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Using neural network to forecast stock index option price: a new hybrid GARCH approach

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Abstract This study aims to apply a new hybrid approach to estimate volatility in neural network option-pricing model. The analytical results also indicate that the new hybrid method can be used to forecast the prices of derivative securities. Owing to combines the grey forecasting model with the GARCH to improve the estimated ability, the empirical evidence shows that the new hybrid GARCH model outperforms the other approaches in the neural network option-pricing model.

Keywords Option-pricing model · GARCH · Grey forecasting model · Volatility

1 Introduction

Accurate measures and good forecasts of volatility are essential for option pricing theories. Volatility is a measure of price movement that is frequently used to determine risk and signal large movements in underlying markets. The predictability of market volatility is important for options practitioners to forecast closing prices and determine expected market return. Estimating stock market volatility has received considerable attention by both academics and practitioners. Some studies have been specified the stochastic volatility of the underlying asset as a deterministic function of time and the price of the underlying asset (Scott 1987; Hull and White 1987; Heston 1993; Bates 1996; Watanabe 1999; Kim and Kim 2004), and other related research compared the ability of various members of the GARCH family to estimate market volatility (Duan 1995; Sabbatini and Linton 1998; Chung and Hung 2000; Heston and Nandi 2000; Duan and Zhang 2001; Szakmary et al. 2003; McMillan and Speight 2004).

Although in the past, GARCH models have been used as estimates of market volatility, there is a growing body of evidence that suggests that the use of volatility predicted from more sophisticated models will lead to more accurate forecasting time-series models.

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Therefore, based on the characteristic of Grey Forecasting Modeling, GM(1,1), we provide the GM(1,1)-GARCH to reduce the stochastic and nonlinearity of the error term sequence and then to improve the predicted ability of option-pricing model further.[1]

This study proposes an alterative data-driven method for pricing options, an artificial neural network model, in which the data itself can provide direct evidence supporting the model estimation of the underlying generation process. The neural network model was widely used as a promising alternative approach to time series forecasting (Zhang and Berardi 2001). This approach is effective for input and output relationship modeling even for noisy data, and has been demonstrated to effectively model nonlinear relationships. Studies on derivative securities pricing using neural network have attracted researchers and practitioners, and they applied the neural network model and obtained better results than using the traditional stochastic model (Hutchinson et al. 1994; Malliaris and Salchenberger 1996; Qi 1999; Yao et al. 2000; Amilon 2003; Binner et al. 2005; Lin and Yeh 2005).

Firstly, we employs the forecasting property of the GM(1,1) model to continually modify the squared error terms sequence, and the traditional symmetric GARCH model and GM(1,1) model are combined, GM(1,1)-GARCH, to construct the conditional volatility. Moreover, we uses different estimated approaches, historical volatility, implied volatility, GARCH, and GM(1,1)-GARCH, to estimate market volatilities which these estimated volatilities provide an input in backpropagation neural network model in order to compare the performance of option pricing. The remainder of this paper is organized as follows. Section 2 outlines the different approaches to estimate volatility and demonstrates how each is calculated. Next, Sect. 3 describes the neural network model as prediction model. Moreover, Sect. 4 presents the data set and presents the empirical results. Finally, Sect. 5 presents the summary and conclusions.

2 Methodology

2.1 Calculating volatilities

The seminal work by Black and Scholes on pricing options assumed that stock prices follow the standard lognormal diffusion:

$$dS_t/S_t = \mu d_t + \sigma dW_t$$

where S_t denotes the current stock price, μ represents the constant drift, σ is the constant volatility, and W_t denotes a standard Brownian motion. The standard Black–Scholes option pricing formula for calculating the equilibrium price is

$$C_t = S_t N(d_1) - X e^{-rt} N(d_2) \tag{1}$$

where $d_1 = \left[\ln(S_t e^{-rt}/X) + (r + \sigma^2/2)t\right] \Big/ \left(\sigma\sqrt{T}\right)$; $d_2 = d_1 - \sigma\sqrt{t}$; C is the call price; S denotes the current underlying asset price; X is the exercise price; t denotes the time to maturity (in years); σ represents the volatility of the underlying asset; r is the short-term risk free interest rate and $N(d_i)$ is the cumulative probability function for d_i, $i = 1, 2$. Equation 2 presents the Black–Scholes function (BS) in simple form.

$$\text{Option price} = \text{BS}\ (S, X, t, \sigma, r) \tag{2}$$

[1] Grey Systems (Deng 1982), including grey relation analysis, grey prediction model, grey decision-making and grey control, are concerned with the analysis, handling and interpretation of indeterminate or uncertain information sequence, aiming to tackle a model where the parameters or information are not entirely known.

