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A Markov regime-switching model for crude-oil markets: Comparison of composite likelihood and full likelihood

Wei ZOU[1]* and Jiahua CHEN[2]

[1] *School of Statistics, Central University of Finance and Economics, Beijing 100081, P.R. China*
[2] *Department of Statistics, University of British Columbia, Vancouver, BC, Canada V6T 1Z2*

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Abstract: We use the two-state Markov regime-switching model to explain the behaviour of the WTI crude-oil spot prices from January 1986 to February 2012. We investigated the use of methods based on the composite likelihood and the full likelihood. We found that the composite-likelihood approach can better capture the general structural changes in world oil prices. The two-state Markov regime-switching model based on the composite-likelihood approach closely depicts the cycles of the two postulated states: fall and rise. These two states persist for on average 8 and 15 months, which matches the observed cycles during the period. According to the fitted model, drops in oil prices are more volatile than rises. We believe that this information can be useful for financial officers working in related areas. The model based on the full-likelihood approach was less satisfactory. We attribute its failure to the fact that the two-state Markov regime-switching model is too rigid and overly simplistic. In comparison, the composite likelihood requires only that the model correctly specifies the joint distribution of two adjacent price changes. Thus, model violations in other areas do not invalidate the results. *The Canadian Journal of Statistics* 41: 353–367; 2013 © 2013 Statistical Society of Canada

Résumé: Les auteurs utilisent le modèle de Markov à changement de régime avec deux états pour expliquer les fluctuations du prix au comptant du pétrole brut WTI de janvier 1986 à février 2012. Ils examinent des méthodes basées sur la vraisemblance composite et la vraisemblance complète et concluent que la méthode de vraisemblance composite saisit mieux les changements structurels généraux dans les cours mondiaux du pétrole. Ce modèle de Markov utilisé avec la méthode de vraisemblance composite reflète étroitement les cycles des deux états considérés : une tendance à la baisse ou à la hausse. Ces deux états persistent en moyenne huit et quinze mois respectivement, ce qui correspond aux cycles observés durant la période. D'après le modèle ajusté, la volatilité du prix du pétrole est plus élevée durant les périodes de baisse que de hausse. Les auteurs estiment que cette information peut être utile aux agents financiers œuvrant dans des domaines connexes. Le modèle basé sur la méthode de vraisemblance complète est moins satisfaisant. Les auteurs attribuent son échec au fait que le modèle de Markov à changement de régime avec deux états est trop rigide et exagérément simpliste. En revanche, la méthode de la vraisemblance composite nécessite seulement que le modèle spécifie correctement la distribution conjointe de deux changements de prix adjacents. Ainsi, la violation d'hypothèses dans le reste du modèle n'invalide pas les résultats. *La revue canadienne de statistique* 41: 353–367; 2013 © 2013 Société statistique du Canada

1. INTRODUCTION

Oil is essential for industry and has a great impact on the world economy. Because of the distribution imbalance, price fluctuations in the world crude-oil market have always attracted attention from both industry and academia. During the 1980s and 1990s, world oil prices were around

* *Author to whom correspondence may be addressed.*
E-mail: zouwein1@163.com

10–20 dollars per barrel. However, both the price and its volatility have increased dramatically since the end of the 1990s. The price fluctuations reached new peaks in 2008, when the West Texas Intermediate (WTI) crude-oil spot price reached a record 133.88 dollars per barrel. It then showed "roller coaster" fluctuations. After a sharp drop in early 2009, it quickly climbed to 100 dollars. These fluctuations are a concern given the importance of oil in the world economy. When the price and volatility are high, the production cost and consumer demand are directly affected, and the result could be a recession. Therefore, if the dynamic process of price fluctuations can be identified, governments and firms will be better able to hedge various risks and to avoid excessive exposure to risk, thus promoting stable and harmonious economic development.

Empirical studies (e.g., Abosedra & Laopodis, 1997; Morana, 2001; Bina & Vo, 2007) suggest that crude-oil time series, like other financial time series, exhibit fat-tail distributions, volatility clusters, asymmetry, and mean reversion. Moreover, some studies find strong evidence of regime switching. For example, Wilson, Aggarwal, & Inclan (1996) find that there are structural changes in volatility in the daily returns of oil futures. They argue that exogenous factors such as severe weather conditions, political turmoil, and changes in OPEC oil policies may cause these structural changes. Fong & See (2002) also report significant regime shifts in the volatility of the WTI daily oil-future prices. Klaassen (2002) accounts for the high persistence of shocks generated by changes in the variance process via a regime-switching GARCH model. Wei, Chen, & Wang (2006) use a three-state Markov-switching model to examine the spot price fluctuations of Brent oil. They conclude that this model is superior to the linear autoregressive model in describing changes in oil prices. Vo (2009) claims that the regime-switching stochastic volatility model performs well in capturing major events affecting the oil market. Kang, Cheong, & Yoon (2011) find that ignoring structural changes may distort the direction of information inflow and volatility transmission between crude-oil markets.

