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Markov Switching Dynamics in REIT Returns: Univariate and Multivariate Evidence on Forecasting Performance

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We document the presence of Markov switching regimes in expected returns, variances and the implied reward-to-risk ratio of real estate investment trust (REIT) returns and compare them to properties of stocks and bonds. Our evidence suggests that regime switching techniques are more successful over the period 1972–2008 than other time-series models are. When the analysis is extended to a multivariate setting in which REIT, stock and bond returns are modeled jointly, we find that the data call for the specification of four separate regimes. These result from the absence of synchronicity among the regimes that characterize univariate REIT, stock and bond returns.

The turmoil that hit most asset markets in 2008–2009 has focused renewed attention on return characteristics that, if better understood, might have been used to hedge against particular risks and to moderate portfolio value swings. This has certainly been true for the publicly traded real estate investment trust (REIT) asset class, in which an unprecedented peak-to-trough decline of 68% was coupled with surges in both volatility and the correlation with other equity assets.

Even before the market crisis, though, researchers in real estate finance had been paying increased attention to the rich dynamic time-series properties of REIT returns. Devaney (2001), Sing (2004), Ewing and Payne (2005), Najand, Lin and Fitzgerald (2006), Bredin, O'Reilly and Stevenson (2007), Kim, Leatham and Bessler (2007), Cheong, Wilson and Zurbruegg (2009), Hung and Glascock (2009), Lin, Rahman and Yung (2009) and Stevenson, Wilson and Zurbruegg (2009) are among recent papers that have examined

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REIT return dynamics. Others, including Winniford (2003), Cotter and Stevenson (2006, 2008), Jirasakuldech, Campbell and Emekter (2009), Liow *et al.* (2009) and Zhou and Kang (2011), have focused on REIT return volatilities, while return correlations between REITs and other assets have been investigated by Chandrashekaran (1999), Clayton and MacKinnon (2001), Conover, Friday and Sirmans (2002), Bley and Olson (2003), Feng, Ghosh and Sirmans (2006), Huang and Zhong (2006), Westerheide (2006), Ambrose, Lee and Peek (2007), Chen (2007) and Case, Yang and Yildirim (2012).

Researchers have employed a variety of empirical techniques to model dynamic REIT return properties. For simple comparisons of sample statistics, Chandrashekaran (1999), Clayton and MacKinnon (2001) and Westerheide (2006) divided their study periods into arbitrarily defined subperiods. Jirasakuldech, Campbell and Knight (2006), Feng, Ghosh and Sirmans (2006) and Ambrose, Lee and Peek (2007) employed exogenously specified structural break points: November 2001 (citing previous researchers) in the first case and October 2001 (the month in which REITs were first included in the S&P 500 and other broad stock market indexes) in the latter two cases. Kim, Leatham and Bessler (2007) and Cheong, Wilson and Zurbruegg (2009) endogenously identified structural break points. Conover, Friday and Sirmans (2002), Bley and Olson (2003), Sing (2004) and Chen (2007) identified no break points but estimated models over exogenously specified rolling periods.

Other researchers have applied several versions of Autoregressive Conditional Heteroskedasticity (ARCH) models to REIT return data to try to capture both time-varying volatilities and correlations in structural time-series frameworks designed to accomplish this task. Cotter and Stevenson (2006), Najand, Lin and Fitzgerald (2006), Bredin, O'Reilly and Stevenson (2007) and Jirasakuldech, Campbell and Emekter (2009), among others, have estimated generalized ARCH (GARCH) models. Devaney (2001), Najand, Lin and Fitzgerald (2006) and Hung and Glascock (2009) employed GARCH in means (GARCH-M) models, while Winniford (2003) used a periodic GARCH (P-GARCH) model. Cotter and Stevenson (2008) and Zhou and Kang (2011) employed fractionally integrated and fractionally integrated exponential GARCH (FIGARCH and FIEGARCH) models.

Far fewer researchers have applied Markov regime switching (for simplicity, MS) frameworks to model REIT returns. Liow *et al.* (2005) examined shifts in returns and volatility in six listed property markets (United States, Australia, Hong Kong, Japan, Singapore and the United Kingdom) between two regimes—one characterized by low returns and high volatility and the other by high returns and low volatility. Huang (2008) examined the effects of monetary policy on U.S. REIT returns during similar low-return/high-volatility and high-return/low-volatility regimes. Lin (2007) compared REIT and stock returns in the United States during three regimes characterized by low, medium and high real interest rates.

Two other papers employed regime switching models to examine asset allocation in a portfolio including REITs. Liow and Zhu (2007) examined optimal allocations across the six listed property markets considered by Liow *et al.* (2005) conditional on the same regimes. Sa-Aadu, Shilling and Tiwari (2010) examined optimal portfolio allocation across eight assets (small cap stocks, medium/large cap stocks, government bonds, corporate bonds, international stocks, commodities, precious metals and equity REITs) in two regimes: the first characterized by low growth in consumption, low returns for most assets and high volatility for most assets, the second by high consumption growth, high returns and low volatility. The authors find that the optimal tangency portfolio composition conditional on the "good" state includes allocations to international stocks (25.4%), small stocks (24.6%), government bonds (21.8%), equity REITs (15.3%), large/medium stocks (9.4%) and precious metals (3.5%), while optimal portfolios in the "bad" state are invested only in government bonds (52.9%), precious metals (27.9%) and equity REITs (19.2%), thus confirming the economic importance of regime shifts for financial decisions.

In this article, we systematically explore the time-series properties of REIT return characteristics by estimating several versions of the (generalized) ARCH and Markov switching models. We conduct the same comparison for the returns on non-REIT stocks and on bonds, and then estimate a trivariate Markov switching model encompassing all three asset classes. This allows us to formulate and test hypotheses on the joint distribution of U.S. REIT, stock and bond returns in the presence of regime switching dynamics. We make use of MS features (such as the number and characterization of regimes) obtained at the univariate level to build a well-specified multivariate MS model that highlights the existence of commonalities but also of heterogeneous dynamics in the regime process characterizing stock, bond and REIT returns. Unlike previous research in real estate finance, we find that four states are required to characterize adequately the joint returns of REITs, stocks and bonds and are endogenously characterized by the maximum likelihood estimates of the parameters of the joint process of asset returns.

Our multivariate estimates have important implications for correlation properties of REIT returns with other asset classes. In a first, "normal" market environment that characterizes almost 50% of any long sample, the correlation between REIT and stock returns is rather high (almost 66%), and this generally high correlation extends to bonds as well (41%). However, such high correlations are of only modest concern to risk and asset managers because this normal regime is also characterized by positive returns on average and moderate volatilities. The same is true of a second bull market regime, characterizing another third of any long sample, in which volatilities are higher but expected returns also very high and positive and the correlation of REITs with stocks is 48%. Interestingly, in this regime REITs offer excellent diversification opportunities for government bond positions, with a zero correlation—important because in this regime bonds do not offer attractive returns.

More interesting are the remaining two states, both marked by high and persistent volatility and jointly representing almost 20% of any long sample. In the high volatility/high expected returns regime REITs yield correlations of only 25% with stocks and zero with bonds. In the panic/crash regime in which volatilities are high and expected returns low, REITs may offer again good hedging properties relative to bonds.

An outline of this article is as follows. In the next section we develop the econometric models and introduce basic notions and descriptions for both univariate and multivariate autoregressive conditional heteroskedastic and Markov regime switching models. The third section presents and discusses the key features of the data. The fourth section conducts in-depth univariate comparative analysis for all the models. Later we introduce and estimate multivariate models of both ARCH and MS types, including mixed MS multivariate GARCH models that feature both regimes and conditional heteroskedasticity. A robustness checks section tests whether MS models, besides providing a different in-sample characterization of regime dynamics, may also support superior predictive accuracy out-of-sample, at least as far the as forecasting of correlations is concerned. Moreover, we test a few economic hypotheses concerning the asset pricing implications of our empirical results. The final section concludes.

Methodology

ARCH Models

Traditional econometric models typically assume that the variances of random shocks are constant. However, economic time-series exhibit time-varying volatility and clustering. Conditional heteroskedastic models, including the ARCH model first proposed by Engle (1982), the GARCH model of Bollerslev (1986) and threshold GARCH (TGARCH) models (Zakoian 1994) all capture the dynamics of volatility and significantly reduce the inefficiencies caused by misspecifications of the second moment of classical models that imposed homoskedastic shocks. The general idea of any conditional heteroskedastic model is that past shocks carry information for current and future second moments (variances as well as covariances, in multivariate applications); the information content of more recent shocks exceeds that of more distant shocks, and thus each innovation is given a declining weight when estimating the dynamic process followed by second moments. For instance, in the univariate case in which only variance can be modeled (the stationary GARCH setting), the conditional variance equation is a weighted average of some constant intercept (reflecting long-run variance), past predicted variance (the generalized ARCH effect) and the square of the most recent shock to returns (the ARCH term). While the GARCH term represents the long-run persistent effect of all past shocks (because any stationary autoregressive component can always be represented as an infinite moving average process), the ARCH term simply represents the short-run impact of any shocks on predicted variance. A simple ARCH model is a special case of a GARCH in which no past predicted variances appear in the conditional variance equation.

Standard GARCH models are "symmetric" in that positive and negative shocks all have the same effect on volatility; financial asset returns, however, often react more severely to "bad" shocks than to "good" shocks. Therefore, Zakoian (1994) proposed a TGARCH model, one type of "asymmetric" GARCH model, to capture this leverage effect by modeling the conditional standard deviation instead of the conditional variance. Glosten, Jagannathan and Runkle (1993) have extended this class of models to frameworks that predict the variance differently based on positive and negative past shocks to returns.

While the volatility dynamics of a single asset class can be easily characterized by simple (threshold) GARCH models, the joint dynamics of multiple asset classes must be modeled as an extension of multivariate (asymmetric) GARCH models with both variance and covariance dynamics being tracked. However, the early models proposed in this literature tended to be very burdensome and difficult to estimate because they were plagued with issues of parameter proliferation. Engle (2002) has instead proposed a tighter and more computationally manageable dynamic conditional correlation GARCH (DCC-GARCH) model to directly fit and conduct inferences on the correlation dynamics of multiple asset classes. DCC-GARCH models not only preserve the ease of interpretation of GARCH models but also retain the consistency in estimation of larger-scale time-varying covariance matrices for multiple assets.

In a typical DCC-GARCH model, in a first stage, the conditional mean residuals are filtered out from a standard VAR(P) model for the multivariate process of asset returns:

$$r_{it} = c_i + \sum_{k=1}^{K} \sum_{j=1}^{J} v_{i,j,k} r_{j,t-k} + \varepsilon_{it},$$

where r_{it} is the return of asset i $(i=1,\ldots,N)$ at time t, ε_{it} is the time t shock to the return of asset i not justified by the effect of previous returns, and $\varepsilon_{i,t}|F_{t-1} \sim N(0, H_t)$ with covariance matrix $H_t = \Lambda_t \Gamma_t \Lambda_t$. In this expression, Λ_t is a diagonal matrix that collects the variances of standardized disturbance, $h_{i,t}$, and Γ_t is the time-varying conditional correlation matrix for pairs of standardized disturbances, that is, $\Gamma_t = [\rho_{ij,t}]$ where

$$\rho_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{ii,t}}\sqrt{h_{jj,t}}}, \quad i \neq j.$$

The diagonal elements of the covariance matrix that refer to each individual asset (conditional variances) follow the robust GARCH specification: $h_{ii,t} = c_i + a_i \varepsilon_{i,t-p}^2 + b_i h_{ii,t-p}$. Finally, the time-varying conditional covariance function is written as follows:

$$h_{ij,t} = w_i + \alpha u_{i,t-1} u_{j,t-1} + \beta h_{ij,t-1},$$

where the standardized disturbance from the first-stage conditional mean estimation is defined as $u_{it} = \varepsilon_{it}/\sqrt{h_{ij,t}}$ and $w_i = (1-\alpha-\beta)\rho_{ij}$, with ρ_{ij} the unconditional correlation between asset (class) i and j. α and β are DCC parameters capturing the effects of past shocks and conditional covariance on current covariance. The closer the sum of α and β is to 1, the stronger the degree of persistence in the covariance process.

The log-likelihood of the estimator can be written as follows using a two-step approach:

$$\begin{aligned}
L &= -\frac{1}{2}\sum_{t=1}^{T}\left(J\log(2\pi) + \log|H_t| + \varepsilon_t' H_t^{-1}\varepsilon_t\right) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(J\log(2\pi) + \log|\Lambda_t\Gamma_t\Lambda_t| + \varepsilon_t'\Lambda_t^{-1}\Gamma_t^{-1}\Lambda_t^{-1}\varepsilon_t\right) \\
&= \left\{-\frac{1}{2}\sum_{t=1}^{T}\left[J\log(2\pi) + 2\log|\Lambda_t| + \varepsilon_t'\Lambda_t^{-1}\varepsilon_t\right]\right\} \\
&\quad + \left\{\sum_{t=1}^{T}\left[\log(\Gamma_t) + u_t'\Gamma_t^{-1}u_t - u_t'u_t\right]\right\}.
\end{aligned}$$

The components in the first set of brackets represent the volatility components related to the GARCH specification. Given the estimated conditional variance Λ_t in the first step, the covariance and correlation component of the likelihood function in the second capital brackets can be separately estimated in the second step.

Markov Regime-Switching Models

In this section, we shall provide a brief introduction to structure and estimation method for multivariate Markov switching models.[1] Of course, everything that is described with reference to the case of n assets/variables can be simply adapted to the univariate case just assuming that $n = 1$. As a result, in many occasions matrices will simply become scalar quantities. Suppose that the $n \times 1$ random vector $\mathbf{r}_t$ follows a K-regime Markov switching (MS) VAR(P) process with heteroskedastic component,

$$\mathbf{r}_t = \boldsymbol{\mu}_{S_t} + \sum_{j=1}^{p} \mathbf{A}_{j,S_t}\mathbf{r}_{t-j} + \boldsymbol{\Sigma}_{S_t}\boldsymbol{\varepsilon}_t \tag{1}$$

with $\boldsymbol{\varepsilon}_t \iota \text{NID}(\mathbf{0},\mathbf{I}_n)$. S_t is a latent state variable driving all the matrices of parameters appearing in (1). $\boldsymbol{\Omega}_{St}$ is an $n \times 1$ vector that collects the n regime-dependent intercepts, while the $n \times n$ matrix $\boldsymbol{\Phi}_{St}$ represents the factor applicable to state S_t in a state-dependent Choleski factorization of the variance covariance matrix of the variables of interest, $\mathbf{I}_{St}$. Obviously, a nondiagonal $\boldsymbol{\Phi}_{St}$ makes the n variables simultaneously cross-correlated. We assume the absence of roots outside the unit circle, thus making the process stationary. In fact, conditional on the unobservable state S_t, (1) defines a standard Gaussian reduced form VAR(P) model. On the other hand, when $K > 1$, alternative hidden states are possible and they will influence both the conditional mean and the volatility/correlation structures characterizing the multivariate process in (1). These unobservable states are generated by a discrete-state, homogeneous, irreducible and ergodic first-order Markov chain:[2]

$$\Pr\left(S_t = j | \{S_j\}_{j=1}^{t-1}, \{\mathbf{y}_j\}_{j=1}^{t-1}\right) = \Pr(S_t = j | S_{t-1} = i) = p_{ij},$$

[1] Additional details can be found in classic textbooks, such as Hamilton (1994).

[2] The assumption of a first-order Markov process is not restrictive, because a higher order Markov chain can always be re-parameterized as a higher dimensional first-order Markov chain, *i.e.*, substitutability exists between the order of the Markov chain driving S_t and the number of regimes K.

where p_{ij} is the generic $[i,\ j]$ element of the $K \times K$ transition matrix $\mathbf{P}$. Ergodicity implies the existence of a stationary vector of (unconditional) state probabilities $\bar{\boldsymbol{\xi}}$ satisfying $\bar{\boldsymbol{\xi}} = \mathbf{P}'\bar{\boldsymbol{\xi}}$. Irreducibility implies that $\bar{\boldsymbol{\xi}} > \mathbf{0}$ meaning that all unobservable states are possible. In practice, $\mathbf{P}$ is unknown and hence $\bar{\boldsymbol{\xi}}$ can be at most estimated given knowledge on $\mathbf{P}$ extracted from the information set.

Multivariate MS VAR models are estimated by maximum likelihood. In particular, estimation and inferences are based on the Expectation-Maximization (EM) algorithm proposed by Dempster, Schatzoff and Wermuth (1977) and Hamilton (1989), a filter that enables the iterative calculation of the one-step ahead forecast of the state vector $\boldsymbol{\Psi}_{t+1|t}$ given the information set and the construction of the log-likelihood function of the data. Because the MLE first-order conditions all depend on the smoothed probabilities (that is, the state probabilities estimated on the basis of the full sample of data), they all present a high degree of nonlinearity in the parameters. As a result, these first-order conditions have to be solved numerically, although convenient iterative methods exist. Under standard regularity conditions (such as identifiability, stability and the fact that the true parameter vector does not fall on the boundaries) Hamilton (1989, 1993) has proven consistency and asymptotic normality of the MLE estimates of the parameters in (1). Although other choices exist—including using the conditional scores or a numerical evaluation of the second partial derivative of the log-likelihood function—in applications it has become typical to employ a White-style “sandwich” sample estimator of the covariance matrix. As a consequence, and with one important exception, standard inferential procedures such as Wald-type and Lagrange Multiplier (LM) tests are available to test statistical hypothesis.

The exception to standard inferential procedures concerns the *number of nonzero rows* of the transition matrix $\mathbf{P}$, *i.e.*, the number of regimes K. In this case, even under the assumption of asymptotic normality of the estimator of the model’s parameter, standard testing procedures suffer from nonstandard asymptotic distributions of the likelihood ratio test statistic due to the existence of nuisance parameters under the null hypothesis. The problem is that under any number of regimes smaller than K there are a few structural parameters of the unrestricted model—the elements of the transition probability matrix associated with the rows that correspond to disappearing states—that can take any values without influencing the resulting likelihood function. We say that these parameters become a nuisance to the estimation.[3] Different alternative ways

[3]The result is that the presence of these nuisance parameters gives the likelihood surface so many degrees of freedom that computationally one can never reject the null that the nonnegative values of those parameters were purely due to sampling variation.

have been proposed to develop sound inferential procedures concerning the number of regimes in multivariate Markov switching models. Hansen (1992) proposes to see the likelihood as a function of the unknown nuisance parameters so that the asymptotic distribution is generated in each case numerically from a grid of transition and regime-dependent nuisance parameters. However in most of the cases a closed-form expression cannot be found and the bound must be calculated by simulation and becomes data-dependent. Also, Davies (1977) bounds the LR test but avoids the problem of estimating the nuisance parameters and derives instead an upper bound for the significance level of the LR test under nuisance parameters:

$$\Pr(LR > x) \leq \Pr\left(\chi_1^2 > x\right) + \sqrt{2x}\exp\left(-\frac{x}{2}\right)\left[\Gamma\left(\frac{1}{2}\right)\right]^{-1}.$$

The bound holds if the likelihood has a single peak. Model specification searches applied to MS models are typically implemented relying on information criteria, such as the Schwartz, Hannan–Quinn and Akaike criteria (see, *e.g.*, Guidolin and Timmermann 2006).

Once a set of Markov switching models has been estimated, the need of further improvements could arise as the result of diagnostic checks. Although the EM algorithm naturally delivers estimates of the parameters besides the smoothed sequence of probability distributions $\{\hat{\xi}_{t|T}\}_{t=1}^{T}$ and would therefore lead to defining the (smoothed) residuals as $\tilde{\mathbf{u}}_t \equiv \mathbf{y}_t - \mathbf{X}_t\widehat{\Psi}\hat{\boldsymbol{\xi}}_{t|T}$, such residuals are not well suited to the use in diagnostic checks as they are full-sample random statistics and hence they structurally overestimate the explanatory power of the MS model. On the contrary the one-step prediction errors $\tilde{\mathbf{e}}_{t|t-1} \equiv \mathbf{y}_t - \mathbf{X}_t\widehat{\Psi}\mathbf{F}\hat{\boldsymbol{\xi}}_{t-1|t-1}$are limited information statistics (being based on filtered probabilities) and uncorrelated with the information set because $E[\mathbf{y}_t|\Im_{t-1}] = \mathbf{X}_t\hat{\mathbf{B}}\mathbf{F}\hat{\xi}_{t-1|t-1}$, and therefore form a martingale difference sequence $E[\tilde{\mathbf{e}}_{t|t-1}|\Im_{t-1}] \equiv \mathbf{0}$. Therefore standard tests of this hypothesis (such as portmanteau tests of no serial correlation) can be used. In the presence of Markov switching heteroskedastic components, researchers in empirical finance (*e.g.*, Kim and Nelson 1999) have also suggested checking whether the smoothed, standardized residuals contain any residual ARCH effects. Standard LM-type as well as Ljiung-Box tests can be applied to check whether Markov switching covariances may be sufficient to capture most of the dynamics in volatility; if not, explicit ARCH-type modeling may be required.

