

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2022-2023

MH1301 – DISCRETE MATHEMATICS

Apr/May 2023

TIME ALLOWED: 2 HOURS

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INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

**QUESTION 1****(20 marks)**

There are 6 adults and 3 children at a party.

- (a) In how many ways can they form 3 groups of 3 people with no restrictions?
- (b) In how many ways can they form 3 groups where each group consists of 2 adults and 1 child?
- (c) In how many ways can they be seated at a circular table with no restrictions?
- (d) In how many ways can they be seated at a circular table such that no two children sit next to each other?

**QUESTION 2****(10 marks)**

Given any string of  $3n$  consecutive integers, show that any subset of  $n + 1$  of the numbers has two members that differ by at most 2.

**QUESTION 3****(15 marks)**

Let  $a_n$  be the number of  $n$ -digit positive integers that do not have two consecutive 1's as digits. Find a recurrence relation for  $a_n$ . You need not solve it.

**QUESTION 4****(10 marks)**

- (a) Find all solutions of the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2} + 5^n \quad \text{for } n \geq 2.$$

- (b) Find the solution of the recurrence relation in Part (a) with initial values  $a_0 = 7/9$  and  $a_1 = 17/9$ .

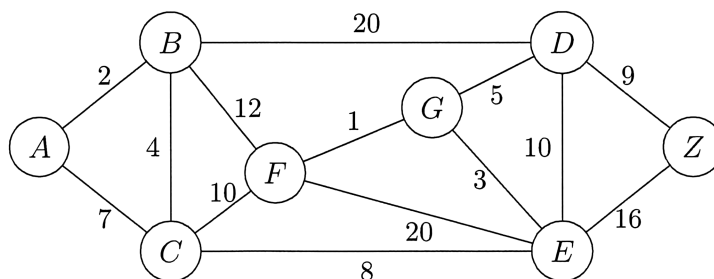
**QUESTION 5****(20 marks)**

Determine whether each of the statements below is true or false. Give an explanation if it is true or provide a counterexample if it is false.

- (a) A connected graph  $G$  with  $n$  vertices ( $n \geq 3$ ) and at least  $n$  edges always contain a cycle.
- (b) If a graph  $G$  with at least 3 vertices has the property that every pair of vertices of  $G$  lies in a cycle, then removing any vertex  $v$  from  $G$  always result in a connected graph.
- (c) If  $T$  is a tree and  $v$  is a vertex of degree  $k$  in  $T$ , then  $T$  contains at least  $k$  leaves.
- (d) All 3-regular graphs with 6 vertices are isomorphic to one another.

**QUESTION 6****(25 marks)**

The following is an undirected weighted graph.



- (a) Find the minimum spanning tree for the graph using Kruskal's algorithm. List the order in which the edges are added, and compute the total weight of the tree.
- (b) Use Dijkstra's algorithm to compute the shortest path between  $A$  and  $Z$ . Illustrate how the values of  $d[v]$  are updated using a table.

**END OF PAPER**

## MH1301 DISCRETE MATHEMATICS

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.