ML for optimal stopping of Brownian motion

Initial problem

$$V_* = \inf_{\tau \in M} E(B_\tau - \max_{0 \le t \le 1} B_t)^2 \xrightarrow{\text{We want to find } z \text{ that describes } B_\tau} V_* = \inf_{\tau \in M} E(z - \max_{0 \le t \le 1} B_t)^2$$

How it can be solved?

Step 1: find explicitly the probability that maximum is less than a threshold

$$P(\max_{0 \le t \le 1} B_t \le z)$$

Results from the article

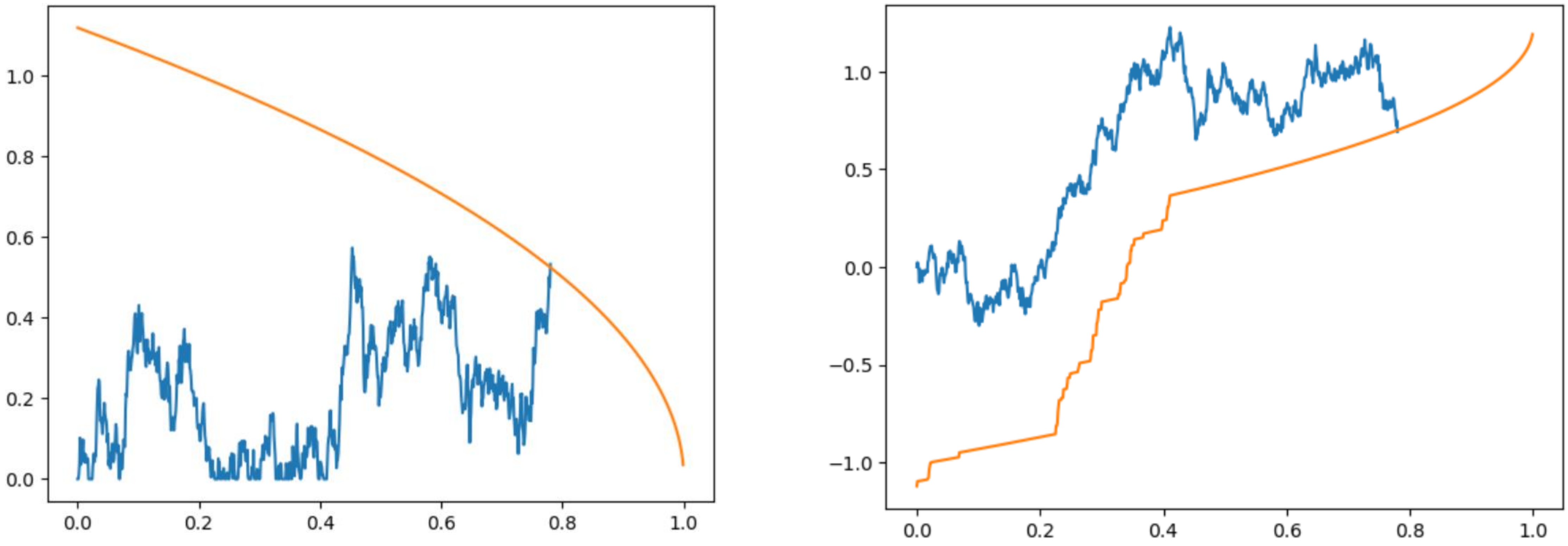
$$P(\max_{0 \le t \le 1} B_t \le z) = 2\phi(z) - 1$$

Using the following equation find needed threshold:

$$4\phi(z_*) - 2\phi(z_*)z_* - 3 = 0 \longrightarrow z_* = 1.12...$$

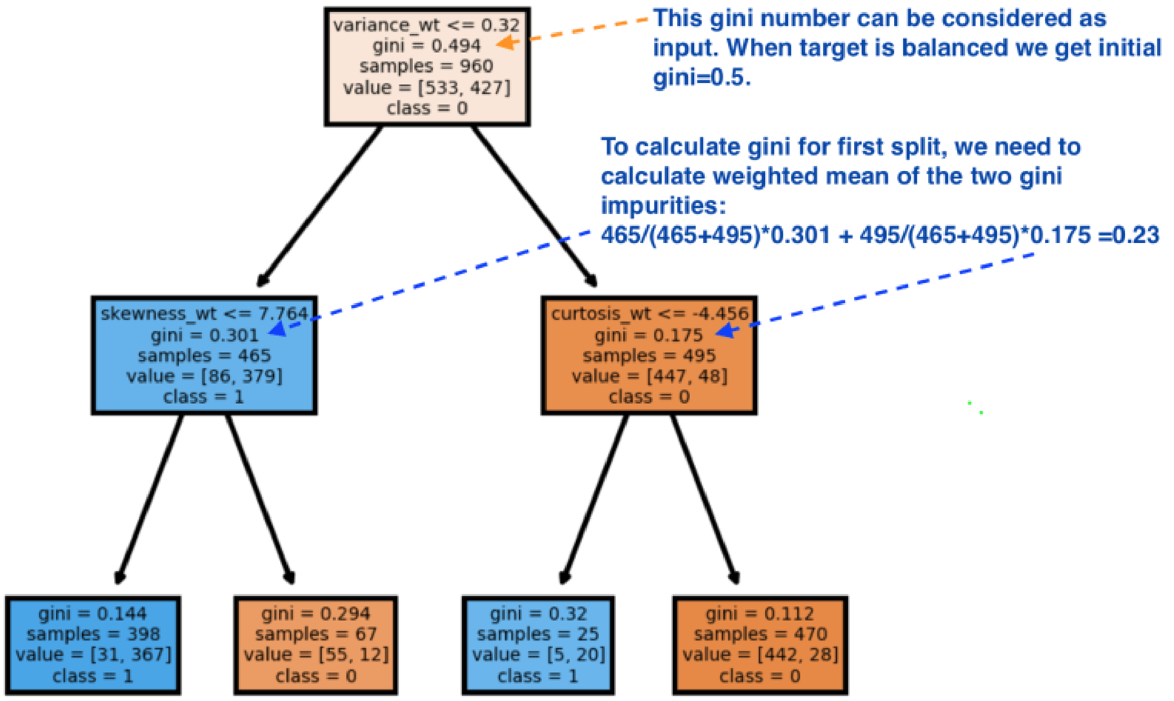
Use found level to form optimal stopping rule

$$\tau_* = inf\{0 \leq t \leq 1 | S_t - B_t \geq z_* \sqrt{1-t}\}$$



Some ML theory

Decision tree



Split criterion

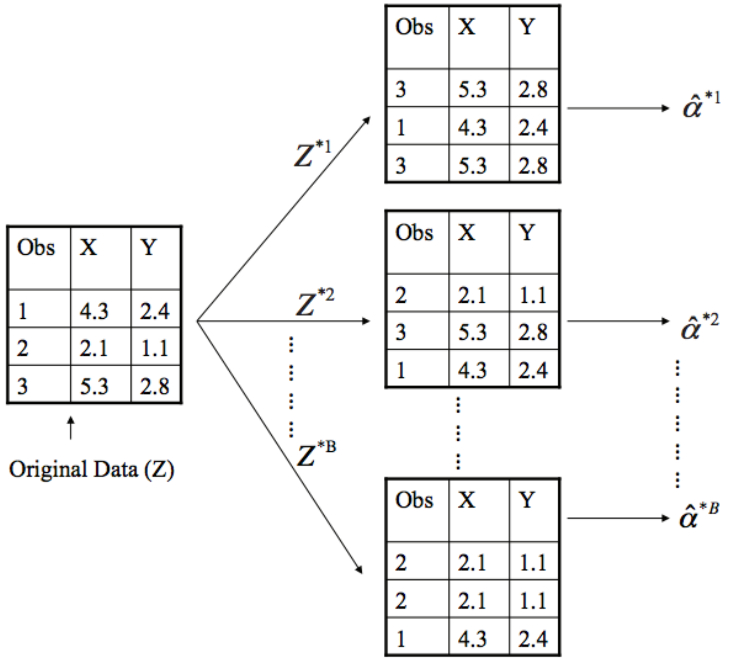
Mathematically, The Gini Index is represented by

$$Gini\ impurity = 1 - \Sigma(p(i)^2)$$

Another commonly used formula is:

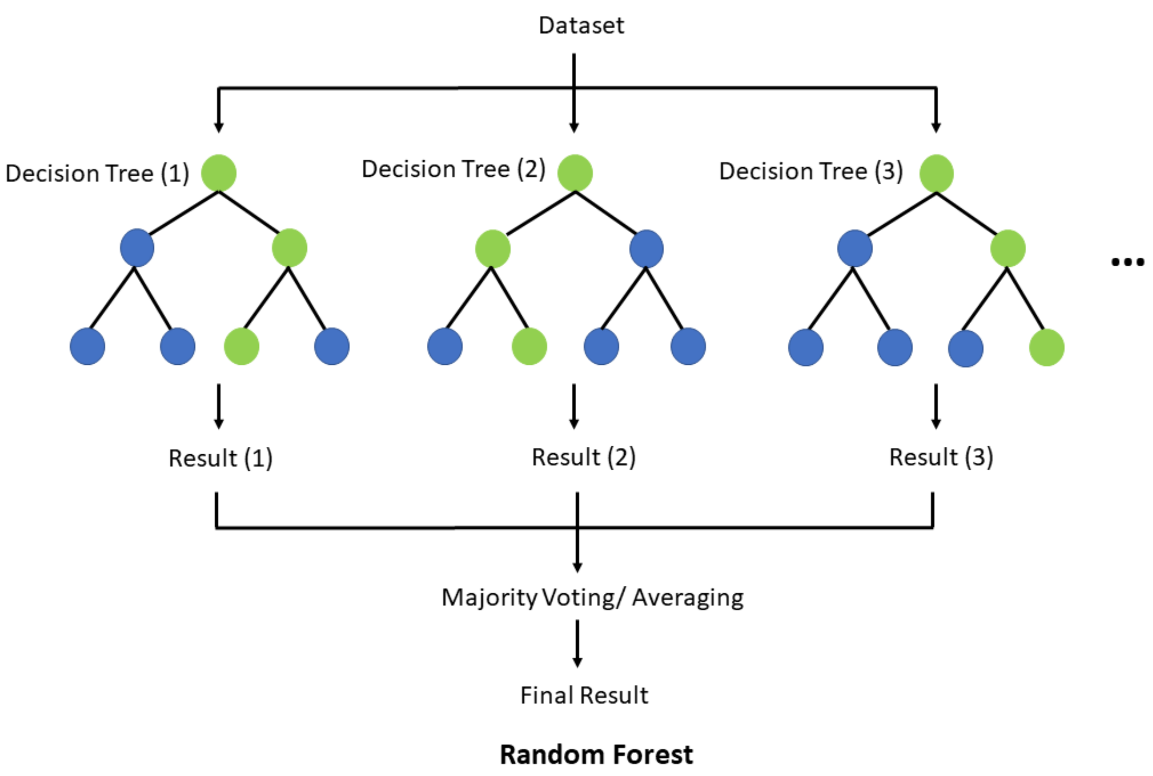
$$Gini\ impurity = 1 - \Sigma\left(p(i) * \left(1 - p(i)\right)\right)$$

