NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2023-2024

BR2207 - QUANTITATIVE ANALYSIS

November 2023

Time Allowed: 3 hours

INSTRUCTIONS

1 This paper contains **ELEVEN (11)** questions and comprises **SEVEN (7)** pages.

2 Answer **ALL** questions.

3 A copy of *Statistical Tables and Selected Distributions* is provided separately.

4 This is a **closed-book** examination.

5 The number of marks allocated is shown at the end of each question.

6 Begin your answer to each question on a separate page of the answer book.

7 Answers will be graded for content and appropriate presentation.

Note: Exam Questions begin on Page 2

Question 1

You are shopping for four gifts for the upcoming Christmas gift exchanging party. A salesman in the store recommends six types of gifts suitable for Christmas, and you have decided to select four gifts out of the salesman's recommendation.

(a) If the store has unlimited number of gifts for each of the six types that he recommends, in how many different ways can you buy Christmas gifts?

(2 marks)

(b) If you want to buy four distinct gifts for the party, in how many different ways can you buy Christmas gifts?

(2 marks)

(c) In the party you will give one gift to each of your four friends:

i. If you have purchased four distinct Christmas gifts: *one cup, one toy, one pen, and one hat*, in how many different ways can you give gifts to your friends?

(2 marks)

ii. If you have purchased the following four Christmas gifts: *two identical cups, and two identical pens*, in how many different ways can you give gifts to your friends?

(2 marks)

(TOTAL: 8 marks)

Question 2

You are playing a game by rolling a six-sided die until you roll the number of 1. All rolls are independent. The winning rule is to have as many rolls as possible.

The outcome of your first play is: 4, 3, 3, 6, 1; i.e. the number of rolls is 5. You intend to play the game again.

(a) Write out the range of the number of rolls in your second play.

(2 marks)

(b) Calculate the probability of beating your first play.

(2 marks)

(c) Calculate the expected number of rolls in your second play.

(3 marks)

(TOTAL: 7 marks)

Question 3

You manage a portfolio of 100 projects. 50 projects are funded by Investor A; each project may receive a "stop funding (SF)" notice with a probability of 20%. 30 projects are funded by Investor B; each project may receive an SF notice with a probability of 60%. 20 projects are funded by Investor C, with a "no SF" guarantee from Investor C.

(a) Calculate the probability that a randomly selected project receives an SF notice.

(4 marks)

(b) If one project receives an SF notice, calculate the probability that it comes from Investor A.

(3 marks)

(TOTAL: 7 marks)

Question 4

You are investigating the distributions of two random variables: X and Y. Having done some experiments on X and Y separately, you have concluded the following marginal distributions of the two random variables:

X	-1	0	1
Probability	0.2	0.5	0.3

Y	-1	0	1
Probability	0.3	0.4	0.3

(a) You assume that X and Y are independent. Determine the joint probabilities of X and Y.

(2 marks)

(b) You decide to analyse the two random variables simultaneously, and you conclude the following joint probabilities of X and Y:

		X		
		-1	0	1
	-1	0	0.1	0.2
Y	0	0.1	0.3	0
	1	0.1	0.1	0.1

i. Determine the conditional probabilities of $X|Y$.

(5 marks)

ii. Calculate the covariance between X and Y, and interpret the result.

(6 marks)

(TOTAL: 13 marks)

Question 5

You are an actuary and have decided to use Poisson distribution to model the number of claims from fire loss insurance. You assume that the expected number of claims per month is 2 from Portfolio A, and 3 from Portfolio B. Fire loss claims from these two portfolios are independent.

(a) Calculate the probability that more than 2 claims from Portfolio A and more than 3 claims from Portfolio B occur in a single month.

(4 marks)

(b) Assuming the monthly number of claims are independent and identically distributed, determine the expectation and variance of the total number of claims from the two portfolios combined in a year.

(4 marks)

(TOTAL: 8 marks)

Question 6

You are modelling the distribution of medical expenses for a portfolio of hospital cover insurance. Let S denote medical expenses. You consider two distributions for medical expenses: Exponential or Lognormal, i.e.

$$\text{Exponential: } S \sim \text{Exp}(\tfrac{1}{\theta}),\ \Pr(S \le x) = 1 - e^{\frac{-x}{\theta}}$$

$$\text{Lognormal: } \ln(S) \sim N(\mu, \sigma^2)$$

From historical data, you find that the sample mean and sample variance of medical expenses are 10 and 100, respectively.

