

Q U A N T U M L A T T I C E M O D E L

Reduced-Action Foundations and Planck-Unit Derivations

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Quantum-Q Research

December 4, 2025

Abstract

This paper develops the Planck-unit foundations of the Quantum Lattice Model (QLM), a deterministic phase-action reconstruction of natural units in which the Planck scale is defined by *action per radian*, rather than by dimensional combinations of $\{G, \hbar, c\}$. The QLM begins from the primitive rule

$$\text{one lattice tick} = 1 \text{ radian of phase}, \quad \text{action per radian} = \hbar,$$

which establishes the fundamental per-radian Planck energy

$$E_P = \frac{\hbar}{t_P}.$$

Once expressed in terms of the triplet (\hbar, ℓ_P, t_P) , all Planck quantities collapse to algebraically minimal forms. Mechanical, electromagnetic, geometric, and thermodynamic units follow directly from this single phase-action identity, with examples such as

$$m_P = \frac{\hbar t_P}{\ell_P^2}, \quad p_P = \frac{\hbar}{\ell_P}.$$

The Bohr identity

$$\hbar = m_e v_B a_0$$

arises naturally within the same framework, showing that hydrogenic structure acts as a direct phase-velocity calibration of the Planck lattice. This provides an exact algebraic bridge between atomic physics and Planck-scale quantities without the introduction of additional constants or fitted parameters.

Electromagnetic units—including the Planck charge, current, voltage, power, and the Planck impedance—emerge from the same substitutions. In the per-radian QLM normalization, the Planck impedance is

$$Z_P = \frac{Z_0}{4\pi},$$

with the corresponding full-loop (loop-aggregated) impedance given by

$$Z_P^{(\text{loop})} = 2\pi Z_P = \frac{Z_0}{2}, \quad Z_P = \frac{Z_P^{(\text{loop})}}{2\pi}.$$

Vacuum electrodynamics is therefore revealed as a derived geometric property of the lattice's phase-action structure.

Through symbolic derivations, dimensional analyses, and CODATA-validated numerical evaluations, this work establishes the complete per-radian Planck-unit system on which the broader Quantum Lattice Model is constructed. It provides the foundational reference for subsequent developments in gravitational impedance, phase-coherent dynamics, and cosmological scaling in companion papers.

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1 Introduction

Planck units are conventionally introduced as dimensional constructs formed from the constants $\{\hbar, G, c\}$, yielding expressions such as $\sqrt{\hbar c^5/G}$ and $\sqrt{\hbar c/G}$ that mix quantum, gravitational, and relativistic structure. In the Quantum Lattice Model (QLM), these square-root forms are not taken as primitive. Instead, all Planck quantities arise from a deterministic phase–action framework in which each lattice tick advances the physical phase by exactly one radian and transports precisely one quantum of reduced action:

$$\text{phase advance per tick} = 1 \text{ radian}, \quad \text{action per radian} = \hbar. \quad (1.1)$$

This postulate immediately defines the fundamental *per-radian* Planck energy,

$$E_P = \frac{\hbar}{t_P}. \quad (1.2)$$

The full-cycle value $h = 2\pi\hbar$ plays no foundational role in what follows; all derivations use the reduced, per-radian form exclusively.

Primitive spacings and exact SI structure. The QLM treats the Planck time t_P and Planck length ℓ_P as the primitive spacetime increments of a discrete four-dimensional lattice. These increments are linked by the exact SI identity

$$c = \frac{\ell_P}{t_P}. \quad (1.3)$$

All numerical values in this paper use CODATA 2022 and the post-2019 SI, in which h , e , k_B , and c are defined constants. The electromagnetic constants ϵ_0 , μ_0 , and Z_0 inherit their uncertainties solely through the fine-structure constant α .

Unified origin of Planck, electromagnetic, and hydrogenic structure. Once $E_P t_P = \hbar$ and $c = \ell_P/t_P$ are taken as primitive, Planck-scale action, vacuum electrodynamics, and the structure of the hydrogen atom all follow from the same phase–action relation. The Bohr identity

$$\hbar = m_e v_B a_0 \quad (1.4)$$

connects the reduced action quantum directly to the mechanical angular momentum of the hydrogen ground state. Combined with $c = \ell_P/t_P$, it gives the purely kinematic form of the fine-structure constant,

$$\alpha = \frac{v_B}{c}. \quad (1.5)$$

Within this unified framework, the Planck charge takes the equivalent set of identities

$$q_P^2 = \frac{4\pi\hbar}{Z_0} = \frac{2\pi\hbar}{Z_P^{(\text{loop})}} = \frac{\hbar}{Z_P} = \hbar Y_P = 4\pi\epsilon_0 \hbar c, \quad (1.6)$$

where the per-radian Planck impedance is

$$Z_P = \frac{Z_0}{4\pi}, \quad (1.7)$$

and the corresponding full-loop (loop-aggregated) impedance is

$$Z_P^{(\text{loop})} = 2\pi Z_P = \frac{Z_0}{2}, \quad Z_P = \frac{Z_P^{(\text{loop})}}{2\pi}. \quad (1.8)$$

These identities naturally assemble into the electromagnetic quartet

$$\{V_P, I_P, P_P, Z_P\}, \quad (1.9)$$

which satisfies the QLM closures

$$P_P = V_P I_P = \frac{V_P^2}{Z_P} = I_P^2 Z_P, \quad (1.10)$$

with the understanding that loop-aggregated power relations involve $Z_P^{(\text{loop})} = 2\pi Z_P$ when full 2π phase cycles are considered.

Scope of this work. This paper develops the complete Planck-unit foundation of the Quantum Lattice Model. Each canonical Planck quantity is first expressed in its traditional square-root form and then collapsed to its minimal QLM representation in terms of the primitive triplet (\hbar, ℓ_P, t_P) . All results include CODATA-verified numerical evaluations and emphasize algebraic consistency across mechanical, electromagnetic, geometric, and hydrogenic sectors. The gravitational, electromagnetic, and cosmological predictions that follow from these foundations are presented in companion papers.

2 Constants and Conventions (CODATA 2022)

Table 1: CODATA 2022 constants used throughout this paper. Exact SI values indicated.

Quantity	Symbol	Value (CODATA 2022)	Source
Speed of light (exact)	c	$2.997\,925 \times 10^8 \text{ m s}^{-1}$	[26]
Planck constant (exact)	h	$6.626\,070 \times 10^{-34} \text{ J s}$	[27]
Reduced Planck constant (exact)	\hbar	$1.054\,572 \times 10^{-34} \text{ J s}$	[27]
Elementary charge (exact)	e	$1.602\,177 \times 10^{-19} \text{ C}$	[26]
Boltzmann constant (exact)	k_B	$1.380\,649 \times 10^{-23} \text{ J K}^{-1}$	[26]
Fine-structure constant	α	$7.297\,353 \times 10^{-3}$	[28, 12]
Planck time	t_P	$5.391\,247 \times 10^{-44} \text{ s}$	[29]
Planck length	ℓ_P	$1.616\,255 \times 10^{-35} \text{ m}$	[30]
Electron mass	m_e	$9.109\,384 \times 10^{-31} \text{ kg}$	[31]
Bohr radius	a_0	$5.291\,772 \times 10^{-11} \text{ m}$	[32]
Vacuum impedance	Z_0	$3.767\,303 \times 10^2 \Omega$	[33]
Vacuum permeability	μ_0	$1.256\,637 \times 10^{-6} \text{ N A}^{-2}$	[34, 12]
Vacuum permittivity	ϵ_0	$8.854\,188 \times 10^{-12} \text{ F m}^{-1}$	[35, 12]
Newtonian gravitational constant	G	$6.674\,300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	[37, 12]

3 Action Constants

The Quantum Lattice Model begins from the foundational identity

$$E_P t_P = \hbar, \quad (3.1)$$

together with the Bohr angular-momentum relation

$$\hbar = m_e v_B a_0. \quad (3.2)$$

In the QLM, *all* action is therefore per-radian action, and the Planck energy is

$$E_P = \frac{\hbar}{t_P}. \quad (3.3)$$

The full-loop action $h = 2\pi\hbar$ is derived and non-primitive, included only when converting to SI forms.

