NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2024-2025

BR2211 Financial and Risk Analytics I

April 2025 Time Allowed: 2.5 Hours

INSTRUCTIONS

1 This paper contains **Five (5)** questions and comprises **SEVEN (7)** pages and **ONE(1)** appendix of **SEVEN(7)** pages.

2 Answer ALL questions.

3 This is a **Closed-book** examination.

4 The number of marks allocated is shown at the end of each question.

5 Begin your answer to each question on a separate page of the answer book.

6 Answers will be graded for content and appropriate presentation.

Note: Exam questions begin on page 2

Question 1

Please briefly describe the following concepts:

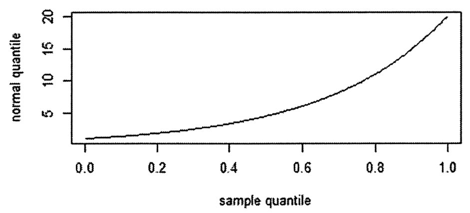
(a) Please list the five aspects of data an analyst should understand at a raw stage of data workflow.

(5 marks)

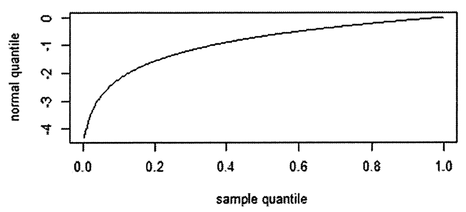
(b) Please map the following distributions to their Normal probability QQ Plot:

1. Distribution Plots

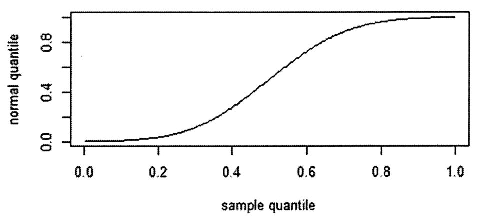
Plot A



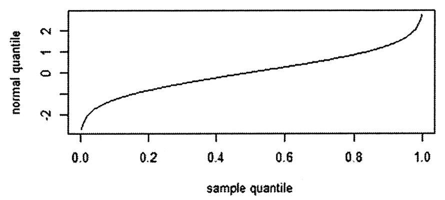
Plot B



Plot C



Plot D



2. Description of shape	
Shape 1 - Convex-concave	Shape 2 – Concave
Shape 3 – Concave-convex	Shape 4 – Convex

3. Characteristic of distribution against normal	
Ch. 1 - Heavy Tails	Ch. 2 - Light Tails
Ch. 3 - Right Skewness	Ch. 4 - Left Skewness

Your answer should be in the following format

Distribution Plot	Description of Shape	Characteristic of distribution
Plot A	(e.g., Shape 1)	(e.g., Ch. 1)
Plot B		
Plot C		
Plot D		

(4 marks)

Note: Question No.1 continues on page 3

Question 1 (Continued)

(c) An analyst is researching the relationship between age and annual salary. Based on a review of published research, the analyst suspects that a polynomial regression is the best model to use. To determine the best form of the model, the analyst employs the forward stepwise regression technique. Based on the results of the first three steps shown below, which model should the analyst select, and briefly explain why?

Step	Model	AIC
0	No predictors	3,202.69
1	Age	3,193.36
	Age^2	3,203.33
	Age^3	3,204.66
	Age^4	3,203.63
	Age^5	3,202.20
2	Age + Age^2	3,125.50
	Age + Age^3	3,123.20
	Age + Age^4	3,126.69
	Age + Age^5	3,132.87
3	Age + Age^3 + Age^2	3,125.19
	Age + Age^3 + Age^4	3,124.77
	Age + Age^3 + Age^5	3,124.50

(3 Marks)

(d) To better tailor its products and advisory services, a private wealth advisor surveyed potential customers about their investment preferences. One set of survey questions asked customers to select the types of investments (stocks, bonds, mutual funds, ETFs, etc.) they held within specific industry sectors (healthcare, defence, travel and leisure, etc.). What would be the most appropriate way to classify the results from these survey questions, and briefly explain why? (Hint: Ordinal, or Nominal)

(3 Marks)

(TOTAL: 15 marks)

Question 2

(a) Suppose now that the spot rate for a one-year treasury bond is 2.5% per annum and that the spot rate for a three-year treasury bond is 5% per annum. (i) Calculate the arbitrage-free forward rate for a two-year treasury bond starting one year from today; (ii) calculate the arbitrage-free spot rate for a two-year treasury bond.

