

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2023-2024

MH1811 MATHEMATICS 2

December 2023

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)
5. Candidates may use calculators. However, they should lay out systematically the various steps in the workings.

QUESTION 1.**(18 Marks)**

- (a) Suppose z is a function of two independent variables x and y implicitly defined by the equation

$$2xe^z = 3y^2z + xy + 1.$$

Find the value of $\frac{\partial z}{\partial y}$ when $x = 1$, $y = 1$ and $z = 0$.

- (b) Find the equation of the normal line (in the form of $y = mx + c$) to the curve $x^2 + 5y^2 = 6$ at the point $(1, 1)$.
- (c) Find all the point(s) on the surface $x^2 + y^2 - 2z^2 = 72$ at which the tangent plane is parallel to the plane $2x + y + z = 4$.

QUESTION 2.**(18 Marks)**

- (a) Suppose that $z = f(x, y)$, where $x = 1+2h$ and $y = 1+3h$. Using Chain Rule, express $\frac{dz}{dh}$ and $\frac{d^2z}{dh^2}$ in terms of the partial derivatives (including higher order if necessary) of $f(x, y)$.
- (b) Suppose that when producing x , y and z units of three different products respectively, the profit of the company is given by the function $P(x, y, z) = 3xz + 6y$. Find the maximum profit for the company subject to the manufacturing constraint $x^2 + 2y^2 + z^2 = 6$.

QUESTION 3.**(15 Marks)**

- (a) Evaluate the double integral $\iint_D (x + 2y) dA$, where D is the region bounded by the lines $y = 0$, $x = 1$ and the curve $y = \sqrt{x}$ in the first quadrant.
- (b) By changing the order of integration, evaluate the following iterated integral:

$$\int_0^1 \int_{-2\sqrt{y}}^{2\sqrt{y}} 12 \cos(12x - x^3) dx dy.$$

QUESTION 4.**(15 Marks)**

Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\sum_{n=1}^{\infty} \cos\left(\frac{n+1}{2n+3}\right)$

(b) $\sum_{n=2}^{\infty} \frac{2^n + n}{n^2 + 3^n}$

(c) $\sum_{n=2}^{\infty} (\ln n) \sin\left(\frac{1}{n^2}\right)$

QUESTION 5.**(16 Marks)**

- (a) Find the interval of convergence of the following power series and, for every $x \in \mathbb{R}$, determine whether the series converges absolutely, converges conditionally or diverges at x . Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt{n^2+1}}$$

- (b) Let $f(x) = \frac{\sin(x)}{(1+x)^2}$. Using Maclaurin series, find the value of $f^{(5)}(0)$.

QUESTION 6.**(18 Marks)**

Find the general solutions of the following ordinary differential equations:

(a)

$$y' - \frac{3y}{x} = -\cos(x^4)y^2.$$

(b)

$$y'' - 2y' = x + \sin x.$$

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Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.