Introduction to PAOFLOW

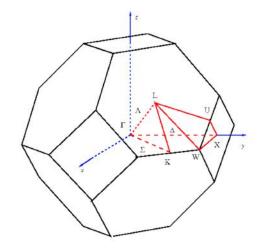
Marco Buongiorno Nardelli University of North Texas and Santa Fe Institute

Some QE post processing

Band Structure of Silicon

Re-run the self-consistent calculation at equilibrium lattice parameter, then run a non-self-consistent (fixed-potential) calculation, with the same input as for scf, but

- variable calculation is set to 'bands';
- the number nbnd of Kohn-Sham states must be explicitly set;
- k-point list is chosen along suitable high-symmetry lines. See sample file **si.bands.in**, containing the $L \Gamma X K \Gamma$ path.



Important: outdir and prefix must be the same in bands and in scf calculations.

\$ pw.x -in si.bands.in > si.bands.out

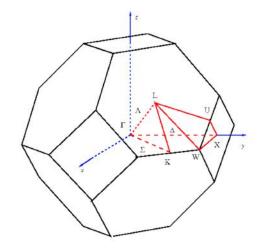
The list of k-points and of Kohn-Sham energies can be found after the line

End of band structure calculation

Band Structure of Silicon

Re-run the self-consistent calculation at equilibrium lattice parameter, then run a non-self-consistent (fixed-potential) calculation, with the same input as for scf, but

- variable calculation is set to 'bands';
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Important: outdir and prefix must be the same in bands and in scf calculations.

\$ pw.x -in si.bands.in > si.bands.out

The list of k-points and of Kohn-Sham energies can be found after the line

End of band structure calculation

Plotting Band Structure

There are various ways to plot the band structure. The simplest: use command bands.x with the following input (outdir and prefix as in previous steps):

```
&bands
  prefix='...', outdir='...', filband = 'sibands.dat', lsym=.false.
/
```

Output file sibands.dat can be plotted using command plotband.x.

Setting option lsym=.true instead, bands.x performs a symmetry analysis. An additional file sibands.dat.rep is generated, containing information on symmetry labels of the various bands.

Beware: the k-point path must be continuous in space; no two consecutive k-points can be the same, or else you may get funny-looking bands

Plotting Band Structure II

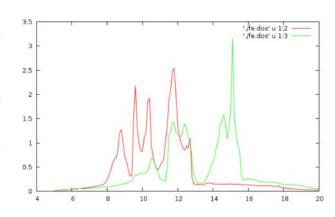
plotband.x prompts for terminal input:

Output is in file **sibands.plot** and can be plotted with gnuplot. If symmetry analysis is performed, output is instead split into several files **sibands.plot.**N.M, where N labels the high-symmetry lines, M labels irreducible representations.

Density of States (DOS)

- 1. Run a non self-consistent calculation (calculation='nscf' in namelist &control), using a previously computed scf potential. Input file fe_fm.nscf.in:
 - has the same prefix and outdir of the previous scf step
 - uses a denser k-point mesh than in the previous scf step
 - uses the *linear tetrahedron method* (variable occupations='tetrahedra')
- 2. Run code dos.x, using input file fe.dos.in: \$ dos.x -in fe.dos.in, where
 - prefix and outdir are the same of the previous non-scf step
 - output is written to file fildos
 - Plot spin-up (column 2) and down (column 3) DOS as a function of E (column 1)
- 3. (step 1 is not strictly needed, but if you want a nice DOS you need tetrahedra and a dense k-point grid)

 Reminder: Absolute values of Kohn-Sham eigenvalues have no direct physical meaning



Localized orbitals

Localized basis sets

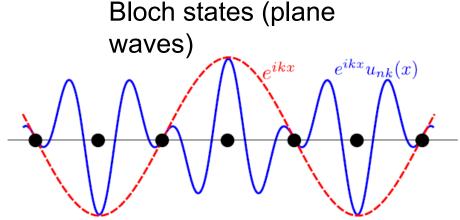
Chemical interpretation of the wavefunction Simple connection with plane-waves via Fourier transforms:

Construction of model Hamiltonians (e.g. strongly correlated systems)

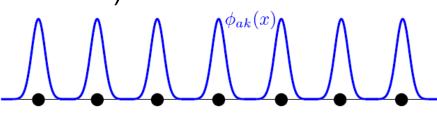
Band structure interpolation

Transport properties

Many more...



Localized states (i.e. atomic orbitals)



Wannier functions

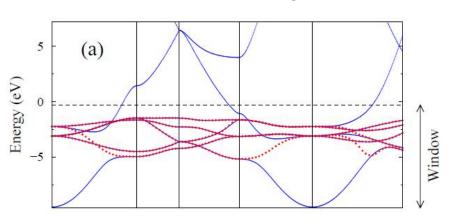
A convenient localized basis set. Maximal localization: Unique basis set $U_{mn}^{(\vec{k})}$

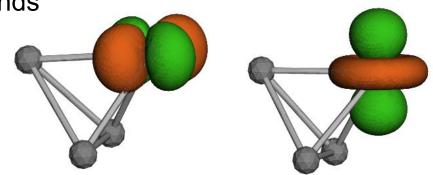
$$\psi_{n\vec{k}}(\vec{r}) = u_{n\vec{k}}(\vec{r})e^{-i\vec{k}\cdot\vec{r}} \qquad \qquad w_n(\vec{r}-\vec{R}) = \int \sum_m U_{mn}^{(\vec{k})} \psi_{n\vec{k}}(\vec{r})e^{-i\vec{k}\cdot\vec{R}}d^3k$$
Bloch — Wannier

Not straightforward (what projectors and how many?)
Internal iterative procedure for disentangling bands (e.g in metals)

Energy window?

