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# Volatility transmission across markets: a Multichain Markov Switching model

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The integration of financial markets across countries has modified the way prices react to news. Innovations originating in one market diffuse to other markets following patterns which usually stress the presence of interdependence. In some cases, though, covariances across markets have an asymmetric component which reflects the dominance of one over the others. The volatility transmission mechanisms in such events may be more complex than what can be modelled as a multivariate GARCH model. In this article, we adopt a new Markov Switching approach and we suppose that periods of high volatility and periods of low volatility represent the states of an ergodic Markov Chain where the transition probability is made dependent on the state of the 'dominant' series. We provide some theoretical background, and illustrate the model on Asian markets data showing support for the idea of dominant market and the good prediction performance of the model on a multi-period horizon.

## I. Introduction

The information technology revolution has had a tremendous impact on the structure of financial markets since the quick diffusion of information and the increased liberalization of capital movements has fostered integration. Cross-listings of financial assets on different exchanges, increase in the flows of trade, foreign direct investment, outsourcing of productive activities have all contributed to the possibility that shocks to a single economy or market may propagate quickly to the others. Beside improving on market efficiency, a likely effect is that the scope for international diversification has decreased. In the cases we want to model, the

transmission mechanism starts from a 'dominant' market with a shock which is transmitted to the other markets: as an effect they will exhibit similar volatility responses. The leading role can be in the hands of a traditionally strong market (such as the USA) or of markets which assume a specific importance due to their position and role following specific events, such as the Mexican Peso crisis in the early 1990s or the Southeast Asian crisis of October 1997. In the latter case, for example, the crisis originated in the Hong Kong stock market as a response of the Thai Bath devaluation and spread over to the other Asian markets very quickly, with apparent turbulence (expressed in terms of high volatility) in the behaviour of stock exchange quotations.

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The analysis of the spillover effects and the detection of the dominant market have been analysed using several econometric approaches. For example, wavelets are adopted by Lee (2004), who investigates the international transmission mechanism of the stock market movements, finding volatility spillover effects from USA to Korea but not vice versa; a two-factors arbitrage pricing theory model before and after the 1997 Asian crisis is applied by Aquino (2005). Several applications refer to GARCH-type models to study transmission mechanisms across markets; for example Kanas (1998) uses an EGARCH model on data for the main European markets, Suliman (2005) studies the presence of contagion from Mexican exchange rate changes on US interest rates and Japanese exchange rates. Multivariate GARCH models with suitable parameterizations (e.g. Engle and Kroner, 1995; Engle, 2002; Tse and Tsui, 2002) allow for studying the dynamics of conditional covariances as representing co-movements in second moments; for example, this approach was used by Higgs and Worthington (2004), who study the transmission of returns and volatility among eight major art markets.

The presence of large switches in the volatility structures of the series has suggested the adoption of Markov Switching features in the GARCH models (Dueker, 1997; Brunetti *et al.*, 2003), whereby two different regimes are isolated, corresponding to the states of low and high volatility ('ordinary' and 'turbulent' regimes). To be sure, the GARCH models and the classical Markov Switching models are not capable to capture the features of the transmission mechanisms across markets when some sort of asymmetric role is present ('dominant' and 'dominated' markets). As an alternative, one could adopt the **Multi-chain Markov Switching (MCMS) model** (Otranto, 2005), in which the state in one dominant market propagates its effects to other markets asymmetrically. In the next section the data set used in this study is introduced by providing some evidence for the existence of regimes. Section III describes the main features of the model proposed discussing how it is suitable to describe dynamics in a crisis situation. The main results of our application are contained in Section IV, where we characterize the features of the markets involved, assuming a leading role for Hong Kong. In particular an interesting aspect is that the MCMS model has markedly better prediction performances relative to a Vector Autoregressive [VAR] model over a two- and three-steps ahead horizon. In Section V we discuss our choice of modelling the phenomena taking the range as dependent variable over a possible alternative of considering the log-range. Some evidence shows that logarithmic

transformations are less suitable to analyse more turbulent periods and the links that may be present across markets in such periods. Concluding remarks follow.

## II. The Data and the Volatility Proxy

The Asian market is a classical example for which the shocks starting from a dominant market are transmitted quickly to the others with different effects; there are many studies in literature that underline this spillover feature of links among markets. A well-known study was conducted by Forbes and Rigobon (2002), who distinguish between contagion and interdependence: both phenomena are characterized by co-movements in stock markets in a bivariate setting, but the former exhibits an increase in the correlation after a shock in one country, provided the correlation coefficient is adjusted for heteroskedasticity. According to this methodology, the two authors show that the shock originating from Hong Kong in October 1997 has not implied a significant increase in the correlation coefficients with the other main Asian markets: the conclusion reached is that the co-movements between the series analysed cannot be considered as a form of contagion from Hong Kong, but only as interdependence.

The co-movements in the volatility of markets can be captured well by bivariate SWARCH models (an extension to the multivariate case of the model of Hamilton and Susmel, 1994), as underlined by Edwards and Susmel (2001, 2003). Their approach shows a strong evidence for common state-varying volatility in Hong Kong and several Latin America markets.

In our approach, we verify the presence of state-varying volatility, studying the mechanisms of its transmission, applying the idea of MCMS model proposed by Otranto (2005) to series representing the volatility of some Asian markets. As shown in the next section, with this approach the Markov Switching model is consistent with the presence of a dominant market from which the contagion starts. The main difference with respect to the Forbes and Rigobon (2002) approach is that the contagion is not represented simply by a linear correlation between the markets, but it depends on asymmetric and nonlinear relationships between the two markets.

