



THE UNIVERSITY  
of NORTH CAROLINA  
*at* CHAPEL HILL

VISTA seminar, June 12, 2024

# **Coupled Quantum Dynamics of Electrons and Protons in Heterogeneous Environments**

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# First-Principles Method Development for Condensed Matter

## Real-time time-dependent density functional theory for non-equilibrium electron dynamics

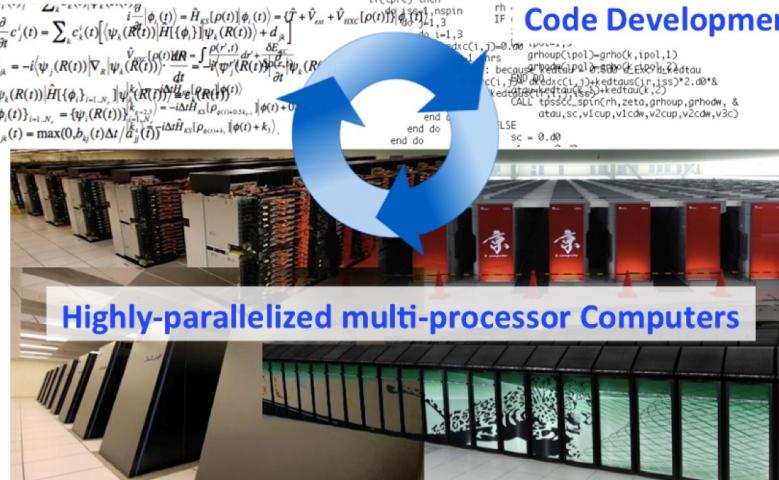
## Many-body Green's function theory for electronic excitation properties

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 $\psi = \psi_0 e^{-iE_0 t} + |\psi_1\rangle\langle\psi_1(t) - \psi_{in}\rangle|\psi_1\rangle$        $\psi_i(r,t+\Delta t) = T e^{-iE_i \Delta t} \psi_i(r,t)$ 
 $\frac{\partial \psi(R,r,t)}{\partial t} = (\hat{H} - E_i) \psi(R,r,t)$        $\psi_i(r,t) = \sum c_i(t) \phi_i(r,R) e^{-iE_i R \omega}$ 
 $(R,t+\Delta t) \leftarrow G(R,R',\Delta t) f(R,t) dR$        $(R,R',\Delta t) = \Psi_i^{-1}(R') \Psi_i(R) R' \exp(-i\int_{R'}^R \frac{\partial \hat{H}}{\partial R} dR) \int_{R'}^R f(R,t) dR$ 
 $(R) = \Psi_i(R) \exp\left(\sum \sum_{\alpha,\beta} \delta_{\alpha\beta} \langle r_\alpha | \phi_i(R) | r_\beta \rangle - \sum_{\alpha,\beta} \delta_{\alpha\beta} \langle r_\alpha | \phi_i(R) | r_\beta \rangle - \sum_{\alpha,\beta} \lambda_{\alpha\beta} \int dr \phi_i(r) \phi_j(r) \delta_{\alpha\beta}\right)$ 
 $\frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$        $\langle \phi_i(r)| \phi_j(r') \rangle = \frac{1}{V} \int_V \sum_{\alpha,\beta} \lambda_{\alpha\beta} \langle r_\alpha | \phi_i(R) | r_\beta \rangle \delta_{\alpha\beta} \delta_{r,r'}$ 
 $E = E_i$ 
```

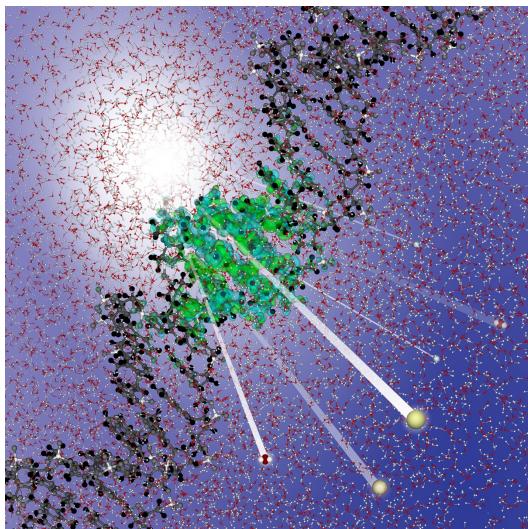
### Analytical Formulations

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 $\tau^{(i)} \sim \frac{1}{\Delta t} \frac{d\phi_i(t)}{dt} = \hat{H}_{\text{xc}}[\rho(t)] \phi_i(t) - i\hat{T} + \hat{V}_{\text{ee}} + \hat{V}_{\text{xc}}[\rho(t)] \phi_i(t)$ 
 $\frac{\partial}{\partial t} c'_i(t) = \sum_k c'_k(t) [\langle \psi_i(R(t)) | H(\phi_i) | \psi_k(R(t)) \rangle + d_{jk}]$ 
 $i_{jk} = -i \langle \psi_j(R(t)) | \nabla_R | \psi_k(R(t)) \rangle - i \frac{d \langle \psi_j(R(t)) | \psi_k(R(t)) \rangle}{dt}$ 
 $\psi_i(R(t)) | \hat{H}[\{\phi_j\}_{j \neq i}] | \psi_i(R(t)) \rangle = -i \Delta \hat{H}_{\text{xc}}[\rho_{j \neq i}] \phi_i(t) + \phi_i(t)$ 
 $\lambda_{jk}(t) = \max(0, b_{jk}(t) \Delta t / \tilde{E}_j) \Delta t \hat{H}_{\text{xc}}[\rho_{j \neq i}] \phi_i(t) + k_{ij}$ 
```

### Code Development

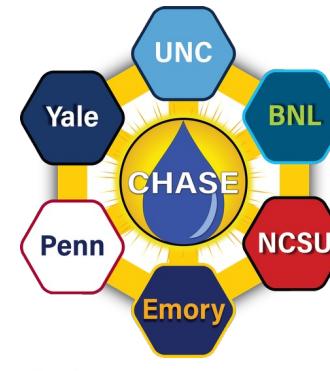


# Phenomena of Current Interest



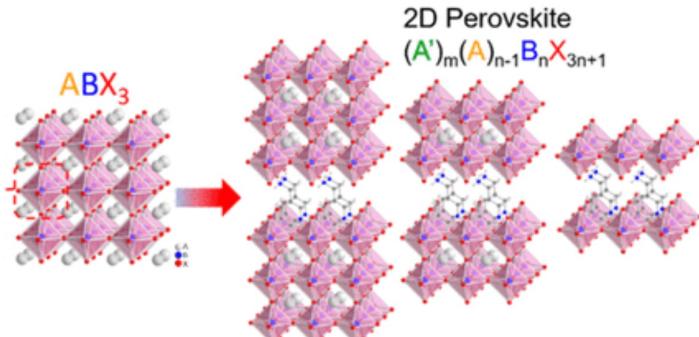
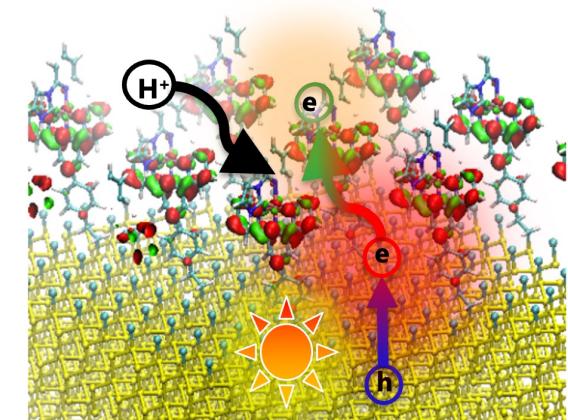
**Electronic Stopping**  
Excitation dynamics under proton irradiation and beyond

