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New Evidence on the Relation between Return Volatility and Trading Volume

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ABSTRACT
In the empirical literature, it has been shown that there exists both linear and non-linear bi-directional causality between trading volumes and return volatility (measured by the square of daily return). We re-examine this claim by using realized volatility as an estimator of the unobserved volatility, adopting a stationary de-trended trading volume, and applying a more recent data sample with robustness tests over time. Our linear Granger causality test shows that there is no causal linear relation running from volume to volatility, but there exists an ambiguous causality for the reverse direction. In contrast, we find strong bi-directional non-linear Granger causality between these two variables. On the basis of the non-linear forecasting modeling technique, this study provides strong evidence to support the sequential information hypothesis and demonstrates that it is useful to use lagged values of trading volume to predict return volatility.

KEY WORDS Return Volatility; Trading Volume; Non-linear Granger Causality; High Frequency Data

JEL CLASSIFICATION NUMBERS: G12, G14

INTRODUCTION

The relationship between return volatility and trading volume in asset markets has long been a subject of financial research. Despite voluminous studies, researchers are still uncertain about their precise relation. For instance, the mixture of distributions hypothesis (MDH) advanced by Clark (1973), Epps and Epps (1975), Harris (1987), and Andersen (1996) implies that the relation is critically dependent on a common underlying (latent) 'mixing' variable, the rate of information flow into the market. This model assumes that the joint distribution of volume and volatility is bivariate normal conditional upon the arrival of information. Under this perspective, all traders receive the new price

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signals simultaneously so that the shift to a new equilibrium is immediate and involves no intermediate partial adjustment. Therefore, under the MDH we should not find information content in lagged return volatility that can be used to forecast trading volume and vice versa, since these variables contemporaneously react to new information (Darrat *et al.*, 2003).

Alternatively, Copeland (1976), Jennings *et al.* (1981), and Smirlock and Starks (1985) present a 'sequential information' hypothesis. This class of models assumes that information is not received simultaneously by all traders; rather it is observed by each trader sequentially and randomly. From an initial position of equilibrium, it is assumed that all traders possess the same information. However, as a new flow of information arrives in the market, traders revise their expectations accordingly. Since traders do not receive the information signals simultaneously, their responses are only partial adjustments, achieving incomplete equilibrium. Once all traders have fully reacted to the information signals, a final equilibrium is re-established. Thus the sequential reactions to information imply that lagged values of volatility could predict current trading volume and vice versa.[1] These theoretical models motivate us to conduct empirical research on the lead and lag relation between return volatility and trading volume.

In his seminal paper, Brooks (1998) argues that the relation between return volatility and trading volume can be both linear and nonlinear. Using daily data from the Dow Jones composite for the period 17 November 1978 to 30 June 1988, he explores the lead–lag relation between trading volumes and return volatility (measured by the square of daily returns) within a linear and a nonlinear Granger causality test framework (Hiemstra and Jones, 1994). His results indicate that there exists both linear and nonlinear bidirectional causality between these two variables, although the relationship is stronger from volatility to volume than the reverse. His finding supports the sequential information theory.[2]

Several points are worth noting. First, in his paper, Brooks uses the square of daily returns as the measure of the volatility of daily stock returns. It is well recognized, however, that this measure is a very noisy estimator for the true latent volatility. Since it is computed by using opening and closing prices, the computed volatility may be underestimated if the opening and closing prices happen to be very close, even though there might be significant intraday price fluctuations. This fact prompts the following question: will the conclusion hold if a more precise estimator of the unobserved volatility is employed? Second, as computing technology advances and market dynamics evolve, can Brooks's argument for empirical regularity hold true over time? Will the conclusions hold true if a more recent data sample and more robust test procedures are employed? To address the above-mentioned questions, we adopt the following approach. First, for the volatility measure, we use realized volatility to replace the square of daily returns (Andersen, 1996; Andersen *et al.*, 2001). Second, to overcome the non-stationarity when applying the causality test, we employ a detrended trading volume instead of raw trading volume. Third, we extend the data to the sample and investigate the sub-period performance as a robustness test. With the increasing availability of high-frequency price data, recent studies have emphasized the use of precise volatility proxies, such as realized volatility, for evaluating the predictive accuracy of volatility models (Andersen and Bollerslev, 1998; Andersen *et al.*, 2003).

