Regime Changes and Financial Markets

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Abstract

Regime-switching models can match the tendency of financial markets to often change their behavior abruptly and the phenomenon that the new behavior of financial variables often persists for several periods after such a change. Although the regimes captured by regime-switching models are identified by an econometric procedure, they often correspond to different periods in regulation, policy, and other secular changes. In empirical estimates, the means, volatilities, autocorrelations, and cross-covariances of asset returns often differ across regimes in a manner that allows regime-switching models to capture the stylized behavior of many financial series including fat tails, heteroskedasticity, skewness, and time-varying correlations. In equilibrium models, regimes in fundamental processes, such as consumption or dividend growth, strongly affect the dynamic properties of equilibrium asset prices and can induce nonlinear risk-return trade-offs. Regime switches also lead to potentially large consequences for investors' optimal portfolio choice.

1. INTRODUCTION

Financial markets often change their behavior abruptly. Although some changes may be transitory (jumps), often the changed behavior of asset prices persists for many periods. For example, the mean, volatility, and correlation patterns in stock returns changed dramatically at the start of, and persisted through, the global financial crisis of 2008–2009. Similar regime changes, some of which can be recurring (recessions versus expansions) and some of which can be unique (breaks), are prevalent in fixed income, equities, and foreign exchange markets, and in the behavior of many macro variables. Regime-switching models can capture these sudden changes of behavior and the phenomenon that the new dynamics of prices and fundamentals persist for several periods after a change.

There are several reasons why regime-switching models have become popular in financial modeling. First, the idea of regime changes is natural and intuitive. Indeed, the original application of regime switching in Hamilton's (1989) seminal work was to business cycle recessions and expansions, and the regimes naturally captured cycles of economic activity around a long-term trend. Hamilton's regimes were closely tied to the notion of recession indicators as identified ex post by the NBER business cycle dating committee.

When applied to financial series, regimes identified by econometric methods often correspond to different periods in regulation, policy, and other secular changes. For example, interest rate behavior markedly changed from 1979 through 1982, during which the Federal Reserve changed its operating procedure to targeting monetary aggregates. Other regimes identified in interest rates correspond to the tenure of different Federal Reserve chairs (see, for example, Sims & Zha 2006). In equities, different regimes correspond to periods of high and low volatility and long bull and bear market periods (Pagan & Sossounov 2003). Thus, regime-switching models can match narrative stories of changing fundamentals that sometimes can only be interpreted ex post, but in a way that can be used for ex ante real-time forecasting, optimal portfolio choice, and other economic applications.

Second, regime-switching models parsimoniously capture stylized behavior of many financial return series including fat tails, persistently occurring periods of turbulence followed by periods of low volatility (ARCH effects), skewness, and time-varying correlations. By appropriately mixing conditional normal (or other types of) distributions, large amounts of nonlinear effects can be generated. Even when the true model is unknown, regime-switching models can provide a good approximation for more complicated processes driving security returns. Regime-switching models also nest as a special case jump models, given that a jump is a regime that is immediately exited next period and, when the number of regimes is large, the dynamics of a regime-switching model approximates the behavior of time-varying parameter models where the continuous state space of the parameter is appropriately discretized.

Finally, another attractive feature of regime-switching models is that they are able to capture nonlinear stylized dynamics of asset returns in a framework based on linear specifications, or conditionally normal or log-normal distributions, within a regime. This makes asset pricing under regime switching analytically tractable. In particular, regimes introduced into linear asset pricing models can often be solved in closed form because conditional on the underlying regime, normality (or log-normality) is recovered. This makes incorporating regime dynamics in affine models straightforward.

The notion of regimes is closely linked to the familiar concept of good and bad states or states with low versus high risk, but surprising and somewhat counterintuitive results can be obtained from equilibrium asset pricing models with regime changes. Conventional linear asset pricing models imply a positive and monotonic risk-return relation (e.g., Merton 1973). In contrast, changes between discrete regimes with different consumption growth rates can lead to increasing, decreasing, flat, or nonmonotonic risk-return relations as shown by, e.g., Backus & Gregory (1993), Whitelaw (2000), Ang & Liu (2007), and Rossi & Timmermann (2011). Intuitively, nonmonotonic patterns arise because "good" and "bad" regimes, characterized by high and low growth in fundamentals and asset price levels, respectively, may also be associated with higher uncertainty about future prospects than more stable, "normal" regimes, which are likely to last longer. The possibility of switching across regimes, even if it occurs relatively rarely, induces an important additional source of uncertainty that investors want to hedge against. Inverse risk-return trade-offs can result in some regimes because the market portfolio hedges against adverse future consumption shocks even though the level of uncertainty (return volatility) is high in these regimes. Further nonlinearities can be generated as a result of investors learning about unobserved regimes.

We begin our review in Section 2 by describing the structure of basic regime-switching models. We discuss how these models can match stylized properties of asset returns in data and show how the presence of regimes economically affects equilibrium risk-return trade-offs. In Section 3 we show how these insights have been used by the now extensive regime-switching literature to model interest rates, equity returns, and exchange rates, and for asset allocation. We conclude in Section 4 by describing some unresolved future research areas for regime-switching model applications.[1]

2. CANONICAL REGIME-SWITCHING MODELS

2.1. Modeling Regimes

Consider a variable y_t, which depends on its own past history, y_{t-1}, random shocks, ε_t, and some regime process, s_t. Regimes are generally modeled through a discrete variable, $s_t \in \{0, 1, \ldots, k\}$, tracking the particular regime inhabited by the process at a given point in time. Although regimes could affect the entire distribution, they are often limited to affect the intercept, μ_{s_t}, autocorrelation, ϕ_{s_t}, and volatility, σ_{s_t}, of the process:[2]

$$y_t = \mu_{s_t} + \phi_{s_t} y_{t-1} + \sigma_{s_t} \varepsilon_t, \quad \varepsilon_t \sim iid(0, 1). \tag{1}$$

To complete the model, the process governing the dynamics of the underlying regime, s_t, needs to be specified. It is common to assume that s_t follows a homogenous first-order Markov chain, $\prod_{[i,j]} = \Pr(s_t = j \,|\, s_{t-1} = i) = p_{ij}$. For example, in the common case with two regimes,

$$\Pr(s_t = 0 \,|\, s_{t-1} = 0) = p_{00} \text{ and } \Pr(s_t = 1 \,|\, s_{t-1} = 1) = p_{11}. \tag{2}$$

More generally, regime transitions could be time-varying and depend on the duration of time spent in the regime (Durland & McCurdy 1994) or on other state variables (Diebold,

[1] Guidolin (2011) also provides a comprehensive review of Markov-switching models and their applications in finance.

[2] More broadly, if other conditioning information, z_{t-1}, affects the mean or volatility, the regime-switching process takes the form $y_t = \mu_{s_t}(y_{t-1}, z_{t-1}) + \sigma_{s_t}(y_{t-1}, z_{t-1})\varepsilon_t, \varepsilon_t \sim iid(0, 1)$.

Lee & Weinbach, 1994; Filardo 1994), in which case $p_{ij}(t) = \Phi(z_t)$, where z_t is conditioning information such as an interest rate spread or a leading economic indicator, and Φ could be a logit or probit model.

2.2. Does History Repeat?

A key issue in regime-switching models is whether the same regimes repeat over time, as in the case of repeated recession and expansion periods, or if new regimes always differ from previous ones. If history repeats and the underlying regimes do not change, all regimes will recur at some time: *Plus ça change, plus c'est la même chose*. With only two regimes this will happen if $p_{ii} < 1$, $i = 0, 1$. Models with recurring regimes have been used to characterize bull and bear markets, calm versus turbulent markets, and recession and expansion periods.

