

MH1100 Calculus I – Revision Notes

Quantitative Research Society @NTU

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Course Overview & Topic Map

Topic Area	Lectures	Key Concepts	ILO
Functions & Graphs	1-2	Definitions, Domain, Range, Even/Odd, Transformations	1
Limits & Continuity	3-4	Limit Laws, Squeeze, ϵ - δ , Continuity Types	1, 2
Differentiation	5-7	Derivative Rules, Chain/Product/Quotient, Linear Approx.	2, 3
Applications of Derivatives	8-10	MVT, Rolle, Optimization, Related Rates, Curve Sketching	3, 4
Asymptotes	11	Vertical, Horizontal, Slant	2
Inverse & Log/Exp Functions	12	Inverse, $\ln x$, e^x , Inverse Trig Derivatives	3
Integration (Intro)	13	Antiderivatives, Area, Integration Concepts	3, 4
Final Review	14	Recurring Question Types, Revision Strategy	4, 5

This revision guide synthesizes lecture material into thematic chapters, enabling efficient review of all ILOs (Intended Learning Outcomes) through definitions, worked examples, and annotated past exam problems. Prioritize understanding connections between topics and practicing with representative exam-style problems.

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Formula Summary

Functions and Basic Properties

- **Domain and Range:**

$$R_f = \{f(x) \in \mathbb{R} \mid x \in D_f\}$$

- **Even and Odd Functions:**

$$\text{Even: } f(-x) = f(x)$$

$$\text{Odd: } f(-x) = -f(x)$$

Any function $f(x)$ can be written as the sum of an odd and even function:

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}$$

Limits

- **Basic Limit Laws:**

$$\lim(f \pm g) = \lim f \pm \lim g$$

$$\lim(fg) = (\lim f)(\lim g)$$

$$\lim \frac{f}{g} = \frac{\lim f}{\lim g}$$

$$\lim f \circ g = \lim f \circ \lim g$$

- **Squeeze Theorem:**

If $f \leq g \leq h$ and $\lim f = \lim h = L$ then $\lim g = L$.

- **Epsilon-Delta Definition:**

$$\lim_{x \rightarrow a} f(x) = L \Leftrightarrow \forall \epsilon > 0, \exists \delta > 0 :$$

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$$

- **Important Inequalities:**

$$||a| - |b|| \leq |a \pm b| \leq |a| + |b|$$

$$x^n - a^n = (x - a)(a^{n-1} + \dots + x^{n-1})$$

- **Special Limits:**

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

- **L'Hôpital's Rule:**

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

for indeterminate forms ($0/0$ or ∞/∞), with differentiability conditions.

Continuity and Theorems

- **Continuity at $x = a$:**

$$f \text{ continuous at } a \Leftrightarrow \lim_{x \rightarrow a} f(x) = f(a)$$

- **Intermediate Value Theorem (IVT):** If f is continuous on $[a, b]$, for any L between $f(a)$ and $f(b)$, there exists $c \in (a, b)$ with $f(c) = L$.

- **Extreme Value Theorem:** A continuous function on $[a, b]$ attains its maximum and minimum.

- **Rolle's Theorem:** If f is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$, then $\exists c \in (a, b) : f'(c) = 0$.

- **Mean Value Theorem (MVT):** If f is continuous on $[a, b]$ and differentiable on (a, b) ,

$$\exists c \in (a, b) : f'(c) = \frac{f(b) - f(a)}{b - a}$$

- **Cauchy's Mean Value Theorem:**

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

for some $c \in (a, b)$ (when $g'(c) \neq 0$).

Differentiation

- **Definition of the Derivative:**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- **Interpretation:** Differentiable \Rightarrow Continuous \Rightarrow Limit exists.

- **Basic Rules:**

$$(f + g)' = f' + g'$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \quad (\text{Chain Rule})$$

- **Implicit Differentiation:** If $F(x, y) = 0$, then

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

- **Higher Order Derivatives:**

$$f^{(n)}(x) = \frac{d^n}{dx^n} f(x)$$

- **Linear Approximation:**

$$f(x) \approx f(a) + f'(a)(x - a)$$

- **Logarithmic Differentiation:**

$$\begin{aligned} \text{For } y &= u(x)v(x) \cdots, \\ \ln |y| &= \ln |u(x)| + \ln |v(x)| + \cdots \\ \Rightarrow \frac{y'}{y} &= \frac{u'(x)}{u(x)} + \frac{v'(x)}{v(x)} + \cdots \end{aligned}$$

Special Derivatives, Antiderivatives, and Integrals

- **Standard Derivatives:**

$$\begin{aligned} \frac{d}{dx} x^n &= nx^{n-1} \\ \frac{d}{dx} e^x &= e^x \\ \frac{d}{dx} a^x &= a^x \ln a \\ \frac{d}{dx} \ln x &= \frac{1}{x} \\ \frac{d}{dx} \sin x &= \cos x \\ \frac{d}{dx} \cos x &= -\sin x \\ \frac{d}{dx} \tan x &= \sec^2 x \end{aligned}$$

- **Derivatives of Inverse Functions:**

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$$

- **Inverse Trig Derivatives:**

$$\begin{aligned} \frac{d}{dx} [\arcsin x] &= \frac{1}{\sqrt{1-x^2}}, \quad |x| < 1 \\ \frac{d}{dx} [\arccos x] &= -\frac{1}{\sqrt{1-x^2}}, \quad |x| < 1 \\ \frac{d}{dx} [\arctan x] &= \frac{1}{1+x^2} \end{aligned}$$

- **Common Antiderivatives:**

$$\begin{aligned} \int x^n dx &= \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \\ \int \frac{1}{x} dx &= \ln |x| + C \\ \int e^x dx &= e^x + C \\ \int \sin x dx &= -\cos x + C \\ \int \cos x dx &= \sin x + C \end{aligned}$$

Asymptotes

- **Vertical Asymptote:** $x = a$ is a vertical asymptote if $\lim_{x \rightarrow a^\pm} f(x) = \pm\infty$.
- **Horizontal Asymptote:** $y = L$ is a horizontal asymptote if $\lim_{x \rightarrow \pm\infty} f(x) = L$.
- **Slant (Oblique) Asymptote:** $y = mx + b$ is a slant asymptote if $\lim_{x \rightarrow \pm\infty} [f(x) - (mx + b)] = 0$.

