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EME 171 Lab #4: The Drowning of Sacramento

Bond Graph and Equations of Motion

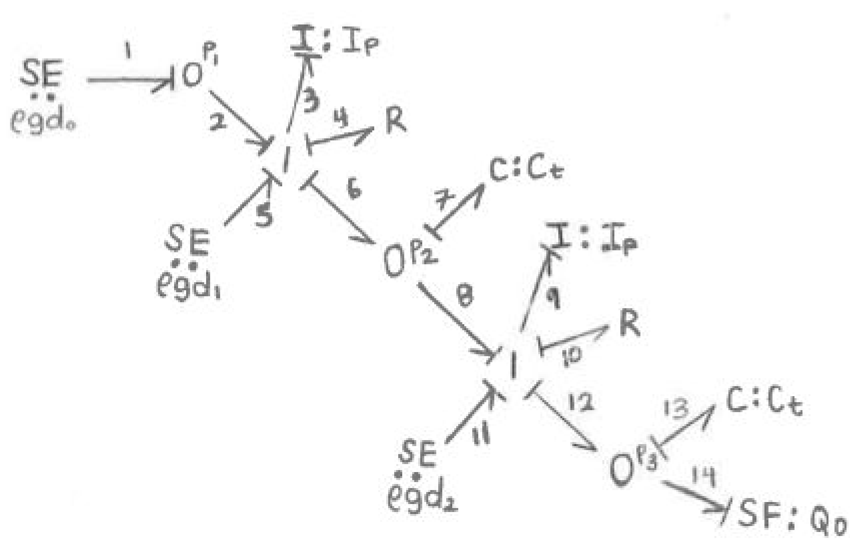

Figure 1: Bond graph with causality and numbered bonds

$$\vec{x} = [P_3 \;\; q_7 \;\; P_9 \;\; q_{13}]^T$$

$$\dot{\boldsymbol{P}}_{\boldsymbol{3}} = e_3 = SE1 + SE5 - C_f\left(\frac{1}{I_3}P_3\right)\left|\frac{1}{I_3}P_3\right| - \frac{1}{C_7}q_7$$

$$\dot{\boldsymbol{q}}_{\boldsymbol{7}} = f_7 = \frac{1}{I_3}P_3 - \frac{1}{I_9}P_9$$

$$\dot{\boldsymbol{P}}_{\boldsymbol{9}} = e_9 = SE11 + \frac{1}{C_7}q_7 - C_f\left(\frac{1}{I_9}P_9\right)\left|\frac{1}{I_9}P_9\right| - \frac{1}{C_{13}}q_{13}$$

$$\dot{\boldsymbol{q}}_{\boldsymbol{13}} = f_{13} = \frac{1}{I_9}P_9 - SF14$$

Initial Conditions

The system begins in equilibrium with an initial turbine flow of Q_o = 1.5 m^3/s. The initial conditions are determined from the equations of motion, with all the state derivatives equaling zero. The initial conditions are listed below:

1) $\dot{q_{13}} = 0 = \frac{1}{I_9} P_9 - SF14$

$$\Rightarrow \quad \boldsymbol{P_{9,initial}} = (SF14) * I_9 = 1.5 * I_p$$

2) $\dot{q_7} = 0 = \frac{1}{I_3} P_3 - \frac{1}{I_9} P_9$

$$\Rightarrow \quad \boldsymbol{P_{3,initial}} = \frac{P_{9,inital}}{I_9} I_3 = \frac{P_{9,inital}}{I_p} I_p = P_{9,inital}$$

3) $\dot{P_3} = 0 = SE1 + SE5 - C_f\left(\frac{1}{I_3} P_3\right)\left|\frac{1}{I_3} P_3\right| - \frac{1}{C_7} q_7$

$$\Rightarrow \quad \boldsymbol{q_{7,initial}} = C_7\left(SE1 + SE5 - C_f\left(\frac{1}{I_3} P_{3,initial}\right)\left|\frac{1}{I_3} P_{3,initial}\right|\right)$$

$$= C_t\left(\rho g d_o + \rho g d_1 - C_f(1.5 * 1.5) * \frac{P_{3,initial}}{I_p}\right)$$

