Applied Economics

ISSN: 0003-6846 (Print) 1466-4283 (Online) Journal homepage: http://www.tandfonline.com/loi/raec20

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To cite this article: Dimos S. Kambouroudis & David G. McMillan (2015): Does VIX or volume improve GARCH volatility forecasts?, Applied Economics, DOI: 10.1080/00036846.2015.1096004

To link to this article: http://dx.doi.org/10.1080/00036846.2015.1096004

Published online: 02 Oct 2015.

APPLIED ECONOMICS, 2015
http://dx.doi.org/10.1080/00036846.2015.1096004

Does VIX or volume improve GARCH volatility forecasts?

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ABSTRACT
This article considers whether the inclusion of two additional variables can improve volatility forecasts over a standard GARCH-based model. We consider three alternative ways of incorporating the volatility index (VIX) and trading volume as exogenous variables within a selection of GARCH models. We are particularly interested in whether these variables have additional incremental forecast power over and above the baseline GARCH specification. Our results suggest that both the VIX and volume do provide some additional forecast power, and this is generally improved when considering both of these series jointly in the model. However, while the results may be statistically significant the gain is marginal and the coefficient values small. Moreover, in a horse race exercise VIX does not outperform the GARCH approach. In answering the question of whether VIX produces better forecasts than the GARCH model, then the answer is no, but the informational content of VIX cannot be ignored and should be incorporated into forecast regressions.

KEYWORDS
GARCH; volatility forecasting; VIX; volume

JEL CLASSIFICATION
C22; G15

I. Introduction

Volatility forecasting remains an important topic within empirical finance, not least in part due to the increase in stock market volatility over the recent past, following the onset of the financial crisis. For example, the annualized SD of US stocks over the past six years is 16.8% compared to 12.7% over the preceding six years, a period which included the fallout from the dot-com bubble.[1] This heightened volatility has obvious implications for asset and derivative pricing as well as the conduct of portfolio and risk management. Volatility plays a key role in market-timing decisions, in the pricing of options and other derivatives and in determining market risk and hedge ratios. Therefore, there remains a need to provide accurate forecasts of volatility, something for which there is an established literature but with no real consensus on the preferred approach (see the review papers of Poon and Granger 2005; while for a general discussion, see Andersen et al. 2005, 2006).

In this article, we explore the effect of including the volatility index (VIX) and trading volume (VO) within the volatility forecasting model, with the explicit purpose of seeing whether their inclusion improves forecast performance. Thus, we are asking the question of whether VIX and VO have incremental information over and above the GARCH specification that is useful in forecasting volatility. Intuitively it is reasonable to consider VIX as an additional variable in our forecast exercise. This is because VIX is defined as a benchmark of expected short-term market volatility and provides a forward-looking measure of volatility. Moreover, VIX provides a benchmark upon which futures and options contracts on volatility can be written (Whaley 2009). Thus, VIX carries benefits for both practitioners and academics alike; first as an updated proxy for the markets expectation of future stock market volatility, and, as such, is regarded of high value for day-to-day trading decisions, and second, it provides an insight into risk and return patterns (Fleming, Ostdiek, and Whaley 1995).

Trading volume also appears to have certain interesting and useful properties for improving the accuracy of volatility forecasts. Volume in terms of either trading value or the number of transactions is caused by the flow of information. Two main theories examine the relationship between volatility and volume. The mixture of distributions hypothesis (MDH) (Clark 1973) and the sequential information

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[1]Based on S&P 500 data.

hypothesis (SIH) (Copeland 1976; Jennings et al., 1981; Smirlock and Starks 1984) look at how information flow is received by the market and hypothesize that a positive relationship between volatility and volume exists (for which empirical evidence is established by Karpoff 1987). Thus, trading volume may provide additional forecast power for volatility, for example, Lamoureux and Lastrapes (1990, 1994).

Therefore, this article considers data from the main stock markets for the US, the UK and France and considers whether VIX and VO has any incremental forecast power over a GARCH model for volatility. Existing work has typically examined the forecasting power of alternative GARCH specifications, such as whether accounting for asymmetry or long-memory improves forecasts (e.g. Hansen and Lunde 2005). Work examining the forecast power of VIX typically does so in comparison with historical volatility models or compares it to the plain vanilla GARCH model (e.g. Blair, Poon, and Taylor 2001; Koopman, Jungbacker, and Hol 2005). Finally, research utilizing volume data typically focuses on the nature of the interactions between volatility and volume and which of the two above-noted hypotheses the data support (e.g. Brooks 1998; Ap Gwilym, McMillan, and Speight 1999; Donaldson and Kamstra 2005; Taylor 2008). Hence, the key contribution of this article is to provide an in-depth analysis of whether VIX and volume has any incremental forecast power over a range of GARCH alternatives. To that end, we consider three alternative ways of incorporating information from VIX and volume into the GARCH forecast results and focus on forecasts metrics designed to see if these additional variables provide incremental information.

The results suggest that both VIX and volume improve on the informational content of the GARCH-type models, with VIX typically providing more information than volume. That said, the results are further improved when both VIX and volume are included together. However, there is a trade-off between the statistical and economic significance of the findings. That is, while on the one hand, statistically, the results are improved, on the other hand, the economic value of that additional information appears minimal. In answering the question whether VIX produces better forecasts than the GARCH genre of models, the answer is no, but the informational content of VIX cannot be ignored, albeit small.

The rest of this article is structured as follows: Sections II provides information on VIX and volume, Section III presents the data and methodology, Section IV presents the empirical results and Section VI concludes.

II. Background information

Volatility index and trading volume

The options market is a good source of information concerning volatility. Engle (2003) and Simons (2003) describe the VIX as the 'fear index'.[2] The original VIX was based on the Chicago Board Options Exchange (CBOE) Market Volatility Index and calculated as an average of S&P 100 option implied volatilities and computed on a real-time basis during the trading day (Fleming, Ostdiek, and Whaley 1995). According to Whaley (2009), VIX was introduced in 1993 for two reasons. First, to provide a benchmark of expected short-term market volatility, and second, to provide an index upon which futures and options contracts on volatility could be written. In trying to understand VIX, it is important to recognize that it is a forward-looking measure of the volatility that investors expect to see.[3] The calculation of the VIX has changed and is now based on the S&P 500 index.[4]

Most recently, defining the VIX according to Mencia and Sentana (2013): 'The VIX index captures the volatility of the S&P 500 over the next month implicit in stock index option prices. It is formally, the square root of the risk neutral expectation of the integrated variance of the S&P 500 over the next 30 calendar days, reported on anannualised basis' (p.

[2]Because its (VIX) level indicates how much market participants are willing to pay in terms of implied volatility to hedge stock portfolios with S&P 100 index put options or to belong by buying S&P 100 index call options. In addition, extreme values of VIX are seen as trading signals, for example, with very high levels of VIX indicating that markets are pessimistic whereas a very low VIX often leads to an increase in stock prices.

[3]Whaley (2009) explains that VIX should be seen the same way as a bond's yield to maturity.

[4]Which is a better known index and because futures contracts on the S&P 500 are actively traded. Furthermore, S&P 500 option contracts are European-style, making them easier to value. Hence, the VIX is implied by the current prices of the S&P 500 index options and represents expected future market volatility over the next 30 calendar days (Whaley 2009).

