NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2013-2014

MH1101 – CALCULUS II

April 2014 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT an OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

Question 1. (15 marks)

(i) Evaluate

$$\int \frac{dx}{\sqrt{x^2+x+1}}.$$

(ii) Hence or otherwise, determine if the improper integral

$$\int_1^{\infty} \frac{dx}{\sqrt{x^2+x+1}}$$

is convergent.

Question 2. (15 marks)

(i) Evaluate

$$\int_0^{1/2} \frac{\cos^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx.$$

(ii) Determine whether the improper integral

$$\int_1^{\infty} \frac{dx}{\sqrt{x\tan^{-1} x}}$$

is convergent.

Question 3. (10 marks)

Find the arc length of the curve $y = \ln(\sec x)$ from $x = 0$ to $x = \dfrac{\pi}{4}$.

Question 4. (15 marks)

(i) How large should we choose n in order to guarantee that the error in approximating the definite integral

$$\int_0^2 x \sin x \, dx$$

by the midpoint rule will be less than 10^{-4}.

(ii) By considering the absolute error, prove that the Simpson's rule is exact when approximating the definite integral of a polynomial function of degree at most three.

Question 5. (15 marks)

(i) Determine if the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent or divergent, where for each n,

$$a_n = \left((-1)^n + 1\right)\left(\frac{n+1}{n}\right).$$

Justify your answer.

(ii) For what values of r does the infinite series

$$1 + 2r + 2r^2 + r^3 + 2r^4 + 2r^5 + r^6 + 2r^7 + 2r^8 + r^9 + \cdots$$

converge, and for what values of r does the series diverge? Find the sum of the series when it converges.

Question 6. (18 marks)

Test the following series for convergence, using any method.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$.

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$.

Question 7. (12 marks)

Let $a_0 > b_0 > 0$ and define the sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ by

$$a_{n+1} = \frac{a_n + b_n}{2}, \qquad b_{n+1} = \sqrt{a_n b_n},$$

for $n = 0, 1, 2, \cdots$. Prove that both sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent.

END OF PAPER

MH1101 CALCULUS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.