HEDGING AND VALUE AT RISK

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In this article, it is shown that although minimum-variance hedging unambiguously reduces the standard deviation of portfolio returns, it can increase both left skewness and kurtosis; consequently the effectiveness of hedging in terms of value at risk (VaR) and conditional value at risk (CVaR) is uncertain. The reduction in daily standard deviation is compared with the reduction in 1-day 99% VaR and CVaR for 20 cross-hedged currency portfolios with the use of historical simulation. On average, minimum-variance hedging reduces both VaR and CVaR by about 80% of the reduction in standard deviation. Also investigated, as an alternative to minimum-variance hedging, are minimum-VaR and minimum-CVaR hedging strategies that minimize the historical-simulation VaR and CVaR of the hedge portfolio, respectively. The in-sample results suggest that in terms of VaR and CVaR reduction, minimum-VaR and minimum-CVaR hedging can potentially yield small but consistent improvements over minimum-variance hedging. The out-of-sample results are more mixed, although there is a small improvement for minimum-VaR hedging for the majority of the currencies considered. © 2006 Wiley Periodicals, Inc. Jrl Fut Mark 26:369–390, 2006

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INTRODUCTION

Minimum-variance hedging is widely used by practitioners, either to offset the risk of a position in the cash or spot market by taking a position in the derivatives market, or as part of an overall investment strategy such as in a hedge fund. The use of variance as a measure of portfolio risk—and hence the use of minimum-variance hedging as a method of minimizing risk—is justified by assuming either that investors have quadratic utility functions or that asset returns are drawn from a multivariate elliptical distribution (see, for example, Ingersoll, 1987). Under the first assumption, asset returns may be skewed or leptokurtic, but this does not affect investors' utility because risk is defined solely in terms of standard deviation. Under the second assumption, asset returns may be leptokurtic, but kurtosis is unchanged when assets are combined into a portfolio (owing to the properties of the multivariate elliptical distribution), and so investors need only consider the standard deviation of the hedge portfolio.

In practice, however, neither of these assumptions is likely to hold. First, short-horizon financial asset returns are characterized by both skewness and leptokurtosis. Moreover, empirical evidence suggests that skewness and leptokurtosis are not preserved when assets are combined into portfolios (see, for example, Agarwal, Rao, & Hiraki, 1989; Simkowitz & Beedles, 1978; Tang & Choi, 1998). Both of these are inconsistent with asset returns being drawn from a multivariate elliptical distribution. Second, Scott and Horvath (1980) show that investors should have preferences for odd moments of the return distribution (such as skewness) and preferences against even moments (such as kurtosis) (see also Arditti, 1967; Kane, 1982). A number of authors have proposed asset-pricing models that explicitly allow for investors' preferences over the higher moments of returns (see, for example, Dittmar, 2002; Fang & Lai, 1997; Kraus & Litzenberger, 1976). Empirically, these studies typically find that the co-skewness and co-kurtosis of asset returns (which measure the contribution that an asset makes to the skewness and kurtosis of a portfolio) are priced by the market. This suggests that investors have preferences over skewness and kurtosis, and hence cannot have quadratic utility functions.

If returns are not drawn from a multivariate elliptical distribution and investors do not have quadratic utility, then standard deviation is no longer an appropriate measure of the risk of the hedge portfolio, because it fails to capture all of the characteristics of portfolio returns that investors consider to be important, including skewness and kurtosis.

Thus, minimum-variance hedging will not in general yield a hedge portfolio with minimum risk. In particular, although minimum-variance hedging unambiguously reduces the standard deviation of portfolio returns, its effect on skewness and kurtosis is ambiguous. Indeed, it is shown below both theoretically and empirically that minimum-variance hedging can potentially *increase* left-skewness and kurtosis, and so the effect of minimum-variance hedging on the risk of the hedge portfolio more generally is uncertain.[1]

This article considers the consequences of minimum-variance hedging in two alternative frameworks that implicitly incorporate portfolio skewness and kurtosis. The first is value at risk (VaR), which is defined as the maximum loss on a portfolio over a certain period of time that can be expected with a certain probability. When returns are normally distributed and the mean return is assumed to be zero (as is commonly the case in practice when dealing with short-horizon returns), the VaR of a portfolio is simply a constant multiple of the standard deviation of the portfolio. However, when the return distribution is nonnormal, the VaR of a portfolio is determined not just by the portfolio standard deviation but by the distribution of returns, including its skewness and kurtosis. VaR is now perhaps the most widely used risk measure among practitioners, largely because of its adoption by the Basle Committee on Banking Regulation (1996) for the assessment of the risk of the proprietary trading books of banks and its use in setting risk capital requirements (see Jorion, 2000).

Increasingly, VaR is also being used as an input to asset allocation and performance evaluation. For example, Alexander and Baptista (2002) propose a mean-VaR framework that is analogous to the traditional mean-variance framework, but in which portfolios are located in mean-VaR space instead of mean–standard-deviation space. Under certain conditions, they prove the concavity of the mean-VaR efficient set and the existence of a CAPM equilibrium when agents have mean-VaR objective functions.[2] A number of authors (Alexander & Baptista, 2004; Campbell, Huisman, & Koedijk, 2001; Huisman, Koedijk, & Pownall, 1999) have suggested combining the two frameworks by solving the portfolio optimization problem in the mean-variance framework but with an additional constraint on the maximum portfolio VaR. In terms of performance evaluation, Alexander and Baptista (2003) introduce a VaR-based measure of portfolio performance that is analogous to the

[1]Hereafter, the term *increase left skewness* is used to mean a reduction in the value of the skewness coefficient.
[2]See also Tokat, Rachev, and Schwartz (2003).

Sharpe ratio. They show that when returns are nonnormal, the risk-taking incentives of a portfolio manager in a VaR-based risk-management system can be substantially different from the incentives in a Sharpe ratio-based system.

Although now widely employed by both financial and nonfinancial institutions, VaR has a shortcoming in that it ignores the expected size of a loss in the event that the VaR of a portfolio is exceeded. One consequence of this is that when returns are not drawn from a multivariate elliptical distribution, VaR is not a *coherent* measure of risk and, in particular, is not subadditive (see, for example, Artzner, Delbaen, Eber, & Heath, 1999). The second measure of risk that is explicitly employed takes this shortcoming of VaR into account. This measure is conditional VaR (CVaR) or expected shortfall, which is defined as the expected loss on the portfolio, conditional on this loss being less than or equal to the portfolio VaR (see, for example, Tasche, 2002). Alexander and Baptista (2004) compare minimum VaR and minimum CVaR portfolios and examine the impact of both VaR and CVaR constraints in the mean-variance model. Under certain conditions, they show that the use of CVaR as a measure to control risk is more effective than the use of VaR.

This article considers two questions. The first is whether the substantial reduction in portfolio standard deviation that can be gained from minimum-variance hedging offers a similarly large reduction in portfolio VaR and CVaR. This question is answered empirically with the use of data on 20 cross-hedged developed-market currency portfolios. Specifically, minimum-variance hedge portfolios are constructed and the percentage reduction in standard deviation is compared with the percentage reduction in VaR and CVaR. *Ex ante*, it is not obvious what to expect: Hedging reduces portfolio standard deviation but can simultaneously increase left skewness and kurtosis, and so the overall effect on the quantile of the hedge portfolio return distribution (which determines VaR), and on the expectation of the returns that are less than or equal to this quantile (which determines CVaR), is uncertain. The effectiveness of the minimum-variance hedging strategy is investigated by considering both in-sample and out-of-sample performance. It is found that minimum-variance hedging yields, on average, a reduction in both VaR and CVaR that is about 80% of the reduction in standard deviation, implying that minimum-variance hedge portfolios are riskier than conventional measures of risk would imply. In some cases, the differences can be substantial. For example, for a long position in the Singapore dollar hedged with a short position in the U.S. dollar, the relative reduction in standard deviation is almost 30%, whereas that in VaR and CVaR is about 5%.

