MEASURING HEDGING EFFECTIVENESS OF INDEX FUTURES CONTRACTS: DO DYNAMIC MODELS OUTPERFORM STATIC MODELS? A REGIME-SWITCHING APPROACH

ENRIQUE SALVADOR* and VICENT ARAGÓ

This study estimates linear and nonlinear GARCH models to find optimal hedge ratios with futures contracts for some of the main European stock indexes. By introducing nonlinearities through a regime-switching model, we can obtain more efficient hedge ratios and superior hedging performance in both an in-sample and an out-sample analysis compared to the other methodologies (constant hedge ratios and linear GARCH). Moreover, nonlinear models also reflect the different patterns followed by the dynamic relationship between the volatility of spot and futures returns during low- and high-volatility periods. © 2013 Wiley Periodicals, Inc. Jrl Fut Mark 34:374–398, 2014

1. INTRODUCTION

Over the past two decades, with the development of the derivatives markets, much of the literature has focused on techniques to reduce investment risk. One simple technique for this purpose is hedging with futures contracts, which despite its simplicity has received extensive research attention (Johnson, 1960; Ederington, 1979; Myers & Thompson, 1989; Cheung et al., 1990; Chen et al., 2003).

There is a great controversy in the literature as to whether dynamic hedging (hedge ratios are updated with the arrival of new information into the market) significantly improves the effectiveness reached with static strategies. Several authors (Myers, 1991; Kroner & Sultan, 1993; Park & Switzer, 1995) show that dynamic hedge ratios outperform constant hedge ratios in terms of reducing the portfolio risk. However, there are some studies where the main conclusion is just the opposite (Lien & Tse, 2002; Cotter & Hanly, 2006; Park & Jei, 2010).

One of the motivations behind this study is to provide empirical evidence on these contradictory results and to analyze whether more complex models better fit the financial

Enrique Salvador and Vicent Aragó are collocated at the Department of Accounting and Finance, Universitat Jaume I, Castellon de La Plana, Spain. The authors are grateful for financial support from the *Ministerio de Educación y Ciencia* project ECO2011-27227, Programa de Fomento de Proyectos I+D Universitat Jaume I P1 1B2012-07 and *Fundació Caixa Castelló Bancaixa* project P1 1B2009-54. They also appreciate the insightful comments from one anonymous referee throughout the refereeing process.

*Correspondence author, Department of Accounting and Finance, Universitat Jaume I, Avda. Sos Baynat s/n, Castellon de La Plana, Spain. Tel: +34-964-387146, Fax: 34-964-728565, e-mail: esalvado@uji.es

Received June 2011; Accepted December 2012

The Journal of Futures Markets, Vol. 34, No. 4, 374–398 (2014)

Published online 14 February 2013 in Wiley Online Library (wileyonlinelibrary.com).
DOI: 10.1002/fut.21598

series patterns and provide superior hedging effectiveness. The results of the study show that considering nonlinearities in the volatility specification leads to differences in the estimations and forecasts of volatility. These differences have an impact on the hedge ratios obtained and the effectiveness reached, causing the nonlinear models to achieve better effectiveness. The results and conclusions reached are robust across countries, independent of the effectiveness measure considered and independent of the consideration of ex-ante or ex-post analysis.

The main contribution of our work consists of considering jointly most of the financial series characteristics analyzed in previous works that could have an effect on the optimal hedge ratio and its effectiveness: (a) cointegration relationships between spot and future markets (Ghosh, 1993; Alizadeh et al., 2008); (b) the asymmetric response of volatility against positive and negative shocks (Brooks et al., 2002); and (c) nonlinearities originated by regime-switches in the spot–future relationship (Lee, 2010). Not considering one of these aspects could influence the estimation and the forecast of the covariance matrix and, therefore, the hedge effectiveness obtained.

No previous study has jointly considered these three characteristics. On the one hand, Alizadeh et al. (2008) do not include the asymmetric volatility term in their model, and Brooks et al. (2002) highlight the importance of allowing optimal hedge ratios to be both time varying and asymmetric. On the other hand, Lee and Yoder (2007a,b), Lee (2009a,b), and Lee (2010) do not reflect on potential cointegration relationships between the spot and future markets. However, the empirical evidence suggests that not considering this aspect leads to misspecified models and underestimation of the optimal hedge ratio (Lien, 1996; Brooks et al., 2002). In this study, we take one step further and consider regime-switching parameters in the error correction term that allows us to analyze whether the speed of adjustment to the long-run equilibrium varies among regimes (Alizadeh et al., 2008).

In our empirical study, we use several multivariate GARCH models. More specifically, we use the traditional BEKK model (Baba, Engle, Kraft, & Kroner, 1990; Engle & Kroner, 1995), and we estimate asymmetric BEKK models (Brooks et al., 2002) to include the well-known "leveraged effect"[1] of volatility. The existence of cointegration relationships between the spot and futures markets leads us to the incorporation of an error correction term (ECT) in the mean equation (Ghosh, 1993; Lien, 1996; Alizadeh et al., 2008). Finally, we also propose more complex models that consider nonlinear relationships with a regime-switching GARCH specification (Lien, 2011). This approach allows us to compare the effectiveness of the linear GARCH models with that of the nonlinear GARCH models. The effectiveness of the hedging strategy is measured through several approaches. Firstly, we compute the variance reduction of the different hedging strategies over the unhedged portfolio (Ederington, 1979). Secondly, we analyze the economic significance of the risk reduction in terms of investor utility (Kroner & Sultan, 1993). Variance reduction is a good risk measure for a hedge strategy if the returns follow a normal distribution, but this assumption is not always satisfied (Park & Jei, 2010). To avoid this problem, we also estimate alternative effectiveness measures based on loss distribution tails such as Value at Risk (VaR) (Jorion, 2000) and Expected Shortfall (ES; Artzner et al., 1999).

The study is performed for several European markets using the main stock index in each case (namely the FTSE for the U.K., DAX for Germany and Eurostoxx50 for Europe) and their future contracts considering an ex-post and ex-ante analysis, with the latter approach being closer to the decision process followed by an investor/hedger. The out-sample analysis also includes the last financial crisis to show the best hedging models for periods of market jitters.

[1]The "leverage effect" is the different response of volatility to shocks of different sign (Nelson, 1991; Glosten et al., 1993).

The outline of the study is as follows. Section 2 reviews the controversy regarding whether static models provide more effective hedging strategies than dynamic models. Section 3 presents the database used in the study. Section 4 introduces the empirical methodology. Section 5 shows the main empirical results of the study, analyzing the optimal hedge ratio estimations and the proposed effectiveness measures. Finally, we present the main conclusions of the study.

2. STATIC VERSUS DYNAMIC MODELS

The pioneering work using constant hedge ratios was performed by Ederington (1979). In this approach, the hedge ratio is ($\mathrm{HR} = \sigma_{sf}/\sigma_f^2$). This hedge ratio is estimated through the slope of the ordinary least squares (OLS) regression between the spot and futures returns.

However, this approach exhibits several problems. One problem is that it does not account for the long-run disequilibrium between the spot and futures markets (Ghosh, 1993; Lien, 1996). Another problem is that it assumes constant conditional second-order moments; therefore, static hedging is not conditional on the arrival of information into the market. There are essentially two approaches to obtain dynamic hedge ratios. The first approach consists of allowing the hedge ratios to be time-varying coefficients and estimating these coefficients directly (Alizadeh & Nomikos, 2004; Lee et al., 2006). The second approach (Kroner & Sultan, 1991; Brooks et al., 2002) uses the conditional second-order moments of the spot and futures returns from multivariate GARCH models, which allow for the estimation of hedge ratios at time t adjusted for the information set available to the investor at $t-1$: ($\mathrm{HR}_t = \frac{\sigma_{s,f}}{\sigma_f^2}|\Omega_{t-1}$).

Most of the literature has focused on this second approach, proposing increasingly complete models that more accurately capture the characteristics of the financial data and thereby overcome the limitations of the simpler GARCH models. One of the limitations of GARCH models is that they are incapable of reliably capturing the patterns of financial data series, specifically the asymmetric impact of news (Engle & Ng, 1993; Glosten et al., 1993; Kroner & Ng, 1998). Negative shocks are widely known to have a greater impact on financial series than positive shocks. This fact should be taken into account when the hedge ratios are estimated. Brooks et al. (2002) conclude that hedging effectiveness is greater when this asymmetric behavior is considered. A further limitation of GARCH models is that they consider high-volatility persistence. This high persistence level suggests the presence of several regimes in the volatility process (Marcucci, 2005). Ignoring these regime shifts could lead to inefficient volatility estimations. Therefore, the consideration of several regimes in the volatility process could lead to more accurate estimations of volatility and, thus, a better performance for hedging strategies. This approach is described in Hamilton and Susmel (1994), who use a switching ARCH (SWARCH) model to introduce regime switches. Susmel (2000) also analyzes the possibility of regime switches but uses an E-SWARCH specification that allows him to consider asymmetry, and he concludes that both ARCH and the asymmetric effects are reduced when regime switches are introduced.

In recent years, regime-switching models have taken on a new dimension with the development of the Markov regime switching (MRS) models. Sarno and Valente (2000) propose a multivariate version of Hamilton's (1989) MRS model. Alizadeh and Nomikos (2004) were the first to use this methodology to estimate time-varying hedge ratios. These authors consider the slope of the OLS regression between spot and futures returns (minimum variance hedge ratio) to be regime-dependent. Chen and Tsay (2011) use a similar methodology including the state-dependent autoregressive terms of the spot and future returns. In all of these studies, the regime-switching is considered in the mean equation, assuming the variance to be constant over time but dependent on the state.

Another way to consider the regime-switching influence on the optimal hedge ratio estimation is through regime-switching-GARCH models (RS-GARCH) (Lee & Yoder 2007a,b; Alizadeh et al., 2008; Lee, 2009a,b; Lee, 2010). Lee and Yoder (2007a) first consider a regime-switching time varying correlation model. Then, Lee and Yoder (2007b) develop a new MRS-BEKK model in which they extend the work of Gray (1996) to the bivariate case. These studies propose a recombining method for conditional covariance matrices that allow the models to be tractable. They focus on modeling the variance and disregard the behavior of the mean. Alizadeh et al. (2008) incorporate an error correction term (ECT) that allows the series characteristics to be related in the short and long run. Regarding the previous studies, these works allow for both time-varying and state-dependent conditional variances.

Besides these studies, Lee (2009a) develops a regime-switching Gumbel–Clayton copula GARCH model where the return series are modeled using a switching copula instead of assuming bivariate normality. An independent switching GARCH process for every state is considered to avoid the path dependency problem. Lee (2009b) also develops a Markov Regime Switching orthogonal GARCH with conditional jumps dynamics. In Lee (2010), it is considered an independent switching dynamic conditional correlation GARCH with more than 2 regimes (2 is the number of regimes commonly used in previous studies).