[1] Predicting return volatility is a very important subject for both academics and practitioners. For example, volatility is often used as a crude measure of the total risk of financial assets and an important component for deriving the prices of traded options. Furthermore, using combinations of options, it is possible to trade volatility as if it were any other commodity.

[2] Darrat *et al.* (2003) report lead–lag relations between the return volatility (measured by the estimated variance from an EGARCH model) and trading volume of DJIA stocks. However, they do not offer robustness tests over time, and their conclusions depend only on a linear Granger causality test, which is shown to be sensitive to the structure break or regime shift in the time series (see, for example, Okunev *et al.*, 2000).

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In this paper, we shall focus on daily realized volatility computed by summing many intraday squared returns. Andersen *et al.* (2001, 2003) show that sampling at sufficiently high frequency leads to a daily volatility estimate that is indistinguishable from the true latent volatility. This estimator is much more efficient compared to traditional standard proxies such as squared returns, absolute returns, or price range, which will mitigate a severe errors-in-variables problem associated with the more noisy volatility estimators used by Brooks (1998). Our linear Granger causality test shows that there is no causal relation running from volume to volatility and shows an ambiguous causality for the reverse direction. In contrast, we find strong nonlinear Granger causality between return volatility and trading volume. The evidence provides significant support to the sequential information theory. Thereafter, we adopt a nonlinear forecasting modeling technique to demonstrate the forecastability of volatility by using the lagged values of volume. Our results also show that it is useful to use lagged values of trading volume to predict return volatility with nonlinear models.

The rest of the paper is organized as follows. The next section reviews the theory of realized volatility. The third section introduces our data sources and provides a preliminary analysis of the data. The fourth section describes the methodology for testing for both linear and nonlinear Granger causality. The fifth section presents the results of the Granger causality tests and shows that there is a strong nonlinear Granger causality relationship from volume to volatility as reported in the previous section. The sixth section demonstrates that lagged values of trading volume help to improve the forecasting of volatility based on the mean squared errors and mean absolute errors. The seventh section contains conclusions.

THEORY ON REALIZED VOLATILITY

High-frequency data can be used to construct *ex post* volatility estimates of continuously compounded returns of assets obeying a standard diffusion. Specifically, assuming the return process satisfies no-arbitrage and has finite instantaneous mean, the log price follows a Brownian semimartingale diffusion process as proved in Back (1991).

Let σ_t^2 denote the instantaneous volatility of returns. We are primarily interested in the variance of returns over discrete horizons, e.g., a day. The true daily variance, conditional on the sample path $\{\sigma_{t-\tau}^2\}_0^1$, is

$$\bar{\sigma}_t^2 = \int_0^1 \sigma_{t-\tau}^2 \mathrm{d}\tau \tag{1}$$

In the literature, $\bar{\sigma}_t^2$ is known as the integrated or quadratic variance.[3] The integrated variance measures the realized sample path variation of the squared return process. A natural estimator of the integrated variance is obtained by summing intraday squared returns over the continual small intervals within the day. This estimator is known as the realized variance. In expression, we write:[4]

[3] The integrated variance is widely used in the options pricing literature where the option price depends on the distribution of the integrated variance of the underlying asset of the life of the option (see, for example, Hull and White, 1987).

[4] Refer to French *et al.* (1987) and Schwert (1989) for the concept of realized volatility in which they compute monthly realized volatilities using daily returns. Theoretical justifications for the realized volatility approach first appeared in Merton (1980) in the context of estimating the diffusion coefficient for continuous time diffusions and later extended to the theory of quadratic variance by Karatzas and Shreve (1991), Barndorff-Nielsen and Shephard (2001, 2002), and Andersen *et al.* (2001).

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$$\tilde{\sigma}^2(m) = \sum_{k=1,2,\ldots,m} r^2_{t+k/m} \tag{2}$$

where m is the interval number within a given trading day. Andersen *et al.* (2001, 2003) and Barndorff-Nielsen and Shephard (2001, 2002), among others, show that $\tilde{\sigma}^2(m)$ converges in probability to the integrated variance as $m \to \infty$. Thus, by summing up high-frequency squared returns, it is possible to construct realized volatility measures that are asymptotically free of measurement errors. Moreover, this result holds even if the underlying price process contains jumps, as long as the price process is a Brownian semi-martingale. Daily squared returns ($m = 1$) are clearly a special case of daily realized volatility. However, daily squared returns are very noisy or they are an inefficient estimator, since it fails to reflect the information contained in intraday stock prices. The same is true for using daily absolute returns and price range.

