Relevant quantities in fluids of light

Average or instantaneous quantities ?

Tangui Aladjidi

Plan

- Introduction
- Adimensional formulation
- Figure of merits
- Temporal evolution
- Conclusion

Introduction

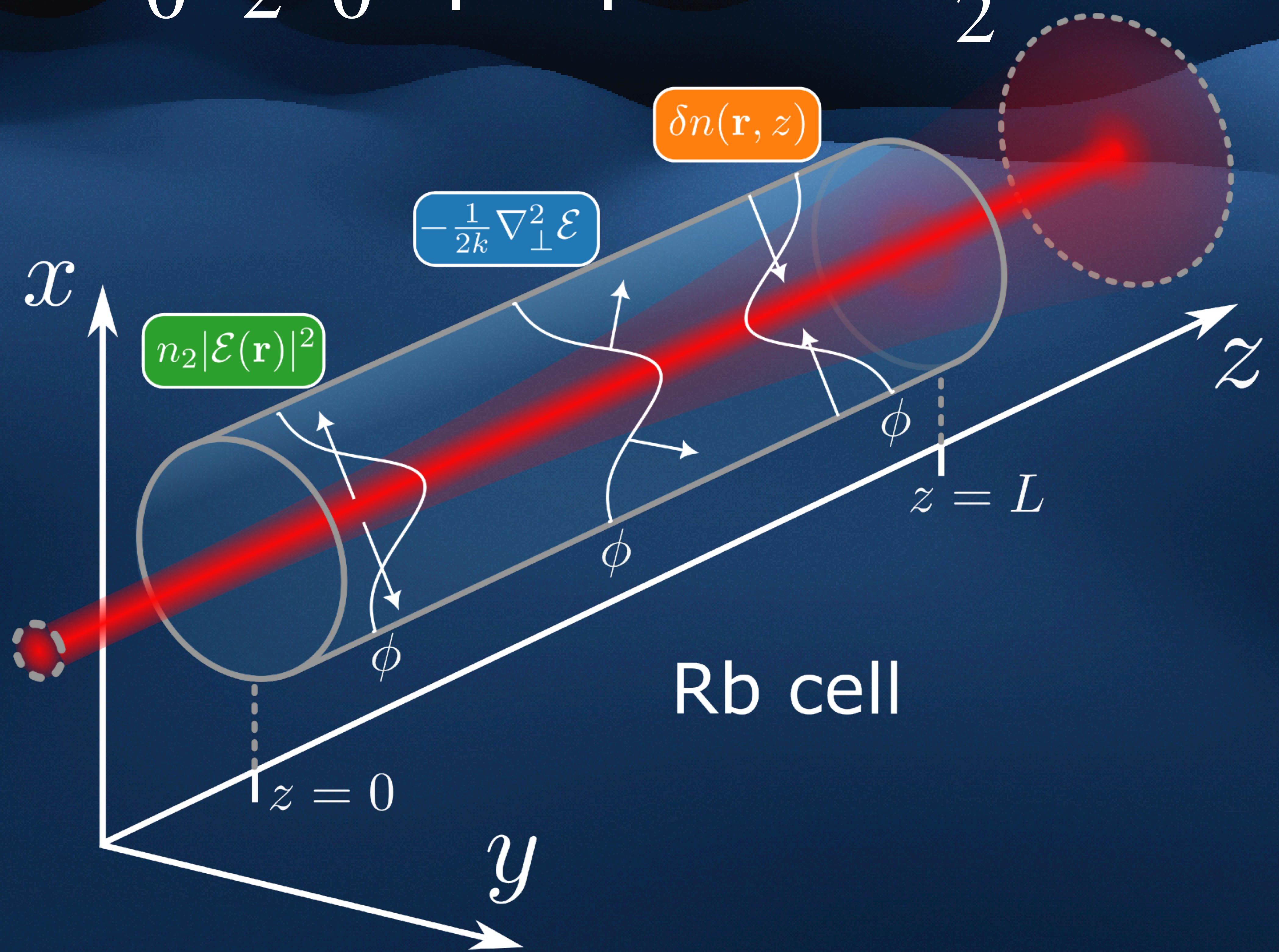
Introduction

Non-linear Schrödinger equation

For the electric field envelope $\mathscr{E}$ in a non-linear medium, we use the **non-linear Schrödinger equation** (NLSE):

$$i\partial_z \mathscr{E} = -\frac{1}{2k_0}\nabla_\perp^2 \mathscr{E} + k_0 \frac{\delta n}{n} \mathscr{E} + k_0 n_2 \epsilon_0 c \,|\, \mathscr{E} \,|^2 \mathscr{E} - i\frac{\alpha}{2}\mathscr{E}$$

- $k_0 \rightarrow$ wavenumber in m^{-1}
- $\delta n \rightarrow$ index variation
- $\alpha \rightarrow$ absorption coefficient in m^{-1}
- $n_2 \rightarrow$ non-linear index in m^2/W



Introduction

Adimensional formulation

- If we want to look at this equation in all generality, one can try to extract **typical scales** in order to obtain an **adimensional equation** that does not depend on **physical units** anymore
- Two “dimensions”: **longitudinal** and **transverse** planes
- One would like to have a natural scale for each of these planes in order to also extract **figure of merits**

Introduction

Figure of merits

Once we have established the typical scales relevant for each dimension, we want to assess three questions:

- **Qualitative**: **what kind of physics can we observe** with the typical range of parameters we have ?
- **Quantitative**: given a certain parameter space, **how can we optimize each parameter** in order to optimize a given figure of merit ?
- **Qualitative**: what is **the impact of the evolution** of these relevant scales along the cell i.e average versus instantaneous quantities ?

