			0.95^{+++} (0.20)	0.07 (0.15)	0.79	1.95		0.94 (0.62)	
−0.12 (0.08)	0.98^{+++} (0.24)	0.15 (0.23)	−0.09 (0.31)	−0.03 (0.13)	0.82	1.88			0.41 (0.98)

Notes: The numbers in brackets are the standard errors. σ_{r} is the realised volatility, σ_{mfi} the model-free volatility computed either without extending the strike price domain ($i = 2$) or with the methodology proposed in Jiang and Tian (2007) ($i = 3$), σ_{BS} the B–S implied volatility and σ_{h} the historical volatility. The χ^{2a} report the statistic of a χ^2 test for the joint null hypothesis $\alpha = 0$ and $\beta = 1$ (p-values in parentheses) in the following univariate regression: $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{mf}i}) + \varepsilon$. The χ^{2b} report the statistic of a χ^2 test for the joint null hypothesis $\gamma = 0$ and $\beta = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{mf}i}) + \gamma \ln(\sigma_{\mathrm{h}}) + \varepsilon$, $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf}i}) + \varepsilon$. The χ^{2c} report the statistic of a χ^2 test in the following encompassing regression: $\ln(\sigma_{\mathrm{r}}) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf2}}) + \delta \ln(\sigma_{\mathrm{mf3}}) + \theta \ln(\sigma_{\mathrm{h}}) + \varepsilon$ for the joint null hypothesis $\delta = 0$, $\gamma = 0$, $\theta = 0$ and $\beta = 1$ (p-values in parentheses).

$^{+}$Slope coefficient is significantly different from zero at the 10%.

$^{++}$Slope coefficient is significantly different from zero at the 5%.

$^{+++}$Slope coefficient is significantly different from zero at the 1%.

the methodology used in Jiang and Tian (2005). Model-free implied volatility computed without extending the strike price domain (σ_{mf2}) obtains the worst performance among option-based volatility forecasts. The overall worst performance is recorded by lagged realised volatility. Second, both in the volatility and in the variance specifications, B–S implied volatility, along with the other option-based forecasts, is efficient, since it subsumes all the information contained in historical volatility.

However, as expected, the unbiasedness results for B–S implied volatility do not hold anymore in the other regression specifications. This is in line with the results of Jiang and Tian (2005): although they prefer a different volatility forecast (in their paper model-free implied volatility is superior to B–S implied volatility), they also find that the best volatility forecast is unbiased after a constant adjustment only in the logarithmic specification and not in the volatility or variance specification.

Given that the aim of the paper is not to have a precise estimate of volatility in order to plug it into an option pricing formula, but rather it is to see if option prices are informative about future realised volatility and to determine which one of the two option-based forecasts (B–S and model-free) is better in predicting future realised volatility, we rely on the logarithmic specification results, since they are the most sound from the econometric point of view.

9.2 *Efficiency of the forecasts with a different proxy for historical volatility*

In order to examine the predictive power of implied volatility versus historical volatility and to assess the efficiency of the option-based forecasts, lagged realised volatility has been used throughout the paper as a proxy for historical volatility. As recalled in the introduction, various are the time series measures proposed for modelling volatility. We distinguish predictions based on past standard deviations, ARCH class conditional volatility models and stochastic volatility models. Among predictions based on past standard deviation, we have the simple random walk hypothesis in which the best estimate of future realised volatility is today volatility; methods based on averages, such as historical averages, moving averages and exponential smoothing moving averages, that try to solve the trade off between having as much observations as possible and sampling close to the present time; simple regression models that regress volatility on its past values. ARCH family conditional volatility models (see Bollerslev, Chou, and Kroner (1992) for a survey) formulate conditional variance as a function of past squared returns via maximum likelihood. ARCH models have the advantage that the next step forecast is available by their very same construction. In Stochastic volatility models (see Ghysels, Harvey, and Renault (1996) for a survey), volatility is driven by a different source of uncertainty from the one of the underlying asset price. Stochastic volatility models are very flexible, but difficult to implement, since they usually have no closed form solution.

In this section, we test the robustness of our results by using different proxies for historical volatility. Given the difficulties in implementing stochastic volatility models, we chose to use two different proxies for historical volatility, which belong to the other two classes of time series volatility forecasts. Among predictions based on past standard deviations, we use a simple AR(1) model. As $\ln(\sigma_{\mathrm{r}})$ has an autoregressive parameter of 0.85 and a constant of -0.24 (Table 2), the historical forecast based on an AR(1) model ($\ln(\sigma_{\mathrm{ar}})$) in month t is defined as

$$\ln(\sigma_{\mathrm{ar},t}) = -0.24 + 0.8\ln(\sigma_{\mathrm{r},t-1}),$$

where $\ln(\sigma_{\mathrm{r},t-1})$ is realised volatility in month $t-1$.

