HEDGING AND VALUE AT RISK: A SEMI-PARAMETRIC APPROACH

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The non-normality of financial asset returns has important implications for hedging. In particular, in contrast with the unambiguous effect that minimum-variance hedging has on the standard deviation, it can actually increase the negative skewness and kurtosis of hedge portfolio returns. Thus, the reduction in Value at Risk (VaR) and Conditional Value at Risk (CVaR) that minimum-variance hedging generates can be significantly lower than the reduction in standard deviation. In this study, we provide a new, semi-parametric method of estimating minimum-VaR and minimum-CVaR hedge ratios based on the Cornish-Fisher expansion of the quantile of the hedged portfolio return distribution. Using spot and futures returns for the FTSE 100, FTSE 250, and FTSE Small Cap equity indices, the Euro/US Dollar exchange rate, and Brent crude oil, we find that the semiparametric approach is superior to the standard minimum-variance approach, and to the nonparametric approach of Harris and Shen (2006). In particular, it provides

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a greater reduction in both negative skewness and excess kurtosis, and consequently generates hedge portfolios that in most cases have lower VaR and CVaR. Jrl Fut Mark 30:780–794, 2010

INTRODUCTION

The motivation for minimum-variance hedging and the benefits that it provides are by now well established in the literature. However, minimum-variance hedging is optimal from a risk reduction perspective only when investors have quadratic utility or when returns are drawn from a multivariate elliptical distribution. In practice, neither of these assumptions is likely to hold.[1] When investors do not have quadratic utility and returns are not elliptically distributed, variance is no longer an appropriate measure of risk since it ignores the higher moments of the return distribution. This has led to the emergence of alternative measures of risk. Of these, perhaps the most widely used are Value at Risk (VaR) and Conditional Value at Risk (CVaR). The importance of VaR stems from its adoption by the Basle Committee on Banking Regulation for the assessment of the risk of the proprietary trading books of banks and its use in setting risk capital requirements (see Basle Committee, 1996; Jorion, 2008). Increasingly, VaR and CVaR are also now used in asset allocation (Huisman, Koedijk, & Pownall, 1999; Campbell, Huisman, & Koedijk, 2001; Alexander & Baptista, 2002, 2004) and portfolio performance measurement (Dowd, 1998; Alexander & Baptista, 2003).

In the context of hedging, Harris and Shen (2006) developed minimum-VaR and minimum-CVaR hedge ratios, estimated nonparametrically using historical simulation. The nonelliptical nature of returns is particularly important in the hedging context because while minimum-variance hedging unambiguously reduces portfolio variance, it can actually increase negative skewness and kurtosis, leading to portfolios that are riskier when measured by VaR or CVaR than when measured by variance. Harris and Shen show that that minimum-VaR and minimum-CVaR hedging generates modest out-of-sample improvements in the VaR and CVaR of the hedge portfolio, respectively, relative to minimum-variance hedging. However, a drawback of the nonparametric approach—in common with the historical simulation approach on which it is based—is its reliance on a large historical sample of data on spot and futures returns. This is unavoidable since by construction it is only possible to

[1] If returns are drawn from an elliptical distribution, the higher moments of returns are preserved when stocks are formed into portfolios. However, the evidence suggests that this is not the case (see, for example, Simkowitz & Beedles, 1978; Agarwal et al, 1989; Tang & Choi, 1998). A number of authors have also shown that investors have preferences over the higher moments of returns, which is inconsistent with quadratic utility (see, for example, Kraus & Litzenberger, 1976; Fang & Lai, 1997; Dittmar, 2002).

accurately measure the empirical frequency of a relatively rare event (such as those that are the focus of VaR and CVaR) by using a sample of data in which there are sufficient occurrences of such events. One is then left with a trade-off between using a large sample to improve the accuracy of the estimated minimum-VaR and minimum-CVaR hedge ratios, and a small sample to capture time-variation in the true hedge ratios.

In this study, we develop an alternative, semiparametric approach for estimating the minimum-VaR and minimum-CVaR hedge ratios that is much less dependent on historical data. The approach is based on the Cornish and Fisher (1937) expansion, which approximates the quantile of a standardized probability distribution by adjusting the corresponding quantile of the standard normal distribution using the higher moments of the distribution. The Cornish–Fisher expansion is already widely used to estimate the VaR and CVaR of long-only portfolios (see, for example, Zangari, 1996; Bali, Gokcan, & Liang, 2007; Jorion, 2008) and so it is natural to extend its use to the case of hedging.