**Quantum Dynamics of Electrons and Protons**  
Elucidating the mechanism for CO<sub>2</sub> conversion



**CHASE**

Center for Hybrid Approaches in  
Solar Energy to Liquid Fuels



**Novel Materials and Dynamics**  
Application of first-principles method to “explore”  
interesting materials and dynamic phenomena

# Kanai Group @ UNC



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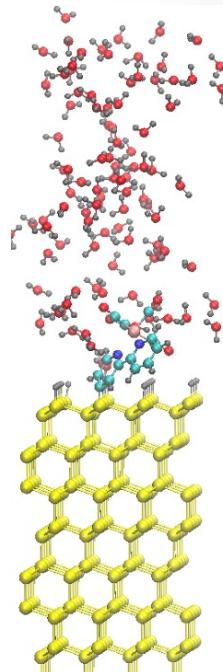
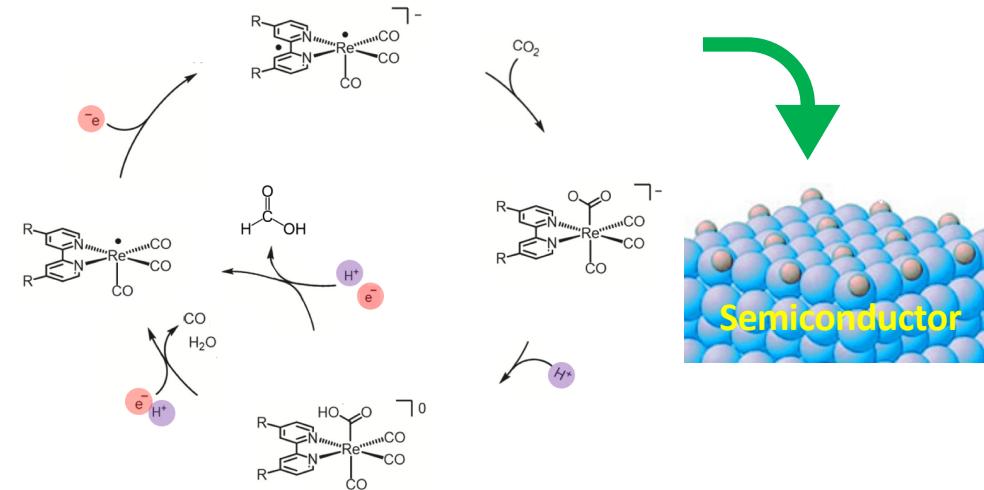
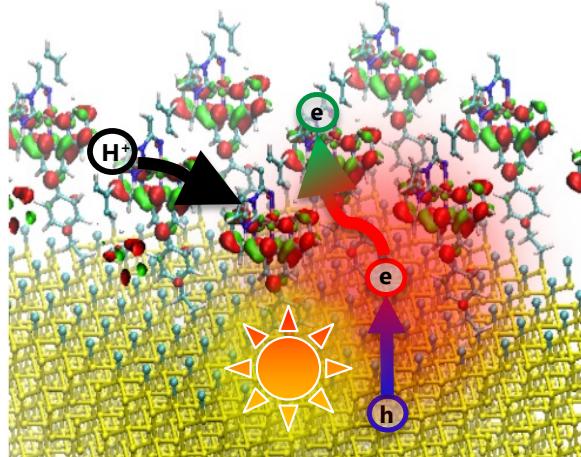


# Theory Challenges in DOE Hub: CHASE



CHASE  
Center for Hybrid Approaches in  
Solar Energy to Liquid Fuels

## Development of CO<sub>2</sub>-catalyst functionalized semiconductor photoelectrodes



How does surface adsorption impact H<sup>+</sup>/e<sup>-</sup> transfer steps?

Complex heterogeneous environments need to be modeled.

To what extent is the interfacial excited e<sup>-</sup> transfer coupled with H<sup>+</sup> addition at an interface?

Coupled quantum dynamics of protons and electrons need to be considered.

## Multicomponent DFT

$$E[\rho^e, \rho^p] = E_{\text{ref}}[\rho^e, \rho^p] + E_{XC}^e[\rho^e] + E_{XC}^p[\rho^p] + E_{epc}[\rho^e, \rho^p]$$

### **“Non-Born–Oppenheimer Density Functional Theory of Molecular Systems”**

J. F. Capitani, R. F. Nalewajski, and R. G. Parr, J. Chem. Phys. 76, 568 (1982)

### **“Multicomponent Density-Functional Theory for Electrons and Nuclei”**

T. Kreibich and E. K. U. Gross, Phys. Rev. Lett. 86, 2984 (2001)

## Nuclear Electronic Orbital (NEO) method

F. Pavosevic, T. Culpitt, S. Hammes-Schiffer, Chem. Rev. **120**, 4222 (2020)

## NEO to KS-DFT for periodic systems

J. Xu, R. Zhou, Z. Tao, C. Malbon, V. Blum, S. Hammes-Schiffer, Y. Kanai, J. Chem. Phys. 156, 224111 (2022)

$$\psi_{i,\mathbf{k}}^e(\mathbf{r}^e) = \sum_{\mu} c_{i\mu,\mathbf{k}} \sum_N e^{i\mathbf{k}\cdot\mathbf{T}(\mathbf{N})} \phi_{\mu}^{e:NAO/GTO} (\mathbf{r}^e - \mathbf{R}_{\mu} + \mathbf{T}(\mathbf{N}))$$

$$\psi_i^p(\mathbf{r}^p) = \sum_m c_{im} \sum_N \phi_m^{p:GTO} (\mathbf{r}^p - \mathbf{R}_m + \mathbf{T}(\mathbf{N}))$$

Atom-centered basis functions

## KS Hamiltonian in Multicomponent DFT

$$\hat{H}_{KS}^e = -\frac{1}{2}\nabla_e^2 + v_{ext}(\mathbf{r}^e) + v_{es}^e(\mathbf{r}^e) - v_{es}^p(\mathbf{r}^e) + \frac{\delta E_{XC}^e[\rho^e]}{\delta \rho^e} + \frac{\delta E_{epc}[\rho^e, \rho^p]}{\delta \rho^e}$$

$$\hat{H}_{KS}^p = -\frac{1}{2M^p}\nabla_p^2 - v_{ext}(\mathbf{r}^p) - v_{es}^e(\mathbf{r}^p) + v_{es}^p(\mathbf{r}^p) + \frac{\delta E_{XC}^p[\rho^p]}{\delta \rho^p} + \frac{\delta E_{epc}[\rho^e, \rho^p]}{\delta \rho^p}$$

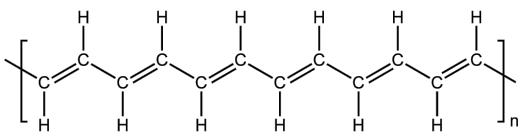
## Electron-Proton Correlation

Multicomponent extensions of **Colle-Salvetti formalism** have been developed. The simplest form is the local density approximation (LDA)

Y. Yang, et al., J. Chem. Phys. **147**, 114113 (2017)

$$E_{epc}[\rho^e, \rho^p] \approx E_{epc17-2} [\rho^e, \rho^p] = - \int \frac{\rho^e(\mathbf{r}) \rho^p(\mathbf{r})}{a + b \rho^e(\mathbf{r})^{\frac{1}{2}} \rho^p(\mathbf{r})^{\frac{1}{2}} + c \rho^e(\mathbf{r}) \rho^p(\mathbf{r})} d\mathbf{r}$$

