PHYSICS 373 Problem Set 10 Due: April 11, 2025.

1. We've solved the 3D isotropic harmonic oscillator in Cartesian coordinates, labeling the energy eigenstates at level n by $|n_1, n_2, n_3\rangle$ where $n = n_1 + n_2 + n_3$. We also solved it in spherical coordinates, labeling the energy eigenstates by $|n; l, m\rangle$. Express the six $|2; l, m\rangle$ states in the $|n_1, n_2, n_3\rangle$ basis. For instance, you should find

$$|2; 2, \pm 2\rangle = \frac{1}{2}\left(|2, 0, 0\rangle - |0, 2, 0\rangle \pm i\sqrt{2}|1, 1, 0\rangle\right)$$

2. Compute Δx and Δp_x in the ground state of the hydrogen atom.

3. Compute the probability that an electron in the $\psi_{2;1,m}$ state of hydrogen is located inside a sphere of radius b.

   a) Express your answer in terms of the dimensionless ratio b/a. (You should see that your answer is independent of the azimuthal quantum number, m, and that it goes to 1 for $b/a \to \infty$.)

   b) Expand your answer in a Taylor series, for small b/a, and show that the probability $\sim c(b/a)^5$, for some constant c. What is c?

4. Consider a hydrogen atom in a uniform magnetic field

$$H = \frac{\hat{\boldsymbol{p}}^2}{2m_e} - \frac{e^2}{4\epsilon_0 r} + \frac{eB}{2m_e}\hat{L}_z$$

   If

$$\Psi(\vec{x}, 0) = \frac{1}{\sqrt{32\pi a^3}}\frac{r}{a}e^{-r/2a}\sin\theta\cos\phi$$

   what is $\Psi(\vec{x}, t)$?