Finally, common sense suggests that correct specification of a Markov switching model should give smoothed probability distributions $\{\hat{\xi}_{t|T}\}_{t=1}^{T}$ that consistently signal switching among states with only limited periods in which the associated distribution is flatly spread out over the entire support and uncertainty dominates. Regime Classification Measures have been popularized as a way to assess whether the number of regimes k is adequate. In simple two-regime frameworks, the early work by Hamilton (1988) offered a rather intuitive regime classification measure:

$$RCM_1 = 100\frac{k^2}{T}\sum_{t=1}^{T}\prod_{j=1}^{k}\Pr(S_t = j|\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T; \tilde{\boldsymbol{\gamma}}),$$

which amounts to a corrected sample average of the products of the smoothed state probabilities. Clearly, when a switching model offers precise indications on the nature of the regime at each time t, for at least one value of $j = 1, \ldots, K$, $\Pr(S_t = j|\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T; \tilde{\boldsymbol{\gamma}}) \cong 1$ so that $\sum_{j=1}^{k}\Pr(S_t = j|\mathbf{y}_1, \mathbf{y}_2, \ldots, y_T; \tilde{\boldsymbol{\gamma}}) \cong 0$ because most probabilities are zero. Therefore a good switching model will imply $RCM_1 \cong 0$. When applied to models such that $K > 2$, RCM_1 has one disadvantage: a model can imply an enormous degree of uncertainty on the current regime, but still have $\sum_{j=1}^{k}\Pr(S_t = j|\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T; \tilde{\boldsymbol{\gamma}}) \cong 0$ for most values of t. As a result, it is rather common to witness that as K exceeds 2, almost all switching models (good and bad) will imply values of RCM_1 that decline towards 0. Guidolin (2009) proposes a few alternative measures that correct this problem. For instance, a measure such as RCM_2,

$$RCM_2 = 100\left\{1 - \frac{K^{2K}}{(K-1)^2}\frac{1}{T}\sum_{t=1}^{T}\sum_{k=1}^{K}\left[\Pr(S_t = k|\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_T; \theta) - \frac{1}{K}\right]^2\right\}$$

appears to be more useful than RMC_1 when the number of regimes is 3 or higher.

Data

We explore return dynamics for three asset classes. Our primary focus is the real estate equity asset class, for which we use monthly total returns for publicly traded equity REITs in the United States as measured by the FTSE NAREIT All Equity REITs Index. We also investigate total returns in the stock and bond asset classes using monthly data for the CRSP NYSE/AMEX/NASDAQ Value-Weighted Market Index and the CRSP U.S. Treasury 10-Year Bond Index. The risk-free rate is represented by the CRSP 30-Day Treasury Bill Returns series.

Table 1 displays summary statistics for all three asset classes with reference to a long sample of monthly data, January 1972–December 2009. Research by Case, Yang and Yildirim (2012), among others, establishes that a “modern

Table 1 ■ Summary statistics for U.S. REIT, stock and long-term government bond returns.

	Mean	Volatility	Sharpe Ratio	Median	Skewness	Excess Kurtosis	Jarque-Bera	LB(12) - Levels	LB(12) - Squares
Full Sample (1972–2009)									
REIT Returns	1.048	4.958	0.118	1.294	−0.783	8.815	1522.9	63.041	237.2
	(0.000)	(0.000)	✍	(0.000)	(0.166)	(0.048)	(0.000)	(0.000)	(0.000)
Stock Returns	0.900	4.644	0.094	1.280	−0.591	2.256	123.2	9.671	24.545
	(0.000)	(0.000)	✍	(0.000)	(0.063)	(0.091)	(0.000)	(0.645)	(0.017)
Govt. Bond Returns	0.665	2.317	0.087	0.615	0.324	1.333	41.730	14.241	89.943
	(0.000)	(0.000)	✍	(0.000)	(0.122)	(0.069)	(0.000)	(0.286)	(0.000)
1-Month T-Bill Returns	0.464	0.257	✍	0.431	0.828	1.349	86.688	3794.8	3092.2
	(0.001)	✍	✍	(0.000)	(0.002)	(0.076)	(0.000)	(0.000)	(0.000)
Pre-Modern Era (1972–1991)									
REIT Returns	1.104	4.144	0.116	1.136	−0.349	2.253	55.619	20.993	18.611
	(0.011)	(0.000)	✍	(0.000)	(0.220)	(0.067)	(0.000)	(0.050)	(0.098)
Stock Returns	1.060	4.867	0.089	1.049	−0.386	2.465	66.718	9.046	7.273
	(0.001)	(0.000)	✍	(0.000)	(0.056)	(0.024)	(0.000)	(0.699)	(0.839)
Govt. Bond Returns	0.769	2.525	0.057	0.604	0.458	1.076	19.979	15.561	57.518
	(0.000)	(0.000)	✍	(0.000)	(0.013)	(0.168)	(0.001)	(0.212)	(0.000)
1-Month T-Bill Returns	0.625	0.228	✍	0.594	1.177	1.573	80.125	1336.3	1175.3
	(0.000)	✍	✍	(0.000)	(0.000)	(0.045)	(0.000)	(0.000)	(0.000)
Modern Era (1992–2009)									
REIT Returns	0.967	5.777	0.167	1.473	−0.897	9.251	795.46	66.212	130.49
	(0.015)	(0.000)	✍	(0.000)	(0.223)	(0.067)	(0.000)	(0.000)	(0.000)
Stock Returns	0.754	4.448	0.170	1.342	−0.911	1.795	58.599	9.646	55.197
	(0.014)	(0.000)	✍	(0.001)	(0.033)	(0.177)	(0.000)	(0.647)	(0.000)
Govt. Bond Returns	0.562	2.055	0.273	0.636	−0.056	1.267	14.493	15.331	15.908
	(0.000)	(0.000)	✍	(0.000)	(0.546)	(0.181)	(0.001)	(0.224)	(0.195)
1-Month T-Bill Returns	0.285	0.145		0.318	−0.409	−1.054	15.947	1635.9	1493.0
	(0.000)	✍	✍	(0.000)	(0.008)	(0.000)	(0.000)	(0.000)	(0.000)

The table reports basic summary statistics for U.S. stock, REIT and long-term government bond returns. For benchmarking purposes the table also includes information on 1-month T-bill yields. The p-value associated with the null hypothesis of a zero value for the parameter or statistic under investigation is shown in parentheses. When possible, the p-values are computed for two-tailed tests of hypothesis. In the case of kurtosis, the null hypothesis is of a kurtosis that equals the Gaussian benchmark of 3. Jarque-Bera is a test of distributional normality based on deviations of skewness and kurtosis coefficients from the null of normality. LB(12) is the Ljung-Box test for zero serial correlation up to order 12 for levels and squared returns, respectively. The null hypothesis of a zero median return is tested using a Wilcoxon signed-rank test. In the case of volatility, the null hypothesis is that the volatility of an asset class is the same as 1-month T-bills, and the hypothesis is tested using a variance ratio test.

REIT era" began in late 1991, so Table 1 also shows summary statistics for the pre-modern era (January 1972–December 1991) and the modern era (January 1992–December 2009). For the full study period, REITs provided the highest returns at 1.048% per month, followed by non-REIT stocks at 0.900% and bonds at 0.665%, with the risk-free rate averaging 0.464% per month. The subperiods, however, show that average returns were substantially higher during the pre-modern era than in the modern era for stocks (1.060% *vs*. 0.754%) and bonds (0.769 *vs*. 0.562), while average returns for REITs differed far less between the two periods (1.104 *vs*. 0.967).

The monthly standard deviation of REIT returns was substantially higher in the modern era relative to the pre-modern era (5.777 *vs*. 4.144), while stock, bond and cash returns were somewhat less volatile during the modern era. The Sharpe ratio indicates that risk-adjusted returns were practically identical for REITs and stocks during the modern era (0.167 *vs*. 0.170) but somewhat better for REITs during the pre-modern era (0.116 *vs*. 0.089); during the modern era bonds provided much stronger risk-adjusted returns than the equity asset classes (0.273). This is unsurprising given that both the dis-inflation of the early 1990s and the recent (2008–2009) expansionary monetary policies implied protracted periods of declining long-term Treasury yields that generated substantial excess bond returns.

REIT returns exhibited greater negative skewness than stocks over the entire period, but the subperiod statistics reveal that stocks were actually slightly more negatively skewed during both the modern era (–0.911 *vs*. –0.897) and the pre-modern era (–0.386 *vs*. –0.349). REIT returns also exhibited significantly greater excess kurtosis than the other three asset classes during the modern era, although slightly less than stocks during the pre-modern era. The Jarque-Bera statistics show that realized returns for all four asset classes appear to have come from non-Gaussian distributions. Finally, as is common in empirical finance studies, Table 1 reveals that stock and bond returns are not serially correlated in levels (of course, T-bill returns are, consistently with the interest rate literature), while REIT returns are, especially in the modern era. However, squared returns for all the asset classes considered appear to be strongly serially correlated, which is a well-known symptom of conditional heteroskedasticity.[4]

[4]For completeness, the correlation coefficients between pairs of return series are 0.59 between REITs and stocks (p-value is 0.000), 0.11 between REITs and bonds (p-value is 0.024), –0.04 between REITs and 1-month bills (0.390), 0.15 between stocks and bonds (0.001), –0.02 between stocks and bills (0.670) and 0.09 between bonds and T-bills (with p-value of 0.045).

Performance Comparison: Univariate Models

We investigate the time-series dynamics of returns to REITs, stocks and bonds by estimating a range of models incorporating different autoregressive and heteroskedastic components in both ARCH and MS frameworks. For all three asset classes we begin with a benchmark single-state model that includes no autoregressive or heteroskedastic components, as well as a simple AR(1) process with no heteroskedasticity. To these simple single-state models we then add heteroskedastic components in the form of ARCH, GARCH and TARCH specifications as described previously; we also compare Gaussian, *t*-Student and GED specifications for the marginal density of disturbance terms.

REIT Returns

Table 2 summarizes the results for the models estimated using REIT returns, with Panel A showing single-state models. The first line presents the baseline model with no autoregressive or heteroskedastic components. Ljung-Box (1978) tests on levels and squares indicate persistence in both conditional means and squared residuals, respectively, and the Lagrange multiplier test indicates the presence of ARCH components. Introducing autoregressive terms of order 1 or 4 (the second and third lines) begins to address the persistence in conditional means, but not the conditional heteroskedasticity, and the LB(squares) and LM tests still reject the null hypothesis under simple ARCH specifications with Gaussian or Student's t error distributions (fourth and fifth lines). More complicated GARCH and TARCH specifications, though, eliminate the problems suggested by the diagnostic tests, especially the TARCH model with error densities modeled using Student's t: the negative log-likelihood measure jumps to more than 809, and all the information criteria—the AIC proposed by Akaike (1974), the BIC proposed by Schwartz (1978) and the H-Q proposed by Hannan and Quinn (1979)—indicate that the improvement in fit outweighs the additional parameters required to estimate these models.

The first part of Panel B of Table 2 shows the results for two-state Markov switching models of REIT returns without and with heteroskedastic components; autoregressive terms are either constrained to be identical in the two regimes or permitted to take different values in the two regimes (unconstrained).[5] Not surprisingly, models without heteroskedastic components continue to exhibit structure of residuals in both mean and variance perspectives (*i.e.*, serial correlation in levels and squares of the residuals as well as

[5]Constraining the autoregressive coefficients to be identical across regimes is indicated in Table 2 with a symbol $\wp$ to follow the parameter p that indicates the autoregressive order.

Table 2 ■ Univariate model selection: REIT returns.

Number of Regimes (K)	AR(P) Order	Heteroskedastic Components	Final Negative Log-Likelihood	Linearity Test	AIC	BIC	H-Q	RCM 1	RCM 2	LB(12) - Levels	LB(12) - Squares	ARCH LM(12) Test	No. Param.	Saturation Ratio
Panel A: Single-State Models														
1	0	No	721.771	✍	−3.1613	−3.1522	−3.1577	✍	✍	63.041***	238.13***	13.380***	2	228.00
1	1	No	722.851	✍	−3.1686	−3.1505	−3.1614	✍	✍	61.547***	250.70***	15.317***	3	152.00
1	4	No	727.930	✍	−3.1988	−3.1533	−3.1809	✍	✍	20.918*	158.68***	10.057***	6	76.00
1	1	Gaussian ARCH(1)	745.539	✍	−3.2595	−3.2233	−3.2452	✍	✍	43.377**	57.647***	4.829***	4	114.00
1	1	t-Student ARCH(1)	793.437	✍	−3.4657	−3.4204	−3.4478	✍	✍	40.605**	66.999***	6.347***	5	91.20
1	1	Gaussian GARCH(1,1)	777.414	✍	−3.3952	−3.3500	−3.3774	✍	✍	15.919	2.762	0.235	5	91.20
1	0	Gaussian GARCH(1,1)	777.033	✍	−3.3905	−3.3543	−3.3762	✍	✍	20.283*	3.201	0.287	4	114.00
1	1	Gaussian TGARCH(1,1)	783.036	✍	−3.4155	−3.3612	−3.3941	✍	✍	14.715	3.100	0.256	6	76.00
1	1	t-Student GARCH(1,1)	806.061	✍	−3.5168	−3.4624	−3.4954	✍	✍	16.007	3.077	0.260	6	76.00
1	1	t-Student TGARCH(1,1)	809.249	✍	−3.5264	−3.4630	−3.5014	✍	✍	14.000	3.301	0.266	7	65.14
1	1	GED−Dist. TGARCH(1,1)	803.293	✍	−3.5002	−3.4368	−3.4752	✍	✍	14.209	3.186	0.259	7	65.14
1	1	t-Student TGARCH(4)	807.640	✍	−3.4973	−3.3887	−3.4545	✍	✍	15.038	7.029	0.588	12	38.00

Table 2 ■ Continued

Number of Regimes (K)	AR(P) Order	Heteroskedastic Components	Final Negative Log-Likelihood	Linearity Test	AIC	BIC	H-Q	RCM 1	RCM 2	LB(12) - Levels	LB(12) - Squares	ARCH LM(12) Test	No. Param.	Saturation Ratio
Panel B: Multistate Models														
Two-State Models														
2	0	No	761.916	80.290***	−3.3198	−3.2746	−3.3020	0.685	**44.385**	41.105***	50.480***	5.877***	5	91.20
2	1	No	780.992	116.282***	−3.3828	−3.2805	−3.3424	63.837	94.157	35.563***	107.98***	8.206***	7	65.14
2	0	Yes	792.356	141.170***	−3.4489	−3.3947	−3.4276	23.321	66.711	14.052	10.668	0.843	6	76.00
2	1	Yes	835.545	225.366***	**−3.6613**	−3.5376	−3.6126	23.085	65.925	10.428	9.867	0.795	7	65.14
2	1	Yes	835.594	225.486***	−3.6169	**−3.5409**	−3.5870	23.324	66.235	10.524	10.230	0.821	8	57.00
Three-State Models														
3	0	No	825.793	208.045***	−3.5570	−3.4621	−3.5196	**0.000**	**59.975**	17.845	31.696***	2.310***	10	45.60
3	0	Yes	851.432	259.321***	−3.6597	**−3.5458**	**−3.6148**	0.057	85.584	19.206*	20.380*	1.833**	12	38.00
3	1	Yes	851.433	257.163***	**−3.6631**	−3.5394	**−3.6144**	0.073	85.368	15.837	20.674*	1.873**	13	35.08
3	1	Yes	844.073	242.445***	−3.6217	−3.4791	−3.5655	3.805	83.019	15.834	20.534*	1.833**	15	30.40
Four-State Models														
4	0	No	826.262	208.981***	−3.5268	−3.3655	−3.4632	**0.000**	90.348	18.156	31.953***	2.329***	17	26.82
4	0	Yes	857.151	270.761***	−3.6478	−3.4579	−3.5730	0.015	68.283	13.970	20.745*	1.787**	20	22.80
4	1	Yes	857.551	261.400***	−3.6613	−3.4602	−3.5821	0.033	82.299	14.835	23.739**	2.485***	21	21.71
4	1	Yes	857.927	270.152***	−3.6405	−3.4118	−3.5504	0.047	78.174	15.794	24.675**	2.389***	24	19.00

The table summarizes outputs for a range of models defined by the number of regimes (K = 1, 2, 3, 4), the autoregressive order (P), and the presence of heteroskedastic components of either ARCH or Markov switching type. For single-state models (K = 1), heteroskedastic components can take the form of ARCH(p), GARCH(l,q) and threshold GARCH or TARCH(l,q) models with the marginal density of error terms distributed Gaussian, Student's t or GED. Final negative log-likelihood is –1 times the maximized log-likelihood. The significance level of linearity tests is computed by applying Davies' (1977) approximation to the asymptotic mixture of noncentral chi-square that characterizes the LR statistic. AIC, BIC and H-Q are information criteria that trade off fit and parsimony. RCM1 and RCM2 are regime classification measures suggested by Hamilton (1988) and Guidolin (2009); values range from 0 to 100 with lower values corresponding to greater classification accuracy. ARCH LM(12) is the Lagrange multiplier test for ARCH. Saturation ratio is the ratio of the total number of observations to the number of parameters estimated. Bold face highlights the models that minimize the information criteria and regime classification measures. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

rejections from ARCH LM tests) and do not perform as well as complex, ARCH-type single-state models according to any of the information criteria. Two-state models with heteroskedastic and autoregressive components, however, perform markedly better than even complex single-state TARCH models: the L-B and LM tests suggest that the models successfully address structure in conditional means and variances, the negative log-likelihood measures indicates a better fit to the data, and all three information criteria suggest a better trade-off between explanatory value and parsimony, as measured by the (inverse of the) number of parameters. The AIC and H-Q measures indicate that the best two-state model of REIT returns is one with constrained autoregressive terms, while the RCM_1 and RCM_2 measures indicate that this model also outperforms in classifying observations between the two regimes.

The remainder of Panel B of Table 2 summarizes three-state and four-state Markov switching models of REIT returns. Again, models without heteroskedastic components leave structure in the residuals according to the L-B and LM tests, and they do not perform as well according to the information criteria. However, the information criteria suggest a slight preference for three-state models with either no autoregressive components or constrained autoregressive components over the best two-state model. In summary, however, the results provide evidence that even a simple two-state Markov switching process with heteroskedastic components provides a better characterization of returns than even a very complex, TARCH-type single-state model.

Stock Returns

Table 3 summarizes the results for models estimated using stock returns, as proxied by the value-weighted CRSP index. Interestingly—and unlike with REIT returns—the L-B levels test for single-state models (Panel A) shows no evidence of structure in conditional means and weak evidence of structure in conditional variances of stock returns even in the absence of modeled autoregressive or heteroskedastic components. Nevertheless, according to the AIC, BIC and H-Q criteria the best-performing single-state models for stock returns, as for REIT returns, are complex GARCH and TARCH specifications with error terms distributed according to a Student's *t*. This implies that GARCH and TARCH specifications fit the realized data well not only (or mostly) because they account for persistence in squared residuals but rather because they successfully capture the shape of the *unconditional* density of stock returns.

Table 3 also shows the results for two-state MS models of stock returns. The information criteria indicate that the best two-state MS models include heteroskedastic components but no autoregressive terms, while the RCM_1 and RCM_2 measures indicate that a two-state MS model with heteroskedastic

Table 3 ■ Univariate model selection: Stock returns.