We analyse the stock market indices of five Asian countries starting from daily data spanning a period between 7 February 1994 and 9 January 2004; the indices are the Hang Seng index (Hong Kong-HS hereafter), the KOSPI index (South Korea-KSE),

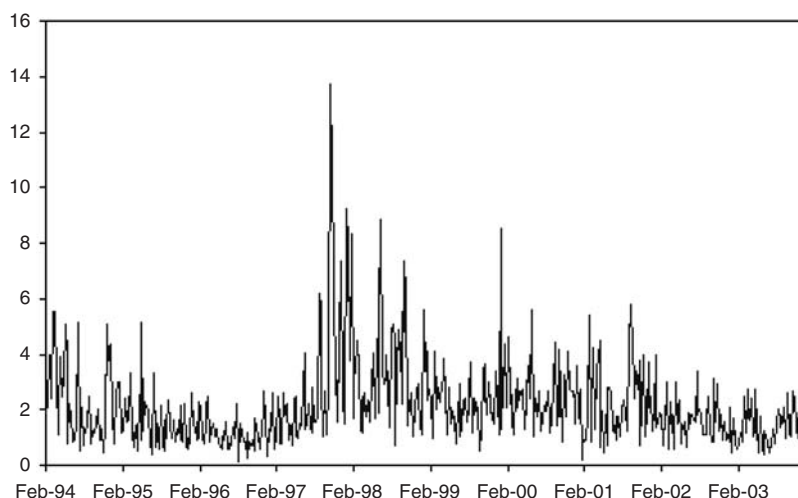


Fig. 1. Hang Seng volatility

the KLSE index (Malaysia-KLSE), the PSE index (Philippines-PSE) and the Straits Times index (Singapore-SGX). To obtain a proxy of the volatility, we compute the range of the logarithms of the closing prices over a week:

$$R_{i,t} = \max_{0 \leq j \leq 5} \ln(I_{i,jt}) - \min_{0 \leq j \leq 5} \ln(I_{i,jt}),$$

$$t = 1, \dots, T$$

where  $i$  is the stock market considered,  $t$  the week and  $j$  the day of the week  $t$  ( $T = 516$ ). Parkinson (1980) showed that the range, rescaled by an appropriate constant, is an unbiased estimator of the volatility parameter in a diffusion process. In addition, Brunetti and Lildholdt (2002) show that the range is superior to the returns as far as volatility proxies are concerned. Following the results of Brunetti *et al.* (2003), we have then computed the volatility proxy:

$$\hat{v}_{i,t} = \sqrt{\frac{\pi}{8}} R_{i,t} \quad (1)$$

Stylized facts suggest that this series is well represented as an Autoregressive Moving Average [ARMA].

The proxy used provides the HS series shown in Fig. 1; the East Asian crises shows its most evident effect in the fourth week of October, in which the volatility increases more than 300%. The dramatically high volatility of this period is a common feature of all the series analysed (Fig. 2), with a different degree of persistence and depth; the PSE volatility increases in the same week of 681%, whereas SGX and KLSE 271 and 129%, respectively;

the KSE seems to have a lagged behaviour, showing a clear increase in volatility only a week after.

The existence of at least two regimes is clear observing the graphs of the series in Fig. 3, in which the scatter plots of each series with respect the HS index are plotted. Note that the periods of ordinary volatility are frequent and form a dense agglomerate of points in a neighbourhood of the origin; at times the periods of turbulence follow a common dynamics (such as the SGX/HS case), in others they are not synchronized. The effect of the transmission of volatility could arise in different ways. In the four pictures the weeks relative to the Hong Kong crash (fourth week of October, maximum peak of the volatility) and the two successive weeks are explicitly marked. It is clear that the Malaysia market absorbs this shock very quickly, without particular turbulence; the Philippines and the Singapore markets suffer for this shock in the first two weeks, the former with a gradual return to normality, the latter with an abrupt change in the third week. The Korean market increases its volatility dramatically in the last two weeks of October, with a persistent effect of the shock.

At any rate, from the graphs it is possible to hypothesize for a bivariate series the presence of co-movements, with possible lags and the presence of regimes characterized by low and high volatility. In particular, the presence of different regimes can be detected by means of the nonparametric Bayesian procedure of Otranto and Gallo (2002). With this approach, using diffuse priors for the distributions of the number of regimes, we obtain the posterior distributions shown in Table 1.<sup>1</sup> All the series shows a strong evidence in favour of

<sup>1</sup> We use the same priors of Otranto and Gallo (2002), with the hyperparameter  $A$ , regulating the prior probabilities of the number of regimes, equal 0.15.

