Received: 27 December 2017
DOI: 10.1002/mma.5539

RESEARCH ARTICLE

WILEY

An IMEX-BDF2 compact scheme for pricing options under regime-switching jump-diffusion models

Yingzi Chen[1] | Aiguo Xiao[1] | Wansheng Wang[2]

[1]School of Mathematics and Computational Science & Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Xiangtan University, Xiangtan, China

[2]Department of Mathematics, Shanghai Normal University, Shanghai, China

Correspondence
Wansheng Wang, Department of Mathematics, Shanghai Normal University, Shanghai, 200234, China.
Email: w.s.wang@163.com

Communicated by: S. Wise

Funding information
National Natural Science Foundation of China, Grant/Award Number: 11771060 and 11671343; Hunan Province Innovation Foundation for Postgraduate, Grant/Award Number: CX2017B267

MSC Classification: 65M06; 60J10; 91B25; 91G60; 65J10

In this paper, an implicit-explicit two-step backward differentiation formula (IMEX-BDF2) together with finite difference compact scheme is developed for the numerical pricing of European and American options whose asset price dynamics follow the regime-switching jump-diffusion process. It is shown that IMEX-BDF2 method for solving this system of coupled partial integro-differential equations is stable with the second-order accuracy in time. On the basis of IMEX-BDF2 time semi-discrete method, we derive a fourth-order compact (FOC) finite difference scheme for spatial discretization. Since the pay-off function of the option at the strike price is not differentiable, the results show only second-order accuracy in space. To remedy this, a local mesh refinement strategy is used near the strike price so that the accuracy achieves fourth order. Numerical results illustrate the effectiveness of the proposed method for European and American options under regime-switching jump-diffusion models.

KEYWORDS
finite difference compact scheme, implicit-explicit backward differentiation formula, option pricing, partial integro-differential equation, regime-switching jump-diffusion model

1 | INTRODUCTION

In recent decades, the widespread success in a series of asset price models from the range of financial markets has been witnessed. The celebrated Black-Scholes model[1,2] is based on assumption that the price of the underlying asset behaves like a geometric Brownian motion with a drift and a constant volatility, which cannot explain the market prices of options with various strike prices and maturities. To explain this behavior, a number of alternative models has appeared in the financial literatures, for example, nonlinear models,[3-8] stochastic volatility models,[9-12] jump-diffusion models,[13-16] regime-switching models,[17,18] and regime-switching jump-diffusion models,[19,20] which are given by coupled partial integro-differential equations (PIDEs). However, these models are more difficult to handle numerically in contrast to the celebrated Black-Scholes model. If we use an implicit method for the time discretization, we should solve a nonlinear system for nonlinear models and a nonsymmetric dense system for jump-diffusion models. We have proposed two classes of splitting methods for solving nonlinear option pricing problems,[7,8] and multigrid methods for dense system resulted from the implicit time discretization of jump-diffusion models.[21,22] In this study, we consider regime-switching jump-diffusion models and formulate an efficient and accurate finite difference (FD) scheme with the fourth-order convergence to price the financial derivatives under the models.

Jump-diffusion models represent a simple way to capture the stylized effects such as the negative skewness, the heavier tail, and the volatility smile effect, thus overcoming the weaknesses of the Black-Schloles model in a wide range of financial markets. There has been much research on pricing options under jump models using FD methods, which are the most common way to discretize the differential operators in the option pricing context (see, for example, Achdou and Pironneau [23] and Tavella and Randall[24]). In 1997, Zhang[25] proposed an implicit-explicit (IMEX) time integral method that treats the integral term explicitly and the differential terms implicitly for American options with Merton's model. This method is a first-order accurate method and has a stability restriction for the time stepsize. Tavella and Randall[24] considered using a fully implicit time stepping method to price European options and a stationary iterative method to solve the resulting dense problems with a full matrix. Andersen and Andreasen[26] proposed an unconditionally stable, second-order accurate alternating direction implicit (ADI)–type operator splitting method with two fractional steps for European options. For American options, d'Halluin et al[27] used a penalty method and the Crank-Nicolson method with adaptive time steps, and an approximate semismooth Newton method for the resulting nonlinear nonsmooth problems. Briani et al[28] proposed a fully explicit time stepping method for European options that leads to a more severe stability restriction. In 2005, on a nonuniform spatial grid, d'Halluin et al [29] developed a method in which to use the fast Fourier transform (FFT) for evaluating the integral term on a uniform grid, they perform interpolations back and forth on nonuniform and uniform grids for European options under Merton's and Kou's model; Almendral and Oosterlee[30] used the BDF2 method for time discretization, FFT for the integrations, and the iterative method proposed in Tavella and Randall[24] for linear systems; Cont and Voltchkova[31] proposed an IMEX time integral method that treats the integral term explicitly and the differential terms implicitly for pricing European options in Exponential Lévy models. Toivanen[32] developed a numerical method for pricing European and American options under Kou's jump-diffusion model by using FD on nonuniform grid for discretizing spatial differential operators, the implicit Rannacher scheme for the time stepping, and a stationary iteration for the resulting dense linear systems. Salmi and Toivanen[33] proposed an iterative method for pricing American options under jump-diffusion models. Pindza et al[34] proposed a spectral collocation method in space in combination with the IMEX predictor-corrector time-marching method for pricing European vanilla and butterfly spread options under Merton's jump-diffusion model. Kadalbajoo et al[35] proposed and analyzed three IMEX time semi-discretizations for solving PIDEs under Merton and Kou jump-diffusion models. Kadalbajoo et al[36] presented a radial basis function–based IMEX-BDF2 to solve the PIDEs under jump-diffusion model. Recently, we considered using discontinuous Galerkin finite element together with FD scheme for solving Merton's jump-diffusion model and designed multigrid methods to solve the dense algebraic system by taking into account the structure of the uniform and nonuniform spatial grids in Wang and Chen[21] and Chen et al,[22] respectively.

To reflect the volatility clustering effect observing in the financial markets, the regime-switching model is introduced by Hamilton[37,38] (see, also, Naik[39]). Then, some numerical methods are proposed to evaluate the financial derivatives when the underlying asset follows a regime-switching model. Huang et al[17] analyzed several methods for pricing American options under a regime-switching stochastic process. They proposed Crank-Nicolson time-stepping method combined with a fixed point policy iteration. Company et al[18] used IMEX $\theta-$methods to price American put option under regime switching by a system of coupled partial differential equations. Egorova et al[40] discussed a coupled free boundary problem of American put option under regime-switching model. Ma and Zhou[41] studied moving mesh implicit FD methods for pricing Asian options with regime-switching.

It is natural to combine the jump-diffusion model and the regime-switching model since they capture different market behaviors (see previous studies[19,20,42-44]). Since numerical valuation has become an important approach to evaluate the financial derivatives, a variety of numerical methods are also proposed to efficiently price options under the mixed models. Lee[19] used IMEX Leap-Frog scheme to solve the PIDEs and applied the operator splitting method to solve the linear complementarity problem (LCP) for the prices of the European and American options at all states of the economy under the regime-switching jump-diffusion models. Bastani et al[20] introduced a radial basis function collocation approach to price American options in a regime-switching jump-diffusion model with less than second-order accuracy. Ramponi[42] presented a Fourier transform method to compute the price of European options within a two-state regime switching version of the Merton jump-diffusion model. Costabile et al[43] proposed an explicit formula and a multinomial approach to evaluate the profit of the underlying asset in regime-switching jump-diffusion models. Dang et al[44] studied the pricing problem of Asian options under regime-switching state-dependent jump-diffusion models.

The purpose of this paper is to establish an efficient and accurate IMEX FD method with higher order convergence accuracy to price the European and American options under regime-switching jump-diffusion models. We use the IMEX BDF2 time discretization method that treats the differential terms implicitly and the integral term and the regime-switching term explicitly, which lead to tridiagonal systems and can significantly reduce the computational cost. In space discretization, we apply compact FD schemes combined with local mesh refinement strategy near the strike price, which is also studied by Lee and Sun[16] for PIDEs without the regime-switching term. Since the regime-switching term involves different state of economy, it is much more complex in both computation and theoretical proof.

The remainder of this paper is organized as follows. In Section 2, we introduce the dynamics of regime-switching jump-diffusion models. In Section 3, we propose the implicit-explicit method with three time levels to solve the PIDEs and analyze the stability of the proposed method. We use fourth-order compact (FOC) difference scheme with local mesh refinement strategy for spatial discretization in Section 4. Numerical results to illustrate the effectiveness of the proposed method for European and American options under regime-switching Merton jump-diffusion models are given in Section 5. The last section, Section 6, is the conclusion.

2 | DYNAMICS OF REGIME-SWITCHING JUMP-DIFFUSION MODELS

In this section, we consider option pricing under regime-switching Merton jump-diffusion model as a stochastic process of an underlying asset. On a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, a continuous-time Markov chain process $X = (X_t)_{t\geq 0}$ is defined to take a value in a finite state space $\mathcal{H} = (e_1, e_2, \dots, e_Q)$ with the following transition property

$$\mathbb{P}(X_{t+\Delta t} = e_i | X_t = e_j) = \begin{cases} q_{ij}\Delta t + o(\Delta t), & \text{for } i \neq j, \\ 1 + q_{ij}\Delta t + o(\Delta t), & \text{for } i = j, \end{cases}$$

where the Q-dimensional column vector $e_j := (e_i^j)$ is given by

$$e_i^j = \begin{cases} 0, & \text{for } i \neq j, \\ 1, & \text{for } i = j. \end{cases}$$

The entries q_{ij} satisfy

$$q_{ij} \geq 0, \text{ if } i \neq j; \qquad q_{jj} = -\sum_{i\neq j} q_{ij}, \ 1 \leq j \leq Q.$$

The Markov chain process can be described as[19]

$$dX_t = \mathcal{A}X_{t-}dt + dM_t, \tag{2.1}$$

where $\mathcal{A} = (q_{ij})_{Q\times Q}$ is the generator of the Markov chain X_t and M_t is a martingale, then $\mathcal{A}e_j = (q_{1j}, q_{2j}, \dots, q_{Qj})^T$ denote the jth column of the matrix $\mathcal{A}$.

