NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2024-2025

MH3700 - NUMERICAL ANALYSIS I

May 2025 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES.

1. This examination paper contains **FIVE (5)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Begin each question on a **FRESH** page of the answer book.

4. This is a **CLOSED BOOK** exam.

5. Candidates may use calculators. However, they should systematically write down the steps in their working.

Question 1 (20 Marks)

(a) Assume that equation `f(`x`)` $= 0$ has a unique solution in [`a`,`b`] and `f(a)f(b)`< 0, where the function `f` has been defined in Python. Write a Python function `bisection(a,b,n)` that reads real numbers `a`, `b` and a positive integer `n` as arguments, and returns the `n`th term $p_{\texttt{n}}$ in the bisection method for solving this equation, with $p_1 = (\texttt{a} + \texttt{b})/2$.

(b) Show that the fixed point iteration for the function $g(x) = \cos(x)$ converges for any first point $p_0 \in (0, 1)$.

(c) Let p be a fixed real number. Let $f(x) = (x - p)^4 h(x)$ where $h(p) \neq 0$. Find m so that the fixed point iteration of

$$g(x) = x - m\frac{f(x)}{f'(x)},$$

if it converges to p, will converge at least quadratically.

Question 2 (15 Marks)

(i) Use divided differences to find the Hermite interpolating polynomial $H_3(x)$ for the data

$$f(0) = 1, f'(0) = -1, f(1) = 1, f'(1) = 2.$$

(ii) For the function $f(x)$ and the Hermite interpolating polynomial $H_3(x)$ in part (i), given that $|f^{(4)}(x)| \leq 1$ for all $x \in [0, 1]$, find an upper bound for

$$|f(x) - H_3(x)|$$

for $x \in [0, 1]$.

Question 3 (23 Marks)

(a) Let $f(x)$ be a smooth function. Use interpolation to derive the central approximation formula

$$f'(x) = \frac{1}{2h}[f(x+h) - f(x-h)] + O(h^2).$$

(b) Use Taylor series to find the constants A and B so that the approximation

$$f'(x) \approx \frac{1}{h}[Af(x+h) + Bf(x-h)]$$

has the error of the highest possible power of h.

(c) Use Richardson's extrapolation to improve the approximation in part (b).

Question 4 (15 Marks)

(i) Use interpolation to derive the trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{(b-a)}{2}(f(a) + f(b)).$$

(ii) Write down the composite trapezoidal rule.

Given that the error of the trapezoidal rule on (a, b) in part (i) is $O((b-a)^3)$, what is the error of the composite trapezoidal rule?

Question 5 **(27 Marks)**

(a) Given the Legendre polynomial $L_2(x) = 3x^2 - 1$, use interpolation to derive the quadrature rule with 2 quadrature points, with the highest possible degree of precision to approximate the integral

$$\int_0^1 f(x)dx.$$

(b) Use the construction of the Gauss-Legendre quadrature rules from the interpolating polynomial $P_n(x)$, show that the degree of precision of a Gauss-Legendre quadrature rule using $n+1$ points is not less than n.

(c) Using the result in part (b) of this question, prove that the degree of precision of a Gauss-Legendre quadrature rule using $n+1$ points is not less than $2n+1$.

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.