NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2023–2024

MH2100 – Calculus III

November 2023 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises of **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)
5. Calculators are allowed.
6. Candidates should clearly explain their reasoning used in each of their answers.

QUESTION 1. **(10 marks)**

Find and classify all the critical points of the function $f(x,y) = x^2+2y^3+3y^2+2xy$.

QUESTION 2. **(10 marks)**

(a) Find the value of the constant C so that the tangent plane to the surface

$$z = x^3 + y^3 + 3x^2 + 2y^2 + C$$

at the point corresponding to $(x, y) = (1, 2)$ goes through the origin.

(b) Given that $u = f(s,t)$ is defined by

$$e^{su} + 3u\cos(t) - st = 1 - 4\pi,$$

use implicit differentiation to evaluate $\dfrac{\partial u}{\partial s}$ at the point $(s,t,u) = (4, \pi, 0)$.

QUESTION 3. **(10 marks)**

Let $u(x,t) = \sin(x - 5t) + \ln(x + 5t)$. The second order partial derivative u_{tt} can be expressed as

$$u_{tt} = Au_x + Bu_t + Cu_{xx},$$

for some constants $A, B, C \in \mathbb{R}$. Find the values for A, B, C. Justify your answer.

QUESTION 4. **(10 marks)**

Let $\mathbf{F} = -yz\mathbf{i} + xz\mathbf{j} + (x+y)\mathbf{k}$ and let C be a curve parametrised by

$$\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + 3t\mathbf{k}; \quad 0 \leqslant t \leqslant 4\pi.$$

Evaluate the line integral $\displaystyle\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$.

QUESTION 5. **(15 marks)**

Evaluate the double integral

$$\iint_T (x+y)\sin(x-y)\mathrm{d}A,$$

where T is the triangle with vertices $(0,0)$, (π,π), and $(\pi,-\pi)$.

QUESTION 6. **(15 marks)**

Let $\mathbf{F} = \langle xy^2 + 7, y(x^2 + z^2) - \sin(x), \cos(x + y) - z \rangle$. Evaluate

$$\iint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S},$$

where S is the positively-oriented boundary of the solid

$$E = \{(x, y, z) \mid 1 \leqslant x^2 + y^2 + z^2 \leqslant 9\}.$$

QUESTION 7. **(15 marks)**

Let C be the curve defined by the intersection of the cylinder $\{(x, y, z) \mid x^2+y^2 = 1\}$ and the plane $\{(x, y, z) \mid y + z = 1\}$. Suppose that C is oriented counter-clockwise when viewed from the positive z-axis. Calculate the work done of the vector field

$$\mathbf{F} = \left\langle 7 + z\ln(1 + y^2), \cos(z^{2023}) - x + \frac{2xyz}{1 + y^2}, x + x\ln(1 + y^2) \right\rangle$$

along the curve C.

QUESTION 8. **(15 marks)**

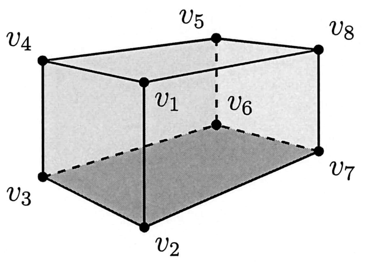

Figure 1: The rectangular box B.

Let B be a rectangular box whose edges are parallel to the x-axis, y-axis, or z-axis. Suppose each of its vertices $v_1, \ldots, v_8$ lies on the ellipsoid $2x^2 + 72y^2 + 18z^2 = 288$. Find the maximum possible volume of B.

END OF PAPER

MH2100 CALCULUS III

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.