All of the variables besides volatility are easily obtainable from the market. σ is the only unknown factor in the formulas, and is frequently assumed to be unchanged when forecasting option prices. An estimate of the asset volatility becomes the focus of attention for both theorists and traders.

Essentially, the historical and implied volatility approaches are fundamental in option-pricing models. The historical volatility approach uses the simple weighted moving average method, and daily volatility is determined by taking the mean of the historical returns of the underlying asset over a 60 days horizon. Therefore, the annualized standard deviation of historical daily returns is defined as the historical volatility.

$$\hat{\sigma}_t = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(R_t - \bar{R})^2}$$

$$\hat{\sigma}^H = \hat{\sigma}_t\sqrt{N} \tag{3}$$

where R_t denotes the stock return at time t and stock returns were calculated as the difference in the logarithms of daily stock prices. $\bar{R}$ represents the mean of the R_t, S_t is the stock closing price, and N denotes the trading day.

However, implied volatility assumes that the option market is effective, and that the call option price can be considered as the actual market price since it reflects the opinions of trading participants regarding future returns. The implied volatility uses the Black–Scholes formula in reverse, and thus can be calculated using the Black–Scholes formula. The simplified approach to estimating implied volatility is as follows:

$$\hat{\sigma}^I = \frac{1}{n}\sum_{j=1}^{n}\sigma \tag{4}$$

where σ_j represents the implied volatility on day j.

As alternative to the historical and implied approach, numerous models are devised that correspond to the stochastic volatility process characteristic. One widespread approach is the ARCH, devised by Engle (1982). This approach sets an unconditional volatility constant, while permitting the conditional volatility to change over time. A generalized approach of ARCH was devised by Bollerslev (1986), and is as well known as the GARCH. This volatility is calculated using observations of historical daily asset prices, considering both the conditional and unconditional variance in the estimation process. The unique feature of the GARCH is that the conditional variance is specified not only as a linear function of past sample variances, but lagged conditional variances are also included in the equation. Although numerous GARCH have been devised, the GARCH (1,1) is the most popular of the GARCH. The simplest GARCH is the GARCH (1, 1), which is expressed as:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{5}$$

In general, the error terms sequence, ε_t, containing a mix of known and unknown information based on the set of past information at time t. The GM(1,1)-GARCH offers a range of techniques for dealing with "grey" information sequence and therefore potentially assists the prediction of GARCH in an uncertainty time sequence. Consequently, this paper adopts the characteristics of GM(1,1) to modify the error terms and propose the GM(1,1)-GARCH. The procedures of error terms sequence's modification are as follows:

1. Define the original error terms, $\varepsilon^{(0)}$, where $\forall\varepsilon^{(0)}(i) \in \varepsilon^{(0)}$, $\varepsilon^{(0)}(i) \in R$, for $i = 1, 2, 3, \ldots t$.

$$\varepsilon^{(0)} = \left\{\varepsilon^{(0)}(1), \varepsilon^{(0)}(2), \ldots, \varepsilon^{(0)}(t)\right\} \tag{6}$$

2. Shift the original error terms sequence by adding the minimum value of the original sequence to meet the non-negative condition and the new sequence $x^{(0)}$ is given by

$$x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(t)\right\} \tag{7}$$

where $x^{(0)}(i) = \varepsilon^{(0)}(i) + \min\left(\varepsilon^{(0)}(1), \ldots, \varepsilon^{(0)}(t)\right)$ and $x^{(0)}(i) \in R^{+}$ for $i = 1, 2, 3, \ldots t$.

3. Obtain the first-order cumulative sum sequence $x^{(1)}$ from $x^{(0)}$ through once of AGO (Accumulated Generating Operation).