Intractable for large number of bands





N. Marzari et. al. Rev. Mod. Phys. **84**, 1419 (2012)

Localized basis sets

Chemical interpretation of the wavefunction Simple connection with plane-waves via Fourier transforms:

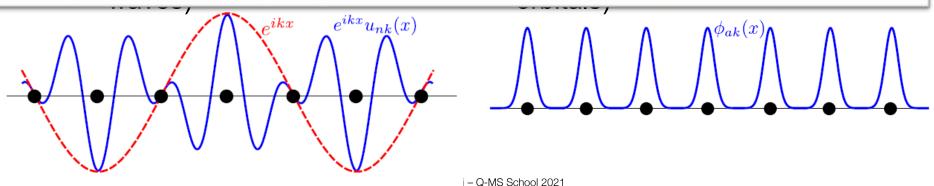
Construction of model Hamiltonians (e.g. strongly correlated systems)

PHYSICAL REVIEW B 88, 165127 (2013)

Effective and accurate representation of extended Bloch states on finite Hilbert spaces

Luis A. Agapito, 1.2. Andrea Ferretti, Arrigo Calzolari, 1.3 Stefano Curtarolo, 2.4 and Marco Buongiorno Nardelli 1.2. Department of Physics, University of North Texas, Denton, Texas 76203, USA ²Center for Materials Genomics, Duke University, Durham, North Carolina 27708, USA 3CNR-NANO S3 Center, Istituto Nanoscienze, I-41125 Modena, Italy

Department of Mechanical Engineering and Materials Science, Duke University, Durham, North Carolina 27708, USA



The PAOFLOW method

Pseudo-atomic orbitals (PAO)

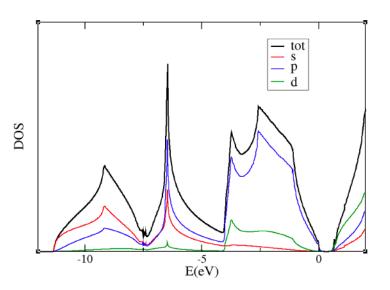
Readily available from pseudo-potentials:

$$\phi_{a\vec{k}}(\vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{R}} e^{-i\vec{k}\cdot\vec{R}} \phi_a(\vec{r} - \vec{R})$$

Connection to Bloch states (i.e Kohn-Sham states from DFT) via a simple projection -- typically done to obtain Projected Density of States (PDOS):

$$\psi_{n\vec{k}}(\vec{r}) = \sum_{a} A_{an}^{\vec{k}} \, \phi_{n\vec{k}}(\vec{r})$$

Silicon



$$DOS = n(\epsilon) = \sum_{n} \delta(\epsilon - \epsilon_{n})$$

$$\sum_{n} \langle \psi_{n} | \psi_{n} \rangle \delta(\epsilon - \epsilon_{n}) =$$

$$\sum_{n} \langle \psi_{n} | (\sum_{a} |\phi_{a}\rangle \langle \phi_{a}|) | \psi_{n} \rangle \delta(\epsilon - \epsilon_{n}) =$$

$$\sum_{n} \sum_{a} \langle \psi_{n} | \phi_{a} \rangle \langle \phi_{a} | \psi_{n} \rangle \delta(\epsilon - \epsilon_{n}) =$$

$$\sum_{n} \sum_{a} A_{an}^{*} A_{an} \delta(\epsilon - \epsilon_{n}) = \sum_{i} n_{i}(\epsilon)$$

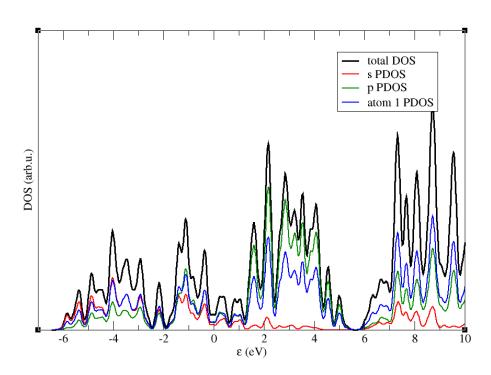
$$PDOS = \sum_{i} n_{i}(\epsilon) = \sum_{n} A_{an}^{*} A_{an} \delta(\epsilon - \epsilon_{n})$$

Projected Density of States (PDOS)

- 1. Run a non self-consistent calculation as for the DOS case
- 2. Run code projwfc.x, using input file proj.in
 \$ projwfc.x < proj.in , where
 - gaussian broadening degauss is used (tetrahedra not yet implemented)
 - prefix and outdir are the same of the previous non-scf step
 - the prefix to all output files in in variable filpdos

