NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER 1 EXAMINATION 2022–2023
MH2814 – PROBABILITY AND STATISTICS

Nov/Dec 2022 TIME ALLOWED: 2 HOURS

Matriculation Number: | | | | | | | | | |

Seat Number: | | | | | |

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **SEVENTEEN (17)** pages, including appendix from page 14 to page 17.
2. Tables of some probability distributions are provided in the Appendix.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.
6. All your solutions should be written in this booklet within the space provided after each question. However, if you write your solutions on other pages, please indicate them clearly.
7. This examination paper is **NOT ALLOWED** to be removed from the examination hall.

For examiners only

Question	Marks
1 (15)	
2 (15)	
3 (20)	

Question	Marks
4 (20)	
5 (15)	
6 (15)	

TOTAL (100)

QUESTION 1. **(15 Marks)**

(a) The members profile of a certain society is such that 80% of the members are male or above 25 years of age. We also know that 50% of the members are male or at most 25 years of age. Find the percentage of members who are male.

(b) In any given month, the probability that two accidents occuring at a certain traffic junction is three times as likely as three accidents occur at the same junction. If the number of traffic accidents occurring at that traffic junction follows a Poisson distribution, what is the expected number of traffic accidents that occur at that junction in a year?

Question 1 continues on page 3.

(c) Suppose $\mu_X = 1$, $\mu_Y = 4$, $\sigma_X^2 = 4$, $\sigma_Y^2 = 9$, and $\rho_{XY} = 0.5$. Find $E(Z)$ where

$$Z = 2X^2 - 3X + XY.$$

End of Question 1.

QUESTION 2. **(15 Marks)**

A candy machine produces candies that are either green or red in colour randomly. It is known that 40% of the candies produced are green in colour.

(a) Suppose 12 candies are selected randomly and arranged in a row. Given that there are 4 red candies and 8 green candies, what is the probability that the first 2 candies in the row are green?

(b) Suppose these candies are randomly packed into bags of 2 candies each and 25 such bags are randomly selected. Using the Poisson approximation to the binomial distribution, find the probability that at most 3 out of the 25 bags contains green candies only.

Question 2 continues on page 5.

(c) Using the normal approximation to the binomial distribution, find the probability that in a box of 300 candies, the number of green candies is between 101 to 110 (both numbers included).

End of Question 2.

QUESTION 3. **(20 Marks)**

The joint density function of X and Y is given as:

$$f(x,y) = \begin{cases} x, & \text{if } 0 < x < y < 1 \quad \text{or} \quad 0 < y < 2 - x < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $P(X < 1)$.

(b) Find the marginal distributions of X and Y, respectively.

Question 3 continues on page 7.

(c) Find $P(Y > 0.2|X = 0.5)$.

(d) Find $E(X)$ and $E(Y)$, the expected value of X and Y, respectively.

(e) Find σ_{XY}, the covariance of X and Y.

End of Question 3.

QUESTION 4. **(20 Marks)**

An electrical company produces light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours.

(a) Suppose a random sample of 50 light bulbs were tested for their lifetime, what is the distribution of the sample mean from this sample? State the distribution and its mean and variance clearly.

(b) In a random sample of 50 light bulbs tested, what is the probability that the mean lifetime of this sample is more than 820 hours?

Question 4 continues on page 9.

(c) Now, suppose the observed sample mean lifetime of a sample of 50 light bulbs is 790, determine a 95% confidence interval for the mean lifetime.

(d) Suppose we want to estimate the true mean lifetime of the light bulbs manufactured by this company, how large a sample is needed if we wish to be 90% confident that our sample mean will be within 8 hours of the true mean lifetime?

End of Question 4.

QUESTION 5. **(15 Marks)**

A mobile phone manufacturer is deciding whether to install lithium batteries manufactured by company A or company B in a particular model of mobile phone that it is producing. A sample of 12 batteries from company A and another sample of 15 batteries from company B were obtained. An employee then designs an experiment to measure the capacity of the batteries (in milliampere hours) from each company. His findings are as follows.

Company A: $n_A = 12,\ x_A = 3680,\ s_A = 22.5,$

Company B: $n_B = 15,\ x_B = 3560,\ s_B = 28.5,$

where n_i, x_i, and s_i are the sample size, sample mean, and sample standard deviation of company i, respectively.