DATA AND PRELIMINARY ANALYSES

This study is based on intraday data on the NASDAQ from the Trade and Quotation (TAQ) database. The TAQ data files contain continuously recorded information on the trades and quotations of securities. The intraday 5-minute scale values for the NASDAQ span the period 1 August 1997, through 27 October 2004. Each trading day contains 78 observations from 9:30 a.m. EST to 4:00 p.m. EST. The market generally operates from 9:30 a.m. EST to 4:00 p.m. EST, so that there are 78 observations each trading day. To conduct the robustness test, we divide the full sample into two approximately equal subsamples: the first subsample covers the period 1 August 1997 to the end of March 2001; and the second one spans the period 2 April 2001 to 27 October 2004.

Realized volatility

Following Andersen *et al.* (2001, 2003), we compute daily realized volatility using intraday returns sampled at 5-minute intervals. Andersen *et al.* (2001) show that, in the limit, sampling at sufficiently high frequency leads to a daily volatility estimate that is indistinguishable from the true latent volatility. Most empirical studies use a 5-minute sampling interval to compute realized volatility. In this

Table I. Descriptive statistics for realized volatility and raw trading volume

	Full sample		Subsample 1		Subsample 2	
	$\tilde{\sigma}^2$	V	$\tilde{\sigma}^2$	V	$\tilde{\sigma}^2$	V
Mean	4.028	1099.673	4.968	881.264	3.059	1324.657
Median	2.474	1058.762	2.856	667.192	2.141	1255.658
Maximum	68.531	3039.556	68.531	2898.860	20.966	3039.556
Minimum	0.136	111.270	0.243	111.270	0.136	285.231
SD	5.057	542.724	6.387	528.187	2.840	459.399
Skewness	4.875	0.485	4.238	0.972	2.104	0.570
Kurtosis	44.183	2.558	31.365	3.031	8.968	3.048
Observations	1821	1821	924	924	897	897

Note: These are descriptive statistics for realized volatility ($\tilde{\sigma}^2$) and raw trading volume (V). To simplify the notations, the subscript t attached to the related variable is dropped in this table and subsequent tables. Full sample covers 1 August 1997 to 27 October 2004; subsample 1 covers 1 August 1997 to 30 March 2001; subsample 2 covers 2 April 2001 to 27 October 2004.

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Table II. Unit root test

Variables	$\tilde{\sigma}^2$	V	v
Full sample			
ADF test statistic	–3.958 (0.000)***	–0.797 (0.371)	–4.964 (0.000)***
Subsample 1			
ADF test statistic	–4.848 (0.004)***	0.724 (0.871)	–2.444 (0.014)**
Subsample 2			
ADF test statistic	–2.473 (0.013)**	–1.126 (0.237)	–6.052 (0.000)***

Note: This table reports ADF unit root test results for realized volatility ($\tilde{\sigma}^2$), raw trading volume (V) and detrended trading volume (v).The null hypothesis of the ADF test is that the variable has a unit root. Asterisks indicate significance at the ***1% and **5% levels. Full sample covers 1 August 1997 to 27 October 2004; subsample 1 covers 1 August 1997 to 30 March 2001; subsample 2 covers 1 April 1 to 27 October 2004.

paper, we follow this practice. Table I reports the descriptive statistics. As may be seen from the skewness coefficients, the series are skewed to the right. The kurtosis coefficients are well above 3.0, implying that the distributions of the series have fat tails compared with the normal distribution. ADF test results in Table II show that the realized volatility is stationary across the full sample and two subsamples.