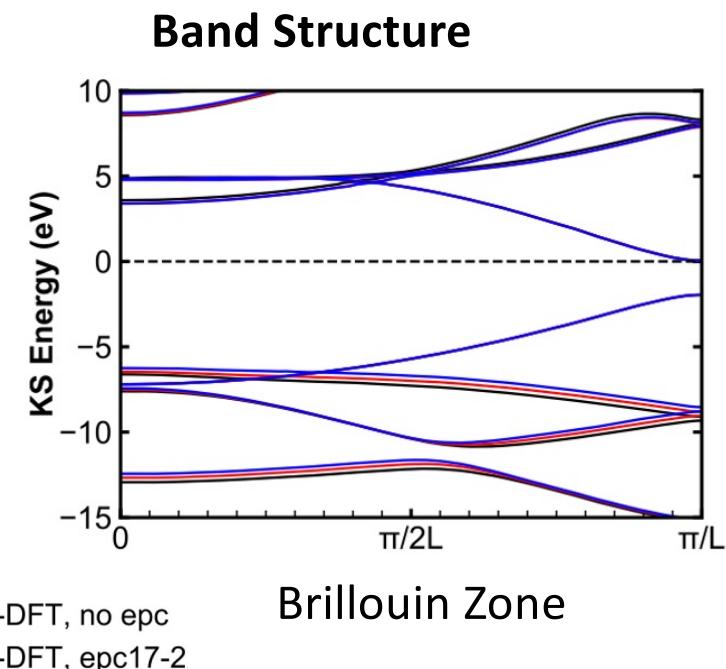
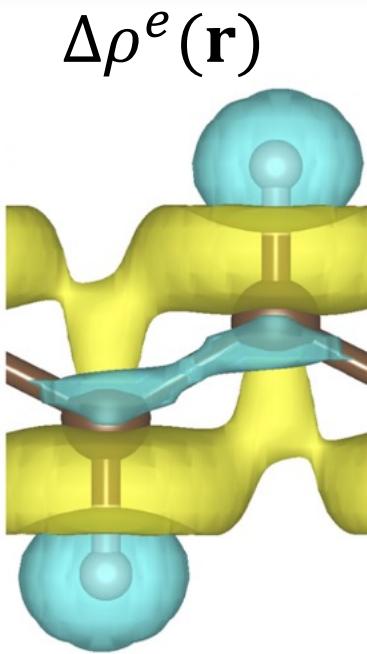
# Proof-of-Principle Demonstration



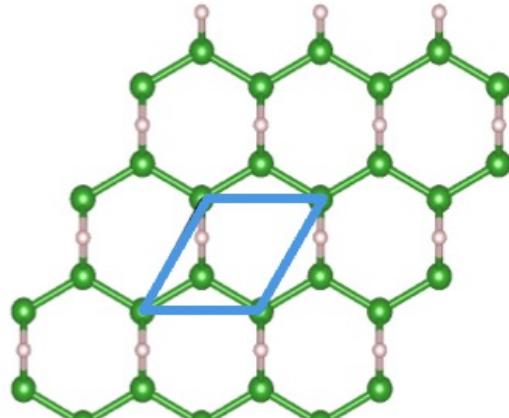
**NEO-induced Electron Density difference**

(+) change  
(-) change

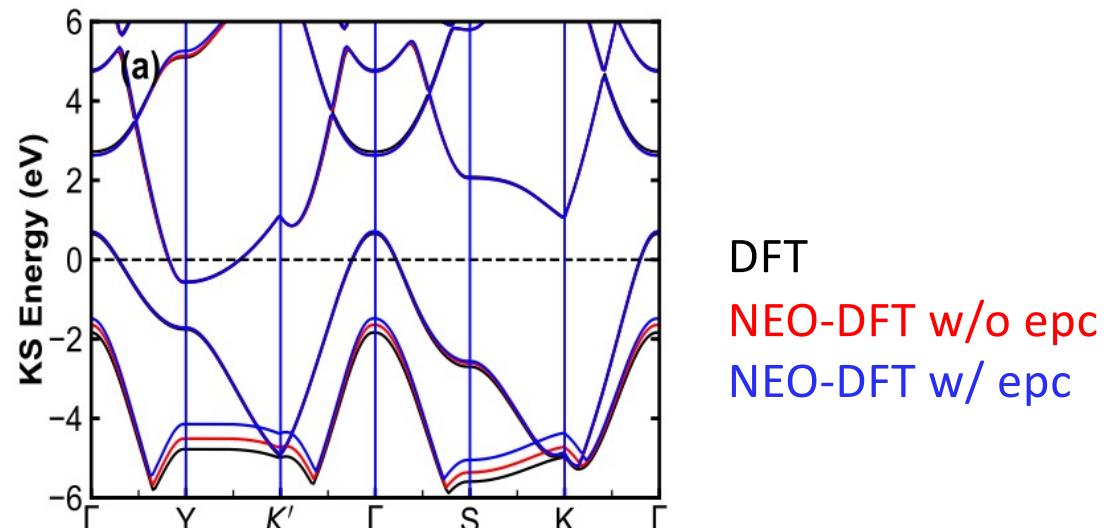
Isosurface @  $1.7 \times 10^{-2} \text{ Ang}^{-3}$



Hydrogen Boride 1D sheet : B-H-B 3-center-2-electron bonds



Y. Jiao, et al. Angewandte Chemie, 55, 35 (2016)



# Dynamics : Lagrangian for our NEO DFT-KS System

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$$\begin{aligned}
L^{NEO}(t) = & \int d\mathbf{r}^e \int d\mathbf{k} \sum_j [\psi_{j\mathbf{k}}^e(\mathbf{r}^e, t)]^* \left[ i \frac{\partial}{\partial t} + \frac{1}{2m^e} \nabla_{\mathbf{r}^e}^2 \right] \psi_{j\mathbf{k}}^e(\mathbf{r}^e, t) \\
& - \frac{1}{2} \iint d\mathbf{r}^e d\mathbf{r}^{e'} \frac{e^2}{|\mathbf{r}^e - \mathbf{r}^{e'}|} \rho^e(\mathbf{r}^e, t) \rho^e(\mathbf{r}^{e'}, t) - E_{XC}^e[\rho^e] \\
& + \int d\mathbf{r}^p \sum_j [\psi_j^p(\mathbf{r}^p, t)]^* \left[ i \frac{\partial}{\partial t} + \frac{1}{2M^p} \nabla_{\mathbf{r}^p}^2 \right] \psi_j^p(\mathbf{r}^p, t) \\
& - \frac{1}{2} \iint d\mathbf{r}^p d\mathbf{r}^{p'} \frac{e^2}{|\mathbf{r}^p - \mathbf{r}^{p'}|} \rho^p(\mathbf{r}^p, t) \rho^p(\mathbf{r}^{p'}, t) - E_{XC}^p[\rho^p] \\
& + \frac{1}{2} \iint d\mathbf{r}^e d\mathbf{r}^p \frac{e^2}{|\mathbf{r}^e - \mathbf{r}^p|} \rho^e(\mathbf{r}^e, t) \rho^p(\mathbf{r}^p, t) - E_{epc}[\rho^e, \rho^p] \\
& + \sum_I \frac{1}{2} M_I [\dot{\mathbf{R}}_I(t)]^2 - \sum_{I < J} \frac{Z_I Z_J e^2}{|\mathbf{R}_I(t) - \mathbf{R}_J(t)|} \\
& - \int d\mathbf{r}^p \rho^p(\mathbf{r}^p, t) \sum_I \frac{Z_I e^2}{|\mathbf{r}^p - \mathbf{R}_I(t)|} + \int d\mathbf{r}^e \rho^e(\mathbf{r}^e, t) \sum_I \frac{Z_I e^2}{|\mathbf{r}^e - \mathbf{R}_I(t)|}
\end{aligned}$$