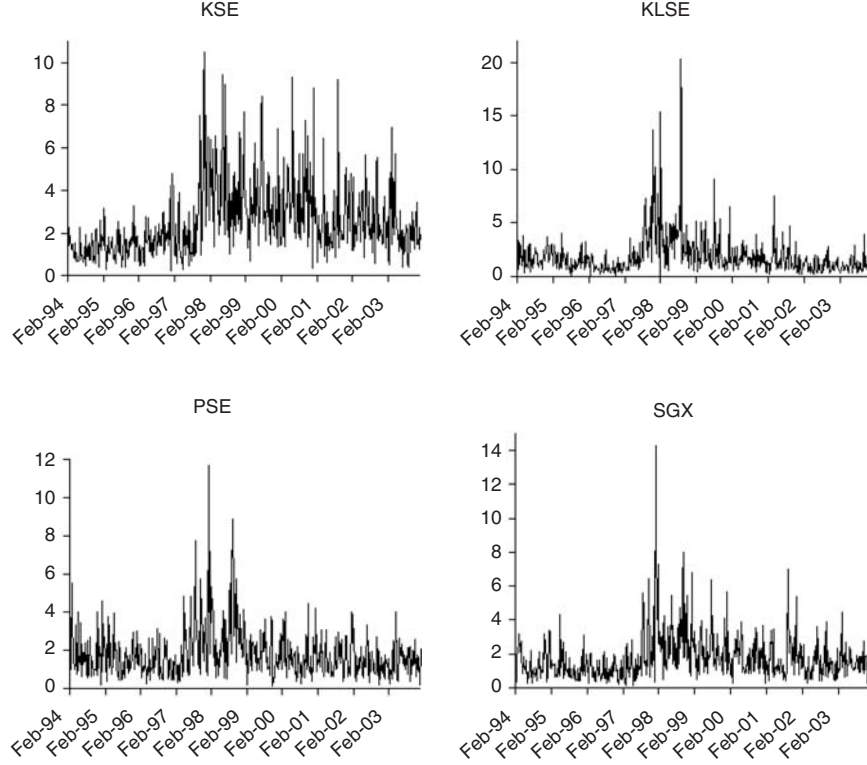


Fig. 2. Volatility of the dominated markets

two regimes (except SGX, which presents very similar probabilities for two and three states). With the exception of HS, the probability of three regimes is larger than the probability of one state for all the series; This result is consistent with the MCMS model, which will be described in the next section.

### III. The Model Proposed

Let us suppose that the  $(2 \times 1)$  vector  $y_t$  follows a VAR(2) model with intercept and variance as switching parameters:

$$\begin{aligned} \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} &= \begin{bmatrix} \mu_{1,s_{1t}} \\ \mu_{1,s_{2t}} \end{bmatrix} + \begin{bmatrix} \beta_{s_{1t},s_{2t}} \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_{11}^1 & \phi_{12}^1 \\ \phi_{21}^1 & \phi_{22}^1 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} \\ &+ \begin{bmatrix} \phi_{11}^2 & \phi_{12}^2 \\ \phi_{21}^2 & \phi_{22}^2 \end{bmatrix} \begin{bmatrix} y_{1,t-2} \\ y_{2,t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \\ \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} &\sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{1,s_{1t}}^2 & \rho\sigma_{1,s_{1t}}\sigma_{2,s_{2t}} \\ \rho\sigma_{1,s_{1t}}\sigma_{2,s_{2t}} & \sigma_{2,s_{2t}}^2 \end{bmatrix} \right) \quad (2) \end{aligned}$$

The second element of  $y_t$  contains the volatility proxy for the dominant market and the first element the volatility proxy for the other market at time  $t$ .

The intercepts and the variances of both indices switch according to the latent random variables  $s_{1t}$

and  $s_{2t}$ ; the value 0 represents the ordinary regime, the value one occurs in the turbulent regime. The difference with respect to the classical multivariate MS models is that  $y_{1,t}$  and  $y_{2,t}$  follow distinct but related state variables. In practice, let us suppose that the *multiple regime*  $(s_{1t}, s_{2t})$  follows an ergodic Markov Chain, so that its realization at time  $t$  depends only on  $(s_{1,t-1}, s_{2,t-1})$ . In addition, let us suppose that the states  $s_{1t}$  and  $s_{2t}$ , conditional on  $(s_{1,t-1}, s_{2,t-1})$ , are independent, so that:

$$\begin{aligned} \Pr[s_{1t}, s_{2t} | s_{1,t-1}, s_{2,t-1}] &= \Pr[s_{1t} | s_{1,t-1}, s_{2,t-1}] \\ &\times \Pr[s_{2t} | s_{1,t-1}, s_{2,t-1}] \quad (3) \end{aligned}$$

The right side of Equation 3 can be parameterized with logistic functions to take into account the influence of the regime relative to the dominant market:

$$\begin{aligned} \Pr(s_{1t} = h | s_{1,t-1} = h, s_{2,t-1}) &= \frac{\exp(\vartheta_{h0}^1 + \vartheta_{h1}^1 s_{2,t-1})}{1 + \exp(\vartheta_{h0}^1 + \vartheta_{h1}^1 s_{2,t-1})} \\ \Pr(s_{2t} = h | s_{2,t-1} = h, s_{1,t-1}) &= \frac{\exp(\vartheta_{h0}^2)}{1 + \exp(\vartheta_{h0}^2)} \\ \Pr(s_{jt} = k | s_{j,t-1} = h, s_{i,t-1}) &= 1 - \Pr(s_{jt} = h | s_{j,t-1} = h, s_{i,t-1}) \\ &h, k = 0, 1, \quad h \neq k, \\ &i, j = 1, 2, \quad i \neq j \quad (4) \end{aligned}$$

