NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2023-2024

MH4200 – Abstract Algebra II

May 2024 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the end of the question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **CLOSED** book examination.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

Question 1 **(10 marks)**

Consider the quotient ring $\mathbb{Q}[X, Y]/(XY)$.

Prove that $\mathbb{Q}[X, Y]/(XY) \simeq \mathbb{Q}[X] \oplus \mathbb{Q}[Y]$.

Question 2 **(20 marks)**

Let R be a commutative ring with identity and assume that all prime ideals in R are finitely generated.

Prove that all ideals in R are finitely generated.

Question 3 **(20 marks)**

Let R be an Artinian ring. Prove the following:

(a) All prime ideals in R are maximal.

(b) There are only finitely many maximal ideals in R.

Question 4 **(15 marks)**

Let R be a principal ideal domain and M be a finitely generated, torsion-free R-module.

Prove that M is a free module.

Question 5 **(15 marks)**

Let R be a commutative ring with identity. Suppose that M is an R-module and N is a submodule of M.

Prove that if both M/N and N are finitely generated, then so is M.

Question 6 **(20 marks)**

Let R be a principal ideal domain and M be an R-module. Prove that M is injective if and only if M is divisible.

END OF PAPER