Set up a statistical test to determine whether there is sufficient evidence to claim that the mean capacity of batteries from company A is larger than the mean capacity of batteries from company B by more than 100 milliampere hours, at 0.05 level of significance.

(a) State the null hypothesis, H_0.

(b) State the alternative hypothesis, H_1.

(c) State the probability of wrongly rejecting your null hypothesis.

Question 5 continues on page 11.

(d) Assuming that the population variances for batteries from both companies can be considered to be equal, determine the test statistics and an approximate p-value, based on the sample data.

(e) State your decision and conclusion.

End of Question 5.

QUESTION 6. **(15 Marks)**

A farmer wishes to investigate how different amounts of a certain nitrogen fertilizer affects the yield of his rice crop. He conducted his own experiment on 22 plots of farmland of size 1 hectare, with different amounts of nitrogen fertilizers added to each farmland. He recorded the amount of fertilizer x_i (in 100 kg) and the yield of rice y_i (in 100,000 kg) for each of the 22 plots of farmland. The summary statistics are given:

$$n = 22, \qquad \bar{x} = 23.52, \qquad \bar{y} = 50.73, \qquad S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 225.4,$$

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = 168.31, \qquad S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = 860.93.$$

(a) Find the estimated simple linear regression line $E(Y) = \beta_0 + \beta_1 \cdot X$, with this dataset.

(b) Based on the simple linear model in Part (a), what would be the average yield of rice of all plots of 1-hectare farmland with 30 kg of nitrogen fertilizer added? Give an estimate of this score with a 95% confidence interval.

Question 6 continues on page 13.

(c) If the farmer wishes to produce at least 5,500,000 kg of rice per hectare of farmland, what is the minimum amount of nitrogen fertilizer that should be added per hectare of farmland? Give a point estimate of this score.

END OF PAPER

• Z-distribution.
Commonly used z_α where $P(Z > z_\alpha) = \alpha$:

$z_{0.05}$	=	1.645	$z_{0.10}$	=	1.283
$z_{0.025}$	=	1.960	$z_{0.01}$	=	2.328
$z_{0.005}$	=	2.575	$z_{0.02}$	=	2.054

• t-distribution.
Commonly used t_α where $P(T > t_\alpha) = \alpha$, for $\nu = 15$ to $\nu = 30$:

	α						
ν	0.20	0.15	0.10	0.05	0.025	0.02	0.015
15	0.866	1.074	1.341	1.753	2.131	2.249	2.397
16	0.865	1.071	1.337	1.746	2.120	2.235	2.382
17	0.863	1.069	1.333	1.740	2.110	2.224	2.368
18	0.862	1.067	1.330	1.734	2.101	2.214	2.356
19	0.861	1.066	1.328	1.729	2.093	2.205	2.346
20	0.860	1.064	1.325	1.725	2.086	2.197	2.336
21	0.859	1.063	1.323	1.721	2.080	2.189	2.328
22	0.858	1.061	1.321	1.717	2.074	2.183	2.320
23	0.858	1.060	1.319	1.714	2.069	2.177	2.313
24	0.857	1.059	1.318	1.711	2.064	2.172	2.307
25	0.856	1.058	1.316	1.708	2.060	2.167	2.301
26	0.856	1.058	1.315	1.706	2.056	2.162	2.296
27	0.855	1.057	1.314	1.703	2.052	2.158	2.291
28	0.855	1.056	1.314	1.701	2.048	2.154	2.286
29	0.854	1.055	1.313	1.699	2.045	2.150	2.278
30	0.854	1.055	1.310	1.697	2.042	2.147	2.250

