Mean and variance of NDVI and the Beta distribution

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Since NDVI ranges $\nu \in [-1, 1] \approx (-1, 1)$, we can apply the linear transformation $\nu_* = \frac{\nu+1}{2}$ so that $\nu_* \in (0, 1)$, and $\nu_* \sim \mathrm{Beta}(a_*, b_*)$. We can re-parameterize the distribution as $\nu_* \sim \mathrm{Beta}(\mu_*, \phi_*)$ in terms of the mean

$$\mu_* = \frac{a_*}{a_* + b_*}$$

and scale

$$\phi_* = \frac{\sigma_*^2}{\mu_* \ (1 - \mu_*)} = \frac{a_* b_*}{(a_* + b_*)^2 (a_* + b_* + 1)} \frac{a_* + b_*}{a_*} \frac{1}{\frac{a_* + b_* - a_*}{a_* + b_*}} = \frac{1}{(a_* + b_* + 1)}.$$

For each parameter, we have the ranges

$$\mathrm{R}(\mu_*) = \left(\frac{0}{0 + \infty}, \frac{\infty}{\infty + 0} \right) = (0, 1),$$

$$\mathrm{R}(\sigma_*^2) = \left(\frac{0}{(0)^2 (0 + 0 + 1)}, \frac{c^2}{4c^2 (2c + 1)} \right) = \left(\frac{0}{(0)^2 (0 + 0 + 1)}, \frac{1}{8(0^+) + 4} \right) = (0, 0.25),$$

and

$$\mathrm{R}(\phi_*) \left(\frac{1}{(\infty + \infty + 1)}, \frac{1}{(0 + 0 + 1)} \right) = (0, 1).$$

For NDVI, the mean is

$$\mu = \mathrm{E}(\nu) = \mathrm{E}(2\nu_* - 1) = 2 \ \mathrm{E}(\nu_*) - 1 = 2\mu_* - 1,$$

the variance is

$$\sigma^2 = \mathrm{Var}(\nu) = \mathrm{Var}(2\nu_* - 1) = 4 \ \mathrm{Var}(\nu_*) = 4\sigma_*^2,$$

and the ranges for μ and σ^2 are $\mathrm{R}(\mu) = (-1, 1)$ and $\mathrm{R}(\sigma^2) = (0, 1)$, respectively. Calculating the scale parameter for NDVI is not necessary because our focus is on the mean and variance of NDVI.