The evidence from the studies including regime switches shows that more robust estimates are generated if volatility is allowed to follow different regimes depending on the market conditions, with the result that the hedge effectiveness will be greater. Lien (2009) analyzes and demonstrates why static models (OLS) can outperform GARCH models. Recently, Lien (2011) derives the form of the hedge ratio when regime-switching GARCH models are considered[2] and compares it to the static and the GARCH models. Lien (2009, 2011) notes that the existence of structural breaks or different regimes of volatility in financial series can improve the performance of dynamic models (RS-GARCH models) or, at least, that the consideration of these in the estimated models improves effectiveness. More specifically, this author determines a theoretical superiority in terms of risk reduction for the RS-GARCH model over the OLS and GARCH hedging strategies.

3. DESCRIPTION OF THE DATA AND PRELIMINARY ANALYSES

The data used in this study include weekly closing prices[3] (Alizadeh & Nomikos, 2004; Alizadeh et al., 2008; Chen & Tsay, 2011) for some of the main European stock indexes and their futures contracts. Specifically, we use information on the U.K. (FTSE100), Germany (DAX30), and Europe (Eurostoxx50). The time horizon includes observations from July 1, 1998 to September 30, 2010. We divide this data into two sub-samples: observations from July 1, 1998 to December 31, 2008 (548 observations) are used for the in-sample analysis and observations from January 1, 2009 to September 30, 2010 (92 observations) are used for the out-sample study. We obtained the index data from Thomson DataStream and the futures information from the Institute of Financial Markets Data Center.

We construct the continuous futures series using the contract closest to maturity.[4] Weekly return series are computed as the logarithmic differences multiplied by 100.

$$r_{i,t} = 100\left(\log\frac{P_{i,t}}{P_{i,t-1}}\right) \text{ for } i = \{s,f\} \tag{1}$$

[2]Lee (2009a) also derives a formula for the two-state regime-switching hedge ratio.

[3]Wednesday closing prices are used as weekly observations. If a Wednesday is not available in a week, it is replaced by the Tuesday in that week.

[4]Carchano and Pardo (2008) show that rolling over the futures series has no significant impact on the resultant series. Therefore, the least complex method can be used for series construction to reach the same conclusions.

Descriptive statistics are presented in Table I. Panel A shows the main summary statistics for the spot and futures indexes. Certain results are noteworthy. For the returns, negative values are present in the third-order moments. There is also excess kurtosis in the returns (fat tails). Finally, note that the Jarque–Bera normality test (1980) is rejected because of the asymmetric and leptokurtic characteristics of the series. The results for the out-sample period differ only slightly from those of the in-sample period.[5] Panel B displays the serial autocorrelation tests for the series in levels and squares. The Ljung–Box statistics for the squared series suggest evidence of conditional heteroskedasticity for both series. There is also evidence of serial correlation for the returns in levels, so it is necessary to include structure (lags) in the mean equation.[6] Panel C reflects the stationarity tests performed over the price series and reveals that the price series are I (1), so we have to work with the returns series for stationarity reasons. Finally, Panel D presents the results of the cointegration tests for the series studied. The results also show that both series are cointegrated. Therefore, these relationships should be introduced in the specification of the model used to calculate the hedge ratios, because otherwise we would obtain inefficient hedges (Lien, 1996).

4. METHODOLOGY

This section explains and develops the empirical models used to estimate the time varying volatilities and hedge ratios. We start with the symmetric and asymmetric linear specifications (BEKK and GJR-BEKK) to model the dynamic relationship between the spot and futures returns. After that, we assume nonlinear dynamics through a regime-switching process, thereby allowing the hedge ratios to be dependent on the state of the market.

4.1. Linear Bivariate GARCH Models

Linear bivariate GARCH models have been widely used in the analysis of dynamic hedge ratios (Baillie & Myers, 1991; Park & Switzer, 1995). One of the most frequently used is the BEKK model (Baba et al., 1990) because it incorporates certain characteristics[7] that make it particularly attractive for this type of study. In this specific case, we incorporate an ECT in the mean equation[8] because both series are cointegrated. Let $r_{s,t}$ and $r_{f,t}$ be the spot and futures returns at period t, respectively; thus, we define the mean equation as follows:

$$r_{s,t} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_3 ECT_{t-1} + e_{s,t}. \tag{2}$$

$$r_{f,t} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_3 ECT_{t-1} + e_{f,t}. \tag{3}$$

$$e_t|\Omega_{t-1} = \left.\begin{pmatrix} e_{s,t} \\ e_{f,t} \end{pmatrix}\right| \Omega_{t-1} \sim BN(0, H_t) \tag{4}$$

[5]The out-sample data run from January 1, 2009 to September 30, 2010 (92 observations). The descriptive statistics, not presented in this study, are available from the authors upon request.

[6]The existence of serial correlation in returns could bias the estimation of conditional second moments. To remove this undesirable feature, we add an AR(1) term in the mean equation.

[7]The main advantage of this model is that it guarantees that the covariance matrix will be a positive definite by construction (quadratic form).

[8]We also include an AR(1) term in the mean equation following previous studies (Lo & McKinley, 1990), which claim that the omission of this autoregressive component has serious implications for the variances, autocorrelations, and cross-autocorrelations of individual stocks as well as portfolios due to the asynchronous trading of some components in the stock indexes. In addition, the model selection criteria (likelihood-ratio test) also recommends the inclusion of the AR(1) term. These results are available upon request.

TABLE I
Summary Statistics for Prices and Returns of Spot and Futures on the Selected European Indexes

	United Kingdom		Europe		Germany	
In-sample[a]	*Spot*	*Futures*	*Spot*	*Futures*	*Spot*	*Futures*
Panel A. Summary statistics						
Mean	−0.0792	−0.0671	−0.0710	−0.0726	−0.0462	−0.0470
Standard deviation	7.0904	6.5581	10.7594	11.1216	13.0599	12.7500
Skewness	−1.2664	−0.4259	−0.8156	−0.4338	−0.6041	−0.3824
Kurtosis (excess)	16.1331	9.1974	10.2306	6.9790	8.0840	6.2083
JB test	4077.29***	891.91***	1252.21***	378.00***	622.37***	247.93***
Panel B. Autocorrelation test						
LB-Q (7)	37.256**	23.541	35.065**	35.457**	33.335**	34.082**
LB-Q^2 (7)	107.73***	124.58***	87.11***	110.64***	137.39**	159.92**
Panel C. Stationarity test						
Dickey–Fuller	−25.170*** (**−0.4435**)	−23.954*** (**−0.4782**)	−25.071*** (**−0.6358**)	−24.760*** (**−0.6514**)	−24.190*** (**−0.3491**)	−23.515*** (**−0.3565**)
Phillips–Perron	−25.171*** (**−0.4435**)	−23.955*** (**−0.4782**)	−25.071*** (**−0.6358**)	−24.760*** (**−0.6514**)	−24.191*** (**−0.3491**)	−23.516*** (**−0.3565**)

In-sample	*Lags*	H_0	*Trace Statistic*	*Eigen Statistic*
Panel D. Cointegration test				
Europe	1	$r \leq 0$	63.999***	62.708***
		$r \leq 1$	1.2935	1.2935
United Kingdom	1	$r \leq 0$	89.394***	87.298***
		$r \leq 1$	2.0961	2.0961
Germany	1	$r \leq 0$	199.648***	197.485***
		$r \leq 1$	2.1585	2.1585

Note. This table presents the descriptive statistics for the in-sample series (spot and futures) for Europe, the U.K., and Germany. The JB test is the Jarque–Bera (1980) test for normality. LB-Q (7) and LB-Q2 (7) are the Ljung–Box (1978) test for series autocorrelation for the series in levels and squares.

where a_i, b_i for $i = \{0, 1, 2, 3\}$ are the parameters to be estimated. The subscripts s and f indicate spot or futures, respectively; $e_{s,t}$ and $e_{f,t}$ indicate innovations; Ω_{t-1} denotes the information set available up to $t-1$; BN refers to the bivariate normal distribution and H_t is a positive definite time-varying 2×2 matrix defined as follows:

$$H_t = \begin{pmatrix} h^2_{s,t} & h_{fs,t} \\ h_{sf,t} & h^2_{f,t} \end{pmatrix} = C'C + A'e_{t-1}e'_{t-1}A + B'H_{t-1}B,$$
$$\text{where} \quad C = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \tag{5}$$

where C is a triangular matrix of constants and A, B are 2×2 square matrices of coefficients to be estimated.

Assuming that the innovations follow a bivariate normal distribution, the unknown parameters $\theta = (a_i, b_i, C^l, A_{2x2}, B_{2x2})$ for $i = \{0, 1, 2, 3\}$ are estimated by maximizing the following likelihood function with respect to θ:

$$f(r_t;\theta) = (2\pi)^{-1}|H_t(\theta)|^{-\frac{1}{2}}\exp\left(-\frac{1}{2}e_t(\theta)'H_t^{-1}e_t(\theta)\right). \tag{6}$$

$$L(\theta) = \sum_{i=1}^{T}\log f(r_t;\theta), \tag{7}$$

where T is the number of observations.

GARCH models allow us to obtain an estimation of the variance–covariance matrix for each period. We obtain the dynamic hedge ratio (HR_t) estimations, according to (8):

$$\frac{HR_t = \hat{h}_{sf,t}}{\hat{h}^2_{f,t}.} \tag{8}$$

This simplest variance specification (shown in (5)) can be used to incorporate other financial series characteristics such as asymmetries in volatility. One of the most popular approaches in the literature is the GJR model of Glosten et al. (1993), which uses specific variables to incorporate this asymmetric behavior.

$$H_t = \begin{pmatrix} h^2_{s,t} & h_{fs,t} \\ h_{sf,t} & h^2_{f,t} \end{pmatrix} = C'C + A'e_{t-1}e'_{t-1}A + B'H_{t-1}B + D'\eta_{t-1}\eta'_{t-1}D \tag{9}$$

where D is a diagonal 2×2 matrix of parameters (similar than matrices A and B) to be estimated and $\eta_t = \min(e_t, 0)$. The remaining parameters and variables are the same as those in (2), (3) and (4), and the estimation procedure is similar to that above.

4.2. Nonlinear Bivariate GARCH Models

In contrast to previous models, in which the dynamic relationship between spot and futures returns is characterized by linear patterns, the model presented by Lee and Yoder (2007a) allows regime shifts, which suggests that one can obtain more efficient hedge ratios and superior hedging performance from this model regarding other methods. These types of nonlinear models open up a new line for dynamic hedging in which the returns process is state-dependent. Let $r_{s,t,st}$ and $r_{f,t,st}$ be the state-dependent spot and futures returns at t, respectively; we define the state-dependent mean equations as follows:

$$r_{s,t,st} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_{3,st} ECT_{t-1} + e_{s,t,st} \tag{11}$$

$$r_{f,t,st} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_{3,st} ECT_{t-1} + e_{f,t,st} \tag{12}$$

$$e_{t,st}|\Omega_{t-1} = \left.\begin{pmatrix} e_{s,t,st} \\ e_{f,t,st} \end{pmatrix}\right|\Omega_{t-1} \sim BN(0, H_{t,st}) \tag{13}$$

where a_i, b_i for $i = \{0, 1, 2, 3\}$ are the parameters to be estimated. Based on the model selection criteria,[9] these parameters are not considered to be state-dependent. However, following Alizadeh et al. (2008), the parameters accompanying the ECT depend on the regime $s_t = \{1, 2\}$.