An alternative to the assumption of recurring regimes is the change point process considered by Chib (1998) and studied in the context of dynamics in stock returns by Pastor & Stambaugh (2001) and Pettenuzzo & Timmermann (2010). In this model, the set of regimes expands over time, each regime is unique, and previous regimes are not visited again:

$$\prod = \begin{pmatrix} p_{00} & 1-p_{00} & 0 & 0 \\ 0 & p_{11} & 1-p_{11} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & p_{kk} \end{pmatrix}. \tag{3}$$

This type of model is likely to be a good representation of regime shifts related to technological change and certain types of legislative, or political, changes that are irreversible or unlikely to repeat. Of course, a combination of recurrent regimes and new regimes is possible.

2.3. Estimation Techniques

Different econometric methods can be used to estimate regime-switching models. Maximum likelihood and EM algorithms are outlined by Hamilton (1988, 1990). The maximum likelihood algorithm involves a Bayesian updating procedure, which infers the probability of being in a regime given all available information up until that time, $\Pr(s_t \mid I_t)$, where I_t is the information set at time t. An alternative to maximum likelihood estimation is Gibbs sampling, which was developed for regime-switching models by Albert & Chib (1993) and Kim & Nelson (1999).

An important issue in estimating regime-switching models is specifying the number of regimes. This is often difficult to determine from data, and as far as possible the choice should be based on economic arguments. Such decisions can be difficult given that the regimes themselves are often thought of as approximations to underlying states that are unobserved. It is not uncommon to simply fix the number of regimes at some value, typically two, rather than basing the decision on econometric tests. The reason is that tests for the number of regimes are typically difficult to implement because they do not follow standard distributions. To see this, consider the simple two-regime model in Equation 1. Under the null of a single regime, the parameters of the other regime are not identified, so there are unidentified nuisance parameters. This means that conventional likelihood ratio tests are not asymptotically χ^2 distributed. Davies (1977), Hansen (1992), Garcia (1998), and Cho & White (2007) further discuss this issue. An alternative is to use residual tests such as Hamilton (1996) does.

Having presented a canonical regime-switching model, we now discuss how this model matches many properties of asset returns, in particular skewness and fat tails, downside risk properties, and time-varying correlations.

2.4. Statistical Properties

To help understand the properties of data generated by regime switching, we next characterize the unconditional moments of such processes. We also derive the autocorrelation of the level and squared value of variables that are affected by regime switching in their mean and variance.

2.4.1. Skewness and fat tails. An attractive feature of regime-switching models is that they capture central statistical features of asset returns. To illustrate this, consider a simple two-regime-switching model,

$$y_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t, \quad \varepsilon_t \sim iid\ N(0,1), \tag{4}$$

where the (unconditional) probability that $s_t = 0$ is π_0 and $s_t = 1$ with probability $1 - \pi_0$. This is a special case of a regime-switching model (Equation 1) with no autoregressive terms.

Figure 1 plots the probability density functions (pdfs) corresponding to this mixture of two normals for ($\mu_0 = 1, \sigma_0 = 1$), ($\mu_1 = -2, \sigma_1 = 2$), and $\pi_0 = 0.8$. The mixture of the two

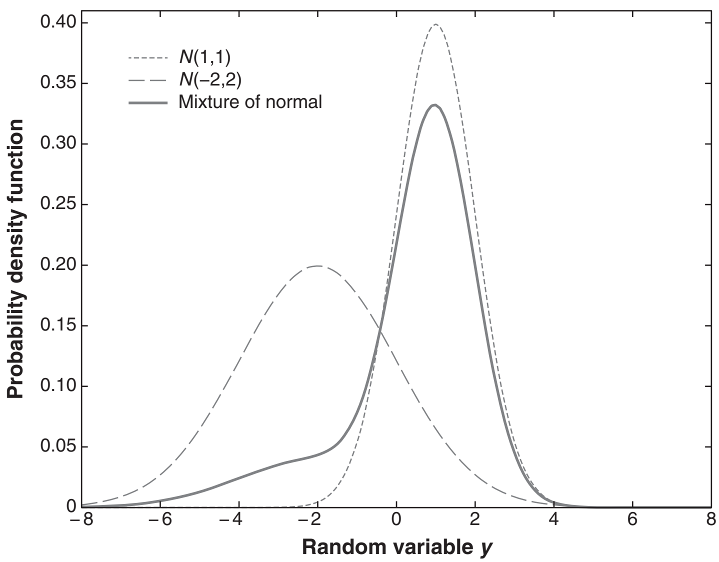

Figure 1

Mixture of normals. The figure plots the probability density functions (pdfs) of an $N(1, 1^2)$ distribution in the light gray dotted line, an $N(-2, 2^2)$ distribution in the light gray dashed line, and a simple mixture of the two distributions that draws from $N(1, 1^2)$ with probability $P = 0.8$ and $N(-2, 2^2)$ with probability $1 - P$ in the heavy dark gray solid line.

normals produces pronounced negative skewness. Timmermann (2000) derives the moments of a general regime-switching process with constant transition probabilities. As a special case, it can be shown that the first four central moments of the process in Equation 4 are given by

$$\begin{aligned} E[y_t] &= \pi_0\mu_0 + (1-\pi_0)\mu_1, \\ Var(y_t) &= \pi_0(1-\pi_0)(\mu_0-\mu_1)^2 + \pi_0\sigma_0^2 + (1-\pi_0)\sigma_1^2, \\ skew(y_t) &= \pi_0(1-\pi_0)(\mu_0-\mu_1)\left[(1-2\pi_0)(\mu_0-\mu_1)^2 + 3\left(\sigma_0^2-\sigma_1^2\right)\right], \\ kurt(y_t) &= \pi_0(1-\pi_0)(\mu_0-\mu_1)^2\left[\left((1-\pi_0)^3+\pi_0^3\right)(\mu_0-\mu_1)^2 + 6\pi_0\sigma_1^2 + 6(1-\pi_0)\sigma_0^2\right] \\ &\quad + 3\pi_0\sigma_0^4 + 3(1-\pi_0)\sigma_1^4. \end{aligned} \tag{5}$$

Differences in means across regimes, $\mu_0 - \mu_1$, enter the higher moments such as variance, skew, and kurtosis. In particular, the variance is not simply the average of the variances across the two regimes: The difference in means also imparts an effect because the switch to a new regime contributes to volatility. Intuitively, the possibility of changing to a new regime with a different mean introduces an extra source of risk. Skew only arises in this model if the means differ across the two regimes $(\mu_0 \neq \mu_1)$. Richer expressions, with similar intuition, apply with regime-dependent autoregressive terms and a full transition probability matrix. For the case shown in **Figure 1**, the mean is -0.50; the standard deviation is 2.18 (bigger than the largest individual standard deviation); and the coefficient of skew is -0.65 whereas the coefficient of kurtosis in this case at 2.85 falls below that of the normal distribution, indicating a platykurtic distribution.

Importantly, differences in means in addition to differences in variances can generate persistence in levels as well as squared values—akin to volatility persistence observed in many return series. For the simple model in Equation 4, we have

$$\begin{aligned} cov(y_t, y_{t-1}) &= \pi_0(1-\pi_0)(\mu_0-\mu_1)^2[p_{00}+p_{11}-1] \\ cov\left(y_t^2, y_{t-1}^2\right) &= \pi_0(1-\pi_0)\left(\mu_0^2-\mu_1^2+\sigma_0^2-\sigma_1^2\right)^2[p_{00}+p_{11}-1]. \end{aligned} \tag{6}$$

Again differences in means play an important role in generating autocorrelation in first moments—without such differences, the autocorrelation will be zero. In contrast, volatility persistence can be induced either by differences in means or by differences in variances across regimes. In both cases, the persistence tends to be greater the stronger the combined persistence of the regimes, as measured by $(p_{00} + p_{11} - 1)$. For the case shown in **Figure 1**, the first-order autocorrelation of the series in levels is 0.28, whereas the first-order autocorrelation for the squared series is 0.13.