367). In an algebraic form, based on the work of Carr and Wu (2006, pp. 14–15), the new VIX is calculated using process from the S&P 500 index options using the formula:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K}{K_i^2} e^{rT} P(K_i, T) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

where T is the common time to maturity for all the options involved in this calculation, F is the forward index level derived from the index option prices, K_i is the strike price of the ith out-of-the-money option in the calculation, $P(K_i, T)$ denotes the mid-quote price of the out-of-the-money option at strike K_i, K_0 is the first strike below the forward index level F, r denotes the risk free rate of maturity T and ΔK_i denotes the interval between strike prices, defined as

$$\Delta K_i = \frac{K_{i+1} - K_i}{2}$$

The calculation of σ^2 is used by the CBOE at two of the most shortest maturities of the available options, T_1 and T_2. Then, the CBOE linearly interpolates between the two σ^2 to obtain a σ^2 at 30-day maturity. The VIX represents the annualized percentage of this 30-day σ.

$$VIX = 100\sqrt{\frac{365}{30}\left[T_1\sigma_1^2 \frac{N_{T_2} - 30}{N_{T_2} - N_{T_1}} + T_2\sigma_2^2 \frac{30 - N_{T_1}}{N_{T_2} - N_{T_1}}\right]}$$

where N_{T_1} and N_{T_2} denote the number of actual days to expirations for the two maturities.

The relationship between stock market volume and volatility has been a subject of interest in the finance literature for several decades. Karpoff (1987) produced a systematic survey of this relationship reviewing a large number of previous studies using a variety of data sets and data frequencies from different markets and arrived at the conclusion 'volume is positively related to the magnitude of the price change and, in equity markets, to the price change *per se*' (p. 109). Hence, there is a general belief that there exists a positive contemporaneous correlation between the absolute price and volume measures.

Given the evidence from a number of studies in a positive relationship, several theoretical models of volume and volatility were proposed to explain this relationship. Following from the work of Clark (1973), the MDH suggests that the volume and volatility should be positively correlated as both originate from the same source; the rate of information flow. Representative studies of this hypothesis include Epps and Epps (1976), Tauchen and Pitts (1983), Harris (1986, 1987) and Andersen (1996). The MDH supports a contemporaneous relationship only and implies that past volume does not contain any useful additional information on the future volatility dynamics.

An alternative theoretical approach to explain the positive relationship between volatility and volume is given by the SIH, which originated from the work of Copeland (1976), Jenning, Starks, and Fellingham (1981) and Smirlock and Starks (1984). This approach advocates that new information enters the market sequentially, implying that a bidirectional causality or positive contemporaneous relationship between volume and volatility could exist. In the same line of thought, the noise trading hypothesis (Harris and Raviv, 1993; Brock and LeBaron 1996) suggests that a causal relationship exists which can be exploited for forecasting purposes. More recently, Abu Hassan ShaariMohd and Chin Wen (2007) studied the dynamic relationships of the realized volatility and trading volume using a bivariate VAR methodology. Their empirical results support the MDH; however, they also discovered significant causal relations between trading volume and return volatility in accordance with the SIH. Nevertheless, it has been argued by Wang (1994) that information asymmetry and investor heterogeneity could also be a factor in the above relationship.[5]

Volatility forecasting with VIX and trading volume

The accuracy of volatility forecasts has been the topic of extensive research and, as an alternative to GARCH forecasts, several academics have proposed the use of implied volatilities from options. Fleming, Ostdiek, and Whaley (1995) find that VIX performs better in forecasting future volatility than other historical measures. In addition, the importance of VIX was highlighted with benefits for both practitioners and academics. Specifically, they mention that as an updated proxy for the future stock

[5]Volume absolute returns relationship, volume is positively correlated with absolute returns and this correlation is increased by information asymmetry.

market volatility it is of high value for day-to-day trading decisions such as asset allocation, portfolio and risk management. Equally, academics have the opportunity for a better insight into risk and return patterns. Furthermore, because VIX contains market expectations it was proven to be a useful instrument for forecasting volatility. Blair, Poon, and Taylor (2001) reached the same conclusion that all relevant information is provided by the VIX index and that the VIX index provides the most accurate forecasts for all forecast horizons and across all performance measures used. As Blair et al. mention, it is reasonable to compare the forecasting ability of GARCH with implied volatility, which are known to co-vary with realized volatility.[6] Further supportive evidence for VIX in providing accurate volatility is provided by Christensen and Prabhala (1998), Fleming (1998), Fleming, Ostdiek, and Whaley (1995), Hol (2003) and Szakmary et al. (2003), all of whom find that option implied volatilities dominate over time-series forecasts. Further still, Corrado and Miller (2005) conclude that although VIX yields upward biased forecasts they are still more accurate than those of other historical models. Dennis, Mayhew, and Stivers (2006) find that daily VIX changes are significant in predicting future index return volatility, while Carr and Wu (2006) report that VIX can predict movements in future realized variance and that GARCH volatilities do not provide extra information over and above VIX; similar findings for implied volatility were confirmed by Yu, Lui, and Wang (2010) and specifically for VIX in Yang and Liu (2012). Giot and Laurent (2007) find that implied volatility has very high information content, even when extended decompositions of past realized volatility are used; this is also confirmed when adding GARCH-type volatility forecasts in the regressions.

In contrast, the results by Becker, Clements, and White (2006) state a differing view to the above-noted findings and report that VIX does not provide an efficient volatility forecast and that other information can improve upon the VIX forecast of volatility. Studies within a GARCH context also produced mixed results. Day and Lewis (1992) report that implied volatilities perform well but not better than the GARCH forecasts, while they also note that forecast combinations outperform the univariate forecasts. In addition, studies by Ederington and Guan (2002) and Martens and Zein (2002) find that GARCH models and historical volatility models perform well in comparison to VIX, while Canina and Figlewski (1993) find that implied volatilities provide poor forecasts and that simple historical models perform better. More recent studies (Ahoniemi 2008; Konstantinidi, Skiadopoulos, and Tzagkaraki 2008; Clements and Fuller 2012; Dunis, Kellard, and Snaith 2013; Fernandes, Medeiros, and Scharth 2014) have looked into the forecasting ability of implied volatility itself and in Haugom et al. (2014), implied volatility from the crude oil volatility index is used to forecast realized volatility in the West Texas Intermediate futures market.

Another more recent stream in the academic literature has attempted with the use of information on the VIX index to estimate GARCH models with the aim of improving the performance of these models. Kanniainen, Lin, and Yang (2014) show that spot volatilities extracted from VIX rather than from returns, using multiple cross sections of options on the S&P 500, improve the models' performance. Hao and Zhang (2013) find that the GARCH implied VIX is consistently and significantly lower than the CBOE VIX for a number of GARCH model variations when only returns are used in the estimation. Based on the premise that VIX approximates the 30-day variance swap rate on the S&P 500 (Carr &Wu 2006), it contains forward-looking information, therefore providing forward-looking parameter estimation when incorporated in the modelling process, whereas asset returns do not.

According to Taylor (2008), a number of studies have demonstrated that the performance of volatility models can be significantly improved with the inclusion of proxies of information flow in their model specification. Trading volume is one of those factors which have shown to lead to significant improvements (Karpoff 1987; Lamoureux and Lastrapes 1990; Bessembinder and Seguin 1993; Bollerslev and Jubinski 1999; Luu and Martens 2003).

The main focus of the majority of studies conducted in this line of research was aimed at assessing trading volume as an information proxy in relation to the volatility in returns (e.g. Hiemstra and Jones

[6]Latane and Rendleman (1976) and Chiras and Manaster (1978).