Hands-on: Silicon – PDOS

- 1. pw.x < scf_Si.in > scf_Si.out
- 2. projwfc.x < proj.in > proj.out
- 3. sumpdos $^{(x)}$ > X.dat (for the x orbitals) or sumpdos silicon.pdos_atm#X1\(^* > atomX.dat (for the X atom)



Hands-on: Silicon – PDOS

Atomic states used for projection (read from pseudopotential files):

```
state # 1: atom 1 (Si ), wfc 1 (I=0 m= 1)
state # 2: atom 1 (Si ), wfc 2 (I=1 m= 1)
state # 3: atom 1 (Si ), wfc 2 (I=1 m= 2)
state # 4: atom 1 (Si ), wfc 2 (I=1 m= 3)
state # 5: atom 1 (Si ), wfc 3 (I=2 m= 1)
state # 6: atom 1 (Si ), wfc 3 (I=2 m= 2)
state # 7: atom 1 (Si ), wfc 3 (I=2 m= 3)
state # 8: atom 1 (Si ), wfc 3 (I=2 m= 4)
state # 9: atom 1 (Si ), wfc 3 (I=2 m= 5)
state # 10: atom 1 (Si ), wfc 4 (I=3 m= 1)
state # 11: atom 1 (Si ), wfc 4 (I=3 m= 2)
state # 12: atom 1 (Si ), wfc 4 (I=3 m= 3)
state # 13: atom 1 (Si ), wfc 4 (I=3 m= 4)
state # 14: atom 1 (Si ), wfc 4 (I=3 m= 5)
state # 15: atom 1 (Si ), wfc 4 (I=3 m= 6)
state # 16: atom 1 (Si ), wfc 4 (I=3 m= 7)
state # 17: atom 2 (Si ), wfc 1 (I=0 m= 1)
state # 18: atom 2 (Si ), wfc 2 (I=1 m= 1)
state # 19: atom 2 (Si ), wfc 2 (I=1 m= 2)
state # 20: atom 2 (Si ), wfc 2 (I=1 m= 3)
state # 21: atom 2 (Si ), wfc 3 (I=2 m= 1)
state # 22: atom 2 (Si ), wfc 3 (I=2 m= 2)
state # 23: atom 2 (Si ), wfc 3 (I=2 m= 3)
state # 24: atom 2 (Si ), wfc 3 (I=2 m= 4)
state # 25: atom 2 (Si ), wfc 3 (I=2 m= 5)
state # 26: atom 2 (Si ), wfc 4 (I=3 m= 1)
state # 27: atom 2 (Si ), wfc 4 (I=3 m= 2)
state # 28: atom 2 (Si ), wfc 4 (I=3 m= 3)
state # 29: atom 2 (Si ), wfc 4 (I=3 m= 4)
state # 30: atom 2 (Si ), wfc 4 (I=3 m= 5)
state # 31: atom 2 (Si ), wfc 4 (I=3 m= 6)
state # 32: atom 2 (Si ), wfc 4 (I=3 m= 7)
```

Our basis or localized orbitals

Hands-on: Silicon – PDOS

$$\rho(\mathbf{r}) = \sum_{n} |\psi|^2 = \sum_{n} (\sum_{a} A_{an}^* \phi_a^*) (\sum_{b} A_{bn} \phi_b) = \sum_{ab} P_{ab} \phi_a^* \phi_b$$

$$P_{ab} = \sum_{n} A_{an}^* A_{bn} \equiv \text{density matrix}$$

$$\text{charge of A} = \sum_{a \in A} P_{aa}$$

Lowdin Charges:

Atom # 1: total charge = 3.9897, s = 0.9061,

Atom # 1: total charge = 3.9897, p = 2.3837, pz= 0.7946, px= 0.7946, py= 0.7946,

```
Atom # 1: total charge = 3.9897, f = 0.0018, fz3= 0.0003, fxz2= 0.0002, fyz2= 0.0002, fzx2-zy2= 0.0002, fxyz= 0.0003, fx3-3xy2= 0.0003, f3yx2-y3= 0.0003,

Atom # 2: total charge = 3.9897, p = 0.9061,
Atom # 2: total charge = 3.9897, p = 2.3837, pz= 0.7946, px= 0.7946, py= 0.7946,
Atom # 2: total charge = 3.9897, d = 0.6981, dz2= 0.0861, dxz= 0.1753, dyz= 0.1753, dx2-y2= 0.0861, dxy= 0.1753,
Atom # 2: total charge = 3.9897, f = 0.0018, fz3= 0.0003, fxz2= 0.0002, fyz2= 0.0002, fzx2-zy2= 0.0002, fxyz= 0.0003, fx3-3xy2= 0.0003, f3yx2-y3= 0.0003,
Spilling Parameter: 0.0026
```

Atom # 1: total charge = 3.9897, d = 0.6981, dz2= 0.0861, dxz= 0.1753, dyz= 0.1753, dx2-y2= 0.0861, dxy= 0.1753,

