

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2022-2023

**MH4500– Time Series Analysis**

April 2023

TIME ALLOWED: 2 HOURS

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INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

**QUESTION 1.****(24 marks)**

- (a) Suppose  $\{e_t\}$  are independent Gaussian random variables with mean zero and variance one. Let  $X_t = e_t + \alpha e_{t-1}^2$ .
- (i) Find the covariance function of  $X_t$ .
  - (ii) Is  $X_t$  stationary ? Justify it.
- (b) Suppose  $X_t \sim AR(1)$ :

$$X_t = 10 + 0.75X_{t-1} + Z_t,$$

where (and below)  $Z_t$  is a white noise with mean zero and variance one. Let  $X_t = 2$ .

- (i) Predict the values of  $X_{t+1}$ .
- (ii) Predict the values of  $X_{t+10}$ .
- (iii) Find  $EX_t$ .

**QUESTION 2.****(29 marks)**

Let  $\{Z_t\}$  be white noise with mean zero and variance one.

- (a) Find the ACFs and PACFs for the model in (i) and the ACFs for the model in (ii).

(i)

$$X_t - 0.3X_{t-2} = Z_t.$$

(ii)

$$X_t = 0.5X_{t-1} + Z_t + 0.7Z_{t-1} + 0.6Z_{t-2}.$$

**Question 2 continues on page 3**

(b) Consider a model

$$X_t = 5 + 1.5X_{t-1} - 0.5X_{t-3} + Z_t - 2Z_{t-1}.$$

- (i) Determine whether  $X_t$  is stationary and justify it.
- (ii) Is  $X_t$  invertible? Justify it.
- (iii) Identify  $X_t$  as a specific ARIMA(p,d,q) process (find the values of p,d and q).
- (iv) Find the values of  $E(\nabla^2 X_t)$  and  $Var(\nabla^2 X_t)$ .

### QUESTION 3.

(23 marks)

(a) From 100 time series observations we calculate its sample PACFs

$$r_{11} = 0.6, \quad r_{22} = 0.5, \quad r_{33} = 0.1,$$

sample mean 1 and sample variance 5. Answer the following questions for such a data:

- (i) Suggest an appropriate model and justify it.
  - (ii) Estimate the parameters involved in the model and write down the explicit expression.
- (b) Suppose that

$$X_t = 0.3X_{t-1} + X_{t-4} - 0.3X_{t-5} + Z_t - 0.3Z_{t-1} - 0.5Z_{t-4} + 0.15Z_{t-5}.$$

Classify the model as a seasonal ARIMA model and write down the explicit values of  $p, d, q, P, D, Q$  and the seasonal period  $s$ .

**QUESTION 4.****(24 marks)**

Some researcher investigated the data set about the number of accidental deaths occurring monthly in the United States during 1973-1978. Figures can be found on pages 5 to 8.

- (i) Identify the possible components it contains (see Fig 1 ).
- (ii) What do the plots Fig 2 and Fig 3 suggest ?
- (iii) Describe in details what the following R statements mean.

```
acf(diff(diff(death,12),1),36)
```

```
pacf(diff(diff(death,12),1),36)
```

- (iv) Which model would you suggest according to Fig 4?
- (v) Fig 5 shows the diagnostic plot for a suggested model. Comment on the quality of the fitted model.

Figure 1: The plot of the original data

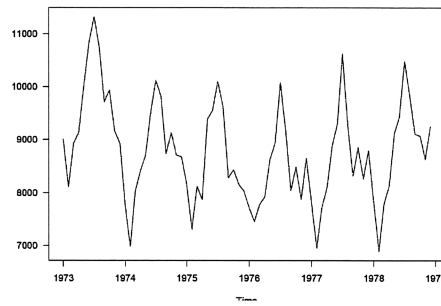


Figure 2: the plot for taking the first differenced data

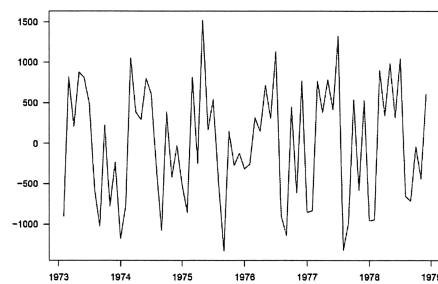


Figure 3: the plot for taking the seasonal differenced data

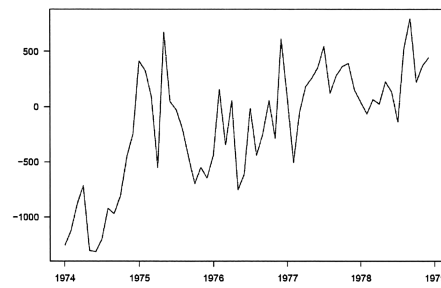


Figure 4: ACF and PACF plots for taking both nonseasonal and seasonal differenced data

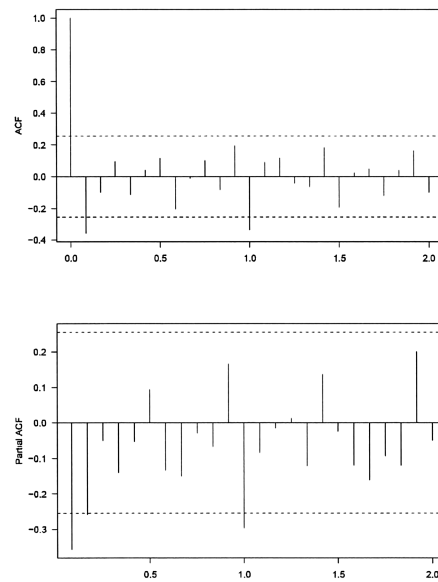
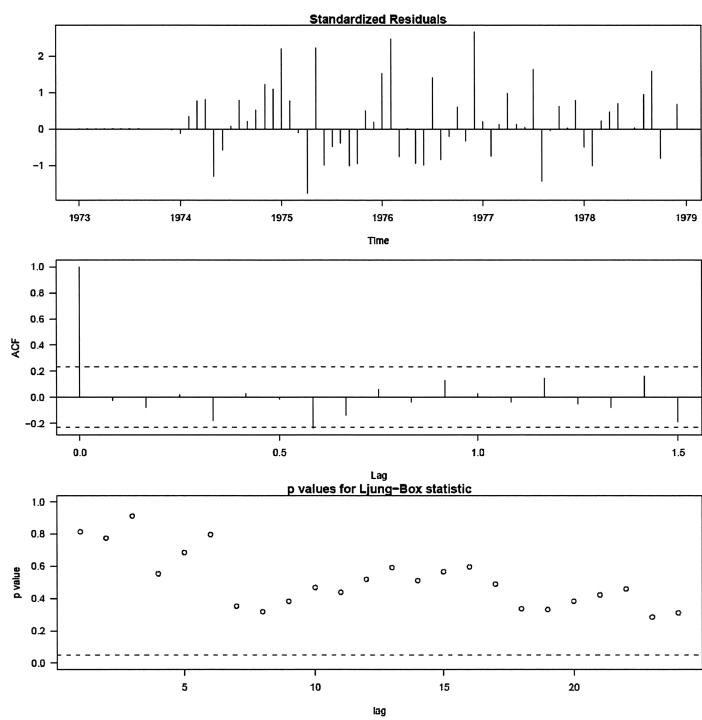


Figure 5: Diagnostic plots



END OF PAPER









## MH4500 TIME SERIES ANALYSIS

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.