Adimensional formulation

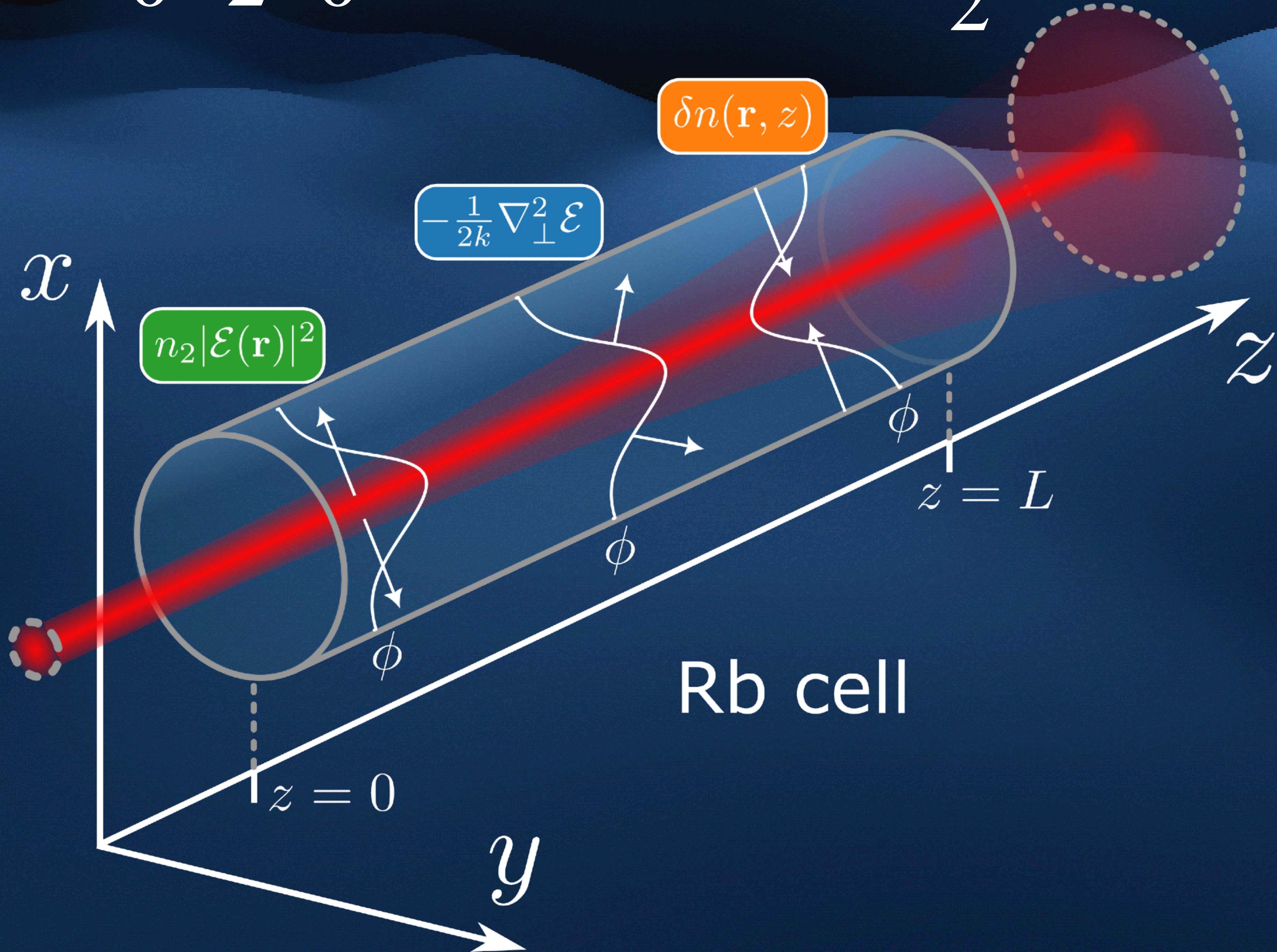
Adimensional formulation

Non-linear Schrödinger equation

How can we extract a **typical length** z_{NL} and a **typical size** ξ ?

$$i\partial_z \mathscr{E} = -\frac{1}{2k_0}\nabla_\perp^2 \mathscr{E} + k_0\frac{\delta n}{n}\mathscr{E} + k_0 n_2 \epsilon_0 c\,|\,\mathscr{E}\,|^2\,\mathscr{E} - i\frac{\alpha}{2}\mathscr{E}$$

- Longitudinal dimension $\leftrightarrow z$
- Transverse dimension $\leftrightarrow \mathbf{r}_\perp$
- Typical **length**
- Typical **size**



Adimensional formulation

Non-linear Schrödinger equation

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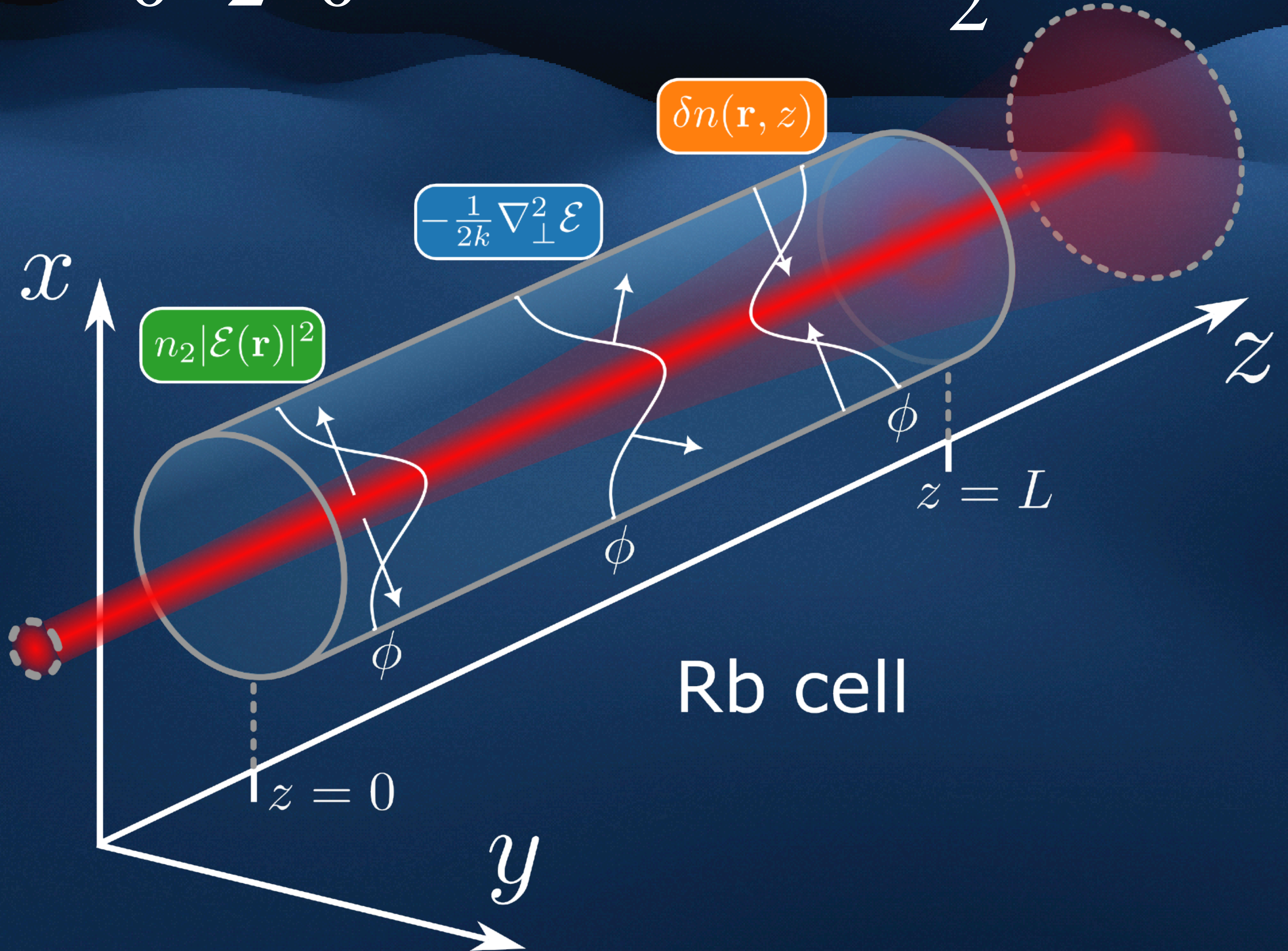
- Longitudinal dimension $\leftrightarrow z$
- Transverse dimension $\leftrightarrow \mathbf{r}_\perp$
- Non-linear evolution $\leftrightarrow$ dephasing $\delta\phi$
- Scale of the beam $\leftrightarrow$ defocusing



Adimensional formulation

Non-linear Schrödinger equation

We anticipate that we need to cook some formulae including n_2 / k_0 / $|\mathscr{E}_0|^2$ ...

We want to obtain a dimensionless propagation equation so let us define adimensional quantities:

- A new field $\tilde{\psi} = \dfrac{\mathscr{E}}{\mathscr{E}_0}$
- A new time $\tau = \dfrac{z}{z_{NL}}$
- A new spatial coordinate system $\tilde{\mathbf{r}} = \dfrac{\mathbf{r}}{\xi}$

Adimensional formulation

Non-linear Schrödinger equation

- If we recall our good old NLSE:

$$i\partial_z \mathcal{E} = -\frac{1}{2k_0}\nabla_\perp^2 \mathcal{E} + k_0 \frac{\delta n}{n}\mathcal{E} + k_0 n_2 \epsilon_0 c\,|\,\mathcal{E}\,|^2\,\mathcal{E} - i\frac{\alpha}{2}\mathcal{E}$$

- $z = z_{NL}\tau \implies \partial_z = \dfrac{1}{z_{NL}}\partial_\tau$
- $\mathbf{r}_\perp = \xi\tilde{\mathbf{r}} \implies \nabla_\perp^2 = \dfrac{1}{\xi^2}\tilde{\nabla}_\perp^2$
- $\mathcal{E} = \mathcal{E}_0\tilde{\psi}$

Adimensional formulation

Non-linear Schrödinger equation

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Board time !

