# NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2024-2025

**MH1802 – Calculus for the Sciences**

Nov/Dec 2024 Time Allowed: 2 Hours

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INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **SIX (6)** pages.

2. Answer **ALL FIVE (5)** questions.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).

1. **(25 marks)** Short questions.

   (a) Find the limit of:

   $$\lim_{x \to 16} \frac{x^2 - 256}{\sqrt{x+9} - 5}$$

   (3 marks)

   (b) Calculate $y'$ where

   $$y = \cos^{-1}(1 - e^x)$$

   (3 marks)

   (c) Find the domain of:

   $$y = \cosh^{-1}(1 + x)$$

   (2 marks)

   (d) Find the slope of the normal line to the curve (up to two decimal places):

   $$y = \ln \sqrt{x^3 - 1}$$

   at $x = 2$.

   (3 marks)

   (e) Find the $\frac{dy}{dx}$ of:

   $$\cos(x^2 + 2y) + xe^{y^2} = 1$$

   (3 marks)

   (f) Find the antiderivative of:

   $$y = \frac{\tanh(\ln x)}{x}$$

   (4 marks)

   (g) Calculate the definite integral of:

   $$\int_1^2 (1 + \sin x)^3 \, dx$$

   (3 marks)

   (h) An object is stationary at time $t = 0$ and its acceleration as a function of $t$ is given by $a(t) = te^t$ (in the unit of meter per second squared). What is the total distance travelled after 2 seconds?

   (4 marks)

2. **(20 marks)** Consider the curve of the following function

$$y(x) = \frac{2 + x^2/2}{1 - x^2}$$

(a) Write down the equation(s) for the vertical and horizontal asymptotes, if there are any. (4 marks)

(b) Find $y'(x)$ and $y''(x)$. (4 marks)

(c) Find the critical point(s) of $f$, and determine if they are the local maxima, local minima or inflection points. (4 marks)

(d) Find the intervals of $x$ where the function is (i) increasing with respect to $x$, (ii) decreasing with respect to $x$. (4 marks)

(e) Determine the concavity of the curve. (4 marks)

3. **(25 marks)** Consider the following function

$$f(x) = \frac{1}{1+\sin x} - 1$$

(a) Treating $\sin x$ as the variable, compute the Taylor polynomial $T_3(\sin(x))$ of $f(x)$ about $\sin x = 0$.

(4 marks)

(b) Find the Taylor polynomial $T_3(x)$ of $f(x)$ about $x = 0$.

(5 marks)

(c) Use linear approximation of $f(x)$ to compute $f(0.02)$ up to five decimal places. Compare your result with that in (b) and comment on which value is more accurate.

(5 marks)

(d) Use $T_3(\sin(0.02))$ to approximate the value of $f(0.02)$ up to five decimal places, and explain why it is different from $T_3(0.02)$.

(5 marks)

(e) Show that there is at least one root of $f(x)$ for $3 < x < 4$, using the Intermediate Value Theorem.

(2 marks)

(f) Using Newton's method, find an approximate root of $f$, accurate up to three decimal places. Choose your own initial guess properly. (4 marks)

4. **(20 marks)**Start with two functions $y_1 = x\sin(x^2)$ and $y_2 = \frac{x}{2}$,

   (a) Find the coordinates of the two intersections of $y_1, y_2$ for $x \geq 0$.
   (2 marks)

   (b) Find the area enclosed by $y_1$ and $y_2$.
   (4 marks)

   (c) Find the circumference of the enclosed area. You can leave your answer as a definite integral.
   (4 marks)

   (d) For the area enclosed by $y_1$ and $y_2$, if we revolve it about the y-axis, find the volume generated. You may need to use the numerical result that $\int_0^{\sqrt{\pi/6}} x^2\sin(x^2)dx = 0.0387$.
   (5 marks)

   (e) For the area enclosed by $y_1$ and $y_2$, if we revolve it about the x-axis, find the volume generated. You may need to use the numerical result that $\int_0^{\sqrt{\pi/6}} x^2\sin^2(x^2)dx = 0.014$.
   (5 marks)

5. **(10 marks)** Miscellaneous questions.

(a) Solve the following equation with the condition $y(0) = 0, y'(1) = 1$:

$$y'' - 10y' + 25y = 0$$

(3 marks)

(b) Find the power series of $f(x) = \frac{1}{x^2+2x-3}$. (3 marks)

(c) (Complex numbers) Solve the equation $x^4 + x^2 + 5/2 = 0$. (4 marks)

- End of Paper -

**MH1802 CALCULUS FOR THE SCIENCES**

**CONFIDENTIAL**

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.