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Abstract

This article describes the primary uses of the VIX index in the financial literature, offering for the first time a joint view of its successes and failures in key financial areas. VIX is a model-free volatility index that measures the investor "fear" gauge due to its significant and negative relationship with S&P 500 return dynamics, which justifies its use as a proxy for market risk and volatility. This article focuses on the most frequent uses of VIX, namely, as (1) a financial product to hedge a portfolio against volatility risk; (2) a market risk measure used to analyze risk flows from financial markets and to relate private and public risks; and (3) a volatility measure to estimate the spot volatility dynamics, the volatility risk premium and volatility jumps. This survey offers an entre for researchers who consider VIX as a proxy for volatility and/or risk.

Key Words: volatility indices, leverage effect, forecast volatility, variance risk premium, volatility derivatives, market risk

JEL Classification: G10, C12, G13, G14

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1. Introduction

The success of VIX, commonly referred to as "the world's barometer of market volatility", has grabbed the attention of practitioners, media and the financial literature. The VIX is a volatility index linked to the US stock market and calculated by the Chicago Board Option Exchange (CBOE) that informs about the expected US market volatility in the upcoming month. The index was first computed in 1993 as a linear combination of eight At-the-Money (ATM) implied volatilities on the S&P 100 options with maturities closest to 30 calendar days, following results by Whaley (1993). The VIX formula changed in 2003 due to the new market conditions (the S&P 500 index became more informative than the S&P 100 so the new methodology uses SPX quotes), and to the growing demand of VIX-based products (options and futures on the old VIX were difficult to price). The new VIX is called model-free because it does not rely on a specific option valuation model (typically Black-Scholes). Instead, the model-free VIX is computed following the results on variance swaps in Carr & Madam (1998), Demeterfi, Derman, Kamal & Zou (1999a), and Britten-Jones & Neuberger (2000), which allow for pricing variance swaps using a weighted portfolio of ATM and Out-of-the-Money (OTM) call and put options.

Financial markets and academic researchers have followed model-free VIX closely since the CBOE disseminates the index in 2003. The success of VIX is reflected in the popularity of VIX derivatives used to trade this index. Futures and options, first and second tradable products based on the VIX, have been so successful that the annual volume in VIX options and futures were ranked sixth and third in 2012, respectively. The high and growing demand of VIX-based products awaken the interest of the CBOE in VIX-related products and the attention of the financial literature, which in turn has intensified the study and use of the VIX (and VIX derivatives) in recent years. Nevertheless, despite the success of model-free volatility indexes in the financial literature, there is no survey that conglomerates main uses of VIX-like volatility indexes by academics. This article fills this gap by describing and relating main research lines using model-free VIX to measure market risk, trade volatility, hedge portfolios, estimate the spot volatility dynamics or anticipate future volatility.

The theoretical model in Britten-Jones & Neuberger (2000), in which the CBOE bases the model-free VIX formula, assumes a diffusive process for the underlying, so some authors debate the "model-free" character of the index. The literature concludes that different assumptions on the underlying process result in differently weighted OTM option schemes in the VIX formula (see Demeterfi, Derman, Kamal & Zou (1999b), Carr & Lewis (2004), Lee (2010), Carr, Lee & Wu (2012) or Martin (2012)). Most weighted functions proposed are decreasing functions of the moneyness. Despite the evidence provided by Martin (2012) in favor of a constant weighted function for the model-free volatility index when the underlying process includes jumps, Jiang & Tian (2005) show that the model-free variance in Britten-Jones & Neuberger (2000) is still valid when the underlying process includes jumps, reporting a negligible approximation error. Since the level, curvature and slope of the (implied) volatility smirk inform about the shape of the underlying returns distribution, we should expect a significant relationship between the stochastic diffusion process assumed for the underlying and the option (implied volatility) weighted scheme. This article summarizes for the first time main results achieved in this research line.

The success of the model-free VIX in the literature and market is mainly due to its significantly negative relationship with the underlying (S&P 500) returns dynamics (the VIX rises on average when the stock index return is negative). Many authors have analyzed this relationship (leverage effect) with the intention of characterizing its magnitude and dynamics, which is necessary to define an adequate hedging strategy (trading VIX) to cover a portfolio against the market volatility risk. The literature finds this relationship to be time-varying, asymmetric and affected by VIX computation errors. Future research projects in this line should consider noise-corrected VIX series to separate the continuous from non-continuous part of the leverage as well as the different time series properties of VIX and S&P 500 returns to explain the time-varying and asymmetric leverage effect, empirically reported.

The literature shows that the model-free VIX relates with the variance swap price formula in such a way that we can use the VIX to approach the local variance (latent variable) dynamics, under different stock return volatility dynamics. This makes it possible to draw a closed-form linear relationship between the squared volatility index and the latent volatility dynamics with a minimum error. This intern could explain the extensive use of the model-free VIX to characterize the underlying spot volatility dynamics. Again, the literature finds that VIX measurement errors affect the spot volatility estimation process (Duan & Yeh (2012)), suggesting a correction of the index formula to minimizes such errors and improve the spot volatility filtering process based on the index.

The documented relationship between model-free VIX and variance swap formulas (given by the spot volatility stochastic dynamics) allows us to estimate the volatility risk premium (VRP) by comparing realized and expected volatilities; see Carr & Wu (2008). Understanding the VRP dynamics is important to improve current asset pricing theories and to better understand the investor fear gauge dynamics, as well as the market reaction to macroeconomic announcements or to longer episodes of uncertainty such as financial crises. The literature on this issue has increased markedly over the last few years focusing on the use of VIX term structure to estimate the VRP term structure, and conclude on the main factors affecting the volatility risk priced by the market over time across different maturities. This task is crucial to understanding how investors price the future risk. Recent literature on the variance risk premium term structure focuses on more accurate techniques to estimate the realized volatility[2] and on studying the VRP (term structure) dynamics, considering different underlying processes and factor model analysis. Future research in this line would certainly help improve and understand the asset pricing market dynamics.

The existence of a significantly negative VRP makes the model-free VIX an imperfect forecaster for realized volatility. Nevertheless, by understanding the VRP dynamics we could use the VIX to forecast the realized volatility. Different articles in the literature have studied the ability of VIX to forecast the realized volatility when we assume a linear relationship between them. The literature agrees on the information content in VIX to forecast the realized volatility; nevertheless, the results differ when we study whether the VIX contains more information than historical volatility-based measures (e.g. GARCH models, historical volatility measures, etc.) to forecast the underlying realized volatility. The time-varying dynamics of the VRP suggest a more complex relation than a simply linear relation between the VIX and the realized volatility, so future research projects should conclude on the ability of VIX to forecast the underlying realized volatility under economically significant non-linear relationship.

The empirical and theoretical evidence in favor of significant financial leverage effect in the stock markets, along with the heavy use of the VIX in the market makes trading the VIX very attractive for investors. Nevertheless, the VIX is not directly tradable. This is only possible by trading VIX futures and options issued on the model-free index, which have been highly traded at the CBOE since these were issued in 2004 and 2006, respectively. Indeed, the CBOE is continuously looking for additional products to answer the growing market demand on more complex VIX derivatives which allow for cheaper volatility hedging strategies under different market scenarios and portfolios. Future research in this line would need to improve the accuracy of VIX derivatives pricing models likely through a better understanding of S&P 500 volatility, VIX level and VIX volatility dynamics.

Other uses of model-free VIX in the literature include the use of this as a market risk factor. Among these, the literature reports a link between the corporate bond price and the VIX, as well as a link between the bond liquidity and the market risk (VIX), which highlights the importance of computing a proper market risk variable to price bonds or corporate yield spreads. On the other hand, the hedge-fund literature extensively uses VIX as a market volatility proxy to improve the forecasting, monitoring and assessment of hedge-fund returns. In addition, the

[2] Free from noise and deterministic dynamics such as market announcements and seasonal patterns

accounting literature uses the VIX as a proxy for market-wide volatility (see Chakrabarty & Moulton (2012)) to study the effect of earnings announcements on stock prices. Furthermore, the VIX level is commonly used to measure market turnover with the intention of characterizing the financial market dynamics at each financial scenario, making the absence of artificial jumps in VIX crucial to improve the accuracy of these studies. International VIX-like volatility indexes are usually compared to determine financial market spillovers and to provide recommendations that help clarify the financial market contagion process.

This article is structured according to the following sections. First, we describe the theoretical background around the old and new VIX formulas to compare the latter with the empirical CBOE model-free VIX formula and describe the different approximation error content in this series. Section three describes the use of VIX as a factor to explain the underlying volatility dynamics. Section four describes articles that study the financial leverage effect using the model-free volatility index, with focus on characterizing the variance risk premium time series properties and dynamics using the index. Section five describes papers using the VIX to forecast the underlying realized variance, and section six summarizes main findings on VIX derivatives pricing. Finally, section eight describes other uses of the model-free volatility index in the literature, most of them related to the index's role as a risk market factor explaining the corporate yield spread and bond liquidity, as well as its role in addressing the market risk in hedge-fund exercises. This review ends with a brief conclusion on past, present and future lines of research that will help improve the information content of the US model-free VIX, VIX-like international volatility indexes and the uses of VIX.

2. The VIX formula: A background

The VIX is a volatility index computed to address the expected short-term US market volatility over the next thirty calendar days. Among the various ways[3] of approaching this concept, Robert E. Whaley developed in 1993 a volatility index called VIX that is computed as a linear combination of eight S&P 100 Black-Scholes (BS) ATM implied volatilities (IVs). Researchers have easily found that the volatility index moves approximately linearly with volatility-induced movements in the S&P 100 option prices[4], in line with results reported by Feinstein (1989), who finds the BS option valuation formula approximately linear in volatility for ATM options. The Chicago Option Board Exchange (CBOE) began computing and disseminating the VIX in 1993. As expected, the strong negative relationship between the VIX changes and S&P 100 returns explain that the index was a great success in the stock markets.

When the VIX relies on the BS pricing model and assumptions, this version of the index is known as the model-based VIX, which currently holds the ticker VXO. We describe the *model based* VIX formula below.

$$\sigma_1 = \left(\frac{\sigma_{C,1}^{X_{1,L}} + \sigma_{P,1}^{X_{1,L}}}{2}\right)\left(\frac{X_{1,L} - S}{X_{1,U} - X_{1,L}}\right) + \left(\frac{\sigma_{C,1}^{X_{1,U}} + \sigma_{P,1}^{X_{1,U}}}{2}\right)\left(\frac{S - X_{1,L}}{X_{1,U} - X_{1,L}}\right) \qquad (1)$$

$$\sigma_2 = \left(\frac{\sigma_{C,2}^{X_{2,L}} + \sigma_{P,2}^{X_{2,L}}}{2}\right)\left(\frac{X_{2,L} - S}{X_{2,U} - X_{2,L}}\right) + \left(\frac{\sigma_{C,2}^{X_{2,U}} + \sigma_{P,2}^{X_{2,U}}}{2}\right)\left(\frac{S - X_{2,L}}{X_{2,U} - X_{2,L}}\right) \qquad (2)$$

$$VIX = \sigma_1\left(\frac{N_2 - 22}{N_2 - N_1}\right) + \sigma_2\left(\frac{22 - N_1}{N_2 - N_1}\right) \qquad (3)$$

Variables σ_1 and σ_2 refer to the estimated S&P 100 ATM IV for the two closest maturities (just below and above) to thirty calendar days. The first and second elements in the pairs ($X_{1,L}$, $X_{1,U}$) and ($X_{2,L}$, $X_{2,U}$) are the available strikes just below and above the underlying price for the first and second (subindex) closest maturities, respectively. Finally, N_1 and N_2 are the number of trading days to the expiration of the first and second nearby contracts, respectively.

In 2003, the CBOE and Goldman Sachs used results by Demeterfi et al. (1999a) and Carr & Madam (1998) to change the VIX formula and make the index (BS) model-free, and to adapt the index to the new option market structure. First, the S&P 500 option market (ticker symbol SPX) became much more active than the S&P 100 option market (ticker symbol OEX), making

[3] For example, Brenner & Galai (1989) suggest using a combination of options and historical volatility to compute a volatility index

[4] See Whaley (1993) for more details

SPX quotes more informative than OEX quotes. Therefore, SPX replaced OEX options in the VIX formula to adjust the index to the new option market structure. Second, investors used OTM put options as insurance against potential drops in the stock index price, so out-of-the-money (OTM) options became more highly traded than ATM options and thus highly informative. Therefore, the new VIX formula must consider the information content in both ATM and OTM options. Theoretical results by Demeterfi et al. (1999a) and Carr & Madam (1998) suggest (i) identifying the squared VIX with the variance swap[5] payoff, which under standard assumptions, is defined as twice the forward price of the log contract refereed to the same underlying asset, and show that (ii) this payoff can be replicated by a weighted continuous sum of OTM call and put vanilla options. The CBOE uses this result to identify the model-free squared VIX with the linear combination of OTM SPX call and put options by adapting the formula in Demeterfi et al. (1999a) and Carr & Madam (1998) to the US option market microstructure. Additionally, market participants started calling for trading VIX to hedge portfolios against the volatility risk. Derivatives on model-based VIX were impossible to price due to the IV character of the index, while we can price derivatives on the model-free VIX. All of this justifies the CBOE decision of making the official US VIX model-free.