# TD-KS equations for RT-**NEO**-TDDFT

$$A = \int L^{NEO}(\mathbf{R}, \dot{\mathbf{R}}, \{\psi_{i,\mathbf{k}}^e\}, \{\psi_I^p\}, t) dt$$

$$\frac{\delta A}{\langle \delta \psi^x |} = 0 \quad \frac{\delta L^{NEO}}{\langle \delta \psi^x |} - \frac{d}{dt} \frac{\delta L^{NEO}}{\langle \delta \dot{\psi}^x |} = 0$$

See, e.g. Kramer and Saraceno, "Geometry of the Time-Dependent Variational Principle" (Springer, Berlin, 1981).

**Electrons:**  $i \frac{\partial}{\partial t} \psi_{i,\mathbf{k}}^e(\mathbf{r}^e, t) = \left[ -\frac{1}{2} \nabla_i^2 + v_{\text{DFT-KS}}^e(\mathbf{r}^e) - v_{\text{es}}^e(\mathbf{r}^e) + \frac{\delta E_{\text{epc}}[\rho^e, \rho^p]}{\delta \rho^e} \right] \psi_{i,\mathbf{k}}^e(\mathbf{r}^e, t)$

**RT-TDDFT**

All-electron real-time and imaginary-time TDDFT within a numeric atom-centered basis function framework J. Hekele, et al. [J. Chem. Phys. 155, 154801 \(2021\)](#)

**Protons:**  $i \frac{\partial}{\partial t} \psi_I^p(\mathbf{r}^p, t) = \left[ -\frac{1}{2M^p} \nabla_I^2 + v_{\text{DFT-KS}}^p(\mathbf{r}^p) - v_{\text{es}}^p(\mathbf{r}^p) + \frac{\delta E_{\text{epc}}[\rho^e, \rho^p]}{\delta \rho^p} \right] \psi_I^p(\mathbf{r}^p, t)$

L. Zhao, et al., J. Phys. Chem. Lett. **11**, 4052 (2020)

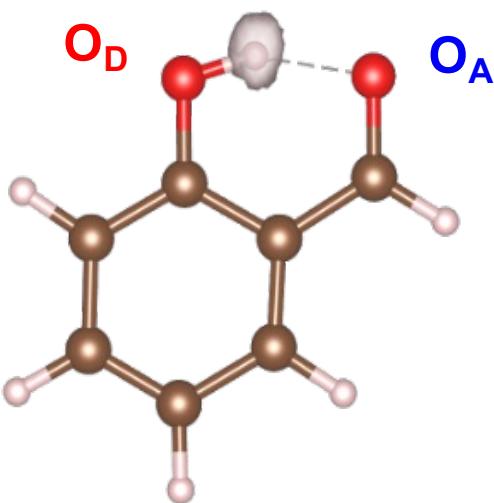
$$\psi_i^p(\mathbf{r}^p, t) = \sum_m c_{im}(t) \sum_N \phi_m^{p:GTO}(\mathbf{r}^p - \mathbf{R}_m + \mathbf{T}(\mathbf{N}))$$

$$\mathbf{C}(t + \Delta t) = \mathbf{S}^{-\frac{1}{2}} \exp \left( -i \Delta t \mathbf{S}^{-\frac{1}{2}} \mathbf{H}_{KS} \left( t + \frac{\Delta t}{2} \right) \mathbf{S}^{-\frac{1}{2}} \right) \mathbf{S}^{\frac{1}{2}} \mathbf{C}(t)$$

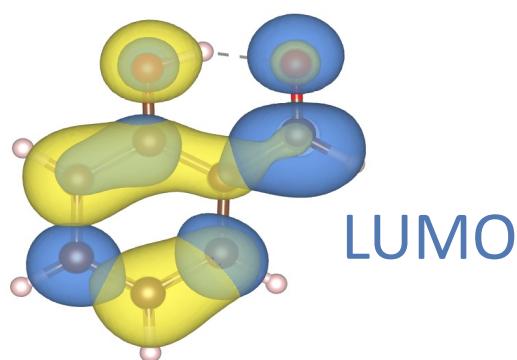
$$\exp(\mathbf{A}) = \mathbf{V} \text{diag}(e^{\lambda_1}, e^{\lambda_2} \dots e^{\lambda_n}) \mathbf{V}^{-1}$$

