NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2022-2023

MH2811– Mathematics II

May 2023 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HAND-WRITTEN OR TYPED ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. (15 marks)

(a) Find the Fourier series of the function $f(x)$ given by

$$f(x) = \begin{cases} 2-x, & \text{if } 0 < x \leq 4, \\ x-6, & \text{if } 4 < x \leq 8. \end{cases}$$

The period of this function is 8.

(b) Using the Fourier series found in part (a), show that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \cdots = \frac{\pi^2}{8}.$$

QUESTION 2. (20 marks)

(a) Find each of the following limits, or show that the limit does not exist.

(i)

$$\lim_{(x,y)\to(2,-1)} \frac{x-y-3}{\sqrt{x-2}-\sqrt{y+1}}, \quad \text{where } x > 2 \text{ and } y > -1.$$

(ii) $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$, where

$$f(x,y) = \begin{cases} y+1, & \text{if } y \leq 0, \\ x, & \text{otherwise.} \end{cases}$$

(b) Let $g(x,y,z) = x^3y + z^2x^2$, where $x = 1 + 4r^3$, $y = r\cos r$ and $z = r^2e^r$. Use the <u>Chain Rule</u> to find $\dfrac{dg}{dr}$ when $r = 0$.

(c) Suppose that z is a function of x, y satisfying

$$x^3 \sin(3y - 2z) = 2 + y^2 \cos(3xz).$$

Find $\dfrac{\partial z}{\partial x}$.

QUESTION 3. (20 marks)

(a) Combine the two double integrals into a single one by reversing the order of integration:

$$\int_0^1 \int_0^{2y} f(x,y)\,dx\,dy + \int_1^3 \int_0^{3-y} f(x,y)\,dx\,dy.$$

(b) Evaluate

$$\iint_R \frac{1}{\sqrt{2y-y^2}}\,dA,$$

where R is the region on the x-y plane in the first quadrant $x > 0, y > 0$ bounded by the curve $x^2 = 4 - 2y$.

QUESTION 4. (15 marks)

(a) Find the general solution of the following ODE:

$$e^{\frac{1}{2}x^2} + \frac{y}{x} + xy = y', \qquad x > 0.$$

(b) Find the general solution of the following ODE:

$$y'' - 3y' + 2y = e^x.$$

QUESTION 5. (12 marks)

(a) Find the divergence and curl of the vector field $\boldsymbol{F}$, where $\boldsymbol{F}(x, y, z) = xye^x\boldsymbol{i} + yze^x\boldsymbol{k}$.

(b) Find the rate of change of $f(x, y, z) = x^2y + x\sqrt{1+z}$ in the direction $\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$ at the point $(1, 1, 1)$. In what direction does the maximum rate of change of f occur at the point $(1, 1, 1)$?

QUESTION 6. (18 marks)

(a) Let $\boldsymbol{F}(x, y, z) = (2xy + z^3)\boldsymbol{i} + (x^2)\boldsymbol{j} + (3xz^2)\boldsymbol{k}$.

(i) Is $\boldsymbol{F}$ conserved? Justify your answer.

(ii) Evaluate $\displaystyle\int_{C_0} \boldsymbol{F} \cdot d\boldsymbol{r}$ where C_0 is the curve $\boldsymbol{r}(t) = \cos(t^2)\boldsymbol{i} + \sin(t^2)\boldsymbol{j} + t^2\boldsymbol{k}$ with $0 \le t \le \sqrt{\pi}$.

(b) Use Green's Theorem to evaluate

$$\oint_{C_1} \left((2x^3 - y^3)\boldsymbol{i} + (x^3 + y^3)\boldsymbol{j}\right) \cdot d\boldsymbol{r},$$

where C_1 is the boundary of the unit circle on the x-y plane $x^2 + y^2 = 1$, oriented anti-clockwise.

END OF PAPER

MH2811 MATHEMATICS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.