Since the model-free VIX was issued, different authors have studied similarities and differences between the indexes. Thus, Carr & Wu (2003a) relate model-based and model-free VIX methodologies, and point out two main differences: (i) the underlying used (S&P 100 vs S&P 500), and (ii) the way each methodology extract information from option prices (non-linear vs linear function of option prices). Table 1 compares main characteristics of model-based (VXO) and model-free (VIX) official volatility indexes. Both indexes are highly correlated but the model-free VIX is on average higher than the model-based VXO, which is explained by the fact that deep OTM put option prices are highly sensitive to jumps. On the other hand, Carr & Lee (2003) show that under general market settings[6], the ATM implied volatility approximates well the volatility swap rate[7], but under this specification of variance swap the volatility swap contract is hardly difficult to replicate and hedge. Moreover, the meaning of ATM implied volatility is not clear, while the squared VIX is a portfolio of options, just what the industry needed, a replicable volatility product that makes it possible to hedge a portfolio against the market volatility risk. The new theoretical framework made it possible to meet the market demands by the calculation of a (model-free) volatility index easily replicable on which the CBOE can specify a derivative contract (i.e. VIX futures and options).

[5] Variance swaps are easier to price and hedge using options, while volatility swaps cannot be replicated using these derivatives. This explains the market preference for variance swaps to trade volatility. The VIX is computed as a discrete truncated version of the square root of a variance swap

[6] Assuming $\frac{dF_t}{F_t} = \sigma_t dW_t$ under Q with diffusion volatility σ_t stochastic and independent of the Brownian Motion W_t in the price

[7] The approach is much better for ATM implied volatilities related to shorter times to maturity

Table 1: Two broad US volatility indexes: VXO and VIX.

	VXO	VIX
Valuation Model	Black-Scholes	--
Options	on S&P100 (OEX)	on S&P500 (SPX)
- Type	European (call and put)	
- Moneyness	ATM	ATM, OTM
- Number	Eight	n(t)*
- Maturity	Two: the ones just below and above 30 calendar days	
- Time to maturity	Number of (trading) days	Number of minutes (calendar days)
Derivatives	--	Futures and Options

(*) See the CBOE cutting-wings rule in the white paper available at www.cboe.com/micro/vix/vixwhite.pdf

Related to the model-free VIX formula, the expressions (4) and (5) illustrate the underlying model and the associated dynamics of the expected risk-neutral variance V_t from t to $\tau > t$ assumed by Demeterfi et al. (1999a). The variable $s_t = \ln(S_t)$ is the log stock price; μ_t and v_t are a return's drift and spot volatility, respectively; $E^*[.]$ is the expected risk neutral operator; r is the risk-free rate; T refers to the option time to maturity; K is the option strike price; S_* denotes the ATM strike price; and $P(K;\, T\,)$ and $C(K;\, T\,)$ are put and call option prices for strike K and maturity T, respectively. Demeterfi et al. (1999a) show that under this underlying model assumption, the risk-neutral expected variance V_t can be replicated by a continuous weighted sum of OTM call and put prices, as in (5). Thus, the CBOE bases the model-free VIX formula on this result, assuming that the S&P 500 returns follow the below-mentioned stochastic process (4).

$$ds_t = \mu_t d_t + v_t dW_t \quad (4)$$

$$V_t = E^*\left[\int_t^T ds_t^2 d_t\right] = E^*\left[\int_t^T v_t^2 dt\right] = 2e^{rT}\left[\int_0^{S_*} \frac{1}{K^2} P(K,T)dK + \int_{S_*}^{+\infty} \frac{1}{K^2} C(K,T)dK\right] \quad (5)$$

The theory behind the model-free VIX is not without its critics. Because the variance swap is priced without any assumption on the volatility process in (5), this is referred to as a *model-free* volatility index. However, many authors are reluctant to the "model-free" character of model-free V, because we still assume a non-jump diffusion process for the underlying returns (ds_t). Especially when the literature reports empirical evidence in favor of a significant jump component in the market returns dynamics; see, for example, Carr & Wu (2003b). Does this imply a limitation for the current VIX formula? There are various answers to this question. Carr

& Lewis (2004), Lee (2010) and Carr, Lee & Wu (2012) consider different underlying processes and show that it is still possible use a certain number of log-contracts to replicate the payoff of the gamma-zero variance swap, but it is true that the option weighting schemes must be changed according to the underlying process assumed[8]. Table 2 summarizes different price weighted schemes proposed to replicate the variance swap payoff for different underlying processes. The literature reports significant links between the level, slope and curvature of the implied volatility smirk and the shape of the underlying distribution (skewness, kurtosis, etc.), which explains different information in option prices across strikes, and different weighted schemes when we assume different underlying price distributions. On the other hand, Jiang & Tian (2005) show that the model-free VIX based on Britten-Jones & Neuberger (2000) is still valid when the underlying process includes jumps, except if the jump size is negatively skewed. In this case the model-free volatility index tends to overestimate V. Further research is needed to characterize the relationship between option weighting schemes and underlying processes and to generate a model-free volatility index formula independent of the underlying process assumption.

2.1 The CBOE model-free VIX

The limited amount of information quoted at the CBOE, the market microstructure of the US option market, and the absence of 30-days-to-maturity options continuously quoted makes the VIX a truncated model-free volatility index.

The CBOE computes the two model-free variances needed to compute the VIX following the formula in (6) that adapts the model-free variance formula in (5) to the discrete market data.

The model-free variance in (6) includes two terms, the first results from the discretization of (5), while the second term controls for empirical discrepancies between the implied forward price F and the ATM strike available at the market (K_0), since $K_0 \leq F$ and $K_0 = F$ does not necessarily happen.[9] Nevertheless, moving from theory to practice the CBOE not only generates the mentioned "corrector error", but also computation errors, that we review in this section.

[8] Martin (2012) suggests using a non-log contract to approximate the variance payoff when the underlying process follows a stochastic diffusion process with jumps. As a result, the weighted scheme is such that each option is weighted at a constant rate inversely proportional to the forward price.

[9] This correction term results from replacing the ITM call price with strike K_0, when $K_0 < F$, with the equivalent OTM put price, according the put-call parity. Note that this term is equal to zero when $F = K_0$. See Carr & Lee (2003) for a detailed explanation.

Table 2: Weighted schemes under different underlying process assumptions. VS denotes the variance swap, CVS is the corridor variance swap, AGS is the arithmetic gamma swap, GS is the gamma swap, SVS is the simple variance swap, and VSL is the variance swap resulting from Levy processes. Parameter Q_S depends on the characteristics driving the Levy process, such that $Q = 2$ when there is no jumps in the underlying diffusion process, $Q > 2$ for high up-jumps, and $Q < 2$ for high down-jumps.

Payoff	**Weighting Scheme ($\omega_{iP} = \omega_{iC}$)**	**Variance Index**	**Review References**
1. Underlying Process: $ds_t = \mu_t dt + v_t dW_t$			
$V_t = \int_t^T ds_t^2 dt$	$\frac{2}{K_i^2}$	VS	Britten-Jones & Neuberger (2000) Demeterfi et al. (1999b)
	$\frac{2}{K_i^2} I_{K\in C}$	CVS	Carr & Lewis (2004)
	2	AGS	Lee (2010)
	$\frac{2}{S_0 K_i}$	GS	
2. Underlying Process: Arbitrary time-changed exponential Levy process			
$V_t = \int_t^T ds_t^2 dt$	$\frac{Q_S}{K_i^2}$	VSL	Carr, Lee & Wu (2012)
3. Underlying Process: $ds_t = \mu_t dt + v_t dW_t + J$			
$V_t = \int_t^T \left(\frac{dS_t}{F_0}\right)^2 dt$	$\frac{2}{F_0^2}$	SVS	Martin (2012)
$V_t = \int_t^T ds_t^2 dt$	$\approx \frac{2}{K_i^2}$	VS	Jiang & Tian (2005)

As (8) reflects, the CBOE computes the VIX (associated to thirty calendar days) interpolating two truncated fair variances V_i related to maturities T_1 and T_2 just below and above thirty calendar days[10], respectively. Each variance is computed according to (6) where r_1 and r_2 are the risk-free rates for each maturity, K_{1L} and K_{2L} are the lowest strike prices considered for first and second maturity variances, K_{1U} and K_{2U} are the highest strike prices considered for first and second maturity var[11]iances, and F_1 and F_2 are the forward prices per each maturity. Finally, $Q(K, T_i)$ is defined as the OTM call or put price (depending on the strike price considered such that $K_{iL} \leq K \leq K_{iU}$), or the ATM consolidated call-put price when $K = K_0$. The squared VIX is a linear combination of the two nearest model-free variance swaps, such that the linear weighted

[10] The CBOE never uses maturities lower than seven calendar days. When the first maturity options reach this threshold, we compute the VIX rolling over the next two available maturities.
[11] Number of minutes

scheme is ruled by α_1 and α_2, which depends on the time between each time to maturity T_1 and T_2 and the thirty calendar days[12] , as in (7).

$$V_i = \underbrace{\frac{2e^{r_i T_i}}{T_i}\left[\sum_{K_{iL}}^{K_{iU}} \frac{\Delta K}{K^2} Q(K, T_i)\right]}_{First\ Part} - \underbrace{\frac{1}{T_i}\left[\frac{F_i}{K_{i0}} - 1\right]^2}_{Correction\ Term} \text{, with } i = 1,2 \qquad (6)$$

$$\alpha_i = \frac{T_i(T_{30} - T_i)}{T_{30}(T_2 - T_1)}(-1)^i \qquad (7)$$

So that,

$$VIX = 100\sqrt{\sum_{i=1}^{2} \alpha_i V_i} \qquad (8)$$

Recently, the CBOE provides both VIX components V_1 and V_2 as (9) and (10), and refers to them as VIN and VIF, respectively. The CBOE rolls over into second and third maturity options to compute the VIX when the first maturity lies below eight calendar days, to reduce the market microstructure error in the volatility index. We probably need to deepen the study of the sensitivity of VIX to this rollover effect[13].

$$VIN = 100\sqrt{V_1 \cdot \frac{T_{365}}{T_1}}$$

$$VIF = 100\sqrt{V_2 \cdot \frac{T_{365}}{T_2}}$$

[12] White paper at the CBOE defines T_1 as the fraction of number of minutes until the option maturity i and the number of minutes in one year. We just define T_i as the number of minutes until maturity, so do not get confuse when you read the official CBOE white paper.

[13] Tzang et al (2011) study the effects of different rollover rules in reducing VIX missing data at 1-minute frequency.

2.2 Sources of error in the CBOE VIX

Jiang & Tian (2005) characterize the differences between the squared V IX in (8) and V in (5), calling them measurement errors. The authors classify these errors as follows: (i) discretization error (δ_{disc}), which is due to a lack of continuous option strikes, making it necessary to include a correction term in the VIX formula; (ii) truncation error (δ_{trunc}), which is generated by the truncated strike range used to compute VIX; (iii) expansion error (δ_{exp}) due to the use of a Taylor series expansion of the log function to approach$V$; and (iv) linear interpolation error (δ_{int}) , which is due to the CBOE linear swap term structure assumption.

$$\delta_{disc} = \sum_i \frac{\Delta K}{K^2} Q(T,K) - \left[\int_{K_L}^{K_0} \frac{P(T,K)}{K^2} dK + \int_{K_0}^{K_U} \frac{C(T,K)}{K^2} dK\right] \qquad (11)$$

$$\delta_{trunc} = \int_0^{K_L} \frac{P(T,K)}{K^2} dK + \int_{K_U}^{+\infty} \frac{C(T,K)}{K^2} dK \qquad (12)$$

$$\delta_{exp} = \frac{2}{T}\left\{\left[\left(\frac{F_0}{K_0} - 1\right) - \frac{1}{2}\left(\frac{F_0}{K_0} - 1\right)^2\right] - \ln(F_0/K_0)\right\} \qquad (13)$$

$$\delta_{int} = \hat{V}_{30} - V_{30}, \text{ where } \hat{V}_{30} = \frac{N_2 - N_{30}}{N_2 - N_1} V_1 + \frac{N_{30} - N_1}{N_2 - N_1} V_2 \qquad (14)$$

$$\delta_{trunc,t} = \int_0^{K_{L,t}} \frac{P(T,K)}{K^2} dK + \int_{K_{U,t}}^{+\infty} \frac{C(T,K)}{K^2} dK \qquad (15)$$

Andersen, Bondarenko & Gonzalez-Perez (2013) identify a new error source in VIX that generates a significant number of jumps in the volatility index unconnected with the underlying volatility process and that weakens the function of VIX as an annualized fair volatility index. This error source is related to the truncation error reported in Jiang & Tian (2005) but differs because it is generated by the CBOE rule that determines the minimum and maximum strikes considered in the volatility index formula (cutting-wings rule). After Andersen, Bondarenko & Gonzalez-Perez (2013) identify this additional error component, the CBOE added in the VIX white paper the following disclaimer: "as volatility rises and falls, the strike price range of options with nonzero bids tends to expand and contract. As a result, the number of options used in the VIX calculation may vary from month-to-month, day-to-day and possibly, even minute-to-minute." Nevertheless, some adjustments or changes in the VIX formula should also be considered to reduce this deficiency. The literature basically suggests (i) to make the range of strikes economically invariant and compute the VIX as a Corridor Implied Volatility (CIV) index (see Andersen & Bondarenko (2007), Andersen & Bondarenko (2010), Andersen, Bondarenko & Gonzalez-Perez (2013)), or (ii) extrapolate option prices in the tails using implied volatility functions. Nevertheless, mispriced deep OTM options difficult the success of the extrapolation exercise.

