NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2017-2018

MH1200– LINEAR ALGEBRA I

December 2017 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED BOOK** exam.
5. Candidates may use calculators.

QUESTION 1. **(30 marks)**

(a) Let

$$A = \begin{bmatrix} 1 & r & 1 & 1 \\ 1 & s & 1 & 2 \\ 1 & t & 1 & 3 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

Compute the determinant of A in terms of the parameters r, s, t.

(b) Let

$$B = \begin{bmatrix} 1 & 1 & 1 & r \\ 1 & 2 & 1 & s \\ 1 & 3 & 1 & t \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

B is the matrix A with the second and fourth columns exchanged. Give a matrix E such that $B = AE$.

(c) True or False: The column space of A is the same as the column space of B. Justify your answer.

(d) True or False: The dimension of the row space of A is the same as the dimension of the row space of B. Justify your answer.

(e) Let

$$C = \begin{bmatrix} 1 & r & 1 & 1 \\ 1 & r & 1 & 2 \\ 1 & r & 1 & 3 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

Find a basis for the column space of C.

(f) Find a *non-zero* vector in the *left nullspace* of C.

QUESTION 2. **(25 marks)**

This question is about the vectors

$$\vec{u} = (1,1,1,1), \quad \vec{v} = (1,-1,-1,1), \quad \vec{w} = (1,-1,1,-1) \ .$$

(a) Compute $\|\vec{v}\|$ and $\langle \vec{u}, \vec{v} \rangle$.

(b) Show that the sequence of vectors $\vec{u}, \vec{v}, \vec{w}$ is linearly independent.

(c) Find a basis for $\mathbb{R}^4$ containing the vectors $\vec{u}, \vec{v}, \vec{w}$. Justify your answer.

(d) Find the projection of $(1,2,3,4)$ onto the line $\{t \cdot \vec{w} : t \in \mathbb{R}\}$.

(e) Let $\vec{z} \in \mathbb{R}^4$ and $\mathrm{proj}(\vec{z})$ be the projection of $\vec{z}$ onto the line $\{t \cdot \vec{w} : t \in \mathbb{R}\}$. Is it always the case that $\|\vec{z}\| \leq \|\mathrm{proj}(\vec{z})\|$? Justify your answer.

QUESTION 3. **(25 marks)**

(a) Let

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} .$$

Show that A is invertible when a, b, c are all distinct and compute $A^{-1}[3,1]$ in this case (the entry in the third row and first column of A^{-1}).

(b) Let

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 1 & 1 \\ 1 & 3 & 9 & 1 & 2 \end{bmatrix} .$$

Determine the rank of B.

(c) Give a basis for the nullspace of B.

(d) Find the least squares solution to the equation

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} .$$

(e) Let C be an invertible 4-by-4 matrix. Determine a basis for the row space of the 4-by-8 matrix

$$D = \begin{bmatrix} C & C \end{bmatrix} .$$

QUESTION 4. **(20 marks)**

For this question let V be the vector space of 3-by-3 matrices with real entries.

(a) Let
$$S = \{A \in V : \det(A) = 0\} \ .$$
Is S a subspace of V? Justify your answer.

(b) Let $T \subseteq V$ be the set of matrices
$$T = \left\{ A \in V : A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} \ .$$
Show that T is a subspace of V.

(c) Find a basis for T and determine its dimension.

(d) Let A be a 3-by-3 matrix such that the row space of A is equal to the column space of A. Does this imply that $A = A^T$? Prove or give a counterexample.

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.