Orbital Angular Momentum entanglement by interaction of optical vortices in a vapor

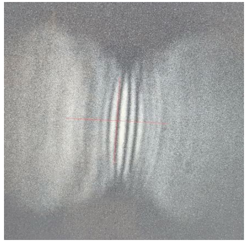
vortex+light in the style of Klimt (AI generated)

Myrann Baker-Rasooli

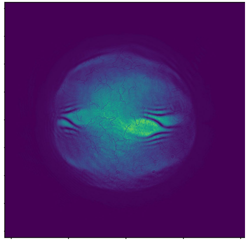
During the first year of my PhD

Vortex-vortex interactions in quantum fluid of light:

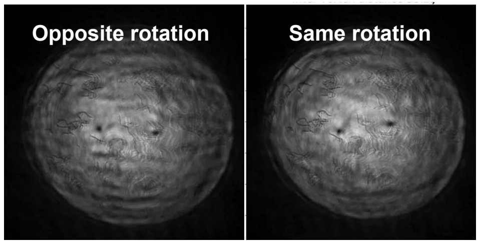
- **Quantum turbulence**
  Many quantum vortices dynamic



- **Kelvin-Helmholtz instabilities**
  Analog Von-Karman experiment

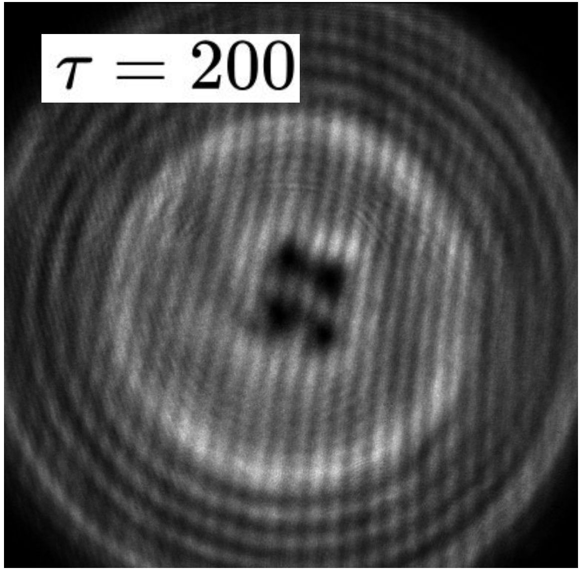


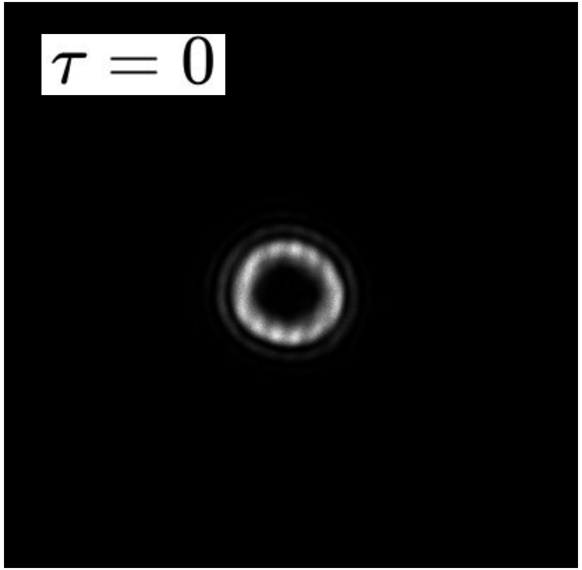
- **Vortex collision**
  Forced collision of two quantum vortex (merging and annihilation)



Do you know what is an optical vortex

vortex + light?





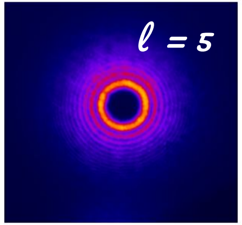
Laguerre-Gauss beams

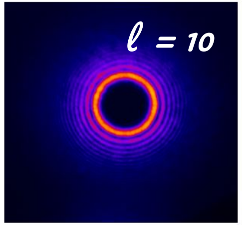
The electric field of an LG mode is given by:

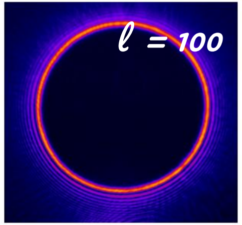
$$\binom{\ell}{p}_z = \overbrace{A^{\ell,p}(r,z)}^{\textbf{Amplitude}} \times \underbrace{e^{i\frac{kr^2}{2R(z)}}}_{\textbf{Curvature}} \times \underbrace{e^{i\ell\theta}}_{\textbf{Azimuthal}} \times \underbrace{e^{-i\phi_G(z)}}_{\textbf{Gouy}} \times \underbrace{e^{-i(kz-wt)}}_{\textbf{Propagation}}$$

Anular intensity of a vortex images[1] recorded with a CCD camera

$$I(r,0) = \left|\binom{\ell}{p}\binom{\ell}{p}^*\right|$$







The intensity ring radius is given by $R = w_0\sqrt{\ell/2}$ with ℓ the **Orbital Angular Momentum** of the optical vortex

1 : A. Chopinaud. Atomes et vortex optiques, Physique Atomique.Université Paris Saclay (2018)

Laguerre-Gauss beams

The electric field of an LG mode is given by:

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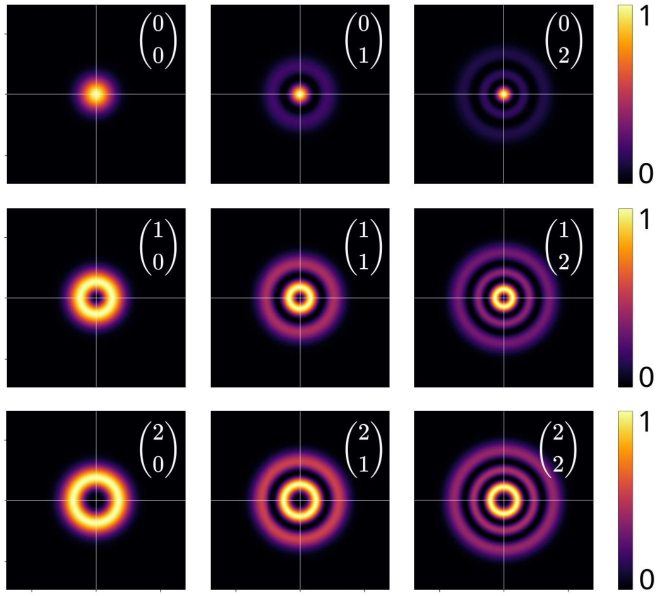
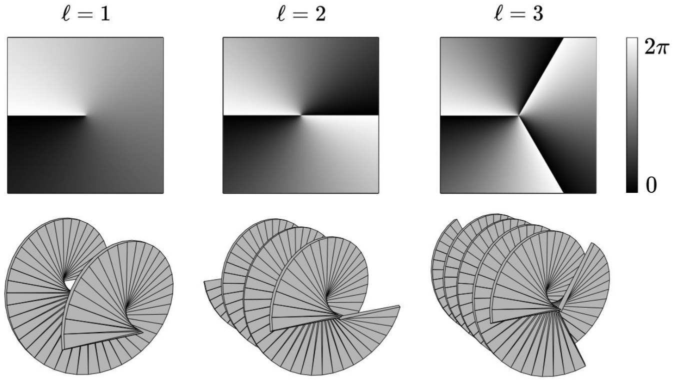
When *p* != 0, modes show *p* + 1 rings in the intensity.

It adds complexity so lets only consider LG modes with *p*=0



Orbital Angular Momentum of the optical vortex

Spatial Light Modulator



- To generate one, a laser beam propagates through an object that has an helical singularity

- Optical vortices are laser beams with an helical wavefront. They carry a quantum variable, the **OAM**[1] ℓ, being a relative integer. It is related to the phase and the handedness

- Usually, information is coded via the light polarization 0 or 1.
  → The OAM can take any value ℓ = …, -3, -2, -1, 0, 1, 2, 3, …

1: L. Allen et al., Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes, Phys.Rev. A **45**, 8185, 1992

Optical vortices development

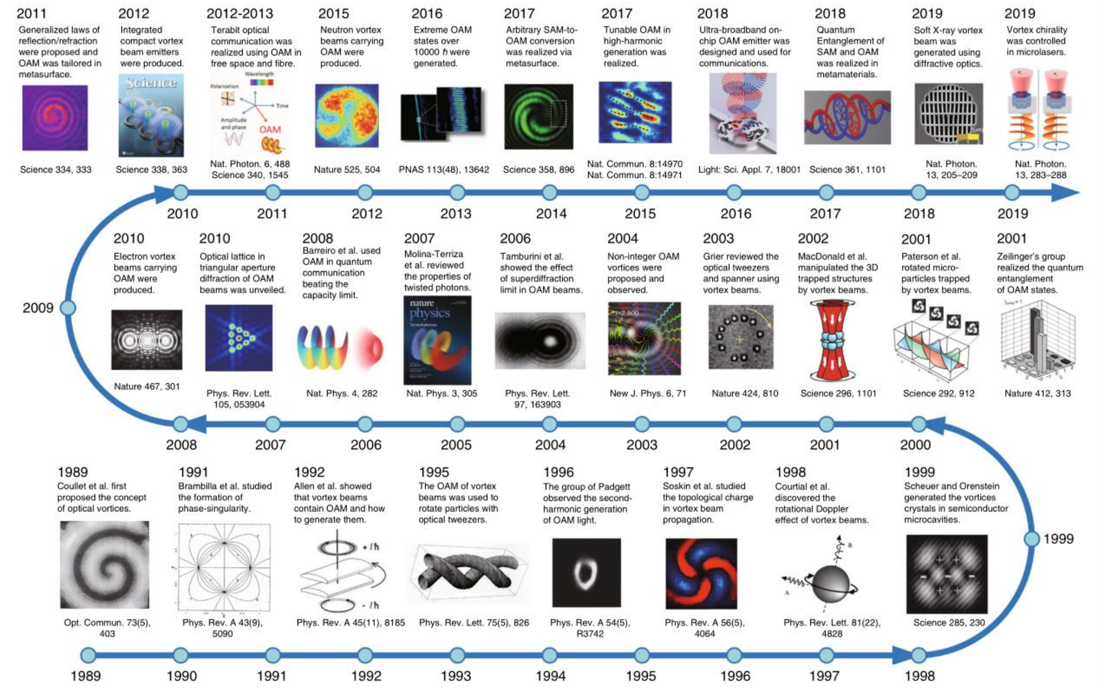


Fig. 1 Roadmap of the 30-year development of optical vortices from 1989 to 2019, including significant theoretical and technical breakthroughs with corresponding references. Reprinted with permission from ref. [1], Copyright (2019), with permission from Elsevier. Reprinted with permission from refs. [5,16,19,25,27,28,38,40]. Copyright (2019) by the American Physical Society. From refs. [29,32,33,41,42,44,46,47]. Reprinted with permission from AAAS. Reprinted by permission from Springer Nature: Nature[30,34,59,60], Nature Physics[31,39], Nature Photonics[43,50,52], Nature Communications[48,49], Light: Science & Applications[51], Copyright (2019). From ref. [57]. Reproduced by permission of IOP Publishing

Shen, Y., Wang, X., Xie, Z. *et al.* Optical vortices 30 years on: OAM manipulation from topological charge to multiple singularities. *Light Sci Appl* **8**, 90 (2019).

