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Open interest, volume, and volatility: evidence from Taiwan futures markets

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Abstract This paper examines the relationships amongst volatility, total trading volume (TVOL) and total open interest (TOI) for three Taiwan stock index futures markets as well as the role of the latter two variables in the dynamics of GARCH modeling and forecasting. From both ex-post and ex-ante perspectives, we study this issue by using the VAR model and augmented GARCH-type models, respectively. For the GARCH-type models, we employ both symmetric and asymmetric models augmented with lagged logs in TOI and/or TVOL. We find that whether addition of these two variables helps the basic GARCH models predict future volatility depends upon the sample period examined for all three sets of futures. Nonetheless, the best three models for out-of-sample volatility forecasting in the MSE sense are generally the augmented models for all sub-intervals and all three futures contracts.

Keywords Open Interest · Trading Volume · Volatility · VAR · GARCH

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1 Introduction

The literature has seen a chunk of studies dedicated to explore the relationship between the volatility of futures contracts and their trading volume and open interest. Several theories predict a positive relationship between return volatility and trading

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volume, and for futures markets open interest is considered to be another important variable. As noted by Bessembinder and Seguin (1993 hereafter BS), for example, a bi-directional causality relationship between volatility and trading volume (or open interest) would support the ‘mixture of distribution’ hypothesis (see Clark 1973). The ‘sequential arrival of information’ theory of Copeland (1976, 1977) would hold if volatility is dependent upon the lagged volume and/or lagged open interest. Also, Admati and Pfleiderer (1988)’s ‘traders with trade timing discretion tend to trade in heavy liquidity’ theory would apply if trading volume affects volatility.

Figlewski (1981) suggests that monthly trading volume and open interest of the GNMA futures help explain its monthly volatility. Karpoff (1987) argues that price changes and volume measures should exhibit positive contemporaneous correlation with each other and reviews eighteen studies that examine evidence with regard to this relationship. Tan and Gannon (2002) find that, apart from the return-volume relationship, the interrelationships between return, volatility and volume upon information arrival are generally consistent with what theory would anticipate.

The literature on modeling the relationship amongst these three variables has seen three main streams of development. First of all, BS suggest that open interest is a proxy for market depth (see Kyle 1985, for the definition of market depth). BS argue that open interest of futures markets apart from trading volume provides an additional measure of trading activity generated by large hedges (or commercial traders in the language of Chartrath et al. 2003). Partitioning the trading volume and open interest, respectively, into their expected and unexpected components, BS study eight physical and financial futures markets based on a model similar to GARCH-in-mean. They find a strong positive relation between contemporaneous volume and volatility. To the contrary, they find volatility to be negatively related to both expected and unexpected open interest. Their results appear to suggest that an unexpected increase in open interest alleviates the effect of an unexpected increase in volume on volatility. BS’s method attracts some subsequent studies on futures volatility based on their model. Amongst others, Ragunathan and Peker (1997) provide evidence that positive volume shocks have a greater impact on volatility than negative shocks and reach a similar conclusion regarding open interest. They suggest that, therefore, both trading volume and market depth, as measured by open interest, do have an effect on volatility. Using BS’s method, Watanabe (2001) also presents evidence of a significant positive relationship between volatility and unexpected volume, as well as a significant negative relationship between volatility and expected open interest for Nikkei 225 stock index futures. However, they find that the relationship amongst price volatility, volume and open interest may vary depending on the market’s regulatory structure.

BS’s study also gives rise to research on the relationship between the futures market and the spot market. For instance, Chang et al. (2000) use BS’s method to examine the S&P 500 futures and find that the open interest of the S&P 500 futures increases as the volatility of its spot index increases. This finding tends to suggest that the increase in open interest of the S&P 500 futures induced by the increase in volatility of the spot index reflects higher hedging demand in index conditions of spot market uncertainty. This conclusion is echoed by Chen et al. (1995) who use an AR(2) model with the current implied volatility as one of the independent variables to study the same S&P 500 index spot and futures markets. Following BS, Chartrath et al. (2003)

re-examine the effects of trading activity for the S&P 500 index futures on the volatility of S&P 500 spot index by classifying futures traders into four categories: commercial traders (large hedgers or institutional traders), non-commercial traders (larger speculators), small traders and spreaders. They find that unexpected change in lagged open interest of commercial traders is positively related to interday and intraday stock market volatility. However, such evidence is not replicated for the other three types of traders. Following, Motladiile and Smit (2003) also use Chen et al. (1995)'s method to examine three South African stock index futures contracts and find that positive shocks in open interest increases the volatility of the spot index.

Another main stream of literature on volatility of futures markets concerns the use of the vector autoregressive (VAR) model. Fung and Patterson (1999) use the VAR model to examine the relationships amongst volatility, volume and open interest for five currency futures markets. They find mixed results: only trading volume is positively and significantly dependent upon volatility and open interest for all five currencies. Stated otherwise, volatility and open interest, respectively, Granger-causes trading volume. Similarly, Ferris et al. (2002) use the VAR model to study the interactions amongst pricing error, change in open interest, change in volume and change in implied volatility for the S&P 500 futures market. They document a bi-directional causal relation between the changes in open interest and trading volume. Nonetheless, the change in implied volatility is not related to either open interest or volume. Interestingly, moreover, the mis-pricing of the S&P 500 futures contract seems to affect all other variables above.

The third main type of models used to discuss the relations amongst volatility, volume and open interest involves the GARCH-type models. Girma and Moutgoué (2002, hereafter GM) add the current or the first lag of volume and/or open interest to the GARCH(1,1) model when studying four series of spread returns for the New York Mercantile Exchange (NYMEX) petroleum markets. They find that contemporaneous volume and open interest help explain the futures spreads volatility, when volume and open interest are added separately. This is also true for the lagged forms of volume and open interest. Their parameter estimates in the GARCH(1,1) model are all positive. However, when these two explanatory variables are entered simultaneously, the results are mixed. Only one spread's volatility is significantly explained by the current volume and none of the four spreads has its volatility related to the current open interest. However, lagged volume is explanatory for two of the four spreads and lagged open interest for three of the four spreads.

Despite the number of studies on the relationship amongst volatility, trading volume, and open interest we in the literature discussed above, they are all conducted from an ex-post perspective. However, in-sample relationships amongst any variables, though significant, do not guarantee to happen again in the future. So one of this paper's goals is to study this issue from an ex-ante perspective, i.e. by testing out-of-sample forecasting performance of the models examined. Given the importance of volatility in modern financial markets, we focus on volatility forecasting using other two variables, i.e. trading volume and open interest.

Turning to the literature on estimation and modeling of financial market volatility, which is also a key issue in econometrics and empirical finance, a plethora of efforts has been dedicated to this area. Amongst others, Chu and Freund (1996) compare the degree of option mispricing resulting from using volatility estimates from existing option prices' implied standard deviations (ISD) and from GARCH and IGARCH models fit to the underlying assets' returns and find that the ISD method is best. Similarly, Poon and Granger (2005) find that the ISD model tends to outperform the volatility forecasts provided by time-series models. However, among time-series models, they find that neither historical volatility nor ARCH models dominate the other in terms of forecasting future volatility, although both types of forecasts appear to be better than those provided by stochastic volatility models.

One of the key areas where volatility forecasts are employed is the futures markets. These markets fulfill two social functions. One function is price discovery, which is the revealing of information about future cash market prices. The other is hedging, which is the prime rationale for futures trading. Hedgers are exposed to some form of preexisting risk that leads them to use futures transactions as a substitute for cash market transactions. However, arbitrage plays a crucial role for pricing index futures contracts. Spot and futures market prices are linked by arbitrage; i.e., participants liquidate positions in one market and take comparable positions at better prices in another market, or choose to acquire positions in the market with the most favorable prices. Index futures markets typically offer investors the opportunity to trade at a substantially lower cost and higher liquidity than trading directly on the spot market. Therefore, one would expect futures markets to exhibit more instantaneous reactions to new information than the equity markets. Because volatility reflects the magnitude of price movement within a given time series, it is an appropriate variable for examining the length of time required for the markets to fully incorporate new information. Consequently, volatility can be considered a measure of information flow in derivative instruments (see Ross 1976) and has always been an essential tool for trading strategies. Thus, the ability to forecast volatility for the futures markets has important implications for investors.

Within the futures markets, stock index futures and options on stock index futures are especially important subjects for research. These financial instruments have very high trading volumes due to hedging, speculative trading, and arbitrage activities. Insight into the behavior of futures price volatility can have important implications for investors using stock index futures contracts, such as a portfolio manager implementing a put-replication portfolio insurance strategy during a period of high market volatility. In calculating the optimal hedge ratio to use in implementing such a strategy, the portfolio manager in this situation would be faced with the issue of whether to use a volatility forecast based on the high level of volatility within the recent past or, alternatively, some other forecast of volatility based on conditioning information appropriate for the insurance horizon.

Related to this issue, Hill et al. (1988) show that unexpected changes in volatility are the most important risk factor in determining the cost of portfolio insurance. Similarly, Chu and Bubnys (1990) use a likelihood ratio test to compare the variance measure of price volatilities of stock market indices and their corresponding futures contracts during the bull market of the 1980s, and find that spot market volatilities are significantly lower than their respective futures price volatilities. Bera et al. (1993) investigate the effectiveness of using conventional OLS estimates of volatility to determine the optimal hedge ratios and find that, compared to ARCH-based hedge ratios, the hedge ratios based on conventional OLS estimates may cause investors to sell short an inappropriate number of futures contracts.

Most studies of volatility forecasting have reiterated the empirical finding that volatility in financial time series is highly persistent with clearly demonstrated volatility clustering behavior. See, for example, Mandelbrot (1963), Bollerslev *et al.* (1994) and Brooks (2002). Numerous models have been proposed to describe the phenomena of heteroscedasticity and volatility clustering. Among these models, the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) processes developed by Engle (1982) and Bollerslev (1986), respectively, appear to be appropriate models for the daily returns of many financial time series. The GARCH(1,1) and Exponential GARCH (EGARCH) models are two of the most successful such parameterizations for characterizing high-frequency financial market volatility. One of the common findings in many empirical applications of these models is a high degree of persistence within the estimated conditional variance process (see Bollerslev et al. 1992).

Since the development of these models, there has been a great deal of research on forecasting volatility, with many different econometric model specifications being compared, yet no single model specification has been shown to be categorically superior. As an example, Akgiray (1989) and Pagan and Schwert (1990) find, using US stock and futures data, that the GARCH models outperform most competitors, while Najand (2002) compares symmetric models with asymmetric models for forecasting the price volatility on S&P 500 Index futures and finds EGARCH to be the best model for forecasting price volatility for stock index futures. However, using data sets from the Japanese and Singaporean markets, respectively, Tse (1991) and Tse and Tung (1992) find that exponentially weighted moving average models provide more accurate forecasts than GARCH models. Braiisford and Faff (1996), on the other hand, find that GARCH models are slightly superior to most simple models for forecasting Australian monthly stock index volatility, and Frances and van Dijk (1996) find that the asymmetric GARCH models perform no better than the standard GARCH model in forecasting the weekly volatility of various European stock markets. Gokcan (2000) finds that, for emerging stock markets, the basic GARCH(1,1) model performs better than EGARCH models, while Wei (2002) presents QGARCH as a better model than either the basic GARCH or the GJR-GARCH model for forecasting the weekly volatility on the China stock market.