1994; Lamoureux and Lastrapes 1994; Richardson and Smith 1994). A limited number of studies have investigated the information content of trading volume in volatility forecasting applications. However, initial results reported here were discouraging, generally concluding that trading volume was not helpful in improving forecasts and that it cannot forecast volatility directly (e.g. Lamoureux and Lastrapes 1994; Brooks 1998). As such, it was argued that trading volume may not be the most accurate measure of information flow because volume could be liquidity motivated or occur as a result of divergent in trader opinion (Taylor 2008).

Notwithstanding the above, a different picture emerges, with some initial signs of success, when trading volume is used within a GARCH-type setting. Lamoureux and Lastrapes (1990) use daily trading volume as a proxy for information arrival within a GARCH(1,1) framework and find that transactions volume has a significant explanatory power on the variance of daily returns and that ARCH effects tend to disappear when volume is included in the variance equation. Wagner and Marsh (2005) successfully managed to explain the heteroscedasticity in returns using trading volume by extending the work of Lamoureux and Lastrapes (1990) and adopting an asymmetric GARCH-in-mean model specification by Golsten et al. (1993).

Brooks (1998) focused onto the causal relationship between volatility and trading volume with the use of a number of different statistical models (and within a GARCH-type setting) on the New York Stock Exchange market. Brooks reported that lagged stock market volume measures play a small role in improving the out-of-sample forecasting performance of volatility models.

In Donaldson and Kamstra (2005) as in Fuertes, Izzeldin, and Kalotychou (2009), it was found that lagged volume has no marginal power to forecast future volatility. However, in Donaldson and Kamstra (2005) using a forecast combination approach and adopting lagged volume with option implied volatility within a GARCH framework, they find that trading volume is significant. More recently in Fuertes, Kalotychou, and Todorovic (2015), who use daily trading volume in addition to intraday and overnight returns in a forecasting equity volatility exercise, they find that trading volume is the most effective predictor in terms of economic but not statistical value.

From the above, it can be seen that the literature has yet to agree on the usefulness of VIX and volume when forecasting volatility. The purpose of this article, therefore, is to address this issue and to assess the usefulness of VIX within a GARCH framework, as originally considered by Blair, Poon, and Taylor (2001), by adding a further information set, trading volume, following the work of Donaldson and Kamstra (2005).

III. Data and methodology

Data

According to Whaley (2009), the CBOE methodology for computing the VIX is not unique to the prices of S&P 500 options but can be applied to any index option market. Examples of indices that have used the CBOE methodology are the VXN based on the NASDAQ 100 and the VXD for the Dow Jones Industrial Average. The NYSE Euronext has applied the same methodology to option indices on European markets, for example, on the AEX, BEL20, CAC40 and the FTSE 100.

Three representative indices were obtained, with the sample constrained by the amount of available data.[7] Data from the US, the UK and France were obtained from Datastream. For the US, the data range from 1 January 1990 until 22 October 2012. However, a smaller number of observations, due to the availability of the relevant VIX data, were obtained for the UK and France. The sample for France and the UK is from 1 January 2000 until 22 October 2012. For each country, the daily closing prices of the main index (S&P 500, FTSE 100 and CAC40) was obtained along with VIX and trading volume data. The measure of trading volume both volume in terms of number of trades and transaction value were considered based on availability. More specifically, for the US and France volume is in terms of traded quantities, while for the UK volume is in terms of transaction value.

We convert the prices into logarithmic returns and report the descriptive statistics in Table 1.

[7]The main constraint was finding VIX data of sufficient length for reliable statistical analysis to be performed.

Table 1. Descriptive statistics for returns FTSE, CAC and S&P.

	RS&P	RFTSE	RCAC
Mean	0.000282	−0.0000374	−0.000146
Median	0.000289	0.000000	0.000000
Maximum	0.109004	0.093843	0.105946
Minimum	−0.094275	−0.092656	−0.094715
SD	0.011602	0.012833	0.015591
Skewness	−0.221782	−0.138691	0.041923
Kurtosis	11.47212	8.954646	7.645724
Jarque–Bera	17840.46	4943.772	3003.679

Note: *R* refers to returns as logarithmic differenced series for each index.

Descriptive statistics for VIX and volume are also presented in Table 2. Examining both of these tables, the markets appear to be similar with only small differences from the reported statistics. That is, all return series exhibit a larger SD relative to the mean and all report positive excess kurtosis and non-normality. Evidence of negative skewness is present for both the US and the UK, although France reports a small positive value. Of further note, the maximum and minimum values for each market are very similar. In Table 2, the VIX values are likewise similar across the markets. That is, in terms of average values, maximum and minimum values as well as the SD, skewness and kurtosis (the latter measure being more noticeably higher for the US). Finally, the volume data are similar across the markets in terms of their distributional properties, despite some difference in definition across the data.[8]

Methodology

GARCH empirical models

A selection of six GARCH models is used with the choice dependent upon the ability of each model to capture different features of the data, such as volatility clustering, asymmetry and long memory. The models considered include the first-generation symmetric GARCH (Bollerslev 1986) model, second-generation asymmetric models including the threshold-GARCH (TGARCH) model (Glosten, Jagannathan, and Runkle 1993), the EGARCH model (Nelson 1991) and the asymmetric power-ARCH (APARCH) model (Ding, Granger, and Engle 1993), and the third-generation long-memory component GARCH (CGARCH) model (Engle and Lee 1999) and the IGARCH model (Engle and Bollerslev 1986). The models are briefly described below; for a detailed literature review on the GARCH genre of model, see Poon and Granger (2003).

The GARCH(1,1) model of Engle (1982) and Bollerslev (1986) is given by

$$h_{t+1}^2 = \omega + \alpha\varepsilon_t^2 + \beta h_t^2 \tag{1}$$

where h_t^2 represents the conditional volatility and ε_t^2 is the volatility news (squared error arising from an autoregressive (AR) conditional mean equation). The TGARCH model of Glosten, Jagannathan, and Runkle (1993) is given by

$$h_{t+1}^2 = \omega + \alpha\varepsilon_t^2 + \gamma\varepsilon_t^2 I_t + \beta h_t^2 \tag{2}$$

where I_t is an indicator variable that equals one of the error term ε_t is negative and zero otherwise. Thus, asymmetry arises if the γ term is statistically significant. The EGARCH model of Nelson (1991) is given by

$$\log(h_{t+1}^2) = \omega + \alpha\left|\frac{\varepsilon_t}{h_t}\right| + \gamma\frac{\varepsilon_t}{h_t} + \beta\log(h_t^2) \tag{3}$$

The CGARCH model of Engle and Lee (1999) is specified as

Table 2. Descriptive statistics for VIX and volume.

