

Volatility behavior of oil, industrial commodity and stock markets in a regime-switching environment

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ABSTRACT

This study supplements previous regime-switching studies on WTI crude oil and finds two possible volatility regimes for the strategic commodity prices of Brent oil, WTI oil, copper, gold and silver, and the S&P 500 index, but with varying high-to-low volatility ratios. The dynamic conditional correlations (DCCs) indicate increasing correlations among all the commodities since the 2003 Iraq war but decreasing correlations with the S&P 500 index. The commodities also show different volatility persistence responses to financial and geopolitical crises, while the S&P 500 index responds to both financial and geopolitical crises. Implications are discussed.

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1. Introduction

Given these volatile times, investors, traders, portfolio managers, commodity-exporting countries and monetary authorities would be interested in understanding the dynamic volatility behaviors of the most widely traded commodities, the stock markets and their dynamic correlations. The commodity and stock classes have increasingly become part of asset portfolio allocations. Most of the previous studies on commodities concentrate on the volatility of West Texas Intermediate (WTI) crude oil and give scant attention to strategic commodities such as Brent crude oil, gold, silver and copper. WTI and Brent are the global oil benchmarks for light, sweet crudes. These commodities are among the mostly traded on the world markets, and are influenced by macro-financial variables. Recently, it has become more evident that commodity (particularly oil) traders concurrently eye both the commodity and stock market movements to determine the directions of commodity prices and stock indices. For example, data could be bullish for oil price to go up, but a concomitant large drop in the stock markets may effectuate a change in the oil price direction to the down side and vice versa. Investment in stock markets provides an alternative to commodities, since the presence of the stock markets in the system provides a mechanism for substitution between stock and commodity classes.

Focusing on one volatility regime in examining volatility of commodities and stock indices may lead to spurious, insignificant

results as factors may average out or offset each other. Assuming one regime or a regime breakdown can disrupt volatility behavior if markets move from a regime of low volatility to another of high volatility or vice versa. An oil option trader, for example, would be interested in knowing how oil volatility changes between different regimes and whether volatility of the high volatility regime would fade out more quickly than volatility of the low volatility state. This information is useful in pricing financial derivatives and determining lengths of contracts in low and high volatility markets. A gold trader would want to figure out the difference in duration of volatility of gold and that of, say, oil or silver during a high volatility state. A commodity portfolio manager would be keen on understanding whether one commodity, such as gold, can be used as a hedge or a safe haven for another commodity such as copper in the portfolio at times of high volatility. A commodity-exporting country such as South Africa, Australia and/or Canada would want to understand the volatility of its commodity-export revenues in order to assess their impact on their economies. A monetary authority would also want to distinguish between the durations of commodity or stock volatilities in order to figure out whether volatility will fade out quickly without causing negative economic side effects or intervention is necessary to combat inflation or lessen market exuberance.

The Generalized Autoregressive Conditional Heteroskedasticity model (GARCH) with one regime has been popular in the literature in examining volatility. Bollerslev (1986) generalized the ARCH model, which was introduced by Engle (1982), to GARCH. These models are widely used in modeling financial variables' volatility.² Hammoudeh and Yuan (2008), Narayan and

² Power GARCH also has been used in modeling commodity conditional volatility. See Tully and Lucey (2006) and Mackenzie et al. (2001).

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Narayan (2007), Sadorsky (2006), Tully and Lucey (2006) and Hammoudeh et al. (2004) have applied the GARCH models to different commodities, particularly oil. These standard models can deal with the volatility persistence, asymmetry and clustering displayed by many time-series for one regime. However, research has also shown that the standard GARCH model leads to over-estimation of volatility persistence, and then a correction procedure is needed to detect the sudden jumps in volatility (Aggarwal et al., 1999; Malik, 2003; Hammoudeh and Li, 2006). A common correction procedure calls for entering dummy variables exogenously to account for the sudden changes, which in turn leads to significant reduction in volatility persistence. Moreover, while volatility may be persistent, there could be frequent and relatively unpredictable regime shifts which the standard model cannot account for (Glosten et al., 1993). Using a rolling AR(1)-GARCH, Watkins and McAleer (2008) show that the conditional volatility for two non-ferrous metals, namely aluminum and copper, is time-varying over a long horizon. Additionally, research has shown that the long-run forecast performance of the standard GARCH model is less satisfactory as the contrast between the in-sample and out-of-sample evaluations is widely observed (Sadorsky, 2006; Balaban, 2004; West et al., 1993).

More recently, the space-state model with a Markov regime-switching process is used to model volatility and shifts in return regimes for oil and stock markets (Hamilton and Susmal, 1994; Kim and Kim, 1996; Kim and Nelson, 1999; Bhar and Hamori, 2004).³ McCarthy and Najand (1995) use this space model to examine the links between the stock markets in Canada, Germany, Japan, UK and USA. Using daily returns data, these authors find those markets to be linked and that the US market exerts the most influence on the other markets. Chu et al. (1996) apply a Markov-switching model to the monthly market returns of the New York Stock Index and examine variations in volatility in different return regimes. They find that volatility is higher in negative return regimes than in positive return regimes. Schaller and van Norden (1997) employ the regime switching model to study volatility switching in the US market and find strong evidence for switching behavior. To our knowledge, there is no study that examines the time-varying volatility over low and high volatility regimes for the strategic commodities, stock markets and their dynamic correlations.

The main advantage of the Markov-switching, space state model over the standard GARCH model is that in the case of the latter the unconditional variance is constant, while in the former the variance changes according to the state of the economy. The Markov regime-switching model can detect switches in the volatility states of the returns, measure lengths of duration in each volatility state and help measure the correlations of movements between markets in each state.

The major focus of this paper is on examining shifts in volatility between two unobserved regimes of five strategic commodity prices and US stock markets, and their dynamic interrelations using Markov regime-switching models. The objectives can be summarized as follows:

- (1) To measure the switch in return volatility between the high and low variance regimes for the five commodity returns and the US stock market using Markov-switching models.
- (2) To measure the duration of the volatility, in terms of trading weeks, of the high and low variance regimes of the commodity and stock returns. If the paper shows that the duration of the high volatility return regime for a certain commodity or stock index is much shorter than that of its low

volatility regime, then the high volatility regime will fade out quickly.

- (3) To identify the evolution of correlations between the commodity and stock markets over time to ascertain whether the dynamic correlations' increases or decreases have implications for managing risk in those markets.
- (4) To compare the patterns of persistence of volatilities and convergence to long-run equilibrium before and after financial and geopolitical events in order to ascertain which commodities and S&P 500 index are more sensitive to this type of events than the other.

The results demonstrate that there exist high and low volatility regimes in the five commodity prices and S&P 500 index. The across-regime persistence is more pertinent to oil and industrial commodities than to S&P 500 index, which has more persistence within a specific regime over time. WTI crude oil, followed by gold, shows the strongest sensitivity to relative changes from low to high growth regimes.

The organization of this paper is as follows. Section 2 presents the data description and characteristics. Section 3 discusses the methodology and results. Section 4 concludes.

