

NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER II EXAMINATION 2024–2025
MH1811 – MATHEMATICS 2

APRIL 2025

TIME ALLOWED: 2 HOURS

SEAT NUMBER:

MATRICULATION NUMBER:

INSTRUCTIONS TO CANDIDATES

- (a) This question cum answer booklet contains **SIX (6)** questions and comprises **SEVENTEEN (17)** pages, including spill over pages.
- (b) Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
- (c) Write your answers in the **space provided** after each question.
- (d) Write your matriculation number on the **cover page**, and at the **bottom right-hand corner of every odd numbered page**.
- (e) The last four pages are spill over pages. Only use them if you run out of space for your answers. If you use them, please indicate clearly which question(s) you are answering.
- (f) This question cum answer booklet **IS NOT** to be removed from the examination hall.
- (g) This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)
- (h) Candidates may use calculators. However, they should write down systematically the steps in the workings.

Question 1.**(15 marks)**

- (a) Let $f(x, y) = x^3 \ln(3 - 2y)$. What is the domain of f ?

Answer:

- (b) Given that $f(x, y) = x^3 \ln(3 - 2y)$, what is the maximal directional derivative $D_{\mathbf{u}}f(-2, 1)$, where \mathbf{u} is a unit vector?

Answer:

Question 1 continues on page 3.

- (c) Use differentials to approximate $(-1.99)^3 \cdot \ln(3 - 2 \cdot 1.02)$.

Answer:

End of Question 1.

MATRIC NO.: _____

Question 2.**(20 marks)**

- (a) Find the Hessian matrix (matrix of second partial derivatives) of the function $f(x, y) = x^3y$.

Answer:

- (b) Explain why the function $f(x, y) = x^3y$ attains a maximum and a minimum subject to the constraint

$$\frac{x^2}{48} + \frac{y^2}{16} = 1.$$

Answer:

Question 2 continues on page 5

- (c) Find the minimum and the maximum values of $f(x, y) = x^3y$ subject to the constraint

$$\frac{x^2}{48} + \frac{y^2}{16} = 1.$$

Answer:

End of Question 2.

MATRIC NO.: _____

Question 3.**(15 marks)**

- (a) Switch the order of integration in the integral

$$\int_0^\pi \int_{x^2}^{\pi^2} \frac{\sin \sqrt{y}}{y} dy dx$$

Answer:

Question 3 continues on page 7

(b) Compute the iterated integral

$$\int_0^\pi \int_{x^2}^{\pi^2} \frac{\sin \sqrt{y}}{y} dy dx$$

Answer:

End of Question 3.

MATRIC NO.: _____

Question 4.**(15 marks)**

Determine, with proof, whether each of the following sequences converges or diverges:

(a) $a_n = \frac{n-42}{n+73}\sqrt{n}.$

Answer:

(b) $b_n = n^4 e^{-n/10}.$

Answer:

Question 4 continues on page 9.

$$(c) \ c_n = \frac{1^2 \cdot 2^2 \cdots n^2}{(1+1^2) \cdot (1+2^2) \cdots (1+n^2)}.$$

Answer:

End of Question 4.

MATRIC NO.: _____

Question 5.**(20 marks)**

- (a) Determine, with proof, whether the following series converges conditionally, converges absolutely, or diverges:

$$\sum_{n=1}^{\infty} \frac{(-5)^n}{n^3}.$$

Answer:

- (b) Determine, with proof, whether the following series converges conditionally, converges absolutely, or diverges:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(5n)^3}.$$

Answer:

Question 5 continues on page 11.

- (c) Find the Maclaurin series of $\arctan x$. Hint: $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$.

Answer:

- (d) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.

Answer:

End of Question 5.

MATRIC NO.: _____

Question 6.**(15 marks)**

(a) Solve the initial value problem

$$\frac{dy}{dx} = \frac{y^4 x}{1 + x^4}, \quad y(0) = 1.$$

Answer:

(b) Solve the initial value problem

$$\frac{dy}{dx} + \frac{4x^3 y}{1 + x^4} = \frac{\cos x}{1 + x^4}, \quad y(0) = 42.$$

Answer:

Question 6 continues on page 13.

(c) Solve the following initial value problem:

$$-2y \sin x \, dx + 2(y + \cos x) \, dy = 0, \quad y(0) = -4.$$

Answer:

If you need additional space for your answer(s) and plan to use this **Spill Over Page**, please indicate this in the designated question area to refer the marker here. Be sure to label the question number and part clearly on this page.

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MH1811 MATHEMATICS 2

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
- 2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.**
- 3. Please write your Matriculation Number on the front of the answer book.**
- 4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.**