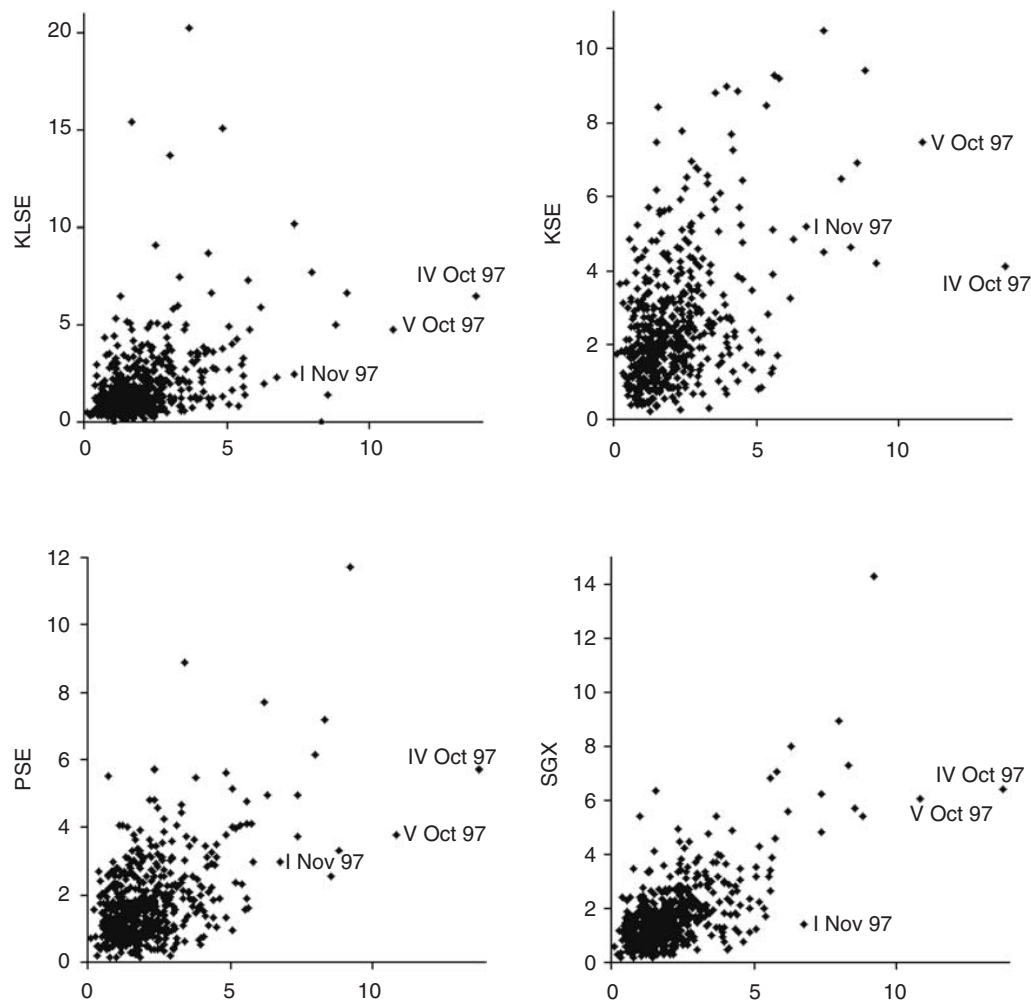


Fig. 3. Scatterplots of the volatility between Hong Kong and other markets

Table 1. Empirical posterior distribution of the number of regimes

	1	2	3	4	5
HS	0.212	0.678	0.100	0.010	0.000
KSE	0.054	0.840	0.102	0.004	0.000
KLSE	0.000	0.502	0.398	0.090	0.010
PSE	0.006	0.800	0.176	0.018	0.000
SGX	0.000	0.454	0.470	0.074	0.002

From the parameterization in Equation 4 we can note that the state of variable two (the dominant market) at time  $t-1$  influences the probability of variable one to stay in the previous regime, but not vice-versa. In this way, the estimations of the probabilities in Equation 4 show how the transition probabilities for variable one changes according to the regime of variable two. We would expect the signs of coefficients  $\vartheta_{01}^1$  and  $\vartheta_{11}^1$  to be negative, respectively, positive.

Disposing the estimated transition probabilities in Equation 3 in a matrix, with rows representing the multiple state at time  $t-1$  and columns the multiple state at time  $t$ , it is possible to evaluate the most probable scenario (a particular combination of  $s_{1t}$  and  $s_{2t}$ ) at time  $t$ , given a certain state at time  $t-1$ .

Let us note that, from model (2), the intercept of the first variable varies with the regime of the dominant market; in particular it will be  $\mu_{1,0}$  when



$s_{1t}=0$  and  $s_{2t}=0$ ,  $\mu_{1,0} + \beta_0$  when  $s_{1t}=0$  and  $s_{2t}=1$ ,  $\mu_{1,1}$  when  $s_{1t}=1$  and  $s_{2t}=0$ ,  $\mu_{1,1} + \beta_1$  when  $s_{1t}=1$  and  $s_{2t}=1$ ; the expected signs of  $\beta_0$  and  $\beta_1$  are both positive. In other terms, for the dominated market, we avoid the possibility that the intercept can assume more than two values; this idea seems consistent with results of Table 1.

The coefficient  $\rho$  in Equation 2 varies in  $[-1, 1]$  and the usual stationarity constraints hold for the autoregressive coefficients.

The model is estimated by maximum likelihood with the EM algorithm following a procedure which is detailed in the Appendix to Otranto (2005): it consists of a simple extension of the filtering algorithm of Hamilton (1990). As it happens in the Markov Switching family of models, the likelihood function is expressed in terms of mixtures of  $m$  normal densities ( $m=4$  in this case) which accommodates substantial departures from unconditional normality.

#### IV. The Results

We have estimated four MCMS models, one for each bivariate series, holding Hong Kong as the dominant market; for the same variables we have estimated classical VAR [Equation 2] models (without switching parameters). In Table 2 the parameter estimations are shown (empty cells signify that we have re-estimated the models excluding the corresponding parameters as statistically insignificant). We can note that the signs of the parameters of the logistic functions,  $\beta_0$  and  $\beta_1$  are consistent with our expectations. The insignificant values of  $\vartheta_{10}^2$  denote a low persistence of the state of turbulence for the HS index across markets. The intercept of the dominated variable when the multiple state changes adjusts gradually: for the KSE index it is equal to 1.054 when  $s_{1t}=0$  and  $s_{2t}=0$ , 1.890 when  $s_{1t}=0$  and  $s_{2t}=1$ , 3.001 when  $s_{1t}=1$  and  $s_{2t}=0$ , 6.134 when  $s_{1t}=1$  and  $s_{2t}=1$ ; for KLSE and SGX there is no difference in the intercept when they are in the turbulent state and HS switches from the ordinary to the turbulent regime; the same holds for PSE when it is in the ordinary regime.

