PHYSICS 373 Problem Set 7 Due: March 7, 2025.

1. Consider the Hamiltonian
$$\hat{H} = \frac{E_0}{18}\begin{pmatrix} 22 & 8 & 8 \\ 8 & 43 & 7 \\ 8 & 7 & 43 \end{pmatrix}$$
acting on the 3-dimensional Hilbert space, $\mathbb{C}^3$. ($\hat{H}$ is related by a similarity transformation, $\hat{H}/E_0 = SMS^{-1}$ to the matrix you studied on the last homework and hence, up to a factor of E_0, it has the same eigenvalues.) Find the general solution to the Schrœdinger equation
$$i\hbar\frac{d}{dt}|\Psi\rangle = \hat{H}|\Psi\rangle$$

2. Given the state $|\Psi\rangle = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, what is the probability distribution for measurements of the Hamiltonian of the previous problem?

3. Consider a 1D harmonic oscillator, whose initial state is
$$\Psi(x,0) = \frac{1}{\sqrt{2}}(\psi_0(x) + \psi_1(x))$$
   a) What is the probability that, at time t, the particle is located to the right of the origin ($x > 0$)?
   b) What is the probability that, at time t, the momentum of the particle is positive ($p > 0$)?

4. In class, we proved the uncertainty relation
$$(\Delta F)^2(\Delta G)^2 \geq \frac{1}{4}\langle i[\hat{F},\hat{G}]\rangle^2$$
by demanding
$$0 \leq \left\|\Big((\hat{F} - \langle F\rangle) + i\alpha(\hat{G} - \langle G\rangle)\Big)|\Psi\rangle\right\|^2$$
for any $\alpha \in \mathbb{R}$. Generalize this to prove the stronger relation
$$(\Delta F)^2(\Delta G)^2 \geq \frac{1}{4}\langle i[\hat{F},\hat{G}]\rangle^2 + \frac{1}{4}\langle\{\hat{F} - \langle F\rangle, \hat{G} - \langle G\rangle\}\rangle^2$$
where the anticommutator $\{\hat{A},\hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$. Hint: demand
$$0 \leq \left\|\Big((\hat{F} - \langle F\rangle) + (\alpha_1 + i\alpha_2)(\hat{G} - \langle G\rangle)\Big)|\Psi\rangle\right\|^2$$
for any $\alpha_1, \alpha_2 \in \mathbb{R}$.