4) $\dot{P_9} = 0 = SE11 + \frac{1}{C_7} q_7 - C_f\left(\frac{1}{I_9} P_9\right)\left|\frac{1}{I_9} P_9\right| - \frac{1}{C_{13}} q_{13}$

$$\Rightarrow \quad \boldsymbol{q_{13,initial}} = C_{13}\left(SE11 + \frac{1}{C_7} q_7 - C_f\left(\frac{1}{I_9} P_9\right)\left|\frac{1}{I_9} P_9\right|\right)$$

$$= C_t\left(\rho g d_2 + \frac{q_{7,initial}}{C_t} - \left(C_f(1.5 * 1.5) * \frac{P_{9,initial}}{I_p}\right)\right)$$

Minimum Tank Area

The minimum possible area of the tank, A_t, that correspond tanks to overflow when the turbine is shut off is determined through simulation. The maximum tank heights before overflow for tanks 1 and 2 are 45 meters and 65 meters, respectively. A tank area of $5A_P = 0.5m^2$ is used initially. The resultant tank heights are then determined by plotting the volumetric displacements divided by the tank area. The max command is then used to find the maximum height. Following this procedure, the tank area was increased by increments of $0.05m^2$ until the smallest possible tank area that satisfies our parameters was found.

The minimum possible area of the tank was determined to be $0.71m^2$ with the max heights of tank 1 and 2 being 42.85 meters and 64.96 meters, respectively.

Time Control Parameters

The time control parameters are determined from the natural frequencies of the system. Due to the nonlinearity of the pipe's resistance, the compliance and inertial properties of the system are solely used, resulting in a rough approximation. In addition to this, it is assumed that there is negligible tank inertia compared to the pipe inertia due to the discrepancy in areas. The general formula used to calculate the natural frequencies of the system is:

$$T = \frac{1}{f_n},\ f_n = \frac{\omega_n}{2\pi},\ \omega_n = \sqrt{\frac{1}{\text{compliance}\times\text{inertia}}}.$$

The system has two natural frequencies, but since the compliance and inertial properties of the tanks are the same (with resistance being ignored), only one calculation for the time period of the simulation is necessary. This calculation is shown below:

$$C_t = \frac{A_t}{\rho g} \qquad I_p = \frac{\rho L}{A_p}$$

$$\omega_n = \sqrt{\frac{A_p g}{A_t L}} \rightarrow f_n = \frac{\sqrt{\frac{A_p g}{A_t L}}}{2\pi}$$

$$\therefore\ T = \frac{2\pi}{\sqrt{\frac{A_p g}{A_t L}}}$$

The natural frequency for the smallest possible area of the tank, A_t, was determined to be 0.166 rad/sec or 0.0265 Hz, with the period being 37.8 seconds. The maximum step size is set to be one-tenth of the vibration period, resulting in a step size of 3.78 seconds.

