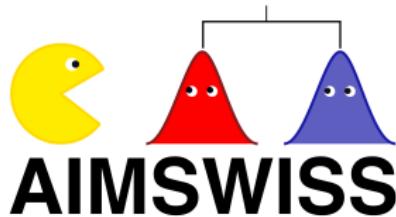


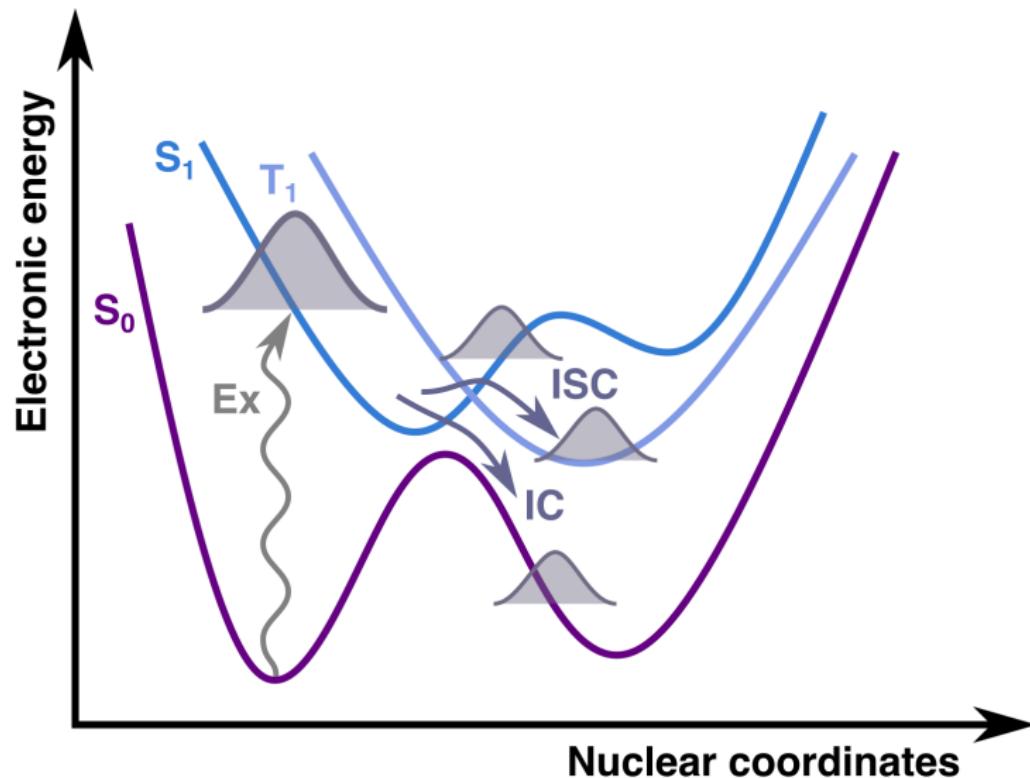
# Ab Initio Multiple Spawning with Informed Stochastic Selections