The second question is whether greater reductions in portfolio VaR and CVaR can be achieved by explicitly minimizing VaR and CVaR, respectively. Increasingly, institutions face constraints on their investments that are specified in terms of VaR or CVaR rather than standard deviation. For example, as noted above, the regulatory capital associated with a bank's trading book depends on its VaR, and traders may face VaR constraints on their individual positions. Consequently, minimum-VaR or minimum-CVaR hedging may be a more appropriate objective than minimum-variance hedging in many situations. A historical simulation approach is used to address this question. Specifically, the hedge ratio that minimizes (the negative of) the appropriate quantile of the empirical distribution of hedged portfolio returns for the minimum-VaR hedging strategy, and (the negative of) the mean of the returns that are less than or equal to this quantile for the minimum-CVaR hedging strategy are estimated. Again, both in-sample and out-of-sample performance are considered. It is found that minimum-VaR and minimum-CVaR hedge ratios are, on average, about 10% lower than minimum-variance hedge ratios, suggesting that smaller short positions are required to minimize VaR or CVaR than to minimize variance. The in-sample results, which show the maximum attainable reduction in VaR and CVaR with the use of a static hedging strategy, suggest that in terms of VaR and CVaR reduction, minimum-VaR and minimum-CVaR hedging can potentially yield small but consistent improvements over minimum-variance hedging. The out-of-sample results are more mixed, although there is a small improvement for minimum-VaR hedging for the majority of the currencies considered.

The outline of this article is as follows. The next section discusses the theoretical distribution of minimum-variance hedge portfolio returns. This is followed by a discussion of the data and methodology. The empirical results for the minimum-variance, minimum-VaR, and minimum-CVaR hedging strategies are then reported. The last section provides a conclusion.

THE DISTRIBUTION OF MINIMUM-VARIANCE HEDGE PORTFOLIO RETURNS

This section summarizes the principles of minimum-variance hedging and derives expressions for the standard deviation, skewness, and kurtosis of the minimum-variance hedge portfolio. The aim of this section is to show that although minimum-variance hedging necessarily reduces the standard deviation of portfolio returns, its effect on skewness and kurtosis—and hence on portfolio VaR and CVaR—is ambiguous. Suppose that there are two assets, Asset 1 and Asset 2, with per-period returns r_1 and r_2, and that a short position in Asset 2 is used to hedge a long position in Asset 1. It is assumed that the mean return for both assets is zero.[3] The hedge portfolio, given by a long position in Asset 1 and a fraction h of a short position in Asset 2, has a return equal to

$$r_p = r_1 - hr_2 \tag{1}$$

The variance of the hedge portfolio return is given by

$$\sigma_p^2 = \sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2 \tag{2}$$

where σ_1^2 is the variance of r_1, σ_2^2 is the variance of r_2, and $\rho_{1,2}$ is the correlation coefficient between r_1 and r_2. The skewness coefficient of the hedge portfolio is given by

$$\begin{aligned} s_p &= \frac{E[r_p^3]}{\sigma_p^3} \\ &= \frac{s_1\sigma_1^3 - 3hs_a\sigma_1^2\sigma_2 + 3h^2s_b\sigma_1\sigma_2^2 - h^3s_2\sigma_2^3}{(\sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2)^{3/2}} \end{aligned} \tag{3}$$

where the skewness and co-skewness coefficients of the two assets are defined by

$$s_1 = \frac{E[r_1^3]}{\sigma_1^3}, \qquad s_2 = \frac{E[r_2^3]}{\sigma_2^3}, \qquad s_a = \frac{E[r_1^2r_2]}{\sigma_1^2\sigma_2}, \qquad s_b = \frac{E[r_1r_2^2]}{\sigma_1\sigma_2^2} \tag{4}$$

The kurtosis coefficient of the hedge portfolio is given by

$$\begin{aligned} k_p &= \frac{E[r_p^4]}{\sigma_p^4} \\ &= \frac{k_1\sigma_1^4 - 4hk_a\sigma_1^3\sigma_2 + 6h^2k_b\sigma_1^2\sigma_2^2 - 4h^3k_c\sigma_1\sigma_2^3 + h^4k_2\sigma_2^4}{(\sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2)^2} \end{aligned} \tag{5}$$

where the kurtosis and co-kurtosis coefficients of the two assets are defined by

$$k_1 = \frac{E[r_1^4]}{\sigma_1^4}, \quad k_2 = \frac{E[r_2^4]}{\sigma_2^4}, \quad k_a = \frac{E[r_1^3r_2]}{\sigma_1^3\sigma_2}, \quad k_b = \frac{E[r_1^2r_2^2]}{\sigma_1^2\sigma_2^2}, \quad k_c = \frac{E[r_1r_2^3]}{\sigma_1\sigma_2^3} \tag{6}$$

[3]This is a common assumption when dealing with daily financial asset returns, but could easily be relaxed to allow for a nonzero mean.

The standard deviation, skewness, and kurtosis of the minimum-variance hedge portfolio are now derived. The minimum-variance hedge ratio is the value of h that minimizes (2), which is

$$h = \rho_{1,2}\frac{\sigma_1}{\sigma_2} \tag{7}$$

(see, for example, Hull, 2003). If (7) is substituted into (2), the standard deviation of the minimum-variance hedge portfolio is given by

$$\sigma_p = \sigma_1(1 - \rho_{1,2}^2)^{1/2} \tag{8}$$

The correlation coefficient, $\rho_{1,2}$, is bounded by plus one and minus one, and so minimum-variance hedging will always reduce the standard deviation of the hedge portfolio relative to that of Asset 1, except in the extreme case that the returns of Asset 1 and Asset 2 are uncorrelated. Substituting (7) into (3) and (5), the skewness and kurtosis coefficients of the minimum-variance hedge portfolio are given by

$$s_p = \frac{s_1 - 3\rho_{1,2}s_a + 3\rho_{1,2}^2 s_b - \rho_{1,2}^3 s_2}{(1 - \rho_{1,2}^2)^{3/2}} \tag{9}$$

$$k_p = \frac{k_1 - 4\rho_{1,2}k_a + 6\rho_{1,2}^2 k_b - 4\rho_{1,2}^3 k_c + \rho_{1,2}^4 k_2}{(1 - \rho_{1,2}^2)^2} \tag{10}$$

