Barra®

Barra Risk Model

Handbook

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About Barra

In recent years the investment management industry has adjusted to continuing changes—theoretical advances, technological developments, and volatility. To address these, investment managers and financial institutions require the most advanced and powerful analytical tools available.

A Pioneer in Risk Management

As the leading provider of global investment decision support tools and innovative risk management technology, Barra has responded to these industry changes by providing quantitative products and services that are both flexible and efficient.

Barra products are a combination of advanced technology and superior analytics, research, models, and data that provide clients around the world with comprehensive risk management solutions.

Barra uses the best data available to develop econometric financial models. In turn, these models are the basis of software products designed to enhance portfolio performance through returns forecasting, risk analysis, portfolio construction, transaction cost analysis, and historical performance attribution.

With more than 80 researchers in offices around the world and products that cover most of the world's traded securities, Barra maintains one of the strongest risk management research practices in the world today.

Contacting Barra

Client Services is available 24 hours a day, Monday through Friday. You can reach the Client Services as follows:

Web:	*http://www.barra.com/support/service_request/new_request.asp*	
Phone:	North America:	888.588.4567
	Europe:	+44 (0)20.7618.2222
	Asia:	+813 5424 5470 (Japanese)

For local access numbers, visit the Client Support web site, *http://support.barra.com.*

Client Services is your first point of contact concerning your Barra product or service. Support desk analysts are available to assist you with general, technical, data, product usage, and model questions.

Other Barra Resources

You can visit the Library at *http://support.barra.com* for more information on the topics discussed in this handbook.

In addition to handbooks and reference guides, Barra offers numerous workshops and seminars throughout the year. For more information, visit the Events Calendar at *http://www.barra.com*.

Introduction

Barra risk models are products of a thorough and exacting model estimation process. This handbook discusses the methods Barra uses to model portfolio risk.

Product	**Sections of Particular Interest**
Aegis	I, II
BarraOne	all
BIMe text files	I, II, IV, V
Cosmos	I, III, IV, V
Equity text files	I, II
TotalRisk	all

Section I. The Theory of Risk

Chapter 1. Forecasting Risk with Multiple Factor Models discusses the application of multiple-factor modeling to the risk analysis problem.

Section II. Equity Risk

Chapter 2. Forecasting Equity Risk takes a historical perspective on equity risk modeling and provides an overview of Barra equity risk models and their factors.

Chapter 3. Barra Equity Risk Modeling details the process of creating and maintaining a Barra equity risk model.

Section III. Fixed-Income Risk

Chapter 4. Forecasting Fixed-Income Risk takes a historical perspective on fixed-income risk modeling and provides an overview of Barra fixed-income risk models and their factors.

Chapter 5. Interest Rate Risk Modeling describes the process of determining the term structure of interest rates for nominal and inflation-protected bonds.

Chapter 6. Spread Risk Modeling explains how the different models describe the spread risk in various markets and discusses the process of estimating three spread risk models.

Chapter 7. Specific Risk Modeling describes the process of creating heuristic specific risk models and detailed transition-matrix based models to account for issue- and issuer-specific risk.

Section IV. Currency Risk

Chapter 8. Currency Risk Modeling describes the process of creating and maintaining Barra currency risk models.

Section V. Integrated Risk

Chapter 9. Integrated Risk Modeling discusses the Barra Integrated Model (BIM)—which is a multi-asset class model for forecasting asset- and portfolio-level risk of global equities, bonds, and currencies—and the innovative methods behind the global model.

Finally, the Glossary and Index are useful resources for clarifying terminology and finding specific topics.

Further references

Barra has a comprehensive collection of articles and other materials describing the models and their applications. To learn more about the topics contained in this handbook, consult the following references or our extensive Publications Bibliography, which is available from Barra offices and from our Web site at *http://www.barra.com*.

Books

Andrew Rudd and Henry K. Clasing, *Modern Portfolio Theory: The Principles of Investment Management*, Orinda, CA, Andrew Rudd, 1988.

Richard C. Grinold and Ronald N. Kahn, *Active Portfolio Management: A Quantitative Approach for Producing Superior Returns and Controlling Risk*, Second Edition, McGraw-Hill Professional Publishing, Columbus, OH, 1999.

The Theory of Risk

This section explains the underlying concepts behind risk forecasting. It includes:

Chapter 1: Forecasting Risk with Multiple-Factor Models

Section One

1. Forecasting Risk with Multiple-Factor Models

The analysis of ***risk***—which is the total dispersion or volatility of returns for a security or portfolio—is a critical element of superior investment performance. The goal of risk analysis is not to minimize risk but to properly weigh risk against return.

Through the years, theoretical approaches to risk analysis have become increasingly sophisticated. With more advanced concepts of risk and return, investment portfolio models have changed to reflect this growing complexity. The multiple-factor model (MFM) has evolved as a helpful tool for analyzing portfolio risk.

What Are Multiple-Factor Models?

Multiple-factor models are formal statements about the relationships among asset returns in a portfolio. The basic premise of MFMs is that similar assets display similar returns. Assets can be similar in terms of quantifiable attributes, such as market information (such as price changes and volume), fundamental company data (such as industry and capitalization), or exposure to other factors (such as interest rate changes and liquidity).

MFMs identify ***common factors***, which are categories defined by common characteristics of different securities, and determine the return sensitivity to these factors.

Multiple-factor models of security market returns can be divided into three types: macroeconomic, fundamental, and statistical factor models. ***Macroeconomic factor models*** use observable economic variables, such as changes in inflation and interest rates, as measures of the pervasive shocks to security returns. ***Fundamental factor models*** use the returns to portfolios associated with observed security attributes such as dividend yield, book-to-market ratio, and industry membership. ***Statistical factor models*** derive their factors from factor analysis of the covariance matrix of security returns.

"Since the factors can represent the components of return as seen by the financial analyst, the multiple-factor model is a natural representation of the real environment."

Barr Rosenberg, 1974
Multiple Factor Models

Barra equity models are fundamental factor models, which outperform the macroeconomic and statistical models in terms of explanatory power.[1] Barra fixed-income models are combinations of fundamental and macroeconomic factor models. Returns of high-quality debt are largely explained by macroeconomic factors such as changes in the default-free or other low-risk yields (that is, in terms of government bond returns or movements of the swap curve). Returns of other forms of debt are accounted for by fundamental factors based on industry and credit quality, in addition to macroeconomic factors.

How Do Multiple-Factor Models Work?

Barra derives MFMs from asset patterns observed over time. The difficult steps are pinpointing these patterns and then identifying them with asset factors that investors can understand.

The asset exposures to these factors are specified or calculated. Then, a cross-sectional regression is performed to determine the returns to each factor over the relevant time period. A history of the factor returns is taken to create the common factor risk model with its variance-covariance matrix and the specific risk model. The resulting models forecast portfolio or asset risk.

Investors rely on risk forecasts to select assets and construct portfolios. They base their decisions on information gleaned from MFM analyses as well as their risk preferences and other information they possess.

Advantages of Multiple-Factor Models

Using multiple-factor models for security and portfolio analysis has many advantages, including:

- MFMs offer a more thorough breakdown of risk and, therefore, a more complete analysis of risk exposure than other methods, such as single-factor approaches.

1. Gregory Connor, "The Three Types of Factor Models: A Comparison of Their Explanatory Power," *Financial Analysts Journal*, May/June 1995.

- Because economic logic is used in their development, MFMs are not limited by purely historical analysis.

- MFMs can be built using methods that can withstand outliers in asset data.

- As the economy and characteristics of individual issuers change, MFMs adapt to reflect changing asset characteristics.

- MFMs isolate the impact of individual factors, providing segmented analysis for better informed investment decisions.

- Lastly, MFMs are realistic, tractable, and understandable to investors.

Of course, MFMs have their limitations. They predict much, but not all, of portfolio risk. In addition, they predict risk, not return; investors must choose the investment strategies themselves.

An Illustration of Multiple-Factor Models

Accurate characterization of portfolio risk requires an accurate estimate of the covariance matrix of security returns. A relatively simple way to estimate this covariance matrix is to use the history of security returns to compute each variance and covariance. This approach, however, suffers from two major drawbacks:

- Estimating a covariance matrix for, say, 3,000 assets requires data for at least 3,000 periods. With monthly or weekly estimation horizons, such a long history may simply not exist.

- It is subject to estimation error: in any period, two assets such as Weyerhaeuser and Ford may show very high correlation—higher than, say, GM and Ford. Our intuition suggests that the correlation between GM and Ford should be higher because they are in the same line of business. The simple method of estimating the covariance matrix does not capture our intuition.

This intuition, however, points to an alternative method for estimating the covariance matrix. Our feeling that GM and Ford should be more highly correlated than Weyerhaeuser and Ford comes from Ford and GM being in the same industry. Taking this further, we can argue that firms with similar characteristics, such as their line of business, should have returns that behave similarly. For example, Weyerhaeuser, Ford, and GM will all have a common component in their returns because they would all be affected by news that affects the market as a whole. The effects of such news may be captured by a stock market component in each stock's return[1] or a yield curve movement component in each bond's return. The degree to which each of the three securities responds to this market component depends on the sensitivity of each security to the stock market or yield curve component.

Additionally, we would expect GM and Ford to respond to news affecting the automobile industry, whereas we would expect Weyerhaeuser to respond to news affecting the forest and paper products industry. The effects of such news may be captured by the average returns of securities in the auto industry and the forest and paper products industry. There are, however, events that affect one security without affecting the others. For example, a defect in the brake system of GM cars, which forces a recall and replacement of the system, will likely have a negative impact on GM's stock and bond prices. This event, however, will most likely leave Weyerhaeuser and Ford security prices unaltered.

In other words, the overall variation in GM's asset returns is the joint result of several sources of variation. The volatility of GM stock returns can be attributed to stock market return, variation in auto industry returns, and any variations that are specific to GM. Similarly, the volatility of bonds issued by GM can be attributed to the movement in the yield curve generally, variation in auto sector and bond ratings, and any variations that are specific to GM. The same can be said about Ford's asset returns, and since the market and industry variations are identical for the two companies, we expect GM and Ford returns to move together to a large degree. Weyerhaeuser and GM, or Weyerhaeuser and Ford, on the other hand, are likely to move together to a lesser degree because the only common component in their returns is the market return. Some additional correlation may arise, however, because the auto industry and paper products industry returns may exhibit some correlation.

1. This common component may be the weighted average return to all U.S. stocks.

This approach of analyzing total variation or risk into its component factors provides insight into many types of assets.

By beginning with our intuition about the sources of co-movement in security returns, we have made substantial progress in estimating the covariance matrix of security returns. What we need now is the covariance matrix of common sources in security returns, the variances of security-specific returns, and estimates of the sensitivity of security returns to the common sources of variation in their returns. Because the common sources of risk are likely to be much fewer than the number of securities, we need to estimate a much smaller covariance matrix and hence a smaller history of returns is required. Moreover, because similar assets are going to have larger sensitivities to similar common sources of risk, similar assets will be more highly correlated than dissimilar assets: our estimated correlation for GM and Ford will be larger than that for Ford and Weyerhaeuser.

The decomposition of security returns into common and specific sources of return is, in fact, a multiple-factor model of security returns.

Model Mathematics

Portfolio risk and return can be decomposed along two dimensions: that which is due to factors which are prevalent throughout the market and that which is due to the idiosyncratic nature of the securities in the portfolio. A multiple-factor model is a powerful tool to shed light on these sources of risk and return within a portfolio.

Single-Factor Model

For single-factor models, the equation that describes the excess rate of return is:

$$r_i = x_i f + u_i \tag{EQ 1-1}$$

where

r_i	=	total excess return over the risk-free rate of security *i*
x_i	=	sensitivity of security *i* to the factor
f	=	rate of return on the factor
u_i	=	non-factor or specific return of security *i*

The rate of factor return (*f*) and the specific return (*u*) are assumed to be uncorrelated, and the *u's* are uncorrelated across different assets.

Multiple-Factor Model

MFMs build on single-factor models by including and describing the interrelationships among factors. We can expand the model to include many factors, as shown in the equation below:

$$r_j = \underbrace{x_1 f_1 + x_2 f_2 + x_3 f_3 + x_4 f_4 \ldots x_K f_K}_{\text{common factor return}} + \underbrace{u_j}_{\text{specific return}} \tag{EQ 1-2}$$

The asset's return is broken out into the return due to individual factors and a portion unique to the asset and not due to the common factors. In addition, each factor's contribution is a product of the asset's exposure or weight in the factor and the return of that factor. The total excess return equation for a multiple-factor model can be summarized with:

$$r_i = \sum_{k=1}^{K} x_{ik} f_k + u_i \quad \text{(EQ 1-3)}$$

where

x_{ik}	=	risk exposure of security i to factor k
f_k	=	rate of return to factor k
u_i	=	non-factor or specific return of security i

Note that when K=1, the MFM equation reduces to the earlier single-factor version.[1]

Exposures (x_{ik})

By observing patterns over time, common factors can be identified and exposures to these factors can be determined. These factors are based on market or fundamental data.

The model's profile of a security responds immediately to any change in the company's structure or the market's behavior. Barra updates the security exposures of most fixed-income models on a daily basis and the security exposures of most equity models on a monthly basis, using the last trading day's information to compute exposures for the coming month.

Factor Returns (f_k)

Factor returns are pure measures of the factor's actual performance net of any other effects. Since factor returns are not readily observable, we must estimate them. Recall that asset exposures are computed at the end of each month. Using our multiple-factor model framework and the observed asset returns over the next month, we can estimate factor returns over the month. This is done with a cross-sectional regression of asset returns over the month on the exposures of the assets to the factors.

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1. For example, a single-factor model in which the market return is the only relevant factor.

Multiple-Asset Portfolio

When a portfolio consists of only one security, Equation 1-3 describes its excess return. But most portfolios comprise many securities, each representing a proportion, or ***weight***, of the total portfolio. When weights h_{P1}, h_{P2},...,h_{PN} reflect the weights of N securities in portfolio P, we express the excess return in the following equation:

$$r_P = \sum_{k=1}^{K} x_{Pk} f_k + \sum_{i=1}^{N} h_{Pi} u_i \tag{EQ 1-4}$$

where

r_P	=	total excess return of portfolio
x_{Pk}	=	$\sum_{i=1}^{N} h_{Pi} x_{ik}$
f_k	=	return of factor k
h_{Pi}	=	weight of security i
u_i	=	non-factor or specific return of security i

This equation includes the return from all sources and lays the groundwork for further MFM analysis.

Risk Prediction with MFMs

A central part of the model is its factor covariance matrix. This matrix contains the variances and covariances of the common factors. To estimate a portfolio's risk, we must consider not only the security or portfolio's exposures to the factors, but also each factor's risk and the covariance or interaction between factors.

Without the framework of a multiple-factor model, estimating the covariance of each asset with every other asset would likely result in finding spurious relationships. For example, an estimation universe of 1,400 assets entails 980,700 covariances and variances to calculate.

$$V(i,j) = Covariance\left[r(i), r(j)\right] \qquad \text{(EQ 1-5)}$$

where

$V(i,j)$ = asset covariance matrix

i,j = individual assets

$$V = \begin{bmatrix} V(1,1) & V(1,2) & \cdots & V(1,n) \\ V(2,1) & V(2,2) & \cdots & V(2,n) \\ V(3,1) & V(3,2) & \cdots & V(3,n) \\ \vdots & \vdots & & \vdots \\ V(n,1) & V(n,2) & \cdots & V(n,n) \end{bmatrix}$$

Figure 1-1

Asset Covariance Matrix

For N=1,400 assets, there are 980,700 covariances and variances to estimate.

A multiple-factor model simplifies these calculations dramatically. This results from replacing individual asset profiles with categories defined by common characteristics (factors). For example, in the Multiple-Horizon U.S. Equity Model, 68 factors capture the risk characteristics of equities. This reduces the number of covariance and variance calculations to 2,346. Moreover, determining fewer parameters results in a smaller chance of finding spurious relationships.

$$F(k,m) = Covariance\left[f(k), f(m)\right] \qquad \text{(EQ 1-6)}$$

where

$F(k,m)$ = factor covariance matrix

k,m = common factors

Figure 1-2

Equity Factor Covariance Matrix

For K=68 factors, there are 2,346 covariances and variances to estimate. Quadrant I includes the covariances of risk indices with each other; quadrants II and III are mirror images of each other, showing the covariances of risk indices with industries; and quadrant IV includes covariances of industries with each other.

$$F = \left[\begin{array}{ccc|ccc} F_{(1,1)} & \cdots & F_{(1,13)} & F_{(1,14)} & \cdots & F_{(1,68)} \\ \vdots & \mathrm{I} & \vdots & \vdots & \mathrm{II} & \vdots \\ F_{(13,1)} & \cdots & F_{(13,13)} & F_{(13,14)} & \cdots & F_{(13,68)} \\ \hline F_{(14,1)} & \cdots & F_{(14,13)} & F_{(14,14)} & \cdots & F_{(14,65)} \\ \vdots & \mathrm{III} & \vdots & \vdots & \mathrm{IV} & \vdots \\ F_{(68,1)} & \cdots & F_{(68,13)} & F_{(68,14)} & \cdots & F_{(68,68)} \end{array}\right]$$

The Covariance Matrix

Barra's risk models use historical returns to create a framework for predicting the future return volatility of an asset or a portfolio. Each month, the ***estimation universe***, which is the set of representative assets in each local market, is used to attribute asset returns to common factors and to a specific, or residual, return.

The monthly returns for the estimation universe are formulated as a single matrix equation of n assets and k factors. Each row represents one of the assets in a portfolio or universe.

The return of each asset at the end of a month, along with its factor exposures at the beginning of the month, is known. The factor returns, which are the values that best explain the asset returns, are estimated via regression. The time series of factor returns are then used to generate factor variances and covariances in the covariance matrix.

Figure 1-3

Factor Return Calculation

Using an MFM greatly simplifies the estimation process. The matrix depicts the multiple factor model.

$$\begin{bmatrix} r_{(1)} \\ r_{(2)} \\ \vdots \\ r_{(n)} \end{bmatrix} = \begin{bmatrix} x_{(1,1)} & x_{(1,2)} & \cdots & x_{(1,k)} \\ x_{(2,1)} & x_{(2,2)} & \cdots & x_{(2,k)} \\ \vdots & \vdots & \cdots & \vdots \\ x_{(n,1)} & x_{(n,2)} & \cdots & x_{(n,k)} \end{bmatrix} \begin{bmatrix} f_{(1)} \\ f_{(2)} \\ \vdots \\ f_{(k)} \end{bmatrix} + \begin{bmatrix} u_{(1)} \\ u_{(2)} \\ \vdots \\ u_{(n)} \end{bmatrix}$$

Deriving the Variance-Covariance Matrix of Asset Returns

We can easily derive the matrix algebra calculations that support and link the above diagrams by using an MFM. We can start with the MFM equation, $r = Xf + u$.

Substituting this relation in the basic equation, we find that:

$$Risk = Var(r) \tag{EQ 1-7}$$

$$Risk = Var(Xf + u) \tag{EQ 1-8}$$

$$Risk = Var(Xf) + Var(u) \tag{EQ 1-9}$$

Using the matrix algebra formula for variance, the risk equation becomes:

$$Risk = XFX^T + \Delta \tag{EQ 1-10}$$

where

X = matrix of factor exposures of *n* assets to *k* factors:

$$\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,k} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,k} \\ \vdots & \vdots & & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,k} \end{bmatrix}$$

F = factor return variance-covariance matrix for *m* factors:

$$\begin{bmatrix} Var(f_1) & Cov(f_1, f_2) & \cdots & Cov(f_1, f_k) \\ Cov(f_2, f_1) & Var(f_2) & \cdots & Cov(f_2, f_k) \\ \vdots & \vdots & & \vdots \\ Cov(f_k, f_1) & Cov(f_k, f_2) & \cdots & Var(f_k) \end{bmatrix}$$

X^T transposition of X matrix

Δ diagonal matrix of specific risk variances

Final Risk Calculation

The covariance matrix is used in conjunction with a portfolio's weight in each asset and the factor exposures of those assets to calculate portfolio risk. The following formula is the underlying form of Barra risk calculations:

$$\sigma_p = \sqrt{h_p (XFX^T + \Delta) h_p^T} \tag{EQ 1-11}$$

where

σ_p = volatility of portfolio returns

h_p = vector of portfolio weights for N assets: $\begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix}$.

Summary

Robust risk analysis can provide insight to all investors. The goal of risk analysis is not to minimize risk but to properly weigh risk against return. In this book, we discuss the methods Barra uses to model portfolio risk. The foundations of risk modeling are relevant for analyzing a wide variety of asset types, including stocks, bonds and other fixed-income securities, currencies, and derivatives.

Equity Risk

Section Two provides an overview of equity risk models and discusses the extensive, detailed process of creating Barra equity models.

Chapter 2: Forecasting Equity Risk

Chapter 3: Barra Equity Risk Modeling

Section Two

2. Forecasting Equity Risk

Many methods exist for forecasting a stock's future volatility. One method is to examine its historical behavior and conclude that it will behave similarly in the future. An obvious problem with this technique is that results depend on the length and type of history used. The security might have changed over time due to mergers, acquisitions, divestitures, or other corporate actions. The past contains information that may be no longer relevant to the present. Yet this is the approach most commonly used for measuring beta (see section on *Barra Predicted Beta* on page 18).

A more informative approach uses insights into the characteristics and behavior of the stock and market as a whole, as well as the interactions between them. We could determine the future behavior of a stock or portfolio by examining its characteristics with respect to the overall market.

A Historical Perspective

Before the 1950s, there was no concept of ***systematic,*** or market-related, return. Return was a rise in the value of an asset and risk was a drop in the value of an asset. The investor's primary investment tools were intuition and insightful financial analysis. Portfolio selection was simply an act of assembling a group of "good" assets.

Financial theorists became more scientific and statistical in the early 1950s. Harry Markowitz was the first to quantify risk (as standard deviation) and diversification. He showed precisely how the risk of the portfolio was less than the risk of its components. In the late 1950s, Leo Breiman and John L. Kelly Jr. derived mathematically the peril of ignoring risk. They showed that a strategy that explicitly accounted for risk outperformed all other strategies in the long run.[1]

1. See, for example, Leo Breiman, "Investment Policies for Expanding Businesses Optimal in a Long-Run Sense," *Naval Research Logistics Quarterly* Volume 7, No. 4, (December 1960): 647–651.

We now know how diversification reduces portfolio risk. It averages factor-related risk (such as industry exposures for equities and credit exposure for bonds) and significantly reduces asset-specific risk. However, diversification does not eliminate all risk because assets tend to move up and down together with the market. Therefore, while non-market-related risk, or ***residual risk***, can be minimized, market or ***systematic risk*** cannot be eliminated by diversification.

Figure 2-1

Diversification and Risk

As a portfolio manager increases the number of assets in a portfolio, residual—or non-market related—risk is diversified or concentrated. Risk is diversified if any position added to the portfolio is less than perfectly correlated with others already in the portfolio and has lower volatility. Market risk is undiversifiable. One benefit of using a multiple-factor model is better understanding of the results of adding and eliminating positions.

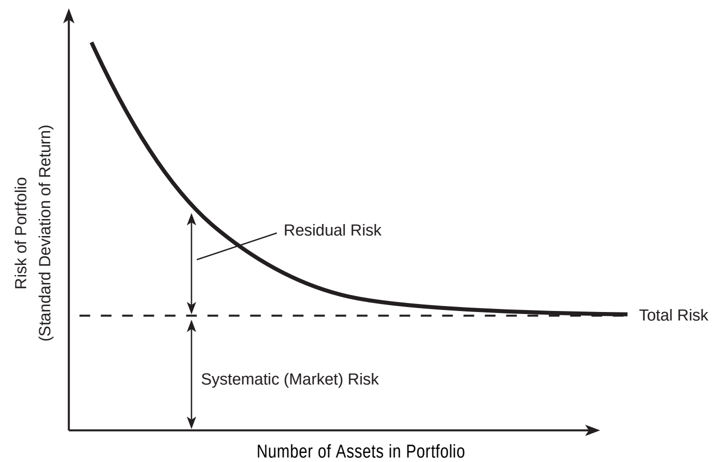


Figure 2-1 shows the balance between residual risk and market risk changing as the number of different assets in a portfolio rises. At a certain portfolio size, all residual risk is effectively removed, leaving only market risk.

"Diversification is good."

Harry Markowitz, 1952

As investment managers became more knowledgeable, there was a push to identify the conceptual basis underlying the concepts of risk, diversification, and returns. The Capital Asset Pricing Model (CAPM) was one approach that described the equilibrium relationship between return and market risk.[1]

1. William Sharpe earned the Nobel Prize in Economics for his development of CAPM.

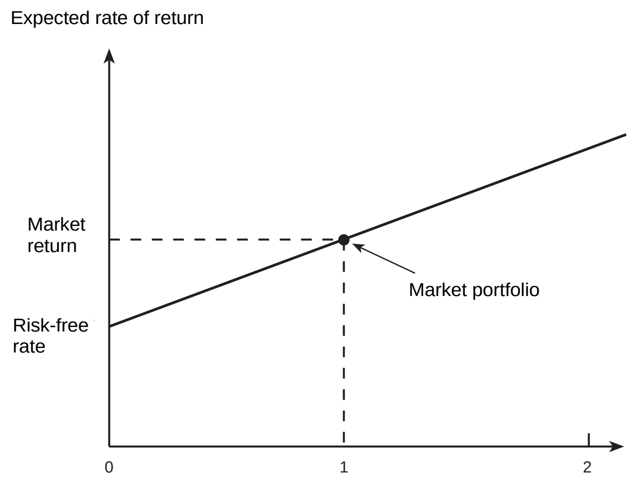


Figure 2-2

Capital Asset Pricing Model (CAPM)

Capital Asset Pricing Model asserts that the expected excess return on securities is proportional to their systematic risk coefficient, or beta. The market portfolio is characterized by a beta of unity.

The central premise of ***CAPM*** is that, on average, investors are not compensated for taking on residual risk. CAPM asserts that the expected residual return is zero while the expected systematic return is greater than zero and is linearly related to an asset's beta with relation to the market portfolio.

The measure of portfolio exposure to systematic risk is called beta (β). ***Beta*** is the relative volatility or sensitivity of a security or portfolio to market moves. Returns, and hence risk premia, for any asset or portfolio will be related to beta, the exposure to undiversifiable systematic risk. Equation 2-1 states this linear relationship.

$$E\left[\tilde{r}_i\right] - r_F = \beta_i E\left[\tilde{r}_M - r_F\right] \qquad \textbf{(EQ 2-1)}$$

where:

$\tilde{r}_i$	=	return on asset i
r_F	=	risk-free rate of return
β_i	=	$\dfrac{Cov\left[\tilde{r}_i, \tilde{r}_M\right]}{Var\left[\tilde{r}_M\right]}$
$\tilde{r}_M$	=	return on market portfolio

Learn more about

Barra Predicted Beta

Beta is a gauge of the expected response of a stock, bond, or portfolio to the overall market. For example, a stock with a beta of 1.5 has an expected excess return of 1.5 times the market excess return. If the market is up 10% over the risk-free rate, then—other things held equal—the portfolio is expected to be up 15%. Beta is one of the most significant means of measuring portfolio risk.

Historical Beta vs. Predicted Beta

Historical beta is calculated after the fact by running a regression (often over 60 months) on a stock's excess returns against the market's excess returns. There are two important problems with this simple historical approach:

- It does not recognize fundamental changes in the company's operations. For example, when RJR Nabisco spun off its tobacco holdings in 1999, the company's risk characteristics changed significantly. Historical beta would recognize this change only slowly, over time.
- It is influenced by events specific to the company that are unlikely to be repeated. For example, the December 1984 Union Carbide accident in Bhopal, India, took place in a bull market, causing the company's historical beta to be artificially low.

Predicted beta, the beta Barra derives from its risk model, is a forecast of a stock's sensitivity to the market. It is also known as fundamental beta, because it is derived from fundamental risk factors. In the Barra model, these risk factors include attributes—such as size, yield, and volatility—plus industry exposure. Because we re-estimate these risk factors monthly, the predicted beta reflects changes in the company's underlying risk structure in a timely manner. Barra applications use predicted beta rather than historical beta because it is a better forecast of market sensitivity of an asset in a portfolio.

"Only undiversifiable risk should earn a premium."

William F. Sharpe, 1964
Capital Asset Pricing Model

CAPM is a model of return. Underlying it are equilibrium arguments and the view that the market is efficient because it is the portfolio that every investor on average owns. CAPM does not require that residual returns be uncorrelated. But it did inspire Sharpe to suggest a one-factor risk model that does assume uncorrelated residual returns. This model has the advantage of simplicity. It is quite useful for back-of-envelope calculations; but it ignores the risk that arises from common factor sources, such as industries, capitalization, and yield.

By the 1970s, the investment community recognized that assets with similar characteristics tend to behave in similar ways. This notion is captured in the Arbitrage Pricing Theory (APT). ***APT*** asserts that security and portfolio expected returns are linearly related to the expected returns of an unknown number of systematic factors.

The focus of APT is on forecasting returns. Instead of equilibrium arguments, Stephen Ross and others used arbitrage arguments to assert that expected specific returns are zero, but expected common factor returns (including the market and other factors) need not be zero. Just like the CAPM, APT inspired a class of risk models: the multiple-factor model (MFM).

"The arbitrage model was proposed as an alternative to the mean variance capital asset pricing model."

Stephen A. Ross, 1976
Arbitrage Pricing Theory

In the mid-1970s, Barr Rosenberg pioneered a new class of risk models based on the idea that assets with similar characteristics should display similar returns. Multiple-factor models assert that many influences act on the volatility of an asset, and these influences are often common across many assets. A properly constructed MFM is able to produce risk analyses with more accuracy and intuition than a simple covariance matrix of security returns or the CAPM.

Barra's Equity Multiple-Factor Model

Barra's equity risk models decompose asset returns into components due to common factors and a specific, or idiosyncratic, factor. The models capture the various components of risk and provide a multifaceted, quantitative measure of risk exposure. Together with specific risk, market membership, industries, and risk indices provide a comprehensive partition of risk.

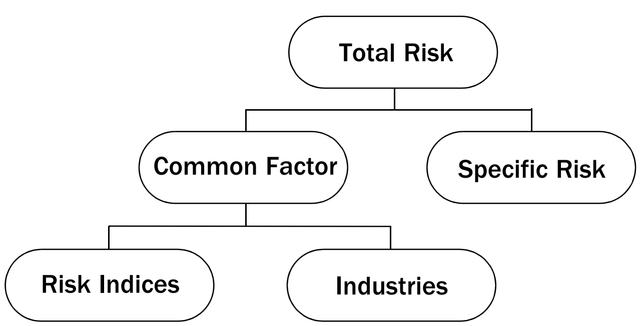


Figure 2-3
Equity Risk Decomposition

Common Factors

Stocks with similar characteristics exhibit similar return behavior. These shared characteristics, or ***common factors***, are excellent predictors of future risk.

Many of the influences that affect the volatility of a stock or portfolio are common factors which are prevalent across the entire market. These common factors—industry membership (and trends in that industry) and risk indices—not only help explain performance, but also anticipate future volatility.

Risk Indices

Barra combines fundamental and market data to create ***risk indices*** that measure risk associated with common features of an asset. Common dimensions of style such as growth/value and smallcap/largecap can be described using risk indices. Each Barra equity risk model has a predefined set of risk indices.

Industries

An ***industry*** is a homogeneous collection of business endeavors. Each Barra equity risk model has a predefined set of industries and sectors appropriate to its environment. Each security is classified into an appropriate industry as defined by its operations, although a number of models support multiple-industry classification for large conglomerates.

Specific Risk

Specific risk forecasting is a three-part process. We first estimate the average specific risk of all assets covered in a model, then the specific risk of each asset relative to the universe of assets. Finally, we combine the average and relative components and scale the product to adjust for average bias. The result is a specific risk forecast for each asset that is generally unbiased.

