Sem 2 AY1819 MH1101 Calculus II Test (1 hour)

Name: ..... **Matriculation Number:** .....

Tutorial Group: .....

Question 1 [16 marks]

(a) Calculate the derivative $\dfrac{d}{dx}\displaystyle\int_{4x}^{2x} \cos(e^{2t})\,dt$.

(b) Write the following limit as a definite integral (do not evaluate it).

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{ni+3i^2}{n^4}}.$$

Answer.

Question 2 [16 marks]

(a) Let R be the region bounded by the curves $y = x^2$ and $y = 4x - x^2$. Compute the volume of the solid formed by revolving R about the line $x = 4$.

(b) Given that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$, evaluate $\int_{-\infty}^{\infty} e^{-kx^2}\,dx$ for $k > 0$. Justify your answer.

Answer.

Question 3 [18 marks] Evaluate the following integrals.

(a) $\displaystyle\int \sin^5(3x)\cos^4(3x)\,dx$

(b) $\displaystyle\int \frac{(\ln x)^2}{x^2}dx$

(c) $\displaystyle\int_0^1 x^a(1-x)^b\,dx$, where a and b are non-negative integers.
Express your answer in terms of a and b.

Answer.