NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

BR3213 Valuation and Risk Models

November 2022 Time Allowed: 2 ½ Hours

INSTRUCTIONS

1 This paper contains **SIX(6)** questions and comprises **SIX(6)** pages and **ONE(1)** appendix of **FOUR(4)** PAGES.

2 Answer **ALL SIX(6)** questions.

3 This is a **Closed-book** examination.

4 The number of marks allocated is shown at the end of each question.

5 Begin your answer to each question on a separate page of the answer book.

6 Answers will be graded for content and appropriate presentation.

Note: Exam Questions begin on Page 2

Question 1

(a) Write down and briefly explain the four properties that needs to be satisfied for a coherent risk measure.

(8 marks)

(b) The return of a one year project has the following probability distribution:

$$X = \begin{cases} -10, & with\ probability\ 0.02 \\ -5, & with\ probability\ 0.13 \\ 6, & with\ probability\ 0.64 \\ 15, & with\ probability\ 0.21 \end{cases}$$

i) Calculate the standard deviation of X

(2 marks)

ii) Calculate the one year Value at Risk (VaR) and the expected shortfall (ES) given that the return is below VaR, when the confidence level is 95%.

(2 marks)

iii) Suppose that there are two independent identical investments X in the portfolio. Calculate the one year VaR and the ES given that the return is below VaR, when the confidence level is 95%.

(5 marks)

iv) Use your results from part (b)(iii), state whether VaR and ES satisfy the coherent risk measure requirements.

(2 marks)

(TOTAL: 19 marks)

Question 2

Within the mean-variance portfolio theory framework,

(a) Draw a graphical representation of the risky-asset only opportunity set in $\mu - \sigma$ space, where μ represents the portfolio mean return and σ represents the portfolio return standard deviation.

(1 mark)

(b) What does efficient portfolio mean?

(1 mark)

(c) Briefly describe and explain the formation of the new efficient frontier when risk-free asset is introduced.

(2 marks)

(TOTAL: 4 marks)

Question 3

Suppose a bank has a loan portfolio that includes n identical loans. Each loan has a total amount of L being borrowed, with the same probability of default p and the recovery rate R in the event of default. The default correlation between each pair of loans is constant as ρ.

Write down the expected loss and standard deviation of losses of this portfolio.

(4 marks)

Question 4

(a) You are given the following information on the spot rates for the next 3 years:

Maturity	Spot rates p.a.	Discount factor	1-year Forward rate
1	3.60%		N.A.
2	3.80%		
3	3.90%		

Calculate the discount factors for the maturity times 1,2 and 3. Calculate the 1-year forward rates for investment between time 1 and 2, and investment between time 2 and 3.

(10 marks)

Note: Question No.4 continues on page 4

Question 4 (continued)

(b) A bond pays annual coupon at a rate of 5% has exactly 2 years to maturity. Given that the par value of the bond is \$1000,

i) Calculate the price of the bond.

(2 marks)

ii) Calculate the carry-roll-down of this bond at time 1 assuming that the forward rates are realized.

(3 marks)

iii) Calculate the yield to maturity of the bond.

(3 marks)

(c) A bond that matures at time 3 pays annual coupons. Let p be the par rate for this bond. Derive a formula for p using the discount factors provided in the table above. Hence or otherwise calculate p for this bond.

(4 marks)

(TOTAL: 22 marks)

Question 5

A stock is currently priced at \$30. After one period at time 1, the stock price either increases by 20% with a probability of 0.6, or decreases by 15%. The risk-free continuously compounding interest rate is 4%.

Consider a one period European call option on this stock with strike price of $K = 32$.

(a) Write down the payoffs of the call option at time 1. Calculate the expected payoff.

(4 marks)

(b) Use the binomial tree method, construct a replicating portfolio that replicates the payoff of the option. Clearly indicate the units of stocks and cash to be held in this replicating portfolio.

(4 marks)

Note: Question No.5 continues on page 5

Question 5 (Continued)

(c) Hence or otherwise, calculate the price of this option at time 0.

(2 marks)

(d) At time 0, how much is the option's intrinsic value and how much is the option's time value?

(2 marks)

Now consider a two period American put option that can be exercised at time 1 or time 2 on this stock with strike price of $K = 32$.

(e) Use the binomial tree method, decided whether it is better for the investor to exercise the option at time 1 and under what condition.

