NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2022-2023

MH1100 – Calculus I

December 2022 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED** book exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 **(16 marks)**

(a) Find the limit or show it does not exist

$$\lim_{x\to\infty} x^{1/3}\sin(\frac{1}{\sqrt{x}}).$$

(b) Find the limit (you can use l'Hospital's Rule)

$$\lim_{x\to 0}\frac{\ln(1+\ln(1+3x^2))}{x^2}.$$

QUESTION 2 **(16 marks)**

Let a be a fixed real number. Use the ϵ-δ definition to prove the following: if we have $\lim_{x\to a} f(x) = f(a)$ and $\lim_{x\to a} g(x) = g(a)$, then we have $\lim_{x\to a}(f(x) - g(x)) = f(a) - g(a)$.

QUESTION 3 **(16 marks)**

Find the explicit expression for $f(x)$ such that

$$f'''(x) = \sin x, \quad f''(0) = 1, \ f'(0) = 2, \ f(0) = 3.$$

\\[0.25em]
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QUESTION 4 **(16 marks)**

Suppose that y is an implicit function of x satisfying that

$$y \ln(x^{2022}) - x \ln(y^{2022}) = 0,$$

find y'.

**Added find y' with reference to solution*

QUESTION 5 **(12 marks)**

Suppose that $n > 0$ is an integer and $a_0, ..., a_n$ are real numbers such that

$$\frac{a_0}{n+1} + \frac{a_1}{n} + \cdots + \frac{a_{n-1}}{2} + a_n = 0.$$

Prove that the function

$$f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n$$

has at least one root in $(0, 1)$.

QUESTION 6 **(12 marks)**

Suppose $f(x)$ has second derivative in $[a, b]$, and $f(a) = f(b) = 0$. Prove that for every $x \in (a, b)$, there exists a $\xi \in (a, b)$, such that

$$f(x) = \frac{f''(\xi)}{2}(x - a)(x - b).$$

QUESTION 7 **(12 marks)**

Suppose we have

$$f(x) = \frac{2x}{1 - x^2},$$

find $f^{(n)}(x)$.
(Note that $f^{(n)}(x)$ is the n-th order derivative of $f(x)$.)

[Solution:] we have

$$f(x) = \frac{1}{1 - x} - \frac{1}{1 + x} = (1 - x)^{-1} - (1 + x)^{-1}.$$

END OF PAPER