QLM Lattice Forms for \hbar (Per-Radian Action)

Using the primitive per-radian identities

$$E_P = \frac{\hbar}{t_P}, \quad f_P = \frac{1}{t_P}, \quad (3.4)$$

and introducing the derived full-loop angular frequency

$$\omega_P = 2\pi f_P = \frac{2\pi}{t_P}, \quad (3.5)$$

we obtain:

$$\hbar = E_P t_P, \quad (3.6)$$

$$\hbar = \frac{E_P}{f_P}, \quad (3.7)$$

$$\hbar = \frac{E_P}{\omega_P} 2\pi. \quad (3.8)$$

An equivalent electromagnetic form follows from

$$q_P^2 = 4\pi\epsilon_0\hbar c, \quad Z_0 = \frac{1}{\epsilon_0 c}, \quad (3.9)$$

giving

$$\hbar = \frac{Z_0}{4\pi} q_P^2 = Z_P q_P^2 = \frac{Z_P^{(\text{loop})}}{2\pi} q_P^2, \quad (3.10)$$

where the per-radian Planck impedance is

$$Z_P = \frac{Z_0}{4\pi}, \quad (3.11)$$

and the loop-aggregated impedance is

$$Z_P^{(\text{loop})} = 2\pi Z_P = \frac{Z_0}{2}. \quad (3.12)$$

Bohr Form (Explicit)

Hydrogen provides the direct angular-momentum interpretation:

$$\boxed{\hbar = m_e v_B a_0} \quad (3.13)$$

This identity is fully consistent with all QLM action forms.

Numeric Validations (CODATA 2022)

Per-radian identity:

$$\begin{aligned} \hbar &= E_P t_P = (1.956\,081 \times 10^9 \text{ J})(5.391\,247 \times 10^{-44} \text{ s}) \\ &= 1.054\,572 \times 10^{-34} \text{ Js.} \end{aligned} \quad (3.14)$$

Bohr form:

$$\begin{aligned} \hbar &= m_e v_B a_0 \\ &= (9.109\,384 \times 10^{-31} \text{ kg})(2.187\,691 \times 10^6 \text{ m s}^{-1})(5.291\,772 \times 10^{-11} \text{ m}) \\ &= 1.054\,572 \times 10^{-34} \text{ Js.} \end{aligned} \quad (3.15)$$

Impedance–charge form (QLM).

$$\begin{aligned}
\hbar &= \frac{Z_P^{(\text{loop})}}{2\pi} q_P^2 \\
&= \frac{1.883\,652 \times 10^2 \Omega}{2\pi} (3.517\,673 \times 10^{-36} \text{ C}^2) \\
&= 1.054\,572 \times 10^{-34} \text{ J s.}
\end{aligned} \tag{3.16}$$

Frequency pairing:

$$\begin{aligned}
\hbar &= \frac{E_P}{\omega_P} 2\pi \\
&= \frac{1.956\,081 \times 10^9 \text{ J}}{1.165\,442 \times 10^{44} \text{ s}^{-1}} (2\pi) \\
&= 1.054\,572 \times 10^{-34} \text{ J s.}
\end{aligned} \tag{3.17}$$

Summary

$$\hbar = E_P t_P = \frac{E_P}{f_P} = \frac{2\pi E_P}{\omega_P} = \frac{Z_0}{4\pi} q_P^2 = Z_P q_P^2 = \frac{Z_P^{(\text{loop})}}{2\pi} q_P^2 = m_e v_B a_0$$

(3.18)

$$h = 2\pi\hbar$$

(3.19)

These identities constitute the complete \hbar -based action structure of the Quantum Lattice Model. All subsequent Planck-unit derivations follow directly from the primitive per-radian identity $E_P = \hbar/t_P$.

4 Gravitational Impedance Equivalence

Gravity in the QLM is expressed entirely in terms of the reduced action quantum \hbar carried per radian of phase advance. Therefore every correct representation of the Newtonian gravitational constant G must collapse to the same (ℓ_P, t_P, \hbar) lattice primitives:

$$E_P = \frac{\hbar}{t_P}, \quad c = \frac{\ell_P}{t_P}. \tag{4.1}$$

The fundamental QLM definition of G is

$$G = \frac{\ell_P^5}{t_P^3 \hbar}$$

(4.2)

and all equivalent forms of the gravitational constant must reduce to Eq. (4.2) when expressed in reduced-action Planck units.

Equivalent Representations of G

Beginning with the standard Planck-time definition

$$t_P^2 = \frac{\hbar G}{c^5}, \quad (4.3)$$

solving for G yields

$$G = \frac{c^5 t_P^2}{\hbar}. \quad (4.4)$$

Bohr-anchored temporal form. Using the hydrogenic identity

$$\hbar = m_e v_B a_0, \quad (4.5)$$

the same expression becomes

$$G = \frac{c^5 t_P^2}{m_e v_B a_0}, \quad (4.6)$$

Pure lattice (length–time) form. Eliminating $c = \ell_P/t_P$ in Eq. (4.4) gives

$$\begin{aligned} G &= \frac{\left(\frac{\ell_P}{t_P}\right)^5 t_P^2}{\hbar} \\ &= \boxed{\frac{\ell_P^5}{t_P^3 \hbar}}, \end{aligned} \quad (4.7)$$

Equivalence of all forms. From Eq. (4.4),

$$\frac{c^5 t_P^2}{\hbar} = \frac{\left(\frac{\ell_P}{t_P}\right)^5 t_P^2}{\hbar} = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (4.8)$$

and since $\hbar = m_e v_B a_0$, the Bohr-anchored form collapses identically.

Numerical Verification (CODATA 2022)

Temporal form.

$$\begin{aligned} G &= \frac{c^5 t_P^2}{\hbar} = \frac{(2.997925 \times 10^8)^5 (5.391247 \times 10^{-44})^2}{1.054572 \times 10^{-34}} \\ &= 6.674300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \end{aligned} \quad (4.9)$$

Bohr-anchored form.

$$\begin{aligned} G &= \frac{c^5 t_P^2}{m_e v_B a_0} \\ &= \frac{(2.997925 \times 10^8)^5 (5.391247 \times 10^{-44})^2}{(9.109384 \times 10^{-31})(2.187691 \times 10^6)(5.291772 \times 10^{-11})} \\ &= 6.674300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}. \end{aligned} \quad (4.10)$$

Pure lattice form.

$$\begin{aligned}
G &= \frac{\ell_P^5}{t_P^3 \hbar} \\
&= \frac{(1.616\,255 \times 10^{-35})^5}{(5.391\,247 \times 10^{-44})^3 (1.054\,572 \times 10^{-34})} \\
&= 6.674\,300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}.
\end{aligned} \tag{4.11}$$

Equivalence of GR, Schwarzschild, and Planck–Lattice Impedance

The gravitational redshift around a static mass M in GR is

$$Z_g(r) = \left(1 - \frac{2GM}{c^2r}\right)^{-1/2}. \tag{4.12}$$

Step 1: Schwarzschild-radius form.

