1. Choose financial series (FX rates or stock index). You will need a 4-year long series (e.g. 10.05.2014-10.05.2018) of daily data. Calculate log returns.

Microsoft data was chosen from Yahoo Finance. The range is 2014-05-20 to 2018-05-20. The series is a daily series. Log returns were calculated using the **dlog** function from the OxMetrics calculator. The data points used were from the Adjusted Close values.

2. Describe the return series including information considering ACF and PACF, normality tests, unit root tests, long memory tests (will be presented next week), ARCH effect test, sign bias test, etc. Use graphs and tests available in OxMetrics.

<u>Log Return</u>

The actual series appears to be stationary, with a mean that exists just above zero. However, there are numerous points in time with large spikes in the price, both positively and negatively. From visual observation, these spikes do not appear to be cyclic/periodic in any way, nor are they always of the same magnitude.

Negative spike examples: 2015 end of Jan (25+), 2016 mid April (20+)
Positive spike examples: 2015 end of Apr (25+), 2015 end of Oct (23+)

<u>ACF</u>

The ACF graph showed auto-correlation to be statistically significant for the 5th and 8th lags of Microsoft's daily log returns. The 8th lag actually exceeds the threshold more than the 5th. One thing to note is that at the 5th and 8th lags, these ACF and PACF values are negative. This is quite a typical.

<u>PACF</u>

The PACF graph also showed lags 5 and 8 to have significant auto correlation. The 8th lag actually exceeds the threshold more than the 5th.

<u>Normality</u>

	Statistic	t-Test	P-Value
Skewness	0.21854	2.8368	0.0045575
Excess Kurtosis	8.7731	56.997	0.0000
Jarque-Bera	3240.7	.NaN	0.0000

For all three of the normality tests, our p-value for the particular test was less than the typical alpha level of 5%. For Skewness, and Kurtosis, this results mean our series is not normal. Since the Jarque-Bera test is built directly using Skew and Kurtosis, it of course falls in line to say that our series is not normally distributed. In fact, the value of the Jarque-Bera statistic, is so far from zero, that we are almost certain it's not from a Chi-Square distribution with 2-degrees of freedom (as it should be if the time-series was normally distributed).

<u>Unit Root Test</u>

```
ADF Test with 2 lags
     No intercept and no time trend
     H0: LogReturn is I(1)
ADF Statistics: -18.2733
     Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)
        1%        5%       10%
  -2.56572  -1.94093  -1.61663
```

Based on the test statistic of les than -18, we can reject the Null hypothesis of the test, and assume that our series has stationarity. In other words, it does not have a unit root.

```
KPSS Test without trend and 2 lags
      H0: LogReturn is I(0)
      KPSS Statistics: 0.0642312
Asymptotic critical values of Kwiatkowski et al. (1992), JoE, 54,1, p. 159-178
      1%       5%        10%
  0.739    0.463      0.347
```

The KPSS test agrees the ADF test. For this test, if the statistics exceeded the critical value we'd reject the null hypothesis of stationarity. However, our value of around 0.06, is less than the critical values for the 10%, 5% and 1% levels. We therefore cannot reject the null, and must assume stationarity or at the least lack of a unit root.

Long Memory Test (link)

Geweke

```
----  Log Periodogram Regression ----
d parameter        -0.0614324 (0.0324513) [0.0583]
No of observations: 1008; no of periodogram points: 504
```

Robinson

```
----  Gaussian semiparametric estimate ----
d parameter        -0.0483983 (0.0222718) [0.0298]
No of observations: 1008; no of periodogram points: 504
```

For a series to have long memory, the d-parameter should be between 0 and ½ . With our d-parameter in both tests being less than zero, we can conclude the series does not have long memory.

ARCH Effect

```
Series #1/1: LogReturn
---------
ARCH 1-2 test:    F(2,1003) =   5.9020 [0.0028]**
ARCH 1-5 test:    F(5,997)  =   3.4806 [0.0040]**
ARCH 1-10 test:   F(10,987) =   1.7497 [0.0657]
---------------
```

The LM ARCH test is showing that between lags 1-2, and lags 1-5, we have an ARCH effect – heteroscedasticity of our residuals. Because this behavior exists, it's useful to use the GARCH process.

3. Consider the ARMA (p,q) specifications for the returns. Show few competing models and indicate the best one. Justify the choice. Show the statistics for the residuals.

Based on the ACF and PACF graphs, we could estimate that there is an Autoregressive process and a Moving Average process of up to order 8 (for each). Our graphs showed lags 5 and 8 significant.