Trading volume

Daily trading volume data for the NASDAQ are obtained from Bloomberg, and these data are the actual daily volume expressed in millions of shares. Consistent with previous studies, our investigation indicates that raw volume series, V_t, is non-stationary as indicated by the insignificance of the ADF test statistics in Table II. Therefore, it is necessary to construct a corresponding stationary series for further analysis (Gallant *et al.*, 1992). Following Gallant *et al.* (1992) and Chen *et al.* (2001), we form a stationary time series of trading volume by regressing the series on a nonlinear trend model as follows:

$$V_t = \alpha + \beta_1 t + \beta_2 t^2 + v_t \tag{3}$$

where V_t is the raw trading volume and v_t is the detrended trading volume, the residual of equation (3). Our estimated results show that β_1 and β_2 are highly significant and the equation has a high R^2 value.[5] This evidence is in agreement with the findings reported by Gallant *et al.* (1992) and Chen *et al.* (2001). Therefore, the detrended trading volume series v_t is appropriate for our analysis. The evidence from the ADF unit root test in Table II indicates that v_t is stationary. Hereafter, we use the volume variable in this study as the detrended trading volume.

LINEAR AND NONLINEAR GRANGER CAUSALITY

This section presents the methodology used to investigate the causal relationship between volatility and volume. The first subsection briefly discusses the linear Granger causality test, and the second subsection presents the nonlinear Granger causality test.

[5] We skip reporting the regression results, but they are available on request.

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Linear Granger causality

To test for a linear lead–lag relation between return volatility and trading volume, we adopt a VAR model represented by equation (4) to test for Granger causality:

$$\begin{aligned}\tilde{\sigma}_t^2 &= c_1 + \sum_{i=1}^{m} \phi_i \tilde{\sigma}_{t-i}^2 + \sum_{i=1}^{m} \theta_i v_{t-i} + \varepsilon_{1t} \\ v_t &= c_2 + \sum_{i=1}^{m} \rho_i \tilde{\sigma}_{t-i}^2 + \sum_{i=1}^{m} \eta_i v_{t-i} + \varepsilon_{2t}\end{aligned} \tag{4}$$

where $\tilde{\sigma}_t^2$ and v_t denote realized volatility and detrended trading volume, respectively, and at time t and both are stationary, $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is the vector of error terms, and m is the optimal lag length, obtained by using the Schwarz information criterion (SIC). The null hypothesis that the trading volume does not Granger cause volatility is equivalent to testing the restriction that $\theta_i = 0$ for all $i = 1, 2, \ldots, m$. The reverse causation is to test the restriction that $\rho_i = 0$ for all $i = 1, 2, \ldots, m$.

Nonlinear Granger causality

The nonlinear Granger causality test developed by Baek and Brock (1992) was modified by Hiemstra and Jones (1994). This approach postulates that by removing the linear predictive power in the VAR shown in equation (4), any remaining incremental predictive power of one residual series on another can be considered to be nonlinear predictive power. A non-parametric statistical method is then proposed, using the correlation integral (a measure of spatial dependence across time) to uncover any nonlinear causal relationship between two time series. Consider two strictly stationary and weakly dependent time series, $\{X_t\}$ and $\{Y_t\}$, for $t = 1, 2, \ldots$. Let X_t^m be the m-length lead vector of X_t, and let $X_{t-L_X}^{L_X}$ and $Y_{t-L_y}^{L_y}$ be the L_x-length and L_y-length lag vectors of X_t and Y_t, respectively. For given values of m, L_x, and L_y and for any distance e, $\{Y_t\}$ does not Granger cause $\{X_t\}$ strictly if

$$\begin{aligned}&\Pr\left(\left\|X_t^m - X_s^m\right\| < e \mid \left\|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\right\| < e, \quad \left\|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\right\| < e\right) \\ &= \Pr\left(\left\|X_t^m - X_s^m\right\| < e \mid \left\|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\right\| < e\right)\end{aligned} \tag{5}$$

where $\Pr(\cdot \mid \cdot)$ and $\|\cdot\|$ denote conditional probability and maximum norm, respectively, for $t \neq s$. In equation (5), the left-hand side is the conditional probability that two arbitrary m-length lead vectors of $\{X_t\}$ are within a distance e of each other, given that the corresponding L_x-length and L_y-length lag vectors of $\{X_t\}$ and $\{Y_t\}$, respectively, are within a distance e of each other. The right-hand side is the conditional probability that two arbitrary m-length lead vectors of $\{X_t\}$ are within a distance e of each other, given that the corresponding L_x-length lag vectors of X_t are within a distance e of each other. The strict Granger non-causality condition in equation (5) can be implemented by expressing it in terms of the corresponding ratios of joint probabilities as follows:

$$\frac{C_1(m+L_x, L_y, e)}{C_2(L_x, L_y, e)} = \frac{C_3(m+L_x, e)}{C_4(L_x, e)}$$

where C_1, C_2, C_3 and C_4 are the correlation-integral estimators of the joint probabilities, which are discussed in detail in Hiemstra and Jones (1994). For given values of m, L_x, $L_y \geq 1$ and $e > 0$, under the assumptions that $\{X_t\}$ and $\{Y_t\}$ are strictly stationary and weakly dependent, if $\{Y_t\}$ does not strictly Granger cause $\{X_t\}$, then

