PHYSICS 373 Problem Set 11 Due: April 18, 2025.

1. Consider a particle of charge q, subject to a constant magnetic field, $\boldsymbol{B} = (0, 0, B_0)$, and a potential $V = \frac{1}{2}m\omega_0^2 z^2$. If the wave function is required to obey periodic boundary conditions in the y-direction, $\psi(x, y, z) = \psi(x, y + L, z)$, what are the allowed energy levels? *Hint: Work in the the gauge where the vector potential is* $\boldsymbol{A} = (0, xB_0, 0)$ *and use separation of variables.*

2. Work out an explicit representation as 4×4 Hermitian matrices for the $\hat{\mathbf{S}}$ in the case $s = 3/2$. *Hint: You might as well take*

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

   Now you just need to work out $\hat{S}_x$ *and* $\hat{S}_y$.

3. Consider a spin-1/2 particle in a 1D infinite square well, subject to the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) + \beta\hat{S}_x$$

   where

$$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise} \end{cases}$$

   a) Solve the Time-Independent Schrœdinger equation for this Hamiltonian.

   b) If

$$\Psi(x, 0) = \sqrt{\frac{2}{L}}\begin{pmatrix} \sin(\pi x/L) \\ 0 \end{pmatrix}$$

   what is $\Psi(x, t)$?

4. In the spin-1/2 representation, $\hat{\mathbf{S}} = \frac{\hbar}{2}\boldsymbol{\sigma}$.

   a) Compute the unitary matrices (we computed $U_y(\alpha)$ in class)

$$U_x(\alpha) = e^{-i\alpha\hat{S}_x/\hbar}, \qquad U_z(\alpha) = e^{-i\alpha\hat{S}_z/\hbar},$$

   b) Show that $U_x(\pi/2)^{-1}U_y(\alpha)U_x(\pi/2) = U_z(\alpha)$, as you'd expect for rotations.