Using the Cornish–Fisher expansion, we first derive an analytical expression for the first order conditions to the minimum-VaR and minimum-CVaR problems. We then solve these first order conditions numerically, using sample estimates of the variance, skewness and kurtosis of spot and futures returns in place of their population counterparts. Although, like the historical simulation approach of Harris and Shen (2006), our approach requires historical data, a much shorter sample is needed because it does not rely solely on the tails of the empirical distribution. We apply the semiparametric approach to dynamically hedge long positions in the FTSE 100, FTSE 250, and FTSE Small Cap equity indices, the Euro/US Dollar exchange rate, and Brent crude oil. As a benchmark, we compare the semiparametric approach with the nonparametric approach of Harris and Shen (2006). We find that the semiparametric minimum-VaR and minimum-CVaR hedging offers significant improvements in VaR and CVaR reduction relative to minimum-variance hedging, and that it works well with relatively short samples of historical data.

The outline of this study is as follows. In the following section we discuss the theoretical background to minimum-variance, minimum-VaR, and minimum-CVaR hedging and introduce the semiparametric approach. The later section discusses the data and methodology. The penultimate section reports the empirical results. The final section concludes.

THEORETICAL BACKGROUND

As in Harris and Shen (2006), suppose that there are two assets, Asset 1 and Asset 2, with per-period returns r_1 and r_2, and that a short position in Asset 2 is used to hedge a long position in Asset 1. We assume that the mean return for both assets is zero.[2] The hedge portfolio, given by a long position in Asset 1 and a fraction, h, of a short position in Asset 2, has a return equal to

$$r_p = r_1 - hr_2 \tag{1}$$

The variance of the hedge portfolio return is given by

$$\sigma_p^2 = \sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2 \tag{2}$$

where σ_1^2 is the variance of r_1, σ_2^2 is the variance of r_2, and $\rho_{1,2}$ is the correlation coefficient between r_1 and r_2. The minimum-variance hedge ratio is the value of h that minimizes (2), which is easily shown to be

$$h_\sigma = \rho_{1,2}\frac{\sigma_1}{\sigma_2} \tag{3}$$

Minimum-variance hedging, while commonly used in practice, ignores the higher moments of the hedge portfolio return distribution, in particular its skewness and kurtosis. The skewness coefficient of the hedge portfolio is given by

$$\begin{aligned} s_p &= \frac{E[r_p^3]}{\sigma_p^3} \\ &= \frac{s_1\sigma_1^3 - 3hs_a\sigma_1^2\sigma_2 + 3h^2s_b\sigma_1\sigma_2^2 - h^3s_2\sigma_2^3}{(\sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2)^{3/2}} \end{aligned} \tag{4}$$

where the skewness and co-skewness coefficients of the two assets are defined by

$$s_1 = \frac{E[r_1^3]}{\sigma_1^3},\ s_2 = \frac{E[r_2^3]}{\sigma_2^3},\ s_a = \frac{E[r_1^2r_2]}{\sigma_1^2\sigma_2},\ s_b = \frac{E[r_1r_2^2]}{\sigma_1\sigma_2^2} \tag{5}$$

The kurtosis coefficient of the hedge portfolio is given by

$$\begin{aligned} k_p &= \frac{E[r_p^4]}{\sigma_p^4} \\ &= \frac{k_1\sigma_1^4 - 4hk_a\sigma_1^3\sigma_2 + 6h^2k_b\sigma_1^2\sigma_2^2 - 4h^3k_c\sigma_1\sigma_2^3 + h^4k_2\sigma_2^4}{(\sigma_1^2 + h^2\sigma_2^2 - 2h\rho_{1,2}\sigma_1\sigma_2)^2} \end{aligned} \tag{6}$$

where the kurtosis and co-kurtosis coefficients of the two assets are defined by

$$k_1 = \frac{E[r_1^4]}{\sigma_1^4},\ k_2 = \frac{E[r_2^4]}{\sigma_2^4},\ k_a = \frac{E[r_1^3r_2]}{\sigma_1^3\sigma_2},\ k_b = \frac{E[r_1^2r_2^2]}{\sigma_1^2\sigma_2^2},\ k_c = \frac{E[r_1r_2^3]}{\sigma_1\sigma_2^3} \tag{7}$$

[2]This is a common assumption when dealing with daily financial asset returns, but could easily be relaxed to allow for a non-zero mean.