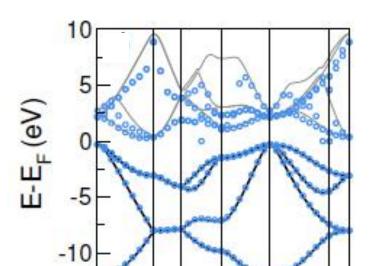
Pseudo-atomic orbitals (PAO)

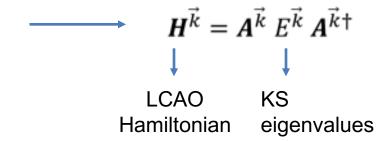
Readily available from pseudo-potentials:

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Connection to Bloch states (i.e Kohn-Sham states from DFT) via a simple projection -- typically done to obtain Projected Density of States (PDOS):

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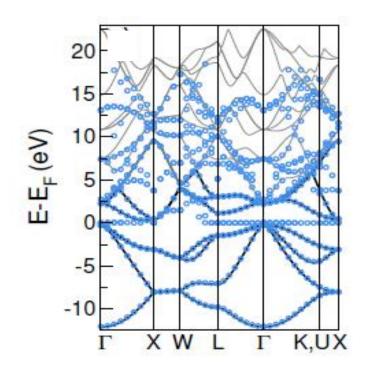




Si (2s²2p² basis set): 8 atomic orbitals Valence bands reproduced to high accuracy Conduction bands deviate strongly from KS eigenvalues.

L. A. Agapito et. al. Phys. Rev. B 88, 165127 (2013)

Projectability of bands



LCAO band structure with *spd* basis set. Some low energy conduction bands improve.

A set of null eigenvalues are inserted Projectability:

$$\mathcal{P}_n = \min\{\sum_{u} a_{un}^{\mathbf{k}*} a_{un}^{\mathbf{k}}, \forall \mathbf{k} \in \mathrm{BZ}\}$$

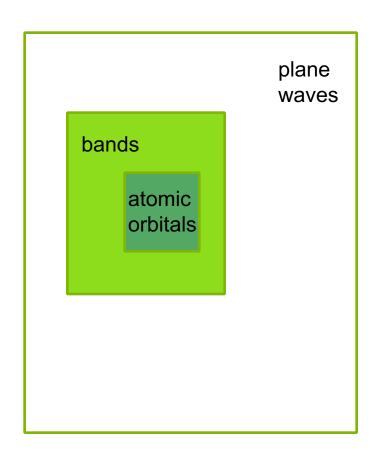
	Si Ω_1		Si Ω_2	
n	\mathcal{P}_n	rms	\mathcal{P}_n	rms
1	0.9927	0.0478	0.9953	0.0792
2	0.9622	0.0572	0.9933	0.0555
3	0.9622	0.0252	0.9952	0.0382
4	0.9622	0.0203	0.9967	0.0266
5	0.4755	1.9200	0.9934	0.0267
6	0.0660		0.9771	0.0316
7	0.0941		0.9728	0.0380
8	0.0652		0.8464	0.1382

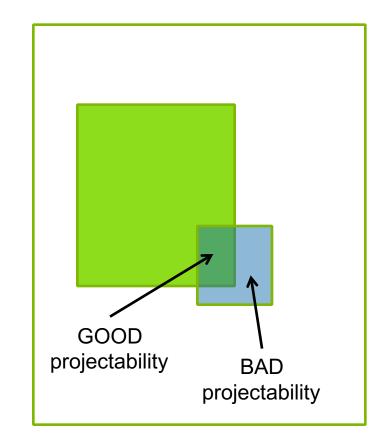
High projectability bands (i.e. $P_n \approx 1$) yield accurate LCAO band structure.

Low projectability bands (i.e. $P_n \ll 1$) yield poor LCAO bands and lead to null eigenstates.

Marco Buongiorno Nardelli - Q-MS School

Accurate PAO Hamiltonians





PHYSICAL REVIEW B 93, 035104 (2016)

Accurate tight-binding Hamiltonian matrices from ab initio calculations: Minimal basis sets

Luis A. Agapito, 1,2 Sohrab Ismail-Beigi, 3 Stefano Curtarolo, 4,5 Marco Fornari, 4,6 and Marco Buongiorno Nardelli 2,4, and 1 Department of Mechanical Engineering and Materials Science, Duke University, Durham, North Carolina 27708, USA

2 Department of Physics, University of North Texas, Denton, Texas 76203, USA

³Department of Applied Physics and Center for Research on Interface Structures and Phenomena (CRISP), Yale University, New Haven, Connecticut 06511, USA

Center for Materials Genomics, Duke University, Durham, North Carolina 27708, USA
 Materials Science, Electrical Engineering, Physics and Chemistry, Duke University, Durham, North Carolina 27708, USA
 Department of Physics, Central Michigan University, Mt. Pleasant, Michigan 48859, USA

$$\bar{H} = AEA^{\dagger}$$

Exclude the bad projectability states from the set of bands to be represented (**filtering**) – orthogonal projection

