NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2023/24 Sem 1 MH5100 Advanced Investigations into Calculus I Week 8

Problem 1. *$f(x) = (x-a)(x-b)(x-c)(x-d)(x-e)$. a, b, c, d, and e are different real numbers. If $f'(k) = (k-a)(k-b)(k-c)(k-d)$, find the value of k.*

Problem 2. *If the function $f(x)$ is differentiable at the number $x = a$, evaluate the limit*

$$\lim_{h\to 0} \frac{f(a+h) - f(a-h)}{h}.$$

Problem 3. *Given that $g(0) = g'(0) = 0$ and let*

$$f(x) = \begin{cases} g(x)\sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Find $f'(0)$.

Problem 4. *Let $f(x)$ is defined on all real numbers. For any x_1 and x_2, the following relationship is satisfied*

$$f(x_1 + x_2) = f(x_1) \cdot f(x_2).$$

If $f'(0) = 1$, show that

$$f'(x) = f(x).$$

Problem 5. *Let*

$$f(x) = \frac{(x+5)^3(x-4)^{\frac{1}{2}}}{(x+2)^5(x+4)^{\frac{1}{2}}} \qquad (x > 4).$$

Find $f'(x)$.

Problem 6. *Let $y = u(x)^{v(x)}$. $u(x) > 0$. Both $u(x)$ and $v(x)$ are differentiable. Find y'.*