(10 marks)

(f) Hence or otherwise, calculate the price of this American put option at time 0.

(4 marks)

(TOTAL: 26 marks)

Question 6

(a) The underlying asset of a European call option is currently selling at \$53 with stock price volatility estimated at 20% per annum. The call option has a strike price of \$55 and matures in three months. Use the Black-Scholes formula to calculate the option price given that the continuously compounding risk free-rate is 3%.

(8 marks)

(b) Given the same underlying asset, strike price and time to maturity, briefly explain the differences between an American call option and European call option. In addition, discuss why the American call option should have the same value as the European call option if the underlying asset does not pay any dividends.

(5 marks)

Note: Question No.6 continues on page 6

Question 6 (Continued)

(c) Write down the definition of the five Greeks, including Delta, Theta, Vega, Rho and Lambda, for analyzing the option values. For an European call option, briefly indicate and explain the sign (positive or negative) of each Greeks.

(10 marks)

(d) What distribution is assumed for the stock price under the Black-Scholes model?

(2 marks)

(TOTAL: 25 marks)

– END OF PAPER –

APPENDIX 1: SELECTED FORMULAE

BINOMIAL PRICING ("TREE") MODEL

Risk-neutral probabilities

Up-step probability $= \dfrac{e^{r\Delta t} - d}{u - d}$,

where $u \approx e^{\sigma\sqrt{\Delta t} + q\Delta t}$

and $d \approx e^{-\sigma\sqrt{\Delta t} + q\Delta t}$.

BLACK-SCHOLES FORMULAE FOR EUROPEAN OPTIONS

Geometric Brownian motion model for a stock price S_t

$$dS_t = S_t(\mu dt + \sigma dz)$$

Black-Scholes partial differential equation

$$\frac{\partial f}{\partial t} + (r-q)S_t\frac{\partial f}{\partial S_t} + \tfrac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 f}{\partial S_t^2} = rf$$

Garman-Kohlhagen formulae for the price of call and put options

Call: $c_t = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$

Put: $p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t e^{-q(T-t)}\Phi(-d_1)$

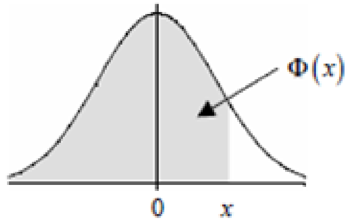
where $d_1 = \dfrac{\log(S_t/K) + (r - q + ½\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$

and $d_2 = \dfrac{\log(S_t/K) + (r - q - ½\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$

Probabilities for the Standard Normal distribution

The distribution function is denoted by $\Phi(x)$, and the probability density function is denoted by $\phi(x)$.