In contrast with expression (8), which shows that hedging unambiguously reduces standard deviation, expressions (9) and (10) do not allow us to easily establish how hedging affects skewness and kurtosis. In particular, setting the first derivatives of (9) and (10) to zero does not yield closed-form solutions for $\rho_{1,2}$, and hence maximal (or, in the case of negative skewness, minimal) values of s_p and k_p. However, it is possible to show that, in general, s_p and k_p are not bounded by s_1 and k_1, respectively, and so it is possible that minimum-variance hedging can potentially *increase* the magnitude of the skewness and kurtosis coefficients relative to those of Asset 1. To see this, note that for uncorrelated assets (i.e., when $\rho_{1,2} = 0$), the skewness and kurtosis coefficients are simply equal to the skewness and kurtosis coefficients of Asset 1, namely, s_1 and k_1. s_1 is assumed to be positive, although an analogous argument can be applied when s_1 is negative. For s_1 and k_1 to be the maxima of s_p and k_p, the first derivatives of (9) and (10) would have to be equal to zero at $\rho_{1,2} = 0$. However, differentiating (9) and (10) with respect to $\rho_{1,2}$ and evaluating for $\rho_{1,2} = 0$ yields $-3s_a$ and $-4k_a$, respectively. For these derivatives to be zero, it would be necessary that $s_a = k_a = 0$, which would be the case, for example, if r_1 and r_2 were stochastically independent, or were drawn from a multivariate elliptical distribution. More generally, however, s_a and k_a will not be zero, and so neither will be the derivatives, in which case, s_1 and k_1 cannot, in general, be the maxima of s_p and k_p, respectively. Intuitively, although the two assets may be highly correlated, there may be instances where a large outlying return on the long asset is not matched by a correspondingly large return on the short asset. In this case, the return on the hedge portfolio will be of the same order of magnitude as the return on the long asset, but because the hedge portfolio has a much smaller standard deviation, this return will be proportionately much larger, leading to an increase in the kurtosis coefficient of the hedge portfolio. If there is a preponderance of either positive or negative returns for which this is true, then the absolute value of the skewness coefficient will rise also.

Thus, although minimum-variance hedging will unambiguously reduce the standard deviation of the hedge portfolio relative to that of Asset 1, the skewness and kurtosis coefficients of the minimum-variance hedge portfolio can be larger in magnitude than those of Asset 1, depending on the joint distribution of r_1 and r_2. Hence the effect of minimum-variance hedging on quantile-based measures of risk that incorporate investors' preferences over skewness and kurtosis will be uncertain.

Value at Risk

One approach to measuring risk that implicitly incorporates agents' preferences over skewness and kurtosis in portfolio returns is value at risk (VaR). VaR, which has its origins in the safety-first criterion of Roy (1952), is defined as the largest loss on a portfolio that can be expected with a particular probability over a certain horizon. When returns are normally distributed, and assumed to have zero mean (as is commonly the case in practice when dealing with short horizon returns), the VaR of a portfolio is simply a constant multiple of the portfolio standard deviation, where the multiple is determined by the VaR confidence level. More generally, however, the VaR of a portfolio is given by

$$\mathrm{VaR}(F_r, p) = -F_r^{-1}(1 - p) \tag{11}$$

where F_r is the cumulative distribution function (CDF) of portfolio returns r, and p is the VaR confidence level. Thus, the VaR of a portfolio depends on both the confidence level and the distribution of returns. Consequently, a portfolio that has a relatively low standard deviation could have a relatively high VaR depending on the skewness and kurtosis of returns and the confidence level.

Conditional Value at Risk

A shortcoming of VaR is that it does not take into account the expected size of a loss in the event that this loss exceeds the VaR of the portfolio. A consequence of this is that when portfolio returns are not drawn from a multivariate elliptical distribution, VaR is not a coherent measure of risk. In particular, it does not have the desirable property of subadditivity, meaning that it is possible for the VaR of a portfolio to exceed the weighted average VaR of the assets that it comprises (see, for example, Artzner et al., 1999; Tasche, 2002). An alternative measure of risk that addresses this shortcoming of VaR is conditional value at risk (CVaR) or expected shortfall. The CVaR of a portfolio is given by

$$\mathrm{CVaR}(F_r, p) = -E[r \mid r \leq -\mathrm{Var}] = -\frac{\int_{-\infty}^{-\mathrm{VaR}} z f_r(z)\, dz}{F_r(-\mathrm{VaR})} \qquad (12)$$

where f_r is the probability density function (PDF) of r. Thus, although VaR measures the maximum loss expected in normal market conditions (where normal market conditions are defined as those that occur a fraction $1 - p$ of the time), CVaR measures the mean loss in abnormal market conditions. As with VaR, CVaR is a function of both the confidence level and the distribution of returns, and so portfolios with low standard deviation can potentially have high CVaR depending on the skewness and kurtosis of returns and the confidence level. A subsequent section investigates, empirically, how the distribution of returns, and in particular their skewness and kurtosis, is affected by minimum-variance hedging and the consequences for portfolio risk as measured by VaR and CVaR. The effectiveness of hedging strategies that explicitly minimize VaR and CVaR are also investigated.

DATA AND METHODOLOGY

Data

In the empirical analysis, the risk reduction for 20 cross-hedged currency portfolios is considered. Specifically, it is assumed that a GBP investor has a long position in each of 10 foreign currencies and hedges the risk exposure of this position with the use of a short position in either one or two of the remaining 10 currencies.[4] Although in many cases it would be more effective to hedge a long currency position with the use of currency futures (see Duffie, 1998, Section 7.5, for tools and difficulties associated with hedging in futures markets), there are situations where currency cross-hedging may be appropriate (see Solnik & McLeavy, 2003, p. 582). For example, many corporations have exposure to two or more currencies simultaneously, and an efficient approach to hedging this exposure is to first exploit the natural cross hedge that arises from the nonzero correlation between the different currency exposures, and then to use derivatives to hedge the residual risk.[5] Moreover, the investigation of cross hedging is relevant to the activities of a currency hedge fund that is, for example, long in an undervalued currency and short in an overvalued currency.

The empirical analysis, uses daily returns provided by Reuters for 10 developed-market currencies (AUD, CAD, EUR, JPY, NZD, NOK, SGD, SEK, CHF, and USD) measured against the GBP for the period 03/01/1994 to 09/07/2004 (a total of 2745 observations), which is the longest common sample available.[6] The mid-rates recorded each day at 4:00 p.m. in London are used. From the quoted mid-rates, 2744 continuously compounded daily returns are calculated. In the empirical evaluation, both in-sample and out-of-sample analyses are conducted. The first 1000 observations are reserved in order to estimate the out-of-sample hedge ratio over the subsequent period. This yields a test period of 03/11/1997 to 09/07/2004 (a total of 1744 observations). The same test period is used for both the in-sample and out-of-sample analyses to enable a comparison between the two. Table I gives summary statistics for the 10 currency return series for the test period, including the mean, standard deviation, skewness, and excess kurtosis coefficients and the Bera-Jarque statistic. For all 10 series, the mean daily return is very close to zero. The standard deviation of returns lies between 0.47% for EUR and 0.77% for JPY. The skewness coefficient ranges from −0.32 for

[4]Standard ISO currency abbreviations are used: AUD is the Australian dollar, CAD is the Canadian dollar, EUR is the Euro, GBP is the British pound, JPY is the Japanese yen, NZD is the New Zealand dollar, NOK is the Norwegian krone, SGD is the Singapore dollar, SEK is the Swedish krona, CHF is the Swiss franc, and USD is the U.S. dollar.
[5]This is borne out by survey evidence. See "Survey of derivatives usage by U.S. nonfinancial firms," conducted by the Weiss Center for International Financial Research, Wharton School, and CIBC World Markets, 1998.
[6]The EUR exchange rate before its inception on 01/01/1999 is a synthetic rate computed by Reuters using the entry weights of the EUR countries.