Markov regime-switching models are also widely used to describe other business cycles (e.g., Hamilton, 1989; Layton & Smith, 2000; Zhao, Wang, & Cai, 2005; Wang, 2007), the stock market (e.g., Turner, Startz, & Nelson, 1989; Cecchetti, Lam, & Mark, 1990; Schaller & Van Norden, 1997), interest rates (e.g., Hamilton, 1988; Garcia & Perron, 1996; Gray, 1996; Ang & Bekaert, 2002; Smith, 2002), and exchange rates (e.g., Engel & Hamilton, 1990; Engel, 1994; Bollen, Gray, & Whaley, 2000; Xie & Liu, 2003; Dueker & Neely, 2007). In these studies, it is assumed that there exists a finite number of unobserved regimes or states that govern the stochastic properties of the times series at any given time. Specifically, the marginal distribution of the series at time t is solely determined by the regime operating at the time, while the dynamics of the regime-switching is determined by a Markov process (Chen & Wang, 2012).

The full likelihood (FL) method is often used to estimate the parameters in Markov regime-switching models, but this approach has some shortcomings. First, the maximization algorithm for maximum likelihood estimation can be easily trapped in local maxima because of the FL's complex analytical form. Second, when the marginal distribution is normal, the maximum FL value is infinite, which implies the existence of many nonsensical maxima. Third, the parameter estimation and prediction precision may be influenced by the departure of the real population from the model assumption.

Given these challenges, Chen & Wang (2012) propose the simpler and likely more robust composite likelihood (CL). They observe that the CL method can address some of the shortcomings of the FL method. First, the CL method requires the correct specification of the joint density of two consecutive observations instead of the joint density of the whole series. Consequently, the resulting statistical inferences are expected to be more robust to model mis-specification. Second, since the time-series observations are ordered in time with the bulk of the serial dependence occurring in adjacent observations, pairs of consecutive observations contain the most useful information. This is especially true when the regime-switching is governed by the underlying first-order Markov chain. Finally, CL is easier to compute than FL. Chen & Wang (2012) also find via simulation studies that the CL method can be more efficient and has better in-sample performance.

In this paper, we show that the two-state Markov regime-switching model based on CL can better capture the general structural changes in world oil prices. The empirical analysis also demonstrates that the model based on CL closely depicts the cycles of the two postulated states: fall and rise. These two states persist for on average 8 and 15 months, which matches the observed cycles during the period. Moreover, drops in oil prices are more volatile than rises. The model based on FL is less satisfactory. In fact, the FL approach requires the model to correctly specify the joint distribution of the whole series. Hence, the two-state Markov regime-switching model based on FL is too rigid and overly simplistic, and model violations are likely to invalidate the results.

We organize the remainder of the paper as follows. In Section 2 we review the Markov regime-switching model and introduce the FL and CL approaches. In Section 3 we discuss the data and perform some exploratory analyses. Section 4 reports the results, and Section 5 presents our conclusions.

2. MARKOV REGIME-SWITCHING MODEL

2.1. N-State Markov Regime-Switching Model

Let the change in the crude-oil price at time t be y_t, for $t = 1, 2, \ldots, T$ over which the data have been collected. Our model postulates the existence of an unobserved discrete variable (denoted s_t) that specifies the marginal distribution of y_t. When $s_t = k$, $k = 1, 2, \ldots, N$, the marginal distribution of y_t is distributed as $N(\mu_k, \sigma_k^2)$. We also postulate that the evolution of s_t forms an N-state first-order Markov chain. In other words, the process of s_t relies only on past realizations of y and s_t through s_{t-1}. The transition probabilities of s_t are denoted

$$Pr(s_t = j | s_{t-1} = i) = p_{ij},$$

where $i, j = 1, 2, \ldots, N$ and $\sum_{j=1}^{N} p_{ij} = 1$ for all i.

2.2. Full Likelihood

According to the above model, given a time series $\{y_t : t = 1, 2, \ldots, T\}$, the FL is given by

$$L(\theta) = \sum_{(s_1, s_2, \ldots, s_T)} \left[p_{s_1, s_2, \ldots, s_T} \prod_{t=1}^{T} \phi(y_t; \mu_{s_t}, \sigma_{s_t}) \right]$$

with

$$\phi(y; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(y-\mu)^2}{2\sigma^2} \right\}.$$

The parameter vector in the FL is $\theta = (A, B, P)$, where $A = (\mu_1, \mu_2, \ldots, \mu_N)^{\mathrm{T}}$, $B = (\sigma_1, \sigma_2, \ldots, \sigma_N)^{\mathrm{T}}$, and $P = \{p_{ij}\}_{1,2,\ldots,N}$.

Generally, the maximum likelihood estimates are obtained as the limit of the iteration through the forward–backward algorithm (Baum et al., 1970), which is a special case of the famous EM-algorithm introduced by Dempster, Laird, & Rubin (1977).

2.3. Composite Likelihood

Under the N-state Markov regime-switching model, given $s_t = u$, $s_{t+1} = \upsilon$, the consecutive values y_t and y_{t+1} are independent of each other. The conditional joint density function of (y_t, y_{t+1})

is given by

$$f(y_t, y_{t+1}|s_t = u, s_{t+1} = \upsilon) = \phi_u(y_t)\phi_\upsilon(y_{t+1})$$

for some regime-specific distribution with normal density function $\phi_u(\cdot)$. Under the assumption that the Markov chain is in equilibrium, the unconditional joint density function of (y_t, y_{t+1}) is given by

$$f(y_t, y_{t+1}) = \sum_{u=1}^{N}\sum_{\upsilon=1}^{N} \pi_{u\upsilon}\phi_u(y_t)\phi_\upsilon(y_{t+1})$$

where $\pi_{u\upsilon} = Pr(s_t = u, s_{t+1} = \upsilon)$, which does not depend on t. Marginally, (y_t, y_{t+1}) is a sample from an N^2-component finite mixture model with the component density functions $\phi_u(x)$ and $\phi_\upsilon(x)$ and component weights $\pi_{u\upsilon}$.

