# README - HMQGM Project

### Overview

This repository contains R scripts and functions for simulation studies, methodological development, and empirical applications of the **Hidden Markov Quantile Graphical Model (HMQGM)** and related baselines (e.g., **HMGlasso**).

The code has been developed for the paper "Hidden Markov Quantile Graphical Model". The code supports the analyses reported in the manuscript (main text §§4.1–4.3, SM §§S2.1–S2.3) and the real data application on PM2.5 concentrations in Northern Italy (main text §§5).

## **Scripts**

#### Simulation Studies

• simulations\_4.1-2\_HMQGM.R

Simulation study for §§4.1–4.2 fitting HMQGM. Focus: edge recovery (ROC curves) and clustering performance (ARI).

- simulations\_4.1-2\_HMGlasso.R
  - Same design as above, but fitting HMGlasso instead of HMQGM.
- simulations 4.3 HMQGM.R

Simulation study for §4.3 fitting HMQGM. Focus: performance of AIC/BIC/ICL for selecting the true number of hidden states.

- simulations 4.3 HMGlasso.R
  - Same design as above, but fitting HMGlasso.
- simulations S2.1 HMQGM.R

SM  $\S$ S2.1: Edge recovery fitting HMQGM when data are generated as in Chun et al. (2016) with K=1 state.

• simulations\_S2.1\_HMGlasso.R

Same as above but for HMGlasso.

• simulations\_S2.3.R

SM  $\S$ S2.3: Sensitivity analysis with smoothly time-varying adjacency matrices, comparing models for  $K=1,\ldots,5$ .

## **Empirical Application**

• RealdataScript HMQGM.R

Application to PM\_{2.5} concentrations in 14 Northern Italian cities (2019–2022). Fits HMQGM with K=1,...,4 and saves results for AIC/BIC/ICL model selection, adjacency structures, selected lambdas, and runtime information for downstream tables/figures.

## **Functions**

• em\_glasso.R

Implements the EM algorithm for HMGlasso (em.glasso).

• EM\_lqgm.Mix\_c.R

Implements the penalized EM algorithm for HMQGM (em.hmm.pen.lqgm).

• MainFunctions.R

Collection of helper functions for data generation and evaluation:

- Theta\_gen: build precision matrices for Scenario 1.
- Ygen\_Chun: data generator as in Chun et al. (2016).
- Ygen1\_sep: conditional-quantile data generator for Scenario 2.
- Ygen1\_dynamic: dynamic data generator with smooth transitions.
- Graph.performance: compute TPR/FPR and related graph metrics.
- Viterbi: classical Viterbi algorithm for HMM state decoding.
- RealdataFun.R

Function realdata to fit HMQGM on real datasets with multi-start and lambda grid, returning selected models, criteria, adjacency structures, and timing.

### Notes

- All scripts are designed to be modular: simulation files source the required functions before running.
- Parallel execution is supported via parallel and doParallel.
- Non-CRAN dependencies (rqPen, LQGM) must be installed from local tarballs.
- Each simulation script saves results as .RData objects (out\_sim, out\_glassoHM, results, or out\_hmqgm\_dyn).
- The real data script saves one .RData file per K (HMLQGMrealdata\_<K>K.RData).