# Pricing Kernels and Risk Premia implied in Bitcoin Options

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21 November 2022

**Abstract**

Bitcoin Pricing Kernels (PK) are estimated using a novel data set from Deribit, the leading Bitcoin options exchange. The PKs, as the ratio between risk-neutral and physical density, dynamically reflect the change of investor preferences. Thus the PKs improve the understanding of investor expectations and risk premium in a new asset class. Bootstrap-based confidence bands are estimated in order to validate the results. Investors are heterogeneous in their risk profiles and preferences with respect to volatility and investment horizon. The empirical PKs turn out to be U-shaped for short-dated instruments and W-shaped for long-dated instruments. We find that investors are willing to pay a substantial risk premium to insure themselves against short-term price movements. The risk premium is smaller for longer-dated instruments and their traders are risk averse. The shape of the empirical PKs reveals the existence of a time-varying risk-premium. The similarity between the shape of empirical PKs for Bitcoin and other markets that represent aggregate wealth shows that Bitcoin is becoming an established asset class.

**Keywords:** Bitcoin, Deribit, pricing kernel, risk aversion, speculation, hedging
**JEL Classification:** C14, C50, G10

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# Introduction

The valuation of digital currencies has been a question that predated its first representative, Bitcoin. Upon the creation of Bit Gold, Szabo (2008) envisioned its value to arise from an interplay between the benefits of the two entities which foremost represent value: cash and metal. The argument put forward was that metal bears an inherent value that is largely independent of trusted third parties. However, the unwieldiness of metal incurs a relatively high transaction cost when used as a means of exchange. (Unbacked) Cash, on the other hand, draws its value largely from the trust in a third party's acceptance. The combination of the two elements, coming in the form of a scarce digital resource that is governed in a trustless system, has been passed on to Bitcoin (BTC) and other Cryptocurrencies (CC).

Since the inception of BTC as proposed by Nakamoto (2009), various arguments have been made on its valuation. Apart from the benefit of a trustless system, the arguments typically include the production cost of a block (Hayes 2019), the benefit of borderless transactions (Deng 2020; Stellar Development Foundation 2022), the value of an inflation hedge (Choi and Shin 2022), network effects (Chen and Vinogradov 2021; Gandal and Halaburda 2016) or expectations about future price developments (Blau 2017; Smaniotto and Neto 2022).

In addition to the former valuation approaches, we offer new insights by means of the BTC derivatives market which has emerged over the course of the last years. Since derivatives markets are particularly rich in information, their evolution provides a unique opportunity to assess the BTC market valuation through the application of proven econometric techniques. Key information about preferences and forward looking decisions are state price densities (SPD), that can be estimated from option prices. SPDs yield risk-neutral probabilities, under which investors price derivatives. They provide the key to pricing exotic or illiquid options in an arbitrage-free manner (Aït-Sahalia and Lo 1998) and offer insight into changing expectations about future developments. In conjunction with the physical density (PD) of Bitcoin returns, the resulting pricing kernels (PK) can be calculated. The shape and evolution of PKs over time discloses investor expectations under different market circumstances and gives insight into time varying preferences. We present the first paper that is based on real Bitcoin option data, inferring and disclosing investor preferences following hypothesizing papers on bootstrap-based confidence bands for empirical PKs (Härdle et al. 2014) and cryptocurrency option pricing under an SVCJ model (Hou et al. 2020).

BTC derivatives markets can already be regarded as efficient information processing mechanisms (Alexander et al. 2022). Among those markets, Deribit is the leading crypto option exchange as measured by open interest and trading volume. As of 2022-07-25, Deribit manages more than 90% of the BTC option volume which translates over a 30 day window into an average 24-hour trading volume of over 331.8 million USD (Skew.com 2022). The competitors LedgerX, OKEx, CME, bit.com and Binance are contributing respective averages of 1.9, 15.7, 13.2, 1.5, 0.582 million USD.

A particularly interesting property of the BTC options market is its decentralized nature. Trading on Deribit is largely dominated by retailers. Despite an ongoing decline of retail market share in favor of institutions (Coinbase 2022), who are cautious to invest in highly volatile and unregulated assets, the options market on Deribit remains driven by retailers: As of 2022-07-25, 86.65% of the volume is attributed to retail, whereas 9.16% and 1.19% are attributed to investors and whales (Deribit 2022a). Deribit classifies market participants according to their share of the circulating BTC supply. According to Deribit's classification, a "whale" is an entity that owns more than 1% of the supply on Deribit, an "investor" owns between 0.1% and 1% and a "retailer" owns less than 0.1%. Considering the retail share in trading volume in conjunction with Deribit's dominating market share over the competition, we figure that the Bitcoin price is mainly driven by retail. Analyzing a retail-driven marketplace renders the study of digital currencies like BTC unique and different from the well studied equity options markets, where retail only has a minor influence (Bloomberg 2021).

Another interesting feature of the market under consideration is not just the dominating presence of retailers, but their easy access to leverage for speculation. Since Deribit is by design a margin-trading platform, levers of up to 100 are available for longs. Easy access to leverage could suggest the existence of the leverage effect if investors were risk affine. However, we cannot confrm a leverage effect to be present in the time frame under consideration.

All underlying code is available on quantlet.com and a corresponding courselet is available on quantinar.com.

# Pricing Kernels

Assume a risky asset with a stochastic price process $\{S_t\}_{t\in\mathbb{N}}$ and a risk-free interest rate $\{r_t\}_{t\in\mathbb{N}}$ in a complete market. Following the second Fundamental Theorem of Asset Pricing, a unique martingale-equivalent measure $Q$ exists in the described setting, under which derivatives can be priced in an arbitrage-free manner (Huynh et al. 2002; Pascucci and Agliardi 2011).

Let $C_t$ be the price at time $t$ of a contingent claim with payoff $\psi(\cdot)$ on the risky asset (underlying), which has a maturity at $T$ and a time-to-maturity $\tau = T - t$. For simplicity assume a constant interest rate $r_t = r$. The price of any such contingent claim can be expressed as the discounted value of expected future payoffs, weighted with their respective probabilities of occurrence. The expectation operator is conditional on the information set at $t$ under the equivalent martingale probability $Q$

$$C_t = e^{-r\tau}\mathsf{E}^Q[\psi(S_T)] = e^{-r,\tau}\int_{-\infty}^{\infty}\psi(u)f_t^Q(u)du \tag{1}$$

Transforming the risk-neutral measure $Q$ to the physical measure $P$ yields the PK by Itô's Lemma.

$$C_t = e^{-r,\tau}\int_{-\infty}^{\infty}\psi(u)q(u)du = e^{-r,\tau}\int_{-\infty}^{\infty}\psi(u)p(u)K(u)du \tag{2}$$

where the PK $K(\cdot)$ is defined as $\frac{q(\cdot)}{p(\cdot)}$ and for simplicity of notation we write $q(\cdot)$ for $\frac{\partial Q}{\partial t}(\cdot)$ and drop the sequence $r_t$.

The PK can therefore be approximated by the ratio between the risk-neutral density and the physical density. This process is discussed and executed in the following sections.

