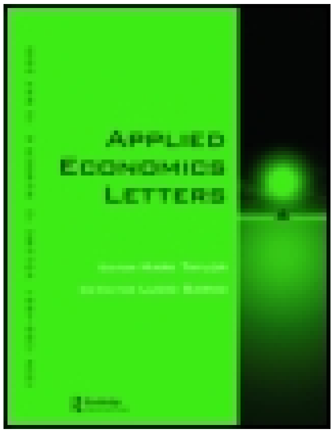

Applied Economics Letters
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/rael20

Detecting jumps and regime switches in international stock markets returns

Julien Chevallier[ab] & Stéphane Goutte[bc]
[a] IPAG Business School, IPAG Lab, 75006 Paris, France
[b] Université Paris 8, LED, 93526 Saint-Denis Cedex, France
[c] ESG Management School, 75013 Paris, France
Published online: 02 Jan 2015.

To cite this article: Julien Chevallier & Stéphane Goutte (2015): Detecting jumps and regime switches in international stock markets returns, Applied Economics Letters, DOI: 10.1080/13504851.2014.995356

To link to this article: http://dx.doi.org/10.1080/13504851.2014.995356

Applied Economics Letters, 2014
http://dx.doi.org/10.1080/13504851.2014.995356

Routledge
Taylor & Francis Group

Detecting jumps and regime switches in international stock markets returns

Julien Chevallier[a,b,*] and Stéphane Goutte[b,c]

[a]*IPAG Business School, IPAG Lab, 75006 Paris, France*
[b]*Université Paris 8, LED, 93526 Saint-Denis Cedex, France*
[c]*ESG Management School, 75013 Paris, France*

This article explores seven international stock markets (DJIA, Euro STOXX 600, Russell 2000, Nikkei, NASDAQ, FTSE and Global Dow) in the quest for jumps and regime switches. The methodological framework borrows from the Markov-switching approach and the stochastic modelling literature based on Lévy processes. The econometric procedure is detailed in a two-step fashion. The data set covers the period from June 2004 to July 2014. The main results uncover changing market dynamics according to economic and/or financial phenomena (e.g., economic crises/ growth, news events) with the occurrence of several episodes characterized by a high jump intensity. We advocate the use of such a jump-robust model modulated by a Markov chain to further study the dependence structure of financial time series.

Keywords: Lévy jumps; Markov switching; equity markets; Nasdaq; Euro STOXX; FTSE

JEL Classification: C32; G15; E44

I. Introduction

Vivid research activity on stock markets has documented the occurrence of jumps due to several factors (Andersen *et al.*, 2002; Chernov *et al.*, 2003; Eraker *et al.*, 2003; Eraker, 2004). Among them, we may cite micro-crashes due to liquidity events, dividend payments or the arrival of macroeconomic and/ or financial news (GDP, quarterly earning reports). Against this background, it appears crucial to advance a methodological framework that is able to capture both jumps and regime switches in international stock market returns.

This article proposes a new statistical method to estimate regime-switching Lévy models that are both efficient and practicable. Our goal lies in estimating a Markov-switching model augmented by jumps, under the form of a Lévy process. This particular class of stochastic processes is entirely determined by a drift, a scaled Brownian motion and an independent pure-jump process (Barndorff-Nielsen and Shephard, 2013).

The estimation strategy relies on a two-step procedure: by estimating first the diffusion parameters in presence of switching, and second the Lévy jump component by means of separate normal inverse Gaussian (NIG) distributions fitted to each regime.

*Corresponding author. E-mail: julien.chevallier04@univ-paris8.fr

Computationally, the expectation–maximization (EM) algorithm is extended to this new class of jump-diffusion regime-switching model.

An empirical application is proposed for seven equity markets with an international scope: the USA, Europe (UK) and Japan. We demonstrate the goodness-of-fit of the regime-switching Lévy model and thereby illustrate the interest to resort to that kind of model in financial economics.

The remainder of the article is structured as follows. Section II develops the stochastic model. Section III details the estimation method. Sections IV provides an empirical application. Section V concludes.