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$$\sqrt{n}\left(\frac{C_1(m+L_x,L_y,e,n)}{C_2(L_x,L_y,e,n)}-\frac{C_3(m+L_x,e,n)}{C_4(L_x,e,n)}\right)\xrightarrow{d}N\left(0,\sigma^2(m,L_x,L_y,e)\right) \tag{6}$$

where $n = T + 1 - m - \max(L_x, L_y)$ and σ^2 (m, L_x, L_y, e) can be estimated.[6] A significant positive value of the test statistic implies that lagged values of $\{Y_t\}$ are able to predict $\{X_t\}$, whereas a significant negative value suggests that lagged values of $\{Y_t\}$ confuse the prediction of $\{X_t\}$. Its asymptotic distribution is the same if the test is applied to the estimated residuals from a VAR model (see Hiemstra and Jones, 1994). This test has well-informed predictive properties against a variety of nonlinear Granger causal and non-causal relations and has been adopted widely in the literature (see, for example, Abhyankar, 1998; Ma and Kanas, 2000; Okunev *et al.*, 2000; Huh, 2002; Chen and Lin, 2004; Qiao *et al.*, 2008, 2009).

GRANGER CAUSALITY RESULTS

The conventional linear Granger test results are reported in Table III. Unlike the finding of Brooks (1998), our testing results reveal no linear causal relation running from volume to volatility. With respect to the reverse causation, the results are ambiguous: there is linear causality running from volatility to trading volume for the first subsample period, but not for the full sample period and the second period. Since linear Granger causality has very low power to detect a possible nonlinear causality relation between variables, we proceed to adopt a nonlinear Granger causality test for further analysis.

Table III. Testing for linear Granger causality between volatility and trading volume

Variables	Test	
	Wald statistics	LB (5)
Full sample		
$\tilde{\sigma}^2 \to v$	0.917 (0.509)	0.560 (0.990)
$v \to \tilde{\sigma}^2$	1.288 (0.238)	0.145 (1.000)
Subsample 1		
$\tilde{\sigma}^2 \to v$	6.803 (0.000)***	0.298 (1.000)
$v \to \tilde{\sigma}^2$	1.412 (0.217)	7.379 (0.194)
Subsample 2		
$\tilde{\sigma}^2 \to v$	2.096 (0.051)	1.822 (0.873)
$v \to \tilde{\sigma}^2$	1.835 (0.089)	6.441 (0.266)

Note: Numbers in parentheses are p-values. LB (5) is the Ljung–Box statistic based on the level of residual series of dependent variables in equation (4), up to the 5th lag. Asterisks indicate significance at the ***1% and **5% levels. Full sample covers 1 August 1997 to 27 October 2004; subsample 1 covers 1 August 1997 to 30 March 2001; subsample 2 covers 2 April 2001 to 27 October 2004.

[6] See the Appendix in Hiemstra and Jones (1994) for more information.

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Before testing for nonlinear Granger causality in the residuals from the linear VAR, we conduct two sets of diagnostic tests on the residuals from the VAR models. First, the Ljung–Box Q-test is used to determine whether any linear dependency remains in the residuals. Second, we perform a formal nonlinear dependence test, known as the Brock, Dechert, Scheinkman (BDS)[7] test, on the residuals (Brock *et al.*, 1996). The BDS approach essentially tests for deviations from identically and independently distributed (i.i.d.) behavior in the time series of residuals. Given x_t ($t = 1, \ldots, T$), they are considered segments of equal size, called m-stories, and form the m-dimensional vector, $x(m)_t = (x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+m-1})$. The BDS test computes the correlation integral $C_{m,T}(l)$ of m dimension and l distance as

$$C_{m,T}(l) = \frac{2}{(T-m+1)(T-m)} \sum_{I \le i \le j \le T-m+1} I_l\left(x(m)_i, x(m)_j\right) \tag{7}$$

where $I_l(\cdot)$ is the indicator function:

$$I_l\left(x(m)_i, x(m)_j\right) = \begin{cases} 1 & if \quad \left\| x(m)_i, x(m)_j \right\| < l \\ 0 & if \quad \left\| x(m)_i, x(m)_j \right\| \ge l \end{cases} \tag{8}$$