$$r_s = \frac{2GM}{c^2}, \tag{4.13}$$

so

$$Z_g(r) = \left(1 - \frac{r_s}{r}\right)^{-1/2}. \tag{4.14}$$

Step 2: Convert the GR form using QLM Planck units. Using Eq. (4.2) and $c = \ell_P/t_P$,

$$\begin{aligned}
Z_g(r) &= \left[1 - \frac{2M}{c^2r} \left(\frac{\ell_P^5}{t_P^3 \hbar}\right)\right]^{-1/2} \\
&= \left[1 - 2 \frac{M\ell_P^3}{\hbar t_P r}\right]^{-1/2}.
\end{aligned} \tag{4.15}$$

To verify consistency:

$$\begin{aligned}
\frac{G}{c^2} &= \frac{G}{(\ell_P/t_P)^2} = G \frac{t_P^2}{\ell_P^2} \\
&= \left(\frac{\ell_P^5}{t_P^3 \hbar}\right) \left(\frac{t_P^2}{\ell_P^2}\right) \\
&= \frac{\ell_P^3}{\hbar t_P}.
\end{aligned} \tag{4.16}$$

Step 3: Collapse to the QLM Planck-mass expression. In the QLM,

$$m_P = \frac{\hbar t_P}{\ell_P^2}, \quad \frac{1}{m_P} = \frac{\ell_P^2}{\hbar t_P}. \quad (4.17)$$

Insert into the impedance definition:

$$Z_g(r) = \left[1 - 2 \left(\frac{M}{m_P} \right) \left(\frac{\ell_P}{r} \right) \right]^{-1/2}. \quad (4.18)$$

Inside the bracket:

$$\left(\frac{M}{m_P} \right) \left(\frac{\ell_P}{r} \right) = \frac{M \ell_P^3}{\hbar t_P r}. \quad (4.19)$$

Substituting back recovers

$$Z_g(r) = \left[1 - 2 \frac{M \ell_P^3}{\hbar t_P r} \right]^{-1/2}, \quad r_s = 2 \frac{M \ell_P^3}{\hbar t_P}. \quad (4.20)$$

Final Equivalence.

$$Z_g(r) = \left(1 - \frac{2GM}{c^2 r} \right)^{-1/2} = \left(1 - \frac{r_s}{r} \right)^{-1/2} = \left(1 - 2 \frac{M \ell_P^3}{\hbar t_P r} \right)^{-1/2}. \quad (4.21)$$

Thus the QLM gravitational impedance is not a modification of general relativity, but a re-expression of the same redshift structure entirely in \hbar -based Planck units.

5 Planck Lattice Foundations

5.1 Planck Time

The Planck time t_P is the fundamental temporal increment of the Quantum Lattice Model (QLM): one lattice tick corresponds to one radian of coherent phase evolution. In conventional dimensional analysis it is introduced through the reduced-action expression

$$t_P = \sqrt{\frac{\hbar G}{c^5}}, \quad (5.1)$$

but in the QLM this is *not* a definition. Instead, t_P is taken as the primitive temporal spacing of the lattice, and spatial and temporal increments are locked by the exact identity

$$c = \frac{\ell_P}{t_P}. \quad (5.2)$$

Spatial–Temporal Ratio of the Planck Lattice. A fundamental structural identity of the QLM is the fixed ratio between one spatial lattice link ℓ_P and one temporal tick t_P . A single phase hop per tick propagates at the invariant wave speed c , forcing the exact relation

$$\boxed{\frac{\ell_P}{t_P} = c \iff \frac{t_P}{\ell_P} = \frac{1}{c}} \quad (5.3)$$

This identity ensures that every null propagation step advances exactly one spatial unit per temporal tick, defining the causal boundary of the discrete spacetime. Any slower advance corresponds to massive or bound configurations. Thus Lorentz symmetry and the causal cone arise deterministically from the lattice update rule.

QLM form. From Eq. (5.2),

$$t_P = \frac{\ell_P}{c}, \quad (5.4)$$

showing that t_P is simply the temporal projection of the Planck phase-update cycle.

Collapse of the square-root form. To verify consistency, we show that the reduced-action expression (5.1) collapses exactly to the QLM identity (5.4). Using the reduced Planck relations

$$E_P = \frac{\hbar}{t_P}, \quad \ell_P^2 = \frac{\hbar G}{c^3}, \quad (5.5)$$

rewrite Eq. (5.1) as

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = \sqrt{\frac{(\ell_P^2 c^3)}{c^5}} \quad (5.6)$$

$$= \sqrt{\frac{\ell_P^2}{c^2}} \quad (5.7)$$

$$= \frac{\ell_P}{c}. \quad (5.8)$$

Thus the conventional form and the QLM form are strictly identical.

Bohr-anchored collapse. Using the hydrogenic identity

$$\hbar = m_e v_B a_0, \quad (5.9)$$

and the QLM gravitational relation

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (5.10)$$

Eq. (5.1) becomes

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \quad (5.11)$$

$$= \sqrt{\frac{\hbar(\ell_P^5/(t_P^3\hbar))}{c^5}} = \sqrt{\frac{\ell_P^5}{t_P^3 c^5}} \quad (5.12)$$

$$= \sqrt{\frac{\ell_P^5}{t_P^3(\ell_P^5/t_P^5)}} = \sqrt{t_P^2} = t_P, \quad (5.13)$$

confirming that the hydrogenic, reduced-action, and QLM forms are fully equivalent.

Numeric evaluation (CODATA 2022).

$$\begin{aligned} t_P &= \sqrt{\frac{(1.054572 \times 10^{-34} \text{ Js})(6.674300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2})}{(2.997925 \times 10^8 \text{ m s}^{-1})^5}} \\ &= 5.391247 \times 10^{-44} \text{ s.} \end{aligned} \quad (5.14)$$

$$\begin{aligned} t_P &= \frac{1.616255 \times 10^{-35} \text{ m}}{2.997925 \times 10^8 \text{ m s}^{-1}} \\ &= 5.391247 \times 10^{-44} \text{ s.} \end{aligned} \quad (5.15)$$

Thus, the gravitational square-root derivation reduces to the QLM lattice identity

$$t_P = \frac{\ell_P}{c}, \quad (5.16)$$

demonstrating the complete internal consistency of the QLM temporal foundation.

5.2 Planck Length

The Planck length ℓ_P is the fundamental spatial increment of the Quantum Lattice Model. In the conventional formulation it is introduced through

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}, \quad (5.17)$$

but in the QLM it is a *primitive* lattice spacing, linked directly to the Planck time by the exact lattice-velocity identity

$$c = \frac{\ell_P}{t_P}. \quad (5.18)$$

Thus the QLM definition is simply

$$\ell_P = c t_P, \quad (5.19)$$

identifying the spatial increment as the spatial projection of a single phase-update tick.

Collapse of the square-root form. To confirm consistency, we show that the reduced-action form (5.17) collapses exactly to the QLM identity (5.19). Using

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (5.20)$$

$$t_P = \frac{\ell_P}{c}, \quad (5.21)$$

rewrite Eq. (5.17) as

$$\ell_P = \sqrt{\frac{\hbar \left(\frac{\ell_P^5}{t_P^3 \hbar} \right)}{c^3}} \quad (5.22)$$

$$= \sqrt{\frac{\ell_P^5}{t_P^3 c^3}} = \sqrt{\frac{\ell_P^5}{t_P^3 (\ell_P^3 / t_P^3)}} = \sqrt{\frac{\ell_P^5 t_P^3}{\ell_P^3 t_P^3}} \quad (5.23)$$

$$= \sqrt{\ell_P^2} = \ell_P. \quad (5.24)$$

Thus the conventional square-root expression is fully consistent with the QLM identity $\ell_P = c t_P$.

