

# 1 Step by step derivation of $F_{R^h}$

Transformation of CDF:

$$\begin{aligned}
 F_{-Y}(y) &= \mathbb{P}(-Y \leq y) \\
 &= \mathbb{P}(Y \geq -y) \\
 &= 1 - \mathbb{P}(Y \leq -y) \\
 (1.1) \quad &= 1 - F_Y(-y)
 \end{aligned}$$

Transformation of Copula:

$$\begin{aligned}
 C_{X,-Y}(u, v) &= F_{X,-Y}(F_X^{-1}(u), F_{-Y}^{-1}(v)) && \text{Sklar} \\
 &= \mathbb{P}(X \leq F_X^{-1}(u), -Y \leq F_{-Y}^{-1}(v)) \\
 &= \mathbb{P}(F_X(X) \leq u, F_{-Y}(-Y) \leq v) \\
 &= \mathbb{P}(F_X(X) \leq u, 1 - F_Y(Y) \leq v) && (1.1) \\
 &= \mathbb{P}(F_X(X) \leq u, F_Y(Y) \geq 1 - v) \\
 &= \mathbb{P}(X \leq F_X^{-1}(u), Y \geq F_Y^{-1}(1 - v)) \\
 &= F_X[F_X^{-1}(u)] - F_{X,Y}[F_X^{-1}(u), F_Y^{-1}(1 - v)] \\
 (1.2) \quad &= u - C_{X,Y}(u, 1 - v) && \text{Sklar}
 \end{aligned}$$

We continue with the result from (1.2):

$$\begin{aligned}
 C_{X,-Y}(u, v) &= u - C_{X,Y}(u, 1 - v) \\
 \frac{\partial C_{X,-Y}(u, v)}{\partial u} &= 1 - \frac{\partial C_{X,Y}(u, 1 - v)}{\partial u} \\
 (1.3) \quad \int_0^1 D_1 C_{X,-Y}(u, v) du &= 1 - \int_0^1 D_1 C_{X,Y}(u, 1 - v) du
 \end{aligned}$$

Now we plug (1.1) and (1.3) into C-Convolution equation:

$$\begin{aligned}
 F_{R^h}(r^h) &= \int_0^1 D_1 C_{R^S, -hR^F}\{w, F_{-hR^F}[r^h - F_{R^S}^{-1}(w)]\}dw \\
 &= 1 - \int_0^1 D_1 C_{R^S, hR^F}^t\{w, 1 - F_{-hR^F}[r^h - F_{R^S}^{-1}(w)]\}dw && (1.3) \\
 &= 1 - \int_0^1 D_1 C_{R^S, hR^F}^t\{w, F_{hR^F}[F_{R^S}^{-1}(w) - r^h]\}dw && (1.1)
 \end{aligned}$$

We proceed with  $F_{hR^F}(x) = F_{R^F}(x/h)$  and  $C_{R^S, hR^F}(w, \lambda) = C_{R^S, R^F}(w, \lambda)$ :

$$F_{R^h}(r^h) = 1 - \int_0^1 D_1 C_{R^S, R^F} \left\{ w, F_{R^F} \left[ \frac{F_{R^S}^{-1}(w) - r^h}{h} \right] \right\} dw$$