2. Data description

This paper uses weekly data for the closing spot prices of five strategic commodities – WTI oil, Brent oil, gold, silver and copper – and the US S&P 500 index. The sample covers the weekly period from January 2, 1990 to May 1, 2006. Crude oil includes the two global light benchmarks: the West Texas Intermediate spot (WTI) and the North Sea Brent spot. WTI spot is traded at NYMEX and delivered at the end of the pipeline at the Cushing, Oklahoma center, while Brent spot is traded at London's Petroleum Exchange (LPE). The WTI and Brent prices are expressed in US dollars per barrel. Although those two oil benchmarks belong to one great pool, they display different dynamics. The precious yellow metal and reserve currency, gold, and the precious and industrial metal, silver, are both traded at COMEX in New York, and are expressed in US dollars per troy ounce.⁴ The industrial metal, copper spot, which is also called Dr. Copper, is traded at London's Metal Exchange (LME) and its price is quoted in dollars per tonne.⁵ The S&P 500 (SP500, thereafter) is an index containing the stocks of 500 large-cap, publicly traded corporations, most of which are American, traded at the New York Stock Exchange and NASDAQ. It is considered to be a bellwether for the US economy and is a component of the US Index of Leading Economic Indicators.

The descriptive statistics reveal that among the five commodities the Brent spot price has the highest annualized average return, followed by WTI and copper over the sample period (see notes of Table 1).

Gold has the lowest annualized average return among those commodities. The yellow metal is known to have notorious extended bear markets. Silver has twice the return of gold, giving support to the notion common among precious metal traders that "if you want to buy gold, buy silver". S&P 500 has a higher annualized return than any of the five commodities, favoring stocks over commodities.

In terms of commodity volatility, oil Brent and WTI consistently with their average returns have the highest volatility,

⁴ Price of silver is usually quoted in cents per troy ounce but we transformed it into dollars per troy ounce.

⁵ It is believed that copper because of its linkages to the overall economy tracks the business cycles well, and thus has some predictability characteristics. See Lahart (2006).

³ For earlier research, see Hamilton (1989) and Turner et al. (1989).

Table 1
The weekly descriptive statistics.

Statistics	Brent	Copper	Gold	Silver	WTI	SP500
Mean	0.0014	0.0012	0.0005	0.0010	0.0013	0.0015
Std. dev.	0.0543	0.0029	0.0187	0.0318	0.0526	0.0218
Skewness	−0.3421	−0.1736	0.7759	−0.0213	−0.1692	−0.1394
Kurtosis	9.2963	4.4657	11.2944	5.3442	7.4208	6.4375
Jarque–Bera	1420.664	80.363	2521.864	194.695	696.228	421.761
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	851	851	851	851	851	851

Notes: The sample period ranges from January 2, 1990 to May 2006. The statistics are for the first differences of the log of the variables (or returns) over the weekly period January 2, 1990–May 1, 2006. The 52-week annualized mean returns are 7.3%, 6.2%, 2.6%, 5.2%, 6.8% and 7.8% for Brent, copper, gold, silver and WTI and S&P 500 index, respectively. The 52-week annualized standard deviations are 2.8, 0.15, 0.97, 1.65, 2.73 and 1.13.

followed by silver, while copper and gold are the least volatile among the commodities. The particularly high volatility of oil prices can complicate the interpretation of market signals and may adversely affect new investment. The lowest volatility for copper has to do with this industrial metal's predictability and the good match between its supply and demand. One of the effectual ways to dampen commodity price volatility is by increasing transparencies through the availability of accurate and timely data.

Despite its highest return, the S&P 500 index has volatility that is slightly higher than the gold volatility, making stocks more profitable on return-risk basis than the five strategic commodities during the sample period. All of the displayed returns have non-symmetric distributions as shown by the skewness and kurtosis statistics. Most of the distributions have a kurtosis significantly higher than 3, implying that extreme market movements in either direction (gains or losses) occur in these commodity and stock markets with greater frequency in practice than would be predicted by the normal distribution.⁶ The Jarque–Bera statistics confirm the non-normal distribution of all the return data series.

Table 2 shows the historic contemporaneous correlation patterns for the returns of the five commodity prices and the S&P 500 index over the sample period.

The returns of these well-diversified commodities have positive correlations, which does not make some of them good substitutes in risk management. Commodities are affected by common macroeconomic variables and are also subject to herding behavior. However, there is a negative but weak, non-significant correlation between WTI crude and S&P 500, contributing to lower portfolio variance. This suggests that higher WTI oil prices are generally associated with lower stock market indices. Oil is an input or feedstock in production, and its price can affect consumer spending and signal inflation, thus it can negatively impact corporate earnings.

Not surprising, the highest return correlation is between the two light, sweet crudes: Brent and WTI crude oil, amounting to 0.814. These two oil grades are the leading benchmarks in the oil market, and thus are not good as a hedge or a safe haven for each other because they belong to one great pool (Hammoudeh et al., 2008). Interestingly, gold and silver show a correlation of only 0.606, implying that these two precious metals have some varying economic uses. Ciner (2001) argues that there has been a separation between gold and silver returns in the 1990s. However, Tully and Lucey (2006) suggest that while there are periods when the relationship between those two precious metals

is weak, overall a stable relationship prevails. Also surprisingly, the lowest return correlation among the commodities is between the copper and crude oil markets, which are 0.075 with WTI and 0.088 with Brent. This is perhaps due to the fact that oil is more of a financial play against risk than copper which moves more closely with the business cycle. It is still not conclusive whether crude oil of any of the two types can be a hedge or a safe haven for copper or vice versa over time because under normal conditions both commodities can be affected by the business cycle.⁷ The correlations between the commodities and S&P 500 are very low because hard assets move less quickly than financial assets.

We use the ADF, Phillips–Perron and ERS tests to check the stationarity of the five commodity prices and the S&P 500 index. The ADF and PP tests show that all variables are I (1), while the results of ERS are slightly mixed. The results are available on request.

3. Models and empirical results

To achieve the objectives stated in the introduction, we present the specifications of two different Markov-switching GARCH models. The first model is a univariate MS heteroskedasticity model with two regimes. The purpose of this model is to measure the switch in return volatility between the high and low variance regimes and to gauge the duration of the volatility of the two regimes. The second is the dynamic conditional correlation (DCC) multivariate GARCH model. The purpose of this multivariate GARCH model is to assess the evolution of correlations between commodity markets and with S&P 500 index over time and their suitability as hedges for each other.

3.1. Markov-switching heteroskedasticity model

Consider the following Markov-switching heteroskedasticity in-the-disturbance model:

$$r_t = \mu + \phi r_{t-1} + e_t, \quad e_t | D_t \sim \text{iid } N(0, \sigma_{D_t}^2) \quad (1)$$

$$\sigma_{D_t}^2 = \sigma_0^2(1 - D_t) + \sigma_1^2 D_t, \quad \sigma_0^2 < \sigma_1^2 \quad (2)$$

where r_t is the rate of return of a commodity price or the S&P 500 index, σ_0^2 and σ_1^2 refer to low and high volatilities, and D_t is a latent variable modeled as a first-order Markov process (for two regimes) with transition probabilities given by

$$P[D_t = 0 | D_{t-1} = 0] = q, \quad P[D_t = 1 | D_{t-1} = 1] = p \quad (3)$$

where $D_t = 0$ refers to the low volatility regime at time t and $D_t = 1$ is related to the high volatility regime, and q and p are the

⁶ For example, in equity markets a 5% daily loss is observed to occur once every 2 years, while if returns were normally distributed such a change would be expected only once in every one thousand years (Johansen and Sornette, 1999). On January 17, 1991, the first full day of the Gulf war, oil prices dropped by about 18% in 1 day.

⁷ For more information on how gold can be a hedge or a safe haven for bond and stock, see Baur and Lucey (2006).