Numeric evaluation (CODATA 2022). Using the QLM identity $\ell_P = c t_P$,

$$\begin{aligned} \ell_P &= (2.997925 \times 10^8 \text{ m s}^{-1})(5.391247 \times 10^{-44} \text{ s}) \\ &= 1.616255 \times 10^{-35} \text{ m}, \end{aligned} \quad (5.25)$$

in exact agreement with the CODATA 2022 recommended value [30].

5.3 Planck Energy

In the Quantum Lattice Model the Planck energy is not defined through dimensional square roots, but as the energy associated with a single radian of coherent phase advance per lattice tick. Since each tick carries one quantum of action \hbar , the fundamental Planck energy follows immediately from

$$E_P = \frac{\hbar}{t_P}. \quad (5.26)$$

This is the only physically fundamental energy scale in the QLM. All other Planck energies (including traditional 2π versions) are derived or interpretive rather than primitive.

Bohr–QLM form. Using the Bohr identity $\hbar = m_e v_B a_0$ gives

$$E_P = \frac{m_e v_B a_0}{t_P}, \quad (5.27)$$

showing that the hydrogenic angular momentum provides a direct mechanical realization of the QLM phase–action quantum.

Numeric evaluation (CODATA 2022).

$$E_P = \frac{1.054\,572 \times 10^{-34} \text{ J s}}{5.391\,247 \times 10^{-44} \text{ s}} = 1.956\,081 \times 10^9 \text{ J.} \quad (5.28)$$

This value matches the reduced per-radian Planck energy determined from CODATA 2022.

5.4 Square-Root Collapse of the Planck Energy

The conventional reduced-action definition is

$$E_P = \sqrt{\frac{\hbar c^5}{G}}. \quad (5.29)$$

Using the QLM gravitational identity

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (5.30)$$

$$c = \frac{\ell_P}{t_P}, \quad (5.31)$$

we compute

$$E_P = \sqrt{\frac{\hbar \left(\frac{\ell_P^5}{t_P^3} \right)}{\frac{\ell_P^5}{t_P^3 \hbar}}} \quad (5.32)$$

$$= \sqrt{\frac{\hbar^2 \ell_P^5 t_P^3}{\ell_P^5 t_P^5}} \quad (5.33)$$

$$= \sqrt{\frac{\hbar^2}{t_P^2}} = \frac{\hbar}{t_P}. \quad (5.34)$$

Thus the conventional square-root definition collapses exactly to the QLM per-radian identity.

5.5 Energy–Mass Relation Collapse

The relativistic identity $E = mc^2$, together with the QLM Planck mass

$$m_P = \frac{\hbar t_P}{\ell_P^2}, \quad (5.35)$$

$$c = \frac{\ell_P}{t_P}, \quad (5.36)$$

yields

$$E_P = m_P c^2 = \frac{\hbar t_P}{\ell_P^2} \frac{\ell_P^2}{t_P^2}, \quad (5.37)$$

$$= \frac{\hbar}{t_P}, \quad (5.38)$$

reproducing again the QLM definition.

Summary.

$$E_P = \frac{\hbar}{t_P} \quad (5.39)$$

All traditional Planck-energy definitions collapse exactly to this per-radian QLM form.

6 Planck Frequency

In the Quantum Lattice Model, each lattice tick advances the coherent phase by exactly *one radian*. Therefore the fundamental temporal cycle rate is the **per-radian frequency**

$$f_P = \frac{1}{t_P} \quad (6.1)$$

which counts radians per second. This is the only primitive temporal frequency in the QLM.

The angular frequency is not fundamental; it is a derived quantity introduced only when discussing angular momentum and full-loop action.

Energy–Frequency Correspondence

Each radian of phase carries \hbar of action. Thus the fundamental (per-radian) Planck energy is

$$E_P = \hbar f_P = \frac{\hbar}{t_P}. \quad (6.2)$$

Collapse of the Square-Root Route

The conventional reduced-action frequency is

$$f_P = \sqrt{\frac{c^5}{\hbar G}}. \quad (6.3)$$

Substituting the QLM gravitational and lattice identities

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (6.4)$$

$$c = \frac{\ell_P}{t_P}, \quad (6.5)$$

gives

$$f_P = \sqrt{\frac{\left(\frac{\ell_P^5}{t_P^3}\right)}{\hbar \left(\frac{\ell_P^5}{t_P^3 \hbar}\right)}} = \sqrt{\frac{1}{t_P^2}} = \frac{1}{t_P}. \quad (6.6)$$

Thus the canonical dimensional expression collapses exactly to the QLM primitive identity (6.1).

Numeric (CODATA 2022)

$$f_P = \frac{1}{5.391\,247 \times 10^{-44} \text{ s}} = 1.854\,858 \times 10^{43} \text{ Hz.} \quad (6.7)$$

Summary.

$$\boxed{f_P = \frac{1}{t_P}, \quad E_P = \hbar f_P = \frac{\hbar}{t_P}.} \quad (6.8)$$

The per-radian frequency f_P is the *only* primitive temporal constant of the Quantum Lattice Model. Angular frequency is not primitive and is introduced only in the next section as part of the angular-momentum structure.

6.1 Planck Wavenumber

In the Quantum Lattice Model the *primitive* spatial cycle rate is the per-radian spatial frequency

$$\boxed{k_P = \frac{1}{\ell_P}} \quad (6.9)$$

counting radians of spatial phase per meter. This is the exact spatial analogue of the primitive temporal frequency $f_P = 1/t_P$.

A full 2π -radian spatial loop defines the derived *angular* wavenumber,

$$k_P^{(\text{loop})} = 2\pi k_P = \frac{2\pi}{\ell_P}, \quad (6.10)$$

directly paralleling the relation $\omega_P = 2\pi f_P$.

Wave-frequency closure. Using the primitive QLM identities

$$f_P = \frac{1}{t_P}, \quad (6.11)$$

$$k_P = \frac{1}{\ell_P}, \quad (6.12)$$

$$c = \frac{\ell_P}{t_P}, \quad (6.13)$$

their ratio becomes

$$\frac{f_P}{k_P} = \frac{\frac{1}{t_P}}{\frac{1}{\ell_P}} = \frac{\ell_P}{t_P} = c. \quad (6.14)$$

Thus the *primitive* QLM wave relation is

$$\boxed{\frac{f_P}{k_P} = c.} \quad (6.15)$$

Using the derived angular quantities reproduces the familiar loop form:

$$\frac{\omega_P}{k_P^{(\text{loop})}} = \frac{2\pi f_P}{2\pi k_P} = \frac{f_P}{k_P} = c. \quad (6.16)$$

Numeric (CODATA 2022).

$$k_P = \frac{1}{1.616\,255 \times 10^{-35} \text{ m}} = 6.186\,390 \times 10^{34} \text{ m}^{-1}, \quad (6.17)$$

$$k_P^{(\text{loop})} = \frac{2\pi}{1.616\,255 \times 10^{-35} \text{ m}} = 3.887\,496 \times 10^{35} \text{ m}^{-1}, \quad (6.18)$$

$$\omega_P = f_P(2\pi) = (1.854\,858 \times 10^{43} \text{ s}^{-1})(2\pi) = 1.165\,442 \times 10^{44} \text{ s}^{-1}, \quad (6.19)$$

$$\frac{f_P}{k_P} = \frac{1.854\,858 \times 10^{43} \text{ s}^{-1}}{6.186\,390 \times 10^{34} \text{ m}^{-1}} = 2.997\,925 \times 10^8 \text{ m s}^{-1} = c, \quad (6.20)$$

$$\frac{\omega_P}{k_P^{(\text{loop})}} = \frac{1.165\,442 \times 10^{44} \text{ s}^{-1}}{3.887\,496 \times 10^{35} \text{ m}^{-1}} = 2.997\,925 \times 10^8 \text{ m s}^{-1} = c. \quad (6.21)$$

Both primitive and loop forms reproduce the invariant QLM wave relation.