Number of Regimes (K)	AR(P) Order	Heteroskedastic Components	Final Negative Log-Likelihood	Linearity Test	AIC	BIC	H-Q	RCM 1	RCM 2	LB(12) - Levels	LB(12) - Squares	ARCH LM(12) Test	No. Param.	Saturation Ratio
Single-State Models														
1	0	No	753.172	✍	−3.2990	−3.2954	−3.2900	✍	✍	9.671	28.751***	1.747*	2	228.00
1	1	No	753.022	✍	−3.3012	−3.2941	−3.2831	✍	✍	7.623	24.614**	1.557	3	152.00
1	0	Gaussian ARCH(1)	759.599	✍	−3.3184	−3.2913	−3.3077	✍	✍	8.519	11.853	0.837	3	152.00
1	0	*t*-Student ARCH(1)	772.703	✍	−3.3715	−3.3353	−3.3573	✍	✍	8.446	11.390	0.812	4	114.00
1	0	*t*-Student GARCH(1,1)	783.990	✍	−3.4166	−3.3714	−3.3988	✍	✍	7.106	5.559	0.416	5	91.20
1	0	*t*-Student TGARCH(1,1)	788.858	✍	−3.4336	−3.3793	−3.4122	✍	✍	7.583	4.579	0.355	6	76.00
1	0	GED-Dist. TGARCH(1,1)	783.035	✍	−3.4081	−3.3583	−3.3867	✍	✍	7.500	4.713	0.365	6	76.00
Two-State Models														
2	0	No	770.883	35.421***	−3.3591	−3.3139	−3.3413	6.307	**49.792**	4.291	23.299**	1.458	5	91.20
2	0	Yes	799.291	92.238***	**−3.5321**	**−3.4645**	**−3.4997**	32.952	71.444	4.017	10.377	0.808	6	76.00
2	1	Yes	799.416	92.788***	−3.4556	−3.3911	−3.4303	32.535	70.969	3.074	9.928	0.774	7	65.14
2	1	Yes	799.902	93.759***	**−3.4974**	**−3.4221**	**−3.4677**	30.083	69.329	3.973	9.938	0.767	8	57.00
Three-State Models														
3	0	No	799.732	93.119***	−3.4447	−3.3527	−3.4085	0.287	72.142	4.131	26.759**	1.988**	10	45.60
3	0	Yes	809.377	112.410***	−3.4779	−3.3674	−3.4344	1.367	79.042	4.814	24.815**	1.749*	12	38.00
3	1	Yes	809.669	113.294***	−3.4736	−3.3538	−3.4264	1.187	78.099	4.014	24.452**	1.750*	13	35.08
3	1	Yes	810.479	114.915***	−3.4148	−3.2789	−3.3613	5.472	89.046	3.511	12.773	0.865	15	30.40
Four-State Models														
4	0	No	808.110	109.876***	−3.4501	−3.2936	−3.3884	0.002	**61.666**	15.299	22.650**	1.467	17	26.82
4	0	Yes	820.610	134.876***	−3.4913	−3.3071	−3.4187	**0.002**	63.483	3.114	27.167***	2.111**	20	22.80
4	1	Yes	820.778	135.511***	−3.4864	−3.2928	−3.4101	**0.001**	63.388	2.325	25.245**	1.938**	21	21.71
4	1	Yes	821.054	136.065***	−3.4654	−3.2441	−3.3782	0.024	74.036	8.778	22.309**	1.674*	24	19.00

The table summarizes outputs for a range of models defined by the number of regimes ($K = 1, 2, 3, 4$), the autoregressive order (P), and the presence of heteroskedastic components of either ARCH or Markov switching type. For single-state models ($K = 1$), heteroskedastic components can take the form of ARCH(p), GARCH(l,q) and threshold GARCH or TARCH(l,q) models with the marginal density of error terms distributed Gaussian, Student's t or GED. Final negative log-likelihood is –1 times the maximized log-likelihood. The significance level of linearity tests is computed by applying Davies' (1977) approximation to the asymptotic mixture of noncentral chi-square that characterizes the LR statistic. AIC, BIC, and H-Q are information criteria that trade off fit and parsimony. RCM1 and RCM2 are regime classification measures suggested by Hamilton (1988) and Guidolin (2009); values range from 0 to 100 with lower values corresponding to greater classification accuracy. ARCH LM(12) is the Lagrange multiplier test for ARCH. Saturation ratio is the ratio of the total number of observations to the number of parameters estimated. Bold face highlights the models that minimize the information criteria and regime classification measures. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

components and unconstrained autoregressive terms performs slightly better at classifying observations between the two regimes.

As with two-state models, the information criteria indicate that the most successful three- and four-state MS models include heteroskedastic components but no autoregressive terms, but the higher-regime models perform no better than the two-state model. Interestingly, however, the Ljung-Box (squares) and Lagrange Multiplier tests suggest the presence of serial correlation in squared residuals. We interpret this to mean that higher-regime Markov mixture models may over-fit the realized data, generating spurious patterns in residual variances.

Bond Returns

Table 4 shows the results for models estimated using returns on long-term government bond returns. The L-B test in levels suggests no persistence in conditional means, but the L-B test in squares and the LM test indicate persistence in squared residuals. According to the AIC, BIC and H-Q criteria, bond returns are modeled most successfully in a two-state MS framework that includes heteroskedastic components with either no or constrained autoregressive terms. Also in this case, there is evidence that MS models are considerably more successful at fitting the data, even when their bigger size and higher number of parameters (*i.e.*, lower saturation ratios) are taken into account, than ARCH-type models are. Unlike the case of REIT and stock returns, in Table 4 there is some evidence from RCM2 that a multistate MS model with as many as four regimes may be required by the data, although the evidence is never stark as the information criteria generally favor simpler two-state models.

Summary of Univariate Models

The results for all three asset classes indicate that the best single-state models are GARCH and TARCH formulations with Student's t disturbance term densities. In all three asset classes, however, simple two-regime Markov switching models with heteroskedastic components but no or constrained (to be identical across regimes) autoregressive terms perform better than the complex GARCH or TARCH specifications. This relative performance—especially coupled with the outcomes of L-B and LM tests for stock returns, which indicate little persistence in either conditional means or conditional variances—suggest that the evidence for ARCH effects in single-state models may be spurious, with the two-state MS models better representing the realized return distributions.[6]

[6]In fact, an early literature (*e.g.*, Lamoureux and Lastrapes 1990) has argued that strong persistence in volatility may be an artifact of changes in the economic mechanism

Table 4 ■ Univariate model selection: Bond returns.

Number of Regimes (K)	AR(P) Order	Heteroskedastic Components	Final Negative Log-Likelihood	Linearity Test	AIC	BIC	H-Q	RCM 1	RCM 2	LB(12) - Levels	LB(12) - Squares	ARCH LM(12) Test	No. Param.	Saturation Ratio
Single-State Models														
1	0	No	1067.672	✍	−4.6902	−4.6811	−4.6866	✍	✍	14.241	95.998***	4.809***	2	228.00
1	1	No	1069.274	✍	−4.6913	−4.6732	−4.6842	✍	✍	11.558	99.064***	4.753***	3	152.00
1	0	Gaussian ARCH(1)	1080.412	✍	−4.7359	−4.7087	−4.7252	✍	✍	10.940	56.185***	3.719***	3	152.00
1	0	t-Student ARCH(1)	1087.256	✍	−4.7616	−4.7253	−4.7473	✍	✍	10.918	55.681***	3.708***	4	114.00
1	0	Gaussian GARCH(1,1)	1099.790	✍	−4.8167	−4.7804	−4.8024	✍	✍	13.111	10.412	0.829	4	114.00
1	1	t-Student GARCH(1,1)	1104.500	✍	−4.8330	−4.7877	−4.8151	✍	✍	13.251	10.305	0.822	5	91.20
1	0	t−Student TGARCH(1,1)	1104.514	✍	−4.8286	−4.7743	−4.8072	✍	✍	13.260	10.485	0.837	6	76.00
1	0	GED−Dist. TGARCH(1,1)	1103.480	✍	−4.8241	−4.7698	−4.8027	✍	✍	13.197	10.598	0.845	6	76.00
Two−State Models														
2	0	No	1086.294	37.243***	−4.7425	−4.6973	−4.7247	3.533	47.283	9.135	30.786**	2.091**	5	91.20
2	0	Yes	1129.693	124.042***	−4.9023	**−4.8471**	**−4.8806**	16.866	58.685	11.739	9.369	0.740	6	76.00
2	1	Yes	1129.708	120.866***	**−4.9086**	**−4.8441**	**−4.8832**	15.884	58.071	6.515	9.013	0.754	7	65.14
2	1	Yes	1130.252	121.956***	−4.9066	−4.8329	−4.8776	15.525	57.930	7.142	9.963	0.788	8	57.00
Three-State Models														
3	0	No	1121.810	108.276***	−4.8501	−4.7580	−4.8138	0.329	72.971	13.757	11.139	1.208	10	45.60
3	0	Yes	1133.244	131.143***	−4.8910	−4.7806	−4.8475	0.846	71.241	16.039	9.007	0.755	12	38.00
3	1	Yes	1135.004	131.459***	−4.9051	−4.7851	−4.8578	0.664	70.944	8.691	10.560	0.877	13	35.08
3	1	Yes	1138.266	137.983***	**−4.9104**	−4.7720	−4.8559	2.021	86.281	7.179	9.602	0.776	15	30.40
Four-State Models														
4	0	No	1133.812	132.280***	−4.8712	−4.7148	−4.8096	**0.000**	**36.096**	8.065	2.531	0.215	17	26.82
4	0	Yes	1142.925	150.505***	−4.8975	−4.7134	−4.8250	0.010	64.812	9.997	15.472	1.112	20	22.80
4	1	Yes	1143.668	148.788***	−4.8983	−4.7048	−4.8221	**0.000**	**43.489**	8.349	7.986	0.618	21	21.71
4	1	Yes	1143.761	148.973***	−4.8809	−4.6597	−4.7938	0.011	69.941	8.275	11.439	0.865	24	19.00

The table summarizes outputs for a range of models defined by the number of regimes ($K = 1, 2, 3, 4$), the autoregressive order (P), and the presence of heteroskedastic components of either ARCH or Markov switching type. For single-state models ($K = 1$), heteroskedastic components can take the form of ARCH(p), GARCH(l, q) and threshold GARCH or TARCH(l,q) models with the marginal density of error terms distributed Gaussian, Student's t or GED. Final negative log-likelihood is –1 times the maximized log-likelihood. The significance level of linearity tests is computed by applying Davies' (1977) approximation to the asymptotic mixture of noncentral chi-square that characterizes the LR statistic. AIC, BIC, and H-Q are information criteria that trade off fit and parsimony. RCM1 and RCM2 are regime classification measures suggested by Hamilton (1988) and Guidolin (2009); values range from 0 to 100 with lower values corresponding to greater classification accuracy. ARCH LM(12) is the Lagrange multiplier test for ARCH. Saturation ratio is the ratio of the total number of observations to the number of parameters estimated. Bold face highlights the models that minimize the information criteria and regime classification measures. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Details of Univariate Models

To characterize the process of returns more completely we compare the estimated coefficients of the best-performing single-state model in each asset class with the two-state Markov switching model.

REIT Returns

Table 5 shows the results for REIT returns, with Panel A displaying the best-performing single-state model, a TARCH(1,1) AR(1) specification with Student's *t* disturbance term distribution. Furthermore, the study period is divided into "pre-modern" and "modern" subperiods, corresponding to the findings of Case, Yang and Yildirim (2012) and others.

For the full-sample TARCH model, the Jarque-Bera test indicates that the standardized residuals are still not normally distributed; furthermore, the implied unconditional variance of 15.6% is somewhat lower than the realized variance of 17.2%, with the result that the implied unconditional Sharpe ratio of 0.534 is higher than the realized Sharpe ratio of 0.408. These deviations between the unconditional moments implied by the estimated process and the corresponding sample moments are certainly possible when a TARCH is estimated by maximum likelihood, but they remain an indication of possible misspecifications. The subsample results reveal some interesting differences: for example, the threshold parameter is statistically significant at 0.215 during the pre-modern era but insignificant during the modern era; the implied unconditional mean return is higher during the modern era even though the realized average return is lower; the implied unconditional variances are nearly identical even though the realized variance was larger in the modern era; and the modern era was characterized by fatter tails in the disturbance term distribution. Finally, although the TARCH coefficients are relatively stable across the pre- and modern periods, the resulting TARCH model is rather odd, as it can be summarized (after setting to zero all coefficients that fail to induce p-values below 0.05) as a simple AR(1) process for the unobservable variance.

Panel B shows the parameter estimates for the simple two-state MS model of REIT returns with constrained autoregressive terms. Regime 1 represents a bear-market state with conditional mean excess returns not significantly different from zero and very large conditional variance, while Regime 2 represents a bull-market state with relatively high conditional mean excess returns and relatively low conditional variance. The estimated transition matrix and ergodic

generating the underlying returns series, because any structural shifts in the unconditional variance is likely to lead to biased estimation of the GARCH parameters to imply too much persistence in volatility.

Table 5 ■ Univariate model results: REIT returns.

Panel A: Single State Model				
	Full Sample			*Unc. Mean & Vol. (annual)*
Conditional mean function	$E_t[r_{\text{REIT},t+1}] = \mathbf{1.043} + \mathbf{0.099} r_{\text{REIT},t-1}$ (0.180) (0.045)			13.891 [Sample: 12.576]
Conditional variance function	$h_{\text{REIT},t} = 1.672 + 0.008 u^2_{\text{REIT},t-1} + \mathbf{0.828} h_{\text{REIT},t-1} + 0.163 I_{\{u^{\text{REIT},t-1}<0\}} u^2_{\text{REIT},t-1}$ (1.039) (0.045) (0.082) (0.088)			15.595 [Sample: 17.175] Implied Sharpe ratio: 0.534
t-Student degrees of freedom parameter	**4693** (0.987)			[Sample: 0.408]
Log-Likelihood	809249	Number of parameters	7	
Akaike information criterion	−3.526	Saturation ratio	65.142857	
Bayes-Schwartz information criterion	−3.463	Jarque-Bera on stdzed res.	396.73***	
Hannan-Quinn information criterion	−3.501	ARCH(12) LM test	0.266	
	Pre-Modern (1972–1991) Sample			*Unc. Mean & Vol. (annual)*
Conditional mean function	$E_t[r_{\text{REIT},t+1}] = \mathbf{0.911} + \mathbf{0.154} r_{\text{REIT},t-1}$ (0.224) (0.061)			12.922 [Sample: 13.253]
Conditional variance function	$h_{\text{REIT},t} = 1.701 + 0.006 u^2_{\text{REIT},t-1} + \mathbf{0.805} h_{\text{REIT},t-1} + \mathbf{0.215} I_{\{u^{\text{REIT},t-1}<0\}} u^2_{\text{REIT},t-1}$ (1.332) (0.051) (0.115) (0.156)			15.826 [Sample: 14.354] Implied Sharpe ratio: 0.343
t-Student degrees of freedom parameter	**3.978** (1.276)			[Sample: 0.401]
	Modern (1992–2009) Sample			*Unc. Mean & Vol. (annual)*
Conditional mean function	$E_t[r_{\text{REIT},t+1}] = \mathbf{1.129} + 0.037 r_{\text{REIT},t-1}$ (0.293) (0.069)			14.069 [Sample: 11.818]
Conditional variance function	$h_{\text{REIT},t} = 1.868 - 0.006 u^2_{\text{REIT},t-1} + \mathbf{0.840} h_{\text{REIT},t-1} + 0.154 I_{\{u^{\text{REIT},t-1}<0\}} u^2_{\text{REIT},t-1}$ (1.770) (0.102) (0.124) (0.136)			15.870 [Sample: 19.988] Implied Sharpe ratio: 0.671
t-Student degrees of freedom parameter	**5.750** (1.714)			[Sample: 0.420]

Table 5 ■ Continued

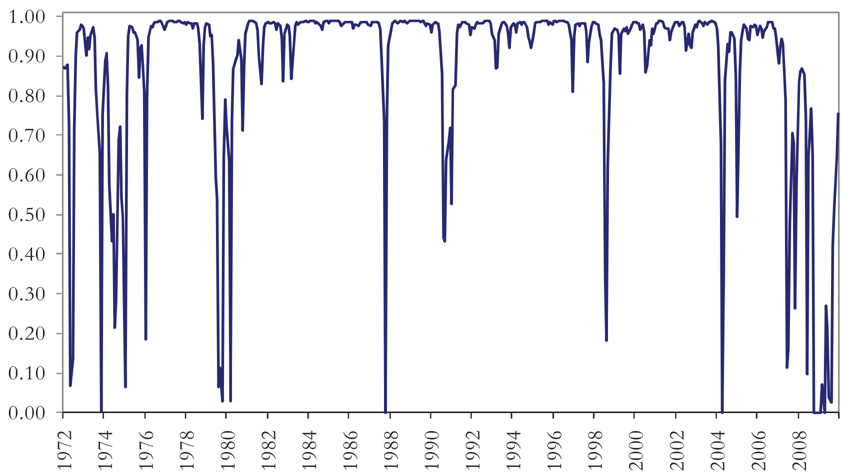
Panel B: Two-State Markov Switching Model					
					Unconditional Mean & Vol. (annual)
		Regime 1:			
Conditional mean function		$E[r_{\mathrm{REIT},t+1}\mid\mathrm{Reg.1}] = \underset{(1.638)}{-1.001} + \underset{(0.041)}{\mathbf{0.082}} r_{\mathrm{REIT},t-1}$			
Conditional variance function		$h_{\mathrm{REIT},t} = \underset{(4.685)}{\mathbf{110.859}}$			14.263 [Sample: 12.576]
		Regime 2:			17.173 [Sample: 17.175]
Conditional mean function		$E[r_{\mathrm{REIT},t+1}\mid\mathrm{Reg.1}] = \underset{(0.191)}{\mathbf{1.401}} + \underset{(0.041)}{\mathbf{0.082}} r_{\mathrm{REIT},t}$			Implied Sharpe ratio: 0.506
Conditional variance function		$h_{\mathrm{REIT},t} = \underset{(0.860)}{\mathbf{10.913}}$			[Sample: 0.408]
Estimated transition matrix		Regime 1	Regime 2		Ergodic Probs.
	Regime 1	**0.633** (0.103)	0.367		0.129
	Regime 2	0.054	**0.946** (0.078)		0.871
Log-Likelihood		835.545	Number of parameters	7	
Akaike information criterion		−3.661	Saturation ratio	65.142857	
Bayes-Schwartz information criterion		−3.538	Jarque-Bera on stdzed res.	7.105**	
Hannan-Quinn information criterion		−3.613	ARCH(12) LM test	0.795	

Table 5 ■ Continued

Panel C: Two-State Markov Switching Model under Restricted Regime Classification					
					Unconditional Mean & Vol. (annual)
		Regime 1:			
Conditional mean function		$E[r_{\mathrm{REIT},t+1}\|\text{Reg. 1}] = -0.819 + 0.082 r_{\mathrm{REIT},t-1}$			
		(1.619) (0.048)			
Conditional variance function		$h_{\mathrm{REIT},t} = \mathbf{140.020}$			
		(5.007)			12.798 [Sample: 12.576]
		Regime 2:			18.845 [Sample: 17.175]
Conditional mean function		$E[r_{\mathrm{REIT},t+1} 1\text{Reg. 1}] = \mathbf{1.263} + 0.082 r_{\mathrm{REIT},t}$			
		(0.212) (0.048)			Implied Shmpe ratio: 0.383
Conditional variance function		$h_{\mathrm{REIT},t} = \mathbf{10.789}$			[Sample: 0.408]
		(0.884)			
Estimated transition matrix		Regime 1	Regime 2		Ergodic Probs.
	Regime 1	**0.690**	0.310		0.141
		(0.152)			
	Regime 2	0.051	**0.949**		0.859
			(0.107)		
Log-Likelihood		791.476	Number of parameters	7	
Aka:ike information criterion		−3.448	Saturation ratio	65.14285714	
Bayes-S chvvartz information criterion		−3.385	Jarque- Bera on stdzed re	7.33s**	
Harman-Quitm information criterion		−3.423	ARCH(12) LM test	0.798	

The table presents estimated coefficients and test statistics for the best-fitting single-state ARCH-type model (panel A), the best two-state Markov switching model (panel B), and a restricted Markov switching model in which the state classification implied by the univariate, two-state model for stock returns is imposed (panel C). In the single-state case, we also report estimates for two subperiods, 1/72–12/91 (pre-modern era) and 1/92–12/09 (modern era), to highlight potential parameter instability. The rightmost column reports unconditional mean, volatility and Sharpe ratio estimates implied by each model. We report in square brackets the corresponding sample period estimates. The Sharpe ratio is computed with reference to the average 1-month T-bill yield. In the table, boldfaced coefficients are significant at 5% or lower size.

