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The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks

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ABSTRACT

We measure the volatility information content of stock options for individual firms using option prices for 149 US firms and the S&P 100 index. We use ARCH and regression models to compare volatility forecasts defined by historical stock returns, at-the-money implied volatilities and model-free volatility expectations for every firm. For 1-day-ahead estimation, a historical ARCH model outperforms both of the volatility estimates extracted from option prices for 36% of the firms, but the option forecasts are nearly always more informative for those firms that have the more actively traded options. When the prediction horizon extends until the expiry date of the options, the option forecasts are more informative than the historical volatility for 85% of the firms. However, at-the-money implied volatilities generally outperform the model-free volatility expectations.

1. Introduction

Volatility forecasts extracted from option prices are often compared with forecasts calculated from historical asset prices.[1] For US stock indices in particular, most of the empirical evidence shows that option prices provide more accurate volatility forecasts than historical methods which rely on daily returns; see, for example, Christensen and Prabhala (1998), Fleming (1998) and Ederington and Guan (2002). Furthermore, option-based forecasts often rank highly against forecasts obtained from the high-frequency realized volatility measures of Andersen et al. (2001, 2003); see Blair et al. (2001), Jiang and Tian (2005) and Giot and Laurent (2007) for US stock indices.[2] In contrast, comparisons of volatility forecasts for the stock prices of individual firms are rare, the most notable being the study of 10 US firms by Lamoureux and Lastrapes (1993). Our first contribution is a comparison of historical and option-based predictors of future volatility for a large sample of US firms, namely all 149 firms that have sufficient option price data included in an OptionMetrics database.

The most recent important innovation in research into volatility forecasts exploits combinations of option prices that do not rely on any pricing formula. These model-free forecasts apply the theoretical results of Carr and Madan (1998), Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000), which we outline in Section 2. For the S&P 500 index, Jiang and Tian (2005) show that the model-free volatility expectation is more highly correlated with future realized volatility than either the at-the-money implied volatility or the latest measurement of realized volatility calculated from 5-min returns; furthermore, in multivariate regressions only the model-free variable has a significant coefficient. Lynch and Panigirtzoglou (2004) also compare the model-free volatility expectation with historical volatility measured by intraday returns. Their results, for the S&P 500 index, the FTSE 100 index, Eurodollar futures and short sterling futures, show that the model-free volatility

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[1] Poon and Granger (2003) and Taylor (2005) provide surveys for equity, foreign exchange and commodity markets. Yu et al. (2009) is a recent relevant study of non-US stock indices.

[2] High-frequency methods rank more highly, however, in recent papers by Bali and Weinbaum (2007), Becker et al. (2007) and Martin et al. (2009).

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expectation is more informative than high-frequency returns, but it is a biased estimator of future realized volatility. Our second contribution is a comparison of model-free and at-the-money predictors for individual firms. We provide an empirical strategy which is able to extract model-free information from the few traded strikes that are usually available for firms.

Model-free volatility expectations have two potential advantages compared with Black–Scholes implied volatilities. Firstly, they do not depend on any option pricing formula so they do not require assumptions about the volatility dynamics. Secondly, they avoid relying on a single strike price which is problematic because implieds are far from constant across strikes. The early version of the VIX volatility index is investigated by Blair et al. (2001) and uses a few near-the-money option prices. The CBOE replaced this index in 2003 by the model-free volatility expectation. Both versions of the VIX index and relevant theory are discussed by Carr and Wu (2006). In related research, Carr and Wu (2009) synthesize variance swap rates, which are equivalent to model-free variance expectations, and show these swap rates are significant when explaining the time-series movements of realized variance.[3]

We compare the information content of three types of volatility forecasts for 149 US firms: historical ARCH forecasts obtained from daily stock returns, the at-the-money (hereafter ATM) implied volatility and the model-free volatility expectation. We initially focus on a 1-day-ahead forecast horizon and subsequently evaluate forecasts whose horizons match the relevant option expiry dates, which we choose to be 1 month into the future on average. Our sample also includes the S&P 100 index, to provide comparisons with the results for our firms and also with previous literature about indices.

Our empirical results show that both the model-free volatility expectation and the ATM implied volatility do contain relevant information about the future volatility of stock prices. In contrast to most previous studies about stock index options, our research into individual stocks shows that for 1-day-ahead prediction neither the ATM implied volatility nor the model-free volatility expectation is consistently superior to a simple ARCH model for all firms. It is often best to use an asymmetric ARCH model to estimate the next day's volatility, particularly for firms with few traded strikes. However, when the estimation horizon extends until the end of the option lives, both the volatility estimates extracted from option prices outperform the historical volatility for a substantial majority of our sample firms.

The ATM implied volatility outperforms the model-free volatility expectation for 87 out of 149 firms when predicting volatility 1-day-ahead, and for 89 firms when the forecast horizon equals the remaining time until the options expire. The relatively unsuccessful performance of the model-free volatility expectation is not explained by either selected properties of the available option data, such as the number of option observations or the range of option moneyness, or by the trading volume of ATM options compared with all other options.

Section 2 introduces three types of volatility forecasts and explains how the model-free volatility expectation is calculated. Section 3 describes the data. Section 4 explains the ARCH and regression methodologies that we use to compare the information content of the historical and option-based volatility estimates. Section 5 presents the empirical results for 1-day-ahead forecasts and option-life forecasts. Section 6 provides cross-sectional comparisons, which identify firm-specific variables which are associated with the best volatility prediction method. Section 7 contains our conclusions.

2. The volatility forecasting instruments

ARCH models provide a vast variety of historical volatility forecasts, obtained from information sets I_t that contain the history of asset prices up to and including time t; Lamoureux and Lastrapes (1993), Blair et al. (2001) and Ederington and Guan (2005) provide examples. The conditional variance of the next asset return, r_{t+1}, denoted by $h_{t+1} = \mathrm{var}(r_{t+1}|I_t)$, is a forecast of the next squared excess return. An advantage of the ARCH framework is that maximum likelihood methods can be used to select a specification for h_{t+1} and to estimate the model parameters. However, historical forecasts rely on past information and are not forward-looking.

Option implied volatilities are forward-looking and essentially contain all the information, including the historical information, required to infer the market's risk-neutral expectation of future volatility. For the risk-neutral measure Q, suppose the price of the underlying asset S_t follows a diffusion process, $dS = (r - q)Sdt + \sigma SdW$, where r is the risk-free rate, q is the dividend yield, W_t is a Wiener process and σ_t is the stochastic volatility. The integrated squared volatility of the asset from time 0 until the forecast horizon T is defined as $V_{0,T} = \int_0^T \sigma_t^2\,dt$; it equals the quadratic variation of the logarithm of the price process because we are here assuming there are no price jumps.[4] The theoretical analysis of Carr and Wu (2006) and Carr and Lee (2008) shows that the Black–Scholes ATM implied volatility for expiry time T represents an accurate approximation of the risk-neutral expectation of the realized volatility over the same time period, namely $E^Q\left[\sqrt{V_{0,T}}\right]$. Consequently, although the ATM implied volatility is model-dependent it has an economic interpretation as an approximation to the volatility swap rate.

The concept of the model-free variance expectation is developed in Carr and Madan (1998), Demeterfi et al. (1999), Britten-Jones and Neuberger (2000) and Carr and Wu (2009), motivated by the development of variance swap contracts. At time 0 a complete set of European option prices is assumed to exist for an expiry time T; the call and put prices are, respectively, denoted by $c(K,T)$ and $p(K,T)$ for a general strike price K. Britten-Jones and Neuberger (2000) show that the risk-neutral expectation of the integrated squared volatility is given by the following function of the continuum of European out-of-the-money (hereafter OTM) option prices:

$$E^Q[V_{0,T}] = 2e^{rT}\left[\int_0^{F_{0,T}} \frac{p(K,T)}{K^2}\,dK + \int_{F_{0,T}}^{\infty} \frac{c(K,T)}{K^2}\,dK\right], \quad (1)$$

where $F_{0,T}$ is the forward price at time 0 for a transaction at the expiry time T. We refer to the right-hand side (RHS) of (1) as the *model-free variance expectation* and its square root as the *model-free volatility expectation*. Dividing, as appropriate, by either T or $\sqrt{T}$ defines the annualized versions of these quantities. As the volatility expectation does not rely on a specific option pricing formula, the expectation is "model-free", in contrast to the Black–Scholes implied volatility.