If market volume, which is used as an exogenous right-hand side variable in the variance equation of the GARCH model, is part of a larger system of equations where volume is itself partly determined by volatility, then failure to appropriately model the system as such will cause a simultaneity bias in the coefficient estimates. One potential solution to this problem is to use lagged measures of volume, which will be predetermined and therefore not subject to the simultaneity problem. Although Lamoureux and Lastrapes (1990) find lagged volume to be a poor instrument for forecasting the monthly volatility of US stock portfolios, Najand and Yung (1991) find it quite acceptable in an analysis of price variability in Chicago Board of Trade futures data. Finally, Brooks (1998) compares many models such as GARCH, EGARCH, GJR-GARCH, etc., which are augmented by the addition of a measure of lagged volume to form more general ex-ante forecasting models. He finds only very modest improvements, if any, in volatility-forecasting performance.

In the present study, we focus on Taiwan stock market futures contracts that are traded on the Taiwan Futures Exchange (TAIFEX). The total trading volume for the TX futures was 10,065,685 and 11,587,758 contracts for the years 2006 and 2007, respectively. These two numbers, respectively, represent a growth rate of 713% and 821% relative to the 1,411,656 contracts traded in 2000. Despite its heavy trading volume, investors in this emerging Taiwan market suffer from a lack of information as well as reliability. Nonetheless, Brooks (1998) suggests the existence of an informational relationship between volatility and both volume and open interest. Open interest represents the number of futures contracts outstanding at any point in time, whereas trading volume captures the number of contracts traded during a specific time period. Open interest supplements the information provided by the trading volume. To investigate whether the total open interest and the total trading volume helps the GARCH models forecast volatility, we include the lagged total open interest and/or the lagged total trading volume in the variance equations. To account for the asymmetry effect, moreover, we compare three asymmetric (EGARCH, GJR and APARCH) against the two symmetric GARCH-class models (GARCH and IGARCH). We compare the volatility-forecasting performances of all these five models with and without lags in total market volume or total open interest included as predictor variables. While Poon and Granger (2005) suggest the dominance of the option-implied volatility (or ISD) models over the GARCH-class models in forecasting volatility, we do not include the ISD models in the comparison since options on Taiwan stock indices were not available until 2001, although the trading volume for options has witnessed tremendous growth since then. Turning to the types of index futures contracts investigated, we focus on the TAIEX futures (hereafter *TX*), the TSE Electronic Sector Index futures (hereafter *TE*), and the TSE Banking Insurance Sector Index futures (hereafter *TF*), which have the greatest liquidity among the futures contracts traded on the TAIFEX.

The goal of this research is to examine the relationship amongst volatility, trading volume and open interest in addition to studying whether total volume and total open interest data help improve the accuracy of the GARCH-type models in Taiwan's futures market. The paper is structured as follows. Section 2 describes the data and sample period used. Section 3 provides the methodology and explains the various models and their formulation. The results of in-sample estimation and out-of-sample forecasting, both with and without lagged total volume and total open interest, are given in Section 4. The conclusions are drawn in the final section.

2 Data and descriptive statistics

The data used in this study include data from the Taiwan Stock Exchange (TSEC)[1] and from the Taiwan Futures Exchange (TAIFEX). Trading on the TAIFEX started on July 21, 1998, with the introduction of TSEC Capitalization Weighted Stock

[1] The TSEC maintains a total of 27 stock price indexes to allow investors to conveniently view overall market movements as well as the performances of the different industrial sectors. The TSEC Capitalization Weighted Stock Index (TAIEX) is the most widely quoted of all TSEC indexes. The base year value as of 1966 was set at 100.

Index (TAIEX) futures (*TX*), a futures contract on the TAIEX[2] stock index. Subsequently, the TAIFEX issued three additional futures products: TSE Electronic Sector Index Futures (*TE*), TSE Banking Insurance Sector Index Futures (*TF*) and Mini-TAIEX Futures. *TE* and *TF* contracts began to be traded on July 21, 1999. We choose only *TX*, *TE*, and *TF* to forecast return volatility, because TAIEX futures and Mini-TAIEX Futures have the same underlying assets. Of the five different contract maturities for each set of futures contracts, we choose the nearest month's contract up until the beginning of the month of delivery when the second nearest contract to expiration begins to be used in order to avoid idiosyncrasies that may be specific to the futures markets during the month of delivery.

The data used are the daily closing prices, total trading volumes, and total open interests for the *TX*, *TE*, and *TF* contracts traded on TAIFEX. We analyze *TX* data for the period from July 21, 1998 to December 31, 2007, and *TE* and *TF* from July 21, 1999 to December 31, 2007. In total, we have 2,394 observations for the *TX* contract and 2,125 observations for both the *TE* and *TF* contracts. To accommodate the variation of market condition and to test the robustness of our results across time, we divide the sample period into three sub intervals for all three contracts, each lasting for roughly 3 years. To be specific, we have 905 observations for the first sub-interval from Jul. 21, 1999 through Dec. 31, 2001 for *TX*, while we have 636 observations for the first sub-interval from Jul 21 1999 through Dec. 31, 2001 for both *TE* and *TF*. The second sub-interval lasts for a period of exact 3 years from Jan 2, 2002 through Dec. 31, 2004 for all three contracts, accounting for a number of 747 observations. Finally, the third sub-interval is also of three years in length from Jan. 3, 2005 to Dec. 31, 2007 accounting for 742 observations for all three contracts. The last 20 observations of each sub-interval are reserved for out-of-sample evaluation of volatility forecast performance by a variety of models, whereas the remaining range of data is used for in-sample model estimation. Fixing the size of in-sample window, we roll it over by one step ahead once one daily volatility is generated by the model for the out-of-sample period. In other words, we estimate the model for 20 times and generate 20 daily volatility forecasts for each sub-interval. used for in-sample model estimation. In total, therefore, we have a 60 daily volatility forecasts for the entire sample period for each of all three contracts.

The daily return series are calculated as the first differences of the logarithms of the daily closing futures prices:

$$r_t = \ln(P_t/P_{t-1}), \tag{1}$$

where P_t is the daily closing price at time t for the relevant *TX*, *TE*, or *TF* futures contract, and r_t is its daily rate of return, assuming continuous compounding. The use of the logarithm of price changes reduces the impact of price-level non-stationarity on the estimated return volatility. Following Chan et al. (1995), Day and Lewis (1992), West and Cho (1995), and Brooks (1998), we measure the daily

[2] The TAIEX covers all of the TSEC-listed stocks, excluding preferred stocks, full-delivery stocks, and stocks that have been listed for less than one calendar month.

volatility by simply the square of the day's return. As Jorion (1995) notes, $\sigma_t^2 = E_{t-1}[r_t - E_{t-1}(r_t)]^2 = E_{t-1}\left[r_t^2\right] - \left[E_{t-1}(r)_t\right]^2$ and $E_{t-1}[r_t] \approx 0$, so

$$\sigma^2 = E_{t-1}\left[r_t^2\right]. \tag{2}$$

Before proceeding to the estimation of the various volatility-forecasting models, we will first examine the distributional properties of the various daily returns series. Descriptive statistics for these return series are reported in Table 1 below.

Over the sample period covered, the mean daily returns for the three futures returns series are all close to zero, though they are all slightly negative (−0.002%, −0.001% and −0.009%). The standard deviations range from 1.77% for the *TX* futures returns to 2.14% for the *TE* futures. As is typical with financial time series, all three series exhibit excess kurtosis, and, as a consequence, the Bera-Jarque skewness-kurtosis test of normality results in a rejection of normality at a 1% level for all three series. Note that these leptokurtic departures from normality come in spite of the tail truncation effects of the 7% daily price limits that exist on the Taiwan financial markets. As shown by Ammermann and Patterson (2003), the effects of these daily

Table 1 Descriptive statistic for the futures contracts returns: *TX* (22 July 1998-31 December 2007), *TE* and *TF* (22 July 1999-31 December 2007)

Futures Contract	*TX*	*TE*	*TF*
Beginning Date	98/07/22	99/07/22	99/07/22
Ending Date	07/12/31	07/12/31	07/12/31
Sample Size (*T*)	2394	2125	2125
Mean (%)	−0.002	−0.001	−0.009
Max (%)	8.49	6.76	7.27
Min (%)	−8.14	−7.65	−7.40
StD (%)	1.77	2.14	1.95
Skewness	−0.13*** (0.05)	−0.05 (0.05)	−0.08 (0.05)
Excess Kurtosis	2.90*** (0.10)	1.99*** (0.11)	2.36*** (0.11)
JB	845.46****	353.10***	493.63***
Q(6)	19.64***	13.92**	22.84***
Q(12)	29.27***	37.81***	28.36***
Q(18)	49.89***	61.08***	37.67***
$Q^2(6)$	326.62***	531.56***	331.06***
$Q^2(12)$	490.73***	914.85***	513.29***
$Q^2(18)$	613.34***	1215.35***	594.53***

1. *TX*, *TE* and *TF* denote TAIEX futures, TSE Electronic Sector Index futures and TSE Banking Insurance Sector Index futures, respectively. StD denotes standard deviation. The standard errors for skewness and kurtosis are in parentheses which are given by $(6/T)^{1/2}$ and $(24/T)^{1/2}$, respectively, where *T* is the sample size. JB denotes the Jarque and Bera (1980) normality test statistic given by $\left[(T/6)b_1^2 + (T/24)(b_2-3)^2\right] \sim \chi_2^2$, where b_1 is the coefficient of skewness, and b_2 is the coefficient of kurtosis. Its critical value at the 1% significance level is 9.21. Q(*n*) and $Q^2(n)$ denote, respectively, the Ljung-Box (1978) Q-statistic with *n* lags for the return and the squared return series. 2. Superscript ***, **, * denote 1%, 5%, and 10% significance levels, respectively

price variation limits not only serve to truncate the tails of the distribution of daily returns, leading to relatively low levels of leptokurtosis as compared to other financial markets throughout the world, but, when Taiwan's markets become especially volatile, the price limits also serve to convert this extant daily return volatility into autocorrelation between the days' returns.

The autocorrelations within each of the three futures returns series and the evolution of their levels of volatility over time are explored via the next set of descriptive statistics, the Ljung-Box (Q) test statistic and the McLeod and Li (Q^2) test statistic. These tests examine the joint significance of the autocorrelations among the first 6, 12, and 18 lags of the return and squared return series, respectively, of the futures contracts. As noted by Taylor (1986), among others, these two sets of autocorrelation functions can be used to explore the degree of predictability of various moments within financial data. For the futures return series examined, significant Q-statistics suggest at least a moderate degree of predictability within the returns of each of the three series, while the much more highly significant Q^2 statistics indicate a very high degree of predictability among the squared returns for all three series. This finding suggests the existence within the futures returns of some form of nonlinear serial dependencies that appear to generate autoregressive conditional heteroscedasticity (ARCH) effects, various possible formulations of which will be explored in Section 3.