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$$



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.50000	**0.40**	0.65542	**0.80**	0.78814	**1.20**	0.88493	**1.60**	0.94520
0.01	0.50399	**0.41**	0.65910	**0.81**	0.79103	**1.21**	0.88686	**1.61**	0.94630
0.02	0.50798	**0.42**	0.66276	**0.82**	0.79389	**1.22**	0.88877	**1.62**	0.94738
0.03	0.51197	**0.43**	0.66640	**0.83**	0.79673	**1.23**	0.89065	**1.63**	0.94845
0.04	0.51595	**0.44**	0.67003	**0.84**	0.79955	**1.24**	0.89251	**1.64**	0.94950
0.05	0.51994	**0.45**	0.67364	**0.85**	0.80234	**1.25**	0.89435	**1.65**	0.95053
0.06	0.52392	**0.46**	0.67724	**0.86**	0.80511	**1.26**	0.89617	**1.66**	0.95154
0.07	0.52790	**0.47**	0.68082	**0.87**	0.80785	**1.27**	0.89796	**1.67**	0.95254
0.08	0.53188	**0.48**	0.68439	**0.88**	0.81057	**1.28**	0.89973	**1.68**	0.95352
0.09	0.53586	**0.49**	0.68793	**0.89**	0.81327	**1.29**	0.90147	**1.69**	0.95449
0.10	0.53983	**0.50**	0.69146	**0.90**	0.81594	**1.30**	0.90320	**1.70**	0.95543
0.11	0.54380	**0.51**	0.69497	**0.91**	0.81859	**1.31**	0.90490	**1.71**	0.95637
0.12	0.54776	**0.52**	0.69847	**0.92**	0.82121	**1.32**	0.90658	**1.72**	0.95728
0.13	0.55172	**0.53**	0.70194	**0.93**	0.82381	**1.33**	0.90824	**1.73**	0.95818
0.14	0.55567	**0.54**	0.70540	**0.94**	0.82639	**1.34**	0.90988	**1.74**	0.95907
0.15	0.55962	**0.55**	0.70884	**0.95**	0.82894	**1.35**	0.91149	**1.75**	0.95994
0.16	0.56356	**0.56**	0.71226	**0.96**	0.83147	**1.36**	0.91309	**1.76**	0.96080
0.17	0.56749	**0.57**	0.71566	**0.97**	0.83398	**1.37**	0.91466	**1.77**	0.96164
0.18	0.57142	**0.58**	0.71904	**0.98**	0.83646	**1.38**	0.91621	**1.78**	0.96246
0.19	0.57535	**0.59**	0.72240	**0.99**	0.83891	**1.39**	0.91774	**1.79**	0.96327
0.20	0.57926	**0.60**	0.72575	**1.00**	0.84134	**1.40**	0.91924	**1.80**	0.96407
0.21	0.58317	**0.61**	0.72907	**1.01**	0.84375	**1.41**	0.92073	**1.81**	0.96485
0.22	0.58706	**0.62**	0.73237	**1.02**	0.84614	**1.42**	0.92220	**1.82**	0.96562
0.23	0.59095	**0.63**	0.73565	**1.03**	0.84849	**1.43**	0.92364	**1.83**	0.96638
0.24	0.59483	**0.64**	0.73891	**1.04**	0.85083	**1.44**	0.92507	**1.84**	0.96712
0.25	0.59871	**0.65**	0.74215	**1.05**	0.85314	**1.45**	0.92647	**1.85**	0.96784
0.26	0.60257	**0.66**	0.74537	**1.06**	0.85543	**1.46**	0.92785	**1.86**	0.96856
0.27	0.60642	**0.67**	0.74857	**1.07**	0.85769	**1.47**	0.92922	**1.87**	0.96926
0.28	0.61026	**0.68**	0.75175	**1.08**	0.85993	**1.48**	0.93056	**1.88**	0.96995
0.29	0.61409	**0.69**	0.75490	**1.09**	0.86214	**1.49**	0.93189	**1.89**	0.97062
0.30	0.61791	**0.70**	0.75804	**1.10**	0.86433	**1.50**	0.93319	**1.90**	0.97128
0.31	0.62172	**0.71**	0.76115	**1.11**	0.86650	**1.51**	0.93448	**1.91**	0.97193
0.32	0.62552	**0.72**	0.76424	**1.12**	0.86864	**1.52**	0.93574	**1.92**	0.97257
0.33	0.62930	**0.73**	0.76730	**1.13**	0.87076	**1.53**	0.93699	**1.93**	0.97320
0.34	0.63307	**0.74**	0.77035	**1.14**	0.87286	**1.54**	0.93822	**1.94**	0.97381
0.35	0.63683	**0.75**	0.77337	**1.15**	0.87493	**1.55**	0.93943	**1.95**	0.97441
0.36	0.64058	**0.76**	0.77637	**1.16**	0.87698	**1.56**	0.94062	**1.96**	0.97500
0.37	0.64431	**0.77**	0.77935	**1.17**	0.87900	**1.57**	0.94179	**1.97**	0.97558
0.38	0.64803	**0.78**	0.78230	**1.18**	0.88100	**1.58**	0.94295	**1.98**	0.97615
0.39	0.65173	**0.79**	0.78524	**1.19**	0.88298	**1.59**	0.94408	**1.99**	0.97670
0.40	0.65542	**0.80**	0.78814	**1.20**	0.88493	**1.60**	0.94520	**2.00**	0.97725

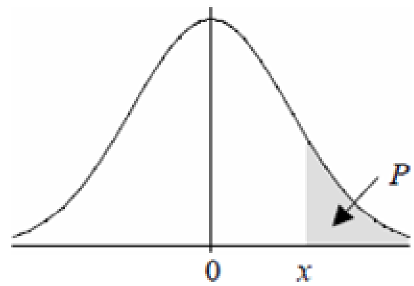
Probabilities for the Standard Normal distribution

