

NANYANG TECHNOLOGICAL UNIVERSITY
SPMS/DIVISION OF MATHEMATICAL SCIENCES

2023/24 Sem 1 MH5100 Advanced Investigations into Calculus I Week 5

Problem 1. Let n be a positive integer. Show that

$$\lim_{x \rightarrow 0} x^n \sin \frac{1}{x} = 0.$$

Problem 2. Consider the Heaviside function

$$H(t) = \begin{cases} 1, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases}$$

Use the precise definition of a limit to prove that $\lim_{t \rightarrow 0} H(t)$ does not exist.

Problem 3. If the function f is defined by

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

Use the precise definition of a limit to prove that $\lim_{x \rightarrow 0} f(x)$ does not exist.

Problem 4. Consider two functions f and g , and real numbers a and L . Assume that $f(x)$ is continuous at the point L , and that $\lim_{x \rightarrow a} g(x) = L$. Use the definitions of “limit” and “continuous” to prove that

$$\lim_{x \rightarrow a} f(g(x)) = f(\lim_{x \rightarrow a} g(x)).$$

Problem 5. Use the precise definition of a limit to prove that if $\lim_{x \rightarrow x_0} f(x) = A$ and $\lim_{x \rightarrow x_0} g(x) = B$, then $\lim_{x \rightarrow x_0} (f(x) + g(x)) = A + B$.

Problem 6. Use the precise definition of a limit to prove that if $\lim_{x \rightarrow x_0} f(x) = A$ and $\lim_{x \rightarrow x_0} g(x) = B$, then $\lim_{x \rightarrow x_0} f(x)g(x) = AB$.

Problem 7. Use the precise definition of a limit to prove that if $\lim_{x \rightarrow x_0} g(x) = B, B \neq 0$, then $\lim_{x \rightarrow x_0} \frac{1}{g(x)} = \frac{1}{B}$.

Problem 8. Use the precise definition of a limit to prove that if $\lim_{x \rightarrow x_0} f(x) = A$ and $\lim_{x \rightarrow x_0} g(x) = B, B \neq 0$, then $\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \frac{A}{B}$.