Before proceeding to the description and estimation of such models for forecasting volatility, the stationarity of the explanatory variables with which we plan to augment such models must first be verified. We used the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Kwaitkowski-Phillips-Schmidt-Shin (KPSS) tests to assess the stationarity of total volume (hereafter TVOL) and total open interest (hereafter TOI) across all the contracts for the given future. Table 2 below shows conflicting results: the null hypothesis of a unit root under both ADF and PP tests is rejected for most of the cases, the only exception being the case of TVOL

Table 2 Unit root and stationarity tests for total volume and total open interest

		ADF		PP		KPSS	
		TVOL	TOI	TVOL	TOI	TVOL	TOI
TX	98/07/22~01/12/31	−3.07**	−6.41***	−3.44***	−4.57***	6.87***	11.36***
	02/01/02~04/12/31	**−2.37**	−12.80***	**−2.33**	−12.61***	**10.31*****	9.07***
	05/01/03~07/12/31	−5.56***	−5.56***	−5.25***	−12.40***	0.71**	5.66***
TE	99/07/22~01/12/31	−2.85*	−7.53***	−2.98**	−6.11***	7.12***	7.58***
	02/01/02~04/12/31	**−2.22**	−12.20***	**−2.60**	−11.95***	**8.65*****	5.36***
	05/01/03~07/12/31	−4.67***	−14.21***	−4.42***	−14.19***	2.26***	1.87***
TF	99/07/22~01/12/31	−2.43	−5.18***	−3.05**	−4.21***	6.54***	5.89***
	02/01/02~04/12/31	**−1.56**	−8.05***	**−2.03**	−6.94***	**10.48*****	7.24***
	05/01/03~07/12/31	−3.09**	−12.34***	−2.94**	−12.32***	6.08***	0.45*

1. TVOL and TOI denote total volume and total open interest, respectively. All three test equations include an intercept but no trend since the inclusion of a time trend does not lead to different results

2. Superscript ***, **, * denote 1%, 5%, and 10% significance levels, respectively

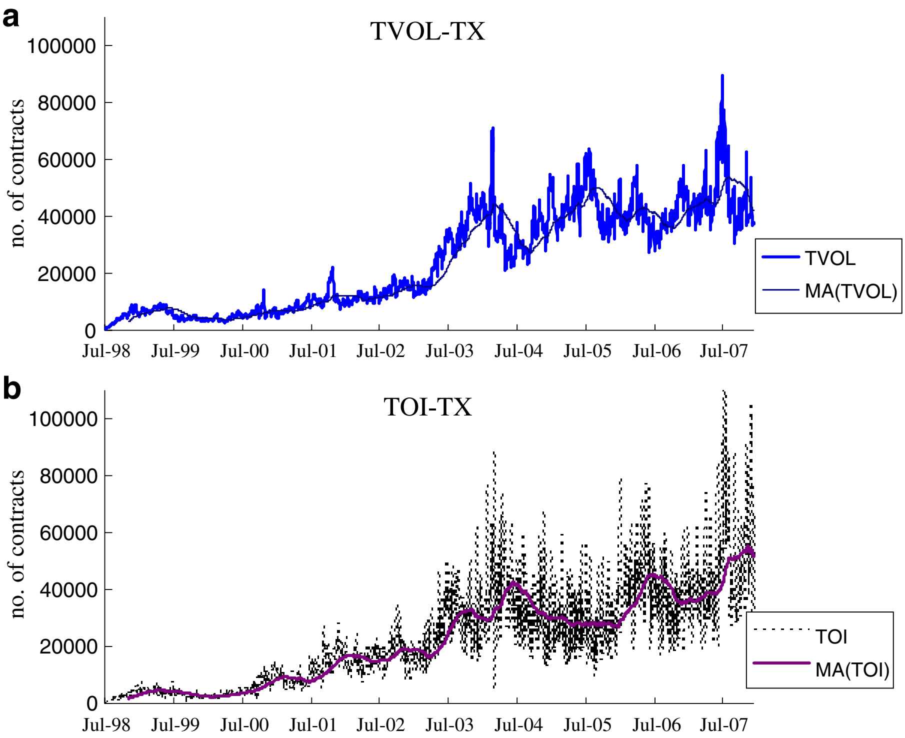

Fig. 1 **a** Charts of TVOL and 100-day MA of TVOL for TX (Jul.21,1998 to Jul. 31, 2007), **b** Charts of TOI and 100-day MA of TOI for TX (Jul.21,1998 to Jul. 31, 2007)

during the second sub-period of time (Jan. 2, 2002 through Dec. 31, 2004)[3] in which both tests indicate non-stationarity for all three contracts. Figures 1a, 2a and 3a confirm the apparent volatility in TVOL during the second sub-period. On the contrary, the null hypothesis of stationarity under the KPSS test is rejected for all sub-periods, contracts and variables. Despite the non-stationarity problem is recognized by all three tests only for the second sub-interval, we tend to use the log of TVOL and TOI in the subsequent modeling for all sample periods to avoid any non-stationarity problem.

3 Models

Before discussing whether the log of TVOL and/or the log of TOI help the GARCH type of models predict volatility of our three futures contracts, we analyze the relationship amongst volatility, the log of TVOL and the log of TOI.

[3] This is not surprising since Taiwan's futures market grew rapidly during that period of time. Taking TX as an example, TVOL increased from 9,401 on Jan. 2, 2002 to 67,316 on Mar. 17, 2004 right before the stock market crash on Mar. 22 and 23 due to a sudden political turmoil in the wake of Taiwan's presidential election on Mar 20. However, a growth of this size was not observed during the other two sub-periods.

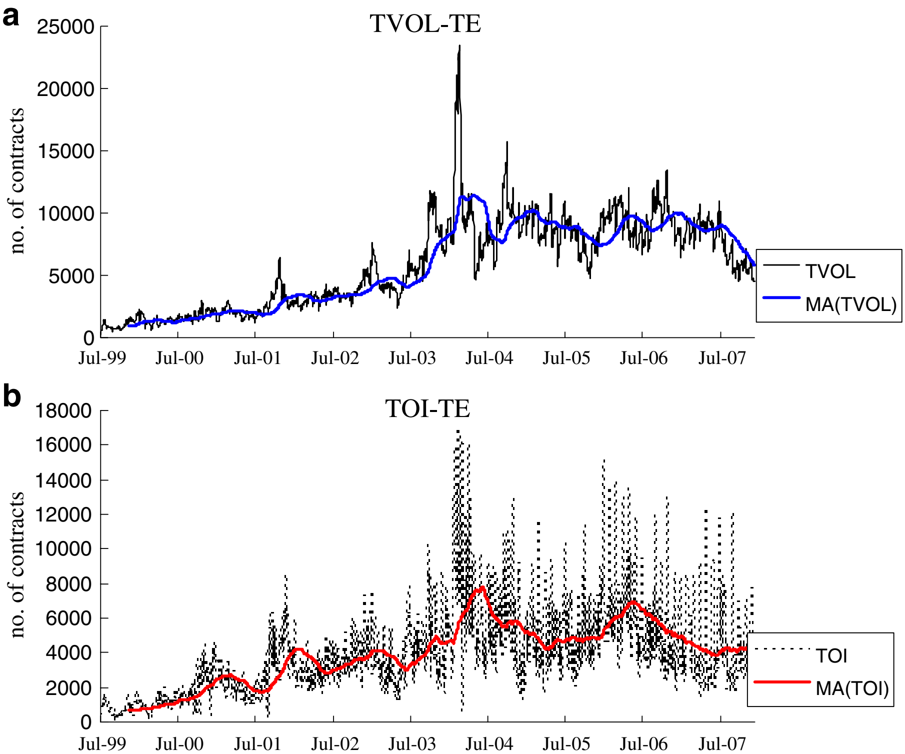

Fig. 2 **a** Charts of TVOL and 100-day MA of TVOL for TE (Jul.21,1999 to Jul. 31, 2007), **b** Charts of TOI and 100-day MA of TOI for TE (Jul.21,1999 to Jul. 31, 2007)

3.1 Ex-post analysis: VAR model

Following Fung and Patterson (1999) and Ferris et al. (2002), we apply the VAR model to our data. Fung and Patterson (1999) use a VAR(15) model for their five currency futures contracts to study this tri-lateral relationship, while Ferris et al. (2002) does not specify the order of their VAR model. To decide the best order of VAR, we use the Akaike's (1974) information criterion (AIC) for the entire sample period and for each of the three sub-periods. But note that, following GM, we adjust the raw daily returns in (1) and (2) for the day-of-the-week effect, if there is any, for all three futures markets by regressing daily returns upon constant plus dummy variables of Monday through Thursday. Although not reported here, the returns for each of the three sub-periods are purged of the day-of-the-week effect. The unobserved daily volatility is therefore measured by the square of adjusted daily return.

Results reported in Table 3 below are the best VAR order for each sample period under each contract as well as the relevant Granger causality test results. The results appear to suggest that the log of TVOL and the log of TOI does not have any lead-lag relationship since there is a bi-directional Granger causality between these two variable for most of the cases. This is in line with the results in Ferris et al. (2002) discussed in Section 1. The only exception occurs for the second sub-interval (02/01/

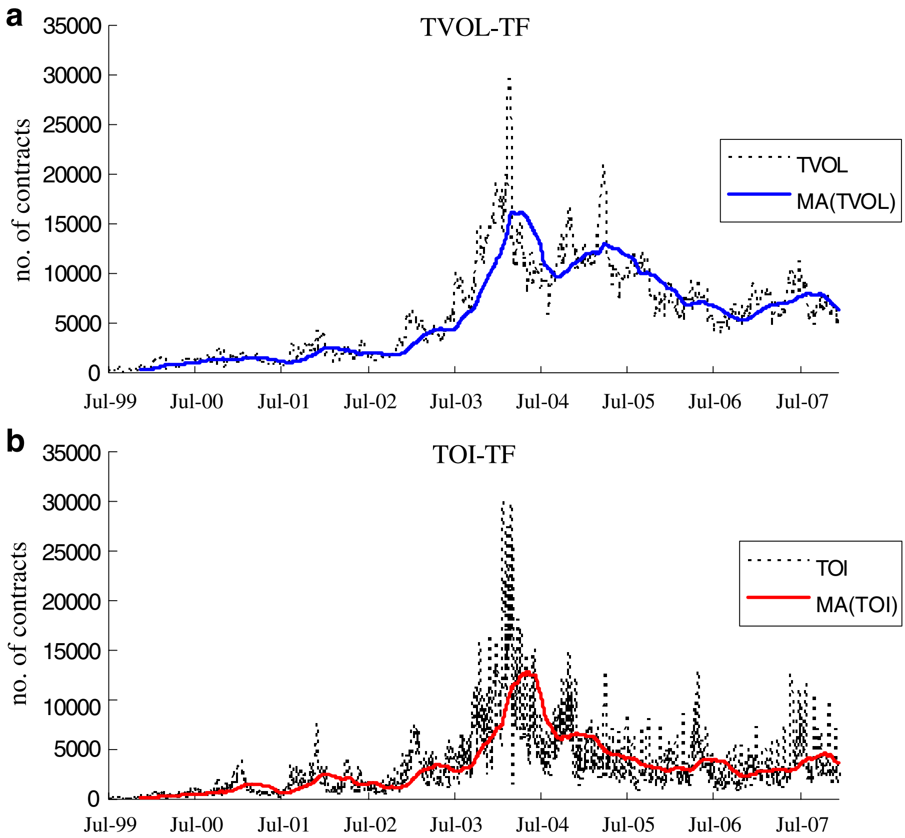

Fig. 3 **a** Charts of TVOL and 100-day MA of TVOL for TF (Jul.21,1999 to Jul. 31, 2007), **b** Charts of TOI and 100-day MA of TOI for TF (Jul.21,1999 to Jul. 31, 2007)

02~04/12/31) for both TX and TE contracts, in which the log of TOI is significantly dependent on the lagged log of TVOL.