Optimization and Applications

- **Critical Points:** Values of x for which $f'(x) = 0$ or $f'(x)$ is undefined.
- **Second Derivative Test:** If $f'(a) = 0$ and $f''(a) > 0$ (min) or $f''(a) < 0$ (max).
- **Related Rates:** If $y = f(x)$ and both y, x depend on t ,

$$\frac{dy}{dt} = \frac{df}{dx} \frac{dx}{dt}$$

- **Newton's Method:**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Other Special Results

- **Concavity:** f is concave up where $f''(x) > 0$, concave down where $f''(x) < 0$.
- **Inflection Point:** f changes concavity at $x = c$ if $f''(c) = 0$ and sign changes.
- **Standard Taylor (Linear Approximation):**

$$f(x) \approx f(a) + f'(a)(x - a)$$

for x near a .

Part I

Functions and Basic Concepts

1 Functions

Function

A function f is a rule that assigns each element x in a set D (domain) to exactly one element in R . Note different values of $x_i \in D$ may map to the same value $f(x_i)$ in R . Hence to check if a function is well defined, we can use a vertical line test.

Domain, Range

The domain of a function f , commonly referred to as D is the set of all possible values of x that the function accepts. The range, commonly referred to as R_f refers to the set of possible values of $f(x)$ for all $x \in D$.

$$R_f = \{f(x) \in \mathbb{R} \mid x \in D_f\}$$

Odd and Even Functions

A function is even $\Leftrightarrow f(-x) = f(x)$ and odd $\Leftrightarrow f(-x) = -f(x)$.

Theorem

Any function $f(x)$ can be written as a sum of an odd and an even function.

Proof

Let $f(x) = E(x) + O(x)$ where E, O are the even and odd functions respectively.

$$f(x) = E(x) + O(x) \tag{1}$$

$$f(-x) = E(-x) + O(-x) \tag{2}$$

$$f(-x) = E(x) - O(x) \tag{3}$$

Solving, we get that

$$E(x) = \frac{f(x) + f(-x)}{2}, \quad O(x) = \frac{f(x) - f(-x)}{2}$$

Part II

Limits and Continuity

2 Limits

2.1 Limit Laws

If f, g are continuous at $x = a$ (limit exists), then we have the following hold true:

1. $\lim(f \pm g) = \lim f \pm \lim g$
2. $\lim \frac{f}{g} = \frac{\lim f}{\lim g}$
3. $\lim fg = \lim f \times \lim g$
4. $\lim f \circ g = \lim f \circ \lim g$. The implication here is that composition of continuous functions is continuous. Prove using epsilon delta (if it comes out just cry).

2.2 Squeeze Theorem

Theorem

If functions f, g, h are such that

$$f \leq g \leq h$$

Then we have that

$$\lim f \leq \lim g \leq \lim h$$

Specifically, if $\lim f = \lim h$, then $\lim g = \lim f = \lim h$.

2.3 Important Inequalities

1. $||a| - |b|| \leq |a \pm b| \leq |a| + |b|$
2. $x^n - a^n = (x - a)(a^{n-1} + a^{n-2}x + a^{n-3}x^2 + \cdots + ax^{n-2} + x^{n-1})$

2.4 Epsilon–Delta Definition of a Limit

Definition

We say that $\lim_{x \rightarrow a} f(x) = L$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

Proof Strategy

To prove a limit using the ϵ – δ definition:

1. Begin with $|f(x) - L|$ and manipulate it into a form involving $|x - a|$.
2. If additional terms appear, bound them appropriately (often by restricting $|x - a| < 1$).
3. Choose δ in terms of ϵ so the inequality is satisfied.

2.4.1 Basic Examples

Example

Example 1. $\lim_{x \rightarrow a} x = a$.

Proof

We want $|x - a| < \epsilon$. Choosing $\delta = \epsilon$ works:

$$|x - a| < \delta \Rightarrow |f(x) - a| = |x - a| < \epsilon.$$

Example

Example 2. $\lim_{x \rightarrow a} x^2 = a^2$.

Proof

We compute

$$|x^2 - a^2| = |x - a||x + a|.$$

If $|x - a| < 1$, then $|x + a| \leq |x - a| + 2|a| < 1 + 2|a|$. Choose

$$\delta = \min\left(1, \frac{\epsilon}{1 + 2|a|}\right).$$

Then $|x - a| < \delta \Rightarrow |x^2 - a^2| < \epsilon$.

2.4.2 Advanced Examples

Example

Example. $\lim_{x \rightarrow 0} x^{2019}(1 + \sin^2(2020x)) = 0$.

Proof

We bound

$$|x^{2019}(1 + \sin^2(2020x))| \leq 2|x|^{2019}.$$

Choose $\delta = \sqrt[2019]{\epsilon/2}$ and the proof follows.

Remark

Infinite limits can be defined analogously: $\lim_{x \rightarrow a} f(x) = \infty$ means for every $N > 0$, there exists $\delta > 0$ such that $0 < |x - a| < \delta \Rightarrow f(x) > N$.

2.5 Limit Laws with ϵ - δ Proofs

2.5.1 Sum Rule

Theorem

If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then

$$\lim_{x \rightarrow a} (f(x) + g(x)) = L + M.$$

Proof

Given $\epsilon > 0$, since $\lim_{x \rightarrow a} f(x) = L$, there exists $\delta_1 > 0$ such that

$$0 < |x - a| < \delta_1 \Rightarrow |f(x) - L| < \frac{\epsilon}{2}.$$

Similarly, since $\lim_{x \rightarrow a} g(x) = M$, there exists $\delta_2 > 0$ such that

$$0 < |x - a| < \delta_2 \Rightarrow |g(x) - M| < \frac{\epsilon}{2}.$$

Let $\delta = \min(\delta_1, \delta_2)$. Then for $0 < |x - a| < \delta$,

$$\begin{aligned} |f(x) + g(x) - (L + M)| &= |(f(x) - L) + (g(x) - M)| \\ &\leq |f(x) - L| + |g(x) - M| \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \end{aligned}$$

Thus the result follows.

