

NANYANG TECHNOLOGICAL UNIVERSITY  
SEMESTER II EXAMINATION 2024-2025  
MH1803 Calculus for Physics

Apr/May 2025

Time Allowed: 2 hours

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INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** pages.
2. Answer **ALL FIVE (5)** questions.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. You should write down systematically the steps in the workings to receive full credit.
5. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).
6. Calculators are **NOT** allowed.

## 1. Short Questions (35 marks)

- (a) Find the equation of the tangent plane to the surface  $x^3y + z^2 = 3$  at the point  $(-1, 1, 2)$ .

(5 marks)

- (b) Compute the divergence and the curl of the vector field  $\mathbf{F}(x, y, z) = (x^2, xyz, yz^2)$ .

(10 marks)

- (c) Solve the differential equation

$$(x^2 + 1) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

by the power series method. Assume the solution can be expressed as a power series with real coefficients (i.e.,  $a_i$ 's  $\in \mathbb{R}$ ,  $\forall i = 0, 1, 2, \dots$ ):

$$y(x) = a_0 + a_1x + a_2x^2 + \dots = \sum_{n=0}^{\infty} a_n x^n.$$

(10 marks)

- (d) Solve the first-order partial differential equation  $5u_t + 6u_x = 0$  with the auxiliary condition  $u(x, 0) = \cos x$  when  $t = 0$ .

(10 marks)

2. Let  $(x, y) = G(u, v) = (u^2 - v^2, 2uv)$ . Let  $A$  be the region defined by  $1 \leq u^2 + v^2 \leq 2$  and  $0 \leq u, 0 \leq v$ . Find the integral of the function

$$f(x, y) = \frac{1}{x^2 + y^2}$$

over  $G(A)$ .

(15 marks)

3. Find the spherical coordinate limits for the solid enclosed by the cardioid of revolution  $r(\theta, \phi) = 1 - \cos \theta$  (which is axisymmetrical about the  $z$ -axis) and then evaluate the integral for its volume explicitly.

(15 marks)

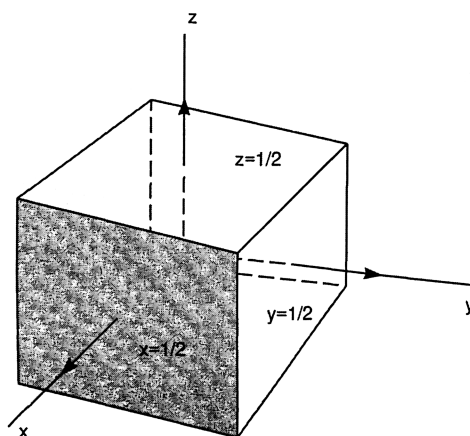
4. Consider the vector field  $\mathbf{F}(x, y, z) = (z - y, x + z, -(x + y))$  and the surface of the paraboloid  $z = 4 - x^2 - y^2$  with  $0 \leq z \leq 4$ . Sketch the paraboloid. Verify Stokes' theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{x} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A},$$

where  $C$  is a circle of radius 2 on the  $xy$  plane and  $S$  is the surface of the paraboloid bound by  $C$ .

(20 marks)

5. Calculate the flux of the vector field  $\mathbf{F} = (x^3y, y^2x, z)$  over a unit cube centered at the origin as shown below.



(15 marks)

- End of Paper -

MH1803 CALCULUS FOR PHYSICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.