The BDS test for a fixed m dimension, an l distance, and a T sample size is

$$\mathrm{BDS}(m,l,T) = \frac{\sqrt{T}\left(C_{m,T}(l) - C_{I,T}(l)^m\right)}{\sigma_{m,T}(l)} \tag{9}$$

where $\sigma_{m,T}(l)$ is the estimated asymptotic standard deviation of $\sqrt{T}\left(C_{m,T}(l) - C_{I,T}(l)^m\right)$.

The properties of this test on finite samples were studied by Brock *et al.* (1991, 1996) and Lee *et al.* (1993), among others. They show that, under the null of the i.i.d. series, the BDS statistic follows a normal distribution. The test is applied for common lag lengths of two to eight lags. The results of these diagnostic tests reported in Table III point out that the VAR models successfully account for linear dependency, as indicated by insignificant values of the Q test. In Table IV, we report the BDS test results. The evidence indicates that all of the residual series from VAR models are nonlinear dependent.

With this evidence, we proceed to conduct the Hiemstra and Jones test (Hiemstra and Jones, 1994). Before employing the test, one has to select the values for the lead length, m, the lag lengths, L_x and L_y, and the scale parameter, e. Following Hiemstra and Jones, we set lead length $m = 1$ and $L_x = L_y$ for all cases, and we use common lag lengths of one to eight lags and a common scale parameter of $e = 1.5\sigma$, where $\sigma = 1$ denotes the standard deviation of standardized series.[8] The results of the nonlinear Granger causality test reported in Table V reveal different relations between return volatility and trading volume as compared with its counterpart of the linear test. For both the full sample and the two subsample periods, there is strong evidence supporting a lead–lag relation between these two variables. Thus our findings provide strong support for the sequential information theory.

[7] The BDS test was suggested by Brock *et al.* (1987) and revised by Brock *et al.* (1996).

[8] For other settings of scale parameters, such as $e = 1.0\sigma$, the results are qualitatively no different. The results are available upon request.

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Table IV. BDS test results for the VAR residuals

Panel A: Full sample		
Dimension	$\tilde{\sigma}^2 \rightarrow v$	$v \rightarrow \tilde{\sigma}^2$
2	11.580 (0.000)***	18.746 (0.000)***
3	14.743 (0.000)***	22.385 (0.000)***
4	16.770 (0.000)***	24.472 (0.000)***
5	18.466 (0.000)***	26.305 (0.000)***
6	20.211 (0.000)***	28.218 (0.000)***
7	22.147 (0.000)***	30.173 (0.000)***
8	24.019 (0.000)***	32.577 (0.000)***
Panel B: Subsample 1		
Dimension	$\tilde{\sigma}^2 \rightarrow v$	$v \rightarrow \tilde{\sigma}^2$
2	11.373 (0.000)***	11.368 (0.000)***
3	13.556 (0.000)***	12.367 (0.000)***
4	14.971 (0.000)***	12.343 (0.000)***
5	15.951 (0.000)***	12.736 (0.000)***
6	16.997 (0.000)***	13.175 (0.000)***
7	18.261 (0.000)***	13.666 (0.000)***
8	19.425 (0.000)***	14.055 (0.000)***
Panel C: Subsample 2		
Dimension	$\tilde{\sigma}^2 \rightarrow v$	$v \rightarrow \tilde{\sigma}^2$
2	6.915 (0.000)***	11.231 (0.000)***
3	8.524 (0.000)***	13.962 (0.000)***
4	9.462 (0.000)***	15.375 (0.000)***
5	10.254 (0.000)***	16.571 (0.000)***
6	11.055 (0.000)***	17.730 (0.000)***
7	12.182 (0.000)***	19.016 (0.000)***
8	13.179 (0.000)***	20.428 (0.000)***

Note: This table reports the results of the normalized BDS test statistics proposed by Brock *et al.* (1996). Each cell contains two numbers: numbers without parentheses are the standardized BDS test statistics based on the level of the residual series of the dependent variable in equation (9); numbers in parenthesis are the corresponding p-values, which are simulated using a bootstrap method with 1000 replications. See Brock *et al.* (1996) for details of this test. Asterisks indicate significance at the ***1% and **5% levels. Full sample covers 1 August 1997 to 27 October 2004; subsample 1 covers 1 August 1997 to 30 March 2001; subsample 2 covers 2 April 2001 to 27 October 2004.

