NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

MH4701 - Mathematical Programming

December 2022 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(25 marks)**

For each real number c, we denote by f_c the real-valued multivariate function

$$x \in \mathbb{R}^2 \mapsto 16x_1^4 + 32x_2^3 + cx_2^2 + 2x_1x_2 + x_2^2$$

(a) Prove that f_c is convex when $c \geq 25$.

(b) Prove that if $c > 19$ and a sequence of iterates $\{x_k\}$ is generated by the steepest descent method when applied to minimize f_c, then $\{x_k\}$ converges to a stationary point.

QUESTION 2. **(25 marks)**

The augmented Lagrangian method with the method of multipliers is applied to minimize f subject to $h(x) = 0$, where

$$f : x \in \mathbb{R}^2 \mapsto x_1^2 - 2x_1x_2 + 3x_2^2 + x_1 + 2x_2$$

and

$$h : x \in \mathbb{R}^2 \mapsto x_1 - 3x_2 + 1.$$

Prove that if a sequence of positive penalty parameters $\{c_k\}$ that strictly increases to ∞ is used and a sequence of iterates $\{x_k\}$ is generated, then $\{x_k\}$ converges superlinearly to a global minimum of f subject to $h(x) = 0$.

QUESTION 3. **(25 marks)**

Prove that $a \in \mathbb{R}^2$ is a stationary point of a continuously differentiable $f : \mathbb{R}^2 \to \mathbb{R}$ over the set

$$X = \{x \in \mathbb{R}^2 : x_1 \geq 0, 2x_1 + x_2 = 1\}$$

if and only if $a \in X$ and

$$\frac{\partial f}{\partial x_1}(a) \begin{cases} \geq 2\dfrac{\partial f}{\partial x_2}(a) & \text{if } a_1 = 0, \\ = 2\dfrac{\partial f}{\partial x_2}(a) & \text{if } a_1 > 0. \end{cases}$$

QUESTION 4. **(25 marks)**

Consider the convex program

$$\begin{array}{rrcrcrcrl} \text{minimize} & x_1 & + & x_2 & + & 3x_3 & + & 2x_4 & \\ \text{subject to} & 2x_1 & + & x_2 & + & 6x_3 & + & 8x_4 & = 9, \\ & x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & = 6, \\ & & & & & x_1, x_2, & x_3, & x_4 & \geq 0. \end{array}$$

Prove that the central path of the above convex program exists and is convergent as the barrier parameter converges to 0, and find its limit.

END OF PAPER