# Electronic Excitation Induced Intra-Molecular Proton Transfer

**oHBA molecule**



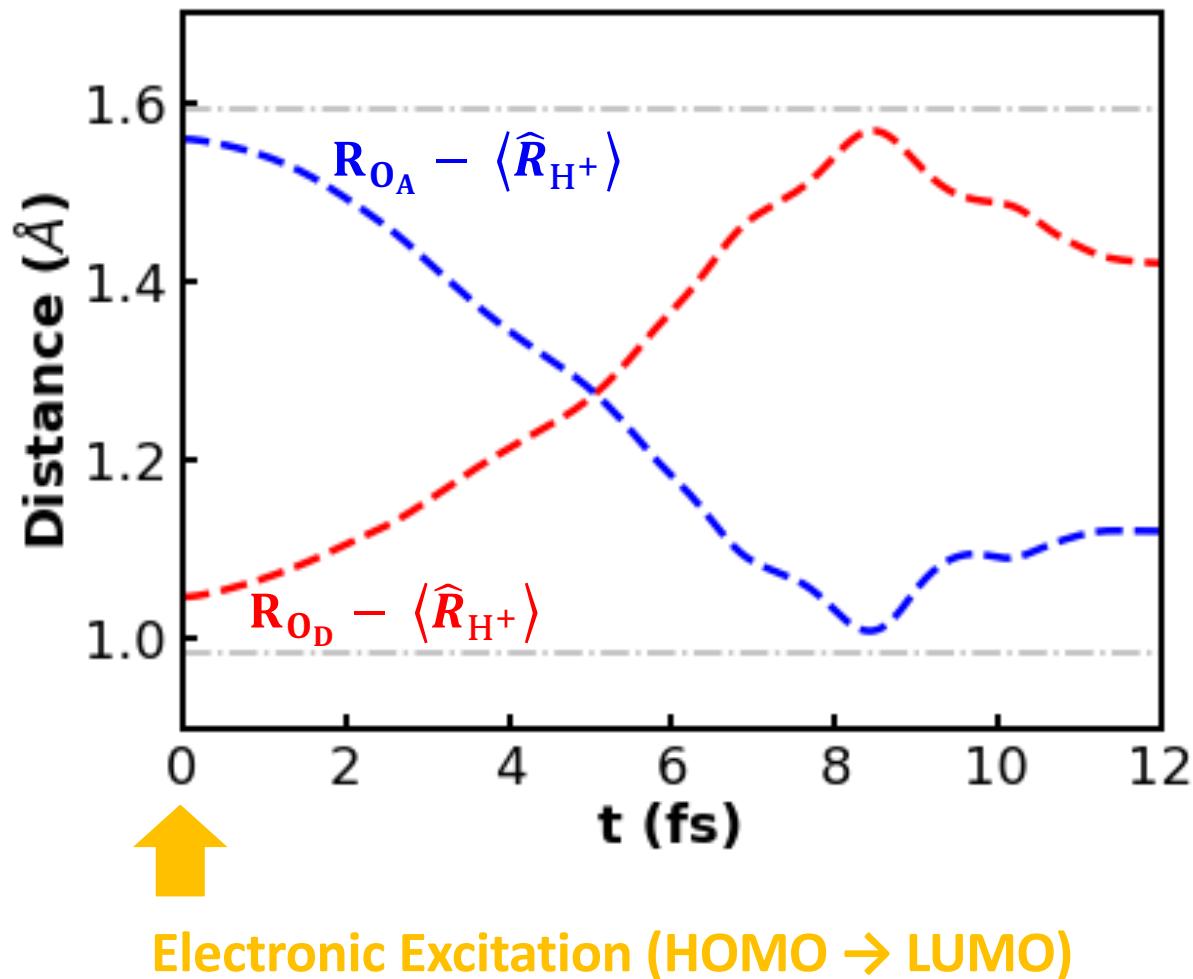
**HOMO**



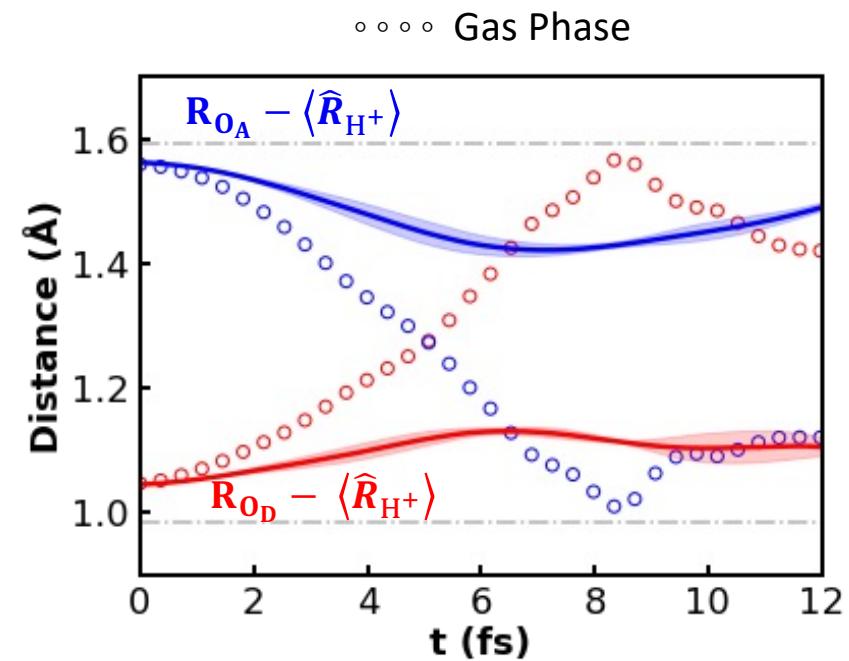
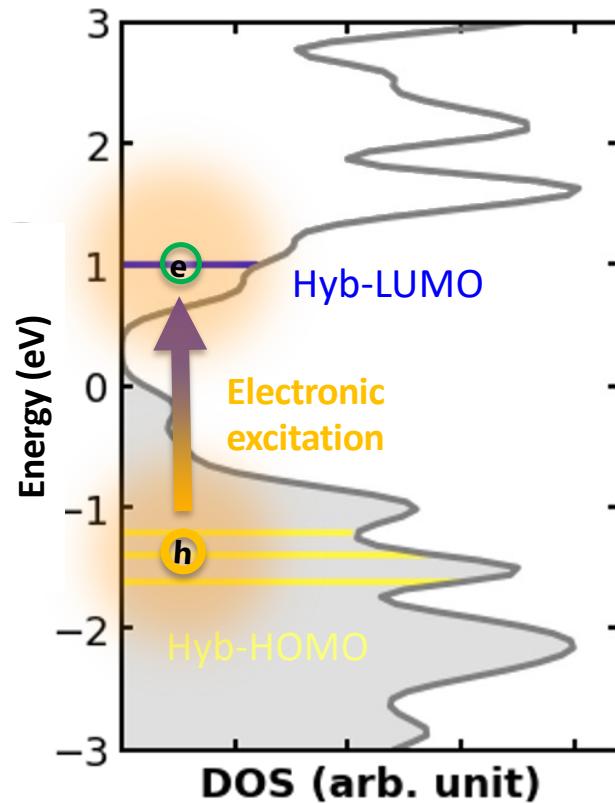
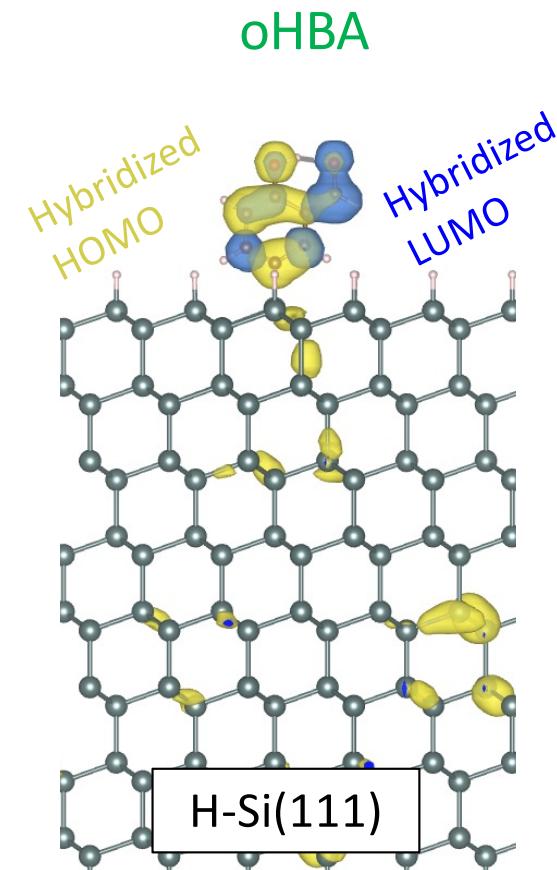
Electrons: PBE + Tier2 NAO basis