Table 2

Spot price and S&P 500 index contemporaneous return correlations.

Correlation	Brent	Copper	Gold	Silver	WTI	SP500
Brent	1.000					
Copper	0.088 (2.565) ^a	1.000				
Gold	0.171 (5.049) ^a	0.134 (3.925) ^a	1.000			
Silver	0.111 (3.258) ^a	0.197 (5.843) ^a	0.606 (22.185) ^a	1.000		
WTI	0.814 (40.737) ^a	0.075 (2.176) ^b	0.148 (4.353) ^a	0.114 (3.330) ^a	1.000	
SP500	0.008 (0.255)	0.022 (0.638)	0.022 (0.665)	0.031 (0.911)	−0.024 (0.911)	1.000

Notes: The sample period ranges from January 2, 1990 to May 2006. Ordinary Pearson correlation method is used for calculating t -statistics which are given in parentheses below the estimates.

^a Denotes rejection of the hypothesis at the 1% level.

Table 3

Estimation of Markov-switching commodity and stock volatility models.

Parameter	Brent	Copper	Gold	Silver	WTI	SP500
μ	0.002 (0.001)	0.001 (0.001)	−0.000 (0.001)	−0.000 (0.001)	0.003 (0.002)	0.002 (0.000) ^a
ϕ	−0.038 (0.034)	0.023 (0.036)	−0.013 (0.035)	0.001 (0.003)	−0.105 (0.036) ^a	−0.1292 (0.035) ^a
σ_0	0.038	0.022	0.011	0.022	0.039	0.013
σ_1	0.081	0.045	0.024	0.046	0.086	0.027
$\Pr(D_t = 0)$	0.986	0.966	0.984	0.980	0.981	0.992
$\Pr(D_t = 1)$	0.966	0.899	0.986	0.961	0.921	0.990
Duration ($D_t=0$)	73.07	29.23	61.50	50.77	51.62	125
Duration ($D_t=1$)	29.51	9.85	72.68	25.64	12.71	100
AIC	−3.175	−4.281	−5.283	−4.212	−3.214	−5.011
HQ	−3.162	−4.268	−5.270	−4.199	−3.201	−4.998
SIC	−3.142	−4.247	−5.250	−4.179	−3.181	−4.978
Log-likelihood	1355.373	1825.427	2251.305	1796.431	1372.219	2135.938

Notes: The sample period ranges from January 2, 1990 to May 1, 2006. Standard deviations are reported in the parenthesis. Duration for the high volatility regime is defined by $E[\Pr(D_t = 1)] = 1/(1-p)$. The duration for the low regime is defined similarly by replacing p with q . The parameters σ_0 and σ_1 represent the volatilities of the low and high regimes, respectively. $D_t=0$ refers to the low volatility regime at time t and $D_t=1$ is related to the high volatility regime.

^a Denotes rejection of the hypothesis at the 1% levels.

corresponding transition probabilities governing the evolution of D_t for those two regimes. The expected duration of the low volatility regime (σ_0) is given by $E(D_t=0)=1/(1-q)$ and for the high volatility regime is given by $E(D_t=1)=1/1-p$.

To estimate this model we derive the joint density of r_t , D_t and D_{t-1} conditional on the past information I_{t-1} :

$$f(r_t, D_t, D_{t-1} | I_{t-1}) = f(r_t | D_t, D_{t-1}, I_{t-1}) \Pr[D_t, D_{t-1} | I_{t-1}] \quad (4)$$

$$= \frac{1}{\sqrt{2\pi\sigma_{D_t}^2}} \exp\left(-\frac{(r_t - \mu - \phi r_{t-1})^2}{2\sigma_{D_t}^2}\right) \Pr[D_t, D_{t-1} | I_{t-1}] \quad (5)$$

Next we use Eq. (5) to derive the marginal density function $f(r_t | I_{t-1})$ as follows:

$$\begin{aligned} f(r_t | I_{t-1}) &= \sum_{D_t=0}^1 \sum_{D_{t-1}=0}^1 f(r_t, D_t, D_{t-1} | I_{t-1}) \\ &= \sum_{D_t=0}^1 \sum_{D_{t-1}=0}^1 f(r_t | D_t, D_{t-1}, I_{t-1}) \Pr[D_t, D_{t-1} | I_{t-1}], \end{aligned} \quad (6)$$

From Eq. (6), we can find the following log likelihood:

$$\ln L = \sum_{t=1}^T \ln \left[\sum_{D_t=0}^1 \sum_{D_{t-1}=0}^1 f(r_t | D_t, D_{t-1}, I_{t-1}) \Pr[D_t, D_{t-1} | I_{t-1}] \right] \quad (7)$$

where $\Pr[D_t = j, D_{t-1} = i | I_{t-1}] = \Pr[D_t = j | D_{t-1} = i] \Pr[D_{t-1} = i | I_{t-1}]$, for $i, j=0, 1$. We can compute the weight term, $\Pr[D_t, D_{t-1} | I_{t-1}]$, in Eq. (7) by updating it, once r_t is observed at time t , as follows:

$$\Pr[D_t = j, D_{t-1} = i | I_t]$$

$$= \frac{f(r_t | D_t = j, D_{t-1} = i, I_{t-1}) \Pr[D_t = j, D_{t-1} = i | I_{t-1}]}{\sum_{D_t=0}^1 \sum_{D_{t-1}=0}^1 f(r_t | D_t = j, D_{t-1} = i, I_{t-1}) \Pr[D_t = j, D_{t-1} = i | I_{t-1}]} \quad (8)$$

with $\Pr[D_t = j | I_t] = \sum_{D_{t-1}=0}^1 \Pr[D_t = j, D_{t-1} = i | I_t]$. By iterating Eqs. (7)

and (8) for $t=1, 2, \dots, T$, we will have appropriate weighting terms in $f(r_t | I_{t-1})$ (see Hamilton (1989) and Kim and Nelson (1999)). We use the algorithm suggested by Kim (1994) to calculate the smoothed probabilities of each regime.

The parameter estimates of the Markov-switching heteroskedasticity models for the five commodity price and S&P 500 index returns are given in Table 3.

Among the commodity prices, WTI spot has the highest volatility in both the low and high variance regimes followed by Brent spot. This finding is in contrast with the historical volatility which places Brent ahead of WTI. Gold has the lowest in both regimes. This yellow metal is different from the other commodities with respect to economic uses and annual production. First, portion of its jewelry supply is hoarded and recycled which smoothes out its volatility. Second, another portion is used as part of international reserves and is employed occasionally by central banks to influence exchange rates, adding to less volatility. Third, the annual production and demand is small relative to its available supply, making volatility small.

In terms of the relative high-to-low volatility ratio, WTI has a ratio of 2.21 showing the strongest sensitivity to switches in growth regimes, followed by gold with a relative volatility of about 2.18. These findings point to the importance of hedging against volatility when market forces change for these

commodities. Copper has the lowest relative switch volatility, hovering around 2.05. Because of its extensive linkages with the overall economy and predictability, this brown metal seems to be able to predict the business cycle that gives traders and buyers enough time to adjust or hedge against volatility. The market in copper may be more competitive and has more substitutes than the oil market, leading to less monopoly power and less volatility. The S&P 500 index has an absolute volatility in both the low and high variance regimes that is slightly higher than Gold price but its relative volatility (2.07) is lower than that of gold (2.18). This result is consistent with the historical unconditional volatility estimate provided in Table 1.

