Analysis of Derivative Instruments

Financial Engineering Project

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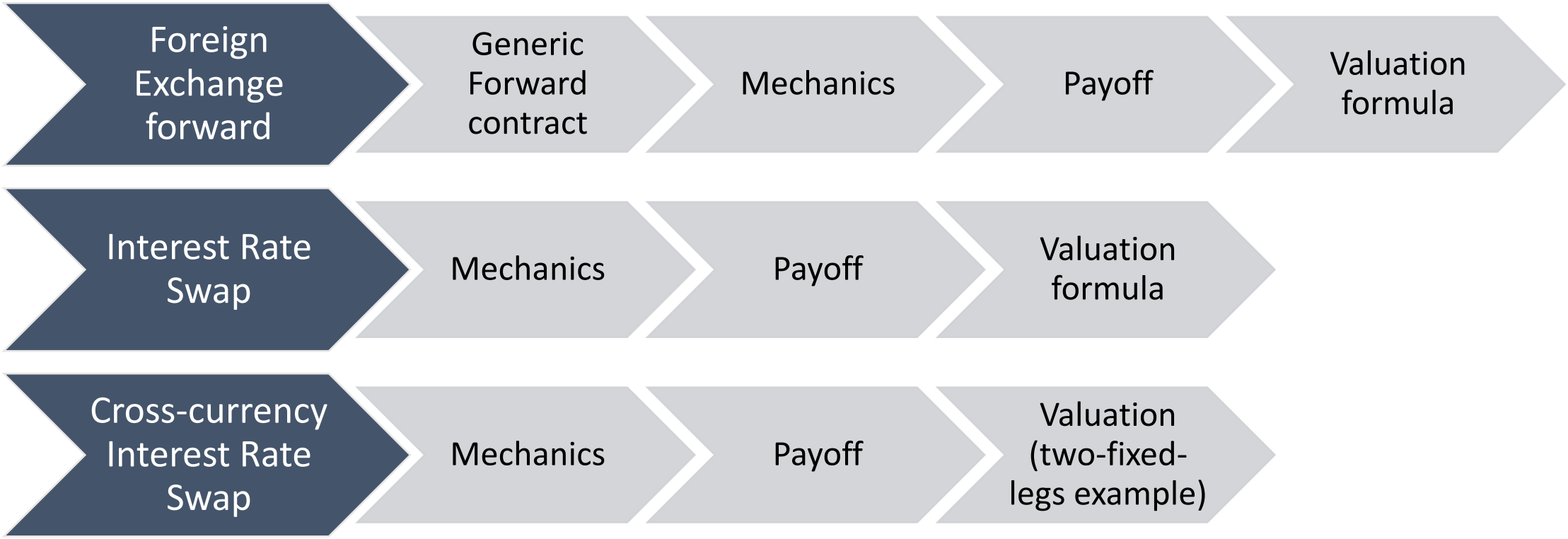
Dobosiewicz, Klaudia
Horzela, Joachim
Ibia, Vincent
Kryvyy, Taras
Zokirkhonov, Fazliddinkhuja

