

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2024-2025

MH1811 MATHEMATICS 2

December 2024

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)
5. Candidates may use calculators. However, they should lay out systematically the various steps in the workings.

QUESTION 1.

(18 Marks)

- (a) Suppose z is a function of two independent variables x and y implicitly defined by the equation

$$x - y + 3z = z^2(x + 2y).$$

Find the value of $\frac{\partial z}{\partial y}$ when $x = 1$, $y = 1$ and $z = 0$.

- (b) Let $F(x, y, z) = -x^2 + 8y^2 + 2z^2$, and $G(x, y, z) = -4x - 2y + 8z$.

- (i) Find the gradient vectors $\nabla F(x, y, z)$ and $\nabla G(x, y, z)$.
- (ii) Find all the point(s) on the surface $F(x, y, z) = 66$ at which the tangent plane is parallel to the plane $G(x, y, z) = 1$.

QUESTION 2.

(18 Marks)

- (a) Suppose that $z = f(x, y)$, where $x = u^2 + v^2$ and $y = u - v$. Using Chain Rule, express $\frac{\partial z}{\partial u}$ and $\frac{\partial^2 z}{\partial u^2}$ in terms of u , v and the partial derivatives (including second order if necessary) of $f(x, y)$. You may assume that $f_{xy}(x, y) = f_{yx}(x, y)$.
- (b) A clinic has a total annual budget of \$900,000 to hire both doctors and nurses. A doctor's annual salary is \$60,000, and a nurse's annual salary is \$30,000. The company's annual revenue (in millions), V , is given as a function of the number of doctors, D , and the number of nurses, N , by

$$V = 10D^{0.8}N^{0.4}.$$

How many doctors and nurses the clinic need to hire so as to **maximize** the annual revenue? Justify your answer.

QUESTION 3.**(15 Marks)**

- (a) Evaluate the double integral $\iint_D 2x \, dA$, where D is the region bounded by the lines $y = 0$ (i.e. the x -axis), $y = x$ and the curve $y = \sqrt{2 - x^2}$ in the first quadrant.
Hint: Use the order $dx \, dy$.

- (b) By changing the order of integration, evaluate the following iterated integral:

$$\int_1^e \int_{\ln x}^1 \sqrt{e^y - y} \, dy \, dx.$$

QUESTION 4.**(16 Marks)**

Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{4n^2 + 1}{2n^2 + n}$

(b) $\sum_{n=2}^{\infty} \frac{4^n + 3^n}{5^n - 2^n}$

(c) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n + \sqrt{n}}\right)$

QUESTION 5.**(16 Marks)**

- (a) Find the interval of convergence of the following power series and, for every $x \in \mathbb{R}$, determine whether the series converges absolutely, converges conditionally or diverges at x . Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\ln(n+1)}$$

- (b) By differentiating a geometric series or otherwise, find a function represented by the following power series on the interval $(-2, 2)$:

$$\sum_{n=1}^{\infty} \frac{nx^{3n+1}}{8^n}.$$

QUESTION 6.**(17 Marks)**

Find the general solutions of the following ordinary differential equations:

- (a)

$$2y' + y \tan x = \frac{6x^2 y^3}{\cos x}.$$

Hint: Bernoulli ODE

- (b)

$$y'' + 4y' + 3y = \frac{2}{1 + e^x}.$$

END OF PAPER

Please read the following instructions carefully:

- 1. Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**
2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.
3. Please write your Matriculation Number on the front of the answer book.
4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.