Notes on hedging cryptos with spectral risk measures

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Abstract

We investigate different methods of hedging cryptocurrencies with Bitcoin futures. A useful generalisation of variance-based hedging uses spectral risk measures and copulas.

1. Setup

Following (Barbi and Romagnoli, 2014), we consider the problem of the optimal hedge ratios by extending commmonly known minimum variance hedge ratio to more general risk measures and dependence structures.

Spectral risk measure (Acerbi, 2002):

$$\rho_\phi = -\int_0^1 \phi(p)\, q_p \,\mathrm{d}p,$$

where q_p is the p-quantile of the return distribution and $\phi(s)$, $s \in [0,1]$, is the so-called *risk aversion function*, a weighting function such that[1]

[1] Note that the treatment in (Acerbi, 2002) is measure-based and therefore slightly different

(i) $\phi(p) \geq 0$,

(ii) $\int_0^1 \phi(p)\,\mathrm{d}p = 1$,

(iii) $\phi'(p) \leq 0$.

Examples: VaR, ES
Dependence through copula (e.g. Student t, Clayton or Gumbel)
Hedge portfolio: $R_t^h = R_t^S - R_t^F$, involving returns of spot and future contract
Optimal hedge ratio: $h^* = \mathrm{argmin}_h\, \rho_\phi(s,h)$, for given confidence level $1-s$.

2. Representation of spectral risk measures

To prevent numerical instabilities involving the quantile function, re-write spectral risk measures as follows:

- Integration by substitution: $\displaystyle\int_a^b g(\varphi(x))\,\varphi'(x)\,\mathrm{d}x = \int_{\varphi(a)}^{\varphi(b)} g(u)\,\mathrm{d}u$.
- Spectral risk measures: $\displaystyle -\int_0^1 \phi(p)\,F^{(-1)}(p)\,\mathrm{d}p$
- Set $\varphi(x) = F(x)$, $g(p) = \phi(p)\,F^{(-1)}(p)$.
- Then:

$$-\int_0^1 \phi(p)\,F^{(-1)}(p)\,\mathrm{d}p = -\int_{-\infty}^{\infty} \phi(F(x))\,x\,f(x)\,\mathrm{d}x.$$

References

Acerbi, C. Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26(7):1505–1518, 2002.

Barbi, M. and S. Romagnoli. A copula-based quantile risk measure approach to estimate the optimal hedge ratio. *Journal of Futures Markets*, 34(7):658–675, 2014.