(5 marks)

(b) You are a financial risk manager considering two investments:

i. Investment *A* offers an investment return with probabilities

S.N.	Return p.a.	Probability
1	-10%	10%
2	2%	30%
3	4%	30%
4	6%	30%

ii. Investment *B* offers an investment return with probabilities

S.N.	Return p.a.	Probability
1	-5%	20%
2	1%	30%
3	2%	30%
4	4%	20%

Which is the riskier investment according to each of the following measures of investment risk: (i) variance (ii) shortfall probability (with a benchmark of 0%)

(10 marks)

(c) Your investment C under your management is \$100m. Please calculate the weekly VaR at the 95% confidence level's return, assuming

i. Investment C's weekly return is normally distributed with $\mu = 1\%$ and $\sigma = 1\%$
ii. Investment C's weekly return is distributed in a t distribution with a $\mu = 1\%$ and $s = 1\%$, $v = 10$

(10 marks)

(TOTAL: 25 marks)

Question 3

You are a real estate data analyst specializing in property valuation. You are provided with a dataset containing 100 residential properties' features and sale prices. You decide to build a linear regression model to predict house prices based on key property characteristics. The model takes the form:

$$Y_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \beta_3 X_{i,3} + \beta_4 X_{i,4} + \epsilon_i$$

Where:

- Y_i is the sale price of the house based on the house size (sq ft) $X_{i,1}$, number of bedrooms $X_{i,2}$, age of the property $X_{i,3}$, Floor Level $X_{i,4}$, and
- ϵ_i is the error term

(a) List and briefly explain the four key assumptions for a classical linear regression model (linearity, independence, homoscedasticity, normality of errors).

(4 marks)

(b) You estimate the following linear regression model based on 100 houses:

$$Hosue\ Price = 45{,}000 + 200 * Size\ (sq\ ft) + 8{,}500 * Bedrooms - 700 * Age + 1{,}000 * Floor\ Level + \epsilon$$

The summary of regression coefficients is below

Predictor	Estimate	Std. Error	t-value	p-value
(Intercept)	45,000	12,000	3.75	<0.001 ***
Size (sq ft)	200	25	8.00	<0.001 ***
Bedrooms	8,500	3,500	2.43	0.017 *
Age	-700	180	-3.89	<0.001 ***
Floor Level	1,000	900	1.11	0.270

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '0.1' ' ' 1

Interpret the results in terms of the impact of each predictor and whether they are significant.

(8 marks)

(c) Use the simplified model (remove the not significant independent variables at 5% level from the above model), please predict the sale price of a house with the following information:

Size = 1,200 sq ft, Bedroom = 3, Age = 12 years and Floor Level = 12

(4 marks)

(d) Suitability for Singapore Properties: given the Singapore housing context, discuss any missing local variables that could affect these estimates.

(4 marks)

(TOTAL: 20 marks)

Question 4

(a) Explain how strict stationarity differs from weak stationarity.

(5 marks)

(b) Let W_t be an i.i.d sequence of random variables with $E[W_t] = 0$ and $Var[W_t] = 1$. Consider the following process:

1) $V_t = 0.5V_{t-1} + W_t$
2) $U_t = t + W_t$

Determine whether each of these two processes is weekly stationary.

- Justify your reasoning by examining the mean and the autocovariance for each process,
- Highlight which aspects violate stationarity (if any).

