NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2021-2022

MH1200 – LINEAR ALGEBRA I

November 2021 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the start of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT an OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(20 marks)**

A polynomial of degree 3 or less is a function $f \colon \mathbb{R} \to \mathbb{R}$ of the form

$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$$

for constants $c_0, c_1, c_2, c_3 \in \mathbb{R}$.

(a) For some given a and b, determine all such polynomials such that $f(1) = a$ and $f(-1) = b$.

(b) Let $S \subset \mathbb{R}^4$ denote the set of 4-tuples (c_0, c_1, c_2, c_3) with the property that the corresponding function $f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$ satisfies

$$f(-x) = -f(x)$$

for all $x \in \mathbb{R}$. Briefly explain why S is a subspace of $\mathbb{R}^4$ and determine its dimension. (You may assume that if $c_0 + c_1 x + c_2 x^2 + c_3 x^3 = 0$ for all $x \in \mathbb{R}$ then $c_0 = c_1 = c_2 = c_3 = 0$.)

QUESTION 2. **(20 marks)**

Consider the following matrix

$$\mathbf{A} = \begin{bmatrix} a+1 & 1+a & 0 \\ a^2-1 & a^2 & 0 \\ 0 & 0 & a \end{bmatrix}.$$

(a) Find all values of the parameter a such that $\mathbf{A}$ is an invertible matrix. Justify.

(b) Find all values of the parameter a such that $\mathbf{A}$ is equal to a product of elementary matrices. Justify.

(c) Assuming a is one of the values you gave in part (b), express $\mathbf{A}$ as a product of elementary matrices.

You may use standard results proved in theorems during the course in your deductions.

QUESTION 3. **(20 marks)**

(a) Calculate the adjoint matrix of the following matrix. Show full working.

$$\mathbf{A} = \begin{bmatrix} -3 & 2 & -5 \\ -1 & 0 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

(b) Is the following statement true or false? Justify your answer.

Statement: If $\mathbf{M}$ is a non-zero square matrix satisfying $\det(\mathbf{M}) = 0$ then the list of rows of the adjoint matrix of $\mathbf{M}$ is linearly dependent.

QUESTION 4. **(20 marks)**

(a) Is the following list of 3 vectors in $\mathbb{R}^4$ linearly dependent or independent? Justify.

$$\vec{w}_1 = (1, 4, 2, -3)\,,\ \vec{w}_2 = (7, 10, -4, -1)\,,\ \vec{w}_3 = (-2, 1, 5, -4).$$

(b) Let n be a positive integer and let $\vec{u}$ and $\vec{v}$ be a linearly independent pair of vectors in $\mathbb{R}^n$. Use the definition of linear independence to carefully prove that if $a, b, c, d \in \mathbb{R}$ satisfy $ad - bc \neq 0$ then the vectors

$$\vec{x} = a\vec{u} + b\vec{v}\ ,\quad \vec{y} = c\vec{u} + d\vec{v}$$

are also linearly independent.

QUESTION 5. **(20 marks)**

Consider the following three vectors in $\mathbb{R}^5$

$$\vec{u} = (1,3,1,3,1),\ \vec{v} = (3,-1,1,-1,1),\ \vec{w} = (-1,2,0,2,0).$$

(a) Compute a basis for $\text{span}\{\vec{u}, \vec{v}, \vec{w}\}$.

(b) Determine a system of linear equations whose set of solutions is precisely $\text{span}\{\vec{u}, \vec{v}, \vec{w}\}$.

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.