Table 5. Descriptive statistics for the AR(1) and GARCH forecasts.

Statistic	σ_{ar}	$\ln(\sigma_{\text{ar}})$	σ_{gar}	$\ln(\sigma_{\text{gar}})$
Mean	0.227	−1.568	0.250	−1.486
Std. dev.	0.101	0.416	0.123	0.436
Skewness	0.996	0.319	1.252	0.606
Kurtosis	3.099	2.245	3.606	2.376
Jarque Bera	9.605	2.364	16.052	4.490
p-Value	0.008	0.306	0.000	0.105

Among ARCH class conditional volatility models, we chose to use a GARCH(1,1) forecast (σ_{gar}), since it is a parsimonious representation that is widely used in the literature. The GARCH(1,1) variance equation is defined as $\sigma^2_{t+1} = a_0 + a_1 R^2_t + b_1 \sigma^2_t$, where R_t is the demeaned DAX-index return on day t (for more details see Bollerslev (1986)). As in Jorion (1995), the GARCH model has been estimated via maximum likelihood over the entire data set. The estimated parameter values are $a_0 = 0.00000139$, $a_1 = 0.088026$, $b_1 = 0.907829$ and they are all highly significant. Following Fleming (1998), the GARCH forecast (σ_{gar}) of the average volatility over the life of the option is defined as

$$\sigma_{\text{gar}} = \sqrt{\frac{1}{T - t - 1} \sum_{j=1}^{T-t} \tilde{\sigma}^2_{t+j|t}},$$

where $\tilde{\sigma}^2_{t+j|t}$ is the forecast at time t of the variance j days into the future, and T is the maturity of the option. We annualise the standard deviation by multiplying it by $\sqrt{252}$. It is important to stress that the GARCH forecast, being estimated over the entire sample period, benefits from information that is not available to other forecasts.

Summary statistics are reported in Table 5. σ_{ar} (σ_{gar}) is, on average, lower (higher) and less volatile than realised volatility. Volatility (log-volatility) series of both AR(1) and GARCH forecasts display positive skewness and are leptokurtic (platykurtic). Given that the natural logarithm of the series conforms more to normality, we use the latter in the regressions.

We run both univariate regression (5) and encompassing regressions (6) and (8) with σ_{h} proxied by both the AR(1) forecast (σ_{ar}) and the GARCH forecast (σ_{gar}). The results are reported in Table 6. We can see that both AR(1) and GARCH forecasts behave significantly better than lagged realised volatility: indeed, AR(1) is unbiased and GARCH is unbiased after a constant adjustment.

However, according to the adjusted R^2, both time series forecasts obtain a worse performance than option-based volatility measures. B–S and model-free implied volatilities remain efficient with respect to both time series volatility forecasts. Moreover B–S implied volatility is still superior to model-free implied volatility since in encompassing regression (8), it subsumes all the information contained in model-free implied volatility and in the time series forecast (either AR(1) or GARCH).

9.3 *The relation between the forecasting performance of option-based measures and the volatility level*

In order to investigate how the information content of option prices changes with the volatility level, we have split the sample into three sub-samples, characterised by high, medium and low

Table 6. OLS regressions for the AR(1) and GARCH forecasts.

Dependent variable: log realised volatility									
Independent variables									
Intercept	$\ln(\sigma_{\mathrm{BS}})$	$\ln(\sigma_{\mathrm{mf}})$	$\ln(\sigma_{\mathrm{ar}})$	$\ln(\sigma_{\mathrm{gar}})$	Adj. R^2	DW	χ^{2a}	χ^{2b}	χ^{2c}
0.00 (0.11)			0.99^{+++} (0.07)		0.70	1.96	0.00 (0.99)		
−0.09 (0.10)				0.99^{+++} (0.06)	0.76	2.02	6.69 (0.04)		
−0.13 (0.07)	1.01^{+++} (0.12)		−0.01 (0.12)		0.83	1.88		0.02 (0.99)	
−0.22 (0.08)		1.05^{+++} (0.16)	−0.03 (0.15)		0.82	1.96		0.17 (0.92)	
−0.15 (0.09)	0.83^{+++} (0.31)	0.21 (0.37)	−0.04 (0.14)		0.83	1.89			0.38 (0.94)
−0.10 (0.06)	0.87^{+++} (0.14)			0.14 (0.12)	0.83	1.96		1.95 (0.38)	
−0.17 (0.06)		0.88^{+++} (0.18)		0.14 (0.16)	0.82	2.04		1.80 (0.41)	
−0.11 (0.07)	0.82^{+++} (0.28)	0.06 (0.32)		0.13 (0.14)	0.83	1.97			2.07 (0.56)