The key assumption required to derive (1) is that the stochastic process for the underlying asset price is continuous. When there are relatively small jumps in the stock price process, Jiang and Tian (2005) and Carr and Wu (2006, 2009), show that the RHS of (1) is an excellent approximation to the risk-neutral, expected quadratic variation of the logarithm of the stock price. However, if there is an appreciable risk of an extreme jump, in particular of default, then the approximation error can be very large.

As the model-free expectation defined by (1) is a function of option prices for all strikes, a potential problem arises from the limited number of option prices observed in practice. This is an

[3] For some recent empirical studies of the VIX index, see Becker et al. (2007, 2009), Konstantinidi et al. (2008) and Martin et al. (2009).

[4] The quadratic variation of log(s) during a time interval is defined as the integrated squared volatility plus the sum of the squared jumps in log(s) during the interval.

important issue when forecasting stock price volatility, because stocks (unlike stock indices) have few traded strikes. To obtain sufficient option prices to approximate the integrals in (1) accurately, we must rely on implied volatility curves which we estimate from small sets of observed option prices; Jiang and Tian (2007) illustrate the importance of constructing these curves when estimating the model-free volatility expectation.

We implement a variation of the practical strategy of Malz (1997a,b), who proposes estimating the Black–Scholes implied volatility curve as a quadratic function of the Black–Scholes delta. The quadratic specification is preferred because it is the simplest function that captures the basic properties of the volatility smile. Furthermore, there are insufficient stock option prices to estimate higher-order polynomials.

Delta is defined here by the equations:

$$\Delta(K) = \partial C/\partial F_{0,T} = e^{-rT}\Phi(d_1(K)), \tag{2}$$

with

$$d_1(K) = \frac{\log(F_{0,T}/K) + 0.5\hat{\sigma}^2 T}{\hat{\sigma}\sqrt{T}}.$$

Following Bliss and Panigirtzoglou (2002, 2004) and Liu et al. (2007), $\hat{\sigma}$ is a constant that permits a convenient one-to-one mapping between Δ and K. In this study, $\hat{\sigma}$ is the volatility implied by the option price whose strike is nearest to the forward price, $F_{0,T}$.

The parameters of the quadratic implied volatility function, denoted by θ, are estimated by minimizing the sum of squared errors function:

$$\sum_{i=1}^{N} w_i (IV_i - \widehat{IV}_i(\Delta_i, \theta))^2, \tag{3}$$

where N is the number of observed strikes for the firm on the observation day, IV_i is the observed implied volatility and $\widehat{IV}_i(\Delta_i, \theta)$ is the fitted implied volatility for a strike price K_i which defines Δ_i by (2). The minimization is constrained to ensure that the fitted implied curve is positive for all Δ between 0 and e^{-rT}. The squared errors are weighted by $w_i = \Delta_i(1 - \Delta_i)$, so that the most weight is given to near-the-money options; consequently, extreme strikes (which can have extreme implied volatilities) cannot distort the fitted curves.

One thousand equally spaced values of Δ, that cover the range from 0 to e^{-rT}, are used to calculate strikes K from (2) then implied volatilities and finally OTM option prices.[5] These OTM prices give accurate approximations to the integrals in (1).

3. Data

The option data are obtained from the IvyDB database of OptionMetrics, which includes price information for all US listed index and equity options, based on daily closing quotes at the CBOE. The database also includes interest rate curves and dividend information. Daily stock price data are from CRSP. Our sample period is the 1009 trading days from January 1996 until December 1999.

We use the implied volatilities provided by the IvyDB database directly in our study, as do Carr and Wu (2009), Cremers et al. (2008) and Xing et al. (forthcoming). Each implied volatility is computed from the midpoint of the most competitive bid and ask prices, across all exchanges on which the option trades, using a binomial tree calculation which takes account of the early exercise premium and dividends. Whenever matching call and put implieds are available on the same trading day, their average is used which reduces any measurement errors created by nonsynchronous asset and option prices. We exclude all options less than 7 days from maturity.

We obtain interest rates corresponding to each option's expiration by linear interpolation of the two closest zero-coupon rates derived from BBA LIBOR rates. We calculate the forward stock price $F_{0,T}$, that has the same expiry date T as options, from the future value of the current spot price minus the present value of all dividend distributions until time T. Daily stock returns are continuously compounded and adjusted for dividends.

Two criteria are used to select firms. Firstly, only firms that have options traded throughout the whole sample period are included. Secondly, a firm must have sufficient option trading activity to allow us to construct implied volatility curves for at least 98% of the 1009 trading days.

Implied volatilities for at least three strike prices are required to estimate quadratic curves. Nearest-to-maturity options are usually chosen, but if they provide less than three implieds we switch to the second nearest-to-maturity. If, however, it is impossible to estimate the implied volatility curve from the two nearest-to-maturity sets of option contracts, we regard the day as having missing data.[6]

A total of 149 firms pass both filters. The number of option price observations during the sample period varies across firms and years, with the least observations in 1996. The average number for firm i, denoted $\overline{N}_i$, equals the number of strike prices used during the sample period divided by the number of trading days; for those trading days when it is impossible to construct an implied volatility curve, the number of available strike prices is set to zero. The minimum, median and maximum values of $\overline{N}_i$ across firms are 3.7, 5.1 and 12.9, respectively. More than half of the averages $\overline{N}_i$ are between 4.0 and 6.0.

4. Empirical methodology

4.1. Econometric specifications

We investigate the informational efficiency of the model-free volatility expectation and the ATM implied volatility, firstly when the forecast horizon is 1 day and secondly when it is matched with the option's time to maturity. We estimate ARCH models from daily returns and regression models for a data-frequency that is determined by the expiration dates of the option contracts. The primary advantages of ARCH models are the availability firstly of more observations and secondly of maximum likelihood estimates of the model parameters. A disadvantage of ARCH models, however, is that the time interval between price observations is short compared with the forecasting horizon that is implicit in option prices, namely the remaining time until expiry. To learn as much as we can about the accuracy of volatility estimates derived from the option prices, our study therefore evaluates both ARCH specifications for the 1-day-ahead forecasts and regressions that employ a forecast horizon equal to the option's time to maturity.

The general ARCH specification we evaluate for daily returns is as follows, and includes an MA(1) term in the conditional mean equation to capture any first-order autocorrelation in stock returns:

$$\begin{aligned} r_t &= \mu + \varepsilon_t + \theta\varepsilon_{t-1}, \\ \varepsilon_t &= \sqrt{h_t}z_t, \quad z_t \sim i.i.d.(0,1), \\ h_t &= \frac{\omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 s_{t-1}\varepsilon_{t-1}^2}{1 - \beta L} + \frac{\gamma\sigma_{MF,t-1}^2}{1 - \beta_\gamma L} + \frac{\delta\sigma_{ATM,t-1}^2}{1 - \beta_\delta L}. \end{aligned} \tag{4}$$

[5] If either the least call price or the least put price exceeds 0.001 cents then we extend the range of strike prices to eliminate any error caused by truncating integrals. The extrapolations employ a spacing of 0.01 in moneyness, defined as $K/F_{0,T}$. They continue until the OTM prices are less than 0.001 cents.