TABLE I

Summary Statistics

	Mean	*SD*	*Skewness*	*Kurtosis*	*Bera-Jarque*	*99% VaR*	*99% CVaR*
AUD	0.00%	0.72%	0.02	2.23	368.9	1.69%	2.19%
CAD	0.00%	0.58%	0.20	1.06	138.4	1.34%	1.56%
EUR	0.00%	0.47%	−0.19	1.01	19.1	1.28%	1.49%
JPY	0.00%	0.77%	−0.32	3.99	1065.0	2.06%	2.75%
NZD	0.00%	0.76%	0.11	2.23	395.7	1.74%	2.34%
NOK	0.00%	0.54%	0.40	7.37	4061.8	1.22%	1.64%
SGD	0.01%	0.58%	−0.24	3.14	648.4	1.48%	2.07%
SEK	0.01%	0.55%	−0.07	1.55	153.9	1.29%	1.71%
CHF	0.00%	0.52%	−0.15	1.26	73.1	1.43%	1.70%
USD	0.01%	0.48%	0.11	1.00	105.5	1.15%	1.35%
Average	0.00%	0.60%	−0.01	2.48	702.98	1.47%	1.88%

Note. The table reports the mean, standard deviation, skewness coefficient, excess kurtosis coefficient, Bera-Jarque statistic, and 99% 1-day VaR and CVaR for daily continuously compounded returns. The sample period is 03/11/1997 to 09/07/2004. The Bera-Jarque statistic has a chi-squared distribution with two degrees of freedom under the null hypothesis that returns are normally distributed.

JPY to 0.40 for NOK. For 5 of the 10 series, the skewness coefficient is negative. All 10 series are highly leptokurtic, with excess kurtosis coefficients ranging from 1.00 for USD to 7.37 for NOK. The Bera-Jarque statistic rejects the null hypothesis of normality for all 10 series at conventional significance levels. The last two columns report the historical simulation 99% 1-day VaR and CVaR of each currency. VaR is computed as the negative of the 1% quantile of the historical distribution of returns over the test period, whereas CVaR is computed as the negative of the mean of the returns that are less than or equal to the 1% quantile (see Jorion, 2000). It is interesting to note that for some currencies, the assumption of normality is quite accurate in spite of the nonnormality of returns. For example, for the CAD, the normal-VaR of the portfolio is equal to $2.326 * 0.58\% = 1.35\%$, which almost coincides with the historical simulation VaR. For other currencies, however, the normality assumption leads to a less precise estimate of historical VaR. This illustrates that the relation between skewness, kurtosis, and VaR depends on the distribution of returns. When the confidence level is sufficiently larger than 99%, the normality assumption is not a good approximation even for estimating VaR for CAD, because its distribution is highly leptokurtic, highlighting the fact that the relation between kurtosis and VaR depends on the confidence level.

Methodology

For a long position in each of the 10 currencies, short positions in (a) one other currency and (b) two other currencies are used. This yields a total of 20 cross-hedged currency portfolios. To construct the hedge portfolio for each currency, the one-currency and two-currency short positions that offer the maximum explanatory power in a linear regression over the entire sample of 2744 observations, as measured by the R-squared coefficient, are used. From the discussion above, this ensures, for each currency, the one-currency and two-currency hedge portfolios that yield the maximum reduction in standard deviation relative to the long position within the sample used to estimate the hedge ratio(s). For the 20 selected currency portfolios, the R-squared coefficients (which are not reported) range from 0.33 (for JPY and SGD) to 0.76 (for EUR and CHF) for the two-currency hedge portfolios, and from 0.39 (for JPY, CHF, and SGD) to 0.81 (for EUR, SEK, and CHF) for the three-currency hedge portfolios.

For each of the 10 currencies, the minimum-variance, minimum-VaR, and minimum-CVaR hedge ratios for the 10 two-currency hedge portfolios and the 10 three-currency hedge portfolios are computed. The ordinary-least-squares estimate of the slope coefficient between the long currency and the short currency is used to estimate the minimum-variance hedge ratio,. To estimate the minimum-VaR hedge ratio, an arbitrary hedge ratio is chosen, hedge portfolio returns are calculated, and the historical simulation approach is used to estimate the VaR of the resulting hedge portfolio. Then a numerical optimization procedure is used in order to estimate the value of the hedge ratio that minimizes the hedge-portfolio VaR. This is the minimum-VaR hedge ratio. Although the historical simulation VaR of the hedge portfolio is not generally a globally convex function of the hedge ratio, a grid search approach quickly leads to a global minimum. The minimum-CVaR hedge ratio is calculated by the same numerical procedure, but with simulation CVaR rather than VaR minimized.

In the empirical analysis, two approaches are used to construct the hedged portfolio. The first is a static hedging strategy in which a single hedge ratio (or pair of hedge ratios for the three-currency portfolio) is estimated with the use of the entire test sample and used to construct the hedge portfolio on each day within the test sample. By construction, this gives the maximum attainable reduction in standard deviation, VaR or CVaR, with the use of a static hedge ratio. The second is a dynamic hedging strategy in which the hedge ratio is estimated every 250 days throughout the test sample with the use of the returns from the previous 1000 days.[7] The estimated hedge ratio is then used to construct the hedge portfolio for the following 250 days.

EMPIRICAL RESULTS

Minimum-Variance Hedging

Panel A of Table II reports the in-sample performance of the static minimum-variance hedging strategy for both the two-currency and three-currency hedge portfolios, and Panel B reports the out-of-sample performance of the dynamic hedging strategy. In each case, the table reports the estimated hedge ratio (in the case of the dynamic hedging strategy, the average estimated hedge ratio), the skewness and excess kurtosis coefficients of the hedged portfolio, and the percentage change in standard deviation, 99% VaR and 99% CVaR relative to the unhedged currency. The table also gives the averages of each of these values.

For the in-sample hedging strategy, minimum-variance hedging yields a substantial reduction in portfolio standard deviation in all cases, owing to the relatively high correlation between the currencies. For the two-currency hedge portfolio, this reduction ranges from 20.2% for the JPY/SGD to 54.5% for the EUR/CHF, and for the three-currency hedge portfolio, it ranges from 21.8% for the JPY/CHF/SGD to 60.1% for the EUR/CHF/SEK. In general, the addition of a third currency to the hedge portfolio leads to a modest improvement in hedge portfolio performance. In particular, the average reduction in standard deviation is 35.6% for the two-currency hedge portfolio, and 39.6% for the three-currency hedge portfolio. Minimum-variance hedging increases left-skewness for 10 of the 20 portfolios, and increases kurtosis for 18 portfolios. In some cases the increase in skewness and/or kurtosis is substantial, such as for the SGD and NOK. The average skewness coefficient is −0.01 for the two-currency hedge portfolio and 0.02 for the three-currency hedge portfolio, compared with −0.01 for the unhedged currency. The average excess kurtosis coefficient is 6.10 for the two-currency hedge portfolio and 5.57 for the three-currency hedge portfolio, compared with 2.48 for the unhedged currency. These results confirm the findings of the previous section that show that minimum-variance hedging, while unambiguously reducing standard deviation, can potentially increase left skewness and kurtosis.

[7]Window lengths of 250, 500, and 1500 observations were also tried, but the results, which are available from the authors, are qualitatively very similar.