2.4.2. Time-varying correlations. A stylized fact of asset returns is that correlations increase during market downturns as shown by Longin & Solnik (2001), Ang & Chen (2002), and others. Regime-switching models are able to match these patterns well through persistence in the probability of staying in a regime with low means, high volatilities, and high correlations. To illustrate this, **Figure 2** plots Longin & Solnik's (2001) exceedances correlations on US and UK equity returns following Ang & Bekaert (2002a).

An exceedance correlation is defined as follows. Consider observations $\{(y_1, y_2)\}$ drawn from a bivariate variable $Y = (y_1, y_2)$. Suppose the exceedance level θ is positive (negative).

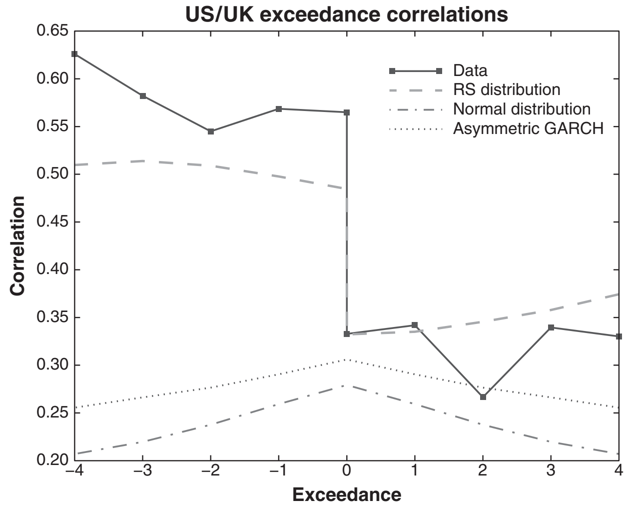

Figure 2

The figure shows Longin & Solnik's (2001) exceedance correlations of US and UK returns following Ang & Bekaert (2002a), which are correlations conditional on exceedances θ. Exceedances are given in percentages away from the empirical mean, so for an exceedance $\theta = +2$, we calculate the correlation conditional on observations greater than three times the US mean, and three times the mean of the UK. For $\theta = -2$, we calculate the correlation conditional on observations less than -1 times the US mean, and –1 times the mean of the UK. The implied exceedance correlations from a regime-switching (RS) model are shown as dashed lines, and the correlations from the data are represented by gray squares. The exceedance correlation for a normal distribution and an asymmetric GARCH model calibrated to the data are drawn as dotted-dashed and dotted lines, respectively.

Consider observations where values of y_1 and y_2 are greater (or less) than θ percent of their empirical means, i.e., the subset of observations $\{(y_1 y_2) \mid y_1 \geq (1+\theta)\bar{y}_1 \text{ and } y_2 \geq (1+\theta)\bar{y}_2\}$ for $\theta \geq 0$ and $\{(y_1 y_2) \mid y_1 \leq (1+\theta)\bar{y}_1 \text{ and } y_2 \leq (1+\theta)\bar{y}_2\}$ for $\theta \leq 0$, where $\bar{y}_j$ is the mean of y_j. The correlation of this subset of points is termed the exceedance correlation.

Figure 2 shows that the exceedance correlations of US/UK returns in the data exhibit a pronounced asymmetric pattern, with negative exceedance correlations higher than positive exceedance correlations. A bivariate regime-switching model of US/UK returns matches this pattern closely. A GARCH model with asymmetry cannot match this pattern. In the regime-switching model, one regime is characterized by low means but high correlations and volatility. This bad regime is persistent, so a draw from this regime makes a draw next period from the same regime more likely. Ang & Chen (2002) show that a model that combines normally distributed returns with transitory negative jumps also fails to reproduce the Longin-Solnik figure.

2.5. Asset Pricing with Regimes

The regime-switching model is not simply an empirical model that can closely match stylized statistical properties of financial returns. When regimes are embedded in an equilibrium specification, they generate realistic and interesting dynamics in risk-return relations. Indeed, the ability to capture various elements of higher moment dynamics, particularly nonlinear time-series patterns, is highlighted when regimes are considered in equilibrium.

To illustrate this, we start with the conventional asset pricing model based on a representative agent with utility, U, over consumption, C_t, and subjective discount factor, β. Let P_t be period-t price of equity, which pays out a dividend, D_t. Following Lucas (1978),

$$P_t U'(C_t) = \beta E_t[U'(C_{t+1})(P_{t+1} + D_{t+1})], \tag{7}$$

where E_t is the conditional expectation. The quantity $M_{t+1} = \beta U'(C_{t+1})/U'(C_t)$ is called the stochastic discount factor. We assume power utility, $U(C) = C^{1+\gamma}/(1+\gamma)$, where $\gamma \neq -1$. We further assume that consumption is equal to dividends each period, $C_t = D_t$. Thus, the Euler equation (Equation 7) becomes

$$P_t D_t^{\gamma} = \beta E_t\left[D_{t+1}^{\gamma}(P_{t+1} + D_{t+1})\right]. \tag{8}$$

We consider a generalization of Cecchetti, Lam & Mark (1990) where the dividend process is subject to switches in both the mean and volatility:

$$D_{t+1} = D_t \exp(\alpha_0 + \alpha_1 s_{t+1} + (\sigma_0 + \sigma_1 s_{t+1})\varepsilon_{t+1}), \quad \varepsilon_t \sim iid\ N(0,1), \tag{9}$$

where $s_t = \{0, 1\}$ follows the two-regime process in Equation 2 and is independent of all current, future, and past values of ε_t. This type of model with regime switching in either consumption or dividend growth appears to be strongly supported by empirical evidence (see, e.g., Cecchetti, Lam & Mark 1990; Whitelaw 2000; Bekaert & Liu 2004; and Lettau, Ludvigson & Wachter 2008).

Investors are assumed to know s_t at time t (but not s_{t+1}) and so set prices conditional on the prevailing regime. Let $\pi_{t0} = 1$ if $s_t = 0$; otherwise $s_t = 1$, so that π_{t0} is an indicator tracking the current regime. Using the transition probabilities in Equation 2, the conditionally expected dividend becomes the current dividend times the weighted average of dividend growth in the two regimes:

$$\begin{aligned} E_t[D_{t+1} \mid s_t] &= \sum_{s_{t+1}} E[D_{t+1} \mid s_{t+1}] \Pr(s_{t+1} \mid s_t) \\ &= D_t \exp\left(\alpha_0 + \sigma_0^2/2\right)\left[\pi_{t0} p_{00} + \left(1 - \pi_{t0}\right)\left(1 - p_{11}\right)\right] \\ &\quad + D_t \exp\left((\alpha_0 + \alpha_1) + (\sigma_0 + \sigma_1)^2/2\right)\left[\pi_{t0}(1 - p_{00}) + (1 - \pi_{t0}) p_{11}\right]. \end{aligned} \tag{10}$$

Even though future dividend growth is not log-normally distributed, by conditioning on the future regime and weighting appropriately by the regime transition probabilities, a closed-form expression for the expected future dividend is obtained.