[6] The GARCH specification introduced in Section 4.1 requires daily values of the forecasting instruments. When it is impossible to estimate the implied volatility curve, we assume both the model-free volatility expectation and the ATM implied volatility are unchanged from the previous trading day.

Here L is the lag operator, h_t is the conditional variance of the return in period t and s_{t-1} is 1 if $\varepsilon_{t-1} < 0$ and it is 0 otherwise. The terms $\sigma_{MF,t-1}$ and $\sigma_{ATM,t-1}$ are, respectively, the daily estimates of the model-free volatility expectation and the ATM implied volatility, computed at time $t-1$ by dividing the annualized values by $\sqrt{252}$.

We focus on three different volatility models based upon different information sets. First, the GJR(1,1)–MA(1) model of Glosten et al. (1993) only uses historical returns, so $\gamma = \beta_\gamma = \delta = \beta_\delta = 0$. Second, the model that uses the information given by model-free volatility expectations alone has $\alpha_1 = \alpha_2 = \beta = \delta = \beta_\delta = 0$. Third, the model that uses information provided by ATM implied volatilities alone has $\alpha_1 = \alpha_2 = \beta = \gamma = \beta_\gamma = 0$.

The parameters are estimated by maximizing the quasi-log-likelihood function, defined by assuming that the standardized returns z_t have a normal distribution. To ensure that the conditional variances of all models remain positive, the constraints $\omega > 0$, $\alpha_1 \geqslant 0$, $\alpha_1 + \alpha_2 \geqslant 0$, $\beta \geqslant 0$, $\beta_\gamma \geqslant 0$ and $\beta_\delta \geqslant 0$ are placed on the parameters. Inferences are made through t-ratios, constructed from the robust standard errors of Bollerslev and Wooldridge (1992). The three special cases listed above are ranked by comparing their log-likelihood values; a higher value indicates that the information provides a better description of the conditional distributions of daily stock returns. As we rank specifications which have the same conditional mean equations, a higher rank indicates a more informative specification of the conditional variance and hence more appropriate forecasts of the next day's realized variance.

Similar models with implied volatility incorporated into ARCH models are estimated by Day and Lewis (1992) and Blair et al. (2001) for the S&P 100 index and by Lamoureux and Lastrapes (1993) for individual stocks. The GJR(1,1) model is adopted here because asymmetric volatility effects have been found for individual US firms in previous studies such as Cheung and Ng (1992) and Duffee (1995).

We estimate univariate and encompassing regressions for the realized volatility of each firm, as in the index studies by Canina and Figlewski (1993), Christensen and Prabhala (1998), Ederington and Guan (2002) and Jiang and Tian (2005). While a univariate regression can assess the information content of one volatility estimation method, the encompassing regression addresses the relative importance of competing volatility estimates.

The explained variable in the regression analysis could be volatility, variance or the logarithm of volatility, as in Jiang and Tian (2005). We find that each of these three variables provides the same ranks for the forecasting instruments. Here we focus on the logarithm of volatility, which Andersen et al. (2001) show has the advantage of an approximately normal distribution and hence more reliable inferences.

The most general regression equation is specified as follows:

$$\log(100\sigma_{RE,t,T}) = \beta_0 + \beta_{His}\log(100\sigma_{His,t,T}) + \beta_{MF}\log(100\sigma_{MF,t,T}) + \beta_{ATM}\log(100\sigma_{ATM,t,T}) + \varepsilon_{t,T}. \quad (5)$$

The forecast quantity is the realized volatility from time t to time T, denoted by $\sigma_{RE,t,T}$ and defined by (6) in Section 4.2. The historical volatility forecast, $\sigma_{His,t,T}$, is calculated from the GJR(1,1)–MA(1) model using the information up to time t. The terms $\sigma_{MF,t,T}$ and $\sigma_{ATM,t,T}$ are non-overlapping measures of the model-free volatility expectation and the ATM implied volatility. Inferences are made using OLS estimates and the robust standard errors of White (1980), which take account of any heteroscedasticity in the residual terms $\varepsilon_{t,T}$.

4.2. Volatility measures and forecasts

We use daily values of the ATM implied volatility and the model-free volatility expectation in the estimation of the ARCH models. The ATM implied volatility corresponds to the available strike price closest to the forward price. We calculate the model-free volatility expectation after extracting a large number of OTM option prices from an implied volatility curve as described in Section 2.

To implement regression analysis for option-life forecasts of volatility, we select both the model-free volatility expectation and the ATM implied volatility on the trading date that follows the previous maturity date, so that we obtain non-overlapping samples of volatility expectations. We are able to use sets of 49 monthly observations, with option expiry months from January 1996 to January 2000, for the S&P 100 index and 129 of the 149 firms. For each of the remaining 20 firms, the number of observations is 46, 47 or 48 because of the occasional illiquidity of option trading for some firms. To match the horizon of all the variables in the regressions with the approximately 1-month horizon of the options information, realized volatility measures and historical volatility forecasts are required for the remaining lives of the options.

The annualized realized volatility from a day t until the option's maturity date T is calculated by applying the well-known formula of Parkinson (1980) to daily high and low stock prices, such that:

$$\sigma_{RE,t,T} = \sqrt{\frac{252}{H}\sum_{i=1}^{H}\frac{[\log(h_{t+i}) - \log(l_{t+i})]^2}{4\log(2)}}, \quad (6)$$

where h_t and l_t are, respectively, the highest and lowest stock price for day t, and H is the number of days until the options expire. The squares of daily price ranges used in (6) provide more accurate measures of volatility than squared returns, and consequently they define more appropriate target quantities when comparing forecasts.[7]

The historical information I_t at day t provides the conditional variance h_{t+1} for day $t+1$ and a forecast of the aggregate variance until the expiry time T given by:

$$h_{t+1} + \sum_{j=2}^{H} E[h_{t+j}|I_t], \quad (7)$$

where H is the forecast horizon. For the GJR(1,1) model, the annualized historical volatility forecast simplifies to:

$$\sigma_{His,t,T} = \sqrt{\frac{252}{H}\left[H\sigma^2 + \frac{1-\phi^H}{1-\phi}(h_{t+1} - \sigma^2)\right]}, \quad (8)$$

where $\phi = \alpha_1 + 0.5\alpha_2 + \beta$ and $\sigma^2 = \omega/(1-\phi)$ are, respectively, equal to the persistence parameter and the unconditional variance of the returns.

The parameters of the ARCH models used to define the historical forecasts are estimated by maximizing the log-likelihood of a set of n returns that do not go beyond time t. Ninety of the 149 firms have continuous price histories from January 1988 until January 2000. For these firms, we initially use $n = 2024$ returns for the trading days between 4 January 1988 and 4 January 1996, as our first forecasts are made on 4 January 1996; the subsequent forecasts use parameters estimated from the 2024 most recent returns. For each of the other firms, whose histories commence after January 1988, we use all the daily returns until the observation day t (although we stop adding to the historical sample if n reaches 2024).

[7] The ranking of the forecast instruments and their combinations is the same if squared returns replace squared ranges in the definition of the forecast target but, as is theoretically predicted, the values of R^2 are reduced.

5. Empirical results

5.1. Descriptive statistics

Table 1 presents summary statistics for daily estimates of the model-free volatility expectation, the ATM implied volatility and their difference. The mean and standard deviation are first obtained for each firm and the index from time series of volatility estimates. Then the cross-sectional mean, lower quartile, median and upper quartile values of each statistic, across the 149 firms, are calculated and displayed.