The state-dependent innovations $e_{t,st}$ follow a bivariate normal distribution that depends on the state $s_t = \{1, 2\}$. This state variable follows a two-state first-order Markov process with transition probabilities:

$$P = \begin{pmatrix} \Pr(s_t = 1|s_{t-1} = 1) = p & \Pr(s_t = 1|s_{t-1} = 2) = (1-q) \\ \Pr(s_t = 2|s_{t-1} = 1) = (1-p) & \Pr(s_t = 2|s_{t-1} = 2) = q \end{pmatrix}, \tag{14}$$

where p represents the probability of continuing in state 1 if it was previously in state 1 and q represents the probability of continuing in state 2 if it was previously in state 2.

The state-dependent conditional second-order moments $H_{t,st}$ follow an asymmetric BEKK[10] specification model that takes different values depending on the value of $s_t = \{1, 2\}$. Because of this state dependence, the model will become intractable as the number of observations increases. To resolve this problem, we apply the recombining method used in Gray (1996) where the path dependency problem is solved for univariate models. Lee and Yoder (2007a) extend this recombining method for the bivariate case. Thus, the variance specification in each state is defined as follows:

$$H_{t,st} = \begin{pmatrix} h^2_{s,t,st} & h_{fs,t,st} \\ h_{sf,t,st} & h^2_{f,t,st} \end{pmatrix} = C'_{st}C_{st} + A'_{st}e_{t-1}e'_{t-1}A_{st} + B'_{st}H_{t-1}B_{st} + D'_{st}\eta_{t-1}\eta'_{t-1}D_{st} \tag{15}$$

where $h^2_{s,t,st}$ and $h^2_{f,t,st}$ are the conditional variances of the spot and futures in period t for each state s_t, and $h_{sf,t,st}$ is the conditional covariance in t for each s_t. C_{st}, A_{st}, B_{st}, and D_{st} are the matrices of parameters to be estimated as in the previous models.

The consideration of several states leads to a noteworthy increase in the number of parameters to estimate. To reduce this over-parameterization, the difference between states is

[9]Following the insightful suggestions from an anonymous referee, we perform the entire in-sample analysis for the non-linear models using different mean equation specifications: (a) allowing all parameters to switch between regimes; (b) not allowing any parameter to switch between regimes; and (c) not including the ECT. However, based on the hedging effectiveness results and the selection criteria between models (likelihood-ratio test), none of the alternative models improved the model presented in the text. Results are available from the authors upon request. Thus, these results, which are consistent with studies such as Alizadeh et al. (2008), lead us to only consider the parameters measuring the speed of adjustment to the long-run equilibrium between spot and future prices as the switching parameters.

[10]We also present the results for the symmetric MRS-BEKK model. This model is similar to that presented in the study except for the variance equation, where the last summation $D'_{st}\eta_{t-1}\eta_{t-1}D_{st}$ is not considered.

defined by four new parameters, *sa*, *sb*, *sc*, and *sd*, that properly weight the estimations obtained in one state for the other state[11] (Capiello & Fearnley, 2000). Therefore, the state-dependent covariance matrices in our model are as follows:

$$H_{t,s_t=1} = \begin{pmatrix} h^2_{s,t,1} & h_{fs,t,1} \\ h_{sf,t,1} & h^2_{f,t,1} \end{pmatrix} = C'_1 C_1 + A'_1 e_{t-1} e'_{t-1} A_1 + B'_1 H_{t-1} B_1 + D'_1 \eta_{t-1} \eta'_{t-1} D_1. \tag{16a}$$

$$H_{t,s_t=2} = \begin{pmatrix} h^2_{s,t,2} & h_{fs,t,2} \\ h_{sf,t,2} & h^2_{f,t,2} \end{pmatrix} = C'_2 C_2 + A'_2 e_{t-1} e'_{t-1} A_2 + B'_2 H_{t-1} B_2 + D'_2 \eta_{t-1} \eta'_{t-1} D_2, \tag{16b}$$

where $C_2 = sc \cdot C_1$, $A_2 = sa \cdot A_1$, $B_2 = sb \cdot B_1$, $D_2 = sd \cdot D_1$, A_1, and B_1 are 2×2 matrices of the parameters, C_1 is a 2×2 lower triangular matrix of the constants and D_1 is a diagonal 2×2 matrix of the parameters.

The basic equations of the recombining method[12] that is used to collapse the variances and covariances of the spot and futures errors and to ensure the model is tractable are described below:

$$h^2_{i,t} = \pi_{1,t}(r^2_{i,t,1} + h^2_{i,1,t}) + (1 - \pi_{1,t})(r^2_{i,t,2} + h^2_{i,2,t}) - (\pi_{1,t} r_{i,t,1} + (1 - \pi_{1,t}) r_{i,t,2})^2 \ \text{ for } \ i = \{s, f\}. \tag{17}$$

$$e_{i,t} = \Delta S_t - (\pi_{1,t} r_{i,t,1} + (1 - \pi_{1,t}) r_{i,t,2}). \tag{18}$$

$$\begin{aligned} h_{sf,t} = {} & \pi_{1,t}(r_{s,t,1} r_{f,t,1} + h_{sf,1,t}) + (1 - \pi_{1,t})(r_{s,t,2} r_{f,t,2} + h_{sf,2,t}) \\ & -(\pi_{1,t} r_{s,t,1} + (1 - \pi_{1,t}) r_{s,t,2}) - (\pi_{1,t} r_{f,t,1} + (1 - \pi_{1,t}) r_{f,t,2}), \end{aligned} \tag{19}$$

where $h^2_{i,t}$, $h_{sf,t}$ are the state-independent variances and covariances aggregated by the recombining method and $h^2_{i,st,t}$, $h_{sf,st,t}$ are the state-dependent variances and covariances for $s_t = \{1, 2\}$.

The terms $r_{i,t,st}$ represent the state-dependent mean equations, and $\pi_{1,t}$ is the probability of being in state 1 at time t, which is obtained by the following expression:

$$\pi_{1,t} = p\left(\frac{g_{1,t-1}\pi_{1,t-1}}{g_{1,t-1}\pi_{1,t-1} + g_{2,t-1}(1 - \pi_{1,t-1})}\right) + (1 - q)\left(\frac{g_{2,t-1}(1 - \pi_{1,t-1})}{g_{1,t-1}\pi_{1,t-1} + g_{2,t-1}(1 - \pi_{1,t-1})}\right), \tag{20}$$

where

$$g_{i,t} = f(r_t | s_t = i, \Omega_{t-1}) = (2\pi)^{-1} \left|H_{t,i}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} e'_{t,i} H^{-1}_{t,i} e_{t,i}\right\} \ \text{for} \ \ i = \{1, 2\}, \tag{21}$$

and p and q are as described in Equation (14).

Thus, the parameters of the model can be estimated using the following maximum likelihood function, where each state-dependent likelihood function is properly weighted by

[11]The economic interpretation of the parameters *sc*, *sa*, *sb*, and *sd* is how much the constant term, the weight of the shocks, the weight of the past variance, and the impact of negative shocks on the volatility formation differ between each state, respectively.

[12]For further details on the recombining method, see Gray (1996) and Lee and Yoder (2007a).

the filtered probability of being in state 1 at time t ($\pi_{1,t}$) and the filtered probability of being in state 2 ($\pi_{2,t}$).

$$
\begin{aligned}
f(r_t|\theta) = \pi_{1,t}\left[(2\pi)^{-1}\left|H_{t,1}\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}e'_{t,1}H_{t,1}^{-1}e_{t,1}\right\}\right] \\
+\,\pi_{2,t}\left[(2\pi)^{-1}\left|H_{t,2}\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}e'_{t,2}H_{t,2}^{-1}e_{t,2}\right\}\right].
\end{aligned}
\tag{22}
$$

$$
L(\theta) = \sum_{i=1}^{T}\log f(r_t;\theta). \tag{23}
$$

Based on the estimations obtained, we calculate the optimal hedge ratio from the results of the state-independent covariance matrix given by the recombining method, substituting the resulting second-order moments in (8).

5. EMPIRICAL RESULTS

This section presents the main empirical results of the study. Section 5.1 shows the parameter estimation results for all of the models proposed. Section 5.2 describes the volatility evolution and the hedge ratios estimated using each model. Section 5.3 proposes several effectiveness measures to analyze the performance of the different hedging policies. Finally, Section 5.4 performs specification tests over the estimation residuals to detect any problems related to a potential misspecification of the empirical model.

5.1. Model Estimation

In this section, we show the evolution of the patterns followed by the volatility[13] in the linear and nonlinear frameworks proposed in the study. The estimations of the models are presented in Table 2 for all of the markets considered. A two-state specification is used for the MRS models. This specification allows the states to be associated with high- and low-volatility regimes[14] using the median of the estimated state-dependent volatilities for the stock indexes,[15] which present a value of 6.766 *(6.510)* for state 1 and 8.188 *(7.021)* for state 2 in Europe, 7.192 *(2,426)* for state 1 and 2.588 *(5.693)* for state 2 in the U.K. and 8.882 *(7.660)* for state 1 and 9.359 *(8.153)* for state 2 in Germany.[16] Therefore, the state with the highest value of estimated conditional variance in each model corresponds to the high-volatility state.

Moreover, Figure 1 shows the smooth probability of being in the low-volatility state in each data series used.[17] The figure corresponding to Eurostoxx is governed essentially by this state, which corresponds with a calm period in the financial markets (2003–2007). When the

[13]We focus mainly on the interpretation of the variance equation parameters because this determines the estimated covariance matrix and, therefore, the optimal hedge ratio.

[14]Sarno and Valente (2000) use a three-state process, but the third state appears to capture spurious state changes that are not related to market regime switches. However, the selection of the two-state process is primarily due to economic interpretations.

[15]The estimated volatility for the futures indexes follow the same order, and they are not displayed to save space. The results are available from the authors upon request.

[16]Values in parentheses refer to medians in the asymmetric models.

[17]The estimation process itself determines whether state 1 corresponds to high- or low-volatility states. Depending on the country, state 1 could refer to a high-volatility state in one market and to a low-volatility state in another market. The figure represents the probability of a low-volatility state.