Bootstrap

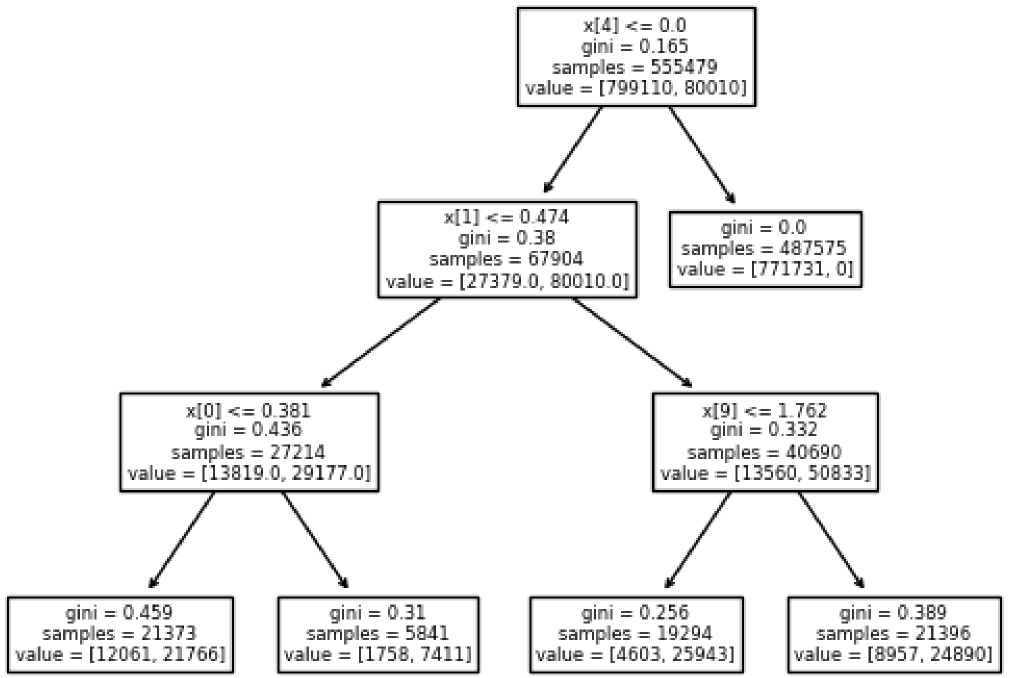


Random forest

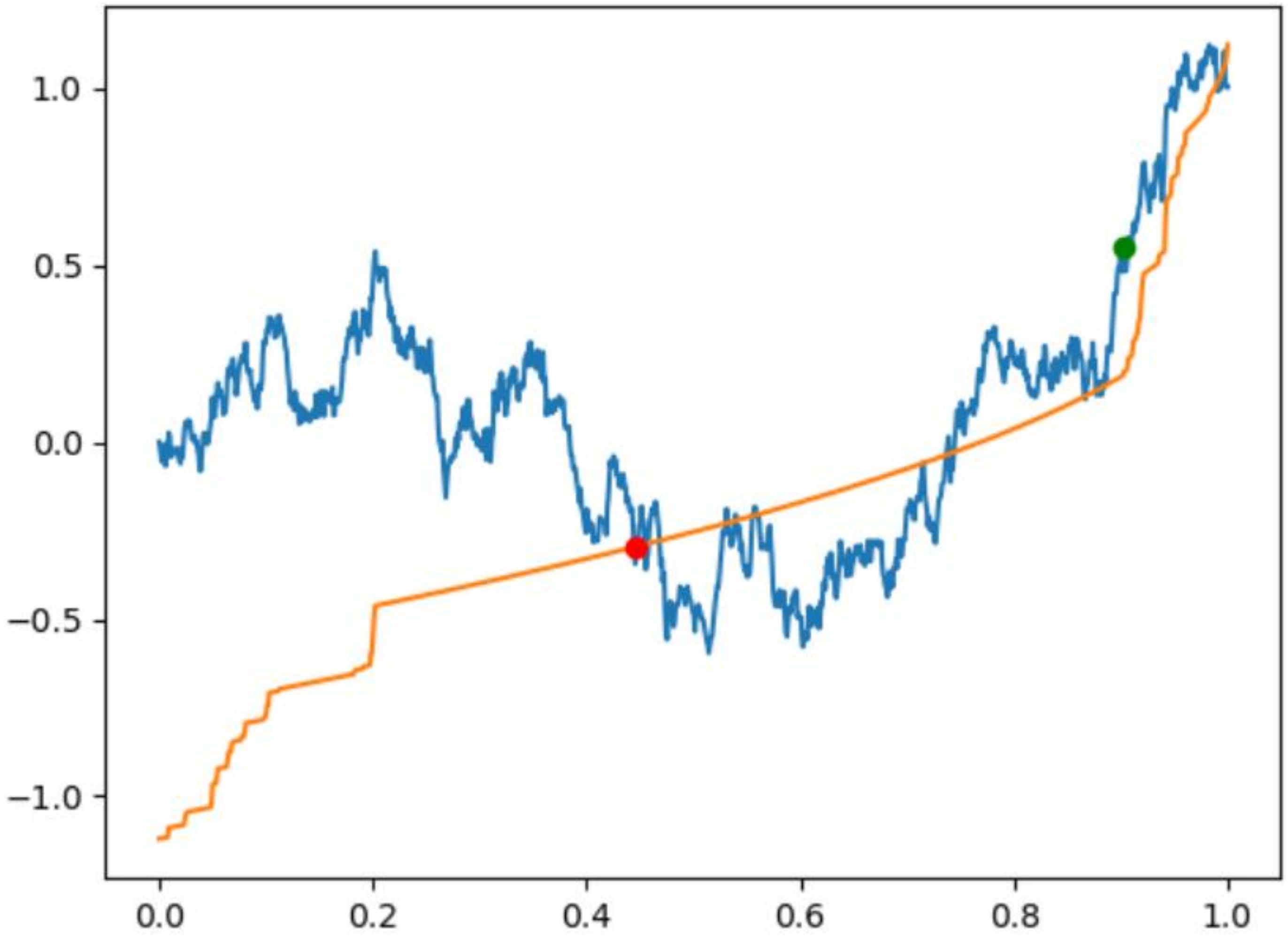
Scheme



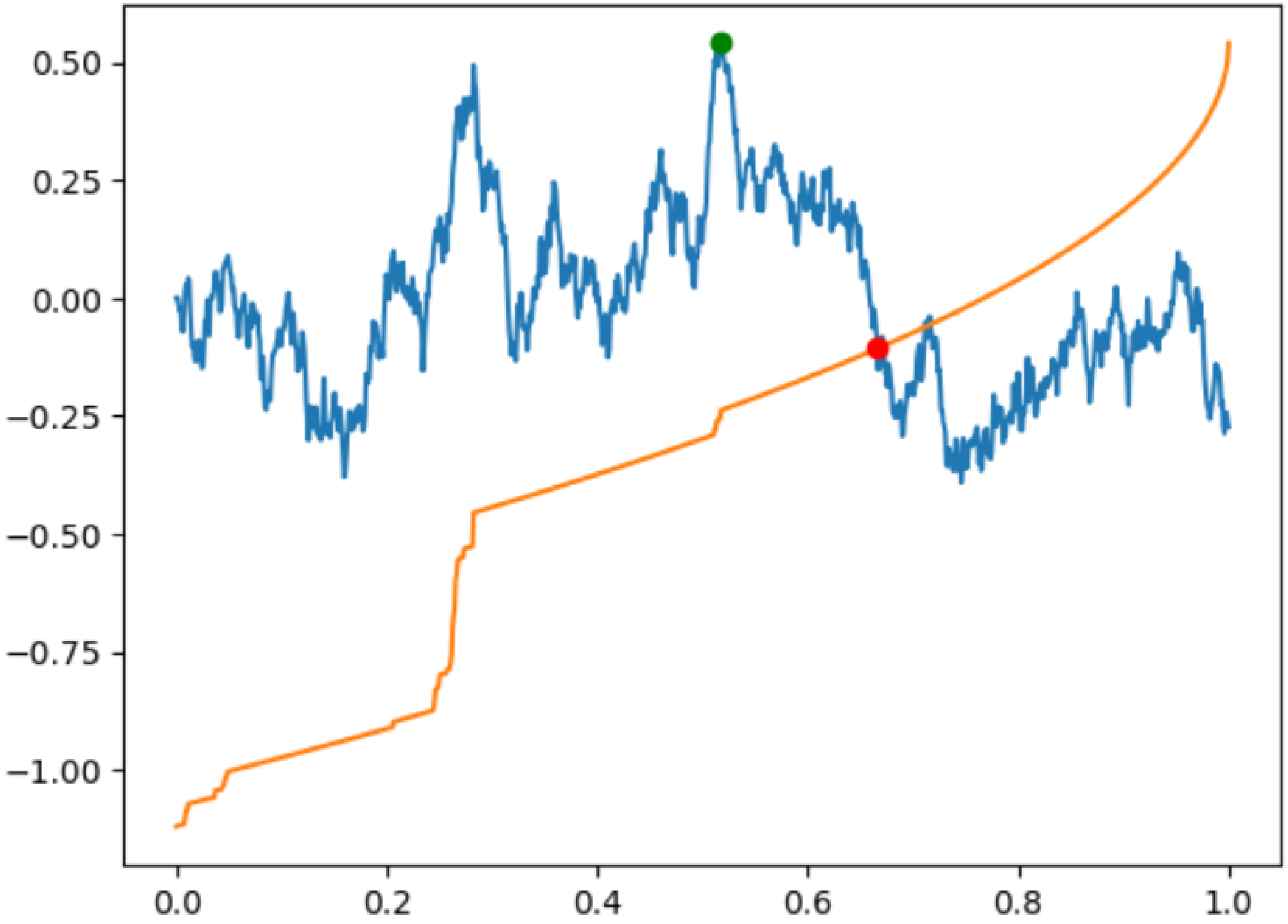
Actual example



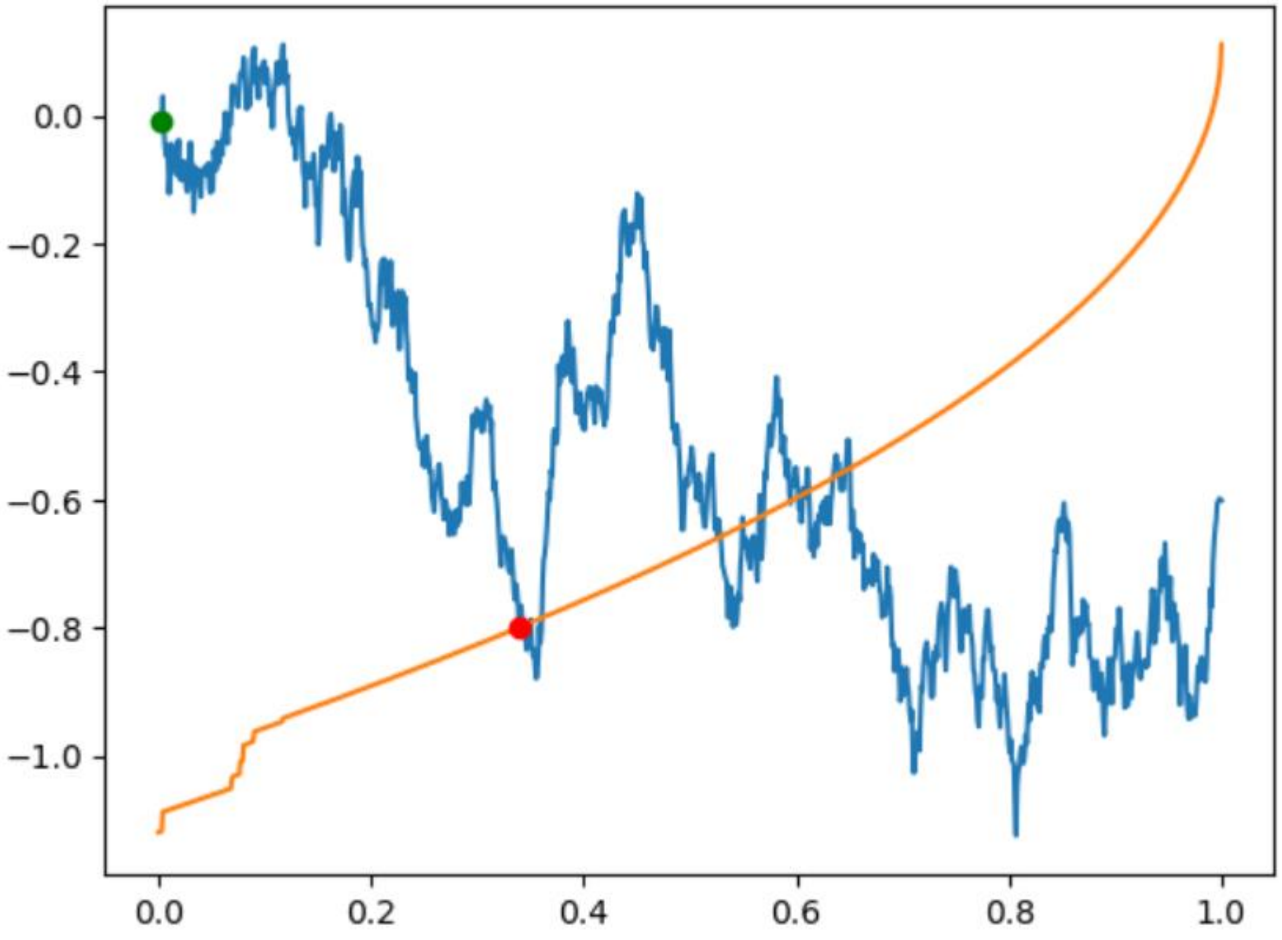