Figure 1 ■ Smoothed state probabilities from two-state models: REITs, bull-market state.



probabilities suggest that the bull-market regime predominates, characterizing 87% of months with spells lasting 18.5 months on average; the bear-market regime occurs during only 13% of months, with spells lasting just 2.7 months on average. The implied unconditional mean return of 14.3% is slightly greater than the realized mean of 12.6%, but the implied unconditional variance of 17.2% is essentially identical to the realized variance. The Jarque-Bera test suggests that the standardized residuals for the simple two-state MS model are still not normally distributed, but the sharp decline in the test statistic from 396.7 to 7.1 suggests that the two-state model has addressed much of the normality problem compared to the TARCH model.

Figure 1 depicts the two regimes identified by the MS model, showing the smoothed probabilities of being in the bull-market state (Regime 2). REIT bull markets prevailed from July 1972 through April 1974 (with a short bear market in November 1973); from February 1975 through July 1979 (with a short bear market in January 1976); from November 1979 through July 1990 (with bear markets in March 1980 and October 1987); from October 1990 through June 1998; from September 1998 through March 2004; from June 2004 through May 2007 (with a bear market in January 2005); and from September 2007 through September 2008 (with bear markets in November 2007 and June 2008). Bear markets prevailed during August–October 1979, August–September 1990, July–August 1998, April–May 2004, June–August 2007 and October 2008–September 2009.

The periods from August 1989 through October 1990 and from December 1997 through November 1999 deserve special attention. These were the third- and fourth-most severe and long-lasting equity REIT downturns on record, with total returns declining by 23.9% over 14 months in the earlier period and by 23.7% over 23 months in the later period. The two-state Markov switching model, however, identifies REITs as being in the bear-market regime during only August–September 1990 in the earlier period (during which the estimated probability of being in the bear-market state reached only 56.9%) and July–August 1998 in the later period. The reason for this is that volatility remained relatively low during these two downturns, with a standard deviation of realized returns of just 1.92% during the 8/89–10/90 downturn and 3.05% during the 12/97–11/99 downturn.

Similarly, the second-most severe downturn—a decline of 37.0% over 27 months from September 1972 through December 1974—is identified by the MS model as a period of oscillation between bull-market regimes (7/72–10/73, 12/73–4/74, 6/74 and 9/74–11/74) and bear-market regimes (11/73, 5/74, 7/74–8/74 and 12/74–1/75) rather than as an extended bear market. Finally, the most severe REIT downturn on record—with total returns declining by 68.3% over 25 months from January 2007 through February 2009—is clearly identified by the MS model but with a later and shorter duration (October 2008–September 2009) because the period of high volatility differed from the period of negative returns.

Over the entire study period, during months identified by the MS model as likely episodes of the bull Regime 2, realized equity REIT returns averaged 1.44% with a standard deviation of only 3.28%, while during the bear Regime 1 realized returns averaged –3.32% with a standard deviation of 12.73%.

Stock Returns

Table 6 compares the results for the best-performing single-state model for stock returns—a *t*-Student TARCH(1,1), AR(0) specification—with the simple two-state MS model. The Jarque-Bera test indicates a non-Gaussian distribution of standardized residuals; moreover, the implied unconditional mean is somewhat higher than the realized average (13.0% vs. 10.8%) and the implied unconditional variance is somewhat lower than the realized variance (15.3% vs. 16.1%), with the result that the implied unconditional Sharpe ratio is significantly higher than the realized Sharpe ratio. The subsample models reveal considerable parameter instability between the pre-modern and modern eras, with the modern era suggesting a smaller intercept in the conditional variance equation (insignificant 0.854 vs. significant 1.552), larger impact of shocks on conditional variance (insignificant +0.094 vs. significant –0.097), less precisely measured threshold impact (insignificant 0.147 vs. significant 0.148) and

Table 6 ■ Univariate model results: Stock returns.

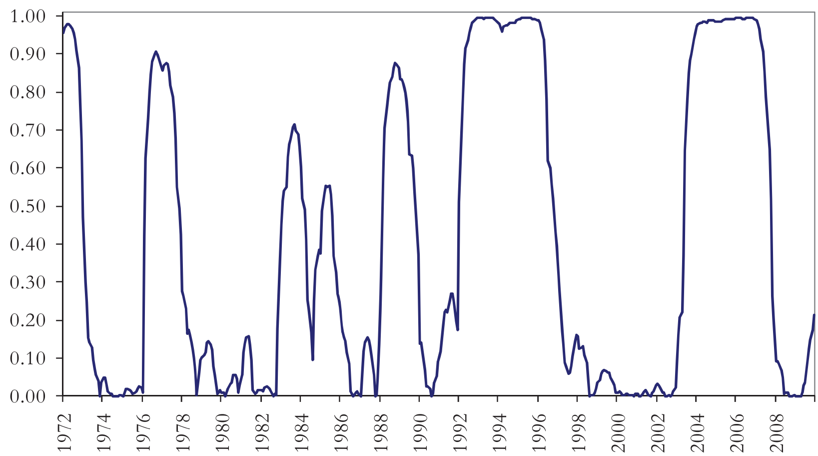
Panel A: Single State Model				
	Full Sample			*Unc. Mean & Vol. (annual)*
Conditional mean function	$r_{\text{Stock},t} = \underset{(0.185)}{\mathbf{1.080}}$			12.964 [Sample: 10.794]
Conditional variance function	$h_{\text{Stock},t} = \underset{(0.957)}{\mathbf{1.968}} - \underset{(0.053)}{0.017} u^2_{\text{Stock},t-1} + \underset{(0.078)}{\mathbf{0.807}} h_{\text{Stock},t-1} + \underset{(0.084)}{\mathbf{0.218}} I_{\{u^{\text{Stock},t-1}<0\}} u^2_{\text{Stock},t-1}$			15.291 [Sample: 16.088]
t-Student degrees of freedom parameter	$\underset{(1.695)}{\mathbf{7.040}}$			Implied Sharpe ratio: 0.484 [Sample: 0.325]
Log-Likelihood	788.858	Number of parameters	6	
Akaike information criterion	−3.434	Saturation ratio	76	
Bayes-Schwartz information criterion	−3.379	Jarque-Bera on stdzed res.	291.787***	
Hannan-Quinn information criterion	−3.412	ARCH(12) LM test	0.355	
	Pre-Modern (1972–1991) Sample			*Unc. Mean & Vol. (annual)*
Conditional mean function	$r_{\text{Stock},t} = \underset{(0.269)}{\mathbf{1.181}}$			14.168 [Sample: 12.413]
Conditional variance function	$h_{\text{Stock},t} = \underset{(0.789)}{\mathbf{1.552}} - \underset{(0.035)}{\mathbf{0.097}} u^2_{\text{Stock},t-1} + \underset{(0.041)}{\mathbf{0.952}} h_{\text{Stock},t-1} + \underset{(0.057)}{\mathbf{0.148}} I_{\{u^{\text{Stock},t-1}<0\}} u^2_{\text{Stock},t-1}$			16.196 [Sample: 16.723]
t-Student degrees of freedom parameter	$\underset{(2.422)}{\mathbf{6.807}}$			Implied Sharpe ratio: 0.412 [Sample: 0.294]
	Modern (1992–2008) Sample			*Unc. Mean & Vol. (annual)*
Conditional mean function	$r_{\text{Stock},t} = \underset{(0.252)}{\mathbf{0.969}}$			11.623 [Sample: 8.996]
Conditional variance function	$h_{\text{Stock},t} = \underset{(0.641)}{0.854} + \underset{(0.156)}{0.094} u^2_{\text{Stock},t-1} + \underset{(0.115)}{\mathbf{0.789}} h_{\text{Stock},t-1} + \underset{(0.141)}{0.147} I_{\{u^{\text{Stock},t-1}<0\}} u^2_{\text{Stock},t-1}$			19.632 [Sample: 15.375]
t-Student degrees of freedom parameter	$\underset{(7.005)}{\mathbf{14.083}}$			Implied Sharpe ratio: 0.418 [Sample: 0.363]

Table 6 ■ Continued

Panel B: Two-State Markov Switching Model					
					Unconditional Mean & Vol. (annual)
		***Regime 1*:**			
Conditional mean function		$r_{\text{Stock},t} = 0.698$ (0.418)			
Conditional variance function		$h_{\text{Stock},t} = \mathbf{31.192}$ (1.963)			10.681 [Sample: 10.794]
		***Regime 2*:**			16.238 [Sample: 16.088]
Conditional mean function		$r_{\text{Stock},t} = \mathbf{1.194}$ (0.290)			Implied Sharpe ratio: 0.315
Conditional variance function		$h_{\text{Stock},t} = \mathbf{7.234}$ (0.730)			[Sample: 0.325]
Estimated Transition Matrix		Regime 1	Regime 2		Ergodic Probs
	Regime 1	**0.970** (0.081)	0.030		0.613
	Regime 2	0.047	**0.953** (0.042)		0.387
Log-Likelihood		799.291	Number of parameters	6	
Akaike information criterion		−3.532	Saturation ratio	76	
Bayes-Schwartz information criterion		−3.465	Jarque-Bera on stdzed res.	13.980***	
Hannan-Quinn information criterion		−3.500	ARCH(12) LM test	0.980	

The table presents estimated coefficients and test statistics for the best-fitting single-state ARCH-type model (panel A) as well as the best two-state Markov switching model. In the single-state case, we also report estimates for two subperiods, 1/72–12/91 (pre-modern era) and 1/92–12/09 (modern era), to highlight potential parameter instability. The rightmost column reports unconditional mean, volatility and Sharpe ratio estimates implied by each model. We report in square brackets the corresponding sample period estimates. The Sharpe ratio is computed with reference to the average 1-month T-bill yield. In the table, boldfaced coefficients are significant at 5% or lower size.

Figure 2 ■ Smoothed state probabilities from two-state models: Stocks, bull-market state.



thinner tails in the disturbance distribution (14.083 vs. 6.807, recalling that the tails of a *t* density are thicker the lower the number of degrees of freedom).

The regimes identified by the two-state MS model for stock returns with no autoregressive terms (Panel B) are somewhat different than for REIT returns, with both the "bear-market" environment (Regime 1) and the "bull-market" environment (Regime 2) more moderate than the corresponding REIT regimes in terms of both returns and volatility. Also unlike with REITs, both regimes are strongly persistent, with the "bear-market" state lasting 33.3 months on average and prevailing 61% of the time while the bull-market state lasts 21.3 months on average and prevails during the remaining 39% of months. As with the REIT MS model, for stocks the Jarque-Bera statistic suggests that standardized residuals still have a non-Gaussian distribution but are much closer to normal than with the TARCH model.

Figure 2 shows the smoothed probabilities of being in the during months identified as likely bear state, realized stock returns averaged just 0.78% with a standard deviation of 5.62%, while during the bull state realized returns averaged 1.10% with a standard deviation of 2.54%.

Bond Returns

Finally, Table 7 shows the results for bond returns, with the best-performing single-state model again (as with stocks) a *t*-Student TARCH(1,1), AR(0) specification. As with stocks, the subsample models provide evidence of parameter instability in the estimates of the conditional variance intercept, of the impact of shocks on conditional volatility, of persistence in volatility and of the thickness of the tails of the distribution of shocks, as captured by the number of degrees of freedom of the *t* distribution. As for both REITs and stocks, the Jarque-Bera statistic rejects a Gaussian distribution of the standardized residuals.

As Panel B shows, the simple two-state MS model is much more intuitive, with Regime 1 identifying a low-return, low-volatility state that can be characterized as representing periods of economic expansion and stable or increasing interest rates, high-return, lower-volatility "bull-market" state (Regime 2), which prevailed from the beginning of the study period through December 1972 and during March 1976—October 1977, February 1983—February 1984, March—July 1985, March 1988—October 1989, January 1992—October 1996, and June 2003—October 2007, while Regime 2 identifies a higher-return, higher-volatility state that can be characterized as recessionary periods of declining interest rates.[7] Both regimes are highly persistent, with expansionary states lasting 83.3 months on average and prevailing 65% of the time while recessionary states last 43.5 months on average and prevailing during 35% of months. Finally, the Jarque-Bera test reveals that for bonds, the two-state model deals successfully with the non-Gaussian distribution of residual terms.

Figure 3 shows the smoothed probabilities of being in the high-return, high-volatility "recession" state, which prevailed during September 1979—March 1988, November 2001—February 2004, and September 2008 through the end of the study period. Realized returns on long-term government bonds averaged 0.6% annualized during the "expansion" state with a very modest standard deviation of 1.69%, compared to a moderately higher 0.8% annualized mean realized return with a standard deviation of 3.27% during the "recession" state.

Association of Regimes between Asset Classes

The graphs in Figure 4 portray the association between smoothed state probabilities derived from univariate two-state MS models for each pair of assets. The graphs on the left depict the state probabilities over time for each of the three pairs of assets, with grey regions indicating periods during which assets were in phase in terms of common bull or bear regimes; the graphs on the right depict the state probabilities for each pair plotted as Cartesian (*X*, *Y*) coordinates in a scatter diagram.

[7]Here it may be useful to recall that, because of the peculiar link between the price of long-term bonds and short-term (policy) rates, periods of stable or increasing short-term rates translate to low or even negative realized bond returns, while periods of declining interest rates imply positive and possibly high realized returns.

Table 7 ■ Univariate model results: Bond returns.

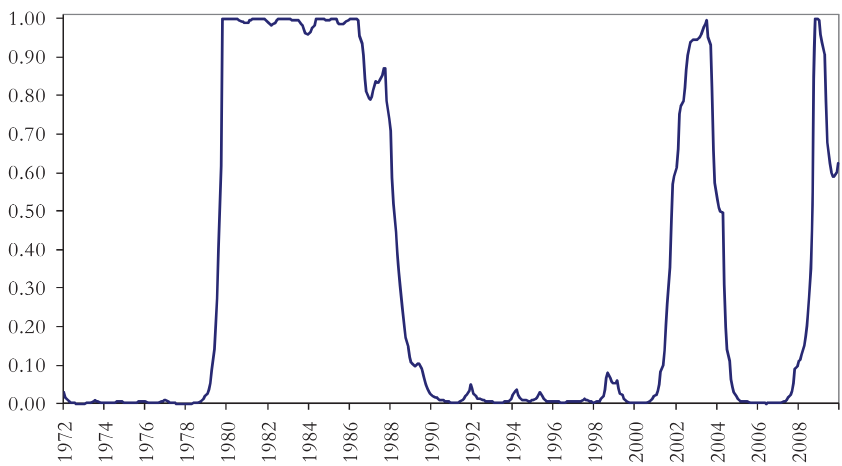
Panel A: Single State Model			
	Full Sample		*Unc. Mean & Vol. (annual)*
Conditional mean function	$r_{\text{Bond},t} = \underset{(0.091)}{\mathbf{0.545}}$		6.541 [Sample: 7.974]
Conditional variance function	$h_{\text{Bond},t} = \underset{(0.072)}{0.137} + \underset{(0.041)}{\mathbf{0.081}} u^2_{\text{Bond},t-1} + \underset{(0.035)}{\mathbf{0.900}} h_{\text{Bond},t-1} - \underset{(0.052)}{0.006} I_{\{u\text{Bond},t-1<0\}} u^2_{\text{Bond},t-1}$		8.645 [Sample: 8.025] Implied Sharpe ratio: 0.113
t-Student degrees of freedom parameter	$\underset{(3.904)}{\mathbf{9.550}}$		[Sample: 0.296]
Log-Likelihood	1104.514	Number of parameters: 6	
Akaike information criterion	−4.829	Saturation ratio: 76	
Bayes-Schwartz information criterion	−4.774	Jarque-Bera on stdzed res.: 22.047***	
Hannan-Quinn information criterion	−4.807	ARCH(12) LM test: 0.837	
	Pre-Modern (1972–1991) Sample		*Unc. Mean & Vol. (annual)*
Conditional mean function	$r_{\text{Bond},t} = \underset{(0.131)}{\mathbf{0.469}}$		5.622 [Sample: 9.233]
Conditional variance function	$h_{\text{Bond},t} = \underset{(0.047)}{0.064} + \underset{(0.038)}{\mathbf{0.025}} u^2_{\text{Bond},t-1} + \underset{(0.033)}{\mathbf{0.909}} h_{\text{Bond},t-1} + \underset{(0.071)}{0.110} I_{\{u\text{Bond},t-1<0\}} u^2_{\text{Bond},t-1}$		8.356 [Sample: 8.745] Implied Sharpe ratio: –0.228
t-Student degrees of freedom parameter	$\underset{(6.704)}{\mathbf{10.900}}$		[Sample: 0.198]
	Modern (1992–2009) Sample		*Unc. Mean & Vol. (annual)*
Conditional mean function	$r_{\text{Bond},t} = \underset{(0.098)}{\mathbf{0.568}}$		6.817 [Sample: 6.744]
Conditional variance function	$h_{\text{Bond},t} = \underset{(0.117)}{0.263} + \underset{(0.317)}{\mathbf{0.366}} u^2_{\text{Bond},t-1} + \underset{(0.282)}{\mathbf{0.281}} h_{\text{Bond},t-1} - \underset{(0.320)}{0.442} I_{\{u\text{Bond},t-1<0\}} u^2_{\text{Bond},t-1}$		7.373 [Sample: 7.119] Implied Sharpe ratio: 0.461
t-Student degrees of freedom parameter	$\underset{(2.662)}{\mathbf{6.326}}$		[Sample: 0.467]

Table 7 ■ Continued

Panel B: Two-State Markov Switching Model				
				Unconditional Mean & Vol. (annual)
	Regime 1:			
Conditional variance function	$r_{\mathrm{Bond},t} =$ **0.562** (0.099)			
	$h_{\mathrm{Bond},t} =$ **2.835** (0.436)			8.103 [Sample: 7.974]
	Regime 2:			
Conditional mean function	$r_{\mathrm{Bond},t} =$ **0.883** (0.282)			Implied Sharpe ratio: 0.309
Conditional variance function	$h_{\mathrm{Bond},t} =$ **10.679** (0.701)			[Sample: 0.296]
Estimated transition matrix	Regime 1	Regime 2		Ergodic Probs.
Regime 1	**0.988** (0.073)	0.012		0.647
Regime 2	0.023	**0.977** (0.036)		0.353
Log-Likelihood	1129.693	Number of parameters	6	
Akaike information criterion	−4.902	Saturation ratio	76	
Bayes-Schwartz information criterion	−4.847	Jarque-Bera on stdzed res.	0.715	
Hannan-Quinn information criterion	−4.881	ARCH(12) LM test	0.840	

The table presents estimated coefficients and test statistics for the best-fitting single-state GARCH models (panel A) as well as the best two-state Markov switching model (panel B). In the single-state case, we also report estimates for two subperiods, 1/72–12/91 (pre-modern era) and 1/92–12/09 (modern era), to highlight parameter instability. The rightmost column reports unconditional mean, volatility and Sharpe ratio estimates implied by each model. We report in brackets the corresponding sample period estimates. The Sharpe ratio is computed with reference to the average 1-month T-bill yield. Boldfaced coefficients are significant at 5%.

