

PHYSICS 373
Midterm Examination 1

Time allowed: **1 hour** (Calculator permitted)

1. True / False

For each statement below, circle **true** or **false**.

- a) A **unitary** operator has eigenvalues that are purely imaginary. [**true**] [**false**]
- b) The ground-state wave-function of *any* 1-D quantum system is an *even* function of x .
[**true**] [**false**]
- c) Eigenfunctions of a Hermitian operator with different eigenvalues are orthogonal.
[**true**] [**false**]
- d) $\psi(x) = \frac{1}{\sqrt{2L \cosh(x/L)}}$ is a valid $t = 0$ wave-function for the 1-D harmonic oscillator.
[**true**] [**false**]
- e) In the ground state of a 1-D quantum system, the momentum uncertainty Δp is zero.
[**true**] [**false**]

2. Operator algebra

Let

$$\hat{O}_1 = i e^{-\beta x} \frac{d}{dx}, \quad \beta > 0,$$

acting on the Hilbert space $\mathcal{H} = L^2(\mathbb{R})$.

- a) What is the adjoint operator \hat{O}_1^\dagger ?
- b) For what (possibly complex) values of a is

$$\hat{O} = \hat{O}_1 + a e^{-\beta x}$$

Hermitian?

- c) Compute $\langle \hat{O} \rangle$ in the *ground state* of the 1-D harmonic oscillator.

3. Two-state system

A certain 2-state system is governed by the Hamiltonian

$$\hat{H} = E_0 \begin{pmatrix} 2 & 1+i \\ * & 3 \end{pmatrix}.$$

- a) What is the missing entry $(*)$ in \hat{H} ?
- b) Consider the state $|\psi\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. What is the expectation value $\langle H \rangle$ and the uncertainty ΔH in this state?
- c) When you *actually perform* an energy measurement on the system, what *possible values* can be obtained?
- d) What is the probability that a measurement in the state $|\psi\rangle$ yields the value E_0 ?

4. Table of integrals (given)

$$\int_0^\pi \sin^2 \theta \, d\theta = \frac{\pi}{2}, \quad \int_{-\infty}^\infty e^{-y^2/a^2} \, dy = a\sqrt{\pi}, \quad \int_{-\infty}^\infty y^2 e^{-y^2/a^2} \, dy = \frac{a^3}{2}\sqrt{\pi}.$$

End of exam — Good luck!