The probability of low volatility regimes (q) is higher than the probability of high volatility regime (p) for all the commodity and stock markets, with the exception of the Gold market (Table 3). This result indicates that the low regime is more stable and markets spend more time in this regime than the high regime for the stock market and most of the commodities, which is more comfortable for risk-averse investors in both the commodity and stock markets. Fig. 1 provides the smoothed probabilities of the high volatility return regime ($D_t=1$) for the five commodity prices and the S&P 500 index. These figures capture the timing of changes in all the high volatility regimes very well. They also display the persistence of the high volatility return regimes

visually. For example, the oil price return shows strong persistence for the high volatility regime. After 2004 and in contrast to the 1990s, the high volatility regime is very persistent over most years for copper, gold and silver but not for the S&P 500 index. This is probably due to the weak dollar and rising global demand for commodities.

Gold shows a highly persistent regime after 2001, the year of the attack on New York. The smoothed probability of the high volatility return regime for the S&P 500 index is very different from those for the five commodity prices. It shows very strong volatility persistence during the period 1996–2004. This finding suggests that at certain times the commodities and S&P 500 index can provide hedges against risk for each other and can be used in a portfolio to diversify risk over high and low regimes.

The durations of the low volatility state for Brent, copper, gold, silver and WTI are 73.07, 29.23, 61.50, 50.77 and 51.62 weeks, respectively, having an average of 53 weeks. The long duration for gold is due to its small annual demand relative to its available supply, the close association with the exchange rate and traders' perception that gold is the first safe haven among the precious metals during crises. Oil is also considered a hedge against inflation and weak dollar because it is priced in dollar. It is also affected by production and capacity constraints and by increases in demand above supply.

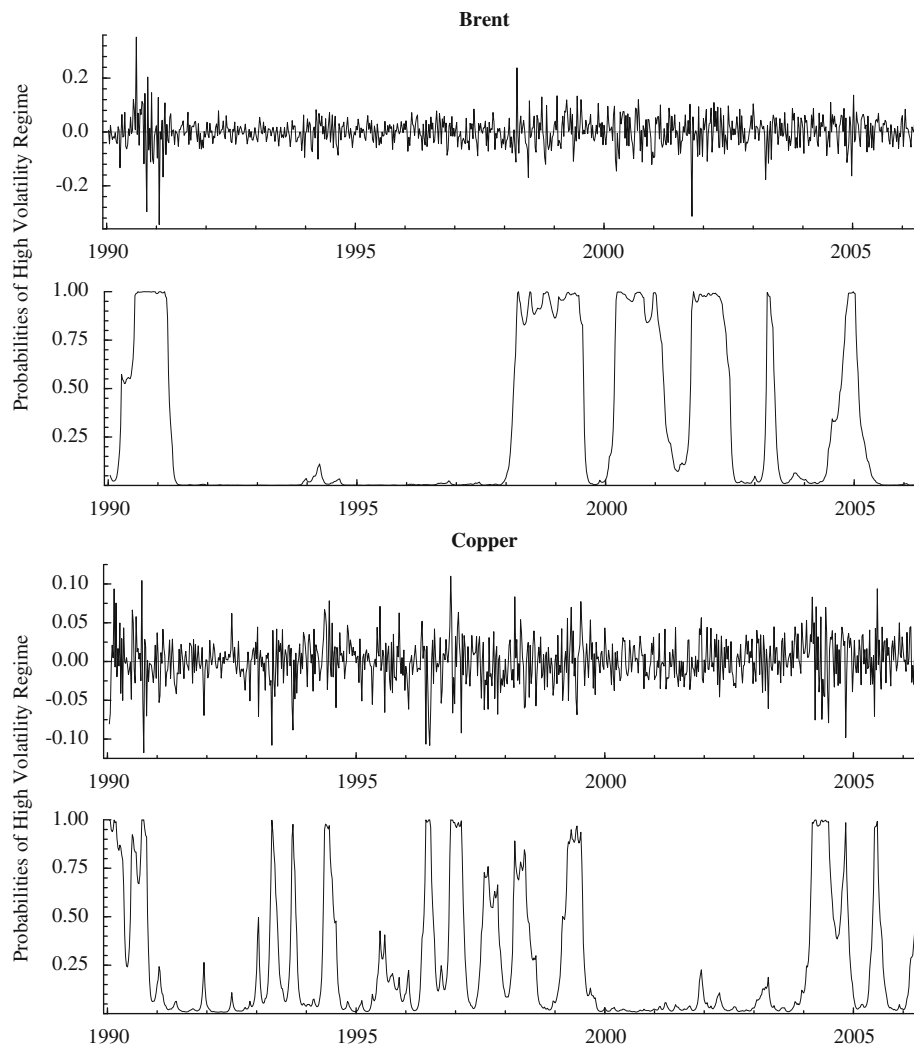


Fig. 1. Probability of high volatility regime.

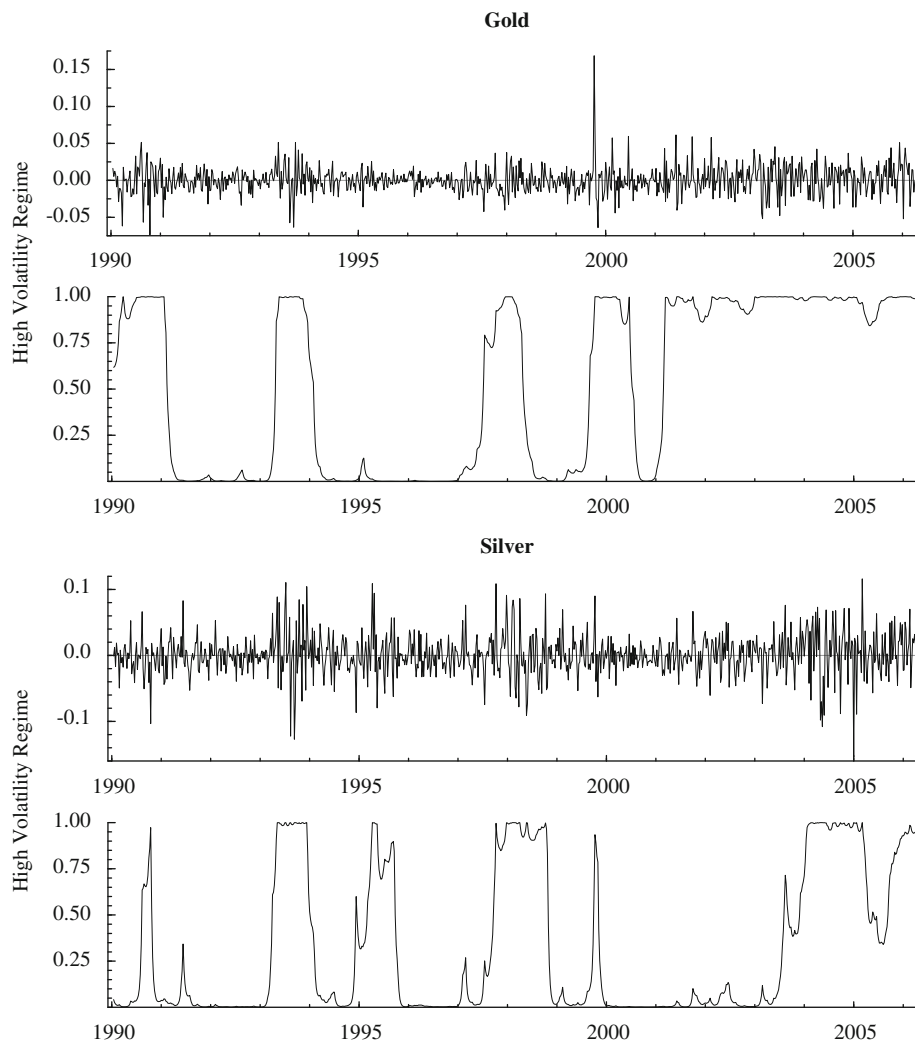


Fig. 1. (Continued)

Copper's lowest duration is due to better balance between its supply and demand and the great moderation brought about by lower output growth volatility and interest rate volatility.