Agenda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Generic Forward contract
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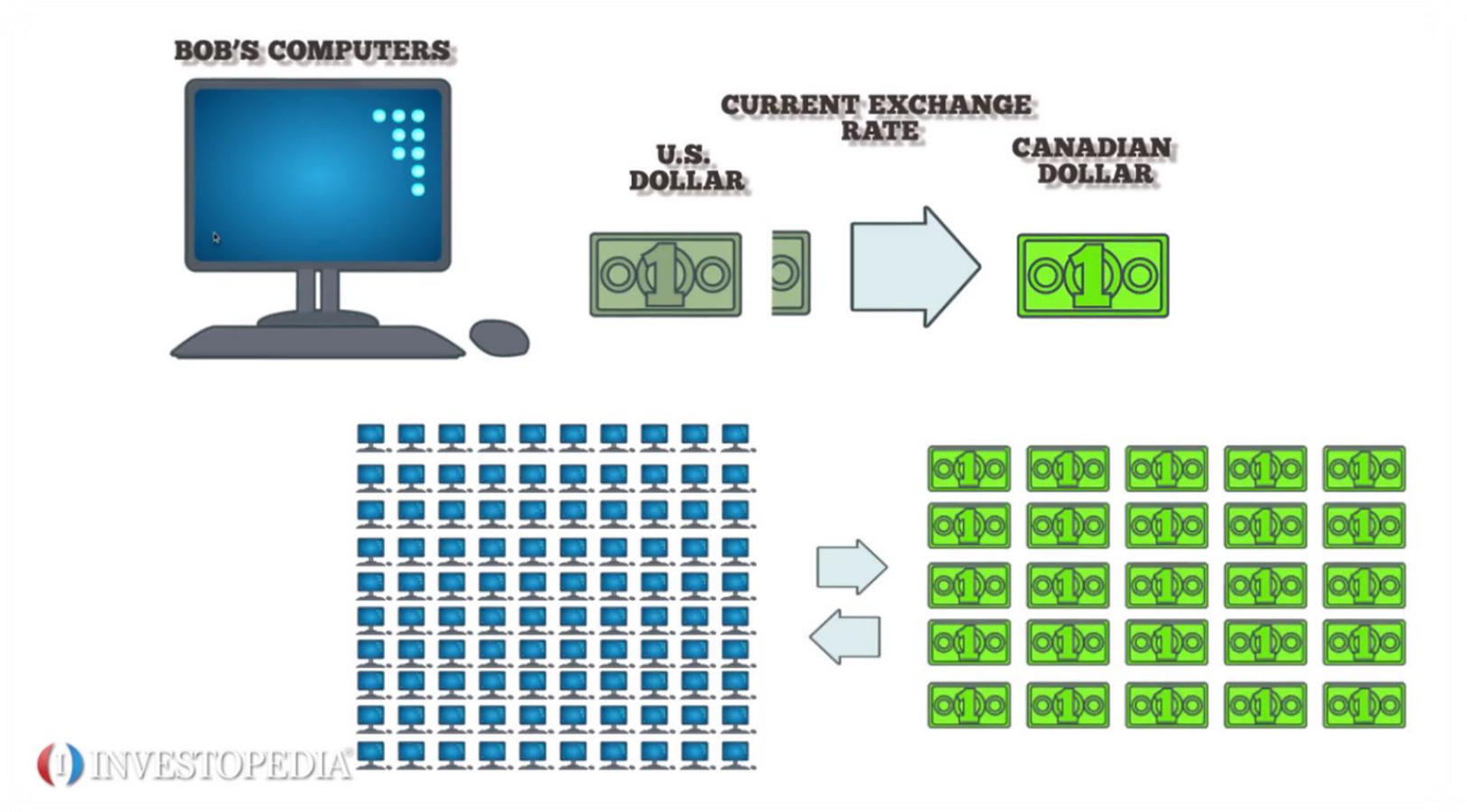
Foreign Exchange forward

Generic Forward contract

- The simplest OTC-traded derivative
- One counterparty, after negotiation, is obliged to buy a particular asset at a particular moment at agreed in advance price from the second counterparty
- Things to agree on:
  - Base instrument (stock, FX rate, interest rate, etc.)
  - Notional
  - Price
  - Date of settlement
  - Type of delivery
    - Physical delivery
    - Cash settled (non deliverable forward)