Yorick Lassmann  
VISTA Seminar 27  
11.11.2021

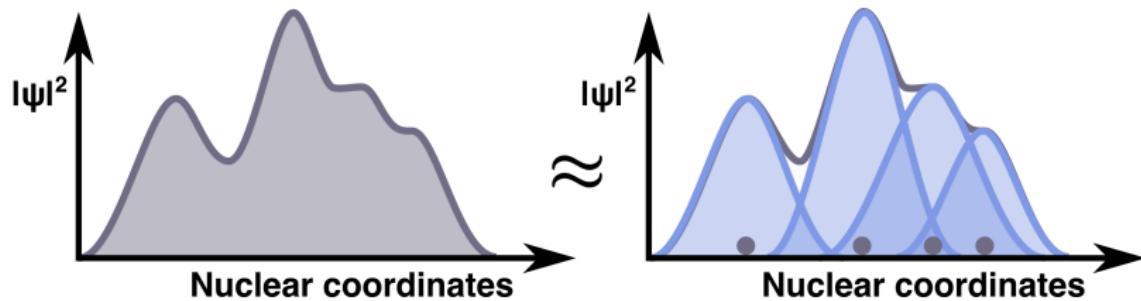


# Standard Model of Photochemistry



# The Threefold Way of Multiple Spawning

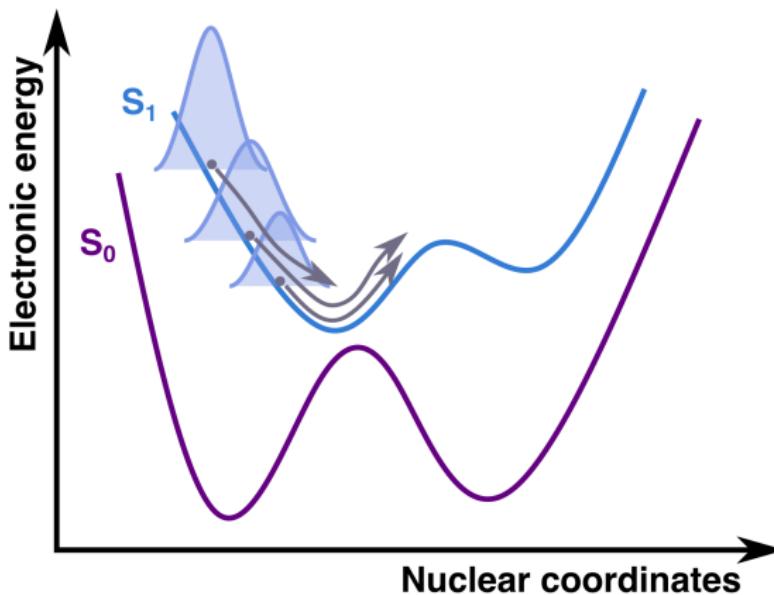
1. Expand nuclear wavefunctions in a basis set of frozen Gaussians



T. J. Martinez, M. Ben-Nun and R. D. Levine, J. Phys. Chem. 1996, **100** (19), 7884–7895.

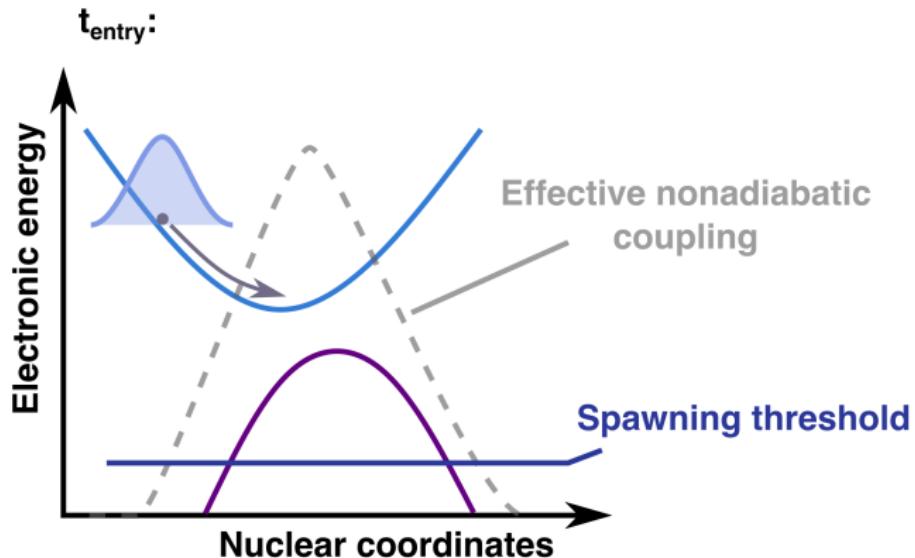
# The Threefold Way of Multiple Spawning

1. Expand nuclear wavefunctions in a basis set of frozen Gaussians
2. Propagate phase space centres of Gaussians classically on single adiabatic surfaces  $\Leftrightarrow$  trajectory basis functions (TBFs)