Summary.

$k_P = \frac{1}{\ell_P}, \quad k_P^{(\text{loop})} = \frac{2\pi}{\ell_P}, \quad \frac{f_P}{k_P} = c, \quad \frac{\omega_P}{k_P^{(\text{loop})}} = c.$

(6.22)

The spatial and temporal increments of the Planck lattice are therefore perfectly matched, and both the primitive and angular formulations yield the same deterministic lattice wave speed c .

6.2 Planck Mass

QLM definition (per-radian). In the Quantum Lattice Model the Planck mass is defined directly from the per-radian Planck energy,

$$E_P = \frac{\hbar}{t_P}, \quad (6.23)$$

through the inertial relation

$$m_P = \frac{E_P}{c^2} = \frac{\hbar}{t_P c^2}. \quad (6.24)$$

Using the lattice identity $c = \ell_P/t_P$, this becomes

$$m_P = \frac{\hbar}{t_P} \frac{t_P^2}{\ell_P^2} = \frac{\hbar t_P}{\ell_P^2}, \quad (6.25)$$

which is the fundamental QLM form.

Collapse of the conventional square-root expression.

$$m_P = \sqrt{\frac{\hbar c}{G}}, \quad (6.26)$$

must collapse to Eq. (6.25).

Using the QLM substitutions

$$c = \frac{\ell_P}{t_P}, \quad (6.27)$$

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (6.28)$$

we obtain

$$m_P = \sqrt{\frac{\hbar \left(\frac{\ell_P}{t_P} \right)}{\frac{\ell_P^5}{t_P^3 \hbar}}} \quad (6.29)$$

$$= \sqrt{\frac{\hbar^2 \ell_P t_P^3}{\ell_P^5 t_P}} = \sqrt{\frac{\hbar^2 t_P^2}{\ell_P^4}} \quad (6.30)$$

$$= \frac{\hbar t_P}{\ell_P^2}, \quad (6.31)$$

exactly matching the QLM definition.

Thus the QLM-native Planck mass is

$$m_P = \frac{\hbar t_P}{\ell_P^2}$$

(6.32)

Consistency with $E = mc^2$. Using $m_P = \hbar t_P / \ell_P^2$ and $c = \ell_P / t_P$,

$$m_P c^2 = \left(\frac{\hbar t_P}{\ell_P^2} \right) \left(\frac{\ell_P}{t_P} \right)^2 = \frac{\hbar}{t_P} = E_P, \quad (6.33)$$

showing the QLM definition is fully consistent with the per-radian Planck energy E_P .

Bohr-anchored form. Using the hydrogenic identity $\hbar = m_e v_B a_0$,

$$m_P = \frac{(m_e v_B a_0) t_P}{\ell_P^2}, \quad (6.34)$$

demonstrating that the Planck mass arises from the same action quantum that governs the Bohr ground state.

Numeric evaluation (CODATA 2022).

$$\begin{aligned} m_P &= \frac{(1.054\,572 \times 10^{-34} \text{ J s})(5.391\,247 \times 10^{-44} \text{ s})}{(1.616\,255 \times 10^{-35} \text{ m})^2} \\ &= 2.176\,434 \times 10^{-8} \text{ kg.} \end{aligned} \quad (6.35)$$

Summary.

$$m_P = \frac{\hbar}{t_P c^2} = \frac{\hbar t_P}{\ell_P^2} = \frac{m_e v_B a_0 t_P}{\ell_P^2} \quad (6.36)$$

All forms are algebraically identical in the QLM and represent the single per-radian Planck mass.

6.3 Planck Mass as Confinement Closure

Mass in the QLM is defined as closed momentum inventory normalized by the lattice propagation speed:

$$m \equiv \frac{p_{\text{loop}}}{c}. \quad (6.37)$$

Confined momentum at spatial scale r satisfies

$$p_{\text{loop}} = \frac{\hbar}{r}. \quad (6.38)$$

Combining Eqs. (6.37) and (6.38) gives the general confinement relation

$$m = \frac{\hbar}{rc}. \quad (6.39)$$

Solving for r yields

$$r = \frac{\hbar}{mc}, \quad (6.40)$$

identifying the reduced Compton wavelength as the characteristic confinement scale of mass.

For the Planck mass,

$$r(m_P) = \frac{\hbar}{m_P c} = \ell_P, \quad (6.41)$$

showing that the Planck mass corresponds to confinement at exactly one lattice spacing. The Planck mass therefore represents the self-consistent closure of the lattice under maximal confinement.

6.4 Planck Momentum

QLM definition. In the Quantum Lattice Model the Planck momentum is defined as the transport of the per-radian Planck energy across the invariant lattice velocity:

$$p_P = \frac{E_P}{c}, \quad E_P = \frac{\hbar}{t_P}. \quad (6.42)$$

Using $c = \ell_P/t_P$ (Eq. (5.19)) this collapses to

$$p_P = \frac{\left(\frac{\hbar}{t_P}\right)}{\left(\frac{\ell_P}{t_P}\right)} = \frac{\hbar}{\ell_P}, \quad (6.43)$$

which is the fundamental QLM form.

Collapse of the conventional square-root expression. The reduced-dimensional expression is

$$p_P = \sqrt{\frac{\hbar c^3}{G}}, \quad (6.44)$$

which must collapse to \hbar/ℓ_P .

Substituting the QLM substitutions

$$c = \frac{\ell_P}{t_P}, \quad (6.45)$$

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (6.46)$$

gives

$$p_P = \sqrt{\frac{\hbar \left(\frac{\ell_P^3}{t_P^3} \right)}{\frac{\ell_P^5}{t_P^3 \hbar}}} \quad (6.47)$$

$$= \sqrt{\frac{\hbar^2 \ell_P^3}{\ell_P^5}} = \sqrt{\frac{\hbar^2}{\ell_P^2}} \quad (6.48)$$

$$= \frac{\hbar}{\ell_P}. \quad (6.49)$$

Thus the canonical square-root definition collapses exactly to the QLM identity.

Wavenumber form (primitive per-radian lattice). The QLM treats the spatial cycle rate exactly analogously to the temporal cycle rate: the primitive wavenumber counts *radians* of spatial phase per meter, not full 2π loops. Thus

$$k_P = \frac{1}{\ell_P}, \quad (6.50)$$

and the Planck momentum follows directly from the per-radian quantum relation

$$p_P = \hbar k_P = \frac{\hbar}{\ell_P}. \quad (6.51)$$

This expression is the spatial dual of the fundamental energy relation $E_P = \hbar/t_P$ and completes the (E_P, p_P) pair generated by the primitive lattice increments (t_P, ℓ_P) .

Numeric verification (CODATA 2022). All independent routes give the same per-radian Planck momentum:

$$p_P = \frac{E_P}{c} = \frac{1.956\,081 \times 10^9 \text{ J}}{2.997\,925 \times 10^8 \text{ m s}^{-1}} = 6.524\,786 \times 10^1 \text{ kg m s}^{-1}, \quad (6.52)$$

$$p_P = \frac{\hbar}{\ell_P} = \frac{1.054\,572 \times 10^{-34} \text{ Js}}{1.616\,255 \times 10^{-35} \text{ m}} = 6.524\,786 \times 10^1 \text{ kg m s}^{-1}, \quad (6.53)$$

$$p_P = \hbar k_P = (1.054\,572 \times 10^{-34} \text{ Js}) (6.186\,800 \times 10^{34} \text{ m}^{-1}) = 6.524\,786 \times 10^1 \text{ kg m s}^{-1}. \quad (6.54)$$

Summary.

$$p_P = \frac{E_P}{c} = \frac{\hbar}{\ell_P} = \hbar k_P = \frac{m_e v_B a_0}{\ell_P}$$

(6.55)

All expressions are algebraically identical and represent the unique per-radian Planck momentum of the Quantum Lattice Model.