Alternatively, and related to market microstructure, Zhang, Taylor & Wang (2013) study the sensitivity of VIX to measurement errors in option prices. They simulate the underlying realized volatility and option prices, and conclude that (i) the informational efficiency of the simulated model-free index increases monotonically with the number of out-of-the-money options considered (this is, with the strike range), and that (ii) the model-free volatility index is less informative with the reduction of the number of options (strike range). The authors suggest using implied volatility curves before implementing CBOE procedure, to reduce the measurement error of option prices and improve the ability of VIX to forecast the underlying realized volatility. However, this almost never works in practice due pricing errors of deep OTM options that generate abnormal implied volatility curves. The range of strikes used to compute the index varies over time, so the measurement error reported by Zhang, Taylor & Wang (2013) is time-varying.

Despite the presence of implementation and measurement errors in VIX, academics and practitioners use model-free VIX as a volatility asset, a market risk measure or a relevant factor to explain the spot volatility dynamics. This paper intends to establish the basis for understanding the results given in the literature of using the CBOE model-free VIX in the researcher's analysis.

3. The VIX as a factor to explain S&P 500 returns and spot volatility dynamics

There is strong evidence in favor of the cross section of option prices providing information about current and future stock returns and volatility dynamics. Attending to current returns, Ang, Hodrick, Xing & Zhang (2006) show that the VIX dynamics are a significant factor in pricing the cross section of asset returns. Banerjee, Doran & Peterson (2007) confirm this result in a broader setting (controlling for the four Fama-French factors and sorting portfolios by book-to-market equity, size and beta). Moreover, Zhu (2013) recently studies the dependence of the US stock and bond returns, and finds the VIX a significant factor that helps to forecast the US stock returns distribution.

Starting with Breeden & Litzenberger (1978), we are able to characterize the risk-neutral density (RND) function for stock returns from the cross section of option prices (see Bakshi, Cao & Chen (1997) and Bates (2000) for more details), while Neuberger (1994), Carr & Madam (1998), Britten-Jones & Neuberger (2000) and Figlewski (2008) relate the conditional return variance expectation (under the risk-neutral measure) with the cross section of option prices. Most importantly, the portfolio of options in the model-free VIX formula (8) is able to address the risk neutral integrated variance of stock returns, making it possible to draw a closed-form linear relationship between the squared volatility index and the latent volatility dynamics, with a minimum error bias, whenever the underlying returns follow an affine-drift stochastic volatility process (see Jones (2003), Dotsis, Psychoyios & Skiadopoulus (2007) or Duan & Yeh (2010), among others). This relationship, described in (16), has resulted in numerous articles that use the CBOE VIX to address the local variance (latent variable) dynamics under different stock return volatility dynamics.

The theoretical result in (17) is then used to estimate the latent spot volatility, by using the stock price dynamics and the squared VIX to filter the spot volatility. We know that the model-free squared VIX approximates the 30-day variance swap rate on the S&P 500 index, and hence, the expected value of the 30-day quadratic variation (QV) under a risk-neutral measure Q. Therefore, under certain assumptions it is possible relate the squared VIX with the spot stock return volatility.

$$\tau_0 \cdot VIX_{t+\tau_0}^2 = \underbrace{\mathrm{E}_t^{\mathrm{Q}}\left[QV_{[t,t+\tau_0]}\right]}_{\substack{Expected\ Value\ of\ the \\ Quadratic\ Variation}} = \underbrace{\mathrm{E}_t^{\mathrm{Q}}\left[V_{[t,t+\tau_0]}^{Q}\right]}_{\substack{Risk\ Neutral \\ Integrated\ Variance}} \quad (16)$$

$$VIX_t^2 = E^Q\left[\frac{1}{\tau_0}\int_t^{t+\tau_0} V_S^Q ds\right] \approx a + b \cdot V_t^Q (17)$$

Different stochastic volatility model assumptions drive different linear relationships between the squared VIX and the instant variance, resulting in different specifications for the a and b parameters in (17). Duan & Yeh (2010) consider a general CEV stochastic volatility model with jumps in asset prices and uncorrelated stock price and volatility innovations, which are consistent with the models of Hull & White (1987), Heston (1993), Bates (2000) and Pan (2002), and filter the latent volatility using squared VIX, as suggested in (18) and (19), obtaining different a and b specifications from different SV parameter restrictions. The Hull & White (1987) model arises when $\lambda^* = 0$, $\gamma = 1$, and $\theta = 0$; the Heston (1993) model corresponds to $\lambda^* = 0$, $\delta = -1/2$, and $\gamma = 1/2$; and the Bates (2000) and Pan (2002) models require $\gamma = 0$.

$$d\ln S_t = \left(r - q - \frac{V_t}{2} + \lambda^*\left(\mu_J^* + 1 - e^{\mu_J^* + \frac{\sigma_J^2}{2}}\right)\right)dt + \sqrt{V_t}dW_t^* + J_t^* dN_t^* - \lambda^* \mu_J^* dt \quad (18)$$

$$dV_t = \kappa(1 - \delta_V V_t)dt + \nu V_t^{\gamma} dB_t^* (19)$$

$$a = 2\phi^* + \frac{(1+2\phi^*)(\kappa+\delta_V)\theta}{(\kappa+\delta_V)}\left(1 - \frac{1-e^{-(\kappa+\delta_V)\tau}}{(\kappa+\delta_V)\tau}\right)(20)$$

$$b = \frac{(1+2\phi^*)(1-e^{-(\kappa+\delta_V)\tau})}{(\kappa+\delta_V)\tau} (21)$$

$$\phi^* = \lambda^*(e^{\mu_J^* + \frac{\sigma_J^2}{2}} - 1 - \mu_J^*) (22)$$

Duan & Yeh (2012) allow for time-varying intensity jumps in stock prices ($\lambda = \lambda_0 + \lambda_1 V$) and VIX measurement errors to filter the spot volatility dynamics. They conclude that the squared-root volatility process is mis-specified and that VIX measurement errors affect the spot volatility estimation process. This result leads to the conclusion that we should first correct VIX from the measurement errors and then use it to filter the spot volatility dynamics.

The literature also uses the relation between squared VIX and σ^2 to explains the relation between VIX volatility, and the volatility of volatility. This is particularly important to price VIX derivatives. Among these papers, Mencia & Sentana (2013) propose a stochastic vol of vol process assuming a Levy process independent of the VIX index dynamics, while Kaeck & Alexander (2013) conclude that including VIX in the vol of vol dynamics improves the estimation results. Going deeper into the relation between VIX and spot volatility dynamics, Dotsis, Psychoyios & Skiadopoulus (2007) find evidence in favor of affine Merton-type jumps models to capture the dynamics of a volatility index, while Psychoyios, Dotsis & Markellos (2010) find necessary the inclusion of jumps in the SV process to model the volatility of volatility and replicate the VIX futures term structure and the VIX option surface. Although these articles fall mainly into the VIX derivatives pricing literature, reported results that relate vol of vol and VIX dynamics could be use to improve our understanding of the vol of vol dynamics.

Finally, the VIX has been considered as a transition variable in multivariate GARCH models, with positive results in almost all cases. See Poon & Granger (2003) for a review.

[- Insert Table 3 here -]

4. The VIX and the financial leverage effect

The financial leverage effect refers to the negative correlation between stock returns and spot volatility (see Black (1976), Christie (1982), Engle & Ng (1993)). There is a vast amount of literature interested in characterizing the leverage effect to improve the specification of stochastic volatility models[14]. This would imply a more accurate option valuation and asset pricing models.

The VIX measures the squared-root of the risk-neutral expectation of the S&P 500 variance over the next thirty calendar days, and, as the previous section suggests, under certain assumptions there is a linear relationship between the squared VIX and the S&P 500 return variance V. Thus, it would be possible to compare the squared VIX changes with S&P 500 returns to characterize the leverage effect phenomenon in the US stock market. Following this reasoning, Ishida, McAleer & Oya (2011) report evidence in favor of the use of VIX to improve the estimation accuracy of the leverage parameter in the Heston stochastic volatility model.

The empirical evidence in favor of a negative correlation between VIX changes and S&P 500 returns explains why we refer to the VIX as the “fear index”, or the “investor fear gauge”; see Whaley (2000) or Whaley (2009) for more details[15]. The literature finds this (linear) correlation to be greater for negative returns (asymmetric). Nevertheless, the relationship between VIX and S&P 500 dynamics is more complex than linear due to the type of VIX heteroscedasticity[16]. The model-free volatility index is more volatile at higher VIX levels and less volatile at lower VIX levels, while the S&P 500 returns exhibit a different type of heteroscedasticity. Thus, we would expect a time varying asymmetric leverage effect.

It is not easy to define “low and high volatility levels” when the volatility series show a local-mean-dependent heteroscedasticity, so identifying the asymmetric time varying dynamics is not an easy task. Bandi & Reno (2012) propose a nonparametric theory of leverage (defined as the negative relationship between shocks in stock market returns and volatility). They use the VIX as a market volatility proxy and find a time-variation in leverage “with more negative values associated with higher variance levels.” Delisle, Doran & Peterson (2011) find that sensitivity to VIX innovations is negatively related to asset returns when volatility is rising but is unrelated when it is falling. They conclude that only the increase in VIX is a priced risk factor.

The negative relationship between VIX and S&P 500 returns[17] makes the VIX a perfect tool to hedge a portfolio of volatility risk. Furthermore, the asymmetry in this relationship makes the

[14] The literature also examines the financial leverage effect regarding the direct relationship between spot volatility dynamics resulting from a stochastic volatility model with the stock returns. For more details on this line of research see Yu (2012)

[15] Official white papers for the main international model-free volatility indexes have found similar results.

[16] Recent papers find evidence in favor of this finding, e.g. Kaeck & Alexander (2013).

[17] The S&P 500 returns can be directly computed from data (see Ishida, McAleer & Oya (2011)), or using a GARCH model to control for the returns heteroscedasticity (see Hilal, Poon & Tawn (2011)).

VIX capable for protecting a portfolio value against the downside risk[18]. This makes the VIX a suitable hedging instrument. In particular, investors can replicate VIX (by replicating the SPX option portfolio that defines the index) and use this for hedging, or trade VIX futures ($FVIX$), which movements are also negatively related with S&P 500 returns. Regardless of this decision, the investor must estimate the optimal (minimum variance) hedging ratio h that determines the number of VIX or $FVIX$ contracts necessary to hedge the value of the portfolio S against the volatility risk, see (23) where ρ is the correlation coefficient between stock returns and the hedging instrument returns (leverage coefficient), and F can be VIX (replicated portfolio) or $VIXF$.

$$h = \frac{\sigma_{\Delta \ln S, \Delta F}}{\sigma^2_{\Delta F}} = \rho \frac{\sigma_{\Delta \ln S}}{\sigma_{\Delta F}} \quad (23)$$

If the investor decides to use VIX, we need to control for artificial jumps and computation errors before estimating h. Otherwise, the estimated leverage parameter $\widehat{\rho}$ is inefficient. On the other hand, the relation we assume between the underlying returns and the hedging instrument also affects the ρ distribution. Practitioners and researchers assume this relationship to be linear and mainly consider three methods to estimate it:

- Ordinary Least Squares (OLS):

$$\Delta S_t = \alpha + \beta \Delta F_t + \varepsilon_t,$$

Where: $\hat{h} = \hat{\beta} = \frac{\sigma_{\Delta S, \Delta F}}{\sigma^2_{\Delta F}}$

- Error Correction Models (ECM): we first estimate the cointegration regression

$$S_t = \alpha + \beta F_t + \varepsilon_t \rightarrow \{u_t\} = \{\widehat{\varepsilon_t}\}$$

$$\Delta S_t = \mu + \lambda u_{t-1} + \beta \Delta F_t + \nu_t$$

Where: $\hat{h} = \hat{\beta} = \frac{\sigma_{\Delta S, \Delta F}}{\sigma^2_{\Delta F}}$

- Auto Regressive Distributed Lag (ARDL) cointegration regression, based on proposed by Pesaran (1997):

$$\Delta S_t = \alpha + \beta_1 S_{t-1} + \beta_2 F_{t-1} + \beta_3 \Delta F_t + \nu_t$$

[18] Some authors study the extremal dependence between daily returns on S&P 500 and VIX futures using extreme value theory (see Hilal, Poon & Tawn (2011)), and estimate the optimal downside risk hedge ratio. This is considered an important field of research for hedge-fund managers.

$$\text{Where: } \hat{h} = \hat{\beta}_3 = \frac{\sigma_{\Delta S,\Delta F}}{\sigma^2_{\Delta F}}$$

While there is no clear consensus about the best model to estimate $\hat{h}$ when we use VIX futures, it is known that the OLS method offers worst results when hedging against extreme stock index movements. It is worth noting that the time series properties of S&P 500 returns and VIX affect the ability of each model to relate S&P 500 and VIX fairly, determining the distribution of h around its estimation $\hat{h}$, and then the accuracy of the hedging ratio estimation. Future research on volatility hedging ratios should care about VIX dynamics (artificial jumps and computation errors), distribution and time series properties. If the investor uses VIXF to hedge the portfolio against the volatility risk, she must take into account the different dynamics of VIXF at different maturities (VIXF term structure). In so doing, we may improve our knowledge about the hedge ratio distribution, and the efficiency of the hedge ratio estimator.