The standard Markov-chain theory (Hamilton, 1994) shows that $\pi_{u\upsilon}$ is given by

$$\pi = (\eta^\top\eta)^{-1}\eta^\top e_{N+1},$$

where e_{N+1} denotes the $(N+1)$th column of the identity matrix I_{N+1} and $\eta = \begin{bmatrix} I_N - P \\ I^\top \end{bmatrix}$.

By "incorrectly" regarding $T-1$ pairs of consecutive observations in $\{y_t : t = 1, 2, \ldots, T\}$ as independent bivariate random variables, we obtain the likelihood function

$$L_{\text{cl}}(\theta) = \prod_{t=1}^{T-1} f(y_t, y_{t+1}) = \prod_{t=1}^{T-1}\sum_{u=1}^{N}\sum_{\upsilon=1}^{N} \pi_{u\upsilon}\phi_u(y_t)\phi_\upsilon(y_{t+1}).$$

Chen & Wang (2012) propose the use of CL under the two-state Markov regime-switching model. For an N-state hidden Markov model with a normal regime-specific distribution, the penalized log CL is given by

$$l_{\text{pcl}}(\theta) = \sum_{t=1}^{T-1} \log\left\{\sum_{u=1}^{N}\sum_{\upsilon=1}^{N} \pi_{u\upsilon}\phi_u(y_t)\phi_\upsilon(y_{t+1})\right\} - T^{-1/2}\sum_{j=1}^{N}\left\{\frac{\hat{\sigma}_0^2}{\sigma_j^2} + \log\left(\frac{\sigma_j^2}{\hat{\sigma}_0^2}\right)\right\},$$

where $\hat{\sigma}_0^2$ is the sample variance of the time series, that is, the penalty is equivalent to assigning a Gamma prior distribution to σ_j^{-2}.

An EM-algorithm can be used to compute the maximum point of $l_{\text{pcl}}(\theta)$. Specifically, let

$$\xi_t(u, \upsilon) = \begin{cases} 1 & \text{when } (s_t, s_{t+1}) = (u, \upsilon), \\ 0 & \text{otherwise} \end{cases}$$

for $u, \upsilon = 1, 2, \ldots, N$. If the $\xi_t(i, j)$ values were observed, we would have the following complete-data log CL

$$l = \sum_{t=1}^{T-1}\sum_{i=1}^{N}\sum_{j=1}^{N} \xi_t(i, j)\log\{\pi_{ij}\phi(y_t; \mu_i, \sigma_i)\phi(y_{t+1}; \mu_j, \sigma_j)\}$$
$$= \sum_{t=1}^{T-1}\sum_{i=1}^{N}\sum_{j=1}^{N} \xi_t(i, j)\log\{\pi_{ij}\}$$
$$+ \sum_{t=1}^{T-1}\sum_{i=1}^{N}\sum_{j=1}^{N} \xi_t(i, j)\log\{\phi(y_t; \mu_i, \sigma_i)\phi(y_{t+1}; \mu_j, \sigma_j)\}.$$

In the E-step, we compute the conditional expectation via Bayes' formula:

$$w_t^{(k+1)}(i, j) = E\{\xi_t(i, j)|y_t, y_{t+1}; \theta^{(k)}\}$$
$$= \frac{\pi_{ij}\phi(y_t; \mu_i^{(k)}, \sigma_i^{(k)})\phi(y_{t+1}; \mu_j^{(k)}, \sigma_j^{(k)})}{\sum_{u=1}^{N}\sum_{\upsilon=1}^{N} \pi_{u\upsilon}\phi(y_t; \mu_u^{(k)}, \sigma_u^{(k)})\phi(y_{t+1}; \mu_\upsilon^{(k)}, \sigma_\upsilon^{(k)})}.$$

Replacing $\xi_t(i, j)$ by $w_t^{(k+1)}(i, j)$ in the complete-data likelihood, we obtain the so-called Q-function:

$$Q(\theta|\theta^{(k)}) = \sum_{t=1}^{T-1}\sum_{i=1}^{N}\sum_{j=1}^{N} w_t^{(k+1)}(i, j)\log\{\pi_{ij}\}$$
$$+ \sum_{t=1}^{T-1}\sum_{i=1}^{N}\sum_{j=1}^{N} w_t^{(k+1)}(i, j)\log\{\phi(y_t; \mu_i, \sigma_i)\phi(y_{t+1}; \mu_j, \sigma_j)\}.$$

In the M-step, we maximize the penalized Q-function

$$\tilde{Q}(\theta|\theta^{(k)}) = Q(\theta|\theta^k) - T^{-1/2}\sum_{j=1}^{N}\left\{\frac{\hat{\sigma}_0^2}{\sigma_j^2} + \log\left(\frac{\sigma_j^2}{\hat{\sigma}_0^2}\right)\right\}$$

with respect to a parameter vector θ containing $\pi_{u\upsilon}$, μ_u, and σ_u.

