NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2022-2023

MH1803 Calculus for Physics

Apr/May 2023 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL FOUR (4)** questions.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. You should write down systematically the steps in the workings to receive full credit.
5. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN ON THE A4 PAPER** (no sticky notes/post-it notes on the reference sheet).
6. Calculators are **NOT** allowed.

1. **Short questions (35 marks)**

(a) Calculate the gradient ∇f of the function $f = x^3 + 2y^3$ at the point $\mathbf{A} = (1,1)$ and the directional derivative $D_{\mathbf{u}}f$ at the point $\mathbf{A}$, in the direction $\mathbf{u} = \mathbf{i} - \mathbf{j}$.

(10 marks)

(b) Compute the divergence and the curl of $\mathbf{V} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$.

(10 marks)

(c) Draw a picture of the region of integration of

$$\int_0^1 \int_x^{2x} dy dx.$$

Do not evaluate the integral.

(5 marks)

(d) Solve the first-order equation $4u_t + 3u_x = 0$ with the auxiliary condition $u(x,0) = \cos x$ when $t = 0$.

(10 marks)

2. **SIS model (15 marks)**

The ODE describing the SIS model is given by

$$\frac{di}{dt} = \beta\langle k\rangle i(1-i) - \mu i,$$

where β, $\langle k\rangle$, and μ are real constants, i is the fraction of infected population and t is time. Assume the initial condition $i(t=0) = i_0$.

(a) Use method of separable-variable to solve for $i(t)$ and show that

$$i(t) = \left[1 - \frac{\mu}{\beta\langle k\rangle}\right] \frac{Ce^{(\beta\langle k\rangle-\mu)t}}{1+Ce^{(\beta\langle k\rangle-\mu)t}},$$

where

$$C = \frac{i_0}{1-\frac{\mu}{\beta\langle k\rangle} - i_0}.$$

(10 marks)

(b) When $\mu < \beta\langle k\rangle$, the endemic state $i(\infty)$ exists. Find the expression for $i(\infty)$.

(5 marks)

3. **Line integral (25 marks)**

Let S be the part of the spherical surface $x^2+y^2+z^2 = 2$ with $z > 1$. The normal of the surface S points outward. The bounding circle, C, of the surface S lies in the plane $z = 1$.

(a) Parametrize the circle C and hence evaluate the line integral

$$I = \oint_C xz\,dx + y\,dy + y\,dz.$$

(10 marks)

(b) Compute the curl of the vector field $\mathbf{F} = xz\mathbf{i} + y\mathbf{j} + y\mathbf{k}$.

(5 marks)

(c) Show that

$$\iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{A} = \iint_S \frac{x+xy}{\sqrt{2}}dA.$$

Hence evaluate this integral.

(10 marks)

4. **Flux! (25 marks)**

Let S be the surface formed by the part of the paraboloid $z = 1 - x^2 - y^2$ lying above the xy-plane. The normal of the surface S points outward.

(a) Sketch the surface S and draw the normal vector at a point of the surface.
(5 marks)

(b) Consider the vector field given by

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + 2(1-z)\mathbf{k}.$$

Calculate the flux of $\mathbf{F}$ across S. Do this in two ways:

i. by direct calculation of $\iint_S \mathbf{F} \cdot d\mathbf{A}$.
(10 marks)

ii. by computing the flux of $\mathbf{F}$ across a simpler surface and using the divergence theorem.
(10 marks)

- End of Paper -

MH1803 CALCULUS FOR PHYSICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.