Adimensional formulation

Typical scales

In the end, we should find:

- $z_{NL} = \dfrac{1}{k_0 \Delta n}$ where $\Delta n = \dfrac{n_2 \epsilon_0 c}{2} |\mathscr{E}_0|^2$
- $\xi = \dfrac{1}{k_0 \sqrt{\Delta n}}$
- **Everything is controlled by** Δn !

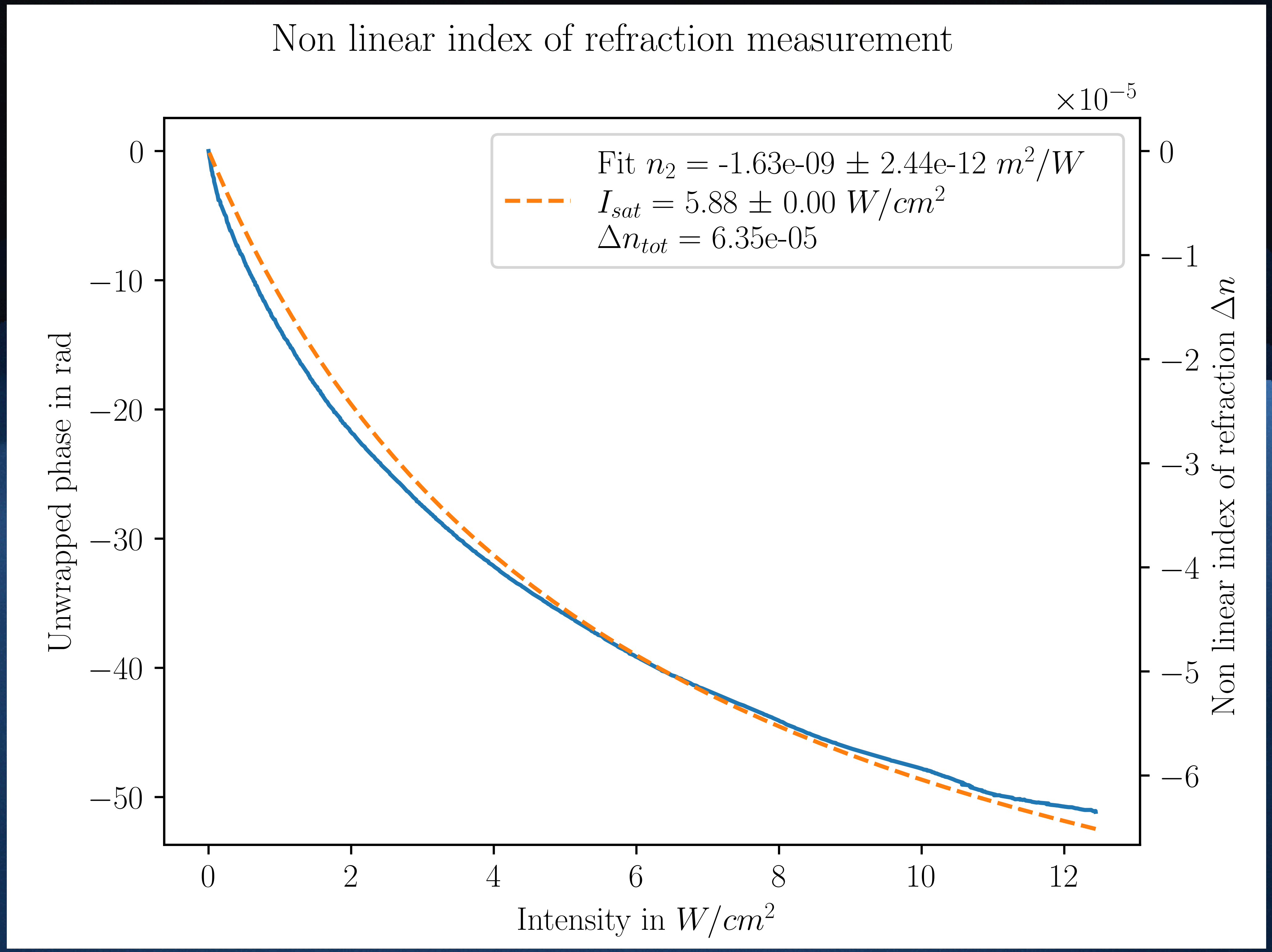
Adimensional formulation

Typical scales

But what do these scales mean ?

- Non-linear length z_{NL}: the length after which there is 1 rad of non-linear dephasing i.e $\delta\phi = \frac{z}{z_{NL}}$

- Healing length ξ: the scale under which the fluid cannot “recover” i.e the end of the sonic regime



Adimensional formulation

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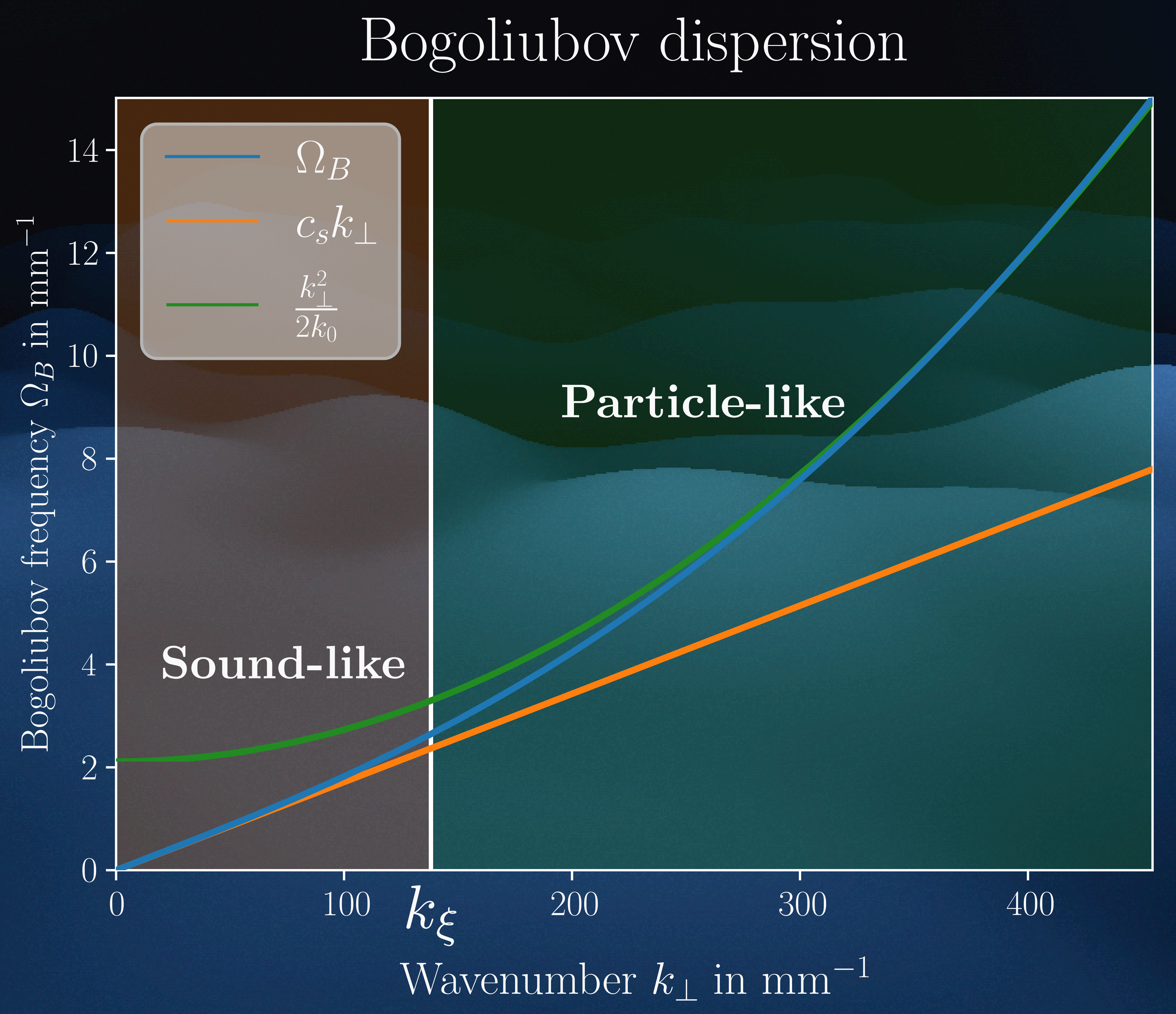