3. Barra Equity Risk Modeling

The creation of a comprehensive equity risk model is an extensive, detailed process of determining the factors that describe asset returns. Model estimation involves a series of intricate steps that is summarized in Figure 3-1.

Model Estimation Overview

The first step in model estimation is acquiring and cleaning data. Both market information (such as price, trading volume, dividend yield, or capitalization) and fundamental data (such as earnings, sales, industry information, or total assets) are used. Special attention is paid to capital restructurings and other atypical events to provide for consistent cross-period comparisons.

Descriptor selection follows. This involves choosing and standardizing variables which best capture the risk characteristics of the assets. To determine which descriptors partition risk in the most effective and efficient way, the descriptors are tested for statistical significance. Useful descriptors often significantly explain cross-sectional returns.

Risk index formulation and assignment to securities is the fourth step. This process involves collecting descriptors into their most meaningful combinations. A variety of techniques are used to evaluate different possibilities. For example, cluster analysis is one statistical tool that might be used to assign descriptors to risk indices.

Along with risk index exposures, industry allocations are determined for each security. In most Barra models, a single industry exposure is assigned, but multiple exposures for conglomerates are calculated in a few models, including the U.S. and Japan models.

Next, through cross-sectional regressions, we calculate factor returns to estimate covariances between factors, generating the covariance matrix used to forecast risk. The factor covariances are computed for most models by exponentially weighting historical observations. This method places more weight on recent observa-

Model Estimation Process

1. Data acquisition
2. Descriptor selection and testing
3. Descriptor standardization
4. Risk index formulation
5. Industry allocation
6. Factor return estimation
    a. Covariance matrix calculation
    b. Exponential weighting
    c. Covariance matrix scaling: DEWIV and GARCH
7. Specific risk forecasting
    a. Average specific risk estimation
    b. Relative specific risk estimation
    c. Average and relative forecasting
8. Model updating

tions and allows the model to capture changes in risk in a timely fashion. We may further modify the matrix with either generalized auto-regressive conditional heteroskedasticity (GARCH) techniques or daily exponentially weighted index volatility (DEWIV) methods to make it more responsive to changing market conditions.

Specific returns are separated out at this stage of return estimation and ***specific risk*** is forecast. This is the portion of total risk that is related solely to a particular stock and cannot be accounted for by common factors. The greater an asset's specific risk, the larger the proportion of return variation attributable to idiosyncratic, rather than common, factors.

Lastly, the model undergoes final testing and updating. Risk forecasts are tested against alternative models. Tests compare *ex ante* forecasts with *ex post* realizations of beta, specific risk, and active risk. New information from company fundamental reports and market data is incorporated, and the covariance matrix is recalculated.

Figure 3-1 summarizes these steps.

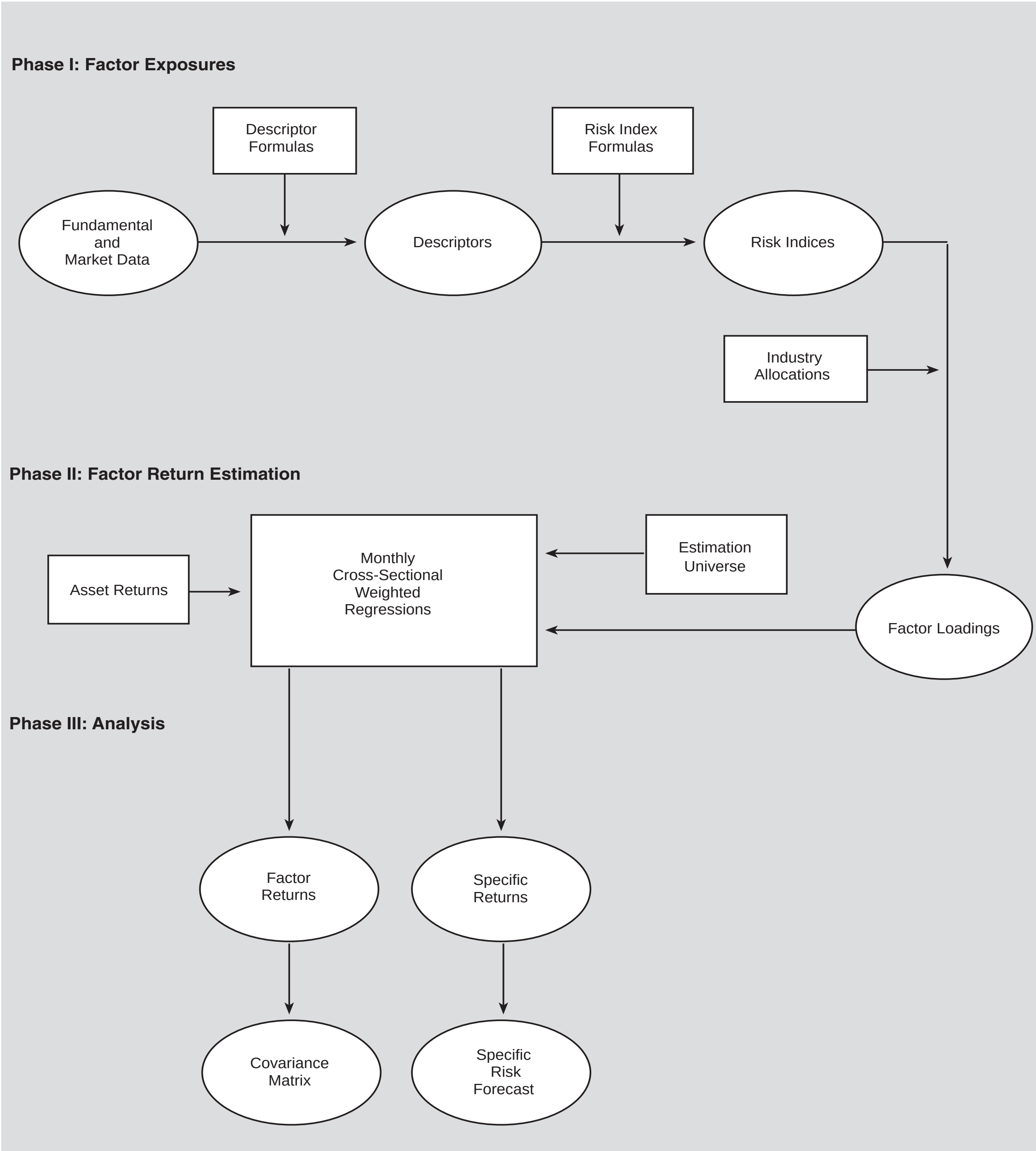


Figure 3-1

Data Flow for Model Estimation

Data Acquisition

The first step in model estimation is acquiring and standardizing data. Market data and fundamental data for each equity risk model are gleaned, verified, and compiled from more than 100 data feeds supplied by 56 data vendors.

Market information is collected daily. Fundamental company data is derived from quarterly and annual financial statements.

After data is collected, it is scrutinized for inconsistencies, such as jumps in market capitalization, missing dividends, and unexplained discrepancies between the day's data and the previous day's data. Special attention is paid to capital restructurings and other atypical events to provide for consistent cross-period comparisons. Information then is compared across different data sources to verify accuracy.

Our robust system of checks and our data collection infrastructure, which has been continuously refined for more than 25 years, ensure that Barra's risk models utilize the best available data.

Descriptor Selection and Testing

Descriptor candidates are drawn from several sources. For some descriptors, market and fundamental information is combined. An example is the earnings to price ratio, which measures the relationship between the market's valuation of a firm and the firm's earnings.

Descriptor selection is a largely qualitative process that is subjected to rigorous quantitative testing. First, we identify preliminary descriptors. Good descriptor candidates are individually meaningful; that is, they are based on generally accepted and well-understood asset attributes. Furthermore, they divide the market into well-defined categories, providing full characterization of the portfolio's important risk features. Barra has more than two decades of experience identifying important descriptors in equity markets worldwide. This experience informs every new model we build.

Selected descriptors must have a sound theoretical justification for inclusion in the model. They must be useful in predicting risk and based on timely, accurate, and available data. In other words, each descriptor must add value to the model. If the testing process shows that they do not add predictive power, they are rejected.

Descriptor Standardization

The risk indices are composed of descriptors designed to capture all the relevant risk characteristics of a company. The descriptors are first ***normalized***, that is, they are standardized with respect to the estimation universe. The normalization process involves setting random variables to a uniform scale. A constant (usually the mean) is subtracted from each number to shift all numbers uniformly. Then each number is divided by another constant (usually the standard deviation) to shift the variance.

The normalization process is summarized by the following relation:

$$[\textit{normalized descriptor}] = \frac{[\textit{raw descriptor}] - [\textit{mean}]}{[\textit{standard deviation}]}$$

The descriptors are then combined into meaningful risk factors, known as ***risk indices.***

Risk Index Formulation

Asset returns are regressed against industries and descriptors, one descriptor at a time, after the normalization step. Each descriptor is tested for statistical significance. Based on the results of these calculations and tests, descriptors for the model are selected and assigned to risk indices.

Risk index formulation is an iterative process. After the most significant descriptors are added to the model, remaining descriptors are subjected to stricter testing. At each stage of model estimation, a new descriptor is added only if it adds explanatory power to the model beyond that of industry factors and already assigned descriptors.

Industry Allocation

Industry allocation is defined according to what is appropriate to the local environment. Each security is classified into an industry by its operations. Barra either uses a data vendor's allocation scheme or creates one that better categorizes the assets in the estimation universe.

For most equity models, companies are allocated to single industries. For the United States, Mexico, and Japan, however, sufficient data exists to allocate to multiple industries.

Learn more about

Multiple Industry Allocation—U.S. and Japan

For the United States and Japan, industry exposures are allocated using industry segment data. For Japan, it's sales; for the U.S. it's operating earnings, total assets, and sales. For any given multi-industry allocation, the weights will add up to 100%. Walt Disney Co., for instance, is allocated to 65% media and 35% entertainment.

Multiple industry allocation provides more accurate risk prediction and better describes market conditions and company activity. Barra's multiple-industry model captures changes in a company's risk profile as soon as new business activity is reported to shareholders. Alternative approaches can require 60 months or more of data to recognize changes that result from market prices.

Factor Return Estimation

The previous steps have defined the exposures of each asset to the factors at the beginning of every period in the estimation window. The factor excess returns over the period are then obtained via a cross-sectional regression of asset excess returns on their associated factor exposures:

$$r_i = X_i f_i + u_i \qquad \text{(EQ 3-1)}$$

where

r_i	=	excess returns to each asset
X_i	=	exposure matrix of assets to factors
f_i	=	factor returns to be estimated
u_i	=	specific returns

The resulting factor returns are robust estimates which can be used to calculate a factor covariance matrix to be used in the remaining model estimation steps.

Covariance Matrix Calculation

The simplest way to estimate the factor covariance matrix is to compute the sample covariances among the entire set of estimated factor returns. Implicit in this process is the assumption that we are modeling a stable process and, therefore, each point in time contains equally relevant information.

A stable process implies a stable variance for a well-diversified portfolio with relatively stable exposures to the factors. However, considerable evidence shows that correlations among factor returns change. In some markets, the volatility of market index portfolios changes. For example, periods of high volatility are often followed by persistent periods of high volatility; in other words, periods of high volatility cluster. The high level of volatility eventually stabilizes to a lower level of volatility. The changing correlations among factor returns and the changing volatility of market portfolios belie the stability assumption underlying a simple covariance matrix.

For certain models, we relax the assumption of stability in two ways. First, in computing the covariance among the factor returns, we assign more weight to recent observations relative to observations in the distant past. Second, we estimate a model for the volatility of a market index portfolio—for example, the S&P 500 in the United States and the TSE1 in Japan—and scale the factor covariance matrix so that it produces the same volatility forecast for the market portfolio as the model of market volatility.

Exponential Weighting

Suppose that we think that observations that occurred 60 months ago should receive half the weight of the current observation. Denote by T the current period, and by t any period in the past, $t = 1,2,3,\ldots,T-1,T$, and let $\lambda = .5^{1/60}$. If we assign a weight of λ^{T-t} to observation t, then an observation that occurred 60 months ago would get half the weight of the current observation, and one that occurred 120 months ago would get one-quarter the weight of the current observation. Thus, our weighting scheme would give *exponentially declining weights* to observations as they recede in the past.

Our choice of sixty months was arbitrary in the above example. More generally, we give an observation that is *HALF-LIFE* months ago one-half the weight of the current observation. Then we let:

$$\lambda = (0.5)^{\frac{1}{HALF-LIFE}} \quad \text{(EQ 3-2)}$$

and assign a weight of:

$$w(t) = \lambda^{T-t}. \quad \text{(EQ 3-3)}$$

The length of the half-life controls how quickly the factor covariance matrix responds to recent changes in the market relationships between factors. Equal weighting of all observations corresponds to *HALF-LIFE* $=\infty$. Too short a half-life effectively throws away data at the beginning of the series. If the process is perfectly stable, this decreases the precision of the estimates. Our tests show that the best choice of half-life varies from country to country.

Hence, we use different values of half-life for different single-country models.

Covariance Matrix Scaling: Computing Market Volatility

In some markets, market volatility changes in a predictable manner. As stated before, we find that returns that are large in absolute value cluster in time, or that volatility persists. Moreover, periods of above normal returns are, on average, followed by lower volatility, relative to periods of below-normal returns. Finally, we find that actual asset return distributions exhibit a higher likelihood of extreme outcomes than is predicted by a normal distribution with a constant volatility.

Variants of daily exponentially weighted index volatility (DEWIV) and generalized autoregressive conditional heteroskedasticity (GARCH) models capture these empirical regularities by allowing volatility to increase following periods of high realized volatility, or below-normal returns, and allowing volatility to decrease following periods of low realized volatility, or above-normal returns.

Variants of these systematic scalings are applied as appropriate to Barra local models over time.[1] See the *Barra Equity Risk Model Reference Guide* for specific information on what scaling—if suitable for the market—is applied to the equity model.

Before we implement DEWIV or GARCH scaling on any model, we first test and validate its application for that model. If we can satisfactorily fit DEWIV or GARCH to the volatility of a market proxy portfolio, we use the model to scale the factor covariance matrix so that the matrix gives the same risk forecast for the market portfolio as the DEWIV or GARCH model. Only the systematic part of the factor covariance matrix is scaled.

DEWIV Model

DEWIV is applied as appropriate to local models over time. The model is expressed as:

1. Some markets, such as the emerging markets, are not scaled. Neither GARCH nor DEWIV is appropriate for the 26 emerging market models.

$$\sigma_{r,t}^2 = 21 \cdot (1-\lambda)\sum_{s=1}^{\infty}\lambda^{s-1}(r_{t-s}-\bar{r})^2 \tag{EQ 3-4}$$

where

$\sigma_{r,t}^2$	=	variance of market return at time t
21	=	approximate number of trading days in a month
λ	=	$(0.5)^{\frac{1}{HALF-LIFE}}$
r_{t-s}	=	return of the index portfolio over the period t–s–1 and t–s
$\bar{r}$	=	mean of the return of the index portfolio

The DEWIV model has only one parameter: the weighting coefficient; that is, the half-life. Before scaling the covariance matrix, the monthly DEWIV variances are first calculated by multiplying the daily variance forecast by the approximate number of trading days in a month (21 days). The monthly DEWIV variances can be calculated only if daily market index data is available for the relevant country model.

GARCH Model and Its Variants

Variants of GARCH[1] are applied as appropriate to some Barra single-country models over time. Denote by r_t the market return at time t, and decompose it into its expected component, $E(r_t)$, and a surprise, ε_t, thus:

$$r_t = E(r_t) + \varepsilon_t \tag{EQ 3-5}$$

The observed persistence in realized volatility indicates that the variance of the market return at t can be modeled as:

$$\sigma_{r,t}^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{r,t-1}^2 \tag{EQ 3-6}$$

1. The form of the variance forecasting function distinguishes the GARCH models from one another.

where

$\sigma^2_{r,t}$	=	variance of market return at time t
ω	=	forecasted mean volatility of the market
α	=	sensitivity to recent realized volatility
ε^2_{t-1}	=	recent realized volatility at time t–1
β	=	sensitivity to previous forecast of volatility

This equation, which is referred to as a GARCH(1,1) model, says that current market volatility depends on recent realized volatility via ε^2_{t-1}, and on recent forecasts of volatility via $\sigma^2_{r,t-1}$.

If α and β are positive, then this period's volatility increases with recent realized and forecast volatility. GARCH(1,1) models fit many financial time series. Nevertheless, they fail to capture relatively higher volatility following periods of below-normal returns. We can readily extend the GARCH(1,1) model to remedy this shortcoming by modeling market volatility as:

$$\sigma^2_{r,t} = \omega + \alpha\varepsilon^2_{t-1} + \beta\sigma^2_{r,t-1} + \theta\varepsilon_{t-1} \tag{EQ 3-7}$$

where θ is sensitivity to surprise return. If θ is negative, then returns that are larger than expected are followed by periods of lower volatility, whereas returns that are smaller than expected are followed by higher volatility.

Scaling

Scaling the covariance matrix involves taking volatility forecasts for a market index and scaling the less dynamic factor covariance matrix with the volatility forecasts. Barra starts with a pre-existing positive definite factor covariance matrix and a diagonal matrix of specific risks.

The forecast for the variance of the market from the unscaled model is:

$$\sigma^2_m = h^T_m XFX^T h_m + \sigma^2_{sp} \tag{EQ 3-8}$$

The monthly specific risk of the monthly market index is defined as:

$$\sigma_{sp}^2 = h_m^T \Delta h_m \tag{EQ 3-9}$$

where

h_m	=	market index portfolio holdings $(n \times 1)$
Δ	=	specific variance diagonal matrix $(n \times n)$

The factor covariance matrix is then scaled using a forecast for a market index variance. The number of assets is n and the number of factors is k. Given a monthly scaled variance forecast for the market index portfolio, σ_s^2, we construct a new $(k \times k)$ factor covariance matrix, F_s, thus:

$$F_s = F + \frac{(\sigma_s^2 - \sigma_m^2)}{(\sigma_s^2 - \sigma_{sp}^2)} F X^T h_m h_m^T X F \tag{EQ 3-10}$$

where

F_s	=	factor covariance matrix with scaling
F	=	original factor covariance matrix $(k \times k)$
σ_s^2	=	monthly scaled variance forecast for the market index, which is based on either DEWIV σ_d^2 or GARCH σ_g^2 models
σ_m^2	=	monthly total variance of the market index, calculated from the pre-existing factor model
σ_{sp}^2	=	monthly specific risk of the market index
h_m	=	market index portfolio holdings $(n \times 1)$
X	=	risk factor exposure matrix $(n \times k)$

Specific Risk Modeling

Referring to the basic factor model:

$$r_i = \sum_{k=1}^{K} x_{ik} f_k + u_i \quad \text{(EQ 3-11)}$$

The specific risk of asset *i* is the standard deviation of its specific return, u_i. The simplest way to estimate the specific risk matrix is to compute the historical variances of the specific returns. This, however, assumes that the specific return variance is stable over time. Rather than use historical estimates, we build a forecasting model for specific risk to capture fluctuations in the general level of specific risk and the relationship between specific risk and asset fundamental characteristics.

Conceptually, an asset's forecast specific risk may be viewed as the product of two factors: the forecast *average* level of specific risk across assets during a given month and the riskiness of each asset *relative* to the average level of specific risk. Our research has shown that the average level of specific risk can be forecast using historical average levels and, occasionally, lagged market return. Additionally, our research has shown that the relative specific risk of an asset is related to the asset's fundamentals. Thus, developing an accurate specific risk model involves a model of the average level of specific risk across assets and a model that relates each asset's relative specific risk to the asset's fundamental characteristics.

Methodology

For robustness reasons, we first construct a model to forecast an asset's expected absolute specific return. This forecast is generated as the product of forecasts for the average level of absolute specific return and the asset's relative level of absolute specific return. We then calculate the forecast specific risk (that is, the standard deviation of specific return) as the product of the forecast for the expected absolute return and a scaling factor. Thus, specific risk is a combination of three components—the average level of absolute specific return, the relative level of absolute specific return, and the scaling factor. These produce the final asset-specific risk forecast:

$$\hat{\sigma}_{it} = \kappa_{it} \cdot \left(1 + \hat{V}_{it}\right) \cdot \hat{S}_t \qquad \text{(EQ 3-12)}$$

where

$\hat{\sigma}_{it}$	=	specific risk of asset i at time t
κ	=	scaling factor that converts absolute return forecasts into standard deviation units
$\hat{V}_{it}$	=	forecast relative level of absolute specific return of asset i at time t
$\hat{S}_t$	=	forecast average level of absolute specific return across all assets at time t

$\hat{S}_t$ is estimated via time-series analysis, in which the average level of realized absolute specific return is related to its lagged values and, in some models, to lagged market returns. The general form is:

$$\hat{S}_t = \alpha + \sum_{i=1}^{k} \beta_i S_{t-i} \qquad \text{(EQ 3-13)}$$

where

$\hat{S}_t$	=	forecast average absolute return at time t
α, β	=	estimated parameters
S_t	=	is the realized average of absolute specific return of estimation universe assets in month t

The Multiple-Horizon U.S. Equity Model and the Mexico Equity Model (MXE1) have an additional lagged market return term.[1]

To model the relative level of absolute specific return, we first identify factors that may account for the cross-sectional variation in specific risk among assets. Having identified these factors, we forecast an asset's real level of absolute specific return using the following model:

1. For specific model details, see the *Equity Risk Model Reference Guide* or the relevant model data sheet on http://support.barra.com.

$$\hat{V}_{it} = \sum_{k=1}^{K} Z_{ikt}\gamma_k \tag{EQ 3-14}$$

where

$\hat{V}_{it}$	=	forecast relative absolute specific return for asset i at time t
Z_{ikt}	=	the exposure of asset i to characteristic k at time t
γ_k	=	characteristic k's contribution to relative specific risk

Updating the Model

Model updating is a process whereby the most recent fundamental and market data is used to calculate individual stock exposures to the factors, to estimate the latest month's factor returns, and to recompute the covariance matrix.

The latest data is collected and cleaned. Descriptor values for each company in the database are computed, along with risk index exposures and industry allocations. Next, a cross-sectional regression is run on the asset returns for the previous month. This generates factor returns, which are used to update the covariance matrix, and the specific return, which are used to calculate the relative absolute specific return and the average absolute specific return. The relative absolute specific return and the average absolute specific return are combined with a scaling factor to forecast specific risk. Finally, this updated information is distributed to users of Barra's software.

Fixed-Income Risk

Section Three provides an overview of fixed-income risk models and discusses the extensive, detailed process of creating Barra fixed-income risk models.

Chapter 4: Forecasting Fixed-Income Risk

Chapter 5: Interest Rate Risk Modeling

Chapter 6: Spread Risk Modeling

Chapter 7: Specific Risk Modeling

Section Three

4. Forecasting Fixed-Income Risk

Through the years, theoretical approaches to fixed-income investment analysis have become increasingly sophisticated. With more advanced concepts of risk and return, investment portfolio models have changed to reflect this growing complexity.

A Historical Perspective

Until the last few decades, investors perceived high-grade bonds largely as a safe haven. But as interest rates spiked upwards in the 1970s and early 1980s, investors learned quickly that even Treasury bonds are not immune from risk. Traditionally, a bond's riskiness was measured by its ***duration***—the sensitivity of a bond's price to changes in yield. Assuming all bond yields were perfectly correlated and equally volatile, the riskiness of a high-grade bond portfolio could be measured in terms of the aggregate duration. Given the absence of awareness and availability of suitable analytical models, bond durations (and durations of other traditional fixed-income securities, such as mortgage-backed securities) were computed on the assumption that they would provide deterministic cash flows.

It is now generally recognized that neither assumption is adequate. Interest rate risk includes risk due to changes in yield curve slope and curvature, not just overall shifts. Portfolios with bonds from multiple markets are exposed to multiple interest rate factors. And most fixed-income securities are not really fixed: they may be callable or putable, subject to prepayment, or have variable interest rates. Thus, fixed-cashflow duration is not a valid measure of risk exposure for such securities.

In addition to interest rate risk, most bonds are also subject to credit risk. The holder of a corporate bond is exposed to the risk of a general change in credit spreads—as what happened dramatically in the second half of 1998[1]—and to issuer-specific credit events, which might lead to default.

1. On August 17, 1998, the Russian government defaulted on domestic debt, declared a 90-day moratorium on payment to foreign creditors, and devalued the ruble.

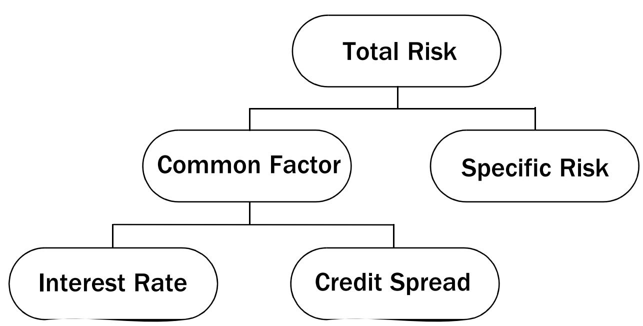
The key to building a successful risk model is to provide both an accurate decomposition of the market risk factors driving bond returns and accurate forecasts of their covariances. Fundamental models of risk for fixed-income securities decompose bond returns into contributions arising from changes in interest rates, credit spreads and, to a lesser degree, liquidity premia, volatility uncertainty, and prepayment expectations. Returns to portfolios of high credit quality securities are typically affected primarily by variation in interest rates, while returns on lower quality portfolios may be determined mostly by credit factors.

Barra's Multiple-Factor Model

Barra's fixed-income risk models decompose asset returns into components due to common factors and a specific, or idiosyncratic, factor.

For a given security, a valuation algorithm enables the calculation of the predictable return component attributable to the passage of time.[1] The remaining "excess" return is due to a combination of changes in default-free interest rates (the benchmark yield curve), changes in market spread levels, changes in market volatility and prepayment expectations, and the specific return. Barra's fixed-income risk model incorporates many of these factors together with models of specific risk.

Figure 4-1
Fixed-Income Risk Decomposition



1. This might include short-term interest rates and time value decay of embedded options.

Common Factors

Each bond is assigned to a single market. The common factor risk of a bond is determined by the volatilities of the term structure of interest rates and spread risk factors in that market, the correlations between factors, and the bond exposures to the risk factors.

Except for the euro block, each market has three nominal interest rate risk factors. These are the first three principal components of a key rate covariance matrix: shift, twist, and butterfly (STB)—so called because of their shapes. The euro market has three term structure factors for each country and three that describe average changes in rates across the euro zone. In addition, the euro, sterling, U.S. dollar, and Canadian dollar blocks include one or more real yield factors applicable to inflation-protected bonds. The U.S. dollar block also includes interest rate factors for municipal bonds.

Every market[1] has a swap spread factor. The only instruments not exposed to swap spread are sovereign issues. The U.S. dollar, sterling, yen, and euro markets have detailed credit spreads, which are measured to the swap spread, in addition to the swap spread factor.

1. A market is defined by its currency. For example U.S. corporate and U.S. sovereign bonds belong to one market.

Table 4-1

Risk Factor Exposure

The table below summarizes the rules for exposing assets to risk factors.

Risk Factor Type	Exposure of Asset to Risk Factor
Term structure	Always. The local market is determined by the currency of the bond's denomination.[1]
Swap spread	Always, unless the asset is issued by a sovereign issuer.
Credit spread	• When the asset is denominated in U.S. dollar, sterling, yen or euro, AND • The asset's sector and rating match an existing local market credit spread sector and rating, AND • The asset is not a sovereign issue, AND • The asset is not exposed to an emerging market factor.
Emerging market	• When the issuer is an emerging market country or a corporation domiciled in an emerging market country, AND • The asset is denominated in a currency other than that of the bond issuer's country of incorporation (or domicile).
Currency	When the asset is denominated in a currency other than the numeraire.

1. Euro-denominated sovereign bonds from European Monetary Union (EMU) member countries are exposed to the term structure factors of the country; all other euro-denominated bonds are exposed to the EMU local market term structure factors.

Interest Rate Risk

The largest source of within-market risk[1] for typical investment-grade portfolios is interest rate variation. Barra models this risk in terms of the shift, twist, and butterfly factor movements of the benchmark interest rate curve in each market.[2] Together, these three principal components capture between 90% and 98% of interest rate variation (as measured by an 8-factor key rate model) in most developed markets.[3]

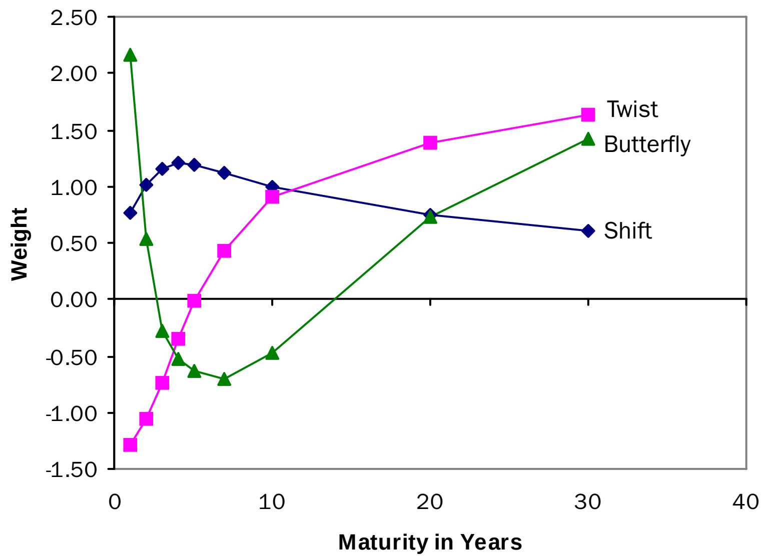


Figure 4-2

Shift, Twist, and Butterfly in the Germany Market

Like most markets, the shift factor in the Germany market tends to be slightly humped at the short end because short rates tend to be more volatile than long rates.

The factors reflect the way term structures actually move, and their characteristics persist across markets and time periods. Re-

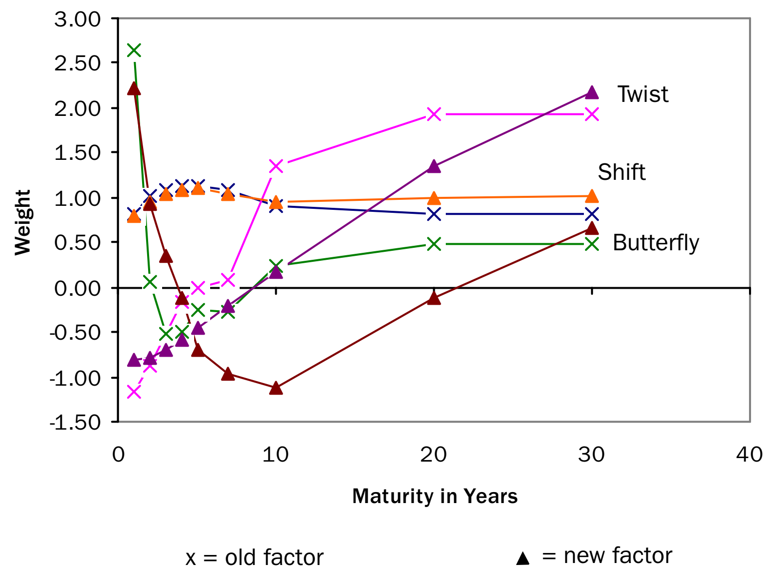
1. Within-market risks excludes currency risk.
2. Interest rate movements are expressed in terms of changes in zero-coupon bond yields inferred from a combination of money-market rates and coupon bond yields. The zero-coupon yield curve is also referred to as the "term structure of interest rates," or sometimes just the "term structure."
3. Approximately 98% of the monthly variation in U.S. government bond interest rates from 1 to 30 years can be expressed in terms of these three factors.

estimation of factors is required only when the market structure changes.

Figure 4-3

Re-estimation is Required Only When Market Structure Changes

The introduction of long bonds in the Spain market in February 1998 changed the length of the market from 15 to 30 years. The new factors, especially twist and butterfly, look quite different from the old.