	US		UK		France	
	VIX	Volume	VIX	Volume	VIX	Volume
Mean	20.45641	1574747	21.65846	1341530	24.61352	4032493
Median	18.94000	1389653	19.74800	1312655	22.72000	3677100
Maximum	80.86000	9510412	75.54000	4461012	78.05000	16017700
Minimum	9.310000	2079.000	9.099000	66188.00	9.242000	164218.0
SD	8.141122	1463307	9.100885	508346.1	9.511489	1635654
Skewness	1.966183	1.120233	1.630534	0.547748	1.474273	1.494323
Kurtosis	9.890915	4.160271	6.984776	4.115044	5.946315	7.678653
Jarque–Bera	15605.91	1578.217	3689.725	340.0448	2417.256	4288.083

[8]The correlation coefficient matrices are also produced as part of the initial analysis. It was found that the coefficients are small, suggesting no correlation and hence multicollinearity will not be present.

$$h_{t+1}^2 = q_{t+1} + \alpha(\varepsilon_t^2 - q_t) + \beta(h_t^2 - q_t) \quad (4)$$

where q_t represents long-run volatility. The APARCH by Ding, Granger, and Engle (1993) is specified as

$$h_t^\delta = \omega + \alpha_1(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta_1 h_{t-1}^\delta \quad (5)$$

Finally, the IGARCH is specified as an extension to the GARCH model for which the following condition must hold, $\alpha + \beta = 1$ for the conditional variance to be clearly nonstationary.

VIX and volume

In order to examine whether the inclusion of VIX and/or volume improves the GARCH forecast, we consider three exercises. First, we include the lagged value of VIX and volume directly into the forecast equation (discussed below). This implicitly assumes that the best forecast arising from these variables for tomorrow's value is today's values and hence the series act like random walk variables. Second, we include VIX and volume as exogenous variables into the GARCH equations outlined above. Here we believe that VIX and volume contain information that aids the forecasting of the conditional variance that is not otherwise captured by the GARCH specification. Finally, we conduct direct estimation of the variables themselves.[9] That is, we estimate appropriate models for VIX and use the forecast from this model as our forecast of stock return volatility and to compare with the GARCH forecast.

Forecasts evaluation

To examine the accuracy of the volatility forecasts, the testing procedure of Mincer–Zarnowitz (MZ, 1969) and its extension to encompassing tests is used. This is because we are interested in whether the VIX and volume measures provide incremental information over the GARCH approach in forecasting volatility. In the basic MZ regression, the true volatility value is regressed on a constant and the forecast value. This is given by

$$\sigma_t^2 = \alpha + \beta h_t^2 + \varepsilon_t \quad (6)$$

where ${\sigma_t}^2$ is the measure of true volatility and ${h_t}^2$ is the forecast of volatility. The coefficient of determination, R^2, is obtained for comparison purposes. For the measure of true volatility, we follow the procedure of Pagan and Schwert (1990), where the proxy for true volatility is given by the squared error from the conditional mean model for returns estimated over the whole sample. The forecast performance for GARCH-type models has been reported extensively in the literature; see, for example, Akgiray (1989), Boudoukh, Richardson, and Whitelaw (1997), Brailsford and Faff (1996), Dimson and Marsh (1990), Frennberg and Hansson (1996), Figlewski (1997), Heynen and Kat (1994), Jorion (1995), and Schwert and Seguin (1990). This research often reports low R^2 values, for which Andersen and Bollerslev (1998) have demonstrated that regression methods will typically give low R^2 values, even for optimal GARCH forecasts, as daily squared returns are a noisy measure of true volatility. Andersen and Bollerslev argue that the use of intraday returns construct realized volatility can eliminate the noise in daily volatility. Notwithstanding this, as our interest here is not the level of the R^2 value but rather whether the R^2 value increases following the introduction of VIX and volume, the above issue is less of a consideration here.

To examine forecast power, first we obtain volatility forecasts from the above alternate GARCH models. In particular, the in-sample estimation period for the US is from 1 January 1990 till 31 December 2005 with the out-of-sample forecast period being from 1 January 2006 till 22 October 2012. For the UK and France, the in-sample estimation period begins from 1 January 2000 and ends on 31 December 2007, while the out-of-sample period is from 1 January 2008 till 22 October 2012. In all cases, we obtain the one-step-ahead forecasts for volatility. Following this, we introduce the VIX and volume variables as described above. First, lagged values of VIX and volume are included only on the right-hand side of the MZ regression of Equation 6. Second, we include values of VIX and volume in the GARCH models. Third, we estimate a conditional

[9]This is done for VIX series only.

mean model for VIX and use the forecast arising from that in Equation 6 to provide a comparison with the GARCH model.

Forecast encompassing

The above MZ regression allows us to examine whether the volatility forecasts provide a reasonable ability to forecast stock return volatility. However, and particularly useful for our purposes here, we can use this forecast regression in order to identify whether the forecasts that include VIX or volume provide additional information over and above the GARCH only forecasts through forecast encompassing. That is, we can examine the relative forecasting performance in order to identify whether the additional components carry information that is not captured by the base (GARCH) forecast.

The method of forecast encompassing was originally developed by Chong and Hendry (1986) and allows consideration of whether a competing forecast carries additional information over a base model forecast. Specifically, if the competing model carries no additional information, then the base forecast model is said to 'encompass' the former. To test for such forecast encompassing, we consider the following extension to the regression model in Equation 6:

$$\sigma_t^2 = \alpha + \rho_1 h_{1,t}^{2f} + \rho_2 h_{2,t}^{2f} + \varepsilon_t \tag{7}$$

where the subscripts 1, 2 denote the forecast models (1 refers to the base model and 2 refers to the competing model). The null hypothesis associated with this test is that model 1 encompasses model 2, in which case ρ_2 is equal to zero, while if ρ_2 is significant and greater than zero then model 2 contains information that model 1 does not, such that model 2 is not encompassed by model 1.

In application of the test in Equation 7, we consider three variations of the regression: first, adding the forecast that includes VIX; second, adding the forecast that includes volume; and third, adding the forecast that includes VIX and volume. If the coefficient on the additional variable is positive and statistically significant, this suggests that this variable has forecast information over and above the base GARCH model.

Value-at-risk

Value-at-risk (VaR) is an important tool used in risk management since it is a measure of risk exposure associated with a particular portfolio of assets. The VaR of a portfolio is defined as the maximum loss occurring within a specified time and with a given probability. This measure is of practical relevance for financial institutions not only for minimizing the possibility of financial distress, but banks are required by regulators to assess their market risk using internally generated risk measurements (Basel Committee on Banking Supervision 1988, 1995, 1996, 2006, 2009). The validity of the VaR measure is then 'backtested' by comparing the number of 'exceptions', for example, the number of days the VaR estimate is insufficient to cover actual trading losses, for penalties to be imposed on poorly performing measures. Our exercise is designed to match that of practice and the regulatory environment. Therefore, for all our models we provide both 1% and 5% VaRs for all three markets. This method is often used to assess the economic significance of the different forecasting models.

IV. Empirical results

GARCH volatility forecasts

Table 3 presents the results for the simple MZ regression equation (Equation 6) for the GARCH baseline forecasts. The coefficient of determination (R^2) is reported for regressing the true volatility on a constant and volatility forecast for each of the different GARCH-type model. The greater the R^2

Table 3. Coefficients of determination for Mincer–Zarnowitz.

