Course II Quantum Computing: From qubits to qudits

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Quantum computing

UNIVERSIDAD NACIONAL DE COLOMBIA

Agradecimientos:

Ilke Ican, Seki Seskir, Lia Yeh -

SeGQuRo (Strengthening and Entangling Global Quantum Roots) by experts from TU Delft, Institute for Technology Assessment and System Analysis – KIT, QWorld, PIQUE and World Quantum Day

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Physicist, Ph.D Computer Science, IEEE Senior Member

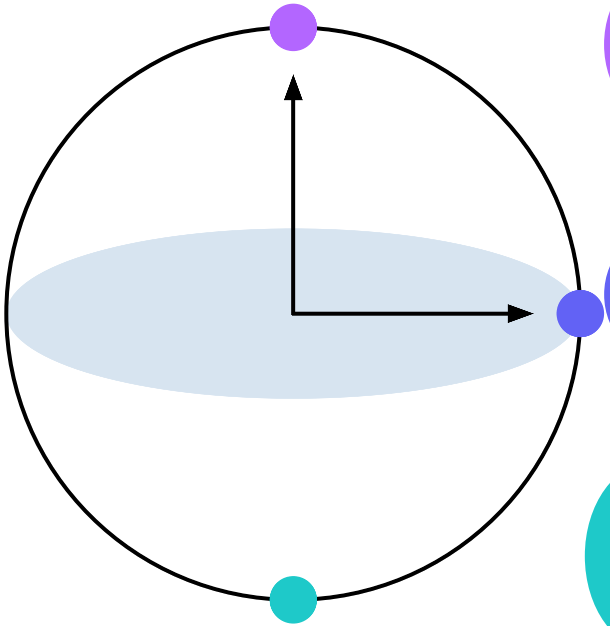
Universidad Nacional de Colombia Sede Medellín - Physics School

Center of Excellence in Quantum Computing and Artificial Intelligence

Director

Quantum Computing Research Group

My story in Quantum Computing



Z

1990 - 2004

Physicist, Quantum Mechanics, Quantum Dynamics, Classical Mechanics, Digital Signal Processing and QoS

Y

2004 - 2012

Cloud computing, AWS, Python, Quantum Computing Basics, IoT, Multi-Agent Systems

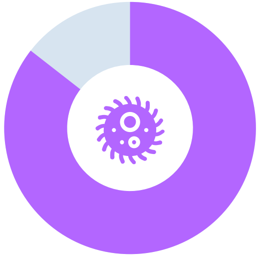
-Z

2016 – 2024

Quantum computing courses, IBM Zurich (Alexandro Curioni), Center of excellence on Quantum Computing and IA, Germany Research Groups in QC, Quantum Netherlands

About Colombia QC

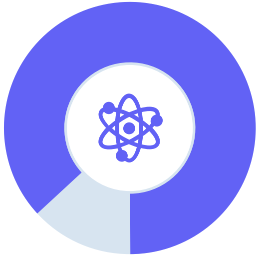
Quantum Computing Research Group



+7

Research Areas

Mathematics, Chemistry, Algorithms, Astrophysics, Metrology, artificial intelligence...



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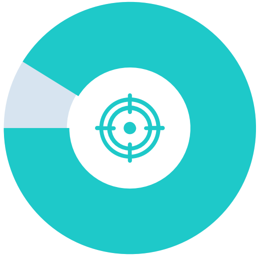
Courses

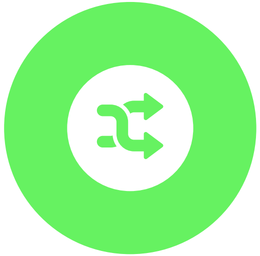
Online, free, in Spanish, certified. Largest community in Colombia and Latin America.



40%

Inclusion

35% of our members are women and vulnerable people (we are working to improve this figure).



+100

Team

Members from different universities, professions and academic levels around the world.

POINTS OF INTEREST IN QUANTUM COMPUTING

COLOMBIA

Bibliography

arXiv > quant-ph > arXiv:2008.00959

Quantum Physics

[Submitted on 30 Jul 2020 (v1), last revised 11 Nov 2020 (this version, v4)]

Qudits and high-dimensional quantum computing

Yuchen Wang, Zixuan Hu, Barry C. Sanders, Sabre Kais

arXiv > quant-ph > arXiv:2406.00792

Quantum Physics

[Submitted on 2 Jun 2024]

Qudit inspired optimization for graph coloring

David Jansen, Timothy Heightman, Luke Mortimer, Ignacio Perito, Antonio Acín

arXiv > quant-ph > arXiv:2302.02966

Search...

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Quantum Physics

[Submitted on 6 Feb 2023 (v1), last revised 1 Jul 2024 (this version, v3)]

Universal quantum computing with qubits embedded in trapped-ion qudits

Anastasiia S. Nikolaeva, Evgeniy O. Kiktenko, Aleksey K. Fedorov

https://arxiv.org/abs/2008.00959

Topics

Session 1: From qubits to qudits

A **Classical harmonic oscillator**
Introduction and basic equations

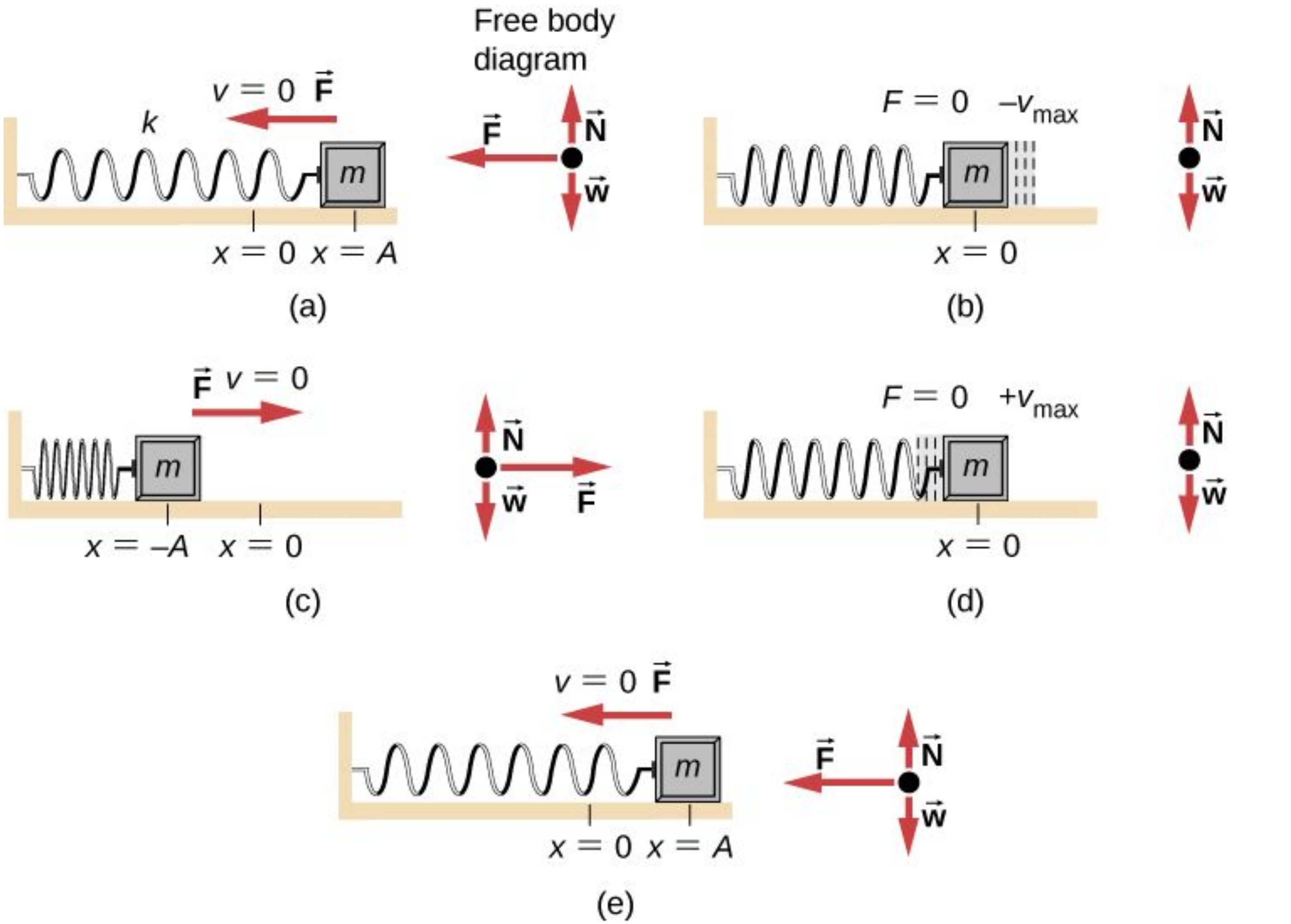
B **Quantum harmonic oscillator**
Introduction, some relations

C **Qubits, Qutrits, Qudits**
Why?, qudits applications

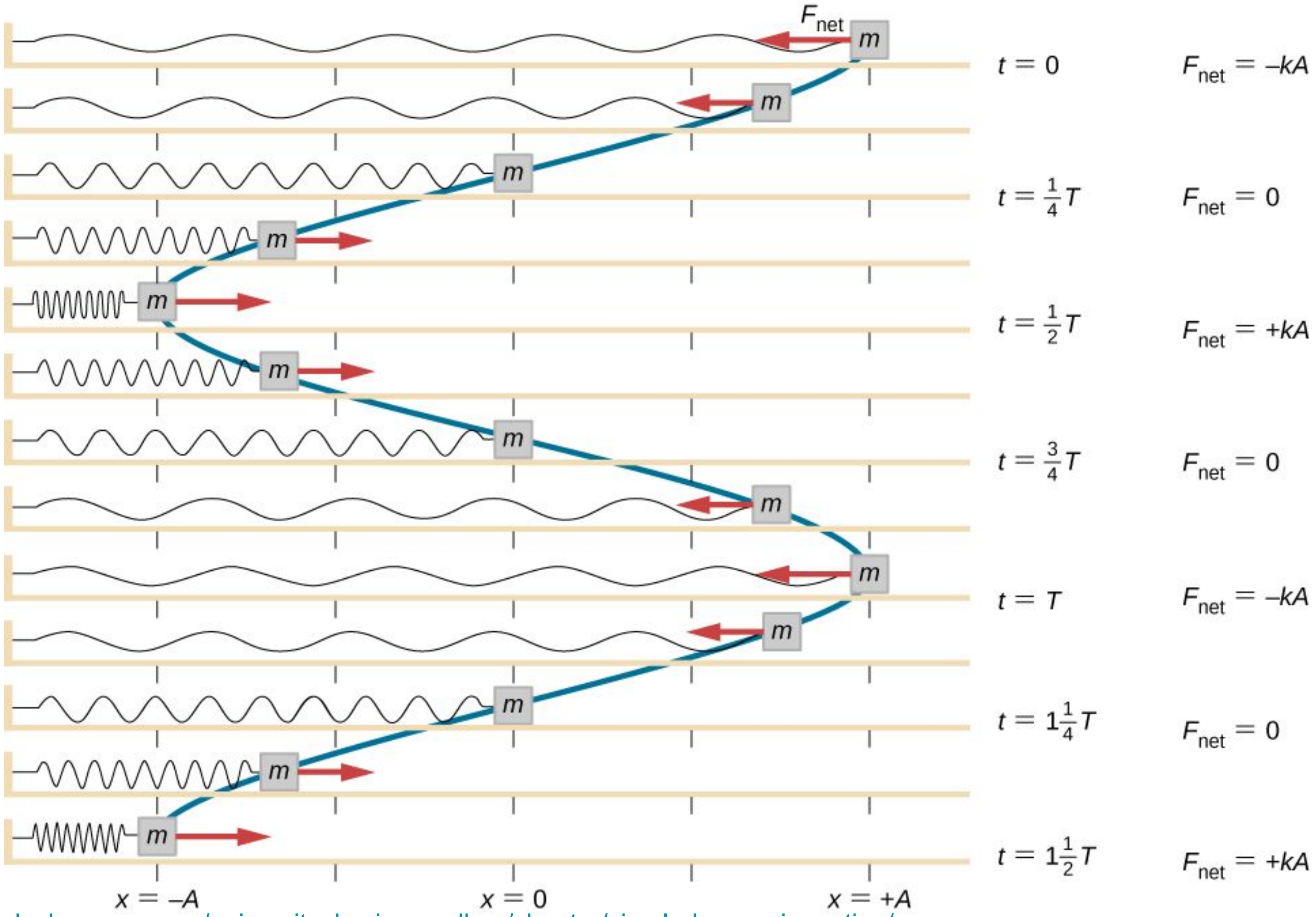
D **Qubits and Bloch Sphere**
Some basic calculus on qubits

E **Classical and quantum gates**
From classical to quantum gates

Waves, superposition and simple harmonic equations



https://pressbooks.bccampus.ca/universityphysicssandbox/chapter/simple-harmonic-motion/