TABLE II
Estimations of the Linear and Nonlinear GARCH Models

$r_{s,t} = a_0 + a_1 r_{s,t-1} + a_2 r_{f,t-1} + a_{3,s_t} ECT_{t-1} + e_{s,t} r_{f,t} = b_0 + b_1 r_{f,t-1} + b_2 r_{s,t-1} + b_{3,s_t} ECT_{t-1} + e_{f,t}$

$H_{t,s_t} = \begin{pmatrix} h^2_{s,t,s_t} & h_{fs,t,s_t} \\ h_{fs,t,s_t} & h^2_{f,t,s_t} \end{pmatrix} = C'_{s_t} C_{s_t} + A'_{s_t} \varepsilon_{t-1} \varepsilon'_{t-1} A_{s_t} + B'_{s_t} H_{t-1} B_{s_t} + D'_{s_t} \eta_{t-1} \eta'_{t-1} D_{st}$, *where* $C_1 = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}$, $A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B_1 = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, $D_1 = \begin{pmatrix} d_{11} & 0 \\ 0 & d_{22} \end{pmatrix}$

and $A_2 = sa \cdot A_1$; $B_2 = sb \cdot B_1$; $C_2 = sc \cdot C_1$; $D_2 = sd \cdot D_1$

	Europe				*U.K.*				*Germany*			
	Linear		*Nonlinear*		*Linear*		*Nonlinear*		*Linear*		*Nonlinear*	
	Sym	*Asym*	*Sym*	*Asym*	*Sym*	*Asym*	*Sym*	*Asym*	*Sym*	*Asym*	*Sym*	*Asym*
c_{11}	**0.1505** *(0.1094)*	**0.2783***** *(0.0552)*	**0.4862***** *(0.0764)*	**0.2738*** *(0.1601)*	**0.2137**** *(0.0952)*	**0.9132***** *(0.2160)*	**2.1490***** *(0.2536)*	**0.9566***** *(0.1565)*	**0.4509***** *(0.0881)*	**0.9530***** *(0.1533)*	**0.4609*** *(0.2459)*	**0.5139***** *(0.1111)*
c_{12}	**0.2944***** *(0.1037)*	**0.2518***** *(0.0588)*	**0.4555***** *(0.0922)*	**0.3136**** *(0.1468)*	**0.1457** *(0.1017)*	**0.9060***** *(0.2211)*	**2.1572***** *(0.2602)*	**1.0047***** *(0.1603)*	**0.6210***** *(0.0922)*	**1.0090***** *(0.1488)*	**0.4977*** *(0.2765)*	**0.5508***** *(0.1140)*
c_{22}	**0.0029** *(0.0347)*	**0.1020** *(0.0883)*	**0.2700***** *(0.0312)*	**0.3105***** *(0.0409)*	**0.0001** *(0.0467)*	**0.00001** *(0.0072)*	**−0.241***** *(0.0361)*	**−0.061**** *(0.0294)*	**0.0012** *(0.0651)*	**0.2841**** *(0.1129)*	**0.2990***** *(0.0215)*	**0.2615***** *(0.0188)*
a_{11}	**0.2934***** *(0.0585)*	**0.2957***** *(0.0788)*	**−0.3466*** *(0.1935)*	**−0.2065** *(0.1447)*	**0.5796***** *(0.1734)*	**0.7023** *(0.6661)*	**1.3691**** *(0.3371)*	**−0.748***** *(0.1309)*	**0.6349***** *(0.0262)*	**0.4229** *(0.2736)*	**−0.0520** *(0.2210)*	**−0.627***** *(0.1270)*
a_{12}	**−0.0579** *(0.0589)*	**−0.049***** *(0.0184)*	**−0.2796*** *(0.1894)*	**−0.1424** *(0.1066)*	**0.0050** *(0.0857)*	**0.3018** *(0.6949)*	**0.9174**** *(0.3278)*	**−0.839***** *(0.1361)*	**0.3095***** *(0.0775)*	**−0.0733** *(0.2905)*	**0.0217** *(0.2489)*	**−0.609***** *(0.1304)*
a_{21}	**−0.0589** *(0.0584)*	**−0.065***** *(0.0139)*	**0.7107***** *(0.1972)*	**0.2336** *(0.1556)*	**−0.393**** *(0.1867)*	**−0.1483** *(0.6619)*	**−0.702**** *(0.3132)*	**0.7702***** *(0.1348)*	**−0.405***** *(0.0741)*	**−0.758***** *(0.2567)*	**0.3903** *(0.2769)*	**0.6744***** *(0.1266)*
a_{22}	**0.2946***** *(0.0586)*	**0.3038***** *(0.0837)*	**0.6578***** *(0.1954)*	**0.1899** *(0.1273)*	**0.2015**** *(0.0945)*	**−0.7824** *(0.6960)*	**−0.3027** *(0.3006)*	**0.847***** *(0.1405)*	**−0.075**** *(0.0312)*	**−0.2620** *(0.2802)*	**0.2923** *(0.3172)*	**0.6207***** *(0.1313)*
b_{11}	**1.1410***** *(0.1714)*	**1.1020***** *(0.2273)*	**0.6743**** *(0.3233)*	**0.3914** *(0.4995)*	**1.0626***** *(0.1291)*	**1.1491***** *(0.3789)*	**0.4974**** *(0.3047)*	**1.1819***** *(0.1579)*	**1.2587***** *(0.0460)*	**0.2916** *(0.3582)*	**0.4491** *(0.8253)*	**0.5200** *(0.6984)*
b_{12}	**0.2340** *(0.1697)*	**0.2556** *(0.2196)*	**0.0710** *(0.3242)*	**0.1444** *(0.4508)*	**0.2340** *(0.1850)*	**0.2722** *(0.3987)*	**−0.3481** *(0.2861)*	**0.3373** *(0.2191)*	**0.3823***** *(0.0050)*	**−0.2707** *(0.2329)*	**0.2870** *(0.9093)*	**0.5721** *(0.7269)*
b_{21}	**−0.1649** *(0.1715)*	**−0.1488** *(0.2179)*	**0.2066** *(0.3116)*	**0.5420** *(0.4746)*	**−0.0828** *(0.1294)*	**−0.3639** *(0.3643)*	**0.1307** *(0.2739)*	**−0.645***** *(0.2140)*	**−0.294***** *(0.0094)*	**0.5387** *(0.3439)*	**0.4639** *(0.8312)*	**0.3947***** *(0.6883)*
b_{22}	**0.7457***** *(0.1694)*	**0.7062***** *(0.2205)*	**0.8173***** *(0.3100)*	**0.79375*** *(0.4324)*	**0.7540***** *(0.1781)*	**0.52237** *(0.3795)*	**0.97530***** *(0.2580)*	**0.1620** *(0.2726)*	**0.58113***** *(0.0041)*	**1.09757***** *(0.2147)*	**0.64209** *(0.9167)*	**0.35606** *(0.7183)*
d_{11}		**0.2769** *(0.2268)*		**0.4370***** *(0.0893)*		**−0.2224** *(0.1225)*		**0.1985** *(0.1172)*		**0.4223***** *(0.0489)*		**0.4837***** *(0.0629)*
d_{22}		**0.2302**** *(0.0967)*		**0.4547***** *(0.0911)*		**−0.271**** *(0.1158)*		**0.2154*** *(0.1268)*		**0.4220***** *(0.0481)*		**0.4763***** *(0.0612)*
sc			**19.519**** *(6.2998)*	**6.1490*** *(3.7525)*			**0.1944***** *(0.0513)*	**3.2259***** *(0.4429)*			**8.3639***** *(2.1276)*	**2.8758***** *(0.3576)*
sa			**1.7966***** *(0.4837)*	**5.9437***** *(0.0416)*			**0.5991***** *(0.1739)*	**3.9729***** *(1.0768)*			**1.9292*** *(1.0491)*	**4.9120***** *(1.0742)*
sb			**0.5353**** *(0.2224)*	**0.8965** *(1.6250)*			**1.8473***** *(0.0856)*	**0.9862***** *(0.0702)*			**0.7523***** *(0.1169)*	**0.8384***** *(0.0585)*
sd				**0.6713**** *(0.2665)*				**2.72048** *(1.6807)*				**0.9241***** *(0.2646)*
p			**0.978**	**0.966**			**0.976**	**0.965**			**0.965**	**0.966**
q			**0.972**	**0.962**			**0.969**	**0.962**			**0.954**	**0.961**

Note. Estimated parameters for all models and indexes (robust standard errors in parenthesis). ***. ** and * represent significance at 1%. 5%, and 10% levels.

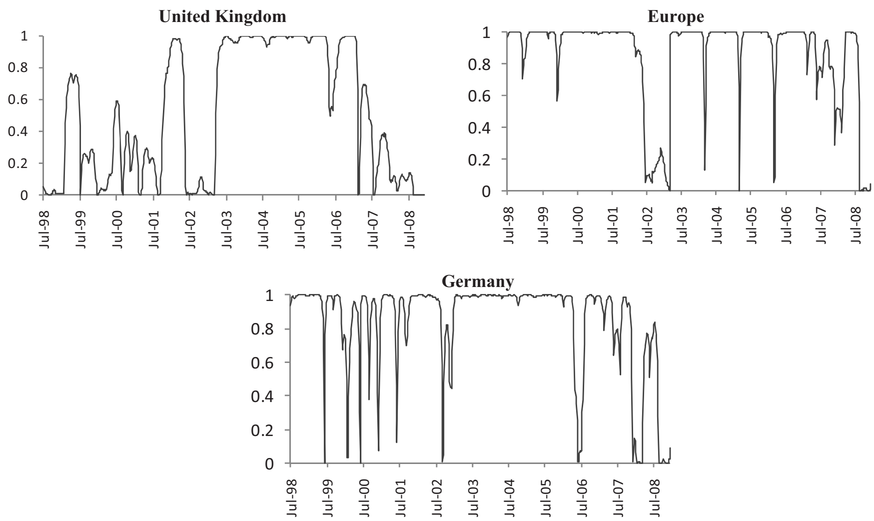

FIGURE 1
Smooth probabilities for low-volatility states. This figure shows the smooth regime probabilities of being in a low-variance state (Hamilton & Susmel, 1994) for all the countries considered.

state governing the process is state 2, this corresponds to periods of market jitters such as the dot-com bubble (2002–2003) and the last financial crisis (2008). The probabilities for the rest of the markets share these periods of high-volatility states and, moreover, present other high-volatility periods that are likely related to their own country's idiosyncratic market evolution.

For each market in Table II, the first two columns in each market show the parameter estimations for the linear models (BEKK and ASYM-BEKK). We can observe that the linear models in most cases reflect a weak significance of the parameters representing the persistence of the impact of shocks in volatility (a11, a22). Furthermore, the impact of one market's shocks on the other markets' volatility is generally not significant (a12, a21).

The evidence for a significant influence of past volatility on volatility formation is more evident both for the spot (b11) and futures markets (b22), but this is not observed for the cross parameters (b12, b21). Generally, there is also an asymmetric response for volatility against negative shocks, although in markets such as Eurostoxx and the U.K., this evidence is only observed in the futures markets (d22).

Finally, there is another remarkable result regarding volatility dynamics: the persistence level in linear models is relatively high. This result suggests the presence of several regimes in the volatility process and, therefore, potential nonlinearities and the adequacy of using MRS-GARCH models.