Following Cecchetti, Lam & Mark (1990), we conjecture that the solution for the asset price takes the form

$$P_t = \rho(s_t) D_t, \quad s_t = \{0, 1\}. \tag{11}$$

The price-dividend ratio is constant within each regime—although it depends in a highly nonlinear way on the parameters of the consumption/dividend process and investor preferences—and takes only a finite number of values equivalent to the number of different regimes:

$$\begin{pmatrix} \rho(0) \\ \rho(1) \end{pmatrix} = \frac{\tilde{\beta}_0}{\Delta} \begin{pmatrix} 1 - \tilde{\beta}_0 \tilde{\alpha}_1 p_{11} & \tilde{\beta}_0 \tilde{\alpha}_1 (1 - p_{00}) \\ \tilde{\beta}_0 (1 - p_{11}) & 1 - \tilde{\beta}_0 p_{00} \end{pmatrix} \begin{pmatrix} p_{00} + \tilde{\alpha}_1 (1 - p_{00}) \\ (1 - p_{11}) + \tilde{\alpha}_1 p_{11} \end{pmatrix}, \tag{12}$$

where

$$\tilde{\beta}_0 = \beta \exp\Big((1+\gamma)\alpha_0 + (1+\gamma)^2\sigma_0^2/2\Big)$$
$$\tilde{\alpha}_1 = \exp\Big((1+\gamma)\alpha_1 + (1+\gamma)^2\big[\sigma_0\sigma_1 + \sigma_1^2/2\big]\Big)$$
$$\Delta = \big(1-\tilde{\beta}_0 p_{00}\big)\big(1-\tilde{\beta}_0\tilde{\alpha}_1 p_{11}\big) - \tilde{\beta}_0^2\tilde{\alpha}_1(1-p_{11})(1-p_{00}).$$

To gain intuition for this result, consider the case with persistent high-growth and low-growth regimes. Starting from the high-growth regime, investors expect high future endowment growth. This lowers the relative price of future endowments, raises current savings and demand for the risky asset, and thereby increases the current stock price. Conversely, investors' desire to intertemporally smooth their consumption leads them to consume more today, sell off their risky asset holdings, and thus reduces the current stock price. Which effect dominates depends on the degree of concavity of the utility function. If $\gamma = -1$ (log utility), the two effects cancel out and the price-dividend ratio is independent of the underlying regime. If $\gamma > -1$, the intertemporal relative price effect dominates and the price-dividend ratio is highest in the high-growth regime, whereas if $\gamma < -1$, the intertemporal consumption smoothing effect dominates, so the price-dividend ratio is highest in the low-growth regime.

With the solution to the asset price in place, (gross) returns are easily computed:

$$R_{t+1} \equiv \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{(\rho(s_{t+1})+1)}{\rho(s_t)} \times \exp(\alpha_0 + \alpha_1 s_{t+1} + (\sigma_0 + \sigma_1 s_{t+1})\varepsilon_{t+1}). \qquad (13)$$

This expression is consistent with the empirical evidence reviewed in the next section of strong regime dependence in asset returns. In this model, return variations arise from two sources. First, there is the usual variation due to uncertainty about future dividend growth, which in this case becomes compounded by the dependence of such growth on the unknown future regime. Second, there is variation over time in the price-dividend ratio. This second source is induced by the presence of regimes and arises because realized returns depend on both current and next-period regimes. If the parameters of the dividend process are sufficiently different across the two regimes and preferences are different from log-utility ($\gamma \neq -1$), price-dividend ratios can be highly regime dependent, and regime switches will have large effects on returns.

The presence of persistent regimes in consumption growth means that the conditional expected return depends on the current regime and hence becomes time-varying:

$$\begin{aligned}
E_t[R_{t+1}\,|\,s_t=0] &= p_{00}\frac{\rho(0)+1}{\rho(0)} \times \exp\big(\alpha_0 + \sigma_0^2/2\big) \\
&\quad + (1-p_{00})\frac{\rho(1)+1}{\rho(0)} \times \exp\Big(\alpha_0 + \alpha_1 + (\sigma_0+\sigma_1)^2/2\Big), \\
E_t[R_{t+1}\,|\,s_t=1] &= p_{11}\frac{\rho(1)+1}{\rho(1)} \times \exp\Big(\alpha_0 + \alpha_1 + (\sigma_0+\sigma_1)^2/2\Big) \\
&\quad + (1-p_{11})\frac{\rho(0)+1}{\rho(1)} \times \exp\big(\alpha_0 + \sigma_0^2/2\big).
\end{aligned} \qquad (14)$$

To better understand these expressions, consider again the model with persistent high-growth and low-growth regimes. Assuming that the price-dividend ratio is not too regime dependent, i.e., γ is close to minus one, expected returns will be higher when starting from the regime associated with the highest expected growth in dividends. This result can be overturned, however, when the dividend growth rate in one regime has a high mean but a low variance and $\gamma > -1$, so the price-dividend ratio is highest in the high-growth regime.

Similar expressions can be derived for the variance of returns conditional on the current regime. In fact, the conventional finding of a monotonic and linear relation between the equity premium and the conditional variance of returns need not hold in this model. For example, when γ is close to minus one, so the price-dividend ratio does not vary much across the two regimes, the mean return can be highest in the high-growth regime, while simultaneously the variance of returns may be highest in the low-growth regime, e.g., as a result of higher dividend growth volatility in this regime (i.e., $\mu_1 < 0$, $\sigma_1 > 0$).

This analysis shows that regimes in the consumption/dividend process endogenously generate differences across regimes in expected returns and return volatility. Combined with our earlier results in Equations 5 and 6, this simple model is further capable of generating skews, kurtosis, serial correlation, and volatility clustering in equilibrium returns.

The findings for this simple model—that introducing regimes in consumption growth can result in time-varying expected returns, skewness, regime-dependent volatility, and an inverted equilibrium risk-return relation—extend to more complex settings. Hung (1994); Garcia, Meddahi & Tedongap (2008); and Bonomo et al. (2011) generalize the analysis to a setting where investors have either Epstein-Zin recursive preferences or generalized disappointment aversion and where the dividend and consumption processes need not be identical. Calvet & Fisher (2007) use an equilibrium regime-switching model to introduce shocks that last from less than a day to several decades. They find that their model can generate a large volatility feedback and produces a trade-off between skewness and kurtosis in asset returns. In all these models, as well as in the solution in Equation 12, the price-dividend ratio can only take one of $k + 1$ different values, corresponding to the number of regimes. By introducing lagged consumption as in Whitelaw (2000) or other state-variable dependence of consumption, price-dividend ratios and expected returns can depend not only on regimes, but also vary continuously as a function of other variables.

2.6. Rare Events and Disasters

Several studies have argued that rare disasters can have a major impact on equilibrium asset prices. These disasters are usually modeled as a transitory jump where consumption levels drop substantially. These types of jumps are special cases of a more general regime-switching process where one regime has very high exit probabilities, or always exits next period, depending on the frequency at which data are modeled. In the two-regime transition probabilities of Equation 2, $s_t = 1$ would correspond to a disaster regime if p_{10} is close to one and the mean of consumption conditional on $s_t = 1$ is set to a very low number. Thus, a rare disaster event is a particularly bad and transitory regime.

In Rietz (1988), consumption follows a first-order Markov process with three regimes, two of which correspond to normal regimes and the third to a crash regime. The model has zero probability of staying in the crash regime and equal probabilities of moving to the normal regimes. In Barro (2006), there is a regular consumption process and a disaster process, which has a small probability of occurring every period. This is a special case of a simple switching model similar to Equation 4. Both studies find that the possibility of rare disasters can significantly raise the equity premium.

Even if rare disasters are not directly observed in data, they may affect price dynamics if agents take into account the probability of a rarely occurring regime, which has yet to be realized. Thus, regime-switching models are well suited to capturing peso problems, where prices reflect possible discrete changes in the future distribution of shocks. Evans (1996) provides a summary of regime-switching applications to various peso problems.

2.7. Learning About Unobserved Regimes

The simple equilibrium regime-switching process of the previous section assumed that agents know which regime applies at each time. This is a valid assumption in many cases, such as a credible policy change, e.g., a switch in currency or monetary policy regime. In other cases, regimes cannot be identified in real time. Then, the underlying regime is treated as a latent variable that is unobserved by economic researchers and possibly also by agents in the economy. This introduces a filtering problem as agents learn about regimes. The filtering algorithm uses Bayes' rule to update beliefs according to how likely new observations are drawn from different regimes, which are weighted by prior beliefs concerning the previous regimes. The higher the persistence of the regimes, the greater the weight on past data.

