

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2015-2016

MH1101 - Calculus II

April 2016

TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. Each candidate is allowed to bring a hand-written A4 size help sheet.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

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QUESTION 1. (15 marks)

Consider the following expression

$$\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{\pi}{4n} \tan^3 \left(\frac{i\pi}{4n} \right) \right). \quad (*)$$

- (a) Rewrite the expression (*) as definite integral.
- (b) Use part (a) to evaluate the expression (*).

QUESTION 2. (15 marks)

- (a) Let $f(x) = (1 - x^3)^{\frac{5}{2}}$. Find $f''(x)$ and show that $|f''(x)| \leq \frac{195}{4}$ for all $x \in [0, 1]$.
- (b) Use part (a) to find a number n such that the error E_M of the approximation of the definite integral

$$\int_0^1 (1 - x^3)^{\frac{5}{2}} dx$$

using Midpoint Rule is not more than 10^{-4} .

QUESTION 3. (15 marks)

Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences. Suppose that $\{a_n\}_{n=1}^{\infty}$ is bounded and $\lim_{n \rightarrow \infty} b_n = 0$.

- (a) Prove that $\lim_{n \rightarrow \infty} a_n b_n = 0$.
- (b) Is $\sum_{n=1}^{\infty} a_n b_n$ necessarily convergent? Justify your answer.

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QUESTION 4. (15 marks)

Let $f(x)$ be a function and r be a real number. Prove that $f(x)$ is continuous at $x = r$ if and only if

$$\lim_{n \rightarrow \infty} f(a_n) = f(r)$$

for any sequence $\{a_n\}_{n=1}^{\infty}$ which converges to r .

QUESTION 5. (20 marks)

Find the radius and interval of convergence of the following series, and identify the values of x for which the series converges absolutely or conditionally.

$$\sum_{n=2}^{\infty} \frac{(-1)^n (x-1)^n}{\sqrt{n} \ln(n)}.$$

QUESTION 6. (20 marks)

(a) Show that

$$\int_0^x \sin(t^2) dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(4n+3)} x^{4n+3}$$

for all $x \in (-\infty, \infty)$.

(b) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{x^2 \arctan(x) - 3 \int_0^x \sin(t^2) dt}{x^5}.$$

END OF PAPER

MH1101 CALCULUS II

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.