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# Forecasting the Volatility of Stock Market Index Using the Hybrid Models with Google Domestic Trends

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In order to improve the forecasting accuracy of the volatilities of the markets, we propose the hybrid models based on artificial neural networks with multi-hidden layers in this paper. Specifically, the hybrid models are built using the estimated volatilities obtained from GARCH family models and Google domestic trends (GDTs) as input variables. We further carry out many experiments varying the number of layers and activation functions to obtain the accurate hybrid model for forecasting volatility. The proposed models are applied to forecast weekly and monthly volatilities of S&P 500 index to verify their accuracy. The performance comparison results show that the hybrid models with GDTs outperform clearly the predicted results with GARCH family models and the hybrid models without GDTs in forecasting the volatility of actual market. We also provide the experiment results with graphs to illustrate the efficiency of models.

*Keywords*: Artificial neural network; google domestic trends; GARCH models; volatility forecasting; hybrid model.

## 1. Introduction

Forecasting of financial data volatility is one of the most important tasks in the financial market. The exact forecasting of volatility is very important to the investors since the accurate forecast helps traders, investors and risk managers who want to

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obtain definite benefits. That is, volatility forecast is an essential key factor in portfolio selection, risk management, monetary policy and option pricing. For this reason, improvement of the forecasting model for the financial data volatility remains still an important issue, and many researchers are investigating to improve the accuracy of volatility forecasts continually.

In order to forecast volatility of the financial data, various statistical models are developed. Engle [1] introduced the ARCH (Autoregressive Conditional Heteroscedasticity) to model the heteroscedasticity of the financial time series. Bollerslev [2] suggested the GARCH (generalized ARCH) model with the moving average terms. Since GARCH model can capture the persistence of volatility, the so-called volatility clustering, GARCH model has become the popular models for forecasting the financial data volatility. However, the GARCH model has the limitation that fails to describe the asymmetric volatility (leverage effect). In order to overcome this limitation, the EGARCH (Exponential GARCH) model by Nelson [3] and the GJR-GARCH (the threshold GARCH) by Glosten *et al.* [4] were proposed. GARCH family models have been developed by many researchers to ensure accurate forecasting in finance and economics for long periods. However, the GARCH family models usually assume a linear correlation among the time-varying data. That is, GARCH family models are hard to capture nonlinear correlation and complex fluctuation of financial time-varying data. Thus, nonparametric method such as ANN (Artificial Neural Network) has been adopted for better forecasting of various complex financial data.

ANN has been developed for accurate forecasting of financial time series by many researchers. Recently, hybrid models based on various models for forecasting the time series data have been proposed to improve the forecast of financial data. Specifically, the hybrid ANN-GARCH models have often been used to increase the forecasting ability of the financial time series such as stock market index, volatility of market index, exchange rate, option price, oil price and gold price, etc. [5–15]. The results of the studies have showed that the proposed hybrid models provide better forecasts of financial time series data. In addition, Qiu *et al.* [16] used a hybrid approach based on a genetic algorithm and simulated annealing to improve the prediction of returns of the Japanese Nikkei 225 index, and Liu and Fu [17] suggested a hybrid algorithm using grey model and extreme learning machine for efficient forecasting of volatility of China Interbank Offered Rate.

There are many studies with the hybrid models for forecasting of financial data volatility in recent years. Roh [18] proposed hybrid models with ANN and GARCH models for forecasting the volatility of KOSPI 200 time series data. Bildirici and Ersin [19] used APGARCH model with ANN to evaluate the volatility. The ANN-APGARCH model was applied to Istanbul stock market and showed the improvement of forecasting performance. Hajizadeh *et al.* [20] proposed two hybrid models to forecast the volatility of the S&P 500 index. They used various explanatory variables related to the S&P 500 and the simulated volatility time series by the GARCH family models as input variables. The proposed hybrid model based on EGARCH model showed better performance than the classic GARCH models in forecasting realized volatility. Lahmiri [21] also used the hybrid model based on EGARCH model to improve the forecasting accuracy of the stock market indices in Morocco and Saudi Arabia. Lahmiri *et al.* [22] presented an ensemble prediction system based on the hybrid EGARCH-ANN models to improve the forecasting accuracy. Kristjanpoller *et al.* [23] adopted a hybrid ANN-GARCH model for forecasting of three Latin-American stock exchange indices. The results indicated that the ANN improves the forecasting ability of the GARCH models when studied in the three Latin-American stock markets.

The search engine Google has been accumulating the volume of quires related to various searches since 2004. From this database, Google provides the publicly available service for analyzing of finance and economic as the Google domestic trends (GDTs). Recently, by using Google trends, Preis *et al.* [24] found the patterns that may know early warning sign of stock market moves. Xiong *et al.* [25] used a Long Short-Term Memory (LSTM) neural network to forecast the S&P 500 volatility based on GDTs with market data. Additionally, by using Google search volume, Hamid and Heiden [26] proposed an empirical similarly method for more accurate forecast of the stock market volatility, and Dimpfl and Jank [27] investigated a movement of the realized volatility and the volume of search queries. France and Shi [28] provided a heuristic algorithm for analyzing Google trends data in economic research. Siliverstovs and Wochner [29] also showed that the prediction based on Google trends highly provides an approximation of Swiss tourism demand. In this paper, we utilize Google trends data for accurate volatility forecasts. Specifically, we propose the hybrid models integrated artificial neural networks (ANNs) with multi-hidden layers and GARCH family models to provide more accurate forecasts of the realized volatility of the S&P 500 index.