It is also interesting to analyze the differences in the volatility parameters between states of the market in nonlinear models. For example, the constant term is usually lower in low-volatility states than it is in high-volatility states,[18] which is understandable because the constant term in our model reflects the unconditional volatility, and this is supposed to be higher in high-volatility states. Second, the influence of shocks on volatility formation is greater in high-volatility states than it is in low-volatility states.[19] However, the impact of past

[18]In the models where state 1 corresponds to low-volatility periods, this fact is observed because the scale *sc* for state 2 (high-volatility) is higher than 1; in the cases where state 1 corresponds to high-volatility periods, the scalar *sc* is lower than 1.

[19]Similar to footnote 16 but using scalar *sa* instead of *sc*.

variance on the formation of volatility is lower in high-volatility states than it is in low-volatility states.[20] There appears to be a trade-off between the impact of shocks and past variance on the formation of volatility between states. In low-volatility states, there is a greater past variance persistence and a lower presence of shocks in volatility. In high-volatility states, there is a higher presence of shocks but a lower impact of past variance. These results are similar to those of Marcucci (2005), who explains these differences in volatility dynamics between low- and high-volatility periods by arguing that there is a greater amount of news during high-volatility periods. Therefore, the continuous arrival of new information into the market causes volatility formation to occur largely because of the impact of these shocks rather than the past variance observed in the market, as occurs in low-volatility periods when less news affects the markets. Finally, we find that the asymmetric response of volatility is significant in the spot and futures markets in the nonlinear specification. We also find that there is a different asymmetric response of volatility in the low- and high-volatility periods. However, there is no common result for how the asymmetric response changes with the volatility regime. In Europe and Germany, this asymmetric response is higher in the low-volatility periods, whereas it is less acute in the high-volatility periods. In the U.K., the opposite occurs.

We also consider interesting to determine the average durations of the different states in the economy. This duration value can be obtained according to the transition probability estimates p and q in Equation (14). For example, Europe presents a value of $p = 0.966$ and $q = 0.962$; this means that once in state 1, the probability of remaining in that state is 96.6%, whereas the probability of remaining in state 2 is 96.2%. Therefore, the average duration of being in state 1 when the volatility process is governed by this state will be approximately 29 weeks $(1/(1 - 0.966))$. A similar duration can be calculated in the high-volatility regime state $(1/(1 - 0.962))$, which indicates that the regime switches present a smooth evolution, keeping the process in each state during relatively long periods. For the rest of markets, these values are very similar.

5.2. Volatility and Hedge Ratios

At this point, it is time to analyze the evolution and differences in the estimated variances obtained in each model, which will then lead us to the differences in the estimated hedge ratios. The estimated covariance matrix for the linear models is obtained using (5) for the symmetric case and (9) for the asymmetric case. For a proper comparison between models, we use the estimation for the independent covariance matrix ((17) and (19)) for the nonlinear models. Figure 2 shows the estimated variance for the spot market[21] for all of the markets considered.

All figures appear to exhibit similar patterns, although there are obvious differences between them. Common to all of the estimations, there are two periods corresponding to 2001–2003 and 2008 that present higher estimations of volatility. These periods of high-volatility coincide with the dot-com bubble and the last financial crisis, which are periods of market jitters. Figure 1 shows that the mentioned periods correspond with periods governed by high-volatility states, and the rest of the sample is often governed by low-volatility states. The volatility estimations in the high-volatility periods using nonlinear models are higher than are those obtained with linear models, but in the rest of the sample coinciding with calm periods, the volatility estimations using linear models are higher than are those obtained with

[20]In this case, when state 1 corresponds to low-volatility periods the scale *sb* for state 2 (high-volatility) is lower than unity; in the cases where state 1 corresponds to high-volatility periods, the scalar *sb* is higher than 1.

[21]For brevity, only the spot market volatility is shown. The estimated volatilities for the futures markets and the covariance between the spot and futures markets are similar. The results are available from the authors on request.

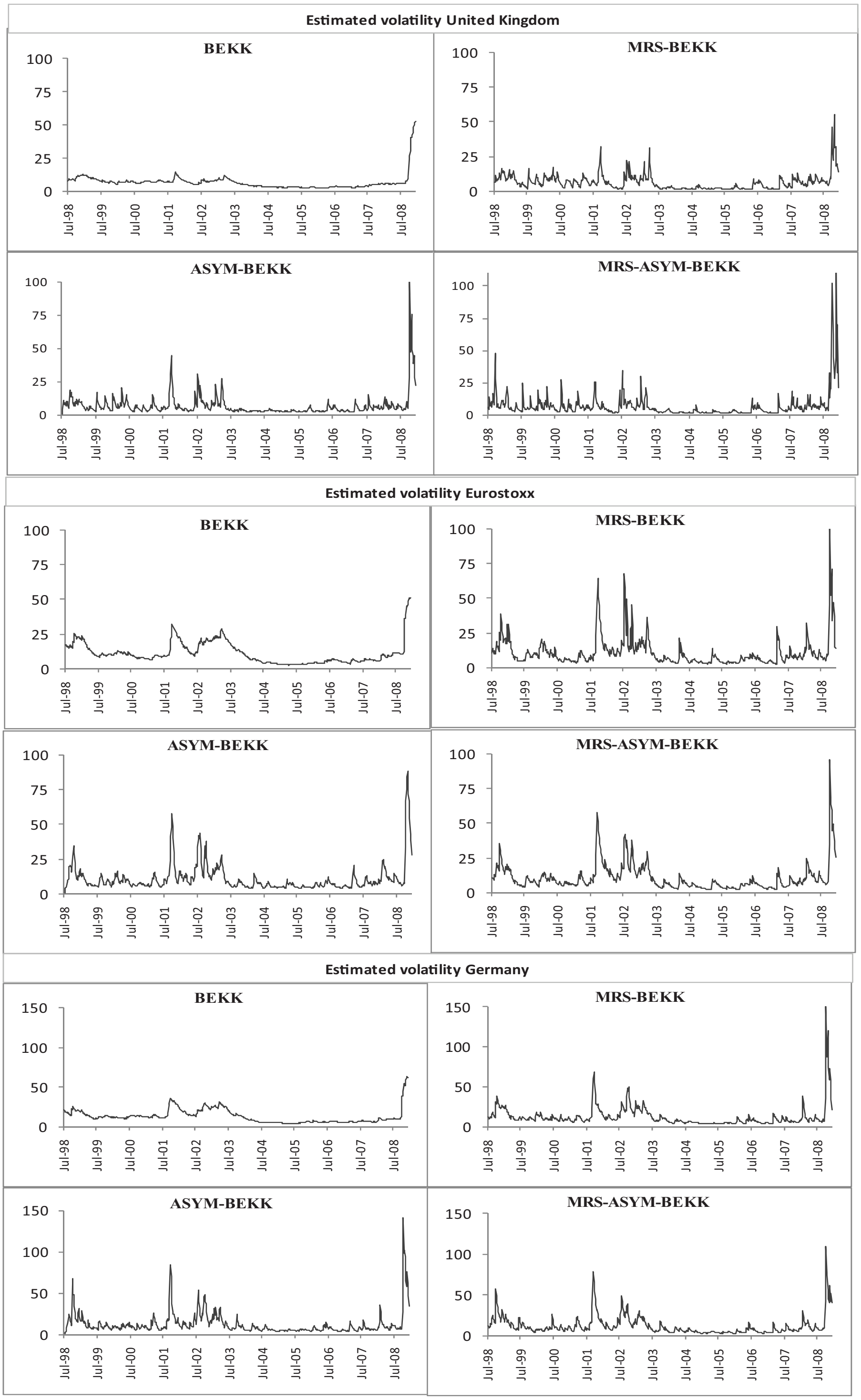

FIGURE 2

Estimated weekly volatility. This figure shows the estimated volatilities for the spot market in the different markets considered. Linear GARCH models (symmetric and asymmetric) are displayed on the left-hand side, whereas the nonlinear specifications are on the right-hand side.

nonlinear models.[22] If we do not distinguish between states, one state would define the volatility process, and this might not properly reflect the patterns during turbulent periods, which exhibit different dynamics than calm periods. Therefore, the volatility estimations tend to be underestimated using linear GARCH models in the periods corresponding to the high-volatility states and overestimated in the low-volatility periods, and this might influence the effectiveness of the hedge policy.

Finding the optimal hedge ratios for the in-sample analysis is simple. For linear GARCH models, we use (8) and the covariance matrix estimates at each moment t (Kroner & Sultan, 1993). For nonlinear models, we also use (8) and the state-independent estimations of the covariance matrix.

Finding hedge ratios for the out-sample period is more complex and differs by model. Common to all models is the construction of a rolling window in which the model is re-estimated for each window period, removing the first observations and adding new ones as the window advances. The parameter values are found for each estimation period, which allows us to make forecasts one period ahead of the covariance matrix. Note that this procedure is performed for the linear BEKK models both with and without asymmetries.

The process of forecasting the covariance matrix for the nonlinear BEKK models with (and without) asymmetries is more complex because of the existence of two possible states. This forecast is performed in a three-stage process (Alizadeh et al., 2008). In the first stage, we use the estimations of the transition matrix in t as in (14) and the smoothed probabilities in t to obtain the prediction of the probability of being in each one of the two states $s_t = 1, 2$ in the period $t + 1$.

$$E\begin{bmatrix} \pi_{1,t+1} \\ \pi_{2,t+1} \end{bmatrix}' = \begin{pmatrix} \hat{\pi}_{1,t} \\ \hat{\pi}_{2,t} \end{pmatrix} \begin{pmatrix} \hat{p} & (1-\hat{q}) \\ (1-\hat{p}) & \hat{q} \end{pmatrix}. \tag{24}$$

In the second stage, we make a prediction one period ahead of the state-dependent mean and variance Equations [(11), (12), (13), and (15)] using the parameters estimated. In the third stage, the recombining method is used as in (17) (18), and (19) to obtain the predictions of the state-independent covariance matrix. Once we have the prediction for one period ahead of the covariance matrix for each model, we obtain the predicted hedge ratio using (8) for $t + 1$.

Figure 3 presents the hedge ratios obtained for both the in-sample and the out-sample period, together with their evolution.[23] The top figures show the evolution for the in-sample analysis and the bottom graphs reflect the forecasts performed for each model. We compare the symmetric (MRS-BEKK) against the asymmetric (MRS-ASYM-BEKK) nonlinear models on the left-hand side and the linear (BEKK) against the nonlinear (MRS-BEKK) on the right-hand side, with the continuous line, the MRS-BEKK model and the alternatives in each case plotted with dashed lines.

The differences between models are evident both between linear (dashed line) and nonlinear (continuous line) specifications and between symmetric (continuous line) and asymmetric (dashed line) specifications.

Differences in the averages and in the variability of the estimated and forecasted ratios also exist (Table III). Therefore, it appears the omission or inclusion of one of these

[22]Using the filtered probability in each market, we find that the average for high-volatility states using symmetric linear models in Europe, the U.K., and Germany are 8.83, 21.14, and 18.51, respectively, while for the non-linear case, they are 9.10, 22.33, and 22.93, respectively. For low-volatility states, the average estimated volatility is 4.51, 9.12, and 11.92 for linear models against 3.14, 8.55, and 10.98 for non-linear models.

[23]For brevity, the estimated hedge ratios of the remaining models are not presented here but are available from the authors upon request.