TABLE II

Hedging Effectiveness of Minimum-Variance Hedge Portfolio

		Two-currency hedge portfolio						*Three-currency hedge portfolio*							
Long	*Short*	*HR*	*Skew*	*Kurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*	*Short*	*HR1*	*HR2*	*Skew*	*Kurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*
Panel A: In-sample effectiveness															
AUD	NZD	0.75	−0.11	1.93	−39.6	−31.2	−34.0	NZD, CAD	0.67	0.26	−0.13	1.84	−42.6	−35.2	−37.1
CAD	USD	0.86	0.06	1.25	−30.5	−22.2	−22.4	USD, AUD	0.74	0.23	0.07	1.18	−36.0	−29.6	−30.1
EUR	CHF	0.81	0.54	5.12	−54.5	−57.6	−57.0	CHF, SEK	0.66	0.23	0.19	2.71	−60.1	−61.9	−60.5
JPY	SGD	0.81	−0.56	4.55	−20.2	−15.5	−14.8	CHF, SGD	0.24	0.77	−0.51	4.00	−21.8	−14.6	−16.4
NZD	AUD	0.84	0.18	2.48	−39.6	−31.8	−35.6	SGD, AUD	0.10	0.81	0.21	2.56	−40.0	−33.3	−36.4
NOK	EUR	0.82	0.99	20.35	−29.0	−29.6	−23.0	SEK, EUR	0.38	0.50	1.19	21.90	−34.3	−38.8	−28.2
SGD	USD	0.83	−0.95	13.87	−27.9	−4.8	−5.7	NZD, USD	0.20	0.74	−0.68	9.74	−32.2	−14.5	−16.3
SEK	EUR	0.83	0.17	3.12	−29.8	−21.3	−27.0	NOK, EUR	0.38	0.52	0.13	2.75	−35.1	−26.9	−32.3
CHF	EUR	0.98	−0.54	6.90	−54.5	−55.3	−46.2	EUR, AUD	1.00	−0.04	−0.54	6.87	−54.8	−55.2	−46.6
USD	CAD	0.60	0.14	1.39	−30.5	−28.7	−22.7	CAD, SGD	0.40	0.35	0.31	2.18	−39.3	−31.3	−33.5
	Average	0.81	−0.01	6.10	−35.6	−29.8	−28.8	Average	0.48	0.44	0.02	5.57	−39.6	−34.1	−33.7
Panel B: Out-of-sample effectiveness															
AUD	NZD	0.77	−0.09	1.82	−39.4	−29.5	−32.8	NZD, CAD	0.65	0.29	−0.12	1.79	−42.4	−35.5	−37.2
CAD	USD	1.00	0.01	1.36	−30.5	−25.5	−20.0	USD, AUD	0.89	0.15	0.02	1.35	−35.3	−28.5	−27.7
EUR	CHF	0.76	0.41	4.74	−52.6	−55.1	−54.9	CHF, SEK	0.63	0.22	0.04	2.71	−58.5	−60.3	−57.4
JPY	SGD	0.78	−0.55	4.75	−20.5	−15.1	−15.6	CHF, SGD	0.32	0.71	−0.39	3.49	−20.8	−11.7	−16.4
NZD	AUD	0.77	0.10	2.86	−38.5	−28.7	−33.8	SGD, AUD	0.20	0.68	0.20	2.91	−38.5	−30.1	−34.2
NOK	EUR	0.87	0.98	18.46	−28.4	−29.6	−23.8	SEK, EUR	0.34	0.59	1.06	20.36	−31.8	−34.9	−25.5
SGD	USD	0.87	−0.91	13.99	−27.9	−3.6	−5.5	NZD, USD	0.22	0.72	−0.68	9.84	−26.3	−8.7	−10.7
SEK	EUR	0.80	0.15	3.14	−29.7	−19.2	−26.6	NOK, EUR	0.43	0.43	0.09	2.74	−34.7	−27.2	−32.5
CHF	EUR	1.04	−0.54	6.21	−53.4	−54.6	−45.4	EUR, AUD	1.06	−0.05	−0.53	6.23	−53.3	−54.6	−45.3
USD	CAD	0.67	0.13	1.48	−30.0	−26.4	−21.5	CAD, SGD	0.47	0.32	0.42	3.58	−36.0	−34.5	−29.5
	Average	0.83	−0.03	5.88	−35.1	−28.7	−28.0	Average	0.52	0.41	0.01	5.50	−37.8	−32.6	−31.6

Note. The table reports the minimum-variance hedge ratio(s) [for the out-of-sample results, the average hedge ratio(s)]; the skewness coefficient; the excess kurtosis coefficient and the percentage change in standard deviation; 1-day 99% VaR; and 1-day 99% CVaR, relative to the long currency, for each minimum-variance hedge portfolio and for the average across the 10 two-currency hedge portfolios and across the 10 three-currency hedge portfolios. Panel A reports the in-sample results, which are based on a static hedging strategy that uses a single hedge ratio used to construct the hedge portfolio on each day during the test period, 03/11/1997 to 09/07/2004. Panel B reports the out-of-sample results, which are based on a dynamic hedging strategy that uses a hedge ratio estimated every 250 days, with the use of the previous 1000 days' observations.

The effect of minimum-variance hedging on the VaR and CVaR of the hedge portfolio is now considered. For 14 of the 20 portfolios, the reduction in VaR is less than the reduction in standard deviation, and in some cases the difference is large. For example, for the SGD/USD, minimum-variance hedging reduces standard deviation by 27.9%, but reduces VaR by only 4.8%, and CVaR by 5.7%. The average reduction in VaR is 29.8% for the two-currency hedge portfolios and 34.1% for the three-currency hedge portfolios. The reduction in CVaR is lower than the reduction in standard deviation for 18 of the 20 hedge portfolios. The average reduction in CVaR is 28.8% for the two-currency hedge portfolio and 33.7% for the three-currency hedge portfolio.

For the out-of-sample hedging strategy, the picture is similar. Minimum-variance hedging yields an average reduction in standard deviation of 35.1% for the two-currency portfolio and 37.8% for the three-currency portfolio, which is marginally lower than the in-sample reduction in standard deviation. Similarly, minimum-variance hedging increases left skewness in 11 out of 20 cases and increases kurtosis in 17 out of 20 cases. The average skewness coefficient is −0.03 for the two-currency hedge portfolio and 0.01 for the three-currency hedge portfolio, and the average excess kurtosis coefficient is 5.88 for the two-currency hedge portfolio and 5.50 for the three-currency hedge portfolio. The reduction in VaR is lower than the reduction in standard deviation for 14 of the 20 portfolios, and the reduction in CVaR is lower than the reduction in standard deviation for 19 of the 20 portfolios. For the two-currency portfolios, VaR is reduced, on average, by 28.7% and CVaR by 28.0%. For the three-currency portfolios, VaR is reduced by 32.6% and CVaR by 31.6%.

The conclusion of this section, therefore, is that although minimum-variance hedging inevitably reduces portfolio standard deviation, in many cases it tends to increase left skewness and/or increase kurtosis. The consequence is that the reduction in VaR and CVaR is in most cases less than the reduction in standard deviation. For the two-currency hedge portfolio, the average reduction in VaR and CVaR is about 80% of the average reduction in standard deviation, whereas for the three-currency hedge portfolio, the average reduction in VaR and CVaR is about 85% of the average reduction in standard deviation.