Model and MZ regression/ coefficient of determination	US	UK	France
	R^2	R^2	R^2
$\sigma_t^2 = \alpha + \beta h_t^2 + \varepsilon_t$			
GARCH(1,1)	0.213953	0.190451	0.135607
TGARCH	0.254524	0.249139	0.187805
EGARCH	0.267007	0.260899	0.199600
APARCH	0.255507	0.246202	0.187973
CGARCH	0.223846	0.195538	0.139123
IGARCH	0.192519	0.184465	0.129331

Notes: The table presents the results of the simple MZ regression equation (Equation 6) for the GARCH baseline forecasts. The in-sample estimation period is from 1 January 1990 to 31 December 2005 for the US and 1 January 2000 to 31 December 2007 for the UK and France. The out-of-sample forecast period is from 1 January 2006 to 22 October 2012 for the US and from 1 January 2008 to 22 October 2012 for the UK and France. In all cases, we obtain the one-step-ahead forecasts for volatility.

values, then the more accurate the forecast is. As can be seen for all three indices, the EGARCH model gives the best forecast, albeit only marginally so over other asymmetric GARCH models. Furthermore, each of the asymmetric GARCH models outperforms both the first-generation symmetric GARCH and the third-generation long memory GARCH models, suggesting the importance of asymmetry for stock index returns volatility. Of note, the IGARCH model provides the worst forecasts, with the lowest R^2 value. The above results are in agreement with those of Hansen and Lunde (2005), who find that when forecasting the volatility of stock returns, models that accommodate the leverage effect are superior. Of particular interest for our purposes here, these results serve as a base for the results in the succeeding sections.

Adding VIX and volume I: as independent variables

Table 4 presents the results of the coefficients of determination for the MZ regressions where lagged VIX and volume are included directly on the right-hand side of the regressions as independent variables. As noted above, this implicitly assumes that the current values of these measures provide the best forecasts such that they can be regarded as random walk-type variables. For ease of comparison, Table 4 presents the baseline GARCH forecasts again as well as the forecasts from including (lagged) VIX, volume and both.

We observe that the inclusion of lagged VIX and volume components increases the R^2 value of the regressions with the largest value reported when both VIX and volume are included together in the regression. When comparing the impact of VIX and volume individually on R^2, a pattern emerges based

Table 4. VIX and volume as independent variables.

	USA	UK	France
Model and MZ regression/coefficient of determination	R^2	R^2	R^2
GARCH(1,1)			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.213953	0.190451	0.135607
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$	0.258759	0.209366	0.172138
$\sigma_t^2 = a + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$	0.230772	0.207470	0.155548
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$	0.237802	0.219875	0.198227
TGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.254524	0.249139	0.187805
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$	0.258759	0.250461	0.192588
$\sigma_t^2 = a + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$	0.264728	0.258773	0.199886
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$	0.264803	0.259046	0.208455
EGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.267007	0.260899	0.199600
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$	0.267625	0.260901	0.200434
$\sigma_t^2 = a + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$	0.273547	0.265401	0.208681
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$	0.274079	0.265487	0.212030
APARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.255507	0.246202	0.187973
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$	0.259164	0.246943	0.191261
$\sigma_t^2 = a + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$	0.266156	0.258454	0.200286
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$	0.266156	0.258502	0.207235
CGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.223846	0.195538	0.139123
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$	0.240106	0.213467	0.174019
$\sigma_t^2 = a + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$	0.239584	0.212399	0.160509
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$	0.243897	0.223807	0.198884
IGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.192519	0.184465	0.129331
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$	0.227982	0.203577	0.170937
$\sigma_t^2 = a + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$	0.212413	0.205561	0.154597
$\sigma_t^2 = a + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$	0.229048	0.216233	0.198472

Notes: The table presents coefficients of determination for the MZ regressions equation (Equation 6), where lagged VIX and volume are included directly on the right-hand side of the regressions as independent variables. The in-sample estimation period is from 1 January 1990 to 31 December 2005 for the US and 1 January 2000 to 31 December 2007 for the UK and France. The out-of-sample forecast period is from 1 January 2006 to 22 October 2012 for the US and from 1 January 2008 to 22 October 2012 for the UK and France. In all cases, we obtain the one-step-ahead forecasts for volatility.

on the forecast model used. More specifically, for the standard symmetric GARCH forecast model and the long memory model (CGARCH and IGARCH[10]), the inclusion of VIX gives a higher increase in the coefficient of determination than the inclusion of volume. On the other hand, a higher increase is reported for the inclusion of volume in the MZ regressions when the forecast model is from the asymmetric GARCH models (TGARCH, EGARCH and APARCH). Overall, the best forecast model, in terms of the highest R^2 value, remains the EGARCH model; likewise, the worst forecast model is still the IGARCH model.

Adding VIX and volume II: as exogenous variables in GARCH model

Following in the steps of Blair, Poon, and Taylor (2001), we include a lagged VIX component in the variance equation of the GARCH models and extend their methodology to include a lagged volume component as well as VIX and volume together in the variance equation. As before, we run series of MZ regressions and obtain the coefficients of determination for comparison purposes, with the results reported in Table 5. The table shows that on the whole when a lagged VIX component is included in the variance equation of the GARCH forecasts the coefficient of determination increases, suggesting that VIX adds to the informational content of the model, increasing the accuracy of the forecast. However, in five cases the base MZ regression was adequate, that is, including VIX did not increase the R^2. When volume is included in the variance equation, only in seven cases this produced a higher R^2 than the baseline GARCH model, while finally the inclusion of VIX and volume in the variance equation results in a higher R^2in only three cases.

Table 5. VIX and volume as exogenous variables in GARCH model.

Country		US	UK	France
Model and MZ regression/coefficient of determination		R^2	R^2	R^2
GARCH				
Base MZ regression	$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.213953	0.190451	0.135607
Variables included in variance equation	VIX_{t-1}	0.225188	0.189851	0.141212
	VO_{t-1}	0.216167	0.107286	0.056450
	VIX_{t-1} and VO_{t-1}	0.226599	0.107238	0.056445
TGARCH				
Base MZ regression	$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.254524	0.249139	0.187805
Variables included in variance equation	VIX_{t-1}	0.271858	0.266705	0.200106
	VO_{t-1}	0.260775	0.131202	0.077982
	VIX_{t-1} and VO_{t-1}	0.064174	0.132315	0.077998
EGARCH				
Base MZ regression	$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.267007	0.260899	0.199600
Variables included in variance equation	VIX_{t-1}	0.231010	0.225855	0.189602
	VO_{t-1}	0.266119	0.262347	0.199822
	VIX_{t-1} and VO_{t-1}	0.236395	0.225097	0.188909
APARCH				
Base MZ regression	$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.255507	0.246202	0.187973
Variables included in variance equation	VIX_{t-1}	0.275694	0.255721	0.190754
	VO_{t-1}	0.245569	0.119608	0.000196
	VIX_{t-1} and VO_{t-1}	0.226599	0.119521	0.000216
CGARCH				
Base MZ regression	$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.223846	0.195534	0.139123
Variables included in variance equation	VIX_{t-1}	0.247781	0.210298	0.149744
	VO_{t-1}	0.227067	0.192652	0.141225
	VIX_{t-1} and VO_{t-1}	0.251323	0.208708	0.051643
IGARCH				
Base MZ regression	$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.192519	0.184465	0.129331
Variables included in variance equation	VIX_{t-1}	0.215064	0.022055	0.135810
	VO_{t-1}	0.216167	0.031113	0.009385
	VIX_{t-1} and VO_{t-1}	0.000120	0.031115	0.009386

Notes: The table shows the results of the inclusion of a lagged VIX and volume component in the variance equation (Equation 6) of the GARCH forecasts. The in-sample estimation period is from 1 January 1990 to 31 December 2005 for the US and 1 January 2000 to 31 December 2007 for the UK and France. The out-of-sample forecast period is from 1 January 2006 to 22 October 2012 for the US and from 1 January 2008 to 22 October 2012 for the UK and France. In all cases, we obtain the one-step-ahead forecasts for volatility.