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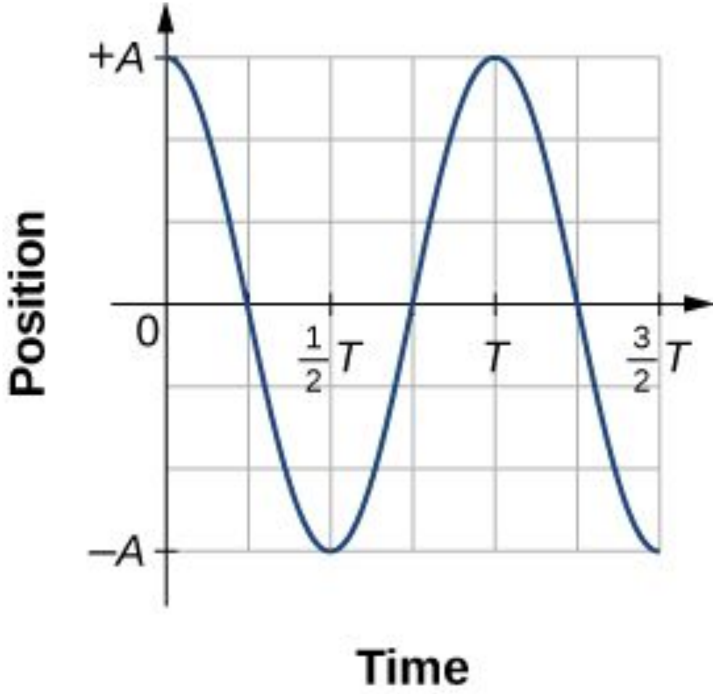
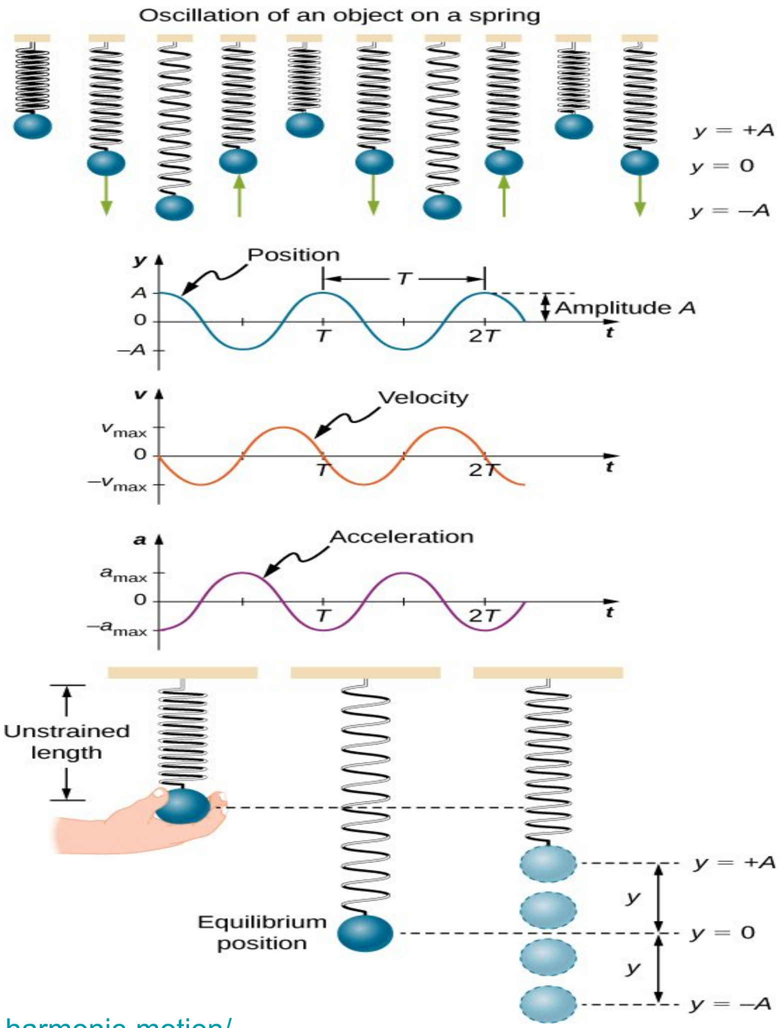
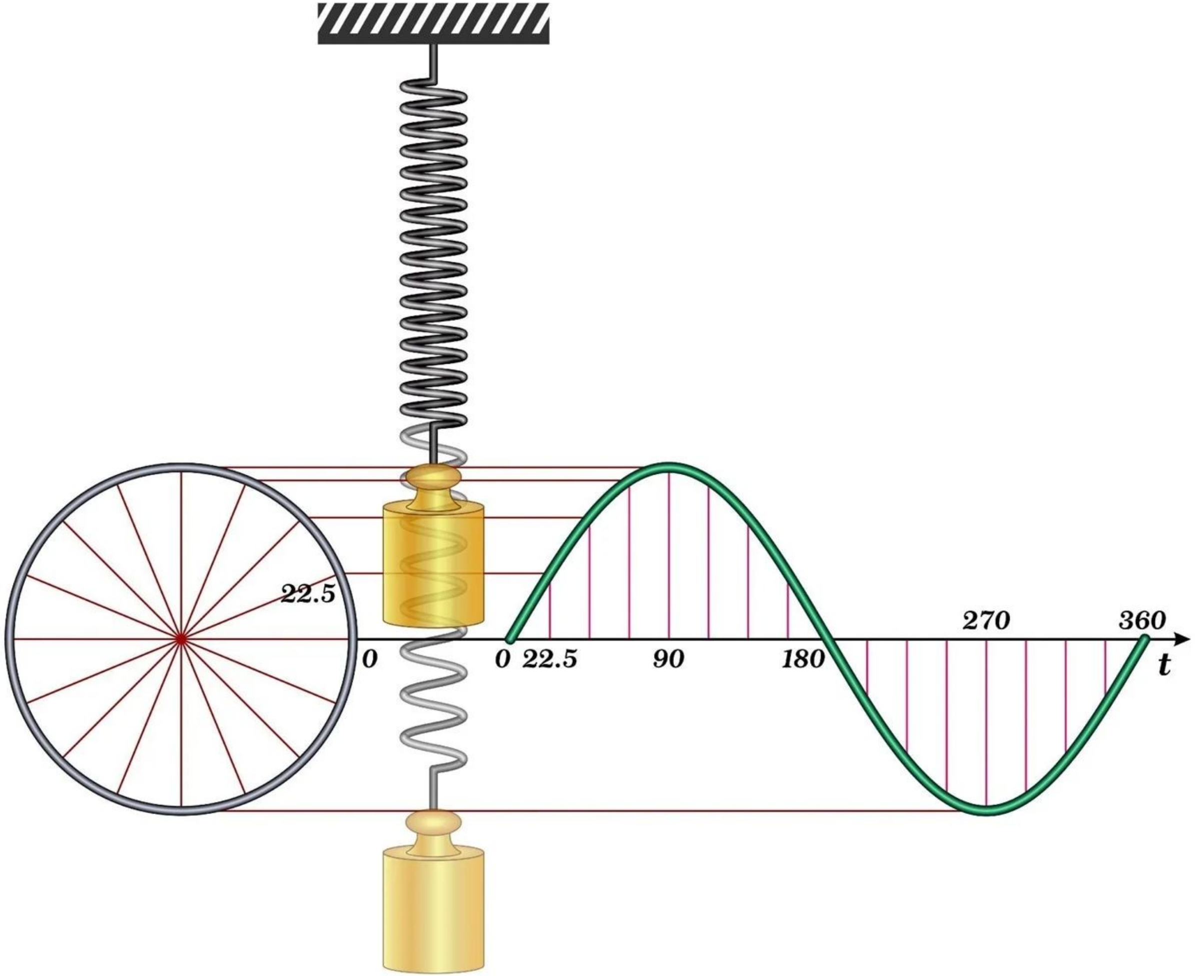




https://pressbooks.bccampus.ca/universityphysicssandbox/chapter/simple-harmonic-motion/

Harmonic motion, classical harmonic oscillators, and quantum harmonic oscillators.



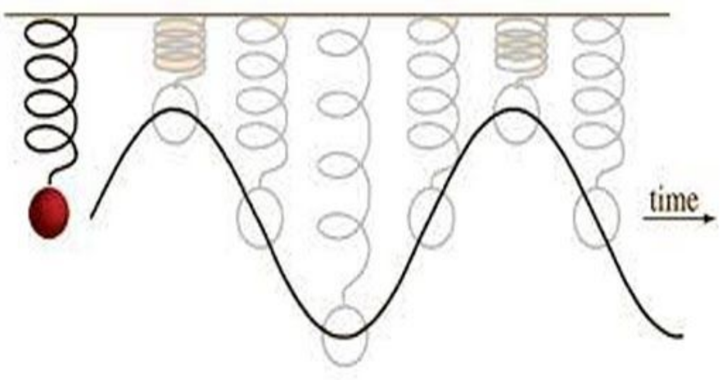
Simple Harmonic Motion

Equations

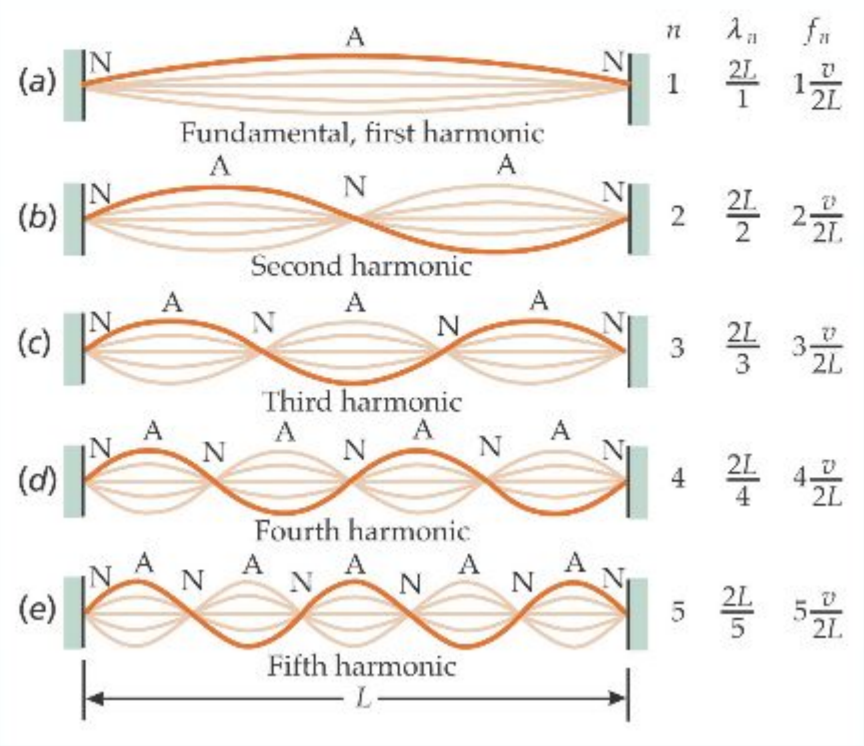
$$x(t) = A\cos(\omega t)$$

$$v(t) = -A\,\omega\sin(\omega t)$$

$$a(t) = -A\,\omega^2\cos(\omega t)$$



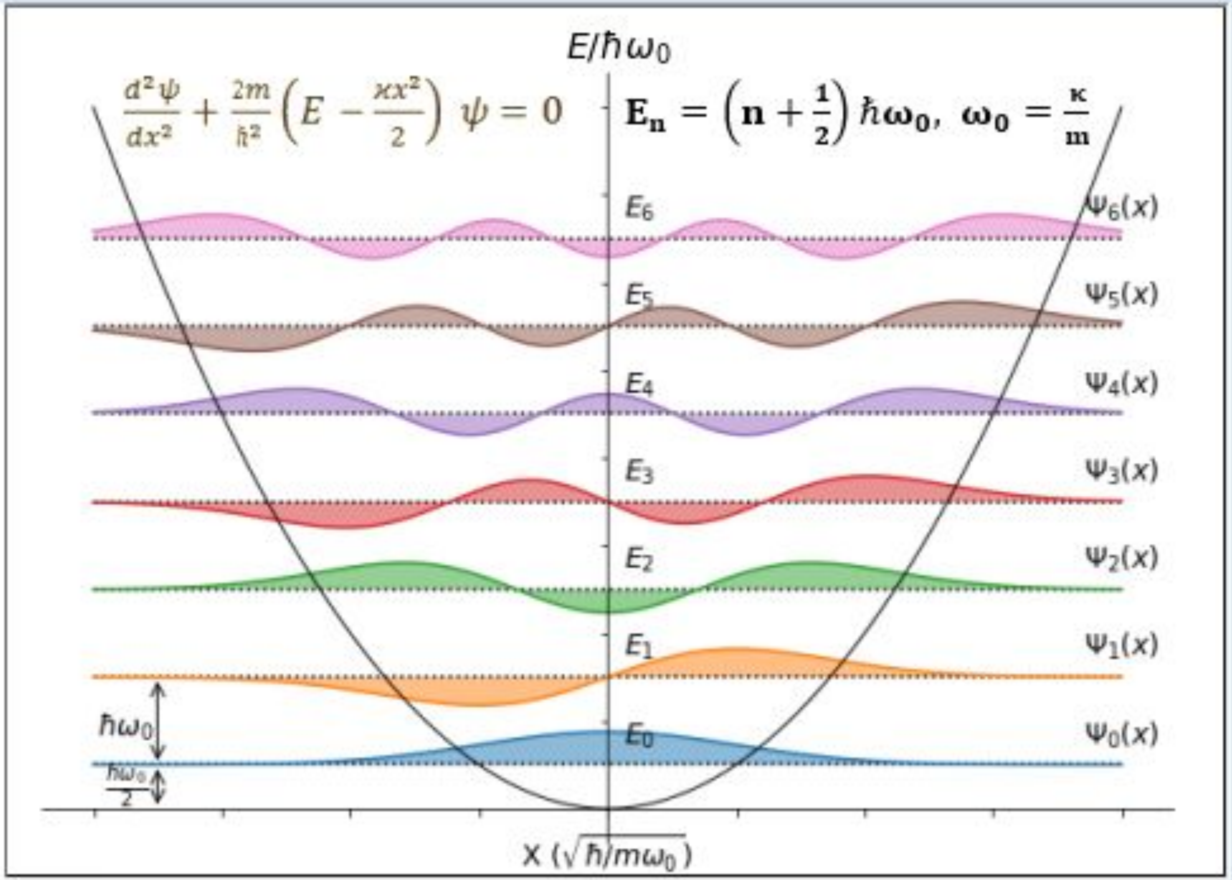
ONDAS ESTACIONARIAS



$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\Psi}{\mathrm{dx}^2}+\frac{1}{2}mw^2x^2\Psi = E\Psi$$