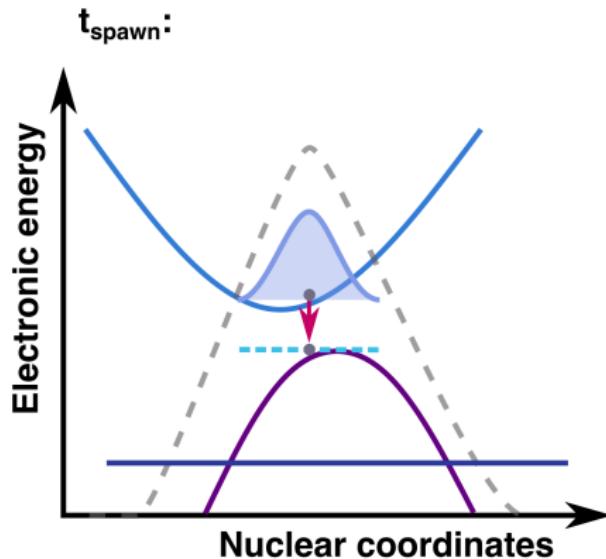
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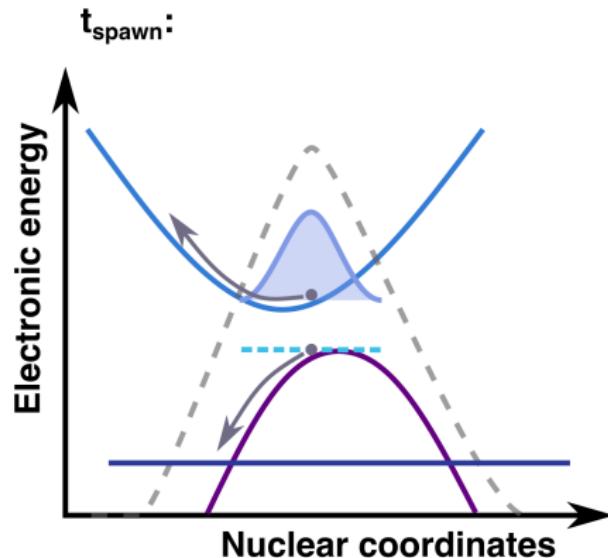
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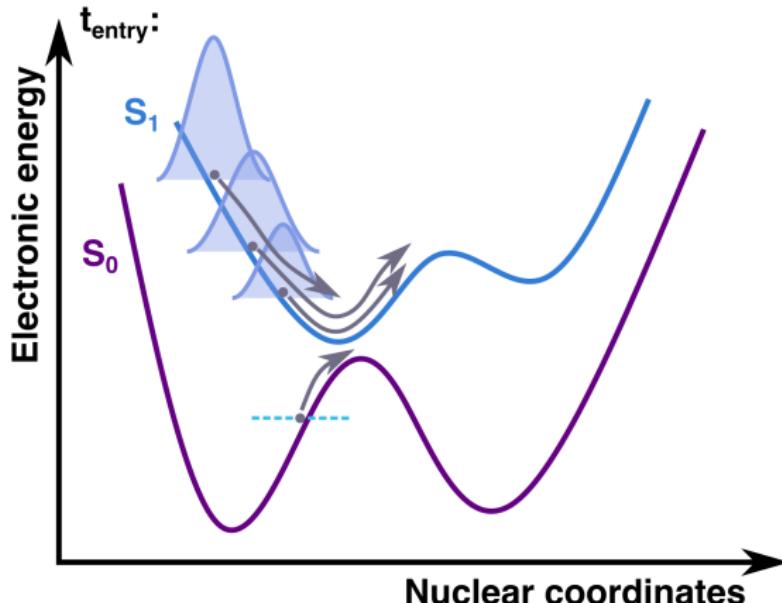
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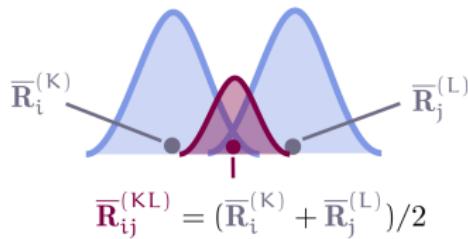
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# Ab Initio Multiple Spawning