Because of its parsimony, the shift-twist-butterfly (STB) model is Barra's preferred approach for interest rate forecasts.

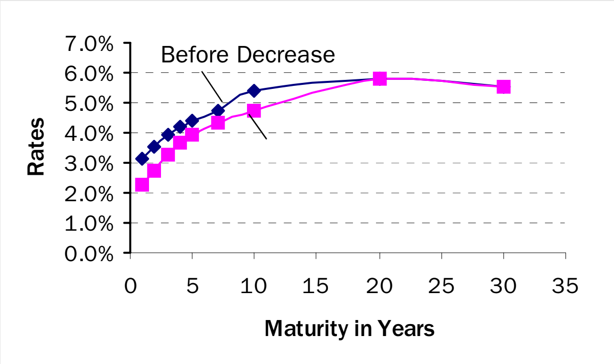
Learn more about

Shift, Twist, and Butterfly Movements

Shift describes approximately parallel yield curve movements; that is, all key rates are moving by approximately the same amount. **Twist** describes yield curve movements with short and long ends moving in opposite directions. **Butterfly** describes a flexing motion of the yield curve.

An example

Suppose the Federal Reserve System attempts to stimulate the U.S. economy by decreasing short-term interest rates. The change in the U.S. term structure can be seen in a comparison of the spot rate curve before and after the decrease. The actions of the Federal Reserve System are seen in the downward movement of most of the spot rates. Each spot rate decreased, but with different magnitude.



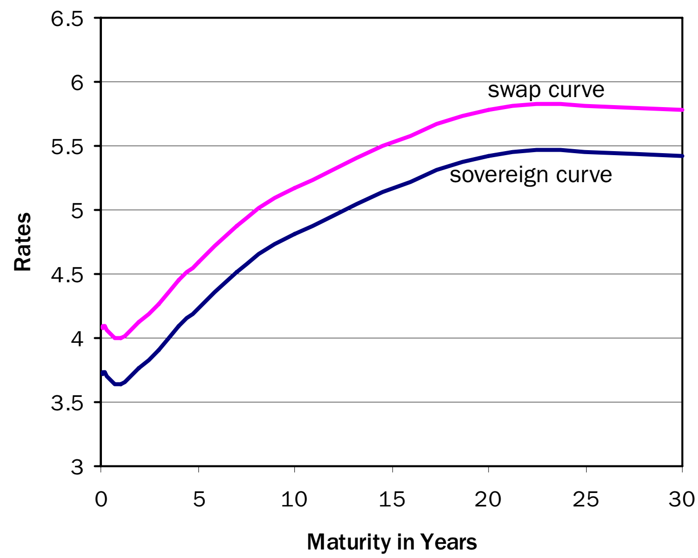
The movement of the curve can clearly be decomposed into a shift movement (the level of interest rates went down) and a twist movement (the slope of the curve went up).

Spread Risk

In addition to the STB interest rate factors, the risk model includes spread factors to accommodate credit-sensitive bonds.

Barra takes a layered approach to modeling spread risk. In each local market, a single factor accounts for changes in the difference between the swap and sovereign curves. Spreads on high-quality issuers are highly correlated with the swap spread, so this has been a reasonable and parsimonious approach to accounting for credit exposure.

Figure 4-4
Swap Spread Model



In markets with detailed credit models (such as the United States, United Kingdom, Japan, and euro zone), additional factors capture the risks due to changes in credit spreads over the swap curve. In these markets, spread risk is decomposed into swap spread risk and risk due to credit spreads over the swap curve.

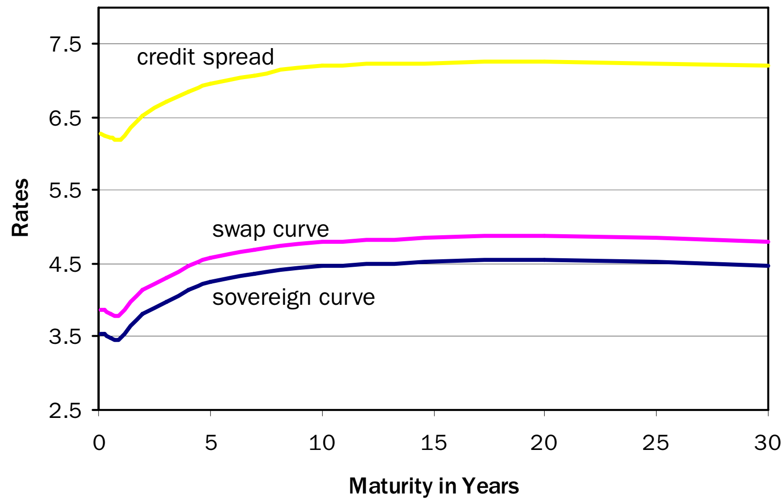


Figure 4-5

Credit Spread Model

In emergent markets, spread risk is decomposed into swap spread risk and risk due to emerging market spreads over the sovereign curve of the market where the bonds are issued.

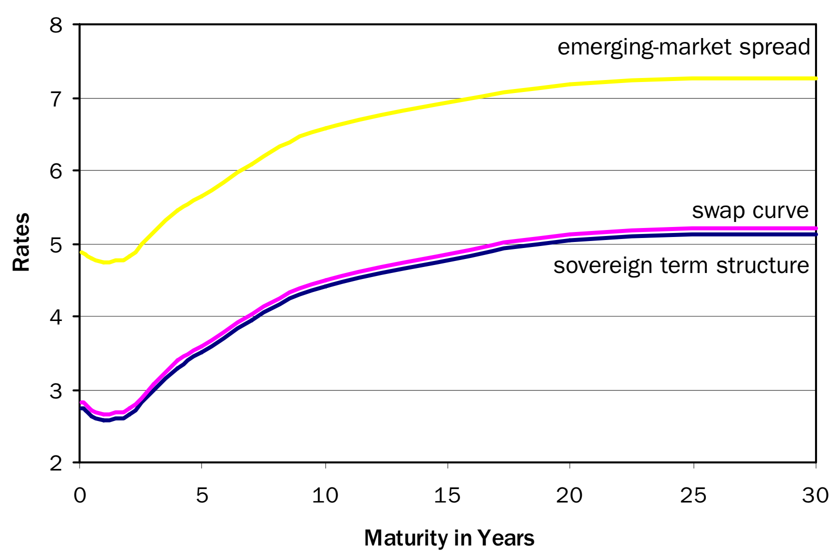


Figure 4-6

Emerging Market Model

Learn more about

Alternative Risk Model: Key Rates

The multiple-factor **key rate model** first described in the 1990s uses changes in rates at key maturities as the factors. Each interest rate scenario predicted a probable set of cash flows, from which the present value of a bond is calculated.

At the core of the model is the covariance matrix that represents the price sensitivity of a security to key rate changes. Using a sufficiently large set of key rates, the maturity dependence of interest rate variation can be described with as much detail as desired. Since the full maturity dependence is represented, this model completely describes term structure variation.

However, the key rate model uses more factors than necessary. There is a high degree of dependency among factors. Neighboring key rates are typically 90% correlated, while even the longest and shortest maturity rates are nearly 60% correlated.

	1 Year	**2 Years**	**3 Years**	**4 Years**	**5 Years**	**7 Years**	**10 Years**	**20 Years**
1 Year	1.00	0.91	0.86	0.83	0.81	0.72	0.63	0.56
2 Years		1.00	0.97	0.95	0.93	0.84	0.75	0.69
3 Years			1.00	0.99	0.98	0.92	0.84	0.77
4 Years				1.00	0.99	0.94	0.88	0.81
5 Years					1.00	0.97	0.91	0.84
7 Years						1.00	0.97	0.89
10 Years							1.00	0.87
20 Years								1.00

Barra uses a succinct multiple-factor model that has far fewer factors than a key rate model yet has virtually the same explanatory power.

Specific Risk

The specific risk model comes in two versions. A simpler version, applicable to markets where credit risk is modeled using the swap spread factor alone, forecasts issue-specific spread volatility as a linear function of a bond's spread. A more detailed version, applicable to U.S. dollar, euro, and sterling credit markets, forecasts specific risk based on a transition matrix.

We individually calibrated specific risk to each market and combined it with the market common factor risk to determine total risk.

Summary

The search for yield has increasingly led portfolio managers to take larger positions in credit-risky assets. The risk profile of these portfolios can no longer be adequately understood purely in terms of interest rate movements. Nor are "slice-and-dice" representations of sub-portfolio exposures to different sector and rating groups adequate, as these pictures fail to reflect the impact of partial diversification between sub-portfolios, and fail to capture the potentially large effects of spread volatility and credit migration events. Barra's global fixed-income risk model provides an effective solution for analyzing fixed-income portfolios in a world of many risks.

Consisting of interest rate risk models for nominal and real markets, detailed and independently constructed models for four major markets, simpler swap-based models for the remaining developed markets, and emerging market models, Barra's global fixed-income risk model covers a large fraction of public credit markets. The model provides asset managers with valuable forecasts of portfolio risk due to changes in market-wide and issuer-specific credit spreads.

5. Interest Rate Risk Modeling

Accurate interest rate risk modeling depends on a term structure of interest rates. The ***term structure*** is a curve that describes the rate of interest that an issuer must pay today to borrow for each term.

In developed markets, yields of high-grade or low default-risk bonds are closely linked to those of similar government bonds. Changes in the term structure of government interest rates therefore imply changes in the pricing of such bonds.

Term structures can change in any number of ways: yields of bonds of all maturities may rise or fall, or yields of bonds of one end of the maturity pole may rise or fall while the other end remains unchanged. These movements are a major contributor to portfolio risk for high-grade bonds.

Estimation Process Overview

Barra's asset valuation models incorporate a model of interest rates, which are inferred from observed prices of bonds and other financial instruments. The basic objective of the term structure estimation algorithm is to find rates that minimize the difference between model and market prices.

We use constraints to smooth the term structure and the LIBOR-driven specifications for short rates.[1] We do this to eliminate kinks that are idiosyncratic to the estimation period.

Next, we apply principal components analysis to a key-rate covariance matrix estimated from a history of term structure changes. The three principal components, or eigenvectors with the largest variance, are the shift, twist, and butterfly factors.

1. LIBOR rates are used in markets that do not have short-term government securities such as treasury bills. The London Interbank Offered Rate is the interest rate offered to banks in the London interbank market and is well known as a reference for short-term rates.

Model Estimation Process

1. Data acquisition
2. Term structure specification:
   a. Interpolation
   b. Estimation algorithm implementation
3. Factor shape determination: key rate covariance matrix
4. Factor exposure calculation
5. Factor return estimation
6. Covariance matrix estimation
7. Covariance matrix correlation
8. Model updating

We calculate factor returns from a cross-sectional regression of bond excess returns onto factor exposures.

The principal components analysis amounts to a "rotation" of the factors and covariance matrix to a representation in which the covariance matrix is diagonal. By keeping only the leading three factors, we retain the dominant sources of risk while reducing the potential for spurious correlations. The shift, twist, and butterfly factors underlying the rotated matrix are orthogonal, weighted combinations of changes in interest rates. The diagonal elements of the covariance matrix are the in-sample variances of the factors.

We generate the 3x3-interest rate risk block of the model with the time series of monthly shift, twist, and butterfly returns. The factor volatilities (expressed as one standard deviation) and correlations determine the interest rate block.

Data Acquisition

We obtain daily price data on local government bonds issued in 29 markets, inflation-protected bonds (IPBs) issued in four markets,[1] AAA-rated tax-exempt municipal bonds issued in the United States, and LIBOR/swap curves in 23 markets.

1. Barra collects daily price data on IPBs issued by the governments of the United States, Great Britain, Canada, and France.

Learn more about

Inflation-Protected Bonds

An **inflation-protected bond** (IPB) is a fixed-income security whose principal is periodically adjusted to provide a fixed return over inflation. The adjustment lags a pre-specified measure of inflation by an amount of time determined by the issuer.[1] The coupon is a fixed rate applied to the adjusted principal.

In the case of U.S. Treasury inflation-protected bonds, the Treasury pays a fixed rate of return over inflation (as measured by the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for All Urban Consumers or CPI). The adjustment lags the CPI release by two months to remove the ambiguity of the nominal amount of the next coupon.

To the extent that IPBs comprise a significant portion of a portfolio, a risk model that accurately and reliably distinguishes the movements of the real yield curve (on which these securities are directly dependent) from those of the nominal curve is needed.

Barra Risk Forecasting

To forecast the risk of the nominal return of an IPB on a monthly horizon, its payoffs are expressed in **real**, or inflation-adjusted currency. The IPB can then be treated as a garden-variety fixed-principal bond.

IPB prices, which are quoted in real terms in every market but the United Kingdom, are used to estimate the real yield curve. The real risk factors and real return risk forecasts are then determined by applying standard principal components analysis, which is the same methodology used in nominal markets. Finally, the real returns are approximately related to nominal returns through $r_N = r_R + r_I$, where r_N is the nominal return of an IPB, r_R is the real return, and r_I is the return due to the inflation adjustment factor. Due to the lag between the inflation index and the inflation adjustment of the bond, the r_I is known with certainty more than a month in advance. So, for the corresponding risk forecasts, $\sigma_N = \sigma_R$, where σ_N is the nominal return risk and σ_R is the real return risk.

1. For details on IPBs, see Barra white paper, "Barra's Real Yield Model." It is available on http://www.barra.com.

Learn more about

U.S. Municipal Bonds

Municipal bonds are debt obligations issued by states, counties, cities, and other local governmental entities to support general governmental needs or special projects for the public good.

The municipal bond market is affected by financial factors somewhat distinct from the primary drivers of the taxable bond market. Aside from the obvious difference in tax treatment, municipal bonds, or *munis*, include:

- A very large number (about two million) of relatively small, illiquid issues
- Additional option features, such as pre-refunding.

Barra's U.S. municipal bond model[1] is based on histories of four yield curves for national general obligation (GO) bonds rated AAA (uninsured), AA, A, and BBB. Histories of these yields, which go back to 1994, are the basic information used in constructing the model.

The risk model structure is similar to the taxable U.S. model. Like the investment-grade portion of the taxable model, the dominant contribution to risk arises from market-wide interest rate levels. These are captured in the municipal bond model by eight key rate factors: 1, 2, 3, 5, 7, 10, 20, and 30 years. Rather than following the usual practice of calculating spot rates from yields by "bootstrapping,"[2] we calculate the current levels for the key rates by means of a modified version of our standard spot-rate estimation mechanism, which minimizes root-mean-squared pricing error of a universe of bonds. This is done because market yields for maturities beyond 10 years are quoted in yields of callable bonds. The root-mean-squared minimization method allows us to properly handle the callability of the longer maturity yields. Were we to do standard bootstrapping and then use the resulting spot rate curve to value callable bonds, we would not be able to reproduce the market yields we started from.

In addition to the non-taxable interest rate factors, the model includes three credit spread factors—one each for AA-rated, A-rated, and BBB-rated bonds. These are calculated as average spreads of the corresponding spot rate curves over the general-obligation AAA curve. In total, then, the municipal bond risk model contains 11 common factors: eight key rates and three rating spreads. In addition to common factor risk, the municipal bond model incorporates a modified version of the taxable issuer credit-risk model. The issuer credit-risk model uses historical information about credit migration rates together with current spread levels to forecast risk due to upgrades or downgrades and default.

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1. For details on municipal bonds, see Barra white paper, "The U.S. Municipal Bond Risk Model." It is available on http://www.barra.com.
2. Bootstrapping is a procedure for recursively calculating successively longer spot rates based on market yields.

Term Structure Specification

Since interest rates are neither bought nor sold, a term structure is a derived quantity. Term structures differ across borrowers and can change dramatically in the course of a day. We estimate term structures for valuation and exposure calculation on a daily basis.

Interpolation

We determine the term structure by a numerical procedure that sets spot rates at predefined vertices to minimize the differences between observed and theoretical bond prices. The term structure is specified by spot rates at a set of vertices.[1] A higher density of vertices is used at the short end to reflect the greater amount of short-end information available.

Interpolation is used to compute rates of maturities that are between the vertices. The interpolation rule assumes that forward rates are constant between vertices. We constrain optimization to keep these forward rates positive. We then continuously compound the rates.

The forward rate between t_{i-1} and t_i is first obtained with:

$$f_{i-1,i} = \frac{s_i t_i - s_{i-1} t_{i-1}}{t_i - t_{i-1}} \tag{EQ 5-1}$$

where

$f_{i-1,\,i}$	=	forward rate of period t_{i-1} to t_i
s_i	=	spot rate of maturity i
t_i	=	length of time before maturity i

Then spot rates between vertices are determined by interpolating with the following formula:

1. For each market, a subset of standard maturities (1, 3, and 6 months, and 1, 2, 3, 4, 5, 6, 7, 10, 15, 20, and 30 years) is selected. The number of vertices used to estimate the term structure depends on the availability of price data on the maturities of bonds.

$$s_i = \frac{f_{i-1,i}(t \; - t_{i-1}) + s_{i-1}t_{i-1}}{t} \qquad \text{(EQ 5-2)}$$

where

s_i	=	spot rate of vertex i
f_i	=	forward rate of period t_{i-1} to t
t	=	length of time before maturity

Suppose the ten-year spot rate, s_{10}, is 3%, and the 20-year spot rate, s_{20}, is 5%. Then the 10-to-20 forward rate is:

$$\begin{aligned} f_{10,20} &= \frac{(5\% \cdot 20) - (3\% \cdot 10)}{20 - 10} \\ &= \frac{70\%}{10} \\ &= 7\% \end{aligned} \qquad \text{(EQ 5-3)}$$

The 15-year spot rate is then:

$$\begin{aligned} s_{15} &= \frac{7\% \cdot (15 - 10) + (3\% \cdot 10)}{15} \\ &= \frac{35\% + 30\%}{15} \\ &= 4.33\% \end{aligned} \qquad \text{(EQ 5-4)}$$

Learn more about

The Benchmark Yield Curve

Interest rates are modeled globally based on sovereign issuers' domestic bonds. For the euro zone, in addition to estimating interest rate factors for each "legacy" sovereign issuer, we also estimate a euro sovereign term structure of interest rates and factor series from the aggregate of all euro government bonds weighted by GDP.

Because interest-rate risk is the dominant source of day-to-day variation in value for investment-grade securities, security prices have traditionally been quoted relative to a market-wide benchmark yield curve. Until recently, in most markets, this curve has been based on the yields of government bonds—generally the highest credit quality issuer in the domestic currency.

This situation has changed, for several reasons. First, in the euro zone, there is no natural government bond yield curve. French and German bonds generally trade at the lowest yields, with other issuers at comparable or higher levels. This lack of an obvious standard has led to the emergence of the LIBOR/swap curve[1] as the valuation benchmark.

Second, the U.S. and other government bond markets have experienced a number of technical, supply-related distortions, particularly since 1998. These are the results of a global flight to quality at the time of the Russian default and LTCM collapse, the debt buybacks of 2000 and early 2001, and the changing issuance patterns, such as the termination of 30-year bond issuance. These market distortions have led to a decoupling of the U.S. Treasury bond market from the mortgage-backed securities (MBS), asset-backed securities (ABS), and bond markets. To varying degrees, traders have shifted to the swap curve as a benchmark in the United States as well.[2]

Finally, the growing liquidity and transparency of swap curves have led to increased acceptance of these financial rates as a reference for the broader debt markets.

This acceptance is, however, not universal. The U.K. and Japan markets, for example, continue to trade primarily relative to government benchmarks. Barra therefore devised a hybrid scheme that admits alternative views to create a factor structure for interest rate risk.

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1. The LIBOR/swap curve, hereafter abbreviated to "swap curve," is derived from deposit rates out to one year and par fixed/floating swap rates at longer maturities. Liquid swap rates are now routinely available up to maturities of 30 years in most developed markets.
2. There was brief speculation that U.S. agency debt would replace Treasury debt as a trading benchmark—speculation that was encouraged by the agencies' large "global benchmark" bond issues. However, it was quickly realized that the agencies are subject not only to liquidity risk but also political uncertainty, and their use as benchmarks has largely fizzled.

Estimation Algorithm Implementation

We estimate the term structure with an objective function composed of three pieces. The objective function is expressed as:

$$E = E_1 + E_2 + E_3 \tag{EQ 5-5}$$

The first term minimizes the differences between market and fitted prices in the term structure; the second term subjects the term structure to a smoothing constraint; the third term uses LIBOR-driven specification to determine short-end vertices of term structures of markets where there are no short sovereign issues. The terms in the objective function incorporate weights that serve to control the relative impact of different effects. For example, placing large weights on the second term (E_2) would force the resulting term structure to be very smooth.

Term structures for real, or inflation-adjusted, markets are estimated with only the first term of the objective function. The smoothing term (E_2) and the short-end correction (E_3) are both zero.

Barra Research Methods

Diagnostics on Term Structure Estimation

The estimation algorithm identifies bonds with large pricing errors and eliminates them. This is done by an iterative process. First, a term structure which includes all bonds is estimated; bonds with pricing errors above a threshold are discarded. Then the estimation runs again. This procedure is repeated until there are no more bonds with large pricing errors.

We use a set of automated diagnostics to identify potential problems with the term structure estimation. The simplest of these measures flags the deviation between the newly estimated term structure and the previous day's estimated term structure. Large daily or monthly changes are investigated.

The root mean square pricing error is also computed. All other things being equal, this statistical quantity tends to increase with the number of bonds in the universe. In the U.S. Treasury term structure estimation, for example, there are approximately 110 bonds and the root mean square error ranges between 30 to 50 basis points. Although the number itself has no definitive interpretation, abrupt changes in its value are useful for flagging problems in the estimation.

Sum of Squared Relative Pricing Errors (E_1)

The first and dominant term in the objective function is the sum of squared relative pricing errors of bonds in the estimation universe. It minimizes the differences between market and fitted prices in the term structure.

The pricing error term E_1 is given by:

$$E_1 = \sum_{i=1}^{n\,bonds} \omega_i^2 \varepsilon_i^2 \quad \text{(EQ 5-6)}$$

where

ω_i	=	weights
ε_i	=	relative pricing error

The relative pricing error, ε_i is given by:

$$\varepsilon_i = \frac{P_i - Q_i}{P_i} \quad \text{(EQ 5-7)}$$

where

P_i	=	market price of bond i
Q_i	=	fitted price of bond i

The weighting scheme (ω_i) can, for example, be used to down-weight callable bonds. Since the model price Q_i depends on the term structure, changes in term structure give rise to changes in E_1. The fitting routine works by moving rates until the minimum difference between fitted price and market price is found.

Smoothing Function (E_2)

Because term structures produced with only E_1 in the objective function may have idiosyncrasies due to noise in the data or a mismatch between the data and the location of vertices, a second term, E_2, is included.

This term is based on a three-parameter family of equations that acts as a smoothing function. The family of functions is expressed as:

$$\Delta(T) = \theta + (r_0 - \theta)e^{-\kappa T} \quad \textbf{(EQ 5-8)}$$

where

θ	=	long rate
r_0	=	short rate
κ	=	decay constant
T	=	term

The parameters θ, r_0, and κ are jointly estimated with the spot rates.

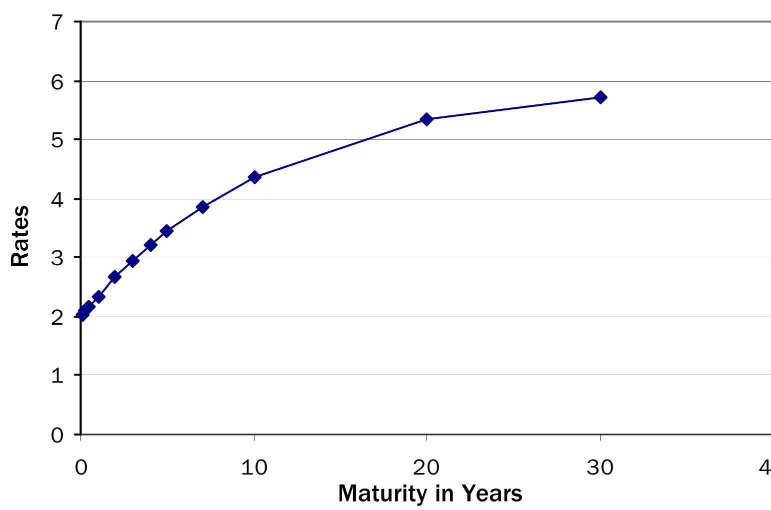


Figure 5-1

Typical Smoothing Curve

The three parameters (which are r_0 = 2%, θ = 6%, κ = .09), along with the rates at the vertices, are outputs of the fitting routine. Typically, all the parameters are positive and $\theta > r_0$.

A discrete version of the smoothing function generates the second term of the objective function, which is expressed as:

$$E_2 = \sum_{j=1}^{n\ vertices} \alpha_j^2 \left(\psi\left(T_j\right) - s\left(T_j\right)\right)^2 \quad \textbf{(EQ 5-9)}$$

where

α_j	=	weight of smoothing term

ψ	=	smoothing curve
T_j	=	maturity of vertex j
$s(T)$	=	spot rate curve

In the equation above, the α_j's are weights that control the relative importance of the smoothing terms at different maturities. α_j is smaller for shorter maturities, so the smoothing function has less influence on the 1-year vertex than on the 30-year vertex.

Short-End Shape Correction (E_3)

The universe of bonds used to determine the term structure excludes bonds with remaining time to maturity under one year. These bonds tend to be relatively illiquid, hence their prices do not reflect current rates. As a result, there is generally not enough information in the objective function to reliably determine key rates under one year.

This problem can be handled in some markets by adding treasury bills or equivalent assets to the estimation.[1] However, many important markets do not include short domestic government issues. Therefore, the shape of the LIBOR term structure is used as an indicator of the shape of the short end of the government term structure.[2]

Since LIBOR rates are not default free, they are not directly included in the government term structure estimation. Instead, we impose the assumption that the ratio between the government and one-year LIBOR rates is roughly constant for the short end of the term structure. This assumption is imposed with the addition of a third term to the objective function, which is:

$$E_3 = \sum_{i=1}^{nLIBOR} \tau_i^2 \left(\frac{SOV_i}{LIBOR_i} - \mu \right)^2 \qquad \textbf{(EQ 5-10)}$$

1. For example, Japan and the United States have treasury bills.
2. The 1-, 3-, and 6-month, and 1-year LIBOR rates are used to determine the short end of the term structure in the fixed-income risk models.

where

τ_i	=	weight of ith short-end constraint term
μ	=	constant ratio between LIBOR and sovereign rates

The constant μ is typically between 0.7 and 1.05. Along with the spot rates and smoothing parameters, μ is found by the estimation routine. The weights, τ_i, can be individually calibrated to each market. The weights on E_1 are larger than the weights on E_3, so if short bonds are available, they will be used to determine the short end of the term structure. For example, if treasury bill information is available, the resulting low E_1 weights would de-emphasize the E_3 term in the estimation.

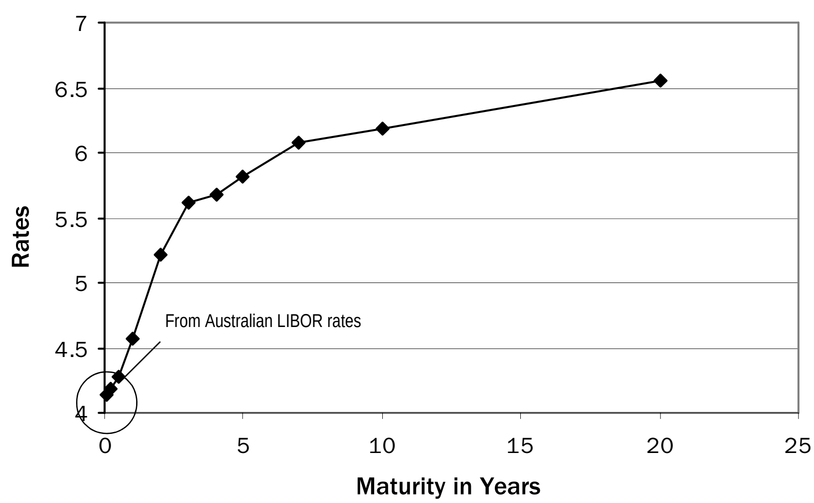


Figure 5-2

Estimated Australian Treasury Term Structure

In these estimated interest rates, the shape of the short end is given by the LIBOR curve.

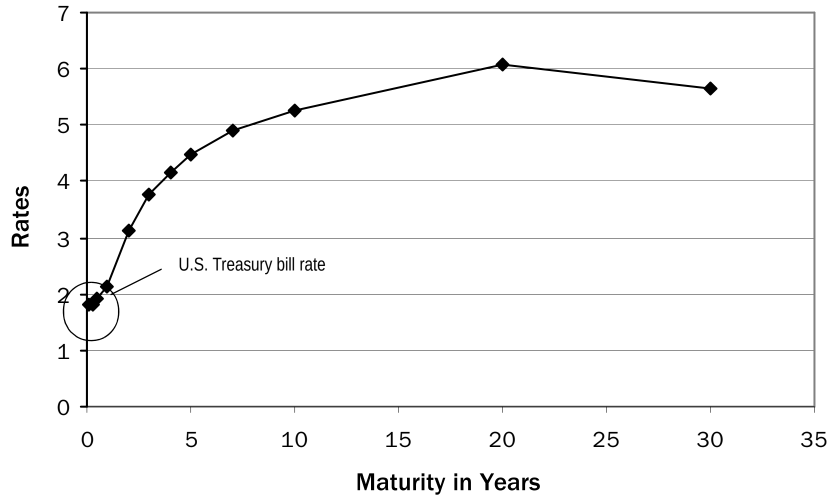


Figure 5-3

Estimated U.S. Treasury Term Structure

Because treasury bill information is available for the U.S. market, the weight in the third term (E_3) of the objective function is small.

Factor Shape Determination

The spot rate covariance matrix can now be generated from the historical term structure. A time series of month-over-month differences in spot rates is first created for each of the standard vertices. From these time series, we obtain the key rate covariance matrix. The eigenvectors of this matrix, or principal components, correspond to uncorrelated movements of the term structure. The three largest contributors—called shift, twist, and butterfly (STB)—typically account for about 95% of the empirical volatility.

In markets with a large number of bonds representing a broad spectrum of maturities (such as the nominal U.S. market), these three factors are significant. In IPB markets, fewer factors are relevant. Generally speaking, IPB markets consist of a small set of bonds with long maturity. For simplicity, we treat the shift factor in these real market as a uniform, parallel shift of the term structure. The twist factor—if present—is then the leading source of risk residual to the parallel shift.[1]

1. Only the U.S. and U.K. IPB markets have a twist factor in addition to the shift factor.

Barra Research Methods

Normalizing Term Structure Factors

Principal components analysis of the key rate covariance matrix, which generates the shift, twist, and butterfly factors, determines the relative weights of the factors at the key maturities. The absolute sizes of the factors are not determined.

If a factor is scaled by a constant *c*, the exposure of a portfolio to this factor is scaled by *c* as well. The returns to the factor are scaled by $1/c$ so that the scale factor cancels out of the risk calculation.

The shift factor is roughly a parallel change in interest rates. For non-callable bonds, therefore, the exposure to shift should be a number comparable to effective duration. The magnitude of the shift exposure can be controlled by changing the size of the shift factor. By convention, the factors are normalized so that their mean-squared value is the number of vertices (a true parallel shift is normalized at a constant 1) and so that they are positive at long maturities. The base shift is normalized so that the shift exposure is comparable to effective duration. The base twist and butterfly factors are normalized to have the same magnitude as the shift.[1]

This normalization is not preserved in the transformation of the STB factors.[2] However, since the magnitude of the transformation is relatively small, the revised shift exposures is still comparable to effective duration.

1. For more details see, for example, Richard A. Johnson and Dean W. Wichern, Applied Multivariate Statistical Techniques, 4th ed., (Upper Saddle River, NJ: Prentice Hall, 1998) 458-497.
2. Transformation of STB factors is discussed in detail in *Covariance Matrix Correlation on page 66*.

