Calculation of Expected Exposure Profiles for Derivatives

Financial Engineering Project

30 April 2019

Dobosiewicz, Klaudia
Horzela, Joachim
Ibia, Vincent
Kryvyy, Taras
Zokirkhonov, Fazliddinkhuja

Agenda

- Calculation of values of the derivatives
  - Valuation of FX Forward, IRS and CIRS at moment 0
  - Valuation at each time-step
- Key features of Expected Exposure
  - FX Forward
  - Interest Rate Swap
  - Cross-Currency Interest Rate Swap
- Sensitivity analysis – impact of increased volatility on Expected Exposure of the derivatives in question

Calculation of values of the derivatives

FX Forward at time zero

$$V^{long} = N\left(Se^{-\tilde{r}_F T} - F_0 e^{-\tilde{r}T}\right) = N\left(Sd_F(T) - F_0 d(T)\right)$$

- S – current FX spot rate,
- F_0 – FX forward rate,
- T – moment of contract settlement
- $\tilde{r}_F$ – foreign continuously-compounded interest rate,
- $\tilde{r}$ – domestic continuously-compounded interest rate
- $d_F(T)$ – foreign discount factor from time T to now
- $d(T)$ – foreign discount factor from time T to now
- N – contract notional

Source: own research; Bartkowiak & Echaust, 2014

Interest Rate Swap at time zero

Spot rate represented as a forward rate.

$$V^{receiver} = Nr_{IRS}(d(1) + d(2) + d(3)) - N\left(\frac{f_{0,0.25}}{4}d(0.25) + \frac{f_{0.25,0.5}}{4}d(0.5) + \cdots + \frac{f_{2.75,3}}{4}d(3)\right)$$

- $d(0.25)$ – present value of 1 PLN in 3 months
- N – contract notional
- $f_{0.25,0.5} = \frac{\left(\frac{d(0.25)}{d(0.5)} - 1\right)}{0.5 - 0.25}$ – current forward rate that starts in 3 months and matures in 6 months
- r_{IRS} – fixed interest rate

Source: own research; Bartkowiak & Echaust, 2014

Cross-Currency Interest Rate Swap at time zero

$$V^{payer}$$

$$= N\left(-1 + \frac{f_{0,0.25}}{4}d(0.25) + \cdots + \left(1 + \frac{f_{2.75,3}}{4}\right)d(3)\right)$$

$$- X^{EUR\backslash PLN}N_f\left(-1 + r_{CIRS}d_f(1) + r_{CIRS}d_f(2) + (1 + r_{CIRS})d_f(3)\right)$$

- $d(0.25)$ – present value of 1 PLN in 3 months
- $d_f(1)$ – present value of 1 EUR in 1 year
- N – contract notional in domestic currency
- N_f – contract notional in foreign currency
- $f_{0.25,0.5} = \frac{\left(\frac{d(0.25)}{d(0.5)} - 1\right)}{0.5 - 0.25}$ – current forward rate that starts in 3 months and matures in 6 months
- r_{CIRS} – fixed interest rate
- $X^{EUR\backslash PLN}$ – current exchange rate

Notional exchange in the beginning and in the end.

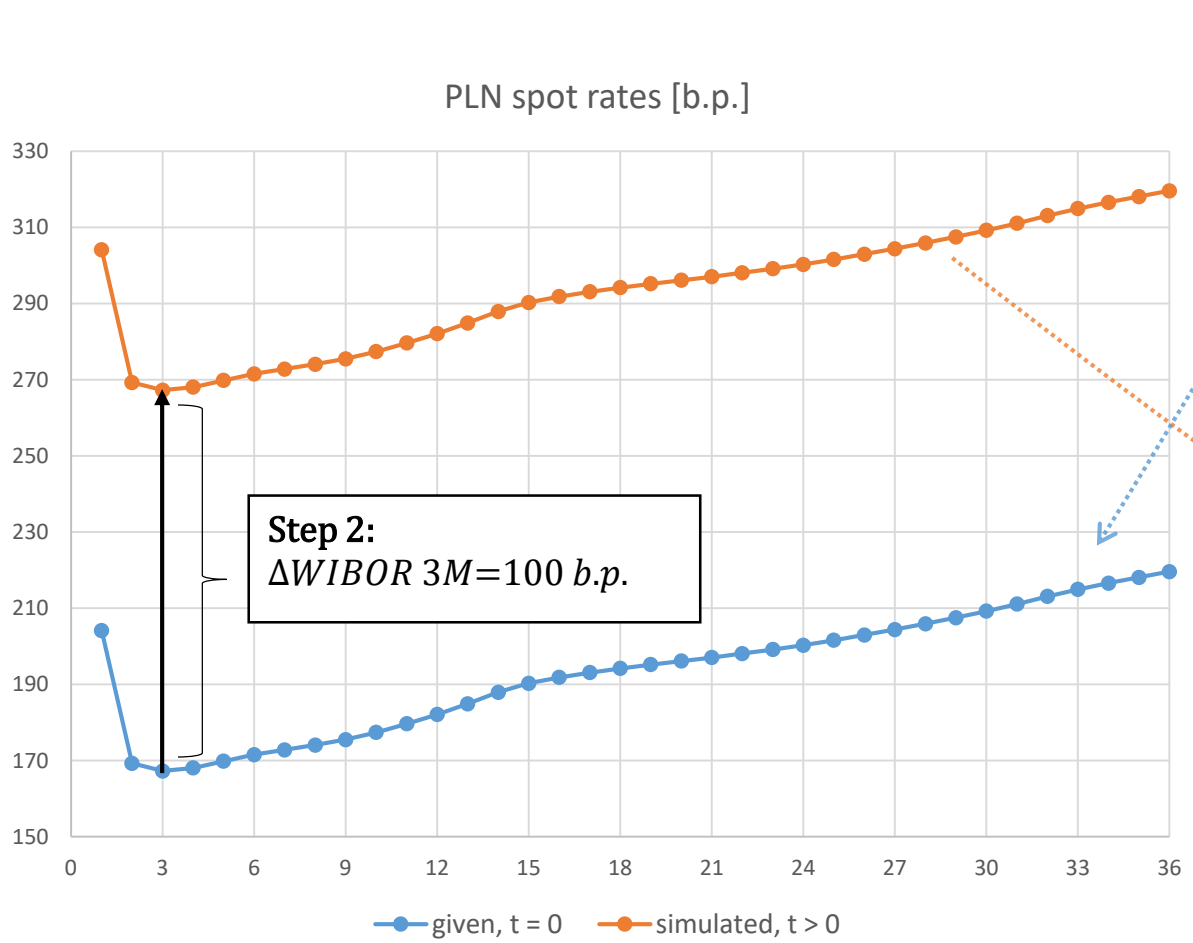
Source: own research; Bartkowiak & Echaust, 2014