Define the scalar product $\langle\cdot,\cdot\rangle$ in $\mathbb{R}^Q$ as

$$v_t = \langle v, X_t\rangle,$$

where $v := (v_1, v_2, \dots, v_Q)^T$ is a Q-dimensional column vector. We assume that the dynamic of the underlying asset S_t follows the regime-switching jump-diffusion model. In a risk-neutral world, the stochastic differential equation of S_t is given by

$$\frac{dS_t}{S_{t-}} = (r_t - \lambda_t\kappa_t)dt + \sigma_t dW_t + \eta_t dN_t, \tag{2.2}$$

where r_t is the risk-free interest rate, and σ_t is the volatility with $r_j, \sigma_j \geq 0$ for $1 \leq j \leq Q$, W_t is a Wiener process, $N_t = \langle \mathbf{N}_t, X_t\rangle$ with $\mathbf{N}_t = (N_t^1, N_t^2, \dots, N_t^Q)^T$ is a Poisson process with intensity λ_t, η_t is an impulse function giving a jump from S_{t-} to S_t, and κ_t is a Q-dimensional column vector with $\kappa_j = \mathbb{E}[\eta_j]$ for $1 \leq j \leq Q$.

When the underlying asset S_t follows the regime-switching jump-diffusion model in (2.2), the price of a European option $u(\tau, x, e_j)$ satisfies the following PIDEs:

$$\frac{\partial u}{\partial \tau}(\tau, x, e_j) - \mathcal{L}u(\tau, x, e_j) = 0, \quad (\tau, x, e_j) \in [0, T] \times \mathbb{R} \times \mathcal{H}, \tag{2.3}$$

where

$$\begin{aligned} \mathcal{L}u(\tau, x, e_j) = {} & \frac{1}{2}\sigma_j^2 \frac{\partial^2 u}{\partial x^2}(\tau, z, e_j) + \left(r_j - \frac{1}{2}\sigma_j^2 - \lambda_j\kappa_j\right)\frac{\partial u}{\partial x}(\tau, z, e_j) - (r_j + \lambda_j)u(\tau, z, e_j) \\ & + \lambda_j \int_{\mathbb{R}} u(\tau, z, e_j) f(z - x, e_j)dz + \langle \mathbf{u}, \mathcal{A}e_j\rangle, \end{aligned} \tag{2.4}$$

$\mathbf{u} = (u^1, u^2, \dots, u^Q)^T$ is a Q-dimensional column vector with $u^j(\tau, x) = u(\tau, x, e_j)$ for $1 \leq j \leq Q, x = \ln(S_t/K)$ is the log asset price with respect to an strike price K, $\tau = T - t$ is the time to the expiration date T, and $f(x, e_j)$ is the probability density function. For Merton jump-diffusion model, $f(x, e_j)$ can be written as

$$f(x, e_j) = \frac{1}{\sqrt{2\pi}\gamma_j} \exp\left(-\frac{(x-\mu_j)^2}{2\gamma_j^2}\right),$$

where μ_j is the mean of the normal distribution at the j^{th} state of economy, and γ_j is the standard deviation.

The boundary conditions are given by

$$u(\tau, x, e_j) = 0, \text{ as } x \to -\infty, \quad u(\tau, x, e_j) = Ke^x - Ke^{-r_j\tau}, \text{ as } x \to +\infty,$$

for European call option, and

$$u(\tau, x, e_j) = Ke^{-r_j\tau} - Ke^x, \text{ as } x \to -\infty, \quad u(\tau, x, e_j) = 0, \text{ as } x \to +\infty,$$

for European put option. The initial condition is given by

$$u(0, x, e_j) = g(x) := \begin{cases} \max(Ke^x - K, 0), & \text{in the case of a call option,} \\ \max(K - Ke^x, 0), & \text{in the case of a put option.} \end{cases}$$

For an American option under the regime-switching jump-diffusion model, its value satisfies the LCP

$$\begin{cases} \frac{\partial u}{\partial \tau}(\tau, x, e_j) - \mathcal{L}u(\tau, x, e_j) \geq 0, \quad (u(\tau, x, e_j) - g(x)) \geq 0, \\ \left(\frac{\partial u}{\partial \tau}(\tau, x, e_j) - \mathcal{L}u(\tau, x, e_j)\right)(u(\tau, x, e_j) - g(x)) = 0, \end{cases} \tag{2.5}$$

for all $(\tau, x, e_j) \in [0, T] \times \mathbb{R} \times \mathcal{H}$, where $\mathcal{L}$ is the integro-differential operator in (2.4).

The boundary conditions are given by

$$u(\tau, x, e_j) = 0, \text{ as } x \to -\infty, \quad u(\tau, x, e_j) = Ke^x - K \text{ as } x \to +\infty,$$

for American call option, and

$$u(\tau, x, e_j) = K - Ke^x, \text{ as } x \to -\infty, \quad u(\tau, x, e_j) = 0, \text{ as } x \to +\infty,$$

for American put option. For the LCP (2.5) derived from an American put option, we consider the penalty method. After introducing a penalty function

$$\beta_\varepsilon(u(\tau, x, e_j) - g(x)) := \begin{cases} 0, & u(\tau, x, e_j) - g(x) \geq 0, \\ (u(\tau, x, e_j) - g(x))/\varepsilon, & u(\tau, x, e_j) - g(x) \leq 0, \end{cases}$$

the LCP (2.5) can be approximated by the following formula:

$$\frac{\partial u}{\partial \tau}(\tau, x, e_j) - \mathcal{L}u(\tau, x, e_j) + \beta_\varepsilon(u(\tau, x, e_j) - g(x)) = 0. \tag{2.6}$$

3 | IMEX-BDF2 DISCRETIZATION IN TIME

In this section, we consider the time semi-discretization system. In order to construct the discrete equation, the integro-differential operator $\mathcal{L}$ in (2.4) is separated into three parts; for all $(\tau, x, e_j) \in [0, T] \times \Omega \times \mathcal{H}$, the PIDE of European option can be written as the following form:

$$\frac{\partial u}{\partial \tau}(\tau, x, e_j) = \mathcal{D}u(\tau, x, e_j) + \lambda_j \mathcal{I}u(\tau, x, e_j) + \mathcal{M}u(\tau, x, e_j), \tag{3.1}$$

where

$$\begin{aligned} \mathcal{D}u(\tau, x, e_j) &= \frac{1}{2}\sigma_j^2 \frac{\partial^2 u}{\partial x^2}(\tau, z, e_j) + \left(r_j - \frac{1}{2}\sigma_j^2 - \lambda_j\kappa_j\right)\frac{\partial u}{\partial x}(\tau, z, e_j) - (r_j + \lambda_j)u(\tau, z, e_j), \\ \mathcal{I}u(\tau, x, e_j) &= \int_{\mathbb{R}} u(\tau, z, e_j) f(z - x, e_j) dz, \\ \mathcal{M}u(\tau, x, e_j) &= \langle \mathbf{u}, \mathcal{A}e_j \rangle, \end{aligned}$$

$\mathcal{D}$ is the differential operator, $\mathcal{I}$ is the integral operator, and $\mathcal{M}$ is the regime-switching operator.

Let us consider uniform time grid on $[0, T]$. For a given number N, let $\Delta\tau = T/N$ be a time grid size. Then we set up time grid points $\tau_n = n\Delta\tau$ for $n = 0, 1, \dots, N$. Let $u^{n,j}$ denote the approximation of $u^j(\tau, x)$ at $\tau = \tau_n$. We apply the IMEX scheme where the differential part is treated implicitly, and the integral and regime-switching part is treated explicitly. Thus, Equation 3.1 will be discretized as the following form:

$$\frac{3u^{n+1,j} - 4u^{n,j} + u^{n-1,j}}{2\Delta\tau} = \mathcal{D}u^{n+1,j} + \lambda_j \mathcal{I}(Eu^{n,j}) + \mathcal{M}(Eu^{n,j}), \tag{3.2}$$

with the initial condition $u^{0,j} = g(x)$, where $Eu^{n,j} = 2u^{n,j} - u^{n-1,j}$, and

$$\mathcal{I}(Eu^{n,j}) = 2(u * f)^{n,j} - (u * f)^{n-1,j},$$
$$\mathcal{M}(Eu^{n,j}) = 2\langle \mathbf{u}^{\mathrm{n}}, \mathcal{A}e_j\rangle - \langle \mathbf{u}^{\mathrm{n}-1}, \mathcal{A}e_j\rangle,$$

$\mathbf{u}^{\mathrm{n}} = (u^{n,1}, u^{n,2}, \dots, u^{n,Q})^T$, $\langle \mathbf{u}^{\mathrm{n}}, \mathcal{A}e_j\rangle = \sum_{k=1}^{Q} u^{n,k} q_{k,j}$.

The above discretization method is called IMEX-BDF2 method. In order to use the proposed method, we need two initial values on the zeroth and first time level. The value $u^{0,j}$ is given by initial condition of the model problem, and the value $u^{1,j}$ can be applying the IMEX backward difference method of order one

$$\frac{u^{1,j} - u^{0,j}}{\Delta\tau} = \mathcal{D}u^{1,j} + \lambda_j \mathcal{I}u^{0,j} + \mathcal{M}u^{0,j}. \tag{3.3}$$

Suppose that $u^{n,j}$ is the solution of Equation 3.2, and $\tilde{u}^{n,j}$ is the solution of perturbed equation

$$\frac{3\tilde{u}^{n+1,j} - 4\tilde{u}^{n,j} + \tilde{u}^{n-1,j}}{2\Delta\tau} = \mathcal{D}\tilde{u}^{n+1,j} + \lambda_j \mathcal{I}(E\tilde{u}^{n,j}) + \mathcal{M}(E\tilde{u}^{n,j}) + \delta^{n+1,j}, n \geq 1. \tag{3.4}$$

It can be easily shown that for all $u(\cdot, t, e) \in L^2(\Omega), t \in (0, T)$, the integral operator satisfies the condition

$$\|\mathcal{I}u(\cdot, t, e)\| \leq C_I \|u(\cdot, t, e)\|$$

for some constant C_I independent of t, where $\|v\| := (\int_\Omega |v(x)|^2 dx)^{1/2}$. The regime-switching term $\langle \mathbf{u}^{\mathrm{n}}, \mathcal{A}e_j\rangle$ can be controlled by the inequality

$$|\langle \mathbf{u}^{\mathrm{n}}, \mathcal{A}e_j\rangle| = \left| \sum_{k=1}^{Q} u^{n,k} q_{k,j} \right| \leq \sum_{k=1}^{Q} \max_{k=1,\dots,Q} |u^{n,k}||q_{k,j}|.$$

Let $u^{n,k_n} := \max_{k=1,\dots,Q} |u^{n,k}|$. Then

$$|\langle \mathbf{u}^{\mathbf{n}}, \mathcal{A}e_j\rangle| \leq \left| u^{n,k_n} \sum_{k=1}^{Q} |q_{k,j}| \right| = 2|q_{j,j}| \left| u^{n,k_n} \right|. \tag{3.5}$$

Now we define the error term $e^{n,j} := \tilde{u}^{n,j} - u^{n,j}$.

