NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2023-2024

MH1802 – Calculus for the Sciences

Nov/Dec 2023   Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **SIX (6)** pages.

2. Answer **ALL FIVE (5)** questions.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).

5. This examination paper is **NOT ALLOWED** to be removed from the examination hall.

1. **(25 marks)** Short questions.

   (a) Find the range of:
   $$y = \ln x$$

   (2 marks)

   (b) Find dy/dx for the following:
   $$y = \sin\left(\sin^{-1}\left(\sqrt{xe^x + x^2}\right)\right)$$

   (4 marks)

   (c) Find the limit of:
   $$\lim_{x \to \infty} x^{100} e^{-x}$$

   (2 marks)

   (d) Find the slope of the normal line to the curve (up to two decimal places):
   $$y = e^{x^2 + x}$$

   at $x = 1$.

   (3 marks)

   (e) Find the $\frac{dy}{dx}$ of:
   $$\sin\left(y^2\right) e^y + xy = \cos(x + 1)$$

   (3 marks)

   (f) Calculate the second derivative y'' of:
   $$y = \cosh\left(x^2 + 1\right)$$

   (4 marks)

   (g) Calculate the definite integral of:
   $$\int_1^2 \frac{\left(1 + \ln x + (\ln x)^3\right)^2}{x} dx$$

   (3 marks)

   (h) Find the antiderivative of:
   $$\frac{\tan^3\left(\tan x\right)}{\cos^2(x)}$$

   (4 marks)

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2. **(25 marks)** Suppose that the velocity of a body moving along a coordinate line at time t is $v(t) = \cos 5t + 2 \sin 5t$.

(a) Give the expression of the displacement s as a function of time, assuming at $t = 0$ the displacement is zero. Note that the time derivative of the displacement of the body is its velocity.

(3 marks)

(b) Give the expression of the acceleration of the body as a function of time.

(3 marks)

(c) The rate of change of the acceleration is called the jerk. Show that the jerk of the body is proportional to its velocity.
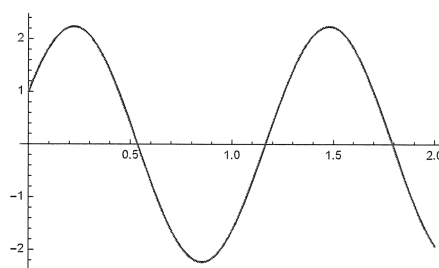
(5 marks)

(d) Give the maximum and minimum of the displacement of the body in the interval $0 \leq t \leq 1$.

(7 marks)

(e) Below is a plot of $v(t)$, with the x-axis being t and the y-axis being v. Give the total distance travelled by the body between $t = 0$ and $t = 1.5$. Note the total distance is different from its displacement.

(7 marks)

3. **(20 marks)** Consider the following function

$$f(x) = \frac{\sin x}{1+x}$$

(a) Compute the Taylor polynomial $T_3(x)$ of $f(x)$ about $a = 0$, and use it to approximate the value of $f(0.02)$, up to five decimal places.

(5 marks)

(b) Use linear approximation of $f(x)$ to compute $f(0.02)$ up to five decimal places, compare your result with that in (a) and comment on which value is more accurate.

(5 marks)

(c) Show that there is at least one root of $f(x)$ for $3 < x < 4$, using the intermediate value theorem.

(2 marks)

(d) Using Newton's method, find an approximate root of f, accurate up to three decimal places. Choose your own the initial guess properly.
(4 marks)

(e) Compute the value of $T_3(x)$ at the approximate root of $f(x)$. Explain why it is so different from the value of $f(x)$ at the same point (which should be very close to zero since it is an approximate root).

(4 marks)

4. **(20 marks)** Start with the cubic function $y = ax^3$, we generate the volume by rotating about the x-axis the area enclosed by the cubic function, the x-axis and $x = b$. Note that a, b are constants.

(a) Find the volume V from the rotation as a function of a and b. (4 marks)

(b) Find the arc length s of the cubic function from $x = 0$ to $x = b$. Just leave your answer as a definite integral, there is no need to evaluate it.
(4 marks)

(c) Find the surface area A of the volume in (a), excluding the circular base at $x = b$.
(4 marks)

(d) Let us fix the volume $V = 1$, find the area A as a function of b.
(4 marks)

(e) Then calculate the instantaneous rate of change of A with respect to b at $b = 1$.
(4 marks)

5. **(10 marks)** Miscellaneous questions.

(a) Find the general solutions $y(x)$ of the differential equation:

$$y' - \frac{3y}{x+1} = (x+1)^4$$

(3 marks)

(b) Let $I_n = \int x^n e^{2x} dx$, derive its reduction formula, and show your steps.

(3 marks)

(c) Solve the equation $\left(\frac{z+i}{4i}\right)^4 = 1$.

(4 marks)

- End of Paper -

MH1802 CALCULUS FOR THE SCIENCES

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.