Notes: The numbers in brackets are the standard errors. The χ^{2a} reports the statistic of a χ^2 test for the joint null hypothesis $\alpha = 0$ and $\beta = 1$ (p-values in parentheses) in the following univariate regression: $\ln(\sigma_r) = \alpha + \beta \ln(\sigma_j) + \varepsilon$, where σ_r is the realised volatility and σ_j the volatility forecast, $j =$ ar, gar. The χ^{2b} reports the statistic of a χ^2 test for the joint null hypothesis $\gamma = 0$ and $\beta = 1$ (p-values in parentheses) in the following encompassing regressions: $\ln(\sigma_r) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_j) + \varepsilon$, where σ_r is the realised volatility, σ_i the implied volatility, $i =$ BS, mf and σ_j the volatility forecast, $j =$ ar, gar. The χ^{2c} reports the statistic of a χ^2 test for the joint null hypothesis $\delta = 0$, $\gamma = 0$ and $\beta = 1$ (p-values in parentheses) in the following encompassing regression: $\ln(\sigma_r) = \alpha + \beta \ln(\sigma_{\mathrm{BS}}) + \gamma \ln(\sigma_{\mathrm{mf}}) + \delta \ln(\sigma_j) + \varepsilon$, where σ_r is the realised volatility, σ_{BS} the B–S implied volatility, σ_{mf} the model-free implied volatility and σ_j the volatility forecast, $j =$ ar, gar.

$^{+}$Slope coefficient is significantly different from zero at the 10%.

$^{++}$Slope coefficient is significantly different from zero at the 5%.

$^{+++}$Slope coefficient is significantly different from zero at the 1%.

volatility, respectively. As in Andersen and Bondarenko (2007), the three sub-samples of approximately equal length (19 observations for low and high volatility; 20 observations for medium volatility) are obtained by sorting the logarithm of B–S implied volatility in ascending order. Given the limited number of observations, in order to gauge the forecasting performance, we do not resort to regression-based methods, but to popular evaluation measures. As indicators of the goodness of fit, we use several performance measures: the MSE, the RMSE, the MAE, the MAPE and the MISP, which are defined as follows (for consistency with previous analysis, we have used the logs):

$$\text{MSE} = \frac{1}{m}\sum_{i=1}^{m}(\ln(\sigma_i) - \ln(\sigma_{\text{r}}))^2$$

$$\text{RMSE} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\ln(\sigma_i) - \ln(\sigma_{\text{r}}))^2}$$

$$\text{MAE} = \frac{1}{m}\sum_{i=1}^{m}|\ln(\sigma_i) - \ln(\sigma_{\text{r}})|$$

$$\text{MAPE} = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{\ln(\sigma_i) - \ln(\sigma_{\text{r}})}{\ln(\sigma_{\text{r}})}\right|$$

$$\text{MISP} = \frac{\sum_{i=1}^{m}(\ln(\sigma_i) - \ln(\sigma_{\text{r}}))/\ln(\sigma_{\text{r}})}{\sum_{i=1}^{m}|(\ln(\sigma_i) - \ln(\sigma_{\text{r}}))/\ln(\sigma_{\text{r}})|},$$

where $\ln(\sigma_i)$ is the log of the volatility forecast (i = B–S, mf), $\ln(\sigma_{\text{r}})$ is the log of subsequent realised volatility and m is the number of observations.

The MSE, the RMSE and the MAE are indicators of absolute errors, while the MAPE indicates the percentage error. The MISP is a miss-prediction indicator that ranges from -1 to 1 and indicates if the forecast has understated (index close to 1) or overstated (index close to -1) future realised volatility.

The results are reported in Table 7, both for the entire sample and for the three sub-samples. The results for the entire sample bring further evidence about the superiority of B–S implied volatility and corroborate the results of regression-based performance evaluation, presented in Section 7. In addition, B–S implied volatility is superior to model-free implied volatility in each sub-sample, according to all the indicators. Overall, the MISP indicates that both implied volatility measures

Table 7. Predictive accuracy of B–S and model-free implied volatilities for the whole sample and across sub-samples characterised by low/medium/high volatility.