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2.00	0.97725	**2.40**	0.99180	**2.80**	0.99744	**3.20**	0.99931	**3.60**	0.99984	**4.00**	0.99997
2.01	0.97778	**2.41**	0.99202	**2.81**	0.99752	**3.21**	0.99934	**3.61**	0.99985	**4.01**	0.99997
2.02	0.97831	**2.42**	0.99224	**2.82**	0.99760	**3.22**	0.99936	**3.62**	0.99985	**4.02**	0.99997
2.03	0.97882	**2.43**	0.99245	**2.83**	0.99767	**3.23**	0.99938	**3.63**	0.99986	**4.03**	0.99997
2.04	0.97932	**2.44**	0.99266	**2.84**	0.99774	**3.24**	0.99940	**3.64**	0.99986	**4.04**	0.99997
2.05	0.97982	**2.45**	0.99286	**2.85**	0.99781	**3.25**	0.99942	**3.65**	0.99987	**4.05**	0.99997
2.06	0.98030	**2.46**	0.99305	**2.86**	0.99788	**3.26**	0.99944	**3.66**	0.99987	**4.06**	0.99998
2.07	0.98077	**2.47**	0.99324	**2.87**	0.99795	**3.27**	0.99946	**3.67**	0.99988	**4.07**	0.99998
2.08	0.98124	**2.48**	0.99343	**2.88**	0.99801	**3.28**	0.99948	**3.68**	0.99988	**4.08**	0.99998
2.09	0.98169	**2.49**	0.99361	**2.89**	0.99807	**3.29**	0.99950	**3.69**	0.99989	**4.09**	0.99998
2.10	0.98214	**2.50**	0.99379	**2.90**	0.99813	**3.30**	0.99952	**3.70**	0.99989	**4.10**	0.99998
2.11	0.98257	**2.51**	0.99396	**2.91**	0.99819	**3.31**	0.99953	**3.71**	0.99990	**4.11**	0.99998
2.12	0.98300	**2.52**	0.99413	**2.92**	0.99825	**3.32**	0.99955	**3.72**	0.99990	**4.12**	0.99998
2.13	0.98341	**2.53**	0.99430	**2.93**	0.99831	**3.33**	0.99957	**3.73**	0.99990	**4.13**	0.99998
2.14	0.98382	**2.54**	0.99446	**2.94**	0.99836	**3.34**	0.99958	**3.74**	0.99991	**4.14**	0.99998
2.15	0.98422	**2.55**	0.99461	**2.95**	0.99841	**3.35**	0.99960	**3.75**	0.99991	**4.15**	0.99998
2.16	0.98461	**2.56**	0.99477	**2.96**	0.99846	**3.36**	0.99961	**3.76**	0.99992	**4.16**	0.99998
2.17	0.98500	**2.57**	0.99492	**2.97**	0.99851	**3.37**	0.99962	**3.77**	0.99992	**4.17**	0.99998
2.18	0.98537	**2.58**	0.99506	**2.98**	0.99856	**3.38**	0.99964	**3.78**	0.99992	**4.18**	0.99999
2.19	0.98574	**2.59**	0.99520	**2.99**	0.99861	**3.39**	0.99965	**3.79**	0.99992	**4.19**	0.99999
2.20	0.98610	**2.60**	0.99534	**3.00**	0.99865	**3.40**	0.99966	**3.80**	0.99993	**4.20**	0.99999
2.21	0.98645	**2.61**	0.99547	**3.01**	0.99869	**3.41**	0.99968	**3.81**	0.99993	**4.21**	0.99999
2.22	0.98679	**2.62**	0.99560	**3.02**	0.99874	**3.42**	0.99969	**3.82**	0.99993	**4.22**	0.99999
2.23	0.98713	**2.63**	0.99573	**3.03**	0.99878	**3.43**	0.99970	**3.83**	0.99994	**4.23**	0.99999
2.24	0.98745	**2.64**	0.99585	**3.04**	0.99882	**3.44**	0.99971	**3.84**	0.99994	**4.24**	0.99999
2.25	0.98778	**2.65**	0.99598	**3.05**	0.99886	**3.45**	0.99972	**3.85**	0.99994	**4.25**	0.99999
2.26	0.98809	**2.66**	0.99609	**3.06**	0.99889	**3.46**	0.99973	**3.86**	0.99994	**4.26**	0.99999
2.27	0.98840	**2.67**	0.99621	**3.07**	0.99893	**3.47**	0.99974	**3.87**	0.99995	**4.27**	0.99999
2.28	0.98870	**2.68**	0.99632	**3.08**	0.99896	**3.48**	0.99975	**3.88**	0.99995	**4.28**	0.99999
2.29	0.98899	**2.69**	0.99643	**3.09**	0.99900	**3.49**	0.99976	**3.89**	0.99995	**4.29**	0.99999
2.30	0.98928	**2.70**	0.99653	**3.10**	0.99903	**3.50**	0.99977	**3.90**	0.99995	**4.30**	0.99999
2.31	0.98956	**2.71**	0.99664	**3.11**	0.99906	**3.51**	0.99978	**3.91**	0.99995	**4.31**	0.99999
2.32	0.98983	**2.72**	0.99674	**3.12**	0.99910	**3.52**	0.99978	**3.92**	0.99996	**4.32**	0.99999
2.33	0.99010	**2.73**	0.99683	**3.13**	0.99913	**3.53**	0.99979	**3.93**	0.99996	**4.33**	0.99999
2.34	0.99036	**2.74**	0.99693	**3.14**	0.99916	**3.54**	0.99980	**3.94**	0.99996	**4.34**	0.99999
2.35	0.99061	**2.75**	0.99702	**3.15**	0.99918	**3.55**	0.99981	**3.95**	0.99996	**4.35**	0.99999
2.36	0.99086	**2.76**	0.99711	**3.16**	0.99921	**3.56**	0.99981	**3.96**	0.99996	**4.36**	0.99999
2.37	0.99111	**2.77**	0.99720	**3.17**	0.99924	**3.57**	0.99982	**3.97**	0.99996	**4.37**	0.99999
2.38	0.99134	**2.78**	0.99728	**3.18**	0.99926	**3.58**	0.99983	**3.98**	0.99997	**4.38**	0.99999
2.39	0.99158	**2.79**	0.99736	**3.19**	0.99929	**3.59**	0.99983	**3.99**	0.99997	**4.39**	0.99999
2.40	0.99180	**2.80**	0.99744	**3.20**	0.99931	**3.60**	0.99984	**4.00**	0.99997	**4.40**	0.99999