Valuation at each time-step

Main idea

1. Pick variables that determine present value of future cash flows (i.e. risk factors)
2. Find out in which direction risk factors move and how unstable they are across time (i.e. calibrate drift and volatility parameters)
3. Simulate risk factors
4. Calculate remaining cash flows
5. Calculate PV of each remaining cash flow
6. Sum those up

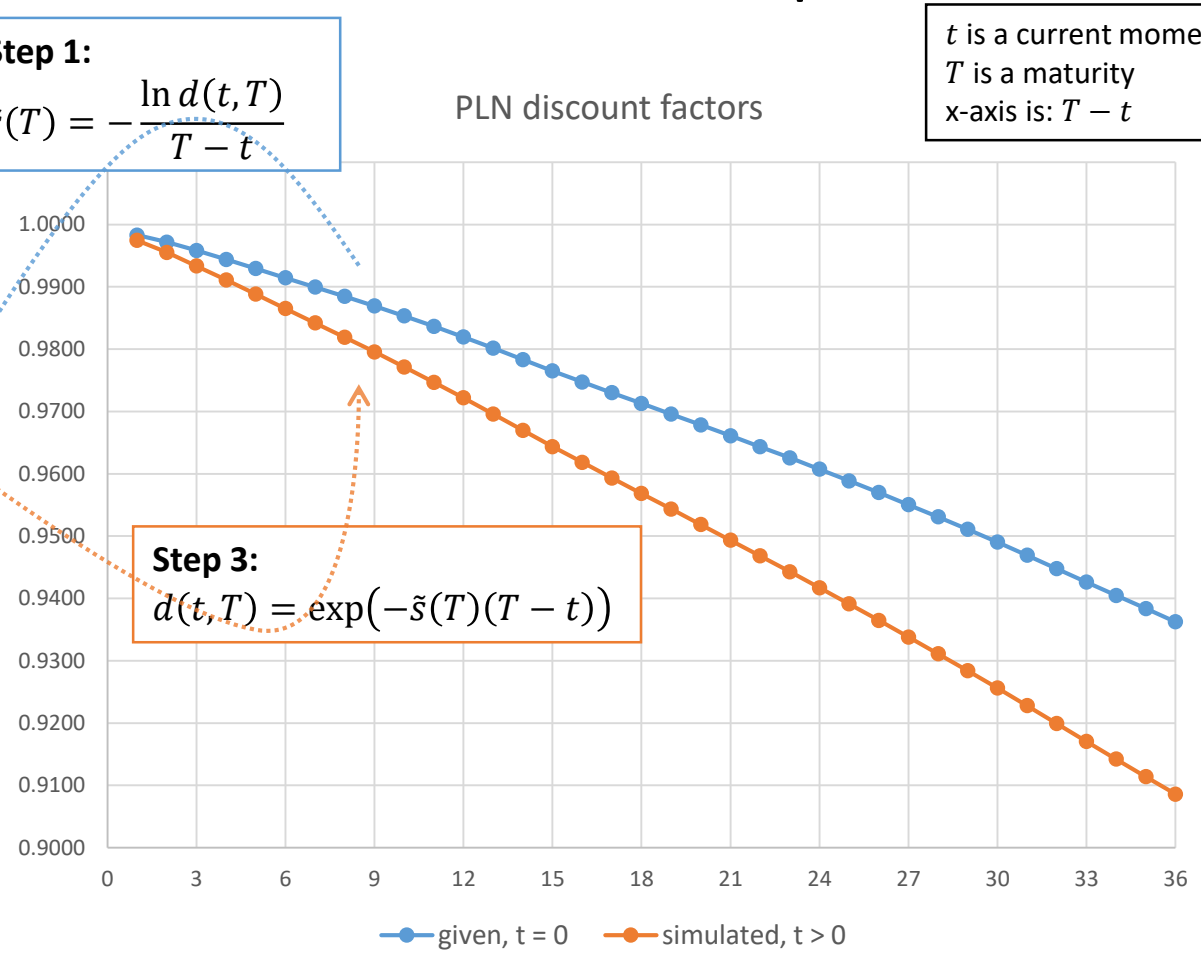
Assumption: shape of spot rates term structure is shifting with simulated xIBOR 3M. PLN example



Step 2:
$\Delta WIBOR\ 3M{=}100\ b.p.$

Step 1:
$$\tilde{s}(T) = -\frac{\ln d(t,T)}{T-t}$$

t is a current moment
T is a maturity
x-axis is: $T - t$



Step 3:
$d(t,T) = \exp\left(-\tilde{s}(T)(T-t)\right)$

Source: own research

Key features of Expected Exposure

Definition

$$Exposure = \max(V_t, 0)$$

(Gregory, 2010)

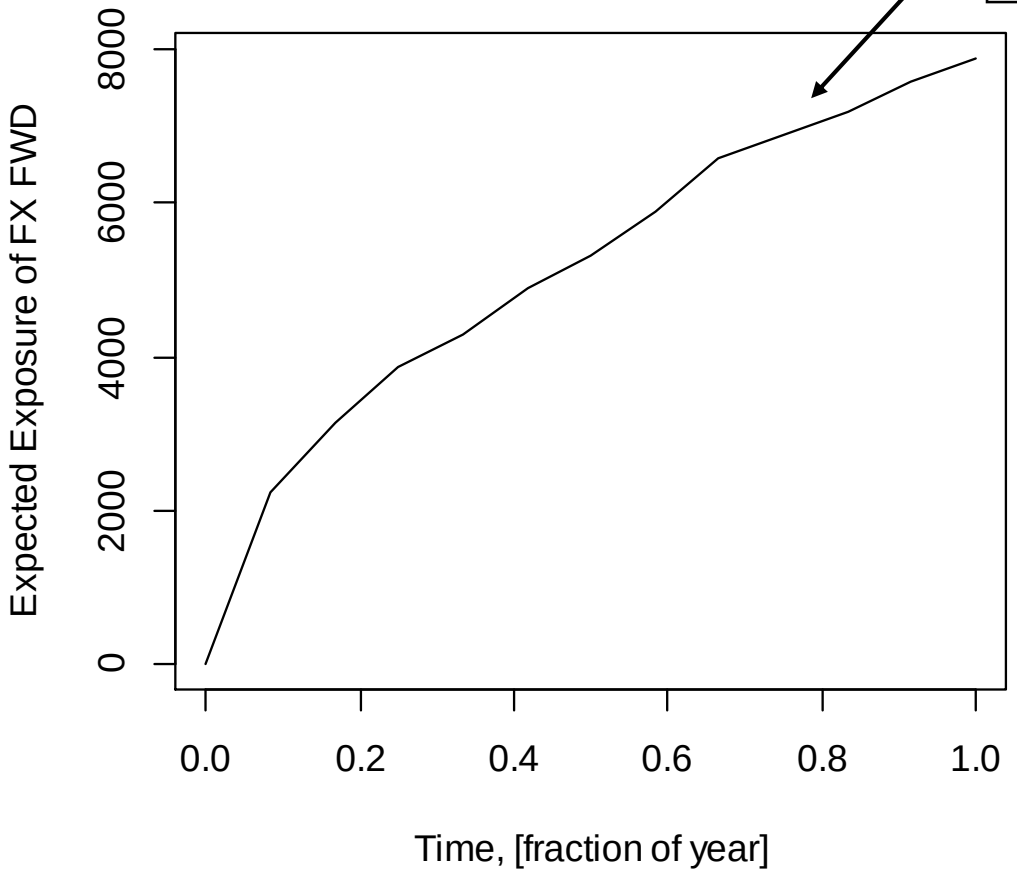
$$Expected\ Exposure = \frac{\sum_1^M V_t}{M}$$