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(t)\right\} \tag{8}$$

where the generating series for the cumulative summation will be

$$x^{(1)}(i) = \left\{\sum_{k=1}^{i} x^{(0)}(k), i = 1, 2, \ldots, t\right\} \tag{9}$$

4. If the original error terms series $x^{(0)}$lacks any apparent trend, the generating series $x^{(1)}$ would then have an apparent trend with an absolute value increasing one-by-one. This provides a basis for establishing a calculus using differential equations. When the differential equation model is of order one and includes just one variable, the model is referred as to as GM(1,1). The general form of GM(1,1) has the following form:

$$\frac{dx^(1)}{dt} + ax^{(1)} = b \tag{10}$$

In Eq. 10, a is the development coefficient and $b$is the grey control parameter. From the time response function of the first derivative, the general solution to Eq. 10 is:

$$x^{(1)}(i+1) = (x^{(0)}(1) - b/a)e^{-ai} + b/a \tag{11}$$

According to the definition of differential equation:

$$\frac{dx^{(1)}(i)}{di} = \lim_{\Delta i \to 0} \frac{x^{(1)}(i+1) - x^{(1)}(i)}{\Delta i} \tag{12}$$

If $\Delta i = 1$, then Eq. 12 can be written as:

$$\frac{x^{(1)}(i+1) - x^{(1)}(i)}{1} = x^{(0)}(i) \tag{13}$$

Then the original differential equation can be described by:

$$x^{(0)}(i) + az^{(1)}(i) = b \tag{14}$$

where $z^{(1)}(i)$ is the background value, and $z^{(1)}(i) = \delta x^{(1)}(i) + (1-\delta)x^{(1)}(i-1), i \geq 2$. δ denotes a horizontal adjustment coefficient, and $0 < \delta < 1$. Parameters a and b in Eq. 14 can be obtained from

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = \left(B'B\right)^{-1} B'Y,$$

where $Y = \left[x^{(0)}(2) \quad x^{(0)}(3) \ldots x^{(0)}(t)\right]'$ and

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & 1 \\ -z^{(1)}(t) & 1 \end{bmatrix}.$$

5. Putting a and b obtained from the grey differential equation back into the general equation with $\hat{x}^{(1)} = x^{(0)}(1) = x^{(1)}(1)$. Since the prediction model is not constructed with original sequence but modes from one accumulative addition, reverse addition is required to recover the predicted sequence. From $\hat{x}^{(0)}(i+1) = \hat{x}^{(1)}(i+1) - \hat{x}^{(1)}(i)$, one can obtain Eq. 15, which is the dynamic situation of future values generated by the GM(1,1).

$$\hat{x}^{(0)}(i+1) = (1-e^{a})(x^{(0)}(1) - b/a)e^{-ai} \tag{15}$$

Finally, forecasted original error at time $t+1$ is given by

$$\hat{\varepsilon}^{(0)}(t+1) = (1-e^{a})(x^{(0)}(1) - b/a)e^{-at} - \min\left(\varepsilon^{(0)}(1), \ldots, \varepsilon^{(0)}(t)\right) \tag{16}$$

After acquiring the forecasted error of time $t+1$ by GM(1,1) model, we put this value in GARCH model to estimate the conditional variance at time $t+1$. Hence, the one-step-ahead variance forecasts are generated by the above-mentioned procedures and the multiple conditional variance forecasts for evaluation period can be obtained by repeating this procedure.

2.2 Evaluation of price forecasts

This study uses the root of mean square error (RMSE) to determine which neural network model has good performance; if RMSE is small then it has good forecasting performance.

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left[C_j^{MP} - C_j(\sigma_i)\right]^2}, \quad i = 1, 2 \tag{17}$$

Then, the neural network model developed here uses the different volatility models and compares them using the mean absolute error (MAE) and mean absolute percentage error (MAPE) to test the forecasting ability of the different models. The mean absolute errors and mean absolute percentage errors are calculated using the following formula:

$$\text{MAE} = \frac{1}{n}\sum_{j=1}^{n}\left|C_j^{MP} - C_j\right| \tag{18}$$

$$\text{MAPE} = \frac{1}{n}\sum_{j=1}^{n}\left|\frac{C_j^{MP} - C_j}{C_j^{MP}}\right| \tag{19}$$

where C_j^{MP} denotes the market real price, C_j represents the evaluate price in the neural network model.

3 Backpropagation neural networks

Neural networks are an information processing technology for modeling mathematical relationships between inputs and outputs. Based on the architecture of the human brain, a set of processing elements or neurons (nodes) are interconnected and organized in layers (Malliaris and Salchenberger 1996). The neural network model makes few assumptions and is an emerging and challenging computational technology that provides a new avenue for exploring the dynamics of various financial applications.