The remainder of this paper is organized as follows. Section 2 introduces the models used in this study to construct the hybrid models. Section 3 describes the data characteristics used in this study. Section 4 provides the hybrid models with GDTs for improving of forecasting the volatility of S&P 500 index and presents experimental results of comparison studies. Section 5 gives concluding remarks.

## 2. GARCH Volatility Models and Artificial Neural Networks

In this section, we present the overview of models employed in this study. We firstly review GARCH family models. Concretely, GARCH, EGARCH and GJR-GARCH which are used to construct the hybrid models are described. We also present ANN with multi-hidden layers.

### 2.1. *GARCH model*

The ARCH model was the first model of the volatility models with conditional heteroscedasticity, but the GARCH model has been the most known model for forecasting the volatility of time series since the GARCH model is able to capture the volatility clustering and has a simple parametric representation to describe the volatility evolution. In the GARCH model, the forecast of volatility is obtained using a constant depicting constant variance throughout trading days, a sum of $p$ weighted products of last periods' forecast and the sum of $q$ weighted products of last periods' squared residual term.

The GARCH $(p, q)$ model is defined as

$$\begin{aligned} \varepsilon_t &= \sigma_t Z_t, \\ \sigma_t^2 &= w + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2, \end{aligned} \tag{1}$$

where $\varepsilon_t$ is a residual term of time series which is a product of a time-dependent volatility, and a random variable $Z_t$ is a standardized sequence of independent and identically distributed (i.i.d) with zero mean and unit variance. The parameters $w, \alpha_i, \beta_i$ of the model are nonnegative coefficients, the parameter $p$ is the degree of the term $\sigma_t$ and the parameter $q$ is the degree of the term $\varepsilon_t$.

### 2.2. *EGARCH model*

The exponential GARCH (EGARCH) model was proposed to capture asymmetric behavior of the returns and describes the volatility leverage effect by Nelson [3]. The GARCH model is restrictive because of the nonnegative constraints in the linear GARCH model. That is, the GARCH model has nonnegative constraints on parameters $w, \alpha_i$ and $\beta_i$ in Eq. (1), however, EGARCH model has no parameter restrictions and allows the negative parameters. Moreover, volatility of the EGARCH model has an explicit multiplicative function of lagged innovations unlike the GARCH model.

The EGARCH $(p, q)$ model is given by

$$\log \sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i \left[ \frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}} + \gamma \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right] + \sum_{i=1}^{q} \beta_i \log \sigma_{t-i}^2, \tag{2}$$

where the parameters $\alpha_i$ and $\beta_i$ have no restrictions to ensure nonnegativity of the conditional variance and $\gamma$ is the asymmetric leverage coefficient to reflect the leverage effect of volatility.

### 2.3. *GJR-GARCH model*

The GJR-GARCH model proposed by Glosten *et al.* [4] is one of the asymmetry-type volatility models in common with EGARCH model and nonlinear GARCH family models. This model expresses the leverage effect in the quadratic form while EGARCH model expresses the leverage effect in the exponential form and captures the potential larger impact of negative shocks which have a larger impact on volatility than positive shocks of the same magnitude. Moreover, the GJR-GARCH model has showed better forecasting performance than other GARCH models [30]. In the GJR-GARCH model, the conditional variance is defined as a linear piecewise function.

The conditional variance of GJR-GARCH $(p, q)$ model is expressed as

$$\sigma_t^2 = w + \sum_{i=1}^{p}[\alpha_i + \gamma_i \mathbf{1}_{\{\varepsilon_{t-i}<0\}}]\varepsilon_{t-i}^2 + \sum_{i=1}^{q}\beta_i\sigma_{t-i}^2, \tag{3}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function that equals 1 if the previous period residual is negative and zero otherwise. The model must satisfy the following conditions:

$$w \geq 0, \quad p \geq 0, \quad q \geq 0, \quad \alpha_i \geq 0, \quad \beta_i \geq 0, \quad \alpha_i + \gamma_i \geq 0 \quad \text{and}$$

$$\sum_{i=1}^{p}\alpha_i + \sum_{i=1}^{q}\beta_i + \frac{1}{2}\sum_{i=1}^{q}\gamma_i < 1,$$

where $\gamma_i$ means the asymmetric leverage coefficient is similar to the coefficient $\gamma$ in the EGARCH model.

### 2.4. *Artificial neural networks* (*ANNs*)

Many financial models assume a linear correlation structure among the time series data while there exist nonlinear patterns in such data. Approximations of such real world problems which have the nonlinear features by linear models have not been satisfactory. Artificial neural networks (ANNs) are a computing system inspired by the biological neural network and non-parametric nonlinear models which overcome the limitations of the linear models. They are used to solve problems in the same way as the human brain. Based on the architecture of human brain, a set of processing elements or neurons is interconnected and organized in layers. These layers can be organized hierarchically with input layer, middle layers and output layer [20]. That is, ANNs consist of an input layer, hidden layers (middle layers) and an output layer with adjoining interlayer nodes linked by acyclic connections. ANNs also do not need any hypothesis about the underlying model since they have a flexible nonlinear modeling capability. In addition, the models are constructed adaptively based on the features extracted from the data.