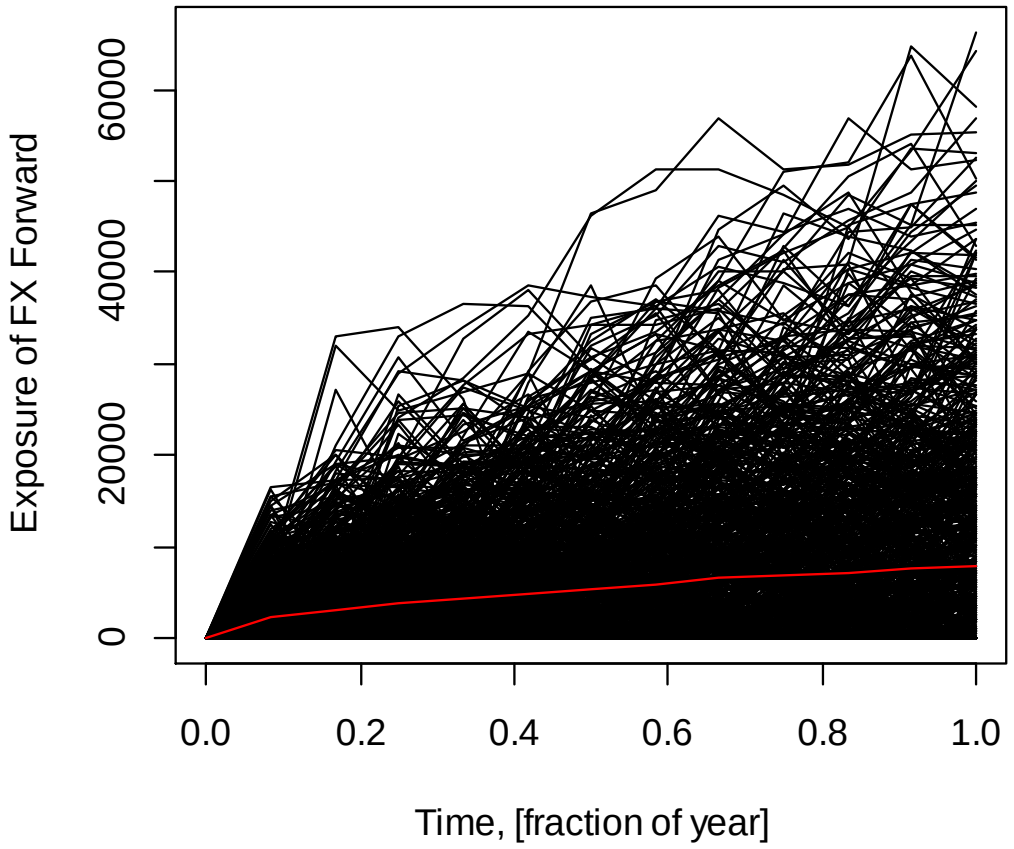
- V_t - value of derivative at moment t
- M - number of simulations

The aim is to quantify how much a counterparty can owe us given market conditions and assumptions.

FX Forward

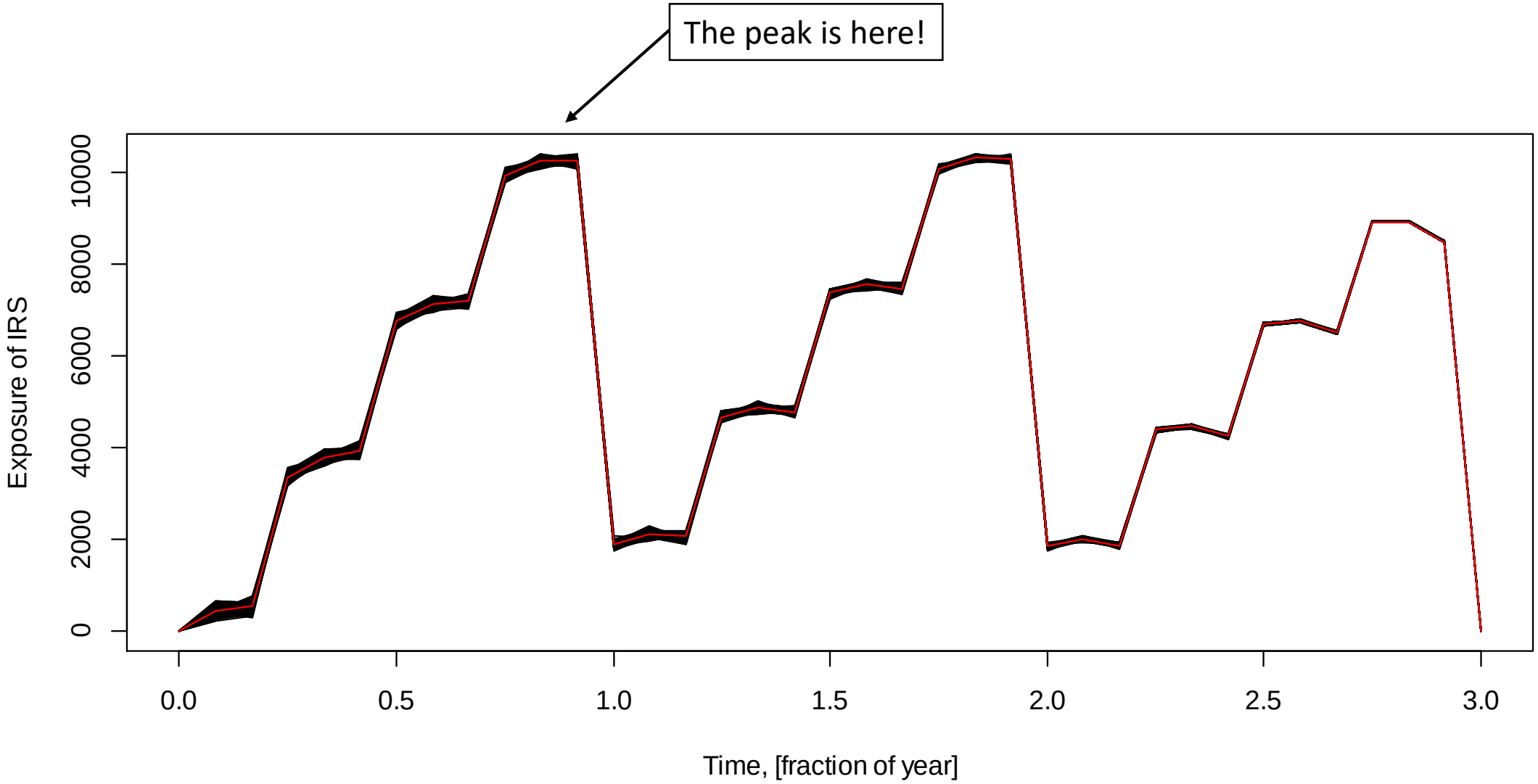
Taking only positive payoffs and discounting of the future payoffs introduces non-linearity