Protons: HF + 4s4p basis w/ 3 "ghost" centers

EPC: epc-17-2 , dt=4.8 attoseconds



# oHBA attached on Semiconductor Surface

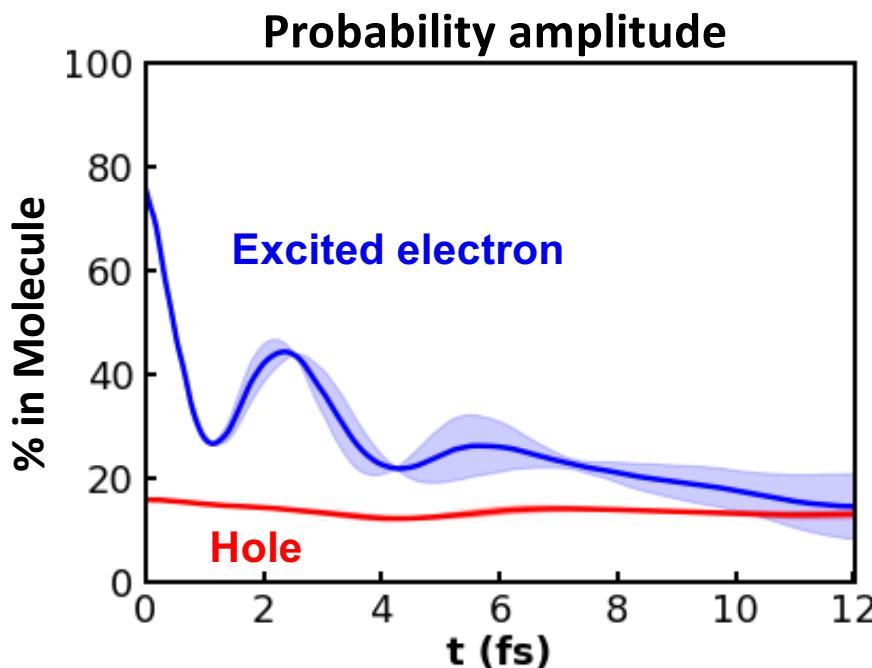
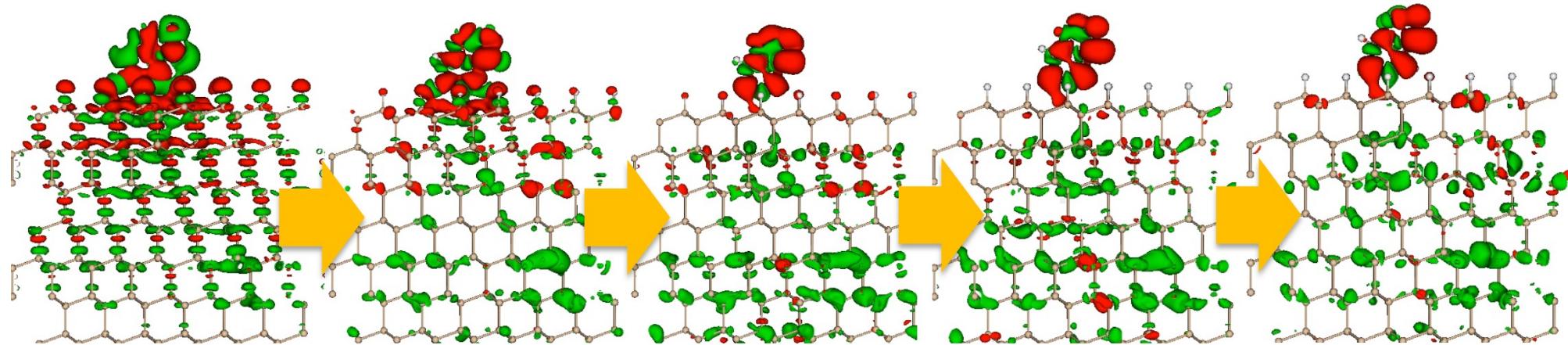


~3700 electrons.

$H^+$  transfer does NOT take place!

# Electron Density Changes at Interface

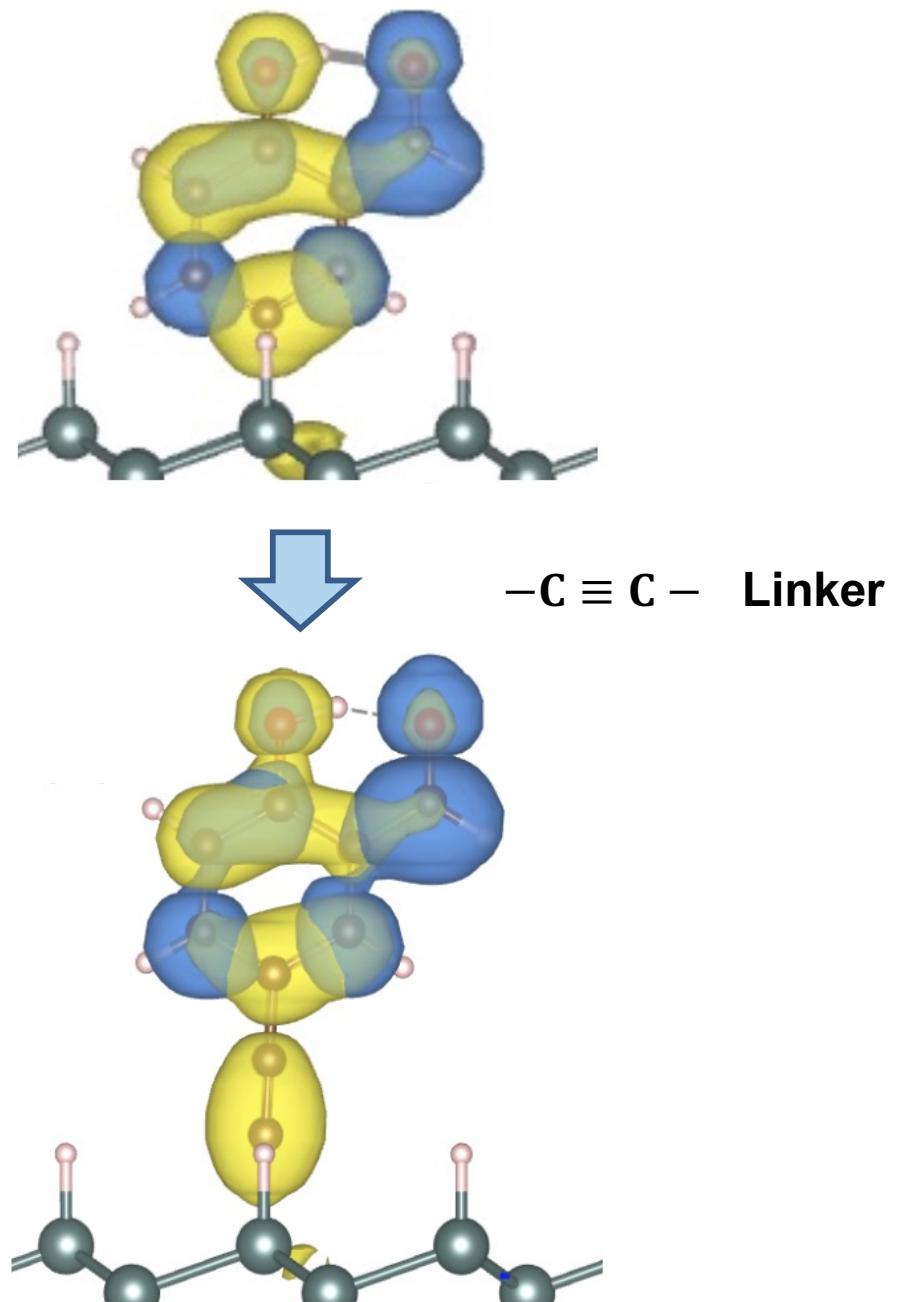
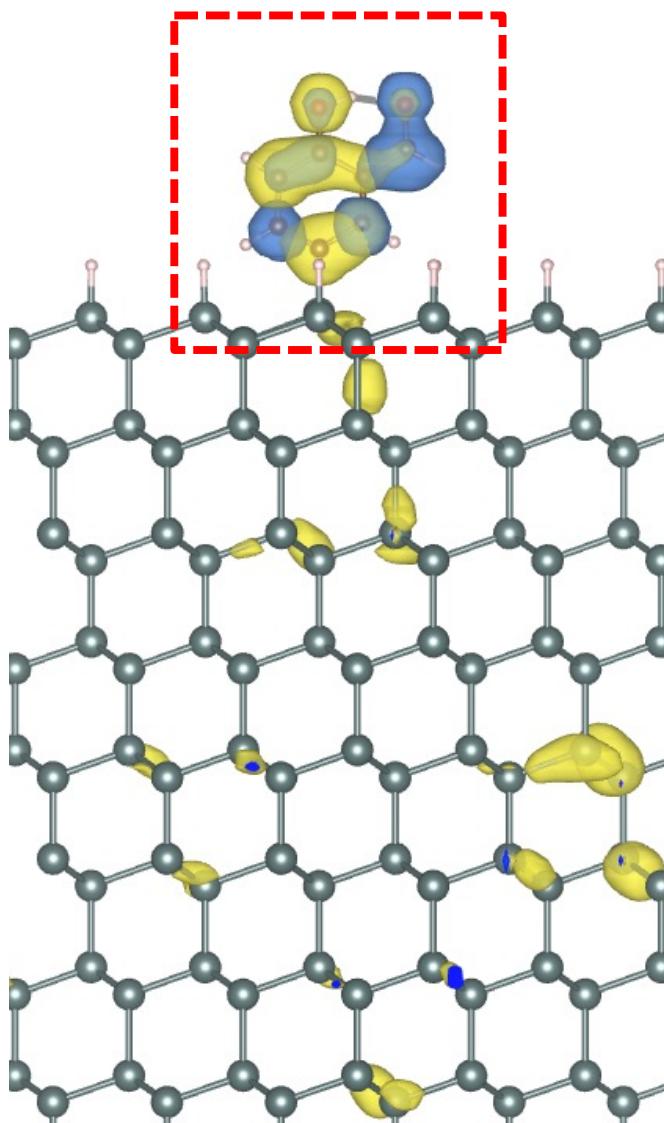
Electron density change :  $\Delta\rho^e(\mathbf{r}, t)$  Increases / Decreases