OAMs correlations study

The OAM is used in several applications such as for quantum memories[1]

A quantum memory for orbital angular momentum photonic qubits

A. Nicolas, L. Veissier, L. Giner, E. Giacobino, D. Maxein, J. Laurat*
Laboratoire Kastler Brossel, Université Pierre et Marie Curie,
Ecole Normale Supérieure, CNRS, 4 place Jussieu, 75252 Paris Cedex 05, France

Or use this variable to make and study OAMs entanglement[2]

Entanglement of Orbital Angular Momentum States of Photons

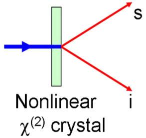
Alois Mair,[1] Alipasha Vaziri, Gregor Weihs, and Anton Zeilinger

Institut für Experimentalphysik, Universität Wien
Boltzmanngasse 5, 1090 Wien, Austria

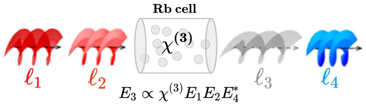
→ Thanks non-linear effects, it is possible to build correlations between pairs of OAMs

Plan

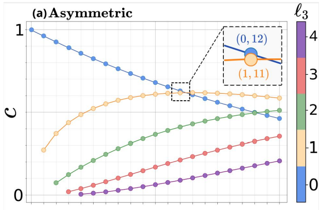
- **Entangled photon sources**



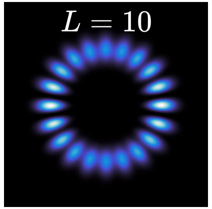
- **Vortex mixing and OAM entanglement**

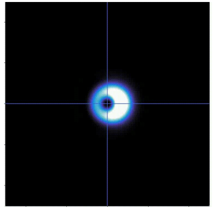


- **Analytical model** (the project I developed during my second year of PhD)



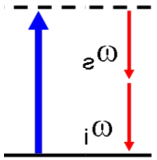
- **Applications**



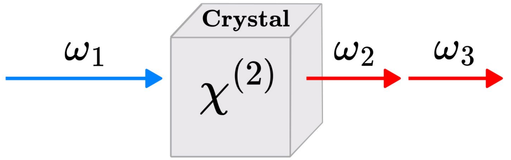


Entangled photon sources

Spontaneous Down Conversion (SPDC)



- **Spontaneous Down Conversion (SPDC)**
  - In crystals
  - One input laser (continuous or pulsed)
  - To be spectrally selective, the crystal must be well prepared



$$\begin{cases} \mathrm{P_{NL}}(\omega_1) = 6\epsilon_0\chi^{(2)}\mathrm{A}_2^{\phi_2}\mathrm{A}_3^{\phi_3}e^{i(k_1z-\omega_1t)} \\ \mathrm{P_{NL}}(\omega_2) = 6\epsilon_0\chi^{(2)}\mathrm{A}_1^{\phi_1}\mathrm{A}_3^{\phi_3^*}e^{i(k_2z-\omega_2t)} \\ \mathrm{P_{NL}}(\omega_3) = 6\epsilon_0\chi^{(2)}\mathrm{A}_1^{\phi_1}\mathrm{A}_2^{\phi_2^*}e^{i(k_3z-\omega_3t)} \end{cases} \longrightarrow \begin{cases} \nabla_\perp^2\mathrm{A}_1 + 2ik_1\dfrac{\partial\mathrm{A}_1}{\partial z} = \dfrac{-6\chi^{(2)}\omega_1^2}{c^2}\mathrm{A}_2\mathrm{A}_3e^{i\Delta kz}e^{i\Delta\phi} \\ \nabla_\perp^2\mathrm{A}_2 + 2ik_2\dfrac{\partial\mathrm{A}_2}{\partial z} = \dfrac{-6\chi^{(2)}\omega_2^2}{c^2}\mathrm{A}_1\mathrm{A}_3^*e^{i\Delta kz}e^{i\Delta\phi} \\ \nabla_\perp^2\mathrm{A}_3 + 2ik_3\dfrac{\partial\mathrm{A}_3}{\partial z} = \dfrac{-6\chi^{(2)}\omega_3^2}{c^2}\mathrm{A}_1\mathrm{A}_2^*e^{i\Delta kz}e^{i\Delta\phi} \end{cases}$$

$$\Delta k = k_1 - k_2 - k_3 \text{ and } \Delta\phi = \phi_1 - \phi_2 - \phi_3$$

Entangled photon sources

Spontaneous Down Conversion (SPDC)

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

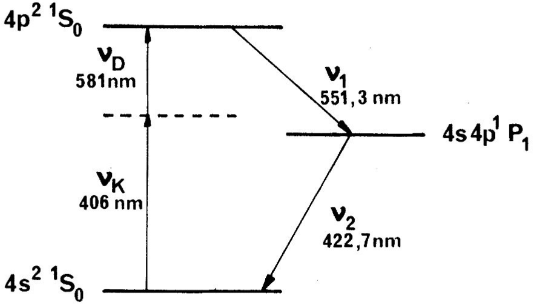
Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique, Université Paris-Sud, F-91406 Orsay, France

(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

PACS numbers: 03.65.Bz, 35.80.+s



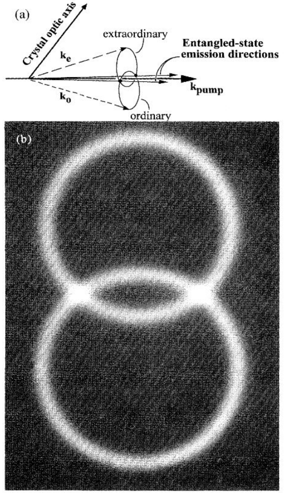
New High-Intensity Source of Polarization-Entangled Photon Pairs

Paul G. Kwiat,* Klaus Mattle, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, 6020 Innsbruck, Austria

Alexander V. Sergienko and Yanhua Shih
Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21228

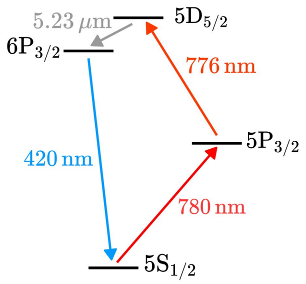
(Received 5 July 1995)

We report on a high-intensity source of polarization-entangled photon pairs with high momentum definition. Type-II noncollinear phase matching in parametric down conversion produces true entanglement: No part of the wave function must be discarded, in contrast to previous schemes. With two-photon fringe visibilities in excess of 97%, we demonstrated a violation of Bell's inequality by over 100 standard deviations in less than 5 min. The new source allowed ready preparation of all four of the EPR-Bell states.

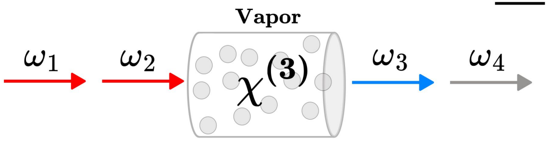


Entangled photon sources

Spontaneous Four Wave Mixing (SFWM)



- **Spontaneous Four Wave Mixing (SFWM)**
  - In vapors
  - Two input lasers (continuous or pulsed)
  - Realizable with different atomic schemes



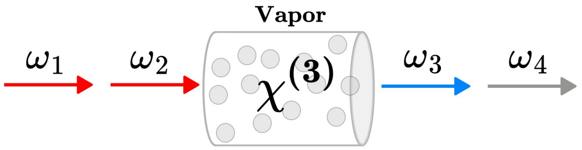
$$\begin{cases} \mathrm{P_{NL}}(\omega_1) = 6\epsilon_0\chi^{(3)}\mathrm{A}_3^{\phi_3}\mathrm{A}_4^{\phi_4}\mathrm{A}_2^{\phi_2^*}e^{i(k_1 z-\omega_1 t)} \\ \mathrm{P_{NL}}(\omega_2) = 6\epsilon_0\chi^{(3)}\mathrm{A}_3^{\phi_3}\mathrm{A}_4^{\phi_4}\mathrm{A}_1^{\phi_1^*}e^{i(k_2 z-\omega_2 t)} \\ \mathrm{P_{NL}}(\omega_3) = 6\epsilon_0\chi^{(3)}\mathrm{A}_1^{\phi_1}\mathrm{A}_2^{\phi_2}\mathrm{A}_4^{\phi_4^*}e^{i(k_3 z-\omega_3 t)} \\ \mathrm{P_{NL}}(\omega_4) = 6\epsilon_0\chi^{(3)}\mathrm{A}_1^{\phi_1}\mathrm{A}_2^{\phi_2}\mathrm{A}_3^{\phi_3^*}e^{i(k_4 z-\omega_4 t)} \end{cases} \longrightarrow \begin{cases} \nabla_\perp^2\mathrm{A}_1 + 2ik_1\dfrac{\partial \mathrm{A}_1}{\partial z} = \dfrac{-6\chi^{(3)}\omega_1^2}{c^2}\mathrm{A}_3\mathrm{A}_4\mathrm{A}_2^* e^{i\Delta k z}e^{i\Delta\phi} \\ \nabla_\perp^2\mathrm{A}_2 + 2ik_2\dfrac{\partial \mathrm{A}_2}{\partial z} = \dfrac{-6\chi^{(3)}\omega_2^2}{c^2}\mathrm{A}_3\mathrm{A}_4\mathrm{A}_1^* e^{i\Delta k z}e^{i\Delta\phi} \\ \nabla_\perp^2\mathrm{A}_3 + 2ik_3\dfrac{\partial \mathrm{A}_3}{\partial z} = \dfrac{-6\chi^{(3)}\omega_3^2}{c^2}\mathrm{A}_1\mathrm{A}_2\mathrm{A}_4^* e^{i\Delta k z}e^{i\Delta\phi} \\ \nabla_\perp^2\mathrm{A}_4 + 2ik_4\dfrac{\partial \mathrm{A}_4}{\partial z} = \dfrac{-6\chi^{(3)}\omega_4^2}{c^2}\mathrm{A}_1\mathrm{A}_2\mathrm{A}_3^* e^{i\Delta k z}e^{i\Delta\phi} \end{cases}$$

$$\Delta k = k_1 + k_2 - k_3 - k_4 \text{ and } \Delta\phi = \phi_1 + \phi_2 - \phi_3 - \phi_4$$

Comparison to our case

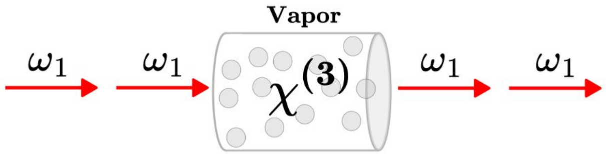
Degenerate Four Wave Mixing (DFWM)

Non Degenerate Four Wave Mixing

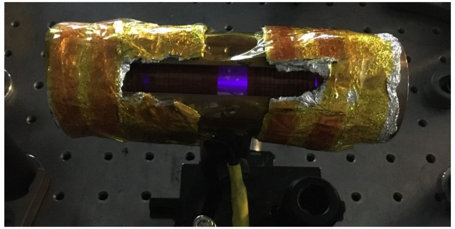


$$\begin{cases} \nabla_\perp^2 \mathrm{A}_1 + 2ik_1 \dfrac{\partial \mathrm{A}_1}{\partial z} = \dfrac{-6\chi^{(3)}\omega_1^2}{c^2} \mathrm{A}_3\mathrm{A}_4\mathrm{A}_2^* e^{i\Delta kz} e^{i\Delta\phi} \\ \nabla_\perp^2 \mathrm{A}_2 + 2ik_2 \dfrac{\partial \mathrm{A}_2}{\partial z} = \dfrac{-6\chi^{(3)}\omega_2^2}{c^2} \mathrm{A}_3\mathrm{A}_4\mathrm{A}_1^* e^{i\Delta kz} e^{i\Delta\phi} \\ \nabla_\perp^2 \mathrm{A}_3 + 2ik_3 \dfrac{\partial \mathrm{A}_3}{\partial z} = \dfrac{-6\chi^{(3)}\omega_3^2}{c^2} \mathrm{A}_1\mathrm{A}_2\mathrm{A}_4^* e^{i\Delta kz} e^{i\Delta\phi} \\ \nabla_\perp^2 \mathrm{A}_4 + 2ik_4 \dfrac{\partial \mathrm{A}_4}{\partial z} = \dfrac{-6\chi^{(3)}\omega_4^2}{c^2} \mathrm{A}_1\mathrm{A}_2\mathrm{A}_3^* e^{i\Delta kz} e^{i\Delta\phi} \end{cases}$$

Degenerate Four Wave Mixing



$$i\frac{\partial E(\mathbf{r}_\perp, z)}{\partial z} = \left[-\frac{1}{2k}\nabla_\perp^2 - i\frac{\alpha_a}{2} - \frac{3}{8}\frac{k}{n_0^2}\chi^{(3)}|E(\mathbf{r}_\perp, z)|^2 \right] E(\mathbf{r}_\perp, z)$$



Entangled photon sources

Spontaneous Four Wave Mixing (SFWM)