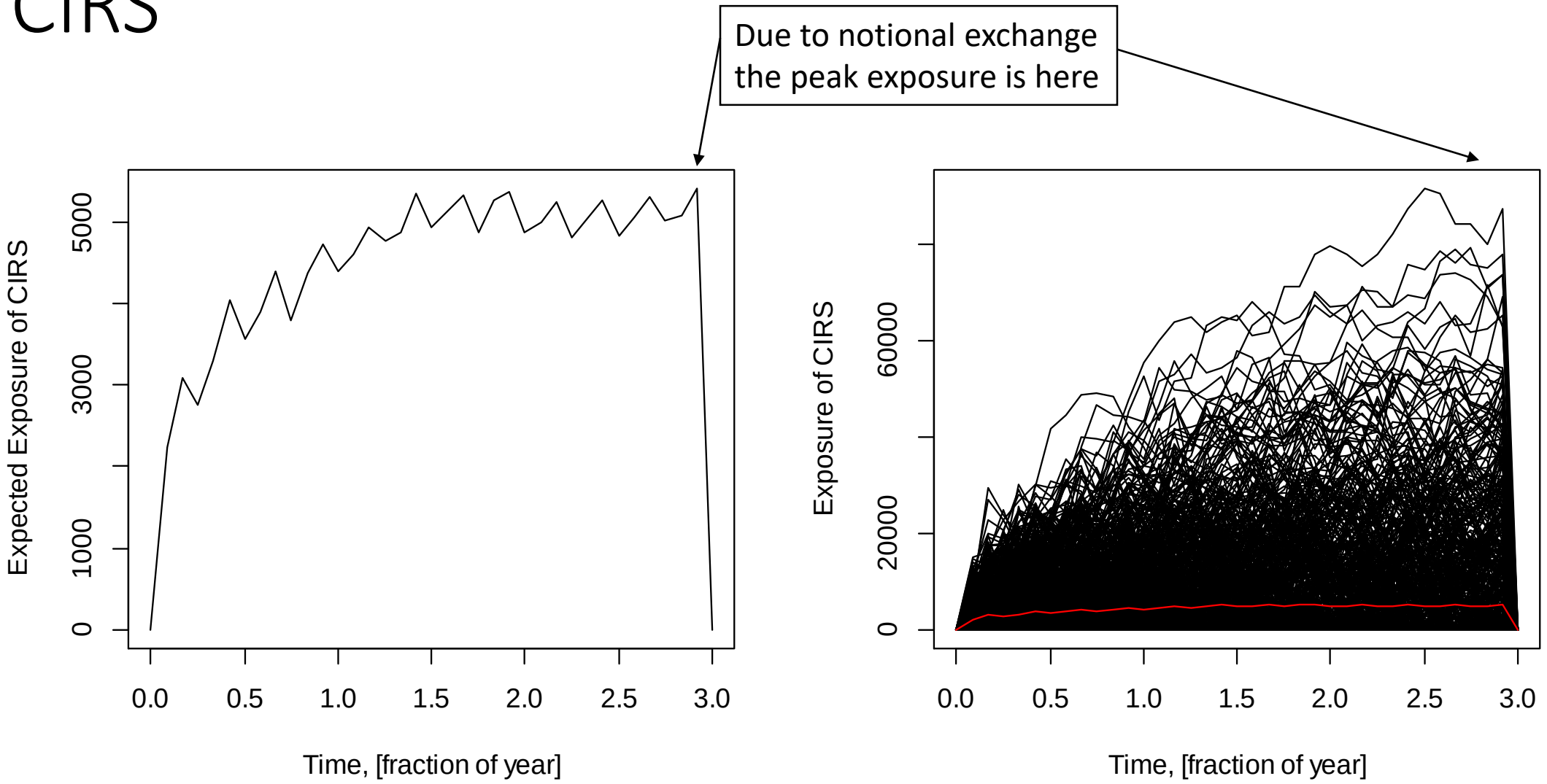
Source: own research

IRS



Source: own research

CIRS

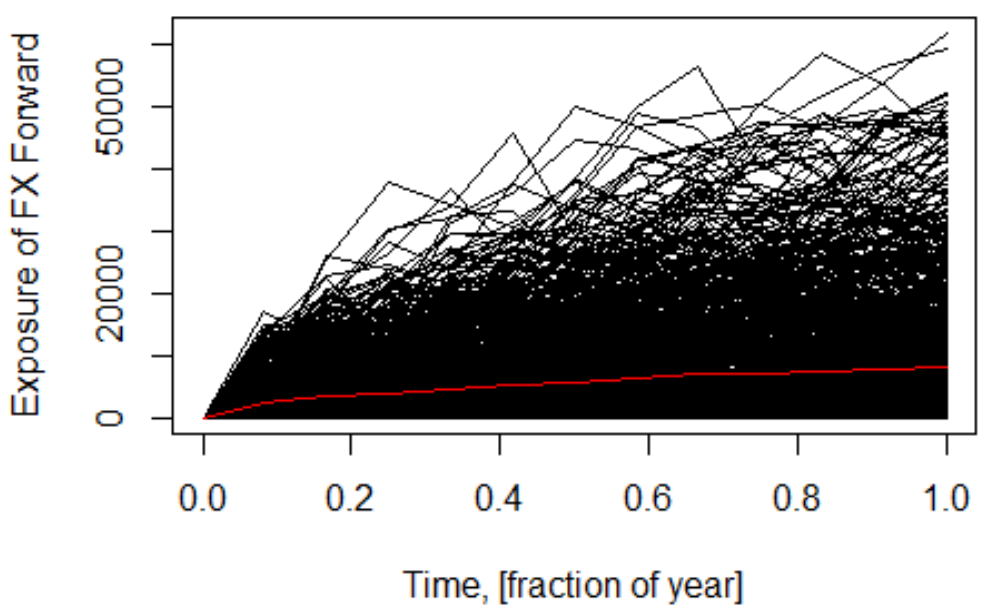


Source: own research

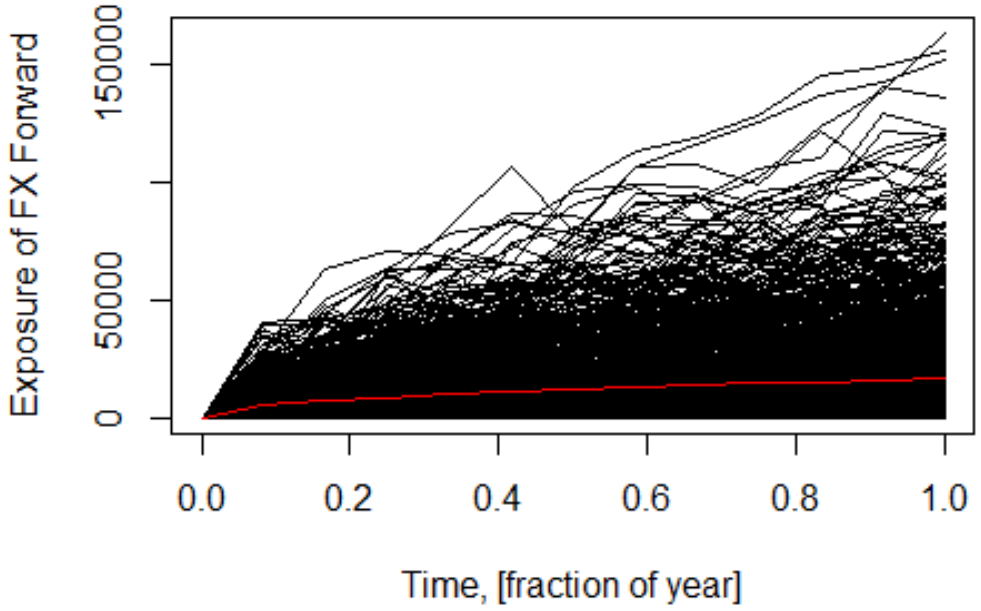
Sensitivity Analysis

FX Forward Exposures: Sensitivity to FX volatility

Baseline volatility of FX



