# NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2023–2024

**MH1811 – MATHEMATICS 2**

APRIL 2024 TIME ALLOWED: 2 HOURS

**Matriculation Number:** | | | | | | | | | |

**Seat Number:** | | | | | |

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## INSTRUCTIONS TO CANDIDATES

(a). This examination paper contains **SIX (6)** questions and comprises **FIFTEEN (15)** pages.

(b). Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

(c). This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)

(d). Candidates may use calculators. However, they should write down systematically the steps in the workings.

(e). All your solutions should be written in this booklet within the space provided after each question. However, if you write your solutions on other pages, please indicate them clearly.

(f). Question Paper is **NOT ALLOWED** to be removed from the exam hall.

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For examiners only

| Question | Marks |
|---|---|
| **1** (15) | |
| **2** (15) | |
| **3** (15) | |

| Question | Marks |
|---|---|
| **4** (15) | |
| **5** (20) | |
| **6** (20) | |

| **TOTAL** (100) |
|---|
| |

**Question 1.** **(15 marks)**

(a). Let $f(x, y) = \ln(x^2 - y)$. Evaluate the directional derivative $D_{(0.6,0.8)}f(4, 15)$.

(b). Approximate the value of $\ln(3.99^2 - 15.02)$ using differentials.

Question 1 continues on page 3

(c). Let

$$g(x,y) = \begin{cases} \frac{x^2y-y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}.$$

Evaluate the partial derivatives $g_x(0,0)$ and $g_y(0,0)$ or prove that they don't exist.

End of Question 1.

**Question 2.** **(15 marks)**

(a). Which of the following matrices is (are) the Hessian matrix of some continuously differentiable function $f(x, y)$ evaluated at a specific point?

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 1 & -42 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & -42 \end{bmatrix}$ d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Choose all that apply.

(b). Find an equation of the tangent plane to the surface $x^2 \cos(y - z) + z^2 = 25$ at the point $(3, 4, 4)$.

Question 2 continues on page 5

(c). Given that the equation $x^2 \cos(y - z) + z^2 = 25$ implicitly defines a function $z(x, y)$, determine all critical points of the function $z(x, y)$.

End of Question 2.

**Question 3.** **(15 marks)**

(a). Switch the order of integration in the integral

$$\int_0^1 \int_{y^2}^1 y^3 e^{-\frac{x^3}{3}}\, dx\, dy$$

Question 3 continues on page 7

(b). Compute the iterated integral

$$\int_0^1 \int_{y^2}^1 y^3 e^{-\frac{x^3}{3}}\, dx\, dy$$



End of Question 3.

**Question 4.** **(15 marks)**

Evaluate the limit of each of the following sequences as $n \to \infty$:

(a). $a_n = \dfrac{n^2 - 42n + 17}{11 + 21n + 5n^2}$.

(b). $b_n = \dfrac{2^{\cos n + n}}{n!}$.

Question 4 continues on page 9

(c). $c_n = d_1 \times d_2 \times \cdots \times d_n$, where $d_k$ is the $k$ th decimal digit of $\pi$ for $k = 1, 2, 3, \ldots$. For instance, $c_1 = 1$, $c_2 = 4$, $c_3 = 4$, $c_4 = 20$ etc.

End of Question 4.

**Question 5.** **(20 marks)**

(a). Give an example of a sequence $a_1, a_2, \ldots$ such that the series $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} (a_n)^2$ diverges.

(b). Does the series $\sum_{n=3}^{\infty} (-1)^n \frac{n+1}{n-2}$ converge absolutely, converge conditionally, or diverge?

Question 5 continues on page 11

(c). What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-42)^n}{n^n} x^n$?

(d). Write the Maclaurin series for the function $f(x) = x^2 \cdot \ln(1 - x^2)$.

End of Question 5.

**Question 6.** **(20 marks)**

(a). Solve the following initial value problem: $\dfrac{dy}{dx} = e^{x-2y}$, $y(0) = -42$.

(b). Solve the following initial value problem: $\dfrac{dy}{dx} + \dfrac{xy}{1+x^4} = \dfrac{\cos x}{\sqrt{e^{\arctan(x^2)}}}$, $y(0) = 0$.

Question 6 continues on page 13

(c). Verify that the following differential equation is exact and solve it:

$$\left(\frac{2x}{x^2-y^3}+\cos x\right)dx-\frac{3y^2}{x^2-y^3}\,dy=0.$$

(d). Solve the following initial value problem:

$$y''+42y'+y=\sqrt{\pi},\quad y(0)=\sqrt{\pi},\quad y'(0)=0.$$

End of Question 6.

FOR ROUGH WORK

FOR ROUGH WORK



END OF PAPER

**MH1811 MATHEMATICS 2**

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.