[- Insert Table 4 here -]

5. The VIX and the financial leverage effect

The return variance (volatility) risk premium on an asset is defined as the investor's uncertainty about the asset return variance (volatility). Carr & Wu (2008) propose a new methodology to estimate this risk compensation component based on variance swap valuation theory, using the squared model-free VIX. They define the VRP as the difference between the expost annualized realized variance[19] and the squared VIX, which proxies the 30-day synthetic variance swap; see (24). Alternatively, Carr & Wu (2008) also define the log variance risk premium (LVRP) as the difference between the log-realized volatility and the log-VIX to reduce the heteroscedasticity of both variance series, see (25). The authors analyze the dynamics of both variance risk variables on five stock indexes and 35 individual stocks, finding the VRP to be strongly negative for the stock indexes and finding a systematic variance risk factor in the stock market that asks for a highly negative risk premium. The LVRP confirms this finding. Using the VRP definition based on the squared VIX, Todorov (2010) confirms the time-varying dynamics of the VRP previously reported by Bollerslev, Gibson & Zou (2005) and Brandt & Wang (2003), and they conclude that characterizing this dynamic allows for a semiparametric stochastic volatility model and anticipated market jumps. Todorov (2010) finds these factors significant for explaining the stock return dynamics. Duan & Yeh (2010) confirm the significant role of the jump component to price the variance risk and estimate the stock price dynamics, this time

[19] See McAleer & Medeiros (2008) for an extended review of literature on realized volatility

using the CBOE VIX to filter the local stochastic volatility of S&P 500 returns. The squared VIX is a risk neutral variance expectation that assumes nonjump in the underlying price process, and previous results suggest the significance of jumps in the stock dynamics process.

$$VRP_{t,t+30} = 100 \cdot (RV_{t,t+30} - VIX^2) \quad (24)$$

$$LRVT_{t,t+30} = \ln\left(\frac{RV_{t,t+30}}{VIX^2}\right) \quad (25)$$

The misspecification of VIX due to the lack of jumps in the assumed underlying process is also highlighted in Ait-Sahalia, Karaman & Mancini (2013). The authors focus on the expected variance of the S&P 500 at different maturities and compare the term structure of VIX with the term structure of S&P 500 variance swaps, knowing that these are not linked to the absence of jumps in the underlying process. In line with previous results, Ait-Sahalia, Karaman & Mancini (2013) conclude in favor of a significant jump risk component in the term structure of S&P 500 variance swaps, which is consistent with the significant role of jumps in the stock return dynamics as already documented in the literature. Moreover, the authors analyze the term structure of the VRP and conclude that the short-term VRP using the squared VIX mainly reflects the investors' fear of a market crash. In addition, the authors find VRP term-structure downward sloping in quiet times, and upward sloping in turbulent times. Another paper that relates VIX and VRP term structure is Fusari & Gonzalez-Perez (2013), who study the determinants of the VRP term structure and conclude in favor of the business cycle and the short-lived economic uncertainty affecting the slope of the VRP term structure such that this is downward in quite times and upward in turbulent times. The authors also find evidence in favor of the VRP term structure predicting future excess equity returns. Future research in this line could stress out the implications of this result and conclude on the VRP dynamics and its role in the future market returns.

[- Insert Table 5 here -]

6. The VIX as a factor to forecast the underlying variance

Option prices provide information about the RND of the underlying price. The ability of option prices to forecast the underlying variance is linked to the ability of the mean path implied by the option price to forecast the realized path of the underlying return. The model-free squared VIX is a linear combination of option prices that cover a significant part of the volatility smile. Thus the ability of the squared VIX to forecast the underlying realized variance depends on the ability of the specific weighted combination of option prices (that make up the VIX) to match the future variance. This section analyzes the performance of VXO and VIX, or a linear combination of these indexes with other volatility measures, to forecast the stock price volatility and hence, the volatility risk. We start describing the performance of VXO as a forecaster and continue with the forecasting capacities of the model-free VIX.

The model-based VIX is a linear combination of BS ATM implied volatilities in eight option prices. If the option market is informationally efficient, the IV (under the correct option valuation model) provides information about the risk neutral expectation of volatility over the next thirty calendar days, including the historical volatility, allowing it to generate efficient volatility forecasts. Nevertheless, reported results on the role of VXO as a forecaster are mixed. There is strong evidence in favor of significant information subsumed in the IV to forecast the future volatility of an asset, but do we need additional measures (such as model-based historical volatility measures) to improve such forecasting? Poon & Granger (2003) offer a complete review of articles that analyze whether the individual IV and/or the VXO generate efficient and unbiased forecasts of the underlying return volatility. First, attending to individual stocks volatility, since Latane & Rendleman (1976) almost all related papers in the literature conclude in favor of the forecasting capability of the ATM IV; however, the results regarding its efficiency are mixed, differing across the type of asset, pre- or post-crisis periods, frequency and methodology used to compute the realized volatility and the ranking methodology. Moreover, the literature finds that the combination of historical and IV measures are best forecasting the volatility of individual stocks, suggesting that the stock option market inefficiency, since the IV does not subsume all the information. Second, when we intend to forecast the *stock market* volatility, the forecasting capability of VXO is mixed. Blair, Poon & Taylor (2001), Christensen & Prabhala (1998), Fleming (1998), Fleming, Ostdiek & Whaley (1995), Koopman, Jungbacker & Hol (2005) and Szakmary, Ors, Kim & Davidson-III (2002) conclude in favor of the model-VIX efficiency, while Day & Lewis (1992) and Ederington & Guan (2002) find historical volatility models more efficient than VXO for forecasting volatility. Third, although the strong predictive power of IV is confirmed in the currency markets, the efficiency of IV measures to forecast the exchange rate volatility is also mixed, mainly varying with the forecasting horizon; see Poon & Granger (2003) for a review.

By linking the information content in the ATM IV with the rest of the volatility smile, Ederington & Guan (2005) find that the IV from low and ATM strikes is biased and less efficient than the IV related to higher strikes. Because the model-free VIX is defined as a linear combination of option prices over the whole smile, the previous results lead the following question: is the model-free VIX a less efficient or more biased forecast than the model-based VIX? The answers to this question given in the literature are mixed. Jiang & Tian (2005) use tick by tick data and find the model-free VIX to be more efficient than the historical realized volatility and the ATM IV, as well as biased in line with the non-zero VRP found in the literature. The authors use tick by tick data and forecast the S&P 500 volatility. Bollerslev & Zhou (2006) and Becker, Clements & White (2006) also conclude that the model-free VIX is inefficient with respect to alternative volatility measures. Martin, Reidy & Wright (2009) correct the S&P 100 and S&P 500 volatility of market microstructure noise, and evaluate the forecasting performance of different GARCH-related models considering the model-free VIX and the ATM IV as benchmarks. The authors find the model-free VIX to be excessively noisy[20] relative to the ATM, and the model-free VIX tends to exceed the ATM IV forecast, although both model-free and IV volatilities overestimate the realized volatility in low and high volatility periods. In the same line, Taylor, Yadava & Zhanga (2010) analyze individual stocks and find the ATM IV and model-free VIX to be more informative than historical volatility for long prediction horizons and that ATM IV is more informative than the model-free VIX. The higher noise in the model-free VIX than in the ATM IV explains this result. Is this extra-noise generated by the lack of economic coherence in the model-free VIX? The CBOE computes this index considering an economically variant range of strikes, generating artificial jumps in the volatility index. The answer to this question is a matter for a future research, because it depends on the option market microstructure, the type of asset, the forecasting horizon, and the frequency analyzed. Related to this question we must compare the VRP resulting from an *economically invariant* model-free VIX (CIV) with that derived from the CBOE model-free VIX. This comparison will improve the specification of the jump component in the underlying variance and the VRP dynamics.

Indeed, the literature reports evidence in favor of a model-free VIX containing a significant jump component; see Carr & Wu (2003b). Bailey, Zheng & Zhou (2012) suggest that a significant portion of VIX variability is related to trader behavior and macroeconomic fundamentals, and Becker, Clements & McClelland (2009) find VIX to be informative about past and future jump activity in the underlying returns, suggesting that the market generates its volatility forecast considering past jump activity and that this shows a pattern that option prices are able to anticipate. The literature also suggests a noisy model-free VIX with an artificial jump component that bears no relation to its dynamics.

[- Insert Table 6 here -]

7. The VIX derivatives: futures and options

[20] In line with results reported by Andersen, Bondarenko & Gonzalez-Perez (2013), who find artificial jumps in the model-free VIX due to the random range of strikes used to compute the index.

In 2003, the CBOE created the Chicago Futures Exchange (CFE) to manage VIX futures and options on the cash VIX, which become tradable in May 2004 and February 2006, respectively. The VIX futures and options quickly became highly successful among traders. The ability of VIX to explain the US stock index volatility and the negative relationship between the VIX and the US stock index dynamics made the volatility index (i) a good indicator of the US fear gauge and (ii) an adequate tool for trade volatility. These characteristics are crucial to protecting the portfolio value against the volatility risk and/or making a profit while anticipating the volatility dynamics in the same way as with a regular asset.

Starting with Brenner & Galai (1989) several articles analyze different methodologies regarding price volatility derivatives and hedging the volatility risk (Grunbichler & Longstaff (1996), Detemple & Osakwe (2000), Heston & Nandi (2000), Daouk & Guo (2004), among others). This article focuses on VIX derivatives21 , or derivatives on the US stock market volatility, making it necessary to define not only the spot market volatility dynamics but also the evolution of the volatility of the squared VIX. The literature finds different relationships between the SPX and VIX options: (i) the VIX option underlying is the square-root of a forward-starting variance swap, (ii) puts on S&P 500 and calls on VIX protect against similar market dislocations, (iii) and the upper bound on variance options depends on SPX option prices. Moreover, the VIX option price provides information about the risk neutral distribution of forward-starting S&P500 variance swaps. Therefore, VIX options and futures are priced looking for consistency between SPX and VIX options pricing models. However, pricing VIX derivatives (futures and options) requires the inclusion of empirical stylized facts in the model-free VIX dynamics, such as (i) the heteroscedasticity of VIX (significant dependency of VIX volatility on the VIX level revealed in the fast mean-reversion at higher VIX levels), (ii) the long-memory of this series, (iii) the significant jump component of the index, and (iv) seasonal patterns such as a “week of the day effect” (see Gonzalez-Perez & Guerrero (2013)).

Regarding the VIX future pricing models, Zhu & Zhang (2007) report a significant relationship between the variance (square VIX) term structure and the drift term of risk-neutral dynamics for instantaneous variance, and suggest the use of this result to price VIX futures. Zhang, Zhu & Brenner (2010) compare the cash VIX with VIX futures using a square root mean-reverting process with a stochastic long-term mean level for the VIX variance. They find both series to be highly correlated, and the stochastic process adequate to forecast VIX futures prices under a normal market situation. The lack of treatment for the VIX heteroscedasticity explains that the variance model is inadequate to forecast the VIX futures under abnormal market situations. Dupoyet, Daigler & Chen (2010) use a constant elasticity of variance (CEV) model and a Cox-

[21] See VIX options specifications at http://www.cboe.com/Products/indexopts/vixoptions_spec.aspx, and VIX futures specifications at http://cfe.cboe.com/Products/Spec_VIX.aspx1

Ingersoll-Ross (CIR) model (with and without jumps) to fit the VIX dynamics and price VIX futures, concluding in favor of CEV. One more time, the type of heteroscedasticity of VIX (higher VIX, higher VIX volatility) explains the success of CEV versus the CIR model with jumps. Nevertheless, VIX futures are preferred to VIX options to hedge a portfolio against changes in volatility mainly because VIX futures are more sensitive to S&P 500 delta than VIX options.