II. The Stochastic Model

Let $(\omega, \mathcal{F}, P)$ be a filtered probability space and T be a fixed terminal time horizon. We propose in this article to model the dynamic of a sequence of historical values of price using a regime-switching stochastic jump-diffusion. This model is defined using the class of Lévy processes.

Lévy process

Definition 1: A Lévy process L_t is a stochastic process such that

(1) $L_0 = 0$.

(2) For all $s > 0$ and $t > 0$, we have that the property of stationary increments is satisfied; that is $L_{t+s} - L_t$ as the same distribution as L_s.

(3) The property of independent increments is satisfied; that is for all $0 \leq t_0 < t_1 < \ldots < t_n$, we have that $L_{t_i} - L_{t_{i-1}}$ are independent for all $i = 1, \ldots, n$.

(4) L has a Cadlag paths. This means that the sample paths of a Lévy process are right continuous and admit a left limits.

Remark 1: In a Lévy process, the discontinuities occur at random times.

Markov switching

Definition 2: Let $(Z_t)_{t \in [0,T]}$ be a continuous time Markov chain on finite space $\mathcal{S} := \{1, 2, \ldots, K\}$. Denote $\mathcal{F}_t^Z := \{\sigma(Z_s); 0 \leq s \leq t\}$, the natural filtration generated by the continuous time Markov chain Z. The generator matrix of Z, denoted by Π^Z, is given by

$$\Pi_{ij}^Z \geq 0 \quad \text{if } i \neq j \text{ for all } i, j \in S \quad \text{and} \\ \Pi_{ii}^Z = -\sum_{j \neq i} \Pi_{ij}^Z \quad \text{otherwise} \tag{1}$$

Remark 2: The quantity Π_{ij}^Z represents the switch from state i to state j.

Regime-switching Lévy

Let us define the regime-switching Lévy Model:

Definition 3: For all $t \in [0, T]$, let Z_t be a continuous time Markov chain on finite space $\mathcal{S} := \{1, \ldots, K\}$ defined as in Definition 2. A regime-switching model is a stochastic process (X_t) which is solution of the stochastic differential equation given by

$$\mathrm{d}X_t = \kappa(Z_t)\left(\theta(Z_t) - X_t\right)\mathrm{d}t + \sigma(Z_t)\,\mathrm{d}Y_t \tag{2}$$

where $\kappa(Z_t)$, $\theta(Z_t)$ and $\sigma(Z_t)$ are functions of the Markov chain Z. Hence, they are constants which take values in $\kappa(\mathcal{S})$, $\theta(\mathcal{S})$ and $\sigma(\mathcal{S})$. Thus, $\kappa(\mathcal{S}) := \{\kappa(1), \ldots, \kappa(K)\} \in \mathbb{R}^{K^*}$, $\theta(\mathcal{S}) := \{\theta(1), \ldots, \theta(K)\}$ and $\sigma(\mathcal{S}) := \{\sigma(1), \ldots, \sigma(K)\} \in \mathbb{R}^{K^+}$. And finally, Y is a stochastic process which could be a Brownian motion or a Lévy process.

Remark 3: The following classic notations apply:

- κ denotes the mean-reverting rate;
- θ denotes the long-run mean;
- σ denotes the volatility of X.

Remark 4:

- In this model, there are two sources of randomness: the stochastic process Y appearing in the dynamics of X, and the Markov chain Z. There exists one randomness due to the market information which is the initial continuous filtration $\mathcal{F}$ generated by the stochastic process Y; and another randomness due to the Markov chain Z, $\mathcal{F}^Z$.
- In our model, the Markov chain Z infers the unobservable state of the economy, that is expansion or recession. The processes Y^i estimated in each state, where $i \in \mathcal{S}$, capture: a different level of volatility in the case of Brownian motion (i.e. $Y^i \equiv W^i$), or a different jump intensity level of the distribution (and a

possible skewness) in the case of Lévy process (i.e. $Y^i \equiv L^i$).