# Nonparametric Estimation of State Price Densities

## Derivation

As stated by Breeden and Litzenberger (1978), a SPD can be estimated via the second derivative of the call price function $\psi(S_T) = \max(S_T - K, 0)$ with respect to the strike price $K$.

$$\left.\frac{\partial^2 C_t}{\partial K^2}\right|_{K=S_t} = q(S_T)e^{-r\tau} \tag{3}$$

A variety of call prices $C$ with different strikes $K$ is required in order to calculate the complete SPD $q$. The present value of a call can be priced in implied volatility (IV). In conjunction with the vector (time-to-maturity $\tau$, strike $K$, spot $S$, interest rate $r$), the market call price can be calculated.

IV is estimated as a function of time-to-maturity and moneyness in the following section. Collapsing spot price $S$ and strike $K$ into a single variable $M = \frac{K}{S}$ reduces the effect of the curse of dimensionality. Similarly, we collapse $S$ and the interest rate $r$ into a Futures price.

## Local Polynomial Estimation of the IV Surface

Assume that the implied volatilites have some noise added (Huynh et al. 2002; Rookley 1997).

$$\sigma(M,\tau) = g(M,\tau) + \sigma^*(M,\tau)\varepsilon \tag{4}$$

with a standardized error random variable $\varepsilon$, moneyness $M$, $\tau$ and $\varepsilon$ independent and $\sigma^*(M, \tau)$ being the scaling of the error term given $M$ and $\tau$. Suppose $g$ is smooth, i.e. it can be approximated using Taylor's Theorem.

Taylor expansion of $g$ in a neighborhood of $(M_0, \tau_0)$:

$$\begin{aligned} g(M,\tau) = g(M_0,\tau_0) + \frac{\partial g}{\partial M}(M-M_0) + \frac{1}{2}\frac{\partial^2 g}{\partial M^2}(M-M_0)^2 + \\ \frac{\partial g}{\partial \tau}(\tau-\tau_0) + \frac{1}{2}\frac{\partial^2 g}{\partial \tau^2}(\tau-\tau_0)^2 + \\ \frac{\partial^2 g}{\partial M \partial \tau}(M-M_0)(\tau-\tau_0) \end{aligned} \tag{5}$$

The functional relationship between the IV surface $\sigma$ and $M$ and $\tau$ can now be approximated using a Weighted Least Squares Estimator (WLSE), minimizing the objective function

$$\arg\min_{\beta}(\sigma - X\beta)^\top W(\sigma - X\beta) \tag{6}$$

where $W = \text{diag}(\mathcal{K}_{h_m,h_\tau}(M_j - M_0, \tau_j - \tau_0))$ for a Gaussian kernel $\mathcal{K}$ with bandwidths $h_m$ and $h_\tau$. $\sigma$ is the $n \times 1$ vector of observed implied volatilities, $\beta$ is the vector of the local polynomial coefficients.

$$X = \begin{pmatrix} 1 & (M_1 - M_0) & (M_1 - M_0)^2 & (\tau_1 - \tau_0) & (\tau_1 - \tau_0)^2 & (M_1 - M_0)(\tau_1 - \tau_0) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & (M_n - M_0) & (M_n - M_0)^2 & (\tau_n - \tau_0) & (\tau_n - \tau_0)^2 & (M_n - M_0)(\tau_n - \tau_0) \end{pmatrix}$$

The resulting WLSE is

$$\hat{\beta} = (X^\top W X)^{-1} X^\top W \sigma \tag{7}$$

Following Härdle et al. (2014), a window of the last 500 daily returns (based on $\tau$) is used to calculate a nonparametric Kernel Density Estimator for the PD.

Chapter 24

# Literature Review

Breeden and Litzenberger (1978) derive SPDs using Arrow-Debreu prices and Butterfly Spreads. Their paper was the cornerstone for the now existing vast literature on estimation of SPDs. Without requiring a parametric form for SPDs, Rookley (1997) who developed a nonparametric estimation method. IV skews are estimated by decomposing the functional relationship between IV, moneyness and time-to-maturity $\tau$. In this manner it is possible to derive the SPD at every point in a robust way. Aıt-Sahalia et al. (2001) estimate PKs from S&P500 options data and the according return series in order to assess the efficiency of the options market. Departures from SPD and PD are used to identify inefficient pricing. They design a trading strategy, exploiting the skewness and kurtosis of the densities. The strategy is shown to have a high Sharpe Ratio. Grith et al. (2009) propose a systematic modeling approach to study the evolution of PKs over time. With European DAX data, a series of empirical PKs is estimated from 2003 until 2006. While the risk-neutral density is inferred using Rookley's method, the PD is estimated with a GARCH model. A common shape is identified and deviations of the time varying EPKs are studied. The deviations between individual PKs are described using a set of parameters for horizontal and vertical shifts. The relationship between PKs and Arrow-Pratt measure of Absolute Risk Aversion (ARA) give insight into investor's risk aversion. In a related research, Härdle et al. (2014) derive bootstrap-based confidence bands for nonparametrically estimated PKs.

Inverse Options, meaning options settled in kind of the underlying, are dominant in the crypto world. Settlement of BTC options in terms of BTC changes the payoff function of the option from $\max(S_T - K; 0)$

to $\max(\frac{S_T - K}{S_T}; 0)$ and thus changes the contract's net present value. Alexander and Imeraj (2021) adjust Black-Scholes prices and hedge ratios to inverse options. However, it is argued that traders are erroneously applying vanilla Black-Scholes valuation instead of the corrected prices - perhaps because of being unaware of the concept. We have decided to use with the standard, non-inverse Black Scholes pricing for multiple reasons. Alexander and Imeraj (2021) argue that traders are perhaps unaware of the difference. Since we are inferring investor expectations, we figure that it is more appropriate to infer pricing kernels under the same Black Scholes prices that the investors use. Additionally, it is likely that many traders actually hedge their overall Bitcoin exposure. This would negate the "inverse" part of the option. Furthermore, reviewing the differences in pricing compared to the adjusted prices, we find that the difference is small in the absence of extreme moves in the underlying. This was not the case in the time period under consideration.

Hou et al. (2020) price BTC options under an SVCJ model. Their results emphasize the tail risk introduced by jumps in the underlying. Jumps in particular introduce market incompleteness, but anonymity of transactions may be relevant as well. Chen and Vinogradov (2021) derive PKs and the impact of market incompleteness on risk premia. They state that a key property of cryptocurrency valuation is the user's anonymity (or pseudonymity), which is simultaneously the source of value and incompleteness in the respective market. They argue that hedging an anonymous transaction would require an identity disclosure. This contradiction may introduce an effectively unhedgeable event.

# Deribit

Deribit, a margin-trading platform for futures and options, has been launched in June 2016 in the Netherlands and is currently incorporated in Panama. As of time of the writing, Deribit is the largest BTC option exchange (Skew.com 2022). For both types of derivatives, BTC and ETH are the underlying as well as the currency in which settlement is conducted. This allows to view them as inverse options.