Regarding the VIX options pricing models, the literature agrees on the importance of including jumps in VIX to model the volatility index dynamics and price VIX options. The standard option valuation theories used to price options on different assets are now used to price derivatives on VIX under different VIX dynamic specifications. Some model rankings are based on simulation exercises, but more often the rankings attend to the ability of each model to approach real VIX option market prices. Wang & Daigler (2011) compare the ability of standard option models[22] to price VIX options based on the ability of each option valuation model to fit real VIX option prices, and conclude that the top model in the ranking varies with the option strike price (OTM strikes are the most difficult to price). On the other hand, Psychoyios, Dotsis & Markellos (2010) suggest the use of the mean reverting logarithmic diffusion process of Detemple & Osakwe (2000) with jumps to fit the VIX dynamics and price VIX options. The log-transformation induces homoscedasticity in the VIX series, beating the non-log model. Recently, Lin (2013) considers the joint use of VIX futures and VIX termstructure to price VIX options and conclude in favor of exponential volatility functions to price VIX calls, and in favor of hump volatility functions to price VIX puts. Gonzalez-Perez & Guerrero (2013) report that the log-transformation is not enough to make the VIX series homoscedastic. This implies that the jump component in log models may be misspecified which may explain differences in the volatility VIX activity at different VIX levels reported in Todorov, Tauchen & Grynkiv (2013). Finally, recent research on VIX options finds the volatility to be increasing on the strike and decreasing with the time to maturity. Hao & Zhang (2013) also link volatility and VIX estimating different GARCH-type models for the underlying returns[23]. The authors assume $VIX_t^2 = a + b \cdot \sigma_{t+1}^2$, where a and b depend on the GARCH parameters, and extract the squared VIX implied by each model. Nevertheless, the authors find that these models do not incorporate a risk premium for the volatility risk and are not able to fit CBOE VIX statistical properties.

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[22] Assuming that the VIX dynamics fit Whaley (1993) (the underlying process follows the Geometric Brownian Motion), Grunbichler & Longstaff (1996) (the underlying process follows a mean-reverting square root process), Carr & Lee (2007) (a model-free approach that does not require estimation of the parameters of the underlying process) and Lin & Chang (2009) (correlated jump-diffusion processes for the S&P 500 index and S&P 500 volatility)

[23] Square-root stochastic autoregressive volatility (SR-SARV) models TGARCH(1,1), AGARCH(1,1) and CGARCH(1,1).

Hilal, Poon & Tawn (2011) show a significant tail dependence between S&P 500 and VIX distributions, which makes joint-dynamics estimation methodologies desirable. Futures research in pricing VIX options should concentrate on the robust estimation of the elasticity parameter in CEV models to fit VIX option prices in a way that is consistent with the SPX pricing model. Indeed, it would be possible to use this VIX option pricing model to suggest the spot volatility dynamics that best fit the SPX option prices in the market. Relative to VIX futures, we should improve our knowledge of the VIX dynamics at the settlement to maximize (minimize) profits (losses) from trading VIX derivatives.

[- Insert Table 7 here -]

8. The VIX as a market risk factor

One of the main uses of VIX in financial literature is related to its role as a market risk proxy. This section describes main uses of the index with this purpose.

The credit spread refers to the yield between different bonds, due to different credit quality. Collin-Dufresne, Goldstein & Martin (2001) study the determinants of credit spread changes and conclude that high-grade bonds move following Treasury bond dynamics, while low-grade bonds are more sensitive to stock returns. Thus, credit and market risks determine the bond yields in different proportions depending on the bond credit rating. In general, corporate bond prices (corporate yield spreads[24]) depend on the credit risk associated with the bond as well as the bond market liquidity[25] , which Bao, Pan & Wang (2011) find linked to the market risk (VIX). In this line, Bhar & Handzic (2011) find VIX to be one of three significant factors explaining the corporate yield spreads, relating peaks in to VIX with declines in the bond market liquidity. Similarly, Azizpour, Giesecke & Kim (2011) find the VIX to be a significant factor to price the risk in the CDX market (CDX is a CDS index class). Gerlach, Schulz & Wolff (2010) find the VIX to be highly correlated not only with the corporate bond yield but also with the sovereign bond yield spread. Wang, Yang & Yang (2013) confirms this result for sovereign debt CDS in Latin America. This reported link between the corporate bond price and the bond liquidity risk, and the link between the bond liquidity and the market risk (VIX) highlight the importance of computing a proper market risk variable to price bonds or corporate yield spreads. Finally, Connolly, Stivers & Sun (2005) uses the VIX to address the stock market volatility and find that the US. stock-bond return correlation is inversely related to VIX.

[- Insert Table 8 here -]

[24] The excess return on a corporate bond over the return on an equivalent Treasury bond (free of credit risk).
[25] See Amihud, Mendelson & Pedersen (2005) for a survey on liquidity affecting asset prices.

The literature also uses the VIX level to measure the market turnover. Hence, Chiarella, He, Huang & Zheng (2012) use the VIX level to define boom and bust periods in the US stock market and show the heterogeneous behavior of the financial market at each stage. Chordia, Roll & Subrahmanyam (2011) study the potential determinants of NYSE turnover across two subperiods (1993-2000 and 2001-2008). Among the three standard variables assumed to run the turnover (differences in analyst's forecasts, return volatility - uncertainty -, and money flow into funds), they use the VIX to approach the return volatility and the uncertainty concluding that it has not been a significant determinant of the sharp upturn in the NYSE turnover, and that only the decline in trading costs and the growing trade by institutional investors explain the rise in turnover.

Another use of model-free volatility indexes is as a market risk measure to address financial market spillovers. Ait-Sahalia, Andritzky, Jobst, Nowak & Tamirisa (2012) study the relationship between the market perception of macroeconomic prospects and the financial market volatility (measured by VIX) and conclude in favor of international spillovers of risk. Similarly, Christopher, Kim & Wu (2012) use VIX (jointly with FX volatilities) to control for common financial and economic uncertainties across stock markets. Chudik & Fratzscher (2011) use VIX as a market risk proxy to determine the wider determinants of global transmission of the 2007-2009 financial crisis (liquidity and/or risk shocks). The authors use a Global VAR approach and study advanced and emerging markets separately[26], concluding that developed countries have been more affected by liquidity shocks, while contagion in emerging markets has mainly been driven by risk shocks[27]. Eichengreen, Mody, Nedeljkovic & Sarno (2012) uses VIX to measure economic volatility and corporate default risk (addressed by HYS) to identify and study the dynamics of common factors in banks' CDS spreads. They find a higher sensitivity of CDS spreads to economic and financial variables after Lehman Brother's failure ignited the global subprime crisis. Corradi, Distaso & Fernandes (2012) study volatility transmission between stocks markets in China, Japan, the UK and the US. They include VIX as a control for serial dependence in the integrated variance[28], concluding in favor of volatility transmission among them[29].

The hedge-fund literature uses VIX as a market volatility proxy to improve the forecasting, monitoring and assessment of hedge-fund returns. Ang, Gorovyy & van Inwegen (2011) report a correlation of 0.89 between VIX and investment bank CDS protection. They conclude that

[26] Total number of 26 economies worldwide
[27] The authors also find emerging economies in Asia to be greatly affected by US-specific liquidity shocks relative to other emerging economies
[28] According to results reported by Bandi & Perron (2006)
[29] Spillovers from China to the US are found to be more significant for jumps.

when the VIX increases, the hedge-fund leverage tends to decrease over the next month (a 1% movement in VIX predicts that gross leverage declines by 0.9% over the next month). Avramov, Kosowski, Naik & Teo (2011) highlight that “some hedge-fund investment styles outperform in times of high market volatility while others perform better in times of low market volatility”, and they conclude that the VIX is crucial for hedge-fund return predictability.

JP Morgan computes a Liquidity, Credit and Volatility Index (LCVI), a combination of seven variables including the VIX, as a proxy for the investor fear gauge and market sentiment. Brown & Cliff (2004) study the information content of several market sentiment direct and indirect proxies, considering the VXO in their measure of expected volatility relative to current volatility. They conclude that these proxies are highly correlated with direct proxies of market sentiment such as surveys and that both direct and indirect variables have little predictive power for near-term future stock returns.

The accounting literature also uses VIX to measure the *market-wide volatility* (see Chakrabarty & Moulton (2012) for the effect of earnings announcements on stock prices). The same can be found in the literature on commodities, where Cochran, Mansur & Odusami (2012) conclude in favor of high sensitivity of return volatility of four metals to VIX (*market volatility*) and suggest the use of this volatility index to model metal returns and volatility.

[- Insert Table 9 here -]

9. Conclusion

This article reviews for the first time the main uses of model-free volatility indexes in the literature to offer an entre for researchers who consider VIX as a proxy for volatility and/or risk. Among the common uses of VIX, we highlight the use of VIX as a market risk factor, its role as a significant spot volatility factor, the success of the volatility index to characterize the leverage effect level and dynamics, and the relationship of VIX with the market realized volatility as a forward-looking volatility measure and VRP factor.

The strong evidence in favor of the relation between VIX and the market returns and volatility dynamics and the negative and significant relationship between VIX movements and S&P 500 returns (financial leverage effect) explains the relevance of VIX in the industry and financial literature.

The significant negative relationship between the VIX and the stock market returns incorporated within the significant leverage effect makes the trading index suitable for implementing volatility-risk hedging strategies. The VIX is tradable by its derivatives (VIX options and futures), and this created a demand for improving models to price VIX options and futures.

Assumptions on the VIX dynamics turns crucial in this research line, and the literature should concentrate on characterizing and finding the stochastic model that better fits the VIX (term structure) dynamics. In particular, pricing models should consider the type of heteroscedasticity of VIX and include a re-defined jump component in the VIX dynamics that is consistent with the SPX options and S&P 500 dynamics.

The role of VIX as a factor that determines the spot volatility dynamics is explained by the risk-information component content in the group of SPX options that make up the VIX. Understanding the role of VIX in the spot volatility dynamics depends on, among other factors, our ability to understand the VIX dynamics. We know that the CBOE VIX is a function, to a greater or lesser degree, of the underlying process we assume, and that the index is contaminated by computation errors that generate artificial jumps and changes in the VIX level. The VIX is sensitive to the underlying process assumed when this includes jumps, so we need to find a theoretical formula for the model-free volatility index that does not depend on the underlying return process. On the other hand, there is evidence in favor of the SV parameters being affected by the computation errors that contaminate the VIX. More research is needed towards the definition of a formula for the model-free VIX that characterizes and minimizes random reported computation errors.

The information content in different international model-free volatility indexes can be used to determine international stock market spillovers and stock market reactions to macroeconomic news and unexpected events. This is especially relevant to understand the convergence process of financial markets (globalization) and how systemic risk flows across countries. Different model-free volatility indexes methodologies are similar, so we can interpret these recommendations such as recommendations towards reducing the computation error of any model-free international volatility indexes to improve their information content. That would definitely help us to understand the flow of risk-related variables across markets and countries, which is of crucial importance to design markets and banking rules.

In short, understanding the composition of the index and studying the time-series properties of VIX is crucial to understand its role in the financial sector. In particular, an adequate detection of VIX jumps and the adequate treatment of the heteroscedasticity of VIX (responsible for a higher VIX volatility at higher VIX level) will improve the estimation of the VRP and spot volatility dynamics and the accuracy of VIX derivative pricing models, among many other uses of VIX in the stock markets. The VIX index and VIX-related products will continue being widely used in financial markets and also analyzed in the financial literature. Future research on VIX dynamics and properties will help us to understand its information content, and to make a good use of the index in the financial markets.

References

Ait-Sahalia, Y., J. Andritzky, A. Jobst, S. Nowak & N. Tamirisa. 2012. "Market Response to Policy Initiatives during the Global Financial Crisis." *Journal of International Economics* 87:162–177.

Ait-Sahalia, Y., M. Karaman & L. Mancini. 2013. "The Term Structure of Variance Swaps, Risk Premia and the Expectation Hypothesis." *Working Paper*.

Amihud, Y., H. Mendelson & L.H. Pedersen. 2005. "Liquidy and Asset Prices." *Foundations and Trends in Finance* 1:269–364.

Andersen, T.G. & O. Bondarenko. 2007. "Construction and Interpretation of Model-Free Implied Volatility." pp. 141–181. *Volatility as an Asset Class*, Edited by I. Nelken, London: Risk Books

Andersen, T.G. & O. Bondarenko. 2010. "Dissecting the Pricing of Equity Index Volatility." *Working Paper*, Northwestern University and University of Illinois at Chicago

Andersen, T.G., O. Bondarenko & M.T. Gonzalez-Perez. 2013. "Uncovering Novel Features of Equity-Index Return Dynamics via Corridor Implied Volatility." *Available at SSRN*: *http://ssrn.com/abstract=1787528 or http://dx.doi.org/10.2139/ssrn.1787528* .

Ang, A., R. Hodrick, Y. Xing & X. Zhang. 2006. "The Cross Section of Volatility and Expected Returns." *Journal of Finance* 51:259–299

Ang, A., S. Gorovyy & G.B. van Inwegen. 2011. "Hedge Fund Leverage." *Journal of Financial Economics* 102:102–126.