To briefly illustrate the effect of learning, assume that the regime process follows the two-regime model of Equation 2. Assume that the two distributions are $N(1, 1^2)$ and $N(-2, 2^2)$ for $s_t = 0$ and $s_t = 1$, respectively, which are the same distributions as those in **Figure 1**. We assume that $p_{00} = 0.95$ and $p_{11} = 0.80$. **Figure 3** shows a particular path drawn from this model. The true regime path is shown as the solid gray line, and the inferred (filtered) regime probability is graphed as the dashed gray line. The updated regime probabilities track the underlying regime quite accurately, but at times miss an important regime change (as in the case of the third regime change) and at other times

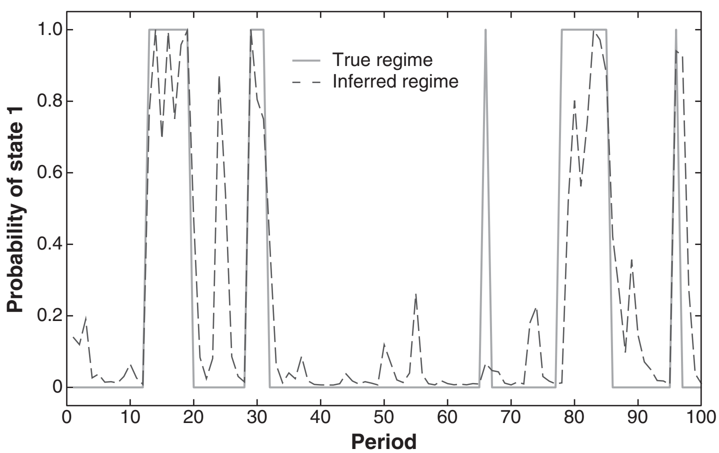

Figure 3

Mixture of normals. The figure plots a simulation from a regime-switching process with two states $s_t = 0$ and $s_t = 1$, with distributions $N(1, 1^2)$ and $N(-2, 2^2)$, respectively. The transition probabilities are $p_{00} = 0.95$ and $p_{11} = 0.80$. The true regime path is shown as the solid gray line, and the inferred (filtered) regime probability is graphed as the dashed darker gray line.

issue false alarms. This emphasizes the difficulty associated with real-time tracking of the underlying regime.

Learning of the type illustrated in **Figure 3** can have a significant effect on equilibrium asset prices as shown by Veronesi (2000), Timmermann (2001), Calvet & Fisher (2007), David & Veronesi (2009), and Cenesizoglu (2011). Because the underlying regime is rarely known with certainty and can undergo abrupt shifts, agents' beliefs and the dynamics of learning will affect equilibrium asset prices even if the underlying model parameters are known. Veronesi (1999) considers a model where the drift of the dividend process changes between two regimes. Agents update their beliefs about the underlying regime by observing past and current dividends, and the equilibrium stock price is a convex function of their posterior estimate of the regime probability. Veronesi shows that agents' attempts to hedge against their own uncertainty about the underlying regime can lead to patterns of over- and underreaction in how news is incorporated into asset prices. This model leads to higher asset price volatility during times with high uncertainty about the underlying regime, which typically occurs around recessions, thus matching the stylized finding that stock return volatility is countercyclical. Calvet & Fisher (2007) and Cenesizoglu (2011) further show how regimes can account for time-varying, state-dependent and asymmetric reaction of equilibrium stock prices to news.

Learning can also induce nonlinearities in risk-return trade-offs and volatility clustering. David & Veronesi (2009) consider a three-regime model with transitory good and bad regimes and a more persistent normal regime in fundamentals. Asset prices are dominated by directional information and so are lowest in the bad regime and highest in the good regime. Conversely, uncertainty is highest in the good and bad regimes, due to the low probability of remaining in these regimes, and lowest in the normal regime. This creates a V-shaped relation between return volatility and valuation measures, which in turn can give rise to an inverse V-shaped relation between volatility and expected returns. Timmermann (2001) shows how regime switching in the dividend growth process and agents' learning about the underlying regime can give rise to volatility clustering in asset returns, volatility being particularly high after a break in the dividend growth process at which point uncertainty about fundamentals is at its highest.

3. APPLICATIONS IN FINANCE

In this section we survey regime-switching applications in finance. We begin by characterizing the salient features of regime-switching estimations that are shared by almost all applications in the literature. To do this we present estimates of regime-switching models applied to equity returns, interest rates, and foreign exchange returns. We then discuss how the literature has added to these benchmark specifications. In each case, we highlight how the empirical estimates bring out both the statistical and economic properties summarized by the previous section.

3.1. Typical Estimations

We estimate the regime-switching model (Equation 1) on equity excess returns, which are total returns (dividend plus capital gain) on the S&P500 in excess of T-bills; interest rates, which are three-month T-bill yields; and foreign exchange excess returns (FX returns), which are returns from converting one USD into deutsch marks or euros, earning the

Table 1 Parameter estimates[a]

	Equity returns		Interest rates		FX returns	
	Estimate	Std. error	Estimate	Std. error	Estimate	Std. error
μ_0	0.3326	0.2354	−0.0170	0.1273	0.4592	0.8921
μ_1	0.8994	0.2614	0.0355	0.0094	0.0143	0.0766
ϕ_0	0.0633	0.0460	0.9888	0.0018	−0.0392	0.1718
ϕ_1	−0.0426	0.0928	1.0000	0.0000	0.3033	0.0447
σ_0	4.8867	0.3448	0.8127	0.0512	4.3035	0.6314
σ_1	2.4462	0.4637	0.1760	0.0104	2.4842	0.1293
P	0.9770	0.0196	0.8789	0.0370	0.8692	0.1011
Q	0.9512	0.0206	0.9499	0.0147	0.9805	0.0179
	p-value		p-value		p-value	
Test $\mu_0 = \mu_1$	0.1057		0.7483		0.6203	
Test $\phi_0 = \phi_1$	0.0406		0.0000		0.0001	
Test $\sigma_0 = \sigma_1$	0.0160		0.0000		0.0026	

[a]We report parameter estimates of the regime-switching model (Equation 1) applied to equity excess returns, which are total returns (dividend plus capital gain) on the S&P500 in excess of the T-bills; interest rates, which are three-month T-bill yields; and foreign exchange excess returns (FX returns), which are returns from converting one USD into deutsch marks or euros, earning the German T-bill return, and then converting back to USD, in excess of the US T-bill return. All returns are at the monthly frequency. Estimations are done by maximum likelihood. The sample period is 1953:01 to 2010:12 for equities and interest rates and 1975:01 to 2010:12 for foreign exchange returns.

German T-bill return, and then converting back to USD, in excess of the US T-bill return. That is, the foreign exchange return is the uncovered interest rate parity return. **Table 1** reports the parameter estimates and reveals some common properties of regime-switching estimations. We assume two regimes which are ordered so that $s_t = 0$ represents the high-volatility regime. The data frequency is monthly in all cases.

First, regimes are mostly identified by volatility. For these cases, conditional on there being two regimes, we cannot reject that the regime-dependent means are equal to each other, $\mu_0 = \mu_1$, but overwhelmingly reject that $\sigma_0 = \sigma_1$. Estimating means of returns is difficult even in a setting without regimes, as the unconditional mean is best pinned down by long time series (see Merton 1980). Thus, it is not surprising that the means conditional on each regime are harder to identify, as the number of observations of each regime must necessarily be less than the total number of observations in the sample.[3] Studies based on longer samples, multiple assets, and/or states have shown sufficient power to reject that mean returns are identical across regimes (e.g., Guidolin & Timmermann 2006).