The duration in the low volatility regime for S&P 500 index is 125 weeks, more or less double the durations of the commodities. Weaknesses or strengths in corporate earnings can last for several quarters. This implies that shocks in the low volatility regimes have much longer impact on the stock markets than on the strategic commodity markets.

The durations of the *high* volatility regime (which is the more relevant state) for the five commodities Brent, copper, gold, silver and WTI are 29.51, 9.85, 72.68, 25.53 and 12.71 weeks, respectively, having an average regime persistence of about 30 weeks. Gold has the highest duration in this regime because it is highly news-driven relative to other industrial commodities. When we exclude the gold market, the average duration of this high volatility regime is 17.34 weeks. Despite the fact that the gold market has the lowest volatility in both regimes, its high volatility return regime is the most regime persistent, compared to those of the other commodity markets. Gold's high persistence is related to duration of world crises, and depreciation/appreciation of the dollar which has long persistence. Interestingly, the high regime's duration for S&P 500 index is more than three times the average duration for the five commodities. This implies that high volatility shocks have much more persistence in the stock markets than in the strategic commodity markets as is the case for

the low volatility shocks. It is also worth noting that Hammoudeh (2007) found that the dollar exchange rate to have the greatest persistence or the slowest convergence to long-run equilibrium relative to the copper, silver and oil.

It is also interesting to assess the high volatility regime's probability correlations among the commodities and with S&P 500. These probability correlations are provided in Table 4.

Surprisingly, the highest probability correlation next to the one between WTI and Brent is between Brent and S&P 500 (0.539), which is higher than the correlation between WTI and S&P 500 (0.411). This indicates that chances of regime-switching or persistence between Brent and S&P 500 over time are very similar. The probabilities for silver and the two oil markets have a negative correlation, pointing to the oil–silver suitability in risk management than oil–gold suitability. Gold and silver show the fourth highest probability correlation, implying that the chances of regime-switching or persistence over time are somewhat similar for those two precious metals. These two precious metals have increasingly different economic uses.

In regime-switching models like ours, the occurrence of a regime is observed by the market. The econometrician infers regimes from the model. Thus, a source of uncertainty to regime-switching models is the ex-post determination of regimes. Until recently, the quality of regime classification is based on the smoothed ex-post regime probabilities. An innovation in this area is the regime classification measure (RCM) by Ang and Bekaert

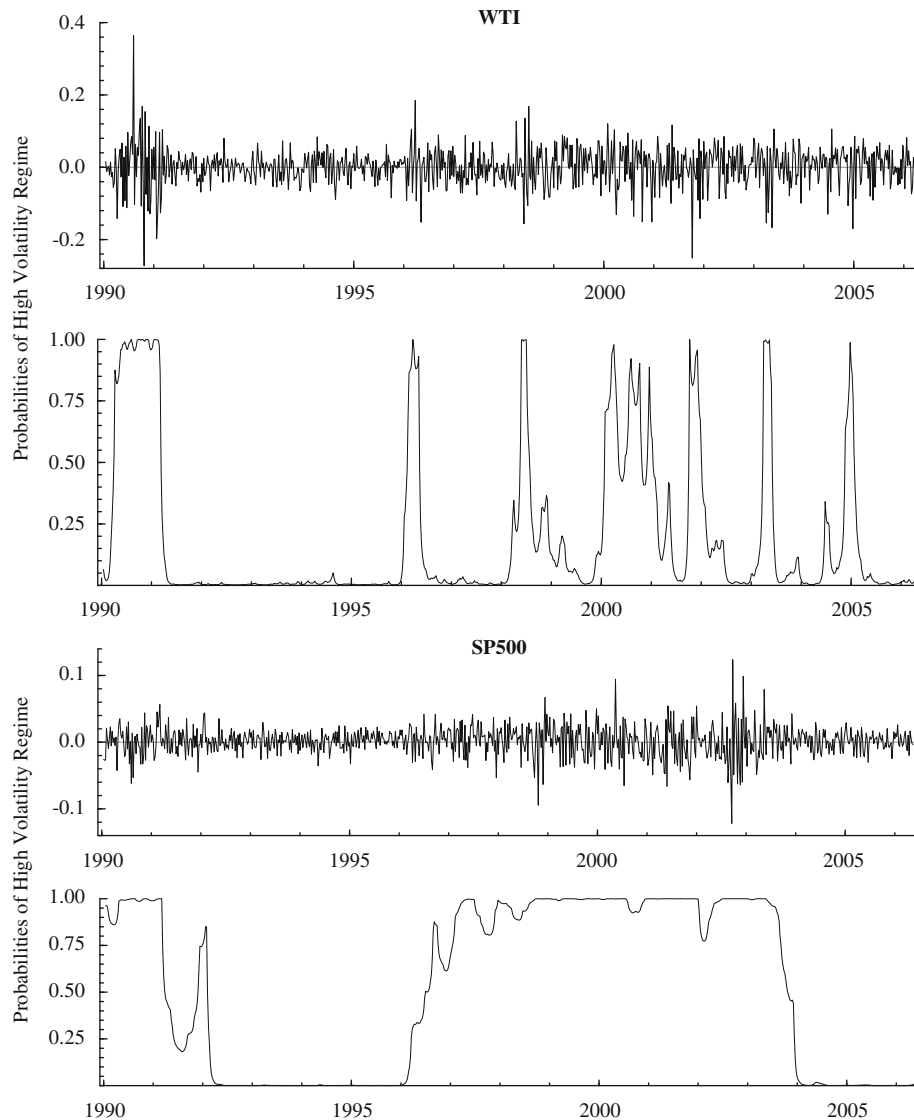


Fig. 1. (Continued)

Table 4
High volatility regime's probability correlations.

Correlation (<i>t</i> -statistic)	Brent	Copper	Gold	Silver	WTI	SP500
Brent	1.000					
Copper	0.037 (1.092)	1.000				
Gold	0.112 (3.296) ^a	0.082 (2.399) ^a	1.000			
Silver	−0.071 (−2.077) ^a	0.264 (7.981) ^a	0.378 (11.872) ^a	1.000		
WTI	0.697 (28.298) ^a	0.001 (0.040)	0.175 (5.189) ^a	−0.137 (−4.041) ^a	1.000	
SP500	0.539 (18.631) ^a	−0.044 (−1.308)	0.192 (5.700) ^a	−0.379 (−11.943) ^a	0.411 (13.148) ^a	1.000

Notes: Ordinary Pearson correlation method is used to calculate the *t*-statistics which are given in parentheses below the estimates.

^a Denotes rejection of the hypothesis at the 1%.