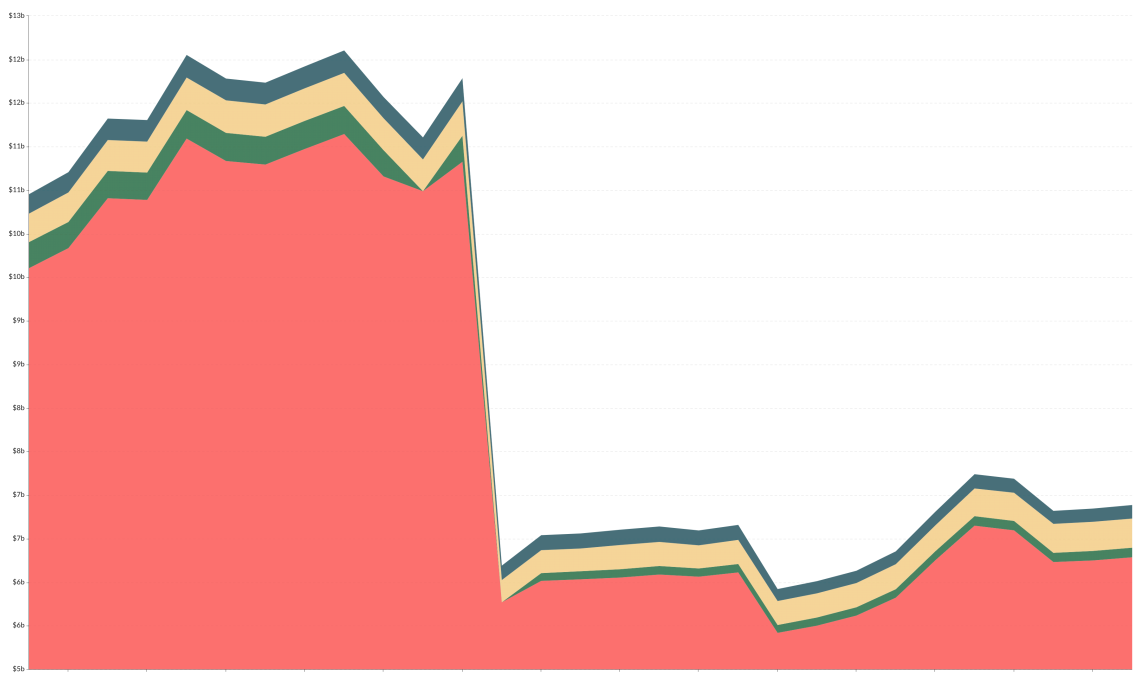


Figure 1: Open Interest in BTC Options per Exchange. Snapshot from skew.com on 2022-01-16. Deribit, LedgerX, CME, bit.com.

### Bitcoin Option Contract Specifications

Deribit offers cash-settled European-style options. The underlying is a synthetic index, whose exact composition is described in the data section. Settlement is first calculated in USD and then conducted in kind of the underlying. E.g. a BTC call with a strike of 10000 and a settlement value of 12000 at maturity will result in a cashflow of 2000 USD, which is equivalent to $\frac{1}{6}$ BTC (excluding transaction cost). The settlement value is defined as the average of the underlying BTC index for the last 30 minutes before settlement. Each contract has a lot size of 1 BTC and is settled at 8am UTC on the respective maturity date. Maturity dates have a daily, monthly, quarterly and annual frequency. Deribit is the only derivatives exchange to offer daily options. Instruments are trading 24/7 with a tick size of 0.0005 BTC. Due to the automatic usage of margin-trading, margin-based liquidation of positions is possible. Initial margin and maintenance margins are both zero for long positions. For short positions, initial margin is required (Deribit 2022b). In case of liquidation, a penalty is applied to the defaulting party, whose proceeds are paid into the Deribit Insurance Fund (Deribit 2020a).

Trading and deliveries on Deribit are subject to fees (Deribit 2022c). The applied fees generally vary depending on whether the order was a liquidity maker or taker. However, for Bitcoin options they are equal. Trading fees are 0.03% of the underlying or 0.0003 BTC per options contract. The fees constitute a maximum of 12.5% of the contract's value.

## Data

### Data Structure

The dataset consists of 8,444,664 order book snapshots, which have been collected from Deribit in the time from 2021-04-01 until 2022-04-01. The snapshots were captured via the Deribit API V2. The data base is available on the Blockchain Research Center (BRC). The full data set includes all parameters that the Deribit API V2 returns at the time of collection under the methods

- *{public/get_last_trades_by_instrument_and_time}*
- *{public/get_order_book}*

Most importantly the results include

- Timestamps
- Greeks
- Implied Volatility
- Tick Direction
- Order Type
- Volume
- Instrument Price
- Strike
- Spot

High-frequent order book changes and executed trades are captured for options and futures whose underlying is BTC. Values of the underlying synthetic BTC USD Index are also saved. The underlying is calculated as an equally-weighted BTC/USD price of eleven major crypto exchanges, namely Binance, Bitfinex, Bitstamp, Bittrex, Coinbase Pro, FTX, Gemini, Huobi, Itbit, Kraken, LMAX Digital, OKEx. Individual feeds can be excluded due to administrative decisions or in case if invalid data. Remaining feeds are sorted, truncated around the median price and weighted (Deribit 2020b).

### Data Integrity

Data integrity plays a crucial role in crypto markets. Mark Carney, the chair of the Financial Stability Board and head of the Bank of England, warned that wash trading, pump and dumps and spoofing, known as outlawed manipulation techniques in equity markets, are also present in crypto and pose a risk to financial stability (Rodgers 2019). Wash trading is used to increase trading volume and thus artificially increase demand. A spoofer submits market non-bona fide price quotes in order to cause artificial price volatility (Sar 2017). Pump & Dump is a form of securities fraud in which a group of traders rapidly and artificially inflate a price in order to offload their previously acquired inventory. Since Pump & Dump schemes require low liquidity (in the underlying), we can exclude this possibility for Bitcoin (La Morgia et al. 2020).

Aloosh and Li (2019) find evidence for exchange-driven inflated volume, generated by wash trades, to market themselves under the guise of liquidity. Cong et al. (2019) report that on average 70% of volume on decentralized exchanges is fake due to the use of wash trading. According to Bitmex (2019), up to 95% of trading volume on unregulated exchanges is generated by wash trading. In order to ensure data integrity, we exclusively use orderbooks instead of executed trades in order to filter possible cases of wash trading.

To address the potential issue of spoofing, we adopt an approach related to Tuccella et al. (2021). They attempt to identify spoofing on cryptocurrency exchanges using a GRU model. Their predictors are a function of the amount of cancelled orders relative to the cumulative order size on each side of the order book. Since Bitcoin derivatives order books do not have the same amount of depth as spot or futures markets, we employ a stricter method and exclude all orders whose lifetime does not exceed two seconds. We find that less than 5% of orders are filtered due to short order lifetime. We figure that traditional spoofing techniques, based on the rapid submission and cancellation of orders, are difficult to implement due to Deribit's rate limits (Deribit 2023). Users with less than 1 million USD in 7-day-turnover can only post five matching engine requests. These include request for a buy or sell order or a corresponding cancellation. Although the limits for a market maker (who could be allowed to post 30 requests per second given a turnover of 25 million USD in 7 days) are considerably higher, the market maker must still meet the quoting requirements for a large range of instruments. This effectively restricts the influence that single entities can have on certain instruments via the rapid submission and cancellation of orders and explains the rarity of such events in our dataset.

