

**NANYANG TECHNOLOGICAL UNIVERSITY**  
**SEMESTER I EXAMINATION 2023–2024**  
**MH4320 – COMPUTATIONAL ECONOMICS**

Nov 2023

Time Allowed: 2 hours

**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **FOUR (4)** questions and comprises **FOUR (4)** printed pages.
2. Answer each question beginning on a **FRESH** page of the answer book.
3. This is a **CLOSED BOOK** examination.
4. Calculators may be used. However, you should write down systematically the steps in the workings.

**QUESTION 1****(25 marks)**

Two players bargain over how to split \$10. Each player  $i \in \{1, 2\}$  simultaneously chooses a number  $s_i \in [0, 10]$  (which does not need to be an integer). Each player's payoff is the money he receives. We consider two allocation rules. In each case, if  $s_1 + s_2 \leq 10$ , each player gets his chosen amount  $s_i$ , and the rest is destroyed.

- (a) In the first case, if  $s_1 + s_2 > 10$ , both players get zero. Find all *pure strategy* Nash equilibria.
- (b) In the second case, if  $s_1 + s_2 > 10$  and  $s_1 \neq s_2$ , the player who chose the smaller amount receives this amount and the other player gets the rest. If  $s_1 + s_2 > 10$  and  $s_1 = s_2$ , both players get \$5. Find all *pure strategy* Nash equilibria.
- (c) In case (b), if we require that  $s_1$  and  $s_2$  must be integers, again find all *pure strategy* Nash equilibria.

**QUESTION 2****(25 marks)**

Consider the following preference profile of 55 voters on 5 candidates:

- 18 voters voted:  $A \succ E \succ D \succ C \succ B$
- 12 voters voted:  $C \succ D \succ E \succ B \succ A$
- 10 voters voted:  $B \succ C \succ D \succ E \succ A$
- 9 voters voted:  $E \succ B \succ D \succ C \succ A$
- 4 voters voted:  $D \succ C \succ E \succ B \succ A$
- 2 voters voted:  $D \succ B \succ E \succ C \succ A$

- (a) Determine the winner of this preference profile according to each of the following voting rules.
  - (i) Plurality rule.

- (ii) Plurality with a single run-off: a run-off election (i.e., pairwise election) is conducted between the two candidates with the most first-place votes.
  - (iii) A different kind of run-off election: first have a run-off election between the candidates that rank second and third in first-place votes. The winner of that election faces the candidate with the most first-place votes in another run-off to decide the final outcome of the election.
- (b) Is there a Condorcet winner in this preference profile? If so, which candidate is a Condorcet candidate?
- (c) Assume the candidates are positioned in alphabetical order from left to right. Among these six types of preferences, list the ones that are single-peaked preferences.

**QUESTION 3****(20 marks)**

In a market, there are four buyers  $\{1, 2, 3, 4\}$  and three items  $\{a, b, c\}$ , where each buyer desires at most one item and each item has unit supply. The table below summarizes the valuations for each buyer getting each item.

	$a$	$b$	$c$
bidder 1	5	10	0
bidder 2	8	8	8
bidder 3	3	4	6
bidder 4	0	2	7

- (a) Determine the allocation produced by the VCG mechanism and the payment for each buyer.
- (b) Assuming that each buyer can receive more than one item, the valuation for a buyer for receiving a bundle of items is the sum of this buyer's valuations for each item in the bundle. Determine again the allocation produced by the VCG mechanism and the payment for each buyer.

**QUESTION 4****(30 marks)**

Alice (player 1) and Bob (player 2) would like to go on a date. They have two options: watching a football game (F) or watching an opera (O). Alice first chooses where to go, and knowing where Alice went, Bob also decides where to go. Alice prefers the opera, and Bob prefers the football game. A player gets 3 from his/her preferred date, 1 from his/her unpreferred date, and 0 if they end up at different places. All of these are common knowledge.

- (a) Find all Nash equilibria in pure strategies of this game. Identify which of those is/are subgame perfect equilibria.

Let us modify this game a little bit: Bob does not automatically know where Alice went, but he can choose to learn this information without any cost. That is, again Alice first chooses where to go, and without knowing where Alice went, Bob chooses between Learn (L) and Not Learn (NL). If he chooses Learn, then he knows where Alice went and then decides where to go; otherwise, he chooses where to go without learning where Alice went. The payoffs depend only on where each player goes and are the same as before. That is, a player gets 3 from his/her preferred date, 1 from his/her unpreferred date, and 0 if they end up at different places.

- (b) Model this new game as an extensive-form game and draw its game tree.  
 (c) Find all subgame perfect equilibria in pure strategies of this new game.

**END OF PAPER**







## MH4320 COMPUTATIONAL ECONOMICS

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.