Under a normality assumption, $\tilde{Q}(\theta|\theta^{(k)})$ is maximized at $\theta = \theta^{(k+1)}$ with

$$\pi_{ij}^{(k+1)} = \frac{1}{T-1}\sum_{t=1}^{T-1} w_t^{(k+1)}(i, j),$$
$$\mu_u^{(k+1)} = \frac{\sum_{t=1}^{T-1}(w_t' y_t + w_t'' y_{t+1})}{\sum_{t=1}^{T-1}(w_t' + w_t'')},$$
$$(\sigma_u^2)^{(k+1)} = \frac{\sum_{t=1}^{T-1}\{w_t'(y_t - \mu_u^{(k)})^2 + w_t''(y_{t+1} - \mu_u^{(k)})^2\} + 2T^{-1/2}\hat{\sigma}_0^2}{\sum_{t=1}^{T-1}(w_t' + w_t'') + 2T^{-1/2}},$$

where $w_t' = \sum_{\upsilon=1}^{N} w_t^{(k+1)}(u, \upsilon)$ and $w_t'' = \sum_{\upsilon=1}^{N} w_t^{(k+1)}(\upsilon, u)$ for $u, \upsilon = 1, 2, \ldots, N$.

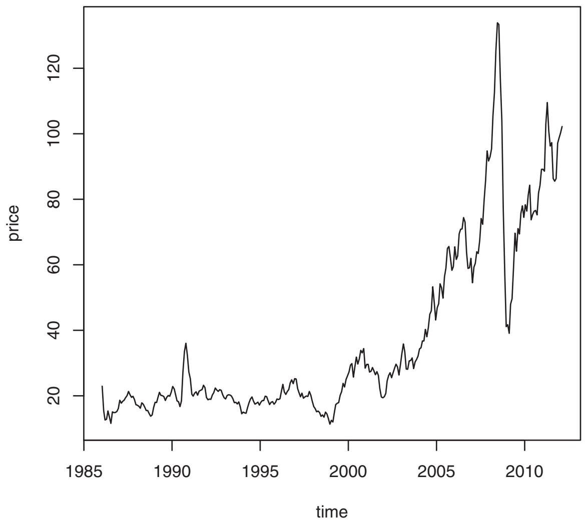

FIGURE 1: WTI crude-oil monthly price movements.

For any feasible choice of the initial parameter values, the EM-iteration increases the penalized log likelihood function. This suffices to ensure the convergence of the EM-algorithm. In general, there is no guarantee that the EM-algorithm will locate a global maximum. In applications, multiple initial values are used to increase the chance of locating the global maximum.

Both the above EM-algorithm for the CL and the forward-backward algorithm for the FL by Baum et al. (1970) are linear in T in terms of computational complexity. However, one crucial step of the forward-backward algorithm is the computation of the forward probabilities, namely the conditional joint density function of $(y_1, \ldots, y_t)$ given an initial state. This quantity can be so small that it may lead to underflow, particularly when T is very large. This and other problems have been extensively investigated in the literature (Zucchini & MacDonald, 2009) and there are ways to overcome these difficulties. In comparison, the EM-algorithm for penalized CL works nicely without taking any special numerical measures.

3. DATA

The data span a continuous sequence of 314 months from January 1986 to February 2012, showing the crude-oil spot prices of the WTI contracts traded on the New York Mercantile Exchange. Figure 1 shows the monthly price movements.

To get a preliminary understanding of the volatility, we first give some descriptive statistics of the changes in the oil price. We calculate the change in the price as $y_t = 100 \times [\ln(\text{price}_t) - \ln(\text{price}_{t-1})]$, where price_t and price_{t-1} are the prices for the months t and $t-1$. We find many extreme y-values in the observed time series; they clearly violate the assumption that the state-specific marginal distributions are normal. However, the regime-switching model with a normal component distribution is still useful after the extreme values have been moderated via truncation. Under a normal distribution $p(|x-\mu|) > 2.6\sigma) \leq 0.01$, and we replaced any observed values below $\bar{x} - 2.6 \times sd(x) = -22.35$ by -22.35, and above $\bar{x} + 2.6 \times sd(x) = 22.4$ by 22.46. In Table 1, we show the descriptive statistics of price_t, y_t, and adjusted y_t. Figure 2 gives a time series plot of the y_t values and the truncation limits.

TABLE 1: Statistics for price_t, y_t, and adjusted y_t.

Data	Mean	Maximum	Median	Minimum	SD	Skewness	Kurtosis
price_t	37.032	133.880	23.080	11.350	27.112	1.415	1.062
y_t	0.477	39.219	1.212	−39.419	8.780	−0.428	3.144
Adjusted y_t	0.548	22.427	1.212	−22.350	8.077	−0.237	0.471

4. EMPIRICAL RESULTS

4.1. Model Estimation

We postulate that there are two hidden states, fall and rise, that determine the random behaviour of oil price fluctuations, so $N = 2$. Our unreported analysis indicates that setting $N = 3$ makes the model more complex without an observable improvement to the fit. We apply CL and FL under the two-state Markov regime-switching model to analyze the volatility of world oil prices. We wrote our own R-code for the CL method but used the CARN R-Package *HiddenMarkov* for the FL method; this package was downloaded from http://cran.at.r-project.org/web/packages/HiddenMarkov. The parameter estimates are presented in Table 2.

There are significant discrepancies in the parameter estimates between the CL and FL methods. Specifically, according to CL, the world oil price averages a −1.573% monthly change in state 1 and a 2.028% monthly change in state 2. The corresponding figures for FL are −1.956% and 0.886%. This implies that there are large differences in the mean estimates between CL and FL especially for state 2, and it illustrates that the two states of the world oil price are generally different in mean. Furthermore, the CL estimate of the standard error is 9.883 for state 1 and 6.074 for state 2, while the corresponding figures for FL are 14.275 and 6.735. Hence, the CL results indicate that drops in oil prices are more volatile than rises.

