NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2022–2023

MH2810 – Mathematics A

November 2022 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises **SEVENTEEN (17)** pages. The formulae table is on page 17.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. This is a **RESTRICTED OPEN BOOK** exam. Each candidate is only allowed to bring in **ONE DOUBLE-SDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN ON THE A4 PAPER (no sticky notes/post-it notes on the reference sheet)**.

4. Candidates may use calculators. However, they should write down systematically the steps in their working.

5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

6. This examination paper is **NOT ALLOWED** to be removed from the examination hall.

For examiners only

Questions	Marks
1 (10)	
2 (10)	
3 (10)	
4 (15)	

Questions	Marks
5 (15)	
6 (15)	
7 (12)	
8 (13)	

Total (100)	

QUESTION 1. **(10 marks)**

(a) Consider the functions $f(x) = \cos(x+1)$, $g(x) = x^2 - 1$, $h(x) = x+1$. Find the composite function $h \circ (f \circ g)$ in terms of x.

(b) Find the maximum and minimum of the function $f(x) = x\sqrt{4-x^2}$ on the interval $[1, 2]$. Justify your answer.

QUESTION 2. **(10 marks)**

(a) Compute the limit $\lim_{x\to\infty} \frac{2\sin^2(x)}{\sqrt{x}}$. Justify your answer.

(b) Compute the limit $\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$. Justify your answer.

QUESTION 3. **(10 marks)**

(a) Evaluate the integral

$$\int_0^{\pi^2/4} \sin(\sqrt{x})\ dx.$$

(You may wish to substitute $u = \sqrt{x}$.)

Question 3 continues on Page 5.

(b) Find the indefinite integral

$$\int \frac{8-3x}{(x+1)(x^2-4x+6)}\,dx.$$

QUESTION 4. **(15 marks)**

Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{\ln n}{\sqrt{n}}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3^n \ln n}{n!}$

Question 4 continues on Page 7.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{n^2+1}$

QUESTION 5. **(15 marks)**

(a) By manipulating a geometric series, find a power series centred at $a = 2$ (expressed in the summation notation) representing the function $\dfrac{1}{1+4x}$, and state its radius of convergence.

(b) Find the first four non-zero terms of the Maclaurin series for the function $\dfrac{\cos(x^2)}{1+x^2}$.

Question 5 continues on Page 9.

(c) Find the radius of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{\sqrt{n^3+1}}.$$

QUESTION 6. **(15 marks)**

(a) Find the equation of the tangent plane to the surface

$$\sin(x) - x^2y + e^{xz} + yz = 3$$

at the point $(x, y, z) = (0, 1, 2)$.

(b) Find the directional derivative of the function $f(x, y) = \ln(x^2 + y^3)$ at the point $(1, 0)$ in the direction of the vector $\mathbf{v} = \langle 2, 3 \rangle$.

Question 6 continues on Page 11.

(c) Suppose that $z = \sin(2x - y)$, $x = u + \sin(v)$, $y = uv$. Use the Chain rule to find the values of both $\dfrac{\partial z}{\partial v}$ and $\dfrac{\partial z}{\partial u}$ when $u = \pi$, $v = 0$.

QUESTION 7. **(12 marks)**

(a) Evaluate the double integral $\iint_R x^2\, dA$, where R is the region in the xy-plane bounded by the graphs of $y = 2x^2$ and $y = 2$.

Question 7 continues on Page 13.

(b) By changing the order of integration if necessary, evaluate the following iterated integral:

$$\int_0^2 \int_0^{4-x^2} \frac{2xe^{2y}}{4-y}\, dy\, dx.$$

QUESTION 8. **(13 marks)**

(a) Find the general solution of the first order differential equation

$$y'\cos(x) + y\sin(x) = x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Question 8 continues on Page 15.

(b) Consider the following second order linear ordinary differential equation

$$y'' + 3y' + 2y = \frac{1}{1+e^x}. \tag{A}$$

Answer parts (i), (ii) and (iii) below.

(i) Find the homogeneous solution of the differential equation (A).

(ii) Find a particular solution of the differential equation (A).

Question 8 continues on Page 16.

(iii) Write down the general solution of the differential equation (A).

$y =$ $\boxed{}$

END OF PAPER

MH2810 Formulae Table (Final Exam)

$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	$\int e^x\,dx = e^x + C$
$\int k\,dx = kx + C$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + C$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$	

$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$	$\sin 2x = 2\sin x\cos x$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\sin A\cos B = \dfrac{1}{2}(\sin(A-B) + \sin(A+B))$	$\sin A\sin B = \dfrac{1}{2}(\cos(A-B) - \cos(A+B))$
$\cos A\cos B = \dfrac{1}{2}(\cos(A-B) + \cos(A+B))$	

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$x \in (-1, 1)$
$\dfrac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$	$x \in (-1, 1)$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$	$x \in \mathbb{R}$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$	$x \in \mathbb{R}$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$	$x \in \mathbb{R}$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots$	$x \in [-1, 1]$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$	$x \in (-1, 1]$
$\ln(1-x) = \sum_{n=1}^{\infty} -\dfrac{x^n}{n} = -x - \dfrac{x^2}{2} - \dfrac{x^3}{3} - \dfrac{x^4}{4} - \cdots$	$x \in [-1, 1)$

MH2810 MATHEMATICS A

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.