1 Step by step derivation of F_{R^h}

Transformation of CDF:

$$
\begin{aligned}
F_{-Y}(y) &= \mathbb{P}(-Y \leq y) \\
&= \mathbb{P}(Y \geq -y) \\
&= 1 - \mathbb{P}(Y \leq -y) \\
&= 1 - F_Y(-y) \qquad (1.1)
\end{aligned}
$$

Transformation of Copula:

$$
\begin{aligned}
C_{X,-Y}(u,v) &= F_{X,-Y}(F_X^{-1}(u), F_{-Y}^{-1}(v)) && \text{Sklar} \\
&= \mathbb{P}(X \leq F_X^{-1}(u), -Y \leq F_{-Y}^{-1}(v)) \\
&= \mathbb{P}(F_X(X) \leq u, F_{-Y}(-Y) \leq v) \\
&= \mathbb{P}(F_X(X) \leq u, 1 - F_Y(Y) \leq v) && (1.1) \\
&= \mathbb{P}(F_X(X) \leq u, F_Y(Y) \geq 1 - v) \\
&= \mathbb{P}(X \leq F_X^{-1}(u), Y \geq F_Y^{-1}(1-v)) \\
&= F_X[F_X^{-1}(u)] - F_{X,Y}[F_X^{-1}(u), F_Y^{-1}(1-v)] \\
&= u - C_{X,Y}(u, 1-v) \qquad (1.2) && \text{Sklar}
\end{aligned}
$$

We continue with the result from (1.2):

$$
\begin{aligned}
C_{X,-Y}(u,v) &= u - C_{X,Y}(u,1-v) \\
\frac{\partial C_{X,-Y}(u,v)}{\partial u} &= 1 - \frac{\partial C_{X,Y}(u,1-v)}{\partial u} \qquad (1.3) \\
\int_0^1 D_1 C_{X,-Y}(u,v) du &= 1 - \int_0^1 D_1 C_{X,Y}(u,1-v) du \qquad (1.4)
\end{aligned}
$$

Now we plug (1.1) and (1.4) into C-Convolution equation:

$$
\begin{aligned}
F_{R^h}(r^h) &= \int_0^1 D_1 C_{R^S,-hR^F}\{w, F_{-hR^F}[r^h - F_{R^S}^{-1}(w)]\} dw \\
&= 1 - \int_0^1 D_1 C^t_{R^S,hR^F}\{w, 1 - F_{-hR^F}[r^h - F_{R^S}^{-1}(w)]\} dw && (1.4) \\
&= 1 - \int_0^1 D_1 C^t_{R^S,hR^F}\{w, F_{hR^F}[F_{R^S}^{-1}(w) - r^h]\} dw && (1.1)
\end{aligned}
$$

We proceed wih $F_{hR^F}(x) = F_{R^F}(x/h)$ and $C_{R^S,hR^F}(w,\lambda) = C_{R^S,R^F}(w,\lambda)$:

$$F_{R^h}(r^h) = 1 - \int_0^1 D_1 C_{R^S,R^F}\left\{w, F_{R^F}\left[\frac{F_{R^S}^{-1}(w) - r^h}{h}\right]\right\} dw$$

Barbi and Romagnoli's proof of Corollary 2:

$$F_{R^h}(r^h) = 1 - \int_0^1 D_1 C_{R^S,R^F}\left\{w, 1 - F_{hR^F}\left[\frac{r^h - F_{R^S}^{-1}(w)}{h}\right]\right\} dw$$