Using the Time-series ARFIMA model, we tested several different ARMA specifications (using MPL) from AR=8, MA=8, down to ARMA(1,1). Based on the OxMetrics progression results, the best model was ARMA(6,6). It had the best AIC, and the highest log-likelihood.

```
Progress to date
Model          T    p         log-likelihood         SC          HQ         AIC
A           1008   18  M        2854.9917     -5.5412     -5.5956     -5.6290
A           1008   16  M        2857.3383     -5.5595     -5.6079     -5.6376
A           1008   14  M        2858.2163     -5.5750     -5.6173     -5.6433<
A           1008   12  M        2853.7403     -5.5799     -5.6161     -5.6384
A           1008   10  M        2853.6186     -5.5933     -5.6236     -5.6421
A           1008    8  M        2851.8254     -5.6035     -5.6277     -5.6425
A           1008    6  M        2845.8545     -5.6054     -5.6235     -5.6346
A           1008    4  M        2847.0480     -5.6215<    -5.6336<    -5.6410

---- Modified profile likelihood estimation of ARFIMA(6,0,6) model ----
The estimation sample is:  2014-05-20 - 2018-05-18
The dependent variable is: LogReturn

                  Coefficient  Std.Error  t-value  t-prob
AR-1                 0.432653    0.08884     4.87   0.000
AR-2                -0.630979    0.09034    -6.98   0.000
AR-3                  1.69228     0.1074     15.8   0.000
AR-4                -0.684427    0.06998    -9.78   0.000
AR-5                 0.299013    0.09313     3.21   0.001
AR-6                -0.844398     0.1158    -7.29   0.000
MA-1                -0.455894     0.1062    -4.29   0.000
MA-2                 0.606411     0.1010     6.00   0.000
MA-3                 -1.67492     0.1216    -13.8   0.000
MA-4                 0.688594    0.08548     8.06   0.000
MA-5                -0.286587     0.1069    -2.68   0.007
MA-6                 0.802145     0.1381     5.81   0.000
Constant          0.000976101  0.0004057     2.41   0.016

log-likelihood      2858.21631
no. of observations       1008  no. of parameters          14
AIC.T              -5688.43261  AIC                 -5.64328632
mean(LogReturn)    0.000975218  var(LogReturn)      0.000200931
sigma                0.0139463  sigma^2             0.000194498

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.036199     -0.047955      0.030053     -0.022894     -0.064342     -0.038121      0.011980    -0.0070483
    -0.0050883     -0.056175      0.096198     -0.070733    0.00097522
```

Residuals Tests

We found an ARCH effect even up tot 5 lags in the residuals from the ARMA(6,6) model.

```
ARCH coefficients:
  Lag Coefficient  Std.Error
    1     0.10269    0.03186
    2     0.03496      0.032
    3    0.048575    0.03198
    4    0.041911      0.032
    5  -0.0037491    0.03186
RSS = 0.000382475  sigma = 0.000623137

Testing for error ARCH from lags 1 to 5
ARCH 1-5 test:    F(5,985)  =   3.7442 [0.0023]**
```

4. Consider GARCH (m,s) specifications for conditional variance. Take into account few different models (e.g. GARCH, IGARCH, FIGARCH, GJR-GARCH, GARCH-in-mean) and again choose the best one (based e.g. on the information criteria and the statistical significance of the parameters). You may change the specification in the mean equation from previous step at this point. Check the conditions for the parameters as well. Use tests to verify if the residuals are white noise and models are well-fitted.

We continued to use the ARMA(6,6) specification, and used onlgy GARCH(1,1). From selecting the different modeling methods, the FGARCH-Chung model process ended up with the best results among our tests. This meant it had the lowest Information Criteria values for Schwarz, Akaike, and Hannan-Quinn. It also had the highest log-likelihood.

```
Progress to date
Model            T     p          log-likelihood          SC          HQ         AIC
G             1008    17   B           3047.9304     -5.9308     -5.9823     -6.0138
G             1008    19   B           3044.5170     -5.9104     -5.9678     -6.0030
G             1008    18   B           3055.3007     -5.9386     -5.9930     -6.0264
G             1008    19   B           3068.0703     -5.9571     -6.0145     -6.0497
G             1008    16   B           3055.6096     -5.9529     -6.0013     -6.0310
G             1008    18   B           3079.1006     -5.9858<    -6.0403<    -6.0736<

Tests of model reduction (please ensure models are nested for test validity)
     G --> G      : Chi^2( 3) =      24.921 [0.0000] **
```

However, the FIGARCH model had no convergence. Only GARCH and GJR converged. In comparing these two, GJR had a better estimation.

```
Model            T     p          log-likelihood          SC          HQ         AIC
G             1008    17   B           3047.9304     -5.9308     -5.9823     -6.0138
G             1008    18   B           3055.3007     -5.9386<    -5.9930<    -6.0264<
```

Tests and Verification

We saved the Residuals, Squared Residuals, Standardized Residuals, and Conditional Variance to the database. After running the GJR model again, we ran the various set of tests on the model.

