NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2023-2024

MH4701 - Mathematical Programming

December 2023 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **CLOSED BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(25 marks)**

The steepest descent method is applied to minimize the function

$$f : \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \to 2(2+x-y)^2 + 3(2+x+y)^2.$$

Prove that if the solutions $\begin{bmatrix} x_k \\ y_k \end{bmatrix}$ and $\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}$ are generated in two consecutive iterations, then

$$f\left(\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}\right) = f\left(\begin{bmatrix} x_k \\ y_k \end{bmatrix}\right) - \frac{(4(2+x_k-y_k)^2 + 9(2+x_k+y_k)^2)^2}{8(2+x_k-y_k)^2 + 27(2+x_k+y_k)^2} \le \frac{1}{25} f\left(\begin{bmatrix} x_k \\ y_k \end{bmatrix}\right).$$

QUESTION 2. **(25 marks)**

Prove that when the Newton descent method is applied to the function

$$f : x \in \mathbb{R}^2 \mapsto \log(1 + x_1^2 + 2x_2^2),$$

it generates the next iterate if and only if the initial iterate $\bar{x}$ satisfies

$$0 < \bar{x}_1^2 + 2\bar{x}_2^2 < 1.$$

QUESTION 3. **(25 marks)**

Use the first- and second-order necessary/sufficient conditions for a local minimum to find all local minimum solutions of

$$f : x \in \mathbb{R}^2 \mapsto 3x_1^2 + 2x_2^2 - 2x_1x_2 + 3x_1 + 4x_2$$

subject to $h = 0$, where

$$h : x \in \mathbb{R}^2 \mapsto 3x_1^2 + 4x_2^2 - 4x_1x_2 + 2x_1 + 4x_2 - 1.$$

QUESTION 4. **(25 marks)**

Consider the convex program

$$\begin{array}{rrcrcrcrcl} \text{minimize} & x_1 & + & 3x_2 & + & 4x_3 & + & x_4 & & \\ \text{subject to} & 2x_1 & + & 6x_2 & + & 6x_3 & + & 5x_4 & = & 8, \\ & x_1 & + & 3x_2 & + & 6x_3 & + & 5x_4 & = & 6, \\ & & & & & & x_1, x_2, & x_3, x_4 & \geq & 0. \end{array}$$

Prove that the central path of the above convex program exists and is convergent as the barrier parameter converges to 0, and find its limit.

END OF PAPER