• Binomial Probability Sums $\sum_{x=0}^{r} b(r; n, p)$ for $n = 17, 18, 19, 20$

		p									
n	r	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**
17	**0**	0.1668	0.0225	0.0075	0.0023	0.0002	0.0000				
	1	0.4818	0.1182	0.0501	0.0193	0.0021	0.0001	0.0000			
	2	0.7618	0.3096	0.1637	0.0774	0.0123	0.0012	0.0001			
	3	0.9174	0.5489	0.3530	0.2019	0.0464	0.0064	0.0005	0.0000		
	4	0.9779	0.7582	0.5739	0.3887	0.1260	0.0245	0.0025	0.0001		
	5	0.9953	0.8943	0.7653	0.5968	0.2639	0.0717	0.0106	0.0007	0.0000	
	6	0.9992	0.9623	0.8929	0.7752	0.4478	0.1662	0.0348	0.0032	0.0001	
	7	0.9999	0.9891	0.9598	0.8954	0.6405	0.3145	0.0919	0.0127	0.0005	
	8	1.0000	0.9974	0.9876	0.9597	0.8011	0.5000	0.1989	0.0403	0.0026	0.0000
	9		0.9995	0.9969	0.9873	0.9081	0.6855	0.3595	0.1046	0.0109	0.0001
	10		0.9999	0.9994	0.9968	0.9652	0.8338	0.5522	0.2248	0.0377	0.0008
18	**0**	0.1501	0.0180	0.0056	0.0016	0.0001	0.0000				
	1	0.4503	0.0991	0.0395	0.0142	0.0013	0.0001				
	2	0.7338	0.2713	0.1353	0.0600	0.0082	0.0007	0.0000			
	3	0.9018	0.5010	0.3057	0.1646	0.0328	0.0038	0.0002			
	4	0.9718	0.7164	0.5187	0.3327	0.0942	0.0154	0.0013	0.0000		
	5	0.9936	0.8671	0.7175	0.5344	0.2088	0.0481	0.0058	0.0003		
	6	0.9988	0.9487	0.8610	0.7217	0.3743	0.1189	0.0203	0.0014	0.0000	
	7	0.9998	0.9837	0.9431	0.8593	0.5634	0.2403	0.0576	0.0061	0.0002	
	8	1.0000	0.9957	0.9807	0.9404	0.7368	0.4073	0.1347	0.0210	0.0009	
	9		0.9991	0.9946	0.9790	0.8653	0.5927	0.2632	0.0596	0.0043	0.0000
	10		0.9998	0.9988	0.9939	0.9424	0.7597	0.4366	0.1407	0.0163	0.0002
19	**0**	0.1351	0.0144	0.0042	0.0011	0.0001					
	1	0.4203	0.0829	0.0310	0.0104	0.0008	0.0000				
	2	0.7054	0.2369	0.1113	0.0462	0.0055	0.0004	0.0000			
	3	0.8850	0.4551	0.2631	0.1332	0.0230	0.0022	0.0001			
	4	0.9648	0.6733	0.4654	0.2822	0.0696	0.0096	0.0006	0.0000		
	5	0.9914	0.8369	0.6678	0.4739	0.1629	0.0318	0.0031	0.0001		
	6	0.9983	0.9324	0.8251	0.6655	0.3081	0.0835	0.0116	0.0006		
	7	0.9997	0.9767	0.9225	0.8180	0.4878	0.1796	0.0352	0.0028	0.0000	
	8	1.0000	0.9933	0.9713	0.9161	0.6675	0.3238	0.0885	0.0105	0.0003	
	9		0.9984	0.9911	0.9674	0.8139	0.5000	0.1861	0.0326	0.0016	
	10		0.9997	0.9977	0.9895	0.9115	0.6762	0.3325	0.0839	0.0067	0.0000
20	**0**	0.1216	0.0115	0.0032	0.0008	0.0000					
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000				
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002				
	3	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000			
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003			
	5	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000		
	6	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003		
	7	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0000	
	8	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0001	
	9	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0006	
	10		0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0026	0.0000

• Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$

	μ								
r	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**
0	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8		1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9			1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10				0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11				1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12					1.0000	0.9999	0.9997	0.9992	0.9980
13						1.0000	0.9999	0.9997	0.9993
14							1.0000	0.9999	0.9998
15								1.0000	0.9999
16									1.0000

- Standard Normal Cumulative Probability Distribution

$$P(Z < z) \text{ for } z > 0.$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

- Note that for $z > 0$, we can calculate $P(Z < -z)$ by

$$P(Z < -z) = P(Z > z) = 1 - P(Z < z).$$

MH2814 PROBABILITY & STATISTICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.