NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2018-2019

MH1101/MH111S – Calculus II

May 2019 Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages. A formulae table is provided on page 4 of the paper.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. Each candidate is allowed to bring **ONE (1)** hand-written, double-sided A4 size help sheet.
5. Candidates may use calculators. However, they should lay out systematically the various steps in the workings.

Question 1. (15 marks)

(a) Use the Trapezoidal Rule with $n = 4$ to approximate the integral $\int_3^5 \ln x \, dx$.

(b) Using the Error Bound formula, find an appropriate n such that the approximation T_n of the integral $\int_3^5 \ln x \, dx$ using the Trapezoidal Rule has an absolute error of at most 10^{-3}.

Question 2. Evaluate the following integrals. (15 marks)

(a) $\int 2x(\ln x)^2 \, dx$.

(b) $\int \frac{2x^3 + 18x - 1}{(x^2 + 9)^2} \, dx$.

Question 3. Determine whether each of the following series converges or diverges. Justify your answer. (15 marks)

(a) $\sum_{n=1}^{\infty} \ln \left(\frac{n^4 + 2n^3}{2n^4 + n^3} \right)$.

(b) $\sum_{n=1}^{\infty} \frac{7^n + n^2}{8^n - n}$.

(c) $\sum_{n=1}^{\infty} n! e^{-n^2}$.

Question 4. Let $a_1 = c$ for some positive real number $c > 1$. For $n \geq 1$, define

$$a_{n+1} = 2 - \frac{1}{a_n}.$$ (15 marks)

(i) Compute the value of a_2 and a_3 in terms of c.

(ii) Show that $a_3 - a_2 < 0$.

(iii) Show that the sequence $\{a_n\}_{n=2}^{\infty}$ is bounded and decreasing.

(iv) Show that $\lim_{n\to\infty} a_n$ exists and find its value.

Question 5. (20 marks)

(a) Find the interval of convergence of the following power series, and for every $x \in \mathbb{R}$, determine whether the series converges absolutely, converges conditionally or diverges at x. Justify your answer.

$$\sum_{n=2}^{\infty} \frac{(3x-7)^n}{n \ln n}$$

(b) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $\lim_{n\to\infty} n^2 a_n = M$ for some real number $M \in \mathbb{R}$. Does the series $\sum_{n=1}^{\infty} a_n$ converge absolutely? Justify your answer.

Question 6. (20 marks)

(a) Find a power series with center $c = 1$ that represents the function $f(x) = \dfrac{1}{3-x}$.

(b) Using the Maclaurin series, find the fourth derivative $f^{(4)}(0)$ at the point 0 of the function $f(x) = \sin(2x)\sqrt{1+x}$.

(c) Using power series or otherwise, find the sum $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+2)!}$.

Formulae Table

$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	$\int e^x\,dx = e^x + C$
$\int k\,dx = kx + C$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \tan x\,dx = \ln\lvert\sec x\rvert + C$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$	

$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$	$\sin 2x = 2\sin x\cos x$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin A\cos B = \dfrac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$	$\sin A\sin B = \dfrac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$
$\cos A\cos B = \dfrac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$	

$\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$R = 1$
$(1+x)^k = \displaystyle\sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$	$R = 1$

END OF PAPER