Minimum-VaR Hedging

This section evaluates the performance of the minimum-VaR hedging strategy for the 20 hedged currency portfolios. Panel A of Table III reports the in-sample results. Comparing Table III with Table II, it can be seen

TABLE III

Hedging Effectiveness of Minimum-VaR Hedge Portfolio

	Two-currency hedge portfolio							Three-currency hedge portfolio							
Long	*Short*	*HR*	*Skew*	*Kurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*	*Short*	*HR1*	*HR2*	*Skew*	*Kurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*
Panel A: In-sample effectiveness															
AUD	NZD	0.64	−0.08	2.12	−38.5	−33.3	−34.9	NZD, CAD	0.55	0.43	−0.10	1.77	−40.7	−37.0	−36.2
CAD	USD	0.59	0.16	1.01	−27.0	−26.2	−22.8	USD, AUD	0.58	0.16	0.15	1.03	−33.5	−37.3	−29.8
EUR	CHF	0.91	0.60	6.49	−53.1	−57.6	−53.7	CHF, SEK	0.63	0.23	0.17	2.45	−59.9	−62.6	−60.9
JPY	SGD	0.67	−0.52	4.66	−19.6	−16.7	−15.7	CHF, SGD	0.34	0.89	−0.48	3.43	−20.3	−18.5	−16.4
NZD	AUD	0.76	0.18	2.68	−39.1	−33.0	−35.2	SGD, AUD	0.10	0.80	0.21	2.57	−40.0	−33.4	−36.4
NOK	EUR	0.72	0.98	20.70	−28.4	−31.2	−22.8	SEK, EUR	0.40	0.50	1.19	21.51	−34.3	−39.0	−28.0
SGD	USD	0.45	−0.66	8.77	−21.1	−8.0	−5.8	NZD, USD	0.27	0.79	−0.55	7.38	−31.1	−20.4	−18.5
SEK	EUR	0.86	0.18	3.11	−29.8	−23.6	−27.1	NOK, EUR	0.31	0.25	0.07	2.54	−28.6	−30.2	−27.3
CHF	EUR	0.91	−0.47	6.62	−54.1	−56.2	−46.8	EUR, AUD	1.01	−0.11	−0.52	6.33	−53.8	−57.9	−46.7
USD	CAD	0.59	0.14	1.38	−30.5	−29.2	−23.0	CAD, SGD	0.38	0.23	0.20	1.26	−37.4	−36.5	−33.9
	Average	0.71	0.05	5.75	−34.1	−31.5	−28.8	Average	0.46	0.42	0.03	5.03	−38.0	−37.3	−33.4
Panel B: Out-of-sample effectiveness															
AUD	NZD	0.66	−0.10	2.15	−38.9	−30.8	−34.6	NZD, CAD	0.51	0.32	−0.09	1.74	−41.0	−35.6	−37.0
CAD	USD	0.98	0.01	1.38	−30.3	−21.0	−20.1	USD, AUD	0.78	0.16	0.01	1.36	−35.1	−31.1	−26.6
EUR	CHF	0.77	0.43	5.59	−52.7	−56.8	−53.9	CHF, SEK	0.71	0.17	0.23	4.54	−57.4	−62.0	−56.7
JPY	SGD	0.74	−0.48	4.66	−18.8	−15.9	−16.0	CHF, SGD	0.37	0.70	−0.35	2.93	−20.8	−11.3	−17.3
NZD	AUD	0.69	0.16	2.93	−37.1	−28.0	−33.1	SGD, AUD	0.21	0.70	0.16	2.59	−36.5	−24.9	−31.3
NOK	EUR	0.83	0.94	18.18	−27.0	−27.8	−23.0	SEK, EUR	0.37	0.55	1.02	20.25	−30.1	−30.6	−24.7
SGD	USD	0.62	−0.73	10.91	−24.0	−4.7	−5.9	NZD, USD	0.23	0.42	−0.61	8.87	−7.2	−5.5	−8.7
SEK	EUR	0.65	0.12	3.04	−27.7	−20.4	−23.4	NOK, EUR	0.29	0.40	0.11	2.82	−30.5	−28.4	−28.8
CHF	EUR	1.06	−0.48	5.40	−52.2	−53.5	−45.6	EUR, AUD	1.13	−0.21	−0.56	5.19	−45.5	−53.7	−40.8
USD	CAD	0.66	0.14	1.49	−29.3	−26.5	−22.3	CAD, SGD	0.44	0.32	0.33	2.61	−38.9	−38.0	−34.1
	Average	0.77	0.00	5.57	−33.8	−28.6	−27.8	Average	0.51	0.35	0.03	5.29	−34.3	−32.1	−28.9

Note. The table reports the minimum-VaR hedge ratio(s) [for the out-of-sample results, the average hedge ratio(s)]; the skewness coefficient; the excess kurtosis coefficient and the percentage change in standard deviation; 1-day 99% VaR; and 1-day 99% CVaR, relative to the long currency, for each minimum-VaR hedge portfolio and for the average across the 10 two-currency hedge portfolios and across the 10 three-currency hedge portfolios. Panel A reports the in-sample results, which are based on a static hedging strategy that uses a single hedge ratio used to construct the hedge portfolio on each day during the test period, 03/11/1997 to 09/07/2004. Panel B reports the out-of-sample results, which are based on a dynamic hedging strategy that uses a hedge ratio estimated every 250 days, with the use of the previous 1000 days' observations.

that the estimated minimum-VaR hedge ratios are generally lower than the corresponding minimum-variance hedge ratios, suggesting that minimizing VaR typically involves taking a smaller short position. In some cases, the differences are quite large. For example, for the SGD/USD, the minimum-variance hedge ratio is 0.83, and the minimum-VaR hedge ratio is only 0.45. For the two-currency hedge portfolios, the average minimum-VaR hedge ratio is 0.71 compared to the average minimum-variance hedge ratio of 0.81. For the three-currency portfolios, the average minimum-VaR hedge ratios are 0.46 and 0.42, compared with average minimum-variance hedge ratios of 0.48 and 0.44.

Compared with minimum-variance hedging, minimum-VaR hedging yields a smaller increase in left-skewness for 12 of the 20 portfolios and a smaller increase in kurtosis for 14 of the 20 portfolios. The average skewness coefficient is 0.05 for the two-currency hedge portfolio and 0.03 for the three-currency portfolios (compared with −0.01 and 0.02 for minimum-variance hedging), whereas the average excess kurtosis coefficient is 5.75 and 5.03, respectively (compared with 6.10 and 5.57 for minimum-variance hedging). This is consistent with the idea that minimum-variance hedging attaches no importance to skewness or kurtosis and hence potentially does not lead to a minimum-VaR portfolio. Indeed, the VaR of the minimum-VaR hedge portfolio is in every case lower than that of the minimum-variance hedge portfolio. In many cases, the differences are not particularly large, but for several of the assets, the reduction is more substantial. For example, for the CAD/USD/AUD portfolio, minimum-variance hedging leads to a 29.6% reduction in VaR, and minimum-VaR hedging leads to a reduction of 37.3%. In several cases, such as AUD/NZD and EUR/CHF, minimum-VaR hedging yields a larger increase in kurtosis than minimum-variance hedging, but a smaller increase in left-skewness. In these cases, either the skewness effect outweighs the kurtosis effect to yield a larger reduction in VaR, or the confidence level is such that increasing kurtosis *reduces* the quantile of the distribution. The average reduction in VaR is 31.5% for the two-currency portfolios and 37.3% for the three-currency portfolios (compared with 29.8% and 34.1%, respectively, for minimum-variance hedging). Thus, based on the in-sample results, minimum-VaR hedging potentially leads to small improvements in risk reduction in terms of VaR. By construction, these improvements are the maximum attainable using a static minimum-VaR hedge ratio.