Moreover, the CL and FL methods give noticeably different estimates of the transition probabilities in the two states. Here p_{11} is the probability that the oil price continues to fall next month if it fell this month; p_{12} is the probability that the price will rise next month if it fell this month; p_{22} is the probability that the price continues to rise next month if it rose this month; and p_{21} is the

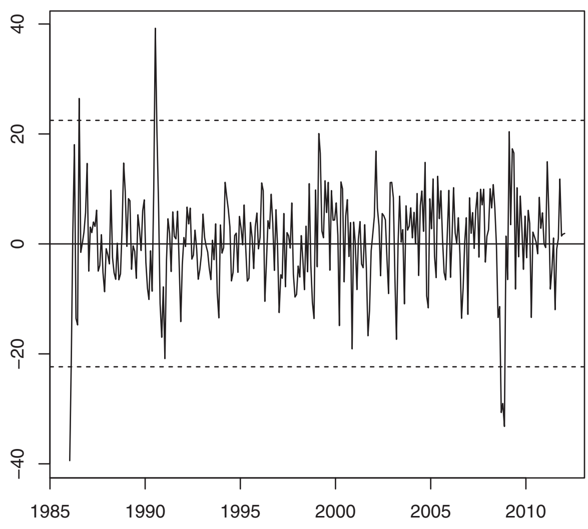

FIGURE 2: WTI crude-oil monthly price change rate y_t.

TABLE 2: Estimation results of two-state Markov regime-switching model.

	CL			FL		
	Estimate	Standard error	T-statistic	Estimate	Standard error	T-statistic
μ_1	−1.573	2.348	−0.670	−1.956	2.752	−0.711
μ_2	2.028	1.022	1.984	0.886	0.429	2.065
σ_1	9.883	1.600	6.177	14.275	2.035	7.015
σ_2	6.074	1.048	5.796	6.735	0.347	19.409
p_{11}	0.876	0.181	4.840	0.869	0.076	11.434
p_{22}	0.934	0.157	5.949	0.986	0.011	89.636

probability that the price will fall next month if it rose this month. The point estimates for p_{11} are 0.876 for CL and 0.869 for FL, while the estimates for p_{22} are 0.934 and 0.986. These probabilities indicate that if the oil price is in either the fall state or the rise state, it is likely to stay in that state. We can also calculate the average expected duration of state 1 or state 2 as $(1 - p_{ij})^{-1}$. For CL the average expected duration is 8.065 months for regime 1 and 15.152 months for regime 2, while the corresponding figures for FL are 7.634 and 71.429 months. Hence, the CL results suggest more frequent regime switching. If we directly examine the price trends, we find that prices rise for at most 17 months (from January 2007 to May 2008) and fall for at most 8 months (from June 2008 to January 2009). These findings show that the actual price fluctuation is relatively large and it is difficult to maintain the same state (up or down) for a long time because of the frequent regime switching. Therefore, the CL results are more accurate than the FL results.

4.2. State Smoothed Probability

This section presents the state smoothed probability. This is the probability distribution over hidden states for a point at time t in the past, relative to time T. Mathematically, given the full sample of available ex-post information $(y_1, y_2, \ldots, y_T)$, the smoothed probability at some date t is $p(s_t = i | y_1, y_2, \ldots, y_T; \theta)$ for $i = 1, 2$.

Figure 3 shows the state smoothed probabilities in the two-state Markov regime-switching model for CL and FL. The solid (black) line is the estimated probability of state 1, and the dashed (red) line is the estimated probability of state 2. Using the smoothed probability, we can clearly observe the state at every moment and its duration. As can be seen from Figure 3, the state smoothed probabilities for CL and FL are radically different. If the smoothed probability of state 1 is greater than 0.5, the price is in the rise state; otherwise, it is in the fall state. According to this classification, FL determines most cases to be in the fall state, which is significantly different from the actual situation. In contrast, the wave crest of the rise state determined by CL is significantly narrower, although CL also finds more rise states than fall states. The CL smoothed results suggest that there are frequent regime shifts rather than the upward trend suggested by FL. The CL outcome matches reality much more closely.

4.3. Testing

One might be interested in many other aspects of the fitted model. For instance, is there a periodic structure in the hidden Markov chain? That is, assuming the existence of exactly two states, we explore whether the transition probabilities satisfy

$$H_0' : p_{11} = p_{21}, \quad \mu_1 \neq \mu_2 \text{ and } \sigma_1 \neq \sigma_2.$$

