NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2024-2025

MH4701 - Mathematical Programming

December 2024 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **CLOSED BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(25 marks)**

The Newton descent method, using the Armijo rule with $s = 1$ and $\beta = \sigma = 0.6$, is applied to the function

$$f : x \in \mathbb{R}^2 \mapsto \|x\|^3 .$$

Prove that if the initial iterate is non-zero, then a sequence of iterates that converges linearly to $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is generated, and determine its rate of convergence.

QUESTION 2. **(25 marks)**

Prove that for any differentiable convex functions $f, g : \mathbb{R}^n \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$, the function $(x, y) \in \mathbb{R}^n \times \mathbb{R} \mapsto g(x) + h(y)$ has a global minimum over the epigraph

$$epi(f) = \{(x, y) \in \mathbb{R}^n \times \mathbb{R} : f(x) \le y\}$$

at $(a, b) \in epi(f)$ if and only if

$$h'(b) \ge 0, \quad h'(b) f(a) = h'(b) b, \quad \text{and} \quad h'(b) \nabla f(a) + \nabla g(a) = 0.$$

QUESTION 3. **(25 marks)**

The augmented Lagrangian method with the method of multipliers is applied to minimize f subject to $h(x) = 0$, where

$$f : x \in \mathbb{R}^2 \mapsto 2x_1^2 - 2x_1x_2 + x_2^2 + x_1$$

and

$$h : x \in \mathbb{R}^2 \mapsto 2x_1 - x_2 + 3.$$

Prove that if a sequence of positive penalty parameters $\{c_k\}$ that strictly increases to ∞ is used and an infinite sequence of iterates $\{x_k\}$ is generated, then $\{x_k\}$ converges superlinearly to a global minimum solution of f subject to $h(x) = 0$.

QUESTION 4. **(25 marks)**

Use the first- and second-order optimality conditions to find all local minima of $f : x \in \mathbb{R}^2 \mapsto 3x_1 - 2x_2$ subject to $g(x) \leq 0$, where $g : x \in \mathbb{R}^2 \mapsto -2x_1^2 + x_2^2 + 2$.

END OF PAPER