Panel B of Table III reports the out-of-sample results for the minimum-VaR hedging strategy. As with the in-sample results, minimum-VaR hedge ratios are generally lower than minimum-variance hedge ratios. Minimum-VaR hedging leads to a smaller increase in left-skewness in 11 of the 20 portfolios and a smaller increase in kurtosis in 12 of the 20 portfolios. The average skewness and excess kurtosis coefficients are 0.00 and 5.57, respectively, for the two-currency portfolios (compared with −0.03 and 5.88 for minimum-variance hedging) and 0.03 and 5.29 for the three-currency portfolios (compared with 0.01 and 5.50 for minimum-variance hedging). This leads to a greater reduction in VaR for 6 of the 10 two-currency portfolios and 5 of the 10 three-currency portfolios. However, where there are improvements, they tend to be less marked than for the in-sample strategy. In some isolated cases, such as AUD/NZD, minimum-VaR hedging yields a larger increase in both kurtosis and left-skewness (although the differences are generally small) and, simultaneously, a larger reduction in VaR. In these cases, it is possible that hedging has an effect on the higher moments of returns that outweighs both the skewness and kurtosis effects. On average, the reduction in VaR is 28.6% for the two-currency portfolios and 32.1% for the three-currency portfolios (compared with 28.7% and 32.6%, respectively, for minimum-variance hedging). Thus, in summary, minimum-VaR hedging yields hedge portfolios that are typically less negatively skewed and/or less leptokurtic than minimum-variance hedging, both in sample and out of sample. This generates a modest improvement in VaR reduction for the majority of the currencies considered.

Minimum-CVaR Hedging

Panel A of Table IV reports the in-sample results for the minimum-CVaR hedging strategy. The results are similar to those for the minimum-VaR hedging strategy. Minimum-CVaR hedge ratios are typically lower than minimum-variance hedge ratios, although marginally higher than minimum-VaR hedge ratios. Compared with minimum-VaR hedging, minimum-CVaR hedging yields an even smaller increase in kurtosis for all 20 portfolios, and for the three-currency hedge portfolios, it yields a smaller increase in left-skewness also. For the three-currency hedge portfolios, the average skewness and excess kurtosis coefficients are 0.08 and 4.75, compared to 0.03 and 5.03 for minimum-VaR hedging, and 0.02 and 5.57 for minimum-variance hedging. The average reduction in CVaR is 29.5% for the two-currency hedge portfolios and 35.1% for the three-currency hedge portfolios, compared with 28.8% and 33.7%, respectively, for minimum-variance hedging. Thus, based on the in-sample results, minimum-CVaR hedging yields small improvements in the reduction of CVaR compared to minimum-variance hedging. Again,

TABLE IV

Hedging Effectiveness of Minimum-CVaR Hedge Portfolio

		Two-currency hedge portfolio						*Three-currency hedge portfolio*							
Long	*Short*	*HR*	*Skew*	*Kurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*	*Short*	*HR1*	*HR2*	*Skew*	*Kurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*
Panel A: In-sample effectiveness															
AUD	NZD	0.67	−0.09	2.06	−39.0	−32.9	−35.1	NZD, CAD	0.64	0.23	−0.12	1.97	−42.5	−34.4	−37.2
CAD	USD	0.72	0.11	1.10	−29.6	−25.3	−24.0	USD, AUD	0.64	0.26	0.10	1.08	−35.5	−32.7	−30.7
EUR	CHF	0.77	0.49	4.41	−54.3	−57.0	−57.3	CHF, SEK	0.60	0.23	0.12	2.12	−59.4	−61.0	−61.2
JPY	SGD	0.65	−0.52	4.67	−19.4	−16.3	−15.8	CHF, SGD	0.28	0.60	−0.45	4.01	−20.8	−15.5	−17.1
NZD	AUD	0.96	0.17	2.17	−38.6	−30.4	−35.9	SGD, AUD	0.17	0.89	0.19	2.24	−38.9	−30.5	−37.2
NOK	EUR	0.87	0.98	19.66	−28.8	−29.3	−23.0	SEK, EUR	0.36	0.45	1.20	22.57	−34.1	−38.6	−28.6
SGD	USD	0.63	−0.85	11.96	−26.1	−6.2	−5.9	NZD, USD	0.44	0.35	−0.24	2.60	−21.9	−14.1	−24.7
SEK	EUR	0.85	0.18	3.11	−29.8	−23.5	−27.2	NOK, EUR	0.30	0.64	0.17	2.83	−34.7	−26.9	−32.7
CHF	EUR	0.90	−0.46	6.54	−53.9	−56.0	−46.8	EUR, AUD	0.92	−0.06	−0.46	6.44	−54.0	−55.8	−47.2
USD	CAD	0.47	0.14	1.18	−28.7	−27.3	−24.1	CAD, SGD	0.46	0.28	0.25	1.65	−38.9	−33.4	−34.7
	Average	0.75	0.01	5.69	−34.8	−30.4	−29.5	Average	0.48	0.39	0.08	4.75	−38.1	−34.3	−35.1
Panel B: Out-of-sample effectiveness															
AUD	NZD	0.68	−0.07	1.84	−39.1	−32.8	−35.1	NZD, CAD	0.62	0.25	−0.14	2.55	−40.1	−33.4	−34.2
CAD	USD	0.94	0.02	1.53	−30.5	−24.5	−19.1	USD, AUD	0.84	0.19	0.04	1.40	−35.4	−29.6	−27.2
EUR	CHF	0.76	0.47	4.79	−54.1	−57.5	−56.7	CHF, SEK	0.66	0.18	0.16	3.52	−59.3	−61.2	−58.1
JPY	SGD	0.65	−0.50	5.29	−18.7	−15.3	−14.4	CHF, SGD	0.41	0.67	−0.37	3.42	−19.9	−12.9	−16.7
NZD	AUD	0.83	0.05	2.79	−37.3	−28.8	−32.4	SGD, AUD	0.20	0.74	0.14	2.73	−38.0	−27.9	−33.0
NOK	EUR	0.97	0.89	15.52	−25.7	−29.4	−22.3	SEK, EUR	0.42	0.48	1.10	20.15	−32.4	−31.8	−26.3
SGD	USD	0.69	−0.82	12.41	−25.6	−4.1	−5.2	NZD, USD	0.35	0.45	−0.43	5.02	−16.8	−4.3	−11.0
SEK	EUR	0.80	0.10	3.16	−29.2	−18.8	−25.1	NOK, EUR	0.37	0.47	0.08	3.04	−32.9	−29.1	−30.1
CHF	EUR	1.12	−0.43	4.86	−49.8	−52.8	−43.5	EUR, AUD	1.12	−0.07	−0.49	5.46	−46.3	−51.0	−40.2
USD	CAD	0.64	0.13	1.49	−30.4	−28.5	−22.3	CAD, SGD	0.49	0.27	0.35	2.71	−36.6	−35.4	−32.0
	Average	0.81	−0.02	5.37	−34.1	−29.2	−27.6	Average	0.55	0.36	0.04	5.00	−35.8	−31.7	−30.9

Note. The table reports the minimum-CVaR hedge ratio(s) [for the out-of-sample results, the average hedge ratio(s)]; the skewness coefficient; the excess kurtosis coefficient and the percentage change in standard deviation; 1-day 99% VaR; and 1-day 99% CVaR, relative to the long currency, for each minimum-CVaR hedge portfolio and for the average across the 10 two-currency hedge portfolios and across the 10 three-currency hedge portfolios. Panel A reports the in-sample results, which are based on a static hedging strategy that uses a single hedge ratio used to construct the hedge portfolio on each day during the test period, 03/11/1997 to 09/07/2004. Panel B reports the out-of-sample results, which are based on a dynamic hedging strategy that uses a hedge ratio estimated every 250 days, with the use of the previous 1000 days' observations.

by construction, these improvements are the maximum attainable using a static minimum-CVaR hedge ratio.