Adimensional formulation

Link with physical parameters

If both scales are controlled by Δn, there are then **only two relevant physical parameters**:

- The non-linear index of refraction n_2
- The saturation intensity I_{sat}

Since, in real life, $\Delta n = n_2 \dfrac{I}{1+\dfrac{I}{I_{sat}}}$, that **saturates** at $n_2 I_{sat}$

But is it the whole story ?

Figure of merits

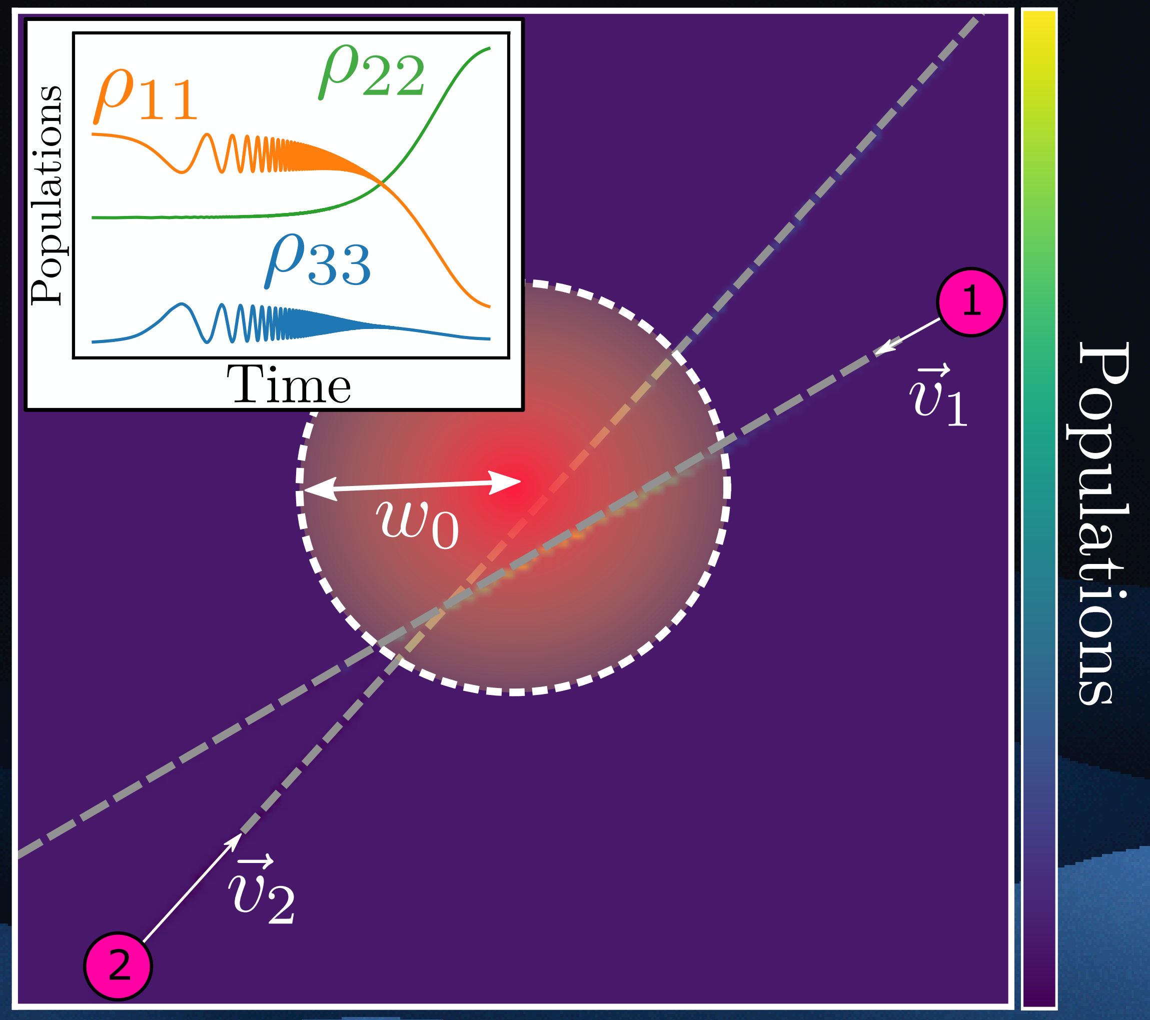
Figure of merits

Be careful what you wish for !

We established that the **two relevant scales** of the propagation equation are controlled by a **single parameter** Δn so what gives ?

- Assuming wanting to **maximize non-linearity**, we cannot do better than $n_2 I_{sat}$
- n_2 varies with the beam waist w_0
- I_{sat} varies with the inverse of the beam waist w_0