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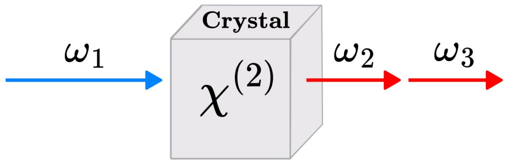
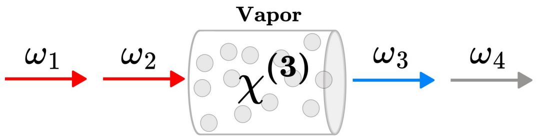
  - In crystals
  - One input laser (continuous or pulsed)
  - To be spectrally selective, the crystal must be well prepared



- **Spontaneous Four Wave Mixing (SFWM)**
  - In vapors
  - Two input lasers (continuous or pulsed)
  - Realizable with different atomic schemes



We realize SFWM with vortex beams to study both:

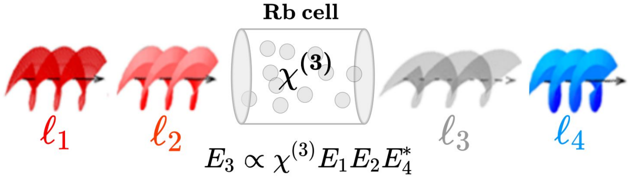
- colors entanglement
- OAMs entanglement

ℓ = OAM

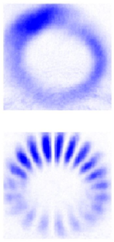
OAMs and Spontaneous Four Wave Mixing

- If the two input colors carry an OAM, the output vortices have **correlated OAMs**
- SFWM allows an unambiguous study of the output OAMs
- The blue light intensity and phase have been measured in experiments done in Glasgow[1], Williamsburg[2] and Paris[3]

Develop a model to study the output



1



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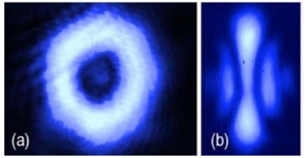
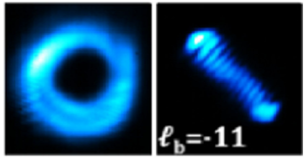


3



1: G. Walker, A. S. Arnold, and S. Franke-Arnold, Trans-Spectral Orbital Angular Momentum Transfer via Four-Wave Mixing in Rb Vapor, Phys. Rev. Lett.**108**, 243601 (2012)
2: A.M. Akulshin, I. Novikova, E.E. Mikhailov, S.A. Suslov, and R.J. McLean. "Arithmetic with optical topological charges in stepwise-excited Rb vapor, Opt. Lett.**41**, 1146 (2016)
3 : A. Chopinaud, M. Jacquey, B. Viaris de Lesegno, and L.Pruvost. High helicity vortex conversion in a rubidium vapor. Phys. Rev. A **97**, 063806 (2018)

ℓ = OAM

Working region and vortex overlap

Compute the overlap of the fields

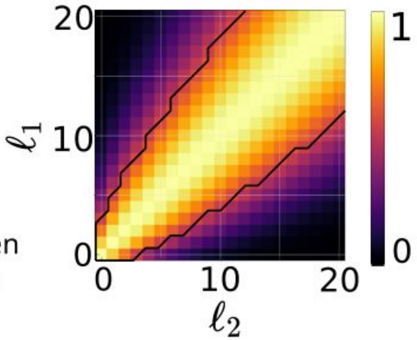
Input

$$J^{in} = \iint A_{\ell_1} A_{\ell_2}\, r dr d\theta = \frac{((\ell_1+\ell_2)/2)!}{\sqrt{\ell_1!\,\ell_2!}}$$

Overlap condition of the input LG modes gives

$$\begin{cases} \ell_2 + \sqrt{\ell_2} > \ell_1 - \sqrt{\ell_1} \\ \ell_2 - \sqrt{\ell_2} < \ell_1 + \sqrt{\ell_1} \end{cases}$$

SFWM is most effective when the input OAMs are close



Output

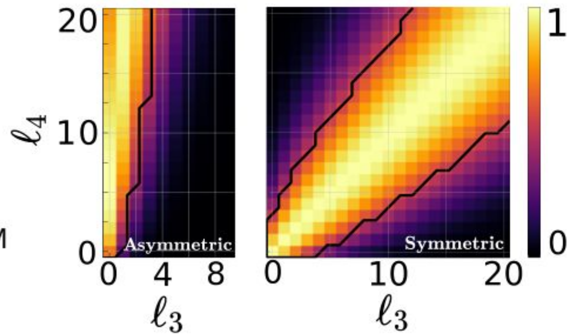
$$J^{out} = \iint A_{\ell_3} A_{\ell_4}\, r dr d\theta = \frac{((\ell_3+\ell_4)/2)!}{\sqrt{\ell_3!\,\ell_4!}} \sqrt{\frac{\lambda^{\ell_3+\ell_4+2}}{\lambda_3^{\ell_3+1}\,\lambda_4^{\ell_4+1}}}$$

Depends on the ratio λ_3/λ_4

Asymmetric $\lambda_3/\lambda_4 \sim 12$

Symmetric $\lambda_3/\lambda_4 \sim 1$

According to the ratio we can predict the output pairs of OAM



The output is expressed as $\sum_{\ell_3,\ell_4} c(\ell_3,\ell_4) \times LG_{p_3}^{\ell_3} \times LG_{p_4}^{\ell_4}$

How the total OAM is distributed?

Laguerre-Gauss basis

Hypothesis: $\ell_1 \geq 0$, $\ell_2 \geq 0$ and $p_1 = p_2 = 0$

Conservation of the total **OAM** and the **Gouy phase**

$$\left.\begin{aligned} L &= \ell_3 + \ell_4 = \ell_1 + \ell_2 \\ |\ell_3| + |\ell_4| + 2p_3 + 2p_4 &= |\ell_1| + |\ell_2| \end{aligned}\right\}$$

All radial number ***p*** are null

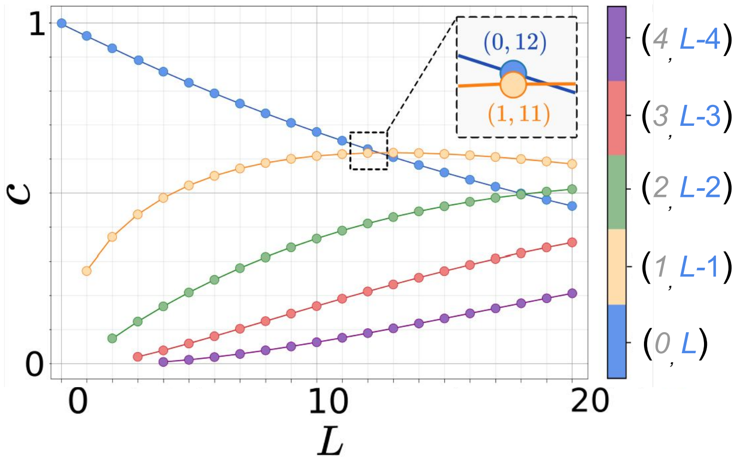
***L*+1** pairs (ℓ_3, ℓ_4) at the output:

(0,*L*), (1,*L*-1), (2,L-2) … (L-1,1), (*L*,0)

Boyd's criterion assumption: beams have same Rayleigh range

$$c(\ell_3, \underbrace{L-\ell_3}_{\ell_4}) \propto \iint LG^{\ell_1} LG^{\ell_2} LG^{\ell_3} LG^{\ell_4} r dr d\theta$$

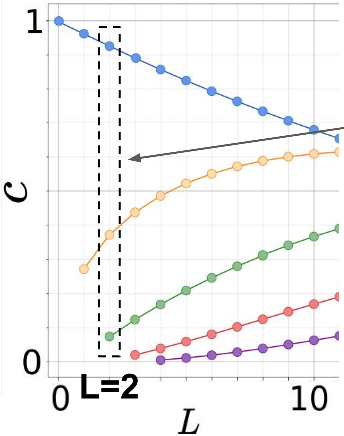
(overlap of 4 modes)



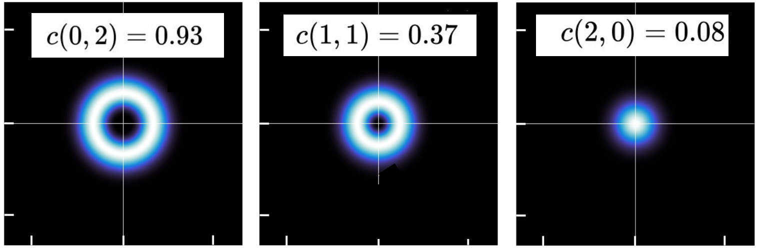
- Each curve represents a family of pairs (ℓ_3, ℓ_4)
- With ***L*** increasing**,** there is more than one pair at the output

Blue output for L=2

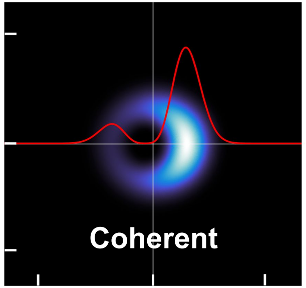
$$E_4 = \sum_{\ell_4}^{2} c(L-\ell_4, \ell_4) \times LG^{\ell_4}$$



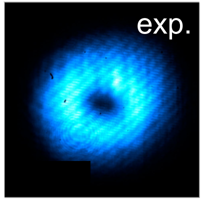
Three modes at the output, one predominant

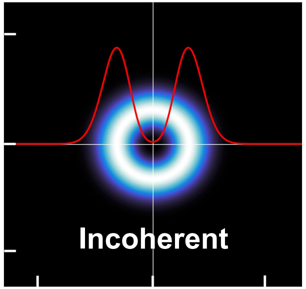


The **coherent** sum gives a crescent moon intensity



Annular intensity if **incoherent**

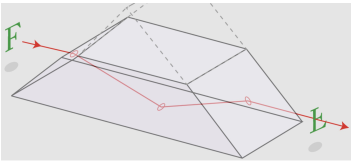




Comparison to experiment for L=2

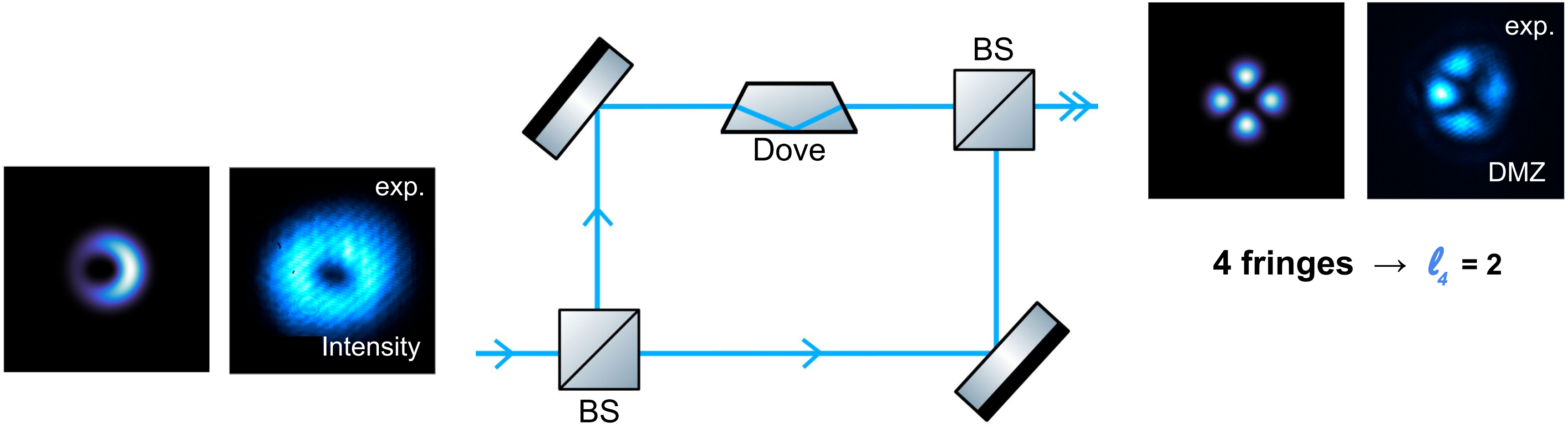
Does information on the OAM remains?