Value at Risk

One approach to measuring risk that implicitly incorporates the higher moments of portfolio returns is VaR. VaR is defined as the largest loss on a portfolio that can be expected with a particular probability over a certain horizon. If returns are drawn from a location-scale family distribution, when the mean return is zero, the $(1 - \alpha)$ percent VaR of a portfolio can be written as

$$VaR_p(1 - \alpha) = -\sigma_p q_p(\alpha) \tag{8}$$

where $q_p(\alpha)$ is the α percent quantile of the standardized distribution (i.e. zero mean and unit variance) of hedge portfolio returns and σ_p is the standard deviation of the hedge portfolio. When returns are normally distributed, the VaR of a portfolio is simply a constant multiple of the standard deviation of portfolio returns adjusted by the portfolio mean return. An analytical expression for the minimum-VaR hedge ratio can be derived from the Cornish–Fisher expansion, which approximates the quantile, $q_p(\alpha)$using the higher moments of the distribution of hedge portfolio returns. In particular, using the skewness and kurtosis of the return distribution, the Cornish–Fisher expansion approximates $q_p(\alpha)$ by

$$\begin{aligned}\tilde{q}_p(\alpha; s_p, k_p) &= c(\alpha) + \frac{1}{6}[c(\alpha)^2 - 1]s_p + \frac{1}{24}[c(\alpha)^3 - 3c(\alpha)](k_p - 3) \\ &\quad - \frac{1}{36}[2c(\alpha)^3 - 5c(\alpha)]s_p^2\end{aligned} \tag{9}$$

where $c(\alpha)$ is the α percent quantile of the standard normal distribution, and s_p and k_p are given by Equations (4) and (6) above, respectively. The Cornish–Fisher approximation for the VaR of the hedge portfolio is then given by

$$VaR_p(1 - \alpha) = -\sigma_p \tilde{q}_p(\alpha; s_p, k_p) \tag{10}$$

To derive the minimum-VaR hedge ratio, we differentiate (10) with respect to the hedge ratio, h, and set the first derivative equal to zero. This yields the following first order condition:

$$\begin{aligned}&\frac{\partial \sigma_p}{\partial h}(A_1 + A_2 s_p + A_3 k_p + A_4 s_p^2) \\ &\quad + \sigma_p\left(A_2 \frac{\partial s_p}{\partial h} + A_3 \frac{\partial k_p}{\partial h} + 2A_4 s_p \frac{\partial s_p}{\partial h}\right) = 0\end{aligned} \tag{11}$$

where $A_1 = c(\alpha) - \frac{1}{8}[c(\alpha)^3 - 3c(\alpha)]$, $A_2 = \frac{1}{6}[c(\alpha)^2 - 1]$, $A_3 = \frac{1}{24}[c(\alpha)^3 - 3c(\alpha)]$, and $A_4 = -\frac{1}{36}[2c(\alpha)^3 - 5c(\alpha)]$. We replace the population moments in (11) with their sample estimates, and solve for the minimum-VaR hedge ratio, h_{VaR}, numerically.

Conditional Value at Risk

A shortcoming of VaR is that it does not take into account the expected size of a loss in the event that this loss exceeds the VaR of the portfolio. An alternative measure of risk that addresses this shortcoming is CVaR or Expected Shortfall (see, for example, Tasche, 2002). The CVaR of a portfolio is given by

$$\begin{aligned} CVaR_p(1-\alpha) &= \frac{1}{\alpha}\int_{1-\alpha}^{1} VaR(x)dx \\ &= -\frac{\sigma_p}{\alpha}\int_{1-\alpha}^{1} q_p^x dx \end{aligned} \tag{12}$$

where the second equality holds for any location-scale family distribution. Using the Cornish–Fisher expansion, this can be approximated as

$$\begin{aligned} CVaR_p(1-\alpha) = &-\sigma_p\bigg(M_1 + \frac{1}{6}(M_2-1)s_p \\ &+ \frac{1}{24}(M_3 - 3M_1)k_p - \frac{1}{36}(2M_3 - 5M_1)s_p^2\bigg) \end{aligned} \tag{13}$$

where $M_i = \frac{1}{\alpha}\int_{-\infty}^{c(\alpha)} x^i f(x)dx$ and $f(\cdot)$ is the standard normal probability density function. To derive the minimum-CVaR hedge ratio, we differentiate (13) with respect to the hedge ratio, h, and set the first derivative equal to zero. This yields the following first order condition:

$$\begin{aligned} &\frac{\partial\sigma_p}{\partial h}(B_1 + B_2 s_p + B_3 k_p + B_4 s_p^2) \\ &+ \sigma_p\bigg(B_2\frac{\partial s_p}{\partial h} + B_3\frac{\partial k_p}{\partial h} + 2B_4 s_p\frac{\partial s_p}{\partial h}\bigg) = 0 \end{aligned} \tag{14}$$

where $B_1 = M_1 - \frac{1}{8}[M_3 - 3M_1]$, $B_2 = \frac{1}{6}[M_2 - 1]$, $B_3 = \frac{1}{24}[M_3 - 3M_1]$, and $A_4 = -\frac{1}{36}[2M_3 - 5M_1]$. We again replace the population moments in (14) with their sample estimates, and solve for the minimum-CVaR hedge ratio, h_{CVaR}, numerically.

DATA AND METHODOLOGY

Data

In the empirical analysis, we dynamically hedge long positions in the FTSE 100, FTSE 250, and FTSE Small Cap equity indices, the Euro/US Dollar exchange rate, and Brent crude oil. To hedge each long position, we use a short position in the corresponding futures contract, except for the FTSE Small Cap index (for which there is no corresponding futures contract), where we use either the FTSE 100 or FTSE 250 futures contract. This yields a total of six hedged portfolios. In each case, we use the nearest futures contract to delivery, rolling over to the next nearest contract to delivery on the first day of the delivery month. Daily data were obtained from DataStream for the period February 2, 1994 to August 8, 2008 (the longest common sample available), yielding a total of 3,778 observations. We use the daily closing price for the spot indices and the corresponding daily settlement price for the futures contracts. We use simple returns to estimate the minimum-variance, minimum-VaR, and minimum-CVaR hedge ratios.

Panel A of Table I gives summary statistics of the five spot and four futures series, including the mean, standard deviation, skewness and kurtosis coefficients, and the Bera-Jarque statistic. All the series are leptokurtic, and all except the Euro/US Dollar exchange rate are negatively skewed. The Bera-Jarque statistic rejects the null hypothesis of normality for all nine series at conventional significance levels. The largest departure from normality is for the Brent crude oil spot and futures series, which display very significant leptokurtosis. The Brent crude oil spot and futures series are also significantly more volatile than the other series. The last two columns report the 99% one-day VaR and CVaR of each currency. These are estimated using historical simulation. Panel B of Table I reports the correlations for each pair of spot and futures series that are used in the empirical analysis. The highest correlations between spot and futures returns are for the FTSE 100 and FTSE 250 indices, while the lowest is between the FTSE Small Cap spot returns and the FTSE 100 futures returns.

To motivate the use of the semiparametric approach, we evaluate the VaR forecasting performance for the nine spot and futures series. Specifically, we calculate 99% VaR using (i) the parametric approach, assuming that returns are normally distributed, (ii) the nonparametric approach based on the historical distribution of returns, and (iii) the semiparametric approach based on the Cornish–Fisher expansion given by Equations (9) and (10) above. On each day t, we estimate the 99% 1-day VaR using the previous 250 days' observations from t-250 to t-1. Following Bali (2003), we count the number of VaR exceptions

TABLE I
Summary Statistics and Correlations

	Mean	*SD*	*Skewness*	*Kurtosis*	*Bera-Jarque*	*VaR*	*CVaR*
Panel A: Summary Statistics							
Spot							
FTSE100	0.01%	1.07%	−0.20	5.98	1,418	3.04%	3.88%
FTSE250	0.02%	0.81%	−0.48	7.21	2,923	2.65%	3.21%
FTSE SC	0.01%	0.58%	−1.44	12.55	15,648	1.93%	2.79%
EUR/USD	0.00%	0.62%	0.12	5.45	706	1.56%	1.97%
Oil	0.02%	2.43%	−0.91	20.99	70,565	6.56%	9.59%
Futures							
FTSE100	0.01%	1.12%	−0.13	5.58	1,058	3.22%	3.87%
FTSE250	0.02%	0.83%	−0.43	6.82	2,413	2.61%	3.20%
Euro/Dollar	0.00%	0.63%	−0.07	4.58	294	1.68%	2.09%
Oil	0.02%	2.34%	−1.3	27.95	135,930	6.33%	9.45%