Theorem 3.1. (L2-stability). *For sufficiently small $\Delta\tau$ such that $\Delta\tau < \frac{1}{4\rho_j + 2\lambda_j C_I + 4|q_{j,j}| + 2}$, we have*

$$\|e^{\mathcal{N},j}\|^2 \leq C(\max_{1\leq k\leq Q} \|e^{0,k}\|^2 + \max_{1\leq k\leq Q} \|e^{1,k}\|^2 + \max_{2\leq n\leq \mathcal{N}} \|\delta^{n,j}\|^2), \quad \forall 2 \leq \mathcal{N} \leq \frac{T}{N}, \tag{3.6}$$

where $\rho_j = \left| \frac{\left(r_j - \frac{1}{2}\sigma_j^2 - \lambda_j\kappa_j\right)^2 - 2(r_j + \lambda_j)\sigma_j^2}{2\sigma_j^2} \right|$, *and C is a constant depending on the parameter C_I, r, σ, λ, and T.*

Proof. The error term $e^{n,j}$ satisfies the following relations:

$$\frac{3e^{n+1,j} - 4e^{n,j} + e^{n-1,j}}{2\Delta\tau} = \mathcal{D}e^{n+1,j} + \lambda_j \mathcal{I}(Ee^{n,j}) + \mathcal{M}(Ee^{n,j}) + \delta^{n+1,j}. \tag{3.7}$$

Taking the inner product of Equation 3.7 with $e^{n+1,j}$, we obtain

$$\begin{aligned}\left(\frac{3e^{n+1,j}-4e^{n,j}+e^{n-1,j}}{2\Delta\tau}, e^{n+1,j}\right) &= \left(\mathcal{D}e^{n+1,j}+\lambda_j\mathcal{I}(Ee^{n,j})+\mathcal{M}(Ee^{n,j})+\delta^{n+1,j}, e^{n+1,j}\right)\\ &= -\frac{\sigma_j^2}{2}\|e_x^{n+1,j}\|^2+\left(r_j-\frac{1}{2}\sigma_j^2-\lambda_j\kappa_j\right)(e_x^{n+1,j}, e^{n+1,j})\\ &\quad -(r_j+\lambda_j)\|e^{n+1,j}\|^2+\lambda_j\left(\mathcal{I}(Ee^{n,j}), e^{n+1,j}\right)\\ &\quad +\left(\mathcal{M}(Ee^{n,j}), e^{n+1,j}\right)+\left(\delta^{n+1,j}, e^{n+1,j}\right).\end{aligned}$$

By simplifying the above results, we can obtain

$$\left(\frac{3e^{n+1,j}-4e^{n,j}+e^{n-1,j}}{2\Delta\tau}, e^{n+1,j}\right) \le \rho_j\|e^{n+1,j}\|^2+\lambda_j\left(\mathcal{I}(Ee^{n,j}), e^{n+1,j}\right)+\left(\mathcal{M}(Ee^{n,j}), e^{n+1,j}\right)+\left(\delta^{n+1,j}, e^{n+1,j}\right),$$

where $\rho_j = \left|\frac{\left(r_j-\frac{1}{2}\sigma_j^2-\lambda_j\kappa_j\right)^2-2(r_j+\lambda_j)\sigma_j^2}{2\sigma_j^2}\right|$.

Using the relation $2(3a-4b+c, a) = \|a\|^2-\|b\|^2+\|2a-b\|^2-\|2b-c\|^2+\|a-2b+c\|^2$, we have

$$\begin{aligned}&\frac{1}{4\Delta\tau}\left[\|e^{n+1,j}\|^2-\|e^{n,j}\|^2+\|2e^{n+1,j}-e^{n,j}\|^2-\|2e^{n,j}-e^{n-1,j}\|^2\right]\\ &\le \rho_j\|e^{n+1,j}\|^2+\lambda_j\left(\mathcal{I}(Ee^{n,j}), e^{n+1,j}\right)+\left(\mathcal{M}(Ee^{n,j}), e^{n+1,j}\right)+\left(\delta^{n+1,j}, e^{n+1,j}\right)\\ &\le \rho_j\|e^{n+1,j}\|^2+\lambda_jC_I\|Ee^{n,j}\|\|e^{n+1,j}\|+\left(\sum_{k=1}^{Q}(Ee^{n,k})q_{k,j}, e^{n+1,j}\right)+\|\delta^{n+1,j}\|\|e^{n+1,j}\|.\end{aligned}$$

Applying Cauchy-Schwarz inequality and (3.5) yields

$$\begin{aligned}&\frac{1}{4\Delta\tau}\left[\|e^{n+1,j}\|^2-\|e^{n,j}\|^2+\|2e^{n+1,j}-e^{n,j}\|^2-\|2e^{n,j}-e^{n-1,j}\|^2\right]\\ &\le \rho_j\|e^{n+1,j}\|^2+\lambda_jC_I\|Ee^{n,j}\|\|e^{n+1,j}\|+2|q_{j,j}|\|Ee^{n,k_n}\|\|e^{n+1,j}\|+\|\delta^{n+1,j}\|\|e^{n+1,j}\|\\ &\le (\rho_j+\frac{\lambda_jC_I}{2}+|q_{j,j}|+\frac{1}{2})\|e^{n+1,j}\|^2+\frac{\lambda_jC_I}{2}\|Ee^{n,j}\|^2+|q_{j,j}|\|Ee^{n,k_n}\|^2+\frac{1}{2}\|\delta^{n+1,j}\|^2\\ &\le (\rho_j+\frac{\lambda_jC_I}{2}+|q_{j,j}|+\frac{1}{2})\|e^{n+1,j}\|^2+\lambda_jC_I(4\|e^{n,j}\|^2+\|e^{n-1,j}\|^2)\\ &+2|q_{j,j}|(4\|e^{n,k_n}\|^2+\|e^{n-1,k_{n-1}}\|^2)+\frac{1}{2}\|\delta^{n+1,j}\|^2.\end{aligned}$$

Multiplying $4\Delta\tau$ on both sides of the equation, we get

$$\begin{aligned}&\|e^{n+1,j}\|^2-\|e^{n,j}\|^2+\|2e^{n+1,j}-e^{n,j}\|^2-\|2e^{n,j}-e^{n-1,j}\|^2\\ &\le \Delta\tau\left[(4\rho_j+2\lambda_jC_I+4|q_{j,j}|+2)\|e^{n+1,j}\|^2+16\lambda_jC_I\|e^{n,j}\|^2+4\lambda_jC_I\|e^{n-1,j}\|^2\right.\\ &\quad\left.+32|q_{j,j}|\|e^{n,k_n}\|^2+8|q_{j,j}|\|e^{n-1,k_{n-1}}\|^2+2\|\delta^{n+1,j}\|^2\right].\end{aligned}$$

After summing up for n between 1 to $\mathcal{N}-1$, for $1\le\mathcal{N}\le N$, we get

$$\begin{aligned}&\|e^{\mathcal{N},j}\|^2-\|e^{1,j}\|^2-\|2e^{1,j}-e^{0,j}\|^2\\ &\le \Delta\tau\left[4\lambda_jC_I\|e^{0,j}\|^2+20\lambda_jC_I\|e^{1,j}\|^2+(4\rho_j+22\lambda_jC_I+4|q_{j,j}|+2)\sum_{n=2}^{\mathcal{N}-1}\|e^{n,j}\|^2+2\sum_{n=1}^{\mathcal{N}-1}\|\delta^{n+1,j}\|^2\right.\\ &\quad\left.+(4\rho_j+2\lambda_jC_I+4|q_{j,j}|+2)\|e^{\mathcal{N},j}\|^2+32|q_{j,j}|\sum_{n=1}^{\mathcal{N}-1}\|e^{n,k_n}\|^2+8|q_{j,j}|\sum_{n=1}^{\mathcal{N}-1}\|e^{n-1,k_{n-1}}\|^2\right].\end{aligned}$$

Case 1. If $e^{n,k_n} = \max_{k=1,\dots,Q} |e^{n,k}| = e^{n,j}$, for $1 \le n \le \mathcal{N}$, then

$$\begin{aligned}
&\|e^{\mathcal{N},j}\|^2 - \|e^{1,j}\|^2 - \|2e^{1,j} - e^{0,j}\|^2 \\
&\le \Delta\tau \Bigg[(4\lambda_j C_I + 8|q_{j,j}|)\|e^{0,j}\|^2 + (20\lambda_j C_I + 40|q_{j,j}|)\|e^{1,j}\|^2 + (4\rho_j + 22\lambda_j C_I + 44|q_{j,j}| + 2)\sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2 \\
&\quad + (4\rho_j + 2\lambda_j C_I + 4|q_{j,j}| + 2)\|e^{\mathcal{N},j}\|^2 + 2\sum_{n=1}^{\mathcal{N}-1} \|\delta^{n,j}\|^2 \Bigg].
\end{aligned}$$

Now, we consider $\Delta\tau$ sufficiently small such that $1 - \Delta\tau(4\rho_j + 2\lambda_j C_I + 4|q_{j,j}| + 2) > 0$, that is, $\Delta\tau < \frac{1}{4\rho_j + 2\lambda_j C_I + 4|q_{j,j}| + 2}$, then the above relation implies that

$$\begin{aligned}
\|e^{\mathcal{N},j}\|^2 &\le C(\|e^{0,j}\|^2 + \|e^{1,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}} \|\delta^{n,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2) \\
&\le C(\|e^{0,j}\|^2 + \|e^{1,j}\|^2 + \mathcal{N}\Delta\tau \max_{2\le j\le \mathcal{N}} \|\delta^{n,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2).
\end{aligned}$$

Since $\mathcal{N}\Delta\tau \le T$, we have

$$\|e^{\mathcal{N},j}\|^2 \le C(\|e^{0,j}\|^2 + \|e^{1,j}\|^2 + \max_{2\le n\le \mathcal{N}} \|\delta^{n,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2).$$

Applying the discrete Gronwall's inequality leads to the result

$$\|e^{\mathcal{N},j}\|^2 \le C(\|e^{0,j}\|^2 + \|e^{1,j}\|^2 + \max_{2\le n\le \mathcal{N}} \|\delta^{n,j}\|^2), \tag{3.8}$$

where C is a constant, which is independent of mesh length.