A Dove prism return the field



For single OAM beams, we use a Mach-Zehnder interferometer added by a Dove prism (DMZ)

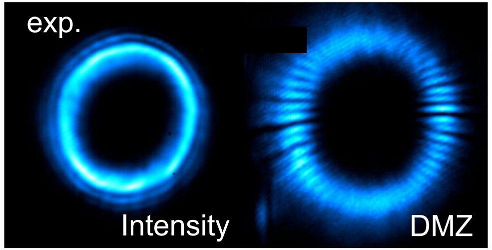
The number of radial interference fringes is equal to twice the OAM



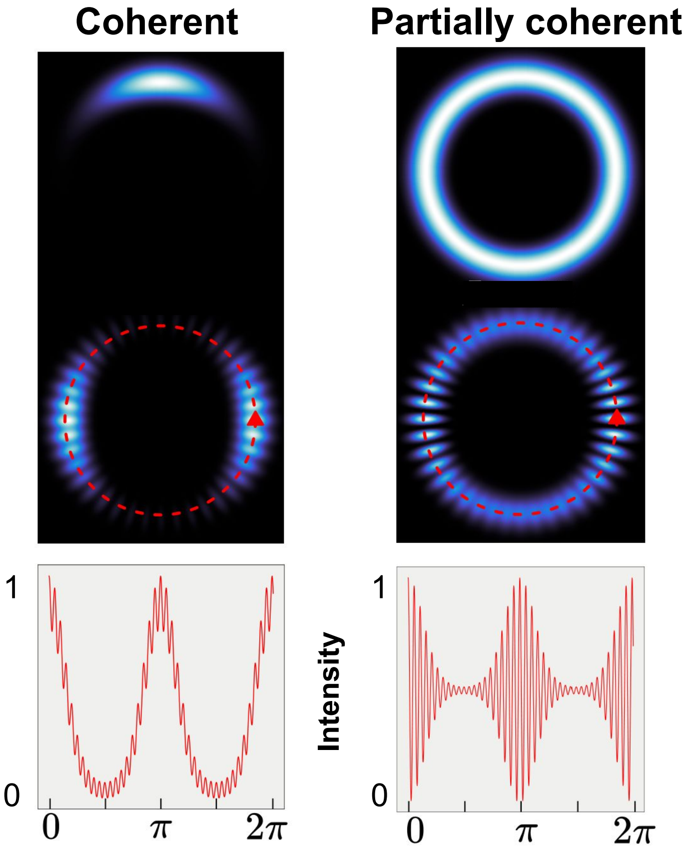
4 fringes → ℓ_4 **= 2**

Experimental data from A. Chopinaud. Atomes et vortex optiques, Physique Atomique.Université Paris Saclay (2018)

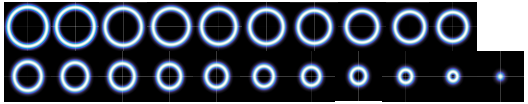
If L=20 at input: 21 modes in output



- It becomes hard to count the fringes
- Theory-experiment agreement
- **Explained by a partially coherent superposition of modes?** Work in progress



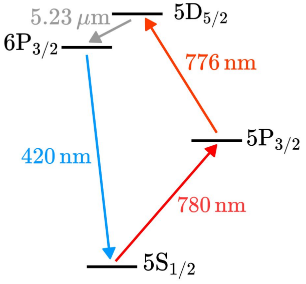
Intensity of the 21 modes involved

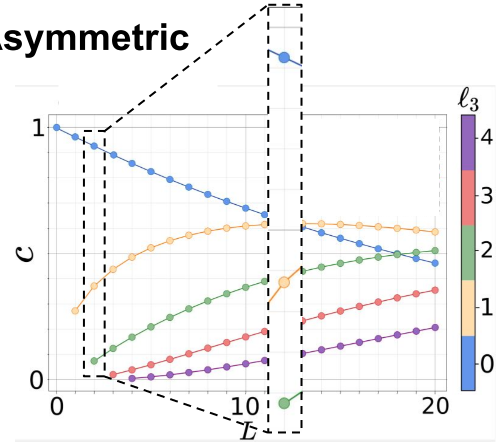


Prediction for the symmetric scheme

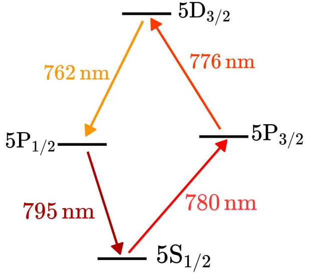
The distribution of the OAM is equiprobable

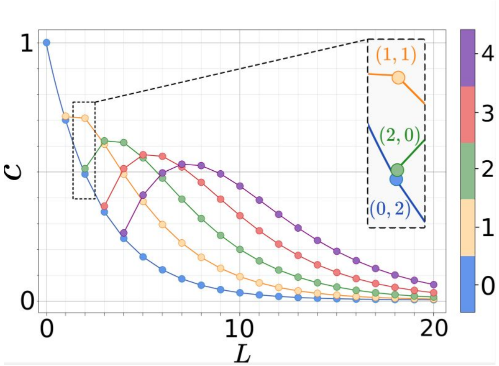
Asymmetric

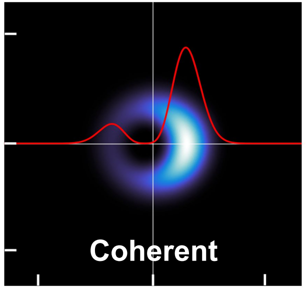


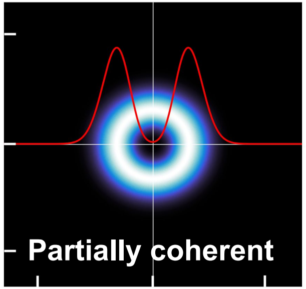


Symmetric (Strong correlations)

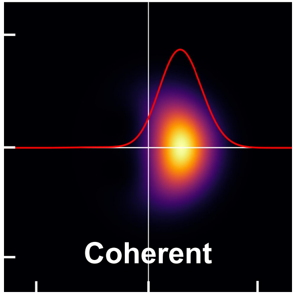


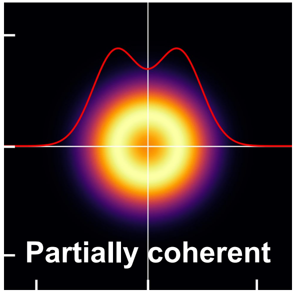






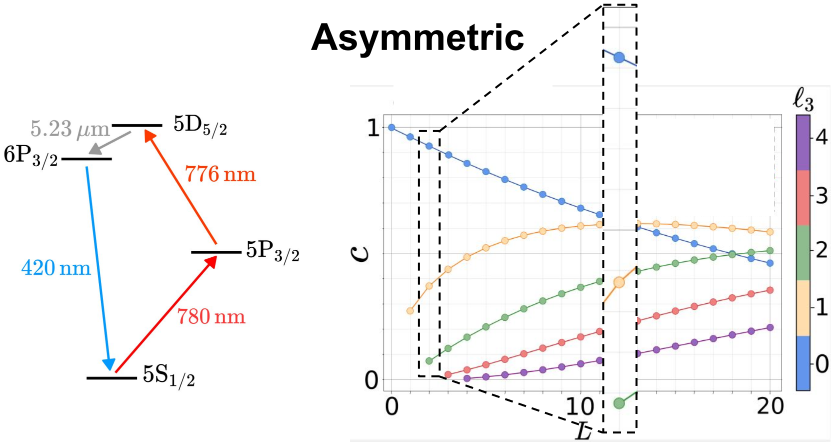
L=2

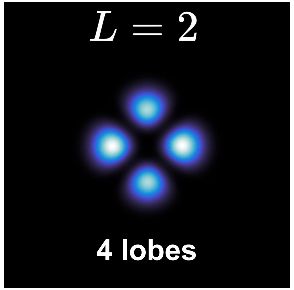


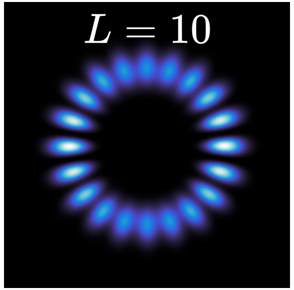


Different expected signatures

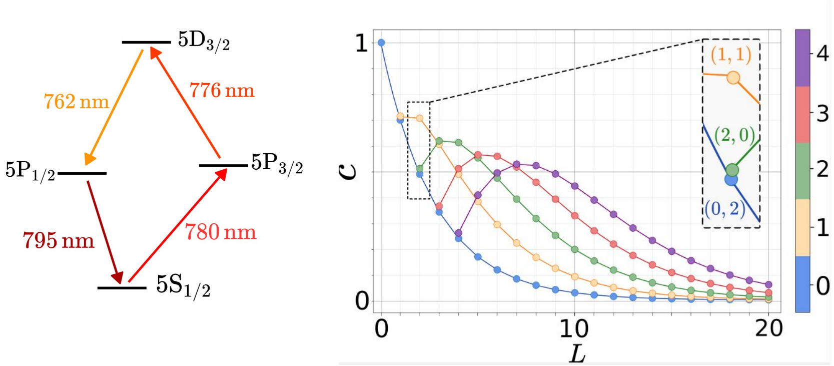
Asymmetric

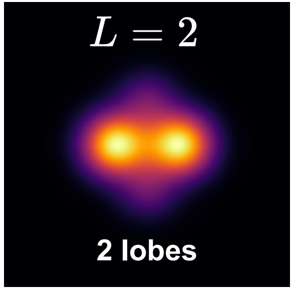


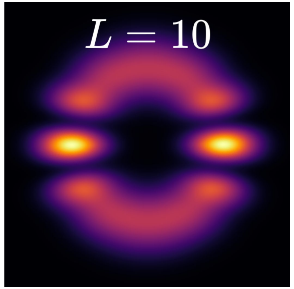




Symmetric (Strong correlations)







Opposite sign vortex mixing

$$\sum_{\ell_3,\ell_4} c(\ell_3,\ell_4) \times LG_{p_3}^{\ell_3} \times LG_{p_4}^{\ell_4}$$

Laguerre-Gauss basis

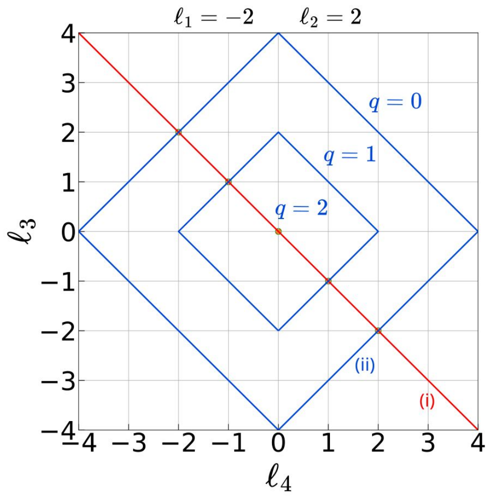
Hypothesis: $\ell_1 \geq 0$, $\ell_2 < 0$ and $p_1 = p_2 = 0$

Conservation of the total **OAM** and the **Gouy phase**

$$\left.\begin{aligned} L &= \ell_3 + \ell_4 = \ell_1 + \ell_2 \\ |\ell_3| + |\ell_4| + 2p_3 + 2p_4 &= |\ell_1| + |\ell_2| \end{aligned}\right\}$$