$$\frac{\mathrm{d}^2\Psi}{\mathrm{dx}^2}+\left(\frac{2\mathrm{mE}}{\hbar^2}-\frac{\mathrm{m}^2\mathrm{w}^2\mathrm{x}^2}{\hbar^2}\right)\Psi = 0$$

https://owlcation.com/stem/schrodinger-equation-simple-harmonic-oscillator



Quantization of Energy

Quantum Harmonic Oscillator

Hamiltonian

$$H = \hbar\omega_0\left(A^\dagger A + \tfrac{1}{2}\right)$$

Eigen-solutions

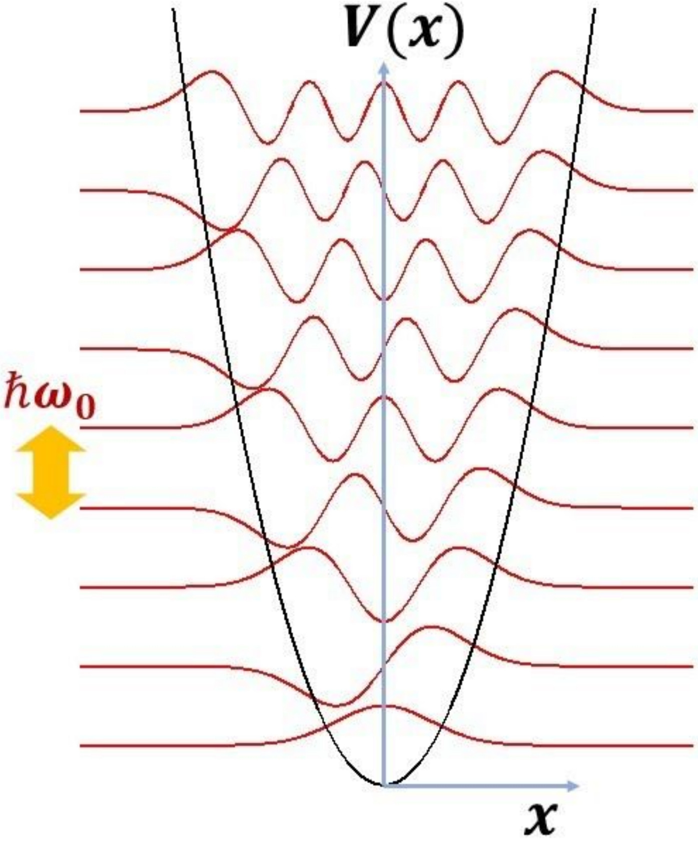
$$|\varphi_n\rangle = \frac{1}{\sqrt{n!}}(A^\dagger)^n|\varphi_0\rangle$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$$

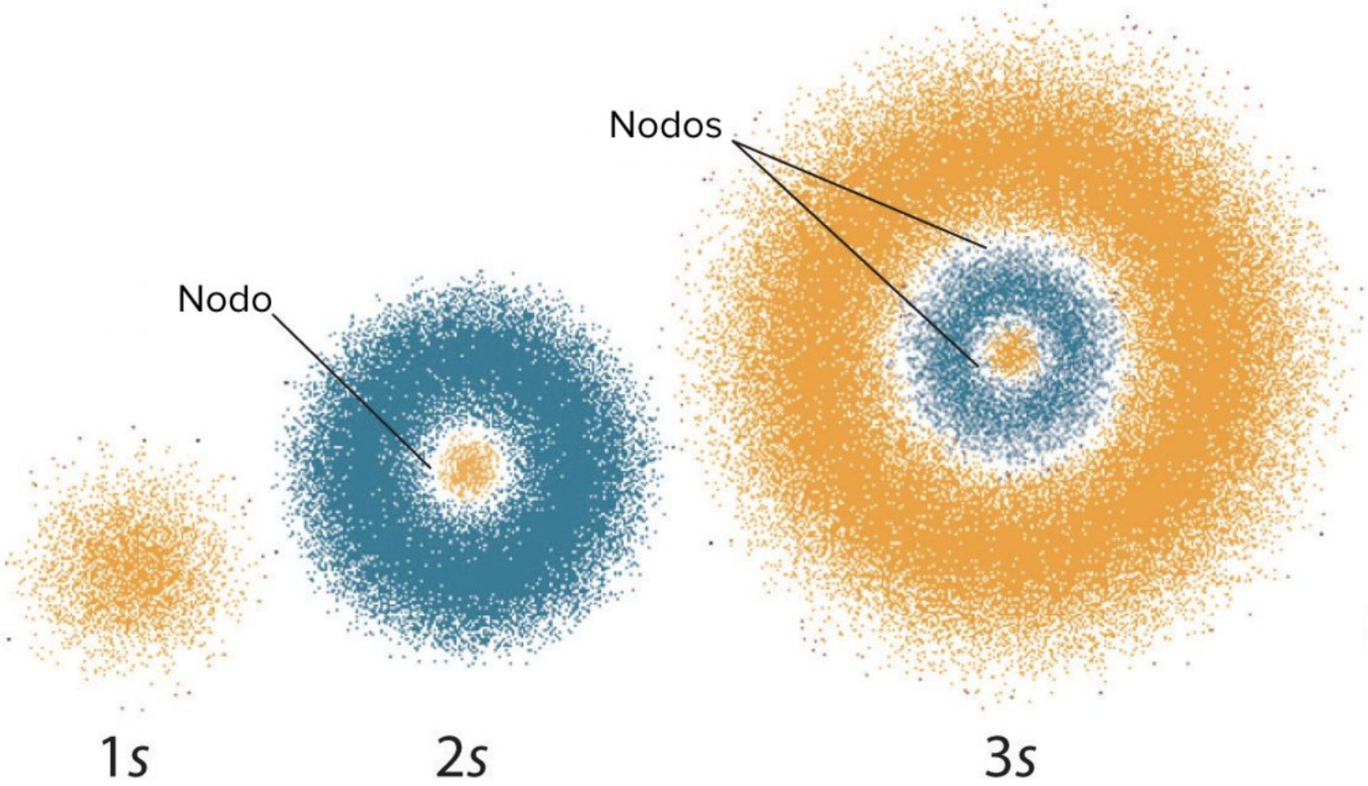
Ladder operators

$$A^\dagger|\varphi_n\rangle = \sqrt{1+n}|\varphi_{n+1}\rangle$$

$$A|\varphi_n\rangle = \sqrt{n}|\varphi_{n-1}\rangle$$

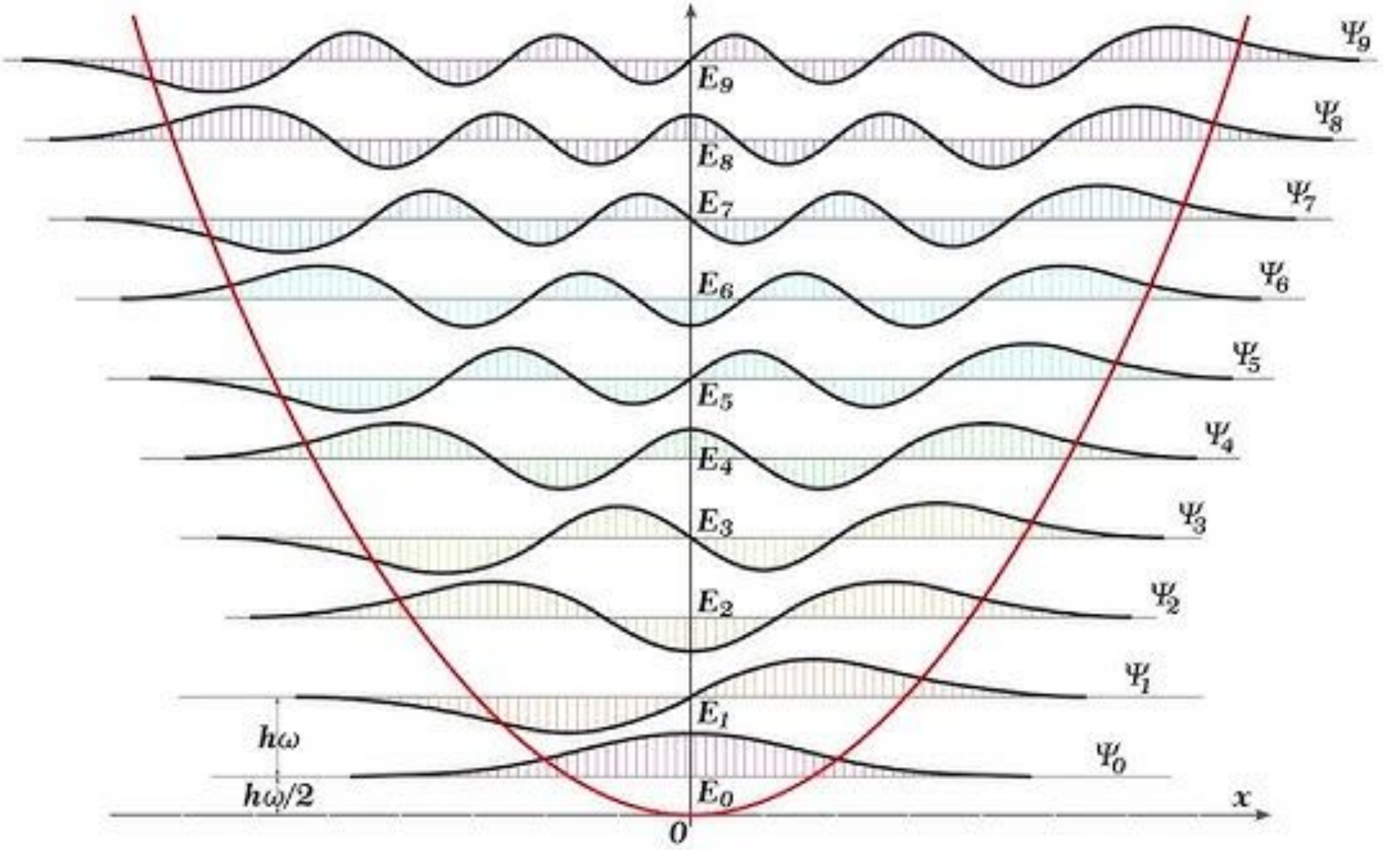


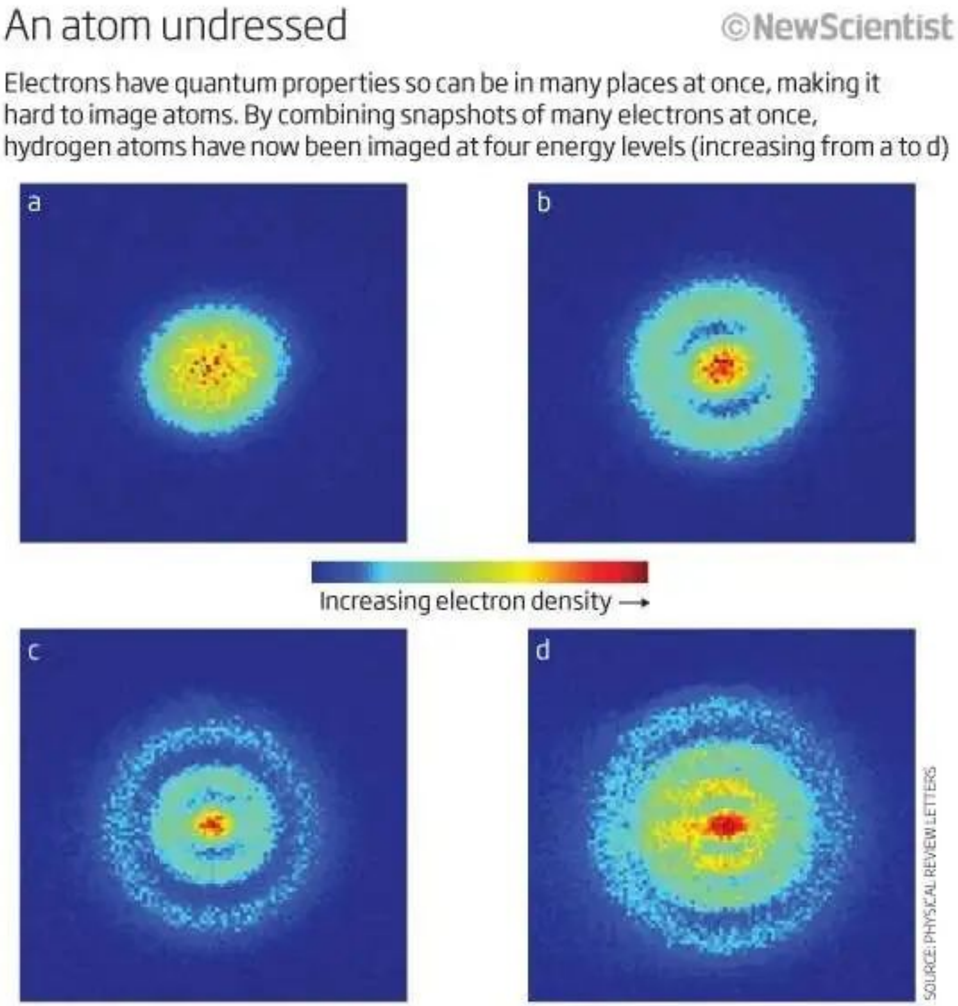
https://youtu.be/jHjebJtBXUs?si=YheJFpMFxA58SF3g