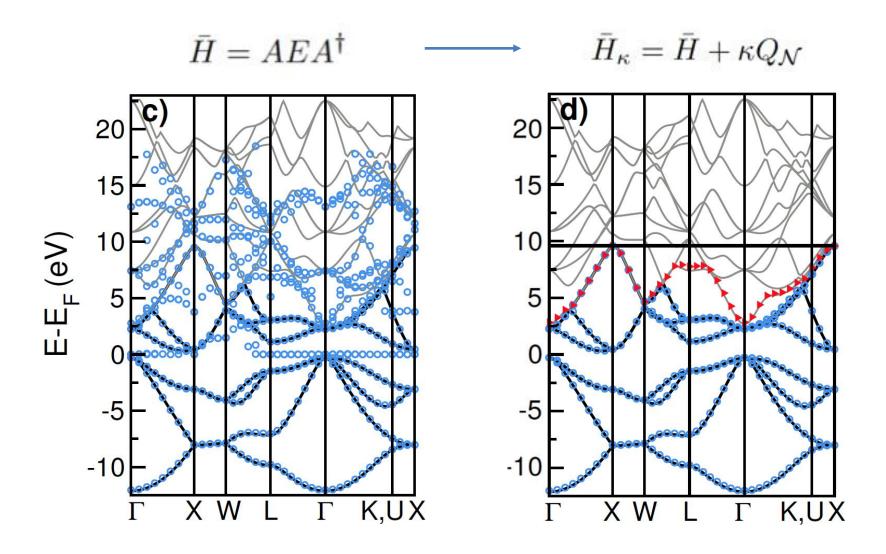
$$Q_{\mathcal{N}} = I_M - A(A^{\dagger}A)^{-1}A^{\dagger}$$

Introduces a spurious null space that must be eliminated

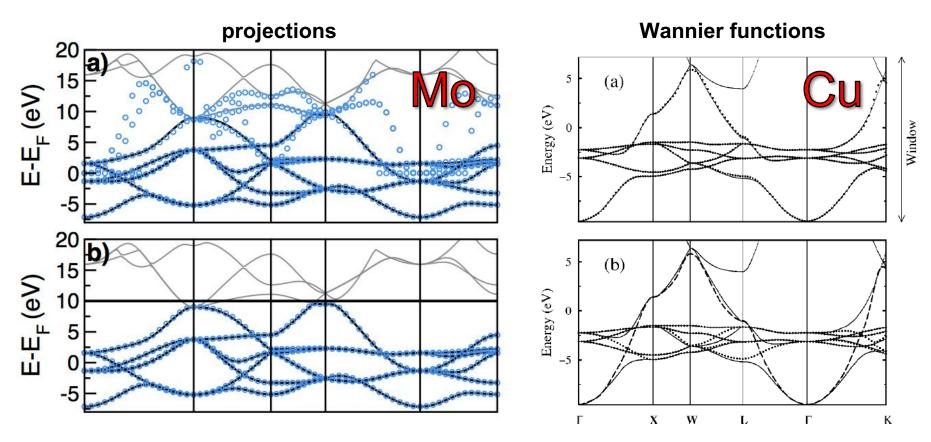
Shift by a fixed energy outside of the region of good projectability bands (shifting)

$$\bar{H}_{\kappa} = \bar{H} + \kappa Q_{\mathcal{N}}$$

Accurate PAO Hamiltonians



Accurate PAO Hamiltonians



Effective description of entangled bands in metals

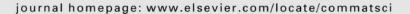
PAOFLOW

- 1. Go to the Example1 directory on the tutorial repository
- 2. Run scf.in and nscf.in
- 3. Go to the jupyter notebook!



Contents lists available at ScienceDirect

Computational Materials Science





PAOFLOW: A utility to construct and operate on *ab initio* Hamiltonians from the projections of electronic wavefunctions on atomic orbital bases, including characterization of topological materials



Marco Buongiorno Nardelli ^{a,e,*}, Frank T. Cerasoli ^a, Marcio Costa ^f, Stefano Curtarolo ^{b,e,h}, Riccardo De Gennaro ^g, Marco Fornari ^{c,d,e}, Laalitha Liyanage ^a, Andrew R. Supka ^{c,d}, Haihang Wang ^a

Computational Materials Science 200 (2021) 110828



Contents lists available at ScienceDirect

Computational Materials Science





Advanced modeling of materials with PAOFLOW 2.0: New features and software design

Frank T. Cerasoli ^a, Andrew R. Supka ^{b,c}, Anooja Jayaraj ^a, Marcio Costa ^d, Ilaria Siloi ^e, Jagoda Sławińska ^f, Stefano Curtarolo ^g, Marco Fornari ^{b,c,g}, Davide Ceresoli ^h, Marco Buongiorno Nardelli ^{a,g,*}