	All sample		Low volatility		Medium volatility		High volatility	
	$\ln(\sigma_{\text{mf}})$	$\ln(\sigma_{\text{BS}})$	$\ln(\sigma_{\text{mf}})$	$\ln(\sigma_{\text{BS}})$	$\ln(\sigma_{\text{mf}})$	$\ln(\sigma_{\text{BS}})$	$\ln(\sigma_{\text{mf}})$	$\ln(\sigma_{\text{BS}})$
MSE	0.10	0.06	0.10	0.06	0.12	0.09	0.07	0.03
RMSE	0.31	0.24	0.32	0.24	0.35	0.30	0.26	0.18
MAE	0.27	0.19	0.28	0.20	0.30	0.23	0.22	0.15
MAPE	0.19	0.14	0.13	0.09	0.22	0.18	0.22	0.15
MISP	−0.73	−0.52	−0.99	−0.88	−0.36	−0.12	−0.92	−0.76

overstate future realised volatility: for model-free implied volatility, the indicator is close to one, pointing to the fact that model-free implied volatility is always higher than realised volatility.

As for the sub-samples analysis, we note that both implied volatility forecasts are much closer, on average, to realised volatility in the high and in the low volatility sub-samples than in the medium volatility one. Moreover, in the high volatility sub-sample, both implied volatility forecasts obtain the best performance as pointed out by the MSE, RMSE and the MAE. The only indicator that points to a better performance in the low volatility sub-sample is the MAPE. However, given that we are working with natural logarithms, the denominator of the MAPE is higher in absolute terms in the low volatility sub-sample, and therefore the latter indicator is biased towards having lower errors in the latter sub-sample. Moreover, the MISP indicates that the best performance of both implied volatility measures is in the medium volatility sub-sample, because in this sub-sample, positive and negative prediction errors tend to cancel out. The MISP is much severe in the low volatility sub-sample than in the high volatility one.

Therefore we can conclude that option-based measures are able to better predict volatility when volatility is unusually low or high. It is clear that the forward-looking nature of implied volatilities provides a timely signal in order to capture abnormally high or low levels of volatility. In particular, high volatility is better predicted than low volatility. A possible explanation could be that if volatility is high, investors use options for portfolio insurance strategies more than if volatility is low. Also, traders that want to benefit from high volatility usually invest in the options market rather than in the stock markets, for lower transaction costs and higher leverage. As implied volatility is a forward-looking estimate of future realised volatility, we expect actively traded options to be more informative of future realised volatility than less-traded options. This has been documented in various papers that have analysed index options markets (Donaldson and Kamstra 2005; Sarwar 2005).

10. Conclusions

In this paper, we have investigated the forecasting performance of both option-based volatility forecasts: B–S implied volatility and model-free implied volatility. In order to pursue a fair comparison with model-free implied volatility, which is based on a cross section of option prices, for B–S implied volatility we have used a weighted average of implied volatilities backed out from different option classes. Moreover, for model-free implied volatility, three different implementation techniques, based on a different extrapolation scheme of the strike price domain, have been analysed. Unbiasedness and efficiency of the different volatility forecasts have been tested in the DAX-index option market, by using synchronous prices matched in a 1 min interval.

Our results suggest that both option-based volatility forecasts are efficient since they subsume the information contained in historical volatility. The efficiency results do not change if we measure historical volatility by an AR(1) or by a GARCH(1,1) forecast. Option-based forecast are not unbiased (since they contain a substantial risk premium), but in the logarithmic specification they can be considered unbiased after a constant adjustment, given by the intercept of the regression.

The comparison between the two option-based volatility forecasts has highlighted that B–S implied volatility subsumes all the information contained in model-free implied volatility. These results are in line with the findings of Andersen and Bondarenko (2007) that stress that B–S implied volatility, being less sensitive to the time variation in the volatility risk premium, is a better forecast than model-free implied volatility. The better performance of B–S implied volatility, with respect to previous papers, can be attributed to the fact that it has been computed by using synchronous prices, from a vast set of options in a 1 h window. On the other hand, the performance of model-free implied volatility has probably been affected by some noise added by the extrapolation of a continuum range of strike prices. In particular, the lack of liquid options in the tails of the strike price domain, and the assumption of a constant volatility outside the available strike price range may have induced some measurement errors in the computation of model-free implied volatility.

One possible solution would be either to use only the existing strike price domain, as it is done for the computation of the VIX index or to extend the strike price domain by using a function that matches the slope of the smile at the bounds of the existing strike prices. However, both tactics have proved to be inferior, in the present dataset, to the flat extrapolation methodology proposed by Jiang and Tian (2005).