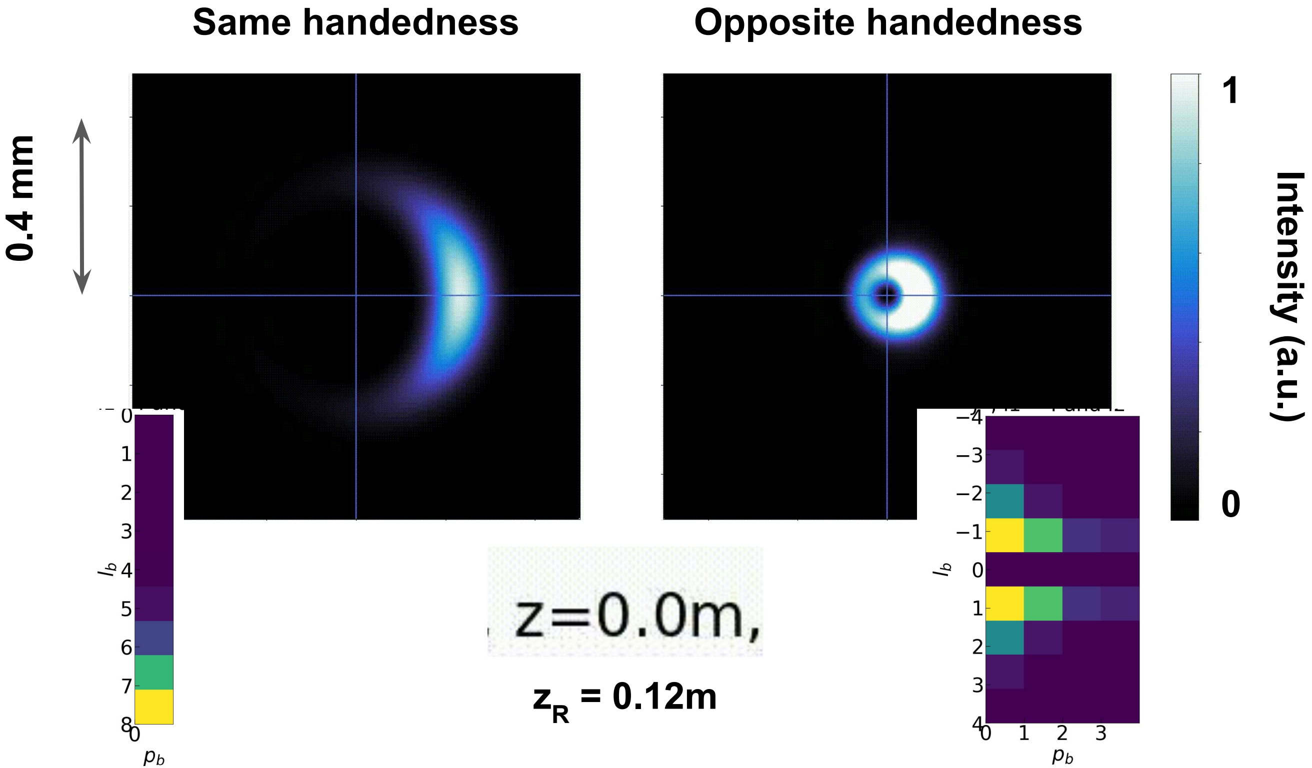
All radial number p are not null (ℓ_3, p_3, ℓ_4, p_4) at the output

(ℓ_1,ℓ_2) : $(-2,2)$		
(ℓ_3,ℓ_4)	$q = p_3 + p_4$	(p_3,p_4)
$(2,-2)$	$q=0$	(0,0)
$(1,-1)$	$q=1$	(1,0)
	$q=1$	(0,1)
(0,0)	$q=2$	(2,0)
	$q=2$	(1,1)
	$q=2$	(0,2)
$(-1,1)$	$q=1$	(0,1)
	$q=1$	(1,0)
$(-2,2)$	$q=0$	(0,0)



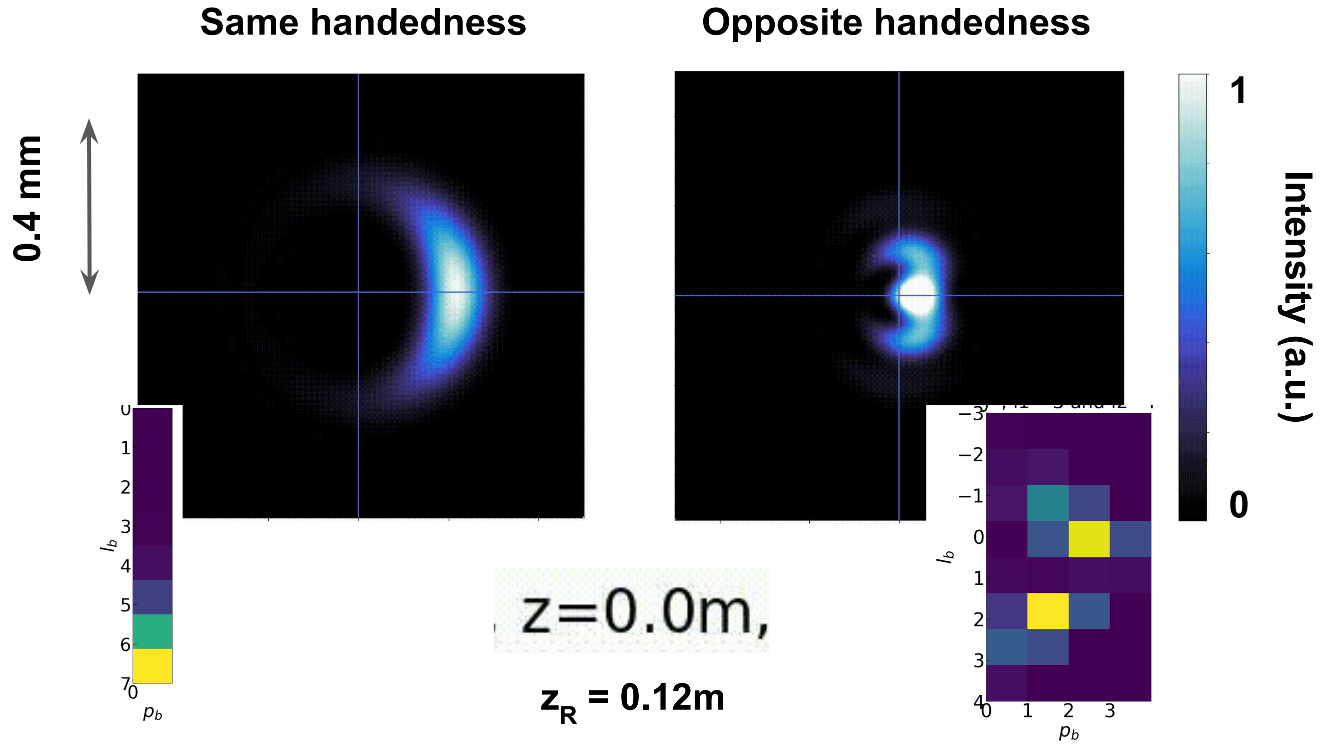
Blue vortex propagation

$\ell_1 = \pm 4$ and $\ell_2 = 4$ $(p_1 = p_2 = 0)$



Blue vortex propagation

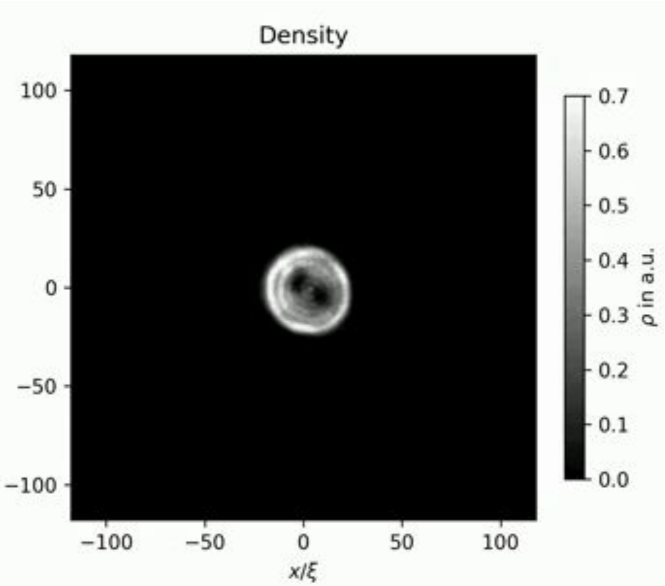
$\ell_1 = \pm 3$ and $\ell_2 = 4$ $(p_1 = p_2 = 0)$



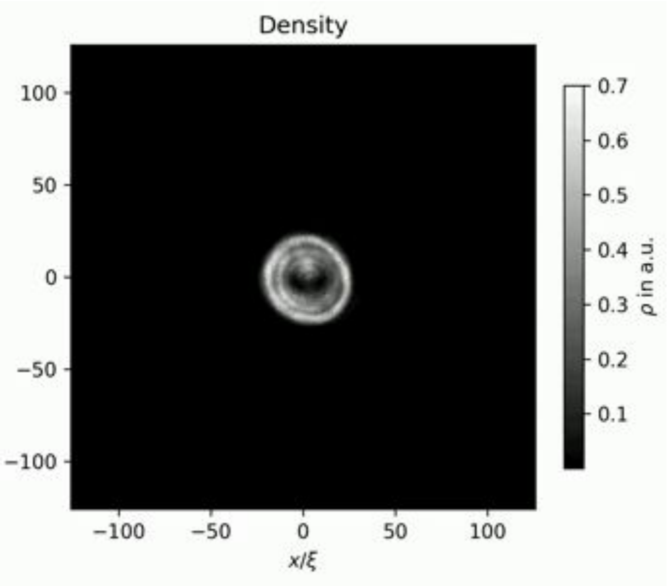
$z_R = 0.12m$

A different point of view on vortex collision

Same handedness



Opposite handedness



Conclusion

- Thanks to Angular Orbital Momentum, vortex optics are a major challenge for quantum technologies.

- Photon pair generation can be combined with OAM to study entanglement

- My 2nd year of PhD here →

Orbital Angular Momentum entanglement by Spontaneous Four Wave Mixing realized in a vapor by vortex beams of same handedness

Myrann Baker-Rasooli, Laurence Pruvost

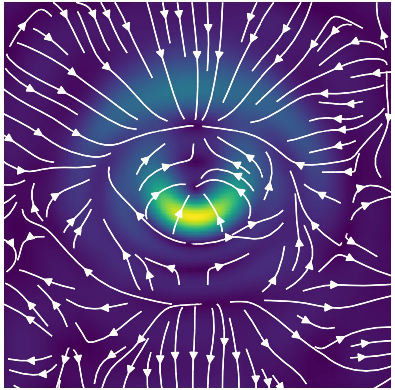
HAL Id: hal-04306168

https://hal.science/hal-04306168

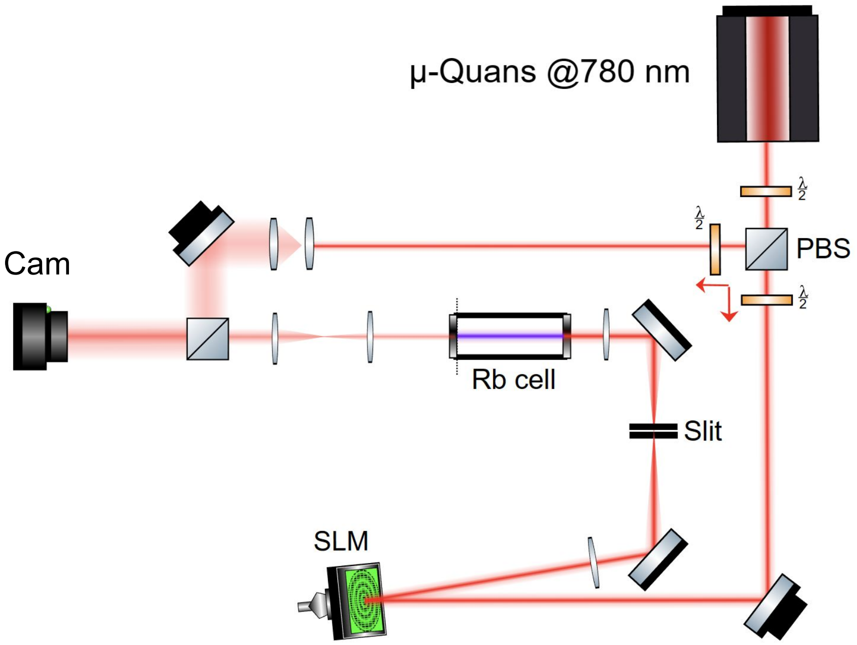
Preprint submitted on 24 Nov 2023

- Possible link with the vortex collider

My actual project:
Vortex collision and quasi-solitons generation



Vortex turbulence (150 vortices interaction)