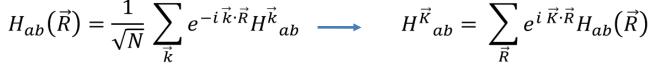


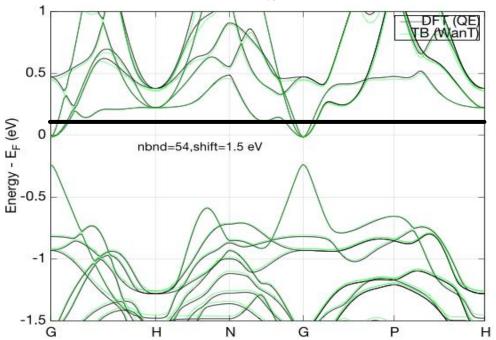
PAOFLOW capabilities

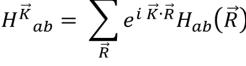
- Construction of PAO Hamiltonians from the DFT wavefunctions onto pseudo atomic orbitals
- Construction of PAO Hamiltonians from analytical tight binding models
- Hamiltonian data for further processing (ACBN0, PAOtransport, etc.)
- External fields and non scf ACBN0 correction
- Spin orbit correction of non SO calculations
- Bands along standard paths in the BZ
- Real space electronic charge density
- Interpolation of Hamiltonians on arbitrary Monkhorst and Pack k-meshes
- Adaptive smearing for BZ and Fermi surface integration
- Density of states (and projected DOS)
- Fermi surfaces and spin textures
- Boltzmann transport (conductivity, Seebeck coefficient, electronic contribution to thermal conductivity
- dielectric function (absorption coefficients and EELS)
- Berry curvature and anomalous Hall conductivity (including magnetic circular dichroism spectra)
- spin Berry curvature and spin Hall conductivity (including spin circular dichroism spectra)
- Band topology (Z2 invariants, Berry and spin Berry curvature along standard paths in BZ, critical points

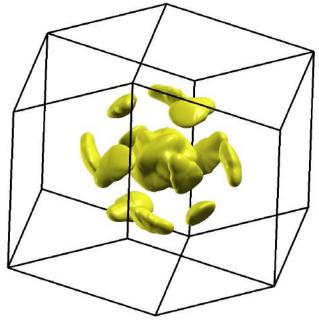
K-point interpolation

$$H_{ab}(\vec{R}) = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{-i \vec{k} \cdot \vec{R}} H^{\vec{k}}_{ab} \longrightarrow$$









nature materials

ARTICLES

PUBLISHED ONLINE: 5 OCTOBER 2015 | DOI: 10.1038/NMAT4430

Convergence of multi-valley bands as the electronic origin of high thermoelectric performance in CoSb₃ skutterudites, Y. Tang, Z.M. Gibbs, L.A. Agapito, G. Li, H. Kim, M. Buongiorno Nardelli, S. Curtarolo and G.J. Snyder

Transport properties

The standard approaches to electron transport in bulk semiconductors are based on the **semiclassical Boltzmann theory**.

Dynamics of carriers and response to external fields follow classical motion equation, whereas scattering events are included in a perturbative approach, via quantum mechanical Fermi Golden Rule.

Semiclassical transport

• probability density distribution function: $f(\mathbf{r},\mathbf{p},t)$

$$\ \, \textbf{equation of motion:} \ \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{p}} \cdot \mathbf{F} + \frac{\partial f}{\partial \mathbf{r}} \cdot \mathbf{v} = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

• relaxation-time approximation:
$$\left(\frac{\partial f}{\partial t}\right)_{coll} \approx -\frac{f - f_{eq}}{\tau_{relax}}$$

■ Fermi Golden Rule:
$$\frac{1}{\tau_{relax}(\varepsilon)} = \frac{n_{imp}}{\hbar} \int d\mathbf{p}' |V_{\mathbf{p}\mathbf{p}'}|^2 (1 - cos\theta) \delta(\varepsilon - \varepsilon_{\mathbf{p}})$$

• Boltzman conductivity:
$$\sigma = \frac{Ne^2 \tau_{relax}}{m^*}$$

Boltzmann Transport Equations

Accurate evaluation of Boltzmann transport integrals:

$$I^{\alpha}{}_{ij} = \frac{1}{V} \sum_{n \vec{k}} (\epsilon_{n \vec{k}} - \mu)^{\alpha} v_i(n, \vec{k}) v_j(n, \vec{k}) \tau_{n \vec{k}} \left(-\frac{\partial f(\mu, T)}{\partial \epsilon_{n \vec{k}}} \right)$$

Band velocities:

Scattering time:

Fermi-Dirac distribution:

$$v_i(n, \vec{k}) = \frac{1}{\hbar} \frac{\partial \epsilon_{n\vec{k}}}{\partial k_i}$$

