Sem 2 AY1718 MH1101 Calculus II Test (1 hour)

Name: ..... **Matriculation Number:** .....

Tutorial Group: .....

Question 1 [16 marks]

(a) Calculate the derivative $\dfrac{d}{dx}\displaystyle\int_x^{x^2} \sin(t^2)\,dt$.

(b) Write the following limit as a definite integral (do not evaluate it).

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(1+\frac{4i}{n}\right)^3.$$

Answer.

Question 2 [16 marks]

(a) Let R be the region bounded by the lines $y = x$, $y = 4 - x$ and $y = 0$ in the first quadrant. Compute the volume of the solid formed by revolving R about the line $x = -1$.

(b) Describe a solid of revolution whose volume is given by the definite integral

$$\int_{-2}^{4} 2\pi(y+3)(9-(y-1)^2)\,dy.$$

Answer.

Question 3 [18 marks] Evaluate the following integrals.

(a) $\displaystyle\int \sec^5(2x)\tan^3(2x)\,dx$

(b) $\displaystyle\int \frac{\ln(2x)}{x^2}\,dx$

(c) $\displaystyle\int_0^1 \frac{\sin x}{\sin(1-x)+\sin x}\,dx$ (Hint: $u = 1 - x$)

Answer.

Formulae Table

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int k\,dx = kx + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \tan x\,dx = \ln|\sec x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin 2x = 2\sin x \cos x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$$

$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$$

– End of Test –