[3]For the regime-switching model applied to three-month T-bill yields, $\mu_0 = -0.0170$, which is statistically insignificantly different from zero and also statistically insignificantly different from μ_1, even though T-bill yields have never been negative in the sample. The regime-switching model is estimated by maximum likelihood and matches unconditional moments closely. Intuitively, in regime $s_t = 0$, interest rates are more mean-reverting and volatility is high. The changes in interest rates, which enter the computation of conditional volatility and are mainly what the algorithm uses to identify the regimes, have a small negative mean during these periods. If μ_0 is imposed to be equal to zero or a small positive number, estimates of the other parameters are very similar.

Despite means being hard to pin down, there are some natural economic properties of the mean estimators. For excess equity returns, there is a high-volatility regime that has, on average, low returns. This regime naturally corresponds to bear markets. This pattern has been observed since the earliest studies of regime switches on equity returns, such as Turner, Startz & Nelson (1989) and Hamilton & Susmel (1994). It may at first seem puzzling that the high-volatility regime has the lowest expected return. However, as we have seen in Section 2.4, equilibrium asset pricing models are consistent with a negative risk-return trade-off in some regimes. Further, these are not ex ante expected returns and ex ante volatility estimates, given that they do not account for the probability of switching across regimes or learning in real time about the regime. High conditional return volatility can be induced by high levels of uncertainty about future states. Regime switches can also reduce the correlation between stock returns and the marginal rate of substitution between current and next period's consumption and may lower the risk premium if stocks act as a hedge against adverse consumption shocks.

Ignoring the mild evidence of serial correlation in equity returns, the model-implied coefficient of skewness and kurtosis from Equation 5 is −0.09 and 3.62, respectively. If the means are not different across regimes, the model-implied skewness and kurtosis would be zero and 3.63. Interestingly, the model-implied first-order autocorrelation in the squared return series induced by regime switching is very high at 0.77, demonstrating the ability of the model to generate persistence in squares without inducing serial correlation in levels (the first-order serial correlation induced by regimes in returns is essentially zero).

Second, for persistent processes such as interest rates, mean reversion coefficients often differ across regimes. In fact, even for returns that are close to i.i.d., such as equity returns and foreign exchange returns, we reject that $\phi_0 = \phi_1$. In **Table 1**, the three-month T-bill yield behaves like a random walk when volatility is low. Ang & Bekaert (2002c) interpret this as arising from the smoothing efforts of activist monetary policy during normal times. When the Federal Reserve intervenes aggressively, volatility of short rates increases but given that these periods of assertive interventions tend to be temporary, mean reversion in the high-volatility regime is lower. An attractive feature of regime-switching models is that although the interest rate is nonstationary in one regime, as long as a recurrent regime is sufficiently mean-reverting, the overall process remains stationary as shown by Holst et al. (1994) and Ang & Bekaert (1998).

Finally, the regimes are persistent with p_{00} and p_{11} both being close to one. This persistence of regimes plays an important role in generating volatility clustering, so periods of high volatility are followed by high volatility and periods of low volatility are followed by low volatility, as shown in Equation 6. In **Figure 4**, we plot the smoothed regime probabilities of the high-volatility regime, $\Pr(s_t = 0 \mid I_T)$, conditioning on the whole sample of the regime-switching models estimated on each asset return. For equity returns, panel *a* shows that the regimes are largely identified by volatility. For example, the period between 1997–2003 is classified as a high-volatility regime and encompasses both the bull market of the late 1990s and the subsequent crash of Internet stocks and the market decline in the early 2000s. For interest rates in panel *b*, the high-volatility regime includes the volatile interest rates in the early 1970s due to the OPEC oil shocks, the high and very volatile interest rates during the monetary targeting experiment over 1979–1982, and more recently the pronounced decrease in interest rates during the early 2000s and during the financial crisis post-2007. The high-volatility regime is least persistent for foreign exchange returns in panel *c*. There, the high-volatility regime closely corresponds

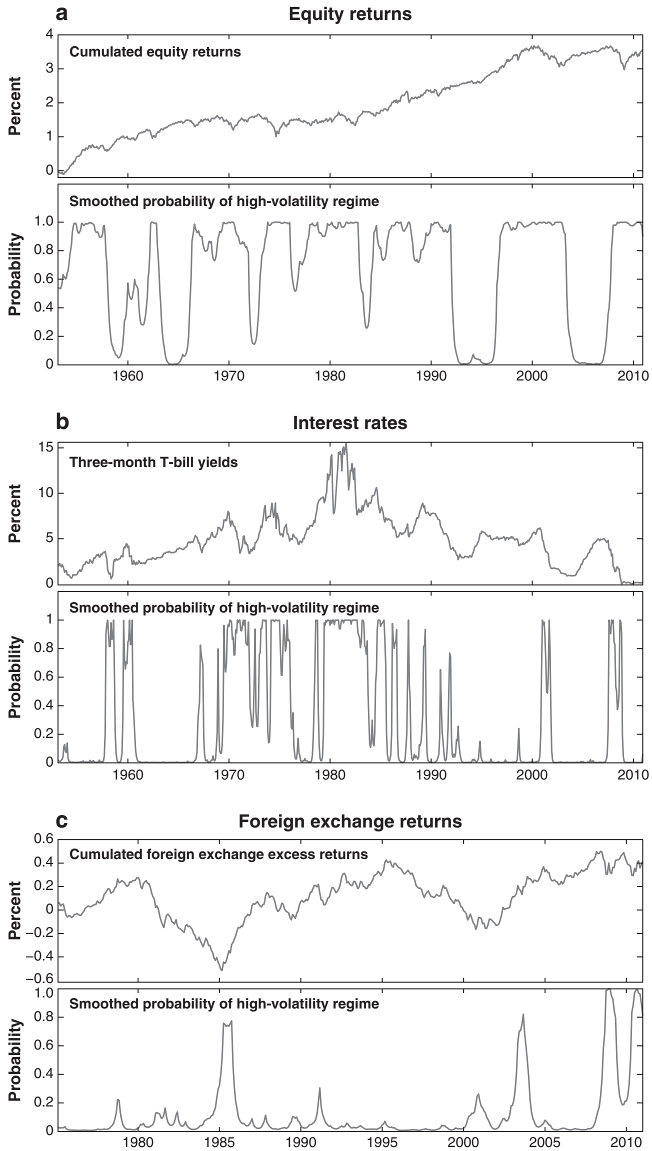

Figure 4

Smoothed probabilities. (*a*) Equity returns. (*b*) Interest rates. (*c*) Foreign exchange returns. In the bottom of each panel, we plot smoothed probabilities of being in regime $s_t = 0$, $p(s_t = 0 \mid I_T)$, conditional over the full sample computed following Hamilton (1990) and Kim (1994) from the regime-switching model (Equation 1) applied to equity excess returns, which are total returns (dividend plus capital gain) on the S&P500 in excess of the T-bills in panel *a*; interest rates, which are three-month T-bill yields in panel *b*; and foreign exchange excess returns (FX returns), which are returns from converting one USD into deutsch marks or euros, earning the German T-bill return, and then converting back to USD, in excess of the US T-bill return in panel *c*. The top of each panel shows cumulated sums of equity and FX returns in panels *a* and *c* and the three-month T-bill yield in panel *b*. All returns are at the monthly frequency. The sample period is 1953:01 to 2010:12 for equities and interest rates and 1975:01 to 2010:12 for foreign exchange returns.

to sudden depreciations of the USD (**Table 1** shows that $\mu_0 = 0.46\%$ per month compared to $\mu_1 = 0.01\%$ per month).

With the properties of a typical regime-switching estimation in mind, we now discuss significant, specific contributions of the literature, beginning with equity returns.