As regards the relationship between volatility and the log of TVOL, results in Table 3 are mixed. For the TX contract, there exists a bi-directional causality between these two variables. For the TE contract, however, the relationship is not consistent across time. For the most recent sub-interval and the entire interval, the relationship is bi-directional, whereas the log of TVOL is Granger caused by volatility for the first sub-interval and vice versa for the middle sub-interval. Inconsistent results over time are also observed for the TF contract in that the two variables have an impact upon each other for the first sub-interval and the entire sample period. For the first and second sub-intervals, the log of TVOL is Granger caused by volatility.

Turning to the relationship between volatility and the log of TOI, we find from Table 3 that the results are also mixed. For the case of TX contract, the log of TOI is dependent upon lagged volatility for the first sub- and entire intervals, whereas a bi-directional relationship is observed for the recent two sub-intervals. For the case of

Table 3 Granger causality of volatility, log of volume and log of open interest, F-value with p-value in parentheses

	Range of data	Model order (by AIC)	Variable explained: Volatility			Variable explained: log of TVOL			Variable explained: log of TOI		
			Volatility	log of TVOL	log of TOI	Volatility	log of TVOL	log of TOI	Volatility	log of TVOL	log of TOI
TX	98/07/22 ~ 01/12/31	VAR(6)	7.35*** (0.00)	2.27** (0.04)	0.19 (0.98)	2.86*** (0.01)	1285.04*** (0.00)	2.78** (0.01)	7.04*** (0.00)	6.18*** (0.00)	131.00*** (0.00)
	02/01/02~ 04/12/31	VAR(6)	3.81*** (0.00)	1.95* (0.07)	2.92*** (0.01)	2.55** (0.02)	1453.44*** (0.00)	0.77 (0.59)	3.68*** (0.00)	12.58*** (0.00)	29.48*** (0.00)
	05/01/03~ 07/12/31	VAR(18)	4.34*** (0.00)	2.16*** (0.00)	1.46* (0.10)	1.71** (0.03)	213.08*** (0.00)	3.81*** (0.00)	1.63** (0.05)	4.72*** (0.00)	24.38*** (0.00)
	98/07/22 ~ 07/12/31	VAR(12)	11.87*** (0.00)	2.63*** (0.00)	1.24 (0.25)	2.49*** (0.00)	2354.18***(0.00)	1.91** (0.03)	7.63*** (0.00)	12.64*** (0.00)	159.79*** (0.00)
TE	99/07/22 ~ 01/12/31	VAR(5)	9.74*** (0.00)	1.05 (0.39)	0.67 (0.65)	2.29** (0.04)	643.47*** (0.00)	2.46** (0.03)	6.29*** (0.00)	6.25*** (0.00)	44.18*** (0.00)
	02/01/02~ 04/12/31	VAR(3)	3.90*** (0.01)	2.69** (0.05)	2.08 (0.10)	2.00 (0.11)	4511.24*** (0.00)	0.90 (0.44)	5.60*** (0.00)	26.48*** (0.00)	62.38*** (0.00)
	05/01/03~ 07/12/31	VAR(4)	11.82*** (0.00)	2.49** (0.04)	1.58 (0.18)	3.83*** (0.00)	1275.45*** (0.00)	17.90*** (0.00)	0.71 (0.59)	11.68*** (0.00)	79.69*** (0.00)
	99/07/22 ~ 07/12/31	VAR(21)	10.71*** (0.00)	1.96*** (0.01)	1.13 (0.31)	1.60** (0.04)	1125.35*** (0.00)	1.66** (0.03)	3.24*** (0.00)	6.56*** (0.00)	47.38*** (0.00)
TF	99/07/22 ~ 01/12/31	VAR(8)	5.72*** (0.00)	2.36** (0.02)	0.59 (0.78)	3.33*** (0.00)	386.06*** (0.00)	3.89*** (0.00)	5.12*** (0.00)	7.02*** (0.00)	55.12*** (0.00)
	02/01/02~ 04/12/31	VAR(5)	3.85*** (0.00)	0.84 (0.52)	2.59** (0.02)	1.93* (0.09)	1981.70*** (0.00)	2.51** (0.03)	3.38*** (0.01)	11.96*** (0.00)	61.75*** (0.00)
	05/01/03~ 07/12/31	VAR(3)	10.09***(0.00)	1.11 (0.34)	3.71** (0.01)	2.38* (0.07)	3473.58*** (0.00)	17.03*** (0.00)	0.48 (0.70)	11.94*** (0.00)	111.45*** (0.00)
	99/07/22 ~ 07/12/31	VAR(8)	13.51*** (0.00)	2.97*** (0.00)	1.35 (0.21)	3.69*** (0.00)	3074.35*** (0.00)	4.28*** (0.00)	4.75*** (0.00)	14.78*** (0.00)	180.89*** (0.00)

***, ** and * denote 1%, 5% and 10% significance level, respectively

TE contract, the log of TOI is Granger caused by volatility for the first two sub- and the entire intervals, whereas there is no relationship between these two variables for the most recent sub-interval. For the TF contract, finally, the relationship is more inconsistent across time. The log of TOI is dependent on lagged volatility for the first sub- and the entire intervals and vice versa for the last sub-interval. However, a bi-directional relationship is observed for the second sub-interval. Lack of consistent results, unfortunately, indicates none of the three theories justifying the relationship amongst the three variables is supported.

Focusing on the relationship between volatility and other two variables, we summarize the results in Table 3 by concluding that volatility is dependent upon lagged log of TVOL but not upon lagged log of TOI for most of the cases according to the Granger causality test. Since there is a significant bi-directional relationship between the log of TVOL and the log of TOI, however, the log of TOI would eventually affect volatility. This explains why we use lags in both the log of TVOL and the log of TOI to augment our chosen GARCH-type models in forecasting volatility.

3.2 Ex-ante analysis: basic and augmented GARCH models

Having discussed the tri-lateral relationship amongst volatility, the log of TVOL and the log of TOI, we focus on whether adding trading volume and/or open interest improves the predictive power for volatility models. Following GM, we adopt the GARCH type of models rather than the VAR type of models to estimate volatility since the former allows for volatility heteroscedasticity whilst the latter does not. Moreover, BS's model is also very close to the specification of a GARCH in mean. Note that, in contrast to this study, GM use the levels of volume and open interest of their three sets of futures spreads as explanatory variables in the GARCH(1,1) model since they argue that both variables are stationary. However, they achieve the conclusion of stationarity for these two variables via the ADF and PP tests without using the KPSS test which directly tests the stationarity of variables under the null. We report all three test results to avoid biased conclusion based on the first two tests, an issue already discussed in Section 2.

Our basic model for forecasting the conditional mean of the return series for each futures contract is then the autoregressive moving average (ARMA) model for either the raw returns (if there is no day-of-the-week effect) or the adjusted returns (if there is significant day-of-the-week effect):

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t, \tag{3}$$

where r_t is the raw return or the adjusted return on each contract at time t, and a_0, a_i and b_j are constant parameters. For each set of futures contracts, we use the likelihood ratio (LR) test to select the best-fitting autoregressive (AR), moving average (MA) or ARMA model. If $L(\theta_r)$ and $L(\theta_u)$ are the maximum log-likelihood function values under the null (restricted model) and the alternative (unrestricted model) hypotheses, respectively, then the statistic -2[$L(\theta_r)$-$L(\theta_u)$] will be asymptotically χ^2 distributed with the number of degrees of freedom equaling the number of restrictions under the null. More details will be discussed in Section 4.1.

To forecast the volatility of the returns series for each of the three sets of futures contracts, we consider the following symmetric and asymmetric GARCH(p,q) model specifications below and, following Akgiray (1989), again use the *LR* test approach to determine the appropriate orders for p and q.

3.2.1 Symmetric GARCH models:

GARCH model

As will be discussed in Section 4.1 later, MA(3) is the most appropriate specification for the conditional means of both TX and TE contracts, while MA(4) for TF contract. Combining this with a GARCH(1,1) specification (proposed by Bollerslev 1986) for the variance leads to an MA(3)-GARCH(1,1) (or MA(4)-GARCH(1,1) model, given by:

$$\begin{aligned} r_t &= \theta_0 + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \varepsilon_t, \\ \varepsilon_t|\Psi_{t-1} &\sim t_v(0, h_t), \\ h_t &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \end{aligned} \tag{4}$$

where $\Psi_{t\text{-}1}$ denotes all available information at time t–1, and α_0, α_1, and β_1 are constant and non-negative parameters, with $\alpha_1+\beta_1 < 1$.[4]

IGARCH model

The IGARCH model is estimated as a constrained GARCH model, with the GARCH polynomial restricted to equal one:

$$\sum_{j=1}^{p}\beta_j + \sum_{i=1}^{q}\alpha_i = 1, \tag{5}$$

or

$$[1 - \alpha(L) - \beta(L)] = 0, \tag{6}$$

where L is the lag operator. As a reflection of the persistence of volatility shocks to such a process, the IGARCH model is strictly but not weakly stationary.

3.2.2 Asymmetric GARCH models

In addition to the volatility clustering that is described by the symmetric GARCH models, a number of researchers have also found asymmetry in financial time series, such that negative return shocks seem to increase volatility to a greater extent than

[4] Instead of using the commonly adopted Gaussian distribution for the error term, we use the more general Student's t distribution which nests the Gaussian distribution and allows for the degree of leptokurtosis typically found in financial time series. See Bollerslev (1986), Bollerslev et al. (1992) and Mills (1999) for examples of conditionally leptokurtic GARCH models, which have been found to provide a better fit than the Gaussian GARCH model for most financial time series.

positive return shocks of the same magnitude (see Bollerslev et al. 1992; Engle and Ng 1993; and Pagan and Schwert 1990). Despite the success of the symmetric GARCH models, they cannot capture the asymmetry and skewness of the financial time series. This is the advantage of asymmetric GARCH models, which include the Exponential GARCH (EGARCH) model, the GJR-GARCH model, and the Asymmetric Power ARCH (APARCH) model.