FX Forward: Mechanics



Source: Investopedia.com

Payoff

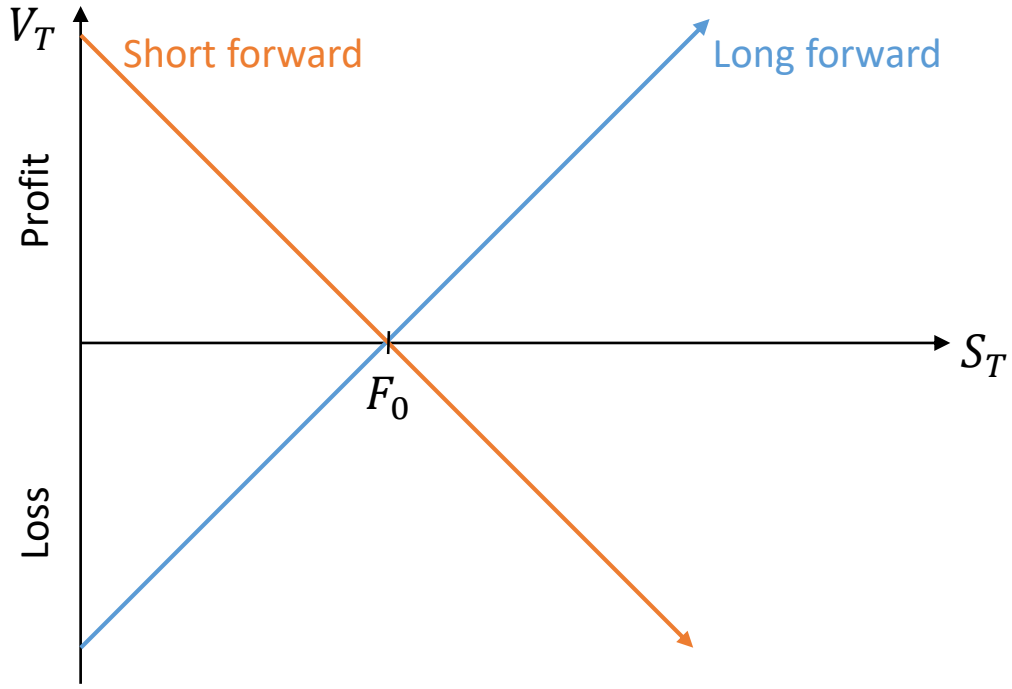
Formula

$$V_T = S_T - F_0$$

- For long position
- S_T – the value of the FX spot rate at maturity
- F_0 – agreed exchange rate
- $-V_T$ for short position

(Bartkowiak & Echaust, 2014)

Profile



Valuation – discrete compounding

$$V_t^{long} = \frac{S_t}{1+r_F\tau} - \frac{F_0}{1+r\tau} \quad ; \quad V_t^{short} = -V_t$$

- S_t – FX spot rate,
- F_0 – FX forward rate,
- r_F – foreign simply-compounded interest rate,
- r – domestic simply-compounded interest rate,
- τ – time remaining to maturity, $\tau = T - t$,
- t – current moment, $t \in [0, T]$,
- T – moment of contract settlement

(Bartkowiak & Echaust, 2014)

Valuation – continuous compounding

- Long position:

$$V_t = S_t e^{-\tilde{r}_F \tau} - F_0 e^{-\tilde{r}\tau}$$

- $\tilde{r}_F$ – foreign continuously-compounded interest rate,
- $\tilde{r}$ – domestic continuously-compounded interest rate
- Short position:

$$V_t^{short} = -V_t$$

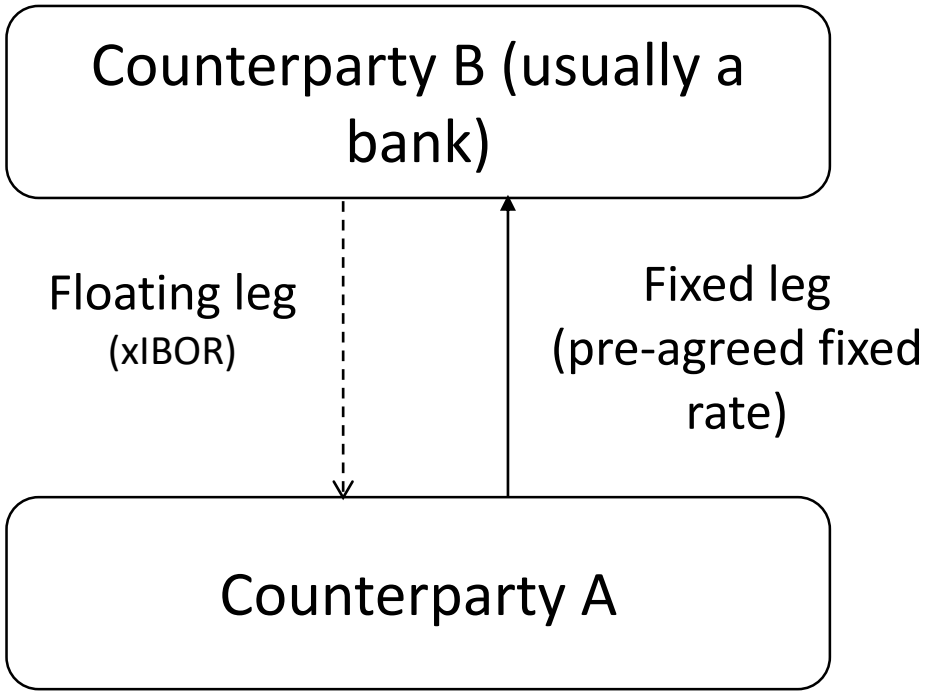
(Bartkowiak & Echaust, 2014)