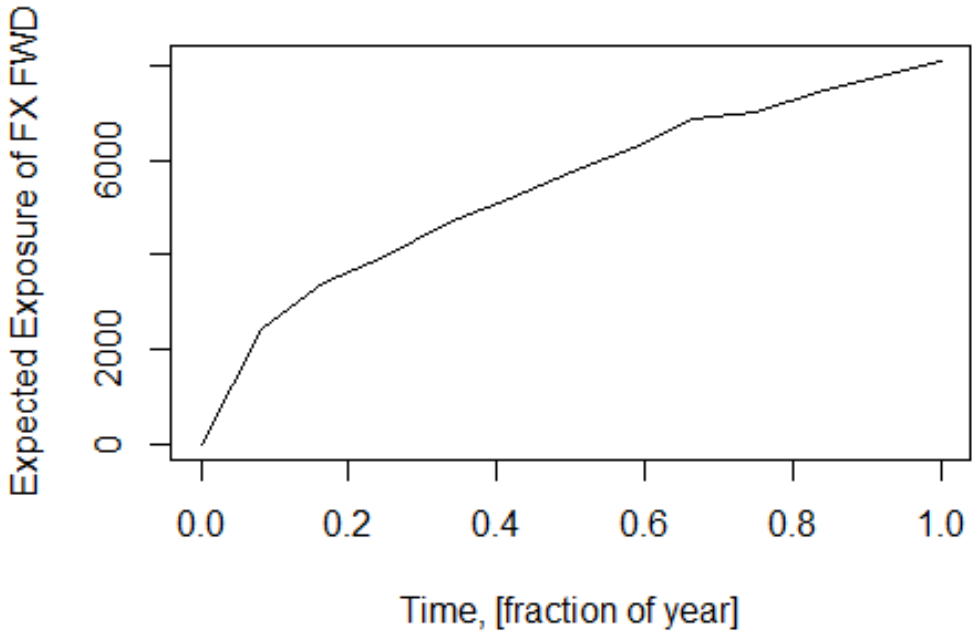
2 times baseline volatility of FX



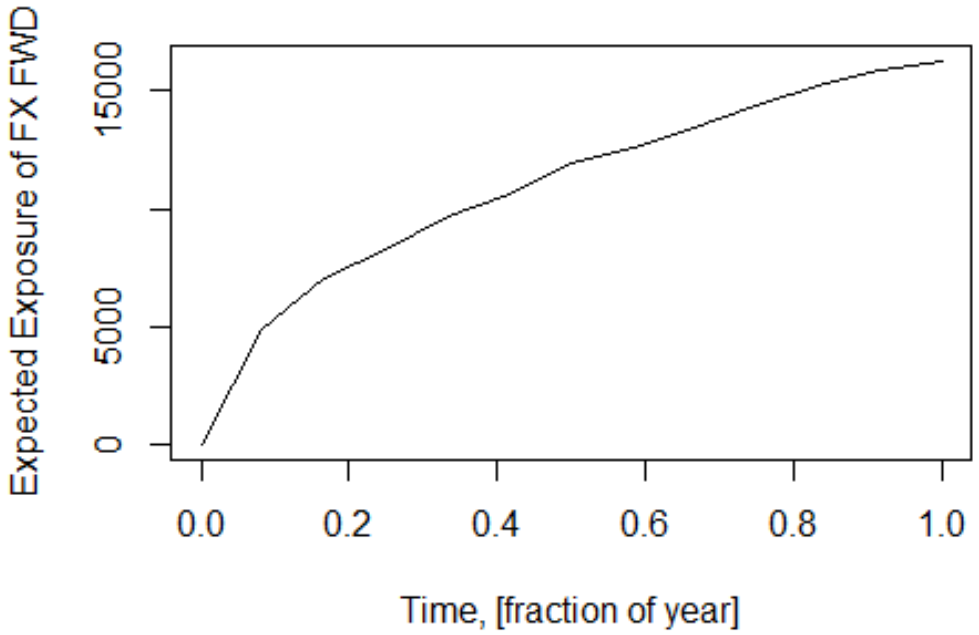
Source: own research

FX Forward Expected Exposure: Sensitivity to FX volatility

Baseline volatility of FX



2 times baseline volatility of FX

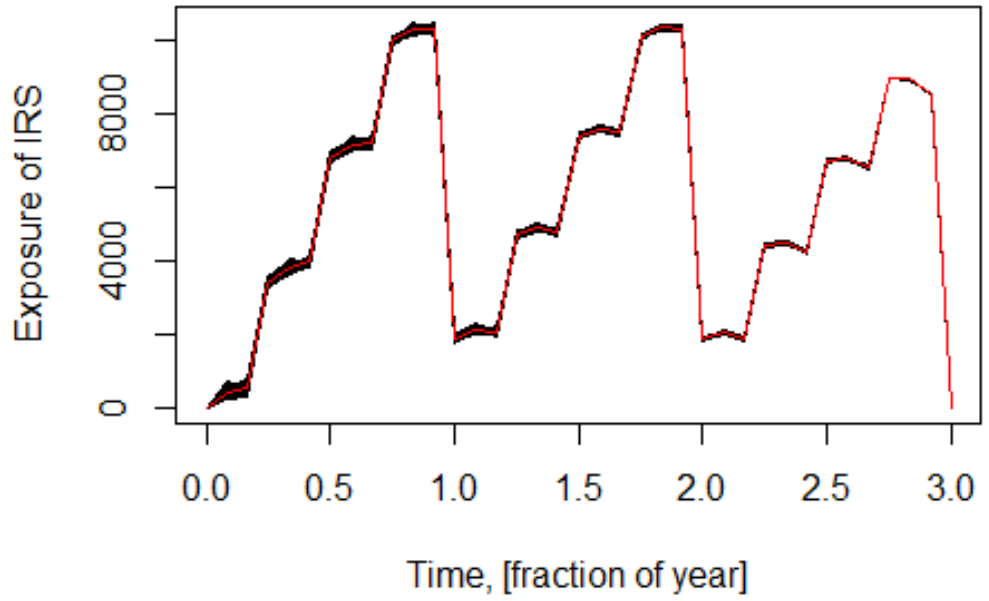


The EE increase is linear.