The results of this paper are very important for the understanding of the information content of option-based measures of volatility and therefore have potential implications for all the fields in which these measures can be used: portfolio selection models, derivatives pricing models, hedging, risk measurement (value at risk and expected shortfall) and risk management techniques, in general. Moreover, these results also have a potential influence on the way market volatility indexes are computed (see e.g. the VIX or the VDAX-New that are computed by truncating the strike price domain).

The present study's answer to the initial question about the more usefulness in predicting future realised volatility of the price of the most traded and thus most liquid at-the-money options, (converted into a volatility measure via the B–S model) or the whole cross section of options prices (converted into a volatility measure via the 'model-free' variance definition) is clearly in favour of the first. The B–S formula, although criticised since based on unrealistic assumptions such as a frictionless market and a constant volatility, is commonly considered as a self-fulfilling prophecy, explained by the widespread use of the formula. Despite its practical limitations, it may well be the case that, thanks to the growing use of model-free implied volatility in both the computation of the new volatility indexes and in the pricing of contracts that depend on general market risk, the latter will impose itself as a new self-fulfilling prophecy.

This paper lends itself to be extended in many directions. High on the research agenda is the study of other possible remedies in order to improve the performance of model-free implied volatility and a comparison with the VDAX-New, the new volatility index of the German equity market. Future research should also address very important questions that the current financial crisis has brought to the forefront, in particular, the need for timely indicators of market stress. Can we use implied volatility measures as warning signals of a coming financial turmoil? An increase in implied volatility can be seen as an increase in market fear and therefore as a sign of a coming period of financial turmoil. On the other hand, a financial crisis should also be preceded by a period in which investors are very optimistic, creating a speculative bubble (as it has been for the September 2001 crash). Therefore, it could well be that it is not the implied volatility level itself that can be used to gain evidence of a coming financial crisis, but the difference between implied and realised volatility, i.e. the volatility risk premium (Bollerslev, Tauchen, and Zhou 2009). Further investigation on these important issues is left for future research.

Acknowledgements

The author thanks the Editor, the two anonymous referees and the members of CEFIN, in particular Marianna Brunetti, Giuseppe Marotta, Chiara Pederzoli and Costanza Torricelli, for helpful comments and suggestions. The author gratefully acknowledges financial support from MIUR. Usual disclaimer applies.

Notes

1. The data source for DAX-index options and DAX-index is the Institute of Finance, Banking, and Insurance of the University of Karlsruhe (TH). The risk-free rate is available on Data-Stream.
2. Nonetheless, as the computation of B–S and model-free implied volatilities has involved some methodological choices deeply described in Section 5, we pursue an EIV procedure in order to see if there is any error in variables in the B–S implied volatility or in the model-free implied volatility. The instruments used for B–S implied volatility (model-free implied volatility) are both historical volatility and past B–S implied volatility (model-free implied volatility) as they are possibly correlated with the true B–S implied volatility (model-free implied volatility), but unrelated to the measurement error associated with B–S implied volatility (model-free implied volatility) 1 month later. As an indicator of the presence of errors in variables, we use the Hausman (1978) specification test statistic (m), which is defined as $m = (\hat{\beta}_{\rm IV} - \hat{\beta}_{\rm OLS})^2/({\rm Var}(\hat{\beta}_{\rm IV}) - {\rm Var}(\hat{\beta}_{\rm OLS}))$, where $\hat{\beta}_{\rm IV}$ is the beta obtained through the two stage least squares procedure, $\hat{\beta}_{\rm OLS}$ is the beta obtained through the OLS procedure and Var(x) is the variance of the coefficient x. The Hausman specification test is distributed as a $\chi^2(1)$.
3. In the regressions that include as an explanatory variable lagged realised volatility, the Durbin's alternative confirmed the non-autocorrelation of the residuals. The results of the Durbin's alternative and of the Breusch–Godfrey LM test are available on request.
4. The non-normality of the residuals is caused by one outlier that corresponds to the September 2001 crash. In order to eliminate the effect of the outlier, regressions (5)–(8) have been re-estimated on the sample period 26 September 2001 to 31 December 2005 and the results, which are available on request, are consistent with the ones reported for the entire sample period.
5. In order to see if B–S implied volatility or model-free implied volatility have been measured with errors, we adopt an instrumental variable procedure. The Hausman (1978) specification test reported in the last column of Table 2 indicates that the errors-in-variables problem is not significant both in univariate and encompassing regressions (In encompassing regression (3), the results are reported for the instrumental variable procedure applied to $\ln(\sigma_{\rm BS})$). Therefore we can trust the OLS regressions results.