$\rightarrow$ $n_2 I_{sat}$ is **constant** ! This limits the parameter space …



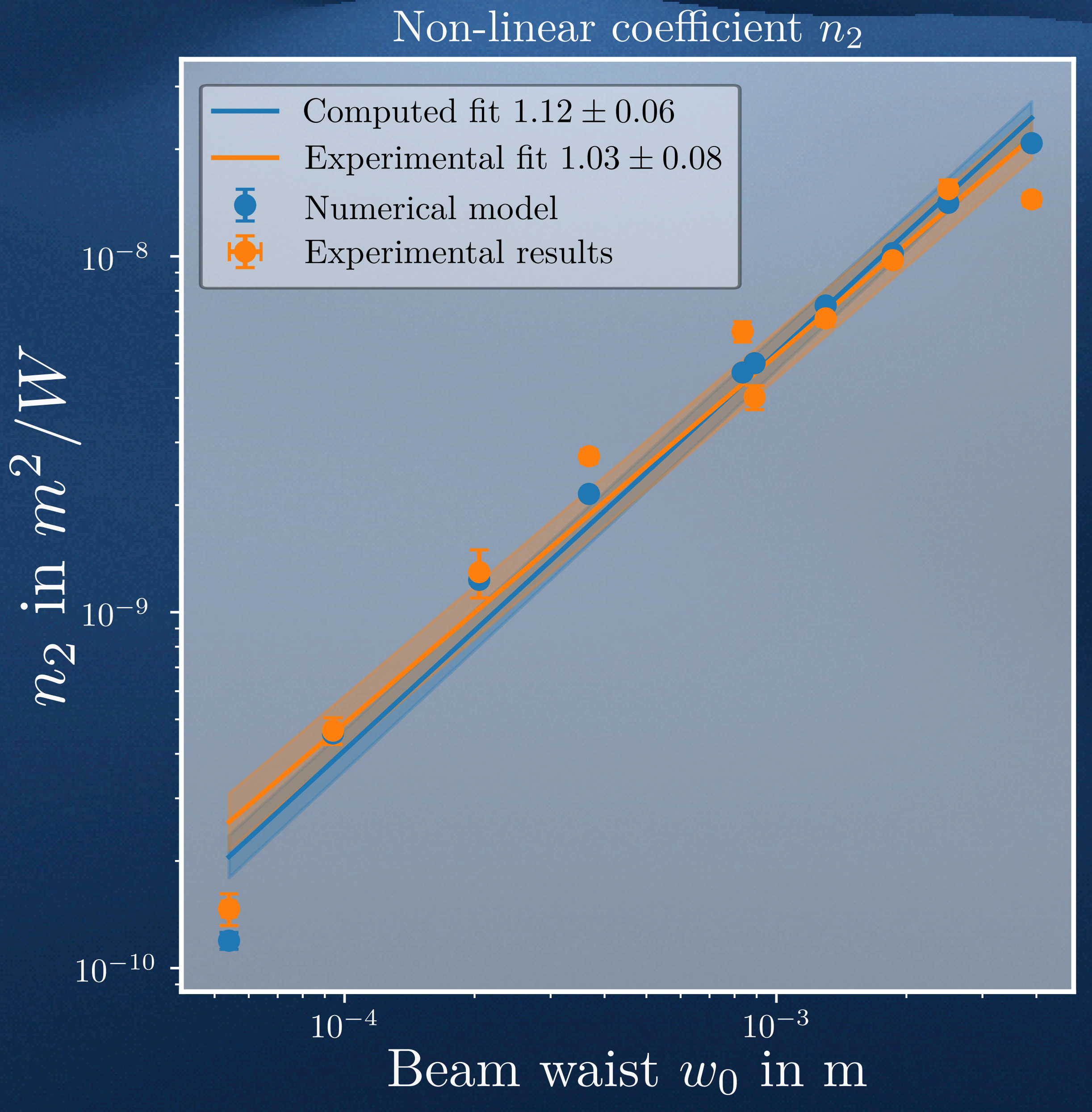


Figure of merits

Temporal evolution

But so far we did not discuss two important limitations:

- **Losses** described by the linear absorption coefficient α in m^{-1} that also saturates following $\partial_z I = -\alpha \frac{I}{1 + \frac{I}{I_{sat}}}$

- **Defocusing** the smaller the beam, the higher $n_2 I_{sat}$, the more it will “explode” and lose intensity fast

→ We want to keep the **intensity as high as possible**, with the **largest beam possible** !

Figure of merits

Temporal evolution

Now that we have established some guidelines, one can try and build a **figure of merit** including the previously introduced relevant quantities:

- $\xi \to$ Difference of input and output ξ ?
- $z_{NL} \to$ Total non-linear dephasing $\delta\phi$?
- $\alpha \to$ Transmission ?

$\Longrightarrow$ *Usefulness of a numerical framework like* `NLSE` *!*

Temporal evolution

Temporal evolution

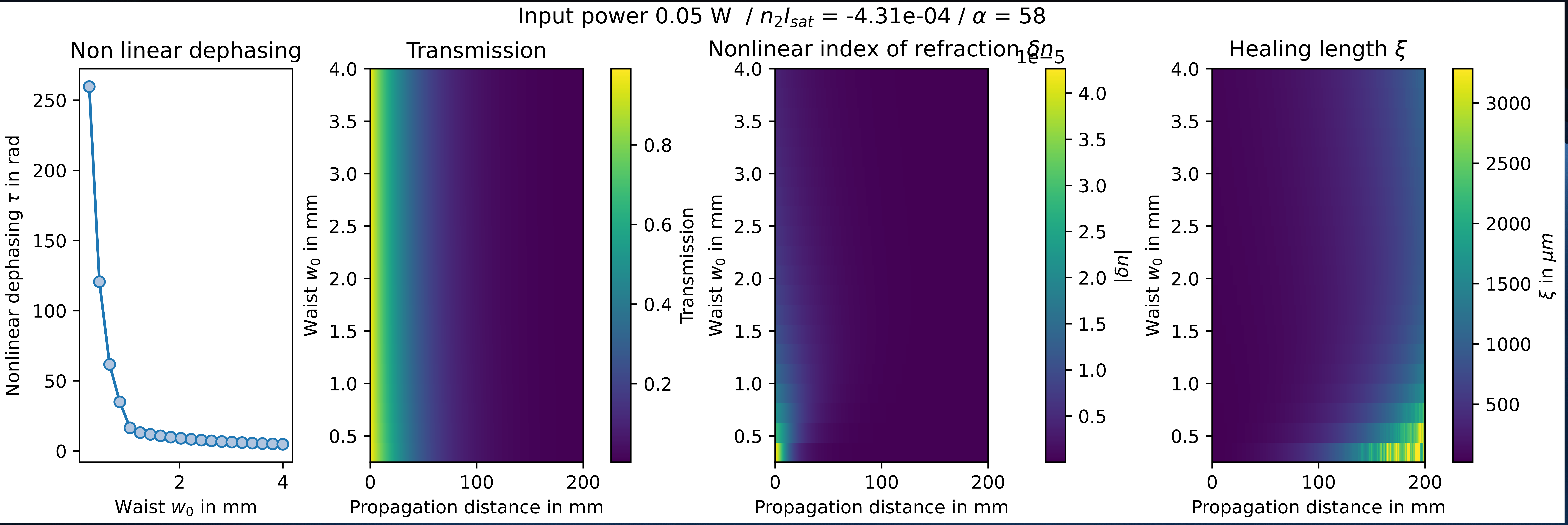
Input, output or average value ?

We saw that the **evolution** of the $n_2 I_{sat}$ product inside the cell will be a **critical** process $\rightarrow$ need for a **numerical campaign** to explore the parameter space

- Plot the evolution of the **instantaneous** quantities for a given power / waist combination
- Extract **2D** maps
- With a given power budget, one can **only use the beam waist** w_0 as an optimization parameter

Temporal evolution

Input, output or average value ?



Temporal evolution

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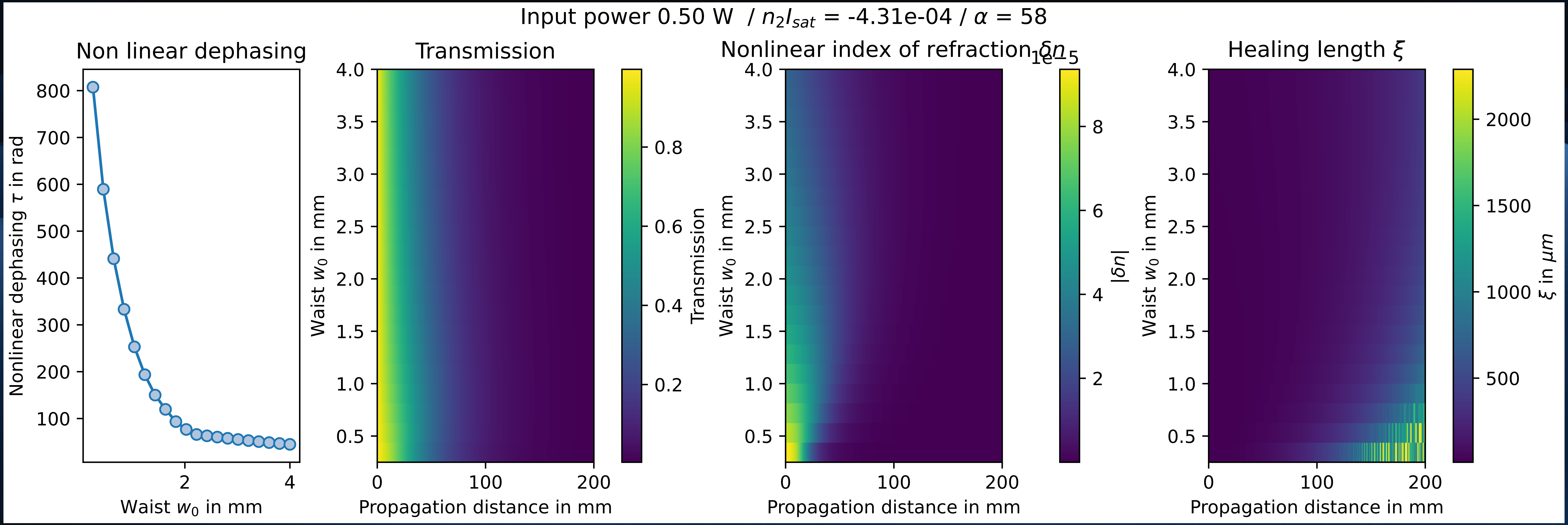

