

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2022-2023

MH1811– Mathematics 2

November 2022

TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HAND-WRITTEN OR TYPED ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1.

(18 marks)

- (a) Evaluate the directional derivative of

$$f(x, y, z) = y \ln(x + y + z^2)$$

at the point $P(-3, 4, 0)$ in the direction of the vector $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$.

- (b) Find the maximum rate of change of

$$f(x, y, z) = y \ln(x + y + z^2)$$

at the point $P(-3, 4, 0)$ among all possible directions.

- (c) Find all points Q on the ellipsoid with equation $2x^2 + 3y^2 + 4z^2 = 49$ at which the plane tangent to the ellipsoid at Q is parallel to the plane $x - 2y + 3z = 5$.

QUESTION 2.

(18 marks)

- (a) Find all critical points of the function

$$f(x, y) = 2xy^2 - 3x^4 - 2y^3.$$

- (b) Find the maximum and the minimum values of the function

$$h(x, y) = 4x^3 + x + y^2$$

subject to the constraint $4x^2 + y^2 = 1$.

QUESTION 3.

(16 marks)

- (a) Evaluate the following iterated integral. You may need to switch the order of integration:

$$\int_0^{2\pi} \int_{15}^{64} e^{-4y^2} \cos x \, dy \, dx.$$

- (b) Consider the following iterated integral:

$$\int_0^6 \int_{\sqrt{\frac{x}{6}}}^1 e^{3y^3} \, dy \, dx.$$

(i) Sketch the region of integration for $\int_0^6 \int_{\sqrt{\frac{x}{6}}}^1 e^{3y^3} \, dy \, dx$.

(ii) Switch the order of integration in the integral $\int_0^6 \int_{\sqrt{\frac{x}{6}}}^1 e^{3y^3} \, dy \, dx$.

(iii) Hence evaluate the iterated integral $\int_0^6 \int_{\sqrt{\frac{x}{6}}}^1 e^{3y^3} \, dy \, dx$.

QUESTION 4.

(15 marks)

Find the general solutions of the following ordinary differential equations:

(a)

$$e^{1-2x} y' = y^2.$$

(b)

$$y'' - 16y = 0.$$

(c)

$$y'' - 16y = 4x \cos x.$$

QUESTION 5.

(15 marks)

- (a) Determine whether the following limit exists.

$$\lim_{n \rightarrow \infty} \frac{e^n \sqrt{3n^2 + 5n}}{n!}$$

If it exists, what is its value? Justify your answer.

- (b) Define the sequence $(a_n)_{n=1}^{\infty}$ recursively by: $a_1 = 3$, and for $n \geq 1$ we have

$$a_{n+1} = \begin{cases} 2a_n, & \text{if } 0 \leq a_n < 5, \\ 2a_n - 10, & \text{otherwise.} \end{cases}$$

- (i) Write out the first six terms of the sequence $(a_n)_{n=1}^{\infty}$.
(ii) Is the sequence $(a_n)_{n=1}^{\infty}$ convergent? Justify your answer.

QUESTION 6.

(18 marks)

- (a) Does the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt[3]{5n+1}}$$

converge absolutely, converge conditionally, or diverge? Justify your answer.

- (b) Consider the following power series:

$$\sum_{n=0}^{\infty} \frac{2022^n}{\sqrt[3]{5n+1}} (x-2)^n.$$

- (i) Find the radius of convergence for the above power series.
(ii) Determine the interval of convergence for the above power series.

END OF PAPER

MH1811 MATHEMATICS 2

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.