Image Registration

Theory and Practice

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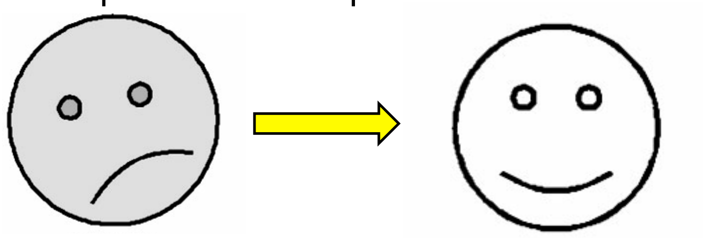
What is Image Registration?

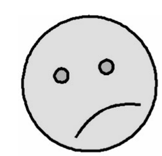
Definition: It is a method to map two different images, which are acquired with the same or different experimental setups.



Moved Image (Source) $I_M(x)$ — Fixed Image (Target) $I_F(x)$

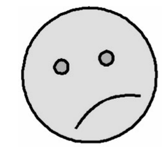
Outlook

- Theory of image registration



Outlook

- Theory of image registration
- Practice:
  - Global registration
  - Local registration



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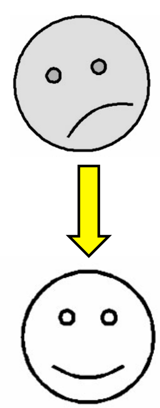

What is Image Registration?

 $I_M(x) \xrightarrow{T:\Omega_F \subset \Re^d \rightarrow \Omega_M \subset \Re^d} I_F(x)$ 

The transformation is defined as a mapping from the moving to the fixed image.

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$$\hat{T} = \arg\min_T C(T; I_F, I_M)$$

$$C(T; I_F, I_M) = -S(T; I_F, I_M) + \gamma P(T)$$

How good is the transformation?

$$C(T;I_F,I_M) = -S(T;I_F,I_M) + \gamma P(T)$$

(Similarity) (Penalty)

$S(T;I_F,I_M)$ evaluate the goodness of the alignment

Criteria:

- Correlation ratio;
- Sum of squared differences (SSD);
- Mutual information (MI).

$P(T)$ penalizes control points displacements that potentially lead to naturally implausible deformations

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- **Mutual information (MI)**.

How good is the transformation?

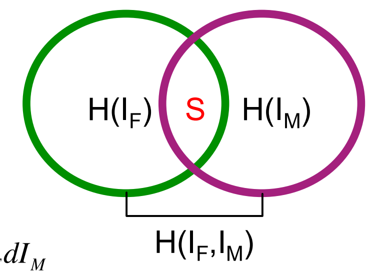
Mutual information (MI)

❖Histogram based;
❖Robust against image degradation;
❖No segmentation required.

$$\boxed{S(T;I_F,I_M) = \frac{H(I_F)+H(I_M)}{H(I_F,I_M)}}$$

$$H(I_{F/M}) = -\int p(I_{F/M})\ln(I_{F/M})dI_{F/M}$$

$$H(I_F,I_M) = -\iint p(I_F,I_M)\ln p(I_F,I_M)dI_F dI_M$$



Optimization problem

Def: Optimization refers to the manner in which the transformation is adjusted to improve the image similarity

(1) *Quasi Newton Broyden-Fletcher-Goldfarb-Shanno (BFGS)*

(2) *Limited-memory BFGS (L-BFGS)*

(3) *Steepest descent method*

Optimization problem

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(1) *Quasi Newton Broyden-Fletcher-Goldfarb-Shanno (BFGS)*, which is a second-derivative line search method, a powerful method to solve unconstrained optimization problem (Shanno et al., 1970)
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(2) *Limited-memory BFGS (L-BFGS)*, which approximates the BFGS algorithm, however using less computer memory and widely used in machine learning.
(3) *Steepest descent method*

Optimization problem

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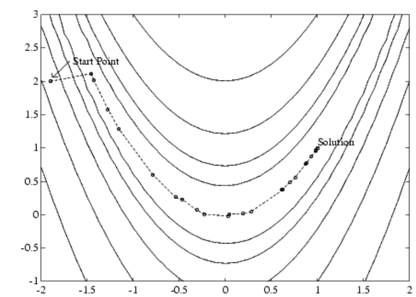