References

Ait-Sahalia, Y., and A.W. Lo. 1998. Non parametric estimation of state-price densities implicit in financial asset prices. *Journal of Finance* 53: 499–547.

Andersen T.G., and T. Bollerslev. 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review* 39: 885–905.

Andersen T.G., T. Bollerslev, F.X. Dieblod, and P. Labys. 2001. The distribution of realised exchange rate volatility. *Journal of the American Statistical Association* 96: 42–55.

Andersen, T.G., and O. Bondarenko. 2007. Construction and interpretation of model-free implied volatility. In *Volatility as an asset class*, ed. Israel Nelken, 141–81. London: Risk Books.

Bandi, F.M., and B. Perron. 2006. Long memory and the relation between implied and realised volatility. *Journal of Financial Econometrics* 4: 636–70.

Bates, D. 1991. The crash of '87: Was it expected? The evidence from options markets. *Journal of Finance* 46: 1009–44.

Becker, R., A.E. Clements, and S.I. White. 2007. Does implied volatility provide any information beyond that captured in model-based volatility forecasts? *Journal of Banking & Finance* 31, no. 8: 2535–49.

Black, F., and M. Scholes. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81: 637–54.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31: 307–27.

Bollerslev, T., G. Tauchen, and H. Zhou. 2009. Expected stock returns and variance risk premia. *The Review of Financial Studies* 22, no. 11: 4463–92.

Bollerslev, T., Y.R. Chou, and K.P. Kroner. 1992. ARCH modelling in finance: A review of the theory and empirical evidence. *Journal of Econometrics* 52: 5–59.

Britten-Jones, M., and A. Neuberger. 2000. Option prices, implied price processes and stochastic volatility. *Journal of Finance* 55: 839–66.

Busch T., B.J. Christensen, and M.Ø. Nielsen. 2008. The role of implied volatility in forecasting future realised volatility and jumps in foreign exchange, stock, and bond markets. Working Papers 1181, Queen's University, Department of Economics.

Campa, J.M., K.P. Chang, and R.L. Reider. 1998. Implied exchange rate distributions: Evidence from OTC option markets. *Journal of International Money and Finance* 17: 117–60.

Canina, L., and S. Figlewski. 1993. The informational content of implied volatility. *Review of Financial Studies* 6, no. 3: 659–81.

Carr P., and L. Wu, 2006. A tale of two indices. *The Journal of Derivatives* 13, no. 3: 13–29.

Christensen B.J., C.S. Hansen, and N.R. Prabhala. 2001. The telescoping overlap problem in options data. Working Paper, University of Aarhus and University of Maryland.

Christensen, B.J., and M.Ø. Nielsen. 2006. Asymptotic normality of narrow-band least squares in the stationary fractional cointegration model and volatility forecasting. *Journal of Econometrics* 133: 343–71.

Christensen B.J., and N.R. Prabhala. 1998. The relation between implied and realised volatility. *Journal of Financial Economics* 50: 125–50.

Day, T.E., and C.M. Lewis. 1992. Stock market volatility and the informational content of stock index options. *Journal of Econometrics* 52: 267–87.

Diebold, F.X., and R.S. Mariano. 1995. Comparing predictive accuracy. *Journal of Business & Economic Statistics* 13, no. 3: 134–44.

Donaldson, R.G., and M. Kamstra. 2005. Volatility forecasts, trading volume and the ARCH vs. option-implied tradeoff. *Journal of Financial Research* 28, no. 4: 519–38.

Ederington, L.H., and W. Guan. 2002. Is implied volatility an informationally efficient and effective predictor of future volatility? *Journal of Risk* 4, no. 3: 29–46.

Fleming, J. 1998. The quality of market volatility forecasts implied by S&P100 index option prices. *Journal of Empirical Finance* 5, no. 4: 317–45.

Ghysels, E., A. Harvey, and E. Renault. 1996. Stochastic volatility. In *Handbook of statistics: Statistical methods in finance*, eds. G. Maddala and C. Rao, vol. 14, 119–91. Amsterdam.

Godbey J.M., and J.W. Mahar. 2007. *The forecasting power of implied volatility: Evidence from individual equities*. B>Quest, a University Journal of Applied Business, Richards College of Business, University of West Georgia.

Hausman, J. 1978. Specification tests in econometrics. *Econometrica* 46: 1251–71.

Heston, S.L. 1993. A closed form solution for options with stochastic volatility with application to bond and currency options. *The Review of Financial Studies* 6, no. 2: 327–43.