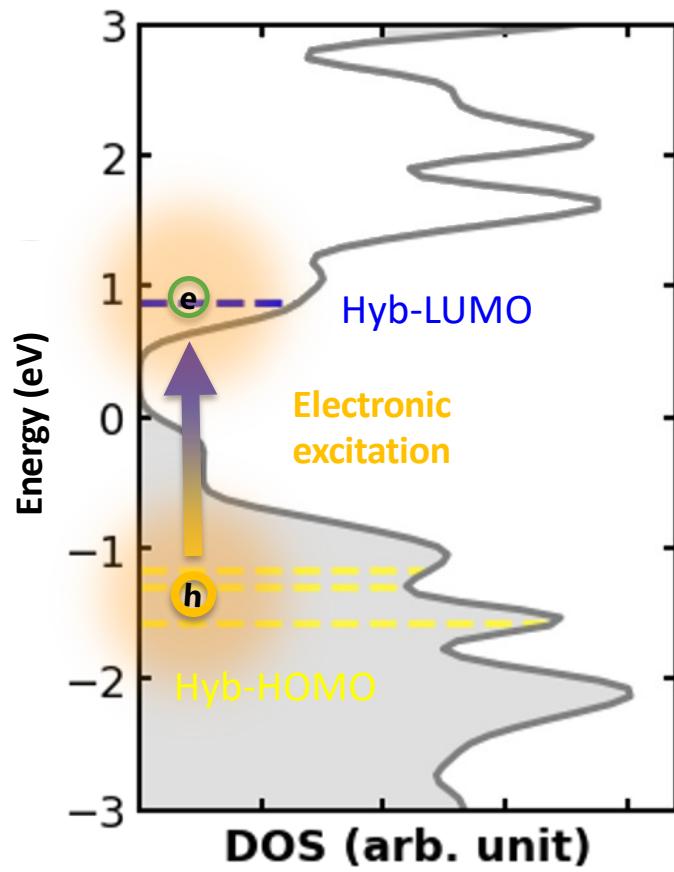
Ultrafast transfer of excited electron to semiconductor surface.

No driving force for the intra-molecular H<sup>+</sup> transfer.