Factor Exposure Calculation

Term structure factor exposures are computed by numerical differentiation. The exposure of a bond or portfolio to a risk factor is the sensitivity of its value to changes in the factor level. For example, ***effective duration*** is the sensitivity of value to a parallel shift of interest rates. The term structure is shocked, or shifted up and down, by a small scalar multiple of the STB factor, and the bond is revalued. In other words, we calculate how the price of a bond changes for a given change in the yield curve. The difference between "up" and "down" values is divided by twice the size of the shift. The mathematical formula is given as:

$$x = \frac{P_- - P_+}{2P_{dirty}\delta} \qquad \text{(EQ 5-11)}$$

where

x	=	factor exposure
P_+ , P_-	=	bond present values obtained by shocking the term structure
P_{dirty}	=	dirty price (price and accrued interests) of the bond
δ	=	size of the shock (typically 25 bps)

Factor Return Estimation

First, we identify the factors that explain the asset return and calculate the exposures to these factors; then, we estimate the factor returns by regressing the computed exposures against the realized bond return.

$$r_i = \sum_{k=1}^{K} x_{ik} f_k + u_i \qquad \text{(EQ 5-12)}$$

where

r_j	=	vector of excess return
x_i	=	matrix of factor exposures
f	=	vector of factor returns
u_i	=	vector of specific returns

The regression universe includes the set of bonds that are in an established market index at the start and end of the month.

Table 5-1

Shift, Twist, and Butterfly Return Volatility in the United Kingdom (November 30, 2000)

Volatilities are reported in basis points per year. The annualized numbers are obtained from the monthly time series by multiplying the monthly estimate by $\sqrt{12}$ *. In a developed market, the typical shift volatility is 65–100 basis points per year (roughly 20–30 basis points per month).*

Term Structure Factors	Annual Volatility
Shift	67.6
Twist	35.7
Butterfly	18.0

Term Structure Covariance Matrix Construction

We use the time series of STB factor returns to generate the covariance matrix.

Table 5-2

U.K. Shift, Twist, and Butterfly Covariance Matrix (November 30, 2000)

The diagonal shows the volatilities, the standard deviation of annual returns.The off-diagonals show the correlations between the factors.

	Shift	Twist	Butterfly
Shift	67.6	0.33	–0.23
Twist		35.7	0.43
Butterfly			18.0

Covariance Matrix Correlation

In principle, factor returns can be estimated from a regression of changes in term structures onto the factors. This simple methodology results in a diagonal covariance matrix in sample.[1] But it does not take into account the uneven distribution of bonds along the maturity spectrum. Therefore, we rejected this method.

Regression based on bond returns, on the other hand, not only accounts for uneven distribution, but also changes dynamically with the distribution. With this regression, the portion of risk that is accounted for by bond-specific factors declines by 20%, while the portion that can be accounted for by common factors increases correspondingly.

1. Nonzero correlations of small magnitude would result from a difference between the risk analysis date and the date at which the factors are fixed.

However, regression based on bond returns results in nontrivial factor correlations that can be as high as 0.5. The bond-return regression used to estimate the factor returns,[1] the initial smoothing process applied to the factors, and the out-of-sample factor returns contribute to the correlations.

Table 5-3

Correlations Between U.K. Shift, Twist, and Butterfly Returns as of November 30, 2000

The correlation comes from the bond-return regressions used to estimate the factor returns, the initial smoothing process applied to the factors, and the out-of-sample factor returns.

	Shift	Twist	Butterfly
Shift	1.00	0.33	–0.23
Twist		1.00	0.43
Butterfly			1.00

These correlations carry information important to risk forecasting. As long as the correlations are not perfect, the explanatory power of the model is not compromised.

For the U.S. municipal bond market, we use a key-rate-based term-structure factor model, derived from a yield curve for national AAA-rated general obligation bonds.

Updating the Model

The most recent fundamental and market data is used to calculate bond exposures to the factors and to estimate the latest month's factor returns.

The term structure is estimated on a daily basis, but the covariance of the risk factors is estimated on a monthly basis. We run a cross-sectional regression on the asset returns for the previous month. This generates factor returns we use to update the covariance matrix.

The precise base forms of the shift, twist, and butterfly factors depend on the estimation period, smoothing technique, exponential weight, and other details. Since the key rate matrices evolve slowly, factor re-estimation requires occasional, rather than frequent, review and re-estimation.

1. Bond-return regression is the main contributor to the non-trivial correlations between the factors.

6. Spread Risk Modeling

In the past, international bond mangers focused largely on government bond issues, which were the constituents of the dominant global fixed-income indexes. Recently, managers have increased the exposure of their global fixed-income portfolios to corporate and agency bonds, foreign sovereigns, supranationals, and credit derivatives. These bonds are subject to spread risk.[1]

Spread risk in bond portfolios arises for two reasons: market-wide spread risks and credit event risks. Market-wide spread risk arises from changes in the general spread level of a market segment. For example, the spreads of BBB-rated telecom bonds might widen. Credit event risk arises when an individual issuer suffers an event that affects it alone. It is the risk associated with changes in company fundamentals. For example, Ford Motor Company's car sales might fall relative to other automakers due to a product recall and bad publicity, or sales might grow through the roof because Ford's engineers invent a car that runs on tap water.

In each market, a single swap-spread factor accounts for changes in the difference between the swap and sovereign curves. For markets with detailed credit models (such as the United States, United Kingdom, Japan, and euro zone), we decompose credit spread risk into two separately modeled components: the swap spread component and the sector-by-rating credit spread component. For emerging markets, we expose the bond to the swap spread factor and the appropriate emerging market spread factor.

1. The market perceives varying levels of creditworthiness among EMU sovereigns, giving rise to spreads between EMU sovereign issuers, so we estimate a term structure for each EMU sovereign.

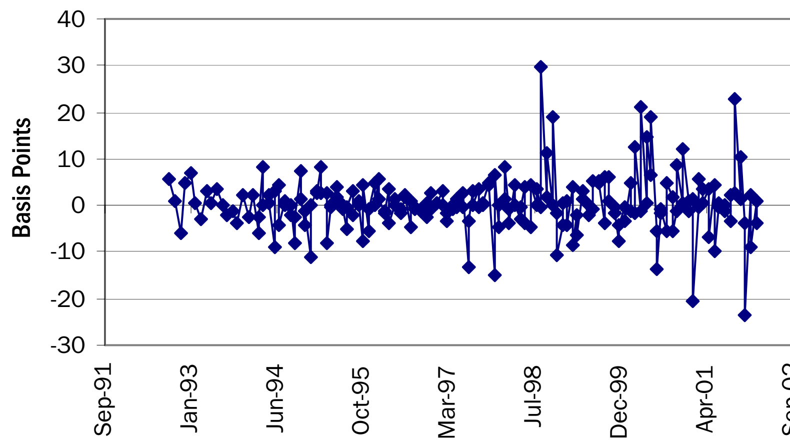
Swap Spread Risk Model

In markets where a detailed credit spread model or an emerging market model is not estimated, credit spread risk is accounted for by exposing bonds to the swap spread.

Figure 6-1

Monthly Changes in BBB Credit Spreads for Bonds Denominated in U.S. Dollars

Volatility spiked during the currency crisis in autumn of 1998 and was followed by a period of persistently high volatility. Risk of this type is modeled with spread factors.



Swap Spread Model Estimation Process (All Markets):

1. Data acquisition
2. Factor return estimation

Data Acquisition and Factor Return Estimation

Barra obtains daily swap rate data for 26 markets from data vendors. In each market, swap spread risk is based on a single factor: the monthly change in the average spread between the swap and treasury curve.

Factor Exposure Calculation

For bonds that are exposed to a credit factor or an emerging-market factor in addition to the swap-spread factor, the swap-spread exposure is equal to the sensitivity of a bond's return to change in the swap spread level. The exposure is equal to effective duration.

For bonds that do not have additional factors to explain their credit risk, the swap-spread factor is used to forecast risk for debts of widely varying credit quality. Since bonds of lower credit quality tend to be more volatile, we scale their exposures to the swap spread by their (higher) option-adjusted spread (OAS), as shown in the formula:

$$x_{swap} = D_{eff} + (\alpha - 1)D_{spr} \quad \text{(EQ 6-1)}$$

where

x_{swap} = swap spread exposure

$$\alpha = \max\left[1, \left(\frac{\max(OAS\_treasury, 0)}{\max(swap\ spread, 10\ \text{bps})}\right)\right]^{\gamma}$$

D_{spr} = spread duration of the bond

For fixed rate bonds, this reduces to $x_{swap}=\alpha D_{eff}$, because for these bonds, $D_{eff} = D_{spr}$. The bond sensitivity to a change in the swap spread level (typically spread duration) is scaled by the ratio of the bond's OAS to the swap-spread level raised to the power γ, which is typically 0.6. Hence, bonds with higher OAS will have correspondingly higher volatility forecasts. The exponent takes into account the nonlinearity in the relationship between OAS and volatility. The exponent γ is fitted in each market with maximum likelihood estimation.

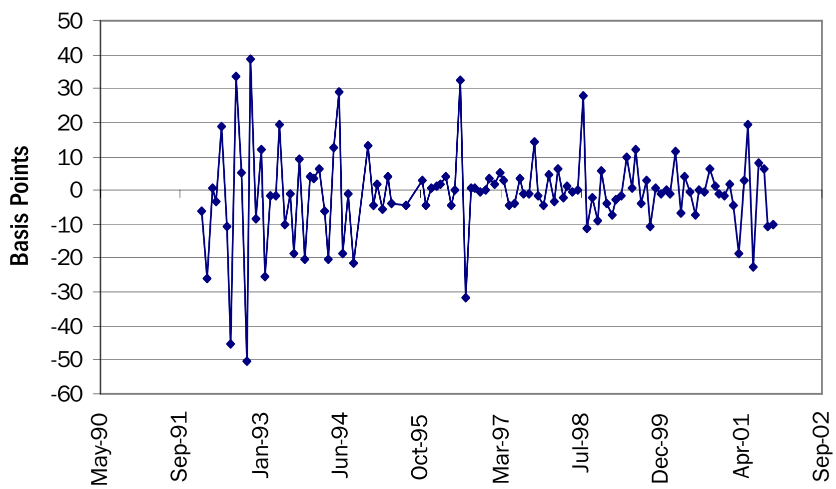


Figure 6-2

Monthly Changes in Average Swap Spread for Bonds Denominated in Australian Dollars

This series and others of the same type are incorporated into the covariance matrix estimation.

Detailed Credit Spread Risk Model

In four of the most active markets, additional factors capture the risks due to changes in credit spreads over the swap curve. The spread risk in the U.S. dollar, sterling, yen, and euro markets is decomposed into swap spread risk and risk due to credit spreads over the swap curve.

The euro zone presents a more complicated picture due to the presence of more than one sovereign issuer. A single set of interest-rate factors is not sufficient to capture the disparate credit qualities of the EMU sovereigns. Consequently, we provide interest rate factors for each sovereign issuer as well as factors for changes in average euro zone rates.[1] Thus, euro-denominated sovereign bonds from EMU member countries will be exposed to the term-structure factors of the country's local market. All other euro-denominated bonds will be exposed to the general EMU term-structure factors.

Currency Dependence

Bond credit spreads are not market independent. To date, high-grade bonds issued in different markets by a single issuer often have no significant correlation between spread changes in the different markets. If Toyota were to issue U.S. dollar-, sterling-, and euro-denominated bonds, the credit spread levels and returns of the bonds will not be equal. The differences will persist even if the bonds were to have an apparent hedge overlay that allows the conversion of a credit exposure in one currency to an equivalent one in another currency.[2]

Factors based on investment-grade bonds with a common currency and a common sector or rating show a very high degree of correlation; factors based on bonds with a common sector and rating but denominated in different currencies show very little correlation. For example, Canadian government bonds in the three

1. These are based on monthly changes in a yield curve estimated from the pool of actively traded EMU sovereign issues. Bonds are weighted by the GDP of the issuer. See Lisa Goldberg and Anton Honikman, "The Euro Yield Curve: Projecting the Future," *Euro* (December 1998).
2. For example, a series of currency forward rate agreements (FRA) could be used, in principle, to convert a sterling-denominated bond into a euro-denominated bond with the same spread over the LIBOR/swap curve.

markets had essentially zero correlation in their spread changes over the two-year period June 1999–May 2001.

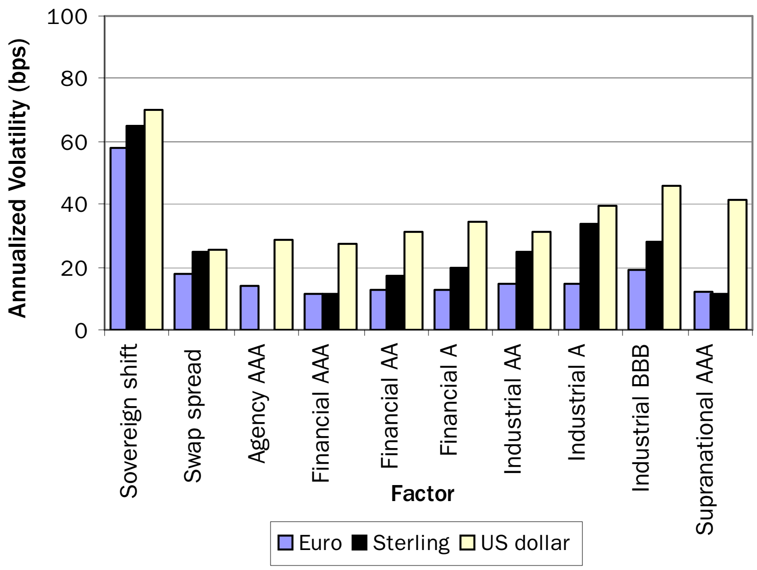


Figure 6-3

Cross-Market Volatility Comparison

A comparison of term structure shift, swap, and several credit spread volatilities for the U.S. dollar, sterling, and euro markets shows that U.S. dollar volatilities are higher than euro volatilities, sometimes by a factor of three. The volatility of the sterling sectors consistently lies between the two extremes. Estimates are based on monthly data weighted exponentially with a 24-month half-life.

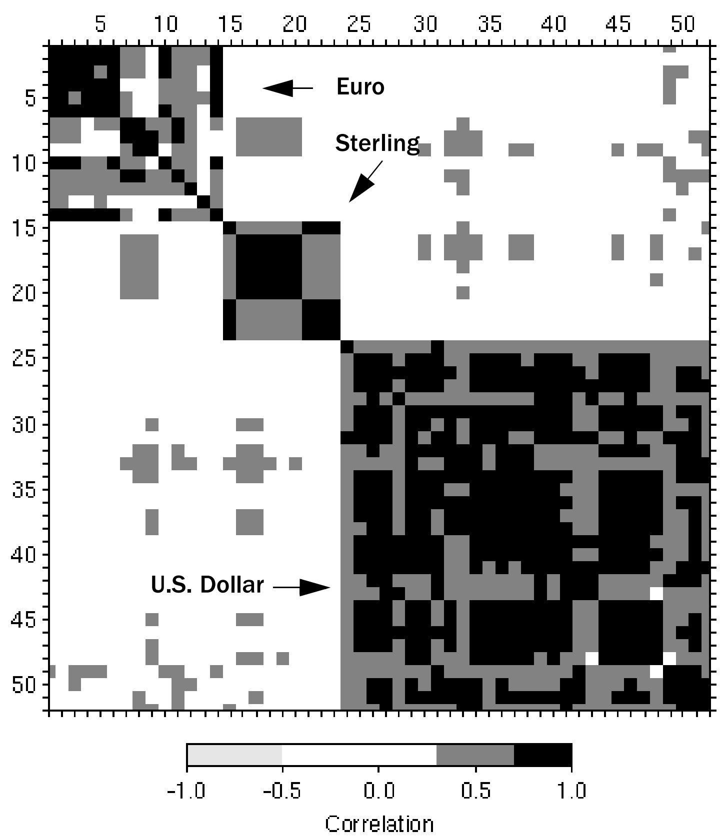


Figure 6-4

Map of Spread Return Correlations for the U.S. Dollar, Sterling, and Euro Markets

The "heat map" shows correlations among credit factors. High correlations (0.7–1.0) are consistently observed within a single market, whereas global market correlations remain mostly between –0.3 and 0.3.

Because empirical evidence indicates that credit spreads are currency dependent, we base our models on currency-specific credit risk factors.[1]

Model Structure

Spreads of different rating categories in different sectors are not perfectly correlated.

For U.S. dollar-, sterling-, and euro-denominated securities, the detailed common-factor spread models are based on a sector and rating breakdown. Non-government securities are exposed to some combination of sector and rating spread risk factors—generally one spread factor for each sector/rating combination (the Japan model works somewhat differently). In addition, each non-government security is exposed to the swap spread for its currency.

The U.S. dollar, sterling, and euro blocks have one factor for each combination of sector and rating. The U.S. dollar block, however, has some exceptions. It has a sector-independent CCC rating factor, as well as an agency and five mortgage-backed security (MBS) factors.

For the yen block, rating dependence is restricted to the Samurai (foreign) and Corporate sectors. In addition, yen bonds are exposed to a premium/discount spread factor, where the exposure is equal to the market premium (positive or negative)[2] of the bond. The exposure to the factor is equal to the difference between the market price and par (market price – par). So a bond that has a par at 100 but is trading at 103 has an exposure of 3. The farther from par the bond is trading at, the greater the exposure to this factor.

In the U.S. dollar, sterling, and euro blocks, factors with sparse or absent data are proxied by a generic rating-based spread factor. For example, if there is no information on BBB-rated supranational securities, we use generic BBB spread data instead. This

1. Further discussion of this point can be found in A. Kercheval, L. Goldberg, and L. Breger, "Currency Dependence in Global Credit Markets: The Need for More Detailed Risk Models," Barra, 2001, in the Client Support Library http://www.barra.com.
2. The spread indicates how much of a premium or discount the bond is trading at.

improves model coverage. If a user wants coverage outside the estimation universe or if new bonds appear in a previously empty sector/rating bucket,[1] the model provides a reasonable forecast.

Barra Research Methods

Sector-by-Rating Framework

Why does Barra use what looks like a tremendous excess of factors within each market, breaking up the market structure into individual sector-by-rating factors, rather than having each bond exposed both to a sector factor and a rating factor (with far fewer total factors)? Using a sector + rating factor structure, rather than a sector-by-rating structure would provide a more parsimonious framework, and, consequently, one might expect, more reliable volatility forecasts.

The explanation is fairly straightforward. Trying to represent bond spread changes as the sum of a sector spread change and a rating spread change fails in many cases. Spreads of different rating categories in different sectors behave independently.

Consider what happens to bond spreads after a positive shock to energy prices. One expects energy company bond spreads to be unaffected, or perhaps even tighten and transport company bonds to widen. The degree of this widening is likely to be strongly dependent on rating (BBB-rated transport bonds will widen much more than AAA-rated ones). In the sector + rating framework, the difference between transport bonds and energy bonds has to be explained entirely by generic transport and energy spreads. There is no place in the model to allow for BBB and AAA energy spreads to stay more or less the same, while the difference between BBB and AAA transport spreads widens significantly. This sort of phenomenon happens often enough for us to reject the sector + rating model as a viable structure. Statistical tests of the added explanatory power of the sector-by-rating model over the sector + rating scheme demonstrate that, in more volatile historical periods, the more detailed model captures genuine effects not seen by the more restrictive model.

After the attacks on New York City and Washington, D.C. on September 11, 2001, the transport company spreads—particularly for lower rated bonds—widened dramatically. The changes for energy and finance bonds, while historically large, were not nearly so significant. In order to accommodate both the sector effect and the rating effect, the restricted sector + rating model actually entails a tightening of both the energy and finance spreads to compensate for the very large BB–A spread widening needed to fit the transports. The result is that the more restrictive model gives a poor fit for the higher rated sectors. Although the magnitude of September 11 is extraordinary, less dramatic instances of the same phenomenon occur frequently.

1. This was recently the case with A-rated supranational bonds.

Table 6-1

Credit Risk Factors in Each Market

In the United States, United Kingdom, and the European Union, most factors are determined by combinations of sector and rating. The U.S. block has some exceptions, specifically, the CCC rating factor, which does not take account of the sector, and the mortgage spreads, which do not depend on rating. In Japan, the model structure is somewhat unique as it is based on local Japanese market conventions. Many factors are issuer-specific, and many more are based only on sectors, rather than on a combination of sectors and ratings.

These factors are subject to change. Updates are available on the Fixed Income Local Market Factors list at http://www.barra.com/support/models/fixed_income/local_markets.asp.

Euro

Sector by Rating: 9 x 4 factors estimated for the nine sectors listed below and four rating categories, AAA to BBB:

- Agency
- Energy
- Financial
- Industrial
- Pfandbrief
- Sovereign
- Supranational
- Utility
- Telecommunications

High Yield: one BB, one B, and one CCC factor

Sterling

Sector by Rating: 7x 4 factors estimated for the seven sectors listed below and four rating categories, AAA to BBB:

- Agency
- Financial
- Industrial
- Sovereign
- Supranational
- Telecommunication
- Utility

High Yield: one BB, one B, and one CCC factor

U.S. Dollar

Agency: One factor for all agency bonds

Sector by Rating: Up to 9 x 6 factors estimated for the nine sectors listed below and six rating categories, AAA to B:

- Canadian-issued bond (any sector)
- Energy (industrial sector–oil and gas subsector)
- Financial
- Industrial (all industrial subsectors other than aerospace and airlines, oil and gas, railway, or shipping)
- Supranational (supranational issuers only)
- Telecommunications (utility sector–telephone subsector)
- Transportation (industrial sector–aerospace and airlines, railways, and shipping subsectors)
- Utility (utility subsectors other than telephone)
- Yankee (non-U.S., non-Canada, non-supranational issuer)

CCC Rating: One factor estimated across all sectors

MBS: Five spreads, one each for conventional 15 year, 30 year, and balloons, and GNMA 15 year and 30 year

Muni: one AA, one A, and one BBB factor

Yen

Sector and Rating: 33 factors

- Government Benchmark
- Bank of Tokyo
- Norinchukin Bank
- Shokochukin Bank
- Corporate AAA
- Corporate AA
- Corporate A
- Corporate BBB
- Corporate BB
- Corporate B and CCC
- Current Yield (government)
- Current Yield (non-government)
- Government Guaranteed
- Highway
- International Bank of Japan
- Finance
- Long-Term Credit Bank
- Government Mid-Term
- Big 5 Municipal (Osaka, Kobe, etc)
- Other Muni
- Tokyo Muni
- Nippon Credit Bank
- NTT
- Non-Government Guaranteed (NHK, TRT)
- Other Electric
- Sinking Fund Bond
- Government Six-Year
- Samurai AAA
- Samurai AA
- Samurai A
- Samurai BBB
- Tokyo Electric
- Zenshiren Bank
- Railway (guaranteed)
- Fiscal Investment and Loan Program

Data Acquisition

We obtain daily bond price, term, rating, and sector data for corporate bonds issued in the U.S. dollar, yen, sterling, and euro markets.

Detailed Credit Spread Model Estimation Process: U.S., U.K., Japan, and EMU

1. Data acquisition
2. Factor return estimation
3. Covariance matrix estimation
4. Model updating

Factor Return Estimation

We calculate monthly credit spread factor return series as weighted average changes in spreads for bonds present in a particular sector-by-rating category at both the start and end of each period.[1] When fewer than a minimum number of bonds are available in a category, we proxy the return for that sector-by-rating bucket with the aggregate spread return for all bonds with the same rating, independent of sector.

Covariance Matrix Estimation

The covariances of risk factors are based on the historical factor-return series. The credit factor covariance forecasts are constructed market by market. We separately estimate local covariance matrices for the U.S. dollar, sterling, euro, and yen credit factors. We obtain the correlations between factors in different local markets with Barra's global integration method.[2]

Exponential Weighting

Since local market factor volatilities change over time, we use an exponential weighting scheme that attaches greater importance to recent events than to older ones. Older returns are weighted by a coefficient, λ^n, where n is the age of the return in months. The value of n is such that $\lambda^n = ½$ is the half-life of the weighting scheme. The mathematical relationship between λ and n is given as:

$$n = -\frac{\log 2}{\log \lambda} \qquad \textbf{(EQ 6-2)}$$

1. The average is duration weighted.
2. For more information on the integration method, see Chapter , "Integrated Risk Modeling," starting on page 109.

The local market factor returns are weighted with a 24-month half-life.[1] This value was chosen on the basis of the results of out-of-sample testing.

Factor Exposure Calculation

A bond is exposed to one of the sector-by-rating credit spreads if all of the following conditions are satisfied:

- The bond is denominated in U.S. dollar, U.K. sterling, European euro, or Japanese yen.
- The bond's sector and credit rating match a spread sector and rating within the applicable local-market factor block.
- The bond is not exposed to an emerging-market spread.

The exposure of a credit instrument to the factor with matching currency, sector, and rating is spread duration. Mathematically, spread duration is given by the following formula:

$$D_{spr} = -\frac{1}{P_{dirty}}\left(\frac{\partial P_{dirty}}{\partial S_{spr}}\right) \qquad \text{(EQ 6-3)}$$

where

P_{dirty}	=	dirty price of bond
S_{spr}	=	parallel spread shift

Emerging-Market Risk Modeling

Much of the debt issued by emerging-market sovereigns and companies is denominated in currencies of developed markets such as the U.S. dollar, sterling, yen, and euro. The dominant source of risk for these bonds tends to come from the creditworthiness of the issuer and not from the local market interest rate factors indicated by the currency of the bond.

1. This corresponds to a weight multiplier $\lambda=0.5^{1/24}$ or 0.9715.

For each emerging market, there is a single, global emerging-market spread factor. This means that there are not, for example, separate factors for Nigerian bonds issued in U.S. dollars and Nigerian bonds issued in British sterling. The effect of the credit quality of the issuer dominates any distinction one might make between the developed-market currencies in which the bonds could be denominated.

Consider for example an Argentine corporate bond issued in sterling. It will be exposed to the local U.K. interest rate factors, the U.K. swap spread factor, the currency factor, and the Argentine emerging-market spread factor.

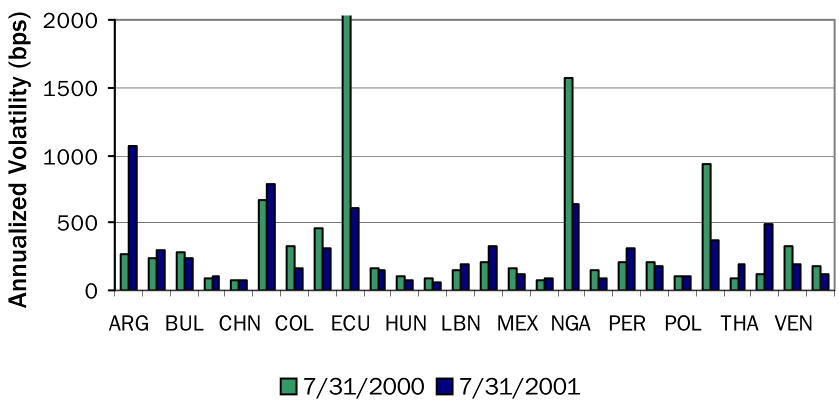


Figure 6-5

Volatility Levels in Emerging Markets

The spread of emerging-market bonds has both higher volatility and greater variability of volatility than investment-grade corporate and developed-market sovereign bonds.

For example, during the period of July 31, 2000, to July 31, 2001, the Ecuadorian sucre (ECU) was dollarized and macroeconomic conditions improved. The resulting large decrease in volatility is obvious. In contrast, the volatility in Argentina (ARG) increased after continuing political and social pressures.

Model Structure

The emerging-market factor block forecasts risk for bonds issued in an external currency (primarily, U.S. dollar and euro) by an emerging-market sovereign or corporation. As with the detailed credit spreads, emerging-market spreads are measured relative to the swap curve.

As of June 2003, the block is composed of 26 factors, one for each emerging market. We estimate the emerging-market block separately from other factors and rely on high-frequency data.

Emerging Market
Model Estimation Process

1. Data acquisition and factor return estimation
2. Covariance matrix estimation
3. Model updating

Data Acquisition and Factor Return Estimation

The changes in the weekly stripped spread[1] levels (which are vendor-supplied[2] basic market data) provide the weekly return history of the risk factors.

Covariance Matrix Estimation

Stripped-spread factors for emerging-market bonds, like the currency factors, can be quite volatile and tend to exhibit variable volatility over time.

The factor return variance and covariance estimates, which are based on weekly data, are weighted exponentially with an eight-week half-life.[3] Next, the exponentially weighted variance and covariances are time-scaled to a one-month horizon. They are multiplied by a weekly-to-monthly conversion constant[4] (52/12). The block is subsequently integrated into the risk model.

---

1. A bond's stripped spread is calculated by stripping or subtracting the present value of the collateralized cash flow (escrowed interest payments and collateralized principal). The adjusted price is then equated to the remaining non-collateralized cash flows, which is discounted at a spread over the base curve. This constant spread over the default-free curve is the stripped spread.
2. The J.P. Morgan EMBI Global is an index of dollar-denominated government debt from 26 markets. The index includes both collateralized restructured (Brady) debt and conventional noncollateralized bonds. For more information on J.P. Morgan's spread estimation methodology, see *Introducing the J.P. Morgan Emerging Markets Bond Index Global* (August 3, 1999), available from J.P. Morgan.
3. The eight-week half-life was chosen on the basis of out-of-sample tests.
4. The numbers in the conversion constant, 52 and 12, are respectively the number of weeks and months in a year.

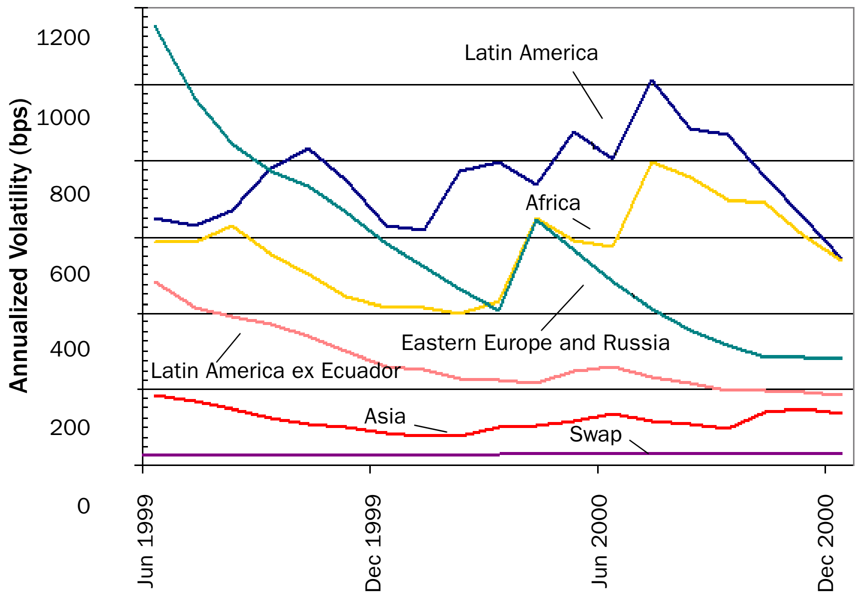


Figure 6-6

Annual Volatility Forecasts for Different Regions of the EMBI Global

Realized volatility was fairly stable from June 1999 to December 2000 in Asia, less so in Africa. It declined by roughly a factor of two in Latin America (because of Ecuador) and a factor of four in Eastern Europe and Russia. The eight-week half-life is considerably shorter than the time scale of volatility decrease in these markets, so the risk forecasts have only slightly lagged the changes in the markets.

Factor Exposure Calculation

An emerging-market bond issued in an external currency is exposed to the spread indicated by the issuer. The exposure is the bond stripped-spread duration.[1] This is expressed as:

$$D_{spr} = -\frac{1}{P_{dirty}}\left(\frac{\partial P_{dirty}}{\partial S_{spr}}\right) \qquad \text{(EQ 6-4)}$$

where

P_{dirty}	=	dirty price of bond
S_{spr}	=	parallel stripped spread shift

Updating the Model

We estimate the latest month's factor returns with the most recent market data. We use monthly factor return data for the swap

1. Stripped-spread duration is the exposure of a bond to the credit spread. It takes into account any collaterization.

spread and detailed credit market spread, and weekly factor return data for the emerging-market spread. We update the covariance matrix with these factor returns every month.