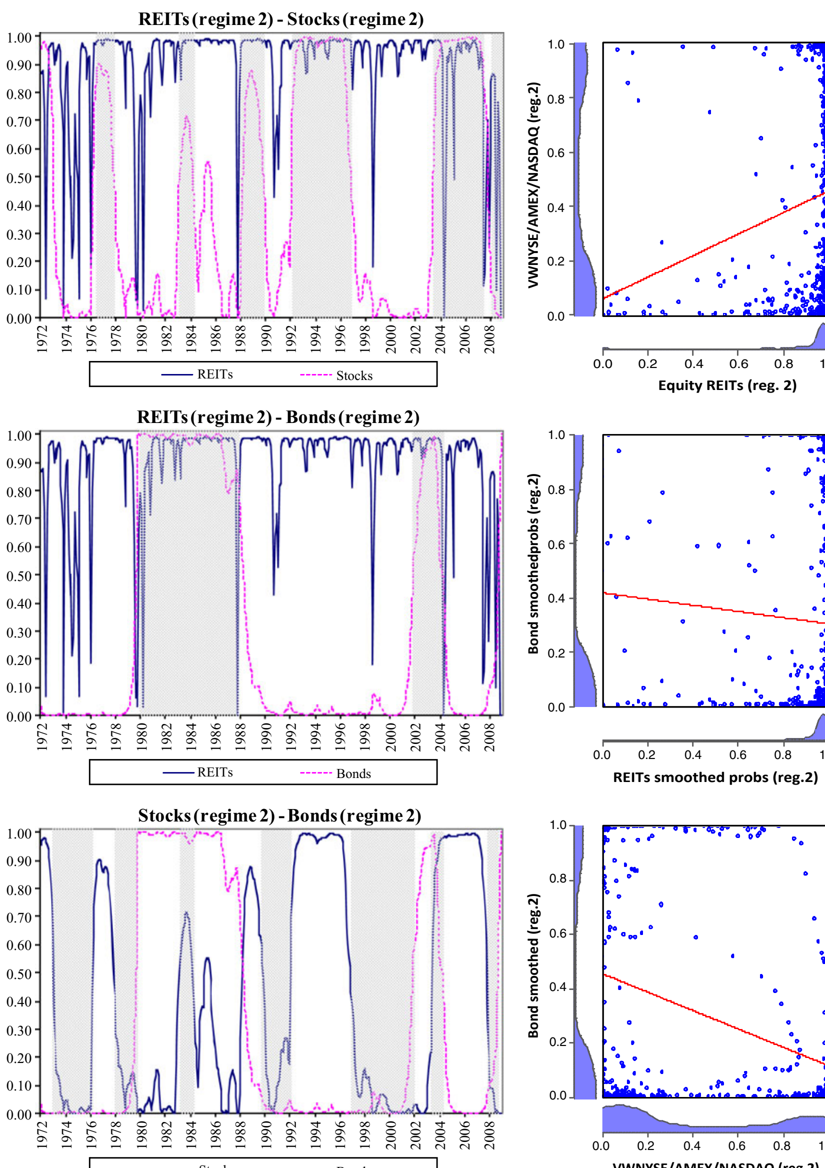
Figure 3 ■ Smoothed state probabilities from two-state models: Long-term government bonds.



As the top pair of graphs shows, regimes in REITs and stocks were generally synchronized as being both in a bull regime from March 1976 through October 1977, from February 1983 through February 1984, from March 1988 through October 1989, from January 1992 through October 1996, from June 2003 through March 2004, and from June 2004 through May 2007; similarly, both assets were in a bear-market state from October 2008 through September 2009. REITs and stocks were generally "out of sync" during November 1977–July 1979, April 1980–January 1983, March 1984–February 1985, August 1985–September 1987, October 1990–December 1991, November 1996–June 1998 and September 1998–May 2003, all periods during which REITs were in their own bull-market state while stocks were in their bear state. The Spearman rank correlation (which is the appropriate association measure because probabilities are bounded between 0 and 1) between smoothed state probabilities is significantly positive at 35.75%, suggests a systematic association between smoothed state probabilities in REITs and stocks, although such an association is much less than perfect.

The middle pair of graphs shows that the high-return, low-volatility (bull) regime in REIT returns was generally in sync with the high-return, high-volatility (recessionary) regime in bond returns during November 1979—March 1988 and November 2001—February 2004; conversely, several short bear regimes in REITs coincided with the low-return, low-volatility expansionary regime in bonds (e.g. May-June 1972, January 1976, July-August 1998, June-August 2007). The two markets were out of sync during most of the bond

Figure 4 ■ Degree of association between smoothed state probabilities from two-state models of U.S. REITs, stock and long-term government bond returns.



expansionary periods before September 1979 and during April 1988—October 2001 and March 2004—August 2008. Overall, the Spearman rank correlation coefficient (not significantly different from zero at 3.5%) suggests that there was no systematic association between states for REITs and bonds.

Finally, the bottom pair of graphs shows that the high-return, low-volatility (bull) regime in stock returns was generally in sync with the high-return, high-volatility (recessionary) regime in bond returns only during short (and weakly identified) stock bull markets of February 1983—February 1984 and March-July 1985 and the transition period of June 2003—February 2004. Conversely, the low-return, high-volatility (bear) regime in stocks was generally in sync with the low-return, low-volatility (expansionary) regime in bonds during January 1973—February 1976, November 1977—August 1979, November 1989—December 1991, November 1996—October 2001, and November 2007—August 2008. The asset classes were generally out of sync during stock mull markets before 1973 and during March 1976—October 1977, April 1988—October 1989, January 1992—October 1996, and March 2004—October 2007, as well as during stock bear markets of September 2979—January 1983, March 1984—February 1985, August 1985—February 1988, November 2001—May 2003, and September 2008 through the end of the study period. Overall, the significantly negative Spearman rank correlation of −29.5% confirms an association in regimes between the two asset classes.

In short, there appear to be systematic associations between REIT and stock regimes and between stock and bond regimes, but REIT and bond regimes appear to be quite independent of each other. Because REITs are traded through the stock market, and are frequently classified as equity rather than real estate investments, this evidence of association raises the possibility whether regime switches in REIT returns may simply provide a signal of generally corresponding regime switches in stock returns (or *vice versa*). We tested this possibility in two ways.

First, Table 5 Panel C presents the results of estimating the same two-state Markov switching model shown in Panel B of Table 5, but in this case we have imposed the filtered state probabilities implied by the Markov switching model estimated for stock returns (see Table 6). If the model under this stock-regime constraint performs nearly as well as the model with REIT-regimes, this result would support the contention that REIT and stock returns define a common pattern of regime switches. In fact, the results shown in Panel C are reasonably similar to the unconstrained results shown in Panel B, identifying a high-return, low-volatility bull-market regime characterizing 86% of months and a low-return, high-volatility bear-market regime prevailing during 14% of months. On the other hand, imposing the stock regime classifications on REIT data leads to a substantial decline in the maximized log-likelihood function (from 835.5 to 791.5) and a very large deterioration in all three information criteria; moreover, in both regimes the coefficient on $r_{\mathrm{REIT},t-1}$, which was (barely) significant under the unconstrained REIT regimes, become (barely) insignificant when constrained to follow stock regimes. This deterioration in model performance suggests that, while REIT and stock regimes are associated as indicated by the Spearman rank correlations, they are not proxies for each other.

Our second robustness test investigated whether the smoothed state probabilities for REITs lead the state probabilities for stocks, or *vice versa*. To test whether REIT regimes are a leading signal of stock regimes we computed the Spearman's rank correlation between smoothed state probabilities for stocks and lagged values of smoothed state probabilities for REITs. The value was 34.24%, slightly less than the 35.75% computed for contemporaneous smoothed state probabilities. Conversely, to test whether stock regimes are a leading signal of REIT regimes we computed the Spearman's rank correlation between stock probabilities and lagged values of REIT probabilities: the value was 34.57%, again a slight decline from the contemporaneous value.[8] In short, although there is some lagged association between REIT and stock regimes (as there is between stock and bond regimes), we found no evidence that stock regimes and REIT regimes proxy for one another or that one may uni-directionally lead or cause the other.

Performance Comparison: Trivariate Models

Given the significant associations between two of the three asset class pairs, we next investigate the design of trivariate models of the joint returns of all three asset classes. We perform a comparative analysis similar to that conducted for each of the asset classes separately, first estimating a comparatively simple model and then adding complexity in terms of autoregressive components, heteroskedastic components and additional states.

Table 8 performs a model specification search similar to Tables 2–4 and summarizes the results for the models estimated using REIT, stock and bond returns, starting with single-state models and including results for MS models with two, three and four regimes. The first line presents the baseline model with no vector autoregressive or heteroskedastic components. Ljung-Box tests on levels and squares indicate the presence of massive persistence in squared residuals for all asset classes, which is evidence of conditional heteroskedastic effects, as well

[8]We conducted a similar test based on the standard Pearson's correlation coefficient, with the same result. Note that the Pearson's correlation coefficient is not a correct statistic because the smoothed state probabilities are random variables defined on (0,1).

Table 8 ■ Multivariate model selection.

Number of Regimes (*K*)	VAR(*P*) Order	Heteroskedastic Components	Final Negative Log-Likelihood	Linearity Test	AIC	BIC	H-Q	RCM 1	RCM 2	LB(12) - Levels RE	LB(12) - Levels S	LB(12) - Levels GB	LB(12) - Own Squares RE	LB(12) - Own Squares S	LB(12) - Own Squares GB	No. Param.	Saturation Ratio
Single-State Models																	
1	0	No	2650.283	✍	−11.5846	−11.5032	−11.5525	✍	✍	63.04***	9.67	14.24	238.1***	28.75***	96.00***	9	152.00
1	1	No	2676.992	✍	−11.6879	−11.5249	−11.6237	✍	✍	61.55***	7.62	11.56	250.7***	24.61**	99.06***	18	75.83
1	4	No	2677.667	✍	−11.6357	−11.2261	−11.4743	✍	✍	20.92*	6.48	9.29	158.7***	22.20**	97.42***	45	30.13
1	0	Gaussian M-GARCH(1,1)	2780.113	✍	−12.1014	−11.9115	−12.0266	✍	✍	20.71*	11.61	18.55*	3.26	7.18	10.46	21	65.14
1	1	Gaussian M-GARCH(1,1)	2802.008	✍	−12.1847	−11.9132	−12.0777	✍	✍	19.26*	10.03	10.22	2.84	8.30	6.29	30	45.50
1	4	Gaussian M-GARCH(1,1)	2789.602	✍	−12.0773	−11.5586	−11.8729	✍	✍	18.92*	8.74	9.00	2.96	8.04	5.02	57	23.79
1	1	Gaussian M-TGARCH(1,1,1)	2808.529	✍	−12.1869	−11.8609	−12.0585	✍	✍	16.53	9.93	11.74	2.97	5.59	5.60	36	37.92
1	1	Gaussian M-DCC/GARCH(1,1)	2789.933	✍	−12.1492	**−11.9137**	−12.0564	✍	✍	24.80**	9.06	10.29	4.24	8.33	5.38	26	52.50
Two-State Models																	
2	0	No	2696.046	91.525***	−11.7634	−11.6368	−11.7135	0.446	44.253	40.65***	5.18	14.71	54.53***	66.76***	104.8***	14	97.71
2	1☞	No	2676.999	0.014	−11.6659	−11.4576	−11.5838	92.845	95.980	48.00***	6.97	8.02	222.4***	29.27***	26.44***	23	59.35
2	1	No	2761.564	169.14***	−11.9981	−11.7083	−11.8839	7.074	50.571	39.29***	5.85	63.69***	71.24***	73.09***	20.55*	32	42.66
2	0	Yes	2756.351	212.14***	−12.0015	−11.8207	−11.9303	29.955	70.412	25.79**	4.62	12.53	132.2***	29.98***	18.45	21	65.14
2	1☞	Yes	2771.182	188.38***	−12.0535	−11.7909	−11.9501	22.368	63.700	19.27*	4.60	26.98***	73.50***	27.98***	38.09***	29	47.07
2	1	Yes	2782.170	210.36***	−12.0623	−11.7182	−11.9267	35.29404	74.862	19.27*	4.57	26.98***	73.49***	27.95***	38.04***	38	35.92

Table 8 ■ Continued

Number of Regimes (K)	VAR(P) Order	Heteroskedastic Components	Final Negative Log-Likelihood	Linearity Test	AIC	BIC	H-Q	RCM 1	RCM 2	LB(12) - Levels			LB(12) - Own Squares			No. Param.	Saturation Ratio
										RE	S	GB	RE	S	GB		
Three-State Models																	
3	0	No	2716.365	132.16***	−11.8218	−11.6319	−11.7470	0.000	61.920	42.94***	4.25	7.64	45.03***	80.86***	40.61***	21	65.14
3	1☞	No	2728.821	103.66***	−11.8630	−11.5913	−11.7559	0.001	62.343	45.51***	4.28	7.94	45.01***	78.18***	36.83***	30	45.50
3	1	No	2789.839	225.69***	−12.0520	−11.6174	−11.8808	0.532	83.494	32.37***	5.75	5.69	77.09***	69.93***	66.48***	48	28.44
3	0	Yes	2779.713	258.86***	−12.0470	−11.7486	−11.9295	1.900	83.257	17.05	5.55	11.95	81.84***	29.39***	82.98***	33	41.45
3	1☞	Yes	2816.092	278.20***	−12.1938	−11.8135	−12.0440	0.543	70.918	20.36*	3.80	7.52	45.24***	34.33***	9.61	42	32.50
3	1	Yes	2822.837	291.69***	−12.1443	−11.6010	−11.9303	2.030	86.141	24.86**	6.36	8.58	48.31***	38.48***	45.63***	60	22.75
Four-State Models																	
4	0	No	2728.179	155.79***	−11.8341	−11.5629	−11.7273	**0.000**	27.359	44.78***	6.36	9.53	41.36***	33.43***	42.96***	30	45.60
4	1☞	No	2749.355	144.73***	−11.9137	−11.5605	−11.7745	**0.000**	**9.481**	40.39***	4.39	10.17	34.49***	45.33***	34.13***	39	35.00
4	1	No	2815.847	277.71***	−12.0872	−11.4896	−11.8518	0.026	78.394	19.24**	12.26	11.98	49.66***	19.85*	1.45	66	20.68
4	0	Yes	2889.052	477.54***	**−12.4555**	**−12.0107**	**−12.2803**	**0.000**	**23.828**	29.47***	14.74	13.74	14.95	11.16	36.97***	48	28.50
4	1☞	Yes	2906.581	459.19***	**−12.5192**	**−11.9899**	**−12.3107**	0.002	**21.382**	12.81	9.36	17.97	5.92	18.84*	19.35*	57	23.95
4	1	Yes	2936.330	518.68***	**−12.5303**	−11.7545	**−12.2247**	0.002	29.676	10.22	10.12	9.60	12.90	12.99	5.15	84	16.25

The table summarizes outputs for a range of models defined by the number of regimes (K = 1, 2, 3, 4), the autoregressive order (P) and the presence of heteroskedastic components of either ARCH or Markov switching type. For single-state models (K = 1), heteroskedastic components can take the form of GARCH(l,q), threshold GARCH and DCC GARCH(l,q) models with the marginal density of error terms always distributed as a multivariate Gaussian. Final negative log-likelihood is –1 times the maximized log-likelihood. The significance level of linearity tests is computed by applying Davies' (1977) approximation to the asymptotic mixture of noncentral chi-square that characterizes the LR statistic. AIC, BIC and H-Q are information criteria proposed by Akaike (1974), Schwartz (1978) and Hannan and Quinn (1979) that trade off fit and parsimony. RCM1 and RCM2 are regime classification measures suggested by Hamilton (1988) and Guidolin (2009); values range from 0 to 100 with lower values corresponding to greater classification accuracy. The Ljiung-Box portmanteau tests for serial correlation are applied to levels and squared residuals from the conditional mean equation for REITs, stocks and bonds separately. The saturation ratio is the ratio of the total number of observations to the number of parameters estimated. Bold face highlights the models that minimize the information criteria and regime classification measures. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

as in the level of residuals for REIT returns. Of course, this is consistent with the evidence in Tables 2–4, as a VAR(0) homoskedastic model with Gaussian shocks is essentially a Gaussian i.i.d model that is likely to imply results that hardly differ from those produced by separate estimation of means and variances for individual asset classes. The second and third row show that simply picking up vector autoregressive components, even of a relatively high order ($P = 4$), hardly helps remove the evidence of conditional heteroskedasticity, which is therefore not simply caused by a misspecified conditional mean function in the first row. Finally, the last four rows of the panel devoted to single-state models show that considerable improvements may be obtained by adopting multivariate GARCH models and that, not surprisingly in the light of the results in Case, Yawei and Yildirim (2012), among them a DCC GARCH(1,1) model offers a considerable degree of parsimony in spite of its excellent fit, which tends to be rewarded by good scores by all information criteria.[9]

However, when we broaden our perspective to the remaining panels of Table 8, those devoted to MS models, we notice that a Davies-style corrected likelihood ratio for the presence of regimes in the data generally rejects the null of absence of multiple regimes with such high values of the test statistic that it is difficult to doubt the statistical significance of the test outcome, independently of the way the LR test is corrected for nuisance parameter problems. The information criteria strengthen this conclusion and two of them reveal that—even when the existence of a trade-off between in-sample fit and parsimony (hence, likely out-of-sample performance) is taken into account—only rather rich four-state MS models ought to be considered, which does mimic similar conclusions reported for portfolios of stocks and bonds only by Guidolin and Timmermann (2006). Given our earlier evidence of considerable synchronization between REIT and stock regimes and between stock and bond regimes (in contrast to the absence of synchronization between REIT and bond regimes), it is unsurprising that a four-state MS outperforms two- and three-state models: if one assumes that two regimes are sufficient to characterize the joint distribution of stock and REIT returns and that two additional regimes are needed to capture the dynamic properties of bond returns across recessions and expansions, one is bound to find that $2 \times 2 = 4$ regimes will be needed in a trivariate modeling exercise.

In general, the AIC and Hannan–Quinn criteria reveal that the three best models are MS models with regime-dependent variances and correlations. The only

[9] Although not reported in Table 8, we have also estimated a few trivariate GARCH(l,q) models with either l or q equal to 2. These are of course relatively large, less parsimonious models than those reported in Table 8 that ended up being heavily penalized by all our information criteria. Interestingly, even the carefully fitted trivariate VAR(1)-GARCH(1,1) and VAR(1)-DCC GARCH(1,1) models seem to have problems at getting rid of the existence of serial correlation in the residuals from the REIT returns equation.

marginal doubts concern whether a VAR(1) component may be useful and, if so, whether it should be regime-specific. The third, and maximally parsimonious, criterion (the Bayes–Schwartz) in fact does give some play to the VAR(1) DCC GARCH(1,1) model, which is however ranked only third, while the model with the best (lowest) BIC is a MS heteroskedastic restricted VAR(1). As it turns out, such rich MS models mark strong improvements in producing residuals that are not serially correlated in either levels or squares.[10] Moreover, the Ljung-Box tests suggest that none of the two-state or three-state models successfully models the structure in mean REIT returns, or the structure in standard deviations for any of the asset classes.

Given this evidence, we now report detailed results for three models that are highlighted by Table 8 as key steps in a modeling exercise of the joint time-series dynamics of REIT, stock and bond returns. Table 9 summarizes the starting point for our comparison of trivariate models, a joint single-state Gaussian VAR(1) model with no heteroskedastic components. The implied unconditional means of 1.047 for REITs, 0.895 for non-REIT stocks, and 0.667 for bonds are all quite close to their realized values of 1.048, 0.900, and 0.665 respectively. The implied unconditional standard deviations of REITs and stocks are somewhat higher than their realized values (5.567 vs. 4.958 for REITs and 4.878 vs. 4.644 for stocks), with the result that the implied unconditional Sharpe ratios are somewhat lower than their realized values (0.105 vs. 0.118 for REITs and 0.088 vs. 0.094 for stocks). The conditional correlations are estimated at 58.8% for REITs with stocks, 12.3% for REITs with bonds, and 17.5% for stocks with bonds.

Table 9 also shows the simple trivariate model estimated separately for the pre-modern and modern eras, revealing considerable parameter instability. Most noticeably, the conditional and implied unconditional volatilities are considerably larger for REITs during the modern era but smaller for stocks and bonds, in keeping with the pattern seen in the realized data. All inter-asset correlations are also lower in the modern than in the pre-modern era, consistent with the results reported by Case, Yawei and Yildirim (2012).

Table 10 details the estimation results for our preferred single-state conditional heteroskedastic model that turns out to yield the lowest values for the information criteria in Table 8, a Gaussian multivariate VAR(1) DCC-GARCH(1,1) model. Interestingly, the estimates of the conditional mean functions implied by the VAR(1) are never substantially affected by our modeling of the dynamic

[10]Further tests, available upon request, confirm that conditioning on the filtered probabilities of the four regimes, the resulting standard residuals have an approximate multivariate Gaussian distribution.