EGARCH model

The MA(3)-EGARCH(1,1) model is given by:

$$\begin{aligned} & r_t = \theta_0 + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \varepsilon_t, \\ & \varepsilon_t|\Psi_{t-1} \sim t_v(0, h_t), \\ & \log(h_t) = \alpha_0 + \alpha_1\left[\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| - \sqrt{\frac{2}{\pi}}\right] + \beta\log(h_{t-1}) + \gamma\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}, \end{aligned} \tag{7}$$

where α_0, α_1, β, and γ are constant parameters.

GJR-GARCH model

An MA(3)-GJRGARCH(*p*,*q*) is given by:

$$\begin{aligned} & r_t = \theta_0 + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \varepsilon_t, \\ & \varepsilon_t|\Psi_{t-1} \sim t_v(0, h_t), \\ & h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma S_t^- \varepsilon_{t-1}^2, \end{aligned} \tag{8}$$

where α_0, α_1, β, and γ are constant parameters.

APARCH model

An MA(3)-APARCH (*p*,*q*) model specification can be expressed as:

$$\begin{aligned} & r_t = \theta_0 + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \varepsilon_t, \\ & \varepsilon_t|\Psi_{t-1} \sim t_v(0, h_t), \\ & \sigma_t^\delta = \omega + \sum_{i=1}^{q}\alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^\delta + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^\delta, \end{aligned} \tag{9}$$

where $\delta > 0$ and $-1 < \gamma_i < 1$ ($i = 1, \ldots, q$).

3.3 Addition of log of total open interest or log of total trading volume to the volatility models

In order to investigate whether the log of TOI and the log of TVOL help the GARCH models forecast future volatility, we add their lagged values as predictor variables to the variance equations of all five GARCH models. GM use the contemporaneous or the first lag of volume and/or open interest to study this issue.

However, the current term of both variables are not able to predict current volatility even though there is any relationship among the three variables. Since one of the main issues of this paper concerns volatility prediction, we adopt GM's method but use the first or second lag of the log of TVOL and/or TOI in our GARCH models. Although the VAR analysis above suggests a possibility of using more lags in these two explanatory variables, we focus on the first two lags to avoid over-fitting of the model. Thus, the augmented version of the basic GARCH(1,1) model specification, for example, could be written as:

$$\begin{aligned} &r_t = \theta_0 + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \varepsilon_t, \\ &\varepsilon_t|\Psi_{t-1} \sim t_v(0, h_t), \\ &h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma\mathbf{TVOL}_{t-i}\ (\text{and/or}+\lambda\mathbf{TOI}_{t-i}), \quad i = 1\ or\ 2 \end{aligned} \tag{10}$$

where TVOL_{t-i} and TOI_{t-i} denote the i^{th} lag of the total volume and the total open interest across all delivery months, respectively. The other GARCH models follow the same approach.

3.4 Evaluation of forecasting ability—Diebold and Mariano (DM) test

In order to assess and compare the relative out-of-sample forecasting abilities of these GARCH specifications that are fit to the three sets of futures contracts returns, we follow an approach introduced by Diebold and Mariano (1995). The DM test statistic is given by:

$$\frac{\overline{d} - \mu}{\left[\sum\limits_{\tau=-\infty}^{\tau=\infty} \gamma_d(\tau)\right] \Big/ \sqrt{T}} \xrightarrow{T\to\infty} N(0, 1), \tag{11}$$

where $\overline{d}$ denotes the sample mean of the loss differential series $\{d_t\}_{t=1}^{T}$ with $d_t \equiv e_{it}$ - e_{jt}, $e_{it} = \left|\sigma_{it}^2 - \sigma_t^2\right|$, and $e_{jt} = \left|\sigma_{jt}^2 - \sigma_t^2\right|$. σ_{it}^2 and σ_{jt}^2 denote the predicted variances by models i and j, respectively. μ denotes the true value of $\overline{d}$ under the null hypothesis, while $\gamma_d(\tau)$ refers to the sample autocovariance of d_t with its τ^{th} lag. In the present context, the null hypothesis assumes that $\mu = 0$. Under the null hypothesis, in other words, both series of forecasts are equally close, on average, to the "true" values for the given daily variances of the futures returns. If we reject the null hypothesis, a finding of $\overline{d} < 0$ would denote that model i is preferred to model j, while $\overline{d} > 0$ would denote that model j is preferred to model i. As we only forecast the volatility one day ahead, $\gamma_d(\tau)$ will be zero for all $\tau \neq 0$, namely, d_t has no autocorrelation. As a result, Eq. (12) reduces to the following form:

$$\frac{d}{\gamma_d(0)/\sqrt{T}} \xrightarrow{T\to\infty} N(0, 1). \tag{12}$$

We use this DM test to compare the variance forecasts of the various GARCH specifications pairwise (i.e., every model is compared with every other model). The DM test results will then be used to rank all of the models, for each of the three sets of futures contracts, in terms of their relative out-of-sample forecasting ability.

4 Empirical results

All the subsequent model estimation and out-of-sample volatility forecasting are performed using WinRATS 7.0, a very flexible software package for time series analysis.

4.1 In-sample estimation

Before estimating the various GARCH model specifications, we need an appropriate model for the conditional means of the data. As mentioned in Section 3, we examine ARMA specifications to describe the conditional means of all three return series. Based upon the *LR* test results reported in Table 4 below, we find the best-fitting conditional-mean model specification to be MA(3) for both TX and TE contracts and MA(4) for TF contract.

As regards the order of the GARCH-type models for the conditional variances, the basic specification of $p = 1$ and $q = 1$ (e.g., GARCH(1,1)) appears to show as good a model fit as other possibilities of (p, q). Higher orders such as GARCH(1,2), GARCH(2,1), and GARCH(2,2) do not appear to make any significant improvements in goodness-of-fit, as measured by *LR* tests. Similar results are also found for other GARCH variants estimated in this study. Thus, although there is little theoretical justification for the use of the (p=1,q=1) model order, we adopt the basic (1,1) order for all five GARCH models used in the following analysis.

Table 5 reports the estimates of the primary parameters of interest for each of the categories of GARCH models that are fitted to each of the three sets of futures returns. The parameter estimates reported include α_0, α_1, and β_1, for the GARCH(1,1) model, the volatility asymmetry parameter γ for the GJR-GARCH, EGARCH, and APARCH models, as well as the volatility scaling parameter δ for the APARCH model.

Table 5 also reports the estimation results for the asymmetry parameters incorporated into the GJR-GARCH, EGARCH, and APARCH models. Note that the estimates of the asymmetry parameter for all three models are statistically significant for the TX and TE contracts. This suggests the existence of a significant leverage or volatility asymmetry effect during the sample period for all these two sets of returns. Namely, bad news (negative return shocks) would have a greater impact on future volatility than good news (positive return shocks). In passing, the estimates of this asymmetry parameter are positive for both GJR-GARCH and APARCH models and negative for the EGARCH model due to the different ways these models incorporate the volatility asymmetry effect. However, data for the TF contract does not support volatility asymmetry under all three asymmetric models.

To investigate whether the forecast performance of the GARCH-type models above can be further improved, we add the lagged values of TVOL and TOI to the right-hand sides of the various variance equations. Following the approach discussed in Section 3.3, both lagged logs of TVOL and lagged logs of TOI are found to have a significant relationship with volatility for all three contracts. The coefficients for the two lags of either the log of TVOL or the log of TOI are significant for most of the cases experimented. This result is in line with previous evidence discussed in Section 1. The detailed results are not reported to save space.

Table 4 The *LR* test to determine the model for the conditional means

$L(\theta_r)$		AR(0)	AR(1)	MA(1)	AR(2)	MA(3)	MA(4)
$L(\theta_u)$							
TX	AR(1)	−254.69					
	MA(1)	3.24					
	ARMA(1,1)	−9.11					
	AR(2)	−7.14					
	MA(2)	4.16					
	ARMA(2,1)	−23.02					
	ARMA(1,2)	−55.41					
	AR(3)	0.09					
	MA(3)	16.62[c]					
	ARMA(1,3)						−3.27
	MA(4)						2.19
TE	AR(1)	−3.8					
	MA(1)	1.4					
	ARMA(1,1)	−43.71					
	AR(2)	−9.39					
	MA(2)	1.91					
	ARMA(2,1)	−9.07					
	ARMA(1,2)	−0.46					
	AR(3)	−3.46					
	MA(3)	9.45[b]					
	ARMA(1,3)						−9.61
	MA(4)						2.68
TF	AR(1)	−1359.73					
	MA(1)	5.93[b]					
	ARMA(1,1)	−38.39		−44.31			
	AR(2)						
	MA(2)			0.46			
	ARMA(2,1)						
	ARMA(1,2)						
	AR(3)						
	MA(3)			8.69[b]			
	ARMA(1,3)					0.05	
	MA(4)					4.89[b]	
	ARMA(1,4)						−6.89
	MA(5)						0.97

The *LR* statistic is defined as: $-2[L(\theta_r) - L(\theta_u)] \sim \chi^2_m$, where m is the number of restrictions. Superscript [c], [b] and [a], respectively, indicate statistical significance at the 1%, 5% and 10% significance levels (the rejection of the restricted model)

Table 5 Estimation results of rate of returns for the three futures and comparison of GARCH, GJR-GARCH, EGARCH and APARCH models

	GARCH(1,1)			GJR(1,1)	EGARCH(1,1)	APARCH(1,1)	
Contracts	α_0	α_1	β_1	γ	γ	γ	δ
TX	0.00 (.00)***	0.08 (.00)***	0.92 (.00)***	0.06 (.00)***	−0.05 (.00)***	0.37 (.00)***	0.91 (.00)***
TE	0.00 (.01)**	0.07 (.00)***	0.93 (.00)***	0.06 (.00)***	−0.06 (.00)***	0.31 (.01)***	1.48 (.00)***
TF	0.00 (.10)*	0.08 (.00)***	0.92 (.00)***	−0.01 (.67)	−0.00 (.91)	0.04 (.67)	0.67 (.02)**

The GARCH specification is: $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}$

The GJR–GARCH specification is: $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma S_t^- \varepsilon_{t-1}^2$

The EGARCH specification is: $\log(h_t) = \alpha_0 + \alpha_1\left[\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| - \sqrt{\frac{2}{\pi}}\right] + \beta_1 \log(h_{t-1}) + \gamma\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$

The APARCH specification is: $\sigma_t^\delta = \omega + \alpha_1(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta_1\sigma_{t-1}^\delta$

1. *p*-values in parentheses. 2. Superscripts ***, **, and * indicate statistical significance at the 1%, 5%, 10% level, respectively. From Table 5, it is clear that the α_1 and β_1 parameters in the GARCH(1,1) model are typically significant at the 1% level; hence, the constant variance model can readily be rejected, at least within sample. Moreover, the α_1 and β_1 parameter estimates are positive and their sum is less than unity for each of the three sets of contracts, so that an IGARCH specification is not indicated, although such specifications will be taken into account in the following section.

Related to this issue, we attempted to fit a FIGARCH(1,1) model to all three return series but found only marginal improvement. This might be due to the conversion of large return shocks into autocorrelation rather than into persistent volatility, a result of the impact of the daily price variation limits imposed on the Taiwan futures markets.