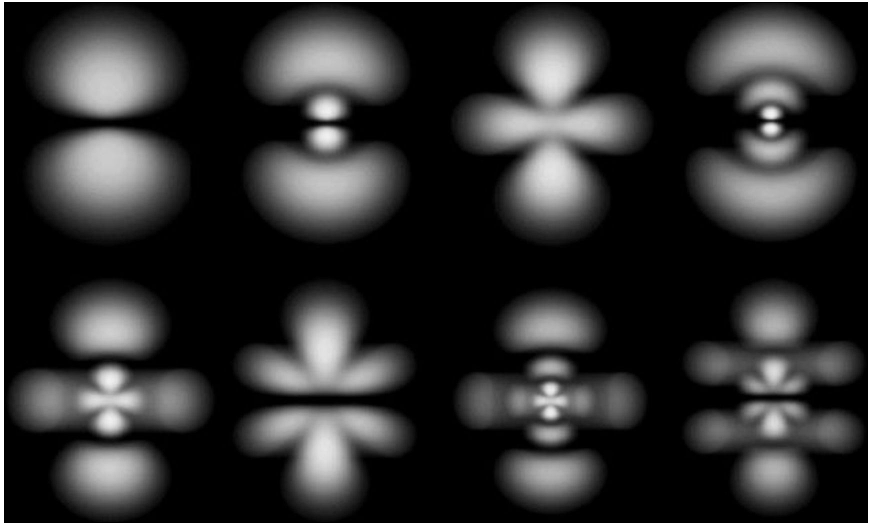


Las distribuciones de probabilidad para los orbitales 1s, 2s y 3s. Mayor intensidad de color indica las regiones en las que hay mayor probabilidad de que los electrones existan. Los nodos indican las regiones en las que un electrón tiene cero probabilidad de encontrarse. Crédito de la imagen: [UCDavis Chemwiki](), [CC BY-NC-SA 3.0 US]()

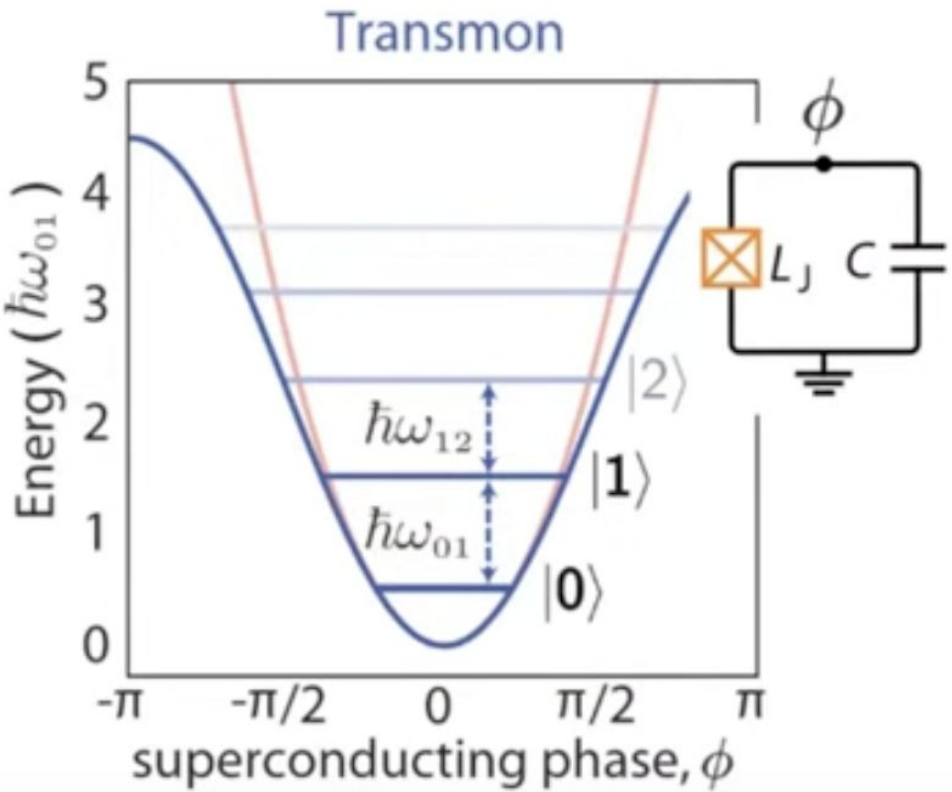
Quantum Harmonic Oscillator



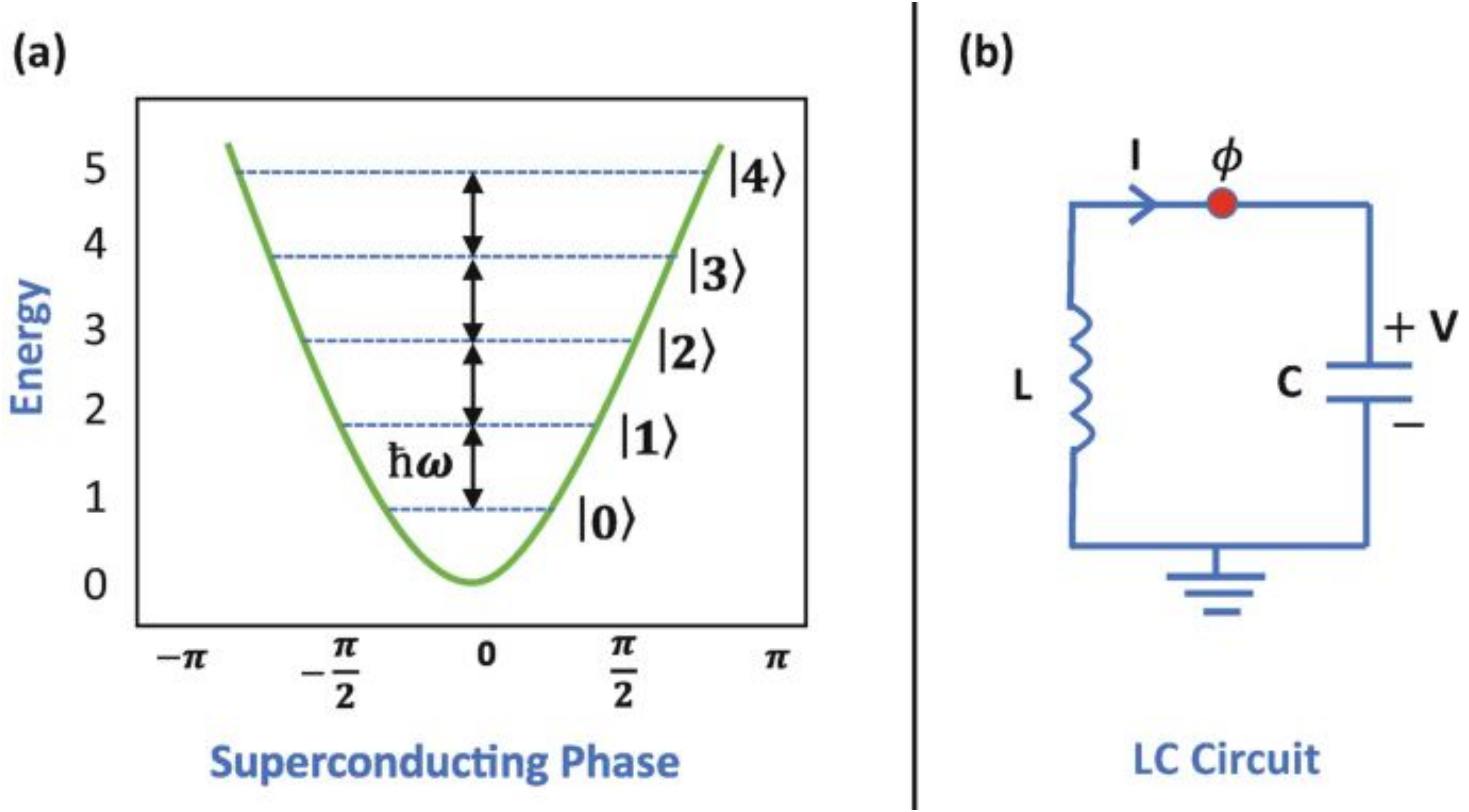




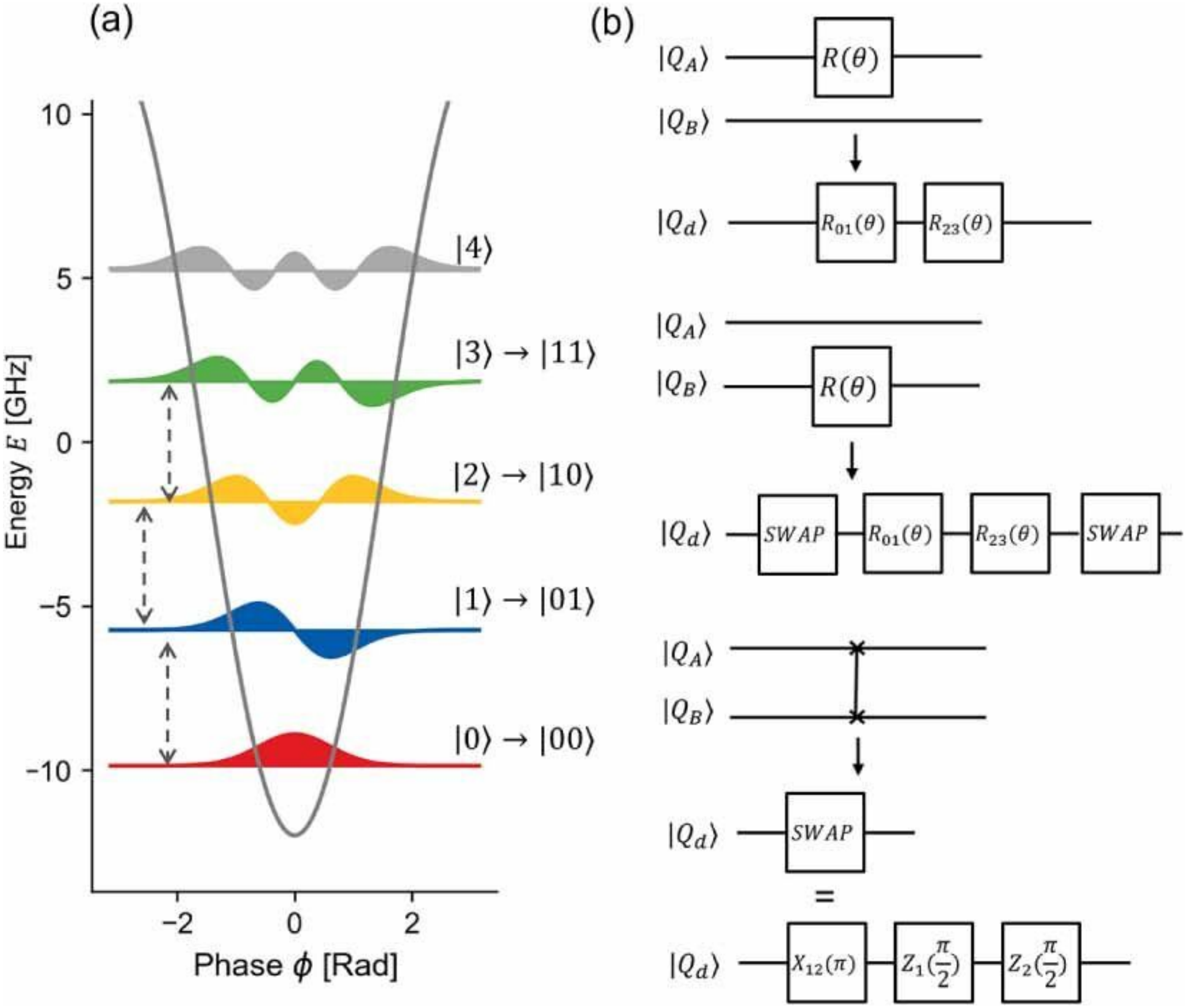
Measurement of hydrogen atom wave functions (Physical Review Letters via New Scientist)