Barndorff-Nielsen (1997) recalls the main properties of the NIG distribution, which is used as the Lévy distribution in this article. The NIG density belongs to the family of normal variance–mean mixtures, that is one of the most commonly used parametric densities in financial economics. The NIG is a good alternative to the normal distribution since (i) its distribution can model the heavy tails, kurtosis and jumps and (ii) the parameters of NIG distribution can be solved in a closed form.

III. Estimation

This section covers the methodology pertaining to the estimation task. The EM algorithm used to estimate the regime-switching Lévy model in this article is a generalization and extension of the EM algorithm developed in Hamilton (1988, 1989).

Our aim is to fit a regime-switching Lévy model such as Equation (2) where the stochastic process Y is a Lévy process that follows a NIG distribution. Thus the optimal set of parameters to estimate is $\hat{\Theta} := \left(\hat{\kappa}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{\mu}_i, \hat{\Pi}\right)$, for $i \in \mathcal{S}$.

We have the three parameters of the dynamics of X, the four parameters of the density of the Lévy process L, and the transition matrix of the Markov chain Z. Because the number of parameters grows rapidly in this class of jump-diffusion regime-switching models, direct maximization of the total log-likelihood is not practicable. To bypass this problem, we propose a method in two successive steps to estimate the global set of parameters.

Discretization

We first take for the stochastic process Y a Brownian motion W. Moreover, suppose that the size of historical data is $M+1$. Let Γ denote the corresponding increasing sequence of time from which the data values are taken:

$$\Gamma = \{t_j; 0 = t_0 \le t_1 \le \dots t_{M-1} \le t_M = T\}, \quad \text{with} \quad \Delta_t = t_j - t_{j-1} = 1$$

The discretized version of model (2) writes

$$X_{t+1} = \kappa(Z_t)\theta(Z_t) + (1-\kappa(Z_t))X_t + \sigma(Z_t)\epsilon_{t+1} \tag{3}$$

where $\epsilon_{t+1} \sim N(0,1)$ (since the process Y is a Brownian motion). We denote by $\mathcal{F}^X_{t_k}$ the vector of historical values of the process X until time $t_k \in \Gamma$. Thus, $\mathcal{F}^X_{t_k}$ is the vector of the $k+1$ last values of the discretized model and therefore, $\mathcal{F}^X_{t_k} = (X_{t_0}, X_{t_1}, \dots, X_{t_k})$.

Remark 5: The filtration generated by the Markov chain Z (i.e. F^Z) is the one generated by the history values of Z in the time sequence Γ.

For simplicity of notation, we will write in the sequel the model (3) as

$$X_{t+1} = \kappa_i\theta_i + (1-\kappa_i)X_t + \sigma_i\epsilon_{t+1}$$

This means that at time $t \in [0, T]$, the Markov chain Z is in state $i \in S$ (i.e. $Z_t = i$) and Z jumps at time $t_j \in \Gamma, j \in \{0, 1, \dots, M-1\}$.

Step 1: Estimation of the regime-switching model (2) in the Brownian case

In the first step based on the EM algorithm, the complete parameter space estimate $\hat{\Theta}$ is split into: $\hat{\Theta}_1 := \left(\hat{\kappa}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\Pi}\right)$, for $i \in \mathcal{S}$, which corresponds to the first subset of diffusion parameters. Recall that we estimate the parameters of the discretized model (3).

Step 2: Estimation of the parameters of the Lévy process fitted to each regime

Using the regime classification obtained in the previous step, we estimate the second subset of parameters $\hat{\Theta}_2 := \left(\hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{\mu}_i\right)$, for $i \in \mathcal{S}$, which corresponds to the NIG distribution parameters of the Lévy jump process fitted for each regime.

IV. Application to International Stock Markets

We apply these statistical methods to detect both regime switches and jumps in the context of international stock markets.

Table 1. Description of the time series

Ticker	Description
Stock markets	
DJIA	Dow Jones Industrial Average
STOXX	EURO STOXX European 600 Index
Russell	Russell 2000 Index
NIKKEI	Nikkei 225 Index
NASDAQ	NASDAQ Composite Index
FTSE	FTSE 100 Index
Global Dow	US Global Dow Jones

The data are retrieved in daily frequency from Thomson Financial Datastream over the period going from June 2004 to 11 July 2014. The characteristics for each time series are given in Table 1.