4.2 Forecasting results

In assessing the out-of-sample forecasting ability of our various candidate models, we calculate the following one-period-ahead forecasting errors for each of the different models:

$$u_{t+1} = \sigma_{t+1}^2 - \sigma_{f,t+1}^2, \tag{13}$$

where u_{t+1} is the forecasting error of the given forecasting model, σ_{t+1}^2 is measured by r_{t+1}^2 (following Eq.(2)), and $\sigma_{f,t+1}^2$ is the forecasted variance given by the variance equations. In order to find the one-day-ahead forecast of the variance for Dec 4, 2001, we use the data from July 22, 1998 to Dec 3, 2001 in *TX* (from July 22, 1999 to Dec 3, 2001 in *TE* and *TF*) as our initial in-sample modeling period for the first sub-interval to estimate the parameters of the models. The sample is then rolled forward by removing the first observation of the sample and adding one to the end, and another one-step-ahead forecast of the next day's variance is made. This forecasting procedure is then repeated for each subsequent trading day during the period from Dec 4, 2002 through December 31, 2002, amounting to a number of 20 daily volatility forecasts for the first sub-interval. Computation of forecasts using a rolling window of data should ensure that the forecasts are made based upon all updated information available at the time. The same approach is taken for the second and third sub-intervals. As a result, there are 60 daily volatility forecasts, 20 for each of the three different sub time periods for each model.

Thus the sequences of one-step-ahead forecasts are generated and evaluated by the mean squared error (MSE) statistic and then by the DM test, the procedure of the latter being described in Section 3.4. In accord with the MSE measure, the variance forecasts of all models are ranked for the entire sample period as well as for each of the three sub-intervals. The results are reported in Table 6 below. While the best model is not consistent across time for all three contracts, we do find that the best three predictors are almost augmented models for all three contracts and all time periods. The only three exceptional cases are: GJR-GARCH (ranked 3rd for the second sub-interval from 02/01/02 to 04/12/31) for TE and IGARCH (ranked 3rd for the entire sample period from 99/07/22 to 07/12/31 and ranked best for the first sub-interval from 99/07/22 to 01/12/31) for TF. However, what we are concerned with is whether adding lagged log of TVOL and/or lagged log of TOI improves the basic GARCH-type models in a statistical sense. To answer this question, we select the best basic model under each MSE ranking from Table 6 and adopt the DM test to investigate whether those of the basic model's variants augmented with log of TVOL and/or log of TOI are real relative outperformers. Table 7 reports the DM test results. For example, GARCH is ranked as the best amongst the five basic models in the entire sample period for TX according to the first column in Panel A of Table 6. We find that all six augmented variants of the basic GARCH model, identified by shadow background, provide more accurate forecasts according to MSE. They are then grouped and moved to the first column of Panel A in Table 7. We test the statistical significance of relative forecast performance between each of these six variants and the basic model. Results in Table 7 show that, while the augmented variants tend to outperform their basic model for all three contracts for the entire sample period, this finding is not consistent across the three sub-intervals for TX and TE futures. In particular, for TX, half the six variants defeat the basic model for the first sub-interval, and all four variants do for the second sub-interval but none for the last sub-interval. For the case of TE, four out of six augmented models beat the basic model for the first sub-interval, none for the second and two out of four for the last sub-interval. However, all variants outperform their basic model for the second and third sub-intervals while none for the first sub-interval. These results tend to suggest that adding lagged log of TVOL and/or log of TOI does not always improve the predictive power of the basic GARCH-type model depending which time period is involved. A relatively more consistent result appears for the second sub-interval from 2002 to 2004 when the levels of both trading volume and open interest for Taiwan's index futures market were most volatile. During this period, all variants outperform the basic model for both TX and TF contracts. However, do we just forget the use of trading volume and open interest to help forecast volatility? The answer might be 'No' since we can find at least one augmented variant outperforming the basic model for most of the cases. It could be the model augmented with log of TVOL, log of TOI or both, depending on which sample period and which contract is concerned.

In passing, we find that the symmetric GARCH or IGARCH models perform best amongst the five basic competitors for the entire sample period according to the MSE ranking. While this is not always the case if we examine their MSE ranking for sub-intervals, this result tend to cast a doubt on the use of asymmetric GARCH models advocated in the literature on volatility modeling for financial markets. At least, we find different evidence for Taiwan's futures market.

Table 6 Out-of-sample forecasting performance ranking based on MSE

Panel A: entire sample period

	TX 98/07/22 ~07/12/31		TE 99/07/22 ~07/12/31		TF 99/07/22 ~07/12/31	
Rank	Model	MSE	Model	MSE	Model	MSE
1	IGAR_LOGTIL(2)	7.664E-07	EGAR_LOGTVOL(2)	1.247E-06	**IGAR_LOGTVOL(1)**	1.250E-06
2	**GAR_LOGTIL(2)**	7.725E-07	APR_LOGTVOL(2)	1.248E-06	**IGAR_LOGTVOL(2)**	1.250E-06
3	**GAR_LOGTOI(2)**	7.730E-07	EGAR_LOGTVOL(1)	1.254E-06	**IGARCH**	1.252E-06
4	IGAR_LOGTVOL(2)	7.730E-07	EGAR_LOGTIL(1)	1.257E-06	IGAR_LOGTOI(1)	1.256E-06
5	**GAR_LOGTIL(1)**	7.730E-07	IGAR_LOGTIL(1)	1.257E-06	IGAR_LOGTOI(2)	1.256E-06
6	IGAR_LOGTIL(1)	7.731E-07	IGAR_LOGTIL(2)	1.258E-06	GAR_LOGTVOL(2)	1.266E-06
7	IGAR_LOGTVOL(1)	7.731E-07	EGAR_LOGTIL(2)	1.267E-06	GAR_LOGTVOL(1)	1.267E-06
8	**GAR_LOGTOI(1)**	7.731E-07	**GAR_LOGTIL(1)**	1.278E-06	IGAR_LOGTIL(1)	1.272E-06
9	IGAR_LOGTOI(2)	7.733E-07	**GAR_LOGTIL(2)**	1.279E-06	GAR_LOGTOI(2)	1.272E-06
10	**GAR_LOGTVOL(2)**	7.737E-07	**GAR_LOGTVOL(2)**	1.280E-06	GAR_LOGTOI(1)	1.273E-06
11	IGAR_LOGTOI(1)	7.738E-07	**GAR_LOGTVOL(1)**	1.280E-06	IGAR_LOGTIL(2)	1.274E-06
12	**GAR_LOGTVOL(1)**	7.739E-07	**GAR_LOGTOI(2)**	1.283E-06	**GJRGARCH**	1.276E-06
13	EGAR_LOGTVOL(1)	7.744E-07	**GARCH**	1.285E-06	**GARCH**	1.279E-06
14	EGAR_LOGTVOL(2)	7.756E-07	GAR_LOGTOI(1)	1.286E-06	GAR_LOGTIL(1)	1.292E-06
15	**GARCH**	7.759E-07	EGAR_LOGTOI(1)	1.286E-06	GAR_LOGTIL(2)	1.293E-06
16	EGAR_LOGTOI(1)	7.808E-07	**IGARCH**	1.289E-06	**APARCH**	1.356E-06
17	EGAR_LOGTIL(2)	7.837E-07	IGAR_LOGTVOL(2)	1.290E-06	EGAR_LOGTVOL(2)	1.357E-06
18	EGAR_LOGTIL(1)	7.837E-07	IGAR_LOGTVOL(1)	1.290E-06	EGAR_LOGTVOL(1)	1.358E-06
19	EGAR_LOGTOI(2)	7.844E-07	IGAR_LOGTOI(2)	1.290E-06	EGAR_LOGTOI(1)	1.370E-06
20	**EGARCH**	7.980E-07	APR_LOGTIL(1)	1.290E-06	EGAR_LOGTOI(2)	1.371E-06
21	APR_LOGTOI(2)	8.013E-07	IGAR_LOGTOI(1)	1.291E-06	**EGARCH**	1.373E-06
22	GJR_LOGTOI(2)	8.054E-07	**EGARCH**	1.292E-06	APR_LOGTOI(2)	1.379E-06
23	GJR_LOGTOI(1)	8.054E-07	APR_LOGTIL(2)	1.300E-06	EGAR_LOGTIL(2)	1.380E-06
24	**APARCH**	8.060E-07	GJR_LOGTVOL(1)	1.302E-06	EGAR_LOGTIL(1)	1.381E-06
25	GJR_LOGTVOL(1)	8.072E-07	GJR_LOGTOI(1)	1.303E-06	GJR_LOGTVOL(2)	1.402E-06
26	GJR_LOGTVOL(2)	8.073E-07	GJR_LOGTVOL(2)	1.303E-06	GJR_LOGTVOL(1)	1.403E-06
27	GJR_LOGTIL(1)	8.083E-07	**APARCH**	1.304E-06	GJR_LOGTIL(1)	1.406E-06
28	GJR_LOGTIL(2)	8.097E-07	GJR_LOGTOI(2)	1.307E-06	GJR_LOGTOI(2)	1.406E-06
29	**GJRGARCH**	8.103E-07	APR_LOGTVOL(1)	1.311E-06	GJR_LOGTOI(1)	1.407E-06
30	APR_LOGTOI(1)	8.111E-07	GJR_LOGTIL(2)	1.313E-06	APR_LOGTVOL(1)	1.407E-06
31	APR_LOGTIL(1)	8.120E-07	GJR_LOGTIL(1)	1.319E-06	GJR_LOGTIL(2)	1.409E-06
32	APR_LOGTVOL(1)	8.138E-07	APR_LOGTOI(1)	1.326E-06	APR_LOGTOI(1)	1.430E-06
33	APR_LOGTIL(2)	8.177E-07	**GJRGARCH**	1.340E-06	APR_LOGTVOL(2)	1.446E-06
34	APR_LOGTVOL(2)	8.266E-07	APR_LOGTOI(2)	1.358E-06	APR_LOGTIL(1)	1.515E-06
35	**IGARCH**	9.170E-07	EGAR_LOGTOI(2)	4.553E+01	APR_LOGTIL(2)	1.546E-06

1. GAR, IGAR, EGAR, GJR and APR denote, respectively, GARCH, IGARCH, EGARCH, GJR-GARCH and APARCH. 2. LOGTIL(i) denotes that the i^{th} lag in both the log of TOI and log of TVOL are included in the corresponding variance model. Similar definition applies to the notation LOGTVOL(i) and LOGTOI(i)

Table 6 (continued)