(10 marks)

(c) Consider the AR(1) (autoregressive of order 1) process

$X_t = \phi + \rho(X_{t-1} - \phi) + \epsilon_t$

- Where $\{\epsilon_t\}$ is a white noise sequence with mean 0 and variance δ^2
- ϕ, ρ are constants

i. Express X_t in terms of X_0: rewrite X_t in terms of the initial value X_0, the constant ϕ and ρ, and the noise term $\{\epsilon_t\}$, assuming $|\rho| < 1$

Hint: $\sum_{i=0}^{t-1} \rho^i = \frac{1-\rho^t}{1-\rho}$ for $|\rho| < 1$

(3 marks)

ii. Conditions for Stationarity: identify the condition ρ required for $\{X_t\}$ to be weekly stationary; discuss what assumption to be held for $E[X_0]$ so that stationarity is preserved from the start.

(4 marks)

iii. Autocovariance Function $\gamma(k)$: derive the autocovariance function $\gamma(k) = Cov(X_t, X_{t-k})$ for $k \geq 0$ in the stationary process; show how $\gamma(k)$ depends on ρ and $\gamma(0)$.

Hint: show $\gamma(0), \gamma(1), \gamma(2), then\ \gamma(k)$

(5 marks)

iv. Autocorrelation Function $\rho(k)$: write down the autocorrelation function $\rho(k) = \frac{\gamma(k)}{\gamma(0)}$; interpret how $\rho(k)$ behaves as k increases, particularly for $|\rho| < 1$.

(3 marks)

(TOTAL: 30 marks)

Question 5

Consider a random variable X that follows an Exponential distribution with rate parameter $\lambda > 0$, i.e. $X \sim Exp(\lambda)$. The probability density function (pdf) X is

$$f_X(x) = \lambda e^{-\lambda x}, x \geq 0.$$

(a) Write down the corresponding cumulative distribution function (CDF), $F_X(x)$.

(3 marks)

(b) Using the Inverse Transformation Method, show how to generate a random variable x from this exponential distribution, given a random number μ drawn from a Uniform (0,1) distribution.

(4 marks)

(c) Illustrate the procedure by generating an example value. Suppose you draw $\mu = 0.35, \lambda = 2$. What value of x does this correspond to?

(3 marks)

(TOTAL: 10 marks)

– END OF PAPER –

Appendix: BR2211 Formulae and Tables

SAMPLE MEAN AND VARIANCE

The random sample $(x_1, x_2, \ldots, x_n)$ has the following sample moments:

Sample mean: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

Sample variance: $s^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right\}$

LINEAR REGRESSION MODEL WITH NORMAL ERRORS

Model

$$Y_i \sim N(\alpha + \beta x_i, \sigma^2), \quad i = 1, 2, \ldots, n$$

Intermediate calculations

$$s_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

$$s_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$$

$$s_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - n\bar{x}\,\bar{y}$$

Sum of squares relationship

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$$

Note: APPENDIX continues on page 9

APPENDIX (continued)

ANALYSIS OF VARIANCE

Single factor normal model

$$Y_{ij} \sim N(\mu + \tau_i, \sigma^2), \quad i = 1, 2, \ldots, k, \quad j = 1, 2, \ldots, n_i$$

where $n = \sum_{i=1}^{k} n_i$, with $\sum_{i=1}^{k} n_i \tau_i = 0$

Intermediate calculations (sums of squares)

Total: $SS_T = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{\cdot\cdot}^2}{n}$

Between treatments: $SS_B = \sum_{i=1}^{k} n_i (\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{k} \frac{y_{i\cdot}^2}{n_i} - \frac{y_{\cdot\cdot}^2}{n}$

Residual: $SS_R = SS_T - SS_B$

Variance estimate

$$\hat{\sigma}^2 = \frac{SS_R}{n - k}$$

where n is sample size, and k is the number of parameters in the model.

Goodness of fit

$$R^2 = 1 - \frac{SS_R}{SS_T}$$

$$\text{Adjusted } R^2 = 1 - \frac{MS_R}{MS_T}$$

AIC & BIC FOR LINEAR REGRESSION MODELS

$$\text{AIC} = n\, ln(\hat{\sigma}^2) + 2(1 + p)$$
$$\text{BIC} = n\, ln(\hat{\sigma}^2) + ln(n)\,(1 + p)$$

where $\hat{\sigma}^2$ is the estimated variance of residuals, p is the number of predictor variables in the model, and n is sample size.