Interest Rate Swap

Mechanics

- **Interest Rate Swap (IRS)** is an exchange of interest rate cash flows
- Depending on perspective:
  - **Receiver** receives fixed rate (sells IRS)
  - **Payer** pays fixed rate (buys IRS)
- IRS is basically a portfolio of FRA contracts with the same fixed rate

Typical IRS



Source: own research

Payoff

Long position

Payer

Pay Fixed rate

Receive Float rate

Float > Fixed ➔ $V_T^{long} > 0$

Short position

Receiver

Receive Fixed rate

Pay Float rate

Float < Fixed ➔ $V_T^{short} > 0$

Valuation – bond method

- $V_t^{long} = \tilde{P}_t - \bar{P}_t$
- $V_t^{short} = \bar{P}_t - \tilde{P}_t$
- $\tilde{P}_t$ – price of a bond with a floating rate,
- $\bar{P}_t$ – price of a bond with fixed rate.

(Bartkowiak & Echaust, 2014)

Valuation – forward rates method

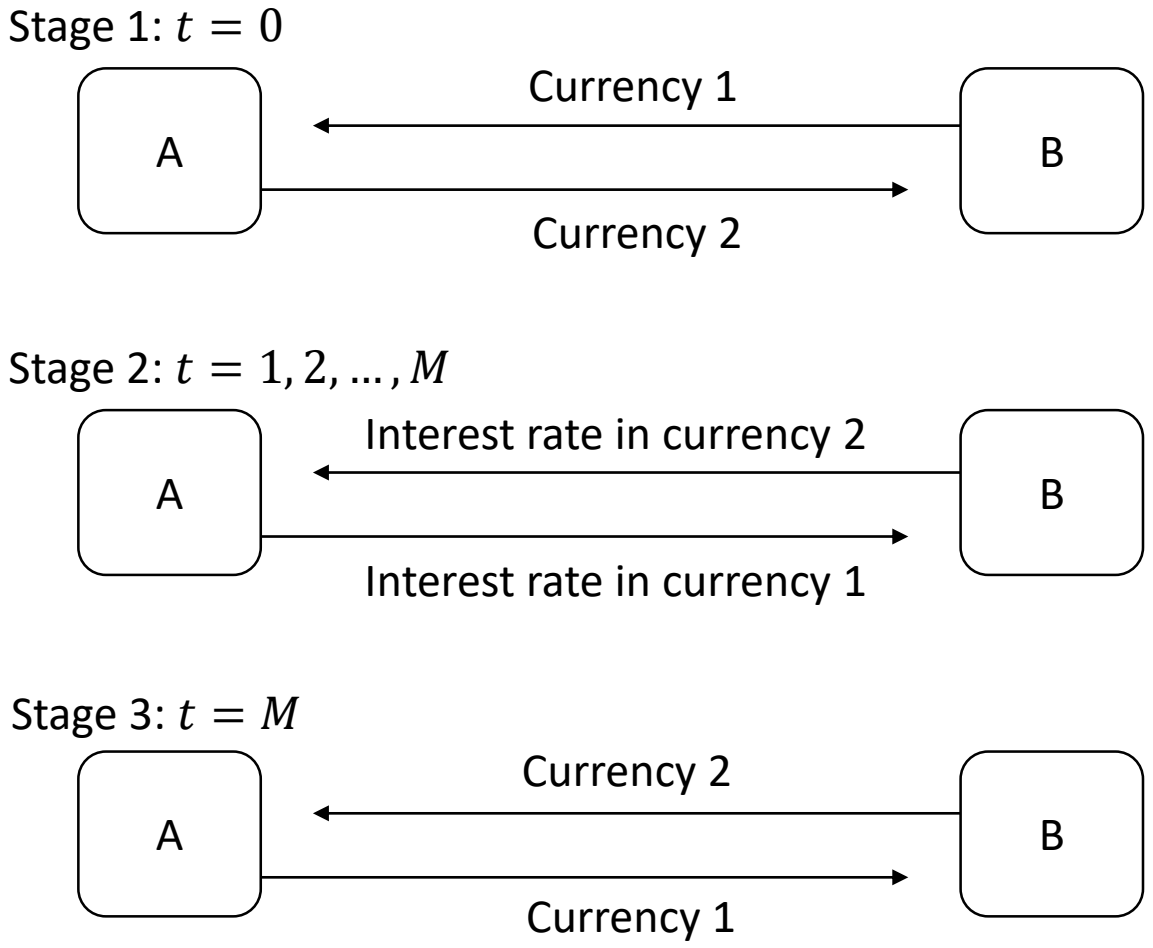
- $V_t^{long} = N\left(\left(\frac{c}{m} - \frac{r_{IRS}}{m}\right)e^{-\tilde{s}_{t_1}t_1} + \left(\frac{f_{t_1,t_2}}{m} - \frac{r_{IRS}}{m}\right)e^{-\tilde{s}_{t_2}t_2} + \cdots + \left(\frac{f_{t_{k-1},t_k}}{m} - \frac{r_{IRS}}{m}\right)e^{-\tilde{s}_{t_k}t_k}\right)$
- $V_t^{short} = -V_t^{long}$
- N – notional amount,
- c – floating interest rate used only in situation when the derivative lives for some time, and it has not price equal to zero,
- m – number of payments per year,
- r_{IRS} – fixed rate,
- $\tilde{s}_{t_k}$ – continuously compounded spot rate at time t for the k^{th} (last) cash flow,
- f_{t_{k-1},t_k} – forward rate at time t for the maturity $< k-1, k >$.