Panel B: first sub-interval

	TX 98/07/22 ~01/12/31		TE 99/07/22 ~01/12/31		TF 99/07/22 ~01/12/31	
Rank	Model	MSE	Model	MSE	Model	MSE
1	IGAR_LOGTIL(2)	1.946E-06	APR_LOGTVOL(2)	2.909E-06	IGARCH	3.517E-06
2	IGAR_LOGTVOL(1)	1.962E-06	APR_LOGTIL(1)	3.070E-06	IGAR_LOGTVOL(1)	3.518E-06
3	IGAR_LOGTVOL(2)	1.962E-06	IGAR_LOGTIL(1)	3.084E-06	IGAR_LOGTVOL(2)	3.519E-06
4	**GAR_LOGTIL(2)**	1.962E-06	IGAR_LOGTIL(2)	3.100E-06	IGAR_LOGTOI(1)	3.536E-06
5	IGAR_LOGTIL(1)	1.962E-06	EGAR_LOGTVOL(2)	3.112E-06	IGAR_LOGTOI(2)	3.537E-06
6	IGAR_LOGTOI(2)	1.963E-06	APR_LOGTIL(2)	3.127E-06	**GJRGARCH**	3.556E-06
7	**GAR_LOGTIL(1)**	1.963E-06	EGAR_LOGTVOL(1)	3.133E-06	GAR_LOGTVOL(2)	3.571E-06
8	IGAR_LOGTOI(1)	1.964E-06	EGAR_LOGTIL(1)	3.147E-06	GAR_LOGTVOL(1)	3.572E-06
9	**GAR_LOGTOI(2)**	1.964E-06	**GAR_LOGTVOL(1)**	3.162E-06	GAR_LOGTOI(2)	3.585E-06
10	**GAR_LOGTOI(1)**	1.964E-06	**GAR_LOGTVOL(2)**	3.163E-06	GAR_LOGTOI(1)	3.585E-06
11	**GAR_LOGTVOL(2)**	1.967E-06	**GAR_LOGTOI(2)**	3.163E-06	IGAR_LOGTIL(1)	3.590E-06
12	**GAR_LOGTVOL(1)**	1.967E-06	**GAR_LOGTIL(1)**	3.168E-06	IGAR_LOGTIL(2)	3.595E-06
13	GARCH	1.971E-06	**GAR_LOGTOI(1)**	3.168E-06	**GARCH**	3.598E-06
14	EGAR_LOGTVOL(1)	1.971E-06	**GAR_LOGTIL(2)**	3.171E-06	GAR_LOGTIL(1)	3.652E-06
15	EGAR_LOGTVOL(2)	1.974E-06	GARCH	3.171E-06	GAR_LOGTIL(2)	3.655E-06
16	EGAR_LOGTIL(2)	1.998E-06	IGAR_LOGTVOL(1)	3.176E-06	**APARCH**	3.839E-06
17	EGAR_LOGTOI(1)	1.998E-06	IGAR_LOGTVOL(2)	3.176E-06	EGAR_LOGTVOL(2)	3.847E-06
18	EGAR_LOGTOI(2)	2.001E-06	EGAR_LOGTIL(2)	3.179E-06	EGAR_LOGTVOL(1)	3.853E-06
19	EGAR_LOGTIL(1)	2.001E-06	**IGARCH**	3.179E-06	EGAR_LOGTOI(1)	3.887E-06
20	**EGARCH**	2.038E-06	IGAR_LOGTOI(1)	3.180E-06	EGAR_LOGTOI(2)	3.889E-06
21	**APARCH**	2.054E-06	IGAR_LOGTOI(2)	3.180E-06	**EGARCH**	3.892E-06
22	APR_LOGTOI(1)	2.054E-06	EGAR_LOGTOI(1)	3.205E-06	APR_LOGTVOL(1)	3.912E-06
23	APR_LOGTOI(2)	2.055E-06	APR_LOGTOI(1)	3.217E-06	APR_LOGTOI(2)	3.915E-06
24	GJR_LOGTIL(2)	2.055E-06	**EGARCH**	3.228E-06	EGAR_LOGTIL(2)	3.916E-06
25	GJR_LOGTOI(2)	2.057E-06	GJR_LOGTVOL(2)	3.238E-06	EGAR_LOGTIL(1)	3.917E-06
26	GJR_LOGTOI(1)	2.059E-06	GJR_LOGTVOL(1)	3.238E-06	GJR_LOGTVOL(2)	3.981E-06
27	APR_LOGTVOL(1)	2.061E-06	GJR_LOGTOI(1)	3.242E-06	GJR_LOGTVOL(1)	3.984E-06
28	GJR_LOGTIL(1)	2.061E-06	GJR_LOGTOI(2)	3.243E-06	GJR_LOGTIL(1)	3.994E-06
29	APR_LOGTVOL(2)	2.061E-06	**APARCH**	3.250E-06	GJR_LOGTOI(2)	3.995E-06
30	GJR_LOGTVOL(1)	2.064E-06	APR_LOGTVOL(1)	3.260E-06	GJR_LOGTOI(1)	3.996E-06
31	APR_LOGTIL(2)	2.066E-06	APR_LOGTOI(2)	3.282E-06	GJR_LOGTIL(2)	4.002E-06
32	GJR_LOGTVOL(2)	2.066E-06	GJR_LOGTIL(1)	3.284E-06	APR_LOGTOI(1)	4.069E-06
33	APR_LOGTIL(1)	2.069E-06	GJR_LOGTIL(2)	3.288E-06	APR_LOGTVOL(2)	4.119E-06
34	**GJRGARCH**	2.070E-06	**GJRGARCH**	3.323E-06	APR_LOGTIL(1)	4.321E-06
35	**IGARCH**	2.392E-06	EGAR_LOGTOI(2)	1.366E+02	APR_LOGTIL(2)	4.359E-06

Table 6 (continued)

Panel C: second sub-interval

	TX 02/01/02~04/12/31		TE 02/01/02~04/12/31		TF 02/01/02~04/12/31	
Rank	Model	MSE	Model	MSE	Model	MSE
1	**IGAR_LOGTOI(2)**	2.05E-08	EGAR_LOGTIL(2)	2.553E-08	IGAR_LOGTIL(1)	2.028E-08
2	**IGAR_LOGTOI(1)**	2.08E-08	EGAR_LOGTIL(1)	2.626E-08	IGAR_LOGTIL(2)	2.034E-08
3	GAR_LOGTVOL(2)	2.11E-08	**GJRGARCH**	3.006E-08	IGAR_LOGTVOL(2)	2.099E-08
4	**IGAR_LOGTVOL(2)**	2.11E-08	GJR_LOGTIL(2)	3.053E-08	IGAR_LOGTVOL(1)	2.115E-08
5	**IGAR_LOGTVOL(1)**	2.13E-08	GJR_LOGTIL(1)	3.054E-08	**EGAR_LOGTVOL(1)**	2.134E-08
6	EGAR_LOGTIL(2)	2.14E-08	GAR_LOGTIL(2)	3.377E-08	**EGAR_LOGTVOL(2)**	2.136E-08
7	GAR_LOGTVOL(1)	2.15E-08	GAR_LOGTIL(1)	3.432E-08	**EGAR_LOGTOI(2)**	2.190E-08
8	EGAR_LOGTVOL(2)	2.18E-08	EGAR_LOGTVOL(2)	3.472E-08	**EGAR_LOGTOI(1)**	2.191E-08
9	EGAR_LOGTIL(1)	2.19E-08	EGAR_LOGTVOL(1)	3.512E-08	GAR_LOGTIL(2)	2.197E-08
10	GAR_LOGTOI(2)	2.20E-08	**APARCH**	3.783E-08	GAR_LOGTIL(1)	2.206E-08
11	EGAR_LOGTVOL(1)	2.22E-08	EGAR_LOGTOI(2)	4.213E-08	IGAR_LOGTOI(2)	2.231E-08
12	GAR_LOGTOI(1)	2.24E-08	EGAR_LOGTOI(1)	4.232E-08	IGAR_LOGTOI(1)	2.253E-08
13	**IGARCH**	2.25E-08	**EGARCH**	4.233E-08	GJR_LOGTIL(1)	2.280E-08
14	APR_LOGTVOL(2)	2.29E-08	IGAR_LOGTIL(2)	4.362E-08	GJR_LOGTVOL(2)	2.284E-08
15	APR_LOGTVOL(1)	2.29E-08	IGAR_LOGTIL(1)	4.494E-08	GJR_LOGTVOL(1)	2.296E-08
16	GAR_LOGTIL(2)	2.30E-08	GAR_LOGTVOL(2)	4.704E-08	GJR_LOGTIL(2)	2.309E-08
17	EGAR_LOGTOI(2)	2.32E-08	GAR_LOGTVOL(1)	4.742E-08	APR_LOGTOI(2)	2.368E-08
18	EGAR_LOGTOI(1)	2.33E-08	GAR_LOGTOI(1)	4.789E-08	APR_LOGTOI(1)	2.384E-08
19	**APARCH**	2.35E-08	GAR_LOGTOI(2)	4.811E-08	GJR_LOGTOI(2)	2.398E-08
20	IGAR_LOGTIL(2)	2.37E-08	**GARCH**	4.820E-08	GJR_LOGTOI(1)	2.414E-08
21	GAR_LOGTIL(1)	2.38E-08	GJR_LOGTVOL(2)	4.988E-08	**EGAR_LOGTIL(1)**	2.483E-08
22	**GARCH**	2.40E-08	**IGARCH**	5.017E-08	**EGARCH**	2.483E-08
23	IGAR_LOGTIL(1)	2.42E-08	IGAR_LOGTVOL(2)	5.050E-08	APR_LOGTVOL(2)	2.649E-08
24	GJR_LOGTVOL(2)	2.47E-08	GJR_LOGTVOL(1)	5.051E-08	GAR_LOGTVOL(2)	2.658E-08
25	GJR_LOGTVOL(1)	2.54E-08	IGAR_LOGTVOL(1)	5.116E-08	**APARCH**	2.694E-08
26	**EGARCH**	2.56E-08	APR_LOGTVOL(1)	5.223E-08	GAR_LOGTVOL(1)	2.697E-08
27	GJR_LOGTIL(2)	2.59E-08	IGAR_LOGTOI(2)	5.267E-08	EGAR_LOGTIL(2)	2.738E-08
28	APR_LOGTIL(1)	2.60E-08	GJR_LOGTOI(2)	5.463E-08	**IGARCH**	2.797E-08
29	APR_LOGTOI(2)	2.71E-08	GJR_LOGTOI(1)	5.491E-08	GAR_LOGTOI(2)	3.123E-08
30	GJR_LOGTOI(2)	2.78E-08	IGAR_LOGTOI(1)	5.552E-08	APR_LOGTIL(1)	3.141E-08
31	GJR_LOGTIL(1)	2.79E-08	APR_LOGTIL(2)	1.142E-07	GAR_LOGTOI(1)	3.161E-08
32	GJR_LOGTOI(1)	2.84E-08	APR_LOGTIL(1)	1.180E-07	**GARCH**	3.995E-08
33	**GJRGARCH**	2.98E-08	APR_LOGTOI(1)	1.581E-07	**GJRGARCH**	5.698E-08
34	APR_LOGTOI(1)	4.20E-08	APR_LOGTOI(2)	1.679E-07	APR_LOGTIL(2)	8.584E-08
35	APR_LOGTIL(2)	5.70E-08	APR_LOGTVOL(2)	1.976E-07	APR_LOGTVOL(1)	1.166E-07

