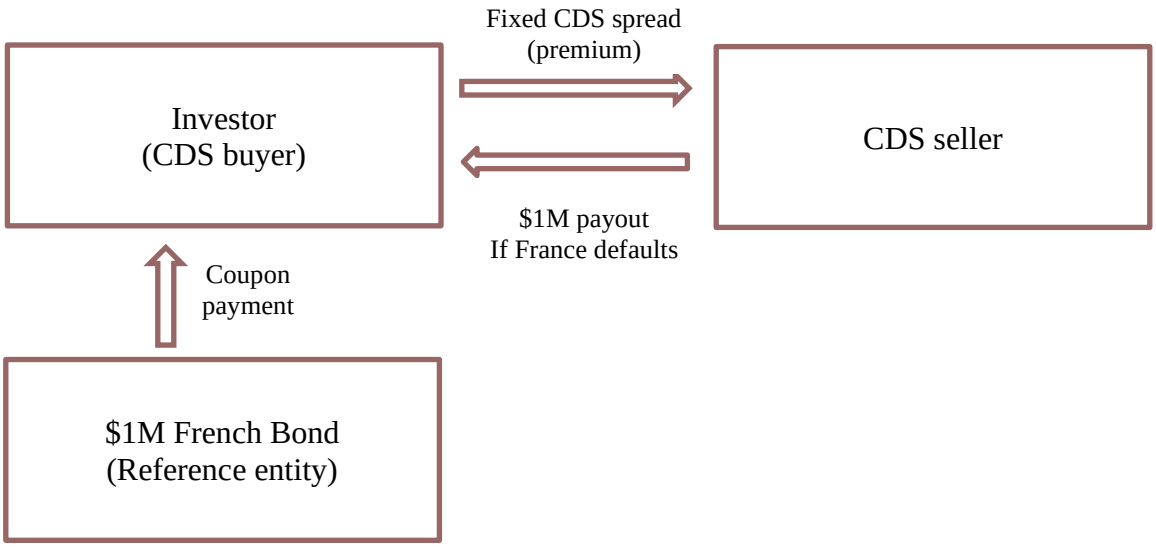
# 1. Correlation Risk

## 1.1 Correlation Risk

Correlation risk measures the risk of financial loss resulting from adverse changes in correlations between financial or nonfinancial assets. Assets that previously had very low or negative correlations suddenly become very highly positively correlated and fell in value together in the financial crisis in 2007. Static financial correlations do not change and measure the relationship between assets for a specific time period (i.e., VaR). Dynamic financial correlations measure the co-movement between assets over time. Examples of dynamic financial correlations are pair trading, deterministic correlation approaches, and stochastic correlation processes.

## 1.2 Credit Default Swap (CDS)

A CDS transfers credit risk from investor (CDS buyer) to a counterparty (CDS seller). Suppose an investor purchases $1 million of French bonds and is concerned about France defaulting. The investor (CDS buyer) can transfer the default risk to a counterparty (CDS seller).




Assume the recovery rate is zero with no accrued interest in the event of default. In practice, there is no consideration about the counterparty risk, the solvency of the CDS seller, for the convenience of CDS valuation. But according to the Hull & White model, for the CDS valuation there needs to be considered the correlation between CDS seller and reference entity. In the academic finding (White), CDS premia and the default correlation is in negative relationship. In other words, the CDS premia are highest when the joint default correlation between CDS seller and reference entity is 0, while smallest when the joint default correlation approaches to 1 or -1.

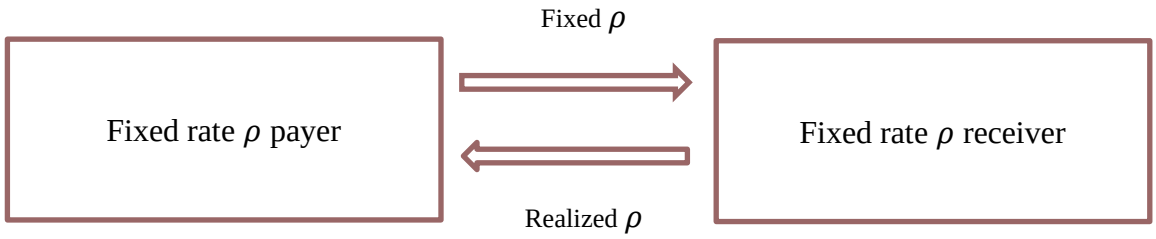
If there is positive correlation between CDS seller and France, the investor has wrong-way risk (WWR). The higher correlation risk, the lower the CDS spread (premium), s. The wrong-way risk means that one counterparty adversely correlated with the other, in this case when CDS seller's credit rating decreases, the investor (CDS buyer) gets loss. Conversely, the right-way risk means one counterparty is friendly correlated with the other.