Panel B: Correlations

	Futures			
	FTSE100	FTSE250	EUR/USD	Oil
Spot				
FTSE100	0.97	–	–	–
FTSE250	–	0.95	–	–
FTSE SC	0.57	0.80	–	–
EUR/USD	–	–	0.80	–
Oil	–	–	–	0.88

Notes. Panel A reports the mean, standard deviation, skewness coefficient, excess kurtosis coefficient, Bera-Jarque statistic, and 99% one-day VaR and CVaR for daily returns. Panel B reports the correlations of each of the spot series with corresponding futures contracts. The Bera-Jarque statistic has a χ^2 distribution with two degrees of freedom under the null hypothesis that returns are normally distributed.

over the whole sample $t = 251$ to 3,778. As a measure of unconditional coverage, we calculate the percentage difference between the empirical exception rate and the nominal exception rate. To test the null hypothesis of correct conditional coverage, we employ the likelihood ratio statistic of Christoffersen (1998).

Table II reports the percentage exception measure and the LR statistic for the five spot and four futures series, for each of the three estimation approaches. In almost all cases, the empirical exception rate exceeds the nominal exception rate, implying that the estimated VaR is typically too low. The under-estimation of VaR is particularly pronounced for the parametric approach. This is to be expected given the non-normality of returns reported in Table I. For seven of the nine series, the empirical exception rate is more than twice the nominal exception rate, and the null hypothesis of correct conditional coverage is rejected for all nine series at the 1% significance level. The nonparametric approach, which is based on the empirical distribution of returns, provides a considerable

TABLE II
VaR Forecast Performance

	Semiparametric		*Nonparametric*		*Parametric*	
	Count	*LR*	*Count*	*LR*	*Count*	*LR*
Spot						
FTSE 100	55.94%	12.93***	41.76%	7.14**	143.83%	60.25***
FTSE 250	13.41%	3.58	53.10%	15.30***	186.37%	110.63***
FTSE SC	0.77%	18.68***	53.10%	27.70***	217.55%	157.11***
EUR/USD	59.91%	8.03**	56.01%	8.21**	63.80%	10.241***
Oil	25.58%	10.01***	55.97%	17.58***	86.34%	42.62***
Futures						
FTSE 100	53.11%	8.68**	38.93%	6.20**	132.49%	51.53***
FTSE 250	13.41%	3.58	38.93%	6.56**	169.35%	86.64***
EUR/USD	56.01%	8.21	40.41%	4.78*	110.61%	26.36***
Oil	35.71%	11.69***	76.22%	28.79***	108.63%	49.74***
Average	34.87%	9.49	50.49%	13.58	135.44%	66.12

Notes. The table reports the percentage difference between the empirical exception rate and the nominal exception rate and the likelihood ratio statistic to test the null hypothesis of correct conditional coverage for the semiparametric approach, the nonparametric approach, and the parametric approach, for the nine spot and futures series. ***, **, and * indicate significance at the one percent, five percent, and ten percent levels, respectively.

improvement over the parametric approach. In particular, the null hypothesis of correct conditional coverage cannot be rejected at the 1% significance level in five of the nine cases, and at the 5% level in one of the nine cases. In all nine cases, the empirical exception rate is much closer to the nominal exception rate. On average, the empirical exception rate exceeds the nominal exception rate by about 50%, compared with 135% for the parametric approach. The use of the semiparametric approach provides a further improvement in VaR performance. On average across the nine series, the empirical exception rate exceeds the nominal exception rate by about 35%. The null hypothesis of correct conditional coverage cannot be rejected for five of the nine series at the 1% significance level, and for two of the nine series at the 5% significance level. Thus, it is clear that both the nonparametric and semiparametric approaches provide a substantial improvement in terms of VaR forecast performance over the parametric approach, and that on balance, the semiparametric approach outperforms the nonparametric approach.