As in other Markov Switching models, it is possible to obtain the inference on the regime for each period in terms of smoothed probabilities, applying the Kim (1994) filter to the bivariate series. In this way we obtain the probability of  $s_{1t}=i$ ,  $s_{2t}=j$ , ( $i, j=0, 1$ ),

given the full information available; summing the probability of  $s_{1t}=0$ ,  $s_{2t}=0$  and  $s_{1t}=0$ ,  $s_{2t}=1$ , we obtain the probability of  $s_{1t}=0$  (and similarly in order to obtain  $s_{2t}=0$ ). In Fig. 4 we show the smoothed probabilities relative to a time-span of 13 weeks, centred on the fourth week of October 1997, for each MCMS model estimated. Clearly the inference on the regime of HS index is not the same, given that it is based on the different interaction between the dominated variable and the dominant variable. At any rate, an aspect in common is the fact that the HS index exhibits a certain degree of turbulence one month before the largest peak, and needs about three weeks after that to show the first signs of reversal to a quiet period. We recall that Thailand announced that the Bath would float on 2 July 1997 and it dropped in value in the next six months, falling to less than half its former value in January 1998. Our models suggest that the Hong Kong market has reacted to the Thailand crises in terms of a strong turbulence two-three months after that announcement and the operators needed several weeks to adapt their expectations to the new situation.

The graphs in Fig. 4 confirm that the dominated markets, excluding KSE, follow the behaviour of HS immediately, showing turbulence in the fourth week of October and the switch to the ordinary volatility regime in the first week of November (KLSE) or the third week of November (PSE and SGX). As previously noted, the KSE index shows turbulence a week after the HS index, returning to the normality in the second week of November. Let us also note that the successive less dramatic switch to turbulence of HS in the third week of November is transmitted to the other markets 1 or 2 weeks after (apart from KLSE).

The better performance of the MCMS models in terms of fitting is documented by the values of the AIC and Schwartz criteria<sup>2</sup> (Table 3). A further step can be taken by comparing the values of the predictions with the actual data. This can be done in a number of ways: we can consider traditional, so-to-speak, loss functions where we perform a comparison between  $\tau$ -steps ahead predictions<sup>3</sup> and data adopting Mean Square Errors (MSE) and Mean Absolute Errors (MAE), defined as

$$\begin{aligned} \text{MSE}_\tau &= T^{-1} \sum_{t=1}^{T-\tau} [y_{t+\tau} - \hat{y}_{t+\tau|t}]^2 \\ \text{MAE}_\tau &= T^{-1} \sum_{t=1}^{T-\tau} |y_{t+\tau} - \hat{y}_{t+\tau|t}| \end{aligned} \quad (5)$$

<sup>2</sup>The AIC and SCH functions are calculated as in Hamilton and Susmel (1994).

<sup>3</sup>In the applications we use  $\tau=1, 2, 3$ .

Table 2. Estimated parameters

	KSE/HS		KLS/HS		PSE/HS		SGX/HS	
	MCMS	VAR	MCMS	VAR	MCMS	VAR	MCMS	VAR
$\mu_{1,0}$	1.054 (0.066)	1.509 (0.091)	0.765 (0.043)	0.900 (0.090)	0.844 (0.052)	1.186 (0.068)	0.604 (0.028)	1.070 (0.066)
$\mu_{1,1}$	3.001 (0.127)		3.864 (0.388)		2.385 (0.102)		2.201 (0.137)	
$\beta_0$	0.836 (0.091)		1.501 (0.056)				1.006 (0.027)	
$\beta_1$	3.133 (0.194)				0.471 (0.140)			
$\mu_{2,0}$	1.020 (0.062)	1.315 (0.082)	1.181 (0.055)	1.246 (0.071)	0.858 (0.067)	1.250 (0.078)	0.909 (0.062)	1.303 (0.073)
$\mu_{2,1}$	3.446 (0.184)		2.685 (0.150)		3.025 (0.162)		2.313 (0.121)	
$\phi_{11}^1$	0.158 (0.025)	0.345 (0.031)	0.167 (0.020)	0.359 (0.031)	0.133 (0.025)	0.276 (0.034)	0.041 (0.018)	0.261 (0.037)
$\phi_{11}^1$	-0.129 (0.024)	0.099 (0.035)	-0.049 (0.016)	0.182 (0.037)	-0.073 (0.021)	0.068 (0.027)	0.033 (0.015)	0.118 (0.033)
$\phi_{21}^1$	0.065 (0.017)	0.058 (0.027)	0.119 (0.022)	0.169 (0.024)	0.072 (0.029)	0.160 (0.039)	0.053 (0.033)	0.175 (0.041)
$\phi_{22}^1$		0.344 (0.032)	0.133 (0.026)	0.294 (0.030)	0.068 (0.023)	0.308 (0.033)	0.079 (0.026)	0.270 (0.036)
$\phi_{11}^2$	0.152 (0.025)	-0.030 (0.005)	0.076 (0.019)	-0.032 (0.006)	0.067 (0.020)	-0.019 (0.005)	0.134 (0.012)	-0.017 (0.005)
$\phi_{12}^2$	-0.004 (0.002)			-0.008 (0.003)	0.035 (0.018)			-0.003 (0.002)
$\phi_{21}^2$	0.085 (0.011)			-0.007 (0.002)	0.097 (0.013)	-0.006 (0.003)	0.091 (0.012)	-0.008 (0.004)
$\phi_{22}^2$		-0.030 (0.005)	-0.004 (0.002)	-0.022 (0.004)		-0.024		-0.018
$\sigma_{1,0}$	0.439 (0.021)	1.110 (0.024)	0.318 (0.011)	1.219 (0.027)		0.839 (0.018)		0.886 (0.019)
$\sigma_{1,1}$	0.696 (0.041)		2.509 (0.169)		0.325 (0.013)		0.211 (0.007)	
$\sigma_{2,0}$	0.458 (0.022)	1.000 (0.022)	0.529 (0.017)	0.979 (0.022)	0.860 (0.030)	0.994 (0.022)	1.296 (0.056)	0.994 (0.022)
$\sigma_{2,1}$	1.295 (0.065)		1.391 (0.061)		0.431 (0.019)		0.428 (0.015)	
$\rho$	0.254 (0.060)	0.388 (0.026)	0.100 (0.043)	0.278 (0.029)	1.157 (0.047)	0.456 (0.025)	1.205 (0.040)	0.608 (0.020)
$\vartheta_{00}^1$	1.376 (0.197)		2.753 (0.298)		0.589 (0.031)		0.422 (0.035)	
$\vartheta_{01}^1$	-0.802 (0.362)		-1.610 (0.390)		0.932 (0.198)		1.916 (0.263)	
$\vartheta_{10}^1$	-0.246 (0.210)		-1.528 (0.788)		-0.930 (0.351)		-1.736 (0.323)	
$\vartheta_{11}^1$	0.631 (0.441)		1.198 (0.945)		-0.310 (0.217)		-2.172 (0.781)	
$\vartheta_{00}^2$	1.657 (0.176)		1.294 (0.149)		0.691 (0.377)		1.752 (0.852)	
$\vartheta_{10}^2$	-0.181 (0.230)		-0.045 (0.064)		1.119 (0.178)		0.654 (0.144)	
					-0.421 (0.218)		0.199 (0.158)	