2.5.2 Product Rule

Theorem

If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then

$$\lim_{x \rightarrow a} f(x)g(x) = LM.$$

Proof

We write

$$f(x)g(x) - LM = (f(x) - L)g(x) + L(g(x) - M).$$

Take $\epsilon > 0$.

Since $\lim_{x \rightarrow a} g(x) = M$, there exists $\delta_1 > 0$ such that

$$0 < |x - a| < \delta_1 \Rightarrow |g(x) - M| < 1.$$

This implies $|g(x)| \leq |M| + 1$ when $|x - a| < \delta_1$.

Now, since $\lim_{x \rightarrow a} f(x) = L$, there exists $\delta_2 > 0$ such that

$$0 < |x - a| < \delta_2 \Rightarrow |f(x) - L| < \frac{\epsilon}{2(|M| + 1)}.$$

Similarly, since $\lim_{x \rightarrow a} g(x) = M$, there exists $\delta_3 > 0$ such that

$$0 < |x - a| < \delta_3 \Rightarrow |g(x) - M| < \frac{\epsilon}{2(1 + |L|)}.$$

Let $\delta = \min(\delta_1, \delta_2, \delta_3)$. Then for $0 < |x - a| < \delta$,

$$\begin{aligned} |f(x)g(x) - LM| &\leq |f(x) - L||g(x)| + |L||g(x) - M| \\ &< \frac{\epsilon}{2(|M| + 1)}(|M| + 1) + |L|\frac{\epsilon}{2(1 + |L|)} \\ &\leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \end{aligned}$$

Thus the product rule holds.

2.5.3 Quotient Rule

Theorem

If $\lim_{x \rightarrow a} f(x) = L$, $\lim_{x \rightarrow a} g(x) = M$, and $M \neq 0$, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{L}{M}.$$

Proof

We want

$$\left| \frac{f(x)}{g(x)} - \frac{L}{M} \right| < \epsilon.$$

Rewrite:

$$\left| \frac{f(x)}{g(x)} - \frac{L}{M} \right| = \left| \frac{Mf(x) - Lg(x)}{Mg(x)} \right| = \frac{1}{|M||g(x)|} |M(f(x) - L) + L(M - g(x))|.$$

Since $\lim_{x \rightarrow a} g(x) = M$, there exists $\delta_1 > 0$ such that for $|x - a| < \delta_1$,

$$|g(x) - M| < \frac{|M|}{2}.$$

Thus $|g(x)| \geq |M| - |g(x) - M| > \frac{|M|}{2}$, so $\frac{1}{|g(x)|} \leq \frac{2}{|M|}$.

Now, since $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, there exist $\delta_2, \delta_3 > 0$ such that

$$|f(x) - L| < \frac{\epsilon|M|}{4(|M| + |L|)}, \quad |g(x) - M| < \frac{\epsilon|M|}{4(|M| + |L|)}.$$

Let $\delta = \min(\delta_1, \delta_2, \delta_3)$. Then for $0 < |x - a| < \delta$,

$$\begin{aligned} \left| \frac{f(x)}{g(x)} - \frac{L}{M} \right| &\leq \frac{2}{|M|^2} (|M||f(x) - L| + |L||g(x) - M|) \\ &< \frac{2}{|M|^2} \left(|M| \cdot \frac{\epsilon|M|}{4(|M| + |L|)} + |L| \cdot \frac{\epsilon|M|}{4(|M| + |L|)} \right) \\ &= \frac{2}{|M|^2} \cdot \frac{\epsilon|M|(|M| + |L|)}{4(|M| + |L|)} \\ &= \frac{\epsilon}{2} < \epsilon. \end{aligned}$$

Thus the quotient rule holds.

2.5.4 Composition Rule

Theorem

If $\lim_{x \rightarrow a} f(x) = L$ and g is continuous at L , then

$$\lim_{x \rightarrow a} g(f(x)) = g(L).$$

Proof

Given $\epsilon > 0$, since g is continuous at L , there exists $\eta > 0$ such that

$$|y - L| < \eta \Rightarrow |g(y) - g(L)| < \epsilon.$$

Since $\lim_{x \rightarrow a} f(x) = L$, there exists $\delta > 0$ such that

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \eta.$$

Then

$$|g(f(x)) - g(L)| < \epsilon$$

for all $0 < |x - a| < \delta$. Hence the composition rule holds.

2.6 L'Hôpital's Rule

Remark

Important: Before applying the rule, you must first *prove differentiability* of the functions involved on an open interval containing the point of interest (except possibly at the point itself).

Suppose f and g are real-valued functions defined on an open interval I containing a , with f and g differentiable on $I \setminus \{a\}$, and with $g'(x) \neq 0$ for all $x \in I \setminus \{a\}$. If

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0 \quad \text{or} \quad \lim_{x \rightarrow a} |f(x)| = \lim_{x \rightarrow a} |g(x)| = \infty,$$

and if the limit $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ exists (finite or infinite), then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

Checking Differentiability Before Applying L'Hôpital

To justify using L'Hôpital's Rule:

- Confirm that f and g are differentiable on an open interval around a (except possibly at a itself).
- Verify that $g'(x) \neq 0$ in this interval.
- Establish the indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ directly from the limits of f and g .

Example

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{x}.$$

First, note that $\ln x$ and x are differentiable on $(0, \infty)$, and $x > 0$ ensures $g'(x) = 1 \neq 0$. Since $\lim_{x \rightarrow 0^+} \ln x = -\infty$ and $\lim_{x \rightarrow 0^+} x = 0$, this is an $\frac{-\infty}{0^+}$ form. Applying L'Hôpital's Rule:

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{x} = \lim_{x \rightarrow 0^+} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x)} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{1} = \lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty.$$

3 Continuity

Continuity

A function is continuous at a point $x = a \Leftrightarrow$

1. $\lim_{x \rightarrow a} f(x)$ exists
2. The limit is equal to $f(a)$

3.1 Intermediate Value Theorem

Theorem

If a function $f(x)$ is continuous in an interval $[a, b]$, then there exists $f(c)$ where $c \in (a, b)$ where $f(c)$ is between $f(a)$ and $f(b)$.