## 1.3 Multi-Asset Options

An **exchange option** is the option that payoff is determined by $\max\{0, S_1 - S_2\}$. The correlation between the two assets $S_1,\ S_2$ is an important factor in determining the price of correlation options. The higher dispersion between $S_1,\ S_2$ is (when two assets are negatively correlated), the higher the valuation of exchange option. For example, the value of an exchange option that sells euro (weight = -1) and buys USD (weight = 1) would be priced high when the volatility of the portfolio is high. Portfolio's volatility is $\sigma_p = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}}$ since the weight of the portfolio is -1 and 1. When the correlation between the euro and USD is high, the portfolio volatility becomes high.

The **quanto option** is another investment strategy using correlation options. A quanto option (short for "quantity-adjusted option") is a type of exotic financial derivative where the option's underlying asset is denominated in one currency, but the payoff is converted into another currency at a pre-specified or fixed exchange rate. Suppose a US investor invests in the Nikkei stock index call option and enters a forward FX rate contract with a rate of 100 USDJPY. When the Nikkei stock index goes up, the investor earns 100 yen but if the spot rate becomes 50 USDJPY, the payoff is 2$. Thus, the lower the correlation (Nikkei stock index and the value of a USD), the higher the price for the quanto option.

A **correlation swap** is used to trade a fixed correlation between two or more assets with the correlation that occurs. The correlation that will occur is unknown and is referred to as the realized or stochastic correlation.




$$Realized\ \rho = \frac{2}{n(n-1)} \sum_{i>j} \rho_{i,j}$$

Q. Suppose a correlation swap buyer pays a fixed correlation rate of 0.2 with a notional value of $1 million for one year for a portfolio of three assets. The realized pairwise correlations of daily log returns at maturity for three assets are $\rho_{2,1} = 0.6$, $\rho_{3,1} = 0.2$, $\rho_{3,2} = 0.04$. What is correlation buyer's payoff?

A. $1 million × (0.28-0.20) = $80,000

Another example of trading correlation is to buy call options on a stock index and sell call options on individual stocks held with the index. Since the index and a specific individual stock (such as TSLA) are relatively uncorrelated, the individual stock return can be seen small or flat changes while the index return considerably changes. An investor can also buy correlation by paying fixed in a **variance swap** on an index and receiving fixed on individual securities within the index. An investor pays fixed on an index and receives fixed on individual securities. An increase in correlation for securities within the index causes the variance to increase. An increase in variance causes the present value of the position to increase for the fixed variance swap payer (i.e., variance swap buyer).

## 1.4 Risk Management

The formula for calculating VaR using variance-covariance method (a.k.a delta-normal method) is shown as follows:

$$VaR_p = \sigma_p \times \sqrt{T} \times z_\alpha$$
$$\sigma_p = \sqrt{\beta^T \times C \times \beta}$$

, where $\beta$ is weight of the portfolio and C is the covariance matrix.

Q. Assume you have a two-asset portfolio with $8 million in asset A and $4 million in asset B. The portfolio is 0.6, and the daily standard deviation of returns for assets A and B are 1.5% and 2%, respectively. What is the 10-day VaR of this portfolio at a 99% confidence level (i.e., $\alpha$ = 2.33) and the capital requirement by the Basel Committee, if green zone multiplier is 3?