## Preprocessing

Although market makers are obliged to quote most instruments for the majority of the time, some may not be quoted at all or only be quoted at the cost of a large spread (Deribit 2020c). For Far-Out-of-Money (FOTM) contracts, when the minimum tick size exceeds the market IV, one can observe bids for 0% IV. Cases of quotes for 0% IV and all duplicates are excluded from the data set. Since the majority of option trading volume concentrates on instruments with short or medium time-to-maturity $\tau$, we restrict $\tau$ to be smaller or equal than one quarter of a year. $\tau$ is normalized on the span of a year. Instruments are grouped according to their time-to-maturity, over a range of 0, 1, 2, 4, 8 weeks. We restrict moneyness $M = \frac{K}{S}$ to the interval $[0.7, 1.3]$ in order to exclude the influence of high-volatility observations which could be unreliable (Grith et al. 2009).

Call options are exclusively used in order to estimate the IV surface in Rookley's method. Put-Call-Parity ensures arbitrage-free call option prices. However, we do not use Put-Call-Parity to increase our data sample. Conversion of option prices via Put-Call-Parity could introduce errors into our dataset due to market microstructure (MMN) noise. Our high-frequent order book data is extracted via an API. Due to rate limitations and a diverse amount of instruments, order book snapshots cannot be taken simultaneously for all instruments. The data scraping application successively iterates over all instruments and captures changes in order books. While the resulting data base provides a clear picture of order book data, it does not allow for a complete reconstruction of all order books at all times. Without simultaneity, volatility of the underlying and order competition within the spread could change the prices of the corresponding instruments (required for Put-Call-Parity) between the individual snapshots. A range of futures may be available on Deribit, but only the perpetual futures contract is actually liquid. The classic (non-perpetual) futures also price in the settlement cost, which often causes them to trade in backwardation close to maturity. But in spite of this, BTC options tend to be traded more frequently when they are close to maturity. The combination of these real-world limitations challenges the simple conversion via Put-Call-Parity and could introduce errors into our dataset. Furthermore, we do not see the necessity to use such a method as we have sufficient call option data to compute the IV surface from which we sample option prices.

Preprocessing

## Volatility

In accordance with Masset (2011) and Eraker (2021), the list of well-established stylised facts on volatility in traditional markets include horizontal dependence of volatility, leverage effect, the volatility premium and extreme events. Horizontal dependence of volatility describes the tendency of local volatility clusters and the tendence to mean-reversion. Both properties can be observed in the 7 day rolling volatility. Sudden spikes in realized volatility form clusters and are often followed by similar drops. The leverage effect can be measured as a negative correlation between returns and volatility. Phases of negative returns coincide with high volatility and vice versa. The Pearson correlation between the underlying and realized volatility is -12.59%. The Pearson correlation between the underlying and the bid (ask) IV is -16.11% (-13.05%). Due to the low absolute values of correlation, the existence of a leverage effect is unlikely. However, the sign matches the direction of a leverage effect. Considering the large share of retailers with easy access to leverage, the lack of significance is a surprising result. It is well known that, in general, the IV exceeds the unconditional annualized standard deviation. This is displayed in Figure **??**. The difference between the implied and realized volatility is commonly referred to as volatility premium. The mean 7 day rolling volatility is 70.96%, whereas the mean IV at the bid and ask are 85.28% and 93.57%. Spikes in realized volatility rarely exceed the ask of the IV. Consequently, the volatility premium is shown to be substantial throughout the regarded time period. Fat-tail events are priced in due to the classic volatility skew. This is depicted in Figure **??**, the empirical volatility skew.

Since the data are high frequency order book snapshots with substantial bid-ask spreads, we must address the issue of Market Microstructure Noise (MMN). A simple estimate of realized volatility would increase with our sample due to order competition within the spread. We resolve the issue of MMN in our calculation of realized volatility by taking the arithmetic mean after aggregating the implied volatility of bids and asks on a daily base. Therefore the frequency is matching the underlying BTC index. The formula for annualized, realized volatility of price $p$ over a window of $w$ days is

$$\sigma_w = \sqrt{\frac{1}{w}\sum_{t=1}^{w}(\log p_t - \log p_{t-1})^2}\sqrt{\frac{365}{w}} \tag{8}$$

Annualization is based on 365 days since Bitcoin options are traded continuously on Deribit.

The term structure itself is easier to assess when regarding ATM-options in a series of dates (Figure 6). ATM-options are defined as those options, whose moneyness falls in the interval $[0.9, 1.1]$. A typical term structure of the Bitcoin options reveals a high level of implied volatility for shorted-dated options. For options with a time-to-maturity of three days or more, implied volatility drops sharply compared to the initial level. Naturally, IV is increasing with time-to-maturity.

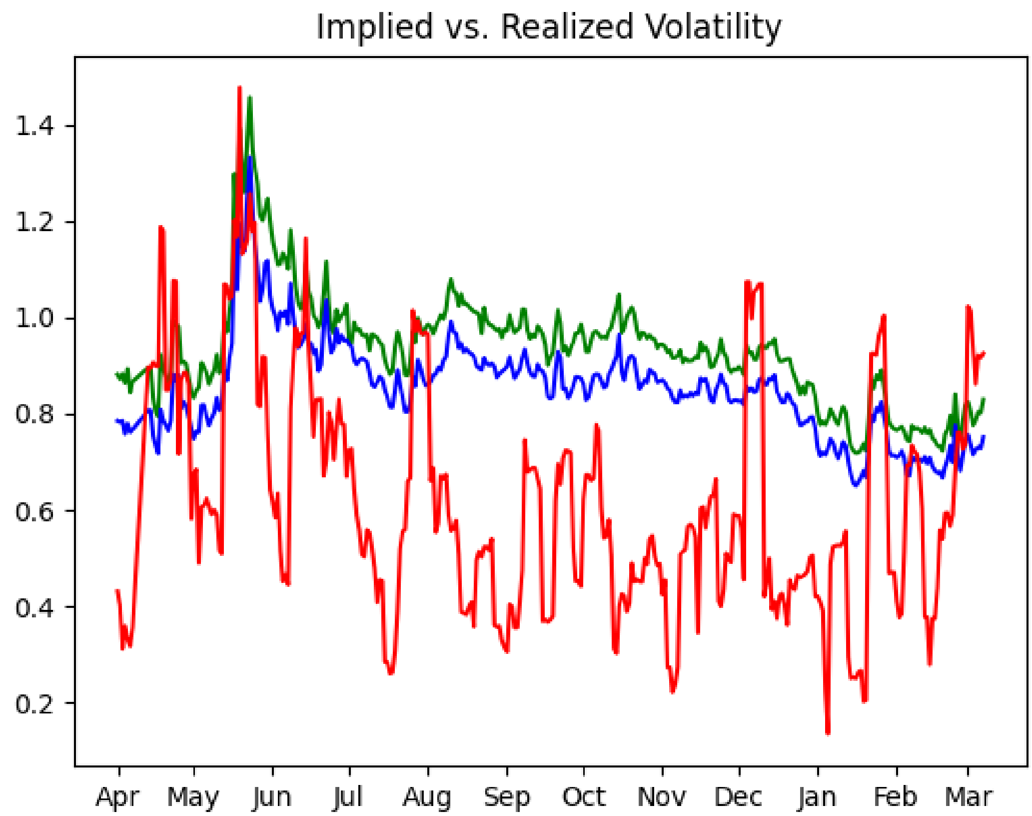


Figure 2: Implied volatility vs. realized volatility. Realized volatility is annualized and regarded in a 7 day window. IV is calculated as the average of observed orderbooks at the bid and ask. Described data filters apply. Ask IV, Bid IV, 7 Day Rolling Volatility.