(a) Use the Method of Moments to estimate θ for the Exponential model.

(2 marks)

(b) Using the estimated parameter in (a), calculate the probability that medical expenses are higher than 30.

(2 marks)

(c) Use the Method of Moments to estimate μ and σ^2 for the Lognormal model.

Hint: $\text{E}(S) = e^{\mu + \frac{\sigma^2}{2}}$, and $\text{Var}(S) = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$

(4 marks)

(d) Using the estimated parameters in (c), calculate the probability that medical expenses are higher than 30.
Hint: $\Pr(S > x) = \Pr(\ln(S) > \ln(x))$

(4 marks)

(TOTAL: 12 marks)

Question 7

You are a risk analyst in a commercial bank, responsible for modelling default risk for a portfolio that consists of a large number of independent and identically distributed loans. The Credit Score of a loan is an important metric of default risk: If the average Credit Score of a portfolio is below 3, this portfolio is considered "Default". The Credit Scores of these loans in this portfolio are independent and identically distributed.

You assume that the mean Credit Score of this loan portfolio is 4, and that the variance of Credit Score is 25. An auditor wants to verify your assumption by randomly selecting 100 loans out of this portfolio to form a sub-portfolio.

(a) Use Central Limit Theorem (CLT) to find the approximate probability that the auditor's sub-portfolio is considered "Default".

(4 marks)

(b) The auditor suspects that your assumption on the mean Credit Score of the portfolio is too high. She observes that the average Credit Score of her sub-portfolio is 3. Construct a statistical test to determine if the auditor should reject your assumption (i.e. the mean Credit Score of this loan portfolio is 4) at the 5% level of significance.

(6 marks)

(TOTAL: 10 marks)

Question 8

You are assessing the quality of robots manufactured by a factory. Let θ denote the probability of being a faulty robot. You want to test the Null Hypothesis that $\theta = 0.1$, against the Alternative Hypothesis that $\theta = 0.3$.

You will randomly select 10 robots (assuming all robots are i.i.d.). Let X denote the number of faulty robots. You will accept the Null Hypothesis if $X \leq 2$; otherwise, you will reject it.

(a) Determine Type I error and Type II error.

(8 marks)

(b) Describe how the two types of error would change if you modify the acceptance region into $X \leq 1$ and keep the sample size fixed.

(2 marks)

(TOTAL: 10 marks)

Question 9

You are exploring to invest into a fund. Your risk tolerance is 30% per year, meaning that the standard deviation of annual returns should not exceed 30% (i.e. variance should not exceed 0.09). You have observed the annual returns of this fund for the past 10 years (i.e. sample size is 10). The calculated sample standard deviation is 40%.

Assuming that annual returns are i.i.d. samples from a Normal distribution, use a statistical test at the 5% level of significance to determine if this fund's performance is within your risk tolerance.

(TOTAL: 8 marks)

Question 10

You are investigating the relationship between extreme temperature and population mortality rate in Country A. You decide to use the following simple regression model:

$$y = \alpha + \beta x + \epsilon,$$

where x is an extreme temperature indicator (representing the proportion of days in a year that have extreme temperatures), and y denotes the annual mortality rate of the population in Country A.

You have observed the following information for the past 7 years:

i	x_i	y_i
1	7.80%	1.29%
2	9.10%	1.42%
3	7.69%	1.26%
4	1.07%	1.08%
5	7.44%	1.30%
6	6.21%	1.22%
7	8.94%	1.38%

(a) Find the fitted values of the two parameters in the simple regression.

(7 marks)

(b) Calculate the coefficient of determination. Interpret your result.

(4 marks)

(TOTAL: 11 marks)

Question 11

You are investigating the distribution of a random variable X. You are given that the moment generating function of X is:

$$M_X(t) = \frac{e^t + 1}{2}, \quad \text{for } -\infty < t < +\infty$$

(a) Show that X's k^{th} moment about the origin is a constant for any k: i.e.

$$\mu'_k = \frac{1}{2}$$

(2 marks)

(b) Use the above result in (a) to determine the mean, variance, and skewness of X.
Hint: $(a-b)^n = \sum_{k=0}^{n}\left[\binom{n}{k}a^{n-k}(-b)^k\right]$

(4 marks)

(TOTAL: 6 marks)

- END OF PAPER -

BR2207 QUANTITATIVE ANALYSIS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.