7. Specific Risk Modeling

Common factors do not completely explain asset return. Return not explained by the common factors—shift-twist-butterfly and spread factors—is called ***specific return***. The risk due to the uncertainty of the specific return is called ***specific risk***. Specific returns of bonds from different issuers are assumed to be approximately uncorrelated with one another, as well as with common factor returns.

Specific risk tends to be relatively small for government, agency, and high-quality corporate bonds. For corporate bonds, specific risk includes ***event risk***, which is the risk that a company's debt may be repriced due to real or perceived changes in the company's business fundamentals. This component of risk can be rather large for bonds with low credit quality.

We forecast specific risk with three models: a heuristic model for sovereign bonds, a heuristic model for corporate bonds, and a transition-matrix-based model for bonds in U.S. dollar, sterling, and euro markets.

Heuristic Models

Except for the U.S. dollar, sterling, and euro corporate markets, all markets—including the yen, U.S. agency, and mortgage-backed security (MBS) markets—use the heuristic models.

The heuristic model for sovereign bonds has only one parameter to account for sovereign market risk. The heuristic model for corporate bonds has an additional parameter to account for the credit riskiness of the corporate issuer. The heuristic models assume that specific risk of the bond is proportional to spread duration.

Heuristic Model Estimation Process

1. Data acquisition
2. Specific risk estimation
   a. Sovereign parameter determination
   b. Corporate parameter determination
3. Model updating

Data Acquisition

We obtain the terms and conditions (TNC) and the daily price data on sovereign and corporate bonds issued in 25 domestic markets, inflation-protected bonds (IPBs) issued by four sovereigns,[1] as well as agency bonds and MBS generics issued in the United States. The option-adjusted spreads (OAS) for all bonds and MBS generics are calculated using local market term structure and daily prices.

Sovereign, U.S. Agency, and MBS Specific Risk Estimation

Underlying the model of specific risk of domestic government bonds, U.S. agency bonds, and U.S. MBS is the assumption that specific risk is constant across assets of the same class, and can be captured by a single parameter. Expressed as price return,[2] specific risk is:

$$\sigma_i = D_i b_a \tag{EQ 7-1}$$

where

σ_i	=	monthly specific risk of bond i
D_i	=	spread duration of bond i
b_a	=	monthly specific-return risk parameter for asset class a or domestic market a

The parameter b_a is the standard deviation of the specific spread returns of bonds of a given class or a given market. A different b_a is calculated for each market or asset class. U.S. agency bonds, U.S. MBS, and sovereign bonds of 25 domestic markets would each have a different value for b_a. The parameter incorporates historical information using an exponential weighting scheme with a 24-month half-life.

1. We collect daily price data on IPBs issued by the governments of the United States, Great Britain, Canada, and France.
2. Price return is related to spread return through spread duration: $r_{price} \approx r_{spr} D_{spr}$, where r_{price} is the price return variance due to spread change, r_{spr} is the spread return, and D_{spr} is the spread duration.

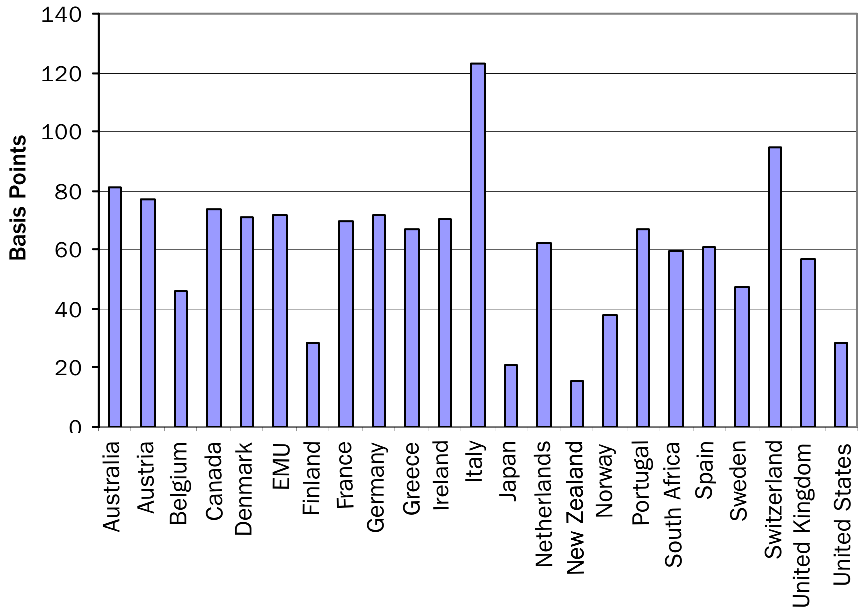


Figure 7-1

Monthly Specific Risk Forecasts for Five-Year Duration Government Bonds

The forecasts have a monthly horizon and are reported in annualized terms.

Corporate Bond Specific Risk Estimation

For non-government bonds outside the U.S. dollar, sterling, and euro markets, an extended issue-specific risk model is used. A second term is added to address the additional risk surrounding credit events. Expressed as return volatility, this specific risk is:

$$\sigma_i = D_i b_a + D_i c_a s_i \qquad \text{(EQ 7-2)}$$

where

σ_i	=	monthly specific risk of corporate bond i
D_i	=	spread duration of bond i
b_a	=	constant spread return risk for government bonds in domestic market a
c_a	=	constant to account for additional specific spread return volatility of corporate bonds in market a
s_i	=	OAS of bond i

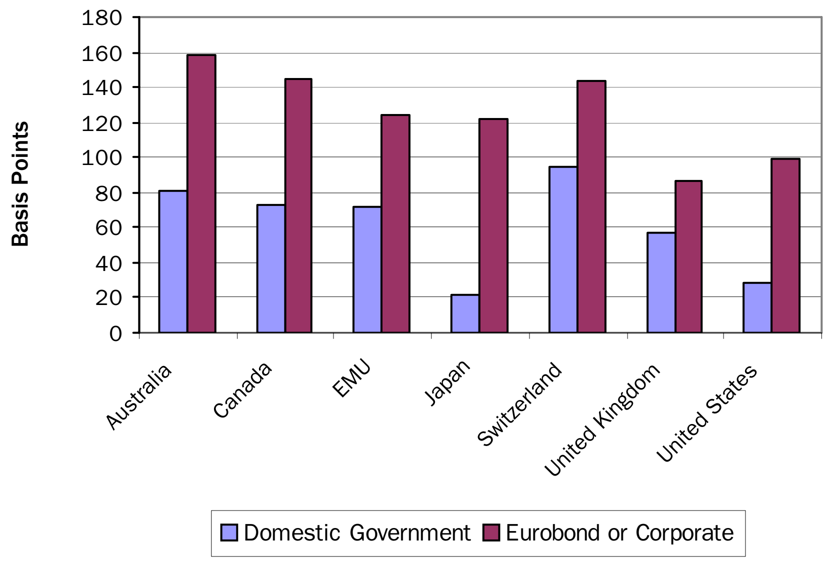
In the estimation of c_a, b_a is considered to be a constant baseline of volatility. The c_a parameter is then scaled by OAS for the same reason that the OAS is used to scale the swap spread exposure. The premise is that the specific return volatility of corporate bonds is proportional to the OAS level. Bonds with lower credit ratings are subject to higher spreads and greater volatility.

The constants b and c are fitted with a maximum likelihood estimation.

Figure 7-2

Monthly Specific Risk Forecasts for Five-Year Duration Bonds

The graph gives a comparison between specific risk forecasts for domestic government bonds and other bonds denominated in the same currency.



Transition-Matrix-Based Model

Transition-Matrix-Based Model Estimation Process

1. Data acquisition
2. Transition matrix generation
3. Rating-specific spread level calculation
4. Credit migration forecasting
   a. Spread migration estimation
   b. Recovery rate estimation
5. Model updating

For the U.S. dollar, sterling, and euro credit markets, a more elaborate model of specific risk is used. We estimate the specific risk with rating spread level differences and with a rating transition matrix, whose entries are probabilities of bonds migrating from one rating to another (from AAA to BB, for example) in a month. We compute the rating spread levels from averages of OAS values of bonds bucketed by market and rating.

Data Acquisition

We obtain bond data from data vendors and historical average issuer credit migration matrices from credit rating agencies.[1]

Transition Matrix Generation

Historical credit migration rates determine a ***transition matrix***, whose entries are estimated probabilities of bonds migrating from one rating to another over a one-year horizon.

Next, we standardize and analyze the data for inconsistencies. First, any firm that transitions to "non-rated" is removed and the matrix elements are rebalanced so that each column once again sums to 100%. Next, we resolve inconsistent ranking data and obtain an improved estimate of the true transition matrix.[2] The transition matrix is finally scaled to a monthly horizon according to the formula:

$$\Lambda_t = e^{Gt} \qquad \textbf{(EQ 7-3)}$$

where

Λ_t	=	transition probability matrix for horizon t
t	=	horizon (in years)
G	=	$\log \Lambda_1$

In Equation 7-3, the matrix exponential and logarithm are power series of matrix multiplications. For example, as long as the right-hand side of the equation is defined and converges, the log Λ_1 is given by:

1. We obtain annual reports showing historical rating changes of issuers from credit rating agencies. The rating agencies monitor business fundamentals in relation to a firm's debt payment obligations. They assign debt ratings based on their estimate of the likelihood of repayment. Event risk includes, but is not limited to, occurrences that may cause rating agencies to change the issuer's debt rating.
2. The best transition matrix is determined by least-square minimization. The valid matrix closest to the original matrix is selected.

$$\log \Lambda_1 \equiv \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(\Lambda_1 - I)^n}{n} \tag{EQ 7-4}$$

Underlying the model is the Markov assumption that the probability of a firm's rating change depends only on its current rating, not on its history.[1]

Table 7-1

The Resulting One-Month Transition Matrix for Corporate Bonds

		Initial Rating						
		AAA	**AA**	**B**	**BBB**	**BB**	**B**	**CCC**
Final Rating	**AAA**	99.36	0.05	0.04	0.02	0.02	0.02	0.02
	AA	0.61	99.28	0.21	0.02	0.01	0.01	0.00
	A	0.02	0.61	99.24	0.49	0.03	0.03	0.03
	BBB	0.00	0.04	0.45	98.94	0.07	0.13	0.13
	BB	0.00	0.00	0.04	0.45	98.33	0.58	0.15
	B	0.00	0.00	0.02	0.06	0.78	98.36	11.73
	CCC	0.00	0.00	0.00	0.00	0.11	0.44	96.22
	D	0.00	0.00	0.00	0.01	0.05	0.44	2.28

Rating Spread Level Calculation

If past experience of rating changes is a reasonable forecast of their future likelihood, then the transition matrix and the estimates of rating spreads can be used to forecast the contribution of changes in credit quality to the volatility of bond spread returns.

To forecast the risk associated with rating migration, the size of the impact that a rating change has on spread level is estimated. To estimate the spread level, bonds are grouped by rating and market on each analysis date. The spread level is the average spread over the benchmark curve of all the bonds within each rating group. This is simply:

$$S_{rating} = \frac{\Sigma w_i S_{rating,i}}{\Sigma w_i} \tag{EQ 7-5}$$

1. For details on Markov's "memory-less process," see D.R. Cox and H.D. Miller, *The Theory of Stochastic Process* (London: Chapman & Hall, 1965).

where

w_i	=	bond weights
$S_{rating,\ i}$	=	OAS of bond i with a given rating in a given market

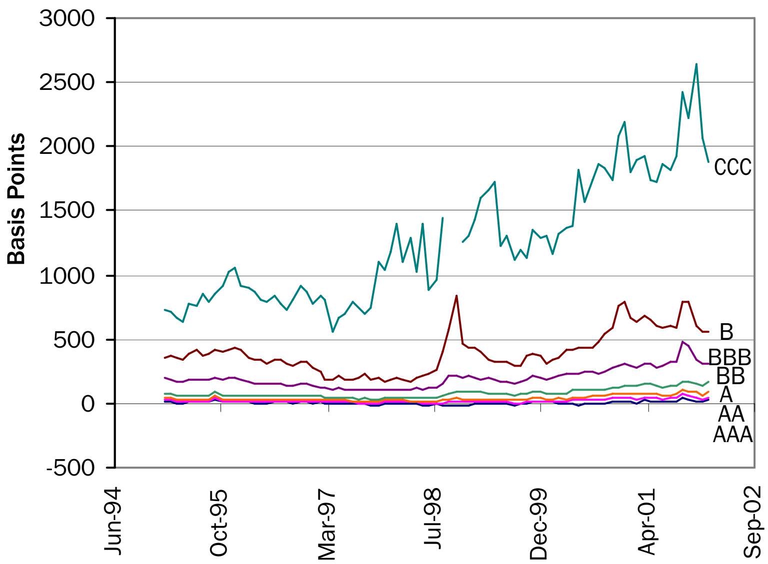


Figure 7-3

U.S. Dollar Rating Spreads to Swap

A time series of rating spread levels for U.S. dollar-denominated bonds shows that the average spread for a bond below investment grade dwarfs the investment-grade spreads.

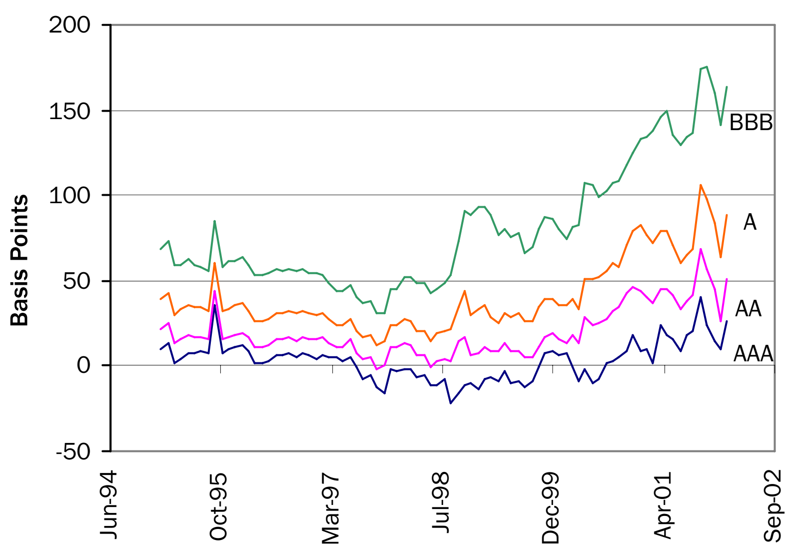


Figure 7-4

U.S. Dollar Investment-Grade Rating Spreads to Swap (Magnified)

Table 7-3

U.S. Dollar Rating Spread Levels, March 31, 2001 (in Basis Points)

Spreads increase as credit quality diminishes.

Rating	Spread Level
AAA	23
AA	45
A	79
BBB	146
BB	313
B	687
CCC	1919

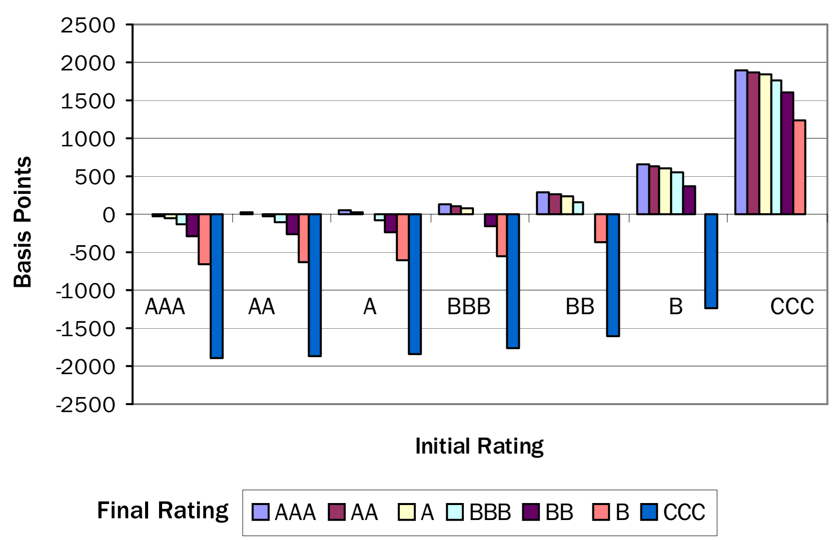
The difference between rating spread levels gives a measure of the impact of a given rating change on spread. For example, an estimate of the spread change for a bond initially rated AAA and then downgraded to AA is computed as the difference in the AAA spread level and the AA spread level.

If the rating at the final period is different from the one at the initial period, one can forecast that the spread of the bond will have changed significantly as well. The spread changes are zero when the initial and final ratings agree; they are negative if the initial rating is higher than the final rating.

Figure 7-5

Returns Generated by Spreads

The chart shows typical rating spread return values for the U.S. corporate market.



Credit Migration Forecasting

The credit migration model predicts that specific risk of corporate bonds arises primarily from events affecting the issuer's credit quality. The impact of such events on return is assumed to be perfectly correlated for all the bonds of an issuer. To calculate the risk, we aggregate the issuer's bonds to form a subportfolio, then the impact of various credit migration events on the subportfolio is calculated.

The approximate return variance for a bond due to credit migration is expressed as:

$$\sigma^2_{credit} = \sum_f \Lambda_{fi}(D_{spr}(s_f - s_i) - \overline{\Delta s_i})^2 + \Lambda_{di}(1 - R - \overline{\Delta s_i})^2 \qquad \text{(EQ 7-6)}$$

where

σ^2_{credit}	=	price return variance due to spread change or default
Λ_{fi}	=	probability of bond rating transition from i to f
D_{spr}	=	the aggregate spread duration of all bonds from a given issuer in the portfolio
s_f	=	final spread level (the final rating)
s_i	=	initial spread level (the initial rating)
$\overline{\Delta s_i}$	=	mean price return due to spread change or default $\sum_f \Lambda_{fi} D_{spr}(s_f - s_i) + \Lambda_{di}(1 - R)$
Λ_{di}	=	probability of a transition from i to default
R	=	recovery rate in default

The formula has two main components. The first component accounts for credit migration from one rating to another. The second accounts for default. The parameter depends on the recovery rate and the current market value.

The default recovery factor, R, is the fraction of the value of the bond that would be recovered in the event of default. If R were 1, then 100% of the bond's value would be recovered, and this component would have no effect. The recovery rate is uncertain and situation dependent. However, studies by Moody's have shown that a typical recovery rate for senior debt is roughly 50%. The recovery value has only a small impact on the risk forecast for investment-grade bonds, but the recovery model may have a pronounced impact on bonds that are below investment grade.

In practice, the effect of the mean price return term, $\overline{\Delta s_i}$, is negligibly small. This allows a simpler form of the equation:

$$\sigma^2_{credit} = \sum_f D^2_{spr} \Lambda_{fi} (s_f - s_i)^2 + \Lambda_{di} (1 - R)^2 \quad \textbf{(EQ 7-7)}$$

Using this form, the two terms of the formula,

$$a_{rating} = \sum_f \Lambda_{fi} \left(s_f - s_i \right)^2 \text{ and } b_{rating} = \Lambda_{di} \left(1 - R \right)^2 ,$$

can be computed separately each month and stored for use in the application. The seven different ratings[1] in the model give seven a_j parameters and seven b_j parameters.

So, the specific risk of a bond with a particular rating is given by:

$$\sigma_{credit} = \sqrt{D^2_{spr} a_{rating} + b_{rating}} \quad \textbf{(EQ 7-8)}$$

Due to the dependence of σ^2_{credit} on current spread levels, the credit risk formula is very responsive to changes in market expectations. In circumstances where low-grade credit spreads significantly widen relative to high-grade ones (due perhaps to expectation of increased default risk), the credit risk forecast would increase immediately by a corresponding amount.

In the parametric context, the main difference in calculation method between the empirical specific risk for sovereign bonds and the specific risk due to credit migration for corporate bonds is that the contributions of the former add independently (that is, are diversified) at the security level, while the contributions of the latter add independently at the issuer level.

1. The seven ratings are: AAA, AA, A, BBB, BB, B, and CCC.

Figure 7-6 and 7-7 show a history of credit risk forecasts using the transition-matrix approach. Note that the risk values have been converted from price-return risk to spread-return risk by dividing by spread duration.

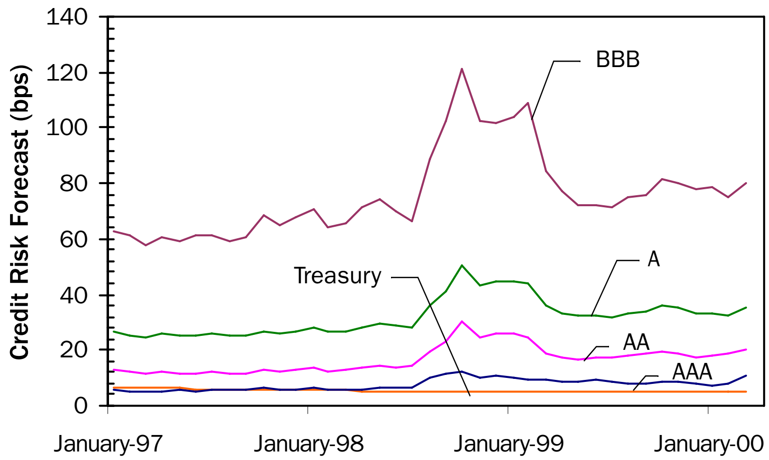


Figure 7-6

Credit Risk Forecasts for a U.S. Dollar-Denominated Five-Year Duration Bond, by Rating, Based on the Transition Matrix (AAA to BBB)

Sharp jumps in risk forecasts for all rating classes are apparent in the credit crash of mid-1998, due to the significant widening of spreads.

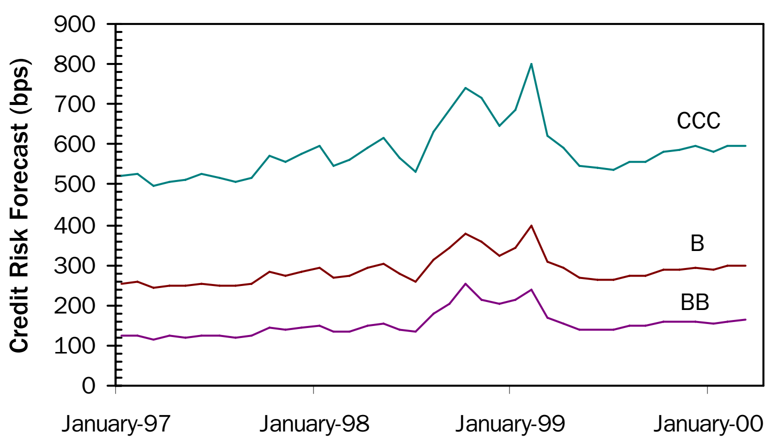


Figure 7-7

Credit Risk Forecasts for a U.S. Dollar-Denominated Five-Year Duration Bond, by Rating, Based on the Transition Matrix (BB to CCC).

Table 7-6

Model Forecasts as of January 31, 2000

The forecasts are expressed as annualized standard deviation of interest rates or spreads. For the rating-based credit risk forecasts, a spread duration of five years is assumed. The first column shows average factor volatility forecasts for factors in different groups. The second column shows the specific or credit risk forecast for a single security.

Type of Issue	Common Factor Risk	Specific/Credit Risk
Treasury/Spot	82	8
Agency	15	14
AA	22	18
A	22	33
BBB	34	79
BB	92	155
B	164	287
CCC	296	582

An interesting observation from this table is that the classification of issuers into investment (BBB and above) and speculative grade (below BBB) neatly corresponds to the split between bonds whose common factor and credit risk are each less than their interest rate risk, and those for which they are greater. That is, the common factor and credit risks for ratings of BBB and above are all less than the 82 basis points of risk due to spot rate volatility, while those of lower-grade bonds are above this level. Interest rate risk is dominant for investment-grade bonds while credit risk is dominant for high-yield bonds.

Updating the Model

The specific risk parameters for the heuristic models and the transition-based models are calculated monthly.

Currency Risk

Section Four discusses the extensive, detailed process of creating Barra currency models.

Chapter 8: Currency Risk Modeling

Section Four

8. Currency Risk Modeling

The fluctuation of currency exchange rates is a significant source of risk faced by international investors.[1] Consider a Norway-based investor with a portfolio of U.S. treasuries. U.S. treasuries belong to the U.S. dollar local market and their interest rate risk comes from changes in U.S. Treasury rates, yet our Norwegian investor is still subject to the risk that comes from changes in the U.S. dollar/Norwegian kroner exchange rate.

Often, more than half the variation of a well-diversified portfolio of global equities or investment-grade bonds can be attributed to currencies. Consequently, for an international investor making currency bets or trying to hedge currency risk entirely, it is essential to have accurate and reliable risk forecasting models that include currency risk.

Model Structure

Because currency risk is time sensitive, the variances and covariances of currency factors are estimated using high-frequency data and models with short memories. The correlations between currencies are derived from weekly data weighted exponentially with a 17-week half-life. A GARCH (1,1) model based on daily data is used to forecast currency volatility.

Since currency risk must be integrated with other sources of risk, Barra also estimates covariances between currency factors and all other Barra risk factors. And because most Barra factor returns are based on monthly data, these "cross-block" covariances are based on monthly data as well.[2]

1. For related discussions, see the Barra Research article, "Forecasting Currency Return Volatility." It is available at http://www.barra.com.
2. Covariances between factors with different frequencies are constrained by the coarser time resolution.

Data Acquisition and Return Calculation

The first step in model estimation is acquiring daily currency prices for more than 50 currencies from data vendors. The inception dates of the histories vary from January 1973 to July 2002, but, for most currencies, Barra has historical data dating from the early 1980s. The exchange rates are not consistent across vendors due to discrepancies in sampling time.

The returns are then calculated with:

$$r_t = \log \frac{P_{t+1}}{P_t} \quad \text{(EQ 8-1)}$$

where

r_t	=	return at time t
P_{t+1}	=	current spot exchange rate
P_t	=	previous exchange rate

Currency exchange rates are quoted in U.S. dollars per local currency. In these terms, a positive return means that the local currency became stronger relative to the U.S. dollar.

Model Estimation Process

1. Data acquisition and return calculation
2. Estimation of covariance matrix:
   a. Correlations
   b. Volatility forecasts: GARCH (1,1) or IGARCH
3. Time-scaling
4. Model updating

Estimation of the Covariance Matrix

The Barra currency risk factor block is a combination of two models: a correlation model and a volatility model. We use weekly returns to estimate correlations between currencies and daily data to estimate currency volatilities. We generate the covariance matrix used to forecast risk with both models.

The final covariance matrix is formed with:

$$Cov(i, j) = Correlation(i, j)\sigma_i \sigma_j \quad \text{(EQ 8-2)}$$

where

Correlation (i,j)	=	correlation estimate of weekly currency data

σ_k = GARCH volatility estimate based on daily return

$$Cov = \begin{bmatrix} (ANG,ANG) & (ANG,ARS) & \cdots & (ANG,ZWD) \\ (ARS,ANG) & (ARS,ARS) & \cdots & (ARS,ZWD) \\ (ATS,ANG) & (ATS,ARS) & \cdots & (ATS,ZWD) \\ \vdots & \vdots & \ddots & \vdots \\ (ZWD,ANG) & (ZWD,ARS) & \cdots & (ZWD,ZWD) \end{bmatrix}$$

Figure 8-1

Currency Covariance Matrix

The diagonal elements in the covariance matrix are the currency variances, while the off-diagonal elements are the covariances between currencies.

Currency Correlation Model

The correlation matrix is estimated using weekly currency return data[1] relative to the U.S. dollar, exponentially weighted with a half-life of 17 weeks. This half-life is fitted with a maximum likelihood estimation[2].

$$Cor = \begin{bmatrix} (ANG,ANG) & (ANG,ARS) & \cdots & (ANG,ZWD) \\ (ARS,ANG) & (ARS,ARS) & \cdots & (ARS,ZWD) \\ (ATS,ANG) & (ATS,ARS) & \cdots & (ATS,ZWD) \\ \vdots & \vdots & \ddots & \vdots \\ (ZWD,ANG) & (ZWD,ARS) & \cdots & (ZWD,ZWD) \end{bmatrix}$$

Figure 8-2

Currency Correlation

The weekly currency return data is used to estimate correlations between currencies.

1. Barra uses weekly data instead of daily data to avoid spurious correlations. Besides being less volatile, weekly data allows Barra to compare different markets with various local holidays and different opening and closing times.
2. Maximum likelihood estimation (MLE) estimates the parameter values that make observed data most likely, given a choice of parametric distribution.

Currency Volatility Model

Currency volatility levels behave like many other economic time series in that they vary dramatically over time. Changes in volatility range from gradual to abrupt.

$$C = \begin{bmatrix} C(ANG,ANG) & C(ANG,ARS) & \cdots & C(ANG,ZWD) \\ C(ARS,ANG) & C(ARS,ARS) & \cdots & C(ARS,ZWD) \\ C(ATS,ANG) & C(ATS,ARS) & \cdots & C(ATS,ZWD) \\ \vdots & \vdots & \ddots & \vdots \\ C(ZWD,ANG) & C(ZWD,ARS) & \cdots & C(ZWD,ZWD) \end{bmatrix}$$

Figure 8-3

Currency Volatility

Currency volatility is stripped from the weekly covariance matrix, then implemented with GARCH to refine daily volatility forecasts. These volatility forecasts (black) are later overlaid on the covariance matrix (gray).

General auto-regressive conditional heteroskedastic (GARCH) models are standard tools used to forecast risk for variable volatility time series.[1]

Barra's currency risk factor block uses a GARCH (1,1) model[2] for currencies against the U.S. dollar. The currency model directly incorporates the previous variance forecast and squared return. The optimal GARCH parameters for each currency are fitted with maximum likelihood estimation.

An underlying assumption of the GARCH (1,1) model is that the currency returns have an unconditional variance. For currencies whose returns against the dollar do not exhibit an unconditional variance, we use a degenerate form for GARCH (1,1) known as integrated GARCH.

GARCH (1,1) Model

When properly calibrated, a GARCH (1,1) model responds quickly to new information. Daily data are used in order to facilitate convergence of GARCH parameters and to minimize standard errors. Daily exchange rate data are available for all currencies.

The functional form of the GARCH (1,1) model is:

1. For more information on GARCH models, see *Covariance Matrix Scaling: Computing Market Volatility* on page 29.
2. A GARCH (p,q) model has terms that depend upon the previous p forecasts and q squared returns.

$$\sigma_t^2 = \omega^2 + \beta\left(\sigma_{t-1}^2 - \omega^2\right) + \gamma\left(r_{t-1}^2 - \omega^2\right) \quad \text{(EQ 8-3)}$$

where

σ_t^2	=	conditional variance forecast for time *t*
ω^2	=	unconditional variance forecast
β	=	persistence constant
σ_{t-1}^2	=	variance forecast for the previous period
γ	=	sensitivity constant
r_{t-1}^2	=	previous period squared return

and

$\beta > 0$

$\gamma > 0$

$\beta + \gamma < 1$

The formula has three parameters. The unconditional variance of the series is denoted by ω^2. If no new information arrives, the variance forecast tends to revert to this value. The sensitivity constant γ measures responsiveness to the most recent (squared) return. The persistence constant β measures the importance of the previous forecast in the current forecast.

The parameters β and γ must be positive, or else the variance forecasts could become negative in response to a sequence of large events. Similarly, $\beta + \gamma > 1$ could lead to a negative variance forecast in the wake of a long sequence of relatively small events. The special case where $\beta + \gamma = 1$ leads to a degenerate form of the GARCH(1,1) model known as an integrated GARCH (IGARCH), which is an exponentially weighted model.