Volatility

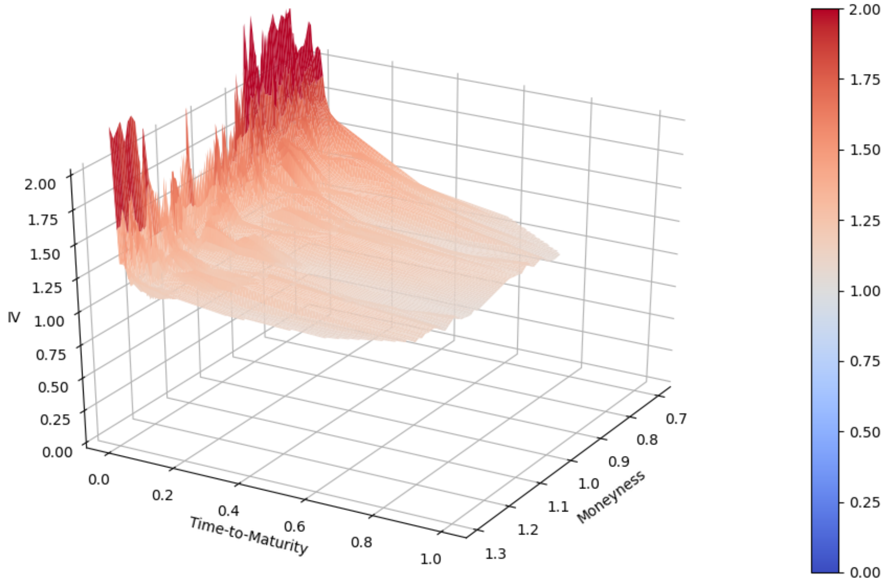


Figure 3: Volatility Skew shown in the empirical Volatility Surface on 2021-05-23

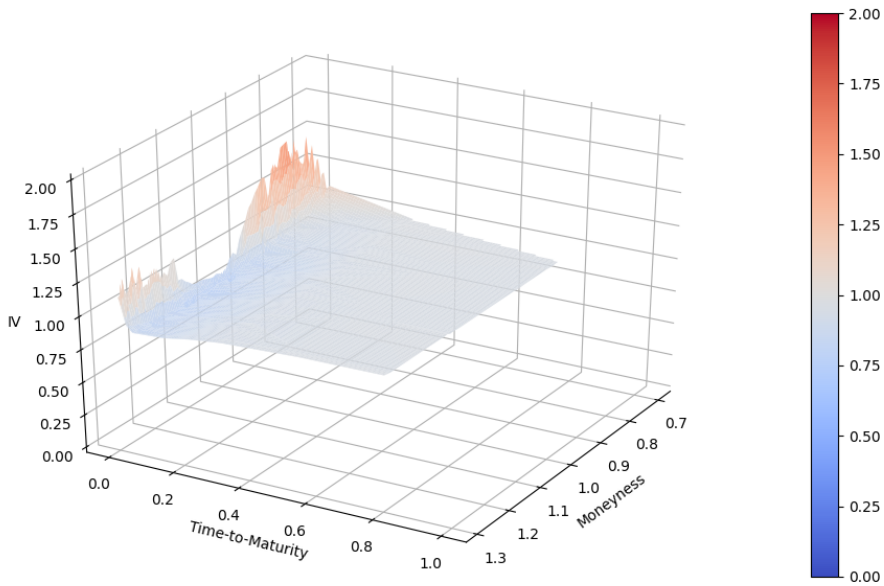


Figure 4: Volatility Skew shown in a calmer period on 2021-09-08

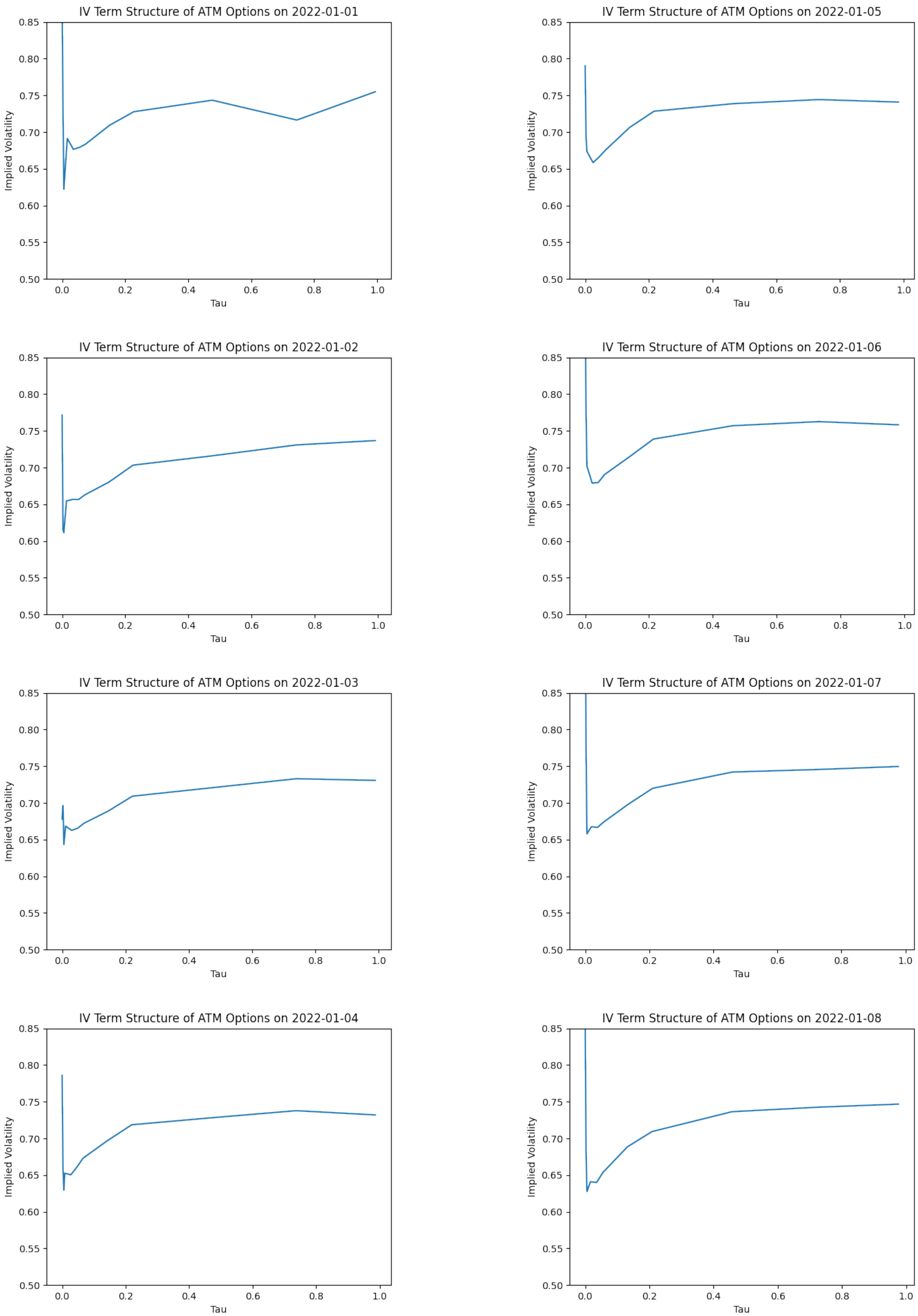


Figure 5: ATM Term Structure from 2022-01-01 until 2022-01-08. Moneyness $M \in [0.9, 1.1]$

Main

## Empirical Pricing Kernels

Instruments are grouped by time-to-maturity in order to summarize the findings. All instruments with less than one week to maturity are classified as having less than one, between one and two, between two and three, between three and four and between five and eight weeks to maturity. We typically observe U-shaped PKs for short-term maturities. In a similar fashion to the oil market, investors perceive short-term price changes in any direction as undesirable and prefer to hedge their risk (Christoffersen et al. 2021). For short-dated instruemnts, option writers are asking for substantial risk premia in order to reflect unhedgeable risks, such as jumps in the underlying (Hou et al. 2020).