Kjaergaard, Schwartz, Braumüller, Krantz, Wang, Gustavsson, Oliver (2020)
10.1146/annurev-conmatphys-031119-050605



(a)
Energy
5
4
3
2
1
0
|4⟩
|3⟩
|2⟩
|1⟩
|0⟩
ħω
−π
−π/2
0
π/2
π
Superconducting Phase
(b)
I
ϕ
L
C
+ V
−
LC Circuit



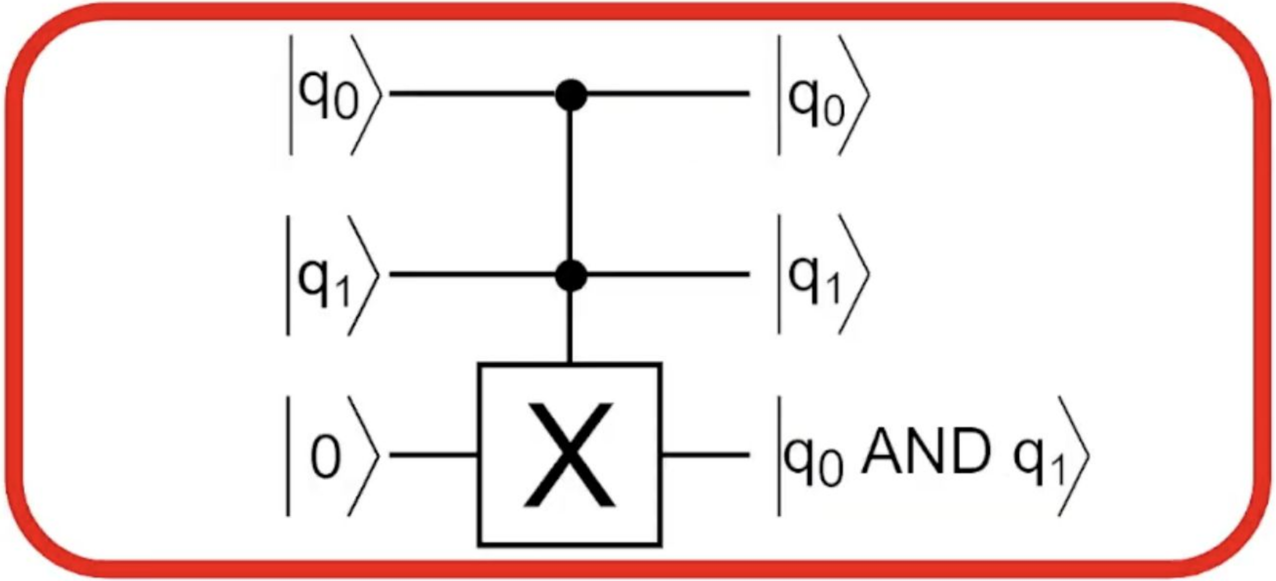
(a)
(b)
Energy E [GHz]
Phase ϕ [Rad]
10
5
0
-5
-10
-2
0
2
$|4\rangle$
$|3\rangle \rightarrow |11\rangle$
$|2\rangle \rightarrow |10\rangle$
$|1\rangle \rightarrow |01\rangle$
$|0\rangle \rightarrow |00\rangle$
$|Q_A\rangle$
$|Q_B\rangle$
$R(\theta)$
$|Q_d\rangle$
$R_{01}(\theta)$
$R_{23}(\theta)$
$|Q_A\rangle$
$|Q_B\rangle$
$R(\theta)$
$|Q_d\rangle$
$SWAP$
$R_{01}(\theta)$
$R_{23}(\theta)$
$SWAP$
$|Q_A\rangle$
$|Q_B\rangle$
$|Q_d\rangle$
$SWAP$
=
$|Q_d\rangle$
$X_{12}(\pi)$
$Z_1(\frac{\pi}{2})$
$Z_2(\frac{\pi}{2})$

Qubits: Why?

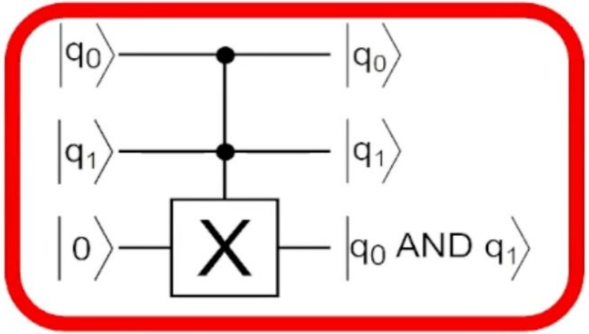
Los qudits (quantum digits) son importantes en la computación cuántica por varias razones:

1. **Mayor Capacidad de Información**: A diferencia de los qubits, que solo pueden representar 0 o 1, los qudits pueden tomar múltiples valores (donde d es la dimensión del sistema). Esto permite almacenar y procesar más información con menos partículas.
2. **Eficiencia Algorítmica**: Los algoritmos cuánticos que utilizan qudits pueden ser más eficientes, ya que pueden reducir el número de operaciones necesarias para realizar cálculos complejos.
3. **Reducción de Errores**: Los qudits ofrecen más estados para codificar información, lo que puede ayudar a reducir la tasa de errores y mejorar la fidelidad de las operaciones cuánticas.
4. **Sistemas Físicos Diversos**: Los qudits son naturales en muchos sistemas físicos, como fotones con múltiples modos, iones atrapados y átomos con múltiples niveles de energía, lo que permite aprovechar mejor las propiedades físicas intrínsecas de estos sistemas.

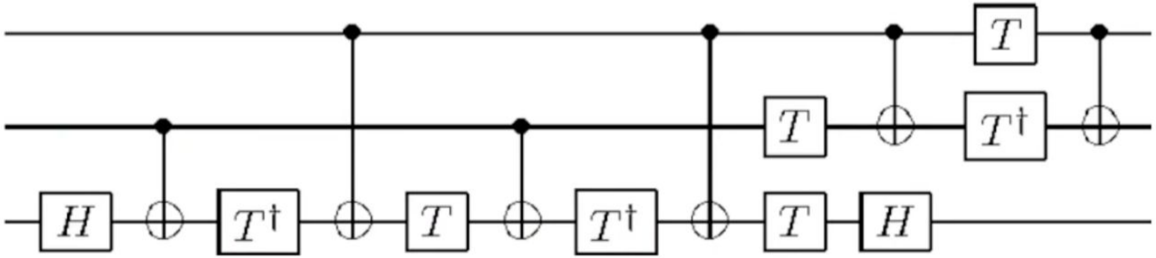
Qubit Circuits



Running Example: Logical Toffoli Circuit

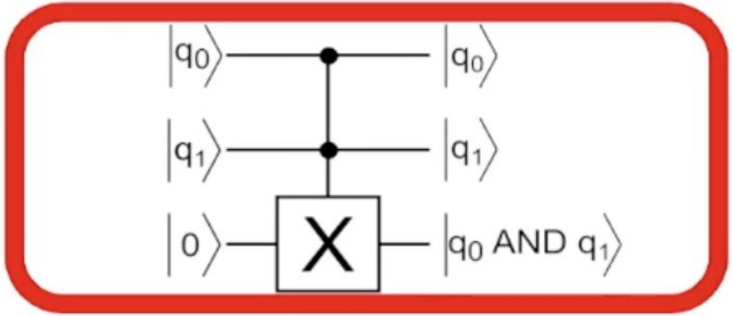


Logical Toffoli Circuit



Implementation on Hardware (Depth: 11
Gates: 15)

Solution: Qudits can reduce circuit depth



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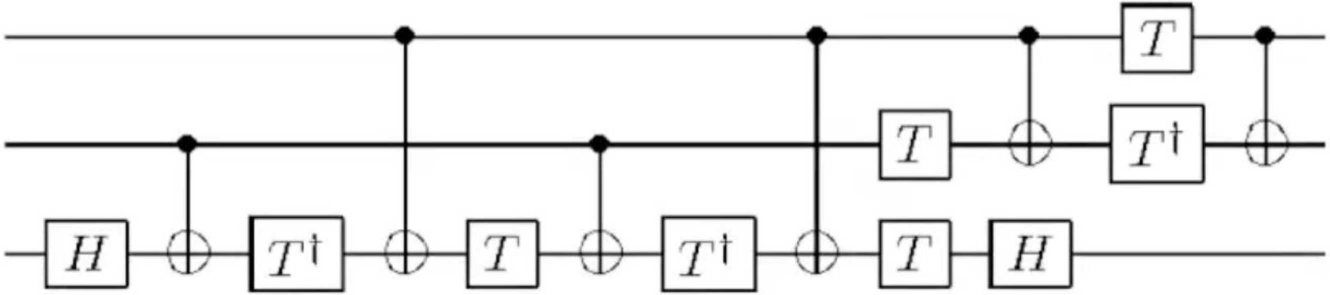
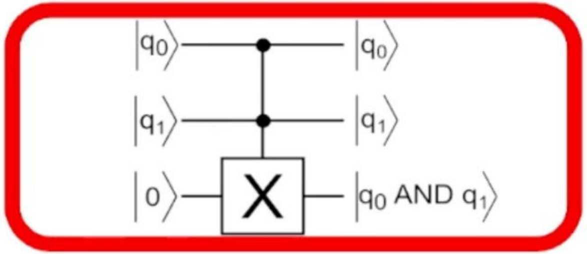


Qubit Circuit (**Depth 11**)

Gokhale et al: Asymptotic improvements to quantum circuits via qutrits (ISCA 2019)

Solution: Qudits can reduce circuit depth



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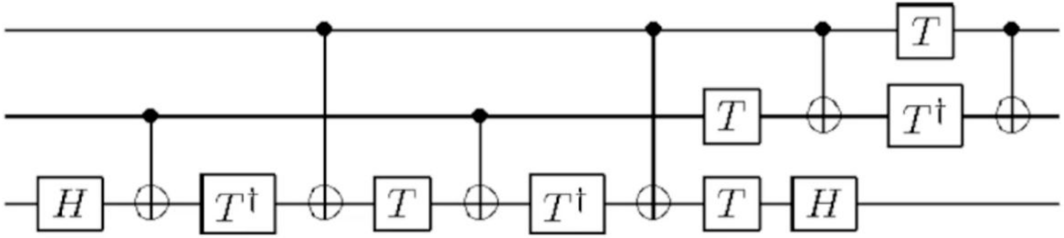

Qubit Circuit (**Depth 11**)

