NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2024–2025

MH4320 – COMPUTATIONAL ECONOMICS

Nov 2024 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **FOUR (4)** printed pages.
2. Answer each question beginning on a **FRESH** page of the answer book.
3. This is a **CLOSED BOOK** examination.
4. Calculators may be used. However, you should write down systematically the steps in the workings.

QUESTION 1 **(20 marks)**

Two bidders, A and B, participate in a first-price auction for a good. They simultaneously choose one of two bids $p_A, p_B \in \{2, 8\}$. Each bidder has the same value for the good, 10. If one bidder submits higher bid than the other, the higher bidder gets the good and pays own bid. If two bids are equal, the winner is chosen randomly with equal probability, and the winner gets the good and pays own bid. Recall that the payoff of the winner is the value of the good minus the payment made, and the payoff of the loser is 0.

(a) Give the strategic-form representation of this game.

(b) Compute all pure strategy Nash equilibria of this game.

(c) Compute all mixed strategy Nash equilibria of this game.

(d) If the value of the good is v instead of 10, when v is high enough then each player has a weakly dominant strategy. Determine the minimum value of v for which this occurs.

QUESTION 2 **(25 marks)**

Consider the following preference profile of 21 voters on 5 candidates:

- 3 voters voted: $A \succ D \succ C \succ E \succ B$
- 6 voters voted: $B \succ A \succ E \succ D \succ C$
- 7 voters voted: $C \succ D \succ B \succ E \succ A$
- 5 voters voted: $E \succ D \succ B \succ C \succ A$

(a) Determine the winner of this preference profile according to each of the following voting rules.

(i) Plurality rule.

(ii) Borda rule.

(iii) Plurality with a single run-off: a run-off election (i.e., pairwise election) is conducted between the two candidates with the most first-place votes.

(b) Is there a Condorcet winner in this preference profile? If so, which candidate is a Condorcet winner?

(c) Assume the candidates are positioned in alphabetical order from left to right on a one-dimensional political spectrum. Among these four types of voter preferences, identify which qualify as single-peaked preferences.

QUESTION 3 **(25 marks)**

Alice and Bob are competing to play a game against Casey. Alice and Bob simultaneously bid p_A and p_B, respectively. The one who bids higher wins; if $p_A = p_B$, then Alice wins by default. The winner of the bidding will

- pay his/her bid amount to Casey, and
- play the following game with Casey:

		Casey: L	Casey: R
Winner	T	(4, 2)	(0, 0)
	B	(0, 0)	(1, 4)

Note that Casey will make his choice between L and R without knowledge of the bids from Alice and Bob.

The payoff to the winner is the payoff of the game with Casey minus the payment made to Casey. The payoff of Casey is the payment received from the winner and the payoff of the game. The payoff of the loser is 0.

(a) Compute all pure strategy subgame-perfect equilibria of this game.

(b) Suppose that, before Alice and Bob submit their bids, Casey can publicly announce his decision to play L or R and commits to this choice regardless of the bids. Compute all pure strategy subgame-perfect equilibria for this scenario.

QUESTION 4 **(30 marks)**

Consider an extensive-form game consisting of a series of auctions for three identical laptops, with four players, A, B, C, and D. The game proceeds as follows:

- In round 1, there is an auction for laptop 1, with players A and B.
- In round 2, there is an auction for laptop 2, with players C and D.
- In round 3, there is an auction for laptop 3, with players B and C.

Each auction is a second-price auction: Each player submits a bid. The player with the higher bid wins the laptop and pays an amount equal to the bid of the losing player. In the event of a tie, the winner is chosen randomly with equal probability.

The values that each player assigns to a laptop are as follows: $v_A = 1$, $v_B = 100$, $v_C = 100$, and $v_D = 99$. Each player values only one laptop; that is, once a player wins a laptop, additional laptops have no extra value to them. The payoff of a player is 0 if he loses in all auctions he participates in, and it is his value for the laptop minus the total payments he makes otherwise.

A strategy for each player specifies, given the outcome of previous auctions, what bid that player submits in each auction he participates in.

(a) Identify all possible sets of winners of the three laptops that can result from pure strategy Nash equilibria in this game.

(b) For each set of winners, describe the strategies of each player in the Nash equilibrium that leads to that particular outcome, and explain why it is a Nash equilibrium.

END OF PAPER

MH4320 COMPUTATIONAL ECONOMICS

CONFIDENTIAL

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.