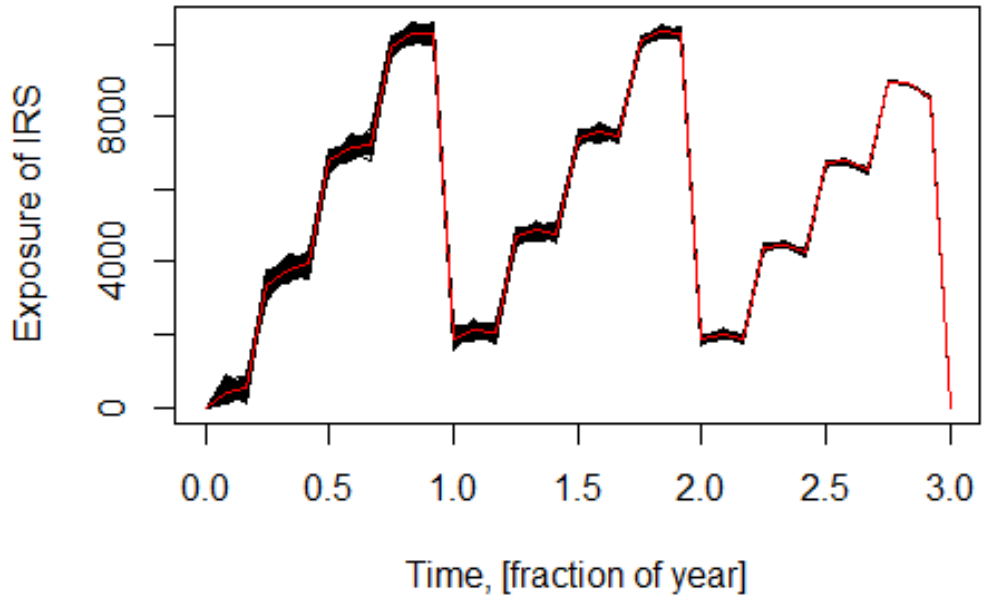
Source: own research

IRS Exposures & Expected Exposure: Sensitivity to WIBOR 3M volatility

Baseline volatility of WIBOR 3M



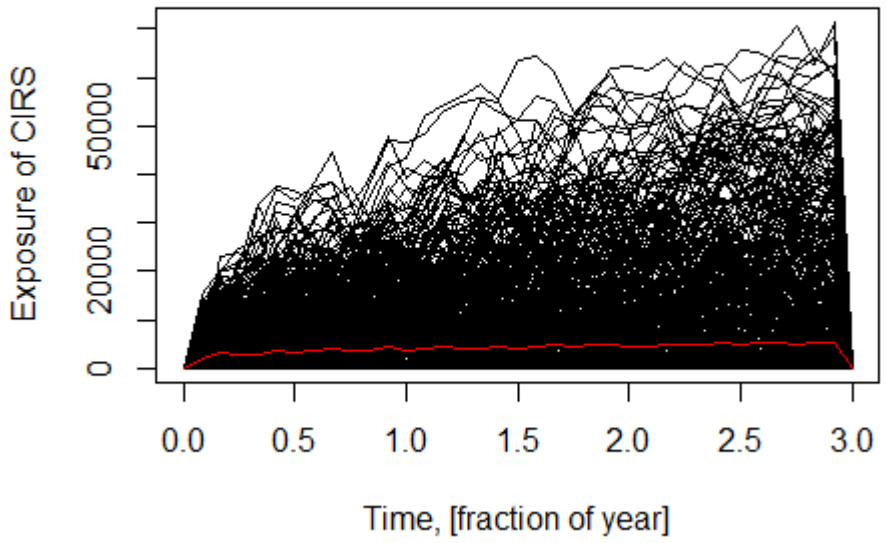
2 times baseline volatility of WIBOR 3M



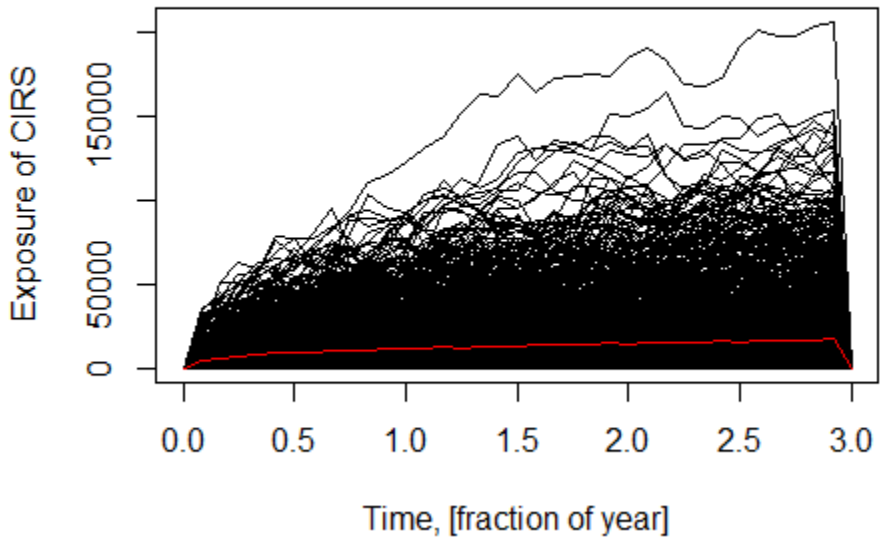
