

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

MH1812 - DISCRETE MATHEMATICS

November 2022

TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

Throughout this examination paper, \mathbb{R} denotes the set of all reals and \mathbb{Q} the set of all rationals. The minuscule letter n is always assumed to be an integer.

QUESTION 1. (15 marks)

Suppose that functions $f, g : D \rightarrow \mathbb{R}$ (where $D \subseteq \mathbb{R}$) are defined as $f(x) = -\sqrt{-x}$ and $g(x) = -\sqrt{1+x}$.

- (a) Find the largest possible D for both f and g to be well-defined, then find the ranges of f and g for that D . (3 marks)
- (b) Find $g \circ f$ and $(g \circ f)^{-1}$, based on the D determined in part (a). (5 marks)
- (c) Find f^{-1} , g^{-1} and $f^{-1} \circ g^{-1}$, based on the D determined in part (a). (5 marks)
- (d) Find $f \circ (g \circ f)^{-1} \circ g$, based on the D determined in part (a). (2 marks)

QUESTION 2. (20 marks)

Define for a finite set A

$$T(A) = \{S \subseteq A \mid |S| \cdot |A - S| = 2|A|\}.$$

- (a) Find $T(\emptyset)$. (4 marks)
- (b) Find $|T(\{1, 2, \dots, 8\})|$. Your answer should be an explicit number. (4 marks)
- (c) Find $|T(\{1, 2, \dots, 9\})|$. Your answer should be an explicit number. (4 marks)
- (d) Find all $n \geq 1$ such that $|T(\{1, 2, \dots, n\})| \geq n$. (8 marks)

You need not justify your answers for parts (a) to (c), but you must justify your answer for part (d).

QUESTION 3.**(15 marks)**

The Fibonacci sequence $\{f_n\}$ is defined by the recurrence relation

$$\begin{aligned} f_n &= f_{n-1} + f_{n-2}, \quad n \geq 3, \\ f_1 &= f_2 = 1. \end{aligned}$$

- (a) Prove by mathematical induction that

$$f_n > \left(\frac{3}{2}\right)^{n-1}$$

for all $n \geq 6$. (10 marks)

- (b) What is the largest β such that $f_n \geq \beta^{n-1}$ for all $n \geq 6$? Justify your answer without using a calculator. (5 marks)

QUESTION 4.**(25 marks)**

We define two relations S and T on \mathbb{R} as follows.

$$\begin{aligned} S &= \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x - y \in \mathbb{Q}\}, \\ T &= \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x + y \in \mathbb{Q}\}. \end{aligned}$$

- (a) Is S symmetric? Is S anti-symmetric? Is S transitive? Is S an equivalence relation? Is S a partial order? You need not state the reason. (5 marks)
- (b) Is T symmetric? Is T anti-symmetric? Is T transitive? Is T an equivalence relation? Is T a partial order? You need not state the reason. (5 marks)
- (c) Is $S \cup T$ an equivalence relation? Justify your answer. (8 marks)
- (d) Find the transitive closure of T . Justify your answer. (7 marks)

QUESTION 5.

(15 marks)

There are 2022 identical horses lined up in a row



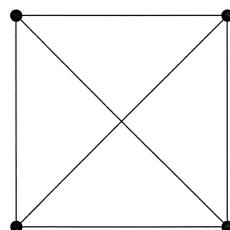
and 10 of them are to be chosen.

- (a) How many choices are there? (5 marks)
- (b) How many choices are there if no two of the chosen horses can be consecutive? (5 marks)
- (c) How many choices are there if there must be at least two horses between each pair of chosen horses? (5 marks)

QUESTION 6.

(10 marks)

Does the following graph contain an Euler path? If it does not, what is the minimum number of edges that must be added to make the graph contain an Euler path? (No vertices can be added.) Justify your answers.



END OF PAPER

MH1812 DISCRETE MATHEMATICS

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.