BFGS Method on Rosenbrock's Function

Type of transformations:

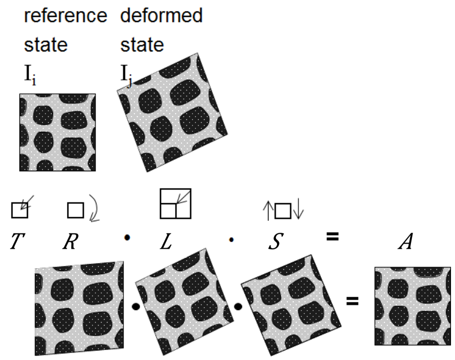
Global transformation: map is composed of a single equation that maps each point in the first image to new location in the second image. The equation is a function of the locations of the first image, but it is the same function for all parts of the image, i.e., the parameters of the function do not depend on the location.

Local transformation: mapping of points in the image depends on their location. The map is composed of several smaller maps (several equations) for each piece of the image that is considered.

Brown, L.G. (1992). A survey of image registration techniques. *ACM Computing Surveys*, 24 (4), 325-376.

Type of transformations:

Affine registration



$$\vec{x}' = \hat{f}(\vec{x}) = \bar{\bar{A}}\vec{x} + \bar{T}$$

$$\bar{\bar{A}} \equiv \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{23} & A_{33} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \bar{\bar{S}} \cdot \bar{\bar{L}} \cdot \bar{\bar{R}}.$$

$$\bar{u}_{i \to j}(\bar{x}) = \bar{x}' - \bar{x} = (\bar{\bar{A}} - \bar{1}_4) \cdot \bar{x},$$

$$\varepsilon_{11} \equiv \frac{\partial u_{1,i \to j}}{\partial x_1} = A_{11} - 1,$$

$$\varepsilon_{22} \equiv \frac{\partial u_{2,i \to j}}{\partial x_2} = A_{22} - 1,$$

$$\varepsilon_{33} \equiv \frac{\partial u_{3,i \to j}}{\partial x_3} = A_{33} - 1,$$

D.Derome et al., Journal of Structural Biology, 2011
A.Patera et al., Journal of Structural Biology, 2013

Optimization problem

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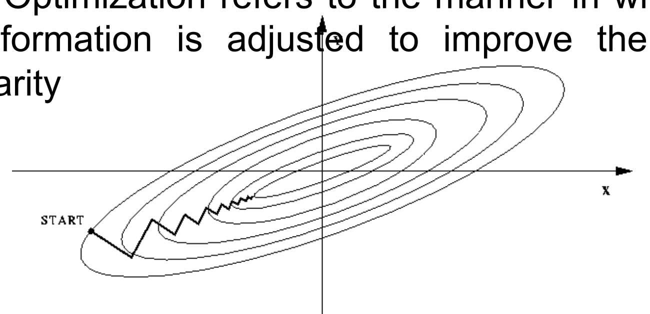

(3) *Steepest descent method*, which is the simplest of the gradient methods. Given a function, the method approaches the minimum in a zig-zag way, in a direction opposite to the function gradient.

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NON-RIGID (CURVED)TRANSFORMATION
considered.

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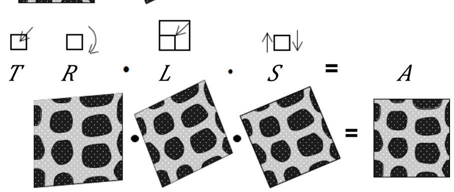
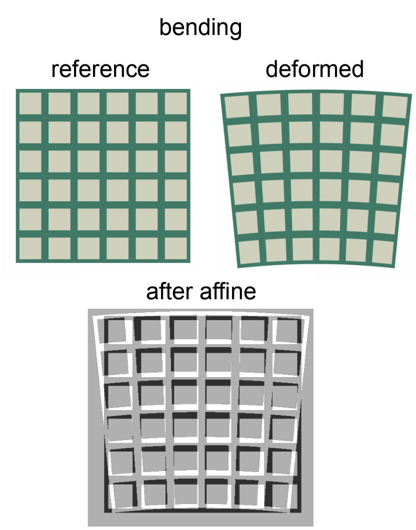
Type of transformations:

Affine registration

reference state I_i deformed state I_j





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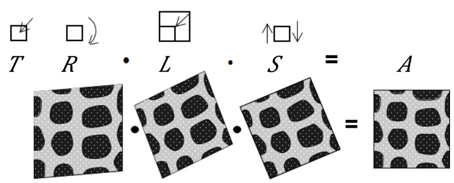

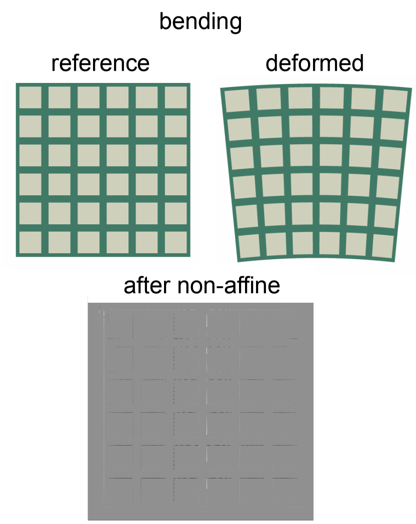

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Type of transformations:

Non-rigid registration: locally warping the moving image to align with the reference image.

Examples: curved transformations (DOF = 100 to 1000)

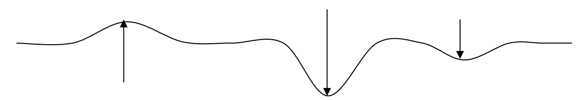
- Thin-plate splines
  Belong to the set of radial-basis functions
- Cubic B-splines
  Belong to the set of B-splines
- Both are better than the polynomial transformations
  Polynomial requires too many terms to produce a “well behaved” transformation.

Type of transformations:

Non-rigid registration:

Thin-plate splines

- Suggested for image registration by Ardi Goshtasby in 1988
- Based on an analogy to the approximate shape of thin metal plates deflected by normal forces at discrete points



Type of transformations:

Non-rigid registration:

Thin-plate splines

- x' and y' have this form (two dimensional example):

$$x' = a_0 + a_1 x + a_2 y + \sum_{i=1}^{N} F_i r_i^2 \ln r_i^2$$

$$y' = b_0 + b_1 x + b_2 y + \sum_{i=1}^{N} G_i r_i^2 \ln r_i^2$$

radial basis

where $r_i^2 = (x - x_i)^2 + (y - y_i)^2$

Type of transformations:

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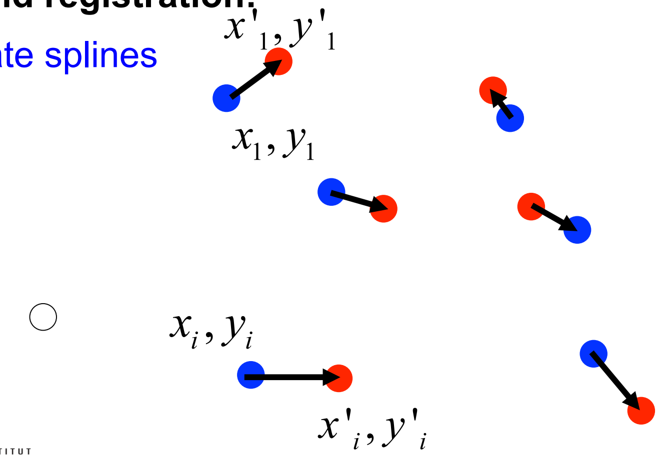
Thin-plate splines



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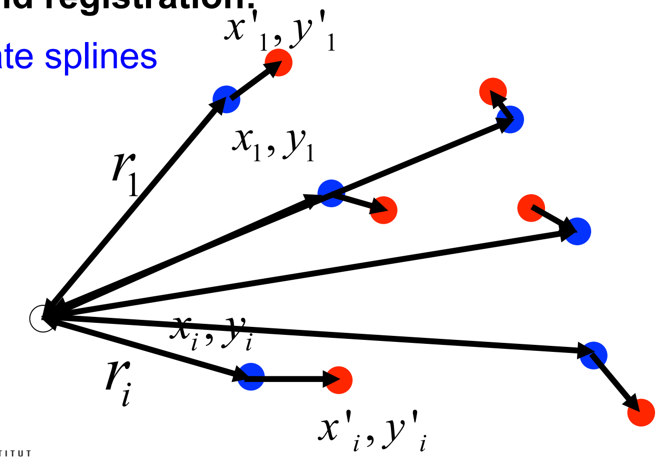