(2002). Therefore, to assess the quality of regime switching in our model, we report the regime classification measure (RCM) proposed by Ang and Bekaert (2002). This is essentially a sample estimate of the variance of the probability series. It is based on the idea that perfect classification of regime would infer a value of 0 or 1 for the probability series and be a Bernoulli random variable. The regime classification measure (RCM) is defined as

$$400 \times \frac{1}{T} \times \sum p_t \times (1 - p_t)$$

where p_t is the probability of being in a certain regime at time t . Good regime classification is associated with low RCM statistic values. A value of 0 means perfect regime classification and a value of 100 implies that no information about the regimes is revealed. Weak regime inference implies that the model cannot successfully distinguish between regimes from the behavior of the data and may indicate misspecification. With the data for the probability series for our study, the RCM measures are reported in Table 5.

We can see that the RCM values for all the series are reasonably low, especially when compared to those reported in Ang and

Bekaert (2002). This shows that the model is able to confidently distinguish which regimes are occurring at each point in time.

3.2. dynamic conditional correlation (DCC) multivariate GARCH model

We investigate the dynamic correlation relationships to identify the commodity and stock markets' correlations over time. It would be interesting and useful to determine whether correlations between commodities increase or decrease over time. This should shed light on whether or not commodities and stock markets can be substitutes in diversified portfolios. The correlations among the five commodity prices and with S&P 500 returns are estimated with the

dynamic conditional correlation (hereafter DCC) multivariate GARCH model which was suggested by Engle (2002). The DCC approach has a number of advantages over the simple contemporaneous correlation analysis for modeling correlations and other GARCH models. First, it is parsimonious compared to many multivariate GARCH models. Second, the DCC models are flexible since they allow for volatility of different assets to be taken into account.

The DCC model is based on the assumption that the N variables in r_t are conditionally multivariate normal, where the expectation is assumed to be zero and covariance matrix is H_t as follows:

$$r_t | I_{t-1} \sim N(0, H_t) \quad (9)$$

$$H_t = D_t R_t D_t \quad (10)$$

$$D_t = \text{diag}[\sigma_{1,t}, \dots, \sigma_{n,t}] \quad (11)$$

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2} \quad (12)$$

$$Q_t = S(1-a-b) + a(\varepsilon_{t-1}\varepsilon'_{t-1}) + bQ_{t-1} \quad (13)$$

$$\varepsilon_t = D_t^{-1} r_t \quad (14)$$

Table 5

Regime classification measure (RCM).

RCM	Brent	Copper	Gold	Silver	WTI	SP500
	17.616	33.831	14.960	23.095	16.613	23.533

Notes: $RCM = 400 \times \frac{1}{T} \times \sum (p_t \times (1-p_t))$.

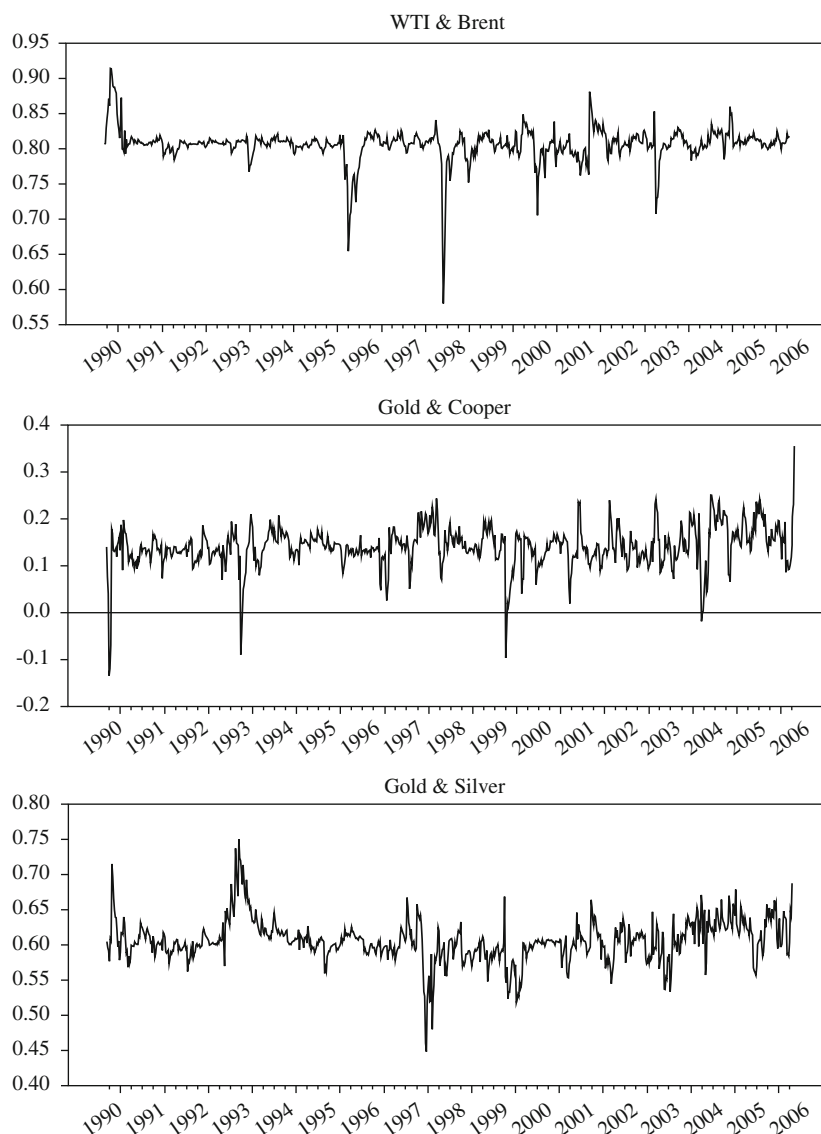


Fig. 2. Dynamic conditional correlation of commodity price returns.