Hedging Methodology

To hedge a long position in a given asset we take a short position in the corresponding futures contract. The exception is for long positions in the FTSE Small Cap index where we hedge by taking a short position in either the FTSE 100

TABLE III
Minimum-Variance Hedging

Long	*Short*	*HR*	*ΔSkew*	*ΔKurt*	*%ΔSD*	*%ΔVaR*	*%ΔCVaR*
FTSE 100	FTSE 100	0.91	−0.23	2.13	−66.80	−49.82	−37.36
FTSE 250	FTSE 250	0.86	0.23	5.12	−66.80	−49.82	−37.36
FTSE SC	FTSE 100	0.27	0.13	−0.45	−17.38	−19.35	−19.24
FTSE SC	FTSE 250	0.53	0.63	−4.62	−37.90	−45.34	−46.74
Oil	Oil	0.92	0.25	6.73	−52.72	−12.95	8.92
EUR/USD	EUR/USD	0.77	−0.12	0.56	−40.56	−33.87	−30.57
Average		0.71	0.15	1.58	−47.03	−35.19	−27.06

Notes. The table reports the average hedge ratio, the change in the skewness and excess kurtosis coefficients, and the percentage change in standard deviation, one-day 99% VaR and one-day 99% CVaR, relative to the long position, for each minimum-variance hedge portfolio. The results are based on a dynamic hedging strategy that uses a hedge ratio estimated every 250 days, using the previous 250 days observations.

or FTSE 250 futures index. This yields six hedge portfolios in total. We reserve the first 250 observations for initial estimation of the minimum-variance hedge ratio using Equation (7), the minimum-VaR hedge ratio using Equation (11), and the minimum-CVaR hedge ratio using Equation (14). These are then used to construct the hedge portfolio for $t = 251, \ldots 500$. We then re-compute the three hedge ratios using data for the period $t = 251, \ldots 500$ and use these to construct the hedge portfolio for $t = 501, \ldots 750$, and so on. This yields a total sample of 3,527 out-of-sample hedge portfolio returns. As a benchmark, we also employ the historical simulation approach of Harris and Shen (2006), which estimates the minimum-VaR and minimum-CVaR hedge ratios by minimizing the in-sample VaR and CVaR, respectively, at each step.

EMPIRICAL RESULTS

Minimum-Variance Hedging

Table III reports the out-of-sample performance of the minimum-variance hedging strategy for the six hedge portfolios. In each case, the table reports the average estimated hedge ratio, the change in the skewness, and excess kurtosis coefficients of the hedge portfolio relative to the unhedged position, and the percentage change in standard deviation, 99% VaR and 99% CVaR. The table also gives the averages of each of these values. In all cases, the standard deviation is significantly reduced reflecting the relatively high correlations between the spot and futures series. The largest reductions are for the FTSE 100 and FTSE 250 equity indices. In contrast with the diversification effects on standard deviation, in two of the six cases, the skewness of the hedge portfolio is more negative than the skewness of the unhedged position, and in four of the six cases, the kurtosis of the hedge portfolio is greater than that of the unhedged position. The largest increase in kurtosis is for the FTSE 250 index and Brent crude oil. Interestingly, the two cases where kurtosis is reduced are those with the lowest correlations, namely the FTSE Small Cap index hedged with the FTSE 100 and FTSE 250 futures contracts. On average across the six hedge portfolios, negative skewness is reduced by 0.15, but kurtosis increases by 1.58. The consequence is that in terms of VaR and CVaR, the reduction in standard deviation that arises from hedging is offset by an increase in negative skewness and/or kurtosis in four of the six cases and hence the reduction in VaR and CVaR is less than the reduction in standard deviation. For Brent crude oil the difference is substantial. Indeed, in this case, minimum-variance hedging actually *increases* CVaR. On average, the reduction in VaR and CVaR relative to the unhedged position is 35.2% and 27.1%, respectively, compared to the reduction in standard deviation of 47.0%. These results are similar to those reported by Harris and Shen (2006), who analyze the risk reduction of cross-hedged currency portfolios.

Nonparametric Minimum-VaR and Minimum-CVaR Hedging

Panel A of Table IV reports the results for the nonparametric minimum-VaR hedging strategy applied to the six hedge portfolios, using the historical simulation approach of Harris and Shen (2006). Compared with minimum-variance hedging, minimum-VaR hedging yields a larger reduction in negative skewness in five of the six cases, and a smaller increase in kurtosis also in five of the six cases. In some cases, the differences are substantial. For example, for the FTSE Small Cap index hedged with the FTSE 100 futures contract, minimum-variance hedging reduces kurtosis relative to the unhedged position by 0.45, while minimum-VaR hedging reduces it by 3.99. On average, minimum-VaR hedging reduces negative skewness by 0.24 (compared with a reduction of 0.15 for minimum-variance hedging) and increases kurtosis by only 0.06 (compared with an increase of 1.58 for minimum-variance hedging). The reduction in standard deviation for minimum-VaR hedging is inevitably lower than for minimum-variance hedging (45.1% vs. 47.0%), but this is more than offset by the reduction in negative skewness and kurtosis, and so minimum-VaR hedging generally provides a greater reduction in both VAR and CVAR. In particular, minimum-VaR hedging reduces VaR by 38.8% on average (compared with 35.2% for minimum-variance hedging) and reduces CVaR by 34.0% (compared with only 27.1% for minimum-variance hedging).