We find a differnet shape for longer-dated instruments. Typically, PKs in traditional markets are monotonically decreasing. This results from human risk aversion; investors are willing to insure themselves against losses and have a preference for smooth consumption curves. However, PKs outside of index options markets are not well studied (Cuesdeanu and Jackwerth 2018). Grith et al. (2009) compartmentalize a pricing kernel in two segments around a possible breakpoint. Testing an unrestricted model against GMM estimates of the restricted one, they reject monotonicity of the empirical PK in four out of five cases (performing a D-test). The phenomenon of having a decreasing slope, although with locally increasing sections in empirical PKs is called the pricing kernel puzzle. We observe a similar behavior and extend the existing literature on the pricing kernel puzzle to a new asset class.

Since the empirical PKs for short-dated instruments are U-shaped, we figure that investors are willing to pay a high risk premium in order to insure immediate price risks. With an increasing maturity of the instrument, the shape of the empirical PK resembles a “W” or “tilde”. This corresponds to a a lesser willingness to pay a risk premium.

The dynamic of the empirical PKs also reveals the existence of a time-varying risk premium in BTC markets. The observed PK shapes are closely aligned with the results reported in Cuesdeanu and Jackwerth (2018). However, their object of study are traditional markets, such as major indices which are highly correlated with aggregate wealth. We conclude that, due to the similarities of the empirical PKs, BTC is well on its way to becoming an established asset class. It may prove to be much rather a store of value than it is currently given credit for.

Apart from the discussion of results, deviations between PD and SPD can be interpreted as trading opportunities. (Blaskowitz et al. 2004) design and evaluate trading strategies based on deviations between PD and SPD. Estimated PKs could be used by traders employing so-called Skewness or Kurtosis trades. Nevertheless, traders must recognize the hidden cost of employing such strategies in terms of substantial hedging cost, e.g. in the form of volatility and large spreads.