Temporal evolution

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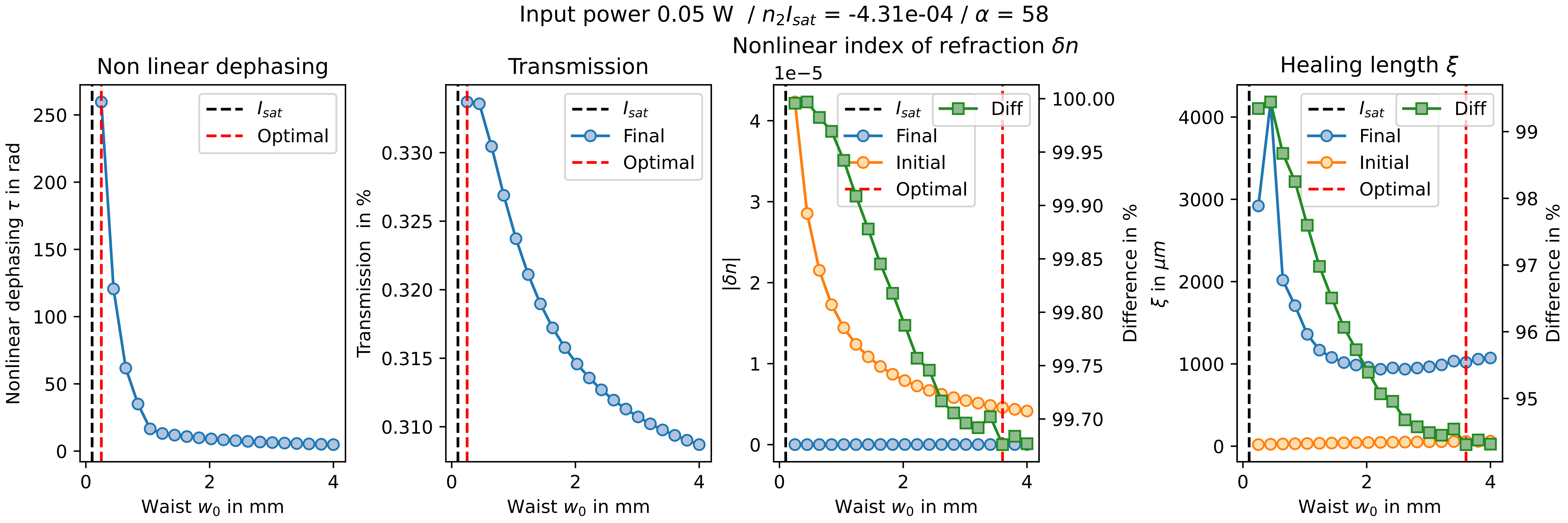

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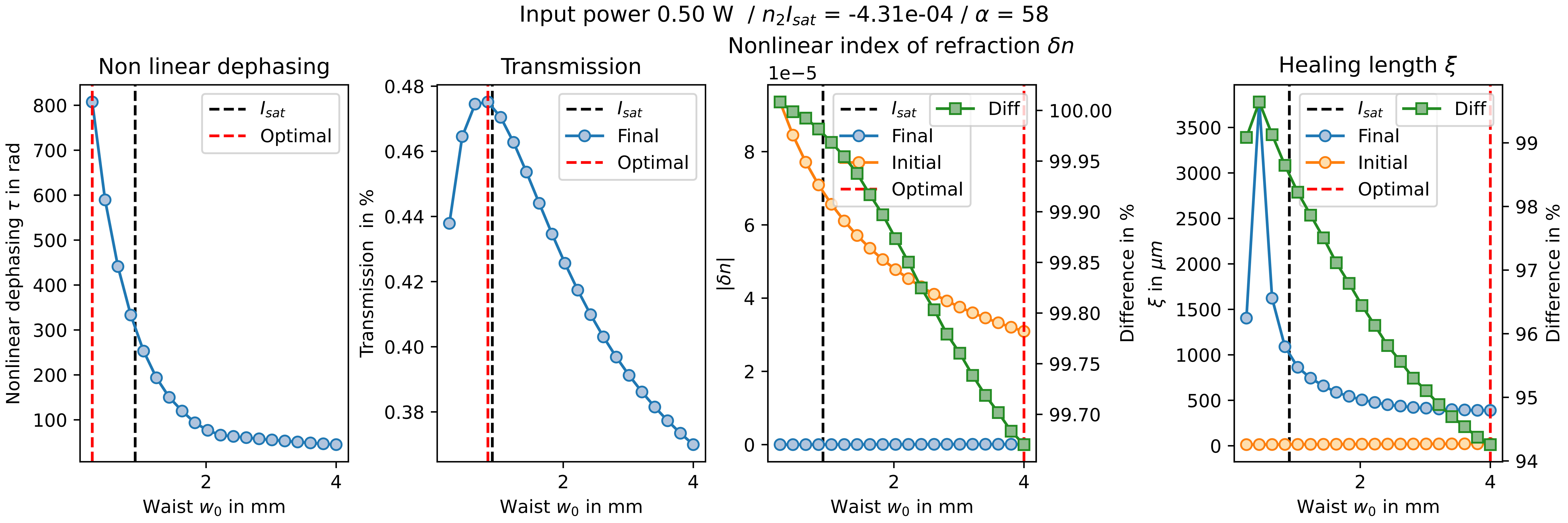

Temporal evolution

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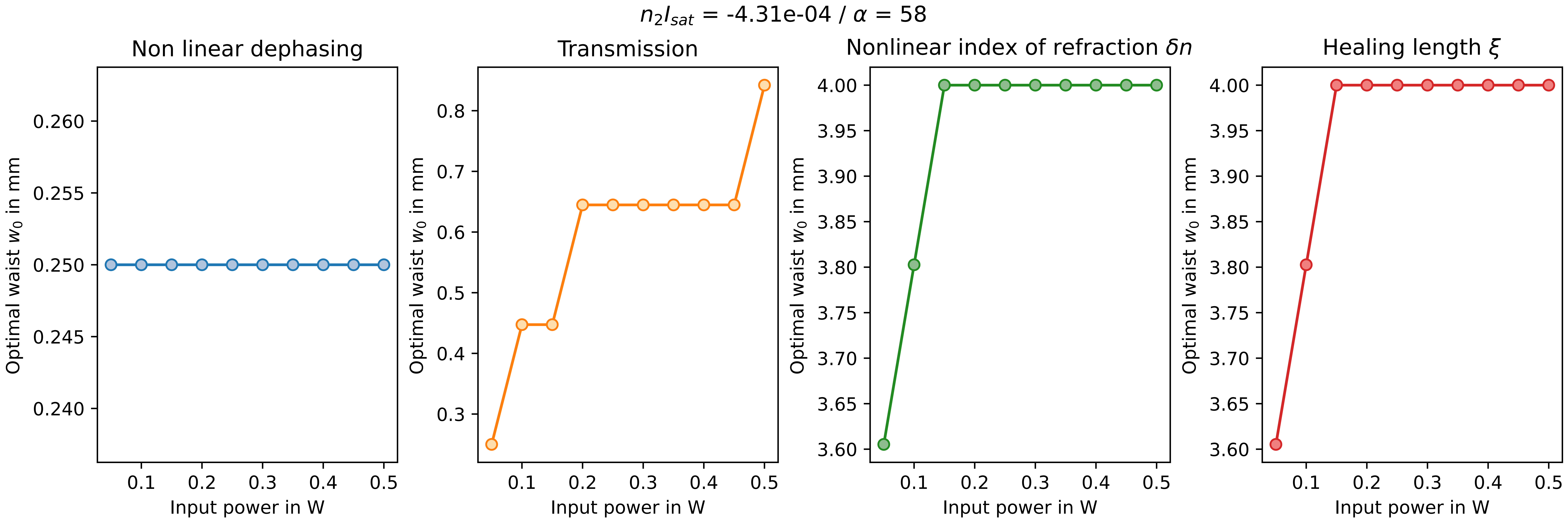

