NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER I EXAMINATION 2024–2025
MH1810 – Mathematics 1

NOVEMBER 2024 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **TEN (10)** questions and comprises **TWENTY-TWO (22)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This is a **CLOSED BOOK** exam. However, a list of formulae is provided in the attachments.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.
6. This examination paper is **NOT ALLOWED** to be removed from the examination hall.

For examiners only

Questions	Marks
1 (10)	
2 (10)	
3 (10)	
4 (10)	

Questions	Marks
5 (10)	
6 (10)	
7 (8)	
8 (8)	

Questions	Marks
9 (12)	
10 (12)	

Total (100)	

QUESTION 1. **(10 Marks)**

(a) In Figure 1, $\mathbf{u} = \overrightarrow{OA}$ and $\mathbf{v} = \overrightarrow{OB}$, where A and B have coordinates $(2, -4)$ and $(4, 2)$ respectively. Let $\mathbf{w} = \mathbf{u} + 2\mathbf{v}$. Mark on the figure the point C so that $\mathbf{w} = \overrightarrow{OC}$. Find the value of $(\mathbf{v} \cdot \hat{\mathbf{w}})$.

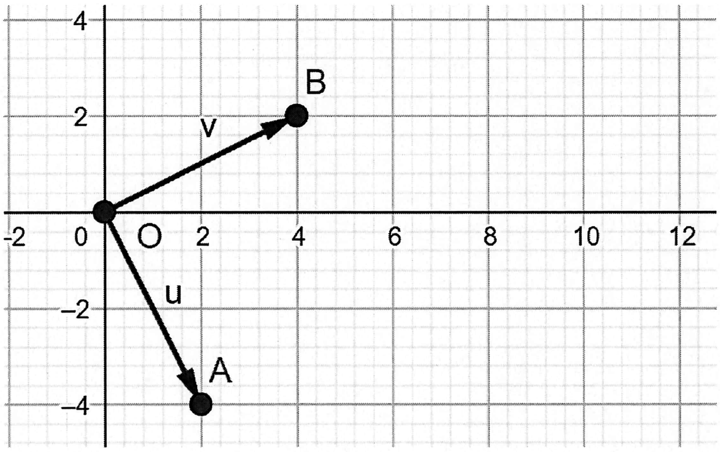

Figure 1.

Question 1 continues on Page 3.

(b) Find a scalar equation of the plane consisting of all points that are equidistant from $(0, 0, 1)$ and $(2, 4, 5)$.

QUESTION 2. **(10 Marks)**

(a) Express $\dfrac{(1+i)^4}{\left(-\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}$ as $re^{i\theta}$, $r>0$, $-\pi<\theta\leq\pi$, where there **exact** values of r and θ are to be determined.

Question 2 continues on Page 5.

(b) Let $z = 1 + 2i$. Plot the points z, z^2 and $\dfrac{10}{z}$ on the Argand diagram below (Figure 2).

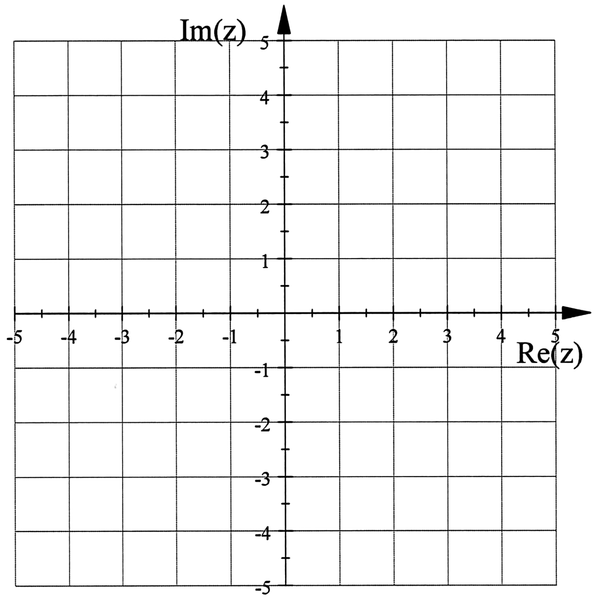

Figure 2

QUESTION 3. **(10 Marks)**

(a) Let $A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -10 \\ 3 & -3 \end{bmatrix}$.

(i) Find A^{-1} and show that ABA^{-1} is a diagonal matrix D.