Panel B of Table IV reports the results for the nonparametric minimum-CVaR hedge ratio. There is again a reduction in negative skewness and kurtosis

TABLE IV

Minimum-VaR and Minimum-CVaR Hedging (Nonparametric)

Long	*Short*	*HR*	*ΔSkew*	*ΔKurt*	*%ΔSD*	*%ΔvaR*	*%ΔCVaR*
Panel A: Minimum-VaR Hedging							
FTSE 100	FTSE 100	0.87	−0.16	1.41	−75.94	−71.84	−69.69
FTSE 250	FTSE 250	0.81	0.27	3.31	−64.41	−51.50	−41.92
FTSE SC	FTSE 100	0.42	0.45	−3.99	−5.94	−16.31	−18.51
FTSE SC	FTSE 250	0.54	0.75	−5.65	−37.37	−46.65	−48.61
Oil	Oil	0.97	0.23	4.53	−50.25	−17.46	0.08
EUR/USD	EUR/USD	0.78	−0.09	0.77	−36.87	−29.20	−25.11
	Average	0.73	0.24	0.06	−45.13	−38.83	−33.96
Panel B: Minimum-CVaR Hedging							
FTSE 100	FTSE 100	0.86	−0.12	1.04	−75.44	−71.99	−70.15
FTSE 250	FTSE 250	0.74	0.33	1.73	−62.11	−54.81	−48.36
FTSE SC	FTSE 100	0.40	0.50	−4.34	−9.08	−21.38	−23.51
FTSE SC	FTSE 250	0.63	0.93	−6.52	−36.03	−47.73	−50.13
Oil	Oil	0.90	0.36	4.58	−47.47	−17.89	−1.10
EUR/USD	EUR/USD	0.68	−0.02	0.44	−35.96	−32.22	−29.95
	Average	0.70	0.33	−0.51	−44.35	−41.00	−37.20

Notes. The table reports the average hedge ratio, the change in the skewness and excess kurtosis coefficients and the percentage change in standard deviation, one-day 99% VaR and one-day 99% CVaR, relative to the long position, for each minimum-VaR hedge portfolio (Panel A) and each minimum-CVaR hedge portfolio (Panel B). The results are based on a dynamic hedging strategy that uses a hedge ratio estimated every 250 days, using the previous 250 days observations. The minimum-VaR and minimum-CVaR hedge ratios are estimated using historical simulation.

for many of the portfolios. On average, the negative skewness of the hedge portfolio is reduced by 0.33 (compared with 0.15 for minimum-variance hedging) and kurtosis is reduced by 0.51 (versus an increase of 1.58 for minimum-variance hedging). Again, the reduction in standard deviation is less than for minimum-variance hedging in all cases, but generally this is not sufficient to outweigh the skewness and kurtosis effects. Consequently, in five of the six cases, CVaR is reduced more by minimum-CVaR hedging than by minimum-variance hedging. On average CVaR is reduced by 37.2% with minimum-CVaR hedging, compared to 34.0% with minimum-VaR hedging and 27.1% with minimum-variance hedging. Interestingly, minimum-CVaR hedging also generates a marginally bigger reduction in VaR than does minimum-VaR hedging (41.0% vs. 38.8%).

Semiparametric Minimum-VaR and Minimum-CVaR Hedging

Panel A of Table V reports the results for the semiparametric minimum-VaR hedging strategy, using Equation (11). Compared with the nonparametric approach, the semiparametric approach offers further improvement. In particular, relative

TABLE V
Minimum-VaR and Minimum-CVaR Hedging (Semiparametric)