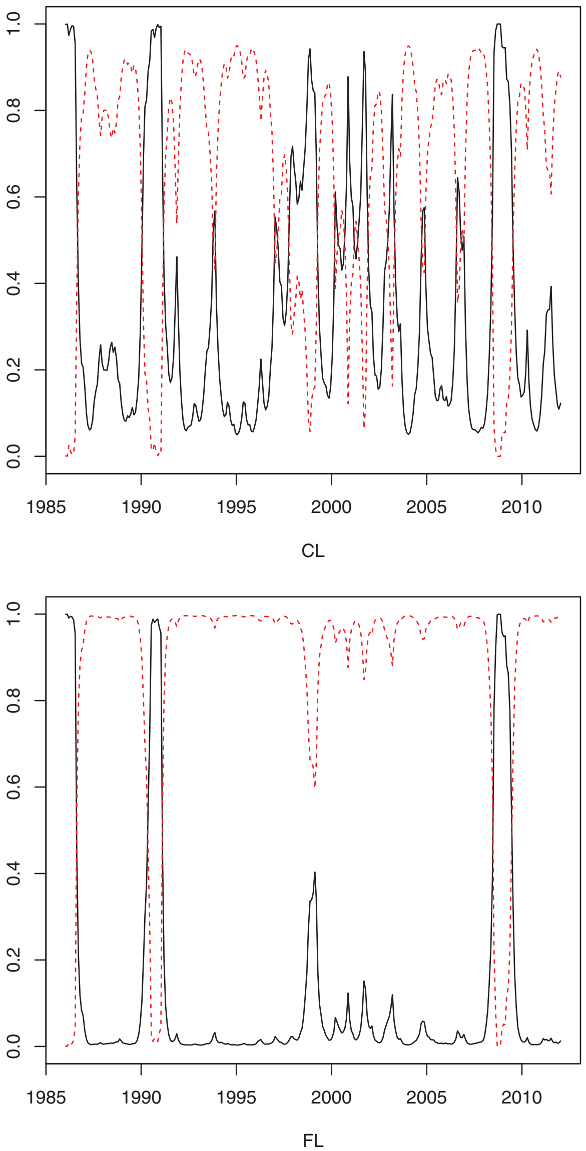

FIGURE 3: Transitional probabilities in two-state Markov regime-switching. *Note*: Figure CL shows the state smoothed probabilities for CL, Fig. FL shows the state smoothed probabilities for FL. The solid (black) line is the estimated probability of state 1, and the dashed (red) line is the estimated probability of state 2. [Colour figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

Note that under H_0', the probability of $s_{t+1} = 1$ is always p_{11}, regardless of whether s_t is the fall state or the rise state. In other words, the distribution of s_{t+1} is independent of s_t, so this hypothesis test examines whether CL and FL can effectively recognize p_{11} and p_{22}.

Another potentially interesting null hypothesis is that the two regime-specific distributions differ in volatility but not in trend. This question could be addressed via a hypothesis test on

$$H_0'' : \mu_1 = \mu_2 \quad \text{and} \quad \sigma_1 \neq \sigma_2.$$

TABLE 3: Likelihood ratio test results.

	$H_0' : p_{11} = p_{21}$		$H_0'' : \mu_1 = \mu_2$	
Method	LR	P-value	LR	P-value
CL	13.441	0.007	9.587	0.047
FL	22.940	0.001	1.367	0.388

Note that under H_0'', the states have the same mean but different variances. That is, the change in oil prices follows a stochastic process.

To date, no rigorous procedures have been developed for hypothesis testing. With two likelihoods available, we conduct likelihood ratio tests as follows. Let

$$LR = 2\{l(\hat{\theta}) - l(\hat{\theta}_0)\},$$

where $\hat{\theta}_0$ denotes the restricted CL or FL estimate of the parameter vector θ, and $\hat{\theta}$ the unconstrained estimate.

We lack theory on the distribution of LR in either case, so we use simulation to suggest a critical value for each version of the likelihood. Specifically, we generated 1,000 sets of time series with $T = 313$ from a normal model fitted under the null hypothesis using CL and FL, respectively. For each time series, we obtained the CL and FL parameter estimates and calculated $l(\hat{\theta})$, $l(\hat{\theta}_0)$, and LR. We used these simulated LR values to approximate the LR distribution. We then computed the P-value of the LR test as the proportion of times that the simulated LR value exceeds the observed LR value under CL and FL, respectively.

Table 3 reports this LRT statistic and the P values for the two tests. The first null hypothesis of $p_{11} = p_{21}$ is rejected by both the CL and FL likelihood ratio tests. This indicates that there is strong evidence that the cycles of the falling and rising regimes are distinct. The second null hypothesis is rejected by the CL likelihood ratio test but not by the FL test at the 5% level. Since the CL model matches reality more closely, this difference is a reflection of the incompetence of the FL method when the regime-switch model assumption is not fully satisfied.

4.4. Forecasting

To further investigate the performance of the two estimators, we calculated in-sample and out-sample performance measures for both CL and FL.

The in-sample fitted values of $\hat{y}_t$ ($t = 1, 2, \ldots, T$) are calculated as linear combinations:

$$\hat{y}_t = w_{t1}\hat{\mu}_1 + w_{t2}\hat{\mu}_2,$$

where w_{tj} ($j = 1, 2$) is the state smoothed probability $Pr(s_t = j | y_1, y_2, \ldots, y_T)$.

Specifically, w_{tj} is calculated by applying the forward-backward recursive formula first proposed by Baum et al. (1970) with the following quantities:

$$a_j(1) = \hat{p}_j \phi(y_1; \hat{\mu}_j, \hat{\sigma}_j), \quad \text{for } j = 1, 2,$$

$$a_j(t) = \sum_{k=1}^{2} \hat{p}_{kj} \phi(y_t; \hat{\mu}_j, \hat{\sigma}_j) a_k(t-1), \quad \text{for } t = 2, \ldots, T,$$

$$b_j(t) = \sum_{k=1}^{2} \hat{p}_{jk}\phi(y_t; \hat{\mu}_k, \hat{\sigma}_k)b_k(t+1), \quad \text{for } t = 1, 2, \ldots, (T-1),$$
$$b_j(T) = 1,$$
$$w_{tj} = \frac{a_j(t)b_j(t)}{a_1(T) + a_2(T)}, \quad \text{for } t = 1, 2, \ldots, T.$$