6.5 Planck Force

QLM definition Planck force is defined as the spatial rate at which the fundamental (per-radian) energy quantum propagates across one lattice spacing ℓ_P :

$$F_P = \frac{E_P}{\ell_P} = \frac{\hbar}{t_P \ell_P}. \quad (6.56)$$

Using the lattice metric relation $\ell_P = c t_P$ (Eq. (5.19)), this becomes

$$F_P = \frac{\hbar}{t_P^2 c}. \quad (6.57)$$

Thus the Planck force is the maximal per-radian momentum flux permitted by a single lattice tick.

The traditional dimensional expression. The conventional Planck force is

$$F_P = \frac{c^4}{G}. \quad (6.58)$$

Using the QLM relations

$$c = \frac{\ell_P}{t_P}, \quad (6.59)$$

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (6.60)$$

substitution into Eq. (6.58) gives

$$F_P = \frac{c^4}{G} = \frac{\left(\frac{\ell_P^4}{t_P^4}\right)}{\frac{\ell_P^5}{t_P^3 \hbar}} \quad (6.61)$$

$$= \frac{\hbar \ell_P^4 t_P^3}{\ell_P^5 t_P^4} = \frac{\hbar}{\ell_P t_P} \quad (6.62)$$

which is exactly the QLM expression (6.56). Thus the dimensional square-root form collapses identically to the \hbar -based QLM definition.

Numeric verification (CODATA 2022). Using $E_P = \hbar/t_P$ and the CODATA Planck length:

$$\begin{aligned} F_P &= \frac{E_P}{\ell_P} = \frac{1.956\,081 \times 10^9 \text{ J}}{1.616\,255 \times 10^{-35} \text{ m}} \\ &= 1.210\,256 \times 10^{44} \text{ N}. \end{aligned} \quad (6.63)$$

This agrees exactly with the invariant dimensional form

$$F_P = \frac{c^4}{G}, \quad (6.64)$$

and is fully consistent with the CODATA 2022 constants.

Summary.

$$F_P = \frac{E_P}{\ell_P} = \frac{\hbar}{t_P \ell_P} = \frac{\hbar}{t_P^2 c} = \frac{c^4}{G} = \frac{m_e v_B a_0 c}{\ell_P^2}$$

(6.65)

All forms are algebraically identical representations of the single QLM Planck force.

6.6 Planck Acceleration

QLM Definition. Planck acceleration is the maximal coherent rate of change of the lattice velocity across one Planck tick. Using only the primitive lattice identity

$$c = \frac{\ell_P}{t_P}, \quad (6.66)$$

the QLM definition is

$$a_P = \frac{c}{t_P} = \frac{\ell_P}{t_P^2}, \quad (6.67)$$

This represents the highest possible coherent acceleration permitted by the discrete phase-action structure of the lattice.

Collapse of the dimensional form. The traditional dimensional combination for Planck acceleration is

$$a_P = \sqrt{\frac{c^7}{\hbar G}}, \quad (6.68)$$

Using the QLM identities

$$c = \frac{\ell_P}{t_P}, \quad (6.69)$$

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (6.70)$$

substitution yields

$$a_P = \sqrt{\frac{\left(\frac{\ell_P^7}{t_P^7}\right)}{\hbar \left(\frac{\ell_P^5}{t_P^3 \hbar}\right)}}, \quad (6.71)$$

$$= \sqrt{\frac{\hbar \ell_P^7 t_P^3}{\hbar \ell_P^5 t_P^7}} = \sqrt{\frac{\ell_P^2}{t_P^4}}, \quad (6.72)$$

$$= \frac{\ell_P}{t_P^2} = \frac{c}{t_P}, \quad (6.73)$$

identical to the QLM definition (6.67).

Bohr-anchored form. Using the hydrogenic relation $v_B = \alpha c$, the Planck acceleration admits the equivalent Bohr representation

$$a_P = \frac{c}{t_P} = \frac{v_B}{\alpha t_P}, \quad (6.74)$$

linking the Planck-scale acceleration to hydrogenic kinematics.

Numeric verification (CODATA 2022).

$$a_P = \frac{2.997\,925 \times 10^8 \text{ m s}^{-1}}{5.391\,247 \times 10^{-44} \text{ s}} = 5.560\,726 \times 10^{51} \text{ m s}^{-2}, \quad (6.75)$$

Bohr-anchored check.

$$v_B = \alpha c = (7.297\,353 \times 10^{-3}) (2.997\,925 \times 10^8 \text{ m s}^{-1}) = 2.187\,691 \times 10^6 \text{ m s}^{-1}, \quad (6.76)$$

$$a_P = \frac{v_B}{\alpha t_P} = \frac{2.187\,691 \times 10^6 \text{ m s}^{-1}}{(7.297\,353 \times 10^{-3}) 5.391\,247 \times 10^{-44} \text{ s}} = 5.560\,726 \times 10^{51} \text{ m s}^{-2}, \quad (6.77)$$

in exact agreement with Eq. (6.75).

6.7 Planck Power

QLM definition. In the Quantum Lattice Model the Planck power is the rate at which the fundamental (per-radian) Planck energy is transported across one lattice tick. Since each tick carries one radian of phase and \hbar of action, the per-radian Planck energy is

$$E_P = \frac{\hbar}{t_P}, \quad (6.78)$$

and the fundamental Planck power is

$$P_P = \frac{E_P}{t_P} = \frac{\hbar}{t_P^2}. \quad (6.79)$$

This is the *only* physically fundamental definition in the QLM, because the lattice advances one radian per tick.

Collapse of the dimensional form. The traditional reduced-action expression for Planck power is

$$P_P = \frac{c^5}{G}. \quad (6.80)$$

Substituting the QLM identities

$$c = \frac{\ell_P}{t_P}, \quad (6.81)$$

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad (6.82)$$

gives

$$P_P = \frac{\left(\frac{\ell_P}{t_P}\right)^5}{\frac{\ell_P^5}{t_P^3 \hbar}} \quad (6.83)$$

$$= \frac{\ell_P^5 t_P^3 \hbar}{\ell_P^5 t_P^5} = \frac{\hbar}{t_P^2}, \quad (6.84)$$

showing that the dimensional square-root form collapses exactly to the QLM definition.

Lattice interpretation. Because the lattice advances one radian of phase per tick, transporting \hbar of action each time, the power associated with one action-quantum per tick must be \hbar/t_P^2 . No additional geometric factors enter the QLM definition.

Relation to Planck force. The \hbar -based Planck force is

$$F_P = \frac{\hbar}{t_P \ell_P}. \quad (6.85)$$

Using the lattice velocity $c = \ell_P/t_P$,

$$P_P = F_P c = \frac{\hbar}{t_P \ell_P} \left(\frac{\ell_P}{t_P} \right) = \frac{\hbar}{t_P^2}, \quad (6.86)$$

identical to Eq. (6.79).

Energy-density representation. Using the \hbar -based Planck energy density

$$u_P = \frac{\hbar}{\ell_P^3 t_P}, \quad (6.87)$$

the same result follows:

$$P_P = u_P \ell_P^2 c = \frac{\hbar}{t_P^2}, \quad (6.88)$$

corresponding to transporting one radian of action across one lattice face per tick.

Bohr-anchored representation. With the identity $\hbar = m_e v_B a_0$,

$$P_P = \frac{m_e v_B a_0}{t_P^2}, \quad (6.89)$$

demonstrating the Planck–hydrogen correspondence.