The output from hidden layers is given by

$$y = f\left(\sum_i x_i w_i\right),$$

where $x_i$ is the set of input variable from node $i$ in the previous layer, $w_i$ is the weight for input variable $x_i$ that connects the node $i$ and $f$ is an activation function. There are many activation functions such as sigmoid function, hyperbolic tangent function and rectified linear unit (ReLU), which are chosen for the better ANNs.

There are two main types of artificial neural networks: feed-forward neural network and back propagation neural network. The feed-forward neural network allows

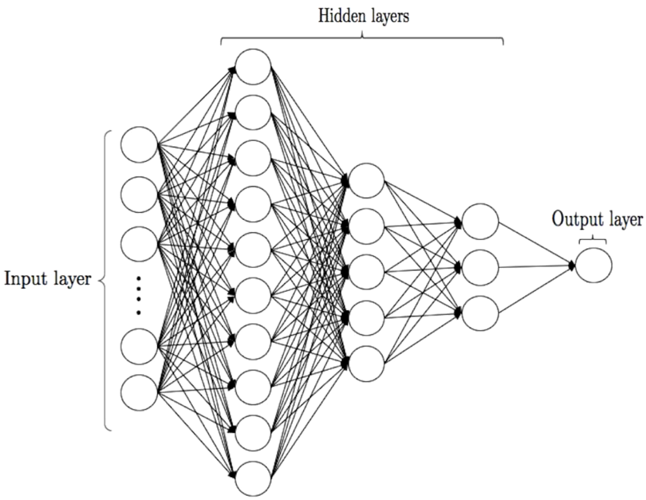



Fig. 1. The structure of neural network.

signals in one way from input to output. On the other hand, back propagation neural network takes inputs only from the previous layer and sends outputs only to the next layer. In the back propagation neural network, the weights of networks are decided using the minimization of the loss function between real values and predicted values. The sum of squares (Euclidean) loss is often used as a loss function. In this study, for better forecasting, we apply back propagation neural network which is a method for supervised learning. The back propagation neural network is generally the most widely used network in financial applications [5].

Figure 1 shows the neural network with multi-hidden layers used in this study for forecasting. Most of researchers agree that many hidden layers lead to the better performance. However, many hidden layers cause the over-fitting problem. In order to overcome this problem, we choose the number of optimized hidden layers and one activation function after many experiments.

## 3. Data

The S&P 500 index prices over the period of 3 January 2007 to 30 December 2016 are downloaded from Bloomberg terminal. And the GDTs data is freely available to the public from the Google finance website.[a] Google has collected the daily volume of searches related to various economic and finance sectors. In this paper, we consider these data as a representation of the public interest in the various macroeconomic factors and environmental variables. That is, Google trends data is used as the input data to the ANNs.

[a] https://finance.google.com/finance/domestic_trends.

Table 1. Google domestic trends used in this paper.

| Google domestic trend | Contractions |
|---|---|
| Bankruptcy | BNKRPT |
| Computers and electronics | COMPUT |
| Credit cards | CRCARD |
| Finance and investing | INVEST |
| Mortgage | MTGE |

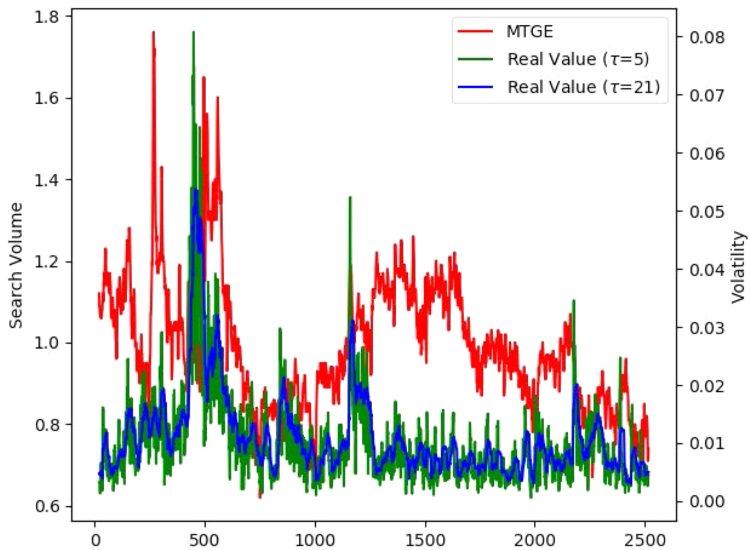



Fig. 2. Relative search volume on Mortgage scaled by the value at the beginning of 1 January 2004.

We use the five trends among the GDTs as the input variables. Selection in GDTs depends on the knowledge of which ones influence on the stock market. Specifically, we choose the explanatory variables based on the work of Xiong *et al.* [25]. Table 1 presents GDTs which are used in this study.

The data set is selected in GDTs from 1 January 2007 to 31 December 2016 and the moving average is calculated as the input variables. In Fig. 2, 7 days moving average of mortgage and the realized volatilities are shown over the given period.

If $p_t$ is the S&P 500 index price at time $t$, the return $r_t$ of the logarithmic price at time $t$ is defined as $r_t = \log p_t - \log p_{t-1}$. We predict the realized volatility[b] ($\mathrm{RV}_t^\tau$) of the log return for $\tau$ days of the return $r_t$. The realized volatility of the log return

[b] We assume five trading days in a week and 21 trading days in a month and focus on weekly volatilities and monthly volatilities. That is, in this study, we only consider $\mathrm{RV}_t^\tau$ when $\tau = 5$ and $\tau = 21$.