On average the model-free volatility expectation is slightly higher than the ATM implied volatility, as also occurs in the study by Jiang and Tian (2005) on S&P 500 index options. Consequently, the squared ATM implied volatility tends to be a downward biased measure of the risk-neutral expected variance. The null hypothesis that the ATM implied volatility is an unbiased estimate of the model-free volatility expectation is rejected for the index and each of the 149 firms, at the 1% significance level, using the standard test. Table 1 also shows that the two volatility estimates are highly correlated, with the mean and median of the correlations, respectively, equal to 0.926 and 0.940. These high values reflect the similar information that is used to price ATM and OTM options.

Table 2 shows similar summary statistics for the 1-month, volatility option-life forecasts and outcomes. The realized volatility measured by daily high and low prices is usually lower than the other volatility estimates because, as Garman and Klass (1980) explain, discrete trading (both in time and in price) induces a downward bias.

Table 3 summarises the correlations between the four 1-month volatility measures, namely the historical forecast of the volatility during the remaining lifetime of a set of option strikes, the at-the-money forecast, the model-free forecast and the realized volatility given by (6). Across all the firms, the average correlation between the realized volatility and the ATM implied volatility, the model-free expectation and the historical forecast, respectively, equals 0.502, 0.492 and 0.344. The highest correlations are between the ATM and the model-free volatilities, and their average value equals 0.937.

Table 1
Summary statistics for daily estimates of volatility from option prices.

	Firms				S&P 100
	Mean	Q_1	Q_2	Q_3	
Panel A: σ_{MF}					
Mean	0.523	0.371	0.521	0.646	0.225
Std. Dev.	0.123	0.078	0.106	0.131	0.056
Panel B: σ_{ATM}					
Mean	0.486	0.351	0.484	0.610	0.200
Std. Dev.	0.099	0.072	0.094	0.114	0.050
Panel C: $\sigma_{MF} - \sigma_{ATM}$					
Mean	0.036	0.024	0.032	0.043	0.025
Std. Dev.	0.051	0.026	0.035	0.048	0.010
Panel D: correlation between σ_{MF} and σ_{ATM}					
Correlation	0.926	0.907	0.940	0.960	0.989

The cross-sectional statistics are calculated from the means and standard deviations of daily observations from time series, which cover the period from January 1996 to December 1999. The cross-sectional mean, lower quartile (Q_1), median (Q_2) and upper quartile (Q_3) values of each statistic are reported in the columns, across 149 firms. σ_{MF} and σ_{ATM} are daily estimates of the model-free volatility expectation and the at-the-money implied volatility. The time-series statistics for the S&P 100 index are reported in the last column.

5.2. One-day-ahead forecasts from ARCH specifications

5.2.1. Estimates of parameters

Table 4 provides the summary statistics of the sets of 149 point estimates (their mean, standard deviation, lower quartile, median and upper quartile) and the point estimates for the S&P 100 index, from the three ARCH specifications defined by (4). All the ARCH parameters are estimated using daily returns from January 1996 to December 1999. The column headed "5%" contains the percentages of the firm-level estimates out of 149 that are significantly different from zero at the 5% level.

Panel A summarises estimates for the GJR(1,1)–MA(1) model, which uses previous stock returns to calculate the conditional variance. The value of α_1 measures the symmetric impact of new information (defined by ε_t) on volatility while the value of α_2 measures the additional impact of negative information (when $\varepsilon_t < 0$). Approximately 75% of all firms have a value of $\alpha_1 + \alpha_2$ that is more than twice the estimate of α_1, indicating a substantial asymmetric effect for individual stocks. The volatility persistence parameter, assuming returns are symmetrically distributed, is $\alpha_1 + 0.5\alpha_2 + \beta$. The median estimate of persistence across the 149 firms equals 0.94.

Panel B provides results for the model which only uses the information contained in the time series of model-free volatility expectations, $\sigma_{MF,t-1}$. This series is filtered by the function $\gamma/(1 - \beta_\gamma L)$. For half of the firms, the estimates of γ are between 0.48 and 0.85. In contrast, most of the estimates of β_γ are near zero. We infer that a conditional variance calculated from the model-free volatility expectation given by the latest option prices cannot be improved much by using older option prices.

Panel C summarises estimates for the model which uses only the information contained in the ATM implied volatility series, $\sigma_{ATM,t-1}$, filtered by $\delta/(1 - \beta_\delta L)$. The interquartile range for δ is from

Table 2
Summary statistics for historical and option-based measures of volatility.

	σ_{MF}				σ_{ATM}				σ_{His}				σ_{RE}			
	Firms			S&P 100	Firms			S&P 100	Firms			S&P 100	Firms			S&P 100
	Q_1	Q_2	Q_3		Q_1	Q_2	Q_3		Q_1	Q_2	Q_3		Q_1	Q_2	Q_3	
Mean	0.37	0.51	0.64	0.22	0.35	0.49	0.61	0.20	0.32	0.50	0.60	0.15	0.29	0.42	0.54	0.15
Std. Dev.	0.07	0.09	0.11	0.05	0.06	0.09	0.11	0.04	0.05	0.08	0.11	0.04	0.09	0.11	0.15	0.05
Skewness	0.43	0.78	1.12	1.08	0.38	0.70	1.10	0.75	0.69	1.28	2.05	1.59	0.60	1.03	1.45	1.95
Kurtosis	3.37	3.25	4.62	4.90	3.61	3.20	4.52	3.62	3.53	5.71	8.89	6.20	3.08	4.21	5.86	8.09
Max	0.59	0.80	0.98	0.40	0.56	0.71	0.90	0.34	0.52	0.73	0.94	0.32	0.57	0.80	0.99	0.36
Min	0.25	0.36	0.45	0.13	0.23	0.34	0.43	0.12	0.22	0.34	0.46	0.09	0.16	0.22	0.31	0.09

The reported statistics are for monthly observations of annualized measures of volatility. σ_{MF}, σ_{ATM}, σ_{His} and σ_{RE} are, respectively, the values of the model-free volatility expectation, the at-the-money implied volatility, the historical forecast from a GARCH model and the realized volatility calculated from daily high and low prices using the formula of Parkinson (1980). All these volatility measures are for the remaining lifetimes of option contracts, which are approximately 1 month.
The cross-sectional statistics are calculated from descriptive statistics for monthly observations from time series, which cover the period from January 1996 to December 1999. The cross-sectional lower quartile, median and upper quartile values of each statistic are reported in the columns, across 149 firms. The time-series statistics for the S&P 100 index are also reported.

Table 3
Summary statistics for the correlations between volatility measures.

	σ_{MF}					σ_{ATM}					σ_{His}				
	Firms				S&P 100	Firms				S&P 100	Firms				S&P 100
	Mean	Q_1	Q_2	Q_3		Mean	Q_1	Q_2	Q_3		Mean	Q_1	Q_2	Q_3	
σ_{ATM}	0.937	0.923	0.952	0.971	0.988										
σ_{His}	0.547	0.373	0.576	0.745	0.892	0.562	0.412	0.577	0.747	0.883					
σ_{RE}	0.492	0.373	0.516	0.627	0.623	0.502	0.377	0.516	0.621	0.624	0.344	0.191	0.343	0.500	0.483

The summarised correlations are between monthly observations of annualized measures of volatility. σ_{MF}, σ_{ATM}, σ_{His} and σ_{RE} are, respectively, the values of the model-free volatility expectation, the at-the-money implied volatility, the historical forecast from a GARCH model and the realized volatility calculated from daily high and low prices using the formula of Parkinson (1980). All these volatility measures are for the remaining lifetimes of option contracts, which are approximately 1 month.
Each correlation is between monthly observations from two time series for the same firm, which cover the period from January 1996 to December 1999. The cross-sectional mean, lower quartile, median and upper quartile of each set of correlation statistics is calculated across 149 firms. The columns headed S&P 100 report the correlations between monthly volatility observations from the index and another time series.

Table 4
A summary of the ARCH parameter estimates obtained from 149 firms.