Case 2. If $e^{n,k_n} = \max_{k=1,\dots,Q} |e^{n,k}| \ne e^{n,j}$, then $|e^{0j}| < |e^{0,k}|, |e^{1j}| < |e^{1,k}|$. Let $e^{n,k} = \max\{e^{1,k_1}, e^{2,k_2}, \dots, e^{n,k_n}\}$. Then we obtain

$$\begin{aligned}
&\|e^{\mathcal{N},j}\|^2 - \|e^{1,j}\|^2 - \|2e^{1,j} - e^{0,j}\|^2 \\
&\le \Delta\tau \Bigg[4\lambda_j C_I\|e^{0,j}\|^2 + 20\lambda_j C_I\|e^{1,j}\|^2 + (4\rho_j + 22\lambda_j C_I + 4|q_{j,j}| + 2)\sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2 + 2\sum_{n=1}^{\mathcal{N}-1} \|\delta^{n+1,j}\|^2 \\
&\quad + (4\rho_j + 2\lambda_j C_I + 4|q_{j,j}| + 2)\|e^{\mathcal{N},j}\|^2 + 32|q_{j,j}|\sum_{n=1}^{\mathcal{N}-1} \|e^{n,k}\|^2 + 8|q_{j,j}|\sum_{n=1}^{\mathcal{N}-1} \|e^{n-1,k}\|^2 \Bigg].
\end{aligned}$$

Given a sufficiently small $\Delta\tau < \frac{1}{4\rho_j + 2\lambda_j C_I + 4|q_{j,j}| + 2}$, we get

$$\begin{aligned}
\|e^{\mathcal{N},j}\|^2 &\le C\left(\|e^{0,j}\|^2 + \|e^{1,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}} \|\delta^{n,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2 + \|e^{0,k}\|^2 + \|e^{1,k}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,k}\|^2 \right) \\
&\le C\left(\|e^{0,j}\|^2 + \|e^{1,j}\|^2 + \mathcal{N}\Delta\tau \max_{2\le j\le \mathcal{N}} \|\delta^{n,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,j}\|^2 + \|e^{0,k}\|^2 + \|e^{1,k}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,k}\|^2 \right) \\
&\le C\left(\|e^{0,k}\|^2 + \|e^{1,k}\|^2 + \max_{2\le n\le \mathcal{N}} \|\delta^{n,j}\|^2 + \Delta\tau \sum_{n=2}^{\mathcal{N}-1} \|e^{n,k}\|^2 \right).
\end{aligned}$$

Applying the discrete Gronwall's inequality, we deduce the result

$$\|e^{\mathcal{N},j}\|^2 \le C(\|e^{0,k}\|^2 + \|e^{1,k}\|^2 + \max_{2\le n\le \mathcal{N}} \|\delta^{n,j}\|^2), \tag{3.9}$$

where C is a constant, which is independent of mesh length.

From (3.8) and (3.9), we get (3.6) and therefore complete the proof. □

4 | FOC DIFFERENCE DISCRETIZATION IN SPACE

In this section, we consider full discretization approximation of the European option under the regime-switching jump-diffusion model. To do this, we truncate the infinite domain $\mathbb{R}$ for x to be finite domain $\Omega = [x_{\min}, x_{\max}]$ with a sufficiently small $x_{\min}$ and a sufficiently large $x_{\max}$. For a given number M, let $h = (x_{\max} - x_{\min})/M$ be a spatial grid size. Then we obtain spatial grid points $x_m = x_{\min} + mh$ for $m = 0, 1, \ldots, M$. We define $u_m^{n,j}$ as the approximation of $u(\tau_n, x_m, e_j)$ and define $f_{m,k}^j := f(x_k - x_m, e_j)$.

In order to approximate numerically the integral term, we divide this term into two parts on Ω and $\mathbb{R} \setminus \Omega$, then the integral operator can be split as

$$\mathcal{I}u(\tau, x, e_j) = \int_\Omega u(\tau, z, e_j) f(z - x, e_j) dz + \int_{\mathbb{R}\setminus\Omega} u(\tau, z, e_j) f(z - x, e_j) dz. \tag{10}$$

The integral over Ω is discretized by the composite Simpson's rule, which gives us the fourth-order accuracy in the spatial variable, and the integral over $\mathbb{R} \setminus \Omega$ is computed by using the corresponding boundary conditions. Then,

$$\mathcal{I}u_m^{n,j} = (u * f)_m^{n,j} + O(h^4),$$

where $(u * f)_m^{n,j}$ at each grid point (τ_n, x_m, e_j) is given by

$$(u * f)_m^{n,j} = h \sum_{l=0}^{M} w_l u_l^{n,j} f_{m,l}^j + R(\tau_n, x_m, e_j)$$

with

$$[w_0, w_1, \ldots, w_M] = \left[\frac{1}{3}, \frac{4}{3}, \frac{2}{3}, \frac{4}{3}, \frac{2}{3}, \ldots, \frac{2}{3}, \frac{4}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{3}\right].$$

Note that $R(\tau_n, x_m, e_j)$ denotes the approximation of the integral over $\mathbb{R}\setminus\Omega$ in (3.2), which can be given by using asymptotic behavior, ie,

$$R(\tau_n, x_m, e_j) = Ke^{x_m + \mu_j + \frac{\gamma_j^2}{2}} \Phi\left(\frac{x_m - x_{\max} + \mu_j + \gamma_j^2}{\gamma_j}\right) - Ke^{-r_j \tau_n} \Phi\left(\frac{x_m - x_{\max} + \mu_j}{\gamma_j}\right)$$

for call option, and

$$R(\tau_n, x_m, e_j) = Ke^{-r_j \tau_n} \Phi\left(\frac{x_{\min} - x_m - \mu_j}{\gamma_j}\right) - Ke^{x_m + \mu_j + \frac{\gamma_j^2}{2}} \Phi\left(\frac{x_{\min} - x_m - \mu_j - \gamma_j^2}{\gamma_j}\right)$$

for put option. Here, $\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{\eta^2}{2}} d\eta$ is the cumulative normal distribution, which can be computed directly.

4.1 | FOC scheme

We briefly introduce how to obtain an FOC scheme for (3.1). For simplicity, we consider the following PIDE:

$$\frac{\partial u}{\partial \tau}(\tau, x, e_j) = a_j \frac{\partial^2 u}{\partial x^2}(\tau, x, e_j) + b_j \frac{\partial u}{\partial x}(\tau, x, e_j) - c_j u(\tau, x, e_j) + \mathcal{J}u(\tau, x, e_j), \tag{4.1}$$

where

$$a_j = \frac{1}{2}\sigma_j^2 > 0, \quad b_j = r_j - \frac{1}{2}\sigma_j^2 - \lambda_j \kappa_j,$$
$$c_j = r_j + \lambda_j, \quad \mathcal{J}u(\tau, x, e_j) = \lambda_j (u * f)(\tau, x, e_j) + \langle \mathbf{u}, \mathcal{A}e_j \rangle.$$

Let u_i^n be the approximation at spatial level x_i and time level τ_n. We consider the following IMEX scheme:

$$-a_j \left(\frac{\partial^2 u}{\partial x^2}\right)_m^{n+1,j} - b_j \left(\frac{\partial u}{\partial x}\right)_m^{n+1,j} + c_j u_m^{n+1,j} = \mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}, \tag{4.2}$$

for $n = 0, 1, \ldots, N$. The difference operators at each grid point (τ_n, x_m, e_j) is approximated by using the central difference formula

$$\delta_x^2 u_m^{n+1,j} = \frac{u_{m+1}^{n+1,j} - 2u_m^{n+1,j} + u_{m-1}^{n+1,j}}{h^2}, \quad \delta_x u_m^{n+1,j} = \frac{u_{m+1}^{n+1,j} - u_{m-1}^{n+1,j}}{2h}.$$

By the Taylor's theorem, we have the following relations:

$$\left(\frac{\partial u}{\partial x}\right)_m^{n+1,j} = \delta_x u_m^{n+1,j} - \frac{h^2}{6}\left(\frac{\partial^3 u}{\partial x^3}\right)_m^{n+1,j} + O(h^4), \tag{4.3}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_m^{n+1,j} = \delta_x^2 u_m^{n+1,j} - \frac{h^2}{12}\left(\frac{\partial^4 u}{\partial x^4}\right)_m^{n+1,j} + O(h^4). \tag{4.4}$$

Putting these approximations into (4.2) gives

$$\begin{aligned} &-a_j\delta_x^2 u_m^{n+1,j} - b_j\delta_x u_m^{n+1,j} + c_j u_m^{n+1,j} + \frac{h^2}{6}b_j\left(\frac{\partial^3 u}{\partial x^3}\right)_m^{n+1,j} + \frac{h^2}{12}a_j\left(\frac{\partial^4 u}{\partial x^4}\right)_m^{n+1,j} \\ &= \mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j} + O(h^4). \end{aligned}$$

Taking the derivative of Equation 4.2, we obtain

$$\left(\frac{\partial^3 u}{\partial x^3}\right)_m^{n+1,j} = -\frac{b_j}{a_j}\left(\frac{\partial^2 u}{\partial x^2}\right)_m^{n+1,j} + \frac{c_j}{a_j}\left(\frac{\partial u}{\partial x}\right)_m^{n+1,j} - \frac{1}{a_j}\frac{\partial}{\partial x}\left[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}\right], \tag{4.5}$$

$$\begin{aligned} \left(\frac{\partial^4 u}{\partial x^4}\right)_m^{n+1,j} &= \left(\frac{b_j^2}{a_j^2} + \frac{c_j}{a_j}\right)\left(\frac{\partial^2 u}{\partial x^2}\right)_m^{n+1,j} - \frac{b_j c_j}{a_j^2}\left(\frac{\partial u}{\partial x}\right)_m^{n+1,j} - \frac{1}{a_j}\frac{\partial^2}{\partial x^2}\left[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}\right] \\ &\quad + \frac{b_j}{a_j^2}\frac{\partial}{\partial x}\left[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}\right]. \end{aligned} \tag{4.6}$$

By substituting the standard second-order central difference operators into (4.5) and (4.6), we obtain second-order accuracy in space approximations for $\frac{\partial^3 u}{\partial x^3}$ and $\frac{\partial^4 u}{\partial x^4}$:

$$\left(\frac{\partial^3 u}{\partial x^3}\right)_m^{n+1,j} = -\frac{b_j}{a_j}\delta_x^2 u_m^{n+1,j} + \frac{c_j}{a_j}\delta_x u_m^{n+1,j} - \frac{1}{a_j}\delta_x[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}] + O(h^2), \tag{4.7}$$

$$\begin{aligned} \left(\frac{\partial^4 u}{\partial x^4}\right)_m^{n+1,j} &= \left(\frac{b_j^2}{a_j^2} + \frac{c_j}{a_j}\right)\delta_x^2 u_m^{n+1,j} - \frac{b_j c_j}{a_j^2}\delta_x u_m^{n+1,j} - \frac{1}{a_j}\delta_x^2\left[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}\right] \\ &\quad + \frac{b_j}{a_j^2}\delta_x\left[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}\right] + O(h^2). \end{aligned} \tag{4.8}$$