Jiang, G.J., and Y.S. Tian, 2005. Model-free implied volatility and its information content. *The Review of Financial Studies* 18, no. 4: 1305–42.

Jiang, G.J., and Y.S. Tian, 2007. Extracting model-free volatility from option prices: An examination of the VIX index. *The Journal of Derivatives* 14, no. 3: 35–60.

Jorion, P. 1995. Predicting volatility in the foreign exchange market. *Journal of Finance* 50, no. 2: 507–28.

Lamourex, C.G., and W.D. Lastrapes. 1993. Forecasting stock-return variance: Toward an understanding of stochastic implied volatilities. *Review of Financial Studies* 6, no. 2: 293–326.

Lynch, D., and N. Panigirtzoglou. 2003. Options implied and realised measures of variance. Working paper Monetary Instruments and Markets Division, Bank of England.

Panetta, F., P. Angelini, G. Grande, A. Levy, R. Perli, P.A. Yesin, S. Gerlach, S. Ramaswamy, and M. Scatigna. 2006. The recent behaviour of financial market volatility. Bank of Italy Occasional Paper no. 2.

Pong, S., M.B. Shackleton, S.J. Taylor, and X. Xinzhong. 2004. Forecasting currency volatility: A comparison of implied volatilities and AR(FI)MA models. *Journal of Banking & Finance* 28, no. 10: 2541–63.

Poon, S., and C.W. Granger. 2003. Forecasting volatility in financial markets: A review. *Journal of Economic Literature* 41: 478–539.

Sarwar, G. 2005. The informational role of option trading volume in equity index options markets. *Review of Quantitative Finance and Accounting* 24: 159–76.

Shimko, D. 1993. Bounds of probability. *Risk* 6: 33–7.

Szakmary, A., E. Ors, J.K. Kim, and W.N. Davidson. 2003. The predictive power of implied volatility: Evidence from 35 futures markets. *Journal of Banking and Finance* 27: 2151–75.

Taylor, S.J., Y. Zhang, and P.K. Yadav. 2006. The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks. http://ssrn.com/abstract=890522.

Tsiaras, L. 2009. The forecast performance of competing implied volatility measures: The case of individual stocks. Finance Research Group Working Papers F-2009-02, University of Aarhus, Aarhus School of Business, Department of Business Studies.

Yu, W.W., E.C.K. Liu, and J.W. Wang. 2010. The predictive power of the implied volatility of options traded OTC and on exchanges. *Journal of Banking and Finance* 34, no. 1: 1–11.

Appendix

In order to examine the performance of model-free implied volatility when we vary the parameters u and ΔK, we provide some simulation results based on the Heston's (1993) stochastic volatility model. In the Heston model, the underlying asset follows the stochastic process:

$$\frac{\mathrm{d}S_t}{S_t} = \mu\,\mathrm{d}t + V_t^{1/2}\mathrm{d}W_t,$$

and the variance of the underlying asset follows the mean reverting process:

$$\mathrm{d}V_t = k_v(\theta_v - V_t)\mathrm{d}t + \sigma_v V_t^{1/2}\,\mathrm{d}W_t^v,$$

with $\mathrm{d}W_t\,\mathrm{d}W_t^v = \rho\,\mathrm{d}t$, where V_t is the variance of the returns of the underlying asset, W_t and W_t^v are the standard Wiener processes for the underlying and the variance with correlation ρ, θ_v is the long run average of variance, k_v is the mean reversion rate and σ_v is the volatility of variance.

For the present implementation, we use the same parameters as in Heston (1993), namely spot price $S = 100$, risk-free rate $r = 0$, time to maturity $\tau = 0.5$, current variance $v = 0.01$, correlation $\rho = -0.5$, mean reversion rate $k_v = 2$, long run average of variance $\theta_v = 0.01$, volatility of variance $\sigma_v = 0.225$, price of volatility risk $\lambda = 0$. The volatility of the stock returns over the lifetime of the option is 0.071. We generate call and put option prices, by using the Heston model for a range of strike prices $K = [80, 120]$ equally spaced by $\Delta K = 0.5$. For this interval of strikes, we analyse both the truncation and the discretisation errors by computing model-free implied volatility with different values of u and ΔK and deriving the percentage error (*PE*) in predicting the true volatility of 0.071, as $\mathrm{PE} = (0.071 - \sigma_{\mathrm{mf}})/0.071$. Recall that the parameter u determines the range of strike prices used, while ΔK sets the spacing between strike prices.