Neural networks can be classified into feedforward and feedback networks. Feedback networks contain neurons that are connected to themselves, enabling a neuron to influence other neurons. Kohonen self-organizing network and Hopfield network are the type of feedforward network. Neurons in feedforward networks is shown in Fig. 1, backpropagation neural network, take inputs only from the previous layer and send outputs only to the next layer.

This study employs a backpropagation neural network, the most widely used network in business applications (Meraviglia 1996; Paik 2000; Maarit and Häkkinen 2000). A three-layer backpropagation neural network is shown in Fig. 2. The backpropagation process determines the weights for the connections among the nodes based on data training, yielding a minimized least-mean-square error measure of the actual, desired and the estimated values from the output of the neural network. The connections weights are assigned initial values. Furthermore, the error between the predicted and actual output values is backpropagated via the network for updating the weights. The supervised learning procedure then attempts to minimize the error between the desired and forecast outputs. Theoretically, neural networks can simulate any kind of data pattern given sufficient training. The neural network must be

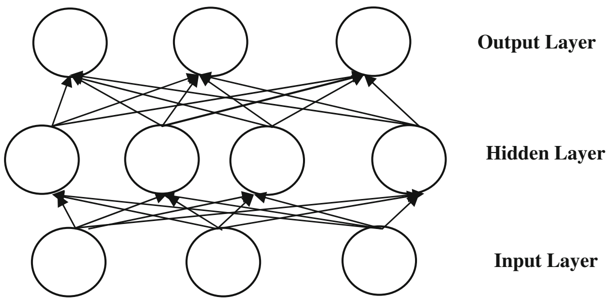
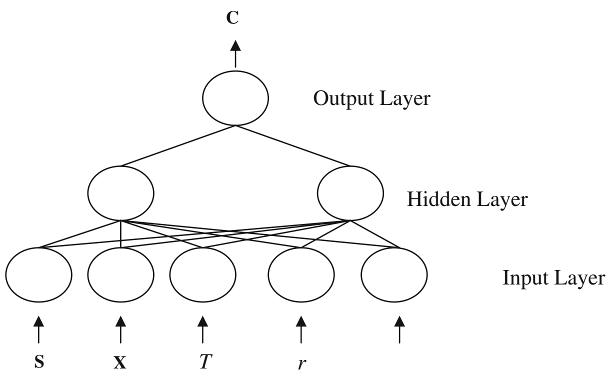


Fig. 1 A backpropagation neural network

Fig. 2 A one hidden layer neural network



trained before being applied for forecasting. During the training procedure, the neural network learns from experience based on the proposed hypotheses. Besides, this study employs one hidden layer for each neural network model and the sigmoid function serves as the activation function.

4 Data and empirical results

This study uses as inputs the primary Black–Scholes model variables that influence the option price, such as, current fundamental asset price, strike price and time-to-maturity, and then defines the option price as the output into which the learning network maps the inputs (Black and Scholes 1973). Given proper training, the network "becomes" the option pricing formula, and used in the same way that formulas obtained from the parametric pricing method are used for pricing.

The data used in this study is transaction data of Taiwan stock index options (TXO) traded on the Taiwan Futures Exchange (TAIFEX). This study investigated 15, 582 call option price data points from 2 January 2003 to 31 December 2004. Only traded prices were used. The trend of Taiwan stock market and return are shown as Fig. 3a, b, respectively. Regarding, the TAIEX has the circulation tendency but the real situation in the Fig. 3a is still presented a rise tendency. Notably, the shape of the TAIEX show clearly random walk and non-normal distribution, and TAIEX returns reveal the volatility clustering, that is the tendency for volatility periods of similar magnitude to cluster in the Fig. 3b. Therefore, usually GARCH models can take into account the time-varying volatility phenomenon over a long period and provide very good in-sample estimates.

When the study began, the time-to-maturity based on the trading and expiration dates was calculated first. The option strike price is the price agreed upon in the option contact. When the contract expires, the market price should be the same as the strike price. If the market price of an underlying asset is below the strike price, the option holder will not exercise the option because to do so would not be profitable. Furthermore, if the market price exceeds the strike price, then exercising option will be profitable.