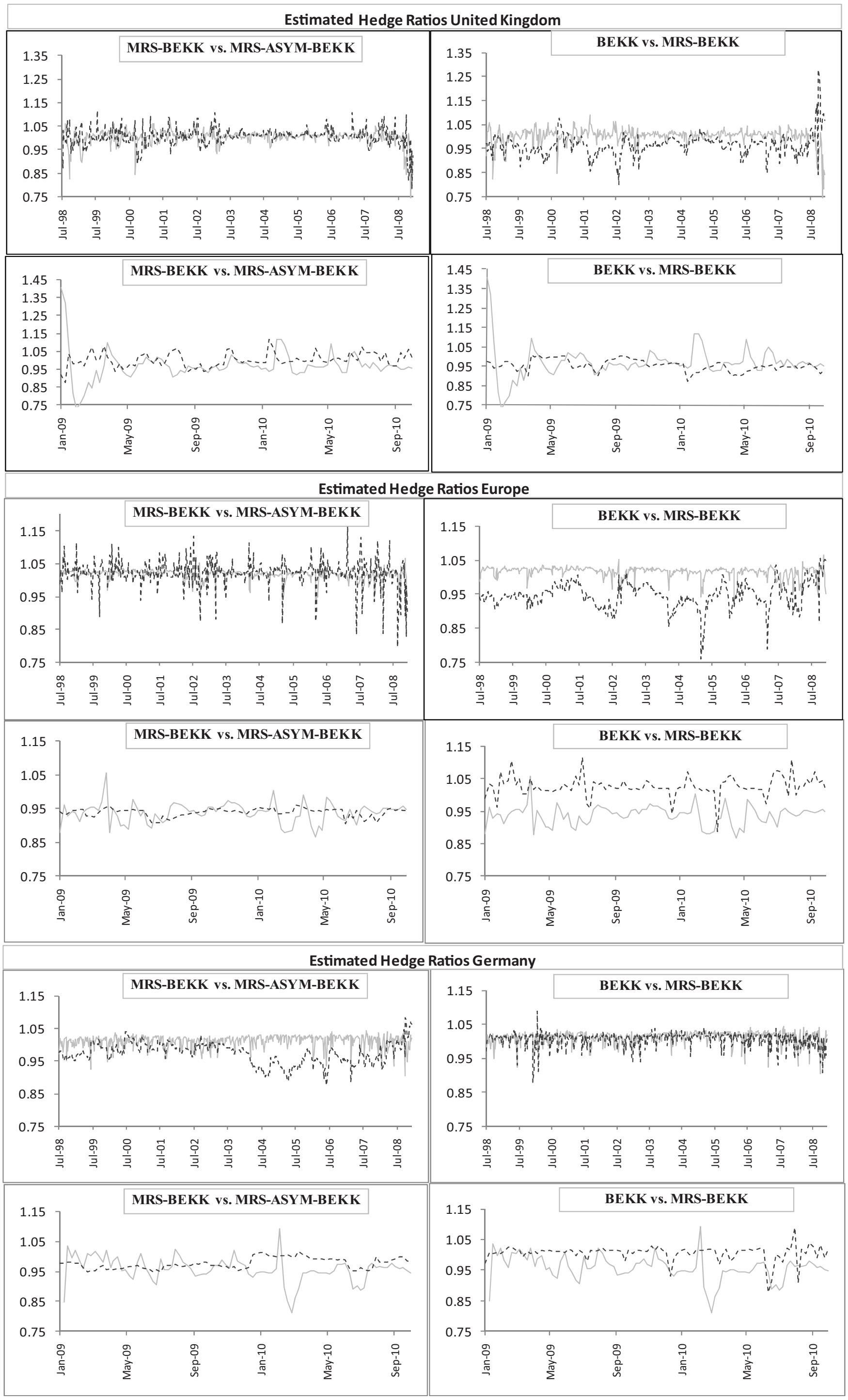

FIGURE 3

Estimated hedge ratios for in-sample and out-sample periods. This figure shows the estimated time-varying hedge ratios for the in-sample period (top figures) and out-sample period (bottom figures). Continuous lines represent the MRS-BEKK model and dashed lines the other models considered in each case.

TABLE III
Summary Statistics for Hedge Ratios

	In-sample (***Out-sample***)				
	Maximum	*Minimum*	*Mean*	*Variance*	*Median*
Europe					
BEKK	1.0570 ***(1.0316)***	0.7603 ***(0.8468)***	0.9473 ***(0.9592)***	0.0014 ***(0.0015)***	0.9464 ***(0.9578)***
ASYM-BEKK	1.0676 ***(1.4682)***	0.7460 ***(0.7089)***	0.9415 ***(0.9803)***	0.0017 ***(0.0280)***	0.9447 ***(0.9800)***
MRS-BEKK	1.0660 ***(1.0570)***	0.9357 ***(0.8669)***	1.0157 ***(0.9383)***	0.0003 ***(0.0009)***	1.0206 ***(0.9451)***
MRS-ASYM-BEKK	1.1717 ***(0.9800)***	0.7973 ***(0.8049)***	1.0218 ***(0.8971)***	0.0014 ***(0.0010)***	1.0229 ***(0.8967)***
U.K.					
BEKK	1.2779 ***(1.1621)***	0.8023 ***(0.9078)***	0.9650 ***(0.9698)***	0.0020 ***(0.0027)***	0.9649 ***(0.9504)***
ASYM-BEKK	1.2125 ***(1.3823)***	0.8212 ***(0.7280)***	0.9678 ***(0.9592)***	0.0014 ***(0.0109)***	0.9662 ***(0.9827)***
MRS-BEKK	1.0902 ***(1.4058)***	0.7404 ***(0.7442)***	1.0019 ***(0.9724)***	0.0011 ***(0.0073)***	1.0074 ***(0.9640)***
MRS-ASYM-BEKK	1.1166 ***(1.1649)***	0.7846 ***(0.9262)***	1.0043 ***(0.9716)***	0.0013 ***(0.0017)***	1.0071 ***(0.9579)***
Germany					
BEKK	1.0822 ***(1.0650)***	0.8749 ***(0.9408)***	0.9728 ***(0.9927)***	0.0010 ***(0.0006)***	0.9770 ***(0.9933)***
ASYM-BEKK	1.0926 ***(1.1599)***	0.8752 ***(0.7187)***	0.9700 ***(0.9455)***	0.0005 ***(0.0060)***	0.9701 ***(0.9380)***
MRS-BEKK	1.0447 ***(1.0917)***	0.9034 ***(0.8113)***	1.0107 ***(0.9575)***	0.0004 ***(0.0017)***	1.0167 ***(0.9594)***
MRS-ASYM-BEKK	1.0875 ***(0.9803)***	0.8789 ***(0.8757)***	1.0064 ***(0.9473)***	0.0004 ***(0.0004)***	1.0136 ***(0.9522)***

Note. This table presents the summary statistics for the hedge ratio series obtained in each European market for the different models proposed from both in-sample (standard type) and out-sample (bold) analysis.

characteristics could lead to significant differences in the estimated hedge ratios and, therefore, in the effectiveness reached. Therefore, concerning the evident differences between the estimated and forecast hedge ratios obtained in each strategy, we try to explain in the next section which hedge strategy allows us to achieve a more effective hedge policy. The study in the next section is especially attractive because the out-sample analysis is performed over the period of the recent financial crises and could thus prove which models work better in periods of market uncertainty.

5.3. Hedging Effectiveness

To analyze hedging effectiveness, we consider four different measures. The first two measures are based on the variance of the loss distribution of the hedge portfolio. The first approach is the variance of the hedged portfolio (Ederington, 1979) for each model compared with an unhedged portfolio, that is $HR_t = 0$ for all t. The variance of the hedged portfolio is as follows:

$$Var(x_t|\Omega_{t-1}) = Var((\Delta S_t - HR_t\Delta F_t)|\Omega_{t-1}). \tag{25}$$

Another commonly used approach is to analyze the economic benefits of the hedging (Kroner & Sultan, 1993) by constructing the investor's utility function based on the return and risk of the hedge portfolio. This measure is motivated by the fact that dynamic strategies are most costly to implement because they require a frequent updating of the hedge portfolio. Consistent with studies such as Park and Switzer (1995) and Meneu and Torró (2003), the utility function is constructed in a mean/variance context:

$$E[U(x_t|\Omega_{t-1})] = E[x_t|\Omega_{t-1}] - \lambda\, Var[x_t|\Omega_{t-1}]. \tag{26}$$

where λ is the investor's level of risk aversion (normally $\lambda = 4$) and the hedged portfolio returns are also assumed to present an expected value equal to 0 (Alizadeh et al., 2008; Lee, 2010).

The third metric proposed is based on the Value at Risk (VaR) measure (Jorion, 2000). The VaR of the hedged portfolio at the confidence level q is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-q)$. In our case, this is calculated by the sample quantiles using the empirical distribution of the hedge portfolio returns.

$$VaR_q = inf\{l \in \Re : P(L > l) \leq 1 - q\}. \tag{27}$$

The last effective measure is based on the Expected Shortfall (ES) of the hedged portfolio (Artzner et al., 1999). ES is an alternative to VaR in that it is more sensitive to the shape of the loss distribution in the tail of the distribution. The ES at the $q\%$ level is the expected return on the portfolio in the worst $q\%$ of the cases.

$$ES_q = E(x|x < \mu). \tag{28}$$

where μ[24] is determined by Pr $(x < \mu) = q$ and q is the given threshold, whereas x is a random variable that represents profit during a specified period.

Table IV summarizes the hedging strategy effectiveness for all of the series used in the study. The table shows the effectiveness measures both for in-sample and out-sample analysis and for all linear and nonlinear models proposed as well as the effectiveness achieved with a constant OLS strategy and by the unhedged portfolio.

Panel A presents the effectiveness analysis for the in-sample period in all countries considered. The highest effectiveness considering the reduction of variance of the hedge portfolio is observed in the MRS-ASYM-BEKK in the U.K., Germany, and Europe. That is, the nonlinear models outperform the effectiveness of the rest of the models in terms of variance reduction. Another interesting result arises here. The effectiveness of the OLS strategy outperforms in all cases (except for the U.K.) the linear GARCH hedging strategies.[25] This result is the same as those found in studies such as Lien (2009), Lien and Tse (2002), Cotter and Hanly (2006), and Park and Jei (2010). Those authors find that constant strategies present better effectiveness than dynamic strategies. However, when we consider nonlinear strategies, these more complex models outperform the rest of the policies. Generally, the introduction of nonlinearities into the models allows us to achieve a greater fit to the data because of the identification of different regimes in the volatility process and the more accurate estimation. Therefore, this nonlinear specification outperforms both the linear models and the constant strategies. The utility analysis reaches a similar conclusion because these first two measures are both based on the variance of the hedge portfolio loss distribution. However, as Park and Jei (2010) remark, this measure could present problems when the return distribution deviates from normality.