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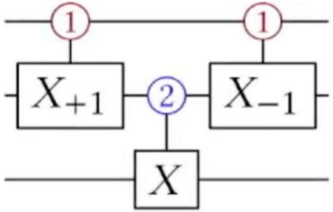
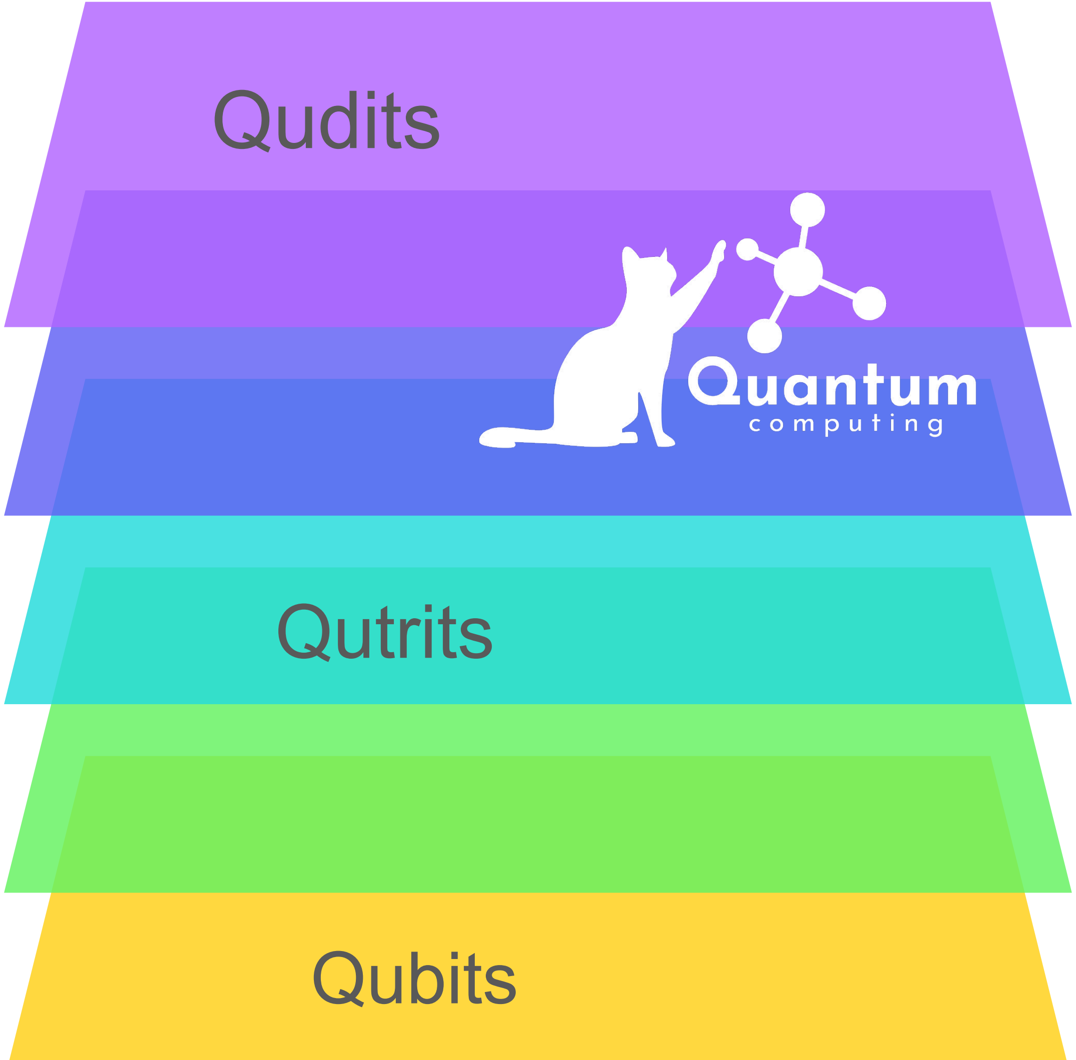
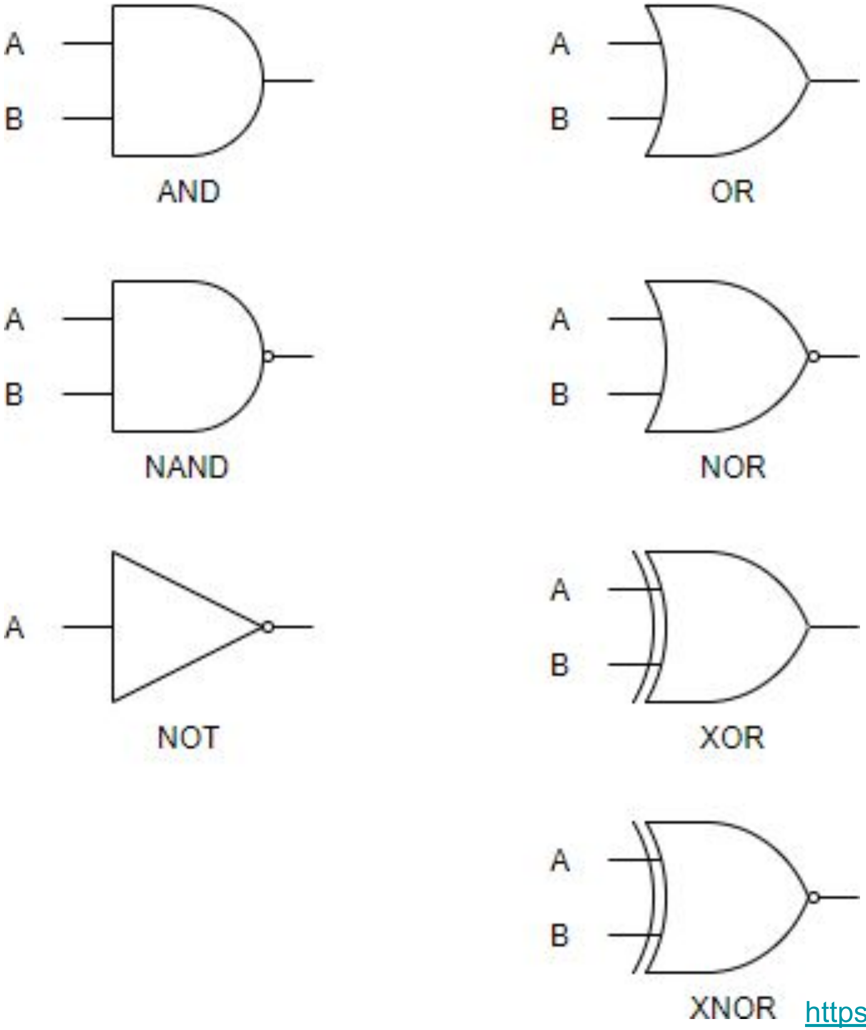
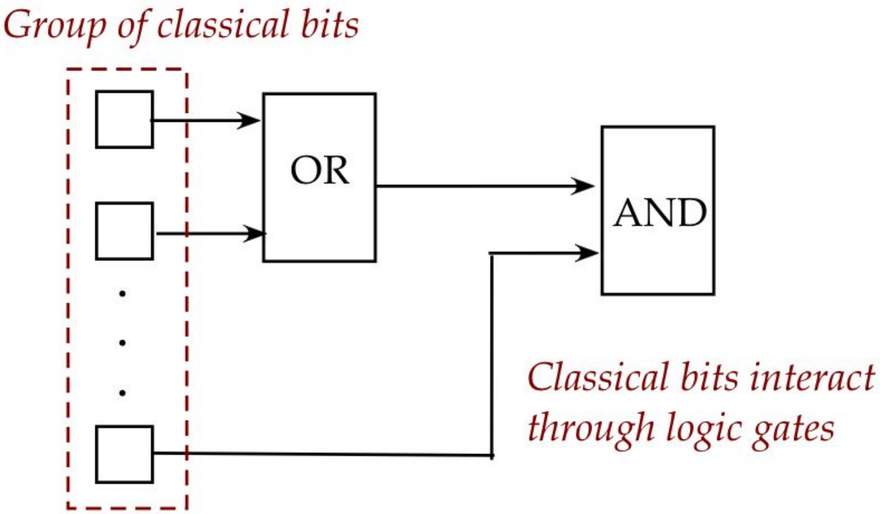


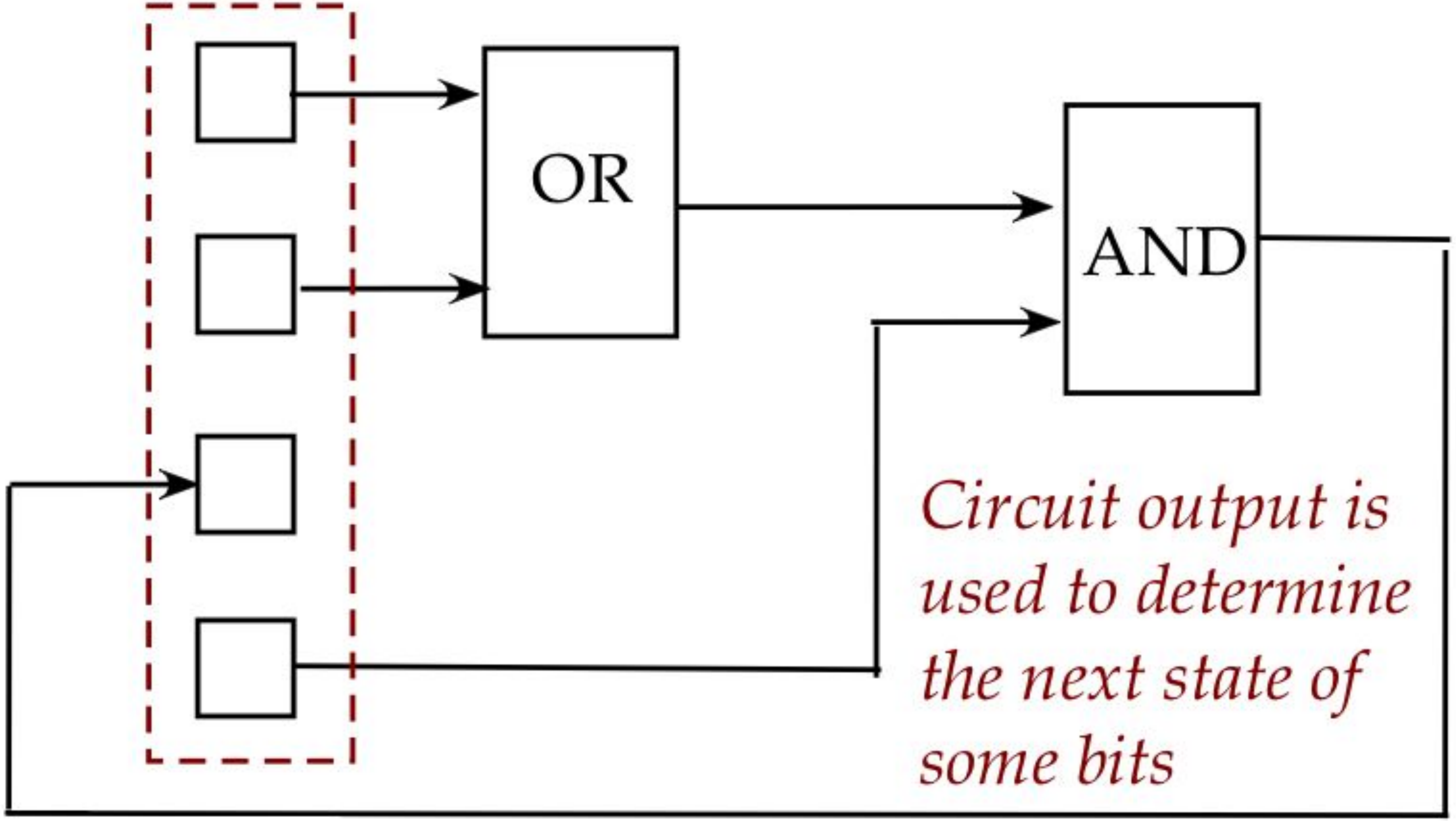
Smaller Qutrit Circuit


Qudits
Qutrits
Qubits





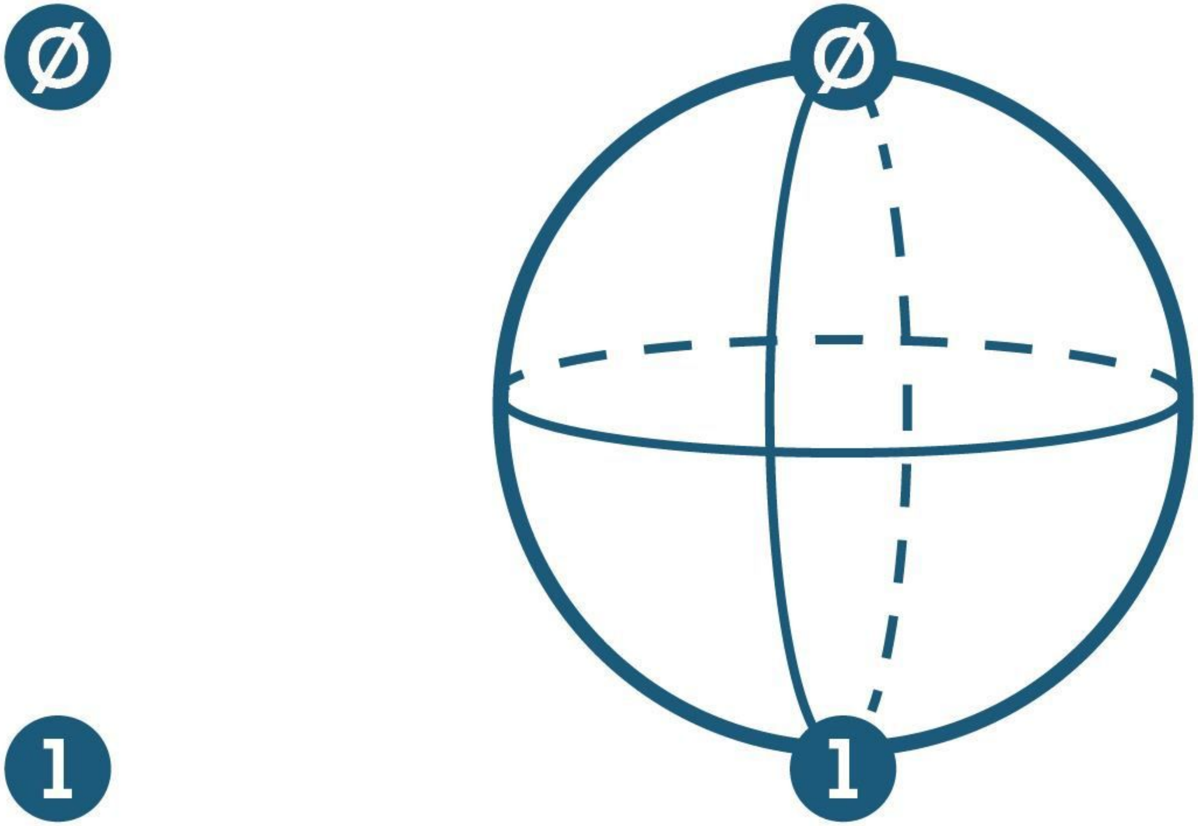
https://www2.seas.gwu.edu/~simhaweb/quantum/modules/module4/module4.html