Source: own research

CIRS Exposures: Sensitivity to FX volatility

Baseline volatility of FX



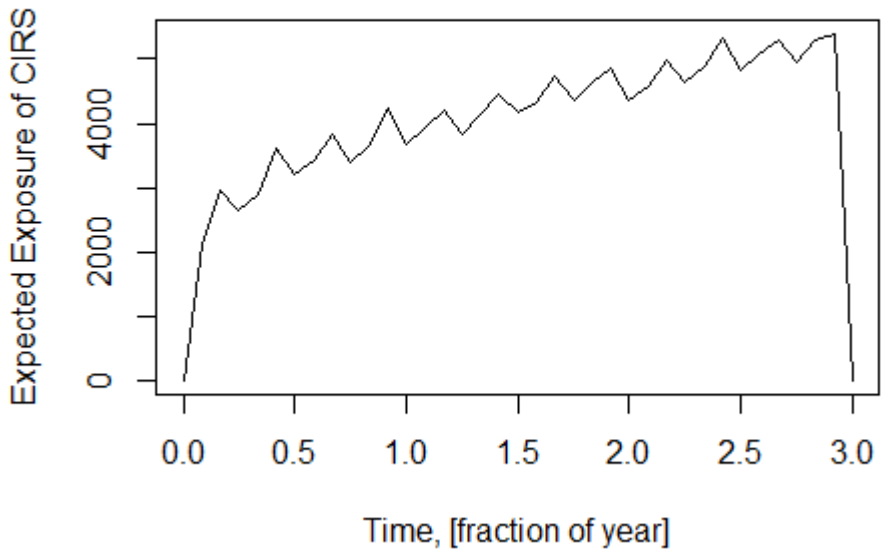
2 times baseline volatility of FX



Source: own research

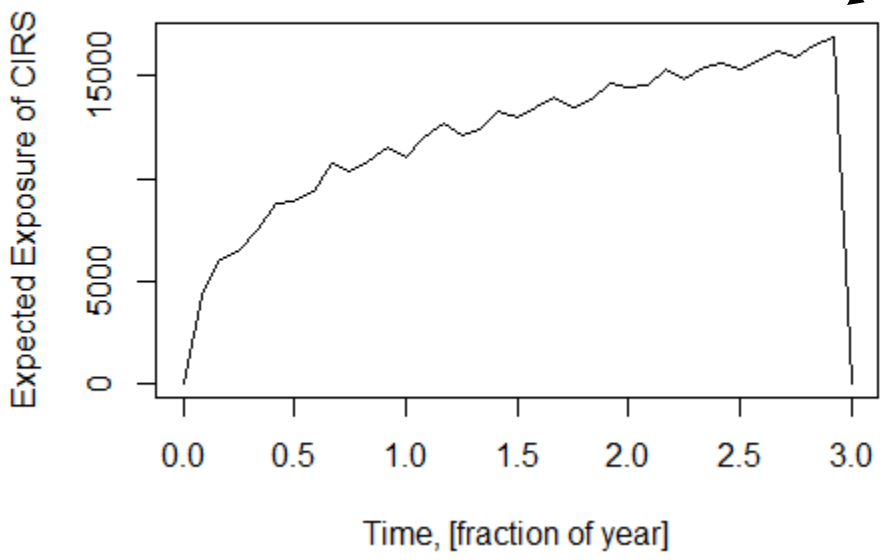
CIRS Expected Exposure: Sensitivity to FX volatility