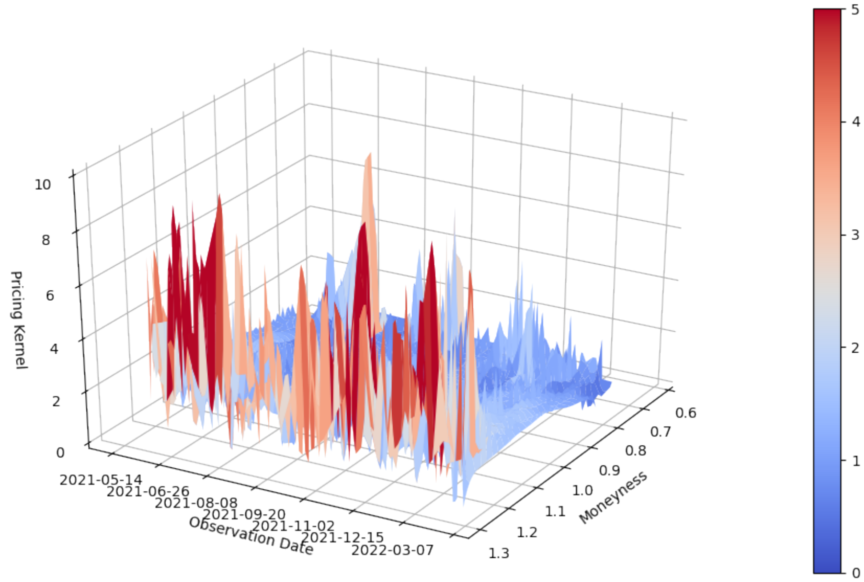


Figure 6: Pricing Kernels with less than one Week to Expiration

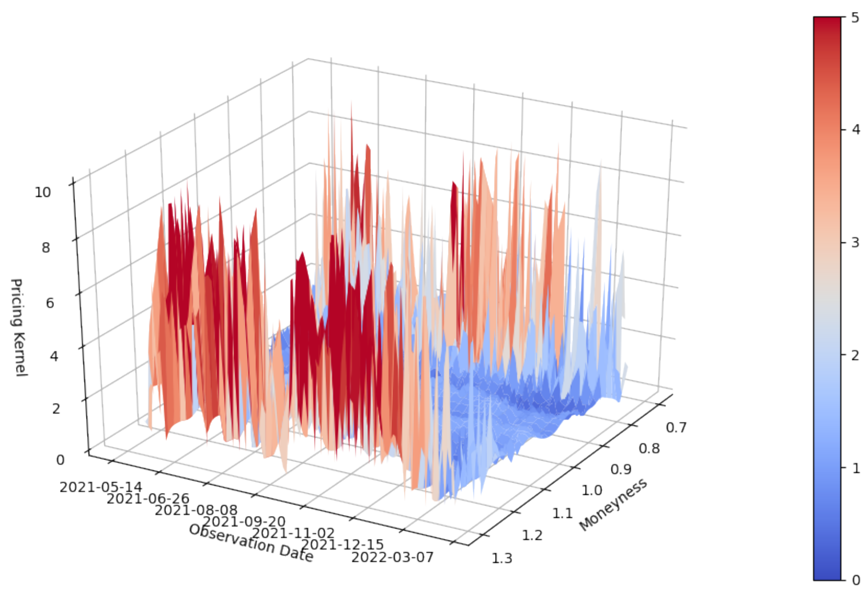


Figure 7: Empirical Pricing Kernel - 1 Week to Expiration

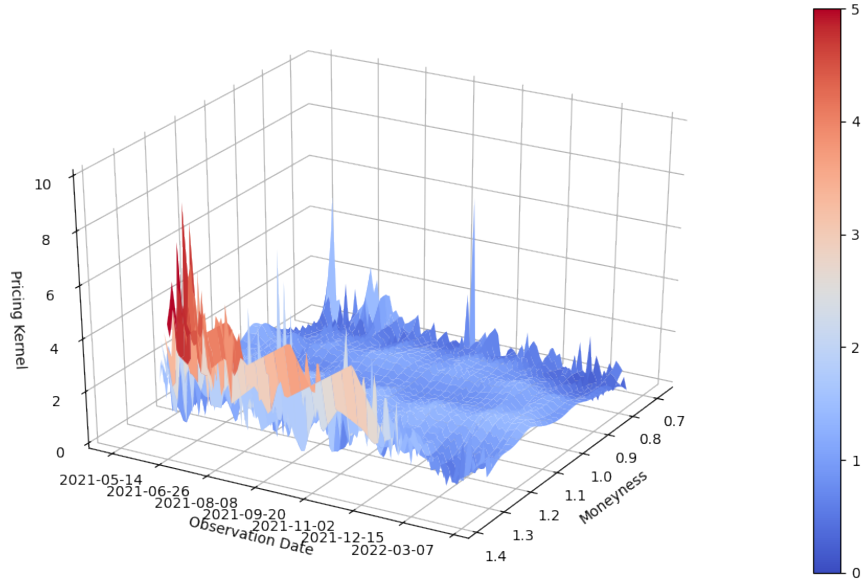


Figure 8: Empirical Pricing Kernel - 2 Weeks to Expiration

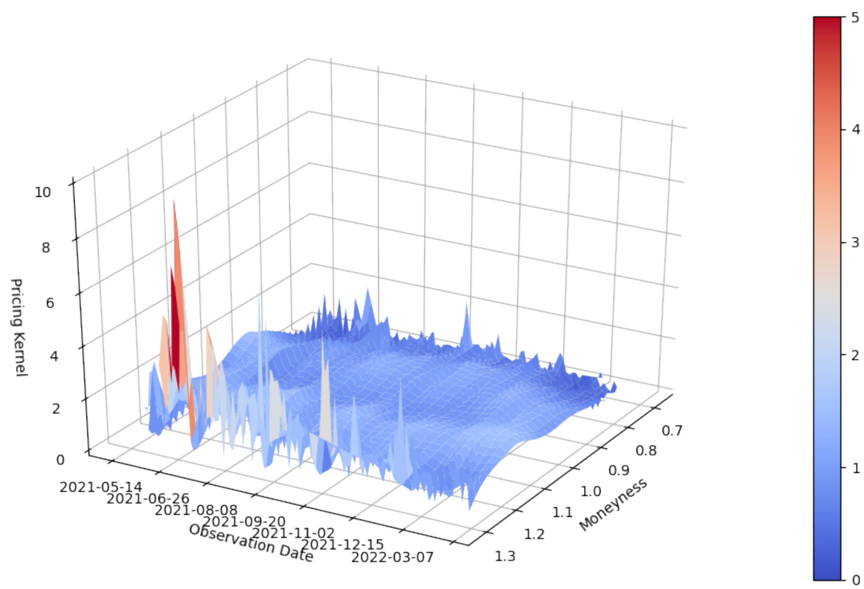


Figure 9: Empirical Pricing Kernel - 3 Weeks to Expiration

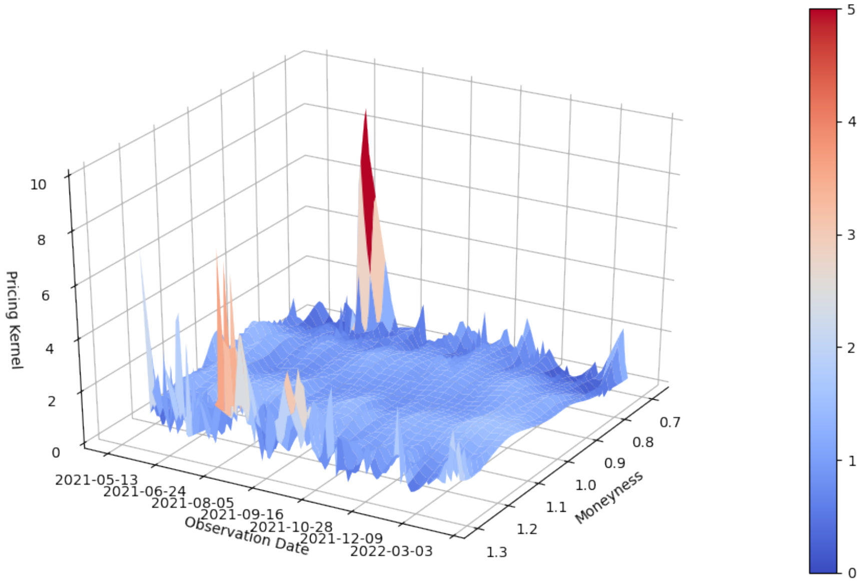


Figure 10: Empirical Pricing Kernel - 4 Weeks to Expiration

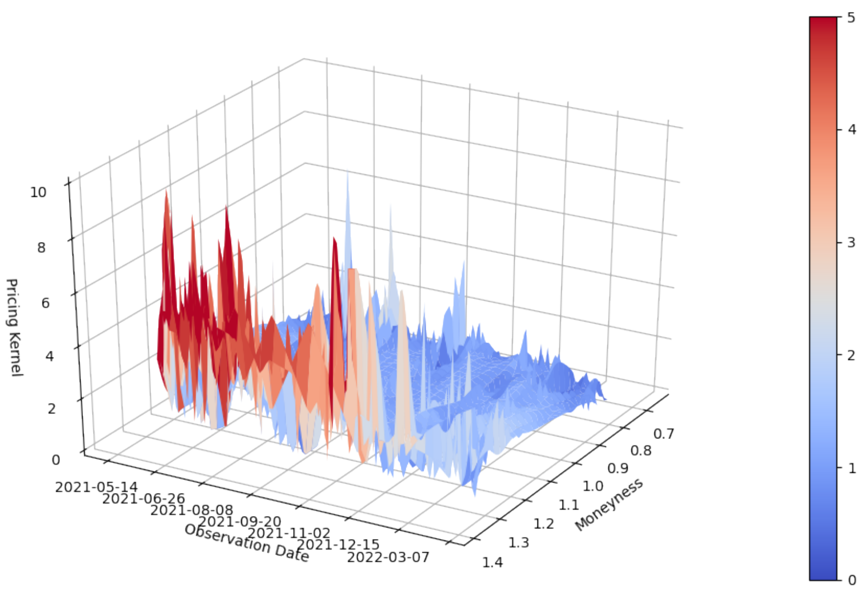


Figure 11: Empirical Pricing Kernel - 5-8 Weeks to Expiration

Main

## Bootstrap-based Confidence Bands

Having computed a large set of empirical PKs with varying maturities, it should be ensured that the observed features are not mere artefacts. A useful tool to assess the validity of our estimated PKs are bootstrap-based confidence bands. Following the derivation of Härdle et al. (2014), confidence bands at any confidence level can be derived via the "wild bootstrap method". Figure **??** until Figure **??** are depicted exemplarily for the whole set. Confidence bands are tighter around the PKs when time to maturity increases. PKs are also flatter with growing time to maturity.