Optical control

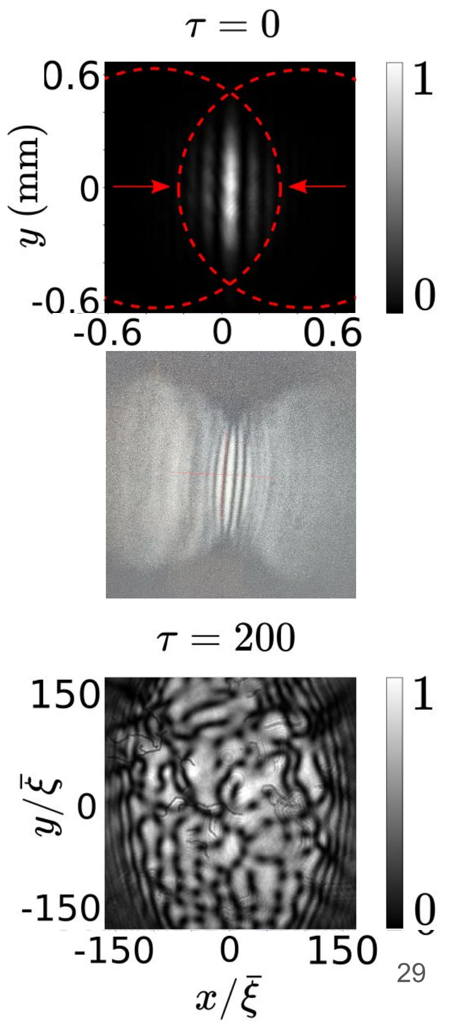
$$\Delta n = n_2 I = \frac{\Delta\theta}{k_0 L}$$

$$v \propto k_\perp = \nabla_\perp \theta$$

$$c_s = c\sqrt{\Delta n}$$

$$\xi = \frac{1}{k_0\sqrt{2\Delta n}}$$

$$\tau = L/z_{NL} = L k_0 \Delta n$$

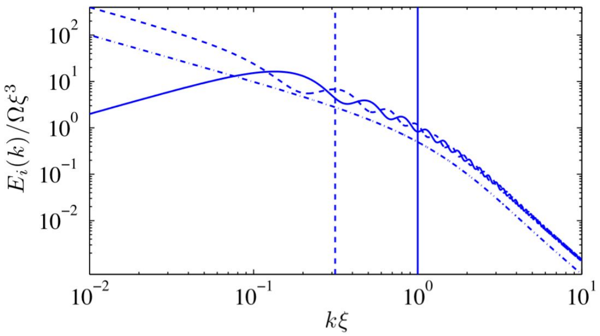


Turbulent dynamics in a two-dimensional paraxial fluid of light. Phys. Rev. A **108**, 063512, 2023 ξ = vortex radius

Microscopic dynamic (2 vortex interaction)

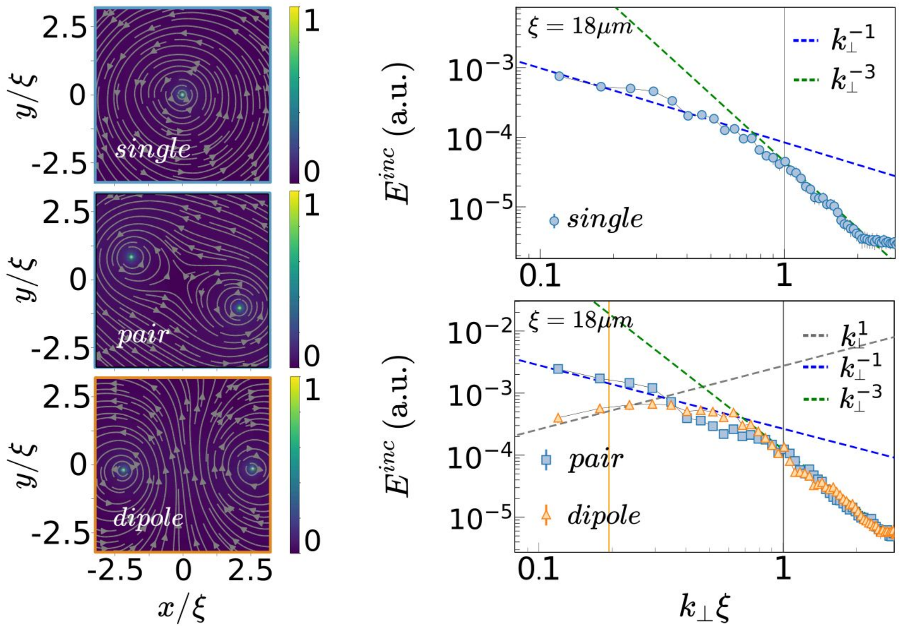
$$\boldsymbol{u}^{\boldsymbol{inc}}(\boldsymbol{r}) \rightarrow \mathbf{U}_{\boldsymbol{i}} = \mathrm{TF}[\mathbf{u}_{\boldsymbol{i}}] \rightarrow \mathrm{E}^{inc}_{kin}(\boldsymbol{k}) = \frac{m}{2}k\int_0^{2\pi}\left(|\mathrm{U}_x(\boldsymbol{k})|^2 + |\mathrm{U}_y(\boldsymbol{k})|^2\right)\mathrm{d}\Omega_k$$

Theoretical prediction:



Bradley Ashton S. and Anderson Brian P.
PRX 2012, **2**, 041001.

Experimental results:



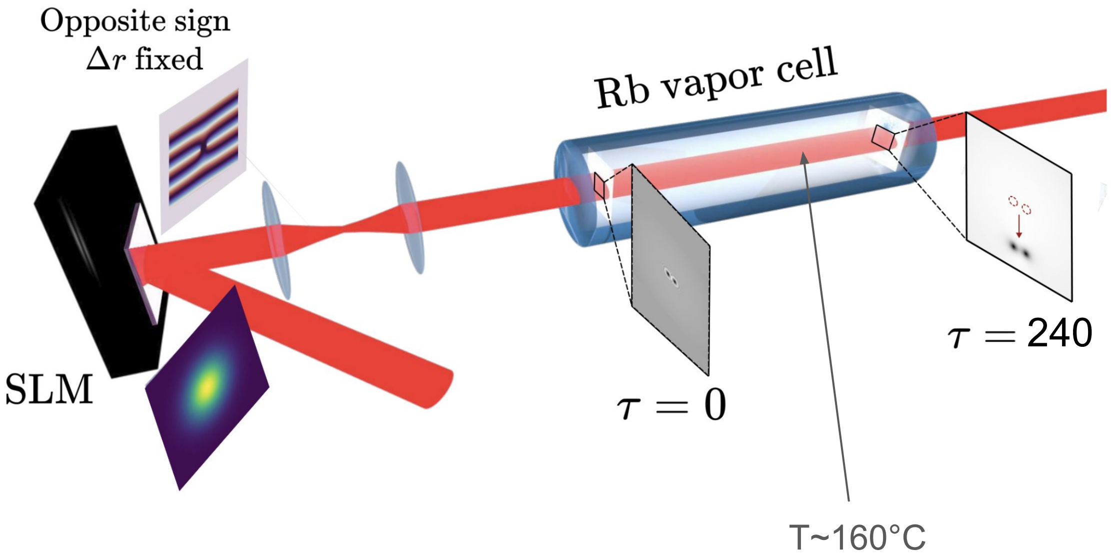
Turbulent dynamics in a two-dimensional paraxial fluid of light. Phys. Rev. A **108**, 063512, 2023

Δr = inter-vortex distance

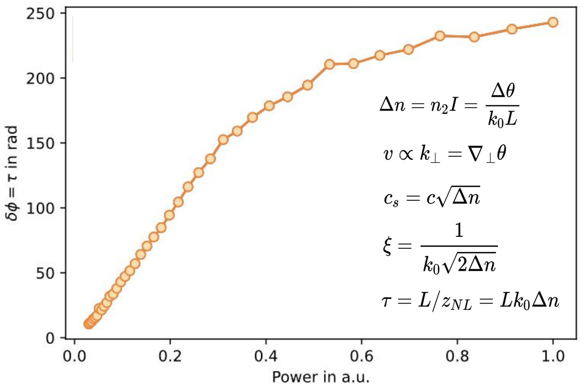
Temporal two vortex interaction

Simplified setup

We print 2 local vortex at the input of the L=20 cm Rb cell at a fixed distance Δr



Ramp τ with the laser power

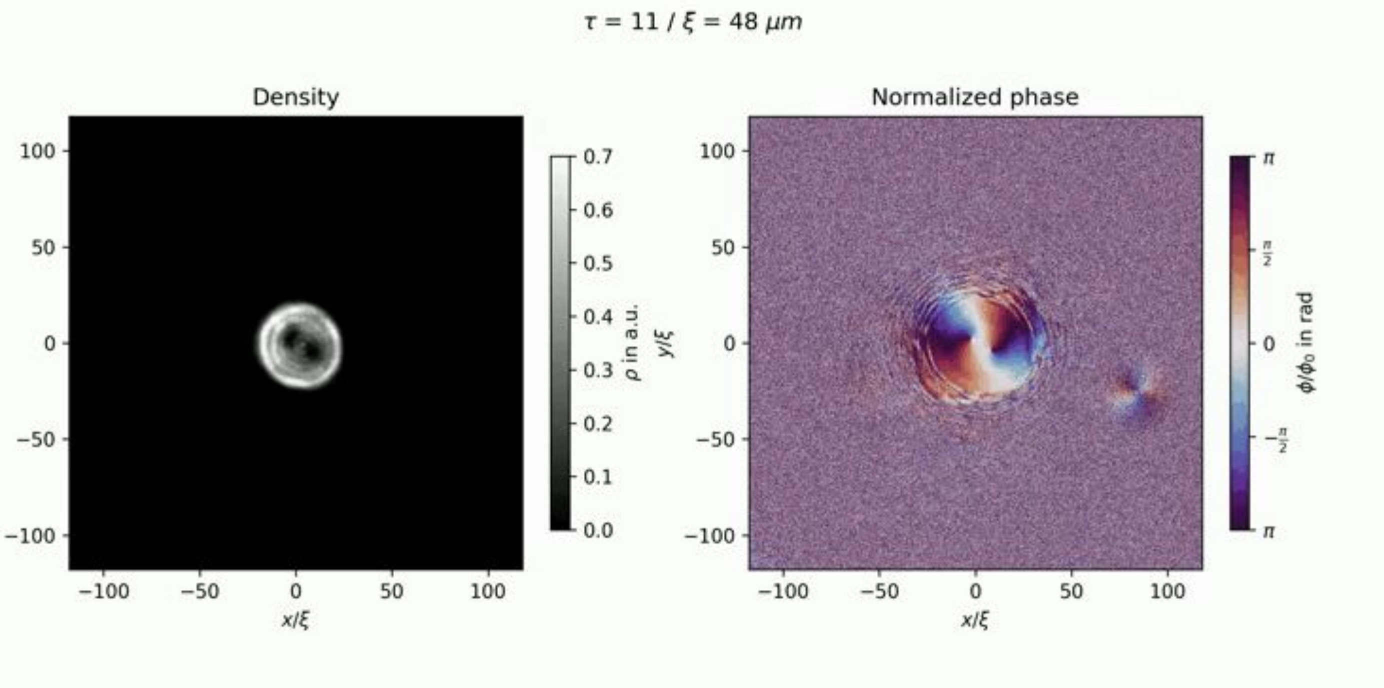


Same sign vortices

Temporal evolution: we keep $\triangle r / \xi$ constant

$\triangle r / \xi = 15$ (ξ = vortex core size; $\triangle r$ = inter-vortex distance)

$$\xi = \sqrt{\frac{L}{k_0 \tau}}$$

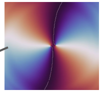


The two vortices make a half-turn

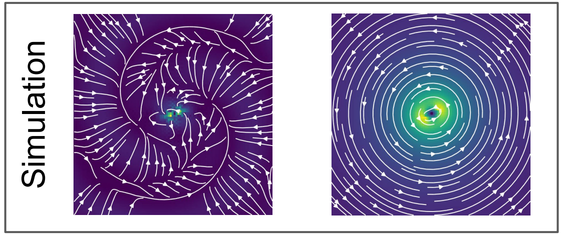
$\Delta r / \xi = 15 \begin{cases} \xi = \text{vortex core size} \\ \Delta r = \text{inter-vortex distance} \end{cases}$