The in-sample mean squared error is computed as

$$\mathrm{MSE}_{\mathrm{in}} = T^{-1}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2.$$

Next, we calculate the out-sample predicted values of $\tilde{y}_t$ $(t = T+1, T+2, \ldots, T+T^*)$, based on the same in-sample estimates $\hat{\theta}$ as follows:

$$\tilde{y}_t = \tilde{w}_{t1}\hat{\mu}_1 + \tilde{w}_{t2}\hat{\mu}_2$$

where $\tilde{w}_{tj}$ $(j = 1, 2)$ is the estimated conditional probability $Pr(s_t = j|y_1, y_2, \ldots, y_{t-1})$ determined by

$$\tilde{w}_{tj} = \sum_{k=1}^{2} \hat{p}_{kj} w_{(t-1)k}.$$

$w_{(t-1)k}$ can be obtained by the following forward-recursive formulas:

$$a_j(1) = \hat{p}_j\phi(y_1; \hat{\mu}_j, \hat{\sigma}_j), \quad \text{for } j = 1, 2,$$
$$a_j(s) = \sum_{k=1}^{2} \hat{p}_{kj}\phi(y_t; \hat{\mu}_j, \hat{\sigma}_j)a_k(s-1), \quad \text{for } t = 2, \ldots, (t-1),$$
$$w_{(t-1)k} = \frac{a_k(t-1)}{a_1(t-1) + a_2(t-1)}, \quad \text{for } t = T+1, T+2, \ldots, T+T^*, k = 1, 2.$$

The out-sample performance measure is then given by

$$\mathrm{MSE}_{\mathrm{out}} = T^{*(-1)}\sum_{t=T+1}^{T+T^*}(\tilde{y}_t - y_t)^2.$$

For the in-sample performance, we use all the data to estimate the two-state Markov regime-switching model with CL and FL. Next, the parameters are re-estimated to forecast the out-sample values over four test periods. Period 1 is from June 2008 to February 2012, period 2 from January 2009 to February 2012, period 3 from January 2011 to February 2012, and period 4 from October 2011 to February 2012. We chose four test periods because a comparison of multi-period out-sample performance is more convincing. Furthermore, the obvious turning point occurred at the beginning of the four periods, and it is useful for forecasting the out-sample values containing turning points for CL and FL.

Table 4 reports the relative performance of CL and FL, indicating that CL is superior to FL for the in-sample. It is not as superior for the out-sample. Figure 4 presents a plot of the actual, estimated, and forecast monthly changes in world crude-oil prices based on CL and FL.

TABLE 4: Forecast performance of CL and FL.

		MSE_{out}			
Method	MSE_{in}	Period 1	Period 2	Period 3	Period 4
FL	64.895	94.917	63.166	45.198	25.031
CL	61.133	92.529	64.372	44.940	23.922
Improvement	6.154%	2.516%	−1.910%	0.571%	4.431%

The solid (red) line is the actual monthly change after the data truncation, the dashed (green) line is the forecast monthly change based on the CL estimates, and the dotted (blue) line is the forecast monthly change based on the FL estimates. Figure 4 suggests that the forecast monthly change based on CL is closer to the actual fluctuation. Our analysis results are consistent with the statistical simulation results of Chen & Wang (2012).

4.5. Rationality of Data Adjustment

We will now discuss the rationality of data adjustment. For the purposes of comparison, we also fitted the two-state Markov regime-switching model based on CL and FL to the original unadjusted data. The parameter estimates are reported in Table 5. These results are different from those obtained after the data truncation (Table 2). The CL and FL estimators now tend to overestimate the negative trends and underestimate the positive trends in the oil prices. The differences are more apparent for CL. On the one hand, this suggests that these extreme values affect the performance of the two-state Markov regime-switching model. On the other hand, this indicates that the CL approach is not robust and is more vulnerable to extreme values.

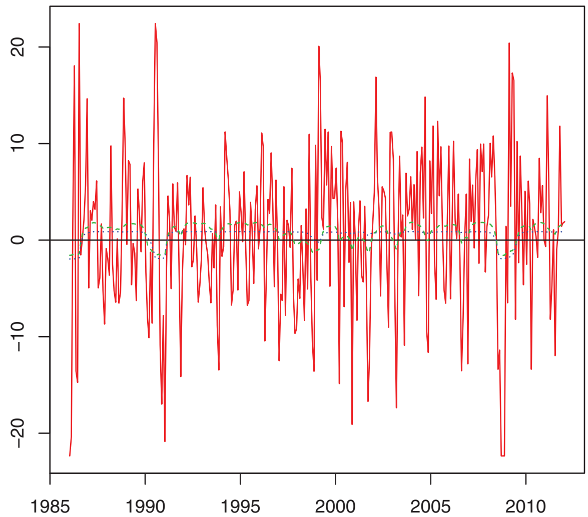

FIGURE 4: Actual, estimated, and forecast monthly changes in world crude-oil price based on CL and FL. *Note*: The solid (red) line is the actual monthly change after the data truncation, the dashed (green) line is the forecast monthly change based on the CL estimates, and the dotted (blue) line is the forecast monthly change based on the FL estimates. [Colour figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

TABLE 5: Results for two-state Markov regime-switching model before data truncation.