[10]There is one exception for the UK.

The results here, therefore, suggest that including VIX and (especially) volume in the GARCH variance equation does not unequivocally increase the forecast power of the GARCH model. A finding different to Blair, Poon, and Taylor (2001), who mention that a mixture of VIX forecasts and index forecasts carry additional forecast information. Aside from this finding, we continue to observe that the asymmetric GARCH-type models perform better than either the symmetric or long memory GARCH models, with the EGARCH model still typically preferred (although the TGARCH model provides the preferred model on occasion). As before, the IGARCH model appears to be the weakest one, with some very low values.

Adding VIX and volume III: forecasting VIX

As a final method of examining whether we can improve the forecast performance over the GARCH model, we consider whether VIX alone can be used for forecasting. In order to consider this, we estimate an AR(1) model[11] for VIX and obtain forecasts from this model:

$$VIX_t = \alpha + \zeta VIX_{t-1} + \varepsilon_t \tag{8}$$

Table 6 reports the MZ results of the VIX only forecasts, the baseline GARCH models and the GARCH and VIX forecasts. This allows us to examine whether the VIX forecast model provides additional information over the GARCH approach or even if the VIX forecast model is preferred. This table confirms our previous findings. As can be seen, although in most cases the inclusion of VIX as an independent variable and as an exogenous variable in the variance equation increases the R^2s and hence improves our forecasts, it appears that in the majority of the cases it does not outperform the GARCH-type forecasts for which higher value R^2s are found especially for the asymmetric GARCH models and more specifically for the EGARCH model which consistently outperforms all other models.

Forecast encompassing

Having examined the overall forecast performance of the different models, in this section we examine the performance a little deeper by presenting the results of the encompassing tests. We present the coefficients and their p-values of the MZ equations where two regressors are included. That is, for the results in Table 4, where VIX and volume are included as lagged regressors, and for Table 6, where the forecast of VIX is included. We present the results for each market in Tables 7–9.

Table 6. VIX.

Model and MZ regression/ coefficient of determination	US R^2	UK R^2	France R^2
VIX			
$\sigma_t^2 = a + \zeta VIX_t + \varepsilon_t$	0.227364	0.192799	0.170885
GARCH(1,1)			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.213953	0.190451	0.135607
$\sigma_t^2 = a + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	0.234920	0.209366	0.172138
TGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.254524	0.249139	0.187805
$\sigma_t^2 = a + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	0.258759	0.250461	0.192588
EGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.267007	0.260899	0.199600
$\sigma_t^2 = a + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	0.267625	0.260901	0.200434
APARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.255573	0.246202	0.187973
$\sigma_t^2 = a + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	0.259164	0.246943	0.191261
CGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.223915	0.195538	0.139123
$\sigma_t^2 = a + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	0.240106	0.213467	0.174019
IGARCH			
$\sigma_t^2 = a + \beta h_t^2 + \varepsilon_t$	0.192519	0.184465	0.129331
$\sigma_t^2 = a + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	0.227982	0.203577	0.170937

Notes: The table reports the MZ results of the VIX only forecasts (Equation 8) the baseline GARCH models and the GARCH and VIX forecasts (Equation 6). The in-sample estimation period is from 1 January 1990 to 31 December 2005 for the US and 1 January 2000 to 31 December 2007 for the UK and France. The out-of-sample forecast period is from 1 January 2006 to 22 October 2012 for the US and from 1 January 2008 to 22 October 2012 for the UK and France. In all cases, we obtain the one-step-ahead forecasts for volatility.

The US

The results for the US are presented in Table 7. Examining the parameters of the MZ regression, we can see that all the β coefficients (GARCH forecast coefficients) are all positive and greater in absolute value than the γ coefficients (the lagged VIX coefficient), the δ coefficients (lagged volume) and the ζ coefficients (forecasted VIX) which, in turn, are very small. Furthermore, all the β parameters are statistically significant, with the exception of the IGARCH model. The VIX coefficients, γ, are all

[11]We choose to use a simple AR(1) model in order to keep the model parsimonious. However, several authors have considered including asymmetric terms (Giot, 2005) or allowing for long memory effects (Konstantinidi, Skiadopoulos, and Tzagkaraki 2008; Dunis, Kellard, and Snaith 2013). While in the context of option pricing, Kanniainen, Lin, and Yang (2014) advocate the use of squared VIX.

Table 7. Encompassing test: the US.

	$\sigma_t^2 = \alpha + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	
Parameters	Coefficient	*p*-Value	Coefficient	*p*-Value	Coefficient	*p*-Value	Coefficient	*p*-Value
GARCH								
β	0.402635	0.0000	0.851609	0.0000	0.492342	0.0000	0.402635	0.0000
γ	1.99E-05	0.0000			1.44E-05	0.0001		
δ			8.52E-11	0.0000	4.40E-11	0.0097		
ζ							2.03E-05	0.0000
TGARCH								
β	0.762622	0.0000	0.901680	0.0000	0.867873	0.0000	0.762622	0.0000
γ	8.94E-06	0.0015			1.46E-06	0.6703		
δ			6.67E-11	0.0000	6.29E-11	0.0001		
ζ							9.10E-06	0.0015
EGARCH								
β	1.322408	0.0000	1.337616	0.0000	1.476681	0.0000	1.322408	0.0000
γ	3.68E-06	0.2217			−4.08E-06	0.2547		
δ			5.44E-11	0.0001	6.45E-11	0.0001		
ζ							3.74E-06	0.2216
APARCH								
β	0.842043	0.0000	0.977099	0.0000	0.978016	0.0000	0.842043	0.0000
γ	8.42E-06	0.0031			−3.63E-08	0.9917		
δ			6.81E-11	0.0000	6.82E-11	0.0000		
ζ							8.57E-06	0.0031
CGARCH								
β	0.507381	0.0000	0.873577	0.0000	0.603808	0.0000	0.507381	0.0000
γ	1.71E-05	0.0000			1.10E-05	0.0015		
δ			8.23E-11	0.0000	5.03E-11	0.0029		
ζ							1.74E-05	0.0000
IGARCH								
β	0.119009	0.2339	0.772427	0.0000	0.181964	0.0913	0.119009	0.2339
γ	2.73E-05	0.0000			2.36E-05	0.0000		
δ			9.28E-11	0.0000	2.71E-11	0.1178		
ζ							2.77E-05	0.0000

Notes: This table presents the coefficients and their *p*-values of the MZ equations where two regressors are included. That is, for the results in Table 4, where VIX and volume are included as lagged regressors, and for Table 6, where the forecast of VIX is included. The in-sample estimation period is from 1 January 1990 to 31 December 2005, and the out-of-sample forecast period is from 1 January 2006 to 22 October 2012.

significant, with the exception of the regression where the EGARCH forecast model is used. This allows us to conclude that the lagged VIX component is not encompassed in the GARCH forecasts and that VIX has some explanatory power. However, although the statistical significance of VIX is verified, due to the very small γ coefficients, its economic significance is questionable. With respect to the lagged volume parameter, δ, these draw a similar conclusion, in that while they are statistically significant the δ coefficients are very small in absolute value. Hence, again, we can say that while the volume variable is not encompassed in the GARCH forecasts, its economic significance is debatable.

