CS场论- IQHE

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1. 基本电磁理论.

$$
\begin{cases}
\nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\
\nabla\times\vec{H} = \dfrac{\partial\vec{D}}{\partial t} + \vec{J} \\
\nabla\cdot\vec{D} = \rho \\
\nabla\cdot\vec{B} = 0
\end{cases}
$$

$$\vec{B} = \nabla\times\vec{A} \quad \rightarrow \quad \nabla\times\left(\vec{E} + \frac{\partial\vec{A}}{\partial t}\right) = 0$$

无旋场, 可以用标势 φ 的梯度来表示.

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$$

$\boxed{\text{Current } J^\mu_\mu}$

δA_μ:
- Electromagnetism background gauge field. NOT DYNAMICAL, Only parameter.
- NOT include original background magnetic field (produce QHE)
- Only perturbations around Hall state (turn on an electric or magnetic field)

J_μ
- Current, DYNAMICAL degrees of freedom.
- Always couple to gauge field

$$S_A = \int d^3x \; J^\mu A_\mu$$

Perturbation – Response pair

$(2+1)d$

$$\vec{J}_\mu = \sigma^{xy}\,\delta\vec{E}\times\vec{z}$$

$\vec{J}_H = \sigma^{xy} \delta \vec{E} \times \vec{z}$

$\delta \vec{E} = -\partial_t \delta \vec{A} - \nabla \delta A_0$

Continuity equation: $\partial_t \rho_H + \nabla \cdot \vec{J}_H = 0$

$$\Rightarrow \partial_t \rho_H = -\nabla \cdot \vec{J}_H = -\sigma^{xy} \vec{z} \cdot (\nabla \times \vec{E})$$

$$= \sigma^{xy} \vec{z} \cdot (\partial_t \nabla \times \delta \vec{A})$$

$$= \sigma^{xy} \partial_t \cdot \delta B_z$$

$$\Rightarrow \rho_H = \sigma^{xy} \cdot \delta B_z$$

$$J_H^\mu = \sigma^{xy} \partial_\nu \delta A_\lambda \varepsilon^{\mu\nu\lambda} \Longleftrightarrow \begin{cases} \rho_H = \sigma^{xy} \delta B_z \\ \vec{J}_H = \sigma^{xy} \delta \vec{E} \times \vec{z} \end{cases}$$

$J_H^\mu = (\rho_H, J_H^x, J_H^y)$

$$\rho_H = \sigma^{xy} (\partial_x \cdot \delta A_y - \partial_y \delta A_x) = \sigma^{xy} \cdot \delta B_z$$

$$J_H^x = \sigma^{xy} (\partial_y \delta A_0 - \partial_0 \delta A_y) = \sigma^{xy} \cdot \delta E_y$$

$$J_H^y = \sigma^{xy} (\partial_0 \delta A_x - \partial_x \delta A_0) = \sigma^{xy} \cdot \delta E_x$$

$$J_H^\mu \overset{\text{def.}}{\equiv} \frac{\delta S[\delta A]}{\delta [\delta A_\mu]}$$

推测 Action $S[\delta A]$

- Say nothing about microscopic: electrons, Landau level
- General assumption: at low energies, there are no degrees of freedom that can affect the physics when the system is perturbed.
  - Gap
  - GSD (FQHE)

Partition function:

$$Z[A_\mu] = \int D(\phi)\, e^{iS[\phi, A_\mu]/\hbar}$$

A_μ: NOT dynamic, source.

ϕ: field, dynamic degrees of freedom,

be integrated out.

$$Z[A_\mu] = e^{iS_{eff}[A_\mu]/\hbar}$$

$S_{eff}[A_\mu]$ depend on parameter! rather than dynamic field!

NO dynimics in A_μ.

$$J^\mu(x) = \frac{\delta S_{eff}[A]}{\delta A_\mu(x)}$$

Assumption of $S_{eff}[A_\mu]$:

1. gauge invariance.

2. symmetry. (rotation, translation)

3. Local. $S_{eff} = \int d^3x \cdots\cdots$

a gap ΔE, non-locality arise at distance $\sim \frac{v\hbar}{\Delta E}$

$\vdots$

$\boxed{\text{The Chern-Simons Term}}$

$$S_{cs}[A] = \frac{k}{4\pi}\int d^3x\, \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

k: level of the Chern-Simons term.

gauge invariance

$$A_\mu \longrightarrow A_\mu + \partial_\mu w$$

$$S_{cs} \longrightarrow \frac{k}{4\pi}\int d^3x\, \varepsilon^{\mu\nu\rho}\,(A_\mu + \partial_\mu w)\,\partial_\nu(A_\rho + \partial_\rho w)$$

$$= S_{cs} + \frac{k}{4\pi}\int d^3x\, \varepsilon^{\mu\nu\rho}\,[\partial_\mu w \cdot \partial_\nu A_\rho + A_\mu \partial_\nu \partial_\rho w + \partial_\mu w\, \partial_\nu \partial_\rho w]$$

$$= S_{cs} + \frac{k}{4\pi}\int d^3k \cdot \partial_\mu(w\,\varepsilon^{\mu\nu\rho}\partial_\nu A_\rho)$$

Second term: throw total derivative. $\longrightarrow$ gauge invariance

HOWEVER ..

Symmetry

rotation symmetry : ✓

parity reversal symmetry: ✗

$$x^0 \rightarrow x^0, \quad x^1 \rightarrow -x^1, \quad x^2 \rightarrow x^2$$

correspondingly $A_0 \rightarrow A_0 \quad A_1 \rightarrow -A_1, \quad A_2 \rightarrow A_2$ (线性矛盾)

$$\Rightarrow \quad \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \longrightarrow -\varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

★ the Chern-Simons effective interaction only arise in systems that BREAK PARITY SYMMETRY.

time reversal symmetry: ✗

J^μ

$$J_i = \frac{\delta S_{cs}[A]}{\delta A_i} = \frac{k}{4\pi}\frac{1}{\delta A_i}\int d^3x\, \varepsilon^{\mu\nu\rho}\,[\delta A_\mu \partial_\nu A_\rho + A_\mu \partial_\nu \delta A_\rho]$$

$$= -\frac{k}{2\pi} \varepsilon_{ij} E_j$$

QHE: $\sigma_{xy} = \frac{k}{2\pi} \xlongequal[k=e^2\nu/h]{} \nu \cdot \frac{e^2}{h}$

$$J_0 = \frac{\delta S_{cs}[A]}{\delta A_0} = \frac{k}{4\pi} \frac{1}{\delta A_0} \cdot \delta \int d^3x\, \varepsilon^{0\nu\rho} A_0 \partial_\nu A_\rho$$

$$= \frac{k}{4\pi} \varepsilon^{\nu\rho} \partial_\nu A_\rho$$

$$= \frac{k}{2\pi} B$$

A_μ is additional gauge field. $\rightarrow$ J^0 is the change in the charge density over and above that already present in the ground state.

Quantisation of the Chern-Simons level k

Wick rotation : τ periodic S^1