Roots finding

Given $f(a) = -4$, $f(b) = 5$, there exists $x \in (a, b)$ such that $f(x) = 0$ by IVT.

Part III

Differentiation

4 Differentiation

Differentiable \Rightarrow Continuity \Rightarrow Limit exists.

Proof

$$\begin{aligned}
 f'(x_0) &= \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \\
 \lim_{x \rightarrow x_0} f(x) - f(x_0) &= \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} (x - x_0) = f'(x_0) \cdot 0 = 0 \\
 &\Rightarrow \lim_{x \rightarrow x_0} f(x) = f(x_0)
 \end{aligned}$$

Formal Definition

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Example

Prove that $\frac{x}{x+1} < \ln(x+1) < x$

Proof

First consider $f(x) = \ln(x+1) - x$, note that

1. $f(0) = 0$
2. $f'(x) = \frac{1}{x+1} - 1 = \begin{cases} > 0 & x < 0 \\ < 0 & x > 0 \end{cases}$

Hence we prove that $\ln(x+1) < x$. A similar method is used for the other half of the inequality and is left as an exercise for the reader.

4.1 Linear Approximation

Before applying linear (or tangent line) approximation, it is essential to verify that f is differentiable at the point of approximation.

Theorem

If f is differentiable at a , then for x near a we may approximate

$$f(x) \approx f(a) + f'(a)(x - a).$$

Remarks

- Differentiability at a implies continuity at a , so no separate continuity check is needed.
- The quality of the approximation depends on the size of $(x - a)$ and higher-order derivatives of f .

Example

For $f(x) = \sqrt{x}$ at $a = 4$,

$$f(4) = 2, \quad f'(x) = \frac{1}{2\sqrt{x}}, \quad f'(4) = \frac{1}{4}.$$

Thus,

$$\sqrt{x} \approx 2 + \frac{1}{4}(x - 4).$$

4.2 Extreme Value Theorem

Theorem

If f is continuous on a closed interval $[a, b]$, then f attains both an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some $c, d \in [a, b]$.

Conditions to Check

- Verify that f is continuous on the entire closed interval $[a, b]$.
- Discontinuities or open intervals invalidate the theorem.

Example

$f(x) = x^2$ on $[-1, 2]$ is continuous.

$$f(-1) = 1, \quad f(2) = 4, \quad f(0) = 0.$$

Hence, $\min f = 0$ at $x = 0$, $\max f = 4$ at $x = 2$.

4.3 Rolle's Theorem

Theorem

Let f be a function such that:

1. f is continuous on $[a, b]$,
2. f is differentiable on (a, b) ,
3. $f(a) = f(b)$.

Then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Conditions to Check

- Continuity on the closed interval $[a, b]$.
- Differentiability on the open interval (a, b) .
- Equal endpoint values: $f(a) = f(b)$.

Example

For $f(x) = \cos x$ on $[0, 2\pi]$, $f(0) = f(2\pi) = 1$. By Rolle's theorem, there exists c with $f'(c) = -\sin(c) = 0$, i.e. $c = \pi$.

4.4 Mean Value Theorem

Theorem

Let f be a function such that:

1. f is continuous on $[a, b]$,
2. f is differentiable on (a, b) .

Then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Conditions to Check

- Continuity on $[a, b]$.
- Differentiability on (a, b) .

Example

For $f(x) = x^2$ on $[1, 3]$,

$$\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4.$$

Since $f'(x) = 2x$, we need $2c = 4$, so $c = 2$ satisfies the theorem.

4.5 Limits at Infinity

Limit at Infinity

We say $\lim_{x \rightarrow \infty} f(x) = L$ if for every $\epsilon > 0$, there exists $N > 0$ such that

$$x > N \Rightarrow |f(x) - L| < \epsilon.$$

Similarly, $\lim_{x \rightarrow -\infty} f(x) = L$ if for every $\epsilon > 0$, there exists $N < 0$ such that

$$x < N \Rightarrow |f(x) - L| < \epsilon.$$

Example

Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x + 1}{x^2 - 5x + 3}$.

Solution. Divide numerator and denominator by x^2 :

$$\lim_{x \rightarrow \infty} \frac{3 + \frac{2}{x} + \frac{1}{x^2}}{1 - \frac{5}{x} + \frac{3}{x^2}} = \frac{3 + 0 + 0}{1 - 0 + 0} = 3.$$

4.6 Special Limits

Theorem

The following special limits are fundamental:

1. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
2. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$
3. $\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$
4. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$

Example

Evaluate $\lim_{x \rightarrow 0} \frac{\sin(5x + 2x^3)}{x^3}$.

Solution. We can rewrite this as:

$$\lim_{x \rightarrow 0} \frac{\sin(5x + 2x^3)}{5x + 2x^3} \cdot \frac{5x + 2x^3}{x^3} = 1 \cdot \lim_{x \rightarrow 0} \left(\frac{5}{x^2} + 2 \right).$$

This limit does not exist (approaches infinity).

5 Advanced Differentiation

5.1 Higher Order Derivatives

Higher Order Derivatives

The n -th derivative of f , denoted $f^{(n)}$, is obtained by differentiating f successively n times.

Example

Find $f''(x)$ for $f(x) = \frac{x \sin(x)}{x + \cos(x)}$.

Solution. First find $f'(x)$ using the quotient rule:

$$f'(x) = \frac{(x + \cos x)(\sin x + x \cos x) - x \sin x(1 - \sin x)}{(x + \cos x)^2}.$$

Then differentiate again to find $f''(x)$ (computation left as exercise).

5.2 Chain Rule

Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).$$

Example

Differentiate $f(x) = \sin(x^2 \cos(x^2 + \tan x) + x^3 \sec x)$.