3.2. Equity Returns

The basic regime-switching specification (Equation 1) applied to equity returns presented in **Table 1** and **Figure 4** models only the equity return as a function of its own lagged value. Several studies find that aggregate stock market returns are predictable. The strength of this predictability, however, has varied considerably over time. The predictable power of many instruments used in the literature to predict excess aggregate equity returns, such as dividend yields, term spreads, and default spreads, declined or even disappeared over the 1990s as documented by Welch & Goyal (2008) and Ang & Bekaert (2007), among others, and formally tested by Pesaran & Timmermann (2002).

One response is that the strength of predictability—or even the unconditional return distribution (Maheu & McCurdy 2009)—changes over time and is subject to breaks and parameter instability (see, e.g., Schaller & van Norden 1997; Perez-Quiros & Timmermann 2001; Paye & Timmermann 2006; Rapach & Wohar 2006; Lettau & van Nieuwerburgh 2008; Johannes, Korteweg & Polson 2011).[4] This is the approach of Henkel, Martin & Nardari (2011) who capture the time-varying nature of return predictability in a regime-switching context. They use a regime-switching vector autoregression (VAR) with several predictors, including dividend yields, and interest rate variables along with stock returns. They find that predictability is very weak during business cycle expansions but is very strong during recessions. Thus, most predictability occurs during market downturns, and the regime-switching model captures this countercyclical predictability by exhibiting significant predictability only in the contraction regime.

Predictability, and its regime-dependent nature, is a form of time-varying first moments of returns. Regime-switching models have also been extensively applied to time-varying second moments. In fact, regime-switching models themselves generate heteroskedasticity (see Equation 6). Under the traditional ARCH and GARCH models of Engle (1982) and Bollerslev (1986), changes in volatility are sometimes found to be too gradual and unable to capture, despite the additions of asymmetries and other tweaks to the original GARCH formulations, sudden changes in volatilities. Hamilton & Susmel (1994) and Hamilton & Lin (1996) developed regime-switching versions of ARCH dynamics applied to equity returns that allowed volatilities to rapidly change to new regimes. A version of a regime-switching GARCH model was proposed by Gray (1996). There have been applications of regime switching to option volatilities and option valuation as well, such as Dueker (1997) and Bollen, Gray & Whaley (2000), among others.

There have been many versions of regime-switching models applied to vectors of asset returns. Ang & Bekaert (2002a) and Ang & Chen (2002) show that regime-switching models provide the best fit out of many alternative models to capture the tendency of many assets to exhibit higher correlations during down markets than in up markets. Ang & Chen (2002) interestingly find that there is little additional benefit to allowing regime-switching

[4]Another response to the lack of predictability is that predictability was never there (see e.g., Bossaerts & Hillion 1999 and Welch & Goyal 2008).

GARCH effects compared to the heteroskedasticity already present in a standard regime-switching model of normals.

It is reasonable to expect that if the market portfolio exhibits regime switches, then portfolios of stocks would also switch regimes, and the regimes and behavior within each regime of the portfolios should be related across portfolios. This is indeed the case. Perez-Quiros & Timmermann (2000), Gu (2005), and Guidolin & Timmermann (2008b), among others, fit regime-switching models to a small cross section of stock portfolios. On the one hand, these studies show that the magnitude of size and value premiums, among other things, varies across regimes in the same direction. On the other hand, the dynamics of certain stock portfolios react differently across regimes, such as small firms displaying the greatest differences in sensitivities to credit risk across recessions and expansions compared to large firms. Factor loadings of value and growth firms also differ significantly across regimes.

3.3. Interest Rates

Regimes in interest rates identified in empirical work by Hamilton (1988), Sola & Driffill (1994), Gray (1996), Bekaert, Hodrick & Marshall (2001), Ang & Bekaert (2002b,c), and others are often linked to underlying monetary policy regimes. Using conventional decompositions of the nominal interest rate, such regimes could reflect dynamics in real rates, inflation expectations, or the inflation risk premium. The literature has found evidence of regimes in all these components, some of which are not directly observable. Ang, Bekaert & Wei (2008) build a model that allows for switches in real rate factors, inflation, and risk premiums. Previously, regimes in real rates (Garcia & Perron 1996) and regimes in inflation (Evans & Wachtel 1993, Evans & Lewis 1995) were only separately considered. By considering two regimes in real rate factors and two regimes in inflation, Ang, Bekaert & Wei expand out the regimes to a total of four regimes.

Ang, Bekaert & Wei find that most of the time real short rates and inflation are drawn from a regime where short rates are relatively low and stable and inflation is relatively high and not volatile. The stable probability of this regime is more than 70%. Their inflation regimes are characterized as normal inflation and regimes of disinflation. During the regimes with decreasing inflation, the real rate curve is downward sloping. These regimes only occur after 1982 and are consistent with activist monetary policy raising real rates through actions at the short end of the yield curve and achieving disinflation. That these regimes only appear after 1982 is consistent with Clarida, Gali & Gertler (2000), Boivin (2006), and others who document a structural break before and after Federal Reserve Chairman Volcker.

Ang, Bekaert & Wei's regime-switching term structure model is able to identify latent factors and regime switches in real rate and inflation components through the cross section of bond yields (the term structure). Their model builds on the popular affine models (see Duffie & Kan 1996) and maintains tractability by maintaining exponential affine forms of bond prices conditional on the prevailing regime. That is, in a standard affine bond pricing model, the time t price of a zero-coupon bond maturing in T periods, $P(t,T)$, can be written as

$$P(t,T) = \exp(A(T) + B(T)'X_t), \tag{15}$$

for some factors X_t. The coefficients $A(T)$ and $B(T)$ are a function of the dynamics of X_t and the specification of bond risk premiums. In the regime-switching models developed by

Ang, Bekaert & Wei (2008), the factors X_t and risk premiums can switch regimes so that conditional on the regime s_t, the bond price can be written as

$$P(t, T \mid s_t) = \exp(A(T, s_t) + B(T)' X_t). \tag{16}$$

Ang, Bekaert & Wei can accommodate switches only in the conditional mean and volatilities of X_t. Dai, Singleton & Yang (2007) present a similar regime-switching model that incorporates regime-dependent mean reversion and regime-dependent probabilities under the real measure, but these parameters still cannot switch regimes under the risk-neutral pricing measure. Bansal & Zhou (2002) and Bansal, Tauchen & Zhou (2004) develop approximate solutions of the form

$$P(t, T \mid s_t) \approx \exp(A(T, s_t) + B(T, s_t)' X_t) \tag{17}$$

when all parameters switch under both the real and risk-neutral measures.

A related literature has tried to endogenize the monetary policy regimes in equilibrium models. Bikbov & Chernov (2008) develop a no-arbitrage term structure model where output shocks, inflation shocks, and monetary policy all change regimes in a macro model. In their model, the response of the monetary authority to output and inflation changes across regimes. Davig & Leeper (2007) and Farmer, Waggoner & Zha (2009) also embed reoccurring policy shifts into macro dynamic stochastic general equilibrium (DSGE) models. All of these authors allow agents to recognize that policy shifts can and do occur, and this recognition of the probability that regimes can change affects equilibrium output and inflation outcomes.

3.4. Exchange Rates

Exchange rates are characterized by highly persistent trends, punctuated by abrupt changes, which regime-switching models capture well (see, for example, panel *c* of **Figure 4**). These regimes have some link with underlying currency policy for some currencies, such as a switch from a free float regime to a target zone, target bands, or an exchange rate peg, as discussed by Froot & Obstfeld (1991), Engel & Hakkio (1996), and Dahlquist & Gray (2000). The carry trade, which is investing in high interest rate currencies by borrowing in currencies with low interest rates, is well known to exhibit long periods of steady gains with sudden periods of high volatility with reversals of the previous regime's gains. More recent papers, such as Ichiue & Koyama (2007), continue to confirm this behavior, which has been documented pervasively in the literature since Engel & Hamilton (1990) and Bekaert & Hodrick (1993). This regime-switching behavior of "going up by the stairs and coming down by the elevator" can result from the action of monetary policy, as shown by Plantin & Shin (2006) and Backus et al. (2010). In Plantin & Shin (2006), a risky asset price can deviate from its fundamental value with a fixed probability but snaps back to its fundamentals price from time to time. This is an example of a two-regime model where one regime represents the long-run fundamentals price, while the other regime allows prices to deviate from their fundamentals.

