NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2022-2023

MH2500 – Probability and Introduction to Statistics

December 2022   TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages including an appendix on page 4.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a RESTRICTED OPEN BOOK exam. You are only allowed to bring in ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER (no sticky notes/post-it notes on the reference sheet).

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

6. The table with values of the standard normal distribution is included on page 4.

QUESTION 1. **(10 marks)**

Consider the discrete random variable X, with outcomes $1, 2, 3, 4, 5$ and corresponding probability distribution

x	1	2	3	4	5
$p(x)$	0.1	0.3	0.4	0.1	0.1

Find the expectation and the variance of X.
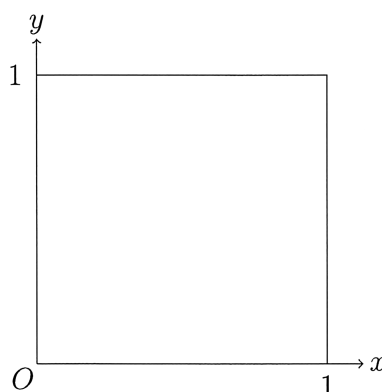
QUESTION 2. **(15 Marks)**

Let X be an exponential random variable with parameter $\lambda = 1$ and Y be a uniform random variable on $[0, 1]$. Suppose that X and Y are independent. Find the probability density function of $Z = X + Y$.

QUESTION 3. **(20 Marks)**

Let X and Y have the joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the region representing the event $X + Y \le 1$.

(b) Find $P(X + Y \le 1)$.

(c) Find the marginal distribution of X.

(d) Are X and Y independent? Justify your answers.

QUESTION 4.   **(20 marks)**

The joint probability density function of X and Y is given by

$$f(x,y) = \frac{1}{y}e^{-(y+\frac{x}{y})} \qquad x > 0, y > 0.$$

Find $\mathbb{E}(X)$, $\mathbb{E}(Y)$, $\mathbb{E}(XY)$ and $Cov(X,Y)$.

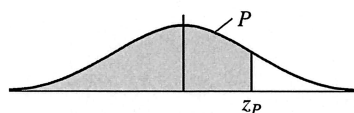**QUESTION 5.**   **(20 marks)**

An urn contains a white and b black balls, where a and b are positive integers. One ball at a time is randomly drawn until the first white ball is drawn. Find the expected number of black balls that are drawn, by considering conditional probabilities.

QUESTION 6.   **(15 marks)**

A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is 30 hours, how many of these components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least 0.95?

END OF PAPER

TABLE   Cumulative Normal Distribution—Values of P Corresponding to z_P for the Normal Curve

z is the standard normal variable.

z_P	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

MH2500 PROBABILITY & INTRODUCTION TO STATISTICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.