While the Black–Scholes function of Eq. 2 holds, S, X, T, σ, r can be used as the inputs, and the option price can be used as the output for establishing a neural network. However, much of the difference in prices may occur on a single trading day. To narrow the data range for improving training, moneyness is used for data partitioning. The moneyness is defined

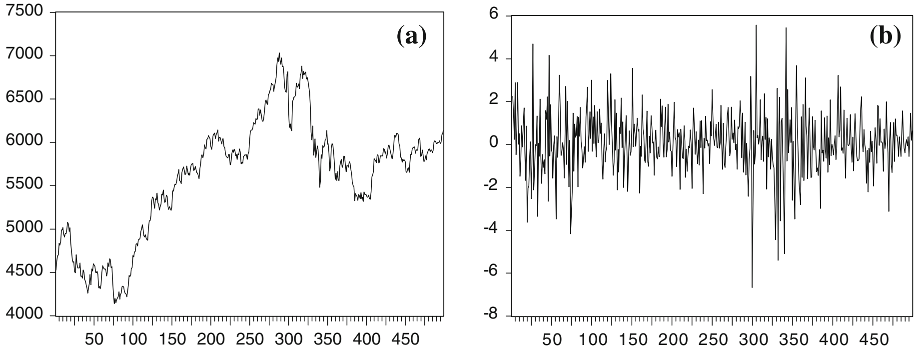


Fig. 3 (**a**) The trend graph of taiex (**b**) The trend graph of taiex returns

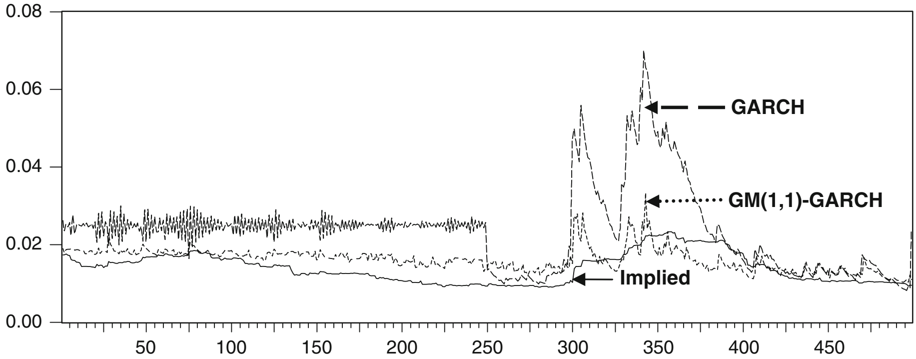


Fig. 4 The trend graph of taiex returns volatities

as the quotient of the stock price and strike price. This approach accelerates neural network training and facilitates the establishment of more stable models. Moneyness is applied to reduce the number of inputs for larger networks and to partition the dataset to clarify which sets the neural network models best. This study partitions the data into four subsets based on its moneyness. The quotient is defined to balance the number of different sets and these are, including in-the-money, at-the-money, out-of-the-money and deep-out-of-the-money. Quotients of stock prices to strike prices of below 0.91, between 0.91 and 0.97, between 0.97 and 1.03 and exceeding 1.03 are used for data partitioning. Table 1 lists all of the sets used.

Table 2 lists the total number of patterns. These patterns are the datasets studied using the neural network models presented here. This study employs 70% of the data from the dataset as the training set. The remaining 30% then comprises the testing set. Given these preparations, this study uses the time to maturity, moneyness, volatility and risk-free rate as inputs and the option price as outputs in the neural network.

Since the historical volatility is an average based on the returns during the preceding 60 days, it makes the estimate smoother and hence less sensitive to daily market fluctuations. Therefore, Fig. 4 shows the implied, GM(1,1)-GARCH and GARCH volatility for January 2, 2003 through December 31, 2004. As can be observed, the implied volatility is markedly lower than the GARCH and GM(1,1)-GARCH volatility, and the GARCH volatility estimates is markedly higher than the implied and GM(1,1)-GARCH volatility. Additionally,

Table 1 Data partition according to moneyness

Subset	Moneyness	Number
In-the-money	$S/X > 1.03$	3,860
At-the-money	$0.97 < S/X \leqq 1.03$	4,185
Out-of-the-money	$0.91 < S/X \leqq 0.97$	3,622
Deep-out-of-the-money	$S/X \leqq 0.91$	3,915