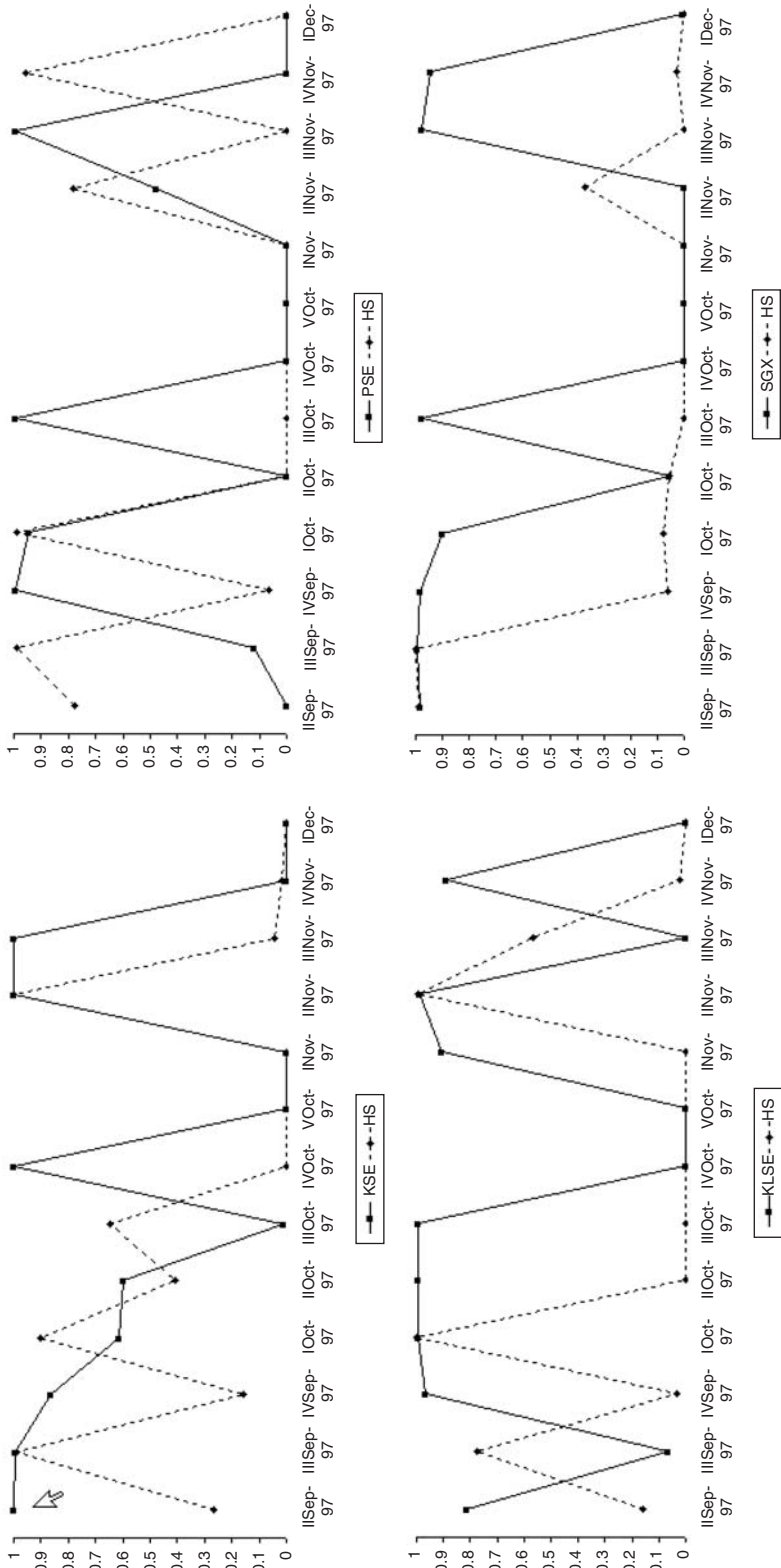


Fig. 4. Smoothed probabilities for the ordinary regimes

Table 3. Likelihood functions

	KSE/HS MCMS	VAR	KLS/HS MCMS	VAR	PSE/HS MCMS	VAR	SGX/HS MCMS	VAR
Log-Like	-1187.4	-2003.6	-1110.7	-2120.9	-981.7	-1672.0	-868.6	-1611.0
AIC	-1210.4	-2014.6	-1132.7	-2133.9	-1004.7	-1684.0	-890.6	-1624.0
SCH	-1259.2	-2038.0	-1179.4	-2161.6	-1053.6	-1709.5	-937.3	-1651.6