	Mean	Std. Dev.	Q_1	Q_2	Q_3	5%	S&P 100
Panel A: the GJR(1,1)–MA(1) model							
$\mu \times 10^3$	0.91	0.96	0.40	0.85	1.42	11.4	0.88*
θ	0.00	0.06	−0.04	0.00	0.04	16.1	0.00
$\omega \times 10^5$	17.61	28.93	1.57	5.91	17.89	63.8	0.58*
α_1	0.05	0.07	0.00	0.03	0.06	20.1	0.00
α_2	0.12	0.20	0.04	0.08	0.13	43.0	0.21*
β	0.77	0.23	0.66	0.86	0.93	93.3	0.85*
Persistence	0.87	0.18	0.81	0.94	0.98		0.96
L_{His}	2141	340	1860	2083	2438		3168
Panel B: the ARCH specification that only uses the model-free volatility expectation							
$\mu \times 10^3$	0.77	0.92	0.33	0.73	1.22	12.8	0.75*
θ	0.01	0.05	−0.03	0.00	0.05	17.4	0.05
$\omega \times 10^5$	10.35	25.16	0.00	0.49	10.15	0.7	0.00
γ	0.65	0.26	0.48	0.71	0.85	50.3	0.58*
β_γ	0.19	0.26	0.00	0.03	0.34	7.4	0.00
$\gamma/(1-\beta_\gamma)$	0.80	0.19	0.72	0.83	0.90		0.58
L_{MF}	2143	339	1872	2075	2454		3165
Panel C: the ARCH specification that only uses the ATM implied volatility							
$\mu \times 10^3$	0.77	0.91	0.35	0.71	1.23	11.4	0.77*
θ	0.01	0.05	−0.03	0.01	0.05	15.4	0.05
$\omega \times 10^5$	9.41	22.27	0.00	0.00	7.27	0	0.00
δ	0.81	0.29	0.62	0.88	1.01	42.3	0.73*
β_δ	0.14	0.24	0.00	0.00	0.20	4.0	0.00
$\delta/(1-\beta_\delta)$	0.92	0.22	0.84	0.96	1.04		0.73
L_{ATM}	2144	340	1870	2078	2453		3170

Daily stock returns r_t are modeled by the ARCH specification: $r_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$, $\varepsilon_t = \sqrt{h_t}z_t$, $z_t \sim i.i.d.(0,1)$, $h_t = \frac{\omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 s_{t-1}\varepsilon_{t-1}^2}{1-\beta L} + \frac{\gamma\sigma_{MF,t-1}^2}{1-\beta_\gamma L} + \frac{\delta\sigma_{ATM,t-1}^2}{1-\beta_\delta L}$, s_t is 1 if $\varepsilon_t < 0$, otherwise s_t is zero. σ_{MF} and σ_{ATM} are, respectively, the daily measure of model-free volatility expectation and the ATM implied volatility. All parameters are estimated by maximizing the quasi-log-likelihood function. Panel A contains the results for the GJR(1,1)–MA(1) model. Panels B and C, respectively, show estimates for models that use information provided by the model-free volatility expectation and the ATM implied volatility. Cross-sectional means, standard deviations, lower quartiles, medians and upper quartiles are presented across 149 firms. The column headed 5% shows the percentages of estimates which are significantly different from zero at the 5% level, using the robust t-ratios of Bollerslev and Wooldridge (1992). The last column reports the parameter estimates for the S&P 100 index; the starred estimates are significantly different from zero at the 5% level. L_{His}, L_{MF} and L_{ATM} are the maximized log-likelihood values of the three models.

0.62 to 1.01. More than half of the estimates of the lag coefficient, β_δ, are zero and few of them are far from zero. On average, δ exceeds γ and β_δ is less than β_γ.

5.2.2. Comparisons of log-likelihoods

A higher log-likelihood value indicates a more accurate description of the conditional distributions of daily stock returns. We use L_{His}, L_{MF} and L_{ATM} to represent the maximum log-likelihoods of the three models defined after (4). The mean of L_{ATM} is slightly higher than the means of L_{His} and L_{MF}, but the differences between the means are small. For the S&P 100 index, the three log-likelihood values are similar with $L_{ATM} > L_{His} > L_{MF}$.

Table 5 shows how often each of the three ARCH specifications has the highest log-likelihood. More than a third of the firms (35.6%) have a log-likelihood for the historical model, L_{His}, which is higher than both L_{MF} and L_{ATM}, which are obtained from the options' information. For the 64.4% of the firms whose log-likelihoods are maximized using option specifications, the ATM specification (36.9%) is the best more often than the model-free volatility expectation (27.5%). Thus there is evidence for the superior efficiency of ATM option implied volatilities when estimating individual stock volatility.

The high frequency for the historical specification providing the best description of 1-day-ahead volatility contrasts with the opposite evidence for options written on stock indices, such as in Blair et al. (2001). There are two plausible explanations for why the GJR model performs the best for so many firms. Firstly, the key difference between the data for individual stock options and the stock index options is that the latter are much more liquid than the former. The illiquidity of individual stock options may cause the relative inefficiency of their implied volatility expectations. When we select the 30 firms with the highest average option trading volume, the historical volatility performs the best for only 2 firms, the model-free volatility expectation for 11 firms and the ATM implied

Table 5
Frequency counts for the variables that best describe the volatility of stock returns.

	For 1-day-ahead forecasts		For options' life forecasts	
Historical is best	53	35.6%	23	15.4%
His > MF > ATM	21	14.1%	12	8.1%
His > ATM > MF	32	21.5%	11	7.4%
Model-free is best	41	27.5%	48	32.2%
MF > ATM > His	38	25.5%	45	30.2%
MF > His > ATM	3	2.0%	3	2.0%
At-the-money is best	55	36.9%	78	52.4%
ATM > MF > His	49	32.9%	61	41.0%
ATM > His > MF	6	4.0%	17	11.4%

The counts and percentages show how many of the 149 firms satisfy the ordering stated in the first column. For 1-day-ahead forecasts, the frequency counts are based on the maximized log-likelihood values of the ARCH specifications that, respectively, use historical daily returns (His), the model-free volatility expectation (MF) and the ATM implied volatility. For option-life forecasts, the frequency counts are based on the explanatory powers of the univariate regressions that contain historical volatility, the model-free volatility expectation and the ATM implied volatility.

volatility for 17 firms. Secondly, our ARCH specifications are estimated with a horizon of 1 day, while volatility estimates from option prices represent the expected average price variation until the end of the option's life. The mismatch between the estimation horizon and the option's time to expiry may reduce the relative performance of both the model-free volatility expectation and the ATM implied volatility when they are compared with the GJR(1,1) model.

5.3. Regression results for option-life forecasts

The regression results are for non-overlapping monthly observations, defined to ensure the estimation horizon matches the option's time to maturity. Table 6 reports the results of both univariate and encompassing regressions that explain realized volatility, defined after (6). As before, Table 6 shows the mean, median, lower and upper quartile of the point estimates across the 149 firms and the point estimates for the S&P 100 index. The bracketed number for each parameter is the percentage of firms whose estimates are significantly different from zero at the 5% level. The last three columns show the summary statistics for the regression R^2, the adjusted R^2 and the sum of squared residuals.

We begin our discussion with the results of the univariate regressions summarised by Table 6. The null hypotheses $\beta_{His} = 0$, $\beta_{MF} = 0$ and $\beta_{ATM} = 0$ are separately rejected for 70%, 89% and 91% of the firms at the 5% level. The values of R^2 are highest for the ATM implied volatility (mean 0.290), but the values for the model-free volatility expectation are similar (mean 0.278); the values for historical volatility, however, are much lower (mean 0.179). This evidence suggests that volatility estimates extracted from option prices are much more informative than historical daily stock returns when the estimation horizons match the lives of the options. The S&P 100 index forecasts rank in the same order as the firm averages but the index values of R^2 are higher than for most firms.