Long	*Short*	*HR*	*ΔSkew*	*ΔKurt*	*%ΔSD*	*%ΔvaR*	*%ΔCVaR*
Panel A: Minimum-VaR Hedging							
FTSE 100	FTSE 100	0.87	−0.12	1.00	−74.28	−70.87	−69.00
FTSE 250	FTSE 250	0.74	0.33	1.64	−61.85	−54.73	−48.39
FTSE SC	FTSE 100	0.45	0.60	−5.13	−5.99	−20.44	−23.52
FTSE SC	FTSE 250	0.62	0.88	−6.31	−36.04	−47.13	−49.39
Oil	Oil	0.91	0.34	3.51	−41.40	−19.03	−6.68
EUR/USD	EUR/USD	0.64	−0.06	0.38	−37.52	−33.54	−31.52
	Average	0.71	0.33	−0.82	−42.85	−40.96	−38.08
Panel B: Minimum-CVaR Hedging							
FTSE 100	FTSE 100	0.86	−0.10	0.73	−73.23	−70.50	−69.03
FTSE 250	FTSE 250	0.71	0.34	1.39	−60.43	−54.25	−48.49
FTSE SC	FTSE 100	0.48	0.66	−5.52	−1.55	−17.78	−21.23
FTSE SC	FTSE 250	0.63	0.91	−6.51	−35.55	−47.13	−49.46
Oil	Oil	0.83	0.28	2.85	−39.54	−21.70	−12.09
EUR/USD	EUR/USD	0.62	−0.05	0.35	−35.97	−32.32	−30.48
	Average	0.69	0.34	−1.12	−41.05	−40.61	−38.46

Notes. The table reports the average hedge ratio, the change in the skewness and excess kurtosis coefficients and the percentage change in standard deviation, one-day 99% VaR and one-day 99% CVaR, relative to the long position, for each minimum-VaR hedge portfolio (Panel A) and each minimum-CVaR hedge portfolio (Panel B). The results are based on a dynamic hedging strategy that uses a hedge ratio estimated every 250 days, using the previous 250 days observations. The minimum-VaR and minimum-CVaR hedge ratios are estimated using Equations (11) and (14) that are based on the Cornish–Fisher expansion.

to the nonparametric approach, the semiparametric approach offers a greater reduction in both negative skewness and kurtosis for all six hedge portfolios. In all cases, the reduction in negative skewness and kurtosis is greater than for minimum-variance hedging. The reduction in VaR is greater than that provided by the nonparametric approach in five of the six cases. The average reduction in VaR is 41.0%, compared with 38.8% for nonparametric minimum-VaR hedging and 35.2% for minimum-variance hedging. The average reduction in CVaR is 38.1%, compared to 34.0% for nonparametric minimum-VaR hedging and 27.1% for minimum-variance hedging.

Panel B of Table V reports the results for the semiparametric minimum-CVaR hedging strategy using Equation (14). The results are similar to those for the minimum-VaR hedging. Relative to the nonparametric minimum-CVaR hedge ratio, the semiparametric minimum-CVaR hedge ratio generates a greater reduction in negative skewness in three of the six cases, and a greater reduction in kurtosis in five of the six cases. Indeed, the average reduction in negative skewness is 0.34 and the average reduction in kurtosis is 1.12, which is the highest among all five hedging strategies. However, the reduction in standard deviation is the lowest of the five strategies, which to some extent offsets the skewness and kurtosis effects. In terms of the reduction in CVaR, the semiparametric minimum-CVaR approach is superior to the nonparametric minimum-CVaR approach for four of the six hedge portfolios, and superior to the minimum-variance approach in all cases.

CONCLUSION

We propose a new semiparametric method of estimating minimum-VaR and minimum-CVaR hedge ratios based on the Cornish–Fisher expansion. We apply this approach to dynamically hedge long positions in the FTSE 100, FTSE 250, and FTSE Small Cap equity indices, the Euro/US Dollar exchange rate, and Brent crude oil. We find that the semiparametric approach works better than both the standard minimum-variance approach, and the nonparametric approach of Harris and Shen (2006). In particular, it provides a greater reduction in both negative skewness and excess kurtosis, and consequently also in VaR and CVaR. The superior performance of the semiparametric approach stems from the fact that it employs information from the entire distribution of hedge portfolio returns, rather than just from the left tail of the distribution. The semiparametric approach is less dependent on historical data and is thus more likely to react to changes in the "true" minimum-VaR and minimum-CVaR hedge ratios. This may make the new approach even more attractive to practitioners. There are a number of interesting avenues for future research. A natural extension would be to explicitly incorporate time-varying volatility into the semiparametric approach. The simple form of the VaR and CVaR expressions that are provided by the Cornish–Fisher expansion are easily adaptable to the application of EWMA or GARCH-type models, as has been done in the case of long-only portfolios. This could be expected to offer further improvements in VaR and CVaR reduction.

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