- Multiple spawning is in principle exact, but in practice intractable
- Two approximations are made to arrive at ab initio multiple spawning (AIMS):
  1. Saddle-point approximation of zeroth order:

$$\langle \chi_i^{(K)} | \Theta^{(KL)}(\mathbf{R}) | \chi_j^{(L)} \rangle_{\mathbf{R}} \approx \Theta^{(KL)}(\bar{\mathbf{R}}_{ij}^{(KL)}) \langle \chi_i^{(K)} | \chi_j^{(L)} \rangle_{\mathbf{R}}$$



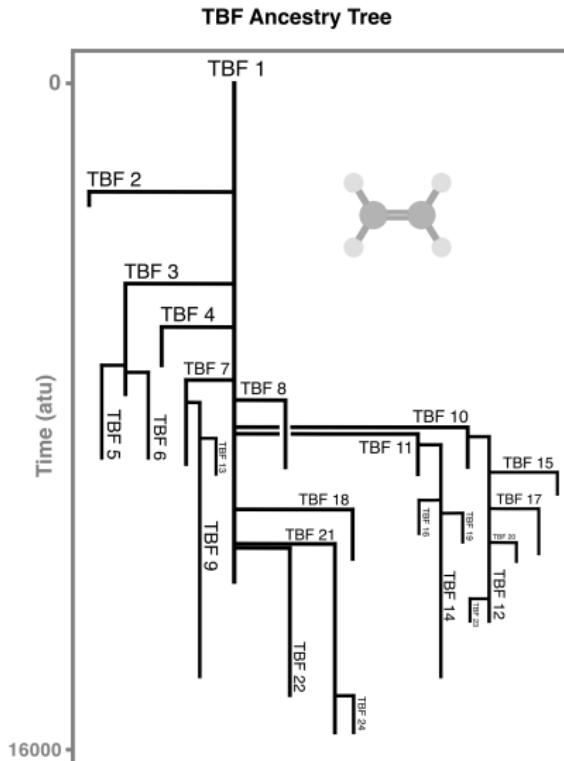
**Complexity of dynamics is then:**  $O(N_{\text{TBF}}^2)$

2. Independent first generation approximation (IFGA)

B. Mignolet and B. F. E. Curchod, J. Chem. Phys. 2018, **148**, 134110.

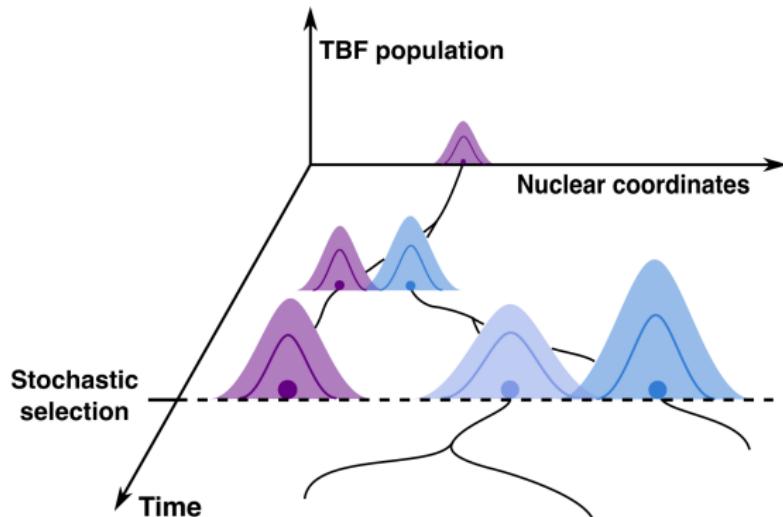
L. M. Ibele and B. F. E. Curchod, J. Chem. Phys. 2021 **155**, 174119