Panel B of Table IV reports the out-of-sample results for the minimum-CVaR hedging strategy. Again, the results are similar to those for the minimum-VaR hedging strategy. Compared with minimum-variance hedging, minimum-CVaR hedging yields a larger reduction in left skewness for 11 of the 20 portfolios, and a larger reduction in kurtosis for 10 of the 20 portfolios. For the three-currency hedge portfolio, the average skewness and excess kurtosis coefficients are 0.04 and 5.00, compared to 0.03 and 5.29 for minimum-VaR hedging, and 0.01 and 5.50 for minimum-variance hedging. For 8 of the 20 portfolios, this yields a larger reduction in CVaR, but for the remaining 12 portfolios, minimum-CVaR hedging yields CVaR that is higher than that obtained by minimum-variance hedging. The average reduction in CVaR is 27.6% for the two-currency hedge portfolios and 30.9% for the three-currency hedge portfolios, compared with 28.0% and 31.6%, respectively, for minimum-variance hedging.

CONCLUSION

This article examines the performance of minimum-variance hedging in the VaR and CVaR frameworks. With the use of data on 20 cross-hedged developed market currency portfolios, it was found that although minimum-variance hedging reduces portfolio standard deviation, it can increase left skewness and kurtosis. Consequently, the reduction in VaR and CVaR from minimum-variance hedging is typically lower than the reduction in standard deviation. The finding that minimum-variance hedging in many cases increases portfolio kurtosis is consistent with, and indeed helps to explain the fact that the returns of hedge funds, many of which take a combination of long and short positions either in the same market or across different markets, tend to be highly leptokurtic (see, for example, Kat & Lu, 2002). Relatedly, Agarwal and Naik (2004) use a mean-CVaR framework in order to analyze the risk of hedge funds and show that the mean-variance framework significantly underestimates tail risk. Again, this is consistent with the finding of this article that hedging generally increases left-skewness and/or kurtosis.

The extent to which minimum-VaR and minimum-CVaR hedging strategies are able to improve upon minimum-variance hedging strategies in the VaR and CVaR frameworks was also investigated with the use of a historical-simulation approach. It was found that minimum-VaR and minimum-CVaR hedge ratios are typically lower than minimum-variance hedge ratios, suggesting that smaller short positions are typically required to minimize VaR or CVaR than to minimize variance. Minimum-VaR and minimum-CVaR hedge portfolios tend to be less left-skewed and/or less leptokurtic than minimum-variance hedge portfolios. In-sample, minimum-VaR and minimum-CVaR hedging strategies yield modest improvements in VaR and CVaR reduction, compared with minimum-variance hedging. However, out of sample, the results are more mixed, although there is a small improvement for minimum-VaR hedging for the majority of the currencies that considered.

Interesting avenues for future research would include the investigation of the consequences of hedging with the use of derivative instruments, such as futures, and the investigation of hedging in other asset markets, such as equities and bonds. Also of interest would be the hedging of derivative portfolios in the VaR and CVaR frameworks. The return distribution of underlying assets such as currencies, equities, and bonds are leptokurtic but approximately symmetric, and so hedging tends to have a larger impact on the kurtosis of the portfolio than on the skewness. In contrast, nonlinear portfolios, such as those that contain options, have return distributions that are typically heavily skewed, and so hedging may have even more impact on these portfolios. Finally, this article has used the historical-simulation approach to analyze the consequences of hedging in the VaR framework. It would be natural to investigate alternative parametric approaches to the estimation of VaR and CVaR and the derivation of analytical minimum-VaR and minimum-CVaR hedge ratios.

BIBLIOGRAPHY

Agarwal, R., Rao, R., & Hiraki, T. (1989). Skewness and kurtosis in Japanese equity returns. Journal of Financial Research, 12, 253–260.

Agarwal, V., & Naik, N. (2004). Risks and portfolio decisions involving hedge funds. Review of Financial Studies, 17, 63–98.

Alexander, G., & Baptista, A. (2002). Economic implications of using a mean-VaR model for portfolio selection: A comparison with mean-variance analysis. Journal of Economic Dynamics and Control, 26, 1159–1193.

Alexander, G., & Baptista, A. (2003). Portfolio performance evaluation using value-at-risk. Journal of Portfolio Management, 29, 93–102.

Alexander, G., & Baptista, A. (2004). A comparison of VaR and CVaR constraints on portfolio selection with the mean-variance model. Management Science, 50, 1261–1273.

Arditti, F. (1967). Risk and the required rate of return. Journal of Finance, 22, 19–36.

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent measures of risk. Mathematical Finance, 9, 203–228.

Basle Committee on Banking Supervision. (1996, January). Overview of the amendment to the capital accord to incorporate market risks.

Campbell, R., Huisman, R., & Koedijk, K. (2001). Optimal portfolio selection in a value-at-risk framework. Journal of Banking and Finance, 25, 1789–1804.

Dittmar, R. (2002). Non-linear pricing kernels, kurtosis preference and the cross-section of equity returns. Journal of Finance, 57, 369–403.

Duffie, D. (1998). Futures markets. Englewood Cliffs, NJ: Prentice Hall.

Fang, H., & Lai, T. Y. (1997). Co-kurtosis and capital asset pricing. Financial Review, 32, 293–307.

Huisman, R., Koedijk, K., & Pownall, R. (1999). Asset allocation in a value at risk framework. Working paper, Erasmus University, Rotterdam.

Hull, J. (2003). Options, futures and other derivatives. New York: McGraw Hill.

Ingersoll, J. (1987). The theory of financial decision making. Lanham, MA: Rowman and Littlefield.

Jorion, P. (2000). Value at risk. New York: McGraw Hill.

Kane, A. (1982). Skewness preference and portfolio choice. Journal of Financial and Quantitative Analysis, 17, 15–25.

Kat, H., & Lu, S. (2002). An excursion into the statistical properties of hedge fund returns. Working paper, University of Reading, Reading, United Kingdom.

Kraus, A., & Litzenberger, R. (1976). Skewness preference and the valuation of risky assets. Journal of Finance, 31, 1085–1100.

Roy, A. (1952). Safety first and the holding of assets. Econometrica, 20, 431–449.

Scott, R., & Horvath, P. (1980). On the direction of preference for moments of higher order than the variance. Journal of Finance, 35, 915–919.

Simkowitz, M., & Beedles, W. (1978). Diversification in a three-moment world. Journal of Financial and Quantitative Analysis, 13, 927–941.

Solnik, B., & McLeavy, D. (2003). International investments. Reading, MA: Addison Wesley.

Tang, G., & Choi, D. (1998). Impact of diversification on the distribution of stock returns: International evidence. Journal of Economics and Business, 22, 119–127.

Tasche, D. (2002). Expected shortfall and beyond. Journal of Banking and Finance, 26, 1519–1533.

Tokat, Y., Rachev, S., & Schwartz, E. (2003). The stable non-Gaussian asset allocation: A comparison with the classical Gaussian approach. Journal of Economic Dynamics and Control, 27, 937–969.