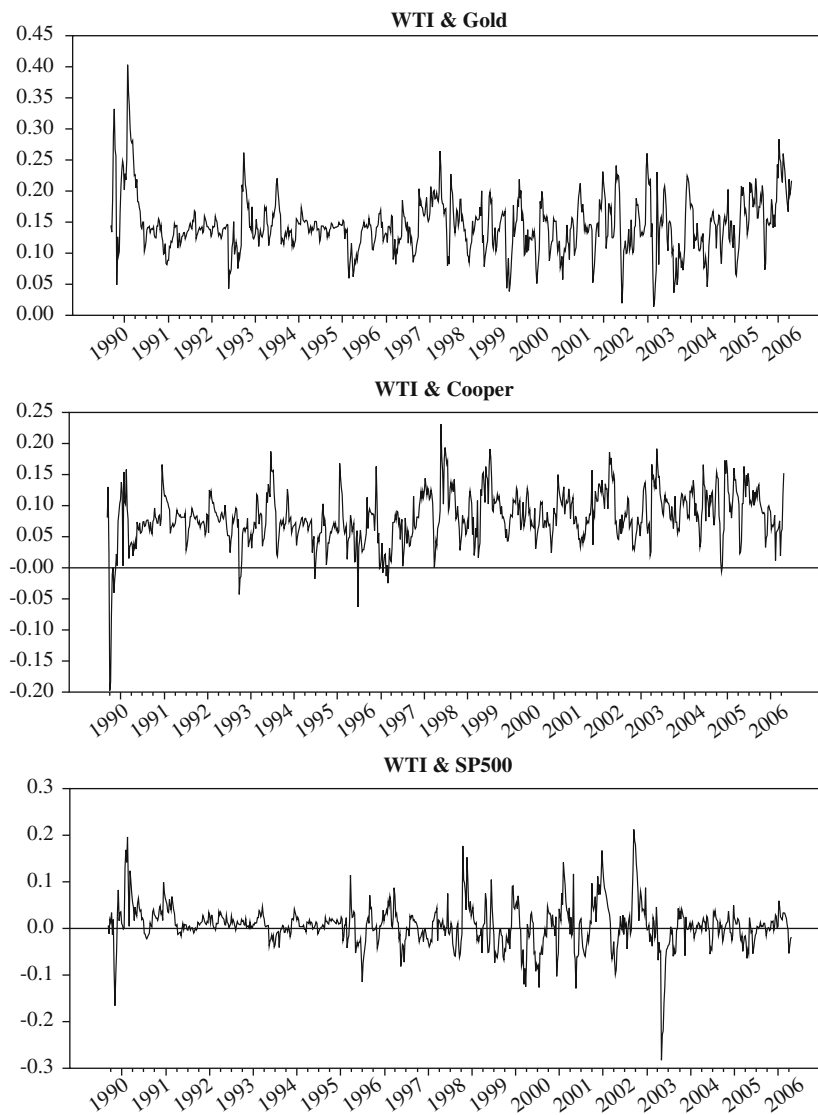


Fig. 2. (Continued)

where r_t is an N dimensional time series of length T , and D_t is the $N \times N$ diagonal matrix with the diagonal components being the square roots of the estimated univariate GARCH variances, R_t is the time-varying correlation matrix and S is the unconditional correlation matrix of ε_t , which is a consistent estimator of the unconditional correlation matrix, and a and b are parameters, which mean that the model is mean-reverting as long as $a+b < 1$. Q_t is a weighted average of a positive-definite and a positive-semidefinite matrices. If $R_t=R$ then the model becomes the constant conditional correlation (CCC) model of Bollerslev (1990).⁸

Engle (2002) suggests the two-step maximum likelihood estimation method of the model. The likelihood function is given by

$$l_t(\theta, \phi) = -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log |D_t| + r_t' D_t^{-1} D_t^{-1} r_t - \varepsilon_t' \varepsilon_t + \log |R_t| + \varepsilon_t' R_t^{-1} \varepsilon_t) \quad (15)$$

where the variables are defined as above. Engle decomposes this likelihood function into the volatility part $L_v(\theta)$ and the correlation part $L_v(\theta, \phi)$.

$$L_v(\theta) = -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + \log |D_t|^2 + r_t' D_t^{-2} r_t) \quad (16)$$

$$L_v(\theta, \phi) = -\frac{1}{2} \sum_{t=1}^T (\log |R_t| + \varepsilon_t' D_t^{-2} \varepsilon_t - \varepsilon_t' \varepsilon_t) \quad (17)$$

Note that in Eq. (16), the likelihood is the sum of the individual GARCH likelihoods. Engle (2002) suggests a two step procedure where the first step is to maximize the volatility part, and the second, given the maximizing value $\hat{\theta}$, is to maximize the correlation part.

The results of some selected DCC estimations for the five commodity prices and the S&P 500 index are quite interesting (results are provided upon request due to space constraint). Fig. 2 indicates that the commodity correlations have been volatile throughout the sample period. They in particular gather greater strength in years hovering around war and crisis times. They have also started increasing again since 2003 for all possible

⁸ We rewrite Eq. (13) as the GARCH (1,1) specification, $q_{ij,t} = \bar{p}_{ij} + \tilde{\alpha}(\varepsilon_{i,t-1} \varepsilon_{j,t-1} - \bar{p}_{ij}) + \tilde{\beta}(q_{ij,t-1} - \bar{p}_{ij})$, where \bar{p}_{ij} is the unconditional expectation of the cross product of $\varepsilon_{i,t} \varepsilon_{j,t}$. The correlation equation is defined as $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}$.

Table 6

Estimation results of DCC-multivariate GARCH(1,1) models.

Parameter	Brent	Copper	Gold	Silver	WTI	SP500
Sample period (1990/1/2–2006/5/01): whole						
μ	−0.004 (0.137)	0.057 (0.094)	−0.016 (0.051)	−0.045 (0.087)	−0.073 (0.137)	0.206 (0.057) ^a
ω	0.559 (0.219) ^a	1.459 (0.480) ^a	0.185 (0.055) ^a	0.262 (0.081) ^a	0.993 (0.327) ^a	0.047 (0.026) ^b
α	0.113 (0.014) ^a	0.151 (0.037) ^a	0.100 (0.021) ^a	0.074 (0.012) ^a	0.127 (0.020) ^a	0.097 (0.020) ^a
β	0.874 (0.017) ^a	0.690 (0.076) ^a	0.854 (0.025) ^a	0.900 (0.016) ^a	0.842 (0.024) ^a	0.896 (0.021) ^a
$\alpha + \beta$	0.987	0.841	0.954	0.974	0.969	0.993
Sample period (1990/1/2–1997/6/30): pre-1997 Asian crisis						
μ	−0.032 (0.157)	0.036 (0.143)	−0.059 (0.059)	−0.075 (0.126)	−0.061 (0.152)	0.228 (0.080) ^a
ω	0.640 (0.294) ^a	2.553 (0.809) ^a	0.173 (0.054) ^a	0.576 (0.206) ^a	1.320 (0.389) ^a	0.034 (0.031)
α	0.157 (0.003) ^a	0.224 (0.072) ^a	0.174 (0.042) ^a	0.068 (0.018) ^a	0.234 (0.046) ^a	0.061 (0.022) ^a
β	0.832 (0.033) ^a	0.528 (0.109) ^a	0.760 (0.047) ^a	0.864 (0.036) ^a	0.735 (0.044) ^a	0.928 (0.026) ^a
$\alpha + \beta$	0.989	0.752	0.934	0.932	0.969	0.989
Sample period (1997/7/1–2006/5/1): post-1997 Asian crisis						
μ	0.368 (0.226)	0.118 (0.114)	0.063 (0.079)	0.014 (0.110)	0.225 (0.213)	0.165 (0.080) ^b
ω	13.357 (4.634) ^a	0.504 (0.303) ^c	1.232 (0.872) ^a	0.099 (0.059) ^c	13.448 (6.063) ^b	0.095 (0.070)
α	0.147 (0.043) ^a	0.104 (0.034) ^a	0.054 (0.031) ^c	0.071 (0.015) ^a	0.133 (0.051) ^a	0.140 (0.033) ^a
β	0.422 (0.165) ^a	0.838 (0.058) ^a	0.660 (0.221) ^a	0.922 (0.015) ^a	0.387 (0.244)	0.853 (0.033) ^a
$\alpha + \beta$	0.569	0.942	0.714	0.993	0.520	0.993
Sample period (1990/1/2–2001/9/10): pre-2001 New York attack						
μ	−0.012 (0.138)	−0.010 (0.107)	−0.084 (0.050) ^c	−0.120 (0.085)	−0.069 (0.135)	0.225 (0.076) ^a
ω	0.308 (0.173) ^c	1.600 (0.656) ^b	0.151 (0.048) ^a	0.253 (0.096) ^a	0.698 (0.238) ^a	0.013 (0.016)
α	0.132 (0.019) ^a	0.144 (0.046) ^a	0.153 (0.034) ^a	0.074 (0.015) ^a	0.164 (0.025) ^a	0.051 (0.017) ^a
β	0.867 (0.016) ^a	0.677 (0.106) ^a	0.824 (0.027) ^a	0.895 (0.021) ^a	0.822 (0.023) ^a	0.948 (0.016) ^a
$\alpha + \beta$	0.999	0.821	0.977	0.969	0.986	0.999
Sample period (2001/9/11–2006/5/1): post-2001 New York attack						
μ	0.007 (0.139)	0.033 (0.086)	−0.014 (0.051)	−0.042 (0.082)	−0.055 (0.141)	0.219 (0.054) ^a
ω	0.546 (0.187) ^a	1.651 (0.530) ^a	0.174 (0.047) ^a	0.265 (0.085) ^a	0.916 (0.294) ^a	0.048 (0.028) ^c
α	0.111 (0.014) ^a	0.158 (0.041) ^a	0.104 (0.021) ^a	0.075 (0.012) ^a	0.119 (0.016) ^a	0.101 (0.020) ^a
β	0.872 (0.015) ^a	0.653 (0.087) ^a	0.854 (0.022) ^a	0.899 (0.017) ^a	0.847 (0.021) ^a	0.892 (0.022) ^a
$\alpha + \beta$	0.983	0.811	0.958	0.974	0.966	0.993