(ii) Show that $B = A^{-1}DA$. Hence find the matrix B^3.

Question 3 continues on Page 7.

(b) Show that Cramer's rule is applicable for the following system of linear equations. Use Cramer's rule to solve for x_2.

$$\begin{aligned} x_1 + 2x_2 &= 1, \\ 2x_2 + x_3 &= 0, \\ x_2 + 2x_3 &= 1. \end{aligned}$$

QUESTION 4. **(10 Marks)**

(a) Suppose that a function f is continuous on the closed interval $[1,2]$ and $1 < f(x) < 2$ for every $x \in [1,2]$. Show that there exists $c \in (1,2)$ so that $f(c) = c$.

Question 4 continues on Page 9.

(b) Consider f which is defined as follows:

$$f(x) = \begin{cases} \dfrac{e^x}{2+x} & \text{if } x \geq 0, \\ xe^{\sin x} & \text{if } x < 0. \end{cases}$$

Is f differentiable at 0? Justify your answer.

QUESTION 5 **(10 Marks)**

(a) The graph of $x^2 + xy + y^2 = 3$ is given in Figure 3 below.

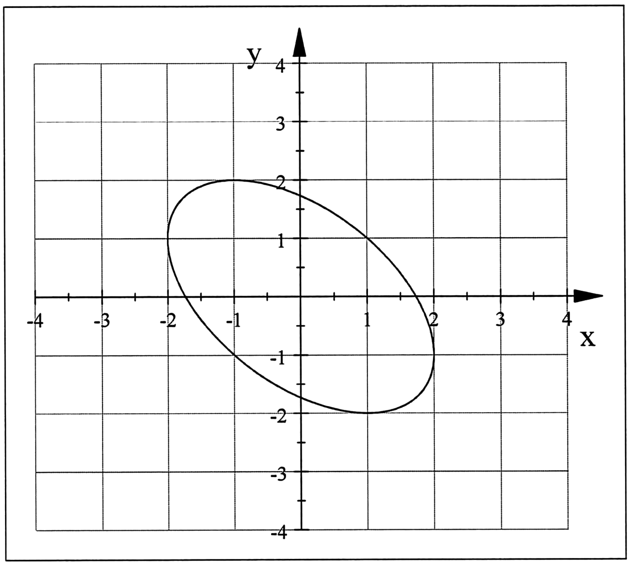

Figure 3.

Find $\dfrac{dy}{dx}$ and determine an equation of normal to the curve at $(1, 1)$. Sketch this normal on Figure 3.

Question 5 continues on Page 11.

(b) Use the definition of derivative to find the first derivative of the function

$$f(x) = \frac{1}{1+x^2}.$$

QUESTION 6. **(10 Marks)**

Compute the following limits.

(a) $\displaystyle \lim_{x\to 9} \frac{(x^2-81)\,e^x}{\sqrt{x}-3}$

Question 6 continues on Page 13.

(b) $\lim_{x\to 0} \dfrac{x^2}{3+\sin\frac{1}{x}}$

QUESTION 7. **(8 Marks)**

Use Newton's Method with $x_1 = 2$ to approximate the value of $\sqrt[5]{20}$ correct to five decimal places.

QUESTION 8. **(8 Marks)**

Show that the equation $2x + 1 + \sin x = 0$ has exactly one real root.

QUESTION 9. **(12 Marks)**

(a) Evaluate the following integrals.

(i) $\displaystyle\int \frac{x+2}{x^2+5x-6}\,dx,$

(ii) $\displaystyle\int x\tan^{-1} x dx.$

Question 9 continues on Page 17.

(b) Find the following limit by expressing the sum as a Riemann sum of a function over the interval $[0, 1]$.

$$\lim_{n\to\infty} \frac{1 + 2^{1/3} + 3^{1/3} + \cdots + n^{1/3}}{2n^{4/3}}.$$

QUESTION 10. **(10 Marks)**

(a) Find the volume of the solid obtained by rotating the region bounded by $y = 2x - x^2$ and $y = 0$ about the vertical line $x = 2$.

Question 10 continues on Page 19.

(b) Use Simpson's rule to approximate $\int_0^1 e^{-x^2}dx$ with $n = 8$. Round your answer to five decimal places.

END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \;\approx\; S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right],$$

where n is even.

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C \qquad \int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C,\ a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C,\ |x| < |a| \qquad \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

MH1810 MATHEMATICS 1

CONFIDENTIAL

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.