Substituting formulas (4.7) and (4.8) into (4.3) and (4.4), we can see that both the first derivative operator $\frac{\partial u}{\partial x}$ and the second derivative operator $\frac{\partial^2 u}{\partial x^2}$ have fourth-order accuracy. Equation 4.2 can be written as

$$\begin{aligned} &-\left[\left(a_j + \frac{h^2}{12}\frac{b_j^2}{a_j}\right)\delta_x^2 + b_j\delta_x\right]u_m^{n+1,j} + c_j\left(1 + \frac{h^2}{12}\delta_x^2 + \frac{h^2}{12}\frac{b_j}{a_j}\delta_x\right)u_m^{n+1,j} \\ &= \left(1 + \frac{h^2}{12}\delta_x^2 + \frac{h^2}{12}\frac{b_j}{a_j}\delta_x\right)\left[\mathcal{J}(Eu_m^{n,j}) - \left(\frac{\partial u}{\partial \tau}\right)_m^{n+1,j}\right], \end{aligned} \tag{4.9}$$

where the difference operators are defined as follows:

$$\delta_x^2\mathcal{J}(Eu_m^{n,j}) = \frac{\mathcal{J}(Eu_{m+1}^{n,j}) - 2\mathcal{J}(Eu_m^{n,j}) + \mathcal{J}(Eu_{m-1}^{n,j})}{h^2}, \quad \delta_x\mathcal{J}(Eu_m^{n,j}) = \frac{\mathcal{J}(Eu_{m+1}^{n,j}) - \mathcal{J}(Eu_{m-1}^{n,j})}{2h}.$$

Combining (4.9), we get, for $n \geq 1$,

$$\left(\frac{3}{2}+c_j\Delta\tau\right)\left[(1-\alpha_j-\bar{\alpha}_j)u_m^{n+1,j}+\alpha_j u_{m-1}^{n+1,j}+\bar{\alpha}_j u_{m+1}^{n+1,j}\right]+\Delta\tau\left[(\beta_j+\bar{\beta}_j)u_m^{n+1,j}-\beta_j u_{m-1}^{n+1,j}-\bar{\beta}_j u_{m+1}^{n+1,j}\right]$$
$$=(1-\alpha_j-\bar{\alpha}_j)\left(2u_m^{n,j}-\frac{1}{2}u_m^{n-1,j}\right)+\alpha_j\left(2u_{m-1}^{n,j}-\frac{1}{2}u_{m-1}^{n-1,j}\right)+\bar{\alpha}_j\left(2u_{m+1}^{n,j}-\frac{1}{2}u_{m+1}^{n-1,j}\right)$$
$$+2\Delta\tau\left[(1-\alpha_j-\bar{\alpha}_j)(u*f)_m^{n,j}+\alpha_j(u*f)_{m-1}^{n,j}+\bar{\alpha}_j(u*f)_{m+1}^{n,j}\right]$$
$$-\Delta\tau\left[(1-\alpha_j-\bar{\alpha}_j)(u*f)_m^{n-1,j}+\alpha_j(u*f)_{m-1}^{n-1,j}+\bar{\alpha}_j(u*f)_{m+1}^{n-1,j}\right]$$
$$+2\Delta\tau\left[(1-\alpha_j-\bar{\alpha}_j)\sum_{k=1}^{Q}u_m^{n,k}q_{k,j}+\alpha_j\sum_{k=1}^{Q}u_{m-1}^{n,k}q_{k,j}+\bar{\alpha}_j\sum_{k=1}^{Q}u_{m+1}^{n,k}q_{k,j}\right]$$
$$-\Delta\tau\left[(1-\alpha_j-\bar{\alpha}_j)\sum_{k=1}^{Q}u_m^{n-1,k}q_{k,j}+\alpha_j\sum_{k=1}^{Q}u_{m-1}^{n-1,k}q_{k,j}+\bar{\alpha}_j\sum_{k=1}^{Q}u_{m+1}^{n-1,k}q_{k,j}\right],$$

where

$$\begin{cases}\alpha_j=\frac{1}{12}-\frac{b_jh}{24a_j}, & \bar{\alpha}_j=\frac{1}{12}+]\frac{b_jh}{24a_j},\\ \beta_j=\frac{b_j^2}{12a_j}+\frac{a_j}{h^2}-\frac{b_j}{2h}, & \bar{\beta}_j=\frac{b_j^2}{12a_j}+\frac{a_j}{h^2}+\frac{b_j}{2h}.\end{cases}$$

Let us consider the numerical approximation U^n of the IMEX method with three levels arranged in a line with all states of the economy on the (n)th time level as the following form:

$$U^n=\left((U_1^{n,1})^T,(U_2^{n,1})^T,\ldots,(U_{M-1}^{n,1})^T,(U_1^{n,2})^T,\ldots,(U_{M-1}^{n,2})^T,\ldots,(U_1^{n,Q})^T,\ldots,(U_{M-1}^{n,Q})^T\right)^T.$$

Then, the linear system of the discrete equations is given by

$$AU^{n+1}=2(C+\Delta\tau D)U^n-\left(\frac{1}{2}C+\Delta\tau D\right)U^{n-1}+\Delta\tau(2(U*f)^n-(U*f)^{n-1})+\varrho^n,$$

where A, B, C is a block diagonal matrix of size $(M-1)Q$,

$$A=\begin{pmatrix}A_{11}&0&\ldots&0\\0&A_{22}&\ldots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\ldots&A_{QQ}\end{pmatrix},\quad B=\begin{pmatrix}B_{11}&0&\ldots&0\\0&B_{22}&\ldots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\ldots&B_{QQ}\end{pmatrix},\quad C=\begin{pmatrix}C_{11}&0&\ldots&0\\0&C_{22}&\ldots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\ldots&C_{QQ}\end{pmatrix},$$

and the submatrices A_{jj}, C_{jj}, and B_{jj} are $(M-1)\times(M-1)$ square matrices, $A_{jj}=\left(\frac{3}{2}+c_i\Delta\tau\right)C_{jj}+\Delta\tau B_{jj}$ for $1\leq j\leq Q$, where

$$C_{jj}=\text{tridiag}[\alpha_j,1-\alpha_j-\bar{\alpha}_j,\bar{\alpha}_j],\quad B_{jj}=\text{tridiag}[-\beta_j,\beta_j+\bar{\beta}_j,-\bar{\beta}_j].$$

D is the square matrix of size $(M-1)Q$ with Q row and column partitions

$$D=\begin{pmatrix}D_{11}&D_{12}&\ldots&D_{1Q}\\D_{21}&D_{22}&\ldots&D_{2Q}\\\vdots&\vdots&\ddots&\vdots\\D_{Q1}&D_{Q2}&\ldots&D_{QQ}\end{pmatrix},$$

and all submatrices $D_{i,j}$ for $1\leq i,j\leq Q$ are $(M-1)\times(M-1)$ scalar matrices of the form

$$D_{ij}=\text{tridiag}[\alpha_j,1-\alpha_j-\bar{\alpha}_j,\bar{\alpha}_j]q_{ji}.$$

We consider the numerical approximation $(U*f)$ of the integral term of the PIDE (3.1) arranged in line with all state as the following form:

$$(U*f)^n=\left(((U*f)^{n,1})^T,((U*f)^{n,2})^T,\ldots,((U*f)^{n,Q})^T\right)^T,$$

and all vectors $(U*f)^{n,j} := ((U*f)_m^{n,j})$ for $1\leq j\leq Q$ are column vectors of M-1 dimensions with entries

$$(U*f)_m^{n,j}=(1-\alpha_j-\bar{\alpha}_j)(u*f)_m^{n,j}+\alpha_j(u*f)_{m-1}^{n,j}+\bar{\alpha}_j(u*f)_{m+1}^{n,j}, m=1,2,\ldots,M-1.$$

ϱ^n is a column vector of size $(M-1)\times Q$ with Q row partitions

$$\varrho^n = \left((\varrho^{n,1})^T, (\varrho^{n,2})^T, \ldots, (\varrho^{n,Q})^T\right),$$

and all column vectors $\varrho^{n,j} := (\varrho_m^{n,j})$ for $1 \le j \le Q$ are of M-1 dimensions with entries

$$\varrho_m^{n,j} = \begin{cases} \left[-\left(\frac{3}{2}+c_j\Delta\tau\right)\alpha_j + \Delta\tau\beta_j\right] U_0^{n+1,j} + \alpha_j\left(2U_0^{n,j} - \frac{1}{2}U_0^{n-1,j}\right) & \\ +\Delta\tau\alpha_j\left(2\sum_{k=1}^{Q} u_0^{n,k} q_{k,j} - \sum_{k=1}^{Q} u_0^{n-1,k} q_{k,j}\right), & \text{for } m=1, \\ 0, & \text{for } 2\le m\le M-2, \\ \left[-\left(\frac{3}{2}+c_j\Delta\tau\right)\bar{\alpha}_j + \Delta\tau\bar{\beta}_j\right] U_M^{n+1,j} + \bar{\alpha}_j\left(2U_M^{n,j} - \frac{1}{2}U_M^{n-1,j}\right) & \\ +\Delta\tau\bar{\alpha}_j(2\sum_{k=1}^{Q} u_M^{n,k} q_{k,j} - \sum_{k=1}^{Q} u_M^{n-1,k} q_{k,j}), & \text{for } m=M-1. \end{cases}$$

For $n = 0$, we can obtain the similar result

$$\begin{aligned} &(1+c_j\Delta\tau)\left[(1-\alpha_j-\bar{\alpha}_j)u_m^{n+1,j} + \alpha_j u_{m-1}^{n+1,j} + \bar{\alpha}_j u_{m+1}^{n+1,j}\right] + \Delta\tau\left[(\beta_j+\bar{\beta}_j)u_m^{n+1,j} - \beta_j u_{m-1}^{n+1,j} - \bar{\beta}_j u_{m+1}^{n+1,j}\right] \\ &= (1-\alpha_j-\bar{\alpha}_j)u_m^{n,j} + \alpha_j u_{m-1}^{n,j} + \bar{\alpha}_j u_{m+1}^{n,j} \\ &+ \Delta\tau\left[(1-\alpha_j-\bar{\alpha}_j)(u*f)_m^{n,j} + \alpha_j(u*f)_{m-1}^{n,j} + \bar{\alpha}_j(u*f)_{m+1}^{n,j}\right] \\ &+ \Delta\tau\left[(1-\alpha_j-\bar{\alpha}_j)\sum_{k=1}^{Q} u_m^{n,k} q_{k,j} + \alpha_j\sum_{k=1}^{Q} u_{m-1}^{n,k} q_{k,j} + \bar{\alpha}_j\sum_{k=1}^{Q} u_{m+1}^{n,k} q_{k,j}\right]. \end{aligned}$$