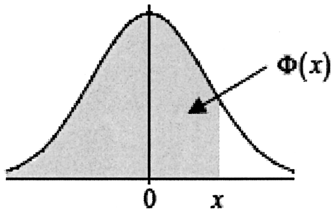
Note: APPENDIX continues on page 10

APPENDIX (continued)

Probabilities for the Standard Normal distribution

The distribution function is denoted by $\Phi(x)$, and the probability density function is denoted by $\phi(x)$.

$$\Phi(x)=\int_{-\infty}^{x}\phi(t)dt=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{1}{2}t^2}dt$$



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.50000	**0.40**	0.65542	**0.80**	0.78814	**1.20**	0.88493	**1.60**	0.94520
0.01	0.50399	**0.41**	0.65910	**0.81**	0.79103	**1.21**	0.88686	**1.61**	0.94630
0.02	0.50798	**0.42**	0.66276	**0.82**	0.79389	**1.22**	0.88877	**1.62**	0.94738
0.03	0.51197	**0.43**	0.66640	**0.83**	0.79673	**1.23**	0.89065	**1.63**	0.94845
0.04	0.51595	**0.44**	0.67003	**0.84**	0.79955	**1.24**	0.89251	**1.64**	0.94950
0.05	0.51994	**0.45**	0.67364	**0.85**	0.80234	**1.25**	0.89435	**1.65**	0.95053
0.06	0.52392	**0.46**	0.67724	**0.86**	0.80511	**1.26**	0.89617	**1.66**	0.95154
0.07	0.52790	**0.47**	0.68082	**0.87**	0.80785	**1.27**	0.89796	**1.67**	0.95254
0.08	0.53188	**0.48**	0.68439	**0.88**	0.81057	**1.28**	0.89973	**1.68**	0.95352
0.09	0.53586	**0.49**	0.68793	**0.89**	0.81327	**1.29**	0.90147	**1.69**	0.95449
0.10	0.53983	**0.50**	0.69146	**0.90**	0.81594	**1.30**	0.90320	**1.70**	0.95543
0.11	0.54380	**0.51**	0.69497	**0.91**	0.81859	**1.31**	0.90490	**1.71**	0.95637
0.12	0.54776	**0.52**	0.69847	**0.92**	0.82121	**1.32**	0.90658	**1.72**	0.95728
0.13	0.55172	**0.53**	0.70194	**0.93**	0.82381	**1.33**	0.90824	**1.73**	0.95818
0.14	0.55567	**0.54**	0.70540	**0.94**	0.82639	**1.34**	0.90988	**1.74**	0.95907
0.15	0.55962	**0.55**	0.70884	**0.95**	0.82894	**1.35**	0.91149	**1.75**	0.95994
0.16	0.56356	**0.56**	0.71226	**0.96**	0.83147	**1.36**	0.91309	**1.76**	0.96080
0.17	0.56749	**0.57**	0.71566	**0.97**	0.83398	**1.37**	0.91466	**1.77**	0.96164
0.18	0.57142	**0.58**	0.71904	**0.98**	0.83646	**1.38**	0.91621	**1.78**	0.96246
0.19	0.57535	**0.59**	0.72240	**0.99**	0.83891	**1.39**	0.91774	**1.79**	0.96327
0.20	0.57926	**0.60**	0.72575	**1.00**	0.84134	**1.40**	0.91924	**1.80**	0.96407
0.21	0.58317	**0.61**	0.72907	**1.01**	0.84375	**1.41**	0.92073	**1.81**	0.96485
0.22	0.58706	**0.62**	0.73237	**1.02**	0.84614	**1.42**	0.92220	**1.82**	0.96562
0.23	0.59095	**0.63**	0.73565	**1.03**	0.84849	**1.43**	0.92364	**1.83**	0.96638
0.24	0.59483	**0.64**	0.73891	**1.04**	0.85083	**1.44**	0.92507	**1.84**	0.96712
0.25	0.59871	**0.65**	0.74215	**1.05**	0.85314	**1.45**	0.92647	**1.85**	0.96784
0.26	0.60257	**0.66**	0.74537	**1.06**	0.85543	**1.46**	0.92785	**1.86**	0.96856
0.27	0.60642	**0.67**	0.74857	**1.07**	0.85769	**1.47**	0.92922	**1.87**	0.96926
0.28	0.61026	**0.68**	0.75175	**1.08**	0.85993	**1.48**	0.93056	**1.88**	0.96995
0.29	0.61409	**0.69**	0.75490	**1.09**	0.86214	**1.49**	0.93189	**1.89**	0.97062
0.30	0.61791	**0.70**	0.75804	**1.10**	0.86433	**1.50**	0.93319	**1.90**	0.97128
0.31	0.62172	**0.71**	0.76115	**1.11**	0.86650	**1.51**	0.93448	**1.91**	0.97193
0.32	0.62552	**0.72**	0.76424	**1.12**	0.86864	**1.52**	0.93574	**1.92**	0.97257
0.33	0.62930	**0.73**	0.76730	**1.13**	0.87076	**1.53**	0.93699	**1.93**	0.97320
0.34	0.63307	**0.74**	0.77035	**1.14**	0.87286	**1.54**	0.93822	**1.94**	0.97381
0.35	0.63683	**0.75**	0.77337	**1.15**	0.87493	**1.55**	0.93943	**1.95**	0.97441
0.36	0.64058	**0.76**	0.77637	**1.16**	0.87698	**1.56**	0.94062	**1.96**	0.97500
0.37	0.64431	**0.77**	0.77935	**1.17**	0.87900	**1.57**	0.94179	**1.97**	0.97558
0.38	0.64803	**0.78**	0.78230	**1.18**	0.88100	**1.58**	0.94295	**1.98**	0.97615
0.39	0.65173	**0.79**	0.78524	**1.19**	0.88298	**1.59**	0.94408	**1.99**	0.97670
0.40	0.65542	**0.80**	0.78814	**1.20**	0.88493	**1.60**	0.94520	**2.00**	0.97725