Numeric verification (CODATA 2022).

$$\begin{aligned} P_P &= \frac{\hbar}{t_P^2} = \frac{1.054\,572 \times 10^{-34} \text{ J s}}{(5.391\,247 \times 10^{-44} \text{ s})^2} \\ &= 3.628\,255 \times 10^{52} \text{ W}. \end{aligned} \quad (6.90)$$

This value agrees exactly with the collapsed dimensional expression c^5/G , confirming the internal consistency of the \hbar -based QLM definition.

7 Planck Temperature

Definition (per-radian QLM form). In the Quantum Lattice Model, the fundamental Planck temperature is defined strictly from the *per-radian* Planck energy,

$$E_P = \frac{\hbar}{t_P}, \quad (7.1)$$

so that

$$T_P = \frac{E_P}{k_B} = \frac{\hbar}{t_P k_B}. \quad (7.2)$$

This represents the thermal energy associated with accumulating one radian of phase per lattice tick.

Bohr-anchored form. With the hydrogenic identity $\hbar = m_e v_B a_0$,

$$T_P = \frac{m_e v_B a_0}{t_P k_B}. \quad (7.3)$$

Phase advance per Kelvin. Since a thermal increment ΔT corresponds to energy $E = k_B \Delta T$, the associated phase advance is

$$\vartheta = \frac{E}{E_P} = \frac{k_B \Delta T}{\hbar/t_P} = \Delta T \left(\frac{t_P k_B}{\hbar} \right). \quad (7.4)$$

Thus the phase advance produced by a temperature change of 1 K is

$$\vartheta_{1K} = \frac{1}{T_P} = \frac{t_P k_B}{\hbar}, \quad (7.5)$$

expressing temperature directly as phase accumulated per Kelvin.

Numeric evaluation (CODATA 2022).

$$E_P = \frac{\hbar}{t_P} = \frac{1.054\,572 \times 10^{-34} \text{ J s}}{5.391\,247 \times 10^{-44} \text{ s}} = 1.956\,081 \times 10^9 \text{ J}, \quad (7.6)$$

$$T_P = \frac{E_P}{k_B} = \frac{1.956\,081 \times 10^9 \text{ J}}{1.380\,649 \times 10^{-23} \text{ J K}^{-1}} = 1.416\,784 \times 10^{32} \text{ K}, \quad (7.7)$$

$$\vartheta_{1K} = \frac{1}{T_P} = 7.058\,239 \times 10^{-33} \text{ rad K}^{-1}. \quad (7.8)$$

Summary.

$$T_P = \frac{\hbar}{t_P k_B} = \frac{m_e v_B a_0}{t_P k_B} \quad (7.9)$$

$$\vartheta_{1K} = \frac{1}{T_P} = \frac{t_P k_B}{\hbar} \quad (7.10)$$

8 Geometric Foundations and Planck Densities of the QLM Lattice

8.1 Primitive Geometric Cells of the Lattice

In the Quantum Lattice Model the Planck length ℓ_P and Planck time t_P are the primitive spacetime increments of the discrete lattice. They define the fundamental geometric cells:

$$A_P = \ell_P^2, \quad \mathcal{V}_P = \ell_P^3, \quad \mathcal{V}_4 = \ell_P^3 t_P. \quad (8.1)$$

- A_P is the *Planck area*, the face of a lattice cell.
- \mathcal{V}_P is the *Planck spatial volume*, the minimal 3D region.
- \mathcal{V}_4 is the *Planck four-volume tick*, the minimal spacetime element updated each lattice tick.

Temporal–spatial locking. The geometry of the lattice enforces the exact causal relation

$$c = \frac{\ell_P}{t_P}, \quad (8.2)$$

so one spatial link ℓ_P is traversed in one temporal tick t_P . This fixes the primitive temporal and spatial frequencies:

$$f_P = \frac{1}{t_P}, \quad k_P = \frac{1}{\ell_P}. \quad (8.3)$$

The angular quantities are derived:

$$\omega_P = 2\pi f_P, \quad k_P^{(\text{loop})} = 2\pi k_P. \quad (8.4)$$

The phase–wave propagation rule then follows identically:

$$\frac{f_P}{k_P} = c, \quad \frac{\omega_P}{k_P^{(\text{loop})}} = c. \quad (8.5)$$

One radian per four–volume cell. Each four–volume cell

$$\mathcal{V}_4 = \ell_P^3 t_P \quad (8.6)$$

supports one radian of coherent phase advance per tick. Because one radian carries exactly one quantum of action,

$$\hbar = E_P t_P, \quad (8.7)$$

the QLM four–volume is the spacetime region in which one unit of reduced action is deposited each cycle.

This establishes the fundamental deterministic rule of the lattice:

one radian of phase, one quantum of action (\hbar) per \mathcal{V}_4 .

(8.8)

8.2 Planck Phase Density (Primitive QLM Quantity)

Because each four–volume $\mathcal{V}_4 = \ell_P^3 t_P$ corresponds to a single radian of phase evolution, the intrinsic phase density of spacetime is

$$\Phi_P = \frac{1}{\mathcal{V}_4} = \frac{1}{\ell_P^3 t_P}. \quad (8.9)$$

Equivalent lattice forms. Using $f_P = 1/t_P$, $\omega_P = 2\pi f_P$, and $\ell_P = ct_P$:

$$\begin{aligned} \Phi_P &= \frac{1}{\ell_P^3 t_P} && \text{(definition)} \\ &= \frac{f_P}{\ell_P^3} && (f_P = 1/t_P) \\ &= \frac{\omega_P}{2\pi \ell_P^3} && (\omega_P = 2\pi f_P) \\ &= \frac{1}{c^3 t_P^4} && (\ell_P = ct_P). \end{aligned} \quad (8.10)$$

Numeric (CODATA 2022).

$$\Phi_P = \frac{1}{(1.616\,255 \times 10^{-35} \text{ m})^3 (5.391\,247 \times 10^{-44} \text{ s})} = 4.393\,202 \times 10^{147} \text{ m}^{-3} \text{ s}^{-1}. \quad (8.11)$$

8.3 Planck Energy Density (Action per Four–Volume)

The fundamental (per–radian) QLM Planck energy is

$$E_P = \frac{\hbar}{t_P}, \quad (8.12)$$

so the energy density associated with one Planck spatial cell is

$$u_P = \frac{E_P}{\ell_P^3} = \frac{\hbar}{\ell_P^3 t_P} = \hbar \Phi_P. \quad (8.13)$$

Thus the Planck energy density is simply the action quantum multiplied by the phase–tick density of spacetime.

Numeric (CODATA 2022).

$$\nu_P = \frac{1.956\,081 \times 10^9 \text{ J}}{(1.616\,255 \times 10^{-35} \text{ m})^3} = 4.632\,947 \times 10^{113} \text{ J m}^{-3}. \quad (8.14)$$

8.4 Planck Mass Density

The fundamental QLM Planck mass is

$$m_P = \frac{\hbar t_P}{\ell_P^2}, \quad (8.15)$$

so the mass density of one Planck cell is

$$\rho_P = \frac{m_P}{\ell_P^3} = \frac{\hbar t_P}{\ell_P^5}, \quad (8.16)$$

Equivalent collapse from gravitational identity. Using the QLM gravitational relation and c

$$G = \frac{\ell_P^5}{t_P^3 \hbar}, \quad c = \frac{\ell_P}{t_P}, \quad (8.17)$$

the dimensional form

$$\rho_P = \frac{c^5}{\hbar G^2} = \frac{\left(\frac{\ell_P}{t_P}\right)^5}{\hbar \left(\frac{\ell_P^5}{\hbar t_P^3}\right)^2} = \frac{\left(\frac{\ell_P^5}{t_P^5}\right)}{\hbar \left(\frac{\ell_P^{10}}{t_P^6 \hbar^2}\right)} = \frac{\hbar^2 \ell_P^5 t_P^6}{\hbar \ell_P^{10} t_P^5} = \frac{\hbar t_P}{\ell_P^5}, \quad (8.18)$$

collapses exactly to Eq. (8.16), confirming internal consistency with the QLM definitions.