The geographical coverage of our data set pertains to various segments of international equity markets, going from the USA (Global Dow, industry with the DJIA, tech values with the NASDAQ, small-caps with the Russel 2000 Index) to Europe (top 600 companies, UK focus with the FTSE) and Japan (Nikkei). We have recovered equity data in order to study the regime-switching and jump properties of financial markets under changing market conditions.

For each time series, we report the results of:

(1) the regime-switching classification with all estimated parameters of the mean-reverting diffusion, and
(2) the NIG density parameters of the Lévy jump process fitted to each regime (when we find an evidence of jumps).

The remaining problem in this work is to specify the number of regimes in the Markov chain. For simplicity, we proceed with two regimes that relate to the 'boom' and 'bust' phases of the business cyle.[1]

We also report a plot where each regime is reported with a different colour (e.g. black (grey) corresponds to regime 1 (regime 2)). To provide the reader with a clearer picture, we have chosen to plug the regimes identified back into the raw (nonstationary) data. Of course, all the estimates were performed on log-returns $r_t := \log(X_t) - \log(X_{t-1})$, for example stationary data. Below this first plot, the smoothed probabilities are displayed.

Presence of regime switches

Tables 2 and 3 contains the parameter estimates for the regime-switching model.

Across the seven international equity markets, we notice that the highest volatility parameter σ is recorded for the Nikkei (state 2, $\sigma = 60114$)

Table 2. Estimated parameters

	DJIA		STOXX		Russell	
Parameters	State 1	State 2	State 1	State 2	State 1	State 2
κ	0.0059	0.0004	0.0012	0.0023	0.0022	0.0038
θ	9196.09	9714.10	740.21	180.86	1147.75	518.15
σ	38972.37	6316.77	25.09	4.49	227.03	0.98
P^Z_{ii}	0.98	0.99	0.97	0.99	54.65	0.99

Table 3. Estimated parameters

	NIKKEI		NASDAQ		FTSE		Global Dow	
Parameters	State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2
κ	0.0037	0.0042	0.0022	0.0041	0.0044	0.0015	0.0017	0.0022
θ	9073.64	15354.61	3821.63	1737.96	7099.61	2751.95	3462.40	777.43
σ	12546.40	60114.46	1838.27	398.53	9266.06	1668.53	918.57	125.58
P^Z_{ii}	0.99	0.99	0.98	0.99	0.98	0.99	0.97	0.98

[1] It is well known that testing for the number of regimes in a Markov chain is a hard problem to tackle, which we leave for further research.

followed by the DJIA (state 1, $\sigma = 38972$). Moving to the long-run mean parameter θ, we notice varying states of the business cycle from one regime to another. For instance, the EURO STOXX 600 Index features a long-run mean that is four times higher during state 1 than during state 2. This means that the regime-switching parameter has clearly captured different dynamics along the two classic boom/bust phases of the economy. Hence, the usefulness of resorting to regime-switching models that are able to capture such inobservable characteristics. The mean reversion parameter κ is close to 0 for all markets.

Figure 1 pictures the regime switches in the two-state setting. For instance in the case of the DJIA, visually, the regime-switching approach captures adequately the dynamics from the two states. Indeed, we identify clearly (i) a bullish market

