NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2022-2023

MH4110 – Partial Differential Equations

April 2023 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 **(20 marks)**

Consider the following partial differential equation

$$u_x - x u_y = u.$$

(a) Determine the general solution of this equation.

(b) Use the initial condition $u(0, y) = 2y$ to determine the particular solution of the equation.

QUESTION 2 **(20 marks)**

Find the solution $u = u(x, y)$ of the partial differential equation

$$u_{xx} - 3u_{xy} + 2u_{yy} = x + 2y.$$

QUESTION 3 **(15 marks)**

Consider the diffusion equation with initial condition and boundary conditions

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + f(x, t), \quad u(x, 0) = g(x), \quad u(0, t) = u(L, t) = 0,$$

where k is a positive constant, $f(x, t)$ and $g(x)$ are given functions. Prove that the solution $u(x, t)$ to this equation is unique on the interval $[0,\ L]$.

QUESTION 4 **(20 marks)**

Let $f(x)$ be a function defined on the interval $[-\pi, \pi]$

$$f(x) = \begin{cases} x + \pi, & -\pi < x < 0 \\ \pi - x, & 0 \leq x < \pi. \end{cases}$$

Question 4 continues on page 3.

(a) Find the Fourier series of $f(x)$ on the interval $[-\pi, \pi]$.

(b) Show that

$$\sum_{n=0}^{\infty} \frac{8}{(2n+1)^2} = \pi^2.$$

QUESTION 5 **(25 marks)**

Consider the initial boundary value problem on the interval [0, 1]

$$\begin{cases} u_t - u_{xx} + 2u_x = 0, & 0 < x < 1, \ t > 0, \\ u(0,t) = u(1,t) = 0, & t > 0, \\ u(x,0) = e^x \sin(2\pi x), & 0 < x < 1. \end{cases}$$

(a) Find the solution $u(t, x)$ to the problem.

(b) Find the maximum and minimum values of the solution. Does it violate the maximum principle? Also discuss the behavior of the solution as $t \to +\infty$.

END OF PAPER

MH4110 PARTIAL DIFFERENTIAL EQUATIONS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.