NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2017-2018

MH1101 – Calculus II

May 2018 Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages. A formulae table is provided at the end of the paper.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. Each candidate is allowed to bring **ONE (1)** hand-written, double-sided A4 size help sheet.

5. Candidates may use calculators. However, they should lay out systematically the various steps in the workings.

Question 1. (15 marks)

(a) Use the Simpson's Rule with $n = 4$ to approximate the integral $\int_0^1 x^2 \, dx$.

(b) Using the Error Bound formula, find an n such that the approximation of the integral $\int_0^1 \sin(x^2) \, dx$ using the Midpoint Rule M_n has an error of at most 10^{-3}.

Question 2. Evaluate the following integrals. (15 marks)

(a) $\displaystyle\int \frac{1}{\sqrt{x^2 + 6x}} \, dx$

(b) $\displaystyle\int \frac{1}{x^2(x^2 + 25)} \, dx$

Question 3. Determine whether each of the following series converges or diverges. Justify your answer. (20 marks)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{\sqrt{n^7 + 1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{n\pi}{4n + \sqrt{n}}\right)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n - 3^n}$

Question 4. (10 marks)

For $n \geq 1$, define $a_{n+1} = \sqrt{a_n + 30}$, where $a_1 = 4$.

(i) Show that $0 < a_n < 6$ for all $n \geq 1$.

(ii) Show that $\{a_n\}$ is increasing.

(iii) Show that $\lim_{n\to\infty} a_n$ exists and find its value.

Question 5. (20 marks)

(a) Find the interval of convergence of the following power series, and identify the values of x for which the series converges absolutely or conditionally. Justify your answer.

$$\sum_{n=0}^{\infty} \frac{n^6(x-5)^n}{n^8+1}.$$

(b) If the series $\sum_{n=1}^{\infty} a_n$ converges absolutely, does the series $\sum_{n=1}^{\infty} a_n^2$ converge? Justify your answer.

Question 6. (20 marks)

(a) Find a power series with center $c = 2$ that represents the function $f(x) = \dfrac{1}{4+3x}$.

(b) Find the first five nonzero terms of the Maclaurin series for $f(x) = \dfrac{1}{1+\sin x}$.

(c) Using power series or otherwise, find the sum $\sum_{n=0}^{\infty} ne^{-n}$.

Formulae Table

$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	$\int e^x\,dx = e^x + C$
$\int k\,dx = kx + C$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \tan x\,dx = \ln\lvert \sec x\rvert + C$	$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$
$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$	
$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$	$\sin 2x = 2\sin x \cos x$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\sin A \cos B = \dfrac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$	$\sin A \sin B = \dfrac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$
$\cos A \cos B = \dfrac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$	

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \dbinom{k}{n} x^n = 1 + kx + \dfrac{k(k-1)}{2!}x^2 + \dfrac{k(k-1)(k-2)}{3!}x^3 + \cdots$	$R = 1$

END OF PAPER

MH1101 CALCULUS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.