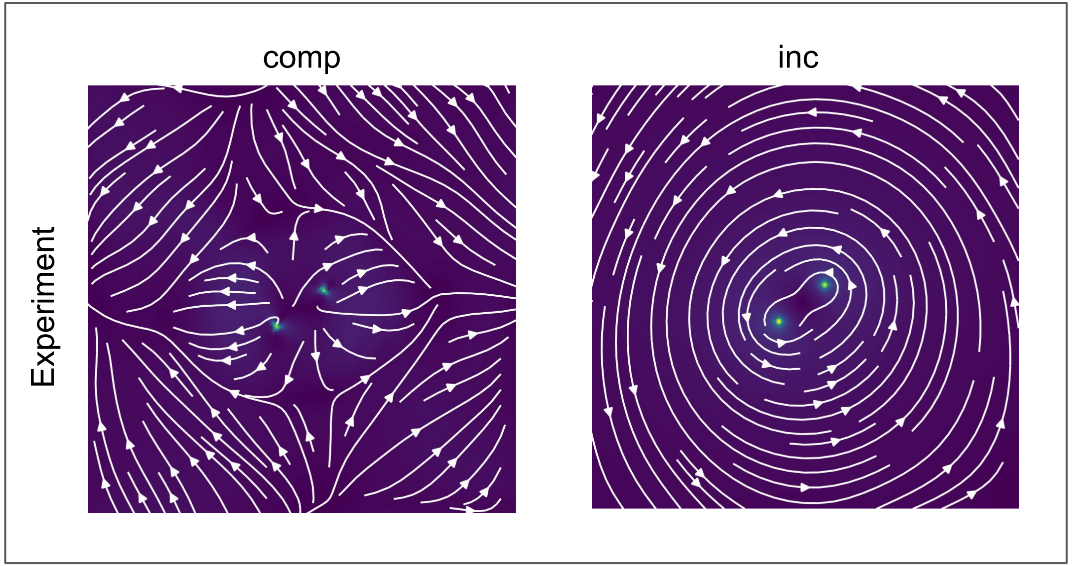
Same sign vortices

Velocity decomposition



$$\boldsymbol{v}^{tot}(\mathbf{r}) \propto \nabla_{\perp}\theta(\mathbf{r}) \longrightarrow \boldsymbol{u}^{tot}(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})}\boldsymbol{v}^{tot}(r) \longrightarrow \boldsymbol{u}^{tot}(\boldsymbol{r}) = \boldsymbol{u}^{comp}(\boldsymbol{r}) + \boldsymbol{u}^{inc}(\boldsymbol{r})$$

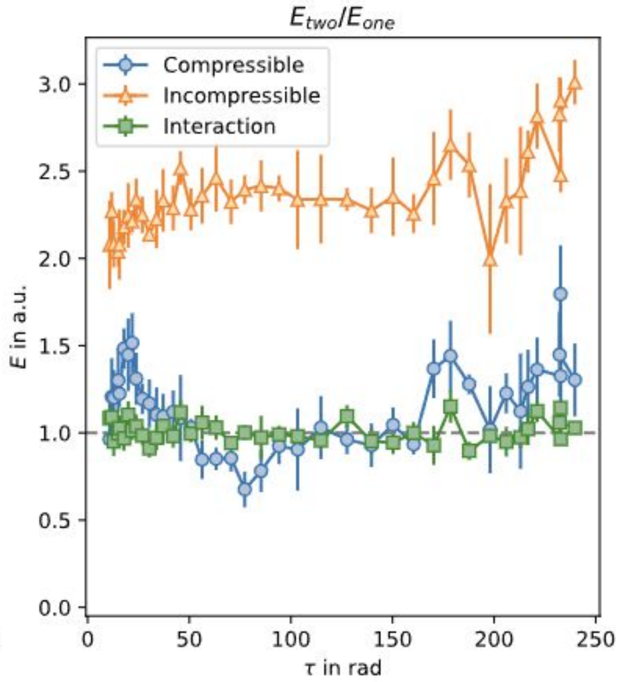




Same sign vortices

Total kinetic energy

$\triangle r$ / ξ = 15 (ξ = vortex core size
$\triangle r$ = inter-vortex distance



Energy computation

$$E_{kin} \propto \int_S |\mathbf{u}^{i,c}(\mathbf{r}_\perp)|^2 \mathrm{d}^2\mathbf{r}_\perp$$

$$E_{int} \propto \int_S \rho^2(\mathbf{r}_\perp) \mathrm{d}^2\mathbf{r}_\perp$$

Incompressible part is close to 2. Bumps when the two vortices get closer

Compressible part is flat. Bumps when the two vortices get closer

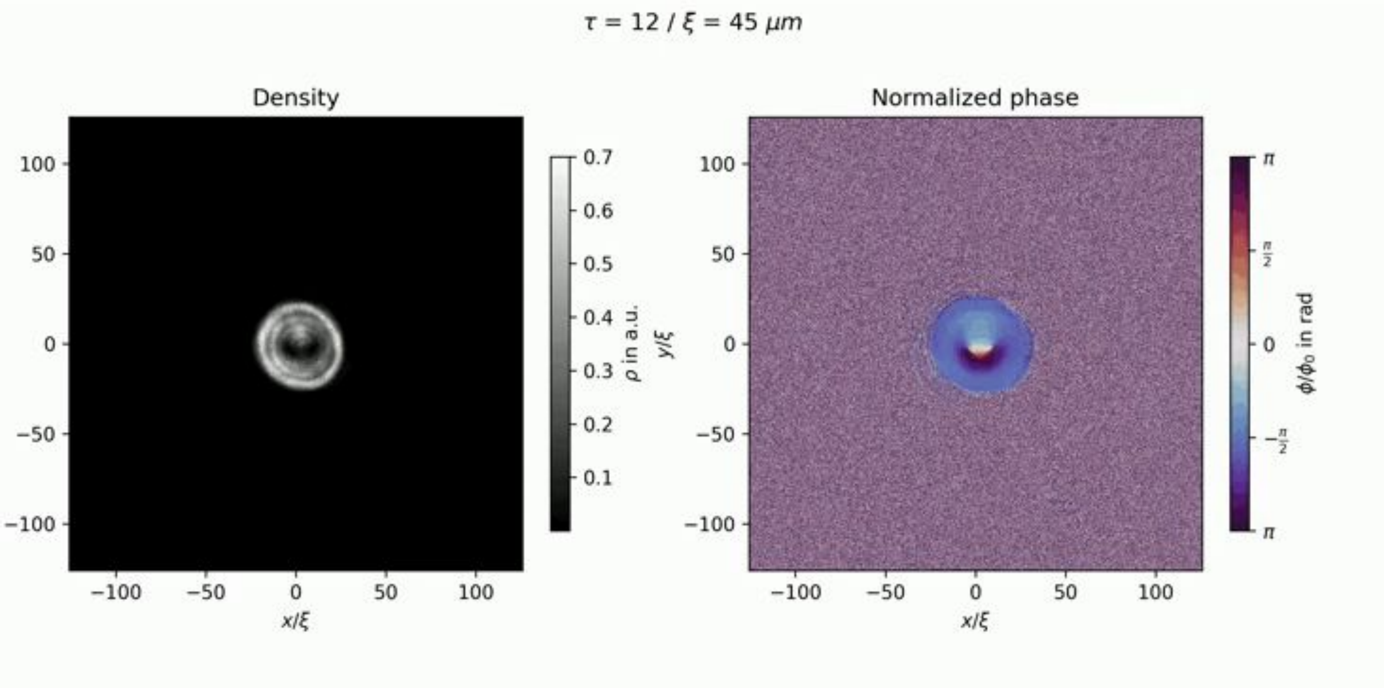
Interaction doesn't depend on the vortex configuration

Opposite sign vortices

Temporal evolution: we keep $\triangle r / \xi$ constant

$\triangle r / \xi = 15$ (ξ = vortex core size
$\triangle r$ = inter-vortex distance

$$\xi = \sqrt{\frac{L}{k_0 \tau}}$$



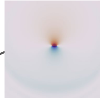
Self attraction of the dipole

The vortices pull and attract each other - Emission of a wave

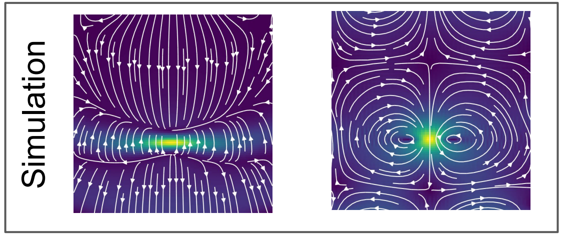
Opposite sign vortices

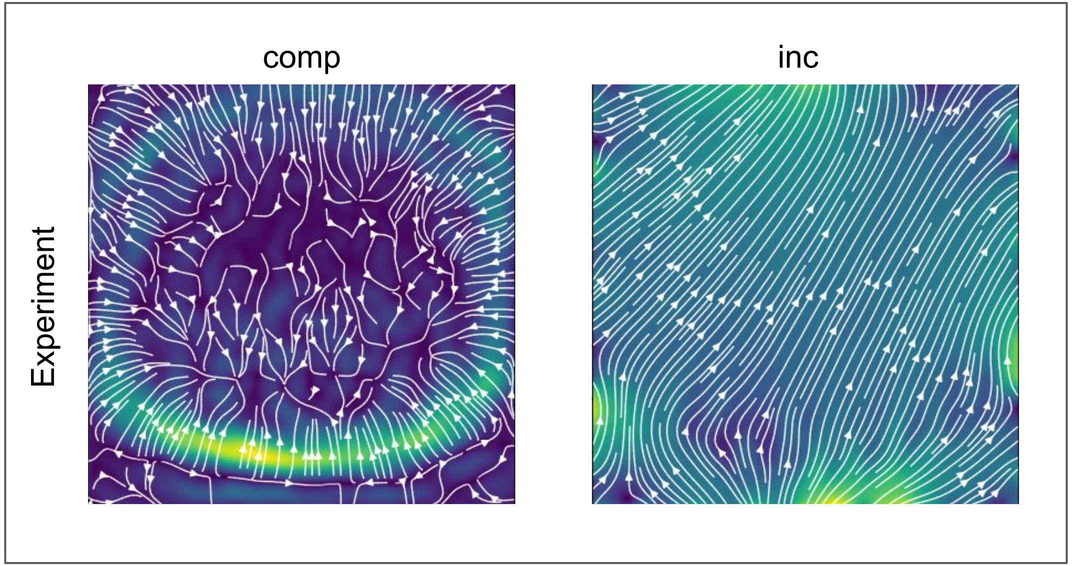
Velocity decomposition

$\triangle r / \xi = 15 \left(\begin{array}{l} \xi = \text{vortex core size} \\ \triangle r = \text{inter-vortex distance} \end{array} \right.$



$$\boldsymbol{v}^{\boldsymbol{tot}}(\mathbf{r}) \propto \nabla_{\perp}\theta(\mathbf{r}) \longrightarrow \boldsymbol{u}^{\boldsymbol{tot}}(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})}\boldsymbol{v}^{tot}(r) \longrightarrow \boldsymbol{u}^{\boldsymbol{tot}}(\boldsymbol{r}) = \boldsymbol{u}^{\boldsymbol{comp}}(\boldsymbol{r}) + \boldsymbol{u}^{\boldsymbol{inc}}(\boldsymbol{r})$$

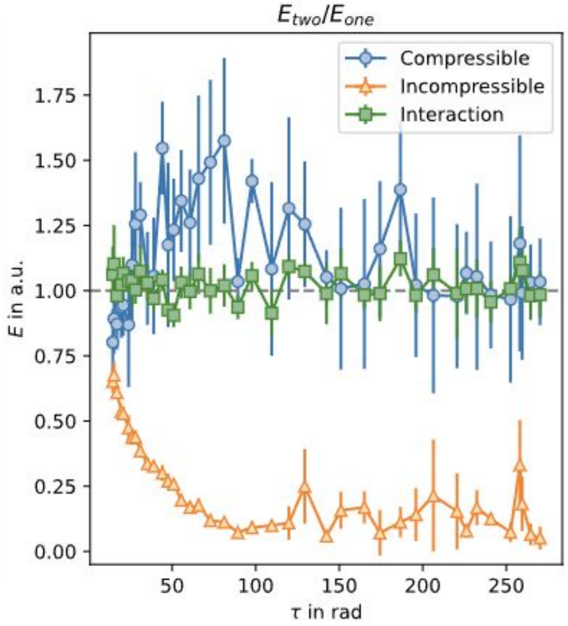




$\triangle r / \xi = 15$ (ξ = vortex core size; $\triangle r$ = inter-vortex distance)