Note: APPENDIX continues on page 11

APPENDIX (continued)

Probabilities for the Standard Normal distribution

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2.00	0.97725	**2.40**	0.99180	**2.80**	0.99744	**3.20**	0.99931	**3.60**	0.99984	**4.00**	0.99997
2.01	0.97778	**2.41**	0.99202	**2.81**	0.99752	**3.21**	0.99934	**3.61**	0.99985	**4.01**	0.99997
2.02	0.97831	**2.42**	0.99224	**2.82**	0.99760	**3.22**	0.99936	**3.62**	0.99985	**4.02**	0.99997
2.03	0.97882	**2.43**	0.99245	**2.83**	0.99767	**3.23**	0.99938	**3.63**	0.99986	**4.03**	0.99997
2.04	0.97932	**2.44**	0.99266	**2.84**	0.99774	**3.24**	0.99940	**3.64**	0.99986	**4.04**	0.99997
2.05	0.97982	**2.45**	0.99286	**2.85**	0.99781	**3.25**	0.99942	**3.65**	0.99987	**4.05**	0.99997
2.06	0.98030	**2.46**	0.99305	**2.86**	0.99788	**3.26**	0.99944	**3.66**	0.99987	**4.06**	0.99998
2.07	0.98077	**2.47**	0.99324	**2.87**	0.99795	**3.27**	0.99946	**3.67**	0.99988	**4.07**	0.99998
2.08	0.98124	**2.48**	0.99343	**2.88**	0.99801	**3.28**	0.99948	**3.68**	0.99988	**4.08**	0.99998
2.09	0.98169	**2.49**	0.99361	**2.89**	0.99807	**3.29**	0.99950	**3.69**	0.99989	**4.09**	0.99998
2.10	0.98214	**2.50**	0.99379	**2.90**	0.99813	**3.30**	0.99952	**3.70**	0.99989	**4.10**	0.99998
2.11	0.98257	**2.51**	0.99396	**2.91**	0.99819	**3.31**	0.99953	**3.71**	0.99990	**4.11**	0.99998
2.12	0.98300	**2.52**	0.99413	**2.92**	0.99825	**3.32**	0.99955	**3.72**	0.99990	**4.12**	0.99998
2.13	0.98341	**2.53**	0.99430	**2.93**	0.99831	**3.33**	0.99957	**3.73**	0.99990	**4.13**	0.99998
2.14	0.98382	**2.54**	0.99446	**2.94**	0.99836	**3.34**	0.99958	**3.74**	0.99991	**4.14**	0.99998
2.15	0.98422	**2.55**	0.99461	**2.95**	0.99841	**3.35**	0.99960	**3.75**	0.99991	**4.15**	0.99998
2.16	0.98461	**2.56**	0.99477	**2.96**	0.99846	**3.36**	0.99961	**3.76**	0.99992	**4.16**	0.99998
2.17	0.98500	**2.57**	0.99492	**2.97**	0.99851	**3.37**	0.99962	**3.77**	0.99992	**4.17**	0.99998
2.18	0.98537	**2.58**	0.99506	**2.98**	0.99856	**3.38**	0.99964	**3.78**	0.99992	**4.18**	0.99999
2.19	0.98574	**2.59**	0.99520	**2.99**	0.99861	**3.39**	0.99965	**3.79**	0.99992	**4.19**	0.99999