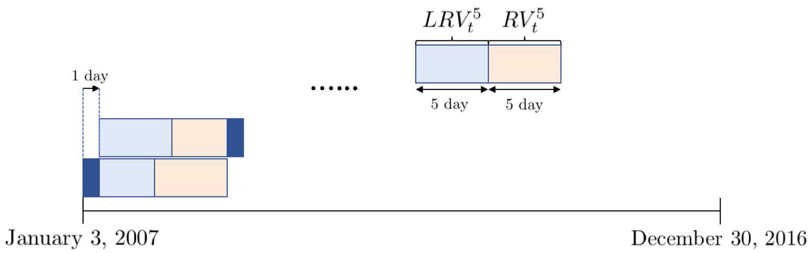



Fig. 3. 1-day lagged realized volatility $\mathrm{LRV}_t^5$.

for $\tau$ days at time $t$ is calculated in $\tau$ days windows into the future from time $t$ and defined as

$$\mathrm{RV}_t^\tau = \frac{1}{\tau}\sum_{i=t+1}^{t+\tau}(r_i - \bar{r}_t)^2,$$

where $\bar{r}_t$ is mean of the log return during $\tau$ days after time $t$. We also use the lagged realized volatility ($\mathrm{LRV}_t^\tau$) as the endogenous variable to improve forecast accuracy. $\mathrm{LRV}_t^\tau$ is calculated by same size of window used in computing $\mathrm{RV}_t^\tau$ and defined as

$$\mathrm{LRV}_t^\tau = \frac{1}{\tau}\sum_{i=t-1}^{t-\tau}(r_i - \bar{r}_t)^2.$$

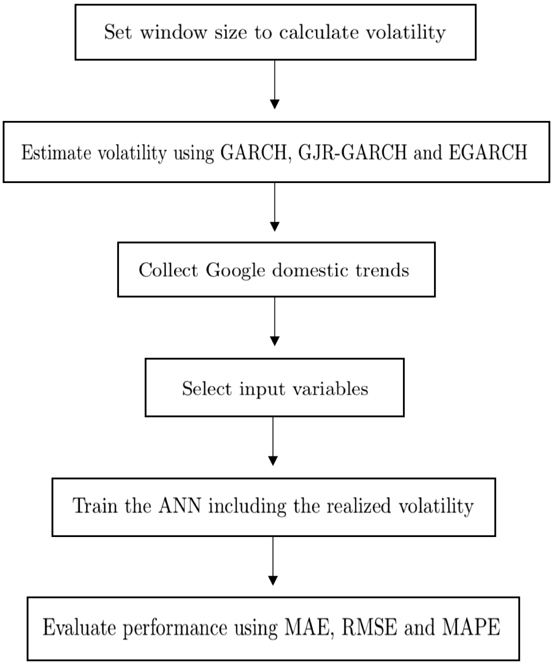



Fig. 4. Flowchart of hybrid model.

Lastly, we use the outputs of the GARCH family models referred in Sec. 2 as the input variables. In short, we use GDTs and GARCH family outputs as the exogenous variables and $\mathrm{LRV}_t^\tau$ as the endogenous variables.

We note that days in windows of $\mathrm{LRV}_t^\tau$ do not have intersection with $\tau$ days in windows of $\mathrm{RV}_t^\tau$. For more details, refer to Fig. 3. We also use averages of GDTs over the window size $\tau$ as inputs to neural networks. Figure 4 presents the flowchart of our model briefly. The data set is divided into two parts: 80% for training and 20% for testing.

## 4. Experiments and Results

In this study, we propose the efficient hybrid models with GDTs to improve the accuracy in forecasting volatility of S&P 500 index. The models are built from input variables to neural network using the outputs of the GARCH family and GDTs. The models are categorized by whether variables are to be included or not. Specifically, we propose GDTs models, GARCH-LRV models and GARCH-GDT-LRV models. As you can guess by names, each model adopts different variables as input variables. GDT models use $\mathrm{LRV}_t^\tau$ and GDTs. GARCH-LRV models and GARCH-GDT-LRV models use, respectively, the following data sets:

$$\{\mathrm{LRV}_t^\tau, g_t^{(\tau,\mathrm{BNKRPT})}, g_t^{(\tau,\mathrm{COMPUT})}, g_t^{(\tau,\mathrm{CRCARD})}, g_t^{(\tau,\mathrm{INVEST})}, g_t^{(\tau,\mathrm{MTGE})}\},$$
$$\{\mathrm{GARCH}_t^{(\tau,\mathrm{outputs})}, \mathrm{LRV}_t^\tau, g_t^{(\tau,\mathrm{BNKRPT})}, g_t^{(\tau,\mathrm{COMPUT})}, g_t^{(\tau,\mathrm{CRCARD})}, g_t^{(\tau,\mathrm{INVEST})}, g_t^{(\tau,\mathrm{MTGE})}\},$$

where $\tau$ is the size of time window used to calculate the volatilities, $g_t^{(\tau,\cdot)}$ is the mean of GDTs '$\cdot$' in $\tau$ days before time $t$ and $\mathrm{GARCH}_t^{(\tau,\mathrm{outputs})}$ is the GARCH forecast in $\tau$ days ahead of time $t$. From these models, we find the best hybrid model for forecasting of volatility of S&P 500 index price.