FORECASTING VOLATILITY

In the previous section we demonstrated that there is no linear causal relationship but there is a nonlinear causal relationship running from volume to volatility. This finding suggests that it may be useful to use lagged values of volume to predict volatility with nonlinear models. In this section we adopt a nonlinear forecasting technique to show that the information from the lagged values of

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Table V. Testing for nonlinear Granger causality

Panel A: Full sample		
Dimension	$\tilde{\sigma}^2 \rightarrow v$	$v \rightarrow \tilde{\sigma}^2$
1	3.132 (0.001)***	2.372 (0.009)***
2	2.792 (0.003)***	2.906 (0.002)***
3	2.798 (0.003)***	2.610 (0.005)***
4	2.383 (0.009)***	2.784 (0.003)***
5	1.604 (0.054)	2.354 (0.009)***
6	1.383 (0.083)	1.767 (0.039)**
7	0.989 (0.161)	1.221 (0.111)
8	0.164 (0.435)	0.894 (0.186)
Panel B: Subsample 1		
Dimension	$\tilde{\sigma}^2 \rightarrow v$	$v \rightarrow \tilde{\sigma}^2$
1	3.173 (0.001)***	2.502 (0.006)***
2	3.577 (0.000)***	2.893 (0.002)***
3	3.966 (0.000)***	3.493 (0.000)***
4	4.058 (0.000)***	3.293 (0.000)***
5	3.665 (0.000)***	3.198 (0.001)***
6	3.092 (0.001)***	2.826 (0.002)***
7	2.721 (0.003)***	2.528 (0.006)***
8	2.277 (0.011)**	2.391 (0.008)***
Panel C: Subsample 2		
Dimension	$\tilde{\sigma}^2 \rightarrow v$	$v \rightarrow \tilde{\sigma}^2$
1	3.397 (0.000)***	3.497 (0.000)***
2	3.329 (0.000)***	4.188 (0.000)***
3	3.347 (0.000)***	3.601 (0.000)***
4	4.437 (0.000)***	3.722 (0.000)***
5	5.087 (0.000)***	3.575 (0.000)***
6	4.611 (0.000)***	3.429 (0.000)***
7	4.225 (0.000)***	3.267 (0.001)***
8	4.256 (0.000)***	2.983 (0.001)***

Note: This table reports the results of the nonlinear Granger causality test proposed by Hiemstra and Jones (1994). Each cell contains two numbers: numbers without parenthesis are the standardized HJ test statistics as per equation (6) and numbers in parenthesis are the corresponding p-values. Under the null hypothesis of nonlinear Granger non-causality, the test statistic is asymptotically distributed N (0, 1) and is a one-tail test. A significant positive test statistic implies that lagged values of $\{Y_t\}$ nonlinear Granger causes $\{X_t\}$. Asterisks indicate positive significance at the ***1% (critical value = 2.326) and **5% (critical value = 1.645) levels. Full sample covers 1 August 1997 to 27 October 2004; subsample 1 covers 1 August 1997 to 30 March 2001; subsample 2 covers 2 April 2001 to 27 October 2004.

volume could be useful in forecasting return volatility. To do so, we split the total sample of 1821 observations into two parts: the first 1500 observations are used for estimation of the forecasting models and the remaining 321 observations are used to evaluate the performance of forecasting. Two popular nonlinear econometric models—an EGARCH model (Nelson, 1991) and a GJR-GARCH

