Problem 1. *Let* $f(x) = \frac{ax+b}{cx+d}$. *Under what conditions* $f(x) = f^{-1}(x)$?

Problem 2. *Prove that*
$$\max\{a,b\} = \frac{1}{2}(a + b + |a - b|), \quad \min\{a,b\} = \frac{1}{2}(a + b - |a - b|)$$

Problem 3. *Let* $f(x)$ *be defined on the closed interval* $[-a, a]$, *prove that* f *can be expressed as the sum of an odd function and an even function. And show that there is only one way to write* f *as the sum of an even and an odd function.*

Problem 4. *Given the equation* $a_0 x^n + a_1 x^{n-1} + \cdots + a_n = 0$, *where* $a_0, a_1, \cdots, a_n$ *are integers and* a_0 *and* $a_n \neq 0$. *Show that if the equation is to have a rational root* p/q, *then* p *must divide* a_n *and* q *must divide* a_0 *exactly.*

Problem 5. *Prove that* $\sqrt{2} + \sqrt{3}$ *cannot be a rational number.*

Problem 6. *If* $a_1, a_2, \ldots, a_n$ *and* $b_1, b_2, \ldots, b_n$ *are any real numbers, prove* **Schwarz's inequality**.
$$(a_1 b_1 + a_2 b_2 + \cdots + a_n b_n)^2 \leq (a_1^2 + a_2^2 + \cdots a_n^2)(b_1^2 + b_2^2 + \cdots b_n^2).$$