NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2012–2013

MH1101 – Calculus II

May 2013 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT** an **OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(10 marks)**

Evaluate the following integrals:

(a) $\int_0^{\pi} \sin(3x)\cos(2x)\,dx.$

(b) $\int_1^2 \sqrt{\frac{4-x}{x}}\,dx.$

QUESTION 2. **(20 marks)**

(a) How large should we take n in order to guarantee that the Simpson's Rule approximation for $\int_0^3 \sqrt{x+1}\,dx$ is accurate to within 0.0001?

(b) Find the value of the constant C for which the integral $\int_0^{\infty} \left(\frac{x}{x^2+1} - \frac{C}{3x+1}\right) dx$ converges. Evaluate the integral for this value of C.

QUESTION 3. **(15 marks)**

(a) Find the radius of convergence of the following power series.

$$\sum_{n=1}^{\infty} \left[n^4 \sin^2\left(\frac{2}{3n^2}\right)\right]^n x^n.$$

(b) Find the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{n(x+2)^n}{5^{n-1}}.$$

QUESTION 4. **(25 marks)**

(a) Test the following series for convergence, using any method.

(i)
$$\sum_{n=1}^{\infty} \frac{(3n)! + 4^{n+1}}{(3n+2)!}.$$

(ii)
$$\sum_{n=1}^{\infty} \left(\frac{2^n}{8^{n+2}} - \frac{1}{2n} \right).$$

(iii)
$$\sum_{n=1}^{\infty} (-1)^n \frac{2n}{\sqrt{n^3+1}}.$$

(b) For what values of $p \in \mathbb{R}$, if any, does the following series converge conditionally?
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^p}.$$

QUESTION 5. **(15 marks)**

(a) Determine a power series representation of $f(x) = \dfrac{x^3}{81 - x^4}$.

(b) Use an appropriate power series to approximate the value of $\dfrac{1}{\sqrt[4]{1.1}}$ to within 10^{-3}.

QUESTION 6. **(15 marks)**

(a) Prove that the following sequence converge by the definition of the limit.

$$\left\{\frac{1}{n^2+3+2}\right\}_{n=1}^{\infty}.$$

(b) Let a_1 be a fixed real number, and define $a_{n+1} = \dfrac{2+a_n}{3}$. Find the general formula for this recursive sequence. What is the limit of this sequence?

END OF PAPER

MH1101 CALCULUS II

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.