If we consider tail-based measures, we obtain most of the greater risk reduction in the nonlinear models but, using this metric, the evidence is less clear than it is with the variance reduction. For VaR metrics, we find that MRS-BEKK performs best for the U.K. at the 1% and 10% significance levels, Germany at 1% and Europe at all levels. The asymmetric nonlinear model (MRS-ASYM-BEKK) performs best for the U.K. at 5% and Germany at 10%. However,

[24]Note that μ is the value at risk.

[25]Lien (2009) shows that variance-based metrics reflect the reduction of the unconditional volatility of the hedge portfolio. Therefore, OLS strategies reach the greatest variance reduction by definition, whereas the linear GARCH strategies achieve a reduction on the conditional variance.

TABLE IV
Effectiveness Analysis for the Different Models Proposed

	Variance reduction (Utility)			Value at risk (1%) (5%) (10%)			Expected Shortfall (1%) (5%) (10%)		
	U.K.	Germany	Europe	U.K.	Germany	Europe	U.K.	Germany	Europe
Panel A. In-sample effectiveness									
Unhedged portfolio	7.1019	13.0830	10.7780	−7.2832	−10.7111	−9.2324	−12.1918	−15.0682	−13.5863
	(−28.407)	(−52.331)	(−43.112)	(−4.2444)	(−5.6611)	(−5.3425)	(−6.4098)	(−8.3671)	(−7.5667)
				−3.0231	−4.4048	−3.9207	−5.1242	−6.8985	−6.2300
OLS	91.475%	96.668%	94.591%	−2.1358	−2.8167	−2.0685	−4.9861	−3.4327	−3.3674
	(−2.421)	(−1.743)	(−2.331)	(−0.8388)	(−0.9381)	(−1.1269)	(−1.8553)	(−1.6813)	(−1.7324)
				−0.5342	−0.6886	−0.8679	−1.3192	−1.3062	−1.4006
BEKK	89.767%	96.243%	94.146%	−2.1636	−2.8207	−2.1794	−5.8531	−3.2691**	−3.2092
	(−2.861)	(−1.966)	(−2.523)	(−0.8033)	(−0.9672)	(−1.2109)	(−2.0409)	(−1.7500)	(−1.7687)
				−0.5599	−0.6658	−0.8656	−1.4166	−1.3241	−1.4377
ASYM-BEKK	91.735%	96.437%	93.461%	−2.7441	−2.8124	−2.2236	−4.8536	−3.2910	−3.4947
	(−2.221)	(−1.864)	(−2.819)	(−0.8415)	(−0.9322**)	(−1.2219)	(−1.8461)	(−1.6942)	(−1.8587)
				−0.5297	−0.6777	−0.9012	−1.3265	−1.2813	−1.5003
MRS-BEKK	89.928%	96.543%	94.245%	−2.1268**	−2.5475**	−2.0156**	−5.6605	−3.6261	−3.5958
	(−2.9069)	(−1.809)	(−2.481)	(−0.8536)	(−1.0126)	(−1.1248)**	(−1.9451)	(−1.7061)	(−1.7885)
				−0.5134**	−0.6464	−0.8582**	−1.3800	−1.2896	−1.4224
MRS-ASYM-BEKK	92.216%**	96.701%**	94.731%**	−2.8041	−2.6631	−2.0687	−4.508**	−3.4202	−2.9199**
	(−2.3476)**	(−1.7262)**	(−2.271)**	(−0.7884)**	(−0.9374)	(−1.1362)	(−1.6996)**	(−1.6685)**	(−1.6957)**
				−0.5159	−0.6270**	−0.8680	−1.2431**	−1.2674**	−1.3943**
Panel B. Out-sample effectiveness									
Unhedged portfolio	14.0564	11.570	14.0565	−11.9260	−9.4690	−11.9260	−11.0761	−9.4690	−11.0761
	(−56.226)	(−46.281)	(−56.226)	(5.3703)	(−5.1069)	(−5.3703)	(−8.4255)	(−7.3291)	(−8.4255)
				−4.0735	−3.9729	−4.0735	−6.6338	−5.8716	−6.6338

TABLE IV
(Continued)

	Variance reduction (Utility)			*Value at risk (1%)* (5%) (10%)			*Expected Shortfall (1%)* (5%) (10%)		
	U.K.	*Germany*	*Europe*	*U.K.*	*Germany*	*Europe*	*U.K.*	*Germany*	*Europe*
OLS	*90.280%*	*95.366%*	*90.859%*	*−2.8404*	*−1.5904*	*−2.5952*	*−2.3674*	*−1.5904*	*−2.4024*
	(**−5.461**)	**(−2.144)**	**(−5.139)**	**(−1.5730)**	**(−1.1950)**	**(−1.4069)**	(**−1.9360)**	**(−1.3755)**	**−1.8939**
				−1.1495	−0.8000	−1.0814	1.6502	−1.1769	−1.5811
BEKK	*90.006%*	*95.280%*	*90.057%*	*−2.7503***	*−1.6095*	*−3.0333*	*−2.6391*	*−1.6095*	*−2.8453*
	(−5.619)	**(−2.184)**	**(−5.590)**	**(−1.6117)**	**(−1.2886)**	**(−1.5012)**	**(−2.0889)**	**(−1.4059)**	**(−2.1588)**
				−1.2745	−0.8836	−1.1619	−1.7573	−1.2079	−1.7536
ASYM-BEKK	*88.596%*	*94.942%*	*82.678%*	*−4.2527*	*−1.7257*	*−4.5691*	*−3.9761*	*−1.7257*	*−4.2994*
	(−6.411)	**(−2.341)**	**(−9.739)**	**(−1.7488)**	**(−1.2620)**	**(−1.9771)**	(**−2.6884)**	(**−1.3842)**	**(−3.0518)**
				−1.0793	−0.9324	−1.1583	−2.0236	−1.2167	−2.3544
MRS-BEKK	*88.882%*	*95.778%***	*91.116%***	*−4.4355*	*−1.5358***	*−2.1602***	*−3.3495*	*−1.5358***	*−2.0762***
	(**−6.251**)	**(−1.953)****	**(−4.994)****	**(−1.5085)**	**(−1.1750)**	**(−1.4208)**	**(−2.4026)**	**−1.3356**	**(−1.8293)****
				−1.3263	−0.7940	−1.1823	−1.9468	−1.1471**	−1.5501**
MRS-ASYM-BEKK	*90.668%***	*95.635%*	*87.602%*	*−2.7115*	*−1.6755*	*−3.3978*	*−2.3596***	*−1.6755*	*−2.9533*
	(−5.257)**	**(−2.019)**	**(−6.970)**	**(−1.4922)****	**(−1.0808)****	**(−1.3975)****	(**−1.9065)****	**(−1.3072)****	**(−2.1160)**
				−1.0608**	−0.7937**	−1.0355**	−1.6045**	−1.1494	−1.6983

Note. This table shows the results for the different effectiveness measures in the different countries considered [risk reduction (Equation 25), economic viability (26), Value at Risk (27), and Expected Shortfall (28)]. Panels A and B display the results for the in-sample (January 01, 1988 to December 31, 2008) and out-sample (January 01, 2009 to September, 2010) periods. ** represents the model with the best performance for each effectiveness measure considered.

for Germany at 5% significance, the asymmetric linear GARCH achieves the best hedging performance. For the ES, which reflects the expected loss when we consider only the worst scenarios, again the nonlinear models perform better than the linear models in most cases. However, there are few cases where the linear models outperform the nonlinear ones, such as Germany at 1% significance. Using these last two metrics, the dominance of nonlinear models is again evident, as they outperform in almost all cases the linear and constant models.[26]

Panel B presents the effectiveness analysis for the out-sample analysis. We evaluate the four effectiveness metrics unconditionally, using 92 forecasted hedged portfolios returns over the 92 periods out-sample. The highest effectiveness considering the reduction of variance of the hedge portfolio is observed in the MRS-ASYM-BEKK in the U.K. and the MRS-BEKK in Europe and Germany. The nonlinear models outperform the effectiveness of the rest of the models in terms of variance reduction in the out-sample analysis. The utility results are similar. With this evidence, it appears clear that the more complex nonlinear models lead to better forecasts of the hedge ratio and a greater risk reduction using variance-based metrics. However, if we compare the linear GARCH models to the constant strategies, we find a greater variance reduction for the constant strategies. This result reveals an issue widely discussed in the empirical literature. Most of the literature comparing dynamic (i.e., linear GARCH models) with constant strategies obtain a better performance from the latter (Lien, 2009; Lien & Tse, 2002; Cotter & Hanly, 2006; Park & Jei, 2010). However, when nonlinear dynamic models through regime switching are introduced, a better performance compared with constant and linear GARCH models is achieved. The tail loss distribution measures also reflect the higher performances of nonlinear models in most cases. The VaR measures show that MRS-BEKK presents the highest effectiveness in Europe and Germany at 1%, whereas the MRS-ASYM-BEKK is the best strategy in the U.K., Germany, and Europe at the 5% and 10% levels. For the U.K. at 1%, the linear BEKK model is most effective. The ES results show similar conclusions to those of the VaR results in the out-sample analysis. This metric also shows the greater effectiveness of the nonlinear models (the symmetric case for Europe at all levels, Germany at 1% and 10%, and the asymmetric model for the U.K. at all levels and Germany at 5%).[27]

These results imply that nonlinear models exhibit a higher hedging effectiveness than the constant and dynamic linear models using variance-based metrics. The evidence with tail loss metrics also supports the more complex models in most cases, although in a few scenarios the linear models beat them. This greater out-sample effectiveness of nonlinear models could occur because they offer more accurate forecasting than more parsimonious models (Marcucci, 2005). When the dynamic relationship between the spot and futures returns is characterized by regime shifts, allowing the hedge ratio to be dependent upon the state of the market, one can obtain more efficient hedge ratios and, hence, superior hedging performance compared with other methods in the literature.

5.4. Specification Test

To test robustness, this section performs several specification tests to check the adequacy of the QML estimations of the multivariate models. For this reason, we analyze the properties of the standardized residuals ($\in_{i,t} = \varepsilon_{i,t}/\sqrt{h_{ii,t}}$) *for* $i = s,f$ and the product of the standardized residuals for the models proposed.

[26]Cotter and Hanly (2006) find that some performance metrics (especially VaR) yield different results in terms of the best hedging model compared with the traditional variance reduction criterion.

[27]Alizadeh and Nomikos (2004) and Alizadeh et al. (2008) also find a general outperforming of regime-switching models regarding other strategies in their studies, but in a few scenarios, the more complex models that they propose are beaten.