# The Problem with Spawning

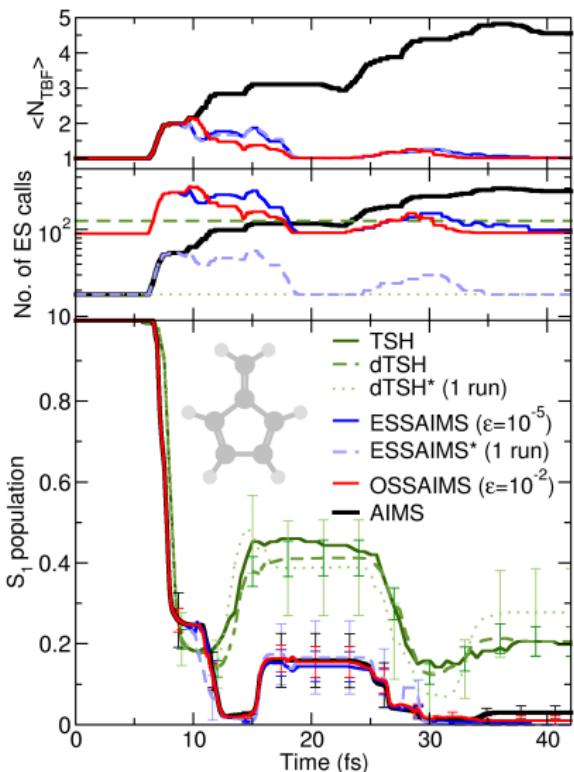


# Stochastic selection: A solution

- Systematically remove TBFs from the simulation as soon as they become separated in phase space.  
⇒ Coupling determined by  $|H_{ij}^{KL}|$  (ESSAIMS) or  $|S_{ij}^{KL}|$  (OSSAIMS).
- If coupling falls  $< \epsilon$  for any two groups of TBFs, pick one at random and renormalize remaining TBF populations.



# It actually works (and is performant)



- SA(2)-CASSCF(6,6)
- 18 initial conditions (Wigner)
- 5 (7) runs per initial condition SSAIMS (dTSH)
- Number of ES calls:
  - (dTSH):  $n_{\text{run}} \times N_{\text{IC}}$
  - (E/OSS)AIMS:

$$\sum_{j=1}^{n_{\text{run}}} \sum_{k=1}^{N_{\text{IC}}} N_{\text{TBF}}^{j,k}(t) \times (N_{\text{TBF}}^{j,k}(t) + 1)/2$$

(worst case)

# Can we do better?

- SSAIMS performs well but relies on user defined thresholds, making it even less of a black box
- Is there some way to remove the selection threshold?
- Overlap of two frozen Gaussians with identical initial conditions should (approximately) decay as a Gaussian in time<sup>1</sup> with