Type of transformations:

Non-rigid registration:

Thin-plate splines

As x moves away from the *N* fiducial points, the terms in the sum

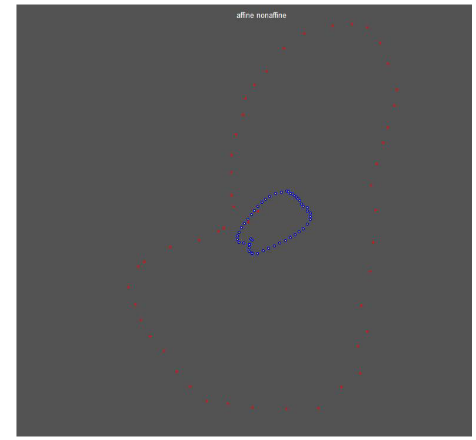
$$\sum_{i=1}^{N} F_i r_i^2 \ln r_i^2$$

begin to cancel out. The sum → 0. The same thing happens for *y*, so

$$x' \to a_0 + a_1 x + a_2 y$$

$$y' \to b_0 + b_1 x + b_2 y$$

(movie)

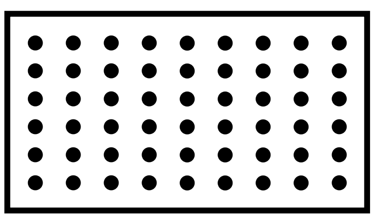


Type of transformations:

Non-rigid registration:

Cubic B-splines

- It determines the motion of all points on few “control” points.

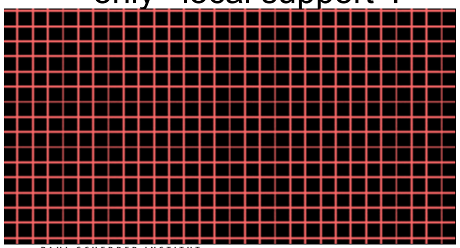


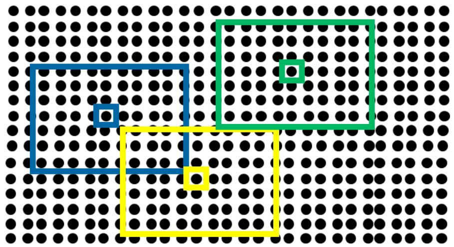
Type of transformations:

Non-rigid registration:

Cubic B-splines

- Good behavior (i.e., small effect from motion of remote control points) is due to the fact that it uses only “local support”:





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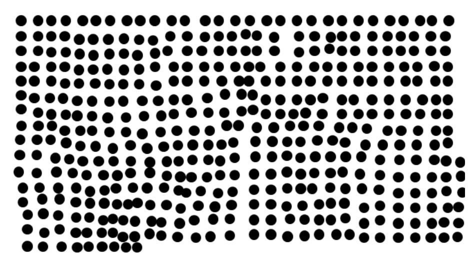
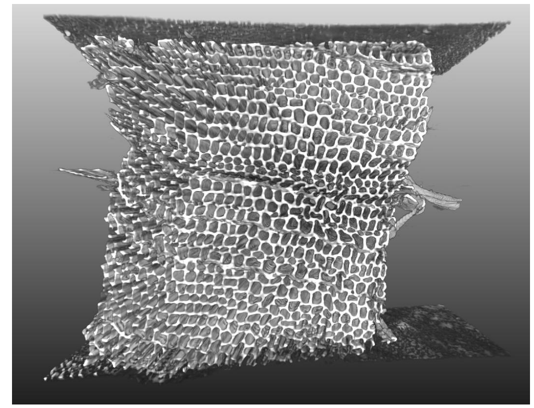


Case study:

Material science: wood

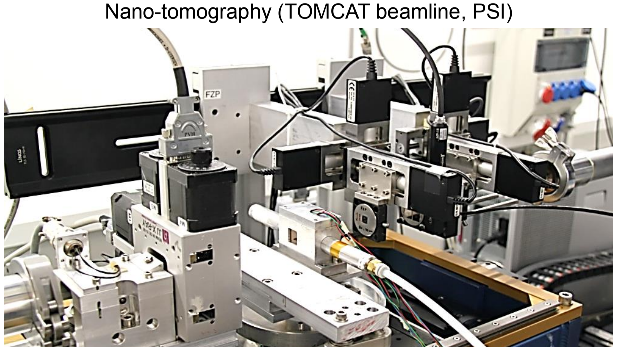
(movie)



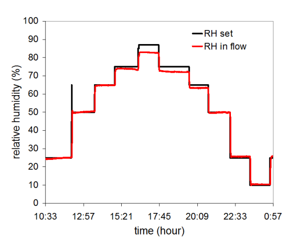
Case study:

Material science: wood

Nano-tomography (TOMCAT beamline, PSI)



hygroscopic loading protocol

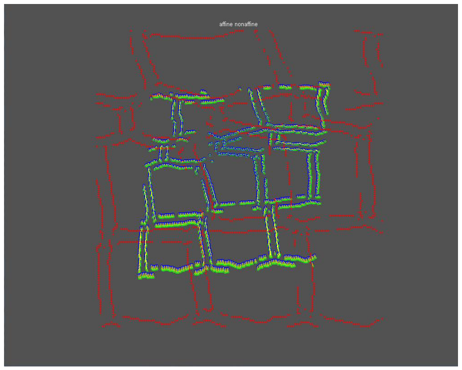


Case study:

Material science: wood

Thin-plate splines

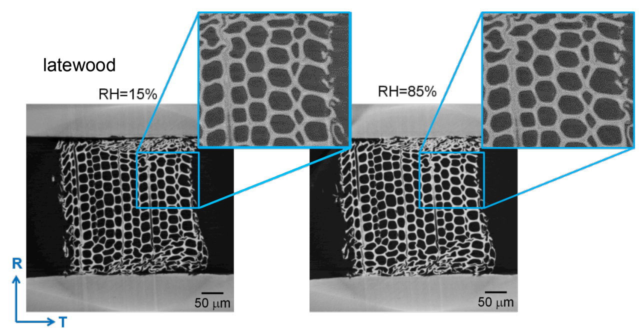
(movie)



Case study:

Material science: wood

Cubic B-splines: Free Form Deformation model



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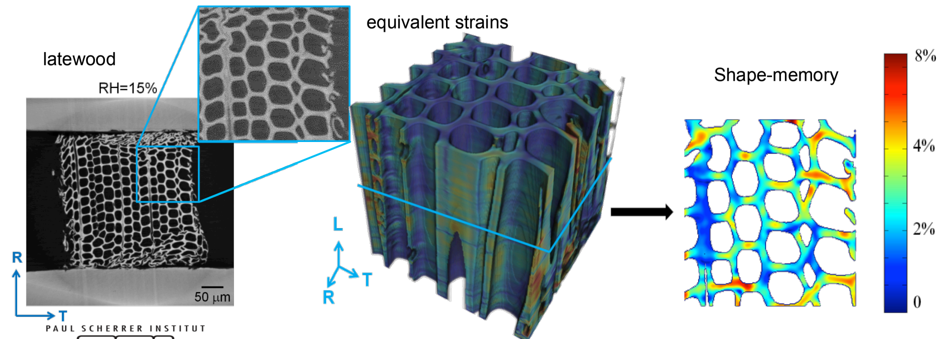

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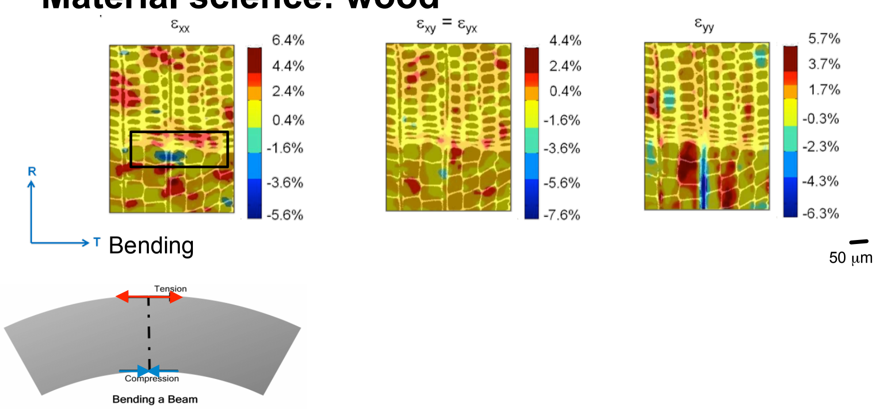
Case study:

Material science: wood



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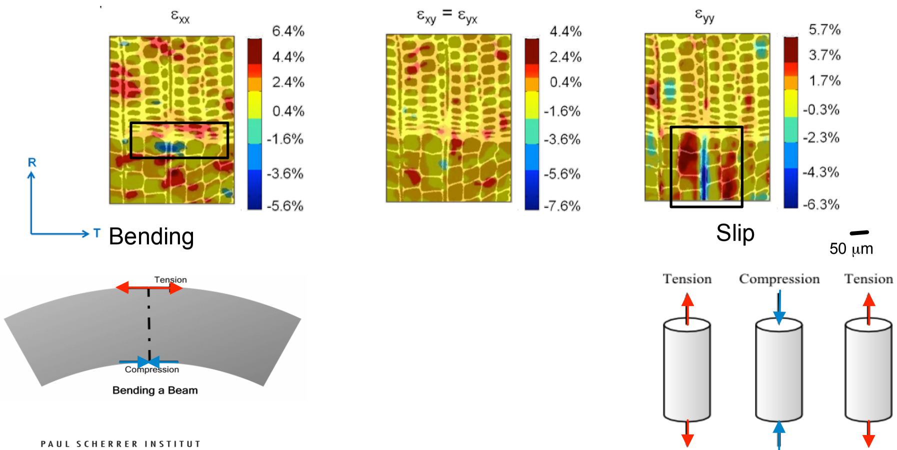
Case study:

Material science: wood



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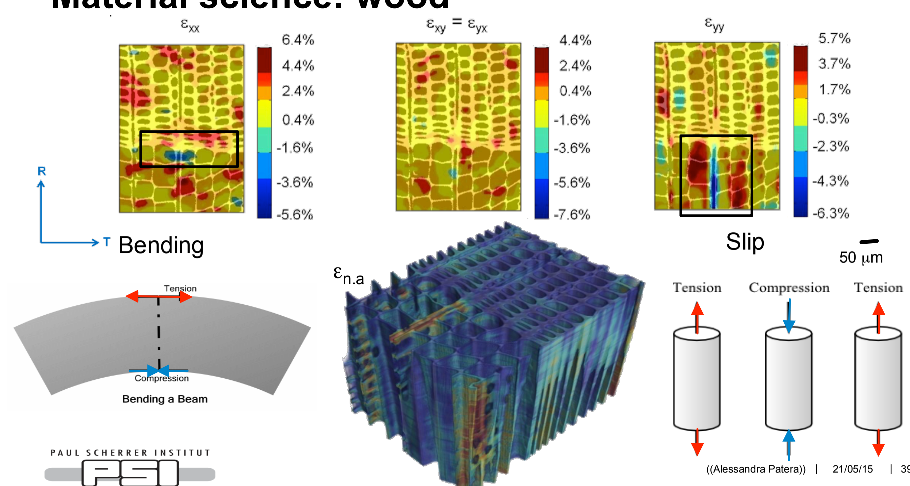
Case study:

Material science: wood



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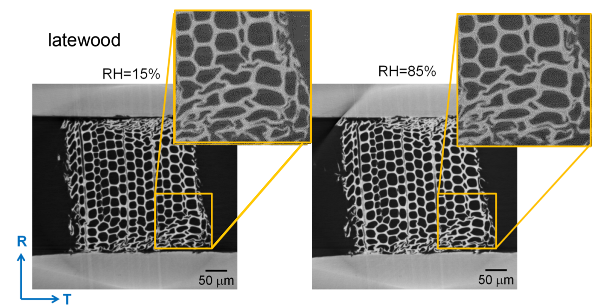
Case study:

Material science: wood



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Take home messages:

- Image registration is a powerful method to study your sample, if subjected to changes
- It can be applied in:
  - Medicine: to study the evolution of certain pathologies;
  - Material science: to understand the deformations of your samples;
  - Astrophysics: to align images taken from space.
- You can find always a good solution!