TABLE V
Specification Test for the Standardized Residuals

BEKK (ASYM-BEKK)	$\hat{\varepsilon}_{s,t}$	$\hat{\varepsilon}_{f,t}$	$\hat{\varepsilon}^2_{s,t}$	$\hat{\varepsilon}_{s,t}\hat{\varepsilon}_{f,t}$	$\hat{\varepsilon}^2_{f,t}$
Panel A. Linear models					
Mean	−0.0444	−0.0302	0.9949	−0.0165	1.0108
	−0.0021	***−0.0338***	***1.0029***	***−0.0205***	***0.9971***
SD	0.9948	1.0117	4.8407	2.4644	3.3990
	1.0047	***0.9978***	***2.8840***	***1.6700***	***3.2769***
Skewness	−0.4708	−0.6055	8.1762	2.0169	4.3518
	−0.4064	***−0.4365***	***3.4986***	***−2.0010***	***4.6834***
Kurtosis	5.8089	4.2504	109.7394	64.6949	28.2937
	3.8588	***4.2340***	***18.2919***	***24.3926***	***33.8424***
J–B test	199.67	68.93	265281.06	86962.68	16278.15
	31.81	***51.98***	***6433.77***	***10775.71***	***23637.00***
L–B (6)	17.8264	22.8514	18.5901	28.6156	14.3485
	25.8294	***19.2729***	***24.1604***	***15.0965***	***10.1548***
Panel B. Nonlinear models					
Mean	−0.0090	−0.0716	1.0208	0.0461	0.9740
	0.0103	***−0.0359***	***0.9778***	***0.0070***	***0.9934***
SD	1.0226	0.9707	3.2994	1.7447	2.9268
	0.9795	***0.9939***	***2.9574***	***1.6894***	***3.2335***
Skewness	−0.3387	−0.4830	4.0646	−1.9229	4.3927
	−0.5074	***−0.5088***	***4.1675***	***−2.3564***	***4.8102***
Kurtosis	4.1488	3.9504	24.1819	23.7857	31.3219
	4.1086	***4.2008***	***26.0987***	***27.0903***	***35.6759***
J–B test	40.47	41.78	11710.71	10165.51	20004.44
	51.39	***56.36***	***13718.81***	***13708.14***	***26396.08***
L–B (6)	28.8913	20.1812	26.2149	15.0549	8.5937
	27.0130	***15.7562***	***24.8734***	***13.1601***	***9.6922***

Note. This table shows the statistics for the standardized residuals. Panel A shows the results for the linear models (BEKK and ASYM-BEKK). Panel B displays the results for nonlinear models (MRS-BEKK and MRS-ASYM-BEKK). The J–B test is the Jarque–Bera test for normality. L–B (6) is the Ljung–Box autocorrelation test including six lags.

Table V displays the main results of these specification tests. The first part of the table shows the summary statistics for the standardized residuals of the estimated models. The mean value is around zero in all cases with a standard deviation close to one. A reduction in the skewness and kurtosis of the residuals is observed compared with the original series. The Ljung–Box test performed over the standardized residuals reveals a lack of serial autocorrelation in both levels and their cross products. The modeling of time-varying variances also removed the heteroskedasticity problem present in the original series.

These results confirm the consistency of the estimations of our models even for deviations from normality (Capiello & Fearnley, 2000).

6. CONCLUSIONS

This study analyzes hedging effectiveness using nonlinear GARCH models in some of the main European stock indexes. It presents MRS-BEKK specifications that assume nonlinear dynamics between spot and futures returns to overcome the traditional linear GARCH limitations and properly reflect the characteristics of the financial data.

The estimation of the models reveals that significant differences exist in the variance equation parameters between states, which indicates that the volatility process is not defined by a unique process, as proposed by the linear GARCH models, but by two different volatility processes observed during high- and low-volatility periods. The consideration of one instead of two volatility processes leads to poor estimations of volatility, which may influence the estimated hedge ratios. Differences in volatility between low- and high-volatility states are observed in terms of the (asymmetric) impact of shocks and the past variance on the volatility formation in each state. Another interesting result is related to the state governing the process in each period. Usually, high-volatility states are present in contexts of market uncertainty such as the dot-com bubble or the last financial crisis.

The volatility estimations and forecasts are also different between linear and nonlinear models. These differences affect the effectiveness achieved by each strategy as demonstrated by our empirical results. The nonlinear models generally outperform the rest of the models in both in-sample and out-sample analysis. The presented results are robust across countries and for most of the effectiveness measures proposed.

Because the out-sample analysis was performed during the last financial crisis, it appears that the nonlinear models improve the rest of the models during these periods of market jitters. The reason for these results is that the use of these more complex models allows us to extract more information from the data (they allow us to distinguish between calm and uncertain periods). Thus, the resulting estimator for the conditional hedge ratios has a simple behavior [MRS-based (out-of-sample) hedge ratios are much more stable than those of BEKK models]. This finding is consistent with the finding that the OLS hedge ratio outperforms any other conditional hedge ratio.

These results also empirically support previous works that find a theoretical superiority of nonlinear models regarding static and linear GARCH models.

REFERENCES

Alizadeh, A., & Nomikos, N. (2004). A Markov regime switching approach for hedging stock indices. The Journal of Futures Markets, 24, 649–674.

Alizadeh, A., Nomikos, N., & Pouliasis, P. K. (2008). A Markov regime switching approach for hedging energy commodities. Journal of Banking & Finance, 32, 1970–1983.

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. Mathematical Finance, 9, 203–228.

Baba, Y., Engle, R. F., Kraft, D. F., & Kroner, K. F. (1990). Multivariate simultaneous generalized ARCH. Mimeo: University of California at San Diego.

Baillie, R. T., & Myers, R. J. (1991). Bivariate GARCH estimation of the optimal commodity futures hedge. Journal of Applied Econometrics, 6, 109–124.

Brooks, C., Henry, O. T., & Persand, G. (2002). The effect of asymmetries on the optimal hedge ratios. Journal of Business, 75, 333–352.

Capiello, L., & Fearnley, T. (2000). International CAPM with regime-switching parameters, FAME Research Paper Series, 2000, rp17. International Center for Financial Asset Management and Engineering, Geneva, Switzerland.

Carchano, O., & Pardo, A. (2008). Rolling over stock index futures contracts. Journal of Futures Markets, 29, 684–694.

Chen, C. C., & Tsay, W. J. (2011). A Markov regime-switching ARMA approach for hedging stock indices. Journal of Futures Markets, 31, 165–191.

Chen, S. S., Lee, C., & Shrestha, K. (2003). Futures hedge ratios: A review. The Quarterly Review of Economics and Finance, 43, 433–465.

Cheung, C. S., Kwan, C. C., & Yip, P. C. (1990). The hedging effectiveness of options and futures: A mean-gini approach. The Journal of Futures Markets, 10, 61–73.

Cotter, J., & Hanly, J. (2006). Reevaluating hedging performance. The Journal of Futures Markets, 26, 677–702.
Ederington, L. (1979). The hedging performance of the new futures markets. The Journal of Finance, 34, 157–170.
Engle, R. F., & Kroner, F. K. (1995). Multivariate simultaneous generalized ARCH. Econometric Theory, 11, 122–150.
Engle, R. F., & Ng, V. K. (1993). Measuring and testing the impact of news on volatility. Journal of Finance, 5, 1749–1778.
Ghosh, A. (1993). Hedging with stock index futures: Estimation and forecasting with error correction model. The Journal of Futures Markets, 13, 743–752.
Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48, 1779–1801.
Gray, S. F. (1996). Modelling the conditional distribution of interest rates as a regime-switching process. Journal of Financial Economics, 42, 27–62.
Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationarity time series and business cycle. Econometrica, 57, 357–384.
Hamilton, J. D., & Susmel, R. (1994). Autoregressive conditional heteroscedasticity and changes in regime. Journal of Econometrics, 64, 307–333.
Jarque, C. M., & Bera, A. K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. Economics Letters, 6, 255–259.
Johnson, L. (1960). The theory of hedging and speculation in commodity futures. Review of Economic Studies, 27, 139–151.
Jorion, P. (2000). Value at risk: The new benchmark for managing financial risk. (2nd ed.). New York: McGraw-Hill.
Kroner, K. F., & Ng, V. K. (1998). Modeling asymmetric comovements of asset returns. The Review of Financial Studies, 11, 817–844.
Kroner, K., & Sultan, J. (1991). Exchange rate volatility and time varying hedge ratios In S. G. Rhee & R. P. Chang (Eds.), Pacific-basin capital markets research, 2 (pp. 397–412). Amsterdam, North-Holland: Elsevier Science Publishers.
Kroner, K., & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. Journal of Financial and Quantitative Analysis, 28, 535–551.
Lee, H. T. (2009a). A copula-based regime-switching GARCH model for optimal futures hedging. The Journal of Futures Markets, 29, 946–972.
Lee, H. T. (2009b). Optimal futures hedging under jump switching dynamics. Journal of Empirical Finance, 16, 446–456.
Lee, H. T. (2010). Regime switching correlation hedging. Journal of Banking and Finance, 34, 2728–2741.
Lee, H., & Yoder, J. K. (2007a). Optimal hedging with a regime-switching time-varying correlation GARCH model. The Journal of Futures Markets, 27, 495–516.
Lee, H., & Yoder, J. K. (2007b). A bivariate Markov regime switching GARCH approach to estimate time varying minimum variance hedge ratio. Applied Economics, 39, 1253–1265.
Lee, H. T., Yoder, J. K., Mittelhamer, R. C., & McCluskey, J. J. (2006). A random coefficient autoregressive Markov regime switching model for dynamic futures hedging. The Journal of Futures Markets, 26, 103–129.
Lien, D. (1996). The effect of cointegration relationship on futures hedging: A note. The Journal of Futures Markets, 16, 773–780.
Lien, D. (2009). A note on the hedging effectiveness of GARCH models. International Review of Economics & Finance, 18, 110–112.
Lien, D. (2012). A note on the performance of regime switching hedge strategy. The Journal of Futures Markets, 32 (4), 389–396.
Lien, D., & Tse, Y. K. (2002). Some recent developments in futures hedging. Journal of Economic Surveys, 16, 357–396.
Lo, A., & McKinley, C. (1990). When are contrarian profits due to stock market over-reaction? Review of Financial Studies, 3, 175–205.
Marcucci, J. (2005). Forecasting volatility with regime-switching GARCH. Studies in Nonlinear Dynamics & Econometrics, 9, Article 6.
Meneu, V., & Torró, H. (2003). Asymmetric covariance in spot-futures markets. The Journal of Futures Markets, 23, 1019–1046.
Myers, R. (1991). Estimating time varying optimal hedge ratios on futures markets. The Journal of Futures Markets, 11, 39–53.
Myers, R. J., & Thompson, S. R. (1989). Generalized optimal hedge ratio estimation. American Agricultural Economics Association, 71, 858–868.
Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. Econometrica, 59, 347–370.

Park, S. J., & Jei, Y. J. (2010). Estimation and hedging effectiveness of time-varying hedge ratio: Flexible bivariate GARCH approaches. The Journal of Futures Markets, 30, 71–99.

Park, T., & Switzer, L. (1995). Bivariate GARCH estimation of the optimal hedge ratios for stock index futures: A note. The Journal of Futures Markets, 15, 61–67.

Sarno, L., & Valente, G. (2000). The cost of carry model and regime shifts in stock index futures markets: An empirical investigation. The Journal of Futures Markets, 20, 603–624.

Susmel, R. (2000). Switching volatility in private international equity markets. International Journal of Finance and Economics, 5, 265–283.