# Role of Excited Electron Transfer

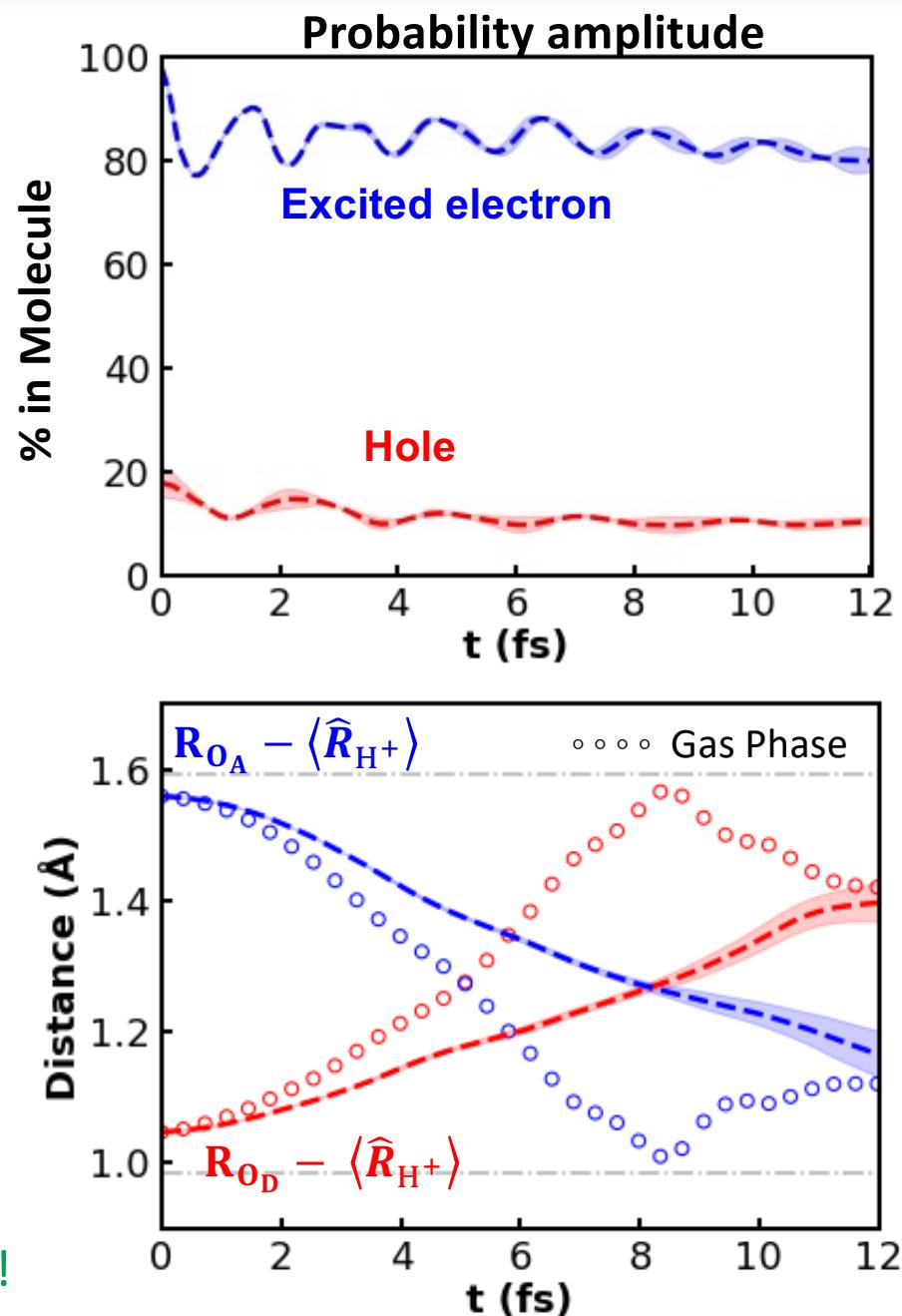


# Controlling Electron Transfer w/ Linker Group

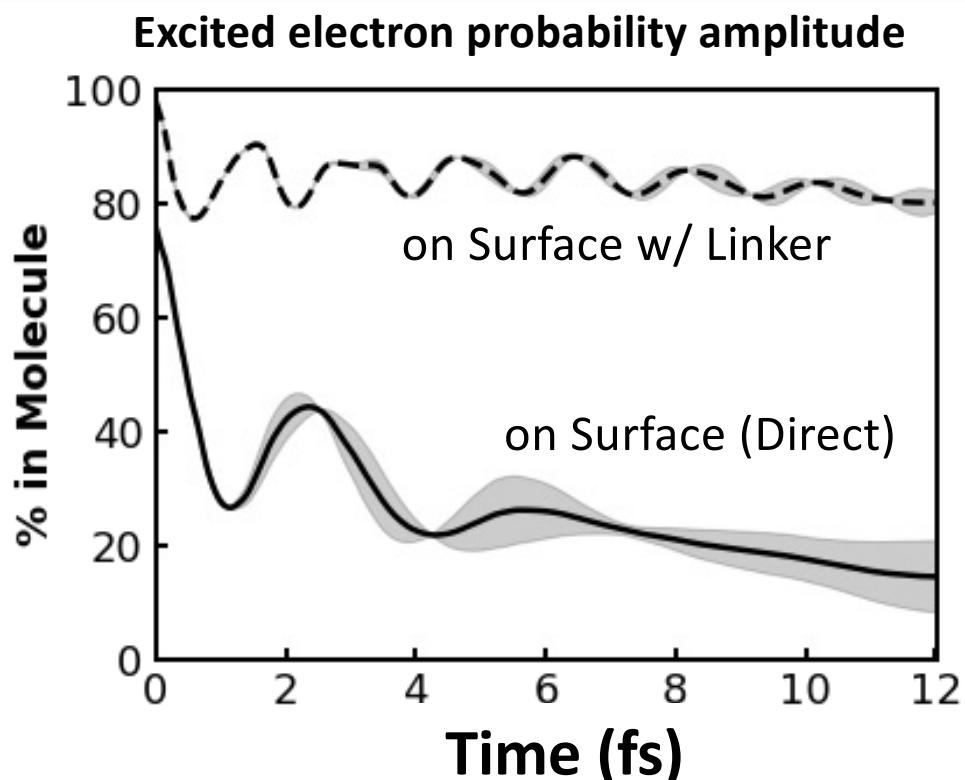
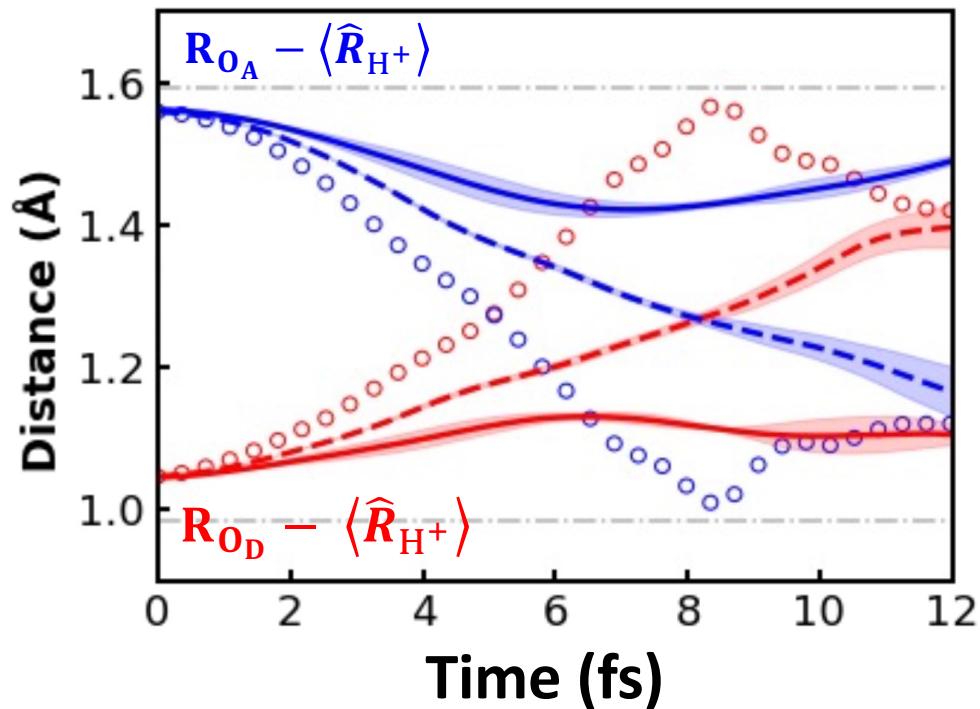


Excited electron transfer to the semiconductor is significantly slower with the linker group.

Excitation-induced  $\text{H}^+$  transfer takes place!

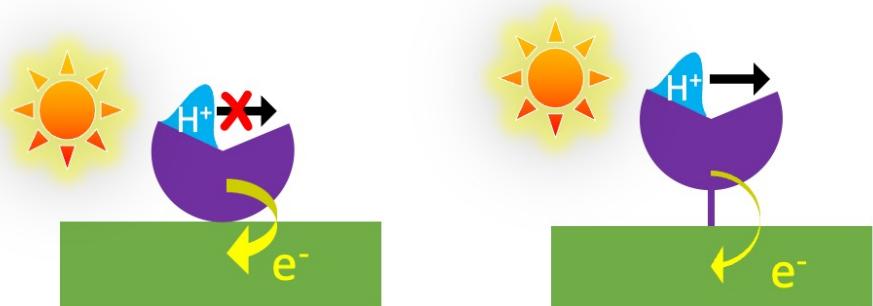


# Dependence on the surface attachment



- Gas Phase
- on Surface (Direct)
- - - on Surface w/ Linker

Surface attachment controls intra-molecular  $\text{H}^+$  transfer via interfacial excited electron transfer.



## Summary and References

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NEO-DFT method for periodic systems.

**Nuclear-Electronic Orbital Approach to Quantization of Protons in Periodic Electronic Structure Calculations**

J. Xu, R. Zhou, Z. Tao, C. Malbon, V. Blum, S. Hammes-Schiffer, Y. Kanai

[J. Chem. Phys. 156, 224111 \(2022\)](#)

RT-TDDFT Implementation in all-electron periodic systems with NAO basis.

**All-electron real-time and imaginary-time time-dependent density functional theory within a numeric atom-centered basis function framework**

J. Hekele, Y. Yao, Y. Kanai, V. Blum, P. Kratzer

[J. Chem. Phys. 155, 154801 \(2021\)](#)

Periodic RT-NEO-TDDFT method for studying coupled electron-proton dynamics in heterogeneous systems

**First-Principles Approach to Coupled Quantum Dynamics of Electrons and Protons in Heterogeneous Systems**

J. Xu, R. Zhou, V. Blum, T. E. Li, S. Hammes-Schiffer, Y. Kanai

[Phys. Rev. Lett. 131, 238002 \(2023\)](#) Editors' Suggestion

Ehrenfest dynamics extension using Lagrangian formulation for traveling proton basis scheme

**Lagrangian Formulation of RT-NEO-TDDFT Ehrenfest Dynamics (temp)**

J. Xu, R. Zhou, T. E. Li, S. Hammes-Schiffer, Y. Kanai

[In preparation](#)

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Thank you for listening!