In order to determine the best model, we carry out the comparative studies with different performance measures. Specifically, we use three performance measures to evaluate the forecast accuracy. They are as follows: mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE).

$$\mathrm{MAE}_\tau = n^{-1}\sum_{i=1}^{n} |\hat{\sigma}_i - \mathrm{RV}_i^\tau|,$$
$$\mathrm{RMSE}_\tau = \left(n^{-1}\sum_{i=1}^{n} (\hat{\sigma}_i^\tau - \mathrm{RV}_i^\tau)^2\right)^{1/2},$$
$$\mathrm{MAPE}_\tau = n^{-1}\sum_{i=1}^{n} \left|\frac{\hat{\sigma}_i^\tau - \mathrm{RV}_i^\tau}{\mathrm{RV}_i^\tau}\right|,$$

where $\hat{\sigma}_i^\tau$ is the predicted value, $\mathrm{RV}_i^\tau$ means weekly ($\tau = 5$) or monthly ($\tau = 21$) actual volatility of the logarithmic return $r_t$ of the S&P 500 index price and $n$ is the number of actual volatilities observed from the test data set.

The models are obtained by different combination of layers and neurons per layers. This means that there are many models depending on the number of layers and neurons. The number of neurons depends on the number of input variable, and the number of layers is based on empirical evidences. Thus, after many experiments, we choose three hidden layers which provides the best performance. Concretely, the number of neurons in the first hidden layer is 10, the number of neurons in the second hidden layer is 5, and the number of neurons in the third hidden layer is 3. We also use the Adam optimizer algorithm to update network weights iterative from training data set and a method for efficient stochastic optimization that requires only first order gradients with little memory requirement. The algorithm generates individual adaptive learning rates for different parameters from estimates of first and second moments of the gradients. The Adam optimizer also is well suited to a wide range of non convex optimization problems in the area of machine learning [31]. In addition, we have to choose an activation function. Many researchers have used smoother functions such as $\tanh(x)$ or $1/(1+\exp(-x))$ as an activation function. However, we choose the ReLU as an activation function after testing of functions. The ReLU is one of the nonlinear activation functions and given by $f(x) = \max(x, 0)$. The ReLU also is well known to be efficient in networks with many hidden layers allowing training of a deep supervised network without unsupervised training [32].

In order to show the accuracy of the proposed hybrid models, we conduct the comparative experiments with three performance measures on forecasting the volatility of S&P 500 index. The GDTs data and the values obtained from GARCH family models are used as input variables to enhance the forecasting ability of ANNs. Specifically, GARCH, EGARCH and GJR-GARCH are used and evaluated, and GARCH forecasts with combinations of $(p, q)$ parameters ranging from (1,1) to (3,3) are applied to ANNs. To show forecast ability of GARCH family models, we first use only three GARCH models to forecast the volatilities for 5 and 21 days ahead. The results are presented in Table 2. According to the results in Table 2, we can see that the GJR-GARCH(2,2) and the GARCH(3,3) are the best models for 5 days ahead and for 21 days ahead, respectively. Table 3 indicates that the GDT model outperforms the GARCH models for each day. This means that the ANNs with GDTs have better predictive ability than GARCH family models and GDTs are meaningful factors which help good forecasts. We also use $\mathrm{LRV}_t$ and the estimated volatilities by the GARCH models as input variables to the ANN. Table 4 presents the results that MAE, RMSE and MAPE increase in most cases with comparison with the results in Table 3. Based on these results, we finally propose the GARCH-GDT-LRV models (ANN-GARCH models with GDTs) which are used the outputs of GARCH models, the GDTs and $\mathrm{LRV}_t$ as input variables in the ANN.

Table 5 shows results obtained by the GARCH-GDT-LRV models. We can find that the performance measures decrease generally compared with results of other models. GJR-GARCH(2,2)-GDT-LRV model and GARCH(3,3)-GDT-LRV model outperform the other models for the window size 5 days and window size 21 days, respectively. The forecasting errors is reduced significantly in the proposed models.

Table 2. Performance results for GARCH models.

| Measure | GARCH(1,1) | GARCH(2,2) | GARCH(3,3) |
|---|---|---|---|
| $MAE_5$ | 0.0054412 | 0.0034339 | 0.0034532 |
| $RMSE_5$ | 0.0123764 | 0.0044474 | 0.0044766 |
| $MAPE_5$ | 1.0762183 | 0.6486662 | 0.6524111 |
| $MAE_{21}$ | 0.0051524 | 0.0034328 | 0.0033150 |
| $RMSE_{21}$ | 0.0106278 | 0.0041228 | 0.0040228 |
| $MAPE_{21}$ | 0.7644555 | 0.5101468 | 0.4857470 |
| **Measure** | **GJR-GARCH(1,1)** | **GJR-GARCH(2,2)** | **GJR-GARCH(3,3)** |
| $MAE_5$ | 6.7674909 | 0.0034248 | 0.3565705 |
| $RMSE_5$ | 79.0580707 | 0.0044078 | 7.6476430 |
| $MAPE_5$ | 877.2793273 | 0.6582663 | 53.1925044 |
| $MAE_{21}$ | 4.7420302 | 0.0035203 | 0.3423461 |
| $RMSE_{21}$ | 63.0943498 | 0.0041752 | 7.1748520 |
| $MAPE_{21}$ | 491.7272873 | 0.5235059 | 48.7805455 |
| **Measure** | **EGARCH(1,1)** | **EGARCH(2,2)** | **EGARCH(3,3)** |
| $MAE_5$ | 0.0035275 | 0.0036532 | 0.0036680 |
| $RMSE_5$ | 0.0044588 | 0.0046099 | 0.0046291 |
| $MAPE_5$ | 0.6840203 | 0.7022439 | 0.7072368 |
| $MAE_{21}$ | 0.0039638 | 0.0041907 | 0.0041929 |
| $RMSE_{21}$ | 0.0045174 | 0.0047817 | 0.0047786 |
| $MAPE_{21}$ | 0.6000719 | 0.6429069 | 0.6436826 |