2.20	0.98610	**2.60**	0.99534	**3.00**	0.99865	**3.40**	0.99966	**3.80**	0.99993	**4.20**	0.99999
2.21	0.98645	**2.61**	0.99547	**3.01**	0.99869	**3.41**	0.99968	**3.81**	0.99993	**4.21**	0.99999
2.22	0.98679	**2.62**	0.99560	**3.02**	0.99874	**3.42**	0.99969	**3.82**	0.99993	**4.22**	0.99999
2.23	0.98713	**2.63**	0.99573	**3.03**	0.99878	**3.43**	0.99970	**3.83**	0.99994	**4.23**	0.99999
2.24	0.98745	**2.64**	0.99585	**3.04**	0.99882	**3.44**	0.99971	**3.84**	0.99994	**4.24**	0.99999
2.25	0.98778	**2.65**	0.99598	**3.05**	0.99886	**3.45**	0.99972	**3.85**	0.99994	**4.25**	0.99999
2.26	0.98809	**2.66**	0.99609	**3.06**	0.99889	**3.46**	0.99973	**3.86**	0.99994	**4.26**	0.99999
2.27	0.98840	**2.67**	0.99621	**3.07**	0.99893	**3.47**	0.99974	**3.87**	0.99995	**4.27**	0.99999
2.28	0.98870	**2.68**	0.99632	**3.08**	0.99896	**3.48**	0.99975	**3.88**	0.99995	**4.28**	0.99999
2.29	0.98899	**2.69**	0.99643	**3.09**	0.99900	**3.49**	0.99976	**3.89**	0.99995	**4.29**	0.99999
2.30	0.98928	**2.70**	0.99653	**3.10**	0.99903	**3.50**	0.99977	**3.90**	0.99995	**4.30**	0.99999
2.31	0.98956	**2.71**	0.99664	**3.11**	0.99906	**3.51**	0.99978	**3.91**	0.99995	**4.31**	0.99999
2.32	0.98983	**2.72**	0.99674	**3.12**	0.99910	**3.52**	0.99978	**3.92**	0.99996	**4.32**	0.99999
2.33	0.99010	**2.73**	0.99683	**3.13**	0.99913	**3.53**	0.99979	**3.93**	0.99996	**4.33**	0.99999
2.34	0.99036	**2.74**	0.99693	**3.14**	0.99916	**3.54**	0.99980	**3.94**	0.99996	**4.34**	0.99999
2.35	0.99061	**2.75**	0.99702	**3.15**	0.99918	**3.55**	0.99981	**3.95**	0.99996	**4.35**	0.99999
2.36	0.99086	**2.76**	0.99711	**3.16**	0.99921	**3.56**	0.99981	**3.96**	0.99996	**4.36**	0.99999
2.37	0.99111	**2.77**	0.99720	**3.17**	0.99924	**3.57**	0.99982	**3.97**	0.99996	**4.37**	0.99999
2.38	0.99134	**2.78**	0.99728	**3.18**	0.99926	**3.58**	0.99983	**3.98**	0.99997	**4.38**	0.99999
2.39	0.99158	**2.79**	0.99736	**3.19**	0.99929	**3.59**	0.99983	**3.99**	0.99997	**4.39**	0.99999
2.40	0.99180	**2.80**	0.99744	**3.20**	0.99931	**3.60**	0.99984	**4.00**	0.99997	**4.40**	0.99999

