NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2023-2024

MH1810 - Mathematics 1

May 2024 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **SIX (6)** printed pages, including an appendix.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **CLOSED BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

6. Some useful formulae are listed on pages 4 - 6.

QUESTION 1. **(15 marks)**

Let $z = 4 - 4\sqrt{3}i$.

(a) Find the modulus and principal argument of z. (7 marks)

(b) Find all distinct fourth roots of z. Leave your answers in the form $re^{i\theta}$. (8 marks)

QUESTION 2. **(15 marks)**

Let Π denote the plane with the equation $x - 2y - z = 1$.

(a) Find a normal vector $\mathbf{n}$ to the plane Π. (5 marks)

(b) Find the distance from $P\,(2, -2, 3)$ to Π. (10 marks)

QUESTION 3. **(15 marks)**

Find the following limits:

(a) $\displaystyle\lim_{x\to 1} \frac{x^2 - 1}{x - 1}$ (5 marks)

(b) $\displaystyle\lim_{x\to -\infty} \frac{e^x + e^{-x}}{e^x - 3e^{-x}}$ (5 marks)

(c) $\displaystyle\lim_{x\to +\infty} \left(\sqrt{x+1} - \sqrt{x}\right)$ (5 marks)

QUESTION 4. **(15 marks)**

Let $a > 0$ and b be a real constant. Prove that the function $f(x) = x^3 + ax + b$ has exactly one root.

QUESTION 5. **(15 marks)**

(a) Prove that $\dfrac{d}{dx}\left(\ln\dfrac{1+\sin x}{1-\sin x}\right) = 2\sec x$. (7 marks)

(b) Find the derivative of $f\,(x) = (\ln\,(x^2+1))^{\sin x}$. (8 marks)

QUESTION 6. **(10 marks)**

Use a linear approximation to find an estimate of $\sqrt[6]{63.99}$.

QUESTION 7. **(15 marks)**

Express the following limit as a definite integral $\displaystyle\int_0^1 f(x)\,dx$ and use it to evaluate the limit:

$$\lim_{n\to\infty}\left\{\frac{e^{1/n}}{n}+\frac{e^{2/n}}{n}+\ldots+\frac{e^{k/n}}{n}+\ldots+\frac{e^{(n-1)/n}}{n}+\frac{e^{n/n}}{n}\right\}.$$

End of Paper

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right],$$

where n is even.

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\text{sech } x) = -\text{sech} x \tanh x \qquad \frac{d}{dx}(\text{csch } x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \tan x \sec x\,dx = \sec x + C \qquad \int \cot x \csc x\,dx = -\csc x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = \ln|\sin x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C,\ a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C,\ |x| < |a| \qquad \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C \qquad \int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

END OF PAPER

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.