Competition

I. ENERGY LEVELS OF THE QUANTUM ISING MODEL

Consider the following Ising model Hamiltonian:

$$H = \sum_{i=0}^{2} Z_i Z_{i+1} + \sum_{i=0}^{2} X_i \tag{1}$$

with periodic boundary conditions. Using the quantum algorithm of your choice, compute all the energy levels of H.

Challenge: Can you get its density matrix at finite temperature?

II. GROUND STATE OF A MOLECULE

Let us assume that for some one-particle states $(|\phi_j\rangle)_{j\in\mathbb{N}}$, a second-quantized and spin-free Hamiltonian can be written:

$$H =(a_0 + a_0^\dagger)(a_1 + a_1^\dagger)(1 - 2a_0^\dagger a_0) + (a_1 - a_1^\dagger)(a_2^\dagger - a_2)(1 - 2a_1^\dagger a_1) + (1 - 2a_2^\dagger a_2)(1 - 2a_3^\dagger a_3) \tag{2}$$

where $(a_j)_{j\in\mathbb{N}}$ and $(a_j^\dagger)_{j\in\mathbb{N}}$ are the corresponding ladder operators, satisfying the fermionic Canonical Commutation Relations. Using the quantum algorithm of your choice, compute the ground state of this Hamiltonian.

Challenge: What is its degeneracy?