Baseline volatility of FX



2 times baseline volatility of FX

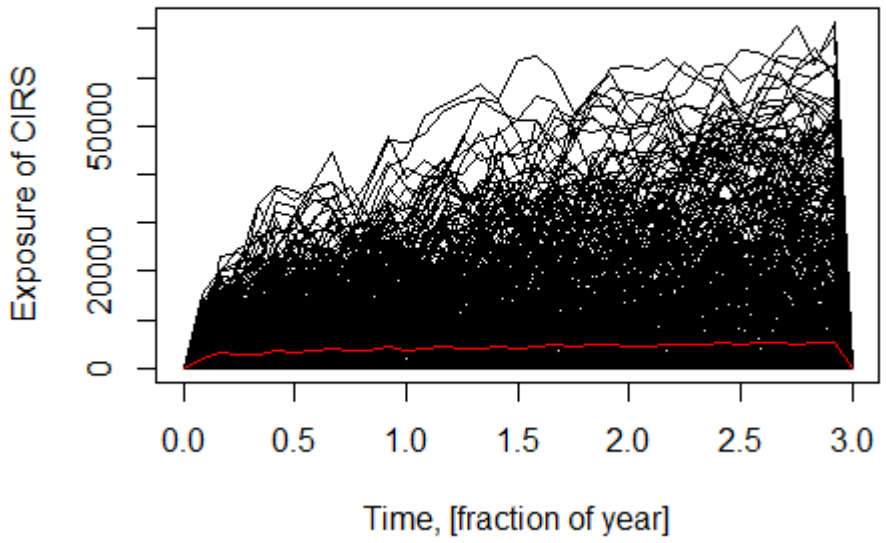
The peak is **more** emphasized



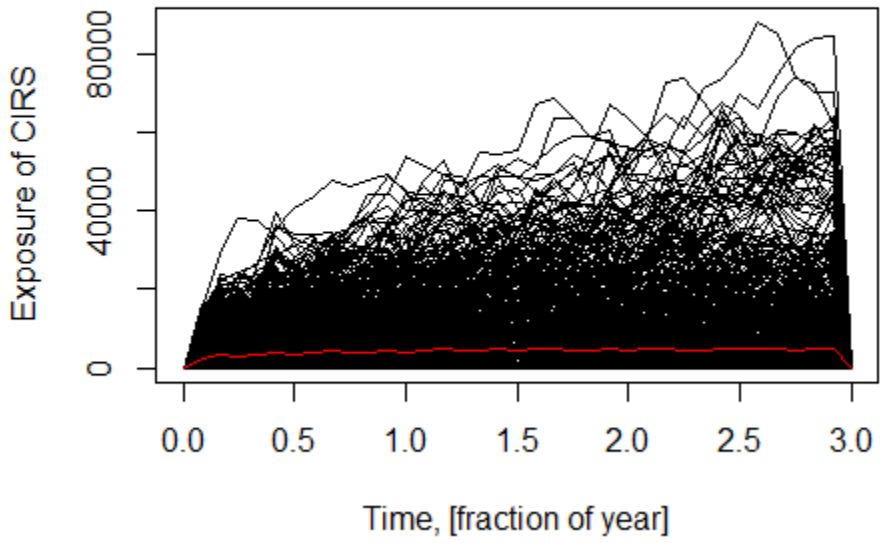
Source: own research

CIRS Exposures: Sensitivity to WIBOR 3M volatility

Baseline volatility of WIBOR 3M



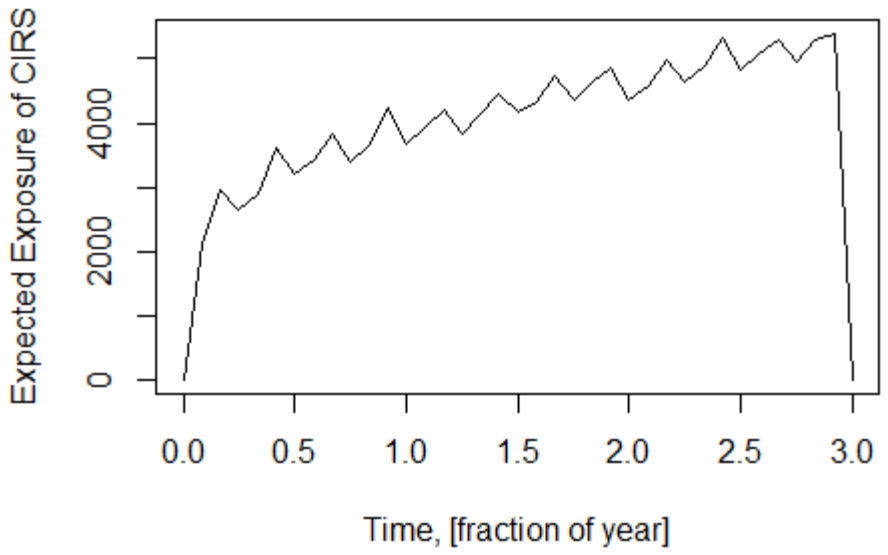
2 times baseline volatility of WIBOR 3M



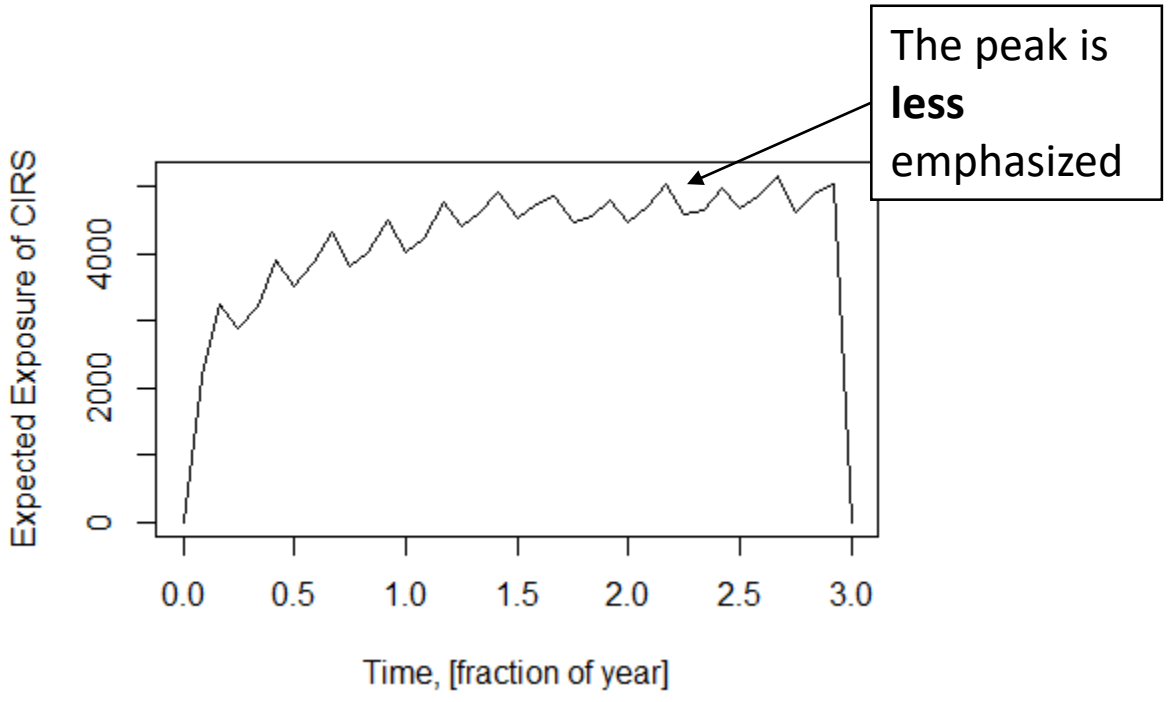
Source: own research

CIRS Expected Exposure: Sensitivity to WIBOR 3M volatility

Baseline volatility of WIBOR 3M



2 times baseline volatility of WIBOR 3M



Source: own research

References

- Bartkowiak, M., & Echaust, K. (2014). *INSTRUMENTY POCHODNE Wprowadzenie fo inżynierii finansowej.* Poznan: Wydawnictwo Uniwersitetu Ekonomicznego w Poznaniu.
- Jon Gregory, *Counterparty Credit Risk: The new challenge for global financial markets*, WileyFinance, 2010