IGARCH Model (Exponentially Weighted)

The case where $\beta+\gamma=1$ is commonly referred to as an integrated GARCH (IGARCH). This model fits a series with variable volatility but no unconditional variance.[1] Replacing γ with $1-\beta$ yields a single parameter:

$$\hat{\sigma}_t^2 = \beta\,\hat{\sigma}_{t-1}^2 + \gamma\, r_{t-1}^2 \quad \text{(EQ 8-4)}$$

$$= \beta\,\hat{\sigma}_{t-1}^2 + \left(1-\beta\right) r_{t-1}^2 \quad \text{(EQ 8-5)}$$

Repeated substitutions for $\hat{\sigma}_{t-1}^2$ yields:

$$\hat{\sigma}_t^2 = \left(1-\beta\right)\sum_{i=1}^{\infty}\beta^{i-1} r_{t-1}^2 \quad \text{(EQ 8-6)}$$

In other words, the i^{th} term in the series is multiplied by the constant $(1-\beta)\beta^{i-1}$. The weighting scheme can also be specified by its half-life.

Volatility Across Markets

Monthly risk forecasts for various markets show the relationship between sensitivity (γ), persistence (β), and currency returns. The higher the value for γ, the more responsive a currency is to recent events. If β is higher, more weight is given to past events.

Figure 8-4 through 8-8 show Barra currency risk forecasts and monthly currency returns for the U.S. dollar against a variety of other currencies. The circular plot points show the ±1 standard deviation risk forecasts. The dark lines with square plot points show the monthly returns. The assumption of conditional nor-

1. Note that ω^2 drops out of Equation 8-3 when $\beta + \gamma = 1$.

mality suggests that we should see roughly two-thirds of the returns between the ±1 standard deviation risk forecasts.

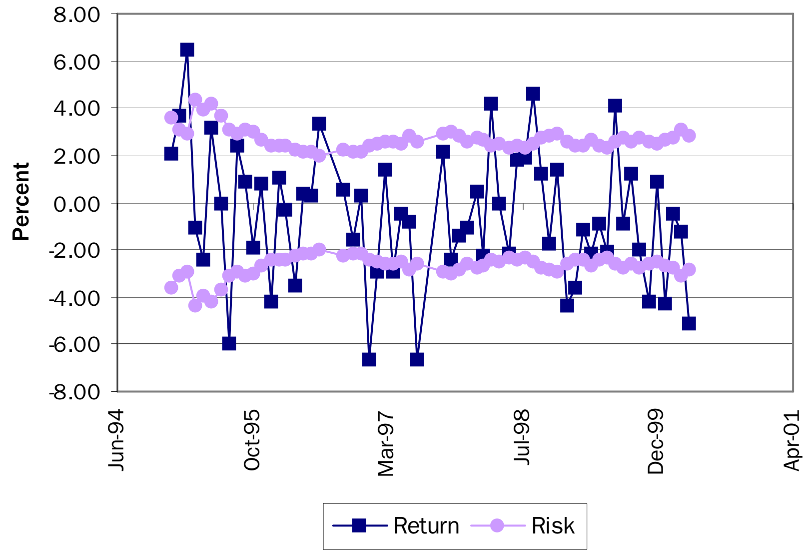


Figure 8-4

Euro Model

The chart depicts monthly risk and returns of the U.S. dollar against the euro.

$\omega^2 = 8.81\%;\ \beta = .97;\ \gamma = .024$

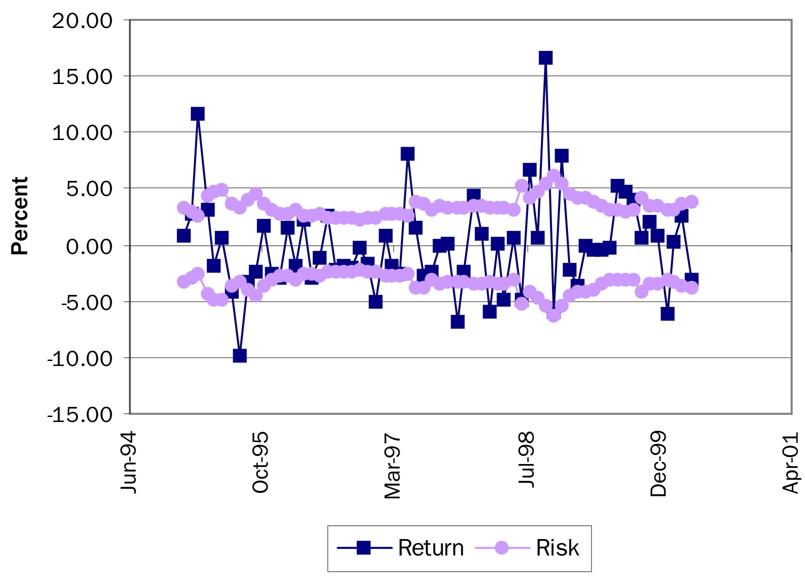


Figure 8-5

Japanese Yen Model

The chart depicts monthly risk and returns of the U.S. dollar against the yen.

$\omega^2 = 11.34\%;\ \beta = .96;\ \gamma = .03$

The yen model has a higher unconditional volatility (11.34% annualized) than the euro (8.81%) and is slightly more reactive.

Figure 8-6

South African Rand Model

The chart depicts monthly risk and returns of the U.S. dollar against the South African rand.

$\omega^2 = 9.97\%$; $\beta = .711$; $\gamma = .272$

The sum $\beta + \gamma$ *is quite close to 1. This means that this GARCH model is nearly degenerate. Since it closely resembles an IGARCH model, the long-term variance plays only a little role in the forecast. As* γ *is relatively large (.272), the forecasts are very reactive.*

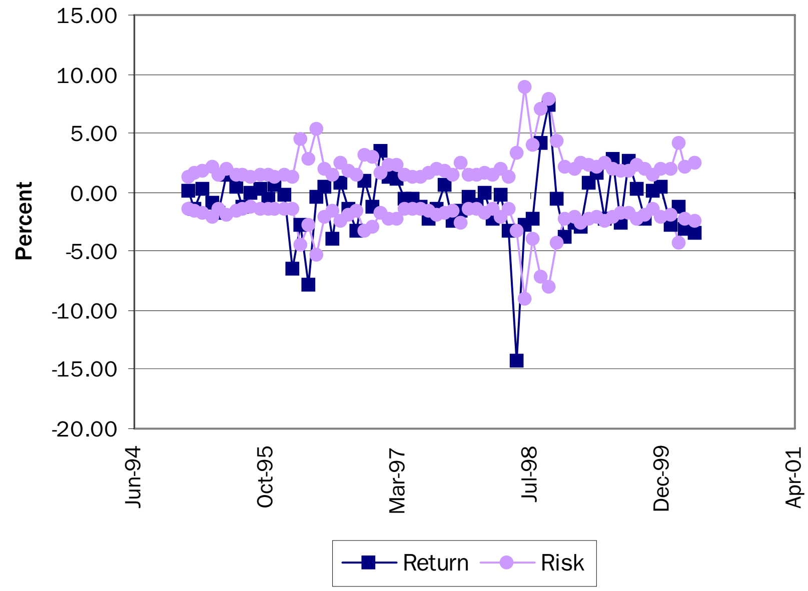


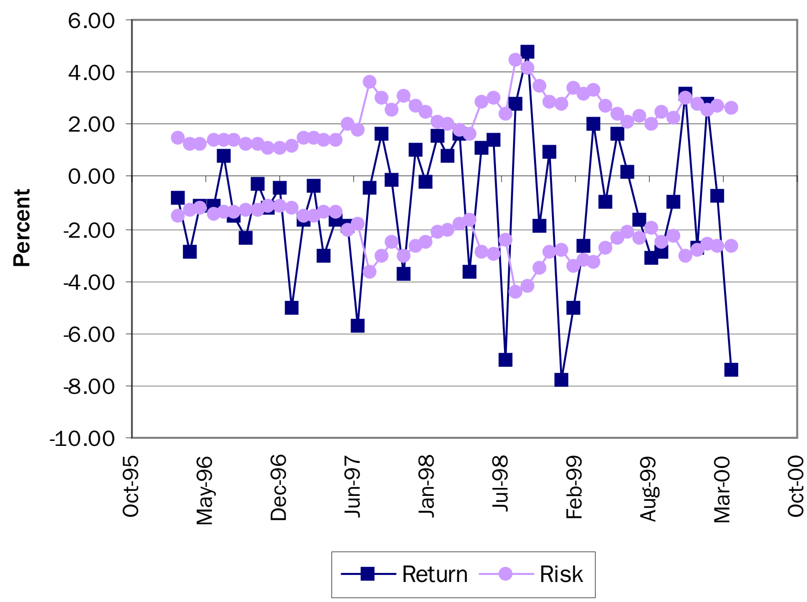
Figure 8-7

Polish Zloty Model

The chart depicts monthly risk and returns of the U.S. dollar against the Polish zloty.

$\beta = .972$; *half-life = 24 days*

The risk and return plot of the Polish zloty has an IGARCH model with a 24-day half-life.



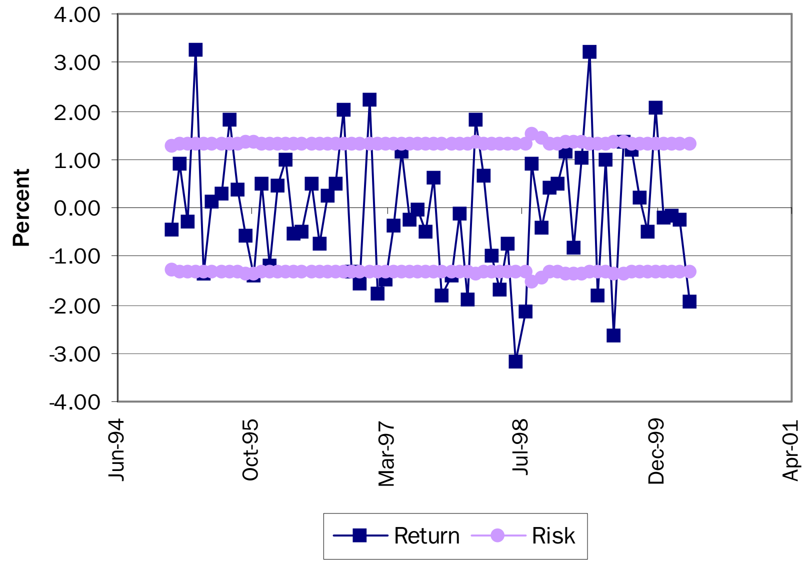


Figure 8-8

Canadian Dollar Model

The chart depicts monthly risk and returns of the U.S. dollar against the Canadian dollar.

ω^2 = *4.53%;* β = *.321;* γ = *.192*

The risk forecast for the Canadian dollar is nearly flat and equal to the unconditional volatility. Here $\beta + \gamma$ *sum to much less than 1, resulting in forecasts that are nearly constant at the level of unconditional volatility.*

Time-Scaling Currency Risk Forecasts

The daily currency forecasts are time-scaled to conform with other Barra risk forecasts, which are based on a monthly horizon. For GARCH (1,1), this is accomplished with an aggregation formula that takes the variance reversion into account. Conditional on today's information, GARCH (1,1) forecasts for the next n periods are calculated with:

$$\hat{\sigma}_{t,n}^2 = n\omega^2 + \frac{1-(\beta+\gamma)^n}{1-(\beta+\gamma)}(\hat{\sigma}_t^2 - \omega^2) \qquad \textbf{(EQ 8-7)}$$

The value of n is usually 20 or 21, which is the number of business days in a month. Note that if the single-period current forecast is close to the unconditional forecast ω^2, the n-period current forecast is close to $n\omega^2$. Similarly, if $\beta+\gamma<1$, the n-period forecast quickly leans towards $n\omega^2$.

Integrated GARCH models have no variance reversion. The n-period forecast is equal to the one-period forecast scaled by n.

$$\hat{\sigma}_{t,n}^2 = n\sigma_t^2 \qquad \textbf{(EQ 8-8)}$$

Updating the Model

The currency risk model is updated monthly to incorporate the latest currency exchange rate volatility estimates.

Integrated Risk

Section Five discusses the innovative methods used to couple broad asset coverage with detailed analysis of single markets.

Chapter 9: Integrated Risk Modeling

Section Five

9. Integrated Risk Modeling

Barra's chief goal is to provide a single model that best forecasts the risk of a wide range of portfolios, from those concentrated in a single market to those diversified over multiple assets across different markets. The model must offer both in-depth analysis and broad coverage. It should be detailed enough to allow portfolio managers to drill down to their assets in a local market and obtain an insightful and accurate analysis, and yet broad enough to let plan sponsors have a high-level view of risk that they face across multiple markets and asset classes.

Unfortunately, these objectives are conflicting. As new markets are added to a global model, the complexity needed to maintain a fine level of detail increases, posing a serious econometric challenge. Until now, this goal has been elusive.

Model Integration Overview

We employed a novel methodology to achieve this goal. First, to provide the needed level of detail, we use Barra factor models of all the local equity and fixed-income markets.[1] These models attribute the explainable portion of an asset's return to the local factors at work in each market. The factors, which may differ from market to market, include risk indices and industries for equities and interest-rate term structure movements and credit spreads for fixed-income securities. By modeling each market individually, we enable investors to see their exposures to the risk factors of each particular market and to have the most accurate forecasts of local market risk.

Next, we combined the local equity models and the local fixed-income models. Since asset returns are driven, in part, by local factors, the key to developing each model is determining the

1. Examples of local models are the Australia Equity Model and Japan Bond Model. While single-country equity or bond models are possible choices for local models, they are not the only possible local models. For example, the Europe Equity Model, a regional model that covers securities listed in 16 Western European markets, is used as the local model for the Western European markets (except the United Kingdom).

covariances between these factors across different markets. The excessive number of factors complicates the estimation of factor covariances. Fortunately, within each asset class, a much smaller set of global factors accounting for much of the cross-market correlation can be identified. By building structural models of how these global factors link local factors across markets, more accurate estimates can be obtained.[1]

The use of structural models provides a new framework for global analysis. These models decompose local factor returns into a part due to ***global factors***, or factors that are shared across markets, and a part due to purely ***local factors***, or factors that only affect the securities within each market. This explains, for example, why the industry risk of a U.S. bank is better hedged with another U.S. bank than with a Japanese bank.

Finally, the global equity, fixed-income, and currency risk models are combined to form the complete Barra Integrated Model (BIM). Leveraging our earlier work, we use the global factors to estimate the correlation structure across asset classes.

Model Estimation Process

1. Local model development
2. Equity: Global factor return estimation
   Fixed-Income: Local factor to global factor exposure estimation
3. Covariance matrix calculation
   a. Estimation of purely local covariances for individual markets
   b. Calculation of global covariance matrix within each asset class
   c. Calculation of cross-asset covariances via core factors
4. Scaling
   a. Scaling to individual asset class models
   b. Scaling to local markets
5. Model updating

Building Global Asset Class Models

Single-market equity models have factors that differ in number, character, and behavior across markets. For example, the U.S. equity market is well characterized by 13 risk index (or style) factors and 55 industries, and is significantly concentrated in technology, finance, and health care. In contrast, the Australian equity market can be captured with nine risk index factors and 14 industries, and is more exposed to basic materials.

Fixed-income models, on the other hand, are more uniform in structure. Each model incorporates three factors to account for interest rate (term structure) movements, and one or more credit spread factors.

1. The term "global" simply refers to the role of these factors in determining cross-market covariances. It does not imply that these factors are applicable to all assets. It just means that these are the factors responsible for the global correlation structure of the covariance matrix.

The Structure of Local Models

The local models, which are the building blocks of the global model,[1] decompose an asset's local excess return into a part due to local factors and a part that is unique to the underlying asset, the specific return.[2]

Let us take Australian equities as an example. The risk of the portfolio is computed with a forecast covariance matrix of Australian asset returns, V_{aus}. Using the factor covariance matrix, F_{aus}, and the matrix of asset-specific variances, Δ_{aus}, the portfolio risk can be expressed as:

$$V_{aus} = \underbrace{X_{aus} F_{aus} X'_{aus}}_{\text{common factor variance}} + \underbrace{\Delta_{aus}}_{\text{specific variance}} \tag{EQ 9-1}$$

Thus, the risk of a portfolio arises from its exposure to factors in the market as well as from the idiosyncratic behavior of individual securities it contains.

Aggregating Local Models

Barra Integrated Model is a multiple-factor risk model that gives the covariances between returns to equities and fixed-income assets in different markets. The factors of this new model are all the local market factors. Further, each asset is exposed only to its own market's factors. Using global equities as an example, the factor X_E, factor covariance, F_E, and specific matrices, Δ_E, of this model can be written as:

$$X_E = \begin{pmatrix} X_{aus} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & X_{usa} \end{pmatrix}, F_E = \begin{pmatrix} F_{aus} & \cdots & F_{aus,usa} \\ \vdots & \ddots & \vdots \\ F_{usa,aus} & \cdots & F_{usa} \end{pmatrix}, \Delta_E = \begin{pmatrix} \Delta_{aus} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Delta_{usa} \end{pmatrix} \tag{EQ 9-2}$$

1. For full details on local models, see the Equity Risk Modeling section in this book, the appropriate handbook (for example, the *United States Equity Risk Model Handbook*), or the single market model data sheets.
2. For equities, the excess return of an asset is $r - r_f$, where r is the asset's total return and r_f is the risk-free rate. For bonds, excess returns are computed as: $\frac{P_{t+1} - F_{t+1/t}}{P_t}$, where P_{t+1} is the price at time t+1, P_t is the price at time t, and $F_{t+1/t}$ is the forward price for time t+1 computed at time t. $F_{t+1/t}$ is computed using a valuation model.

The global equity asset covariance matrix takes the familiar form:

$$V_E = X_E F_E X'_E + \Delta_E \quad \text{(EQ 9-3)}$$

The global fixed income covariance matrix takes a similar form:

$$V_F = X_F F_F X'_F + \Delta_F \quad \text{(EQ 9-4)}$$

Modeling Covariances of Local Factors in Different Markets

To complete the global model, we fully specify the factor covariance matrices F_E and F_F. The diagonal blocks of these matrices contain covariances between the local factors within each market; the local models have already provided these. What remains to be specified is the covariance between local factors in *different* markets. This involves estimating a significant number of covariances since there are more than 1,000 local equity and almost 300 fixed-income factors. The underlying data is available at monthly intervals and, in many cases, goes back fewer than 15 years.

With such a small sample size compared to the number of factors, the covariance forecasts will show a large degree of spurious linear dependence among the factors. One consequence is that it becomes possible to create portfolios with artificially low risk forecasts. The structure of these portfolios would be peculiar. For example, the forecasts might show an overweight in U.K. AA financials apparently hedged by an underweight in U.S. munis and Canadian automobiles.

Rather than computing these covariances directly, we use a structural model for each asset class that establishes a sensible relationship between factors in different markets. In place of local factors, we specify a much smaller number of global factors that are responsible for the correlations between factor groups. The idea is that the behavior of local factors may be accounted for, in part, by a much smaller set of global factors. This method appreciably reduces the degree of spurious linear dependence among the factors in the covariance matrix.

For example, part of the return to the local U.S. and U.K. oil factors is due to an underlying global oil factor which captures global oil prices, cartel activity, and so forth. In similar fashion, spreads on corporate bonds of different credit qualities in the United States and United Kingdom are partly driven by a common spread factor for these countries. These global factors link local factors across markets, accounting for any correlation between them. Thus, estimating the covariance between such local market factors requires a much smaller set of global factor covariances, thereby improving the reliability of the estimates.

Implementing Global Factor Models

The global factor models for equities and fixed-income securities differ in the global factors used and the calculation of local factor exposures to global factors, but the structural models for equities and fixed-income securities have the same form:

$$f_{ac} = Y_{ac} g_{ac} + \phi_{ac} \qquad \text{(EQ 9-5)}$$

where

f_{ac}	=	vector of local asset class (that is, *E* for equities, *F* for fixed-income securities) factor returns across all markets
Y_{ac}	=	a matrix of exposures of the local factors to the global factors
g_{ac}	=	a vector of global factor returns for the asset class
ϕ_{ac}	=	a vector of the purely local factor return

The structural models not only overcome the econometric problem, but also provide a new framework for global analysis. They decompose each local factor return into a part due to global factors and a part that is purely local. These purely local returns are not correlated across markets but may be correlated within each market. This construction allows the factors in different markets to be correlated but not identical.

From the structural models, an estimate of the asset class factor covariance matrix, $\hat{F}_{ac}$, consisting of two parts, can be obtained:

$$\hat{F}_{ac} = \underbrace{Y_{ac} G_{ac} Y'_{ac}}_{\text{Due to global factors}} + \underbrace{\Phi_{ac}}_{\text{Due to purely local factors}} \qquad \text{(EQ 9-6)}$$

where

G_{ac}	=	covariance matrix of global factors
ϕ_{ac}	=	covariance matrix of purely local factors with ϕ = 0 if i and j are not in the same market

Consistency Between Local Models and Global Model

In building $\hat{F}_{ac}$, we simply sought good estimates of the covariance between factors in different markets. The factor covariance matrices from the local models, which have the best estimates, form the diagonal blocks of F_{ac}. Given that the diagonal blocks of $\hat{F}_{ac}$ generally differ from the target, the last step is to form the final covariance matrix of all the local factors, F_{ac}, by altering $\hat{F}_{ac}$ in Equation 9-6 so that the local blocks are consistent with the local models. So, for example, the final covariances of local factors for the Australian market are the same as those in the Australia Equity Model.

Global Equities

Barra's new approach to modeling global equities differs markedly from that of most global equity models, which use a single set of factors to characterize the risk of equities throughout the world.[1] Altogether, Barra uses over 1,000 local factors.

Table 9-1

Local Equity Factor Correlation in Nine Major Markets January 1990 to April 2002

The average pair-wise correlation between the same factors (for example, U.S. Size, U.K. Size) in nine major markets (Australia, Canada, France, Germany, Italy, Japan, Switzerland, the United Kingdom, and the United States) shows that both industries and risk indices across countries share some common behavior. Despite this correlation, however, these factors behave significantly differently from market to market.

Local Equity Factor	Average Correlation
Materials	0.42
Finance	0.43
Information Technology	0.46
Momentum	0.34
Size	0.18
Volatility	0.43

1. Those models assume that all equities are driven by exactly one parsimonious set of factors, implying that returns due to industries and risk indices (styles) move in lockstep across markets.

Global Equity Factors

The global factors that explain covariances of equity factors across local models are:

- A world factor
- Country factors
- Global industry factors
- Global risk index factors: Price Momentum, Size, Value, and Volatility

The world factor captures the global market return, while the industry and country factors reflect the return to global industry and country influences net of other factors.[1] These factors were selected based on their ability to capture common fluctuations across local equity factor returns.

Exposures of Local Equity Factors to Global Equity Factors

Each local industry factor has unit exposure to the world factor, g_{wld}, its own country factor, g_{cnty}, and the global industry to which it belongs, g_{ind}; it has no other global exposures.[2] Each local risk index factor corresponding to one of the four global styles has unit exposure to that style, g_{ri}, and no exposure to other factors. The other local risk index factors have no global exposure.

$$f_{ind} = g_{wld} + Y_{cnty} g_{cnty} + Y_{ind} g_{ind} + \phi_{ind} \quad \text{(EQ 9-7)}$$

$$f_{ri} = g_{ri} + \phi_{\mathrm{ri}} \quad \text{(EQ 9-8)}$$

Estimating Returns to Global Equity Factors

A history of returns to global equity factors is compiled by fitting the structural model in Equation 9-7 and 9-8 to monthly local

1. Both the industry factors and the country factors render the world factor redundant, creating what is known as an identification problem. To resolve this, we follow the standard practice of requiring a weighted combination of the countries and the industry factor returns to sum to zero.
2. A local industry factor may, in some cases, be exposed to more than one global industry factor with fractional weights in each.

factor returns. The global factor returns were estimated using cross-sectional weighted least squares regression subject to constraints. The estimation incudes the period from January 1984 to the present. Because many models did not exist until some time after 1984, proxies replaced these missing returns from single country index returns whenever possible.

Barra Research Methods

European Equities in BIM

Barra Integrated Model (BIM) uses single-country models as the local models for most countries. It, however, uses the Europe Equity Model (EUE2) for equities in Western Europe excluding the United Kingdom.

EUE2 has a structure different from other single-country equity models; in particular, it has a set of country factors. To fit EUE2 excluding U.K. into BIM, it is useful to restate the EUE2 factors. We do this by forming single-county models that are based on the EUE2 factors.

Consider France as an example. Its risk indices are defined to be the same as EUE2's, so that the return to the France Value factor is equal to the return to the EUE2 Value factor, and a French asset's exposure to the France Value factor equals its exposure to the EUE2 Value factor; thus, $f_{ri,France} = f_{ri,EUE2}$.

When we construct industries for the France model, we combine the return of the corresponding EUE2 Continental industry with the EUE2 France country factor return. As such, the return to the France Automobiles industry is the return to the EUE2 Continental Automobiles industry plus the return to the EUE2 France country factor. In other words: $f_{industry}$ $f_{industry,France} = f_{industry,EUE2} + f_{France\_country,EUE2}$. A French asset exposed to the Continental Automobiles industry is now exposed to this France Automobiles industry. More generally, all assets have unit exposure to their local industries.

It is important to note that these derived single-country models are completely consistent with EUE2. Both models produce exactly the same risk forecasts.

The 15 EUE2-based single-country models are used in the estimation of the global factor returns, which are the basis for BIM. The risk indices and industries from these models are exposed to the corresponding global factors in the same way in which other countries' factors are. The factor returns are used in the regression to estimate global factor returns.

Computing Covariances of Global Equity Factors

The covariance matrix of the global equity factors (the G_{ac} in Equation 9-6) is computed from historical estimates of the returns to these factors.

The covariance matrix employs an exponentially weighted scheme with a half-life of 48 months. In other words, this month's factor return is given twice the weight of one four years ago. The covariance matrix of the purely local part of equity factor returns, the $\mathbf{\Phi}$ in Equation 9-6, was computed in a similar manner—also using a half-life of 48 months.

Scaling to Local Markets

The final step is to apply the scaling procedure to make the covariance block for each local market match that of the corresponding local model. The result is a model for global equities that provides analyses consistent with the local equity models and at the same time forecasts cross-market risk.

Barra Research Methods

Missing Data

We often need to estimate variances and covariances directly from data series. Occasions for this include estimation of the global factor covariance matrix within a single asset class as well as the core covariance matrix.

The series used in these estimations are often incomplete for reasons that include differing inception dates of series, data errors, and local holidays. In these circumstances, the most naïve approach—estimating the sample covariance between each pair of factors using the maximum number of data points available in the two series—may result in a matrix with negative eigenvalues. A matrix with negative eigenvalues cannot be a covariance matrix. The opposite extreme—estimating the sample covariances using only data on dates for which there are no missing values—addresses this issue. It generates a matrix that is free of negative eigenvalues; however, it introduces a new problem: far too much data is discarded.

Our approach is to use the method of maximum likelihood estimation. Unlike the case where the data series are complete, there is no closed-form expression for the maximum. Instead, we use a numerical estimation procedure called the Expectation-Maximization (EM) algorithm[1] that converges to the maximum. **EM algorithm**—an iterative method for estimating the covariance matrix of incomplete data sets and inputting missing values—ensures the creation of a positive semi-definite factor covariance matrix even when some of the factors have incomplete histories.

1. For details on the Expectation-Maximization Algorithm, see Geoffrey J. McLachlan and Thiriyambakam Krishnan, *The EM Algorithm and Extensions* (New York: Wiley-Interscience, 1996) or Dempster, Laird, and Rubin, "Maximum Likelihood Estimation from Incomplete Data Using the EM Algorithm," Journal of the Royal Statistical Society, ser. B, 39 (1977), 1–38.

Global Bonds

The approach to building a global fixed-income model parallels that of the equities. The global fixed-income model starts with the construction of factor models for each of the local fixed-income markets. The factors at work in these local markets include shift, twist, and butterfly term structure movements (STB) and a swap spread. Four markets—the United States, the United Kingdom, Japan, and the euro zone—have more detailed credit factors that explain the spread over swap on the basis of sector or sector-by-rating classifications.[1] In addition, emerging-market bonds denominated in an external currency have country-dependent spreads—one spread for all bonds from each emerging market—which allows it to be included in the global model. Altogether, there are over 270 local factors.

Global Bond Factors

The factor model that links local equity models is built first by specifying local equity factors' exposures to global factors, then by estimating the returns to these global factors. In contrast, the factor model that links local bond models is built first by specifying the returns to global factors, then by estimating the exposures of each local bond factor to these global factors through time-series regressions.

The global factors that capture covariances in bond factors across different local markets are:

- The STB factors from each of the local markets
- The swap spread factor from each local market
- An average credit spread factor for each of the U.S., U.K., Japan, and the euro zone markets
- An average emerging-market credit spread factor
- An average U.S. municipal factor
- Shift and, where applicable, twist factors for real interest rates in the U.S., U.K., Canada, and euro zone markets

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1. For the purposes of modeling corporate bonds, we have a single euro-credit model that spans twelve markets. The structure of this model is similar to that of the other markets, so it makes no real difference for our exposition whether we consider the euro zone to be one market or twelve individual markets; we will think of it as one.

STB and Swap Spread Factors

The local term structure and swap factors are themselves global factors as well. By this we mean that no factors act as proxies for them in estimating their covariance with the other local factors. Some of these variables are significantly correlated across markets and the gain from proxying them with a reduced set of variables is negligible.

Table 9-2

Average Pair-wise Factor Correlations Across Markets

The average pair-wise correlations between each of the STB and swap factors across markets and the percentage of significant positive correlations over different time periods clearly show that shift and twist, and to some extent, swaps are correlated significantly across markets.

Period	Shift	Twist	Butterfly	Swap
Jan 1993 to Apr 2002				
Average Correlation	0.42	0.22	0.02	0.09
% Significantly Positive	80%	50%	3%	22%
Jan 1993 to Dec 1996				
Average Correlation	0.40	0.16	0.02	0.07
% Significantly Positive	55%	15%	4%	14%
Jan 1997 to Apr 2002				
Average Correlation	0.45	0.27	0.04	0.12
Significantly Positive	72%	51%	9%	26%

Credit Spread Factors

Credit spread factors are strongly correlated within each of the U.S., U.K., Japan, and euro zone markets. The magnitude of the correlation can be seen in the following tables:

Table 9-3

Average Correlation of U.K. Credit Spread Factors from May 1999 to April 2002

	AAA	AA	A	BBB
AAA	0.94	0.83	0.77	0.72
AA		0.88	0.89	0.85
A			0.95	0.92
BBB				0.96

Table 9-4

Average Correlation of Euro-Zone Credit Spread Factors from June 1999 to April 2002

	AAA	AA	A	BBB
AAA	0.73	0.59	0.39	0.15
AA		0.63	0.59	0.39
A			0.64	0.62
BBB				0.78

Each cell in the table is the average correlation of spread factors, either for a single rating category or between two rating categories, but across different sectors. This is why the diagonals, the average correlation within a rating category, do not equal one.

An average spread factor captures this commonality and helps account for the correlation of local credit spreads with factors in other markets. The return to this factor is defined as:

$$\textit{Average credit spread} = \sum_k w_k f_k \qquad \text{(EQ 9-9)}$$

where $k \in$ local credit factors and each factor's weight, w_k, is inversely proportional to its volatility. The weights mitigate the influence of the lower quality credit factors that tend to have substantially higher volatilities.

Emerging-Market Factors

Emerging-market bond spreads explain the risk of emerging-market debt issues denominated in external currencies. These spreads are strongly correlated. Over the period from January 1998 to April 2002, the average correlation between these factors was 0.38.[1] As with the credit spreads for markets in the United States, the United Kingdom, Japan, and the euro zone, an average emerging-market credit factor reflects the common behavior of these markets, defining it to be:

$$\sum_k w_k f_k \qquad \text{(EQ 9-10)}$$

where $k \in$ emerging-market credit factors, and the weight on each factor is inversely proportional to its volatility.