3.5. Asset Allocation

A natural question given the overwhelming existence of regimes is which portfolios should be optimally held in each regime, and whether there is an optimal portfolio to hedge against the risk of regime changes. The first paper to examine asset allocation with regime changes was Ang & Bekaert (2002a), who examine portfolio choice for a small number of countries. They exploit the ability of the regime-switching model to capture higher correlations during market downturns and examine the question of whether such higher correlations during bear markets negate the benefits of international diversification. They find there are still large benefits of international diversification. The costs of ignoring the regimes is very large when a risk-free asset can be held; investors need to be compensated approximately two to three cents per dollar of initial wealth to not take into account regime changes.

Figure 5, which is a version of figure 3 from Ang & Bekaert (2004), conveys the intuition for the effects of regime shifts on asset allocation. There are two regimes in an international CAPM: The high-volatility regime has the lowest Sharpe ratio and its mean-standard deviation frontier is the closest from the bottom. The low-volatility regime has the highest Sharpe ratio. The unconditional mean-standard deviation frontier averages across the two mean-standard deviation frontiers and is drawn as the solid gray line. An investor who ignores regimes sits on this unconditional frontier. Clearly, an investor can do better by holding a higher Sharpe ratio portfolio when the low-volatility, high Sharpe ratio regime prevails. Conversely, when the bad regime occurs, the investor who ignores regimes holds too high an equity weight and would have been better off shifting into the risk-free asset.

Although **Figure 5** considers mean-variance utility, investors usually care about more than the first two moments. Guidolin & Timmermann (2008a) consider asset allocation over international assets with a regime-switching model by an investor who takes into account skew and kurtosis preferences. Regime-switching models generate skewness and

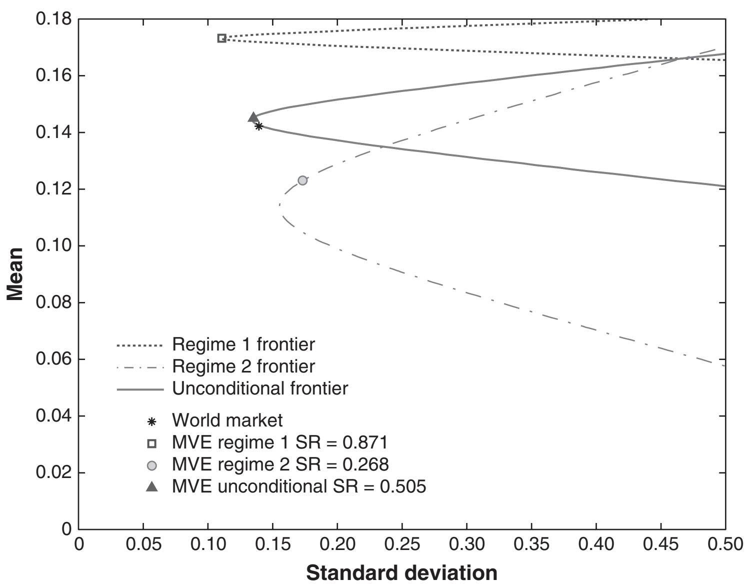

Figure 5

The figure shows regime-dependent mean-standard deviation frontiers following Ang & Bekaert (2004). The mean-standard deviation frontier of the high-volatility regime is shown as the dark gray dotted-dashed line and has the lowest Sharpe ratio. The mean-standard deviation frontier of the low-volatility regime is shown as the gray dotted line and has the highest Sharpe ratio. The unconditional mean-standard deviation frontier is drawn as the gray solid line.

kurtosis (see Equation 5), and so the regime-switching data-generating process is natural to use with utility functions that capture the effect of higher moments.[5] They find that the presence of regimes leads to a substantial home-biased portfolio for a US investor, and the introduction of skew and kurtosis preferences leads to further home biases. The strong persistence of the regimes (see **Table 1**) generates interesting term structures of risk linking the variance and higher order moments to the investment horizon (see Guidolin & Timmermann 2006). Guidolin & Timmermann (2007) show that these can have significant effects on long-term hedging demands.

In **Figure 5**, the risk-return trade-offs are known in each regime. Given the parameters, the investor can infer which regime prevails at each time. This updating of the probability of the current regime, given all information up to time t, can be computed using methods similar to the learning problem in Section 2.7. A further consideration is that the parameters themselves have estimation error. Guidolin & Timmermann (2008a) and Tu (2010) tackle the problem of parameter uncertainty in a regime-switching model applied to asset allocation problems. Tu (2010) finds that even after taking into account parameter uncertainty, the cost of ignoring the regimes is considerable. This is consistent with the finding in Pettenuzzo & Timmermann (2011) that uncertainty about future regimes can have a large effect on investors' optimal long-run asset allocation decisions, which can even change from being upward-sloping in the investment horizon in the absence of multiple regimes to being downward-sloping once uncertainty associated with future regime changes is accounted for.

4. CONCLUSION

We have discussed how regime changes are modeled, their impact on equilibrium asset prices, and the empirical evidence consistent with regimes in a variety of asset return series in fixed income, equities, and currency markets. An important remaining question is, "What gives rise to regimes?" In some instances, the discrete shift from one regime to another may result from a change in economic policy, e.g., a shift in monetary or exchange rate regime. In other cases, a major event, such as the bankruptcy of Lehman in September 2008, or the 1973 oil crisis, may be the trigger. More broadly, however, regimes can approximate swings in the state of the economy, which may not be of a binary nature and build up over time.

Another possibility is that regimes are driven by investor expectations. Branch & Evans (2010) propose a framework with bounded rational investors who use underparameterized models to form expectations. They show that in equilibrium, agents' beliefs and asset prices are jointly determined in a way that can give rise to multiple misspecified equilibria, each with distinct means and variances of returns. Learning dynamics and bounded rationality could thus be some reasons behind why there are regimes.

In addition to the underlying source of regimes, there are many other areas open for future research. Most work in asset pricing incorporating regime switching has considered either a single or a small set of risky assets. Cross-sectional effects of regimes on asset returns have been far less studied. Individual stocks and industry portfolios may differ in terms of their sensitivity and exposure to regime changes. Interesting questions are then whether regime change is a risk factor that is priced in equilibrium and whether differences

[5]Higher moment risk does enter Ang & Bekaert's (2002a) constant relative risk aversion (CRRA) utility, but CRRA utility is locally mean-variance.

in exposure to such a risk factor can help explain cross-sectional variations in expected equity returns.

A second question is whether the regimes inferred from asset return series can be used to shed light on the underlying fundamentals of the economy. Our simple analysis of an equilibrium asset pricing model showed that regimes in consumption or dividend growth translate into regimes in asset returns. Can this relation be reverse engineered? Consumption and dividend data tend to be very smooth, so the question is whether regimes deduced from asset returns (which are less smooth) can help us better infer properties of the underlying fundamentals. In a broader context, can regimes identified from asset prices, which are observed at high frequencies, be used to forecast regimes in macro variables, which are sampled only at low frequencies? A third question is whether we can exploit data measured at different horizons to identify the unobserved regimes. If regimes predominantly affect the volatility of asset returns, then data observed at high frequency would seem to provide important information on the regime. In addition, regime-switching models impose constraints on data aggregated across different horizons, so cross-horizon constraints could also prove helpful in identifying regimes.

DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

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Errata

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