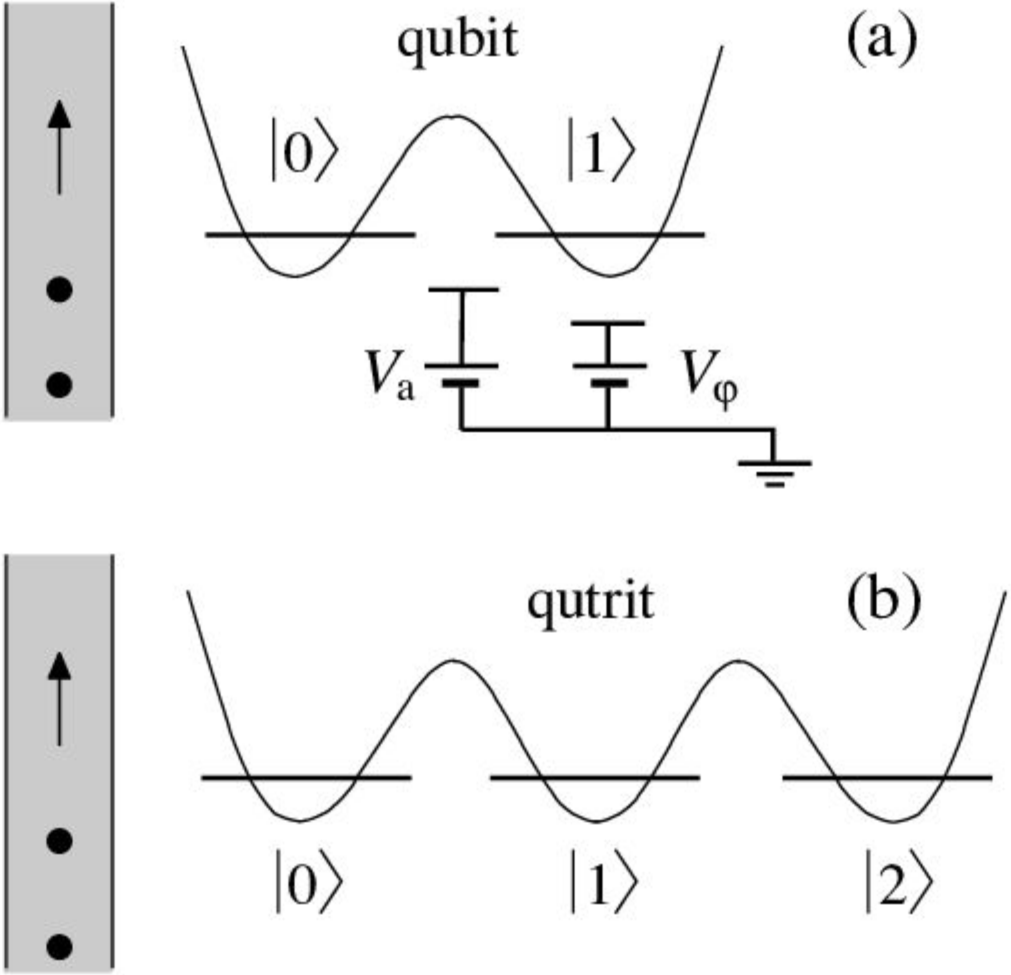


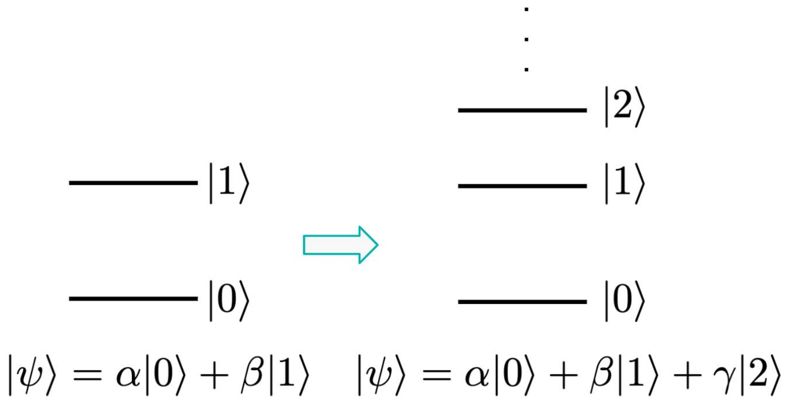
https://www2.seas.gwu.edu/~simhaweb/quantum/modules/module4/module4.html

BIT

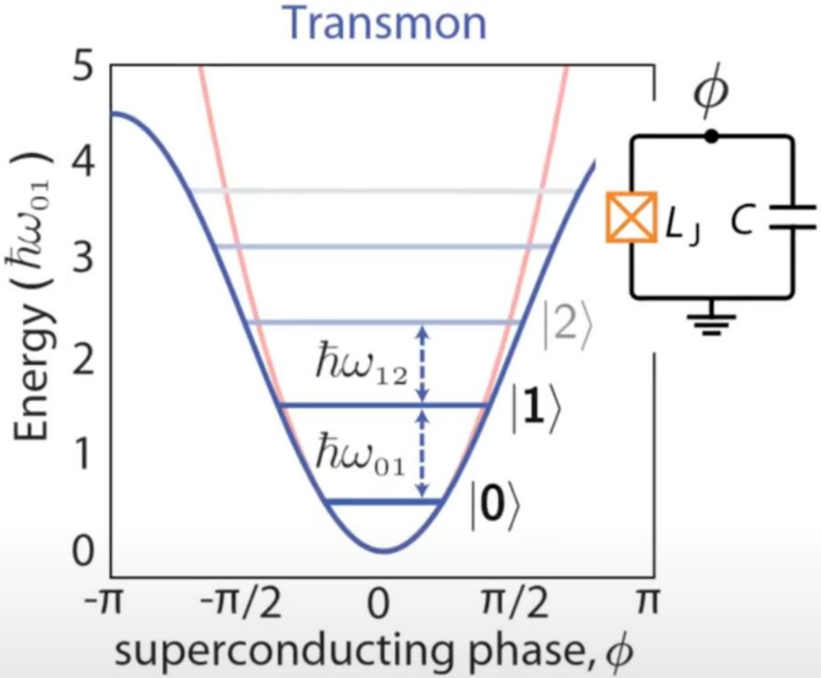
QUBIT




qubit
(a)
$|0\rangle$
$|1\rangle$
V_a
V_φ
qutrit
(b)
$|0\rangle$
$|1\rangle$
$|2\rangle$


$|1\rangle$
$|0\rangle$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$

What is a qutrit?



Kjaergaard, Schwartz, Braumüller, Krantz, Wang, Gustavsson, Oliver (2020)
10.1146/annurev-conmatphys-031119-050605

- The qutrit Z-basis states are $|\mathbf{0}\rangle, |\mathbf{1}\rangle,$ **and** $|\mathbf{2}\rangle$
- The $|\mathbf{2}\rangle$ state is already present in today's qubit computers

Qudits: Sistemas Cuánticos de d Niveles

Definición

- **Qudit**: Un qudit es un sistema cuántico que puede estar en una superposición de d estados base diferentes. El término "qudit" es una generalización del "qubit" (que tiene dos niveles) y el "qutrit" (que tiene tres niveles).

Estado de un Qudit

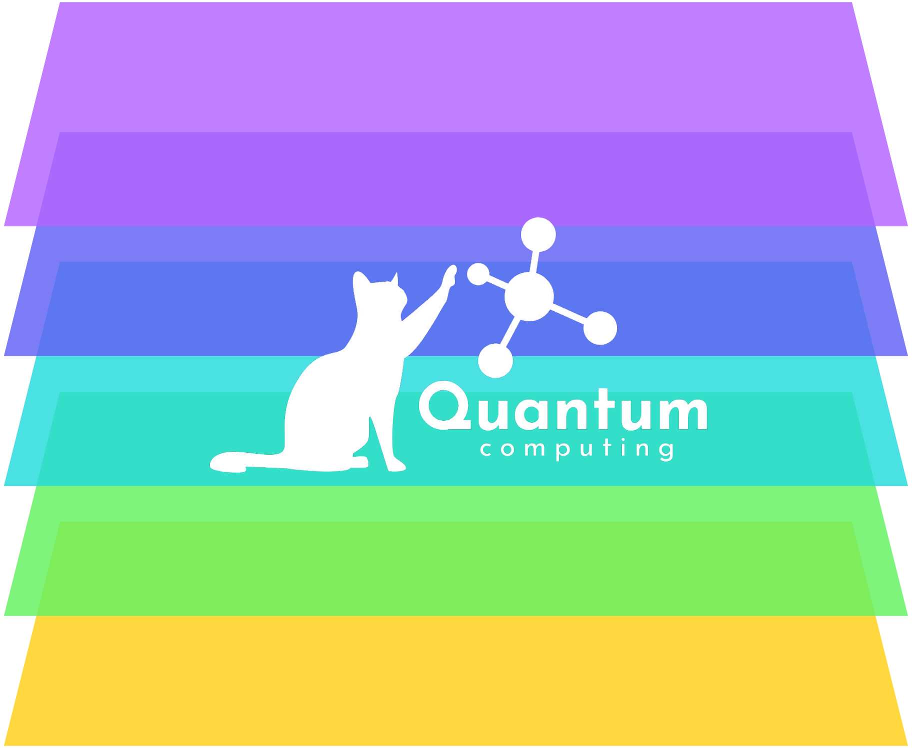
- Un estado general de un qudit se puede expresar como:

$$|\psi\rangle = \sum_{k=0}^{d-1} \alpha_k |k\rangle$$

  donde α_k son coeficientes complejos que satisfacen la condición de normalización:

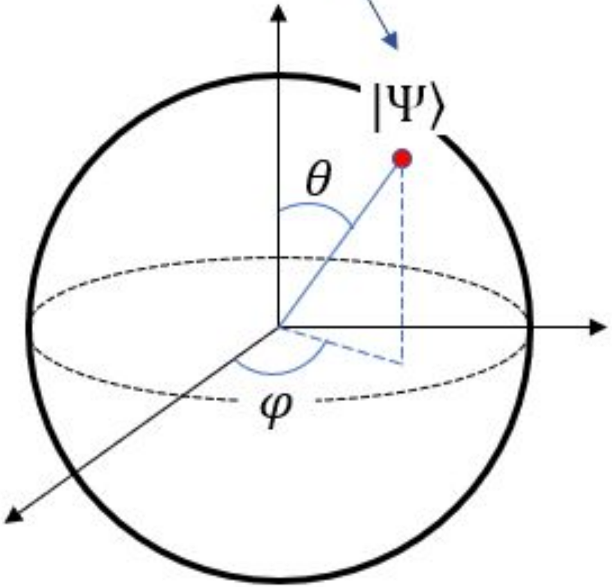
$$\sum_{k=0}^{d-1} |\alpha_k|^2 = 1$$

Qubits: Bloch Sphere



The absolute value (magnitude) of this term is always 1 regardless of the value φ.
(i.e, the magnitude of α and β is determined by θ only)

$$|\Psi'\rangle = \cos(\frac{\theta}{2})|0\rangle + \boxed{e^{i\varphi}}\sin(\frac{\theta}{2})|1\rangle$$