Then, the linear system of the discrete equations is given by

$$\tilde{A}U^{n+1} = (C+\Delta\tau D)U^n + \Delta\tau(U*f)^n + \tilde{\varrho}^n,$$

where $\tilde{A}$ is a block diagonal matrix of size $(M-1)Q$, the submatrices $\tilde{A}_{jj} = (1+c_i\Delta\tau)C_{jj} + \Delta\tau B_{jj}$, C_{jj}, B_{jj} and $(U*f)^n$ are given by the above form, for $1 \le j \le Q$, and $\tilde{\varrho}^n$ is defined by the similar form to ϱ^n, here

$$\tilde{\varrho}_m^{n,j} = \begin{cases} \left[-(1+c_j\Delta\tau)\alpha_j + \Delta\tau\beta_j\right] U_0^{n+1,j} + \alpha_j U_0^{n,j} + \Delta\tau\alpha_j\sum_{k=1}^{Q} u_0^{n,k} q_{k,j}, & \text{for } m=1, \\ 0, & \text{for } 2\le m\le M-2, \\ \left[-(1+c_j\Delta\tau)\bar{\alpha}_j + \Delta\tau\bar{\beta}_j\right] U_M^{n+1,j} + \bar{\alpha}_j U_M^{n,j} + \Delta\tau\bar{\alpha}_j\sum_{k=1}^{Q} u_M^{n,k} q_{k,j}, & \text{for } m=M-1. \end{cases}$$

4.2 | Local mesh refinement

In the case of pricing a European option with regime-switching jump-diffusion, the financial payoff function for each state is given by

$$u(0,x) = \begin{cases} \max(Ke^x - K, 0), & \text{for a call option}, \\ \max(K - Ke^x, 0), & \text{for a put option}, \end{cases}$$

which is nonsmooth around the strike price $\xi^* = 0$.

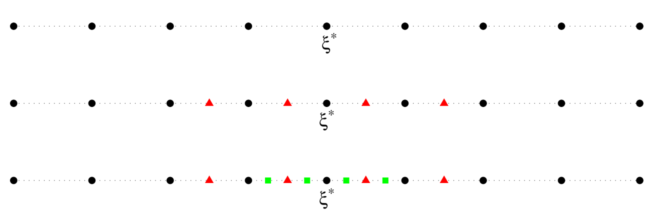

FIGURE 1 A discretized computational domain with local mesh refinement [Colour figure can be viewed at wileyonlinelibrary.com]

Assume that the spatial direction is first discretized by a uniform mesh with initial mesh size

$$h_0 = h = \frac{x_{\max} - x_{\min}}{M},$$

where $M + 1$ is the beginning number of grid points in the x-direction. Choosing ξ^* and four points closest to ξ^*, that is,

$$\xi^* - 2h_0, \quad \xi^* - h_0, \quad \xi^*, \quad \xi^* + h_0, \quad \xi^* + 2h_0,$$

inserting the first four points among them, and letting $h_1 = h_0/2$, we obtain

$$\xi^* - 2h_0, \quad \underline{\xi^* - 3h_1}, \quad \xi^* - h_0, \quad \underline{\xi^* - h_1}, \quad \xi^*, \quad \underline{\xi^* + h_1}, \quad \xi^* + h_0, \quad \underline{\xi^* + 3h_1}, \quad \xi^* + 2h_0.$$

Choosing ξ^* and four points among them, and letting $h_2 = h_1/2$, we obtain

$$\xi^* - h_0, \quad \underline{\underline{\xi^* - 3h_2}}, \quad \underline{\xi^* - h_1}, \quad \underline{\underline{\xi^* - h_2}}, \quad \xi^*, \quad \underline{\underline{\xi^* + h_2}}, \quad \underline{\xi^* + h_1}, \quad \underline{\underline{\xi^* + 3h_2}} \quad \xi^* + h_0.$$

Repeat the above steps until some h reaches the stopping criterion $h \le h_0^2$; see Figure 1.

5 | NUMERICAL EXPERIMENTS

In this section, we present several numerical experiments to evaluate the prices of the European and American options under the regime-switching jump-diffusion models. We discuss three states of the Markov chain under the

TABLE 1 The value of European call option at the first state of the economy obtained by IMEX-BDF2-FOC scheme with uniform grid and the convergence orders of the scheme

	$S = 90$			$S = 100$			$S = 110$			CPU
M	Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32	8.54712688	1.17e-02		15.59375427	2.05e-02		23.84844357	1.03e-02		3.75
64	8.53806069	2.63e-03	2.15	15.50128597	4.80e-03	2.10	23.86016081	1.37e-03	2.92	7.70
128	8.53598345	5.51e-04	2.25	15.61308786	1.16e-03	2.05	23.85847262	3.18e-04	2.11	17.19
256	8.53530607	1.26e-04	2.12	15.61395789	2.90e-04	2.00	23.85886767	7.74e-05	2.04	43.44
512	8.53545681	2.43e-05	2.38	15.61431338	6.58e-05	2.14	23.85877119	1.90e-05	2.02	115.29
1024	8.53543825	5.74e-06	2.08	15.61426369	1.61e-05	2.03	23.85879364	3.40e-06	2.48	331.02
2048	8.53543143	1.08e-06	2.40	15.61425146	3.88e-06	2.05	23.85879108	8.47e-07	2.01	1195.16

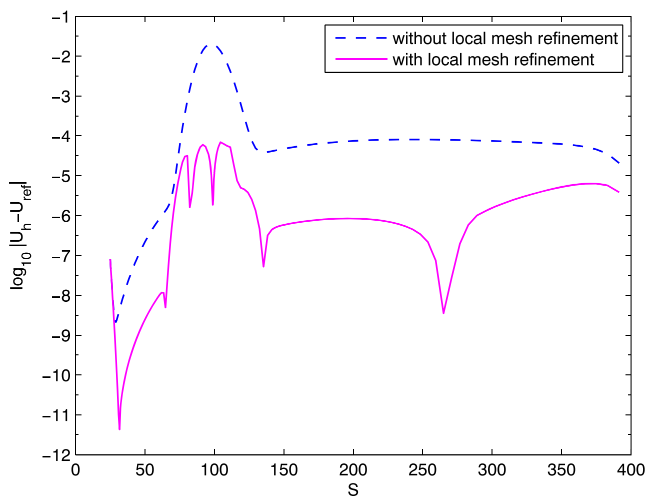

FIGURE 2 Error distribution of fourth-order compact (FOC) scheme for pricing European call option under regime-switching Merton model with $M = 128$ and $N = 1600$ at time $\tau = T$

regime-switching Merton model. The corresponding parameters used in the simulation are

$$\sigma = \begin{pmatrix} 0.15 \\ 0.15 \\ 0.15 \end{pmatrix}, \quad r = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \end{pmatrix}, \quad \mu = \begin{pmatrix} -0.50 \\ -0.50 \\ -0.50 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 0.45 \\ 0.45 \\ 0.45 \end{pmatrix},$$

the rate matrix $\mathcal{A}$ of the Markov chain, and the intensity λ are

$$\mathcal{A} = \begin{pmatrix} -0.8 & 0.2 & 0.1 \\ 0.6 & -1.0 & 0.3 \\ 0.2 & 0.8 & -0.4 \end{pmatrix} \quad \text{and} \quad \lambda = \begin{pmatrix} 0.3 \\ 0.5 \\ 0.7 \end{pmatrix},$$

the strike price is $K = 100$, and the maturity date is $T = 1$. These parameters are also used by Lee.[19] For truncating the infinite spatial domain $[x_{\min}, x_{\max}]$, we select $x_{\min} = -1.5$ and $x_{\max} = 1.5$.

We first give the numerical results of pricing European call options under regime-switching Merton jump-diffusion model. Because there is no exact solution, we need a numerical reference solution. The reference solution u^j_{ref} is computed on the grid with $M = 4096$ and $N = 1600$ for $\tau = T$ at the j-state of the economy. This choice of the reference solution stems from previous research.[16,36] From their numerical research, we know that the approximation error is very small if we take their spatial and temporal steps. The CPU times are given in seconds on a PC with Dell OptiPlex 3020 Intel CORE i3.

TABLE 2 The value of European call option at the first state of the economy obtained by IMEX-BDF2-FOC scheme with local mesh refinement and the convergence orders of the scheme

		$S = 90$			$S = 100$			$S = 110$			CPU
M	L_0	Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32	16	8.54102948	5.60e-03		15.62250215	8.26e-03		23.85523189	3.56e-03		4.20
64	24	8.53498631	4.46e-04	3.65	15.61474272	4.95e-04	4.06	23.85856357	2.27e-04	3.97	8.01
128	32	8.53545784	2.53e-05	4.14	15.61428252	3.49e-05	3.82	23.85880444	1.42e-05	4.00	19.48
256	40	8.53543104	1.47e-06	4.10	15.61424548	2.09e-06	4.06	23.85879113	9.01e-07	3.98	46.13
512	48	8.53543261	9.83e-08	3.91	15.61424772	1.45e-07	3.85	23.85879029	6.23e-08	3.85	117.51
1024	56	8.53543252	6.74e-09	3.87	15.61424758	8.86e-09	4.03	23.85879024	3.88e-09	4.00	334.53
2048	64	8.53543251	4.84e-10	3.80	15.61424758	6.18e-10	3.84	23.85879023	2.69e-10	3.85	1230.45