U.S. Municipal Bond Factors

The risk of U.S. municipal bonds is captured in the local U.S. bond model using muni key-rate factors. We calculate the average U.S. muni factor as the equally weighted mean of all key rate factors—that is, the muni global interest rate factor corresponds to a parallel shift of interest rates calculated as an equal-weighted average of the key rate changes. Municipal bonds may additionally be

1. Omitting the month surrounding the Russian default, August 1998, reduces this correlation to 0.23, which is still substantial.

exposed to a credit spread factor. However, there is no global factor for muni credit spreads.

Shift and Twist Factors for Real Markets

To describe real return risk, we use market prices in real terms to estimate changes in the real yield curve. Standard principal components analysis, which is the same methodology used in nominal markets, is then applied to determine risk factors and forecasts. Finally, the real returns are approximately related to nominal returns. So, for the corresponding risk forecasts, the real return risk can be treated in the same manner as a nominal return risk.

Exposures of Local Bond Factors to Global Bond Factors

The exposures of local bond factors to the global bond factors are defined as follows:

- The local shift, twist, butterfly, and swap factors each have unit exposure to the corresponding global factors.
- The local credit spread factors in the United States, United Kingdom, Japan, and euro markets are exposed to the average credit and global swap factors corresponding to their market. The exposures are the coefficients obtained from regressing the time series of local credit spread factors on the average credit and global swap factors.
- The local emerging-market credit spread factors are exposed to the average emerging-market credit spread factor. The exposures are the regression coefficients obtained from regressing the time series of the local factors returns on this global factor.
- The local U.S. muni key rate factors are exposed to the average U.S. muni factor. The exposures are calculated by running a regression of each local factor on this global factor.

Computing Covariances Of Global Bond Factors

As with equities, the fixed-income covariance matrix is estimated from the global and purely local factor returns and is exponentially weighted with a half-life of 24 months. We use the EM algorithm to cope with missing data.[1]

After scaling in the local factor covariance blocks, we obtain a global fixed-income model that is consistent with the local models, yet is appropriate for risk analysis of portfolios of global fixed-income securities.

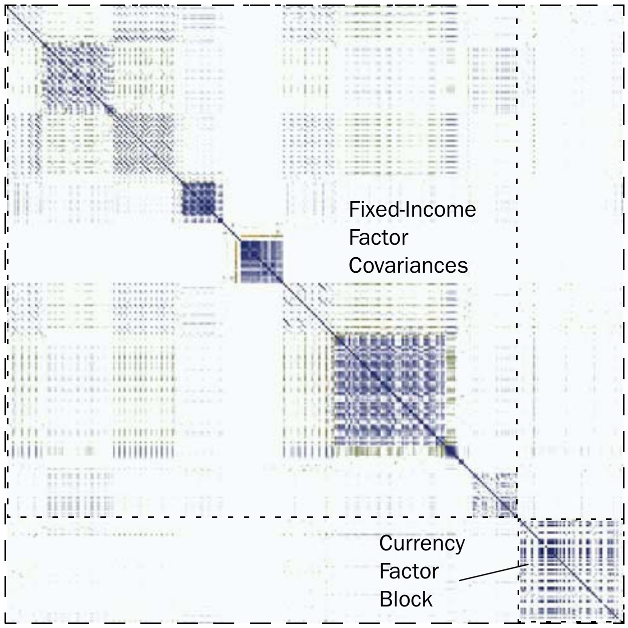

Figure 9-1

Common Factor Blocks in the Fixed-Income Covariance Matrix

Barra's risk model is flexible, allowing the number of currencies, local market factors, and emerging-market factors to increase or decrease.

The Currency Model

We decompose the excess return in the numeraire currency into a part due to currency fluctuations and a part due to the return of the asset in the local market.[2] Consider the excess return from a U.S. dollar perspective of an investment in Sony Corporation on the Tokyo Stock Exchange, $r_{sony/\$}$. We can write this as:

$$\begin{aligned} r_{sony/\$} - rf_{usa} &= \left(1 + ex_{¥/\$}\right)\left(1 + r_{sony}\right) - (1 + rf_{usa}) \\ &\approx \underbrace{r_{sony} - rf_{jpn}}_{\text{local excess return}} + \underbrace{ex_{¥/\$} + rf_{jpn} - rf_{usa}}_{\text{currency return}} \end{aligned} \quad \textbf{(EQ 9-11)}$$

1. See also, *Missing Data* on page 118.
2. For more information on the currency model, see Chapter 8, "Currency Risk Modeling".

where

r_{sony}	=	local return to Sony
rf_{usa}	=	U.S. risk-free rate
rf_{japan}	=	Japan risk-free rate
$ex_{¥/\$}$	=	exchange return to an investment in yen from a dollar perspective

The currency return is the excess return to an investment in a foreign instrument yielding the short-term rate.

Currency returns are local factors. Cash holdings have unit exposure to the appropriate currency factor. Since currencies have substantially fewer factors than equities and fixed-income assets, we do not model the covariance of currencies with a smaller set of variables. However, for ease of exposition later, we place the currencies in the same framework as equities and bonds. We treat currencies as both local (to the currency asset class) and global factors (like the bond term structure factors).

Thus, we can formally write:

$$f_C = Y_C g_C \tag{EQ 9-12}$$

and

$$F_C = Y_C G_C Y_C' \tag{EQ 9-13}$$

where

f_C	=	a vector of local currency returns
Y_C	=	the identity matrix
g_C	=	a vector of global currency returns
G_C	=	the currency covariance matrix

Putting It All Together—A Multi-Asset Class Risk Model

The factors in BIM include all the local equity, fixed-income, and currency factors. The exposure matrix, X_{BIM}, and the specific risk matrix, Δ_{BIM} are of the form:

$$X_{BIM} = \begin{pmatrix} X_E & X_{E,C} & 0 \\ 0 & X_C & 0 \\ 0 & X_{B,C} & X_B \end{pmatrix} and\, \Delta_{BIM} = \begin{pmatrix} \Delta_E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta_B \end{pmatrix} \quad \text{(EQ 9-14)}$$

where $X_{E,C}$, $X_{B,C}$, and X_C are the exposures of equities, fixed-income securities, and currencies to the currency factors. From the decomposition in Equation 9-11, it is clear that each equity and fixed-income security has a unit exposure to the currency of its own market and no exposure to any other currency.[1]

To complete our multi-asset class model, we specify the covariance between factors in different asset classes. Drawing on our earlier work, the natural answer is that these factors are related through the global factors in each asset class. These global factors embody the information that relates markets within an asset class and are therefore likely to capture important links across asset classes. This implies that the BIM factor covariance matrix is:

(EQ 9-15)

$$F_{BIM} = \begin{pmatrix} Y_E & 0 & 0 \\ 0 & Y_C & 0 \\ 0 & 0 & Y_B \end{pmatrix} \underbrace{\begin{pmatrix} G_E & G_{E,C} & G_{E,B} \\ G_{C,E} & G_C & G_{C,B} \\ G_{B,E} & G_{B,C} & G_B \end{pmatrix}}_{\mathrm{G_{BIM}}} \begin{pmatrix} Y'_E & 0 & 0 \\ 0 & Y'_C & 0 \\ 0 & 0 & Y'_B \end{pmatrix} + \begin{pmatrix} \Phi_E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Phi_B \end{pmatrix}$$

where the notation $G_{X,Y}$ denotes the covariance between the global factors of asset classes X and Y.

1. More precisely, a fixed-income or equity investment in a local market incurs an implicit exposure to the currency of that market.

Table 9-9

Average Correlations Between Global Equity Factors and Global Fixed-Income Factors in the Same Market, January 1993 to April 2002

For each market, we computed the correlation between country returns, on the one hand, and term structure factor returns and the average emerging-market credit factor returns, on the other.

Among the term structure factors, shifts are most significantly correlated with the equity factor. The average emerging-market credit factor is strongly correlated with the country factor.[1]

	Shift	Twist	Swap	Average Emerging-Market Credit Spread
Average Correlation	0.13	0.08	0.09	0.36
% Positive and Significant	40%	12%	13%	66%
% Negative and Significant	0%	4%	0%	5%

1. The country return is the country factor return plus the world factor return. The correlation between our measure and the returns to country indices for most countries is greater than 0.9.

The correlations of more than 140 global factors—some of which have short histories or have little data available—are too many to reliably estimate directly. So, rather than using all 140 factors, we designated a subset of the global factors—those most likely to account for correlations between asset classes—as ***core factors***.

The core factors are:

- World factor
- Country equity factors
- Shift factors
- Average credit spreads for the U.S., U.K., Japan, and euro-zone markets
- Average emerging-market credit spread

The correlations between global factors in different asset classes are assumed to be expressed through these core factors. The equation for any set of global factors A is:

$$g_A = \Sigma_{A,core}\Sigma_{core}^{-1}g_{core} + \tau_k \quad \text{(EQ 9-16)}$$

where

g_A	=	a vector of returns to the global factors in A
$\Sigma_{A,core}$	=	a matrix of covariances between the global factors and the core factors
Σ_{core}	=	a matrix of the variances and covariances of the core factors
g_{core}	=	a vector of returns of the core factors
τ	=	a vector of returns of the global factors residual to the core factor returns

The components of τ_k are assumed to be uncorrelated with the g's. We also assume that $\tau_{a,p}$ and $\tau_{a,q}$ are uncorrelated with each other when p and q index global factors are corresponding to different asset classes.

The covariances between any two sets of global factors A and B is calculated with:

$$G_{A,B} = \Sigma_{A,core}\Sigma_{core}^{-1}\Sigma_{core,B} + \Sigma_{\tau_A,\tau_B} \quad \text{(EQ 9-17)}$$

We use this formula to compute to a provisional estimate G_{BIM}.[1] To ensure consistency with the asset class models, the covariance matrices for global bond and equity factors are scaled with the same procedure used for individual asset classes.[2]

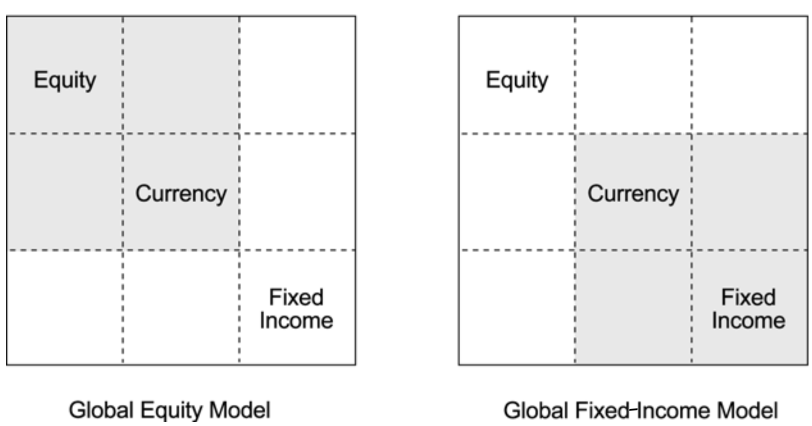

Figure 9-2

Barra Integrated Model Factor Covariance Matrix

The matrix contains the two single asset class models. The global equity model including both equities and currencies is shown in the upper left corner while the global bond model including both bonds and currencies is shown in the lower right corner.

1. The covariance matrices $\Sigma_{A,core}$ and Σ_{core} are estimated from the global factors using an exponentially weighted scheme with a half-life of 36 months.
2. Currencies are included with bonds for the scaling.

Summary

The Barra Integrated Model is a multi-asset class risk model that couples breadth of coverage (global equities, global fixed-income securities, and currencies) with the depth of analysis provided by Barra's local models. Users do not have to choose between granularity of local model analysis, on the one hand, and the broad scope of global model analysis, on the other. The model is suitable for a wide range of investment needs, from analysis of a single-country equity portfolio to a plan-wide international portfolio of equities, fixed-income securities, and currencies.

In-depth, accurate local analysis requires choosing factors that are effective in the market under study and recognizing that the factors developed for one market are not always appropriate for use in other markets. Thus, we started by building individual risk models for each market to best capture the behavior of the local securities.

For broad global analysis, we determined how securities in different markets co-vary. We accomplished this by modeling the relationships between the factors across local markets. The number of correlations between factors rises sharply with the number of factors.[1] There is simply not enough data to estimate so many correlations directly. Fortunately, these relationships may be modeled using a smaller set of global factors. Correlations across markets are expressed through these global factors, requiring less data for accurate estimation.

The model is also flexible in its structure, allowing the incorporation of advances in modeling for different countries or regions. For example, if a group of countries is better modeled as a bloc, then a local model for the bloc may replace local models for the individual countries. Its design also enables us to add additional asset classes (for example, real estate, commodities, or hedge funds) without changing the model's architecture. Its adaptability even allows the number of currencies and local market factors to increase or decrease. Furthermore, assets belonging to different asset classes can be linked. Fixed-income assets, such as convertible bonds or bonds with high option-adjusted spreads, can be exposed to equity factors.

1. The number of correlations is $N(N-1)/2$, where N is the number of factors.

Reference

Chapter A: Glossary

Chapter B: Contributors

Section Six

Glossary

A

ABS See *asset-backed securities*.

accrued interest The dollar amount of interest that accumulates on an asset between the most recent interest payment (or the issue date) and the settlement date. To calculate accrued interest, multiply the coupon rate by the number of days that have elapsed since the last payment, but do not include the settlement date.

active The portion of the portfolio that differs from its *benchmark*, and is therefore attributable to managed assets or portfolio characteristics. For example, if a portfolio's return is 5%, and the benchmark's return is 3%, then the portfolio's active return is 2%. A portfolio's *active risk* is the risk associated with the volatility of *active returns*.

Active weight is the portfolio's weight in an asset minus the benchmark's weight in the same asset. Active exposure is the portfolio's exposure to a factor minus the benchmark's exposure to that same factor.

active return Return relative to a benchmark. If a portfolio's return is 5%, and the benchmark's return is 3%, then the portfolio's active return is 2%.

active risk Also called *tracking error*. It is the risk (annualized standard deviation) of the active return. It is the difference in risk between a managed portfolio and a specified benchmark and is measured as the expected standard deviation of the differential return between the portfolio and the benchmark. Active risk arises from active management; for passively managed portfolios, it is often referred to as *tracking error*.

agency security A security issued by an agency of the federal government whose debt issues are not backed by the full faith and credit of the United States government, but is nonetheless held in high regard because of the presumption of government backing. Major agencies are farm and home lending corporations. The debt of international banks is sometimes placed in this category.

algorithm A procedure for solving a mathematical problem (for instance, finding the minimum combination of risk and return given a utility function) in a finite number of steps that frequently involves repetition of an operation.

alpha (α) The component of an asset's total historical return that is not attributable to market effects or that is solely unique to the particular asset.

The term alpha is borrowed from statistics and originates from the regression of a stock's or portfolio's *excess returns* on the market's excess returns (which also derives a historical beta estimate). Thus, originally alpha was defined as historical residual return. When the historical asset's return is plotted against the historical market return, alpha is the Y-intercept and beta is the slope. Historical alpha is estimated as the constant term in a time series regression of an asset return upon market return. For expository purposes, alpha is usually expressed as percentage annual return; for example, an alpha of 1.25 indicates that a stock is projected to rise 25% in price in a year when the return on the market and the stock's *beta* are both zero. For mathematical purposes, alpha is expressed as an adjustment to proportional return (or logarithmic return), again expressed as an annual rate (for example, 0.01).

When applied to stocks, alpha is essentially synonymous with misvaluation: a stock with a positive alpha is viewed as undervalued relative to other stocks with the same *systematic risk*, and a stock with a negative alpha is viewed as overvalued relative to other stocks with the same systematic risk.

APT See *Arbitrage Pricing Theory.*

arbitrage To profit from the differences in price of a security, currency, or commodity sold in different markets.

Arbitrage Pricing Theory (APT) Theory developed in the late 1970s that asserts that securities and portfolio returns are based on the expected returns attributable to an unknown number of underlying *factors*. APT provides a complementary alternative to its precursor, the *Capital Asset Pricing Model (CAPM)*.

asset-backed security (ABS) A security whose cash flow is backed by assets other than real estate. Such collaterals can be leases, auto loans, and credit lines.

average The arithmetic average is the sum of a group of *n* items divided by *n*.

The geometric average is defined as the *n*th root of the product of *n* values. The geometric average will always be less than or equal to the arithmetic average. All data values must be positive to determine the geometric average.

B

Barra Integrated Model (BIM) A multi-asset class model for forecasting asset and portfolio level risk of global equities, bonds, and currencies.

base currency See *numeraire*.

basis The difference between a bond's price and its delivery price (conversion factor times futures price). Basis has two components: net basis and carry.

basis point (bps) One-hundredth of one percent (.01%); thus, 100 basis points equals 1%. A basis point is the smallest measure used in quoting yields on bills, notes, and bonds. A bond's yield that increased from 8.00% to 8.50% would be said to have risen 50 basis points.

benchmark A reference portfolio or standard of comparison for investment performance and risk control. Benchmarks can be generally accepted market-weighted indexes, customized indexes, liability streams tailored to the cash outflow of a pension fund, or pure bullet payments associated with a guaranteed return over a specified holding period.

The list of assets in a benchmark portfolio represents the investment manager's performance target. The goal of the active manager is to exceed the benchmark return. The benchmark will typically contain assets that fall within a manager's investment style. For example, a largecap growth manager of United States equities may choose the S&P 500 Growth Index as a benchmark portfolio since it represents the type of assets that he or she would hold in the managed portfolio.

benchmark return The total return of the benchmark portfolio.

benchmark risk The total risk of the benchmark portfolio.

benchmark weight Weight of the asset in the benchmark. This allows comparison of the weight of a position in the portfolio against the weight of the same security in the benchmark.

beta (β) A measure of the sensitivity of an asset to movements in the market; thus, a measure of the asset's non-diversifiable or *systematic risk*. A beta of one (1) indicates that, on average, the asset is expected to move in tandem with the market. Beta can be applied to both equity and fixed-income securities.

Any security with a beta higher than one is more volatile than the market; any security with a beta lower than one can be expected to rise and fall more slowly than the market. An investor whose main concern is the preservation of capital should focus on assets with low betas, whereas an investor whose chief goal is to earn high returns should look for assets with high beta. However, a beta of zero does not mean that the asset has no risk, just that its volatility has no linkage with overall market movements. For example, gold stocks have low betas, yet often have high risk. This is because they are driven more by the price of gold than by the direction of the overall stock market.

Beta can be interpreted as an estimate of the average change in rate of return that corresponds to a 1% change in the market. Betas are usually plotted on a scatter diagram which shows the movement of the market as a whole and the return of a particular asset on a daily, weekly, monthly, or quarterly basis.

See also *historical beta* and *predicted beta*.

BIM See *Barra Integrated Model*.

book value A company's total assets minus intangible assets and liabilities, such as debt. A company's book value might be higher or lower than its market value. When referred to as book value per share, it is the ratio of stockholder equity to the average number of common shares.

bootstrapping The process of creating a theoretical spot rate curve using one yield projection as the basis for the yield of the next maturity.

bps See *basis points*.

bulldog bond A sterling-denominated bond issued by a non-British firm or institution. An example of a bulldog bond is one denominated in sterling and issued in England by a U.S.-based company.

butterfly The change in the *term structure* where the short and long ends of the curve move in the same direction, but the intermediate part of the curve moves in the opposite direction. The butterfly movement defines curvature in the term structure.

Historical regressions have been run to define the typical shape and magnitude of a one-standard deviation butterfly movement over a one-year horizon. The butterfly movement is nearly orthogonal to both shift and twist movements within a market.

butterfly risk The part of risk due to exposure to butterfly movements in the *term structure*.

buyback The purchase of bonds by the issuing company in the open market.

C

call option A contract that gives the holder the right to buy a security from the person who writes the option at a pre-specified price.

call price
The agreed price at which a security is traded when a call option is exercised. Also known as the strike price.

callable bond
A bond whose debt the issuer retains the right to retire before the scheduled maturity. This permits the issuer to replace an old bond with a lower-interest cost issue if interest rates fall. Typically, a callable bond includes a schedule of call dates and strike prices (call schedule), allowing the issuer to refinance the issue prior to maturity at specified dates for specified strike prices.

cap
1. The highest level interest rate that can be paid on a floating rate note, expressed as a percentage.
2. Abbreviation of capitalization.

capitalization
See *market capitalization*.

capitalization-weighted
A portfolio, typically an index, in which the weight invested in each asset is proportional to the asset's market capitalization.

Capital Asset Pricing Model (CAPM)
The simplest version states that the expected *excess return* on securities will be exactly in proportion to their *systematic risk* coefficient, or *beta*. CAPM implies that total return on any security is equal to the risk-free return, plus the security's beta, multiplied by the expected market excess return.

CAPM
See *Capital Asset Pricing Model*.

cash flow
1. For bonds, it is the total of the interest payments and principal payments received by a bond owner.
2. For stocks, it is the total dividends received by the stock holder.

category A class of bonds grouped by issuer type or rating for purposes of analysis. All bonds within a category (or all bonds within a cell defined by the intersection of two categories) are customarily considered to be identical insofar as the model is concerned.

clean price The price of a bond without accrued interest. Also called the flat price. The clean price plus accrued interest equals the *dirty price*. Both prices are normally expressed as a percentage of par value.

CMO See *Collateralized Mortgage Obligation*.

coefficient of determination (R^2) See *R-squared*.

collateralized cash flow Payment backed by an asset such as real estate or credit line. In the case of emerging-market debt, the payment is backed by the creditworthiness of the government.

Collateralized Mortgage Obligation (CMO) An instrument entitling the holder to the underlying cash flows from a series of bonds issued with the collateral of long-term mortgage securities. The nature of the cash flows to be received by the holder of a CMO series is uncertain. The magnitude and timing of the cash flows, which are fully defined by the terms of the CMO indenture, are highly dependent upon the future prepayments on the underlying mortgage securities.

common factor A characteristic shared by a group of securities that influences the returns of those securities. Securities with similar characteristics exhibit similar return behavior, which may be distinct from the rest of the market. In Barra multiple-factor risk models, the common factors determine correlations between asset returns. Examples of common factors are industry, style, term structure movements, and spread changes. See also *risk model*.

common factor risk The part of total risk due to exposure to *common factors*.

convexity The degree of curvature of the price-to-yield relation. It describes the rate at which duration responds to a change in interest rates. Positive convexity means that when interest rates decrease, the price of a bond will increase at a faster rate than that predicted by duration alone. For non-optionable securities, convexity can be computed as the second-order Taylor effect of interest rates on bond prices.

core factors A subset of global factors in the Barra Integrated Model that mostly likely accounts for correlations between asset classes. The correlations between global factors in different asset classes are assumed to be expressed through these core factors.

correlation A statistical term giving the strength of a linear relationship between two random variables. It is a pure number, ranging from –1 to +1. A correlation of +1 indicates perfect positive linear relationship; –1, perfect negative linear relationship; 0, no linear relationship. For jointly distributed random variables, correlation is often used as a measure of strength of relationship, but it fails when a nonlinear relationship is present. Low or negative correlation between securities or between common factors leads to portfolio risk diversification.

coupon A periodic interest payment a bond issuer makes to a bond holder. It is normally expressed as a percentage of *par*. The coupon rate is always expressed in annual terms. A semi-annual bond with a coupon rate of 8% pays 4% every six months.

coupon currency The currency denomination of coupon payments.

coupon frequency Number of coupon installments paid annually.

coupon type One of three methods for determining coupon payments:

- Fixed: coupon payments are fixed in frequency and amount.
- Floating: coupon payments match a current benchmark index rate.
- Scheduled: coupon payments that may change in amount (for example, to "step up" from 6% to 7%) or coupon type (for example, fixed rate to floating rate), according to a predefined schedule.

covariance The tendency of different random investment returns to have similar outcomes, or to "co-vary." The greater the covariance, the greater the strength of the common movement of assets. When two uncertain outcomes are positively related, the covariance is positive, and the converse is true. The magnitude of covariance measures the strength of the common movement. Covariance can be scaled to obtain the pure number, or *correlation*, that measures the closeness of the relationship without its magnitude.

covariance matrix The square matrix containing the *variances* (along the diagonal) and *covariances* (off-diagonal) of all the *common factors* in a *risk model*. It is a key component in the forecasting of risk measures.

credit spread The difference in yield between a corporate bond and a comparable government bond. This spread describes bonds in the U.S. dollar, sterling, euro, and yen markets.

cumulative return The investment return cumulated over a number of periods, ordinarily expressed as a proportional return.

currency The currency in which an asset is denominated.

currency risk The predicted risk of an asset (in one-year standard deviation) due to its being invested in a foreign currency. This risk takes into account the exchange rates and short-term interest rates of the foreign country and the *numeraire* country.

current yield The ratio of the annual coupon rate to the clean price of the bond. For example, an 8% coupon bond trading at 91% of par has a current yield of 8/91, or 8.79%.

D

default The failure of a debtor to make interest or principal payments in a timely manner. Default occurs whenever the payment is not made according to the terms of the original issue. When an issue is in default, the holders can make claims against the issuer to salvage as much of their principal as possible. The exercise of these rights often lead the company into reorganization or bankruptcy.

default risk premium The higher yield expected on an issue that is subject to the risk of default. This anticipated return compensates the investor for assuming the default risk of the bond.

descriptor A fundamental or market-related data item that is used as a fundamental building block of risk index or style factors in a Barra equity risk model. Most style factors or risk indices are comprised of several descriptors combined using proprietary formulas. For example, a volatility risk index, which distinguishes high volatility assets from low volatility assets, might consist of several descriptors based on short-term volatility, long-term volatility, systematic volatility, and residual volatility, and so on.

dirty price The bond price plus accrued interest.

discount Amount by which a bond is trading below *par* value.

discount bond See *zero coupon bond*.

discount factor A factor that, when applied to a future payment or cash stream, converts it to its *present value*.

discount function

A complete schedule of discount factors for all future dates. Because each discount factor gives the present value of a payment at a given future date, a complete schedule of these factors gives the *present values* of all future payments. Future interest rates can be expressed in the discount function as continuously or semiannually compounded spot rates, forward rates, or yield-to-par rates. The discount function is therefore one way of representing the term structure.

discounting

A financial concept whereby the values of single or multiple future *cash flows* are computed as of a given date in the past or present. This is diametrically opposite to the concept of compounding, which is used to compute the future value of a present cash flow.

distribution

The function which describes the frequency with which a random variable takes on any given value. The distribution of future values for a random variable is usually described in terms of its mean, standard deviation, skewness, and functions.

diversification

The spreading of risk by investing in a number of different assets whose returns are not perfectly positively correlated. Since the returns are not perfectly correlated, losses of any one asset tend to be offset by gains on other assets. In this manner, the risk of a portfolio may well be less than the average risk of its constituent assets. Diversified portfolios typically contain assets in several categories of investments—stocks, bonds, money market instruments, and precious metals, for instance—or several industries in a stock portfolio.

Diversification is the spreading of risk; hedging is the offsetting of risk.

diversifiable return See *residual return*.

dividend A periodic payment an equity issuer makes to a stockholder.

dividend yield The return of a security or portfolio in the form of dividend cash payments. It is calculated as the annual dividend payment of a security divided by the security's current price. Yield is usually described, for expository purposes, in percentage terms (for example, 7 percent per annum), but for mathematical purposes it is expressed as a decimal fraction (for example, 0.07). Also known as the *yield*.

dollarization A situation in which a country uses foreign currency alongside its own currency, or abandons its own currency entirely and adopts another country's currency as a means of payment and unit of account.

dummy variable A statistical term for a variable that represents a single fixed characteristic; also called an indicator variable for that characteristic. A dummy variable is one for all cases where the characteristic occurs, and zero otherwise. Consequently, the coefficient of the dummy variable in a model tells us the difference between the model value for that characteristic and the model value in the absence of that characteristic.

duration

A summary measure of the price responsiveness of an interest-sensitive asset to changes in interest rates. Duration is a reasonably good predictor as long as the change in interest rates is small and of a parallel nature.

The maturity duration in the Barra model is calculated as the modified *Macaulay duration*: Macaulay duration divided by (1 + yield/2). Barra computes option-adjusted Macaulay and *modified duration* (also called effective duration) by simulating future interest rates and modeling the change in option value for small changes in interest rates.

E

earnings yield

The earnings per share divided by the price per share.

effective duration

See *duration*.

emerging-market spread

The spread associated with bonds issued in an external currency by an emerging market sovereign or by a company domiciled in an emerging market country. In the Barra risk model, emerging- market spread is measured relative to the swap spread.

emerging-market risk

The part of risk due to exposure to *emerging-market spreads*.

equal-weighted

A portfolio in which approximately equal value is invested in all assets.

estimation When a model is fitted to data, the estimated value of the model is the one that best fits the data, or that "maximizes its likelihood." An estimation method, in view of the random nature of the data, finds the parameters of the model that fit the data best. The estimated model is not "true," but is thought of as a closest approach to the underlying or "true" model. The discrepancy between the estimated model values and these underlying but unknown values is called estimation error.

estimation error See *estimation*.

estimation universe Set of assets used in the estimation of factor returns.

Eurobond A bond denominated in a particular currency and issued simultaneously in the capital markets of several nations, including nations with different currencies. The Eurobond market is an important source of capital for multinational companies and foreign governments, including developing countries' governments.

excess return Return in excess of the risk-free rate. The excess return is computed by subtracting the promised risk-free rate from a security's return. If an asset's return is 3% and the risk-free return is 0.5%, then the asset's excess return is 2.5%.

exchange An institution where securities or futures trading takes place. It regulates the processes by which the market operates, particularly market access, formation, settlement of bargains, and dissemination of market intelligence. Exchanges are often organized as associations of major market participants. The New York Stock Exchange and American Stock Exchange are the largest centralized places to trade stocks in the United States.

expected return The average return expected from an asset or portfolio. Expected return is defined over a particular investment horizon. The expected value depends upon one's view of the future, so when used as a tool, the expected value will be that which relates to the user's own expectation of future scenarios. Expected return is the mean of the probability distribution of investment return.

exponential The case in which a number is multiplied to a power, with the power being, in mathematical terminology, the exponent. When any given interest rate is compounded continuously, the *present value* of a payment declines progressively with time along an exponential curve.

exposure A term used to quantify the magnitude of an asset's (or portfolio's) sensitivity to factors.

1. Equity: for style factors or *risk indices*, exposure is expressed in standard deviation; for industries, it is expressed in percent of portfolio value.
2. Fixed-Income: for term structure, spread, and emerging market factors, exposures represent the sensitivity of the bond or portfolio to shocks in those term structure or spread curves, and is therefore related to *duration*.

F

face amount See *face value*.

face value The value of a security as it appears on the certificate of the instrument. This is the amount of principal due the bondholder at maturity and also the amount on which interest payments are calculated. See *par value*.

factor A factor of value or factor of return represents an underlying construct that influences many securities. When the existence of a factor is established, it becomes a convenient way of isolating common elements in securities and of tracking events in financial markets. One important application is to attribute elements of value and elements of investment returns to underlying factors. Examples of fundamental equity factors are: size, value, growth, and earnings variation. Examples of fundamental fixed-income factors are: shift, twist, and butterfly.

factor exposure See *exposure*.

factor return The return attributable to a particular common factor. Barra decomposes asset returns into a common factor component—based on the asset's exposures to common factors times the factor returns—and a *specific return*.

Fannie Mae (FNMA) An acronym for the Federal National Mortgage Association. It refers to the mortgages insured by the Federal Housing Administration but managed by FNMA. FNMA represents the effort of the government to stimulate the development of a secondary market for mortgages in order to enhance market liquidity.

FHLMC An acronym for Federal Home Loan Mortgage Corporation. See *Freddie Mac*.

FNMA An acronym for Federal National Mortgage Association. See *Fannie Mae.*

fit of estimation The degree of closeness between the data and the values predicted by the model. Perfect fit is impossible because of the randomness of data, which gives rise to noise.

fitted prices Prices estimated by a model which best fit the data with minimal expected discrepancies between the estimated values and the values of the underlying random variables.

floating rate notes (FRN) Also called a "floater," an instrument with the coupon rate pegged to a predetermined rate and reset contractually by formula at a stipulated interval. Floor and/or ceiling coupon rates may be specified.