Avramov, D., R. Kosowski, N.Y. Naik & M. Teo. 2011. "Hedge Funds, Managerial Skills and Macroeconomic Variables." *Journal of Financial Economics* 99:672–692.

Azizpour, S., K. Giesecke & B. Kim. 2011. "Premia for Correlated Default Risk." *Journal of Economics, Dynamics and Control* 35:1340–1357.

Bailey, W., L. Zheng & Y. Zhou. 2012. "What Makes the VIX Tick?" *Working paper*.

Bakshi, G., C. Cao & Z. Chen. 1997. "Empirical Performance of Alternative Option Pricing Models." *Journal of Finance* 52:527–566

Balasubramnian, B. & K.B. Cyree. 2011. "Market Discipline of Banks: Why are Yield Spreads on Bank-Issued Subordinated Notes and Debentures not Sensitive to Bank Risk?" *Journal of Banking and Finance* 35:21–35.

Bandi, F.M. & B. Perron. 2006. "Long Memory and the Relation Between Implied and Realized Volatility." *Journal of Financial Econometrics* 4:636–670.

Bandi, F.M. & R. Reno. 2012. "Time-Varying Leverage Effects." *Journal of Econometrics* 169:94–113.

Banerjee, P.S., J.S. Doran & D.R. Peterson. 2007. "Implied Volatility and Future Portfolio Returns." *Journal of Banking and Finance* 31:3183–3199.

Bao, J., J. Pan & J. Wang. 2011. "The Illiquidity of Corporate Bonds." *The Journal of Finance* 66(3):911–946.

Bates, D. 2000. "Post-'87 Crash Fears in S&P 500 Futures Options." *Journal of Econometrics* 94:181–238.

Becker, R., A.E. Clements & A. McClelland. 2009. "The Jump Component of S&P 500 Volatility and the VIX Index." Journal of Banking and Finance 33:1033–1038.

Becker, R., A.E. Clements & S.I. White. 2006. "On the Informational Efficiency of S&P 500 Implied Volatility." The North American Journal of Economics and Finance 17:139–153.

Bhar, R. & N. Handzic. 2011. "A Multifactor Model of Credit Spreads." Asia-Pacific Financial Markets 18:105–127.

Black, F. 1976. "Studies of stock market volatility changes." Proceedings of the American Statistical. Association, Business and Economic Statistics Section pp. 177–181.

Blair, B.J., S-H Poon & S.J. Taylor. 2001. "Forecasting S&P 100 Volatility: the Incremental Information Content of Implied Volatilities and High Frequency Index Returns." Journal of Econometrics 105:5–26.

Bohl, M.T., C.A. Salm & M. Schuppli. 2011. "Price discovery and investor structure in stock index futures." The Journal of Future Markets 31(3):282–306.

Bollerslev, T., G. Tauchen & H. Zhou. 2009. "Expected Stock Returns and Variance Risk Premia." The Review of Financial Studies 22:4463–4492.

Bollerslev, T. & H. Zhou. 2006. "Volatility Puzzles: a Simple Famework for Gauging ReturnVolatility Regressions." Journal of Econometrics 131:123–150.

Bollerslev, T., M. Gibson & H. Zou. 2005. "Dynamic Estimation of Volatility Risk Premia and Investor Risk Aversion from Option-Implied and Realized Volatilities." Working paper, Duke University .

Brandt, M. & K. Wang. 2003. "Time-Varying Risk Aversion and Expacted Inflation." Journal of Monetary Economics 58:1457–1498.

Breeden, D. & R.H. Litzenberger. 1978. "State Contingent Prices Implicit in Option Prices." Journal of Business 51:163–180.

Brenner, M. & D. Galai. 1989. "New Financial Instruments for Hedging Changes in Volatility." Financial Analysts Journal July/August:61–65.

Britten-Jones, M. & A. Neuberger. 2000. "Option Prices, Implied Price Processes, and Stochastic Volatility." Journal of Finance 55:839–866.

Brown, G.W. & M.T. Cliff. 2004. "Investor Sentiment and the Near-Term Stock Market." Journal of Empirical Finance 11:1–27.

Carhart, M. 1997. "On persistence in mutual fund performance." Journal of Finance 52:57–82.

Carr, P. & D. Madam. 1998. "Towards a theory of volatility trading." Volatility Editor. R. Jarrow:417–427.

Carr, P. & K. Lewis. 2004. "Corridor Variance Swaps." Risk February:67–72.

Carr, P. & L. Wu. 2003a. "A Tale of Two Indices." The Journal of Derivatives 13(3):13–29.

Carr, P. & L. Wu. 2003b. "What Type of Process Underlies Options? A Simple RobustTest." The Journal of Finance 58(6):2581–2610.

Carr, P. & L. Wu. 2008. "Variance Risk Premiums." The Review of Financial Studies 22 (3):1311–1341.

Carr, P. & R. Lee. 2003. "At-the-money implied as a robust approximation of the volatility swap rate." Working paper, Bloomberg LP .

Carr, P. & R. Lee. 2007. "Realized volatility and variance: Options via swaps." Risk pp. 76–83.

Carr, P., R. Lee & L. Wu. 2012. "Variance Swaps on Time-Changed Levy Processes." Finance and Stochastics 16(2):335–355.

Chakrabarty, B. & P.C. Moulton. 2012. "Earnings Announcements and Attention Constraints: The Role of Market Design." Journal of Accounting and Economics 53:612–634.

Chiarella, C., X-Z He, W. Huang & H. Zheng. 2012. "Estimating Behavioural Heterogeneity under Regime Switching." Journal of Economic Behavior & Organization (forthcoming).

Chordia, T., R. Roll & A. Subrahmanyam. 2011. "Recent Trends in Trading Activity and Market Quality." Journal of Financial Economics 101:234–263.

Christensen, B.J. & N.R. Prabhala. 1998. "The relation between implied and realized volatility." Journal of Financial Economics 5:125–50.

Christie, A.A. 1982. "The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects." Journal of Financial Economics 10:407–432.

Christopher, R., S-J Kim & E. Wu. 2012. "Do Sovereign Credit Ratings Influence Regional Stock and Bond Market Interdependencies in Emerging Countries?" International Financial Markets, Institutions and Money (forthcoming).

Chudik, A. & M. Fratzscher. 2011. "Identifying the Global Transmission of the 20071709 Financial Crisis in a GVAR Model." European Economic Review 55:325–339.

Cochran, S.J., I. Mansur & B. Odusami. 2012. "Volatility persistence in metal returns: A FIGARCH approach." Journal of Economics and Business 64(4):287–305.

Collin-Dufresne, P., R.S. Goldstein & J.S. Martin. 2001. "The Determinants of Credit Spread Changes." The Journal of Finance 56(6):2177–2207.

Connolly, R., C. Stivers & L. Sun. 2005. "Stock market uncertainty and the stockbond return relation." Journal of Financial and Quantitative Analysis 40:161–194.

Corradi, V., W. Distaso & M. Fernandes. 2012. "International Market Links and Volatility Transmission." Journal of Econometrics (forthcoming).

Daouk, H. & J.Q. Guo. 2004. "Switching Asymmetric GARCH and Options on a Volatility Index." Journal of Futures Markets 24:251–282.

Day, T.E. & C.M. Lewis. 1992. "Stock Market Volatility and the Information Content of Stock Index Options." Journal of Econometrics 52:267–287.

Delisle, R.J., J.S. Doran & D.R. Peterson. 2011. "Asymmetric Pricing of Implied Systematic Volatility in the Cross-Section of Expected Returns." The Journal of Futures Markets 31:34–54.

Demeterfi, K., E. Derman, M. Kamal & J. Zou. 1999a. "A Guide to Volatility and Variance Swaps." Journal of Derivatives 6:9–32.

Demeterfi, K., E. Derman, M. Kamal & J. Zou. 1999b. "More than you ever wanted to know about Volatility Swaps." Quantitative Strategies Research Notes. Goldman Sachs March 1999.

Detemple, J. & C. Osakwe. 2000. "The Valuation of Volatility Options." European Finance Review 4:21–50.

Dotsis, G., D. Psychoyios & G. Skiadopoulus. 2007. "An Empirical Comparison of Continuous-Time Models of Implied VOlatility Indices." Journal of Banking and Finance 31:3584–3603.

Duan, J-Ch. & Ch-Y. Yeh. 2010. "Jump and Volaitlity Risk Premiums Implied by VIX." Journal of Economic Dynamics & Control 34(11):2232–2244.

Duan, J-Ch. & Ch-Y. Yeh. 2012. "Price and Volatility Dynamics Implied by the VIX Term Structure." Working Paper .

Dupoyet, B., R.T. Daigler & Z. Chen. 2010. "A simplified pricing model for volatility futures." Journal of Futures Markets 31(4):307–339.

Ederington, L.H. & W. Guan. 2002. "Is Implied Volatility an Informationally Efficient and Effective Predictor of Future Volatility?" Journal of Risk 4 (3).

Ederington, L.H. & W. Guan. 2005. "The Information Frown in Option Prices." Journal of Banking & Finance 29:1429–1457.

Eichengreen, B., A. Mody, M. Nedeljkovic & L. Sarno. 2012. "How the Subprime Crisis went Global: Evidence from Bank Credit Default Swap Spreads." Journal of International Money and Finance 31(5):1299–1318.

Engle, R.F. & V. Ng. 1993. "Measuring and Testing the Impact of News in Volatility." Journal of Finance 48:1749–1778.

Fama, E.F. & K.R. French. 1993. "Common risk factors in the returns on stocks and bonds." Journal of Financial Economics 25:23–49.

Feinstein, S. 1989. "The Black-Scholes Formula is Nearly Linear in Sigma for At-The-Money Options are Virtually Unbiased. Unpublished manuscriot, Federal Reserve Bank of Altlanta.".

Figlewski, S. 2008. The Implied Risk Neutral Density for the U.S. Market Portfolio. Tim Bollerslev, Jeffrey R. Russell and Mark Watson ed. Oxford, UK: Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle. Oxford University Press.

Fleming, J. 1998. "The quality of market volatility forecasts implied by S&P 100 index option prices." Journal of Empirical Finance 5(4):317–345.

Fleming, J., B. Ostdiek & R.E. Whaley. 1995. "Predicting stock market volatility: A new measure." Journal of Futures Markets 15:265–302.

Fusari, N. & M.T. Gonzalez-Perez. 2013. "Volatility Dynamics and the Term Structure of the Variance Risk Premium." Working Paper .

G ¨ untay, L. & D. Hackbarth. 2010. "Corporate bond credit spreads and forecast dispersion." Journal of Banking & Finance 34:2328–2345.

Gerlach, S., A. Schulz & G.B. Wolff. 2010. "Banking and Sovereign Risk in the Euro Area." Discussion Paper Series 1: Economic Studies, Deutsche Bundesbank, Research Center .

Gilchrist, S., V. Yankov & E. Zakrajsek. 2009. "Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets." Journal of Monetary Economics 56:471–493.

Gonzalez-Perez, M.T. & D.E. Guerrero. 2013. "Day of the Week Effect on VIX. A Parsimonious Representation." The North American Journal of Economics and Finance 25:243–260.

Grunbichler, A. & F. Longstaff. 1996. "Valuing Futures and Options on Volatility." Journal of Banking and Finance 20:985–1001.

Hao, J. & J.E. Zhang. 2013. "GARCH Option Pricing Models, the CBOE VIX, and Variance Risk Premium." Journal of Financial Econometrics (forthcoming).

Heston, S.L. 1993. "A Closed-Form Solution for Options with Stochastic Volatility and Applications to Bond and Currency Options." The Review of Financial Studies 6:327–347.

Heston, S.L. & S. Nandi. 2000. "Derivatives on Volatility: Some Simple Solutions Based on Observables." Working Paper. Federal Bank of Atlanta .

Hilal, A., S-H Poon & J. Tawn. 2011. "Hedging the black swan: Conditional heteroskedasticity and tail dependence in S&P 500 and VIX." Journal of Banking and Finance 35:2374–2387.

Hull, J. & A. White. 1987. "The Pricing of Options on Assets with Stochastic Volatilities." The Journal of Finance 42(2):281–300.

Ishida, I., M. McAleer & K. Oya. 2011. "Estimating the Leverage Parameter of ContinuousTime Stochastic Volatility Models using High Frequency S&P 500 and VIX." Managerial Finance 37(11):1048–1067.

Jiang, G.J. & Y.S. Tian. 2005. "The Model-Free Implied Volatility and Its Information Content." The Review of Financial Studies 18:1305–1342.

Jones, C.S. 2003. "The Dynamics of Stochastic Volatiltiy: Evidence from Underlying and Options Markets." Journal of Econometrics 116:181–224.

Kaeck, A. & C. Alexander. 2013. "Continuous-time VIX dynamics: On the role of stochastic volatility of volatility." International Review of Financial Analysis 28:46–56.

Konstantinidia, E. & G. Skiadopoulos. 2011. "Are VIX futures prices predictable? An empirical investigation." International Journal of Forecasting 27(2):543–560.