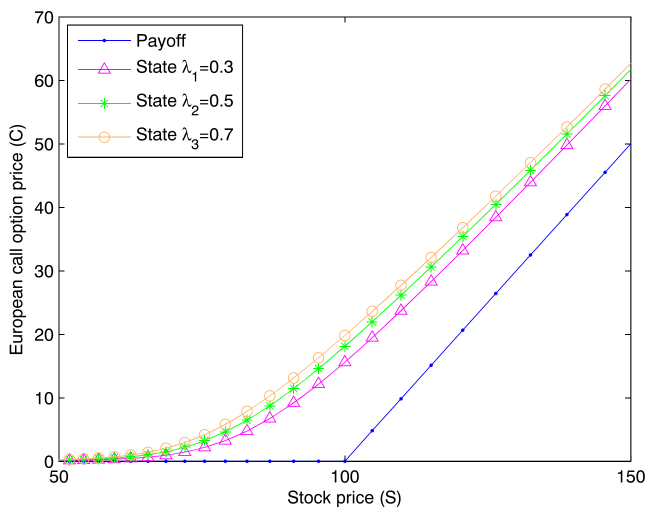

FIGURE 3 The price curve European call option under the regime-switching Merton model with 1600 time steps and 64 spatial meshes at $\tau = T$ [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 3 The value of European call option at the second state of the economy obtained by IMEX-BDF2-FOC scheme and the convergence orders of the scheme. Upper: without local mesh refinement; bottom: with local mesh refinement

		$S = 90$			$S = 100$			$S = 110$			CPU
M		Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32		10.74824369	1.42e-02		18.13914260	2.55e-02		26.26403285	1.82e-02		3.54
64		10.76573949	3.33e-03	2.09	18.11978688	6.13e-03	2.06	26.28655223	4.31e-03	2.08	7.20
12		10.76318045	7.74e-04	2.11	18.11207350	1.58e-03	1.95	26.28331606	1.07e-03	2.01	16.94
256		10.76221810	1.88e-04	2.04	18.11404805	3.91e-04	2.02	26.28250225	2.56e-04	2.06	39.85
512		10.76245267	4.67e-05	2.01	18.11374756	9.05e-05	2.11	26.28218298	6.29e-05	2.03	109.37
1024		10.76241749	1.15e-05	2.02	18.11367591	1.88e-05	2.27	26.28223127	1.46e-05	2.11	336.36
2048		10.76240856	2.55e-06	2.17	18.11365253	4.56e-06	2.05	26.28224235	3.50e-06	2.06	1169.42
		$S = 90$			$S = 100$			$S = 110$			CPU
M	L_0	Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32	16	10.75594856	6.46e-03		18.12204692	8.39e-03		26.28643612	4.19e-03		4.15
64	24	10.76282154	4.16e-04	3.96	18.11312479	5.32e-04	3.98	26.28251136	2.66e-04	3.99	7.85
128	32	10.76243148	2.55e-05	4.03	18.11362446	3.26e-05	4.03	26.28226244	1.66e-05	4.00	18.27
256	40	10.76240457	1.45e-06	4.14	18.11365488	2.21e-06	3.89	26.28224476	1.10e-06	3.92	45.28
512	48	10.76240610	8.63e-08	4.07	18.11365722	1.36e-07	4.03	26.28224579	6.53e-08	4.07	116.65
1024	56	10.76240601	5.38e-09	4.00	18.11365708	8.93e-09	3.92	26.28224586	4.05e-09	4.01	335.25
2048	64	10.76240602	3.24e-10	4.06	18.11365709	5.37e-10	4.06	26.28224585	2.50e-10	4.02	1198.06

TABLE 4 The value of European call option at the third state of the economy obtained by IMEX-BDF2-FOC scheme and the convergence orders of the scheme. Upper: without local mesh refinement; bottom: with local mesh refinement

		$S = 90$			$S = 100$			$S = 110$			CPU
M		Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32		12.33533900	5.26e-02		19.74244557	6.24e-02		27.83084006	2.09e-02		3.53
64		12.37491025	1.31e-02	2.01	19.81968956	1.48e-02	2.08	27.85664872	4.92e-03	2.09	7.17
128		12.38474242	3.28e-03	2.00	19.80857015	3.68e-03	2.01	27.85293917	1.21e-03	2.02	16.59
256		12.38730517	7.18e-04	2.20	19.80394438	9.47e-04	1.96	27.85140539	3.21e-04	1.92	39.15
512		12.38819697	1.74e-04	2.04	19.80465988	2.32e-04	2.03	27.85180249	7.58e-05	2.09	106.97
1024		12.38806583	4.33e-05	2.01	19.80483460	5.72e-05	2.02	27.85174357	1.68e-05	2.17	333.91
2048		12.38803328	1.07e-05	2.01	19.80487781	1.40e-05	2.03	27.85172266	4.08e-06	2.05	1157.42
		$S = 90$			$S = 100$			$S = 110$			CPU
M	L_0	Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32	16	12.39238763	4.37e-03		19.81244557	7.56e-03		27.84843612	3.29e-03		4.03
64	24	12.38827146	2.49e-04	4.13	19.80439335	4.98e-04	3.92	27.85151556	2.11e-04	3.96	7.54
128	32	12.38803722	1.47e-05	4.09	19.80486211	2.97e-05	4.07	27.85171386	1.29e-05	4.04	17.82
256	40	12.38802348	9.14e-07	4.00	19.80489359	1.80e-06	4.04	27.85172595	7.89e-07	4.03	44.12
512	48	12.38802251	5.76e-08	3.99	19.80489190	1.10e-07	4.03	27.85172679	4.94e-08	4.00	114.87
1024	56	12.38802256	3.39e-09	4.09	19.80489178	6.43e-09	4.10	27.85172673	2.98e-09	4.05	334.13
2048	64	12.38802257	2.04e-10	4.06	19.80489179	3.73e-10	4.12	27.85172674	1.63e-10	4.20	1195.49

In Table 1, we present the prices of the European call option at the first state of economy with FOC scheme using uniform grid. We give the prices and their errors at the stock prices $S = 90, S = 100$, and $S = 110$. Also, the given convergence order is defined by

$$\text{Order} = \log_2 \frac{\|u_h^j - u_{ref}^j\|}{\|u_{h/2}^j - u_{ref}^j\|},$$

where the value u_h^j is the numerical solution at $\tau = T$ at the j-state of the economy.

From Table 1, we notice that the result can only achieve second-order convergence accuracy, even though our theory can reach order 4. The reason is that in the option pricing problem, the initial condition is nondifferentiable at the strike price. To solve this problem, we apply the space grid refinement method. Figure 2 shows the effect of local mesh refinement by drawing the error distribution at time T. From the figure, we notice that the error at the strike price $S = K = 100$ is around 10^{-2} without local mesh refinement (blue dotted line), while by local mesh refinement (red solid line) with the added new node number $L_0 = 32$, the error significantly lowers to 10^{-4}. In Table 2, M is the initial number of spatial grid points; L_0 is the added new node number. From the table, we can observe that the FOC scheme with local mesh refinement achieves fourth-order accuracy. Furthermore, we plot the price curve of the European call option under regime-switching Merton models with the intensity $\lambda = 0.3, 0.5, 0.7$ in Figure 3, respectively.

Tables 3 and 4 present the prices of the European call option at the second and third states of the economy, respectively. Table 5 presents the prices of the European put option at the first state of the economy. From Tables 3, 4, and 5, we can also notice that the FOC scheme only achieve second-order convergence accuracy with uniform mesh, while it can reach fourth-order accuracy by local mesh refinement.

TABLE 5 The value of European put option at the first state of the economy obtained by IMEX-BDF2-FOC scheme and the convergence orders of the scheme. Upper: without local mesh refinement; bottom: with local mesh refinement

		$S = 90$			$S = 100$			$S = 110$			CPU
M		Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32		13.34798554	1.80e-01		10.30254189	2.43e-01		8.58318495	1.67e-01		3.82
64		13.48567913	4.27e-02	2.08	10.48785901	5.80e-02	2.07	8.70935825	4.08e-02	2.03	8.06
128		13.51848914	9.85e-03	2.11	10.53173758	1.42e-02	2.04	8.74036916	9.78e-03	2.06	17.82
256		13.52608741	2.25e-03	2.13	10.54239125	3.50e-03	2.02	8.74782458	2.32e-03	2.07	42.12
512		13.52779014	5.52e-04	2.03	10.54502149	8.69e-04	2.01	8.74957436	5.74e-04	2.02	116.34
1024		13.52846812	1.26e-04	2.13	10.54567650	2.14e-04	2.02	8.75028833	1.40e-04	2.03	334.25
2048		13.52837268	3.06e-05	2.04	10.54584051	5.02e-05	2.09	8.75018272	3.48e-05	2.01	1201.47
		$S = 90$			$S = 100$			$S = 110$			CPU
M	L_0	Value	Error	Order	Value	Error	Order	Value	Error	Order	times (s)
32	16	13.51954987	8.79e-03		10.53675697	9.14e-03		8.74350135	6.65e-03		4.15
64	24	13.52778014	5.62e-04	3.97	10.54534531	5.46e-04	4.07	8.74973549	4.12e-04	4.01	8.38
128	32	13.52837612	3.41e-05	4.04	10.54585625	3.45e-05	3.98	8.75012359	2.43e-05	4.08	20.46
256	40	13.52834421	2.15e-06	3.99	10.54589283	2.07e-06	4.06	8.75014636	1.55e-06	3.97	45.37
512	48	13.52834219	1.33e-07	4.01	10.54589063	1.33e-07	3.96	8.75014781	9.90e-08	3.97	118.12
1024	56	13.52834207	8.28e-09	4.01	10.54589075	8.29e-09	4.00	8.75014790	5.74e-09	4.11	337.54
2048	64	13.52834206	5.25e-10	3.98	10.54589076	4.98e-10	4.06	8.75014790	3.52e-10	4.03	1216.75

TABLE 6 The price of American put option at various state of economy with $M = 1024$ and $N = 1600$ at $\tau = T$

	$S = 90$	$S = 100$	$S = 110$
First state of economy	14.28425133	11.11738725	9.22672458
Second state of economy	16.66358246	13.83104538	11.88294541
Third state of economy	18.44635972	15.73549804	13.64587215

TABLE 7 Comparative CPU times (s) of American put option using our method versus those using radial basis function (RBF) collocation method in Bastani et al[20] for the first state of economy at $\tau = T$

		Our method without refinement		Our method with refinement			RBF[20]	
M	N	Value	CPU times	L_0	Value	CPU times	Value	CPU times
128	64	14.3552	0.55	32	14.3370	0.56	14.3128	0.57
256	128	14.3085	2.59	40	14.2843	2.63	14.2842	3.90
512	256	14.2914	15.74	48	14.2649	16.17	14.2664	20.62
1024	512	14.2863	104.26	56	14.2567	106.92	14.2575	218.64
2048	1024	14.2845	746.04	64	14.2537	763.34	14.2539	1683.82