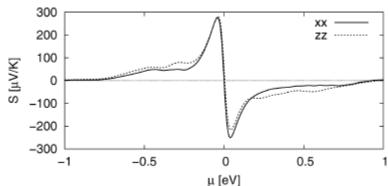
$$\tau_{n\,\vec{k}}$$

$$f(\mu,T)$$

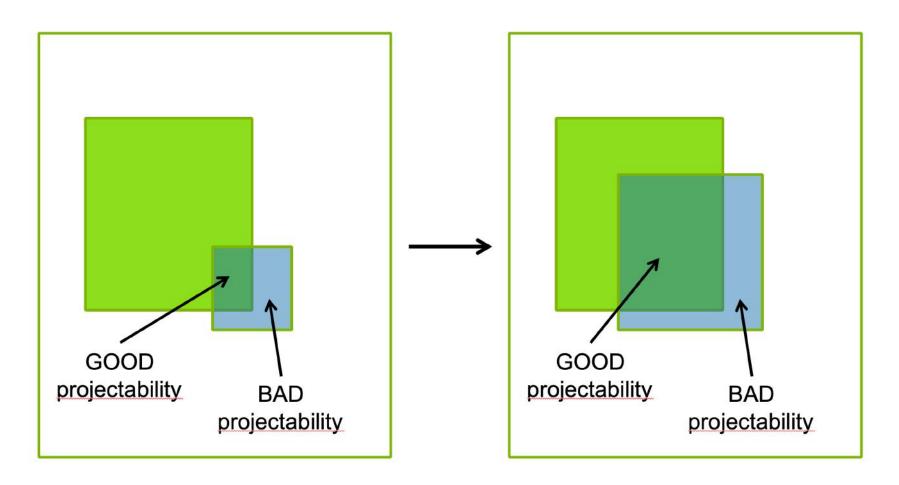
Requires very fine sampling of bands around Fermi energy – projection Hamiltonian!

e.g. Seebeck coefficient of Bi₂Te₃

$$S = -\boldsymbol{I_0}^{-1} \cdot \boldsymbol{I_1}$$



Increase the basis set

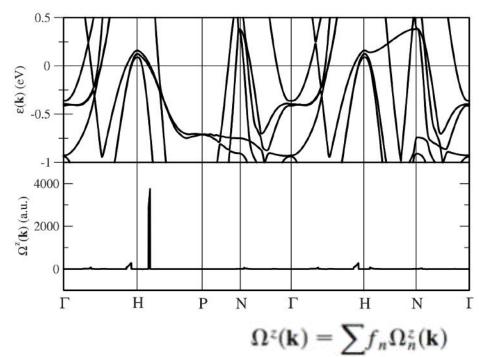


From SZ to DZ to DZP to TZP...etc.

PAOFLOW internal bases (go to jupyter notebook)

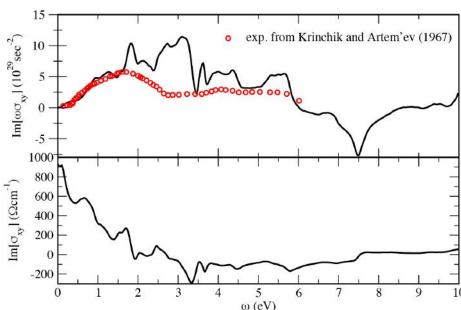
Some more advanced applications (go to jupyter notebook)

Berry curvature and Anomalous Hall conductivity: Fe



Berry curvature from Kubo formula:

$$\Omega_n^z(\mathbf{k}) = -\sum_{n'\neq n} \frac{2\text{Im}\langle \psi_{n\mathbf{k}} | v_x | \psi_{n'\mathbf{k}} \rangle \langle \psi_{n'\mathbf{k}} | v_y | \psi_{n\mathbf{k}} \rangle}{(\omega_{n'} - \omega_n)^2}$$



Anomalous Hall conductivity:

$$\sigma_{xy} = -\frac{e^2}{\hbar} \int_{BZ} \frac{d^3k}{(2\pi)^3} \Omega^z(\mathbf{k})$$

VOLUME 92, NUMBER 3 PHYSICAL REVIEW LETTERS week ending 23 JANUARY 2004

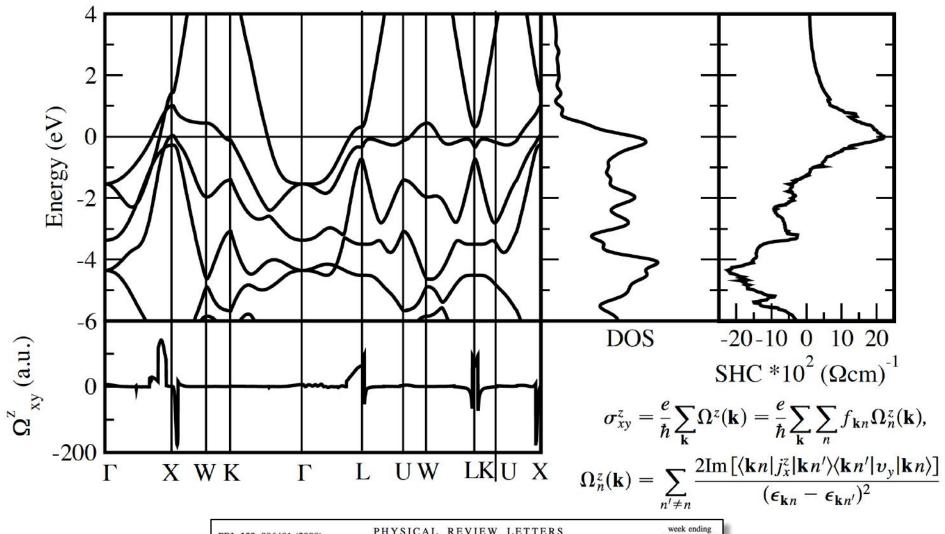
First Principles Calculation of Anomalous Hall Conductivity in Ferromagnetic bcc Fe