Solution. Using the chain rule repeatedly:

$$\begin{aligned} f'(x) &= \cos(x^2 \cos(x^2 + \tan x) + x^3 \sec x) \\ &\quad \times \left[\frac{d}{dx}(x^2 \cos(x^2 + \tan x)) + \frac{d}{dx}(x^3 \sec x) \right]. \end{aligned}$$

Continue applying product and chain rules to each term.

5.3 Implicit Differentiation

Implicit Differentiation

When a relation between x and y is given implicitly (e.g., $F(x, y) = 0$), we differentiate both sides with respect to x , treating y as a function of x , and solve for $\frac{dy}{dx}$.

Example

Find the tangent line to $x^3 + y^3 = 6xy$ at the point $(3, 3)$, and find y'' at this point.

Solution. Differentiate both sides:

$$3x^2 + 3y^2y' = 6y + 6xy'.$$

Solving for y' :

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}.$$

At $(3, 3)$: $y' = \frac{6-9}{9-6} = -1$.

The tangent line is: $y - 3 = -1(x - 3)$, or $y = -x + 6$.

For y'' , differentiate y' implicitly:

$$y'' = \frac{d}{dx} \left[\frac{2y - x^2}{y^2 - 2x} \right].$$

Using the quotient rule and substituting $(3, 3)$ and $y' = -1$, we get $y'' = -\frac{16}{3}$.

5.4 Logarithmic Differentiation

Logarithmic Differentiation

Logarithmic differentiation is a method to differentiate functions of the form $y = f(x)^{g(x)}$, products, or quotients by first taking the natural logarithm of both sides and then differentiating implicitly.

Remark

This technique is especially useful when the exponent, the base, or both are functions of x . It simplifies the differentiation of complicated products or quotients as well.

Theorem

If $y = u(x) \cdot v(x) \cdot w(x) \cdots$, then

$$\ln |y| = \ln |u(x)| + \ln |v(x)| + \ln |w(x)| + \cdots$$

So

$$\frac{y'}{y} = \frac{u'(x)}{u(x)} + \frac{v'(x)}{v(x)} + \frac{w'(x)}{w(x)} + \cdots$$

Example

Differentiate $y = x^x$.

Solution. Take natural logarithms:

$$\ln y = x \ln x$$

Differentiate both sides with respect to x :

$$\frac{1}{y} \frac{dy}{dx} = \ln x + 1$$

Thus,

$$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$$

Example

Differentiate $y = \frac{(x^2+1)^4 \sqrt{\sin x}}{e^{3x}(2x-1)^5}$.

Solution. Let $y = \frac{(x^2+1)^4 (\sin x)^{1/2}}{e^{3x}(2x-1)^5}$. Take \ln on both sides:

$$\ln y = 4 \ln(x^2 + 1) + \frac{1}{2} \ln(\sin x) - 3x - 5 \ln(2x - 1)$$

Differentiate:

$$\frac{y'}{y} = \frac{4 \cdot 2x}{x^2 + 1} + \frac{1}{2} \frac{\cos x}{\sin x} - 3 - \frac{10}{2x - 1}$$

So:

$$y' = y \left(\frac{8x}{x^2 + 1} + \frac{1}{2} \cot x - 3 - \frac{10}{2x - 1} \right)$$

Substitute for y to get the full answer.

Remark

You can also use logarithmic differentiation to find derivatives of functions raised to variable powers, complicated products, or quotients quickly and systematically.

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Part IV

Applications of Derivatives

6 Applications of Derivatives

6.1 Related Rates

Related Rates

Related rates problems involve finding the rate of change of one quantity in terms of the rate of change of another quantity, using the chain rule.

Example

Air is being pumped into a spherical balloon so that its surface area increases at a rate of $100 \text{ cm}^2/\text{s}$. How fast is the radius increasing when the diameter is 10 cm?

Solution. The surface area of a sphere is $S = 4\pi r^2$. We are given $\frac{dS}{dt} = 100$. Differentiate with respect to time:

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}.$$

When the diameter is 10 cm, $r = 5$ cm. Thus:

$$100 = 8\pi(5) \frac{dr}{dt} \implies \frac{dr}{dt} = \frac{100}{40\pi} = \frac{5}{2\pi} \text{ cm/s}.$$

6.2 Increasing and Decreasing Functions

Increasing/Decreasing Test

Let f be continuous on $[a, b]$ and differentiable on (a, b) .

- If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on $[a, b]$.
- If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on $[a, b]$.

Example

Prove that $\sin x < x < \tan x$ for $0 < x < \frac{\pi}{2}$.

Solution. Consider $f(x) = x - \sin x$ on $(0, \frac{\pi}{2})$.

$$f'(x) = 1 - \cos x > 0 \quad \text{for } 0 < x < \frac{\pi}{2}.$$

Since $f(0) = 0$ and f is increasing, $f(x) > 0$, hence $x > \sin x$.

Similarly, let $g(x) = \tan x - x$. Then $g'(x) = \sec^2 x - 1 = \tan^2 x > 0$ for $0 < x < \frac{\pi}{2}$. Since $g(0) = 0$ and g is increasing, $g(x) > 0$, hence $\tan x > x$.

6.3 Concavity and Inflection Points

Concavity

Let f be twice differentiable on an interval I .

- f is **concave up** on I if $f''(x) > 0$ for all $x \in I$.
- f is **concave down** on I if $f''(x) < 0$ for all $x \in I$.

Inflection Point

A point c is an inflection point of f if f is continuous at c and the concavity changes at c .

Example

Find the intervals of concavity and inflection points for $f(x) = 2x^3 + 3x^2 + x + 8$.

Solution. Compute the second derivative:

$$f'(x) = 6x^2 + 6x + 1, \quad f''(x) = 12x + 6.$$

Set $f''(x) = 0$: $12x + 6 = 0 \implies x = -\frac{1}{2}$.

- For $x < -\frac{1}{2}$: $f''(x) < 0$ (concave down).
- For $x > -\frac{1}{2}$: $f''(x) > 0$ (concave up).

Thus, $x = -\frac{1}{2}$ is an inflection point.