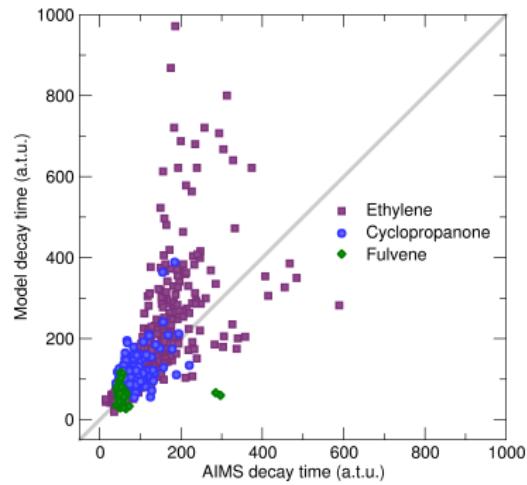
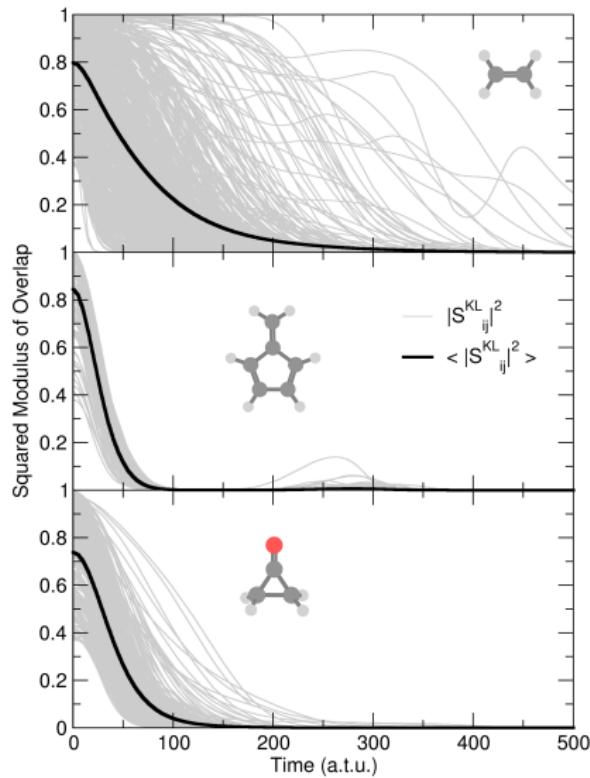
$$\tau_D = \left[ \frac{1}{4} (\mathbf{F}_{i,0}^{(K)} - \mathbf{F}_{j,0}^{(L)})^T \boldsymbol{\alpha}^{-1} (\mathbf{F}_{i,0}^{(K)} - \mathbf{F}_{j,0}^{(L)}) \right]^{-1/2}.$$

- It should consequently also apply to parent-child TBF pairs.

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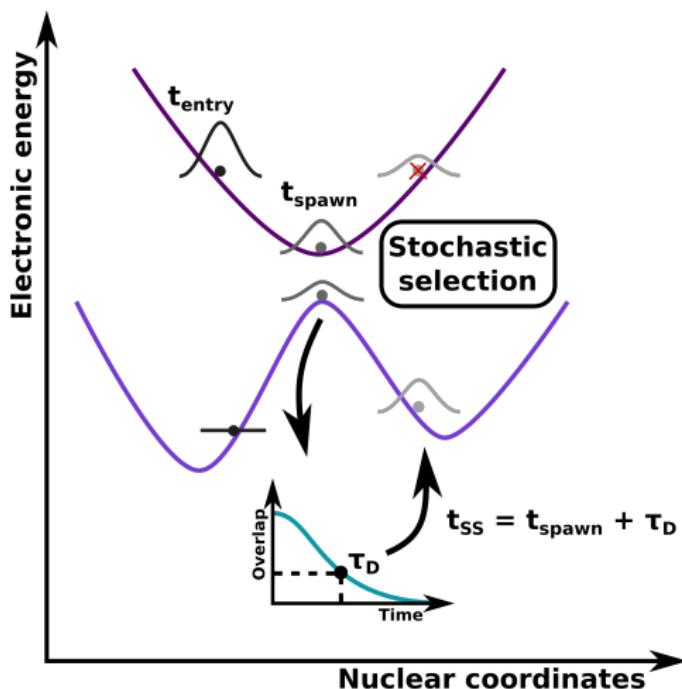
<sup>1</sup>B. J. Schwartz, E. R. Bittner, O. V. Prezhdo, and P. J. Rossky, J. Chem. Phys. **104**, 5942 (1996).