Table 5 provides the frequency counts that show how often each of the three univariate forecasts has the highest value of R^2. There are important differences between the frequencies for 1-day-ahead forecasts (left block) and for option-life forecasts (right block). Only 15.4% of the firms have historical volatility ranking highest for the option-life forecasts, compared with 35.6% in the left block. Thus, only when the estimation horizon is matched do we find that the option prices are clearly more informative than the historical daily returns. Both the model-free volatility expectation and the ATM implied volatility rank as the best more often in the right block than in the left block. The ATM implied volatility has the best regression results for 52.4% of the sample firms, while the model-free volatility expectation performs the best for 32.2%.

We next consider the encompassing regressions with two explanatory variables. Table 6 documents that the bivariate regression models which include the historical volatility variable increase the average adjusted R^2 values slightly from the univariate levels for option specifications; from 0.290 to 0.319 for the ATM implied volatility and from 0.278 to 0.308 for the model-free (MF) volatility expectation. For 73 firms, the adjusted R^2 of the bivariate regression for the historical and MF variables is higher than that of the MF univariate regression, although the null hypothesis $\beta_{His} = 0$ is only rejected for 37 firms at the 5% level. Similarly, for 71 firms the adjusted R^2 of the regression with the historical and ATM variables is higher than that of the ATM univariate regression and $\beta_{His} = 0$ is rejected for 32 firms at the 5% level. Therefore, we do not reject the hypothesis that the historical volatility is redundant when forecasting future volatility for most firms, which may be a consequence of the informative option prices and/or the small number of forecasts that are evaluated.

The MF and ATM bivariate regressions have an average adjusted R^2 equal to 0.311, which is fractionally less than the average for the bivariate regressions involving the historical and the ATM volatilities. This is explained by the very high correlation between the model-free volatility expectation and the ATM implied volatility. For most firms, both the null hypotheses $\beta_{MF} = 0$ and $\beta_{ATM} = 0$ are not rejected at the 5% level, showing that we cannot conclude that one option measure subsumes all the information contained in the other.

Finally, the highest average adjusted R^2, equal to 0.336, occurs when all three volatility estimates define the explanatory variables in the regression model. Consequently, the evidence overall favours the conclusion that every volatility estimate contains some additional information beyond that provided by the other estimates. The mean values of β_{His}, β_{MF} and β_{ATM} are, respectively, 0.118, 0.197 and 0.483, suggesting that the ATM forecasts are the most informative.

6. Cross-sectional comparisons

6.1. Variables

None of the volatility estimation methods is consistently more accurate than the others for most of our sample firms. We now investigate eight firm-specific variables, seeking to identify variables which are associated with the best volatility estimation method. The variables and their definitions are summarised in Table 7.

To explain the variation across firms in the performance of volatility estimates from option prices relative to historical forecasts, we expect the firm's option trading liquidity to be the most important factor. The IvyDB database provides the option trading volume, measured by option contracts, which is a proxy for liquidity. We also consider the firm's stock trading volume and its stock market capitalization, because firms with more liquid stock trading and with larger size tend to have more liquid option trading. We conjecture that firms with option prices more informative than historical volatilities have relatively high average values of option trading volume, stock trading volume and market capitalization.

Theoretically the model-free volatility expectation is superior to ATM implied volatility, as it is model-independent and contains information from a complete set of option prices. We attempt to explain the generally superior empirical performance of the ATM

Table 6
Summary statistics for regression models when the dependent variable is realized volatility.

		β_0	β_{His}	β_{MF}	β_{ATM}	R^2	*adj.* R^2	SSE
His	Mean	1.344	0.620			0.179	0.162	0.059
	Q_1	0.250	0.378			0.050	0.030	0.044
	Q_2	1.288	0.618			0.136	0.117	0.053
	Q_3	2.312	0.889			0.270	0.254	0.067
		(44.3)	(69.8)					
	S&P 100	1.254*	0.538*			0.242	0.226	0.064
MF	Mean	0.834		0.731		0.278	0.262	0.052
	Q_1	0.170		0.594		0.145	0.127	0.038
	Q_2	0.764		0.749		0.263	0.247	0.047
	Q_3	1.374		0.887		0.390	0.377	0.064
		(24.8)		(88.6)				
	S&P 100	0.092		0.852*		0.426	0.414	0.048
ATM	Mean	0.713			0.775	0.290	0.275	0.051
	Q_1	0.016			0.601	0.155	0.137	0.037
	Q_2	0.673			0.777	0.284	0.268	0.047
	Q_3	1.310			0.960	0.396	0.383	0.061
		(22.8)			(90.6)			
	S&P 100	0.179			0.852*	0.438	0.426	0.047
His + MF	Mean	0.607	0.182	0.613		0.308	0.278	0.049
	Q_1	−0.179	−0.016	0.372		0.175	0.140	0.036
	Q_2	0.501	0.187	0.645		0.290	0.259	0.046
	Q_3	1.145	0.418	0.812		0.424	0.399	0.062
		(12.1)	(24.8)	(67.1)				
	S&P 100	−0.062	−0.320	1.180*		0.447	0.423	0.048
His + ATM	Mean	0.601	0.127		0.680	0.319	0.289	0.049
	Q_1	−0.257	−0.074		0.477	0.180	0.144	0.036
	Q_2	0.412	0.154		0.692	0.307	0.277	0.045
	Q_3	1.125	0.389		0.873	0.425	0.401	0.058
		(14.8)	(21.5)		(73.2)			
	S&P 100	0.059	−0.358		1.218*	0.465	0.441	0.046
MF + ATM	Mean	0.714		0.252	0.519	0.311	0.281	0.049
	Q_1	−0.025		−0.254	0.092	0.170	0.133	0.036
	Q_2	0.631		0.214	0.576	0.302	0.271	0.046
	Q_3	1.399		0.676	1.053	0.425	0.400	0.060
		(23.5)		(11.4)	(23.5)			
	S&P 100	0.185		−0.034	0.886	0.438	0.414	0.048
His + MF + ATM	Mean	0.067	0.118	0.197	0.483	0.336	0.292	0.047
	Q_1	0.003	−0.086	−0.280	0.049	0.196	0.142	0.035
	Q_2	0.036	0.153	0.138	0.572	0.330	0.285	0.044
	Q_3	0.157	0.363	0.599	0.983	0.453	0.415	0.057
		(13.4)	(18.1)	(9.4)	(18.1)			
	S&P 100	0.031	−0.363	0.147	1.080	0.465	0.429	0.047

The most general estimated regression model for the logarithm of realized volatility is: $\ln(100\sigma_{RE,t,T}) = \beta_0 + \beta_{His}\ln(100\sigma_{His,t,T}) + \beta_{MF}\ln(100\sigma_{MF,t,T}) + \beta_{ATM}\ln(100\sigma_{ATM,t,T}) + \varepsilon_{t,T}$, where σ_{RE}, σ_{MF}, σ_{ATM} and σ_{His}, respectively, refer to the realized volatility calculated from the formula of Parkinson (1980), the model-free volatility expectation, the at-the-money option implied volatility and the historical forecast obtained from the GJR–GARCH model specified in Table 4. The regressions are estimated by OLS for each of the 149 firms in the sample, from which the cross-sectional mean, lower quartile, median and upper quartile are calculated for each model coefficient. The numbers in parentheses are the percentages of firms whose estimates are significantly different from zero at the 5% level. Inferences are made using standard errors that are robust against heteroscedasticity (White, 1980). The rows headed S&P 100 report the regression results for the index; the starred estimates are significantly different from zero at the 5% level. SSE is the sum of squared errors for the regression.

implied by using variables that proxy for model-free implementation problems. We consider the average number of market available strike prices, because more strike prices will enable a more accurate estimation of the implied volatility curve. According to the simulations in Jiang and Tian (2005, 2007), the estimation error of the model-free volatility expectation is higher firstly when the range of available strike prices is small and/or secondly when the distance between adjacent strike prices is large; we measure these two distance criteria using the moneyness scale.

