NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2023-2024

MH4518 – Simulation Techniques in Finance

Nov/Dec 2023 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **FIVE (5)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)

5. Calculators may be used. However, you should write down systematically the steps in the workings.

QUESTION 1. **(20 marks)**

(a) Write the pseudo-codes that use the inverse-transform method to generate random variates from p.d.f. $f(x)$ given by

$$f(x) = x \exp\left\{-\frac{1}{2}(x^2 - \theta^2)\right\} \quad \text{for } x \in [\theta, \infty),$$

where θ is a positive constant.

(b) Write the pseudo-codes that use the acceptance-rejection method to generate random variates from p.d.f. $h(x)$ that is proportional to

$$\exp\left\{-\frac{1}{2}x^2\right\} \quad \text{for } x \in [\theta, \infty)$$

with the proposal density $f(x)$ in part (a), where both parts use the same θ.

QUESTION 2. **(15 marks)**

Denote by $\{W_t\}_{t\geq 0}$ a standard Brownian motion. Find the stochastic differential equation (SDE), dY_t, for the following quantities:

(a) $Y_t = \exp(tW_t)$.

(b) $Y_t = \ln W_t$.

(c) $Y_t = e^{W_t}\left(e^{\frac{t}{2}} + e^{-\frac{t}{2}}\right)$.

(*Note that the SDE dY_t should be expressed in terms of dt, dW_t, t, and Y_t only.*)

See next page.

QUESTION 3. **(15 marks)**

Suppose that $C_E^{(1)}(t,S)$, $C_E^{(2)}(t,S)$, and $C_E^{(3)}(t,S)$ are three European call option prices at time t, written on the same underlying asset, whose price at time t is S. The options have the same maturity date T and their strike prices are K_1, K_2, and K_3, respectively, while $K_1 < K_2 < K_3$ and $2K_2 = K_1 + K_3$. By no-arbitrage arguments, show that

$$C_E^{(1)}(t,S) + C_E^{(3)}(t,S) \geq 2C_E^{(2)}(t,S).$$

QUESTION 4. **(18 marks)**

Suppose that the price of a stock is modelled by the Black–Scholes model: $dS_t = S_t[\mu dt + \sigma dW_t^{\mathbb{P}}]$, where the parameters (μ, σ) are considered known and the initial price is S_0. We are interested in a quantity κ that is defined by

$$\mathbb{P}(S_T > \kappa) = \alpha,$$

where $T > 0$ and $\alpha \in [0,1]$ are known constants.

(a) Write pseudo-codes that estimate the κ via the Monte-Carlo simulation with antithetic variate approach.

(b) Describe the procedure or write pseudo-codes that estimate the κ via the Monte-Carlo simulation with importance sampling approach (you may denote the density of S_T by $f(\cdot)$ and illustrate the algorithm with an importance sampling density $g(x) = e^{-x}$, $x \geq 0$).

See next page.

QUESTION 5. **(32 marks)**

In Figure 1 shown on the next page, you can find an extracted factsheet of a structured product. Suppose that the initial fixing date is considered as the beginning time of the product and the final fixing date is considered as the maturity of the product. Moreover, assume that *today* is the initial fixing date and thus the **time to maturity is exactly 3 years**. Suppose that we have estimated the parameters (μ, σ) in the Black–Scholes model for the underlying asset price in the real world: $dS_t = S_t[\mu dt + \sigma dW_t^{\mathbb{P}}]$.

(a) What is the payoff function of this structured product for a given future underlying index price path? Express it mathematically.

(b) Write pseudo-codes that estimate the price of this product via Monte-Carlo simulation with control variate approach.

(c) Write pseudo-codes that estimate the gamma of this product.

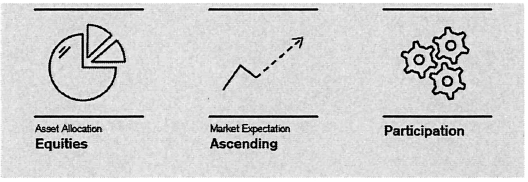
(d) Can we apply stratified sampling to improve the Monte-Carlo simulated price of this product? If yes, write the pseudo-codes (you may simply consider equal sample sizes over B bins for illustration). If no, briefly explain why.

See next page.

USD LastLook Bonus Certificate

S&P 500®

Reference Index / underlying asset(s) incl. CS Analyst Recommendation*	Bloomberg	Initial Level	Barrier
S&P 500® Index Not Rated	SPX	2,789.82	1,645.9938



Key Fixed Terms

Issuer (Debtor)	[redacted]
Participation on Final Fixing Date	100% in the performance of the Reference Index
Minimum Repayment	100% if the Reference Index closes above its Barrier on the Final Fixing Date
Initial Fixing Date	9 April 2020
Final Fixing Date	10 April 2023
LastLook Barrier	59% of the Initial Level, observed on the Final Fixing Date
Currency/Issue Price	USD 1,000
Capital Protection	**No**

How It Works

The LastLook Bonus Certificate (the 'Certificate') offers 100% participation in the positive performance of the Reference Index on the Final Fixing Date. A minimum of USD 1,000 is returned on the Redemption Date as long as the Reference Index closes above its Barrier on the Final Fixing Date. If not, your invested capital will be subject to a loss.

Scenarios Analysis at Redemption

a) The Reference Index closes above its Barrier on the Final Fixing Date (best case)

- You will receive USD 1,000 per Certificate plus 100% of the positive performance of the Reference Index, calculated from its Initial Level, or
- You will receive a Minimum Repayment of USD 1,000 per Certificate even if the performance of the Reference Index is negative.

b) The Reference Index closes at or below the Barrier on the Final Fixing Date

- Your redemption amount will be reduced by 1% for each percentage point the Reference Index closes below its Initial Level.

c) The Reference Index closes at zero on the Final Fixing Date (worst case)

- **You will lose all of your invested capital.**

Sample Returns on Investment (ROI) at Redemption²

Performance of the Reference Index on the Final Fixing Date	Capital redemption	ROI
+30%	130%	+30%
+10%	110%	+10%
-20%	100%	0%
-41%	59%	-41%
Worst case	**0%**	**-100%**

Figure 1: Extracted factsheet of a structured product

END OF PAPER

MH4518 SIMULATION TECHNIQUES IN FINANCE

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.