Notes. S is the current underlying asset price; X is the strike price

Table 2 Entries of each dataset

Subset	All	Train	Test
In-the-money	3,860	2,702	1,158
At-the-money	4,185	2,931	1,254
Out-of-the-money	3,622	2,537	1,085
Deep-out-of-the-money	3,915	2,742	1,173

Notes. Train (Test) denotes the Training (Testing) dataset

Table 3 Rmse results of different volatilities

Moneyness	Volatility							
	Historical		Implied		GARCH		GM-GARCH	
	RMSE							
	TrE	TeE	TrE	TeE	TrE	TeE	TrE	TeE
In-the-money	0.0191	0.0230	0.0176	0.0251	0.0181	0.0263	0.0189	0.0219
At-the-money	0.0285	0.0352	0.0250	0.0291	0.0283	0.0290	0.0263	0.0277
Out-of-the-money	0.0275	0.0438	0.0211	0.0421	0.0286	0.0349	0.0276	0.0354
Deep-out-of-the-money	0.0353	0.0502	0.0327	0.0334	0.0372	0.0222	0.0359	0.0278

Notes. TrE (TeE) denotes the Training (Testing) Error

the GM(1,1)-GARCH volatility employed grey modeling (GM(1,1)) to reduce the stochastic and nonlinearity of the error term sequence, and then to forecast the parameter estimates to further adjust the transformation of the error term sequence.

Table 3 lists the neural network for the different volatility models results in RMSE. First, for the in-the-money option and at-the-money, the GM(1,1)-GARCH for volatility has smaller testing error (0.0219) than the other approaches. For the out-of-the-money and deep-out-of-the-money options, the approach with the best performance (i.e. the smallest error) is the GARCH. Additionally, in terms of historical volatility, train error (0.0191) and testing error (0.0230), the in-the-money call option has smaller error than the at-the-money, out-of-the-money and deep-out-of-the-money options. The same condition also occurred in implied volatility and grey forecasting, except in the case of the GARCH. In the GARCH, the deep-out-of-the-money option has the smallest testing error (0.0222).

The above comparisons involve the neural network outputs. This study next uses the measured and actual prices to compare the best approach presented here with the four volatilities in Table 4. The MAE results indicate that for in-the-money and at-the-money call option, the GM(1,1)-GARCH has smaller error than for the other approaches. Furthermore, for the out-of-the-money and deep-out-of-the-money options the GARCH has better performance than other volatilities. Consequently, the price assessed based on GM(1,1)-GARCH and GARCH volatilities more closely reflects the market price. The results obtained via MAPE are almost the same as those obtained with the MAE method. For in the in-the-money option the GM(1,1)-GARCH yields the smallest error, meanwhile for the at-the-money option all has smallest error among all of the different volatilities. However GARCH beat other volatility approaches in the out-of-the-money and deep-out-of-the-money option. The result means that the GM(1,1)-GARCH volatility used in neural network model measures the in-the-money option price comparatively near the market actual price.

5 Conclusions

This study applies the different volatility modeling approaches to study the predictability of Taiwan stock index price by applying nonlinear neural network forecast models to daily observations of four financial variables. Overall, the in-the-money option is valuable to call option price, the empirical result indicates that the GM(1,1)-GARCH achieves better forecasting performance than the historical, implied volatility and GARCH approaches. GM(1,1)-GARCH employs the forecasting property of the GM(1,1) to continually modify

Table 4 Different volatility approaches results in mae and mape

Moneyness	Indices							
	MAE				MAPE			
	Volatility							
	Historical	Implied	GARCH	GM-GARCH	Historical	Implied	GARCH	GM-GARCH
In-the-money	78.45	89.57	94.18	78.16[a]	0.2182	0.2625	0.2749	0.2155[a]
At-the-money	50.49	39.43	39.79	37.33[a]	2.2038	2.1171	1.8147[a]	1.6843[a]
Out-of-the-money	39.79	37.80	28.41[a]	28.84	7.6190	9.7718	5.2534[a]	5.2865
Deep-out-of-the-money	35.85	21.16	11.12[a]	16.31	12.4638	7.5921	1.3036[a]	2.7178

Notes. [a] Smallest value

the squared error terms sequence, and combines the traditional symmetric GARCH for estimating the volatility. The GM(1,1)-GARCH has been demonstrated to the in-the-money option, GM(1,1)-GARCH has relatively good market forecasting ability. This result reveals that GM(1,1)-GARCH also can be used to forecast derivative securities prices.

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