$$Time\ Step = \frac{T}{10} = \frac{\pi}{5\sqrt{\frac{A_p g}{A_t L}}}$$

Plots

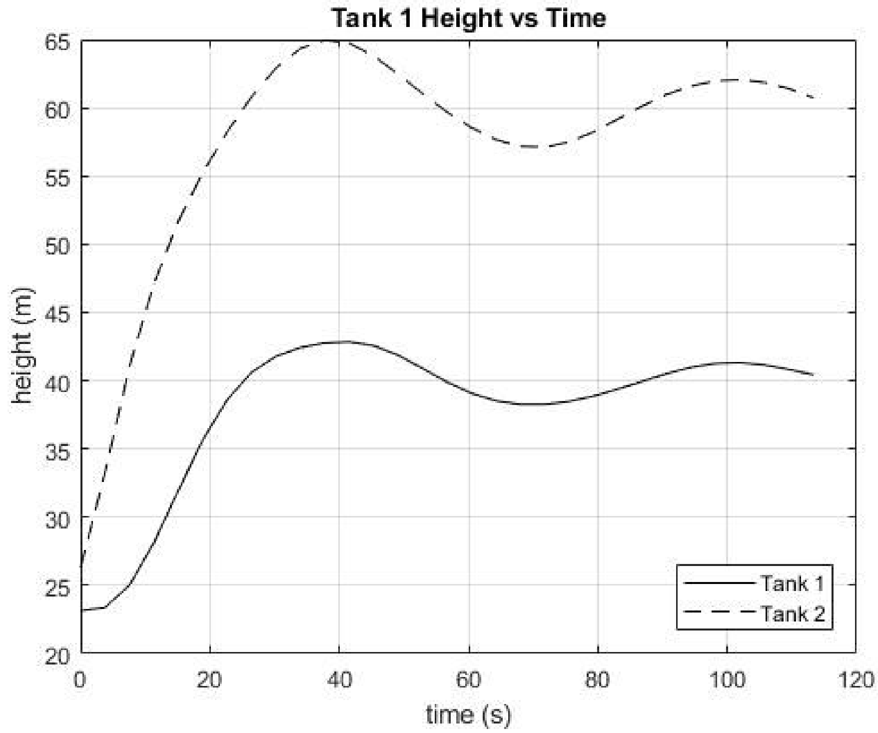

Figure 2: Height vs. Time plot to determine minimum tank area

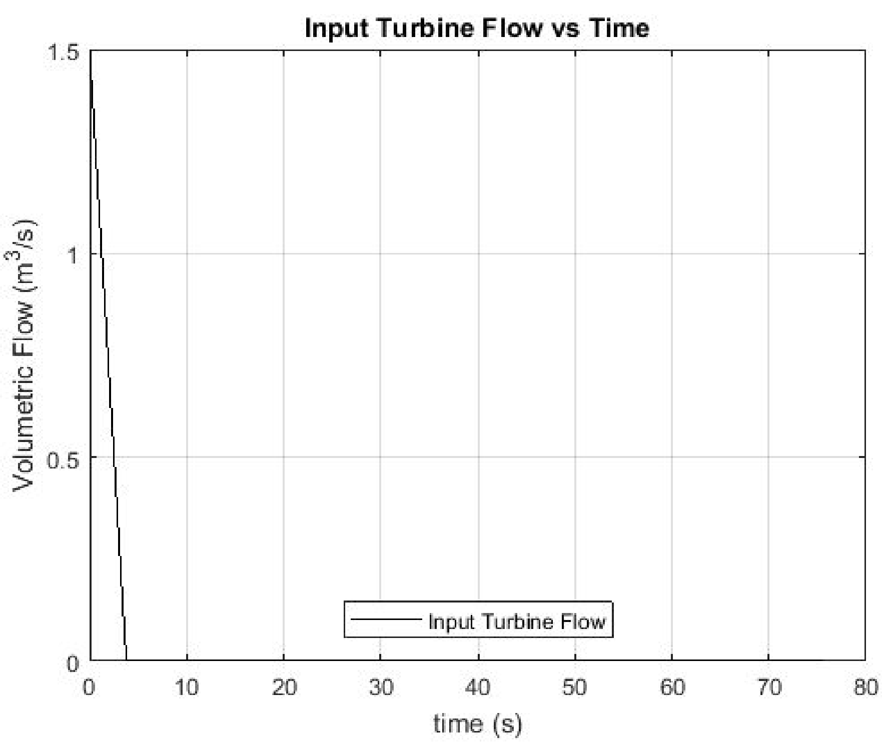

Figure 3: Input Turbine Flow vs Time

Discussion

1) Which tank came closer to overflowing? Why?

   The second tank came closer to overflowing. Both tanks were very close however, but the second tank was filled much faster because the pressure at the bottom of the second tank is much higher than the pressure at the first tank and as the turbine is slowing, the momentum of the water will still be moving downward and would fill the second tank first.

2) How do the calculated natural frequencies compare to the vibration periods seen in your plotten results? Why is there a difference?

