

NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER II EXAMINATION 2024–2025
MH1301 – DISCRETE MATHEMATICS

Apr/May 2025

TIME ALLOWED: 2 HOURS

Matriculation Number:

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Seat Number:

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INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **ELEVEN (11)** pages.
 2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
 3. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).
 4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
 5. All your solutions should be written in this booklet within the space provided after each question. However, if you write your solutions on other pages, please indicate them clearly.
 6. This examination paper is **NOT ALLOWED** to be removed from the examination hall.
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For examiners only

Question	Marks
1 (20)	
2 (15)	
3 (15)	

Question	Marks
4 (15)	
5 (20)	
6 (15)	

TOTAL (100)

QUESTION 1.

- (a) A bakery has 12 loaves of bread, including 2 identical sourdough loaves and 10 identical rye loaves. The bakery wants to arrange 5 loaves in a single row on a display shelf, where the order of placement matters.

How many different ways can the bakery arrange these 5 loaves on the shelf?

- (b) How many positive integers strictly greater than 299,999 and strictly less than 400,000 have the sum of their digits equal to 20?

- (c) For positive integer $n \geq 3$, define the following sum

$$S(n) := \sum_{k=2}^{n-1} C(n-1, k) \sum_{\ell=1}^{k-1} C(k-1, \ell).$$

Describe what $S(n)$ counts in terms of the distribution of distinguishable or indistinguishable objects into distinguishable or indistinguishable boxes, specifying whether the boxes must be nonempty. Provide a combinatorial proof.

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(15 Marks)**

QUESTION 2.

Given a sequence of 20 positive integers x_1, x_2, \dots, x_{20} whose sum satisfies

$$x_1 + x_2 + \cdots + x_{20} \leq 39,$$

prove that there exist consecutive terms whose sum is 5. In other words, show that there exist positive integers i and j with $1 \leq i \leq j \leq 20$.

$$x_i + x_{i+1} + \cdots + x_j = 5.$$

QUESTION 3.

Let a_n be the number of ternary strings of length n that contain either two consecutive 0's or two consecutive 1's.

- (a) Write down the value of a_1 , a_2 , and a_3 .
- (b) Find a recurrence relation for a_n . You need not solve it.

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(15 Marks)**

QUESTION 4.

- (a) Find all solutions of the recurrence relation

$$a_n = 4a_{n-1} - 3a_{n-2} + 2 + 3^n \quad \text{for } n \geq 2.$$

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- (b) Find the solution of the recurrence relation in Part (a) with initial values $a_0 = 4$ and $a_1 = \frac{19}{2}$.

QUESTION 5.

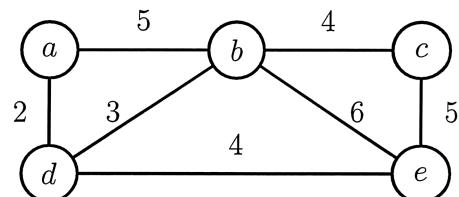
Determine whether each of the statements below is true or false. Give an explanation if it is true or provide a counterexample if it is false.

- (a) Suppose G is a connected simple graph with four vertices. Then G has an Eulerian Path.

(b) There does not exist an 8-regular simple graph that is planar.

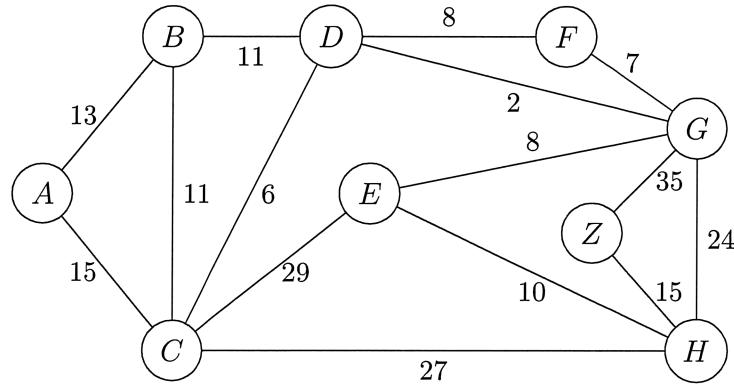
- (c) A connected graph with n vertices and $n - 1$ edges cannot contain any cycles.

- (d) In the weighted graph below, edge $b - d$ is safe.



QUESTION 6.

The following is an undirected weighted graph.

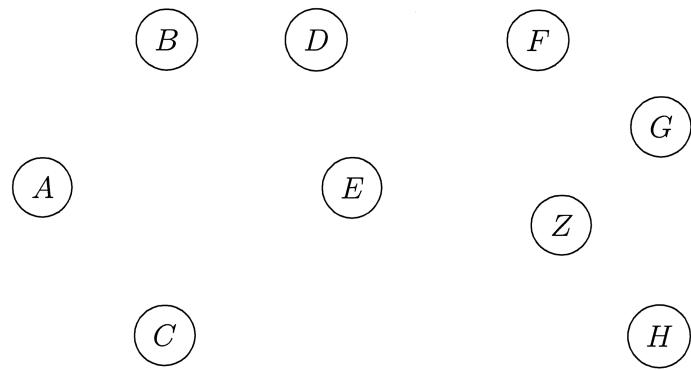


- (a) Find the minimum spanning tree for the graph using Prim's algorithm starting with vertex A. List the order in which the edges are added, and compute the total weight of the tree.

- (b)(i) By using Dijkstra's algorithm to compute the shortest path between A and Z , fill in the table below illustrating how the values of $d[v]$ are updated.

$d[A]$	$d[B]$	$d[C]$	$d[D]$	$d[E]$	$d[F]$	$d[G]$	$d[H]$	$d[Z]$	S
0	∞	\emptyset							
0									
0									
0									
0									
0									
0									
0									
0									

- (b)(ii) Complete the following spanning tree giving the minimum distances from vertex A to the other vertices.



END OF PAPER

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Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
- 2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.**
- 3. Please write your Matriculation Number on the front of the answer book.**
- 4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.**