# Yes we can!

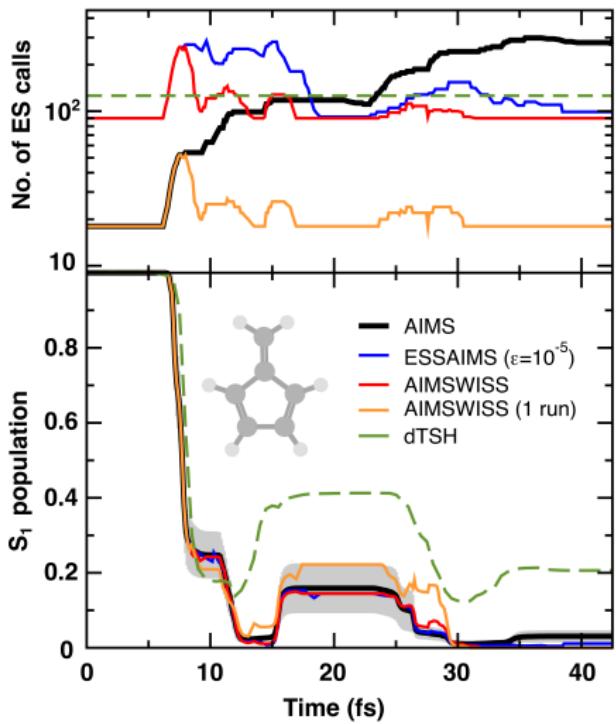


# Introducing AIMSWISS

Use the Schwartz decoherence time to predict when to perform stochastic selection.

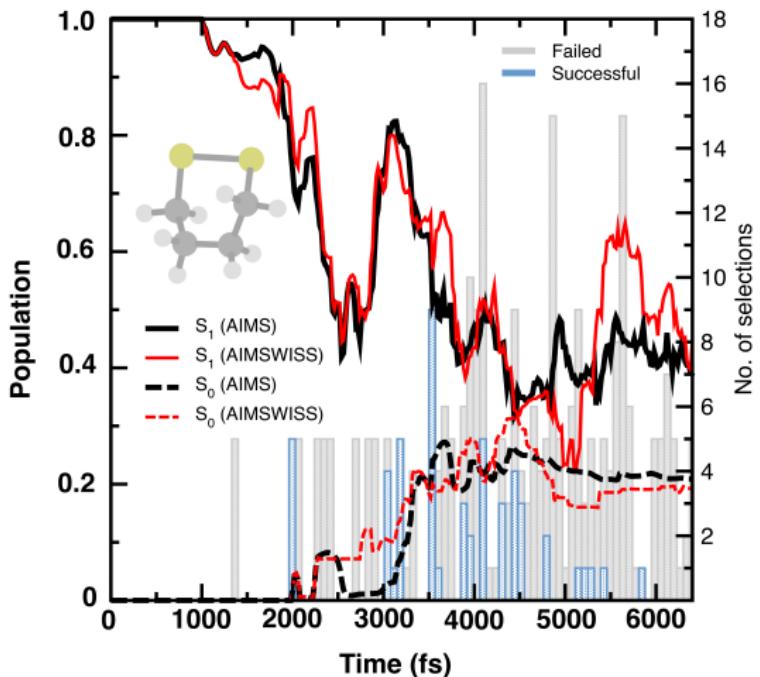


# Back to Fulvene again



# AIMSWISS warning feature

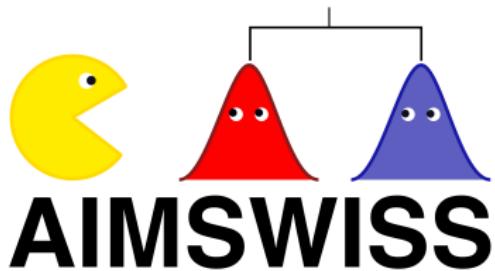
It is possible to notify the user, whenever the main assumption of AIMSWSIISS is not valid. E.g., for dithiane (unpublished work):



- SA(3)-CASSCF(6,4)
- 14 initial conditions (Wigner)
- 5 runs per initial condition

## Conclusion & open questions

- The stochastic selection idea allows us to get AIMS quality dynamics at the computational cost of TSH
- AIMSWISS is the greediest version of SSAIMS possible
- However, a diagnostic is implemented that gauges the trustworthiness of the method.
- We envision AIMSWISS to be used as a
  - cheap benchmark of mixed quantum/classical dynamics methods
  - first step in an application of the multiple spawning methodology



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- We envision AIMSWISS to be used as a
  - cheap benchmark of mixed quantum/classical dynamics methods
  - first step in an application of the multiple spawning methodology
- How does stochastic selection affect other expectation values?
- Does it still work for low-dimensional systems?
- How far can we go when it comes to system size?

# That's all folks!



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