Table 9 ■ Trivariate model results: Single-state Gaussian VAR(1) homoskedastic specification.

Single-State Homoskedastic VAR(1) Model				
	Full Sample	*Unconditional Means, Vols & Correlations (monthly)*		
Conditional mean functions	$r_{\mathrm{REIT},t} = \mathbf{0.608} + 0.001 r_{\mathrm{REIT},t-1} + \mathbf{0.170} r_{\mathrm{Stock},t-1} + \mathbf{0.428} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$ (0.237) (0.056) (0.060) (0.099)		1.047	
	$r_{\mathrm{Stock},t} = \mathbf{0.656} + \mathbf{0.091} r_{\mathrm{REIT},t-1} + 0.019 r_{\mathrm{Stock},t-1} + \mathbf{0.191} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$ (0.228) (0.054) (0.330) (0.094)		0.895	
	$r_{\mathrm{Bond},t} = \mathbf{0.703} \; \mathbf{-0.065} r_{\mathrm{REIT},t-1} - \mathbf{0.048} r_{\mathrm{Stock},t-1} + \mathbf{0.112} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$ (0.112) (0.027) (0.029) (0.046)		0.667	
Conditional variance functions	$h_{\mathrm{REIT},t} = 23.107$ $h_{\mathrm{Stock},t} = 21.240$ $h_{\mathrm{Bond},t} = 5.136$	5.567	4.878	2.304
Conditional correlation functions	ρ[REIT, Stock] = 0.588 ρ[REIT, Gov.bond] = 0.123 ρ[Stock, Gov.bond] = **0.175**	0.703	0.171	0.193
	Implied Sharpe ratios:	0.105	0.088	0.088

Log-Likelihood	2650.283	Number of parameters	18
Akaike information criterion	−11.5846	Saturation ratio	76.00
Bayes-Schwartz information criterion	−11.5032	Jarque-Bera on stdzed res.	1066.36***
Hannan-Quinn information criterion	−11.5525	Mult. Ljung-Box(12) autocorrel.	137.37**

Table 9 ■ Continued

Single-State Homoskedastic VAR(1) Model				
	Pre-Modern (1972–1991) Sample	*Unconditional Means, Vols & Correlations (monthly)*		
Conditional mean functions	$r_{\mathrm{REIT},t} = \underset{(0.273)}{\mathbf{0.696}} + \underset{(0.085)}{0.074} r_{\mathrm{REIT},t-1} + \underset{(0.069)}{0.005} r_{\mathrm{Stock},t-1} + \underset{(0.107)}{\mathbf{0.439}} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$		1.125	
	$r_{\mathrm{Stock},t} = \underset{(0.324)}{\mathbf{0.678}} + \underset{(0.100)}{\mathbf{0.256}} r_{\mathrm{REIT},t-1} - \underset{(0.089)}{0.145} r_{\mathrm{Stock},t-1} + \underset{(0.127)}{\mathbf{0.289}} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$		1.040	
	$r_{\mathrm{Bond},t} = \underset{(0.171)}{\mathbf{0.768}} - \underset{(0.053)}{\mathbf{0.049}} r_{\mathrm{REIT},t-1} \underset{(0.047)}{-0.046} r_{\mathrm{Stock},t-1} + \underset{(0.067)}{\mathbf{0.143}} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$		0.776	
Conditional variance functions	$h_{\mathrm{REIT},t} = 15.974 \quad h_{\mathrm{Stock},t} = 22.543 \quad h_{\mathrm{Bond},t} = 6.265$	4.839	5.548	2.599
Conditional correlation functions	ρ[REIT, Stock] = **0.676** ρ[REIT, Gov.bond] = **0.198** ρ[Stock, Gov.bond] = **0.320**	0.811	0.311	0.380
	Implied Sharpe ratios:	0.103	0.075	0.058
	Modern (1992–2009) Sample	*Unconditional Means, Vols & Correlations (monthly)*		
Conditional mean functions	$r_{\mathrm{REIT},t} = \underset{(0.394)}{0.424} - \underset{(0.077)}{0.051} r_{\mathrm{REIT},t-1} + \underset{(0.100)}{\mathbf{0.362}} r_{\mathrm{Stock},t-1} + \underset{(0.181)}{\mathbf{0.554}} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$		0.949	
	$r_{\mathrm{Stock},t} = \underset{(0.315)}{\mathbf{0.562}} + \underset{(0.062)}{0.015} r_{\mathrm{REIT},t-1} + \underset{(0.080)}{\mathbf{0.135}} r_{\mathrm{Stock},t-1} + \underset{(0.145)}{0.114} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$		0.739	
	$r_{\mathrm{Bond},t} = \underset{(0.141)}{\mathbf{0.643}} - \mathbf{0.067} r_{\mathrm{REIT},t-1} - \underset{(0.028)}{\mathbf{0.071}} r_{\mathrm{Stock},t-1} + \underset{(0.036)}{0.045} r_{\mathrm{Bond},t-1} + \underset{(0.065)}{(h_{\mathrm{REIT},t})^{1/2}} u_{\mathrm{REIT},t}$		0.552	
Conditional variance functions	$h_{\mathrm{REIT},t} = 30.424 \quad h_{\mathrm{Stock},t} = 19.499 \quad h_{\mathrm{Bond},t} = 3.907$	7.570	4.785	1.971
Conditional correlation functions	ρ[REIT, Stock] = **0.531** ρ[REIT, Gov.bond] = 0.073 ρ[Stock, Gov.bond] = − 0.045	0.752	0.092	–0.027
	Implied Sharpe ratios:	0.088	0.095	0.136

The table presents MLE estimates and test statistics for a single-state Gaussian VAR(1) model of asset returns. We also report estimates for two subperiods, 1/72–12/91 (pre-modern era) and 1/92–12/09 (modern era), to highlight potential parameter instability. Standard errors are shown in parentheses. Parameter estimates in bold face are significant at 5% confidence. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table 10 ■ Trivariate model results: Single-state VAR(1) DCC-GARCH(1,1) specification.

Single-State VAR(1) DCC GARCH(1,1) Model	*Full Sample*	*Unconditional Means, Vols & Correlations (monthly)*
Conditional mean functions	$r_{\text{REIT},t} = \underset{(0.197)}{\mathbf{0.700}} + \underset{(0.060)}{0.007} r_{\text{REIT},t-1} + \underset{(0.048)}{\mathbf{0.114}} r_{\text{Stock},t-1} + \underset{(0.095)}{\mathbf{0.353}} r_{\text{Bond},t-1} + \in_{\text{REIT},t} \quad \in_{\text{REIT},t} \sim N(0,(H_t)^{1/2})$	1.013
	$r_{\text{Stock},t} = \underset{(0.216)}{\mathbf{0.676}} + \underset{(0.051)}{\mathbf{0.093}} r_{\text{REIT},t-1} \underset{(0.060)}{-0.013} r_{\text{Stock},t-1} + \underset{(0.090)}{\mathbf{0.218}} r_{\text{Bond},t-1} + \in_{\text{REIT},t} \quad \in_{\text{Stock},t} \sim N(0,(H_t)^{1/2})$	0.885
	$r_{\text{Bond},t} = \underset{(0.103)}{\mathbf{0.593}} - \underset{(0.023)}{\mathbf{0.035}} r_{\text{REIT},t-1} - \underset{(0.025)}{\mathbf{0.065}} r_{\text{Stock},t-1} + \underset{(0.049)}{\mathbf{0.139}} r_{\text{Bond},t-1} + \in_{\text{REIT},t} \quad \in_{\text{Bond},t} \sim N(0,(H_t)^{1/2})$	0.581
Conditional variance functions	$h_{\text{REIT},t} = \underset{(0.378)}{\mathbf{0.769}} + \underset{(0.034)}{\mathbf{0.100}} \in^2_{\text{REIT},t-1} + \underset{(0.044)}{\mathbf{0.870}} h_{\text{REIT},t-1}$	5.063
	$h_{\text{Stock},t} = \underset{(0.493)}{\mathbf{1.063}} + \underset{(0.032)}{\mathbf{0.094}} \in^2_{\text{Stock},t-1} + \underset{(0.042)}{\mathbf{0.861}} h_{\text{Stock},t-1}$	4.860
	$h_{\text{Bond},t} = \underset{(0.068)}{\mathbf{0.171}} + \underset{(0.028)}{\mathbf{0.078}} \in^2_{\text{Bond},t-1} + \underset{(0.036)}{\mathbf{0.890}} h_{\text{Bond},t-1}$	2.312
Conditional covariance functions	$h_t[\text{REIT, Stock}] = \underset{(0.003)}{\mathbf{0.011}} + \underset{(0.000)}{\mathbf{0.022}} u_{\text{REIT},t-1} u_{\text{Stock},t-1} + \underset{(0.001)}{\mathbf{0.959}} h_{t-1}[\text{REIT, Stock}]$	0.586
	$h_t[\text{REIT, Bond}] = \underset{(0.021)}{0.003} + \underset{(0.000)}{\mathbf{0.022}} u_{\text{REIT},t-1} u_{\text{Bond},t-1} + \underset{(0.001)}{\mathbf{0.959}} h_{t-1}[\text{REIT, Bond}]$	0.159
	$h_t[\text{Stock, Bond}] = \underset{(0.004)}{0.003} + \underset{(0.000)}{\mathbf{0.022}} u_{\text{Stock},t-1} u_{\text{Bond},t-1} + \underset{(0.001)}{\mathbf{0.959}} h_{t-1}[\text{Stock, Bond}]$	0.179
	Implied Sharpe ratios:	0.109 0.087 0.051

Log-Likelihood	2789.933	Number of parameters	26
Akaike information criterion	−12.1492	Saturation ratio	52.50
Bayes-Schwartz information criterion	−11.9137	Jarque-Bera on stdzed res.	121.72***
Hannan-Quinn information criterion	−12.0564	Mult. Ljung-Box(12) autocorrel.	84.900

The table presents MLE coefficients and test statistics for a joint single-state Gaussian VAR(1) DCC GARCH(1,1) model of asset returns. Standard errors are shown in parentheses. Parameter estimates in bold face are significant at 5% confidence. For test statistics, *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

conditional correlation GARCH process for the conditional variances and covariances. As a result, the conditional means in Table 10 are similar to those in Table 9 and therefore close to the sample means. The estimated conditional variance processes all confirm what is typically reported in the ARCH literature on the persistence of conditional variances for asset returns, with sums of ARCH and GARCH coefficients being generally close to 1, from a minimum of 0.96 for stock returns to a maximum of 0.97 for REIT returns. The implied unconditional volatilities (5.063, 4.860 and 2.312 for REIT, stock and bond returns, respectively) are also higher than the corresponding sample standard deviations. Finally, the implied conditional covariance process induces a remarkable degree of persistence (as noticed early on, the corresponding coefficients are restricted to be common across assets). The implied unconditional correlations are 0.586 for REITs with stocks, 0.159 for REITs with bonds and 0.179 for stocks with bonds. Interestingly, while the unconditional correlations between REITs and stocks and between stocks and bonds are close their full-sample sample counterparts (0.594 and 0.154), the implied unconditional correlation between REITs and bonds exceeds the sample estimate (0.106).

Details of the Trivariate Markov Switching Model

The statistical evidence in Table 8 shows that the MS VAR(1) model with regime-dependent variances and covariances ends up dominating the VAR(1) DCC GARCH(1,1) model. Table 11 presents the parameter estimates for our preferred specification, the trivariate four-state Markov regime switching model with heteroskedastic components. To favor comparability with the models featured in Table 9 and 10 and because in Table 8 it is only a model with regime-dependent VAR coefficients that can reduce the values of the Ljiung-Box statistics for levels and squared residuals to insignificant levels, in Table 11 we allow the VAR coefficients to be time varying.

Figure 5 plots the smoothed state probabilities and along with the parameter estimates reported in Table 11 helps us provide the following interpretations. Regime 1 represents the “normal” market environment, characterizing 49.4% of market months. This state is only moderately persistent, with episodes lasting 15.9 months on average, but the estimated transition matrix indicates that this regime is readily entered from Regime 3. During Regime 1 REITs and stocks provide “normal” bull-market returns with relatively low equity volatilities. The unconditional mean returns of 0.706% per month for REITs and 0.772% per month for stocks correspond to 8.81% and 9.67% compounded annual averages; in this environment bonds return 0.472 per month on average or 5.81% compounded annual. While stocks have higher unconditional mean returns than REITs they have correspondingly higher unconditional volatilities (3.608% vs. 2.690%), with the result that the Sharpe ratios for REITs and stocks

Table 11 ■ Trivariate model results: Four-state trivariate VAR(1) Markov switching heteroskedastic model.

Four-State VAR(1) Markov Switching Model				
			Unconditional Means, Vols & Correlations (monthly)	
	Regime 1 ("Normal" Bull State):			
Conditional mean functions	$r_{\text{REIT},t} = \mathbf{0.526} + 0.064 r_{\text{REIT},t-1} + 0.023 r_{\text{Stock},t-1} + \mathbf{0.248} r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.229) (0.053) (0.050) (0.075)		0.706	
	$r_{\text{Stock},t} = \mathbf{0.749} + \mathbf{0.099} r_{\text{REIT},t-1} - 0.073 r_{\text{Stock},t-1} + 0.020 r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.311) (0.059) (0.060) (0.088)		0.772	
	$r_{\text{Bond},t} = \mathbf{0.495} - 0.021 r_{\text{REIT},t-1} - \mathbf{0.071} r_{\text{Stock},t-1} + \mathbf{0.098} r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.168) (0.024) (0.025) (0.046)		0.472	
Conditional variance functions	$h_{\text{REIT},t} = \mathbf{6.648}$ $h_{\text{Stock},t} = \mathbf{12.730}$ $h_{\text{bond},t} = \mathbf{3.650}$	2.690	3.608	1.943
Conditional correlation functions	ρ[REIT, Stock] = **0.655** ρ[REIT, Gov.bond] = **0.413** ρ[Stock, Gov.bond] = **0.495**	0.687	0.440	0.495
	Implied Sharpe ratios	0.075	0.074	−0.017
	Regime 2 ("REIT Premium" Bull State):			
Conditional mean functions	$r_{\text{REIT},t} = 0.667 + 0.064 r_{\text{REIT},t-1} + 0.023 r_{\text{Stock},t-1} + \mathbf{0.248} r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.432) (0.053) (0.050) (0.075)		0.858	
	$r_{\text{Stock},t} = 0.349 + \mathbf{0.099} r_{\text{REIT},t-1} - 0.073 r_{\text{Stock},t-1} + 0.020 r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.408) (0.059) (0.060) (0.088)		0.414	
	$r_{\text{Bond},t} = \mathbf{0.509} - 0.021 r_{\text{REIT},t-1} - \mathbf{0.071} r_{\text{Stock},t-1} + \mathbf{0.098} r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.130) (0.024) (0.025) (0.046)		0.512	
Conditional variance functions	$h_{\text{REIT},t} = 22.\mathbf{586}$ $h_{\text{Stock},t} = \mathbf{22.983}$ $h_{\text{bond},t} = \mathbf{2.714}$	4.930	4.843	1.654
Conditional correlation functions	ρ[REIT, Stock] = **0.483** ρ[REIT, Gov.bond] = −0.064 ρ[Stock, Gov.bond] = **−0.206**	0.502	−0.060	−0.205
	Implied Sharpe ratios:	0.104	0.014	0.101
	Regime 3 ("Surge Market" State):			
Conditional mean functions	$r_{\text{REIT},t} = \mathbf{3.115} + 0.064 r_{\text{REIT},t-1} + 0.023 r_{\text{Stock},t-1} + \mathbf{0.248} r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.663) (0.053) (0.050) (0.075)		3.842	
	$r_{\text{Stock},t} = \mathbf{3.411} + \mathbf{0.099} r_{\text{REIT},t-1} - 0.073 r_{\text{Stock},t-1} + 0.020 r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.803) (0.059) (0.060) (0.088)		3.563	
	$r_{\text{Bond},t} = \mathbf{1.785} - 0.021 r_{\text{REIT},t-1} - \mathbf{0.071} r_{\text{Stock},t-1} + \mathbf{0.098} r_{\text{Bond},t-1} + (h_{\text{REIT},t})^{1/2} u_{\text{REIT},t}$ (0.540) (0.024) (0.025) (0.046)		1.609	
Conditional variance functions	$h_{\text{REIT},t} = \mathbf{11.775}$ $h_{\text{Stock},t} = \mathbf{13.818}$ $h_{\text{bond},t} = \mathbf{10.341}$	3.557	3.754	3.244
Conditional correlation functions	ρ[REIT, Stock] = **0.278** ρ[REIT, Gov.bond] = −0.003 ρ[Stock, Gov.bond] = **0.249**	0.300	0.020	0.495
	Implied Sharpe ratios:	0.937	0.814	0.339

Table 11 ■ Continued

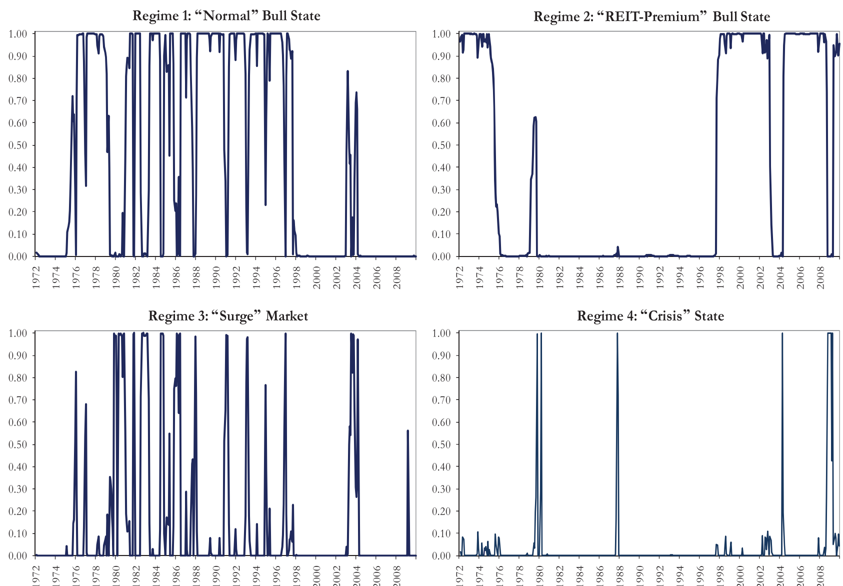
Four-State VAR(1) Markov Switching Model

		Unconditional Means Vols & Correlations (monthly)		
	***Regime 4 ("Crisis State")*:**			
Conditional mean functions	$r_{\mathrm{REIT},t} = -6.096 + 0.064 r_{\mathrm{REIT},t-1} + 0.023 r_{\mathrm{Stock},t-1} + \mathbf{0.248} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$ (4.671) (0.053) (0.050) (0.075)		−0.549	
	$r_{\mathrm{Stock},t} = -\mathbf{5.362} + \mathbf{0.099} r_{\mathrm{REIT},t-1} - 0.073 r_{\mathrm{Stock},t-1} + 0.020 r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$ (3.150) (0.059) (0.060) (0.088)		−0.467	
	$r_{\mathrm{Bond},t} = -0.305 - 0.021 r_{\mathrm{REIT},t-1} - \mathbf{0.071} r_{\mathrm{Stock},t-1} + \mathbf{0.098} r_{\mathrm{Bond},t-1} + (h_{\mathrm{REIT},t})^{1/2} u_{\mathrm{REIT},t}$ (1.118) (0.024) (0.025) (0.046)		0.021	
Conditional variance functions	$h_{\mathrm{REIT},t} = \mathbf{246.867}$ $h_{\mathrm{Stock},t} = \mathbf{85.071}$ $h_{\mathrm{bond},t} = \mathbf{18.417}$	16.302	9.361	4.324
Conditional correlation functions	ρ[REIT, Stock] = 0.807 ρ[REIT, Gov.bond] = 0.015 ρ[Stock, Gov.bond] = − 0.166	0.826	0.020	−0.162
	Implied Sharpe ratios:	−0.052	−0.081	−0.063

Estimated transition matrix	Regime 1	Regime 2	Regime 3	Regime 4	Ergodic Probs.
Regime 1	**0.937**	0.011	0.048	0.004	0.494
	(0.044)	(0.007)		(0.009)	
Regime 2	0.013	**0.973**	0.000	0.014	0.328
	(0.066)	(0.081)		(0.011)	
Regime 3	**0.194**	0.002	0.755	**0.049**	0.138
	(0.033)	(0.009)		(0.011)	
Regime 4	0.086	**0.086**	0.247	**0.664**	0.040
	(0.035)	(0.035)		(0.148)	
Log-Likelihood	2906.581	Number of parameters		57	
Akaike information criterion	−12.5192	Saturation ratio		24.00	
Bayes-Schwartz information criterion	−11.9899	Joint Jarque-Bera on stdzed res.		12.691	
Hannan-Quinn information criterion	−12.3107	Mult. Ljung-Box(12) autocorrel.		130.17*	

The table reports MLE estimates for a MSVAR(1) with regime-dependent variances and correlations. The rightmost column reports unconditional means, volatilities, correlations and Sharpe ratios implied by the model conditional in each of the four possible regimes. The Sharpe ratios are computed with reference to the average 1-month T-bill yield. Unconditional moments are computed under the (counter-factual) assumption that the system never leaves a given state and they should be interpreted with caution. The elements in the last column of the estimate Markov transition matrix are not associated to standard errors because of the presence of an adding-up constraint in estimation (by which the elements in each of row of the transition probability must sum to a total probability of 1). The joint Jarque-Bera test is computed using a Doornik-Hansen type square root of correlation matrix transformation to compute standardized residuals which are independent of the ordering of the asset return variables in the estimated vector system. In the table, boldfaced coefficients are significant at 5% or lower size.