The informational efficiency of the model-free volatility expectation relies on the liquidity of OTM options. Consequently, we also consider the trading volume of ATM options as a proportion of all option trading volume and conjecture that the proportion is higher for those firms for which the ATM implied volatility outperforms the model-free volatility expectation. With the same motivation, we also include the trading volume of intermediate delta options divided by all option trading volume, where intermediate call deltas are here defined as being between $0.25e^{-rT}$ and $0.75e^{-rT}$.

6.2. Results

The means and standard deviations of each variable across the 149 firms are presented in Panel A of Table 8.

In Panel B, we show summary statistics and test results for the 16 firms for which the historical volatility is superior to option prices for both 1-day-ahead and option-life forecasts. These firms define the group called "His > OP". Here OP refers to both the model-free volatility expectation and the ATM implied volatility, and the group "His > OP" consists of all firms that have both "His > MF" and "His > ATM". Based on similar criteria, there are 72 firms satisfying "OP > His", which is equivalent to both "MF > His" and "ATM > His". First, as expected, the averages of the three option liquidity proxies are lower for the group "His > OP" than for the group "OP > His". The standard two-sample t-test rejects the null hypothesis that the two groups provide observations from a common distribution, at low significance levels; the t-statistics are −6.30 (option volume), −4.41 (stock volume) and −4.18 (firm

Table 7
Definitions of eight firm-specific variables.

Variable	Definition
TV_OP	The natural logarithm of the firm's average option trading volume
TV_Stock	The natural logarithm of the firm's average stock trading volume
FirmSize	The natural logarithm of the firm's average size, measured in thousands of dollars, and calculated as the number of shares outstanding multiplied by the closing stock price
Moneyness Range	The maximum moneyness minus the minimum moneyness
Strike Prices	The number of available option strike prices
Average Strike Price Interval	The average interval between adjacent strike prices, measured in moneyness units
TV_ATM/TV_ALL	At-the-money option trading volume divided by total option trading volume
TV_IntermediateDelta/TV_ALL	The trading volume of intermediate delta options divided by total trading volume, where intermediate delta options have deltas within the interquartile range of feasible delta values

This table defines eight firm-specific candidate variables, which may be associated with the best volatility estimation method. The first three variables are plausible variables when comparing the relative advantages of historical and option-based methods, while the remaining five variables may help to explain when the model-free volatility expectation outperforms the at-the-money implied volatility.
All variables for each firm are calculated as time-series averages of daily measures from January 1996 to December 1999. The option-related variables are only calculated from the option contracts described in Section 3. Moneyness is defined as the option strike price divided by the matched forward price.

Table 8
Comparisons of the firm-specific variables for selected groups of firms.

	Firms	TV_OP	TV_Stock	FirmSize	Moneyness Range	Strike Prices	Average Strike Price Interval	TV_ATM/TV_ALL	TV_IntermediateDelta/TV_ALL
Panel A: all firms									
	149	6.18	13.92	22.60	0.39	5.44	0.10	0.42	0.62
		(1.42)	(1.07)	(1.65)	(0.11)	(1.54)	(0.03)	(0.06)	(0.07)
Panel B: OP versus His									
His > OP	16	5.18	13.24	21.81	0.37	4.59	0.11	0.43	0.60
		(0.90)	(1.03)	(1.09)	(0.11)	(0.65)	(0.03)	(0.06)	(0.09)
OP > His	72	6.88	14.46	23.19	0.40	5.96	0.09	0.43	0.64
		(1.29)	(0.85)	(1.57)	(0.12)	(1.77)	(0.03)	(0.06)	(0.04)
t-Statistic		−6.30	−4.41	−4.17	−0.80	−5.15	2.15	0.54	−1.76
p-Value		0.00	0.00	0.00	0.22	0.00	0.02	0.30	0.05
Panel C: MF versus ATM									
MF > ATM	34	6.04	13.85	22.18	0.41	5.17	0.11	0.43	0.63
		(1.43)	(1.04)	(1.50)	(0.11)	(1.22)	(0.03)	(0.06)	(0.06)
ATM > MF	61	6.35	14.05	22.86	0.39	5.51	0.10	0.43	0.63
		(1.32)	(0.97)	(1.63)	(0.12)	(1.48)	(0.03)	(0.06)	(0.07)
t-Statistic		−1.04	−0.92	−2.07	0.88	−1.20	1.49	0.11	0.12
p-Value		0.15	0.18	0.02	0.19	0.12	0.07	0.46	0.45

Averages and standard deviations (shown in parentheses) are tabulated for various firm-specific variables, for all firms in Panel A and for selected groups of firms in Panels B and C. MF, ATM and His, respectively, refer to the model-free volatility expectation, the ATM implied volatility and historical volatility methods. The symbol OP refers to both MF and ATM. Panel B shows results for those firms having both option-based methods performing either worse or better than His, for *both* 1-day-ahead and option-life forecasts. Panel C covers those firms with MF performing either better or worse than ATM for *both* 1-day-ahead and option-life forecasts. The *t*-statistics are for the standard two-sample test of the null hypothesis of equal population means, while the *p*-values are for one-tail tests.

size). Second, the same null hypothesis is also decisively rejected using the average number of traded strike prices, which can also be considered an option liquidity proxy, as the *t*-statistic is −5.15.

Panel C compares the 34 firms satisfying "MF > ATM" for both 1-day-ahead and option-life forecasts with the 61 firms satisfying "ATM > MF" for both horizons. These group sizes emphasize that the ATM implied volatility performs better than the model-free volatility expectation for more firms across both forecast horizons. The *t*-statistic for firm size is significant at the 5% level for one-sided tests, and it is significant for the strike price interval at the 10% level; the firms in the group "MF > ATM" have a lower average size and a wider average strike price interval which reflects a wider strike price range. Thus we find some evidence that the theoretical advantages of the model-free volatility expectation are more likely to be detected when the traded strike prices are more dispersed. We do not find evidence that the relative trading volumes of ATM or intermediate delta options are associated with the relative performance of the ATM implied volatility and the model-free volatility expectation.[8]

[8] Negative conclusions are also obtained for the average realized volatility, the average risk-neutral skewness and estimates of systematic risk.

The cross-sectional analysis shows that the relative performance of option prices and historical methods for forecasting the volatility of individual stocks is strongly associated with the liquidity of option trading. In contrast, the comparisons between the model-free volatility expectation and the ATM implied volatility for individual stocks only provide weak evidence that the dispersion of the strike prices is a relevant explanatory factor.

There are three possible explanations of the relatively unsatisfactory performance of the model-free volatility expectation. Firstly, our options data might contain measurement errors from the bid-ask spread and nonsynchronous trading of options and stocks, which might be more severe for OTM options. The model-free volatility expectation, calculated as a function of option prices across all strikes, might then contain more noise in total than the ATM implied volatility. Secondly, the trading of individual stock options, especially OTM, might have been insufficient to reflect the theoretical advantages of the model-free volatility expectation; the most liquid options in our study have option trading volumes far below the levels observed for stock indices. Thirdly, as noted by the referee, the model-free volatility expectation gives more weight to OTM puts than to OTM calls and it is the puts which reflect default risk; however, our firms all survive and hence the observed forecast targets do not include any exceptional prices when firms fail.

7. Conclusions

This paper provides the first comparison of the information content of three volatility measures for a large set of US stocks, namely a forecast obtained from historical prices, the at-the-money implied volatility and the newly-developed model-free volatility expectation. We find that each of the three volatility estimates contains some, but not all, relevant information about the future variation of the underlying asset returns.