Table 6 (continued)

Panel D: third sub-interval

	TX		TE		TF	
	05/01/03~07/12/31		05/01/03~07/12/31		05/01/03~07/12/31	
Rank	Model	MSE	Model	MSE	Model	MSE
1	**EGAR_LOGTOI(1)**	3.211E-07	**EGAR_LOGTVOL(2)**	5.930E-07	APR_LOGTIL(2)	1.921E-07
2	APR_LOGTOI(2)	3.217E-07	**EGAR_LOGTVOL(1)**	5.955E-07	APR_LOGTVOL(2)	1.923E-07
3	**EGAR_LOGTIL(1)**	3.282E-07	**EGAR_LOGTIL(1)**	5.980E-07	APR_LOGTVOL(1)	1.923E-07
4	GJR_LOGTOI(1)	3.291E-07	**EGAR_LOGTIL(2)**	5.980E-07	APR_LOGTIL(1)	1.942E-07
5	**EGAR_LOGTOI(2)**	3.292E-07	APR_LOGTOI(1)	6.020E-07	EGAR_LOGTIL(2)	1.961E-07
6	IGAR_LOGTIL(2)	3.298E-07	**EGARCH**	6.065E-07	APR_LOGTOI(1)	1.974E-07
7	APR_LOGTIL(2)	3.300E-07	EGAR_LOGTOI(1)	6.111E-07	EGAR_LOGTVOL(1)	1.984E-07
8	**EGAR_LOGTVOL(1)**	3.300E-07	GJR_LOGTOI(1)	6.112E-07	APR_LOGTOI(2)	1.992E-07
9	**EGARCH**	3.307E-07	EGAR_LOGTOI(2)	6.113E-07	GJR_LOGTOI(2)	2.006E-07
10	GJR_LOGTVOL(2)	3.310E-07	GJR_LOGTVOL(1)	6.176E-07	GJR_LOGTOI(1)	2.007E-07
11	EGAR_LOGTVOL(2)	3.310E-07	GJR_LOGTIL(2)	6.199E-07	**GAR_LOGTIL(2)**	2.008E-07
12	**GJRGARCH**	3.311E-07	APR_LOGTVOL(1)	6.222E-07	**GARCH**	2.008E-07
13	GJR_LOGTOI(2)	3.316E-07	GJR_LOGTOI(2)	6.224E-07	GAR_LOGTOI(1)	2.008E-07
14	EGAR_LOGTIL(2)	3.317E-07	GJR_LOGTVOL(2)	6.227E-07	GAR_LOGTIL(1)	2.008E-07
15	GJR_LOGTVOL(1)	3.322E-07	**APARCH**	6.233E-07	GAR_LOGTOI(2)	2.008E-07
16	GAR_LOGTIL(1)	3.324E-07	APR_LOGTOI(2)	6.238E-07	GAR_LOGTVOL(2)	2.010E-07
17	GAR_LOGTIL(2)	3.324E-07	GAR_LOGTVOL(2)	6.291E-07	GAR_LOGTVOL(1)	2.010E-07
18	IGAR_LOGTIL(1)	3.327E-07	GAR_LOGTVOL(1)	6.310E-07	GJR_LOGTVOL(2)	2.010E-07
19	GAR_LOGTOI(1)	3.328E-07	IGAR_LOGTIL(2)	6.313E-07	GJR_LOGTVOL(1)	2.011E-07
20	GAR_LOGTOI(2)	3.329E-07	GAR_LOGTIL(2)	6.319E-07	EGAR_LOGTOI(1)	2.011E-07
21	GAR_LOGTVOL(1)	3.329E-07	GAR_LOGTIL(1)	6.326E-07	EGAR_LOGTOI(2)	2.013E-07
22	GAR_LOGTVOL(2)	3.329E-07	**GARCH**	6.359E-07	GJR_LOGTIL(2)	2.013E-07
23	**GARCH**	3.330E-07	GAR_LOGTOI(2)	6.367E-07	**APARCH**	2.015E-07
24	GJR_LOGTIL(1)	3.363E-07	IGAR_LOGTOI(1)	6.371E-07	EGAR_LOGTVOL(2)	2.017E-07
25	IGAR_LOGTVOL(1)	3.364E-07	**IGARCH**	6.371E-07	GJR_LOGTIL(1)	2.018E-07
26	**IGARCH**	3.364E-07	IGAR_LOGTOI(2)	6.372E-07	EGAR_LOGTIL(1)	2.019E-07
27	IGAR_LOGTVOL(2)	3.364E-07	APR_LOGTVOL(2)	6.377E-07	**EGARCH**	2.020E-07
28	IGAR_LOGTOI(1)	3.366E-07	GAR_LOGTOI(1)	6.404E-07	IGAR_LOGTIL(2)	2.061E-07
29	IGAR_LOGTOI(2)	3.366E-07	IGAR_LOGTVOL(1)	6.418E-07	IGAR_LOGTIL(1)	2.067E-07
30	APR_LOGTOI(1)	3.376E-07	IGAR_LOGTVOL(2)	6.421E-07	IGAR_LOGTOI(2)	2.087E-07
31	APR_LOGTIL(1)	3.408E-07	IGAR_LOGTIL(1)	6.424E-07	IGAR_LOGTOI(1)	2.090E-07
32	**APARCH**	3.409E-07	GJR_LOGTIL(1)	6.427E-07	**IGARCH**	2.104E-07
33	GJR_LOGTIL(2)	3.476E-07	APR_LOGTIL(2)	6.585E-07	IGAR_LOGTVOL(1)	2.105E-07
34	APR_LOGTVOL(1)	3.578E-07	**GJRGARCH**	6.661E-07	IGAR_LOGTVOL(2)	2.106E-07
35	APR_LOGTVOL(2)	3.960E-07	APR_LOGTIL(1)	6.831E-07	**GJRGARCH**	2.152E-07

Table 7 DM test results for relative performance between the outperforming augmented model and the basic model

Model	DM test	Model	DM test	Model	DM test
Panel A: entire sample period					
TX		TE		TF	
98/07/22 ~07/12/31		99/07/22 ~07/12/31		99/07/22 ~07/12/31	
GAR_LOGTIL(2)	$+^{***}$	GAR_LOGTIL(1)	$+^{***}$	IGAR_LOGTVOL(1)	$+^{***}$
GAR_LOGTOI(2)	$+^{***}$	GAR_LOGTIL(2)	$+^{***}$	IGAR_LOGTVOL(2)	$+^{***}$
GAR_LOGTIL(1)	$+^{***}$	GAR_LOGTVOL(2)	$+^{***}$	**IGARCH**	
GAR_LOGTOI(1)	$+^{***}$	GAR_LOGTVOL(1)	$+^{***}$		
GAR_LOGTVOL(2)	$+^{***}$	GAR_LOGTOI(2)	$-^{***}$		
GAR_LOGTVOL(1)	$+^{***}$	**GARCH**			
GARCH					
Panel B: first sub-interval					
98/07/22 ~01/12/31		99/07/22 ~01/12/31		99/07/22 ~01/12/31	
GAR_LOGTIL(2)	$-^{***}$	GAR_LOGTVOL(1)	$+^{***}$	**IGARCH**	
GAR_LOGTIL(1)	$+^{***}$	GAR_LOGTVOL(2)	$+^{***}$		
GAR_LOGTOI(2)	$-^{***}$	GAR_LOGTOI(2)	$-^{***}$		
GAR_LOGTOI(1)	$-^{***}$	GAR_LOGTIL(1)	$+^{***}$		
GAR_LOGTVOL(2)	$+^{***}$	GAR_LOGTOI(1)	$-^{***}$		
GAR_LOGTVOL(1)	$+^{***}$	GAR_LOGTIL(2)	$+^{***}$		
GARCH		**GARCH**			
Panel C: second sub-interval					
02/01/02~04/12/31		02/01/02~04/12/31		02/01/02~04/12/31	
IGAR_LOGTOI(2)	$+^{***}$	**GJRGARCH**		EGAR_LOGTVOL(1)	$+^{***}$
IGAR_LOGTOI(1)	$+^{***}$			EGAR_LOGTVOL(2)	$+^{***}$
IGAR_LOGTVOL(2)	$+^{***}$			EGAR_LOGTOI(2)	$+^{***}$
IGAR_LOGTVOL(1)	$+^{***}$			EGAR_LOGTOI(1)	$+^{***}$
IGARCH				EGAR_LOGTIL(1)	$-^{***}$
				EGARCH	
Panel D: third sub-interval					
05/01/03~07/12/31		05/01/03~07/12/31		05/01/03~07/12/31	
EGAR_LOGTOI(1)	$-^{***}$	EGAR_LOGTVOL(2)	$-^{***}$	GAR_LOGTIL(2)	$+^{***}$
EGAR_LOGTIL(1)	$-^{***}$	EGAR_LOGTVOL(1)	$-^{***}$	**GARCH**	
EGAR_LOGTOI(2)	$-^{***}$	EGAR_LOGTIL(1)	$+^{***}$		
EGAR_LOGTVOL(1)	$-^{***}$	EGAR_LOGTIL(2)	$+^{***}$		
EGARCH		**EGARCH**			

1.The bottom model under each contract is the basic model which performs best amongst the five basic models. 2. *** denotes 1% significance for the DM test, whereas + and – denote that the augmented model outperforms (+) or underperforms (−) the benchmark basic model

5 Conclusions

This paper studies the interrelationship amongst volatility and the log of total trading volume and log of total open interest in three Taiwan's stock index futures markets from both ex-post (in-sample modeling) and ex-ante (out-of-sample forecasting) perspectives. To study the first issue, i.e., the relationship amongst the three variables, we use the VAR model and find significant relationship between each pair of the three variables. We then examine the relationship between volatility of the futures returns and market trading volume and/or open interest by directly adding the lagged total volume and/or lagged total open interest data to the right-hand side of the variance equations. To control for the leverage effect in forecasting volatility, moreover, we employ three asymmetric GARCH models (EGARCH, GJR, and APARCH) as well as the standard GARCH and IGARCH models as the basic models. We find significant in-sample relationships amongst the futures' daily volatilities, the lagged total volume and the lagged total open interest. To investigate whether inclusion of the open interest or the market volume helps the basic models predict future volatility, we use the MSE loss function to compare the relative out-of-sample volatility-forecasting performance amongst all models and the DM test to examine whether the augmented variants really beat their corresponding basic model.

Despite the in-sample significant explanatory power of lagged logs in TVOL and TOI for GARCH models, the DM tests suggest that including these two variables does not guarantee a better out-of-sample forecasting performance for volatility. Whether addition of lagged total volume and/or lagged total open interest helps the basic GARCH models predict future volatility depends upon the sample period examined for all three sets of futures. Nonetheless, the best three models for out-of-sample volatility forecasting in the MSE sense are generally the augmented models for all sub-intervals and all three futures contracts. This result tends to support both the 'sequential information arrival' theory and the 'traders with trade time discretion tend to trade when market is relatively liquid' theory. In addition, the three asymmetric GARCH models are not found to always outperform their two symmetric competitors, i.e. GARCH and IGARCH.

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