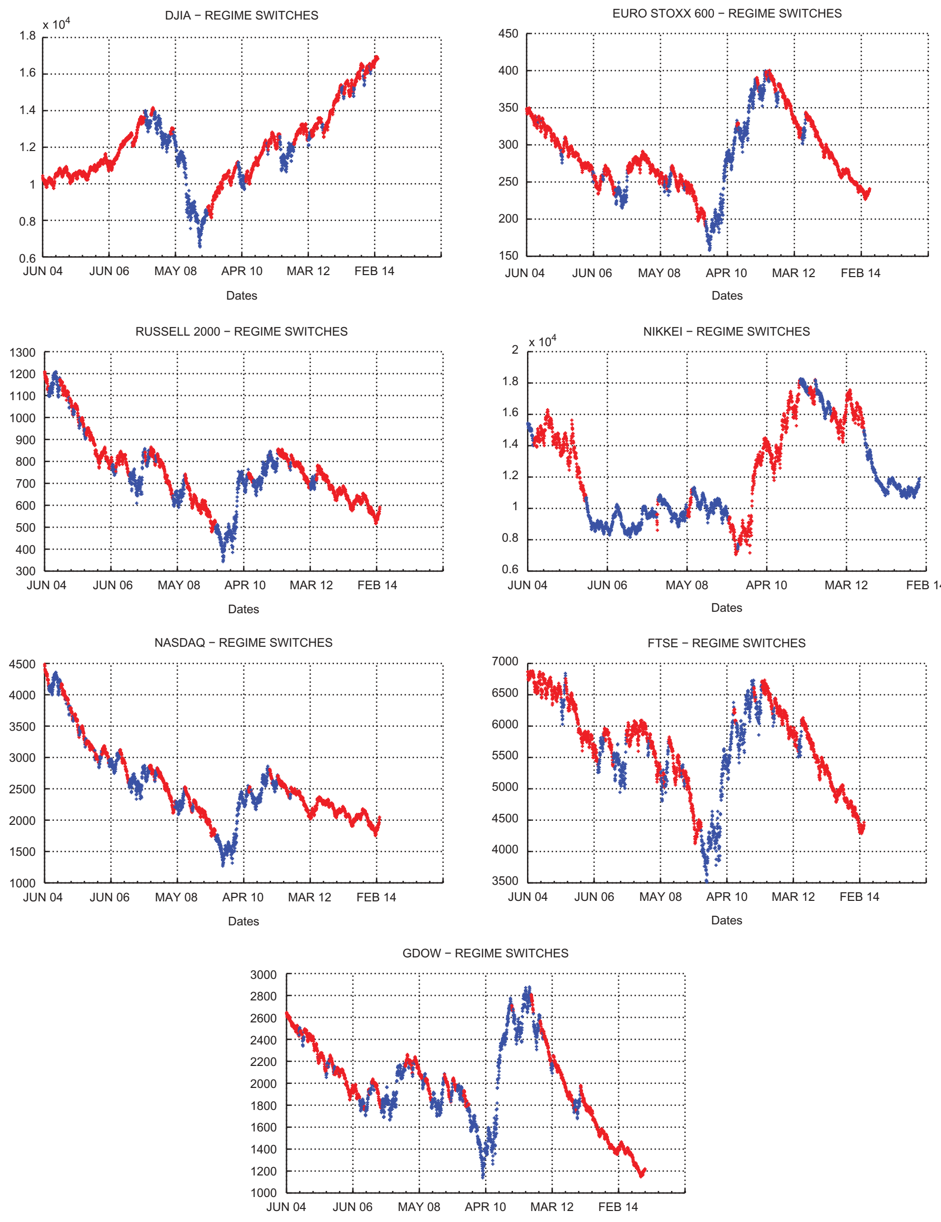

Fig. 1. Regime-switching classification for the seven international stock markets (from top to bottom and left to right: DJIA, Euro StoXX, Russell, Nikkei, Nasdaq, FTSE and Global Dow)
Note: Black (grey) corresponds to regime 1 (regime 2).

trend depicted in grey, and (ii) a bearish market trend depicted in black. Similar comments apply for the remaining panels in Fig. 1.

Regime classification measures

An ideal model is one that classifies regimes sharply and has smoothed probabilities which are either close to 0 or 1. In order to measure the quality of regime classification, we propose two measures:

(1) The *regime classification measure (RCM)* introduced by Ang and Bekaert (2002). Let $K(>0)$ be the number of regimes, the RCM statistic is then given by

$$\mathrm{RCM}(K)=100.\left(1-\frac{K}{K-1}\frac{1}{T}\sum_{k=1}^{N}\sum_{Z_{t_k}}\left(P\left(Z_{t_k}|\mathrm{Y}_{\mathrm{T}};\Theta^{(n)}\right)-\frac{1}{K}\right)^2\right), \quad (4)$$

where the quantity $P\left(Z_{t_k}|Y_T;\Theta^{(n)}\right)$ is the well-known smoothed probability and $\Theta^{(n)}$ is the vector parameter estimation result (see Goutte, 2014, for more details). The constant serves to normalize the statistic to be between 0 and 100. Proper regime classification is associated with low RCM statistic value: a value of 0 means perfect regime classification, whereas a value of 100 implies that no information about the regimes is revealed.

