

**NANYANG TECHNOLOGICAL UNIVERSITY**

**SEMESTER II EXAMINATION 2024-2025**

**MH1803 Calculus for Physics**

Apr/May 2025

Time Allowed: 2 hours

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**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** pages.
2. Answer **ALL FIVE (5)** questions.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. You should write down systematically the steps in the workings to receive full credit.
5. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).
6. Calculators are **NOT** allowed.

**1. Short Questions (35 marks)**

- (a) Find the equation of the tangent plane to the surface  $x^3y + z^2 = 3$  at the point  $(-1, 1, 2)$ .

(5 marks)

- (b) Compute the divergence and the curl of the vector field  $\mathbf{F}(x, y, z) = (x^2, xyz, yz^2)$ .

(10 marks)

- (c) Solve the differential equation

$$(x^2 + 1) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

by the power series method. Assume the solution can be expressed as a power series with real coefficients (i.e.,  $a_i$ 's  $\in \mathbb{R}$ ,  $\forall i = 0, 1, 2, \dots$ ):

$$y(x) = a_0 + a_1x + a_2x^2 + \dots = \sum_{n=0}^{\infty} a_n x^n.$$

(10 marks)

- (d) Solve the first-order partial differential equation  $5u_t + 6u_x = 0$  with the auxiliary condition  $u(x, 0) = \cos x$  when  $t = 0$ .

(10 marks)

2. Let  $(x, y) = G(u, v) = (u^2 - v^2, 2uv)$ . Let  $A$  be the region defined by  $1 \leq u^2 + v^2 \leq 2$  and  $0 \leq u, 0 \leq v$ . Find the integral of the function

$$f(x, y) = \frac{1}{x^2 + y^2}$$

over  $G(A)$ .

(15 marks)

3. Find the spherical coordinate limits for the solid enclosed by the cardioid of revolution  $r(\theta, \phi) = 1 - \cos \theta$  (which is axisymmetrical about the  $z$ -axis) and then evaluate the integral for its volume explicitly.

(15 marks)

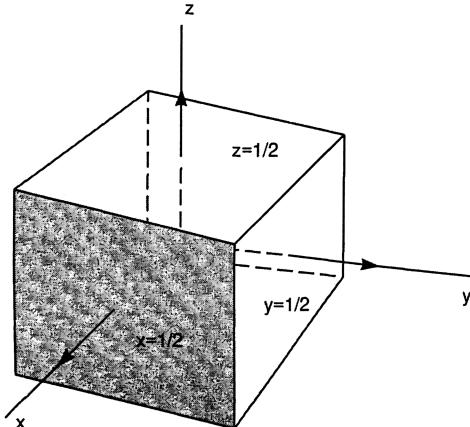
4. Consider the vector field  $\mathbf{F}(x, y, z) = (z - y, x + z, -(x + y))$  and the surface of the paraboloid  $z = 4 - x^2 - y^2$  with  $0 \leq z \leq 4$ . Sketch the paraboloid. Verify Stokes' theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{x} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A},$$

where  $C$  is a circle of radius 2 on the  $xy$  plane and  $S$  is the surface of the paraboloid bound by  $C$ .

(20 marks)

5. Calculate the flux of the vector field  $\mathbf{F} = (x^3y, y^2x, z)$  over a unit cube centered at the origin as shown below.



(15 marks)

- End of Paper -

**CONFIDENTIAL**

**MH1803 CALCULUS FOR PHYSICS**

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
- 2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.**
- 3. Please write your Matriculation Number on the front of the answer book.**
- 4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.**