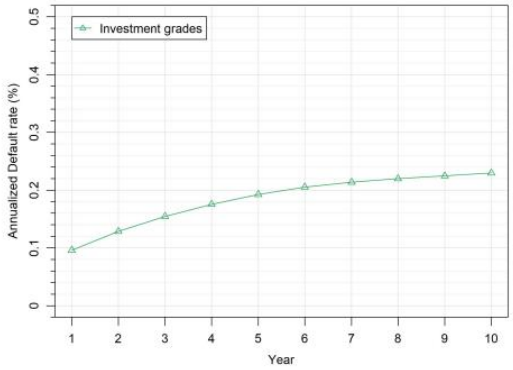
A. 10-day VaR = $1.3248 million, capital requirement = $1.2248 million × 3 = $3.9744 million

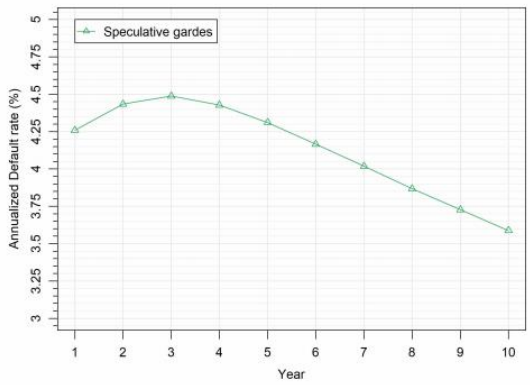
The correlations of assets within and across different sectors and geographical regions were a major contributing factor for the financial crisis of 2007 to 2009. The overly optimistic housing market led individuals to take on more debt on overvalued properties. New structured products known as collateralized debt obligations (CDOs), constant-proportion debt obligations (CPDOs), and credit default swaps (CDSs) helped encourage more speculation in real estate investments. Rating agencies, risk managers and regulators overlooked the amount of leverage individuals and financial institutions were taking on.

CDOs are separated into several tranches based on the degree of default risk. The riskiest tranche is called equity tranche, and investors in this tranche are typically exposed to the first 3% of defaults. The next tranche is referred to as the mezzanine tranche where investors are typically exposed to the next 4% of defaults (above 3% to 7%). A number of large hedge funds were long the CDO equity tranche and short the CDO mezzanine tranche. Unfortunately, huge losses lead to bankruptcy fillings by several large hedge funds because the correlation properties across tranches were not correctly understood. The mezzanine tranche, which was the fully collateralized asset of the CDS, also deteriorated since their correlation was too high.

The CDS seller must be financially stable enough to protect against losses. The global financial crisis revealed that American International Group (AIG) was overextended, selling $500 million in CDSs with little reinsurance. Also Lehman Brothers had leverage 30.7 times greater than equity in September 2008 leading to its bankruptcy.

Migration risk is the risk that the quality of a debtor decreases following the lowering of quality ratings. Lower debt quality ratings imply higher default probabilities. Default correlation is of critical importance to financial institutions in quantifying the degree that occurs at the same time. Most default correlations across industries are positive except for the energy sector. The energy sector has little or no correlation with other sectors and is, therefore, more resistant to recessions. Historical data suggests that default correlations are higher within industries.




The default term structure increases slightly with time to maturity for most investment grade bonds. This is expected because bonds are more likely to default as many market or company factors can change over a longer time period. Conversely, for non-investment grade bonds, the probability of default is higher in the immediate time horizon. If the company survives the near-term distressed situation, the probability of default decreases over time.

Systemic risk refers to the potential risk of a collapse of the entire financial system. From OCT 2007 to MAR 2009, the Dow Jones Industrial Average fell over 50% and only 11 stocks increased, which were consumer stables, educational, entertainment, energy and automotive. During the time period, correlations of stocks with each other increased dramatically from a pre-crisis average correlation level of 27% to 50%. Concentration risk is the financial loss that arises from the exposure to multiple counterparties for a specific group. Concentration risk is measured by the concentration ratio as follows: $Concentration\ ratio = \sum w_i^2$.

Q. Suppose commercial bank Y makes $2,500,000 loan to company A and a $2,500,000 loan to company B. Assuming companies A and B each have a 5% default probability, what is the concentration ratio and expected loss (EL) for commercial bank Y under the worst case scenario? Assume default correlation between companies is 1.0 and loss given default (LGD) is 100%.

A. Concentration ratio = 0.05, EL = $5 million × 5% = $0.25 million