Concretely, compared with the results of Table 2, the MAPE of GJR-GARCH(2,2)-GDT-LRV model for 5 days window is reduced by 36%, and the MAPE of GARCH (3,3)-GDT-LRV model for 21 days window is reduced by 43%. Figures 5 and 6 provide four best results among the GARCH-GDT-LRV models with graphs to show model validity, respectively. Specifically, Figs. 5 and 6 present the predicted values based on the GARCH-GDT-LRV models and the realized volatilities of the actual market. We also apply the Model Confidence Set (MCS) to analyze the robustness of the results [33]. The results show that GJR-GARCH(2,2)-GDT-LRV model is the best model for window size 5 days and GARCH(3,3)-GDT-LRV model the best model for window size 21 days. Table 6 shows more detailed results of the MCS test.

Table 3. Performance results for GDT models.

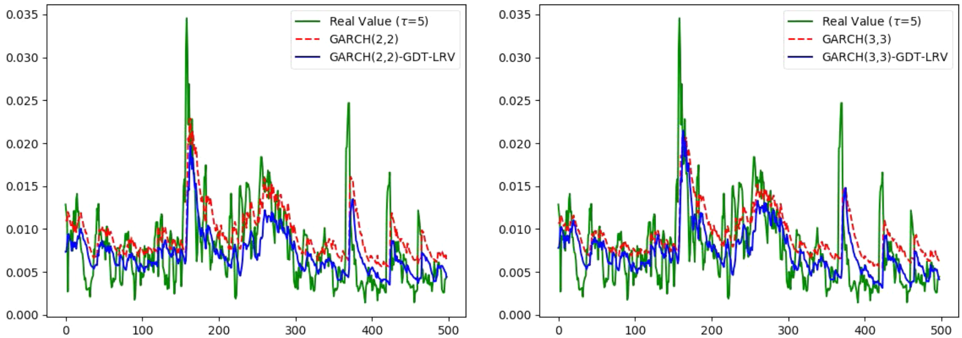
| Measure | GDT model |
|---|---|
| $MAE_5$ | 0.0029892 |
| $RMSE_5$ | 0.0043784 |
| $MAPE_5$ | 0.4525253 |
| $MAE_{21}$ | 0.0024937 |
| $RMSE_{21}$ | 0.0034300 |
| $MAPE_{21}$ | 0.3566237 |

Table 4. Performance results for GARCH-LRV models.

| Measure | GARCH(1,1) | GARCH(2,2) | GARCH(3,3) |
|---|---|---|---|
| $MAE_5$ | 0.0036590 | 0.0030523 | 0.0031029 |
| $RMSE_5$ | 0.0060724 | 0.0042841 | 0.0042482 |
| $MAPE_5$ | 0.6071080 | 0.5170638 | 0.5340934 |
| $MAE_{21}$ | 0.0035434 | 0.0027618 | 0.0027338 |
| $RMSE_{21}$ | 0.0058934 | 0.0037578 | 0.0037212 |
| $MAPE_{21}$ | 0.4769430 | 0.3663534 | 0.3614394 |
| Measure | GJR-GARCH(1,1) | GJR-GARCH(2,2) | GJR-GARCH(3,3) |
| $MAE_5$ | 0.0169583 | 0.0030014 | 0.0115610 |
| $RMSE_5$ | 0.1186654 | 0.0042819 | 0.0605740 |
| $MAPE_5$ | 2.1028007 | 0.5024996 | 1.4742654 |
| $MAE_{21}$ | 0.0093543 | 0.0029573 | 0.0069032 |
| $RMSE_{21}$ | 0.0921995 | 0.0038750 | 0.0540599 |
| $MAPE_{21}$ | 1.0381875 | 0.4051065 | 0.9444580 |
| Measure | EGARCH(1,1) | EGARCH(2,2) | EGARCH(3,3) |
| $MAE_5$ | 0.0031594 | 0.0031881 | 0.0032202 |
| $RMSE_5$ | 0.0043207 | 0.0043398 | 0.0043417 |
| $MAPE_5$ | 0.5453240 | 0.5600085 | 0.5718415 |
| $MAE_{21}$ | 0.0030881 | 0.0031514 | 0.0031676 |
| $RMSE_{21}$ | 0.0039084 | 0.0040002 | 0.0039964 |
| $MAPE_{21}$ | 0.4333269 | 0.4406900 | 0.4491985 |