Figure 5 ■ Smoothed state probabilities from four-state Markov switching VAR(1) heteroskedastic model.



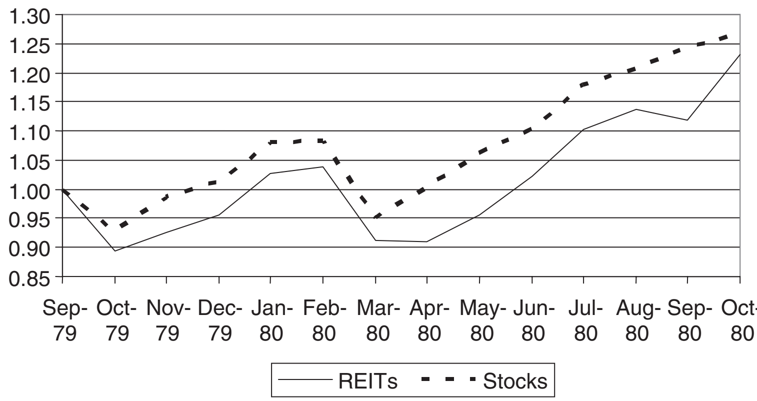
are essentially identical (0.075 vs. 0.074). Bonds are less volatile than equities with an unconditional volatility of 1.943%, but they actually produce a negative Sharpe ratio during the "normal" bull-market environment. The conditional correlation between REITs and stocks is surprisingly high at 65.5% during Regime 1, and both equity assets also have high correlations with bonds at 41.3% and 49.5%, respectively.

Regime 2 represents another bull-market environment, but one with higher (that is, more "normal") volatilities for both REITs and stocks, higher returns for REITs (but lower returns for stocks) and lower correlations among all three asset classes. This regime characterizes 32.8% of market months: episodes are quite persistent, lasting 37.0 months on average, and this state is readily entered from Regime 4 but—partly because Regime 4 is the least common market environment—is the least frequently entered, with just five episodes (totaling 171 months) witnessed during the 38-year study period. Unconditional mean returns are 0.858% per month (10.80% per year) for REITs compared to just 0.414% per month (5.08% per year) for stocks, with bonds at 0.512% per month (6.32% per year). Even with a higher unconditional volatility, the higher returns for REITs produce an unconditional Sharpe ratio of 0.104 in Regime 2 (slightly greater than the figure of 0.101 for bonds), while stocks—with lower returns as well as higher volatility—see an implied unconditional Sharpe ratio of just 0.014. The conditional REIT-stock correlation is 48.3%, with equity-bond correlations not significantly different from zero for REITs and –20.6% for stocks.

Regime 3 represents a dramatic "surge" regime in which all three asset classes produce spectacular returns with moderate volatilities and low inter-asset correlations. This "investor's dream" environment is surprisingly common, characterizing 13.8% of market months—but not persistent, averaging just 4.1 months in duration, though readily entered from both Regimes 1 and 4. This implies that, even if investors quickly realize that a surge regime has begun, the surge may not persist long enough for investors to benefit by reacting to it. Still, during the study period we have witnessed 17 episodes of Regime 3 (totaling 64 months), equal in entry frequency (though not in total months) to the "normal" market environment. Unconditional mean returns in Regime 3 are stunning at 3.842 per month (57.2% per year annualized with compounding) for REITs, 3.563 per month (52.2% per year annualized with compounding) for stocks and 1.609% per month (21.1% per year annualized with compounding) for bonds. With volatilities higher than in the "normal" market environment but lower than in Regime 2 (or Regime 4), the implied unconditional Sharpe ratios are equally stunning at 0.937 for REITs, 0.814 for stocks and 0.339 for bonds. The conditional correlation between REITs and stocks is low at only 27.8%, with no correlation between REITs and bonds and only 24.9% correlation between stocks and bonds.

Finally, Regime 4 presents the opposite scenario: an "investor's nightmare" state in which equity investments produce dramatically negative and highly correlated returns and all asset classes exhibit spectacularly higher volatility. Fortunately this "crisis" environment characterizes only 4.0% of market months, lasts only 3.0 months on average, is readily entered only from Regime 3, and has been seen only six times during the 38-year study period for a total of 11 months—six of which were during the recent period October 2008–April 2009. Unconditional mean equity returns during Regime 4 are horrendous at –0.549 per month (–6.39% per year annualized with compounding) for REITs and –0.467 per month (–5.46% per year annualized with compounding) for stocks; bonds provide an illusory "safe haven" with unconditional mean returns of 0.021% per month (0.25% per year annualized with compounding), but Sharpe ratios are negative for all three asset classes and conditional variances are spectacular. Furthermore, Regime 4 displays the lamented "crisis spike" in equity correlations, with the conditional correlation between REITs and stocks estimated at 80.7%; REIT-bond and stock-bond correlations, though, are not significantly different from zero.

Figure 6 ■ Alternating regimes, September 1979–October 1980.



Note the important implication of the transition probabilities between Regimes 3 and 4: the markets sometimes alternate between brief periods of dramatically high and dramatically low returns, with a "rebound" typically (about one-fourth of the time) following a crisis but occasionally (about one-twentieth of the time) preceding it. Figure 6 illustrates an example, showing REIT and stock returns from September 1979 through November 1980. REIT returns posted "crisis" returns of –10.6% in October 1979 followed by three months of "surge" gains totaling +14.9%; in March 1980 a second "crisis" with returns of –12.2% was then followed by a second "rebound" with gains totaling +35.0%. Stocks showed a similar pattern, with returns of –7.3% in the crisis of October 1979, +16.5% in the rebound of November 1979–January 1980, –12.0% in the crisis of March 1980 and +33.1% in the rebound of April–October 1980.

On the other hand, the graphs in Figure 5 showing the smoothed state probabilities emphasize that most market oscillations are between Regimes 1 and 3 (the "normal" environment and the "surge" environment). Indeed, given the frequency with which Regime 3 is entered, the ergodic probabilities imply that (independent of the current regime identification) any investor who stays out of the REIT or stock market for as few as five months is more than likely to miss at least one episode of spectacularly high returns. In contrast, (again, independent of the current regime identification) an investor would have to stay out of the market for 17 months to have a better than even chance of avoiding the rarely entered Regime 4 "crisis" market state.

It is worth noting that in a Markov regime switching framework the overall variance of monthly returns for a given asset class reflects both the variance of asset returns within a given market environment and the switching behavior across market environments; similarly, the overall correlation between two asset classes reflects both correlation of asset returns within a given environment and switching across environments. While this holds true for mean returns as well, the overall mean return can be estimated simply as the weighted average of implied unconditional mean returns across regimes, with the ergodic probabilities as weights. In contrast, overall variances and correlations can be estimated only by simulating the dynamic behavior of the system, a task that we intend to address in a subsequent paper.

To summarize, the trivariate four-state Markov switching model with heteroskedastic components and autoregressive terms identifies market environments that differ markedly from each other, distinguished by trivariate return densities that can most easily be summarized with reference to different levels of returns, different levels of volatility and different correlations among asset classes. The matrix of transition probabilities, and the dynamic conditional functions describing returns, variances and correlations, provide the basis for developing investment decision-making rules that would potentially enable investors to profit from dynamic asset allocation strategies based on the regime switching framework.

Robustness of the Trivariate Markov Switching Model

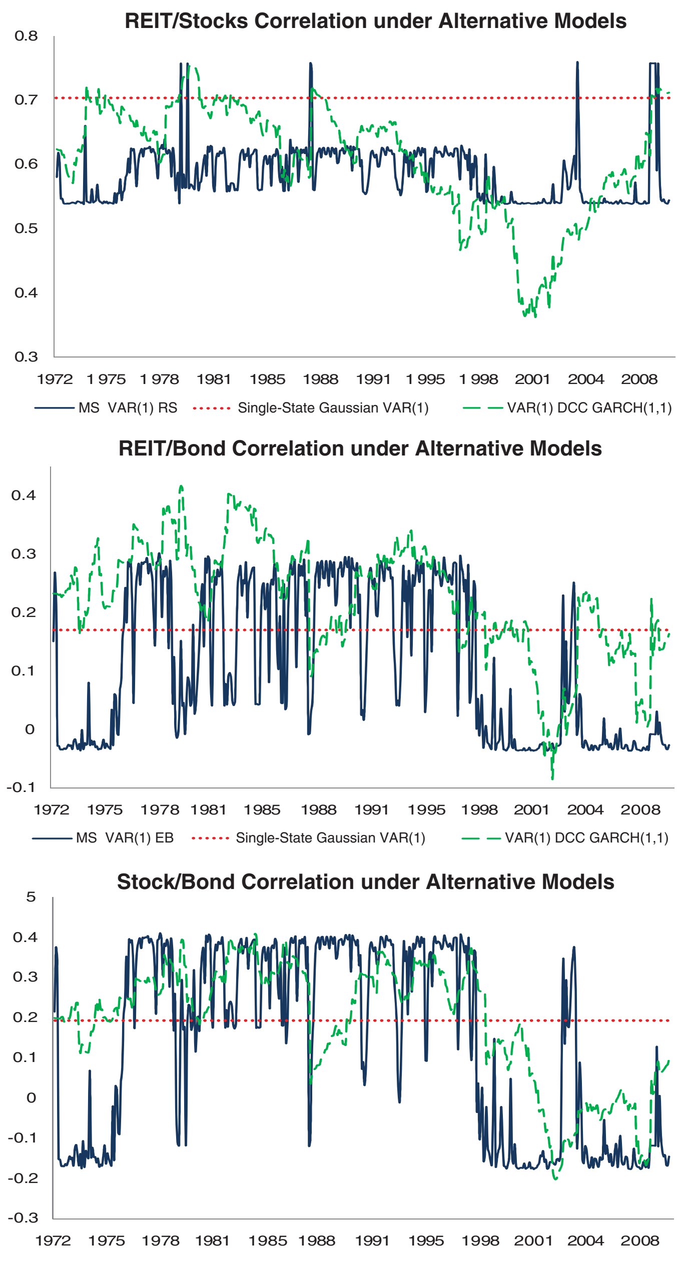
In this section we conduct additional analyses to test the robustness of the four-state Markov switching model. We address three questions in particular.

Predictive Correlation Accuracy

The first questions relates to the strong time variation in REIT-stock correlations observed by Case, Yang and Yildirim (2012), with a sustained decline from late 1991 through September 2001 followed by a sustained increase from September 2001. The three plots in Figure 7 each display conditional correlations predicted by three of the models employed in this article; because of its obvious importance in defining REITs as an asset class, we focus on REIT-stock correlations.[11] As the top graph shows, the single-state Gaussian

[11] Similar comments apply to the plots of correlations between REITs and bonds and stocks and bonds, although for these two additional pairs of assets the departures between single-state DCC GARCH and Markov switching forecasts are moderate and essentially indicating lower estimated correlations under the four-state model than under the single-state model for the periods 1972–1974, 1999–2001 and 2004–2007.

Figure 7 ■ One-step ahead predicted correlations under alternative models.



VAR(1) model generates a constant prediction of 70.3% correlation, as one would expect of any homoskedastic model. The VAR(1) DCC-GARCH(1,1) model employed by Case, Yang and Yildirim generates the pattern of steadily declining correlations during the 1990s, with a minimum value of 36.3% in September 2011, following by steadily increasing correlations to 64.6% in September 2008 followed by a "crisis spike" to as high as 71.6%. Finally, the four-state Markov switching model generates predicted correlations in four general ranges:

- The "crisis" correlation of about 76% is predicted for the five crisis periods (Regime 4) identified in the historical period: October 1979, March 1980, October–November 1987, April 2004 and October 2008–April 2009 (except for March 2009, for which the state probability of the "crisis" regime was 43% compared to 56% for the "REIT-premium" regime).
- The lowest range of correlations, at about 54%, was predicted generally during the "REIT-premium" regimes (Regime 2) that prevailed in five periods: from the beginning of the study period through July 1975, June-September 1979, September 1997–December 2002, May 2004–September 2008 and May 2009 through the end of our sample.
- Another low range of correlations, at about 56%, prevailed during the "surge" or "investor's dream" states (Regime 3), which happened frequently but generally with short duration, the most notable episodes being November 1979–December 1980 (except March 1980), July 1982–April 1983, November 1985–June 1986, November 1990–March 1991 and May-November 2003.
- Finally, a higher range of correlations, at about 62%, prevailed during "normal" bull markets (Regime 1), which alternated frequently with Regime 3 ("surge" or "investor's dream" states) but predominated from August 1975 through August 1997.

Given the marked differences between the REIT-stock correlations predicted by the three modeling approaches, we evaluated their predictive accuracy by computing root mean squared forecast errors for each pair of return covariances. It is important to assess whether these in-sample differences are borne out by the actual subsequent behavior of stock and REIT returns, in this case taken in pairs to measure monthly realized correlation. Not surprisingly, the single-state VAR(1) model generally produced the largest root mean squared forecast error of 0.4487% for REIT-stock covariances (along with 0.1781% for REIT-bond covariances and 0.1567% for stock-bond covariances). The DCC-GARCH model performed substantially better at predicting REIT-stock and stock-bond covariances (0.4273% and 0.1549%, respectively), although surprisingly slightly worse at predicting REIT-bond covariances (0.1796%).

The MS model performed quite significantly better with root mean squared forecast errors of just 0.4033% for REIT-stock covariances and 0.1535% for stock-bond covariances. For REIT-bond covariances, however, the four-state model was only slightly better than the single-state VAR(1) model, with a RMSFE of 0.1779%. In spite of the presence of some interesting patterns across the relative predictive performances of the different models, the fact that for all pairs the Markov switching model turned out to be more accurate than single-state frameworks lends credibility to the patterns displayed in Figure 7.

Asset Pricing Implications: Relationship to Fama–French Factors

The second question we address, raised also by Guidolin and Timmermann (2006), concerns whether there is any relationship between expected returns under the four-state trivariate Markov switching model and the factors found to affect stock and bond returns by Fama and French (1993) and others. To investigate this issue we estimated seven specifications of a model that related expected returns from each of the asset classes—REITs, stocks and bonds—under the four-state Markov switching model to various combinations of the Fama–French factors found to predict cross-sectional variation of stock or bond returns: besides the standard CAPM-style excess returns on the market portfolio, market capitalization/size (as proxied by the SMB portfolio returns), book-to-market value (HML portfolio returns), momentum (as proxied by a portfolio long in momentum winners and short in momentum losing stock), the dividend yield, the default spread and the term spread.[12]

Table 12 summarizes the results from this analysis. As the coefficients indicate, the Fama–French factors do indeed help to explain the expected returns under the Markov switching model—especially HML (significant in all specifications for REIT and stock returns) and the dividend yield (significant in all specifications for all three asset classes). On the other hand, the R^2 values are quite small, reaching a high of just 0.59 (*i.e.*, 59%) for the least parsimonious model estimated on bond returns. In fact, the R^2 values are actually considerably lower for REITs (highest R^2 is 0.44) and especially stocks (0.39). The implication is that the presence of other factors related to the business cycle cannot entirely explain the dynamics in asset returns that are captured by the

[12]The factors have standard definitions common in the literature. Returns on the market portfolio (defined as the value-weighted returns on all stocks traded on the NYSE, the NASDAQ and AMEX), SMB and HML have been downloaded from Ken French's web site at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. The dividend yield is defined as the ratio between trailing 12-month moving average dividends paid on all NYSE stocks and the total NYSE capitalization 12 months before; the default spread is the difference between Baa and Aaa closest to 10-year Moody's corporate bond portfolio yields; the term spread is the difference between yields on 10- and on 3-month U.S. government securities.

Table 12 ■ Explaining Markov switching expected returns using standard linear factor models.

Specifications	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: REIT Expected Returns from Four-State MSVAR(1) Model							
Variable							
Excess Market Return	**0.1215**	**0.1397**	**0.1490**		**0.1186**	**0.1369**	**0.1462**
	(9.91)	(10.33)	(11.31)		(9.96)	(10.53)	(11.49)
SMB Returns		0.0044	0.0051			0.0018	0.0018
		(0.39)	(0.50)			(0.15)	(0.17)
HML Returns		**0.0827**	**0.0976**			**0.0802**	**0.0952**
		(4.46)	(5.50)			(4.51)	(5.54)
Momentum Returns			**0.0400**				**0.0395**
			(3.74)				(3.84)
Dividend Yield				**0.1670**	**0.1456**	**0.1397**	**0.1243**
				(3.11)	(3.47)	(3.65)	(3.57)
Default Spread				−0.1224	−0.1249	−0.0953	−0.0289
				(–0.59)	(–0.81)	(–0.70)	(–0.23)
Term Spread				**0.0866**	**0.0589**	**0.0496**	**0.0540**
				(2.16)	(2.04)	(1.79)	(2.03)
Constant	**0.0090**	**0.0086**	**0.0082**	**0.0041**	**0.0048**	**0.0043**	**0.0036**
	(22.81)	(20.85)	(18.80)	(2.76)	(3.78)	(3.66)	(3.00)
Observations	455	455	455	455	455	455	455
R^2	0.32	0.38	0.41	0.05	0.36	0.41	0.44
Panel B: Stock Expected Returns from Four-State MSVAR(1) Model							
Variable							
Excess Market Return	**0.0333**	**0.0460**	**0.0500**		**0.0290**	**0.0416**	**0.0444**
	(3.09)	(4.15)	(4.49)		(2.85)	(4.27)	(4.49)
SMB Returns		**0.0498**	**0.0501**			**0.0470**	**0.0470**
		(4.57)	(4.53)			(4.88)	(4.88)
HML Returns		**0.0951**	**0.1015**			**0.0906**	**0.0952**
		(5.73)	(6.03)			(6.55)	(6.73)
Momentum Returns			**0.0172**				0.0121
			(2.32)				(1.85)
Dividend Yield				**0.2555**	**0.2503**	**0.2429**	**0.2382**
				(7.02)	(7.54)	(9.48)	(9.15)
Default Spread				**−0.2986**	**−0.2992**	**−0.2842**	**−0.2638**
				(–2.16)	(–2.43)	(–3.23)	(–2.86)
Term Spread				**0.0961**	**0.0893**	**0.0738**	**0.0751**
				(3.84)	(3.94)	(3.71)	(3.94)
Constant	**0.0079**	**0.0074**	**0.0072**	**0.0016**	**0.0018**	**0.0016**	**0.0014**
	(23.15)	(21.79)	(20.81)	(1.73)	(1.87)	(1.92)	(1.59)
Observations	455	455	455	455	455	455	455
R^2	0.05	0.21	0.23	0.20	0.23	0.38	0.39

Markov switching model. Although this result is similar to the one reported by Guidolin and Timmermann (2006) for stock and bond portfolios, it is interesting that this extends to REITs. This means that nonlinear dynamics plays an important role for the asset classes under examination, so that important components are unlikely to be captured by simple linear factor models that essentially resemble the regressions that have been estimated on the Markov switching-implied returns.[13]

[13]For instance, Guidolin and Timmermann (2008) have shown how Markov switching in a linearized stochastic discount factor leads to linear asset pricing models in which

Table 12 ■ Continued

Specifications	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel C: Long-Term Bond Expected Returns from Four-State MSVAR(1) Model							
Variable							
Excess Market Return	**−0.0654**	**−0.0640**	**−0.0610**		**−0.0669**	**−0.0652**	**−0.0616**
	(–14.49)	(–13.72)	(–12.63)		(–15.83)	(–15.14)	(–14.16)
SMB Returns		**−0.0199**	**−0.0197**			**−0.0227**	**−0.0227**
		(–4.53)	(–4.87)			(–4.88)	(–5.56)
HML Returns		−0.0098	−0.0050			−0.0104	−0.0044
		(–1.79)	(–0.99)			(–1.84)	(–0.91)
Momentum Returns			**0.0130**				**0.0156**
			(3.31)				(3.74)
Dividend Yield				0.0249	**0.0370**	**0.0381**	**0.0321**
				(1.09)	(2.60)	(2.77)	(2.67)
Default Spread				0.1047	0.1061	**0.1115**	**0.1378**
				(1.27)	(1.98)	(2.22)	(3.22)
Term Spread				0.0076	**0.0232**	**0.0270**	**0.0287**
				(0.46)	(2.03)	(2.50)	(2.70)
Constant	**0.0066**	**0.0067**	**0.0066**	**0.0042**	**0.0039**	**0.0038**	**0.0035**
	(46.29)	(45.88)	(42.66)	(6.58)	(8.13)	(8.06)	(7.47)
Observations	455	455	455	455	455	455	455
R^2	0.49	0.52	0.53	0.03	0.54	0.57	0.59

Note : Robust *t*-statistics in parentheses below coefficients.

