NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2022–2023

MH2100 – Calculus III

November 2022 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises of **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
5. Calculators are allowed.
6. Candidates should clearly explain their reasoning used in each of their answers.

QUESTION 1. **TOTAL: (6 marks)**

Find the tangent plane to the surface $x^3 - 2xy + \cos(\pi z) = 0$ at $(1, 0, 1)$. Write your answer in the form $Ax + By + Cz = 3$.

QUESTION 2. **TOTAL: (10 marks)**

Describe all points (x, y) such that $w_x + w_y = 0$, where $w = uv$, $u = x^2 - y^2$, and $v = x^2 + y^2$. Write your answer in the form $y = Ax^3 + Bx^2 + Cx + D$.

QUESTION 3. **TOTAL: (15 marks)**

Let

$$f(x,y) = \begin{cases} \dfrac{y^3 - 3x^2y}{x^2+y^2}; & \text{if } x^2+y^2 \neq 0 \\ 0; & \text{if } x^2+y^2 = 0. \end{cases}$$

(a) Is f continuous at $(0, 0)$? Justify your answer.

(b) Find $f_x(0, 0)$ and $f_y(0, 0)$.

(c) Is f differentiable at $(0, 0)$? Justify your answer.

QUESTION 4. **TOTAL: (15 marks)**

Determine the absolute minimum and maximum values of the function

$$f(x,y) = 2x^2 - 2xy + y^2 - y + 3$$

restricted to the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 2)$.

QUESTION 5. **TOTAL: (12 marks)**

Consider the positively-oriented closed curve C consisting of the semicircle given by $y = \sqrt{4 - x^2}$ from $(2, 0)$ to $(-2, 0)$ and part of the x-axis from $(-2, 0)$ to $(2, 0)$. Calculate the work done of the vector field $\mathbf{F} = (x^2 - 2y^2)\mathbf{i} + (2y - 2xy)\mathbf{j}$ along the curve C.

QUESTION 6. **TOTAL: (12 marks)**

Let $\mathbf{F} = \langle -y, x, \cos(x^3 + y^3)\rangle$ and let S be part of the sphere $x^2 + y^2 + z^2 - 2z = 1$ where $z \geqslant 0$. Suppose S is oriented with normal vectors pointing towards the point $(0, 0, 1)$. Evaluate the surface integral

$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot \mathrm{d}\mathbf{S}.$$

QUESTION 7. **TOTAL: (15 marks)**

Find the flux of the vector field $\mathbf{F} = \langle xz^2, y + \sin(z^3), x^{2022}\rangle$ through the positively-oriented closed surface S, where

$$S = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1 \text{ and } z \geqslant 0\} \cup \{(x, y, z) \mid x^2 + y^2 \leqslant 1 \text{ and } z = 0\}.$$

QUESTION 8. **TOTAL: (15 marks)**

Let C be the curve parametrised by $\mathbf{r} = (e^{t^2 - t} - \cos(2\pi t))\mathbf{i} + (2\sin(\pi t^2/2) - t^9)\mathbf{j}$ where $0 \leqslant t \leqslant 1$, oriented to start at the point where $t = 0$ and end at the point where $t = 1$. Evaluate the line integral

$$\int_C (2xy + e^{-x^2})\mathrm{d}x + \left(x^2 + y\cos\left(\pi y^2/2\right)\right)\mathrm{d}y.$$

END OF PAPER

MH2100 CALCULUS III

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.