(Bartkowiak & Echaust, 2014)

Cross-currency Interest Rate Swap

Mechanics

- Hedges both FX and interest rate risks
- Initial capital exchange in two currencies at the current exchange rate.
- Periodic interest exchange payments on swap nominal amounts (possible interest rates: fixed-fixed, fixed-variable, variable-variable).
- Final equity exchange at the exchange rate starting from the swap date.



Source: (Bartkowiak & Echaust, 2014)

Payoff

For A

$$CF_A - CF_B \cdot F_A > 0$$

$$V_A > 0$$

For B

$$CF_B - CF_A \cdot F_B > 0$$

$$V_B > 0$$

Valuation – bond method

$$V_t = S_t \cdot P_t^f - P_t^d$$

(for the counterparty paying in domestic currency)

$$V_t = P_t^d - S_t \cdot P_t^f$$

(for the counterparty paying in foreign currency)

- P_t^d – domestic bond price
- P_t^f – foreign bond price
- S_t – FX rate

(Bartkowiak & Echaust, 2014)

Valuation – forward rate method (two fixed legs example)

$$V_A = \left(CF_A - CF_B \cdot F_{A_1}\right) \cdot e^{-\tilde{s}_{A_1} \cdot 1} + \left(CF_A - CF_B \cdot F_{A_2}\right) \cdot e^{-\tilde{s}_{A_2} \cdot 2} + \cdots + \left((CF_A + N_A) - (CF_B + N_B) \cdot F_{A_M}\right) \cdot e^{-\tilde{s}_{A_M} \cdot M}$$

- $F_{A_t} = C_A \backslash C_B \cdot e^{\left(\tilde{s}_{A_t} - \tilde{s}_{B_t}\right)}$
- $CF_A = N_A \cdot r_{IRS_A}$
- $CF_B = N_B \cdot r_{IRS_B}$
- M – moment of the last cash flow

Source: own research, (Bartkowiak & Echaust, 2014)

Valuation – forward rate method (two fixed legs example)

- V_A – value of CIRS for counterparty A
- C_A/C_B - current exchange rate (eg. PLN/EUR)
- $\tilde{S}_{A\,t}$, $\tilde{S}_{B\,t}$ – spot rates of counterparties A and B at time t
- $F_{A\,t}$ – forward exchange rate at time t
- CF_A, CF_B – cash flows of counterparty A and B
- N_A, N_B – total swap amounts of counterparties A and B
- r_{IRS} – fixed rate

Source: own research, (Bartkowiak & Echaust, 2014)

References

- Bartkowiak, M. & Echaust, K. (2014). *INSTRUMENTY POCHODNE Wprowadzenie do inżynierii finansowej.* Poznan: Wydawnictwo Uniwersytetu Ekonomicznego w Poznaniu.