NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2022-2023

MH1810 - Mathematics 1

May 2023 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **SEVEN (7)** printed pages, including an appendix.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

6. Some useful formulae are listed on page 5 - 7.

QUESTION 1. Let $z = -5 + 5i$. [12 Marks]

(a) Find the modulus and principal argument of z.

(b) Find all distinct fourth roots of z. Leave your answers in the form $re^{i\theta}$.

QUESTION 2. Let $B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 19 & 6 & 0 \end{pmatrix}$. [12 Marks]

(a) Calculate the determinant of B via cofactor expansion along the second row.

(b) Deduce whether B is invertible or not.

QUESTION 3. Find the following limits [12 Marks]

(a) $\displaystyle\lim_{x\to 1} \frac{x^2 + 2x - 3}{x - 1}$.

(b) $\displaystyle\lim_{x\to -\infty} \frac{3e^x + e^{-x}}{e^x - 3e^{-x}}$.

(c) $\displaystyle\lim_{x\to \infty} \left(x - \sqrt{x^2 + 1}\right)$.

QUESTION 4. The function $f(x)$ is defined as follows.

$$f(x) = \begin{cases} x^3 + x + 2a & \text{if } \; x \geq 0 \\ -bx + 4 & \text{if } \; x < 0 \end{cases},$$

where a and b are some constants. [16 Marks]

(a) Find the value of a if f is continuous.

(b) Find the value of b if f is differentiable.

QUESTION 5. Find the derivatives of the following functions [16 Marks]

(a) $f(x) = \ln\left|\frac{1+e^x}{1-e^x}\right|$.

(b) $f(x) = 3\left|\ln x\right|^{\cos x}$.

QUESTION 6. Find the following limits by using L'Hospital's Rule. [16 Marks]

(a) $\lim\limits_{x\to\infty} \dfrac{\ln x}{x}$.

(b) $\lim\limits_{x\to 0^+} x \ln x$.

QUESTION 7. [16 Marks]

(a) Evaluate $\int x^3 \cos(x^4+1)\,dx$.

(b) Use the definition of definite integral (limit of Riemann sums) to prove that

$$\int_0^1 2x dx = 1.$$

(You may assume $\sum_{k=1}^n k = \frac{n(n+1)}{2}$).

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x-a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right],$$

where n is even.

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\text{sech } x) = -\text{sech} x \tanh x \qquad \frac{d}{dx}(\text{csch } x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1 \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C \qquad \int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C, \; a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \; |x| < |a| \qquad \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C \qquad \int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

END OF PAPER

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.