6.4 Cauchy's Mean Value Theorem

Cauchy's Mean Value Theorem

Let f and g be continuous on $[a, b]$ and differentiable on (a, b) , with $g'(x) \neq 0$ for all $x \in (a, b)$. Then there exists $c \in (a, b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}.$$

Example

Prove that $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

Solution. Without loss of generality, assume $x < y$. By the Mean Value Theorem applied to $f(t) = \sin t$ on $[x, y]$, there exists $c \in (x, y)$ such that

$$\frac{\sin y - \sin x}{y - x} = \cos c.$$

Thus $|\sin y - \sin x| = |y - x| |\cos c| \leq |y - x|$ since $|\cos c| \leq 1$.

6.5 Curve Sketching

Remark

To sketch the graph of a function f :

1. Find the domain and any intercepts.
2. Determine symmetry (even/odd).
3. Find asymptotes (vertical, horizontal, slant).
4. Compute $f'(x)$ and find critical points; determine intervals of increase/decrease.
5. Compute $f''(x)$ and determine concavity and inflection points.
6. Plot key points and sketch the curve.

Example

Sketch the function $f(x) = \frac{x^2}{x^2 - 5x + 6}$.

Solution.

- Domain: $x \neq 2, 3$ (vertical asymptotes at $x = 2$ and $x = 3$).
- Horizontal asymptote: $\lim_{x \rightarrow \pm\infty} f(x) = 1$ (so $y = 1$).
- $f'(x) = \frac{2x(x^2 - 5x + 6) - x^2(2x - 5)}{(x^2 - 5x + 6)^2}$.
- Critical points and further analysis left as exercise.

6.6 Optimization Problems

Remark

To solve optimization problems:

1. Identify the quantity to be optimized (maximized or minimized).
2. Express this quantity as a function of one variable.
3. Find the critical points by setting the derivative equal to zero.
4. Use the first or second derivative test to determine the nature of critical points.
5. Check endpoints if the domain is a closed interval.

Example

Find the area of the largest rectangle that can be inscribed in a circle of radius r .

Solution. Let the rectangle have dimensions $2x$ by $2y$, where $x^2 + y^2 = r^2$.

Area: $A = (2x)(2y) = 4xy = 4x\sqrt{r^2 - x^2}$ for $0 \leq x \leq r$.

To maximize A , compute $A'(x)$:

$$A'(x) = 4\sqrt{r^2 - x^2} + 4x \cdot \frac{-x}{\sqrt{r^2 - x^2}} = 4 \frac{r^2 - 2x^2}{\sqrt{r^2 - x^2}}.$$

Set $A'(x) = 0$: $r^2 - 2x^2 = 0 \implies x = \frac{r}{\sqrt{2}}$.

Then $y = \sqrt{r^2 - \frac{r^2}{2}} = \frac{r}{\sqrt{2}}$.

Maximum area: $A = 4 \cdot \frac{r}{\sqrt{2}} \cdot \frac{r}{\sqrt{2}} = 2r^2$.

6.7 Newton's Method**Newton's Method**

To approximate a root of $f(x) = 0$, start with an initial guess x_1 and iterate:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Example

Starting with $x_1 = 2$, find the third approximation x_3 to the root of $x^3 - 2x - 5 = 0$.

Solution. Let $f(x) = x^3 - 2x - 5$, so $f'(x) = 3x^2 - 2$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{8 - 4 - 5}{12 - 2} = 2 - \frac{-1}{10} = 2.1.$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.1 - \frac{(2.1)^3 - 2(2.1) - 5}{3(2.1)^2 - 2}.$$

Computing: $f(2.1) = 9.261 - 4.2 - 5 = 0.061$, $f'(2.1) = 13.23 - 2 = 11.23$.

$$x_3 = 2.1 - \frac{0.061}{11.23} \approx 2.1 - 0.0054 \approx 2.0946.$$

6.8 Antiderivatives**Antiderivative**

A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.

Theorem

If F is an antiderivative of f on I , then the most general antiderivative is $F(x) + C$, where C is a constant.

Example

Find f if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$.

Solution. Integrate $f''(x)$:

$$f'(x) = \int (12x^2 + 6x - 4) dx = 4x^3 + 3x^2 - 4x + C_1.$$

Integrate again:

$$f(x) = \int (4x^3 + 3x^2 - 4x + C_1) dx = x^4 + x^3 - 2x^2 + C_1x + C_2.$$

Use initial conditions:

$$f(0) = C_2 = 4.$$

$$f(1) = 1 + 1 - 2 + C_1 + 4 = 4 + C_1 = 1 \implies C_1 = -3.$$

Thus, $f(x) = x^4 + x^3 - 2x^2 - 3x + 4$.

7 Asymptotes

7.1 Vertical Asymptotes

Vertical Asymptote

The line $x = a$ is a vertical asymptote of the function f if at least one of the following is true:

$$\lim_{x \rightarrow a^+} f(x) = \pm\infty, \quad \lim_{x \rightarrow a^-} f(x) = \pm\infty.$$

Example

Find the vertical asymptotes of $f(x) = \frac{2 \tan(x)}{(1+x)(x-5)}$.

Solution. Vertical asymptotes occur where the denominator is zero or where $\tan(x)$ is undefined:

- From $(1+x)(x-5) = 0$: $x = -1$ and $x = 5$
- From $\tan(x)$: $x = \frac{\pi}{2} + n\pi$ for any integer n

7.2 Horizontal Asymptotes

Horizontal Asymptote

The line $y = L$ is a horizontal asymptote of f if

$$\lim_{x \rightarrow \infty} f(x) = L \quad \text{or} \quad \lim_{x \rightarrow -\infty} f(x) = L.$$

Example

Find the horizontal asymptote of $f(x) = \frac{\sqrt{2x^2+10}}{\sqrt{x^3+x^2+10}}$.