The consensus from studies about the informational efficiency of US stock index options is that these option prices are significantly more informative than historical daily returns when predicting index volatility. Our analysis of 149 firms shows that a different conclusion applies to options for individual firms. For 1-day-ahead estimation, more than a third of our firms do not have volatility estimates, extracted from option prices, that are more accurate than those provided by a simple ARCH model estimated from daily stock returns. When the prediction horizon extends until the expiry date of the options, the historical volatility becomes less informative than either the ATM implied volatility or the model-free volatility expectation for 126 of the 149 firms. Our results also show that both volatility estimates from option prices are more likely to outperform historical returns when the firm has higher option trading activity, as measured either by option volume or the number of strikes traded.

The recent interest shown in the model-free volatility expectation is explained by the few assumptions required to derive the model-free formula. The formula has the limitation, however, that it is an integral function of a complete set of option prices. As few strikes are traded for individual firms, we use a quadratic function of a delta quantity to estimate implied volatility curves from which we can derive the required interpolated option prices. Although Jiang and Tian (2005) find the model-free volatility expectation is the most accurate predictor of realized volatility for the S&P 500 index, the model-free predictor only outperforms both the ATM implied volatility and the historical volatility for about one-third of our sample firms. In contrast, the ATM implied volatility is the method that most often performs the best. When the ATM implied volatility outperforms the model-free volatility for a firm, we find that the relative trading volume of ATM options is not significantly higher than otherwise.

Our paper shows that model-free methods do not realize their theoretical potential for some assets. The relatively unsatisfactory performance of the model-free volatility expectation for individual firms is attributed to the illiquidity of their OTM options, which reduces the aggregate information content of the available option prices. Compared with an index, at the firm level fewer strikes are traded, option bid-ask spreads are wider and recorded option prices are more likely to be nonsynchronous with the price of the underlying asset.

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References

Andersen, T.G., Bollerslev, T., Diebold, F.X., Ebens, H., 2001. The distribution of realized stock volatility. Journal of Financial Economics 61, 43–76.

Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica 71, 579–625.

Bali, T.G., Weinbaum, D., 2007. A conditional extreme value volatility estimator based on high-frequency returns. Journal of Economic Dynamics and Control 31, 361–397.

Becker, R., Clements, A.E., McClelland, A., 2009. The jump component of S&P 500 volatility and the VIX index. Journal of Banking and Finance 33, 1033–1038.

Becker, R., Clements, A.E., White, S.I., 2007. Does implied volatility provide any information beyond that captured in model-based volatility forecasts? Journal of Banking and Finance 31, 2535–2549.

Blair, B.J., Poon, S.-H., Taylor, S.J., 2001. Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns. Journal of Econometrics 105, 5–26.

Bliss, R.R., Panigirtzoglou, N., 2002. Testing the stability of implied probability density functions. Journal of Banking and Finance 26, 381–422.

Bliss, R.R., Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. Journal of Finance 59, 407–446.

Bollerslev, T., Wooldridge, J.M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Reviews 11, 143–179.

Britten-Jones, M., Neuberger, A., 2000. Option prices, implied price processes, and stochastic volatility. Journal of Finance 55, 839–866.

Canina, L., Figlewski, S., 1993. The informational content of implied volatility. Review of Financial Studies 6, 659–681.

Carr, P., Lee, R., 2008. Robust Replication of Volatility Derivatives. Working Paper, New York University.

Carr, P., Madan, D., 1998. Towards a theory of volatility trading. In: Jarrow, R.A. (Ed.), Volatility. Risk Books, London, pp. 417–427.

Carr, P., Wu, L., 2006. A tale of two indices. Journal of Derivatives 13 (3), 13–29.

Carr, P., Wu, L., 2009. Variance risk premiums. Review of Financial Studies 22, 1311–1341.

Cheung, Y.-W., Ng, L.K., 1992. Stock price dynamics and firm size: an empirical investigation. Journal of Finance 47, 1985–1997.

Christensen, B.J., Prabhala, N.R., 1998. The relation between implied and realized volatility. Journal of Financial Economics 50, 125–150.

Cremers, M., Driessen, J., Maenhout, P., Weinbaum, D., 2008. Individual stock-option prices and credit spreads. Journal of Banking and Finance 32, 2401–2411.

Day, T.E., Lewis, C.M., 1992. Stock market volatility and the information content of stock index options. Journal of Econometrics 52, 267–287.

Demeterfi, K., Derman, E., Kamal, M., Zou, J., 1999. A guide to volatility and variance swaps. Journal of Derivatives 6, 9–32.

Duffee, G.R., 1995. Stock returns and volatility a firm-level analysis. Journal of Financial Economics 37, 399–420.

Ederington, L.H., Guan, W., 2002. Measuring implied volatility: is an average better? Which average? Journal of Futures Markets 22, 811–837.

Ederington, L.H., Guan, W., 2005. Forecasting volatility. Journal of Futures Markets 25, 465–490.

Fleming, J., 1998. The quality of market volatility forecasts implied by S&P 100 index option prices. Journal of Empirical Finance 5, 317–345.

Garman, M.B., Klass, M.J., 1980. On the estimation of security price volatilities from historical data. Journal of Business 53, 67–78.

Glosten, L.R., Jagannathan, R., Runkle, D.E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48, 1779–1801.

Giot, P., Laurent, S., 2007. The information content of implied volatility in light of the jump/continuous decomposition of realized volatility. Journal of Futures Markets 27, 337–359.

Jiang, G.J., Tian, Y.S., 2005. The model-free implied volatility and its information content. Review of Financial Studies 18, 1305–1342.

Jiang, G.J., Tian, Y.S., 2007. Extracting model-free volatility from option prices: an examination of the VIX Index. Journal of Derivatives 14 (3), 35–60.

Konstantinidi, E., Skiadopoulos, G., Tzagkaraki, E., 2008. Can the evolution of implied volatility be forecasted? Journal of Banking and Finance 32, 2401–2411.

Lamoureux, C.G., Lastrapes, W.D., 1993. Forecasting stock-return variance: toward an understanding of stochastic implied volatilities. Review of Financial Studies 6, 293–326.

Liu, X., Shackleton, M.B., Taylor, S.J., Xu, X., 2007. Closed-form transformations from risk-neutral to real-world distributions. Journal of Banking and Finance 31, 1501–1520.

Lynch, D.P., Panigirtzoglou, N., 2004. Option Implied and Realized Measures of Variance. Working Paper, Bank of England.

Malz, A.M., 1997a. Estimating the probability distribution of the future exchange rate from option prices. Journal of Derivatives 5 (2), 18–36.

Malz, A.M., 1997b. Option-implied Probability Distributions and Currency Excess Returns. Staff Reports 32, Federal Reserve Bank of New York.

Martin, G.M., Reidy, A., Wright, J., 2009. Does the option market produce superior forecasts of noise-corrected volatility measures? Journal of Applied Econometrics 24, 77–104.

Parkinson, M., 1980. The extreme value method for estimating the variance of the rate of return. Journal of Business 53, 61–65.

Poon, S.-H., Granger, C.W.J., 2003. Forecasting volatility in financial markets: a review. Journal of Economic Literature 41, 478–539.

Taylor, S.J., 2005. Asset Price Dynamics, Volatility, and Prediction. Princeton University Press, Princeton.

White, H., 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica 48, 817–838.

Yu, W.W., Lui, E.C.K., Wang, J.W., 2009. The predictive power of the implied volatility of options traded OTC and on exchanges. Journal of Banking and Finance 34, 1–11.

Xing, Y., Zhang, X., Zhao, R., forthcoming. What does individual option volatility smirk tell us about future equity returns? Journal of Financial and Quantitative Analysis.