Percentage Points for the Standard Normal distribution

The table gives percentage points x defined by the equation

$$P = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{1}{2}t^2} \, dt$$



P	x	**P**	x	**P**	x	**P**	x	**P**	x	**P**	x
50%	0.0000	**5.0%**	1.6449	**3.0%**	1.8808	**2.0%**	2.0537	**1.0%**	2.3263	**0.10%**	3.0902
45%	0.1257	**4.8%**	1.6646	**2.9%**	1.8957	**1.9%**	2.0749	**0.9%**	2.3656	**0.09%**	3.1214
40%	0.2533	**4.6%**	1.6849	**2.8%**	1.9110	**1.8%**	2.0969	**0.8%**	2.4089	**0.08%**	3.1559
35%	0.3853	**4.4%**	1.7060	**2.7%**	1.9268	**1.7%**	2.1201	**0.7%**	2.4573	**0.07%**	3.1947
30%	0.5244	**4.2%**	1.7279	**2.6%**	1.9431	**1.6%**	2.1444	**0.6%**	2.5121	**0.06%**	3.2389
25%	0.6745	**4.0%**	1.7507	**2.5%**	1.9600	**1.5%**	2.1701	**0.5%**	2.5758	**0.05%**	3.2905
20%	0.8416	**3.8%**	1.7744	**2.4%**	1.9774	**1.4%**	2.1973	**0.4%**	2.6521	**0.01%**	3.7190
15%	1.0364	**3.6%**	1.7991	**2.3%**	1.9954	**1.3%**	2.2262	**0.3%**	2.7478	**0.005%**	3.8906
10%	1.2816	**3.4%**	1.8250	**2.2%**	2.0141	**1.2%**	2.2571	**0.2%**	2.8782	**0.001%**	4.2649
5%	1.6449	**3.2%**	1.8522	**2.1%**	2.0335	**1.1%**	2.2904	**0.1%**	3.0902	**0.0005%**	4.4172

-END OF APPENDIX 1-