FNMA See *Fannie Mae*.

foreign bond A bond issued by foreign borrowers in a nation's domestic capital market and denominated in the nation's domestic currency. This will also include foreign currency-denominated issues by foreigners in the domestic bond market.

forward interest rate An interest rate which is determined at the present time for a loan that will occur at a specified future date. The compound interest to any future date can be obtained by successively compounding the forward rate for all intervals between now and that future date. Consequently, the schedule of forward rates determines the present value of all future payments, and so it is one way of representing the term structure. Usually abbreviated to forward rate.

forward rate See *forward interest rate*.

Freddie Mac (FHLMC) Federal Home Loan Mortgage Corporation sells two types of pass-through securities: mortgage participation certificates and guaranteed mortgage certificates. The investor receives prorated principal and interest payments based on the underlying pool and its experienced repayments.

fundamental beta See *predicted beta.*

G

GARCH See *general auto-regressive conditional heteroskedastic model.*

general auto-regressive conditional heteroskedastic model (GARCH) Model used in predicting a time series variance when volatility is serially dependent (heteroskedasticity). The GARCH model links the predicted (conditional) variance with past realizations of the error process and the variance itself.

In conventional econometric models, the variance of the disturbance term is assumed to be constant (homoskedasticity). However when periods of unusually large volatility are followed by periods of relative tranquillity, this assumption is inappropriate.

general obligation (GO) bond Municipal securities secured by the issuer's pledge of its full faith, credit, and taxing power. A general obligation bond, or GO bond, as it is more commonly called, is repaid with the municipal agency's general revenues and borrowings. General obligation bonds are different than municipal bonds, where payments are based on the revenue from a specific facility built with the borrowed funds.

Ginnie Mae (GNMA) An acronym for the Government National Mortgage Association. It creates pools of mortgages and sells participations in these pools to private investors.

global bond A bond issued in two or more countries' markets by organizations such as the World Bank.

global factor Factors responsible for the global correlation structure of the *Barra Integrated Model* covariance matrix. It does not imply applicability to all assets.

GNMA See *Ginnie Mae*.

GO bond See *general obligation bond*.

government bond See *sovereign bond*.

growth stock Stock of a corporation that has exhibited faster-than-average gains in earnings over the last few years and is expected to continue to show high levels of profit growth. Over the long run, growth stocks tend to outperform more slowly growing or stagnant stocks. Growth stocks are riskier investments than average stocks, however, since they usually support higher price/earnings ratios and make little or no dividend payments to shareholders.

H

hedging The process whereby the risks of several opportunities are largely or completely ("perfect hedge") offset. Hedging requires either that the two opportunities be negatively correlated (gold stocks and brokerage firm stocks or a put option and its underlying security), in which case positive amounts are invested in both opportunities, or that the two opportunities are positively correlated (a call option and its underlying security or two very similar securities), in which case one opportunity is short-sold. Hedging is the offsetting of risk; *diversification* is the spreading of risk.

A stockholder worried about declining stock prices, for instance, can hedge his or her holdings by buying a put option on the stock or selling a call option. Someone owning 100 shares of XYZ stock, selling at $70 per share, can hedge his position by buying a put option giving him the right to sell 100 shares at $70 any time over the next few months. This investor must pay a certain amount of money, called a premium, for these rights. If XYZ stock falls during that time, the investor can exercise his option—that is, sell the stock at $70—thereby preserving the $70 value of the XYZ holdings. The same XYZ stockholder can also hedge his position by selling a call option. In such a transaction, he sells the right to buy XYZ stock at $70 per share for the next few months. In return, he receives a premium. If XYZ stock falls in price, the premium income will offset to some extent the drop in value of the stock.

historical beta Historical measure of the response of a company's *excess return* to the market's excess return. It is computed as the slope coefficient in a historical regression (usually 60 months) of the asset's return against the market.

Note that betas for any individual company do change, so one cannot rely on historical betas as a guide for future betas. Many studies have demonstrated that predicted betas significantly outperform historical betas as predictors of future stock behavior.

See also *predicted beta*.

holdings The number of units held in a particular position. The face amounts or *par values* of the issues held in a portfolio.

horizon The investment period of an investor. Investment horizon affects the expected changes in interest rates and returns, thereby influencing the risk/return profile of a portfolio. As a rule of thumb, the longer the horizon, the more volatile the return will be. See also *investment horizon*.

I

idiosyncratic risk See *specific risk*.

IGARCH See *integrated general auto-regressive conditional heteroskedastic model*.

independent risk See *specific risk*.

index A target group of assets against which other portfolios can be tracked and compared. It is also a measure of the value and return to a group of assets. The value and return of the index should be identical to the value and return of an investment portfolio whose weights coincide exactly with the weights of the index. Thus, the value of an index and its return are found by the same accounting rules by which we compute the value of the portfolio and its return.

index rate The rate that serves as a basis for the floating rate note. The options include the following:

- LIBOR: London Interbank Offered Rate
- LIBID: London Interbank Bid Rate
- LIMEAN: Calculated from the mean average of LIBOR and LIBID
- Prime: Rate at which banks lend to their most favored customers
- Fed Funds: Interest rate on Federal Reserve Bank funds. This is a closely watched short-term interest rate; it signals the Fed's view on the state of the money supply.
- COFI: Cost of Funds Index
- T-bill: Yield on short-term obligations of a government. Issued for periods of one year or less.

industry risk The part of risk due to exposure to industries.

inflation-protected bond (IPB)
A fixed-income security whose principal is periodically adjusted to provide a fixed return over inflation. The adjustment lags a pre-specified measure of inflation by an amount of time determined by the issuer.

instrument type
The category of investment vehicle to which a security belongs; for example: equity, bond future, treasury bills.

integrated general auto-regressive conditional heteroskedastic model (IGARCH)
A *GARCH* model that allows for a unit root in the conditional variance. Such inclusion allows for shocks to have a permanent effect on the conditional variance. See also *general auto-regressive conditional heteroskedastic model.*

interest
Technically, a prespecified amount paid to an investor in excess of repayment of the principal. In general, interest can be thought of as the reward to the investor for lending purchasing power to the borrower.

interest rate sensitivity
The responsiveness of a fixed income instrument's price to changes in interest rates.

interest rate swap
A contractual obligation entered into by two parties to deliver a fixed sum of money against a variable sum of money at periodic intervals. The transaction typically involves an exchange of payments on fixed- and floating-rate debt. If the sums involved are in different currencies, the swap is simultaneously an interest rate swap and a currency swap.

investment grade
The quality ratings of bonds, extending from AAA through BBB (Standard & Poor's convention) or Baa (Moody's convention).

investment horizon The period in which a given portfolio is held. For most investment decisions, it is appropriate to establish a horizon for which the portfolio is to be optimized. The expectation is that the portfolio will be thoroughly rebalanced at the end of that horizon. In fact, in active investment management, a portfolio is continually being modified as expectations change, but it is nevertheless useful to retain a moving investment horizon with respect to which forecasts are defined. In the scenario approach to return forecasting, the end of the investment horizon corresponds with the scenario forecast date.

IPB See *inflation-protected bond*.

issue A particular class of security; for example: a bond, a convertible instrument, or a stock. The issue date is the date on which the security is publicly available. The issue amount is the quantity of securities that enter the market.

issuer The entity that issues a particular security.

K

key rates A set of rates at distinguished maturities used to model *term structure risk*.

key rate duration The sensitivity of a bond to a change in a single *spot* or *key rate* on the *term structure*.

kurtosis Characterizes the relative peakedness or flatness of a distribution compared with the *normal distribution*. The kurtosis of a normal distribution is 0. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution.

L

liquidity In its modern usage, the liquidity of an asset is the extent to which it can be readily converted into cash without paying a large spread or moving the market. Thus, the larger the regular volume of trade in an issue, the more likely that a holder can dispose of his position without being forced to accept a low price to induce someone else to buy it. Liquidity is almost synonymous with a large volume of trade. One historical use refers to an asset's ability to hold its value during hard times. From that perspective, a liquid asset was one that could be quickly converted, at or near its *par value*, during a monetary crisis.

Associated with liquidity is the concept of the "spread," which is the difference between the bid and offer price quoted by market makers. The bid price is what the market maker will pay for your shares if you want to sell them. The offer is the price at which you can buy them from him. Large, liquid stocks have narrow spreads (a good thing). Small, illiquid stocks have wide spreads (a bad thing).

local factor Factor that only affect securities within each market. See *global factor*.

local market The country that Barra uses to model a security. For example, a Finnish ADR would have a local market of Finland.

local market risk The part of risk due to exposure to local market factors such as styles, industries, term structure movements, and changes in spreads. Local market risk arises from decisions made with local markets.

M

Macaulay duration
The weighted average time to receipt of a series of cash flows, where the weights are the proportions of the *present values* of the *cash flows* to the present value of the entire series of cash flows. Therefore, the Macaulay duration of a *zero-coupon bond* is equal to its time-to-maturity. The maturity duration in the Barra model is calculated as the modified Macaulay duration: Macaulay duration divided by (1 + yield/2). Barra computes option-adjusted Macaulay and *modified duration* (also called effective duration) by simulating future interest rates and modeling the change in option value for small changes in interest rates.

See also *duration* and *modified duration*.

margin
Yield spread of the floating rate note over the index rate, expressed as a percentage.

market
The economic entity that is constituted by buyers and sellers coming together to effect purchases and sales.

maturity
Almost all fixed-income securities carry a specified maturity date at which time the securities will be redeemed. The maturity of an outstanding issue is the number of years until the maturity date. The exception is a perpetual bond, or consol, which has no pre-announced terminus.

maturity date
The date on which the unpaid principal balance of a security becomes due and payable. For a bond without options, this date is stated in the prospectus. For a bond with options, the maturity date is affected by calls, puts, sinking fund schedules, and so on.

mean The expected average value of a random variable.

median A statistical term denoting the value of a random variable, such that 50 percent of all possible value lies above this value and 50 percent lie below. For example the number 5 is the median between the numbers 1 and 9, since there are four numbers above and below in this sequence.

model A mathematical representation of an economic system or corporate financial application so that the effect of changes can be studied and forecast. In general, a model specifies a set of relationships among constructs.

modern portfolio theory (MPT) The theory of portfolio optimization which accepts the risk/reward tradeoff of total portfolio return as the crucial criterion. Derived from Markowitz's pioneering application of statistical decision theory to portfolio problems, *optimization* techniques and related analysis are increasingly applied to investments.

modified duration The modified duration is the *Macaulay duration* divided by 1 plus periodic yield. Periodic yield is the yield to maturity divided by the discounting frequency per year. Modified duration can be used to measure the sensitivity of a bond's price to changes in yield, for securities whose *cash flows* are not sensitive to changes in interest rates.

momentum Rate of acceleration of an economic, price, or volume movement. An economy with strong growth that is likely to continue is said to have a lot of momentum. In the stock market, technical analysts study stock momentum by charting price and volume trends.

mortgage-backed security (MBS) A fixed-income security backed by collateral of a pool of mortgages. The issuer often services the pool of mortgages and withholds the cost of servicing from the coupon payments to be passed through to the investor.

multiple-factor model (MFM) A model where more than one underlying common factor influences numerous securities. Each factor influences some or all securities in proportion to their responsiveness to that factor (or their loadings upon that factor). Thus, the outcome for any one security is the sum of its responses to each of the multiple factors, with the contribution of each factor being the product of the factor itself times the responsiveness of the security to that factor. The multiple-factor model does not ordinarily assume that all events can be associated with factors. Instead, there is also a unique event associated with each security, called the specific event for that security, which is in addition to the contributions of the factors.

municipal bonds Bonds issued by state or local governments used to pay for special projects such as highways or sewers. The interest that investors receive is exempt from some income taxes.

N

nominal A term often used to refer to prices and returns expressed in current dollars, in contrast to *real* values, which are expressed in constant dollars—that is, in terms of constant purchasing power.

nominal cash flow A bond's promised *cash flow*, ignoring option provisions and default risk.

nominal spread Also referred to as the "spread," this differs from *option-adjusted spread* in that it does not account for the embedded options in a bond.

nominal yield The yield that is computed by determining the *interest rate* that will make the *present value* of the *cash flow* from the investment equal to the price of the investment. The nominal yield is not the coupon rate (see *coupon*).

normal A benchmark portfolio.

normal distribution The familiar bell-shaped curve which is called the "normal" distribution because it is the distribution that occurs when large numbers of independent factors are added together. It is a symmetrical distribution, with approximately two-thirds of all outcomes falling within ± 1 standard deviation and approximately 95 percent of all outcomes falling within ± 2 standard deviations.

normalization The process of transforming a random variable into another form with more desirable properties. One example is standardization in which a constant (usually the mean) is subtracted from each number to shift all numbers uniformly, then each number is divided by another constant (usually the standard deviation) to shift the variance.

numeraire The currency in which an asset or portfolio is valued.

O

OAS See *option-adjusted spread*.

optimization The best solution among all the solutions available for consideration. Constraints on the investment problem limit the region of solutions that are considered, and the objective function for the problem, by capturing the investor's goals correctly, provides a criterion for comparing solutions to find the better ones. The optimal solution is the solution among those admissible for consideration that has the highest value of the objective function. The first-order conditions for optimality express the trade-offs between alternative portfolio characteristics to provide the optimum solution.

option An amount paid for the right to buy or sell a security.

- A call option is an option to buy shares. Call options generally rise in price if the underlying shares rise in price (and vice versa).
- A put option is an option to sell shares. Put options generally rise in price if the underlying shares fall in price (and vice versa).

The holder of the option is not obligated to buy the security, but the amount paid for the option is non-refundable. The main criterion for deciding whether to buy the security is whether the exercise price of the option is higher or lower than the current price of the underlying share.

option-adjusted cash flow A bond's *nominal cash flows*, adjusted for estimated option influence.

option-adjusted convexity Measures a bond's convexity, explicitly accounting for the influence of embedded options.

option-adjusted duration The modified duration of a security, calculated using a model that accounts for embedded *options*.

option-adjusted spread (OAS)
Measures the difference of a bond's yield to maturity above the default-free *term structure*, explicitly accounting for the influence of embedded options. Option-adjusted spread (OAS) is a measure of a security's extra return over the return of a comparable default-free government security after accounting for embedded *options*.

option-adjusted spread to swap
Measures the difference of a bond's yield to maturity above the *swap curve*, explicitly accounting for the influence of embedded *options*.

option-adjusted yield
The yield to maturity adjusted for the value of the embedded *options* (call, put, sinking fund, and so on). Derived by adding the option value to the price and recalculating the yield to maturity.

outlier
A data observation that is very different from other observations. It is often the result of an extremely rare event or a data error.

P

par
1. See *par value*.
2. The typical lot size associated with an instrument type. For example, par for bonds is 1000. Par for equities is 1.

par value
The value of a security as it appears on the certificate of the instrument. This is the amount of principal due the bondholder at maturity. It is the amount on which interest payments are calculated.

payout ratio
The ratio of dividends to earnings. The fraction of earnings paid out as dividends.

PCA
See *principal components analysis*.

performance attribution

The process of attributing portfolio returns to causes. Among the causes are the normal position for the portfolio, as established by the owner of funds or the manager, as well as various active strategies, including market timing, common factor exposure, and asset selection. Performance attribution serves an ancillary function to the prediction of future performance, in as much as it decomposes past performance into separate components that can be analyzed and compared with the claims of the manager.

predicted beta

A forecast of a stock's sensitivity to the market. Predicted beta is also known as fundamental beta, because Barra's risk models are based on fundamental risk factors. These risk factors include industry exposures as well as various "style" attributes called risk indices, such as Size, Volatility, Momentum, and Value. Because we re-estimate a company's exposure to these risk factors frequently, the predicted beta responds quickly to changes in the company's underlying risk structure.

Predicted systematic risk coefficients (predictive of subsequent response to market return) are derived, in whole or in part, from the fundamental operating characteristics of a company.

premium

Amount by which a bond trades above the *par value*. In other words, buyers are willing to pay more for the bond.

present value The present value of a stream of payments is the price at which that stream of payments could be purchased in the marketplace. It is the sum of the present values of the constituent payments in the stream.

The present value of a restless payment is equal to the amount of that payment multiplied by the discount factor for that date (or, equivalently, discounted for compound interest from the present through that date).

pricing error The discrepancy between the market price and Barra's fitted price. Positive pricing error implies that the security is overvalued by the market relative to Barra's valuation model, and vice versa.

principal The value paid to the issuer of a bond at the original issue date, usually indistinguishable from the *par value*. When the bond is redeemed, the payment by the issuer to the holder is treated as a return of principal.

principal component Possibly correlated variables that have been transformed into uncorrelated variables. The first principal component accounts for as much of the variability in the data as possible, and each succeeding component accounts for as much of the remaining variability as possible. See also *principal components analysis*.

principal components analysis (PCA)
A multivariate analysis which maximizes the spread of data by plotting covariance values on sets of axes in multidimensional space. It enables the identification of correlations which may have been hidden in the data. The first *principal component* corresponds to the first axis in multidimensional space and describes the majority of the spread of the data; subsequent higher-order principal component axes are orthogonal to the first axis. Higher-order axes display progressively less variation, where the data is less correlated and more representative of statistical noise.

probability
The expression of the likelihood of occurrence. A probability may be stated as a proportion of one, or sometimes as a percentage of 100%. It may be objective, in the sense of expressing the frequency of times that a random event will occur; or it may be subjective, in expressing the perceived likelihood of occurrence.

put option
An option that gives the holder the right to deliver a security to the person who writes the option at a prespecified price. In the case of fixed-income securities, a putable bond gives the investor the right to sell the bond to the issuer on a specified date for a previously agreed-upon put price.

putable bond
A bond that gives the holder the right to sell the bond back to the issuer on specified date(s) for a specified price (the "put price"). A putable bond includes a schedule of put dates and strike prices allowing the investor, typically, to redeem the principal at one or more specified dates prior to maturity.

Q

quality ratings Bond ratings assigned by rating services, such as Moody's, Standard and Poor's, Duff and Phelps/MCM, and Fitch. Although conventions vary, Barra uses the rating system where the highest quality rating is AAA and the lowest investment grade rating is BBB. Ratings extend downward through C, and agency bonds are unrated.

R

R-squared A statistic usually associated with regression analysis. It describes the fraction of variance in the dependent variable that can be explained by the independent or explanatory variable(s). It is often used to describe the fraction of investment risk in portfolios that can be associated with market risk.

Mathematically, it is the (predicted) market variance divided by (predicted) total variance. To calculate the coefficient of determination, take the portfolio beta (measured against the market) squared times the total market variance forecast, and divide the product by the total portfolio variance forecast. The coefficient of determination is a pure number ranging from 0 to 1, with 1 indicating perfect explanation. R^2's for portfolios typically range from 0.8 up to 1, with a median of about 0.95.

real A term that refers to prices and returns expressed in constant dollars—that is, in terms of constant purchasing power—in contrast to nominal values, which are expressed in terms of current dollars.

real return Return expressed in units of purchasing power.

real yield curve Yield curve based on the prices of *inflation-protected bonds*. Real rates are adjusted by inflation to give nominal rates.

regression A statistical technique which examines the correlation between two or more variables in a mathematical model and attempts to prove whether or not the past relationships will be the same in the future. Regression finds the linear combination of one or more independent variables which best explain the variation in a dependent variable. When there is a single independent variable and the observations of the independent and dependent variables are plotted on a graph, regression draws the best straight line through the data points. Regression analysis is used in the Black-Scholes option pricing model, portfolio theory, and the *Capital Asset Pricing Model*.

residual A statistical term for that part of a variable that is unexplained by some underlying factor. For example, *residual return* is usually defined as that part of return that is not explained by the systematic factor (often the market portfolio) in a model of systematic and residual return. Similarly, *residual risk* is that part of risk that arises in addition to risk from the market factor. The term residual is also often used to refer to the difference between the actual datum and the datum fitted in the model.

residual return The component of return that is uncorrelated with the return on the market portfolio or benchmark. Mathematically, it is the total return minus beta times the market return.

Residual return is also called unsystematic or diversifiable return. All components of active management, except market timing, contribute to residual return at one point in time.

residual risk

Residual risk is the standard deviation of *residual return*. Residual return is return that is not attributable to market influence. It is the return net of the market-related return.

return

The value of an investment at the end of a period, plus any payouts during the period, divided by the initial value. Return is (depending on the length of the period) a number close to 1.0 and represents one plus the rate of return. For expository purposes, return is often given as a percentage by subtracting one and multiplying the result by 100.

risk

The uncertainty of investment outcomes. Technically, risk defines all uncertainty about the mean outcome, including both upside and downside possibilities. Studies of investment return have shown very consistently that when returns are centered about their expected value, there is little difference between the extent of upside and downside variability relative to that value. Thus a measure of total variability in both directions is typically used to summarize risk. The more intuitive concept for risk measurement is the standard deviation of the distribution, a natural measure of the spread. *Variance*, the square of the standard deviation, must be used in comparing independent elements of risk.

On the other hand, when the shapes of the upper and lower tails of the probability distribution are different—as they are in the case of catastrophic default risk, or whenever an option is present—it may be necessary to take these into account and analyze the risky distribution more completely. A more complete analysis can be accomplished by taking into account not only the spread of the distribution (standard deviation or variance) but also the asymmetry or skewness of the distribution.

risk index A common factor in equity models that is typically defined by some continuous measure, as opposed to a common industry membership factor, which is defined as 0 or 1. Risk index factors include Volatility, Momentum, Size, and Value.

risk index exposure A variable computed for each equity asset that determines the asset's exposure to a common factor. Risk indices include Market Variability, Earnings Variability, Size, and Growth. In each case, a higher value of the index exposure implies that a company is more strongly exposed to that common factor. Risk index exposures are expressed as standardized numbers that range (usually) from –5 to +5. The average (capitalization-weighted) stock in the estimation universe has an exposure of zero.

risk model A model that tries to explain asset or portfolio risk through one or more factors. The *Capital Asset Pricing Model* is an example of a risk model. In this model, the asset or portfolio's risk is explained by its sensitivity to the market and the market's risk. See also *multiple-factor model*.

risk premium An increased expected return to compensate for the undesirability of the riskiness of that return. A true risk premium must result in an increase in expected return, not just an increase in promised yield that compensates for expected loss from default and simply preserves expected return over different time periods.

risk-free return The return an investor can lock in with certainty at the beginning of an investment period. Conceptually, such an investment should have guaranteed purchasing power at its termination. In practice, this rate is usually defined by the rate of return on short-term government-issued bonds. For example in the U.S. model, the 90-day U.S. Treasury bill is used.

RMSE See *root mean squared error.*

root mean squared error (RMSE) A measure of goodness of fit to the data. It takes into account both an average error, if any, and the variability of errors. In fact, it is a measure of the typical magnitude of error. It is equal to the square root of the average squared error. The root mean squared error is always larger than the standard error, except in the case where the average error is zero, in which case the two are equal.

S

sector-by-rating Process used by Barra to model credit risk for the most active markets. When possible and appropriate, issues are categorized both by sector and by rating, to more accurately capture risk.

shift A near-parallel movement of the term structure (in *spot rate* space). Historical regressions have been run to define the typical shape as well as magnitude of a one-standard deviation shift over a one-year horizon. Generally, countries with more volatile term structures can be expected to have larger *shift risk*.

shift risk The part of risk due to exposure to shift movements in the *term structure*.

shock Change imposed on the term structure to value a bond's exposures to different factors.

sovereign bond Bond issued by a government.

specific return The part of the *excess return* not explained by *common factors*. The specific return is independent of (or uncorrelated with) the common factors and specific returns of other assets.

specific risk The uncertainty in asset or portfolio return that arises from unpredictability that is specific to that asset. This risk is unrelated to all other assets or common factors. Specific risk is also called idiosyncratic, or independent risk.

spot rate A rate prevailing from the present to any particular future date. There is a different spot rate for every future date. The series of spot rates at the different maturities gives the *term structure*. Spot rates can be calculated using different techniques, such as *bootstrapping*. Barra estimates every local term structure by minimizing the differences between market and fitted prices, subject to a smoothing constraint.

spread The difference between yields on securities with different credit qualities.

The spread used over the sovereign yield curve to price non-government bonds is typically derived from the *swap curve* in each market, although some markets have detailed sector-by-rating spreads available and some emerging markets have emerging spreads. The standard deviation of changes in spread is used to approximately capture systematic *spread risk* for non-government securities.

spread duration The sensitivity of a bond's price to a change in its *option-adjusted spread (OAS)* (with the spot *term structure* held constant).

For corporate bonds, spread duration will be equivalent to *effective duration*. For floating-rate notes (FRN), spread duration is likely to be longer than term structure duration since a change in OAS affects all the *cash flows* over the life of the instrument, yet term structure sensitivity is effectively concentrated in the current coupon period (until reset).

spread factor A risk factor that captures typical movements in interest rate spread level. Spread factors in Barra's risk model include non-government spread (also known as *credit spread* and *swap spread*) and *emerging-market spread*.

spread risk The risk due to exposure to spread movements.

standard deviation A statistical term which measures the spread of variability of a probability distribution. It is the square root of *variance*. Standard deviation is widely used as a measure of risk or volatility of portfolio investments. A higher standard deviation indicates a product with more risk. A product's portfolio is expected to differ positively or negatively from the mean return by no more than the standard deviation amount for approximately 68% of its cycle.

standardization The process of scaling *descriptors* and *risk indices*. Standardization involves setting a common (standard) zero point and scale for measuring a variable. The mean value is subtracted from a distribution, then all the values are divided by the standard deviation. The result is a distribution with mean of zero and standard deviation of one. Standardization is typically done to convert "apples and oranges" to "apples and apples" so that comparisons can be made across data items and across time. Standardization is applied to all Barra descriptors before combining them into risk index values. The risk index values themselves are then standardized.

stochastic A term synonymous with random—that is, having unpredictable events that obey a probability distribution. It is often used in statistics to refer to a random process, with the term random itself being reserved for the simplest forms of probability distributions, which give equal likelihood to all outcomes.

stripped spread A bond's stripped spread is calculated by subtracting the *present value* of the collateralized *cash flow* (escrowed interest payment and collateralized principal). The adjusted price is then equated to the remaining non-collateralized cash flows, which are discounted at a spread over the base curve. This constant spread over the default-free curve is the stripped spread.

swap A financial obligation to exchange the cash flows of two interest-rate instruments. This can happen between a floating-rate instrument and a fixed-rate security, or between bonds of different sectors and maturities. Swaps occur when it is advantageous for both parties to assume the swapped financial obligation because of situations unique to the parties concerned.

swap curve A curve that plots *swap rates* at different maturities.

swap rate *Interest rate* based on a *swap* contract.

swap spread Difference between the *swap* and *sovereign* curves.

systematic return The component of return that is associated with the broad-based market portfolio. Also, the reward expected from the market portfolio and the risk of that reward are referred to as systematic reward and *systematic risk*. More generally, the risk and reward of any asset that can be associated with that asset's exposure to the market are termed systematic. Systematic reward generally refers to the excess return, rather than the total return, associated with the market.

systematic risk That part of risk associated with exposure to the systematic portfolio. Total risk can always be decomposed, for any given definition of systematic portfolio, into the systematic component that is related to that portfolio and the residual component that is unrelated to it.

Systematic risk is the standard deviation of *systematic return*. Systematic return is portfolio return due to the same forces that influence the return on the benchmark. Systematic risk can be thought of as the portfolio risk that can't be diversified away.

T

terms and conditions (TNC) All features of a bond that may be important to the investor and that may influence market value. These include the payment schedule for the bond, options attached to the bond, marketability of the bond, or any other characteristics of the issuer.

term structure A full schedule of *spot rates*, *forward rates*, par yields, or pure discount bond prices. Interest rate movements are expressed in terms of changes in zero-coupon bond yields inferred from a combination of money-market rates and coupon bond yields. The zero-coupon yield curve is also referred to as the "term structure of interest rates," or sometimes just "term structure."

term structure exposure A mapping of bond or portfolio classifications to the Barra *term structure* vertices. The weight at each vertex represents the percentage of bond or portfolio value "thrown off" and is therefore related to *Macaulay duration*.

term structure factor A factor that describes a typical *term structure* movement. For most markets, the Barra risk model defines three movements: shift, twist, and butterfly.

term structure risk The part of risk due to exposure to *term structure* movements.

time to maturity The length of time between the current date and the maturity date of a fixed-income security, expressed in years.

TNC See *terms and conditions*.

total return The total (gross) return to a portfolio including capital gains and dividend income. For global equity models, the total return is calculated with respect to a currency perspective or base currency. The monthly total return is calculated assuming a buy and hold strategy. The holdings at the beginning of the month are assumed to be held until the end of the month with no transactions.

total risk The total (gross) risk to an asset, which is the standard deviation of the asset's total return distribution. We forecast total risk using Barra's multiple-factor model.

tracking error A measure of active portfolio risk which indicates how closely the portfolio return tracks the benchmark return. Tracking error is the standard deviation of the difference of returns between a portfolio and the benchmark position over a specified holding period.

twist The non-parallel change in the *term structure* where the short end of the curve moves in the opposite direction of the long end of the curve. This is commonly referred to as a flattening or steepening of the curve. Historical regressions are run to define the typical shape and magnitude of a one-standard deviation twist over a one-year horizon. By definition, the twist movement is nearly orthogonal to both *shift* and *butterfly* movements within a market.

twist risk The part of risk due to exposure to *twist* movements in the *term structure*.

U

universe The list of all assets eligible for consideration for inclusion in a portfolio.

unsystematic return See *residual return*.

V

value at risk (VaR) A measure that characterizes the potential loss in currency units in a given time period for a given probability level. For example, a VaR of –1,000,000 at the 5% probability level indicates there is a 5% probability one would lose up to 1,000,000 in the coming year.

value stock A stock is considered to be a value stock based on the relationship between its market price and its book price. Value stocks are considered attractive because the company is undervalued.

variance A measure of the variability of variables around the mean. Variance is defined as the expected squared deviation of the random variable from its mean—that is, the average squared distance between the mean value and the actually observed value of the random variable. When a portfolio includes several independent elements of risk, the variance of the total arises as a summation of the variances of the separate components.

volatility A measure of a share's propensity to go up and down in price. A volatile share is one which has a tendency to move drastically across a broad share price range. Mathematically, this is expressed as the standard deviation from the average performance.

In general, high volatility means high unpredictability, and therefore greater risk. Numerous attempts have been made to incorporate volatility into pricing models, but the problem has always been that past volatility is not necessarily a good guide to future volatility.

W

weighting scheme A method of assigning importance to each component of a summation process. Weights denote the relative importance of various items in an average, and the weights are summed up to one.

winsorization A procedure where the outliers are replaced by the two (one positive and one negative) remaining extreme values. That is, the extreme values are moved toward the center of the distribution. This technique is sensitive to the number of outliers, but not to their actual values.

Y

yield The return on a security or portfolio in the form of cash payments. Most yield comes from dividends on equities, coupons on bonds, or interest on mortgages. In general, yield is defined in terms of the component of return that is taxable as ordinary income. Consequently, since the capital gain on a short-term government bond (treasury bill) or other discount note is viewed for tax purposes as a form of interest, it is also included in the definition of yield.

yield curve The plot of *yield-to-maturity* against term-to-maturity. See also *term structure*.

yield spread The spread in yield between a particular bond and a comparable liquid government benchmark issue. Yield spread reflects the difference in yield between a particular bond and its corresponding pricing curve, be it the sovereign spot curve for government bonds or the swap curve for Eurobonds.

yield-to-maturity The yield if the bond is held to maturity. It is the yield that equates the net *present value* of the future *cash flows* of the security (as of the maturity date) to its current market price. This assumes that the redemption date is the maturity date and uses the corresponding redemption price. Calls and puts are not taken into account.

Z

zero-coupon bond A bond that is issued without a coupon. At maturity it is redeemed at its face amount.

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