(2) The *smoothed probability indicator* introduced by Goutte and Zou (2013). A satisfactory classification for the data can be also seen when the smoothed probability is less than 0.1 or greater than 0.9. This then means that the data at time $t \in [0, T]$ is, with a probability exceeding 90%, in one of the regimes at the 10% error level.

Thus, it is important that the RCM statistic be close to 0, and the smoothed probability indicator close to 100%, to ensure that we have detected significantly different regimes.

Table 4 displays these corresponding statistics. We notice that the regime-switching model behaves very well in discriminating the regimes, as the RCM statistics are low (in the range of 9–19), whereas we obtain a high percentage through the smoothed probability indicators. Another clue that the regime switches are well detected can be inferred from Tables 2 and 3 where, on the last row, we observe a high persistence of staying in the current regime with the matrix P_{ii}^{Z}. The interested reader may look at Fig. 2 for additional insights on the timing of regime switches at stake for each of the seven stock markets selected in this study.

Table 4. RCM and smoothed probability indicator

Markets	RCM	P%
DJIA	12.51	86.88
STOXX	13.92	85.71
Russell	18.33	80.43
NIKKEI	9.07	90.35
NASDAQ	18.38	81.13
FTSE	13.12	86.33
Global Dow	19.22	78.92

Evidence of jumps/spikes in stock markets

Next, we estimate the NIG distribution parameters for each regime of the seven stock markets to identify the evidence of jumps/spikes. The results are summarized in Tables 5 and 6.

Let us start with the DJIA. Its high volatility level is identified by our approach, since the two states are modelled with an evidence of jumps. The lower the value of the α parameter, the higher the occurrence of jumps. In Table 5, the jump-intensity parameter α of the DJIA is less than 0.01 in each state. The β parameters are close to 0, which confirms the symmetric distribution of this stock index.

In Fig. 1, for the EURO STOXX 600 index, the regime 1 (depicted in black) corresponds to a huge increase in market value of the index (roughly from 160 to 390). In Table 5, this increase is well captured by the jump-intensity parameter, since $\alpha_1 = 0.990$ against $\alpha_2 = 1.423$ in state 2. Besides, this increase is also reflected in the asymmetric value of the β parameter equal to −0.216 in state 1.

Moving to the Russell 2000 index, there is an evidence of one economic state with jumps/spikes and another one without jumps. In Table 5, the parameter α_1 is equal to 0.028 which indicates a high jump-intensity during state 1. On the contrary, α_2 = 1.391 during the second state, which argues in favour of a continuous diffusion (e.g. Brownian without jumps). For regime 2, we also observe β = 0.408 which indicates a negative asymmetric trend.