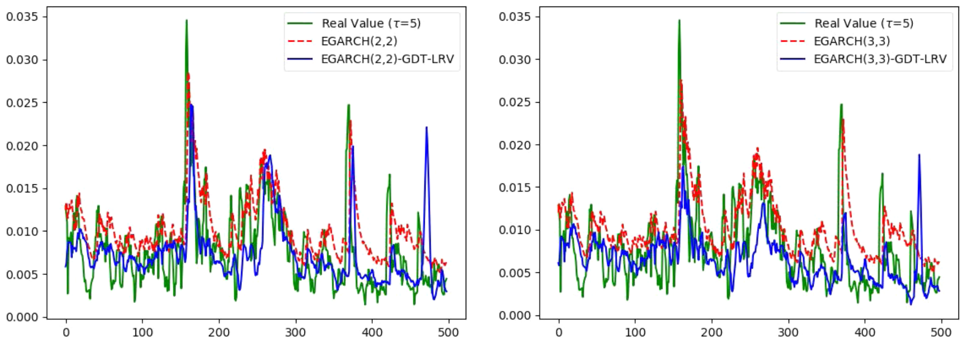
Table 5. Performance results for GARCH-GDT-LRV models.

| Measure | GARCH(1,1) | GARCH(2,2) | GARCH(3,3) |
|---|---|---|---|
| $MAE_5$ | 0.0033817 | 0.0029006 | 0.0029640 |
| $RMSE_5$ | 0.0056052 | 0.0042375 | 0.0042816 |
| $MAPE_5$ | 0.4721284 | 0.4408817 | 0.4631082 |
| $MAE_{21}$ | 0.0028448 | 0.0023929 | 0.0020568 |
| $RMSE_{21}$ | 0.0036936 | 0.0033211 | 0.0026739 |
| $MAPE_{21}$ | 0.3792814 | 0.2778062 | 0.2745517 |
| Measure | GJR-GARCH(1,1) | GJR-GARCH(2,2) | GJR-GARCH(3,3) |
| $MAE_5$ | 0.0097825 | 0.0028438 | 0.0048791 |
| $RMSE_5$ | 0.0632095 | 0.0042104 | 0.0369150 |
| $MAPE_5$ | 1.6054999 | 0.4262675 | 0.7547744 |
| $MAE_{21}$ | 0.0038544 | 0.0023993 | 0.0035819 |
| $RMSE_{21}$ | 0.0050561 | 0.0035170 | 0.0050376 |
| $MAPE_{21}$ | 0.4627807 | 0.2978525 | 0.4238438 |
| Measure | EGARCH(1,1) | EGARCH(2,2) | EGARCH(3,3) |
| $MAE_5$ | 0.0030784 | 0.0031366 | 0.0031824 |
| $RMSE_5$ | 0.0046327 | 0.0046410 | 0.0046522 |
| $MAPE_5$ | 0.4712633 | 0.4695684 | 0.4522580 |
| $MAE_{21}$ | 0.0026842 | 0.0027532 | 0.0027567 |
| $RMSE_{21}$ | 0.0035732 | 0.0035873 | 0.0036890 |
| $MAPE_{21}$ | 0.3256046 | 0.3333536 | 0.3324854 |




(a) GARCH(2,2)-GDT-LRV

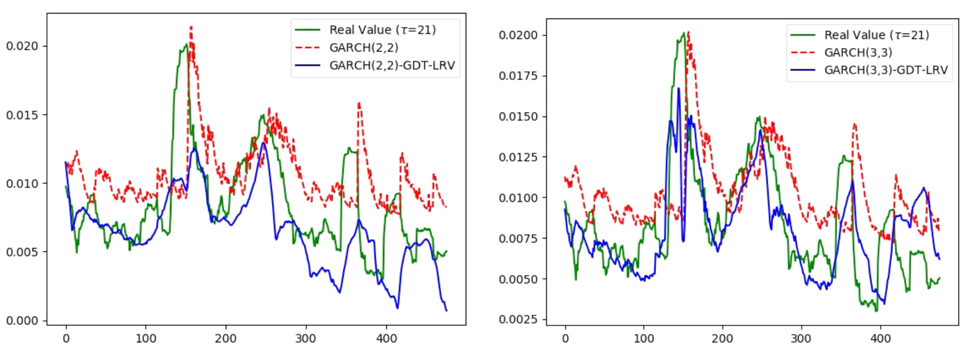
(b) GARCH(3,3)-GDT-LRV




(c) EGARCH(2,2)-GDT-LRV

(d) EGARCH(3,3)-GDT-LRV

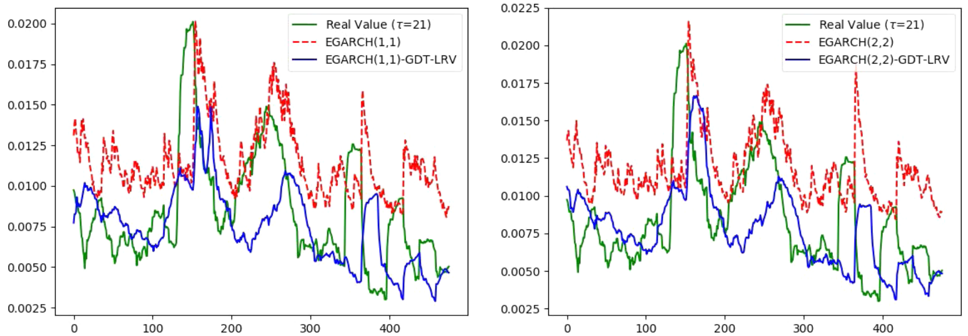
Fig. 5. The realized volatilities $\mathrm{RV}_t^5$ (Green) of actual market and predicted values (Red) from the GARCH-GDT-LRV models.




(a) GARCH(2,2)-GDT-LRV

(b) GARCH(3,3)-GDT-LRV

Fig. 6. The realized volatilities $\mathrm{RV}_t^{21}$ (Green) of actual market and predicted values (Red) from the GARCH-GDT-LRV models.