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model (Glosten *et al.*, 1993)—are adopted here. Under the EGARCH (1, 1) and the GJR-GARCH (1, 1), one-step-ahead forecasts of volatility are given by

$$\log\left(\sigma_{f,t}^2\right) = \omega + \beta \log\left(\sigma_{t-1}^2\right) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-i}^2}} + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{10}$$

and

$$\sigma_{f,t}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d \tag{11}$$

respectively. Here, $\sigma_{f,t}^2 \,|\, \Omega_{t-1}$ from (10) and (11) denotes the one-step-ahead forecast of volatility conditional upon information available at time $t - 1$, and d is a dummy variable with a value of 1 when ε_{t-1} is negative and 0 otherwise. Note that the EGARCH(1, 1) and GJR-GARCH(1, 1) models being used can be augmented by adding the lags of trading volume as predictor variables. In this paper, besides the EGARCH and GJR-GARCH components, we also add a one-period lag of trading volume, λv_{t-1}, to the right-hand side of equations (10) and (11) as an incremental variable. Only one lag is considered here, since it is likely to have a more profound effect on the current value of volatility and the effects from earlier lags are negligible. Thereby we compute the mean squared errors (MSE) and mean absolute errors (MAE) to evaluate the accuracy of the forecasts:[9]

$$\mathrm{MSE} = \frac{1}{321} \sum_{t=1501}^{1821} \left(\sigma_t^2 - \sigma_{f,t}^2\right)^2 \tag{12}$$

$$\mathrm{MAE} = \frac{1}{321} \sum_{t=1501}^{1821} \left|\sigma_t^2 - \sigma_{f,t}^2\right| \tag{13}$$

where σ_t^2 and $\sigma_{f,t}^2$ represent the actual volatility and its forecast value, respectively, at time t. To judge whether volume is helpful in predicting volatility, we could compare the models (either the EGARCH model in equation (10) or the GJR-GARCH model in equation (11)) in pairs, one without

Table VI. Evaluation of alternative models for volatility forecasting

Model description	MSE	MAE
EGARCH (1, 1)	5.006	2.162
EGARCH (1, 1) with one lag of volume	4.534	2.036
GJR-GARCH (1, 1)	20.244	4.043
GJR-GARCH (1, 1) with one lag of volume	18.954	3.922

Note: This table reports the results of volatility forecasting as described in the text. MSE and MAE are defined in equations (12) and (13), respectively. A modified Diebold–Mariano (MDM) statistic (Harvey *et al.*, 1997) is employed to test the difference in MSE and in MAE for alternative models. Corresponding *p*-values for differences in the MSE of Model 1 (without the volume term) and Model 2 (with the volume term) for EGARCH and GJR-GARCH specifications are 0.0488 and 0.0065, respectively. Similarly, corresponding *p*-values for differences in the MAE of Model 1 (without the volume term) and Model 2 (with the volume term) for EGARCH and GJR-GARCH specifications are 0.0062 and 0.0058, respectively.

[9] We appreciate the suggestion made by the editor and an anonymous referee to add this section.

the volume term, v_{t-1} (Model 1) and one with the volume term, v_{t-1} (Model 2). If the MSE(MAE) of Model 2 is smaller than that of Model 1, it means that the volume terms will enhance the capacity to predict the volatility. The forecasting results are reported in Table VI. From the calculated statistics, we find that models that include trading volume as a predictive variable produce smaller values of both MSE and MAE.[10] This finding implies that it is useful to use lagged values of trading volume to predict return volatility with nonlinear models.

CONCLUSIONS

Using realized volatility as an estimator of the unobserved volatility, adopting a stationary detrended trading volume to measure trading volume, and adding more recent data to the sample with robustness tests over time, this paper re-examines the empirical results reported by Brooks (1998). Unlike his findings, our linear Granger causality tests show no causal relation running from volume to volatility and an ambiguous causality relation for the reverse direction. In contrast, we find strong bidirectional nonlinear Granger causality between return volatility and trading volume. Our results provide strong support to the sequential information theory, and our forecasting results show that it is useful to use lagged values of trading volume to predict return volatility with nonlinear models.

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[10] A modified Diebold–Mariano (MDM) statistic (Harvey *et al.*, 1997) is adopted to test for the difference in the MSE and the difference in the MAE. Corresponding *p*-values for differences in the MSE of Model 1 (without the volume term) and Model 2 (with the volume term) for EGARCH and GJR-GARCH specifications are 0.0488 and 0.0065, respectively. Similarly, corresponding *p*-values for differences in the MAE of Model 1 (without the volume term) and Model 2 (with the volume term) for EGARCH and GJR-GARCH specifications are 0.0062 and 0.0058, respectively.

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