Konstantinidia, E., G. Skiadopoulos & E. Tzagkaraki. 2008. “Can the evolution of implied volatility be forecasted? Evidence from European and US implied volatility indices.” Journal of Banking and Finance 32:2401–2411.

Koopman, S. J., B. Jungbacker & E. Hol. 2005. “Forecasting Daily Variability of the S&P 100 Stock Index using Historical, Realised and Implied Volatility Measurements.” Journal of Empirical Finance 12 (3):445–475.

Latane, H.A. & R.J. Rendleman. 1976. “Standard Deviations of Stock Price Ratios Implied in Option Prices.” The Journal of Finance 31:369–381.

Lee, R. 2010. “Gamma Swaps.” Encyclopedia of Quantitative Finance .

Lin, Y-N. 2007. “Pricing VIX futures: Evidence from integrated physical and risk-neutral probability measures.” The Journal of Futures Markets 27(12):1175–1217.

Lin, Y-N. 2013. “VIX option pricing and CBOE VIX Term Structure: A new methodology for volatility derivatives valuation.” Journal of Banking and Finance 37:4432–4446.

Lin, Y-N. & C. Chang. 2009. “VIX option pricing.” The Journal of Futures Markets 29:523–543.

Martin, G.M., A. Reidy & J. Wright. 2009. “Does the option market produce superior forecasts of noise-corected volatility measures?” Journal of Applied Econometrics 24:77–104.

Martin, I. 2012. “Simple Variance Swaps.” Working Paper; Stanford University .

Matsumura, M.S. & J.V. Machado-Vicente. 2010. “The role of macroeconomic variables in sovereign risk.” Emerging Markets Review 11:229–249.

McAleer, M. & M.C. Medeiros. 2008. “Realized volatility: A review.” Econometric Reviews 27(1-3):10–45.

Mencia, J. & E. Sentana. 2013. “Valuation of VIX derivatives.” Journal of Financial Economics 108(2):367–391.

Neuberger, A.J. 1994. “The Log Contract.” Journal of Portfolio Management 20:74–80.

Pan, J. 2002. “The Jump-Risk Premia Implicit in Options: Evidence from an Integrated TimeSeries study.” Journal of Financial Economics 63:3–50.

Pesaran, M.H. 1997. “The role of economic theory in modeling the long run.” Economic Journal 107:178–191.

Poon, S-H & C.W.J. Granger. 2003. “Forecasting Volatility in Financial Markets: A Review.” Journal of Economic Literature 41:478–539.

Psychoyios, D., G. Dotsis & R.N. Markellos. 2010. “A Jump Diffusion Model for VIX Volatility Options and Futures.” Review of Quantitative Finance Accounting 35:245–269.

Sepp, A. 2008. “VIX Option Pricing in a Jump-Diffusion Model.” Risk April 2008:84–89.

Shu, J. & J.E. Zhang. 2012. “Causality in the VIX Futures Market.” Journal of Futures Markets 32:24–46.

Szakmary, A., E. Ors, J.K. Kim & W.N. Davidson-III. 2002. “The predictive power of implied volatility: Evidence from 35 futures markets.” Journal of Banking & Finance 27:2151C2175.

Taylor, S.J., P.K. Yadava & Y. Zhanga. 2010. “The Information Content of Implied Volatilities and Model-Free Volatility Expectations: Evidence from options written on individual stocks.” Journal of Banking & Finance 34(4):871–881.

Todorov, V. 2010. “Variance Risk-Premium Dynamics: The Role of Jumps.” The Review of Financial Studies 23(1):345–383.

Todorov, V., G. Tauchen & I. Grynkiv. 2013. “Volatility activity: Specification and estimation.” Journal of Econometrics (forthcoming).

Tzang, S-W, C-H Hung, C-H Wang & D. S-D Shyu. 2011. “Do liquidity and sampling methods matter in constructing volatility indices? Empirical evidence from Taiwan”. International Review of Economics & Finance, 20 (2):312-324

Wang, A.T., S-Y Yang & N-T Yang. 2013. “Information transmission between sovereign debt CDS and other financial factors - The case of Latin America.” North American Journal of Economics and Finance Article in Press.

Wang, Z. & R.T. Daigler. 2011. “The Performance of VIX Option Pricing Models: Empirical Evidence Beyond Simulation.” The Journal of Futures Markets 31(3):251–281.

Whaley, R.E. 1993. “Derivatives on Market Volatility: Hedging Tools Long Overdue.” Journal of Derivatives 1:71–84.

Whaley, R.E. 2000. “The Investor Fear Gauge.” The Journal of Portfolio Management 26:12–17.

Whaley, R.E. 2009. “Understanding the VIX.” The Journal of Portfolio Management 35:98–105.

Yu, J. 2012. “A Semiparametric Stochastic Volatility Model.” Journal of Econometrics 167:473–482.

Zhang, J.E. & Y. Zhu. 2006. “VIX Futures.” The Journal of Future Markets 26 (6):521–531.

Zhang, J.E., Y. Zhu & M. Brenner. 2010. “The New Market for Volatility Trading.” The Journal of Future Markets 30 (9):809–833.

Zhang, Y., S.J. Taylor & L. Wang. 2013. “Investigating the information content of the modelfree volatility expectation by monte carlo methods.” The Journal of Futures Markets 33(11):1071–1095.

Zhu, M. 2013. “Return distribution predictability and its implications for portfolio selection.” International Review of Economics & Finance 27:209–223.

Zhu, S-P. & G-H Lian. 2012. “An analytical formula for VIX futures and its applications.” The Journal of Futures Markets 32(2):166–190.

Zhu, Y. & J.E. Zhang. 2007. “Variance Term Structure and VIX Futures Pricing.” International Journal of Theoretical and Applied Finance 10(1).

Appendix (Tables)

Table 3: The VIX and the spot returns and volatility dynamics. *Main articles using VIX to better understand the market returns and/or variance dynamics*

Article	Frequency	Variable (Sample)	Results
Jones (2003)	Daily	VXO(Jan86-Jun00)	Assumption: $VIX_t = A + B \cdot V_t + \varepsilon_t$. Estimate SQRT, CEV and 2GAM models to filter V. Main conclusions: [1] CEV; 2GAM >> SQRT ,[2] $\frac{d\sigma_V^2}{dV} > 0$ and bigger when ↑↑ V, [3] $\frac{d\rho}{dV} > 0$
Ang et al. (2006)	Daily	VXO(Jan86-Dec00)	Assumption: VIX ≈ innovations in V, and $r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \beta_{\Delta \mathrm{FVIX}}^i \Delta FVIX_t + \varepsilon_t^i$. Main findings: [1] stocks with ↑↑ V^i →↓↓ r_t^i
Banerjee, Doran & Peterson (2007)	Daily	VXO (Jun86-Jun05)	Assumption: $\lambda = \lambda_V + \gamma$. Model: VIX ≈ innovations in V, and $r_t^{p,T} = \alpha^p + \beta_{MKT}^p MKT_t + \beta_{SMB}^p SMB_t + \beta_{HML}^p HML_t + \beta_{UMD}^p UMD_t + \varepsilon_t^p$, with *T = 30; 60*, and *p = 1,…,12* portfolios. Main fndings: $\frac{dr_t^{p,t}}{dVIX}$ is stronger for ↑↑ beta portfolio.
Duan & Yeh (2010)	Daily	VIX (Jan90-Aug07)	Assumption: $VIX_t = A + B \cdot V_t + \varepsilon_t$. Estimate JCEV model to filter V. Three Jump intensities ($\gamma = \hat{\gamma}, \frac{1}{2}, 1$). Main conclusions: [1] Jumps are critical, [2] SQRT and JSQRT are not adequate
Duan & Yeh (2012)	Daily	VIX (Jan92-Mar09)	Assumption: $VIX_t = A + B \cdot V_t + \varepsilon_t$. Estimate JCEV, HW, SQRT models ($\gamma = \hat{\gamma}, \frac{1}{2}, 1$) considering constant and volatility dependent components in the jump. Main results: [1] SV process is mean reverting, [2] Jump intensity is time-varying, [3] Jump and Vol risks are priced, [4] Measurement errors in VIX affect parameters in the SV model, [5] SQRT and JSQRT are not adequate
Zhu (2013)	Monthly	VIX (Jan90-Feb11)	Assumption: VIX is a broad market variable. Estimate an univariate quantile regression. Main results: VIX results significant to explain the low and high extreme quantiles of stock returns ($\alpha = 0.05, 0.10, 0.7, 0.8, 0.9, 0.95$)

Notes: MKT is the market excess return, *FVIX* is an ex-post factor such that we estimate $\boldsymbol{\Delta VIX_t = a + b' r_t + u_t}$ and $\boldsymbol{FVIX_t = \hat{b}' r_t}$ and *MKT* , *SMB* and *HML* are the three Fama & French (1993) model's market, size, and value factors, respectively; while UMD is a forth factor computed following Carhart (1997). $\boldsymbol{\lambda}$ is the price risk premium,$\boldsymbol{\lambda_v}$ is the volatility risk premium, and $\boldsymbol{\gamma} > 0$ is a constant. MRSRP: Mean-reverting SV square-root process, GBMPJ: Geometric Brownian motion SV process augmented by jumps, MRSRPJ: Mean-reverting square-root SV process augmented by jumps.

Table 4: The VIX in estimating the *Financial Leverage Effect*

Article	Frequency	Variable (Sample)	Results
Whaley (2000)	Weekly	VXO (1995-2000)	Significant and asymmetric leverage. $\rho = 0.47$ when $\downarrow VIX$, and $\rho = -0.71$ when $\uparrow VIX$
Carr & Wu (2003a)	Daily	VIX (01/02/90-10/18/05)	Significant leverage effect ($\rho \approx -0.70$)
Whaley (2009)	Daily	VIX (1983-2008)	Significant and asymmetric leverage. Model: $RVIX_t = \beta_0 + \beta_1 RSPX_t + \beta_2 RSPX_t^- + \varepsilon_t$, $\hat{\beta}_2 = -3.00$, $\hat{\beta}_1 = -1.50$
Hilal, Poon & Tawn(2011)	Daily	VIXF (03/26/04-05/30/08)	The OHR estimated using EV theory outperforms the timevarying minimum variance OLS hedge ratio
Ishida, McAleer &Oya (2011)	Intraday tick(5min)	VIX (09/22/03-12/31/07)	Assumption: Heston's affine-drift square-root SV model. The parametric leverage estimator depends on the price and variance processes. $\rho(realized\ leverage) = -0.51$
Delisle, Doran & Peterson (2011)	Monthly	VIX (Jan86-Dec07)	Models:$R_{it} = \alpha_i + \beta_{MKT,i} MKT_t + \beta_{\Delta \mathrm{VIX,i}} \Delta VIX_t + \varepsilon_{it}$, and $R_{it} = \alpha_i + \beta_{MKT,i} MKT_t + \beta^+_{\Delta \mathrm{VIX,i}} \Delta VIX_t^+ + \beta^-_{\Delta \mathrm{VIX,i}} \Delta VIX_t^- + \varepsilon_{it}$ Main conclusion:$\rho < 0$ when ↑↑ VIX, and $\rho = 0$ when ↓ VIX → " ↑ IV is a priced risk factor."
Bandi & Reno (2012)	Daily	VIX and VIXF (1990-2009)	Model: Kernel estimates of leverage effects (ρ_t). $\rho_t = f(pdf(r, \sigma))$. Empirical findings: for index futures, ρ is stronger at higher variance regimes.

Notes: V IXF : futures on VIX. OHR: Optimal hedge ratio. EV: extreme value theory.

