# NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2014-2015

**MH1101 – Calculus II**

April 2015 TIME ALLOWED: 2 HOURS

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INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **Five (5)** questions and comprises **FOUR (4)** printed pages.
2. Answer all questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT an OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

**Question 1.** (20 marks)

Evaluate the following definite integrals.

(a) $\displaystyle\int_0^1 (2x^2+1)e^{x^2}\,dx$

(b) $\displaystyle\int_1^2 \frac{(\ln x)^2}{x^3}\,dx$

**Question 2.** (20 marks)

(a) Find the volume obtained by rotating the region bounded by the curves $y = \sin^2 x$, $y = 0$, $0 \leq x \leq \pi$ about the $x$-axis.

(b) Find the length of the curve $y = \displaystyle\int_1^x \sqrt{t^3-1}\,dt$ where $1 \leq x \leq 4$.

**Question 3.** (20 marks)

Determine whether the following series converge or diverge. Justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$

**Question 4.** (20 marks)

(a) Find a power series representation of the function $f(x) = \dfrac{1+x}{1-x}$, and determine its interval of convergence.

(b) Use series to approximate the definite integral $\displaystyle\int_0^1 x\cos(x^3)\,dx$ to within three decimal places.

**Question 5.** (20 marks)

Consider the function $f(x) = \begin{cases} \frac{e^x - 1}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$

(i) Use the Taylor series for $f$ and $f'$ about 0 to evaluate $f'(2)$.

(ii) Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1)!}$.

**END OF PAPER**

# MH1101 CALCULUS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.