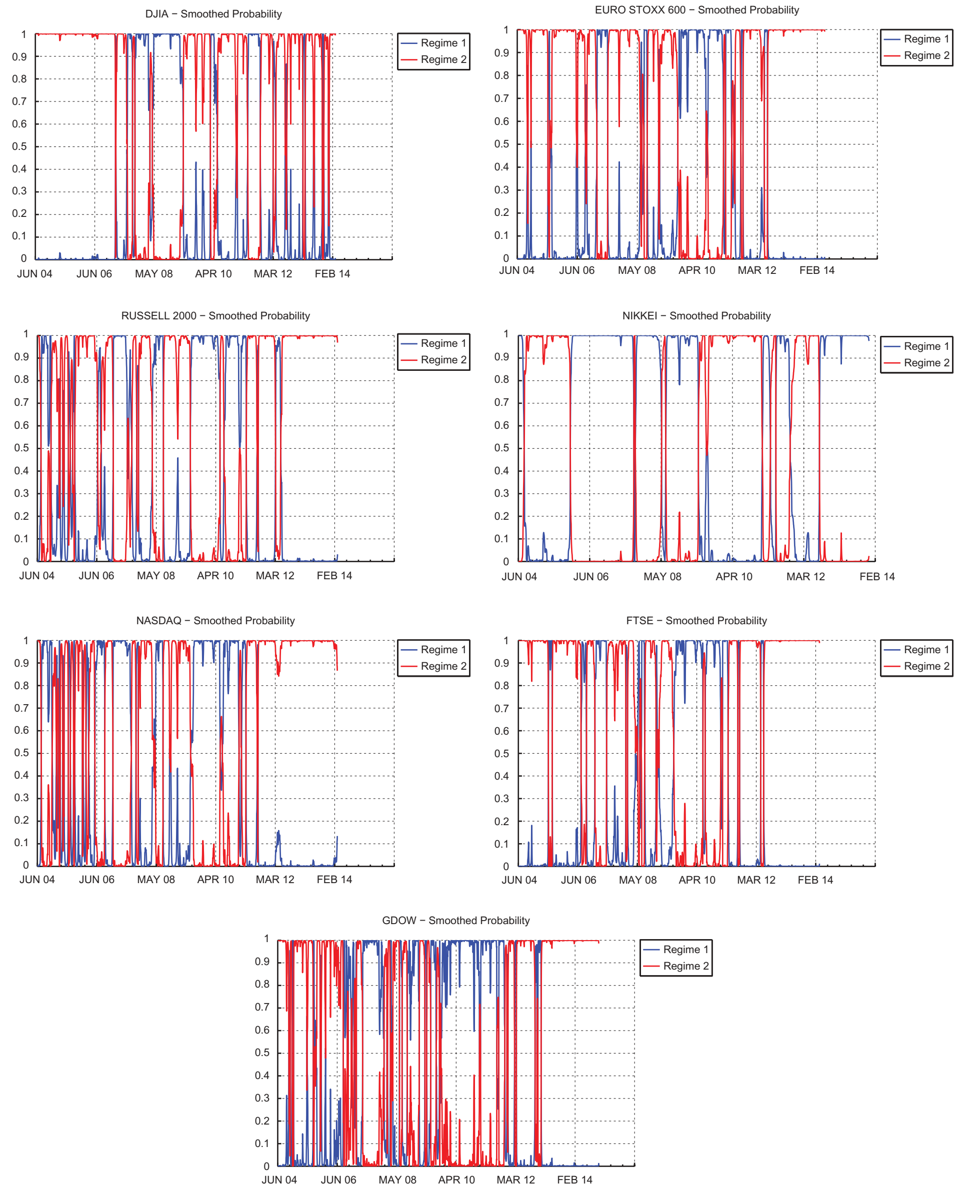

Fig. 2. Smoothed probabilities obtained for the seven international stock markets (from top to bottom and left to right: DJIA, Euro StoXX, Russell, Nikkei, Nasdaq, FTSE and Global Dow)
Note: Black (grey) corresponds to regime 1 (regime 2).

Table 5. Estimated parameters

	DJIA		STOXX		Russell	
Parameters	State 1	State 2	State 1	State 2	State 1	State 2
α	0.002	0.009	0.990	1.423	0.028	1.391
β	0.001	−0.008	−0.216	0.393	−0.014	0.408
δ	108.204	7.577	0.712	0.421	7.197	0.737
μ	−62.116	19.348	0.159	−0.121	4.153	−0.226

Table 6. Estimated parameters

	NIKKEI		NASDAQ		FTSE		Global Dow	
Parameters	State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2
α	0.001	0.002	0.011	0.013	0.005	0.006	0.014	0.010
β	−0.001	−0.000	−0.006	0.009	−0.002	0.004	−0.007	−0.007
δ	76.137	130.265	22.316	4.701	40.348	7.076	15.398	13.311
μ	−98.844	24.732	14.824	−4.422	22.289	−7.845	8.066	−15.832

In Table 6, the investigation of jumps in the Nikkei stock market reveals two distinct jump dynamics. In Fig. 1, we observe that this stock index is highly volatile, as confirmed by the estimated parameters reproduced in Table 6. The same comments apply for the NASDAQ and the FTSE.