Examples of predictions



Examples of predictions

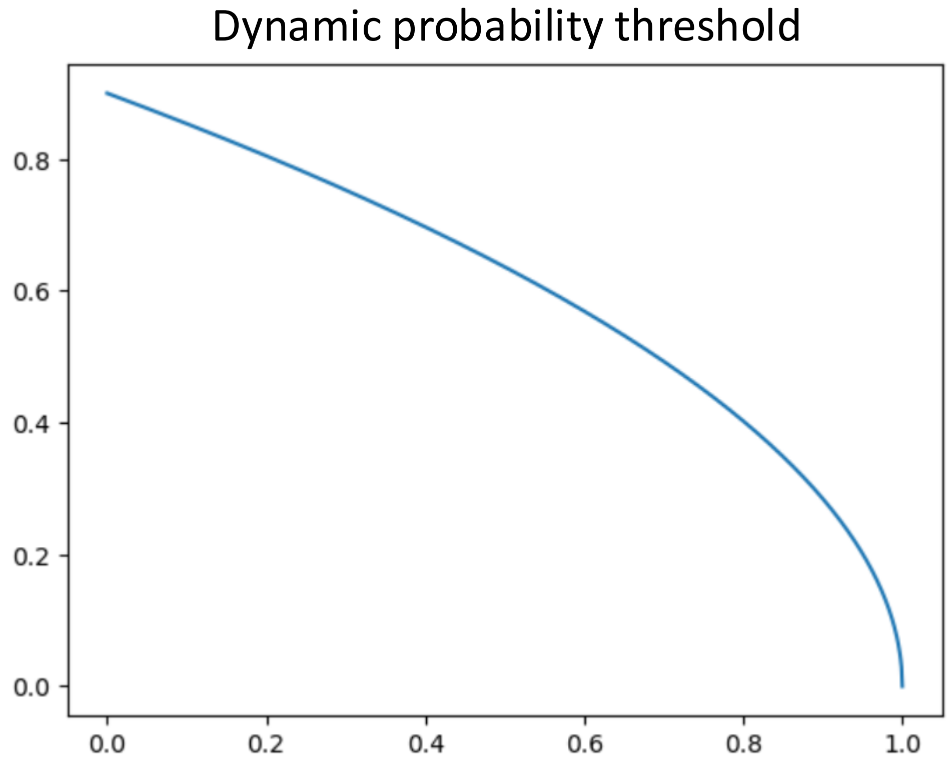


Examples of predictions



Prediction approach

1. Balance the set
2. Tune the tree
3. Train the model
4. Predict probability
5. Compare with the benchmark



$$threshold * \sqrt{1 - t}$$

Left to fix

Falling trajectories are still an issue:

- Add stopping condition to avoid skipped trajectories (new variables, non-ML stopping criterion)
- Adjust the probability threshold
- Use additional model or criterion to have two-sided filter
- Test another models

Results of testing the ML approach