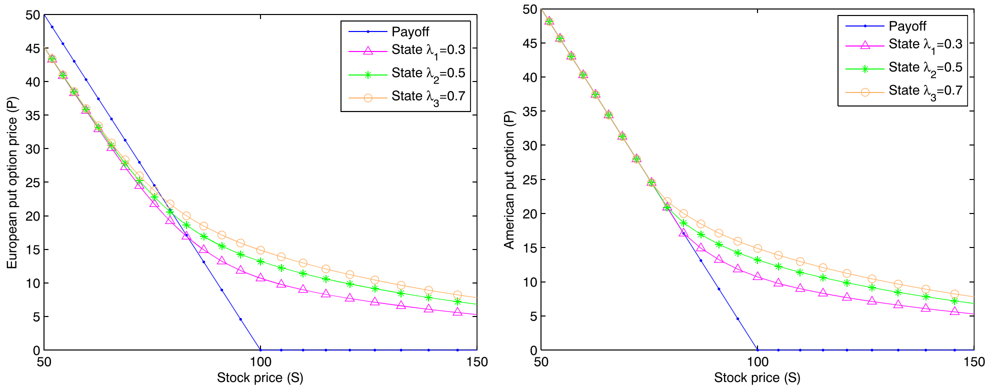

FIGURE 4 The option price curve obtained with 1600 time steps and 64 spatial meshes steps at $\tau = T$ under the regime-switching Merton model. Left: European put option. Right: American put option [Colour figure can be viewed at wileyonlinelibrary.com]

Next, we describe the prices of American put option under Merton regime-switching jump-diffusion model. Table 6 presents options at various states of economy for different values of asset price. The parameters of American put option are the same as those of the European option. As for the penalty method, we use the penalty parameter $\varepsilon = 10^{-4}$. Table 7 present the CPU times (s) of American put option at the first state of the economy obtained by our method and by radial basis function (RBF) collocation method proposed by Bastani et al[20] at $\tau = T$. From Table 7, we observe that our method is much faster than the RBF collocation method, and the numerical valuations using our method with refinement are better approximations of to 14.2502 (which is reported with the Fourier space time-stepping [FST] method[45]) than those obtained by RBF collocation method.[20] In Figure 4, the prices of the European and American put options are plotted at various states of economy.

6 | CONCLUSIONS

In this paper, we used the IMEX-BDF2 method to solve the PIDEs for the prices of the European option under the regime-switching Merton jump-diffusion models. We proved the L^2-stability of the semi-discrete IMEX-BDF2 method. The governing equation was discretized in space by using FOC scheme with local mesh refinement method around the singularity, which effectively improves the overall accuracy. Moreover, we applied the penalty method to solve the LCP derived from the American option. A number of numerical experiments were carried out for European and American options under the regime-switching models and showed that the proposed method are effective.

ACKNOWLEDGEMENTS

The authors would like to thank the referee for comments and suggestions that led to improvements in the presentation of this paper. This work was supported by the Natural Science Foundation of China (Grant No. 11771060 and 11671343) and the Hunan Province Innovation Foundation for Postgraduate (No. CX2017B267).

ORCID

Wansheng Wang https://orcid.org/0000-0002-2128-7501

REFERENCES

1. Black F, Scholes M. The pricing of options and corporate liabilities. *J Polit Econ*. 1973;81:637-654.
2. Merton RC. Theory of rational option pricing. *Bell J Econ Manage Sci*. 1973;4(1):141-183.
3. Leland HE. Option pricing and replication with transaction costs. *J Finance*. 1985;40(5):1283-1301.
4. Lyons TJ. Uncertain volatility and the risk-free synthesis of derivatives. *Appl Math Finance*. 1995;2(2):117-133.
5. Barles G, Soner HM. Option pricing with transaction costs and a nonlinear Black-Scholes equation. *Finance Stochastics*. 1998;2(4):369-397.
6. Frey R, Patie P. Risk management for derivatives in illiquid markets: a simulation study. *Advances in Finance and Stochastics*. Berlin, Heidelberg: Springer; 2002:137-159.

7. Guo JQ, Wang WS. An unconditonally stable, positivity-preserving splitting scheme for nonlinear Black-Scholes equation with transaction costs. *Sci World J*. 2014;2014:1-11. ID 525207.
8. Guo JQ, Wang WS. On the numerical solution of nonlinear option pricing equation in illiquid markets. *Comput Math Appl*. 2015;69:117-133.
9. Ikonen S, Toivanen J. Efficient numerical methods for pricing American options under stochastic volatility. *Numer Methods Partial Differ Equ*. 2008;24(1):104-126.
10. Huang JX, Zhu WL, Ruan XF. Option pricing using the fast Fourier transform under the double exponential jump model with stochastic volatility and stochastic intensity. *J Comput Appl Math*. 2014;263:152-159.
11. Lee J, Lee Y. Stability of an implicit method to evaluate option prices under local volatility with jumps. *Appl Numer Math*. 2015;87:20-30.
12. Liu TM, Muroi Y. Pricing of options in the singular perturbed stochastic volatility model. *J Comput Appl Math*. 2017;320:138-144.
13. Merton RC. Option pricing when underlying stock returns are discontinuous. J. *Financial Econ*. 1976;3(1-2):125-144.
14. Kou SG. A jump diffusion model for option pricing. *Manage Sci*. 2002;48(8):1086-1101.
15. Cont R, Tankov P. *Financial Modelling with Jump Processes*. Boca Raton, FL: Chapman & Hall/CRC; 2004.
16. Lee ST, Sun HW. Fourth order compact scheme with local mesh refinement for option pricing in jump-diffusion model. *Numer Methods Partial Differ Equ*. 2012;28(3):1079-1098.
17. Huang Y, Forsyth PA, Labahn G. Methods for pricing American options under regime switching. *SIAM J Sci Comput*. 2011;33: 2144-2168.
18. Company R, Egorova V, Jódar L, Vázquez C. Computing American option price under regime switching with rationality parameter. *Comput Math Appl*. 2016;72:741-754.
19. Lee Y. Financial options pricing with regime-switching jump-diffusions. *Comput Math Appl*. 2014;68:390-404.
20. Bastani AF, Ahmadi Z, Damircheli D. A radial basis collocation method for pricing American options under regime-switching jump-diffusions. *Appl Numer Math*. 2013;65:79-90.
21. Wang WS, Chen YZ. Fast numerical valuation of options with jump under Merton's model. *Comput Appl Math*. 2017;318:79-92.
22. Chen YZ, Wang WS, Xiao AG. An efficient algorithm for options under Merton's jump-diffusion model on nonuniform grids. submitted.
23. Achdou Y, Pironneau O. *Computational Methods for Option Pricing*, Society for Industrial and Applied Mathematics, vol. 30. Philadelphia, PA: SIAM; 2005.
24. Tavella D, Randall C. *Pricing Financial Instruments: The Finite Difference Method*. Chichester, UK: John Wiley & Sons; 2000.
25. Zhang XL. Numerical analysis of American option pricing in a jump-diffusion model. *Math Oper Res*. 1997;22(3):668-690.
26. Andersen L, Andreasen J. Jump-diffusion processes: volatility smile fitting and numerical methods for option pricing. *Rev Derivatives Res*. 2000;4(3):231-262.
27. d'Halluin Y, Forsyth PA, Labahn G. A penalty method for American options with jump diffusion processes. *Numer Math*. 2004;97(2):321-352.
28. Briani M, Chioma CL, Natalini R. Convergence of numerical schemes for viscosity solutions to integro-differential degenerate parabolic problems arising in finance theory. *Numer Math*. 2004;98:607-646.
29. d'Halluin Y, Forsyth PA, Vetzal KR. Robust numerical methods for contingent claims under jump diffusion processes. *IMA J Numer Anal*. 2005;25(1):87-112.
30. Almendral A, Oosterlee CW. Numerical valuation of options with jumps in the underlying. *Appl Numer Math*. 2005;53(1):1-18.
31. Cont R, Voltchkova E. A finite difference scheme for option pricing in jump-diffusion and exponential Lévy models. *SIAM J Numer Anal*. 2005;43:1596-1626.
32. Toivanen J. Numerical valuation of European and American options under Kou's jump-diffusion model. *SIAM J Sci Comput*. 2008;30(4):1949-1970.
33. Salmi S, Toivanen J. An iterative method for pricing American options under jump-diffusion models. *Appl Numer Math*. 2011;61(7):821-831.
34. Pindza E, Patidar KC, Ngounda E. Robust spectral method for numerical valuation of European options under Merton's jump-diffusion model. *Numer Methods Partial Differ Equ*. 2014;30(4):1169-1188.
35. Kadalbajoo MK, Tripathi LP, Kumar A. Second order accurate IMEX methods for option pricing under Merton and Kou jump-diffusion model. *J Sci Comput*. 2015;65:979-1024.
36. Kadalbajoo MK, Kumar A, Tripathi LP. A radial basis function based implicit-explicit method for option pricing under jump-diffusion models. *Appl Numer Math*. 2016;110:159-173.
37. Hamilton JD. A new approach to the economic analysis of non-stationary time series. *Econometrica*. 1989;57:357-384.
38. Hamilton JD. Analysis of time series subject to changes in regime. *J Econom*. 1990;45:39-70.
39. Naik V. Option valuation and hedging strategies with jumps in the volatility of asset returns. *J Finance*. 1993;48(5):1969-1984.
40. Egorova V, Company R, Jódar L. A new efficient numerical method for solving American option under regime switching model. *Comput Math Appl*. 2016;71(1):224-237.
41. Ma JT, Zhou ZQ. Moving mesh methods for pricing Asian options with regime switching. *J Comput Appl Math*. 2016;298:211-221.
42. Ramponi A. Fourier transform methods for regime-switching jump-diffusions and the pricing of forward starting options. *Int J Theor Appl Finance*. 2012;15(5):1250037.

43. Costabile M, Leccadito A, MassabóI RE. Option pricing under regime-switching jump-diffusion models. *J Comput Appl Math*. 2014;256:152-167.

44. Dang DM, Nguyen D, Sewell G. Numerical schemes for pricing Asian options under state-dependent regime-switching jump-diffusion models. *Comput Math Appl*. 2016;71(1):443-458.

45. Jackson KR, Jaimungal S, Surkov V. Option pricing with regime switching Lévy processes using Fourier space time stepping. In: Proceedings of the Fourth IASTED International Conference on Financial Engineering and Applications. Berkeley, California, USA; 2007:92-97.

How to cite this article: Chen Y, Xiao A, Wang W. An IMEX-BDF2 compact scheme for pricing options under regime-switching jump-diffusion models. *Math Meth Appl Sci*. 2019;1–18. https://doi.org/10.1002/mma.5539