For the GDOW, we notice that there are more jumps in the second regime ($\alpha_2 = 0.010$). There is an absence of asymmetry is both regimes. The plot reported in Fig. 1 shows that the regime switching cuts well the two trends appearing after the bullish/bearish market trends between April 2010 and May 2012.

V. Conclusion

Jumps are intrinsic to the functioning of financial markets, reflecting how the transmission of new information (from various sources) impacts the asset price (Cont and Tankov, 2004; Chevallier and Ielpo, 2014). At the same time, the normal behaviour of economies is occasionally disrupted by periods of crash or recession, which can be accurately tracked by Markov-switching models (Hamilton and Raj, 2002). Therefore, gaining a deeper methodological command on how to measure simultaneously jumps and regime switches appears of interest to academics and practitioners alike.

In this article, we propose to contribute to the literature by advancing the regime-switching Lévy model that combines jump-diffusion under the form of a Lévy process and Markov regime switching. Following a two-step estimation procedure, we demonstrate that the empirical fit of this technique is remarkable for a selection of seven international stock markets across the USA, Europe (UK) and Japan. Broadly speaking, we are able to capture the presence of two contrasted regimes in each time series (reflecting the 'boom-bust' economic cycle or bullish/bearish market trends), with the evidence of a high-jump intensity in at least one (if not two) regime. Therefore, it seems appropriate to model stock markets series with jumps and regimes identified simultaneously with the Lévy regime-switching model.

References

Andersen, T. G., Benzoni, L. and Lund, J. (2002) An empirical investigation of continuous-time equity return models, *The Journal of Finance*, **57**, 1239–84. doi:10.1111/1540-6261.00460

Ang, A. and Bekaert, G. (2002) Regime switches in interest rates, *Journal of Business and Economic Statistics*, **20**, 163–82. doi:10.1198/073500102317351930

Barndorff-Nielsen, O. and Shephard, N. (2013) *Financial Volatility in Continuous Time*, Cambridge University Press, Cambridge, UK.

Barndorff-Nielsen, O. E. (1997) Processes of normal inverse Gaussian type, *Finance and Stochastics*, **2**, 41–68. doi:10.1007/s007800050032

Chernov, M., Gallant, R. A., Ghysels, E., *et al.* (2003) Alternative models for stock price dynamics, *Journal of Econometrics*, **116**, 225–57. doi:10.1016/S0304-4076(03)00108-8

Chevallier, J. and Ielpo, F. (2014) Twenty years of jumps in commodity markets, *International Review of Applied Economics*, **28**, 64–82. doi:10.1080/02692171.2013.826637

Cont, R. and Tankov, P. (2004) *Financial Modelling with Jump Processes*, Chapman & Hall/CRC Financial Mathematics Series, London.

Eraker, B. (2004) Do stock prices and volatility jump? Reconciling evidence from spot and option prices, *The Journal of Finance*, **59**, 1367–404. doi:10.1111/j.1540-6261.2004.00666.x

Eraker, B., Johannes, M. and Polson, N. (2003) The impact of jumps in volatility and returns, *The Journal of Finance*, **58**, 1269–300. doi:10.1111/1540-6261.00566

Goutte, S. (2014). Conditional Markov regime switching model applied to economic modelling, *Economic Modelling*, **38**, 258–69. doi:10.1016/j.econmod.2013.12.007

Goutte, S. and Zou, B. (2013) Continuous time regime switching model applied to foreign exchange rate, *Mathematical Finance Letters*, **8**, 1–37.

Hamilton, J. D. (1988) Rational-expectations econometric analysis of changes in regime, *Journal of Economic Dynamics and Control*, **12**, 385–423. doi:10.1016/0165-1889(88)90047-4

Hamilton, J. D. (1989) A new approach to the economic analysis of non-stationary time series and the business cycle, *Econometrica*, **57**, 357–84. doi:10.2307/1912559

Hamilton, J. D. and Raj, B. (2002) New directions in business cycle research and financial analysis, *Empirical Economics*, **27**, 149–62. doi:10.1007/s001810100115