	CL			FL		
	Estimate	Standard error	T-statistic	Estimate	Standard error	T-statistic
μ_1	−4.351	2.509	−1.734	−3.371	3.77	−0.894
μ_2	0.904	1.229	0.736	0.878	0.42	2.090
σ_1	17.75	1.764	10.062	18.759	2.748	6.826
σ_2	7.396	1.391	5.317	6.82	0.315	21.651
p_{11}	0.583	0.184	3.168	0.869	0.081	10.728
p_{22}	0.977	0.187	5.225	0.99	0.007	141.429

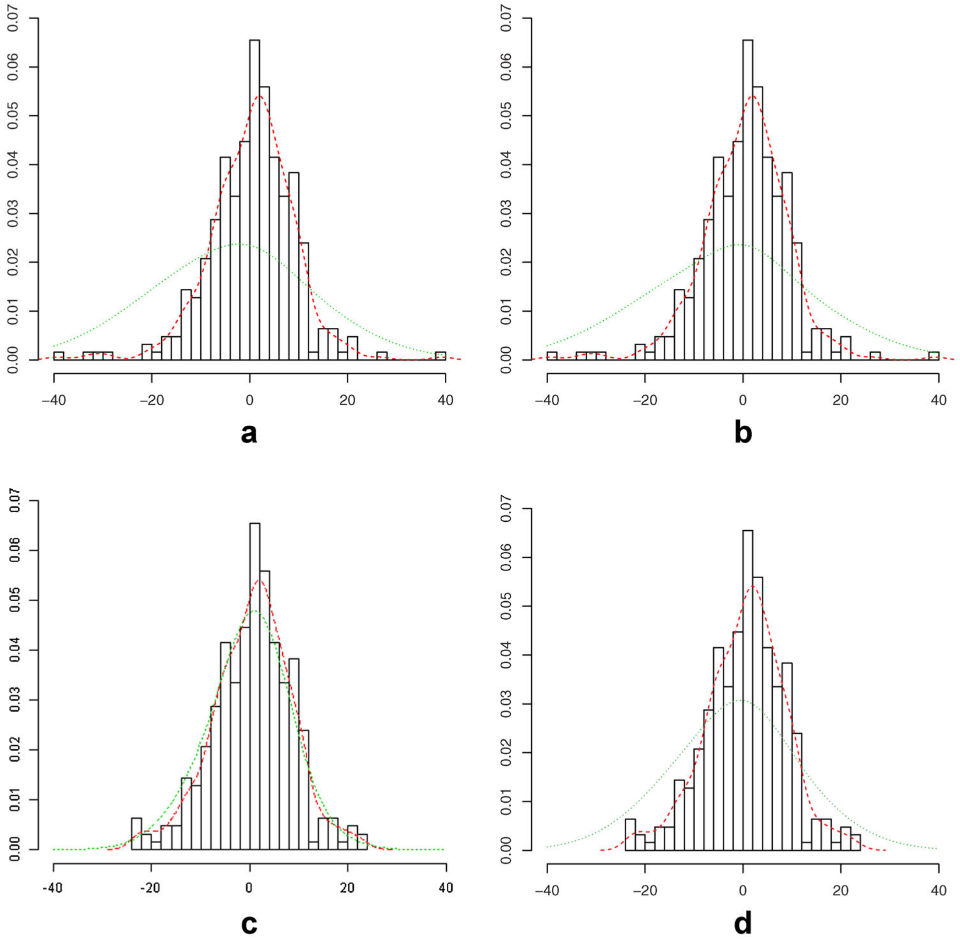

FIGURE 5: Comparison of fitted mixture of two Gaussian distributions before and after data truncation for CL and FL. *Note*: Figure a (b) gives the CL (FL) results before data truncation, and Fig. c (d) gives the CL (FL) results after data truncation. The solid (black) line is the histogram, the dashed (red) line is the kernel density estimate, and the dotted (green) line is the fitted mixture of two Gaussian distributions. [Colour figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

We also fit a mixture of two Gaussian distributions to compare the performance before and after data truncation. We use the kernel density estimate to approximate the actual data distribution. Figure 5 presents a comparison of these results before and after data truncation for CL and FL. Figure 5a,b gives the CL (FL) results before data truncation, and Figure 5c,d gives the CL (FL) results after data truncation. The solid (black) line is the histogram, the dashed (red) line is the kernel density estimate, and the dotted (green) line is the fitted mixture of two Gaussian distributions. We find that after data truncation, both CL and FL perform much better. In particular, for CL the fitted distribution after data truncation is closer to the actual distribution.

This analysis shows that the data truncation improves the fit. The CL results are closer to reality than the FL results when the influence of extreme values is reduced. Therefore, data adjustment is reasonable and effective.

5. CONCLUSION

We have used a two-state Markov regime-switching model to explain the behavior of the WTI crude-oil spot prices from January 1986 to February 2012. We investigated the use of CL and FL methods for the model estimation. Our empirical analysis has demonstrated that CL can better capture the general structural changes in world oil prices. The two-state Markov regime-switching model based on CL closely depicts the cycles of the two postulated states: fall and rise. These two states persist for on average 8 and 15 months, which matches the observed cycles during the period. According to the fitted model, drops in oil prices are more volatile than rises. These empirical findings based on the CL approach may be helpful to financial officers working in related areas.

The fitted model based on FL is less satisfactory. It is inferior to the CL approach for in-sample performance. The FL approach requires the model to correctly specify the joint distribution of the whole series, whereas the CL approach requires the joint distribution of only two adjacent price changes. The two-state Markov regime-switching model based on FL is too rigid and overly simplistic, and model violations are likely to invalidate the results. Hence, we believe that the simpler mathematical structure of CL may be a better choice than FL for examining the behaviour of world oil prices.

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