The inclusion of both VIX and volume in the MZ regression again leads to similar findings. All the GARCH forecast coefficients are positive and significant with the exception of the IGARCH forecast. The lagged VIX component is positive and significant in three models (and not significant coefficient and positive for one and negative for two). The lagged volume component is always positive and significant with the exception of when the IGARCH forecast is used. The findings suggest that jointly VIX and volume carry additional information and are not encompassed in the GARCH-type forecasts. Nevertheless, when comparing the absolute values of the coefficients we find $\beta > \gamma > \delta$, with γ and δ being very small. Finally, for the MZ regression with the GARCH and VIX forecasts, we find that in all cases $\beta > \zeta$, and that all the β and ζ coefficients are significant with one exception for each, the EGARCH model for the VIX and the IGARCH model for the GARCH forecast. Overall, the findings suggest that any contribution to the volatility forecast arising from the VIX and volume variables, while statistically significant, it is economically very small.

The UK

In a similar way for the UK when looking at the coefficients of the MZ regressions with VIX and

Table 8. Encompassing test: the UK.

	$\sigma_t^2 = \alpha + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	
Parameters	Coefficient	*p*-Value	Coefficient	*p*-Value	Coefficient	*p*-Value	Coefficient	*p*-Value
GARCH								
β	0.479605	0.0000	0.785343	0.0000	0.455087	0.0000	0.479605	0.0000
γ	1.67E-05	0.0000			1.38E-05	0.0000		
δ			2.28E-10	0.0000	1.83E-10	0.0000		
ζ							1.69E-05	0.0000
TGARCH								
β	0.900403	0.0000	0.924080	0.0000	0.874062	0.0000	0.900403	0.0000
γ	4.52E-06	0.1376			2.10E-06	0.4970		
δ			1.72E-10	0.0001	1.66E-10	0.0001		
ζ							4.58E-06	0.1376
EGARCH								
β	1.520449	0.0000	1.408170	0.0000	1.453291	0.0000	1.520449	0.0000
γ	−2.23E-07	0.9449			−1.24E-06	0.7026		
δ			1.21E-10	0.0057	1.23E-10	0.0053		
ζ							−2.26E-07	0.9449
APARCH								
β	1.137433	0.0000	1.135336	0.0000	1.107211	0.0000	1.137433	0.0000
γ	3.56E-06	0.2674			9.20E-07	0.7765		
δ			1.92E-10	0.0000	1.90E-10	0.0000		
ζ							3.61E-06	0.2674
CGARCH								
β	0.513258	0.0000	0.788777	0.0000	0.490326	0.0000	0.513258	0.0000
γ	1.58E-05	0.0000			1.29E-05	0.0000		
δ			2.26E-10	0.0000	1.82E-10	0.0000		
ζ							1.60E-05	0.0000
IGARCH								
β	0.401890	0.0000	0.741875	0.0000	0.409990	0.0000	0.401890	0.0000
γ	1.80E-05	0.0000			1.40E-05	0.0000		
δ			2.50E-10	0.0000	2.01E-10	0.0000		
ζ							1.83E-05	0.0000

Notes: This table presents the coefficients and their *p*-values of the MZ equations where two regressors are included. That is, for the results in Table 4, where VIX and volume are included as lagged regressors, and for Table 6, where the forecast of VIX is included. The in-sample estimation period is from 1 January 2000 to 31 December 2007, and the out-of-sample forecast period is from 1 January 2008 to 22 October 2012.

volume, we find that GARCH forecast coefficients are all positive and significant (Table 8). The VIX coefficients are all positive with the exception of when EGARCH model is used and not significant when the asymmetric GARCH models are used in the forecasting process. In contrast, the volume coefficients are all positive and significant. When comparing the values of the coefficients as before, we find $\beta > \gamma$ and $\beta > \delta$, with again with γ and δ being very small (close to zero). Similarly, when VIX and volume are jointly included in the MZ regression, the β and δ coefficients are all positive and significant while the γ coefficients are all positive with one exception (when the EGARCH model is used), and only significant in half the cases, that is, not significant when forecasts from the asymmetric models are used in the regression. In the final MZ regression, a similar pattern as before is observed, $\beta > \zeta$ with very low ζ coefficients, positive and significant βs with one exception and positive and significant ζs with the exception of when the asymmetric GARCH models are used.

France

A similar story emerges when using the CAC index (Table 9). The GARCH forecast coefficients are positive throughout with two exceptions and significant for most cases. The VIX and volume coefficients are also positive and significant with a small number of exceptions for the VIX coefficient. Comparing the coefficients, the following relationships are established as previously: $\beta > \gamma$, $\beta > \delta$ and $\beta > \gamma > \delta$. In the final MZ regression where VIX forecasts are used after modelling VIX as an AR process, the relationship $\beta > \zeta$ is established once more.

V. Discussion

Overall the above findings suggest that VIX and volume help to produce more accurate volatility forecasts in comparison to the GARCH forecasts alone. On the basis of the MZ regressions, the results show that an increase in R^2 can be achieved, and hence the existence of additional information

Table 9. Encompassing test: France.

	$\sigma_t^2 = \alpha + \beta h_t^2 + \gamma VIX_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \delta Volume_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \gamma VIX_{t-1} + \delta Volume_{t-1} + \varepsilon_t$		$\sigma_t^2 = \alpha + \beta h_t^2 + \zeta VIX_t + \varepsilon_t$	
Parameters	Coefficient	*p*-Value	Coefficient	*p*-Value	Coefficient	*p*-Value	Coefficient	*p*-Value
GARCH								
β	0.162489	0.1689	0.811114	0.0000	−0.008268	0.9447	0.162489	0.1689
γ	3.27E-05	0.0000			3.55E-05	0.0000		
δ			7.25E-11	0.0000	8.33E-11	0.0000		
ζ							3.30E-05	0.0000
TGARCH								
β	0.749914	0.0000	0.976849	0.0000	0.541414	0.0001	0.749914	0.0000
γ	1.32E-05	0.0066			1.80E-05	0.0002		
δ			5.70E-11	0.0000	6.66E-11	0.0000		
ζ							1.33E-05	0.0066
EGARCH								
β	1.262561	0.0000	1.349642	0.0000	0.934362	0.0000	1.262561	0.0000
γ	5.96E-06	0.2531			1.24E-05	0.0213		
δ			4.98E-11	0.0002	5.86E-11	0.0000		
ζ							6.03E-06	0.2531
APARCH								
β	0.930419	0.0000	1.166883	0.0000	0.652890	0.0002	0.930419	0.0000
γ	1.17E-05	0.0242			1.74E-05	0.0010		
δ			5.75E-11	0.0000	6.70E-11	0.0000		
ζ							5.25E-06	0.0242
CGARCH								
β	0.239054	0.0295	0.810678	0.0000	0.111656	0.3101	0.239054	0.0295
γ	3.03E-05	0.0000			3.18E-05	0.0000		
δ			7.47E-11	0.0000	8.07E-11	0.0000		
ζ							3.06E-05	0.0000
IGARCH								
β	0.033162	0.7789	0.758659	0.0000	−0.073044	0.5335	0.033162	0.7789
γ	3.67E-05	0.0000			3.77E-05	0.0000		
δ			8.06E-11	0.0000	8.42E-11	0.0000		
ζ							3.71E-05	0.0000

Notes: This table presents the coefficients and their *p*-values of the MZ equations where two regressors are included. That is, for the results in Table 4, where VIX and volume are included as lagged regressors, and for Table 6, where the forecast of VIX is included. The in-sample estimation period is from 1 January 2000 to 31 December 2007, and the out-of-sample forecast period is from 1 January 2008 to 22 October 2012.

content, by including lagged values of VIX and volume. Furthermore, through the use of encompassing tests, it was found that the GARCH forecasts did not encompass the lagged values of VIX and volume or the forecasts of VIX, suggesting again that they contain some information content. However, while statistically significant, the contribution is small in terms of the gain in R^2 and the coefficient values within the encompassing tests.