Opposite sign vortices

Total kinetic energy



Energy computation

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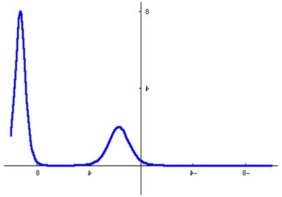
$$E_{int} \propto \int_S \rho^2(\mathbf{r}_\perp) \mathrm{d}^2\mathbf{r}_\perp$$

- Incompressible part tends to 0
- Peaks of compressible energy above 1
- Interaction doesn't depend on the vortex configuration

What is this wave

Solitons in water

Solitary waves that:
- Preserves their shape
- Constant velocity
- Immune to collision



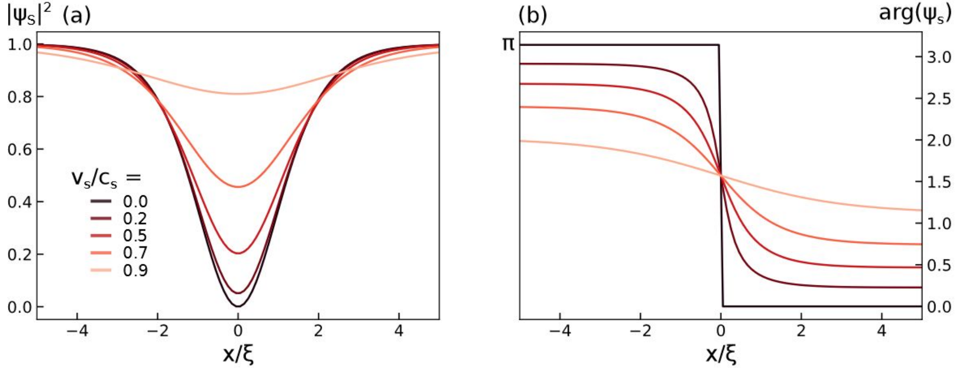
Solitons in quantum gases

Solitons are one class of solution of the GPE

$$\mathrm{i}\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x}\psi(x,t) + g|\psi(x,t)|^2\psi(x,t)$$

$$\psi_S(x - v_s t) = \sqrt{n}\left[\sqrt{1-\frac{v_s^2}{c_s^2}}\tanh\left(\frac{\mathrm{x}-\mathrm{v_s t}}{\xi\sqrt{2}}\sqrt{1-\frac{\mathrm{v_s^2}}{\mathrm{c_s^2}}}\right)+\mathrm{i}\frac{\mathrm{v_s}}{\mathrm{c_s}}\right]$$

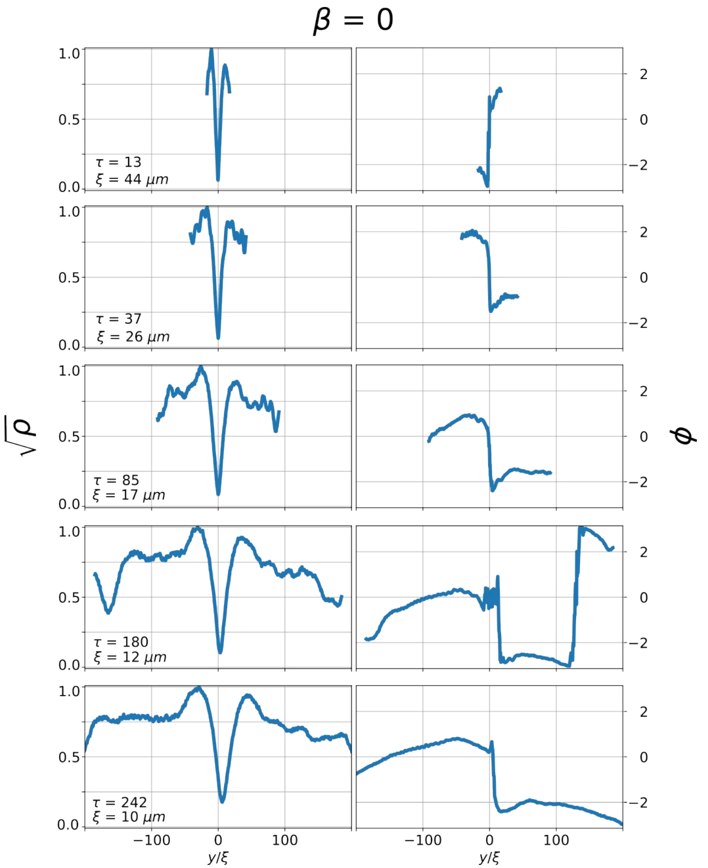
If repulsive interaction, the soliton is "dark"

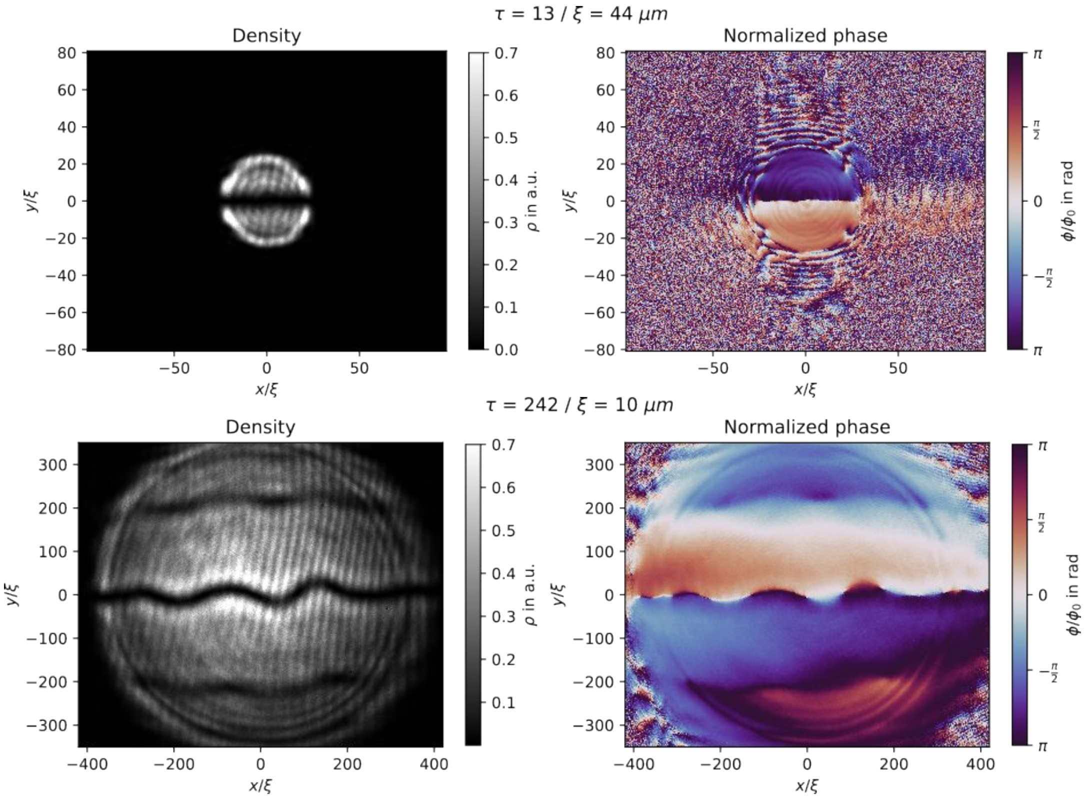


Ferdinand Claude. High resolution coherent spectroscopy of a polariton fluid : Bogoliubov modes, non equilibrium phase transitions and turbulence. Sorbonne Université, 2022

Dark soliton temporal evolution

Mach number β = v_S/c_S





Jones Roberts Solitons (JRS)

PRL 119, 150403 (2017) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2017

Observation of Two-Dimensional Localized Jones-Roberts Solitons in Bose-Einstein Condensates

Nadine Meyer,[1,2] Harry Proud,[1] Marisa Perea-Ortiz,[1] Charlotte O'Neale,[1,3] Mathis Baumert,[1,4] Michael Holynski,[1] Jochen Kronjäger,[1,5] Giovanni Barontini,[1,*] and Kai Bongs[1]
[1]*Midlands Ultracold Atom Research Centre, School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom*
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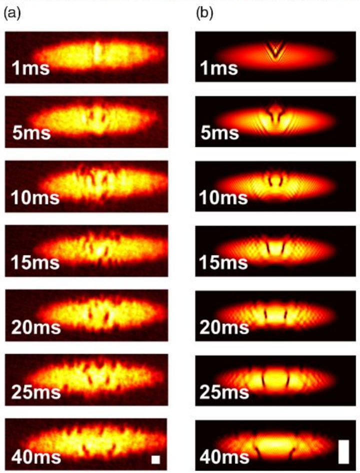
(Received 3 May 2017; published 11 October 2017)

Jones-Roberts solitons are the only known class of stable dark solitonic solutions of the nonlinear Schrödinger equation i … d elliptical shape that allows them to travel … n, we experimentally generate two-dimensio… -Einstein condensate. We monitor their dyna… y dynamic (snaking) or thermodynamic ins… stable in dimensions higher than one. Our … table solutions of the nonlinear Schrödinge… r wave physics, like energy and informatio…

DOI: 10.1103/PhysRevL…



JRS are localized propagating dark solitons

- The shape follows the Kadomtsev-Petviashvili condition
- Not stable if the speed of the wave is slow enough $\frac{v_{wave}}{c_s} < 0.43$

Two pair of opposite sign vortices generates a rarefaction pulse that is a finite solitary wave
→ Is it a JRS?
We can measure the shape and the velocity versus the background
And compare to a planar soliton

Quasi-soliton versus the background density

PHYSICAL REVIEW A **85**, 053620 (2012)

Dynamics of two-dimensional dark quasisolitons in a smoothly inhomogeneous Bose-Einstein condensate

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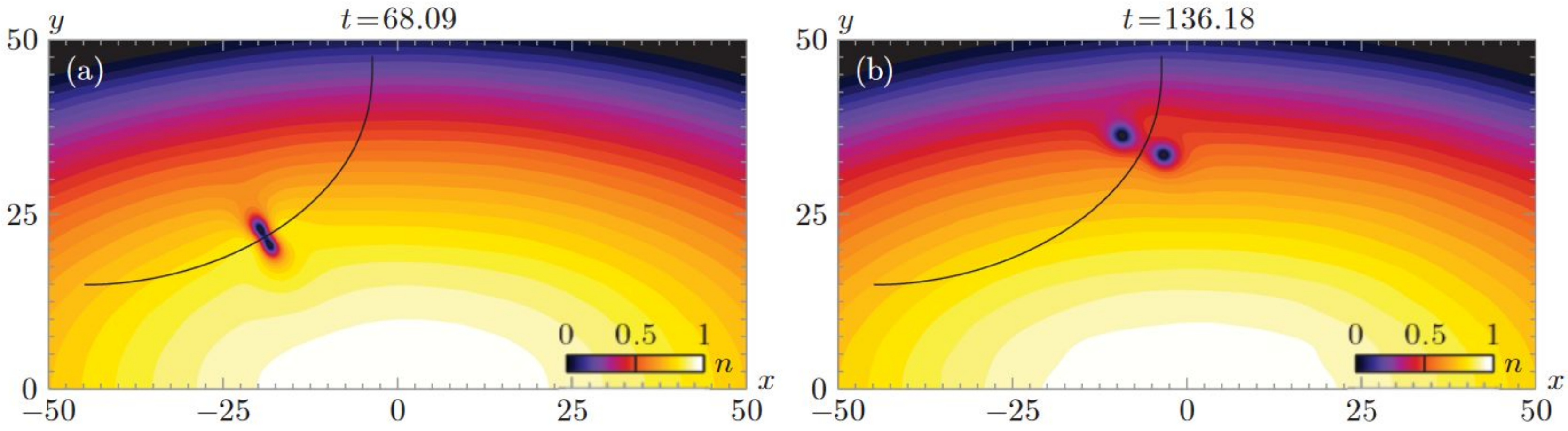

Conclusion

- **From macroscopic to microscopic study of vortex interactions**

- **The vortex collider experiments opened multiple project**

- **Pair (same sign) vortex interactions**
  - Vortex dissipation
  - Multi-charged vortex splitting (Thibaut)

- **Dipole (opposite sign) vortex interactions**
  - The dipole decay into a wave
  - Quasi-soliton versus the background density (Myrann)
  - Quasi-soliton versus the background velocity (Simon)