Table 4. MCMS model relative to VAR model predictions. One, two and three steps ahead loss function ratios

	1-step		2-steps		3-steps	
	KSE	HS	KSE	HS	KSE	HS
MSE	0.99	1.06	0.62	0.77	0.60	0.74
MAE	0.99	0.97	0.78	0.81	0.77	0.79
LSE	0.98	0.93	0.71	0.70	0.69	0.68
LAE	0.99	0.97	0.78	0.81	0.77	0.79
	KLSE	HS	KLSE	HS	KLSE	HS
MSE	0.98	1.02	0.80	0.92	0.80	0.86
MAE	0.99	1.01	0.79	0.96	0.77	0.92
LSE	0.99	1.01	0.74	0.93	0.72	0.88
LAE	0.99	1.01	0.79	0.96	0.77	0.92
	PSE	HS	PSE	HS	PSE	HS
MSE	1.02	1.06	0.87	0.90	0.86	0.87
MAE	1.02	0.98	0.91	0.88	0.90	0.86
LSE	1.01	0.94	0.91	0.80	0.90	0.77
LAE	1.02	0.98	0.91	0.88	0.90	0.86
	SGX	HS	SGX	HS	SGX	HS
MSE	1.02	1.05	0.86	0.94	0.86	0.91
MAE	0.98	0.97	0.83	0.90	0.84	0.89
LSE	0.98	0.94	0.77	0.83	0.77	0.81
LAE	0.98	0.97	0.83	0.90	0.84	0.89

Notes: By row, we report the values of the loss function for the MCMS model as a proportion to the corresponding value of the loss function for the VAR model, for one-, two- and three-steps ahead predictions. MSE is Mean Square Error, MAE is Mean Absolute Error, LSE is the Mean Square Error when variables and predictions are expressed in logs, LAE is the Mean Absolute Error when variables and predictions are expressed in logs.

where the hat indicates the prediction of the variable  $y_t$ , or we can follow Hamilton and Susmel (1994) to suggest alternative measures of performance where predictions and target variables are transformed logarithmically. These loss functions are:

$$\begin{aligned} \text{LSE}_\tau &= T^{-1} \sum_{t=1}^{T-\tau} [\ln(y_{t+\tau}) - \ln(\hat{y}_{t+\tau|t})]^2 \\ \text{LAE}_\tau &= T^{-1} \sum_{t=1}^{T-\tau} |\ln(y_{t+\tau}) - \ln(\hat{y}_{t+\tau|t})|. \end{aligned} \quad (6)$$

The theoretical superiority of one among the loss functions in this framework is not clear cut: on the one hand, the use of squares makes large prediction errors even larger which is not the case for absolute values. We may expect large prediction errors when unusual events such as some values recorded during the 1997 crisis are involved. On the other hand, the logarithmic transformations will flatten the largest

data values (and amplify the smallest), thus offering a rebalancing of the data and a different viewpoint on the quality of the predictions.

We present all combinations in Table 4 where we report the values of the loss functions in Equations 5 and 6 for the MCMS model as a proportion to the corresponding value of the loss function for the VAR model, for predictions one-, two- and three-steps ahead. Thus, values smaller than one point to a better performance of the MCMS model. The results are organized by bivariate models where Hong Kong is the dominant market and the four other markets change, as before.

The results in Table 4 are quite interesting. For one-step ahead predictions the evidence is somewhat neutral in that all values are close to one: by pairs of markets, respectively, in the range 0.93–1.06 for Korea/HK, 0.98–1.02 for Malaysia/HK, 0.94–1.06 for Philippines/HK and 0.94–1.05.

Table 5. Transition probability matrices  $t-1$  side –  $t$  top

KSE/HS					KLSE/HS				
	00	01	10	11		00	01	10	11
00	0.671	0.128	0.169	0.032	00	0.738	0.202	0.047	0.013
01	0.349	0.291	0.196	0.164	01	0.388	0.370	0.124	0.118
10	0.471	0.090	0.369	0.070	10	0.645	0.177	0.140	0.038
11	0.221	0.184	0.324	0.271	11	0.298	0.284	0.214	0.204

PSE/HS					SGX/HS				
	00	01	10	11		00	01	10	11
00	0.541	0.177	0.213	0.069	00	0.574	0.298	0.084	0.044
01	0.302	0.198	0.302	0.198	01	0.245	0.300	0.205	0.250
10	0.435	0.142	0.319	0.104	10	0.591	0.307	0.067	0.035
11	0.245	0.161	0.359	0.235	11	0.272	0.332	0.178	0.218

The situation changes quite substantially when further steps ahead are considered, since the VAR never outperforms the MCMS as shown by the loss functions ratios which range, respectively, between 0.62 and 0.81, 0.74–0.96, 0.80–0.91 and 0.77–0.94 for the two-steps ahead; and between 0.60 and 0.79, 0.72–0.92, 0.77–0.90 and 0.77–0.91 for the three-steps ahead. This can be interpreted with a better performance of the MCMS over longer prediction horizons, since the multi chain structure adopted allows for a more rapid reaction of the model across regimes when relatively short lived crisis episodes are involved.