(c) EGARCH(2,2)-GDT-LRV

(d) EGARCH(3,3)-GDT-LRV

Fig. 6. (*Continued*)

Table 6. Model confidence set.

| Loss function | 5 Days | | |
|---|---|---|---|
| Ranking | Model | MAE | MCS |
| 1 | GJR-GARCH(2,2)-GDT-LRV | 0.0028438 | 1.000 |
| 2 | GARCH(2,2)-GDT-LRV | 0.0029006 | 0.578 |
| 3 | GARCH(3,3)-GDT-LRV | 0.0029640 | 0.578 |
| 4 | GJR-GARCH(2,2)-LRV | 0.0030014 | 0.578 |
| 5 | GJR-GARCH(3,3)-GDT-LRV | 0.0048791 | 0.546 |
| Loss function | 21 Days | | |
| Ranking | Model | MAE | MCS |
| 1 | GARCH(3,3)-GDT-LRV | 0.0020568 | 1.000 |
| 2 | GARCH(2,2)-GDT-LRV | 0.0023929 | 0.540 |
| 3 | GJR-GARCH(2,2)-GDT-LRV | 0.0023993 | 0.540 |
| 4 | GDT | 0.0024937 | 0.472 |
| 5 | EGARCH(1,1)-GDT-LRV | 0.0026842 | 0.472 |

Table 7. The input variables for the ANN-GARCH model without Google domestic trends.

| Selected input variables based on the model of Hajizadeh *et al.* [20] |
|---|
| Dow Jone return |
| NASDAQ return |
| S&P 500 return |
| S&P 500 squared return |
| 1-day lagged volatility |

Table 8. Performance results for ANN-GARCH models without the Google domestic trends.

| Measure | GARCH(1,1) | GARCH(2,2) | GARCH(3,3) |
|---|---|---|---|
| $MAE_5$ | 0.0036168 | 0.0030972 | 0.0031187 |
| $RMSE_5$ | 0.0057721 | 0.0043685 | 0.0047167 |
| $MAPE_5$ | 0.5962047 | 0.5179546 | 0.5211834 |
| $MAE_{21}$ | 0.0029938 | 0.0030208 | 0.0028810 |
| $RMSE_{21}$ | 0.0038794 | 0.0037689 | 0.0038623 |
| $MAPE_{21}$ | 0.4116122 | 0.4237559 | 0.3861349 |
| Measure | GJR-GARCH(1,1) | GJR-GARCH(2,2) | GJR-GARCH(3,3) |
| $MAE_5$ | 0.0101036 | 0.0029940 | 0.0050052 |
| $RMSE_5$ | 0.0282132 | 0.0042111 | 0.0364472 |
| $MAPE_5$ | 1.8860717 | 0.4877833 | 0.7756672 |
| $MAE_{21}$ | 0.0040997 | 0.0029372 | 0.0032506 |
| $RMSE_{21}$ | 0.0063241 | 0.0037966 | 0.0071639 |
| $MAPE_{21}$ | 0.5451638 | 0.3986376 | 0.4442046 |
| Measure | EGARCH(1,1) | EGARCH(2,2) | EGARCH(3,3) |
| $MAE_5$ | 0.0031450 | 0.0031465 | 0.0031034 |
| $RMSE_5$ | 0.0044712 | 0.0043334 | 0.0045677 |
| $MAPE_5$ | 0.5264446 | 0.5517192 | 0.5269804 |
| $MAE_{21}$ | 0.0029123 | 0.0030841 | 0.0029454 |
| $RMSE_{21}$ | 0.0037679 | 0.0039046 | 0.0040166 |
| $MAPE_{21}$ | 0.4018983 | 0.4213633 | 0.4033511 |

Finally, we present the performance of the ANN-GARCH models without the GDTs to show efficiency of GDTs data as input data in the ANNs. Based on the suggestion of Hajizadeh *et al.* [20], the input variables are selected for the hybrid model. (For more details, see Table 7) Table 8 shows the results of the ANN-GARCH models without the GDTs for the window size 5 days and window size 21 days, respectively. From Table 8, one can observe that the ANN-GARCH models with the GDTs give better performances than the ANN-GARCH models without the GDTs.

## 5. Concluding Remarks

In this paper we construct the hybrid models for accurate forecasting of volatility of financial time series. Specifically, we propose the hybrid models with GARCH family models, the lagged realized volatility and GDTs data to forecast the volatility of the S&P 500 index. GDTs data is easily accessible and provide the current state of the economy and insight into future trends in the behavior of economic actors. Thus, we consider GDTs as input variables in the models for improving volatility forecasting.

We carry out many experiments changing the number of hidden layers and activation functions to find the optimized neural networks. Based on the architectures constructed from the experiments, we propose the hybrid models and compare various hybrid models combining GARCH family models under the ANNs with three hidden layers for forecasting weekly volatility and monthly volatility of the S&P 500 index. Most of the proposed hybrid models with the GDTs provide better performances than GARCH family models and the hybrid models without the GDTs. To verify these results, we use three performance measures for comparison the predictive accuracy of the models. As a result, GJR-GARCH(2,2)-GDT-LRV model has the best predictive accuracy for weekly volatility (5 days), and GARCH(3,3)-GDT-LRV model has the best predictive accuracy for monthly volatility (21 days). That is, the results show that the proposed models are suitable for forecasting the volatility of financial time series and improve the forecasting accuracy of GARCH family models.

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