   The calculated periods were much lower compared to the vibration periods seen in the plotted results. This difference is due to neglecting the pipe's resistance in the time control parameter calculations. The pipe's resistance was neglected due to its nonlinearity to obtain simplicity in exchange for accuracy. If the resistance of the pipe was included in the calculations, the natural frequencies would become smaller. Resultantly, this would result in a larger vibration period. Due to this discrepancy, figure 2's time span was changed to 3T from 2T as a time span of 2T did not adequately show the system's response.

   In addition to neglecting the pipe resistance, when calculating natural frequencies, it is assumed that the systems are independent, when in reality they are dependent on each other, or coupled together. This condition was not included in the assumptions and methodology for calculating the natural frequencies, so the results would be different than the theoretical calculations.

Appendix

Matlab Code

Lab4_master.m

```
global row g Ap Cf L Ip At Ct d0 d1 d2
row = 1000; %[kg*m^-3]
At = 0.71; %[m^2]
g = 9.81; %[m*s^-2]
Ap = 0.1; %[m^2]
Cf = 49000; %[kg*m^-7]
L = 50; % [m]
Ip = row*L/Ap; %[kg*m^-4]
Ct = At/(row*g); %[m^5/N]
d0 = 20; %[m]
d1 = 20; %[m]
d2 = 20; %[m]

p9init = 1.5*Ip;
p3init = p9init;
q7init = Ct*(row*g*d0+row*g*d1-Cf*(1.5*1.5)*p3init/Ip);
q13init = Ct*((q7init/Ct)+(row*g*d2)-(Cf*(1.5*1.5)*p9init/Ip));

initial = [p3init q7init p9init q13init];
% s(1) = fluid momentum in section 1
% s(2) = volumetric dispalcement of tank 1
% s(3) = fluid momentum in section 2
% s(4) = volumetric dispalcement of tank 2

T = (2*pi/(sqrt(1/(Ct*Ip))));

tspan = 0:T/10:2*T;

[t, s] = ode45(@lab4_eqns,tspan,initial);
ext = zeros(length(t),2);
ds = zeros(length(t),4);

for i = 1:length(t)
   [ds(i,:), ext(i,:)] = lab4_eqns(t(i), s(i,:));
end

maxh1 = max(s(:,2))/At;
maxh2 = max(s(:,4))/At;
```

```
figure('Name','Tank Heights','NumberTitle','off','Color','white')
plot(t,s(:,2)/At,'k', t,s(:,4)/At,'--k'), grid on
title('Tank 1 Height vs Time')
legend('Location', 'south','Tank 1', 'Tank 2')
ylabel('hight (m)')
xlabel('time (s)')

figure('Name','Flow Input','NumberTitle','off','Color','white')
plot(t,ext(:,1),'k'), grid on
title('Input Turbine Flow vs Time')
legend('Location', 'south','Input Turbine Flow')
ylabel('Volumetric Flow (m^3/s)')
xlabel('time (s)')
```

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Lab4_eqns.m

```
function [ds, ext] = lab4_eqns(t,s)
global row g Ap Cf L Ip At Ct d0 d1 d2

p3 = s(1); % fluid momentum in section 1
q7 = s(2); % volumetric dispalcement of tank 1
p9 = s(3); % fluid momentum in section 2
q13 = s(4); % volumetric dispalcement of tank 2

T1 = .5; %Turbing begins to turn off
T2 = T1 + 0.15;

Qon = 1.5;

if t < T1
   Q0 = Qon;
elseif t >= T1 && t <= T2
   Q0 = Qon - (Qon/0.15)*(t - T1);
elseif t > T2
   Q0 = 0;
```

```
end

p3_dot = row*g*d0 + row*g*d1 - Cf*(abs(p3/Ip))*p3/Ip - q7/Ct;
q7_dot = p3/Ip - p9/Ip;
p9_dot = q7/Ct + row*g*d2 - Cf*(abs(p9/Ip))*p9/Ip - q13/Ct;
q13_dot = p9/Ip - Q0;

ext(1) = Q0;
ext(2) = 0;

ds = [p3_dot; q7_dot; p9_dot; q13_dot];
```