Notes: Standard errors are given in parentheses below the respective parameter estimates.

^a Denote rejection of the hypothesis at 1%, 5% and 10%.^b Denote rejection of the hypothesis at 1%, 5% and 10%.^c Denote rejection of the hypothesis at 1%, 5% and 10%.

relationships. It is worth noting that the DCC of Brent and WTI has two big drops around the financial crises in 1995 and 1998, indicating a change in behavior of the spread between those two grades of crude oil. These years coincide with the 1995 Russian crisis, and the 1997 Asian crisis. This oil spread can widen and narrow, or become positive or negative based on the changes in the fundamentals of WTI and Brent. In 2007, this spread becomes negative and WTI was labeled as the broken benchmark. After 2003, the DCC between these two oil benchmarks fluctuates around 0.85, which is close to the contemporaneous correlation.

Interestingly enough, the DCC of gold and silver has sharply increased after 2003, unsteadily climbing up to 0.70, which is higher than their contemporaneous correlation of 0.61. The DCC also indicates rising correlation between WTI and gold over time since 2003, reaching 0.30 compared to 0.149 in the contemporaneous case. The correlation between WTI and copper improved over the period 2001 and 2004, but in the last 2 years this DCC dropped probably due to interruptions in Chinese purchases of copper. Overall the DCC between the commodities improved recently, which has implications for investors, portfolio managers and monetary authorities.

In contrast, the DCCs between commodities and S&P 500 index have weakened more recently, indicating differing sensitivities to fundamentals and special factors prevailing in the commodity and stock markets. Contrary to stocks, commodities in general are being conceived to be in short supply relative to strong demand coming from developing countries such as China and India. They are also showing different sensitivities to crises and geopolitical

events. The decreasing commodity-stock correlations are more prominent between WTI and S&P 500.⁹

Finally, we examine the volatility persistence in the multivariate DCC GARCH model for the whole sample period and for sub-period splits according to two well-known structural breaks: the 1997 Asian crisis and the November 11, 2001 New York attack (Table 6).¹⁰ The first break is financial and the second is geopolitical. In all the sample and sub-sample periods, with the exception of the post-financial 1997 crisis sub-period, Brent crude has the over time highest volatility persistence and the slowest convergence to equilibrium, while copper has the least persistence and fastest convergence. WTI volatility persistence comes in the middle. This result highlights the greater impacts of imported oil benchmarked to Brent on forming causal relations among the macroeconomic variables in the US economy.¹¹ The Brent volatility seems to be especially prone to geopolitical crises because the Brent benchmark crudes exported from the Middle East and Africa. In contrast, Copper's volatility persistence has increased in the post-financial Asian crisis sub-period, which indicates that this volatility persists and converges less after financial and economic crises.

⁹ Gold and S&P 500, and Cooper and S&P 500 DCC also show similar patterns.¹⁰ The third break marking the start of the 2003 Iraq war did not produce meaningful results, probably because of the short size of the post war sub-period.¹¹ Bhar et al. (2008) show that the causal relations among macroeconomic variables in the US economy increase when Brent price is used instead of that of WTI.

Copper is known to be a predictor of the business cycle and the results confirm its sensitivity to turns in economic activity. On the contrary, Brent, WTI and gold demonstrate a decrease in volatility persistence after the financial 1997 Asian crisis. S&P 500 index' volatility seems to respond to both financial and geopolitical crises.

4. Conclusions

The implications of the results on commodity and stock volatilities, whether in terms of their duration over different regimes or in the evolution of their correlations over time, are pertinent to both policy makers and investors in the oil, industrial commodities and stock markets.

Since this study demonstrates the existence of two regimes in the commodity and stock markets, then a policy recommendation calls on risk-averse investors in those markets to demand higher compensations when the markets switch from a low volatility to a high volatility regime. This recommendation applies particularly to WTI oil, which has the strongest sensitivity to regime switching. Investors in copper, the most affected by Great Moderation (Pannetta et al., 2006), may ask for the lowest regime change compensation because this metal has the lowest sensitivity to regime switching.

On the other hand, risk-averse investors and traders in the gold market should opt for longer term investments than in the other commodity markets in order to ride out the high volatility. Gold, among all the commodities, has the longest duration in the high volatility regime.

Since S&P 500 index shows strong expected duration over the regimes, the policy implication in this case is that risk-averse investors have more time to ride out high volatilities in the stock markets. This perhaps is due to the availability of more risk-management tools, more sophistication and less speculation in the stock markets than in the commodity markets.

S&P 500 index also shows greater duration of volatility from one period to another, over each of the two regimes than the commodities in general. This is perhaps due to growth of stock markets for risk transfer instruments and growth in the proportion of assets held by well informed investors. Investors in the broad market should be inclined to hold stocks longer over time than commodity investors do. Stocks in the broad market are mostly influenced by earnings, while commodity prices are affected by supply and demand as well as by geopolitical and weather events and crises. Among the commodities, gold is the most regime-persistent in the high volatility return regime compared to those of the other commodity markets. Thus, risk-averse investors and traders in the gold market should opt for longer term investments than in the other commodity markets in order to ride out the high volatility. Monetary authorities and financial policy-makers should also be aware of the longer-lasting volatility for those two markets.

More recently the correlations among those different commodities have increased. There is a noticeable rise in commodity correlations since 2003. This evolution in commodity correlations reduces their hedging substitutability in portfolios. Another policy implication is that it is easier for monetary authority to use monetary policy to shepherd them in order to combat commodity inflation.

In regard to the two benchmarks of crude oil, the study indicates that Brent shows more volatility persistence than WTI. This result underscores the stronger impacts of imported oil-benchmarked to Brent can have on economic activity in the United States. WTI and Brent are not perfect substitutes when it comes to volatility in economic activity.

Since the correlations between the commodities and the S&P 500 index have weakened recently, investors and portfolio managers can use them in their risk management strategies. This increased risk diversification as a result of lower correlations is particularly more pertinent between S&P 500 index and each of oil, gold and copper. This flies in the face of the high negative correlation between oil and stocks that prevailed during 2006, which should be considered a very short run phenomenon.

Investors and policy makers should be keenly aware that Brent and WTI crudes have more volatility persistence over time in response to geopolitical crises, while copper is more sensitive to financial crises. S&P 500 index is sensitive to both financial and geopolitical crises.

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