NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2022-2023

MH1805 – Calculus

November 2022 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **CLOSED BOOK** examination.

5. Candidates may use calculators but should clearly explain their reasoning.

QUESTION 1. **(25 marks)**

(a) Suppose $a \in A$ is a limit point of A. State the defined meaning of a function $f : A \to \mathbb{R}$ being *differentiable* at a.

(b) Is the function $f(x) = |x|$ differentiable at $x = 0$? Explain your answer.

(c) For each statement below, determine whether it is true or false. Support your answer with a proof or counterexample.

(i) If $g(x)$ is differentiable at $x = a$, then

$$\lim_{h \to 0} \frac{g(a+h) - g(a-h)}{2h} = g'(a).$$

(ii) If

$$\lim_{h \to 0} \frac{g(a+h) - g(a-h)}{2h} = L,$$

then $g(x)$ is differentiable at $x = a$ and $g'(a) = L$.

QUESTION 2. **(25 marks)**

Let

$$f(x) = x \int_0^x e^{-t^2}\, dt.$$

(a) Is f an even function, an odd function, both even and odd, or neither even nor odd?

(b) Is $x = 0$ a critical point of f? If so, is it a local/global point of maximum/minimum, or none of that?

(c) Find all points of inflection for f (if any).

See next page.

QUESTION 3. **(25 marks)**

Solve the initial value problems for $y = y(x)$:

(a)
$$y'' + 4y = 2 \text{ for all } x \in \mathbb{R}, \quad y(0) = 1,\ y'(0) = 0.$$

(b)
$$y'' + \frac{1}{x}y' = 2 \text{ for all } x \in (0, +\infty), \quad y(1) = 1,\ y'(1) = 0.$$

The substitution $u = y'$ might be a good idea.

QUESTION 4. **(25 marks)**

(a) Evaluate the limit
$$\lim_{x \to 0} \frac{\cos(x^2) - x^{-4}\ln(1 + x^4)}{\sin(x^8)}.$$

Reminders:

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \quad \text{for } x \in (-1, 1],$$
$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad \text{for } x \in \mathbb{R},$$
$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad \text{for } x \in \mathbb{R}.$$

(b) Suppose f has continuous derivatives up to order 3 on an open interval containing a. Show that
$$\frac{f(a+h) - f(a-h)}{2h} - f'(a) = \mathcal{O}(h^2), \quad \text{as } h \to 0.$$

END OF PAPER

MH1805 CALCULUS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.