Solution. For large x , we have:

$$f(x) = \frac{\sqrt{2x^2(1 + \frac{5}{x^2})}}{\sqrt{x^3(1 + \frac{1}{x} + \frac{10}{x^3})}} = \frac{|x|\sqrt{2}}{|x|^{3/2}\sqrt{1 + \frac{1}{x} + \frac{10}{x^3}}}.$$

As $x \rightarrow \infty$: $f(x) \rightarrow \frac{\sqrt{2}}{x^{1/2}} \rightarrow 0$. Thus $y = 0$ is a horizontal asymptote.

7.3 Slant (Oblique) Asymptotes

Slant Asymptote

The line $y = mx + b$ (with $m \neq 0$) is a slant asymptote of f if

$$\lim_{x \rightarrow \infty} [f(x) - (mx + b)] = 0 \quad \text{or} \quad \lim_{x \rightarrow -\infty} [f(x) - (mx + b)] = 0.$$

Example

Find the slant asymptote of $f(x) = \frac{3x^5 + x^4 + 10}{6x^4 + x^3 + 2}$.

Solution. Perform polynomial long division:

$$\frac{3x^5 + x^4 + 10}{6x^4 + x^3 + 2} = \frac{x}{2} + \frac{\text{lower degree terms}}{6x^4 + x^3 + 2}.$$

Thus the slant asymptote is $y = \frac{x}{2}$.

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8 Inverse Functions

8.1 Inverse Functions and Their Derivatives

Inverse Function

A function f has an inverse f^{-1} if f is one-to-one (injective). For $y = f(x)$, the inverse satisfies $x = f^{-1}(y)$.

Derivative of Inverse Function

If f is differentiable at x and $f'(x) \neq 0$, then f^{-1} is differentiable at $y = f(x)$ and

$$(f^{-1})'(y) = \frac{1}{f'(x)} = \frac{1}{f'(f^{-1}(y))}.$$

Example

Find $(f^{-1})'(a)$ for $f(x) = 2x^3 + 3x^2 + 7x + 4$, where $a = 4$.

Solution. First, find x such that $f(x) = 4$:

$$2x^3 + 3x^2 + 7x + 4 = 4 \implies 2x^3 + 3x^2 + 7x = 0 \implies x(2x^2 + 3x + 7) = 0.$$

Thus $x = 0$ (the quadratic has no real roots).

Now compute $f'(x) = 6x^2 + 6x + 7$, so $f'(0) = 7$.

Therefore,

$$(f^{-1})'(4) = \frac{1}{f'(0)} = \frac{1}{7}.$$

8.2 Exponential and Logarithmic Functions

Exponential Function

The exponential function e^x is defined as the inverse of the natural logarithm $\ln x$. It satisfies:

$$\frac{d}{dx}[e^x] = e^x, \quad \frac{d}{dx}[e^{g(x)}] = e^{g(x)}g'(x).$$

Logarithmic Function

The natural logarithm $\ln x$ is the inverse of e^x . Its derivative is:

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad \frac{d}{dx}[\ln(g(x))] = \frac{g'(x)}{g(x)}.$$

Example

Differentiate:

(a) $y = e^{\tan(x)}$

(b) $y = e^{e^x}$

(c) $y = \ln(\sin x + e^x)$

Solution.

(a) $y' = e^{\tan x} \cdot \sec^2 x.$

(b) $y' = e^{e^x} \cdot e^x.$

(c) $y' = \frac{\cos x + e^x}{\sin x + e^x}.$

8.3 Inverse Trigonometric Functions**Derivatives of Inverse Trigonometric Functions**

The derivatives of the inverse trigonometric functions are:

$$\begin{aligned}\frac{d}{dx}[\arcsin x] &= \frac{1}{\sqrt{1-x^2}}, \quad |x| < 1, \\ \frac{d}{dx}[\arccos x] &= -\frac{1}{\sqrt{1-x^2}}, \quad |x| < 1, \\ \frac{d}{dx}[\arctan x] &= \frac{1}{1+x^2}.\end{aligned}$$

ExampleDifferentiate $y = \arctan(e^x) + \arcsin\left(\frac{x}{2}\right).$ **Solution.**

$$y' = \frac{e^x}{1+e^{2x}} + \frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2} = \frac{e^x}{1+e^{2x}} + \frac{1}{\sqrt{4-x^2}}.$$

Part V

Past Examination Analysis and Practice

9 Past 5-Year Final Examination Analysis (2020–2024)

Overview

This section provides a comprehensive analysis of MH1100 Calculus I final examinations from Academic Years 2020/2021 through 2024/2025 (5 years, 35 questions total). The analysis identifies recurring themes, common problem types, and essential skills tested consistently across these examinations.

Key Observations:

The MH1100 final examinations consistently emphasize a strong blend of computational proficiency and rigorous proof-writing skills. Across the five years analyzed, several clear patterns emerge in topic coverage, problem structure, and skill requirements. The examinations typically contain 7 questions per year, with marks distributed to reflect both routine calculations and deeper conceptual understanding.

- **Limits and Continuity** form the foundation, appearing in nearly every exam with substantial weight. Questions on limits often involve special techniques (squeeze theorem, L'Hôpital's rule) and ϵ - δ proofs.
- **Derivatives and Applications** dominate the examination content, comprising roughly 60% of all questions. This includes computing derivatives, proving properties, implicit differentiation, and applying theorems like MVT and Rolle's Theorem.
- **Rigorous Proofs** constitute approximately 60% of all questions, emphasizing the theoretical foundations of calculus. Students must demonstrate fluency with ϵ - δ arguments, mean value theorem applications, and proving inequalities.
- **Epsilon-Delta Definitions** appear regularly (approximately once every 1–2 exams), requiring students to construct formal limit proofs from first principles.
- **Multi-part Questions** are standard, testing multiple related concepts within a single problem (e.g., evaluating limits combined with continuity conditions).
- **Computational Complexity** varies from straightforward limit evaluation to intricate implicit differentiation and optimization problems requiring multiple techniques.

Topic Frequency Distribution

Topic	Frequency	Percentage
Derivatives & Applications	21	60%
Limits & Special Limits	13	37%
Continuity	10	29%
Mean Value Theorem / Rolle's Theorem	7	20%
Epsilon–Delta Proofs	5	14%
Squeeze Theorem	2	6%
L'Hôpital's Rule	2	6%
Implicit Differentiation	2	6%
Optimization	1	3%

Note: Many questions cover multiple topics, so percentages sum to more than 100%.