Yugui Yao, ^{1,2,3} Leonard Kleinman, ¹ A. H. MacDonald, ¹ Jairo Sinova, ^{4,1} T. Jungwirth, ^{5,1} Ding-sheng Wang, ³ Enge Wang, ^{2,3} and Qian Niu¹

$$\sigma(\omega)_{xy} = \frac{e^2}{\hbar} \int_{V_G} \frac{d^3k}{(2\pi)^3} \sum_{n \neq n'} (f_{n,\mathbf{k}} - f_{n',\mathbf{k}})$$

$$\times \frac{\operatorname{Im}\langle \psi_{n\mathbf{k}} | v_x | \psi_{n'\mathbf{k}} \rangle \langle \psi_{n'\mathbf{k}} | v_y | \psi_{n\mathbf{k}} \rangle}{(\omega_{n'} - \omega_n)^2 - (\omega + i\delta)^2}$$

Spin Hall conductivity



PRL 100, 096401 (2008)

PHYSICAL REVIEW LETTERS

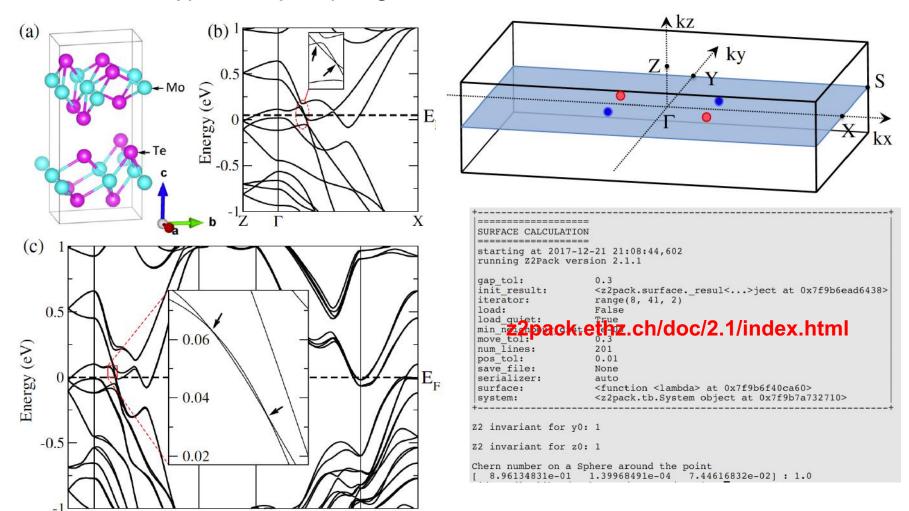
week ending 7 MARCH 2008

Intrinsic Spin Hall Effect in Platinum: First-Principles Calculations

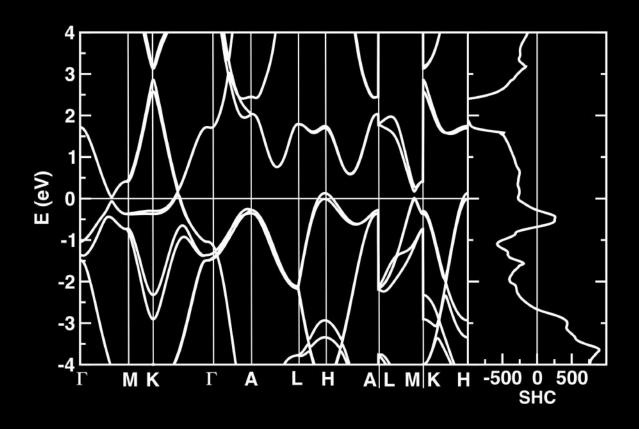
G. Y. Guo, ^{1,*} S. Murakami, ² T.-W. Chen, ¹ and N. Nagaosa^{3,4}

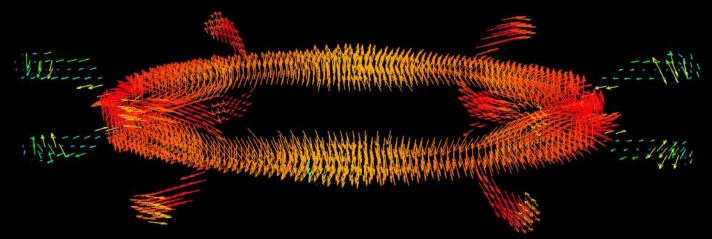
Topological invariants

MoTe₂: A Type-II Weyl Topological Metal



Wang et al., 10.1103/PhysRevLett.117.056805





Nodal chain and Weyl points in HfC

PAOFLOW Examples

example01 : Si with spd pseudpotential

example02 : Al with spd pseudpotential

example03 : Pt with Isda (nspin = 2)

example04: Fe non collinear with spin orbit - Anomalous Hall conductivity

example05: Pt non collinear with spin orbit - Spin Hall conductivity

example06 : AIP with non self consistent ACBN0

example07: Al starting from projections with overlap matrix of the atomic orbital

(prior to orthogonalization)

example08: 2D SnTe - Spin Hall conductivity

example09: Weyl point search in MoP2

example 10: Transport in GaAs with temperature and energy dependent model for

relaxation time