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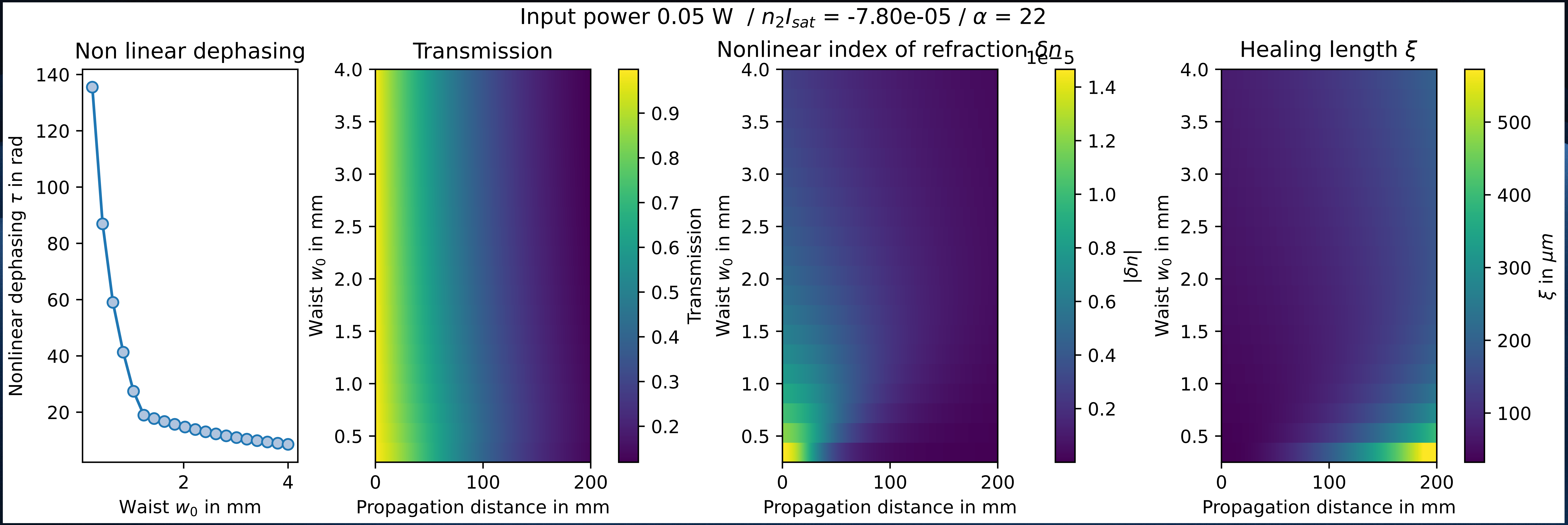

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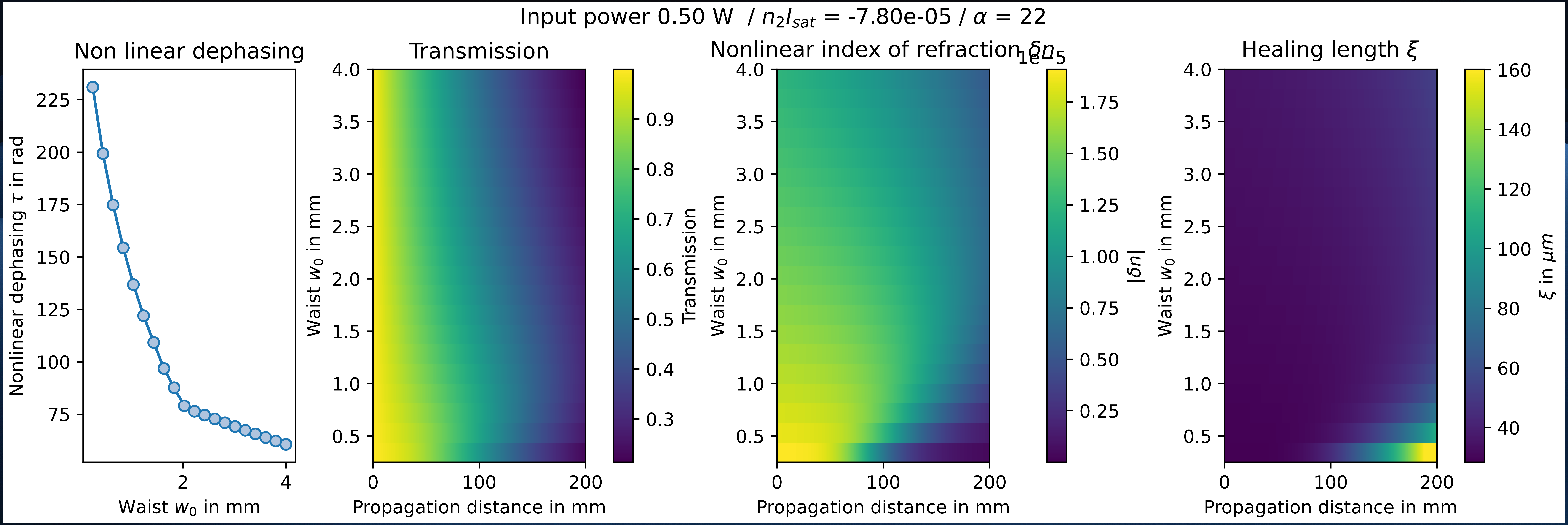