Table 5: The use of VIX to compute the *Variance Risk Premium*

Article	Frequency	Variable (Sample)	Results
Carr & Wu (2003a)	Daily	VIX (Jan/02/90-Oct/18/05)	VRP is time-varying, $VRP < 0, corr(VRP_t, VIX_t) \neq 0, corr(VRPR_t, VIX_t) = 0$
Carr & Wu (2008)	Daily	VIX (Jan96-Feb03)	$RV - SVSR \approx RV - VIX \approx VRP, VRP < 0$
Bollerslev, Tauchen & Zhou (2009)	Monthly	VIX (Jan90-May04)	V RP is time-varying, relates V RP and macro-finance variables dynamics (MacroFinanceVar1)
Todorov (2010)	Daily	VIX (Jan90-Nov02)	V RP dynamics needs including jumps, the occurrence of jumps leads to a persistent $\uparrow VRP$
Duan & Yeh (2010)	Daily	VIX (Jan90-Aug07)	V RP dynamics needs including jumps, the square-root SV model (with and without jumps) is miss-specified
Ait-Sahalia, Karaman & Mancini (2013)	Daily	VIX (Jan/04/96-Sep/02/10)	*VRP < 0*, V RP term-structure is downward sloping in quite times, and upward sloping in turbulent times, $VRP_t(\tau) = VIX_t$ (*VRP* term structure depends on the VIX level)
Fusari & Gonzalez-Perez (2013)	Daily	VIX (Jun08-Jul10)	The *VRP* term structure predicts future excess equity returns, *VRP* term-structure is downward sloping in quite times, and upward sloping in turbulent times ($VRP_t(\tau) = VIX_t$). (VIX level relates V RP and macro-finance variables dynamics (MacroFinanceVar2)

Notes: V RP: Variance Risk Premium, V RPR: VRP defined in excess return terms. RV: realized variance. SV SR: Synthetic Variance Swap Rate. MacroFinanceVar1: Moody AAA bond spread, Housing start, S&P500 P/E ratio, Industrial production, Producer price index, and Payroll employment. MacroFinanceVar2: the spread between BBB and AAA corporate bond yields, and the difference between the bank prime loan rate and the three month T-Bill yield

Table 6: The use of (model-free) VIX to *forecast Future Volatility*

Article	**Frequency**	**Variable (Sample)**	**Results**
Carr & Wu (2003a)	Daily	VIX (Jan/02/90-Oct/18/05)	VIX >> GARCH models to forecast RV
Jiang & Tian (2005)	Daily	VIX (Jan96-May04)	VIX >> VXO to forecast RV
Bollerslev & Zhou (2006)	Monthly	VIX (Jan90-Feb02)	VIX is a downward biased forecaster of RV
Becker, Clements & White (2006)	Daily	VIX (Jan/02/90-Oct/17/03)	$corr(VIX, RV) \neq 0$
Martin, Reidy & Wright (2009)	Daily	VIX (Jun/30/96-Jun/30/06)	ATMIV>>VIX, forecasting the volatility of individual equities and the S&P500 stock index. Main reason mentioned by the authors: VIX exhibits excess variability.
Becker, Clements & McClelland (2009)	Daily	VIX (Jan/02/90-Oct/17/03)	V IX >> VXO for explaining future jump actitity, VIX reflects past jump activity in the S&P 500
Taylor, Yadava & Zhanga (2010)	Daily	VIX (Jan96-Dec99)	ATMIV>>VIX

Notes: RV : realized variance. ATMIV : At-The-Money Implied Volatility

Table 7: Pricing Futures and Options on VIX: *VIX level and VIX derivatives dynamics* (1/2)

Article	**Frequency**	**Variable (Sample)**	**Results**
Grunbichler & Longstaff (1996)	No data	--	Closed-form VIX option and VIX future models (assuming $\sigma_{SP500} \approx$ GBM)
Carr & Wu (2003a)	Daily	VIX (Jan/02/90-Oct/18/05)	$VIXF \leftrightarrow VIXO$. Reasoning: *VIXF* price depends on *VIXO*, *VIXF* informs about the risk-neutral future mean of *VIX*, *VIXF* and SPX option market combined inform about the risk-neutral *VIX* variance. This information can be used to price VIX options.
Zhang & Zhu (2006)	Daily	VIX (Jan/02/90-Jun/06/05)	Relate VIX^2 with σ^2_{SP500} assuming CIR models and different time windows. Main conclusion: "the most recent VIX data should be used to estimate the volatility structural parameters in the VIX futures-pricing model"
Dotsis, Psychoyios & Skiadopoulus(2007)	Daily	VIX, VXO, VXD, VDAX, VX1, VX6 (Oct/14/97-Mar/24/04)	Jump-diffusion SV model for spot volatility to price VIX derivatives
Lin (2007)	Daily	VIX (Apr/21/04-Apr/18/06)	SVJ model to price the short-dated VIXF, including a state-dependent volatility jump improves the pricing of all-dated VIXF (evidence in favor of VIX heteroscedasticity)
Zhu & Zhang (2007)	No data	Weighted Monte Carlo (WMC) simulation	The VRP assumes a Heston stochastic volatility model, and derive an arbitrage-free pricing model for volatility derivatives
Sepp (2008)	Daily	VIXO, VIXF (Jul07)	IV in VIXO are positively skewed, $VIXF = f(\sigma^2_{SP500})$, but $VIXF \neq f(RV)$, *VIXF* dynamics: mean-reverting (reversion speed depending on SV model parameters) with stochastic mean level being a function of own future price.
Zhang, Zhu & Brenner (2010)	Daily	VIX (Mar/26/04-Feb/13/07)	$corr(VIX, SP500) < 0, corr(VIXF, SP500) < 0$, *VIXF* term structure is upward sloping, while the volatility term structure of *VIXF* is downward sloping, a mean-reverting SV model captures the dynamics of *VIXF*.
Dupoyet, Daigler & Chen (2010)	Daily	VIX (Mar04-Aug06), VIXF expire from May04 through Sep06	CEV model is adequate to price VIX futures, CEVJ model does not outperform CEV to price VIXF.

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Table 7: Pricing Futures and Options on VIX: *VIX level and VIX derivatives dynamics* (2/2)

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Article	Frequency	Variable (Sample)	Results
Konstantinidia & Skiadopoulos (2011)	Daily	VIX (Mar/26/04-Mar/17/05)	It is not possible to reject H_0: $VIXF\ market\ is\ informatively\ efficient$
Shu & Zhang (2012)	Daily	VIX (Mar/26/04-May/20/09)	VIX vs VIXF: VIXF market lead spot VIX (Bohl, Salm & Schuppli (2011): the market with more institutional traders will lead the other market), bi-directional causality between VIX and VIXF (suggesting information efficiency in the VIX futures market)
Zhu & Lian (2012)	Daily	VIX (2004-2008)	Provide closed-form solutions to price VIXF under SVJ, SVVJ and SVJJ SV models. Main results: including jumps in the underlying price improves VIXF pricing, while including jumps in the SV model does not.
Mencia & Sentana(2013)	Daily	VIX and VIXF (Feb06-Dec10)	In favor of (SV + central tendency) for the log of VIX to price VIX derivatives

Notes: *GBM* : Geometric Brownian Motion, *VIXF* : VIX futures, *VIXO*: VIX options. *SVJ* : Stochastic Volatility Model with Jumps. *RV* : realized variance. *CEV* : constant elasticity of variance model, *CEVJ*: constant elasticity of variance model with jumps

Table 8: The VIX as a *market risk factor*: studying how the VIX affects *Corporate and/or Sovereign Bond prices and spreads* (1/2)

Article	Frequency	VIX Sample	Dependent Variable	Independent Variables
Collin-Dufresne, Goldstein & Martin (2001)	Annual	Jul88-Dec97	ΔCS	$\Delta Lev, \Delta r^{10}, \Delta slope, \Delta VIX, S\&P, \Delta J$
Gerlach, Schulz & Wolff (2010)	Weekly	Jan99-Feb09	YS(GB)	Lagged YS(GB), τ, BA, VIX, BAs
Bao, Pan & Wang (2011)	Monthly	Jan03-Dec08	Bond Liquidity (γ)	$\Delta VIX, \Delta Bond\ Volatility, \Delta\ CDS\ Index, \Delta Term\ Spread,$ $\Delta Default\ Spread, Lagged\ Stock\ Return, Lagged\ Bond\ return$
Bhar & Handzic (2011)	Monthly	Apr96-Mar03	CS	Kalman Filter (three factors), VIX is related to the first factor
Azizpour, Giesecke & Kim (2011)	Weekly	Aug04-Nov07	CDXHYI, CDXIGI	VIX, CS, LOIS
Balasubramnian & Cyree (2011)	Monthly/ quarterly	Jan94-Dec99	YS(US)	SPRET, TRATE, TSLOPE, VIX, VIXHILO, VIXCLOSE, EURO
Christopher, Kim & Wu (2012)	Daily	Jan94-Jun07	$\Delta SBMC$	ΔFCRatingΔFCOutlook, ΔFXVOL,VIX
Eichengreen et al. (2012)	Weekly	Jul02-Nov08	cov(CDS)	latent factors (VIX, TED, LIBOR)
Gilchrist, Yankov & Zakrajsek (2009)	Monthly	Jan90-Sep00	Economic activity (i.e. VIX)	CCS
Güntay & Hackbarth (2010)	Annual	1987-1998	ΔCS	ΔR(10y), ΔSLOPE, ΔVIX, ΔDISP, ΔVOLEARN, ΔRATINGSQ, ΔVOLRET, RETSP, ΔJUMP

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Table 8: The VIX as a *market risk factor*: studying how the VIX affects *Corporate and/or Sovereign Bond prices and spreads* (2/2)

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Article	Frequency	VIX Sample	Dependent Variable	Independent Variables
Matsumura &Machado-Vicente (2010)	Daily	Feb99-Sep04	SvR(Brasil)	VIX, ln(IBOV ESPA), ln(Real=$), Brazilian Domestic Yields
Wang, Yang & Yang (2013)	Daily	Jul07, Sep08, May10	VIX, CDS, TED, LIBOR, T-bills, LS	Countries analyzed: US, Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela. Concludes on the determinants of CDS premium (VIX is one of them, "Treasury yields, VIX and TED spreads, are important determinants for future sovereign CDS price movements").

Note: YS(GB): yield spread to German Bonds, YS(US): yield spread to US Bonds, τ : time to maturity, BA: bid-ask spread, BAs: Bank assets, VIX: US volatility index, VIXHILO: intra-day change in VIX index, VIXCLOSE: Daily closing value VIX, CDXHYI: CDX High Yield Index, CDXIGI: CDX Investment Grade Index, CS: the spread between the yield on Moody's seasoned 30-year Baa-rated bond and the 10-year Treasury constant maturity rate, LOIS: difference between the 3-month LIBOR and 3-month overnight index swap rate, CS: observed credit spread, SPRET: Average S&P 500 return, TRATE: Closest benchmark treasury rate, TSLOPE: 10 - 2 Year treasury rates, EURO: 30-Day Eurodollar and T-bill rate, SBMC: Stock and bond market returns correlation, FXVOL: Exchange rate volatility, FCRating: the foreign currency sovereign credit ratings, FCOutlook: is the foreign currency sovereign credit outlook, CDS: credit default swap, CCS: corporate credit spread, SvR: Sovereign Rate, Δlev: Change in firm leverage ratio, Δr^{10}: Change in yield on 10-year Treasury, Δslope: Change in 10-year minus 2-year Treasury yields, S&P: Return on S&P 500, DJ : Change in slope of Volatility Smirk. LS: Domestic lending spread. TED: difference between the 3-month LIBOR and 3-month T-Bill rate.

Table 9: The VIX as a *market risk factor: some other uses* (1/2)

Article	**Frequency**	**Variable (Sample)**	**Results**
Chiarella et al. (2012)	Monthly	VIX (Jan00-Jun10)	Use VIX to define 'Boom' and 'Bust' states ('Boom': when ↑ $S\&P$ 500 and the VIX is low. 'Bust': when ↓ S&P 500 and the VIX is high)
Konstantinidia, Skiadopoulos & Tzagkaraki (2008)	Daily	VIX, VXO, VXN, VXD, VDAX, VX1, VX6, VSTOXX (Oct/14/97-Mar/24/04)	Forecasting volatility indexes is not possible. We cannot beat the RW model
Chordia, Roll & Subrahmanyam (2011)	Daily	VIX (1993-2008)	↑ Market Turnover $\neq f(VIX)$
Ait-Sahalia et al. (2012)	Daily	VIX (Ju07-Mar09)	Use VIX to conclude that most policy announcements had international spillovers.
Christopher, Kim & Wu (2012)	Daily	VIX (Jan/01/94-Jul/01/07)	"Stock and bond market co-movements within a region respond heterogeneously to sovereign ratings information."
Chudik & Fratzscher (2011)	Weekly	VIX (Jan/01/05-Aug/07/09)	Contagion in emerging markets has been mainly driven by risk shocks, while developed countries have been more affected by liquidity shocks.
Corradi, Distaso & Fernandes (2012)	Daily and UHF data	VIX (Jan/03/00-Dec/30/05)	China, Japan, UK and US stock markets display significant interconnection mainly through the IVar (one exception: spillovers from China to the US, taking place mainly through price jumps).
Eichengreen et al. (2012)	Daily	VIX (Jul/29/02-Nov/28/08)	Fortunes of international banks rise and fall together (common factors) even in normal times along with short term global economic prospects, the relevance of these common factors raises after the financial crisis
Ang, Gorovyy & van Inwegen (2011)	Monthly	VIX (Dec04-Oct09)	Changes in hedge fund leverage tend to be more predictable by economy-wide factors (VIX) than by fundspecific characteristics.

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Table 9: The VIX as a *market risk factor: some other uses* (2/2)

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Article	**Frequency**	**Variable (Sample)**	**Results**
Avramov et al. (2011)	Monthly	VIX (Jan90-Dec08)	Incorporating predictability (based on the default spread and the VIX) is important in forming optimal portfolios of hedge funds.
Brown & Cliff (2004)	Weekly	VIX (Jul/24/87-Dec/18/98)	The Investor/Market Sentiment has little predictive power for near-term future stock returns.
Chakrabarty & Moulton (2012)	Daily	VIX (Jun06-May07)	Use the VIX to conclude on the effects of earnings announcements on stock prices
Connolly, Stivers & Sun (2005)	Daily	VIX (1998-2000)	Δ(corr(USbonds; USreturns)) = inversely related to VIX level.

Note: RW: Random Walk. IVar: Integrated Variance