This improved accuracy of the volatility forecasts confirms, to some extent, the work by Fleming, Ostdiek, and Whaley (1995), Blair, Poon, and Taylor (2001) and Ahoniemi (2008), who argue that VIX index improves the accuracy of volatility forecasts when it is used as an instrument in the forecasting process. Similarly, with respect to lagged volume, these results could be seen in line with the work by Karpoff (1987), Lamoureux and Lastrapes (1990), Bessembinder and Seguin (1993), Bollerslev and Jubinski (1999) and Luu and Martens (2003) and Taylor (2008). Notwithstanding this, our results could also be seen to confirm those of Brooks (1998), who found that lagged volume within a GARCH setting played little role in improving the out-of-sample performance of the models.

To further illustrate the nature of the results and to consider the economic significance, beyond the strength of the parameter values, we consider a simple VaR exercise. Using both a 1% (Table 10) and 5% (Table 11) cut-off, we record the number of exceptions, where losses exceed the VaR, for each model forecast. In more detail for the 1% VaR, we see that the UK and France follow a similar pattern where in most cases the inclusions of volume and VIX with volume improve the performance of the models with one exception for each where the inclusion of VIX alone improves the model; in the case of the CGARCH model. For the US, in half the cases no further improvement is found with the inclusion of VIX and volume, and in two cases the inclusion of VIX alone improves the model. Overall, it can be seen that the performance of the GARCH models is

Table 10. Summary of 1% VaR failure rates.

	Average failure rate		
Forecast model	US	UK	France
GARCH(1,1)			
GARCH	0.025	0.023	0.021
GARCH VIX	0.037	0.022	0.024
GARCH volume	0.029	0.012	0.013
GARCH VIX volume	0.032	0.012	0.013
TGARCH			
TGARCH	0.025	0.028	0.016
TGARCH VIX	0.029	0.027	0.017
TGARCH volume	0.036	0.012	0.012
TGARCH VIX volume	0.031[a]	0.012	0.012
EGARCH			
EGARCH	0.031	0.029	0.018
EGARCH VIX	0.029	0.025	0.018
EGARCH volume	0.039	0.029	0.019
EGARCH VIX volume	0.019	0.023	0.018
APARCH			
APARCH	0.030	0.029	0.019
APARCH VIX	0.029	0.022	0.020
APARCH volume	0.031	0.012	0.067
APARCH VIX volume	0.032	0.012	0.067
CGARCH			
CGARCH	0.025	0.022	0.021
CGARCH VIX	0.034	0.019	0.020
CGARCH volume	0.026	0.025	0.023
CGARCH VIX volume	0.031	0.021	0.033
IGARCH			
IGARCH	0.028	0.024	0.017
IGARCH VIX	0.024	n/a	0.014
IGARCH volume	0.029	n/a	n/a
IGARCH VIX volume	n/a	n/a	n/a

Notes: For the IGARCH model, the majority of the VaR calculations were not possible due to negative variances. The VaR values are based on the out-of-sample forecast results reported above.
[a]Average were used to calculate the VaR failure rate.

Table 11. Summary of 5% VaR failure rates.

	Average failure rate %		
Forecast model	US	UK	France
GARCH(1,1)			
GARCH	0.061	0.070	0.068
GARCH VIX	0.075	0.068	0.071
GARCH volume	0.063	0.037	0.037
GARCH VIX volume	0.071	0.037	0.037
TGARCH			
TGARCH	0.062	0.072	0.072
TGARCH VIX	0.071	0.067	0.071
TGARCH volume	0.076	0.032	0.038
TGARCH VIX volume	0.032[a]	0.032	0.038
EGARCH			
EGARCH	0.070	0.078	0.072
EGARCH VIX	0.061	0.070	0.070
EGARCH volume	0.079	0.082	0.070
EGARCH VIX volume	0.055	0.068	0.070
APARCH			
APARCH	0.064	0.079	0.075
APARCH VIX	0.067	0.069	0.074
APARCH volume	0.067	0.036	0.098
APARCH VIX volume	0.071	0.036	0.098
CGARCH			
CGARCH	0.062	0.068	0.071
CGARCH VIX	0.075	0.068	0.072
CGARCH volume	0.064	0.070	0.072
CGARCH VIX volume	0.071	0.070	0.085
IGARCH			
IGARCH	0.061	0.068	0.070
IGARCH VIX	0.058	n/a	0.062
IGARCH volume	0.063	n/a	n/a
IGARCH VIX volume	n/a	n/a	n/a

Notes: For the IGARCH model, the majority of the VaR calculations were not possible due to negative variances. The VaR values are based on the out-of-sample forecast results reported above.
[a]Average were used to calculate the VaR failure rate.

improved with the inclusion of VIX and volume and that in the case of asymmetric GARCH models this improvement is more obvious confirming our previous findings. For the 5% VaR, generally similar findings are reported. The inclusion of VIX and volume improves the models especially for the asymmetric GARCH models and for GARCH(1,1). In both the 1% and 5% VaR exercises, the TGARCH model gives the lowest failure rate and on the whole the performance the models was found to be better in the 5% VaR exercise.

Overall, the results here suggest two broad conclusions. First, in terms of the preferred forecast model for stock index return volatility for the three markets of the US, the UK and France, the EGARCH model performs the best in terms of the highest R^2. Second, the inclusion of VIX and volume provides a small but significant increase in that forecast power. Furthermore, the economic significance of this improvement is also confirmed for most cases and especially for the asymmetric GARCH models using a VaR analysis. However, when examining forecasts of VIX alone we can observe that the GARCH approach, in particular, the asymmetric GARCH models are preferred.

VI. Summary and conclusion

After evaluating the performance of GARCH-type models by producing one-step-ahead volatility forecasts for the three markets of the US, the UK and France, the explanatory power of VIX and trading volume is assessed and compared, within an MZ regression framework. The results show that the forecast models perform slightly better when VIX and volume are included as independent or forecasted variables, although the results are less clear-cut when they are included as regressors in the GARCH variance equation. We thus provide evidence that VIX and volume have additional explanatory power in forecasting returns volatility.

When assessing the significance of the added components and their explanatory power through a forecast encompassing exercise, it is revealed that both VIX and volume in the majority of cases have positive and significant coefficients, suggesting that they carry additional information and are not encompassed by the GARCH-type forecast. However, although statistically their added value is confirmed, the coefficients are very small, and in most cases close to zero especially for volume, suggesting that economically their added value is of little importance. The economic significance of our findings is assessed using a VaR framework for 1% and 5% failure rates, replicating the approach followed by practitioners, where it is found that the inclusion of VIX and volume overall improve on the performance of the models, especially for the asymmetric GRACH models.

To summarize, our results suggest that asymmetric GARCH models provide the preferred forecast, while VIX and volume contribute a small but significant additional degree of forecast power and information not contained in the GARCH forecasts.

Disclosure statement

No potential conflict of interest was reported by the authors.

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