The concept of bootstrapping PK can be introduced by linearization of the PK. Stochastic deviation of the PK can be linearised into a stochastic part, containing the estimator of the SPD and a deterministic part containing the expectation of the physical density. Convergence of both parts can be proven separately.

$$\widehat{K}(x) = \frac{\widehat{q}(x)}{\widehat{p}(x)}$$

$$\sup_{x\in E}|\widehat{K}(x) - K(x)| = \sup_{x\in E}|\frac{\widehat{q}(x) - q(x)}{p(x)} - \frac{\widehat{p}(x) - p(x)}{p(x)} * \frac{q(x)}{p(x)} - \frac{\{\widehat{q}(x) - q(x)\}\{\widehat{p}(x) - p(x)\}}{p^2(x)}|$$
$$+\mathcal{O}_p[\max((n_p h_{n_p}/\log)^{-1/2} + h_{np}^2, h_{n_q}^{-2}\{n_q h_{n_q}/\log n_q\}^{-1/2} + h_{n_q}^2)]$$

Consider the leading term of

$$\sup_{x\in E}|\frac{\widehat{q}(x) - q(x)}{p(x)}|$$

Resample data from the smoothed bivariate distribution of strike and moneyness (X,Y)

$$\widehat{f}(x,y) = \frac{\widehat{\sigma_X}}{n_q h_{n_q}^2 \widehat{\sigma_Y}} \Sigma_{i=1}^{n_q} K\{\frac{X_i - x}{h_{n_q}}, \frac{(Y_i - y)\widehat{\sigma_X}}{h_{n_q}\widehat{\sigma_Y}}\}$$

Using the resampled data, calculate the bootstrap analogue

$$\sup_{x\in E}|\frac{\widehat{q}^*(x) - \widehat{q}(x)}{p(x)}|$$

where $\sup_{x\in E}|\widehat{q}^*(x) - \widehat{q}(x)|$ converges to the Gumbel distribution at an unfortunately slow rate of $\frac{1}{\log n_q}$ (Hall 1991).

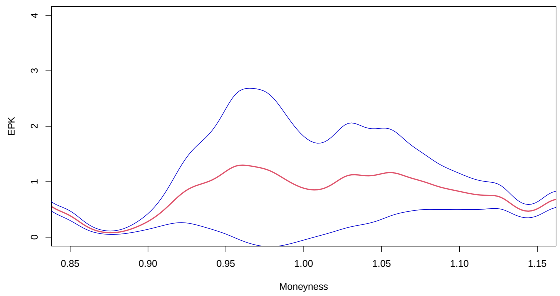


Figure 12: 2 Days to Maturity.

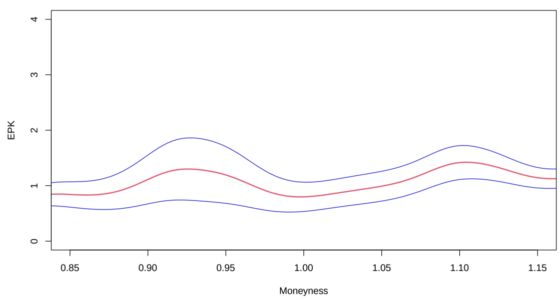


Figure 13: 7 Days to Maturity.

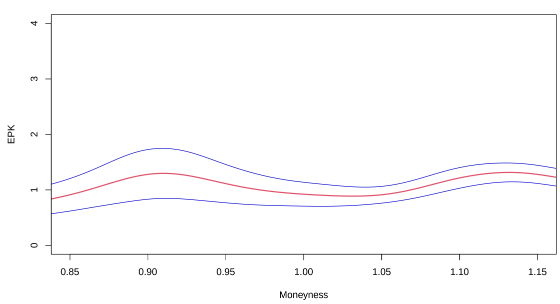


Figure 14: 14 Days to Maturity.

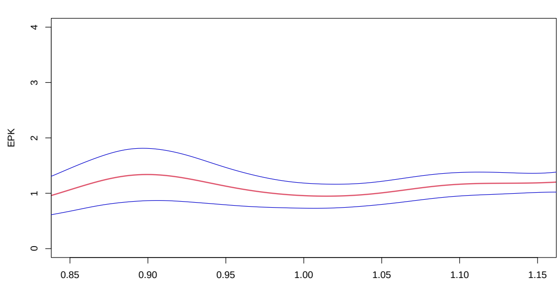


Figure 15: 21 Days to Maturity.

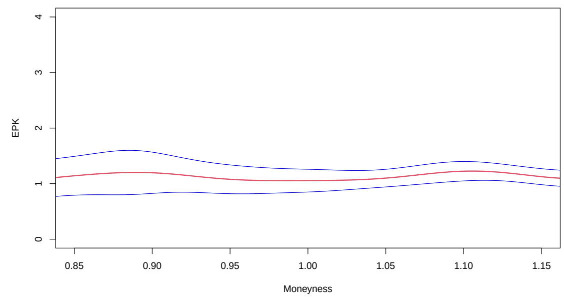


Figure 16: 28 Days to Maturity.

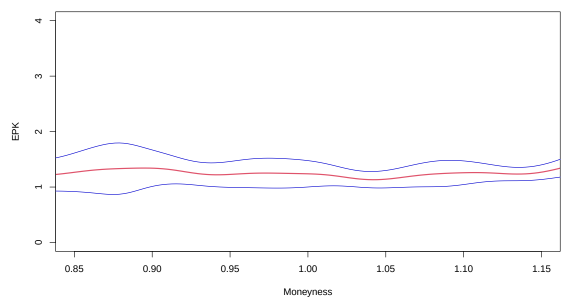


Figure 17: 56 Days to Maturity.

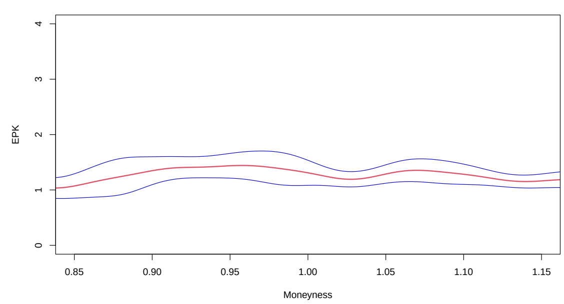


Figure 18: 84 Days to Maturity. Empirical Pricing Kernels with bootstrapped Confidence Bands on 2021-12-31.

Main

# Conclusion

Empirical PKs have been estimated using Rookley's Method on a dataset of order book snapshots from Deribit, the leading BTC options market. Bootstrap-based confidence bands have been estimated in order to validate the results.
We assess the presence of well-established stylized facts of volatility in the BTC market. We find horizontal dependence of volatility, meaning that volatility tends to cluster and revert to the mean. The difference between implied and realized volatility, commonly referred to as the volatility premium, is substantial. Realized volatility rarely exceeds the implied volatility. The IV term structure is found to be traditional. Despite market participant's easy access to leverage, we cannot confirm the existence of a leverage effect.
Our analysis extends the literature to a new asset class and contributes to the pricing kernel puzzle. We are able to replicate empirical results from the analysis of PKs in traditional markets, which are highly correlated with aggregate wealth. Their similarity to PKs estimated from BTC options indicates that BTC is becoming an established asset class.
Furthermore, our analysis sheds light on the BTC valuation and risk-aversion of the retail traders. BTC option traders mostly consist of retailers who are heterogeneous in their risk profile. The price of short-dated instruments includes a high risk premium. Such instruments reflect the anticipation of jumps in the underlying. These instruments would be traded by either very risk-averse or risk-affine traders. Long-dated instruments are employed as classic hedging instruments. While a substantial risk premium is paid by traders in order to insure themselves against falling prices, they are additionally (but to a lesser degree) insuring themselves against sharply rising prices. Thus investors are also hedging their risk of being priced out of a dynamic market.
Several extensions of the presented research are available for the future. Among those, we would find an extended dataset insightful, that could support estimation of PKs and their evolution over a longer time frame and possibly describe the convergence to the PKs of established markets. It would also be interesting to analyze the evolution of PKs around stress events in order to get further insights into investor risk aversion, anticipation of such events and trading opportunities.

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