NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2016-2017

MH1101 – Calculus II

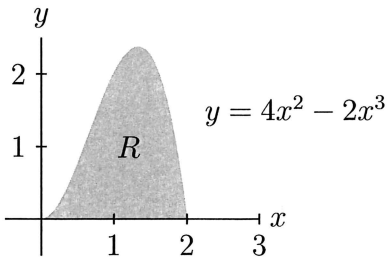
April 2017 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. Each candidate is allowed to bring **ONE(1)** hand-written, double-sided A4 size help sheet.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

Question 1. (15 marks)

Let R be the region bounded by the curve $y = 4x^2 - 2x^3$ and the x axis in the first quadrant (see figure below). Using the **cylindrical shell method**, find the volume of the solid generated by revolving the region R about the vertical line $x = 3$.



Question 2. (15 marks)

Evaluate the following integrals. Express your final answers in terms of x.

(a) $\displaystyle\int \frac{5x^2}{x^3 - x^2 + 4x - 4}\,dx$

(b) $\displaystyle\int \sqrt{3 - 2x - x^2}\,dx$

You may assume that $\displaystyle\int \frac{1}{1+x^2} = \tan^{-1} x + C$, $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$, $\sin 2\theta = 2\sin\theta\cos\theta$.

Question 3. (20 marks)

Determine whether the following series converges or diverges. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^6 - 5n^2 + 7}{4n^6 + n + 1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n + 5^n}{5^n + 6^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{5 + \ln n}{n^2}$

Question 4. (20 marks)

(a) Use **power series** to evaluate

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin 3x}.$$

(b) Find the interval of convergence of the following power series, and identify the values of x for which the series converges absolutely or conditionally.

$$\sum_{n=1}^{\infty} \frac{(4x+3)^{n+1}}{3n+1}.$$

Question 5. (10 marks)

Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$, where $L > 0$ is a real number. Show that the power series $\sum_{n=1}^{\infty} a_n x^n$ converges for $|x| < \frac{1}{L}$.

Question 6. (20 marks)

(a) Find the Taylor series centred at $c = -1$ for the function $f(x) = \dfrac{1}{3x-2}$, and determine its radius of convergence.

(b) Let $F(x) = \displaystyle\int_0^{\pi/2} \sqrt{1 - x^2 \sin^2 t}\, dt$.

Find the Maclaurin series for $F(x)$.

You need to evaluate all integrals completely in your final answer.

END OF PAPER

MH1101 CALCULUS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.