

NANYANG TECHNOLOGICAL UNIVERSITY
SPMS/DIVISION OF MATHEMATICAL SCIENCES

2023/24 Semester 1 MH5100 Advanced Investigations into Calculus I Week 10

Problem 1. Suppose f and g are differentiable functions such that $f(g(x)) = x$ and $f'(x) = 1 + [f(x)]^2$. Show that $g'(x) = \frac{1}{1+x^2}$.

Problem 2. Find $y''(0)$ if $e^y + xy = e$.

Problem 3. Let

$$f(x) = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}.$$

Find $f'(x)$.

Problem 4. Find the derivative function of $F(x)$.

$$F(x) = \begin{vmatrix} x - 1 & 1 & 2 \\ -3 & x & 3 \\ -2 & -3 & x + 1 \end{vmatrix}$$

Problem 5. Show that the sum of the x - and y -interceptors of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c .

Problem 6. Let $P(x, y)$ be a point on the parabola $y^2 = 4px$ with focus $F(p, 0)$. Let α be the angle between the parabola and the line segment FP , and let β be the angle between the horizontal line $y = y_1$ and the parabola as in the figure. Prove that $\alpha = \beta$. (Thus, by a principle of geometrical optics, light from a source placed at F will be reflected along a line parallel to the x -axis.