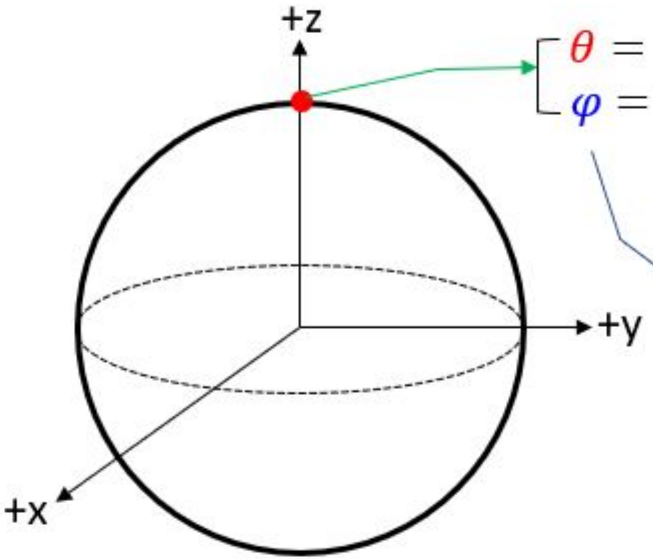
$$0 \leq \theta \leq \pi$$

$$0 \leq \varphi \leq 2\pi$$

$$|\Psi'\rangle = \boxed{\alpha}\,|0\rangle + \boxed{\beta}\,|1\rangle$$

$$|\Psi'\rangle = \boxed{\cos(\frac{\theta}{2})}|0\rangle + \boxed{e^{i\varphi}\sin(\frac{\theta}{2})}|1\rangle$$

https://www.sharetechnote.com/html/QC/QuantumComputing_BlochSphere.html



$$\begin{cases} \theta = 0 \\ \varphi = 0 \end{cases}$$

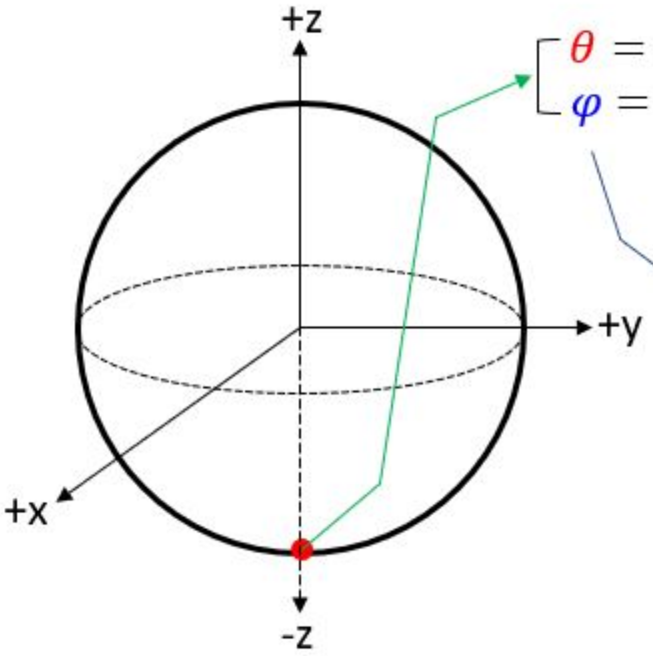
$$\cos(0) + i\,sin(0)$$

$$|\Psi'\rangle = \cos(\tfrac{0}{2})|0\rangle + \boxed{e^{i0}}\,sin(\tfrac{0}{2})|1\rangle$$

$$|\Psi'\rangle = 1|0\rangle + 0\,|1\rangle$$

$$|\Psi'\rangle = |0\rangle$$

https://www.sharetechnote.com/html/QC/QuantumComputing_BlochSphere.html



$$\begin{cases}\theta = \pi \\ \varphi = 0\end{cases}$$

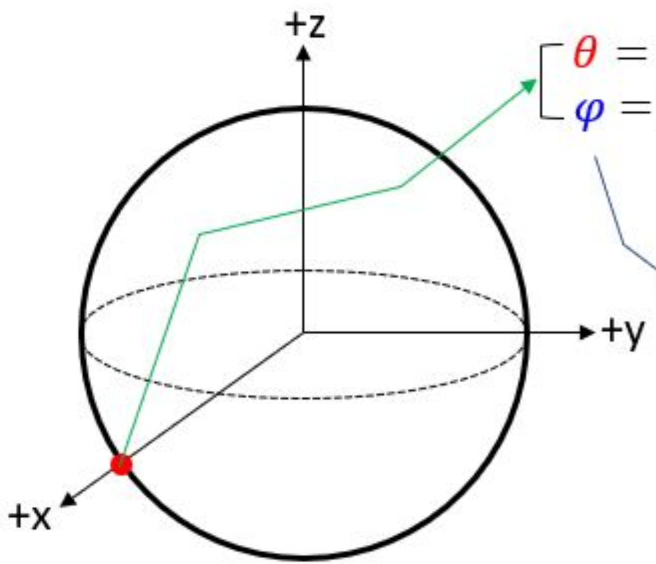
$$\cos(0) + i\,sin(0)$$

$$|\Psi'\rangle = \cos(\tfrac{\pi}{2})|0\rangle + \boxed{e^{i0}}\,sin(\tfrac{\pi}{2})|1\rangle$$

$$|\Psi'\rangle = 0|0\rangle + 1\,|1\rangle$$

$$|\Psi'\rangle = |1\rangle$$

https://www.sharetechnote.com/html/QC/QuantumComputing_BlochSphere.html



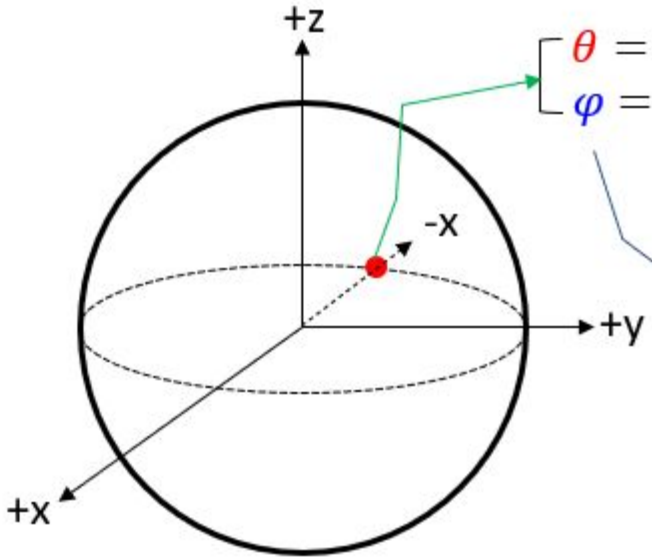
$$\begin{cases}\theta = \pi/2 \\ \varphi = 0\end{cases}$$

$$\cos(0) + i\,sin(0)$$

$$|\Psi'\rangle = \cos\left(\frac{\pi/2}{2}\right)|0\rangle + e^{i0}\,sin\left(\frac{\pi/2}{2}\right)|1\rangle$$

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\Psi'\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \Rightarrow |+\rangle$$



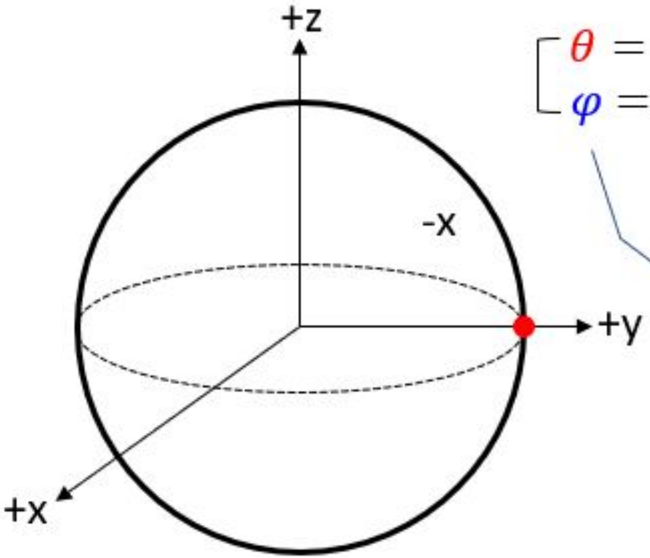
$$\begin{cases} \theta = \pi/2 \\ \varphi = \pi \end{cases}$$

$$\cos(\pi) + i\,sin(\pi)$$

$$|\Psi'\rangle = \cos(\frac{\pi/2}{2})|0\rangle + \boxed{e^{i\pi}}sin(\frac{\pi/2}{2})|1\rangle$$

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|\Psi'\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \Rightarrow |-\rangle$$



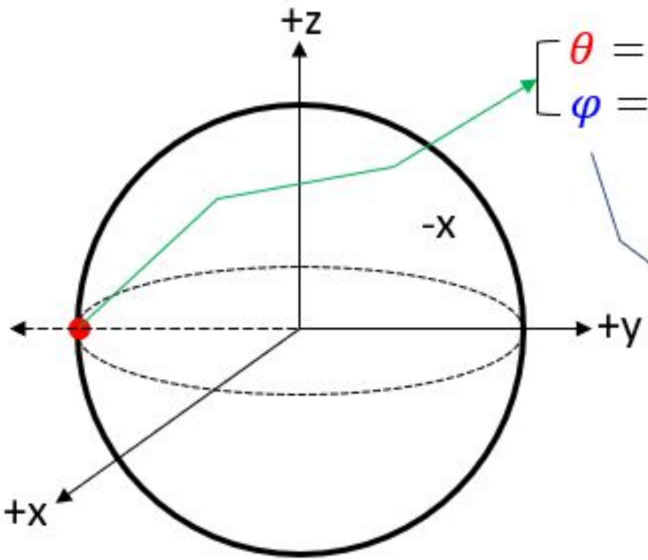
$$\begin{cases} \theta = \pi/2 \\ \varphi = \pi/2 \end{cases}$$

$$\cos(\pi/2) + i\ sin(\pi/2)$$

$$|\Psi'\rangle = \cos(\frac{\pi/2}{2})|0\rangle + \boxed{e^{i\pi/2}} sin(\frac{\pi/2}{2})|1\rangle$$

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}|0\rangle + i\,\frac{1}{\sqrt{2}}|1\rangle$$

$$|\Psi'\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \Rightarrow |+i\rangle$$



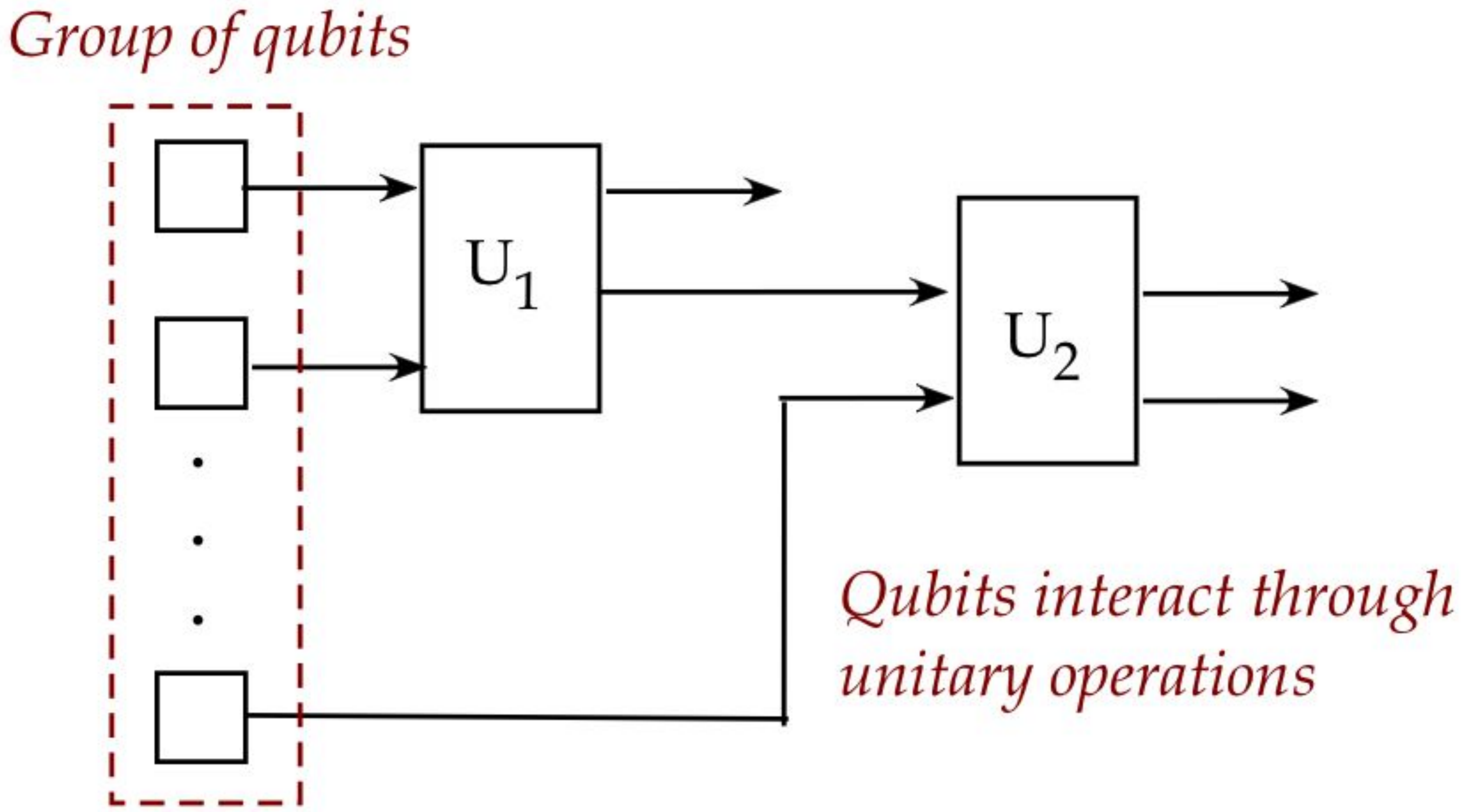
$$\begin{cases} \theta = \pi/2 \\ \varphi = 3\pi/2 \end{cases}$$

$$\cos(3\pi/2) + i\, sin(3\pi/2)$$

$$|\Psi'\rangle = \cos(\tfrac{\pi/2}{2})|0\rangle + \boxed{e^{i\,3\pi/2}} sin(\tfrac{\pi/2}{2})|1\rangle$$

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}|0\rangle - i\,\frac{1}{\sqrt{2}}|1\rangle$$

$$|\Psi'\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \Rightarrow |-i\rangle$$


Group of qubits
U1
U2
Qubits interact through
unitary operations