NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2023-2024

MH4110 – Partial Differential Equations

April 2024 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 (24 marks)

Consider the partial differential equation

$$xu_x + yu_y = u^2.$$

(a) Determine the general solution of this equation. *(Hint: Introduce a new function* $v = -\frac{1}{u}$ *and solve the corresponding partial differential equation for* v*.)*

(b) Use the initial condition

$$u(-1, y) = \frac{1}{y}$$

to determine the particular solution of the equation.

QUESTION 2 (20 marks)

Find the solution $u = u(x, y)$ of the partial differential equation

$$u_{xx} - 2u_{xy} + u_{yy} = \sin x + 2\cos y.$$

QUESTION 3 (24 marks)

Consider the function $f(x) = \sin x$ defined on the interval $[0, \pi]$.

(a) Calculate its Fourier cosine series.

(b) Show that

$$\frac{1}{2} + \sum_{k=1}^{\infty} \frac{(-1)^k}{1 - 4k^2} = \frac{\pi}{4}.$$

QUESTION 4 **(32 marks)**

Consider the initial boundary value problem defined on the interval $[0,\ L]$, where c and L are positive constants:

$$\begin{cases} u_{tt} - c^2 u_{xx} + c u_t = 0, \quad 0 < x < L, \ \ t > 0, \\ u(0,t) = u(L,t) = 0, \quad t > 0, \\ u(x,0) = \sin\left(\dfrac{2\pi x}{L}\right), \ \ u_t(x,0) = \left(2 - \dfrac{2\pi c}{L}\right) \sin\left(\dfrac{2\pi x}{L}\right), \quad 0 < x < L. \end{cases}$$

(a) Prove that the solution $u(x,t)$ to this problem is unique on the interval $[0,\ L]$.

(b) Find the solution.

END OF PAPER

MH4110 PARTIAL DIFFERENTIAL EQUATIONS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.