The table presents regressions of the one-month-ahead expected returns for each asset class on a number of asset pricing factors commonly used in the asset pricing literature: excess market returns (computed as the difference between value-weighted returns on all stocks traded on the NYSE, NASDAQ and AMEX and 1-month Treasury bills), returns on Fama and French's SMB portfolio, returns on Fama and French's HML portfolio, returns on Fama and French's momentum portfolio (long in past 12-month winner stocks and short in past 12-month loser stocks), the dividend yield (defined as the 12-month trailing average of NYSE dividends and the NYSE market capitalization as of t – 12), the default spread (Baa-Aaa yields from Moody's corporate bond portfolios) and the term spread(10-year minus 3-month government bond yields). Robust standard errors are reported in parentheses below the estimated least squares coefficients.

An Alternative Model with Time-Varying GARCH: Markov Switching CCC

The final question that we address is motivated by the observation that, although the superior performance of the MS model suggests the importance of modeling structural change, perhaps incorporating such time-varying parameters into a GARCH framework would provide even stronger modeling performance.[14] In order to investigate this question we estimated the MS dynamic conditional correlation model proposed by Pelletier (2006) as an extension of Bollerslev's

not only the (conditional) covariance with the priced risk factors, but also (conditional) co-skewness and co-kurtosis involving such factors will be priced in the absence of arbitrage opportunities.

[14]For instance, it is well-known that the performance of standard GARCH models can be adversely affected by the presence of structural instability; the observed high persistence in the volatility process may be caused by unaccounted structural breaks (see, *e.g*., Lamoureux and Lastrapes 1990). Besides the MS GARCH option explored in the text, several other modeling approaches should be explored in future research, for instance using smooth transition models (see, *e.g*., Amado and Teräsvirta 2008) or the Spline-GARCH (see, *e.g*., Engle and Rangel 2008).

(1990) constant conditional correlation (CCC) multivariate framework to incorporate Markov switching dynamics in the conditional variance and covariance functions.[15] As in a standard CCC model, Pelletier's regime switching dynamic correlation (RSDC) model decomposes the covariances into standard deviations and correlations, but these correlations are allowed to change over time as they follow a Markov process:

$$r_t = H_t^{1/2}\varepsilon_t \quad \varepsilon_t \mathrm{IID} N(0, I_3) \quad \mathrm{H}_t = D_t \Gamma_{S_t} D_t,$$

where D_t is a diagonal matrix composed of the standard deviations of the asset return series and the regime-dependent matrix Γ_{S_t} contains the correlations, which are assumed to be constant within a regime S_t but different across regimes. If we call $K \geq 2$ the number of regimes allowed, this feature implies that in the evaluation of the likelihood, the correlation matrix can only take K possible values so we have to invert K times a 3×3 matrix (we have three portfolios), which can be a computational advantage over models such as a standard CCC, where a different correlation matrix has to be inverted for every observation. Pelletier shows that his RSDC model has many interesting properties. First, it is easy to impose that the variance matrices are semi-positive definite. Second, it does not suffer from the curse of dimensionality because it can be estimated with a two-step procedure. Finally, by modeling time variation in correlations as a Markov switching process, the variances and covariances are not bounded, which is instead the case when they are the ones following a simpler regime switching model. Estimation is made simpler by adopting a two-step quasi-ML estimation procedure as in Engle (2002): in a first step, we can estimate the univariate volatility models and, in a second step, we can estimate the parameters in the correlation matrix conditional on the first step estimates.

Table 13 presents the results from a four-state trivariate Markov switching VAR(1) CCC-GARCH(1,1) model. Note that the conditional mean functions are identical to those shown in Table 10, by construction: that is, the VAR(1) model is estimated before fitting the conditional heteroskedasticity model to the residuals. Note also that the switching model reveals less volatility persistence than is suggested by the single-state DCC-GARCH model shown in Table 10. For instance, in the case of REIT returns, the sum of the GARCH coefficients in Table 10 was 0.97 and it becomes now 0.79. This is typical of switching versions of ARCH models: the more regime-switching is likely, the less strong

[15] Pelletier's MS DCC seems to be one of the most advanced multivariate regime switching GARCH models proposed in the literature. Older variations based on simpler Markov switching ARCH structures are reviewed in Guidolin (2011). However, Pelletier (2006) reports that his novel framework outperforms standard DCC models as well as simpler MS ARCH models.

Table 13 ■ Trivariate model results: Four-state VAR(1) Markov switching DCC GARCH(1,1).

Four-State VAR(1) CCC GARCH(1,1) Model		
	Full Sample	*Unconditional Means, Vols and Correlations (monthly)*
Conditional mean functions	$r_{\text{REIT},t} = \underset{(0.197)}{\mathbf{0.700}} + \underset{(0.060)}{0.007} r_{\text{REIT},t-1} + \underset{(0.048)}{\mathbf{0.114}} r_{\text{Stock},t-1} + \underset{(0.095)}{\mathbf{0.353}} r_{\text{Bond},t-1} + \in_{\text{REIT},t} \quad \in_{\text{REIT},t} \sim N(0, (H_t)^{1/2})$	1.013
	$r_{\text{Stock},t} = \underset{(0.216)}{\mathbf{0.676}} + \underset{(0.051)}{\mathbf{0.093}} r_{\text{REIT},t-1} - \underset{(0.060)}{0.013} r_{\text{Stock},t-1} + \underset{(0.090)}{\mathbf{0.218}} r_{\text{Bond},t-1} + \in_{\text{REIT},t} \quad \in_{\text{Stock},t} \sim N(0, (H_t)^{1/2})$	0.885
	$r_{\text{Bond},t} = \underset{(0.103)}{\mathbf{0.593}} - \underset{(0.023)}{\mathbf{0.035}} r_{\text{REIT},t-1} - \underset{(0.025)}{\mathbf{0.065}} r_{\text{Stock},t-1} + \underset{(0.049)}{\mathbf{0.139}} r_{\text{Bond},t-1} + \in_{\text{REIT},t} \quad \in_{\text{Bond},t} \sim N(0, (H_t)^{1/2})$	0.581
Conditional variance functions	$h_{\text{REIT},t} = \underset{(0.584)}{\mathbf{4.957}} + \underset{(0.037)}{\mathbf{0.125}} \in^2_{\text{REIT},t-1} + \underset{(0.150)}{\mathbf{0.662}} h_{\text{REIT},t-1}$	4.824
	$h_{\text{Stock},t} = \underset{(0.583)}{\mathbf{6.384}} + \underset{(0.040)}{\mathbf{0.206}} \in^2_{\text{Stock},t-1} + \underset{(0.095)}{\mathbf{0.552}} h_{\text{Stock},t-1}$	5.136
	$h_{\text{Bond},t} = \underset{(0.257)}{\mathbf{1.780}} + \underset{(0.048)}{\mathbf{0.193}} \in^2_{\text{Bond},t-1} + \underset{(0.104)}{\mathbf{0.484}} h_{\text{Bond},t-1}$	2.348

Table 13 ■ Continued

Four-State VAR(1) CCC GARCH(1,1) Model

	Full Sample				*Unconditional Means, Vols and Correlations (monthly)*
Conditional correlations					
Regime 1	ρ_t[REIT, Stock] = **0.846** (0.140)	ρ_t[REIT, Gov.bond] = −0.074 (0.105)	ρ_t[Stock, Gov.bond] = −0.162 (0.085)		
Regime 2	ρ_t[REIT, Stock] = **0.498** (0.076)	ρ_t[REIT, Gov.bond] = **0.405** (0.145)	ρ_t[Stock, Gov.bond] = −0.183 (0.096)		
Regime 3	ρ_t[REIT, Stock] = **0.685** (0.094)	ρ_t[REIT, Gov.bond] = **0.234** (0.122)	ρ_t[Stock, Gov.bond] = **0.547** (0.075)		
Regime 4	ρ_t[REIT, Stock] = **0.316** (0.111)	ρ_t[REIT, Gov.bond] = **0.240** (0.065)	ρ_t[Stock, Gov.bond] = **0.268** (0.071)		
Estimated transition matrix				Implied Sharpe ratios:	0.127 0.094 0.069
	Regime 1	Regime 2	Regime 3	Regime 4	Ergodic Probs.
Regime 1	**0.613** (0.168)	**0.095** (0.038)	0.004 (0.019)	0.288	0.065
Regime 2	0.011 (0.015)	**0.964** (0.131)	0.023 (0.068)	0.002	0.518
Regime 3	**0.032** (0.011)	0.015 (0.010)	**0.879** (0.126)	0.074	0.310
Regime 4	**0.090** (0.045)	**0.073** (0.019)	**0.238** (0.047)	0.599	0.107
Log-Likelihood	2803.005	Number of parameters		45	
Akaike information criterion	−12.2256	Saturation ratio		30.40	
Bayes-Schwartz information criterion	−11.8590	Jarque-Bera on stdzed res.		48.671***	
Hannan-Quinn information criterion	−12.0661	Mult. Ljung-Box(12) autocorrel.		81.802	

The table reports MLE estimates for a MSVAR(1) with regime-dependent variances and correlations modeled through a DCC GARCH(1,1) as in Pellettier (2006). The rightmost column reports unconditional means, volatilities, correlations and Sharpe ratios implied by the model conditional in each of the four possible regimes. The Sharpe ratios are computed with reference to the average 1-month T-bill yield. Unconditional moments are computed under the (counter-factual) assumption that the system never leaves a given state and they should be interpreted with caution. The elements in the last column of the estimate Markov transition matrix are not associated to standard errors because of the presence of an adding-up constraint in estimation. The joint Jarque-Bera test is computed using a Doornik-Hansen type square root of correlation matrix transformation to compute standardized residuals which are independent of the ordering of the asset return variables in the estimated vector system. In the table, boldfaced coefficients are significant at 5% or lower size.

is the evidence of ARCH effects, at least using monthly data such as the series that we use.

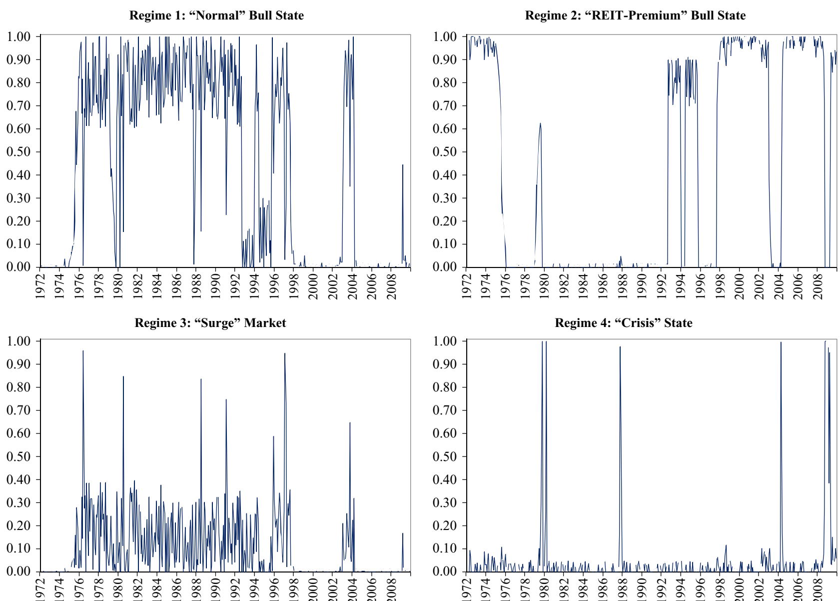
The conditional correlations estimated with the MS CCC-GARCH model are generally quite similar to those estimated using the simpler MS model. For example, the REIT-stock implied unconditional (ergodic) correlations are 68.5% in Regime 1 (compared to 65.5% in the simple MS model), 49.8% in Regime 2 (compared to 48.3%), 31.6% in Regime 3 (27.8%) and 84.6% in the "crisis" Regime 4 (80.7%). The exceptions are the REIT-bond correlations in Regimes 2 and 3, which are much higher (40.5% and 24.0%, respectively) in the MS CCC-GARCH model than in the simpler MS model (–6.4% and –0.3%, in each case not significantly different from zero). Relative to the simpler MS model, however, the MS CCC-GARCH model implies much greater ergodic probability in Regime 2 (the "REIT-premium" market) at 51.8% compared to 32.8%, and much less ergodic probability in Regime 1 (the "normal" bull market) at 31.0% compared to 49.4%.

The log-likelihood of the MS CCC-GARCH model (2803.005) is greater than the corresponding measure for the single-state DCC-GARCH model shown in Table 10 (2789.933), but this improvement comes at the cost of a large increase in the number of parameters, from 26 to 45, and therefore a large decline in the saturation ratio. As a result, while the Akaike information criterion improves (from –12.1492 to –12.2256), the Hannan–Quinn information criterion is essentially unchanged (from –12.0564 to –12.0661) and the Bayes–Schwartz information criterion actually deteriorates (from –11.9137 to –11.8590). Compared to the simple MS model shown in Table 11, the MS CCC-GARCH model is inferior according to the log-likelihood and all three information criteria; moreover, the standardized residuals from the MS CCC-GARCH model still display signs of nonnormality (Jarque-Bera = 48.671 and significant), while this was not the case with the simple MS model (Jarque-Bera = 12.691 and insignificant).

Finally, Figure 8 shows the smoothed state probabilities implied by the MS CCC-GARCH model. There are as many as 378 months (out of a total of 455 observations) during which no regime had a state probability greater than 0.99, an uncertainty that on the opposite occurred over just 160 months according to the smoothed state probabilities implied by a simple MS model with no GARCH effects (Figure 5). Indeed, there are 46 months during which no state has a probability greater than 0.99 or less than 0.01, a mixed outcome that happened only for six months with our simpler model.[16] Given the uncertainty

[16]This weak regime identification is likely to derive from the fact that in Table 13, Regimes 3 and 4 are very similar in terms of the parameter estimates they imply .Equiv-

Figure 8 ■ Smoothed state probabilities from four-state Markov switching VAR(1) CCC GARCH(1,1).



of regime identification, along with the deterioration in measures of model performance, we conclude that at least this version of a regime switching correlation model does not repay its additional complexity. Although the in-sample evidence of GARCH effects in Table 13 survives the addition of a MS component in correlations, it does not appear to be fruitful to perform closer comparisons between either the four-state MS VAR(1) CCC GARCH and the four-state MS VAR(1) or the single-state VAR(1) DCC GARCH because the former model estimated in this additional exercise does not seem to be correctly specified. Of course, it is possible that additional (possibly, more flexible) multivariate models that allow ARCH coefficients to be time varying may yield more compelling evidence. We leave this extension of future research.

alently, this may be an indication of the need for fewer than four regimes. However, internal consistency within our own research design requires estimating a number of regimes identical to the model proposed in Table 10. Unreported regime classification measures confirm that the model in Table 13 suffers from considerable regime identification problems.

Summary and Implications

The empirical results indicate that Markov regime switching techniques may be more successful than even rather complex ARCH specifications in modeling the rich dynamic time-series properties of returns on REITs, non-REIT stocks and bonds during the period 1972–2009. In a Markov switching model the conditional means, variances and covariances depend on an unobservable Markov state variable that can assume a finite number of values characterizing the state of the financial market at each time. Univariate versions of the Markov model suggest that REITs and non-REIT stocks can all be described successfully with two states: "bull-market" states in REITs and stocks characterized by high conditional returns and low conditional volatility and "bear-market" states in the same two equity assets characterized by low conditional returns and high conditional volatilities. Bond returns, too, can be described with two states: a low-return, low-volatility "expansionary" state and a high-return, high-volatility "recessionary" state.

These univariate models suggest that the existence of separate regimes with different conditional expectations and variances should be taken into account in evaluating and managing portfolio risks. Measures of association, however, indicate that while the two regimes are generally in synch for REITs with stocks and for stocks with bonds, they are not synchronized for REITs with bonds. This lack of synchronization suggests that successful risk management must take into consideration up to $2 \times 2 \times 2 = 8$ combinations of regimes for the three assets considered. Further empirical analysis has shown that for practical purposes, simpler models with four regimes do provide a better fit to equity REIT, stock and bond return data than multivariate MS conditional correlation GARCH models do.

One regime represents a "typical" regime with normal returns and relatively low volatilities for stocks and REITs, relatively low returns on bonds and relatively high correlation between the two equity asset classes. A second regime represents a "REIT-premium" state with much higher returns for REITs than for non-REIT stocks, relatively high returns for bonds and relatively low correlations among all asset classes. A third regime represents an "investor's dream" with spectacularly high returns for all three assets (especially REITs and stocks) and dramatically lower conditional correlations, while the fourth regime represents an "investor's nightmare" with spectacularly low returns, especially for the two equity asset classes, along with a dramatic spike in equity asset correlations. Although these findings are certainly new in the literature on the dynamic risk-return properties for REITs, they can be compared to related results reported by Guidolin and Timmermann (2006, henceforth GT) with reference to small stocks, large stocks and bonds. On the one hand, if one assimilates REITs to small stocks (as their sheer market capitalization suggests should be done), the finding of four regimes may be somewhat unsurprising. On the other hand, some interesting differences also exist. For instance, while GT isolate two extreme regimes similar to the dream and nightmare regimes reported here, no regime picks up the "REIT premium" features isolated in our empirical findings.[17] Moreover, our results show a much higher persistence of all regimes and especially the extreme (crash and "dream" ones) when compared to the findings in GT. In principle this ought to make results easier to exploit in risk management and portfolio choice applications.

While the empirical results suggest the possibility of sharply improved risk management from recognition of the underlying state-specific conditional expected asset return moments, it must be recognized that the results shown in this article represent only the characterization of historically realized returns. For the purposes of risk management and dynamic portfolio optimization, it will be important to determine whether the modeling structure developed in this article remains successful in out-of-sample validation tests. In particular, the authors intend to use the multivariate multistate Markov regime switching framework to predict transitions among the four regimes identified, and to perform dynamic portfolio optimization on the basis of the predicted regime-specific conditional return moments. The development of successful trading strategies, of course, will depend not only on the success of predictive models but also on the implied cost of implementing such dynamic portfolio optimization signals.

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[17]GT do notice that their third regime shows high (excess) mean returns for small caps, but this is accompanied by higher regime-dependent volatility. Instead, as we have discussed, in the case of REITs the second regime implies a monthly Sharpe ratio seven times higher than the ratio for stocks.

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