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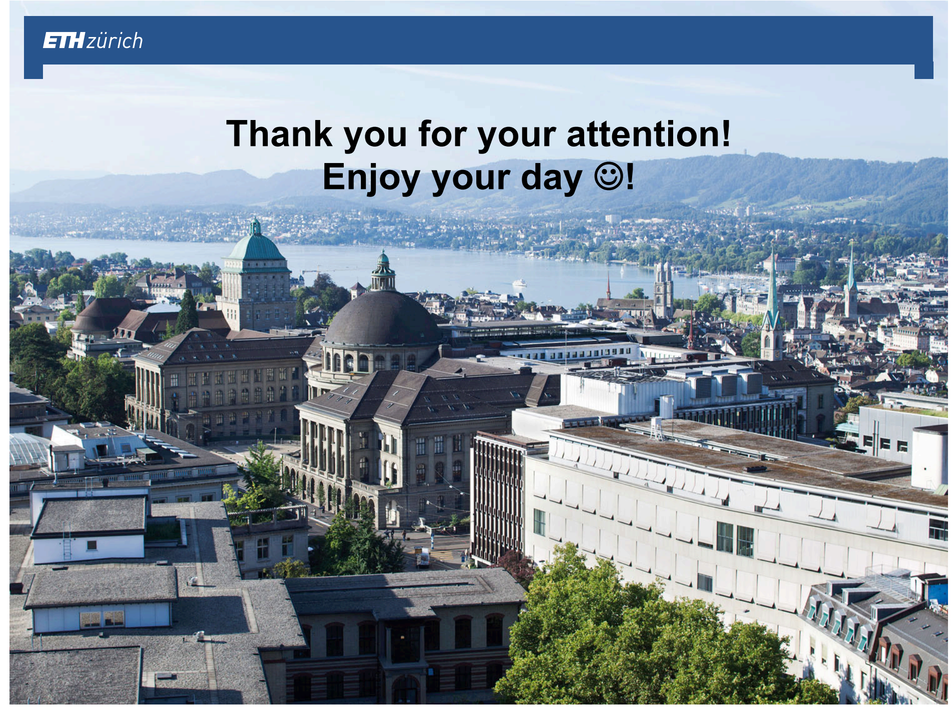

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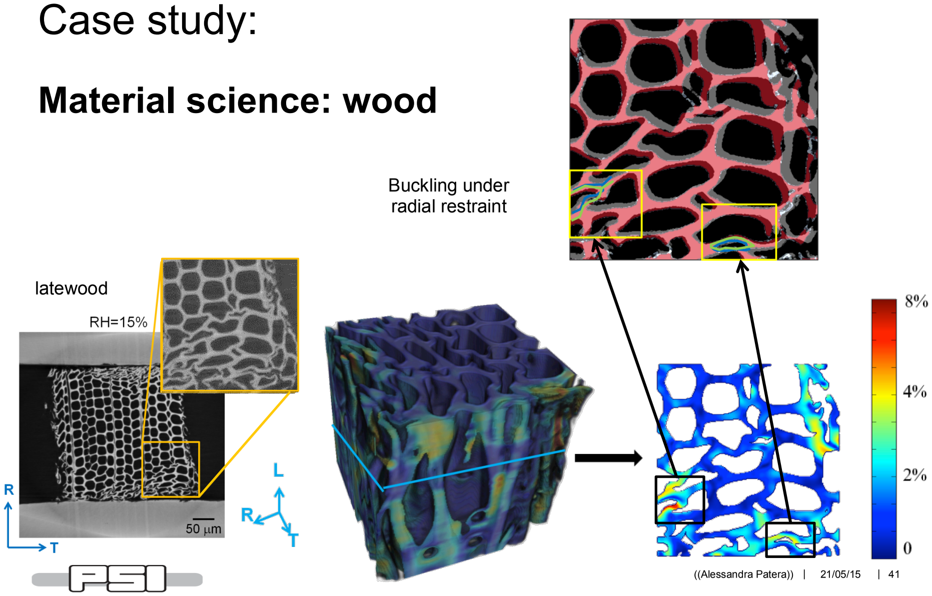
Case study:

Material science: wood



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