Sign Bias – Two-sided: seems significant. Positive and negative shocks have the same effect on errors
Positive/Negative – also significant and matching the two-sided test

```
Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                      Test   P-value
Sign Bias t-Test                   1.05460   0.29161
Negative Size Bias t-Test          0.00079   0.99937
Positive Size Bias t-Test          0.40236   0.68742
Joint Test for the Three Effects   1.50972   0.68003
```

ARCH Test – There's no longer any ARCH effect.

```
---------------
ARCH 1-2 test:     F(2,1001) = 0.034914 [0.9657]
ARCH 1-5 test:     F(5,995)  =  0.10033 [0.9920]
ARCH 1-10 test:    F(10,985) =  0.16089 [0.9985]
---------------
```

5b. Find different series that might be somehow related to one chosen at the beginning (e.g. DJ might be related to S&P, some currencies also behave similarly). Model both series within MGARCH (scalar BEKK, diagonal BEKK, CCC, DCC), find the best model and obtain conditional volatility of both series, conditional covariances and conditional correlations. Indicate the best model and justify your choice.

We selected the DOW Jones Index from the same period. When modeling the returns of Microsoft with the included information of the DOW Jones Index, we found that using an ARMA (5,5) GARCH(1,1) model was the best, and that our results shows the best values under Diagonal-BEKK, (something to do with [semi-]positive definiteness of a matrix built form past observations), using a student distribution.

```
******************************
 ** MG@RCH(1) SPECIFICATIONS **
 ******************************
Conditional Mean : ARMA (5, 5) model. No regressor in the conditional mean.
Conditional Variance : Diagonal BEKK (1, 1).
No regressor in the conditional variance
Multivariate Student distribution, with 3.4328 degrees of freedom.
      Strong convergence using numerical derivatives
      Log-likelihood = 6992.11
 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst1                 0.001204 0.00024175    4.979  0.0000
Cst2                 0.000728 0.00016489    4.416  0.0000
AR_1-1               0.271198    0.21763    1.246  0.2130
AR_2-1               0.342112   0.088746    3.855  0.0001
AR_3-1               0.166272    0.13989    1.189  0.2349
AR_4-1              -0.704121    0.16468   -4.276  0.0000
AR_5-1               0.176552    0.26382   0.6692  0.5035
AR_1-2              -0.366699    0.21012   -1.745  0.0813
AR_2-2              -0.319370   0.080235   -3.980  0.0001
AR_3-2              -0.110785    0.13513  -0.8199  0.4125
AR_4-2               0.734146    0.16106    4.558  0.0000
AR_5-2              -0.303958    0.25673   -1.184  0.2367
MA_1-1               0.028182    0.28126   0.1002  0.9202
MA_2-1               0.346578    0.11871    2.919  0.0036
MA_3-1              -0.110824    0.11057   -1.002  0.3165
MA_4-1              -0.803272    0.11796   -6.810  0.0000
MA_5-1              -0.061939    0.18301  -0.3384  0.7351
MA_1-2              -0.113810    0.27944  -0.4073  0.6839
MA_2-2              -0.339235    0.12067   -2.811  0.0050
MA_3-2               0.145191    0.10530    1.379  0.1683
MA_4-2               0.808259    0.11872    6.808  0.0000
MA_5-2              -0.032426    0.18196  -0.1782  0.8586
C_11                 0.004610 0.00064530    7.145  0.0000
C_12                 0.001341 0.00023683    5.660  0.0000
C_22                 0.001589 0.00023163    6.860  0.0000
b_1.11               0.858383   0.026321    32.61  0.0000
b_1.22               0.901339   0.017901    50.35  0.0000
a_1.11               0.423312   0.053679    7.886  0.0000
a_1.22               0.399790   0.045283    8.829  0.0000
df                   3.432800    0.37473    9.161  0.0000
No. Observations :      1008  No. Parameters  :        30
No. Series       :         2  Log Likelihood  :  6992.113
Elapsed Time : 19.508 seconds (or 0.325133 minutes).
```