In order to examine the truncation error, we compute model-free implied volatility by using different levels of u and a fixed $\Delta K = 0.5$. The results are plotted in Figure A1: u determines the strike price range used $[K_{\min}, K_{\max}]$, and PE is the percentage error in predicting the true volatility. We can see that the truncation error is very low for values of u bigger than 0.3: increasing the strike price interval, the truncation error converges very quickly to zero. As the spot price is 100, a strike price interval of [77, 130] is enough to ensure that the truncation error does not have any impact on the model-free implied volatility calculation. For the implementation on the DAX-index data, we used $u = 0.5$; therefore truncation errors are not likely to have affected our results.

In order to examine the discretisation bias, we compute model-free implied volatility by using different values of ΔK ranging from 0.05 to 3, and a fixed $u = 1$. In Figure A2, we plot the percentage error against ΔK: we can see that the discretisation error is negligible for ΔK bigger than 1. As the spot price is 100, a strike price discreteness of 1% is enough to ensure an insignificant discretisation error. For the implementation on the DAX-index data, we used $\Delta K = 10$, given that the DAX-index values used in the present implementation are typically greater than 2900; discretisation errors are not likely to have affected our results.

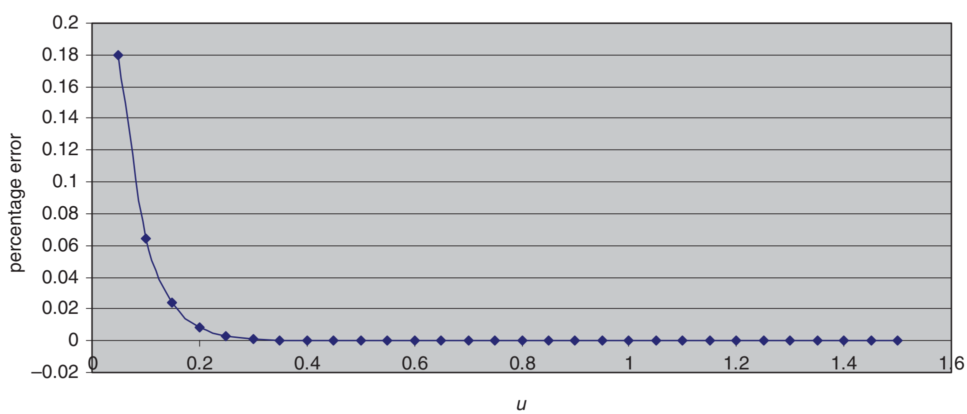

Figure A1. The truncation error for different values of u.

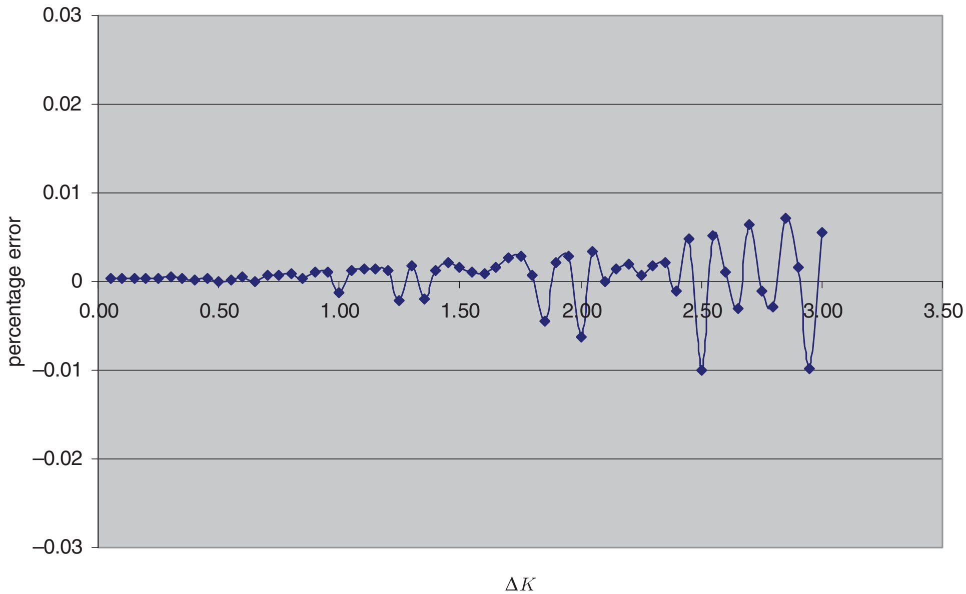

Figure A2. The discretisation error for different values of ΔK.