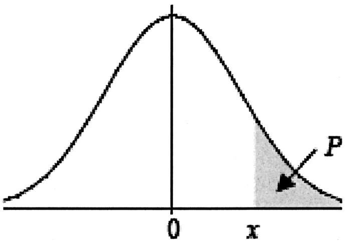
Note: APPENDIX continues on page 12

APPENDIX (continued)

Percentage Points for the Standard Normal distribution

The table gives percentage points x defined by the equation

$$P = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{1}{2}t^2}\, dt$$



P	x	P	x	P	x	P	x	P	x	P	x
50%	0.0000	5.0%	1.6449	3.0%	1.8808	2.0%	2.0537	1.0%	2.3263	0.10%	3.0902
45%	0.1257	4.8%	1.6646	2.9%	1.8957	1.9%	2.0749	0.9%	2.3656	0.09%	3.1214
40%	0.2533	4.6%	1.6849	2.8%	1.9110	1.8%	2.0969	0.8%	2.4089	0.08%	3.1559
35%	0.3853	4.4%	1.7060	2.7%	1.9268	1.7%	2.1201	0.7%	2.4573	0.07%	3.1947
30%	0.5244	4.2%	1.7279	2.6%	1.9431	1.6%	2.1444	0.6%	2.5121	0.06%	3.2389
25%	0.6745	4.0%	1.7507	2.5%	1.9600	1.5%	2.1701	0.5%	2.5758	0.05%	3.2905
20%	0.8416	3.8%	1.7744	2.4%	1.9774	1.4%	2.1973	0.4%	2.6521	0.01%	3.7190
15%	1.0364	3.6%	1.7991	2.3%	1.9954	1.3%	2.2262	0.3%	2.7478	0.005%	3.8906
10%	1.2816	3.4%	1.8250	2.2%	2.0141	1.2%	2.2571	0.2%	2.8782	0.001%	4.2649
5%	1.6449	3.2%	1.8522	2.1%	2.0335	1.1%	2.2904	0.1%	3.0902	0.0005%	4.4172

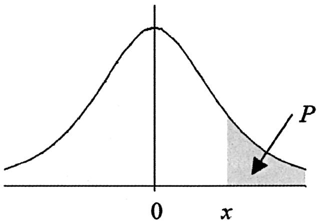
Note: APPENDIX continues on page 13

APPENDIX (continued)

Percentage Points for the *t* distribution

This table gives percentage points x defined by the equation

$$P = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_x^{\infty} \frac{dt}{\left(1 + t^2/\nu\right)^{\frac{1}{2}(\nu+1)}}$$



The limiting distribution of t as ν tends to infinity is the standard normal distribution. When ν is large, interpolation in ν should be harmonic.

$P =$ / ν	40%	30%	25%	20%	15%	10%	5%	2.5%	1%	0.5%	0.1%	0.05%
1	0.3249	0.7265	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Note: APPENDIX continues on page 14

APPENDIX (continued)

Derivative of Common Function

Common Function	Function	Derivative
Constant	c	0
Line	x	1
	ax	a
Square	x^2	$2x$
Square Root	$\sqrt{x} = x^{1/2}$	$\frac{1}{2}x^{-\frac{1}{2}}$
Polynomial	x^n	nx^{n-1}
Exponential	e^x	e^x
	e^{kx}	ke^{kx}
	a^x	$\log(a)\, a^x$
Logarithms (nature)	$\log(x)$	$\frac{1}{x}$

- END OF APPENDIX –

CONFIDENTIAL

BR2211 FINANCIAL & RISK ANALYTICS I

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.