Question Type & Difficulty Analysis

Category	Count	Percentage
<i>Question Type</i>		
Proof-Based	21	60%
Calculation / Computation	14	40%
<i>Difficulty Level</i>		
Hard	18	51%
Medium	17	49%

Recurring Problem Archetypes

- Limit Evaluation with Special Techniques:** Questions requiring squeeze theorem, substitution, or algebraic manipulation to evaluate limits involving trigonometric, exponential, or composite functions.
- Continuity and Piecewise Functions:** Determining values of parameters to ensure continuity, often combined with limit calculations.
- Epsilon–Delta Limit Proofs:** Formal proofs of limits using the definition, typically for polynomial or rational functions.
- Derivative Applications – MVT & Rolle:** Proving inequalities or properties of functions using the Mean Value Theorem or Rolle's Theorem, often requiring auxiliary function construction.
- Implicit Differentiation:** Finding derivatives and tangent lines for implicitly defined curves.
- Higher-Order Derivatives and Leibniz Rule:** Computing second and third derivatives, sometimes using product/chain rules iteratively.
- Proving Inequalities:** Using derivatives to show functions are increasing/decreasing, thereby establishing inequalities.

8. **Multi-Step Composite Problems:** Questions combining multiple concepts, such as finding limits involving derivatives or applying continuity with differentiability conditions.

Essential Skills Tested Consistently

- Proficiency with limit laws, special limits, and indeterminate forms
- Mastery of ϵ - δ techniques for formal limit proofs
- Strong understanding of continuity definitions and theorems (IVT, EVT)
- Computational fluency with differentiation rules (product, quotient, chain, implicit)
- Ability to construct auxiliary functions for MVT/Rolle applications
- Rigorous proof-writing with clear logical flow and justifications
- Combining multiple theorems/techniques to solve complex problems

Recommendations for Revision

Based on this 5-year analysis, students preparing for the MH1100 final examination should:

1. **Master ϵ - δ Proofs:** These appear regularly and carry substantial marks. Practice constructing proofs for various function types (polynomials, rationals, roots).
2. **Practice MVT & Rolle's Theorem Applications:** Focus on problems requiring you to construct auxiliary functions $h(x) = f(x) - g(x)$ or similar, then apply theorems to prove inequalities.
3. **Drill Limit Techniques:** Ensure fluency with squeeze theorem, special limits ($\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, $\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$), algebraic manipulation, and L'Hôpital's rule.
4. **Work Through Multi-Part Problems:** Many exam questions test 2–3 related concepts. Practice linking concepts (e.g., limits \rightarrow continuity \rightarrow differentiability).
5. **Emphasize Proof-Writing:** Approximately 60% of questions are proof-based. Write out full solutions with clear reasoning, not just calculations.
6. **Review Past Papers Under Timed Conditions:** Simulate exam conditions (2–3 hours) to build stamina and time-management skills.
7. **Focus on High-Frequency Topics:** Prioritize derivatives, limits, continuity, and MVT/Rolle – these constitute roughly 85% of all questions.
8. **Don't Neglect Edge Cases:** Questions often test understanding of definitions at boundary cases (e.g., one-sided limits, piecewise continuity).

Important Note: This analysis describes only the patterns observed in past examinations (2020–2024). It does not predict or guarantee what topics will appear in future examinations. Students should prepare comprehensively across all course material.

10 Appendix: Past Examination Questions

Complete Question Mapping Table (2020–2024)

The following table provides a comprehensive overview of all 35 final examination questions from the past five years, categorized by topic, type, and difficulty.

Year	Q#	Primary Topics	Type	Difficulty
2020	1	Limits, Continuity, ...	Calculation	Hard
2020	2	Limits, Epsilon-Delta	Proof	Hard
2020	3	Derivatives	Calculation	Medium
2020	4	Derivatives	Proof	Hard
2020	5	Limits	Proof	Medium
2020	6	Continuity, Derivatives, ...	Proof	Hard
2020	7	Derivatives	Proof	Hard
2021	1	Limits, L'Hopital's Rule	Calculation	Medium
2021	2	Limits, Epsilon-Delta	Proof	Hard
2021	3	Derivatives	Calculation	Medium
2021	4	Continuity, Derivatives, ...	Proof	Hard
2021	5	Continuity, Derivatives, ...	Proof	Hard
2021	6	Derivatives	Proof	Medium
2021	7	Limits, Derivatives	Proof	Hard
2022	1	Limits, L'Hopital's Rule	Calculation	Medium
2022	2	Limits, Epsilon-Delta	Proof	Hard
2022	3	Derivatives	Calculation	Medium
2022	4	Derivatives, Implicit Differentiation	Calculation	Medium
2022	5	Continuity, Derivatives, ...	Proof	Hard
2022	6	Continuity, Derivatives, ...	Proof	Hard
2022	7	Derivatives	Calculation	Medium
2023	1	Limits	Calculation	Medium
2023	2	Limits, Epsilon-Delta	Proof	Hard
2023	3	Derivatives	Calculation	Medium
2023	4	Derivatives, Implicit Differentiation	Calculation	Medium
2023	5	Limits, Squeeze Theorem	Calculation	Medium
2023	6	General	Proof	Medium
2023	7	Continuity, Derivatives, ...	Proof	Hard
2024	1	Limits	Calculation	Medium
2024	2	Limits, Epsilon-Delta	Proof	Hard
2024	3	Continuity, Derivatives, ...	Proof	Hard
2024	4	Derivatives	Calculation	Medium
2024	5	Derivatives	Proof	Medium
2024	6	Continuity, Derivatives, ...	Proof	Hard
2024	7	Continuity	Proof	Hard

Legend

- **Type:** Calculation = computational problem; Proof = requires rigorous proof
- **Difficulty:** Based on conceptual complexity, proof requirements, and multi-step reasoning
- Topics marked with “...” indicate additional topics beyond the two primary ones listed