From the estimated coefficients in Table 2 it is possible to derive the transition probability matrices [applying Equations 3 and 4], which show a behaviour of the dominated variable conforming to the changes in the regime of the dominant one. In Table 5 these four matrices are shown; the element  $(i, j)$  denotes the probability to be in the state  $j$  at time  $t$ , given the state  $i$  at time  $t-1$  (00, 01, 10, 11, where the first digit indicates the value assumed by  $s_1$  and the second the one for  $s_2$ ). The four cases are very similar; when the dominated and the dominant series are in the state 0, it is very likely that both series will stay in the same state at the next period. When the dominant series is in the turbulent regime at time  $t-1$ , while the dominated series is in a quiet regime, the four possible situations at time  $t$  have a similar probability to arise in the SGX/HS case; in the PSE/HS case the most probable scenario is a change of regime for both series or a return to the quiet period for HS, whereas KSE/HS and KLSE/HS show  $s_{1t}=0$ ,  $s_{2t}=0$  or 1 as the most probable scenario. Interestingly, when the dominated market is in a turbulent state, while the HS is in a quiet state, it is very likely that also the dominated market will go

back to the quiet period the next period. When both series are in the turbulent state at time  $t-1$ , it is difficult to predict the next state, depending on the duration of the crisis.

## V. The Choice of the Volatility Proxy

The issue whether the model should be estimated with the range as a dependent variable as a proxy of volatility, or by log-range (which would correspond to log-volatility) is a very interesting one which does not have a clear cut answer in the literature.<sup>4</sup> We repeated the MCMS estimation on the log-transformed variables. The evidence we got was in favour of the presence of Markov Switching regimes but without any discernible structure in the transition probabilities: in detail the statistical significance of the state variable in the dominant market was lost, hiding the spillover effects from Hong Kong we have found during the Asian crisis period. As a matter of fact, when log-range is used, the estimators of the coefficients representing the effect of the volatility transmission ( $\vartheta_{01}^1$  and  $\vartheta_{11}^1$ ) are not significant and in some cases they are identically equal to zero (their estimates are equal to zero and the variance of the estimator is null).

Further exploration of why this could be the case leads us into analysing the properties of the model by sub-periods. To this end, we have tried to see whether excluding the crisis period from the sample would maintain the same results previously shown to be obtained on the whole sample. Thus, we have estimated two separate MCMS models for the *levels* of the volatility proxy, one for the span February 1994 to August 1997 and one for January 1998 to January 2004. In this way we have excluded from the

<sup>4</sup>In the present framework, the adoption of a MCMS rests on an assumption of a mixture of normal densities for the innovation term, the interpretation of which becomes cumbersome should logarithms be adopted as the dependent variable.

analysis the period from September to December 1997, which is characterized by larger spikes in the volatility proxy and is likely to contain the main information about asymmetric effects in the volatility spillovers from one market to the other. Somewhat not surprisingly, the estimation results show that  $\vartheta_{01}^1$  and  $\vartheta_{11}^1$  are not significant in both sub-periods for most cases, the only exceptions being significant  $\vartheta_{11}^1 = 1.802$  in the first subsample and  $\vartheta_{01}^1 = -0.738$  in the second sub sample of the KLSE/HS case; and, again, a significant  $\vartheta_{01}^1 = -1.500$  for the second sub sample for SGX/HS. This evidence clearly points out that the MCMS is capturing the leading role of the Hong Kong market during the crisis period. The fact that the links across Asian markets are time-varying is also confirmed by some results presented in Engle *et al.* (2005).

A second experiment was in a sense the mirror image of the first one: we aimed at detecting whether the model estimated on the log-range would come close to detecting some asymmetry in the transmission of volatility, once the sample period was restricted to the months around the crisis. Unfortunately, in this case, we ran into some severe limitations dictated by the sample size, and we do not think it is advisable to trust the improvement in the results (away from zero estimates).

In synthesis, the significance of the MCMS effects seem to lie in the capability of the model to capture links for the more turbulent periods which, on the other hand, are smoothed out and somewhat flattened by the logarithmic transformations.

## VI. Concluding Remarks

In this article, we have used an MCMS model to represent the volatility transmission mechanisms between pairs of markets, interpreting one market as the dominant one, in the sense that it is the market from which the shock originates (in the present analysis Hong Kong is assumed to be such a market). This way to represent bivariate time series could be seen as a synthesis of two frequent approaches adopted in literature to study the volatility contagion across markets, namely the one by Forbes and Rigobon (2002), who study the contagion in terms of correlations between markets depurated from heteroskedasticity and that of Edwards and Susmel (2001, 2003), who study the co-movements in the second moments of returns between markets with bivariate SWARCH models.

Adopting a MCMS model, we take into account the presence of four regimes, which have a particular

interpretation. In fact, each regime represents a combination of the nonnecessarily coincident states of the series. In other words, the state of high or low volatility could be common or different to the couple of series. In this way we can calculate the probabilities that both the series are in the same regime, but also the probability that the dominant series drags the other series to change the state. This could be considered as a nonlinear way to represent the spillover effects.

In the literature, it is still an open question whether one should model volatility proxies (or their square, variances) as such or after transforming them into logs. The experiment we performed by estimating the current model in logs showed some interesting differences relative to the model estimated here: the presence of regime switching was maintained but some of the evidence of spillover provided by the multi chain specification was lost, possibly due to the smoothing of extreme events. In the present context, therefore, one may not want to flatten out episodes which instead may be responsible for tracking the direction of the spillover. The overall good prediction performance of the MCMS model is a comforting property of the model in this sense.

In this article we have limited our study in the comparison of a classical multivariate approach (VAR) and this new approach; further studies could compare the MCMS approach with other multivariate models, such as the bivariate SWARCH model of Edwards and Susmel (2001, 2003).

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