Temporal evolution

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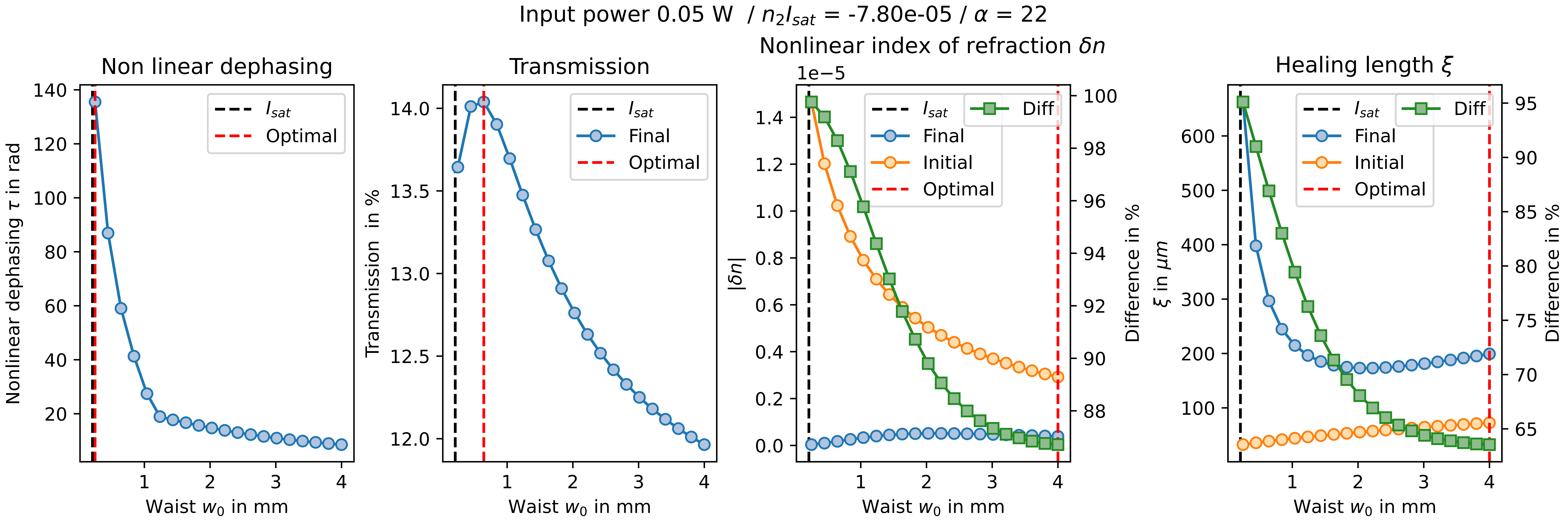

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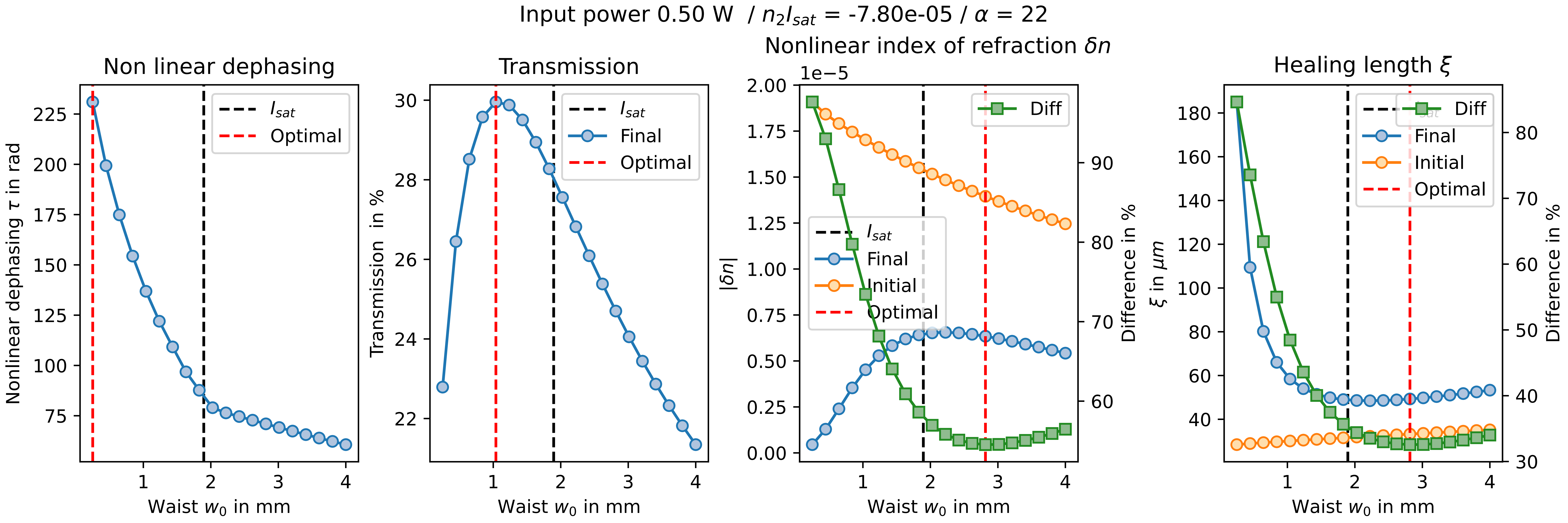

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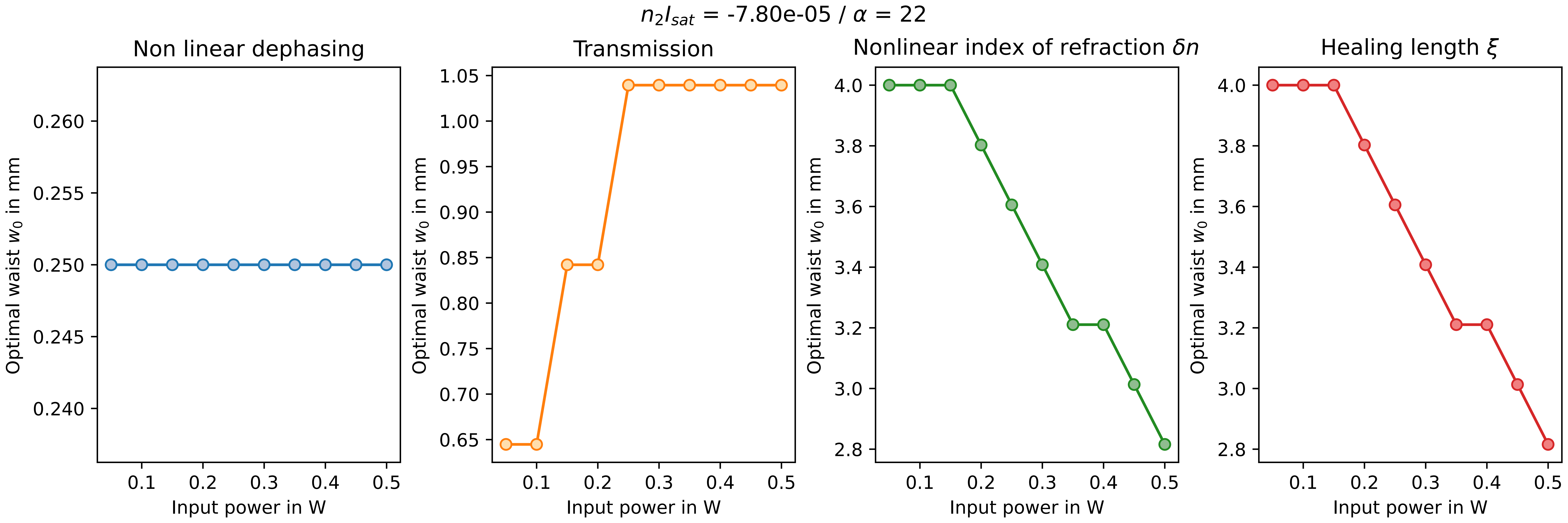

Temporal evolution

Input, output or average values ?

In the end, it seems there is **no general solution**, but rather a **set of observations** we can make, assuming we want to **minimize the variation** of the parameters while maximizing them:

- It is better to be **under** than over I_{sat}
- Higher powers are **always** better
- **Larger waists** look more promising to minimize variations since they defocus less

Temporal evolution

Input, output or average values ?

While there is no easy answer, we can try this strategy (that will change for each detuning):

- Measure $n_2 I_{sat}$
- Aim for a beam waist ensuring an intensity close to I_{sat} to maximize τ
- Fine tune for other desired characteristics

$\Longrightarrow$ *Small beams and super lossy regimes should be avoided ?*

Conclusion

Conclusion

Did we find our magic regime ?

- More exploration of the parameter space is required …
- Preliminary remarks made in this presentation suggest looking towards going away from resonance
- A set of **experimental measurements** would be very beneficial
- All of the changes considered here are **adiabatic** w.r.t to the dynamics of the fluid : the physics follows the changes of parameters