

NANYANG TECHNOLOGICAL UNIVERSITY
SPMS/DIVISION OF MATHEMATICAL SCIENCES

2022/23 Sem 1

MH5100 Advanced Investigations into Calculus I

Week 4

Problem 1. Find all values of a such that f is continuous on \mathbb{R}

$$f(x) = \begin{cases} x+1 & \text{if } x \leq a \\ x^2 & \text{if } x > a \end{cases}$$

Problem 2. If

$$\lim_{x \rightarrow 0} \frac{f(x)}{\sin^2 x} = 2,$$

find the following limits

$$(a) \lim_{x \rightarrow 0} f(x) \quad (b) \lim_{x \rightarrow 0} \frac{f(x)}{x} \quad (c) \lim_{x \rightarrow 0} \frac{f(x)}{\sin x} \quad (d) \lim_{x \rightarrow 0} \frac{f(x)}{x^2}$$

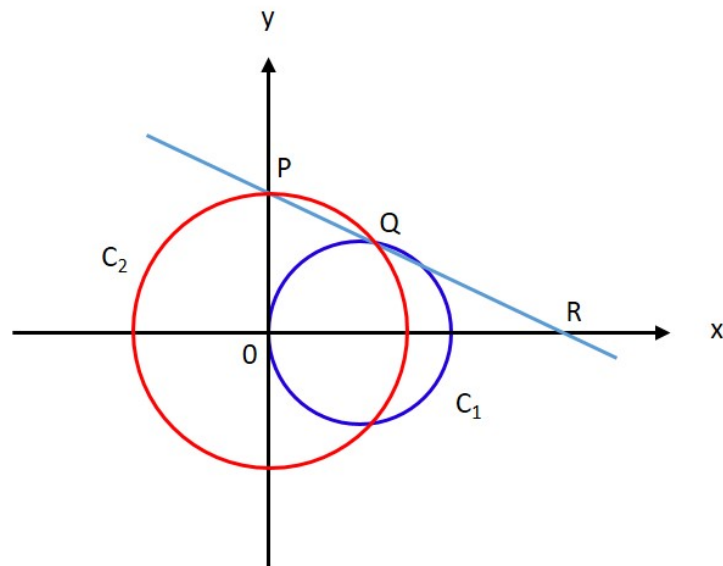
Problem 3. Prove **Bernoulli's inequality** $(1+x)^n > 1+nx$ for $n = 2, 3, \dots$ if $x > -1, x \neq 0$

Problem 4. Determine the limit

$$\lim_{x \rightarrow 0} x \left\lfloor \frac{1}{x} \right\rfloor,$$

where $\lfloor x \rfloor$ is the floor function. $\lfloor x \rfloor$ is the greatest integer that is less than or equal to x .

Problem 5. The figure shows a fixed circle C_1 with equation $(x-1)^2 + y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point $(0, r)$, Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x -axis. What happens to R as C_2 shrinks, that is, as $r \rightarrow 0^+$?



Problem 6. Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y = \tan(2 \sin x)$$

Then find the exact equations of the vertical asymptotes of $y = \tan(2 \sin x)$ on the whole real axis.