Energy–mass closure. Because $E_P = m_P c^2$ and $\ell_P = ct_P$,

$$u_P = \rho_P c^2, \quad (8.19)$$

the expected relativistic identity holds automatically in QLM units.

Numeric (CODATA 2022).

$$\rho_P = \frac{2.176\,434 \times 10^{-8} \text{ kg}}{(1.616\,255 \times 10^{-35} \text{ m})^3} = 5.154\,849 \times 10^{96} \text{ kg m}^{-3}. \quad (8.20)$$

8.5 Summary of Planck Densities

$$\boxed{\Phi_P = \frac{1}{\ell_P^3 t_P}} \quad \boxed{u_P = \hbar \Phi_P} \quad \boxed{\rho_P = \frac{u_P t_P^2}{\ell_P^2}} \quad (8.21)$$

These three quantities form the fundamental density hierarchy of the Quantum Lattice Model: phase density Φ_P , energy density u_P , and mass density ρ_P , all derived directly from the primitive geometric four-volume \mathcal{V}_4 .

9 Charge and Electromagnetism in the Quantum Lattice Model

Electromagnetism in the Quantum Lattice Model (QLM) arises directly from the phase-action structure of the Planck lattice. Each Planck tick advances the physical phase by one radian and transports the reduced action quantum,

$$E_P t_P = \hbar, \quad E_P = \frac{\hbar}{t_P}. \quad (9.1)$$

Because the lattice is defined by the increments (ℓ_P, t_P) and the action per tick \hbar , all electromagnetic quantities are algebraically determined by these primitives.

Planck Charge and Impedance

The Planck charge follows from

$$q_P^2 = 4\pi \epsilon_0 \hbar c, \quad (9.2)$$

while the per-radian Planck impedance is defined by

$$Z_P = \frac{Z_0}{4\pi}, \quad (9.3)$$

with the loop-aggregated impedance

$$Z_P^{(\text{loop})} = 2\pi Z_P = \frac{Z_0}{2}. \quad (9.4)$$

Planck Electromagnetic Quartet

The maximal electromagnetic power transported per lattice tick is

$$P_P = \frac{E_P}{t_P} = \frac{\hbar}{t_P^2}. \quad (9.5)$$

From this, the Planck voltage and current follow algebraically:

$$V_P = \sqrt{P_P Z_P} = \frac{E_P}{q_P}, \quad (9.6)$$

$$I_P = \sqrt{\frac{P_P}{Z_P}} = \frac{q_P}{t_P}. \quad (9.7)$$

Together,

$$(Z_P, V_P, I_P, P_P) \quad (9.8)$$

form the complete Planck electromagnetic quartet, representing the maximal impedance, voltage, current, and power carried by one radian of coherent phase evolution.

Loop-aggregated relations are obtained by replacing Z_P with $Z_P^{(\text{loop})} = 2\pi Z_P$ when full 2π phase cycles are considered.

9.1 Fine-Structure Constant and Hydrogenic Calibration

QLM Interpretation of the Fine-Structure Constant.

In the Quantum Lattice Model the fine-structure constant is not a primitive coupling parameter. It is a dimensionless lattice ratio linking confined rotational motion to invariant photon propagation:

$$\alpha = \boxed{\frac{v_B}{c}}. \quad (9.9)$$

Hydrogenic (Kinematic) Form

The Bohr action identity,

$$\hbar = m_e v_B a_0, \quad (9.10)$$

connects the reduced action quantum directly to the mechanical angular momentum of the hydrogen ground state. Solving for the orbital velocity gives

$$v_B = \frac{\hbar}{m_e a_0}. \quad (9.11)$$

Using the lattice kinematic identity

$$c = \frac{\ell_P}{t_P}, \quad (9.12)$$

the fine-structure constant becomes

$$\alpha = \frac{v_B}{c} = \frac{v_B t_P}{\ell_P}. \quad (9.13)$$

Thus in hydrogen, α represents the geometric ratio converting rotational confinement into linear propagation at the lattice speed.

Electromagnetic (Impedance) Form

The fine-structure constant is also defined in SI units by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \quad (9.14)$$

Using the per-radian impedance normalization

$$Z_P = \frac{Z_0}{4\pi}, \quad Y_P = \frac{1}{Z_P}, \quad (9.15)$$

together with

$$4\pi\epsilon_0\hbar c = \hbar Y_P = \frac{\hbar}{Z_P}, \quad (9.16)$$

one obtains the equivalent QLM impedance form

$$\boxed{\alpha = \frac{e^2}{\hbar Y_P} = \frac{e^2 Z_P}{\hbar}} \quad (9.17)$$

Solving for the elementary charge yields

$$\boxed{e^2 = \alpha \hbar Y_P = \alpha \frac{\hbar}{Z_P} = \alpha q_P^2,} \quad (9.18)$$

so that

$$\boxed{e = q_P \sqrt{\alpha}.} \quad (9.19)$$

Unified Interpretation

The hydrogenic velocity ratio and the electromagnetic impedance ratio are therefore identical representations of the same lattice scaling:

$$\boxed{\frac{v_B}{c} = \frac{e^2}{\hbar Y_P}.} \quad (9.20)$$

The fine-structure constant is thus a dimensionless conversion factor relating rotational confinement, electromagnetic admittance, and Planck-scale action throughput within the same reduced-action lattice framework.

Numeric (CODATA 2022).

$$\alpha = \frac{v_B}{c} = \frac{2.187\,691 \times 10^6}{2.997\,925 \times 10^8} = 7.297\,353 \times 10^{-3}, \quad (9.21)$$

in agreement with CODATA 2022.

10 Rydberg Scale in Bohr and Planck–Lattice Form

The Rydberg scale governs the fundamental wavelengths and frequencies of hydrogenic spectra. In the Quantum Lattice Model (QLM), it is not a dynamical parameter but a *pure geometric ratio* that links Bohr orbital geometry to the Planck lattice's linear and rotational phase rules.

10.1 Lattice form of the Rydberg constant

The QLM identity for the Rydberg constant (spectroscopic wavenumber, in cycles per meter) is

$$R_\infty = \frac{v_B t_P}{4\pi a_0 \ell_P} \quad (10.1)$$

Equivalently, the per-radian (angular) Rydberg wavenumber is

$$k_\infty \equiv 2\pi R_\infty = \frac{v_B t_P}{2 a_0 \ell_P}. \quad (10.2)$$

These expressions contain no reference to electric charge, ϵ_0 , μ_0 , or the Coulomb potential. The Rydberg scale emerges solely from:

- the electron's rotational velocity v_B ,
- the Planck tick t_P that sets linear phase advance,
- the Bohr radius a_0 (hydrogenic spatial coherence length),
- the Planck length ℓ_P (lattice spacing).

Numerical validation (CODATA 2022). Using

$$v_B = 2.187\,691 \times 10^6 \text{ m/s}, \quad t_P = 5.391\,247 \times 10^{-44} \text{ s}, \quad (10.3)$$

$$a_0 = 5.291\,772 \times 10^{-11} \text{ m}, \quad \ell_P = 1.616\,255 \times 10^{-35} \text{ m}, \quad (10.4)$$

compute the numerator:

$$v_B t_P = (2.187\,691 \times 10^6)(5.391\,247 \times 10^{-44}) = 1.179\,438 \times 10^{-37}. \quad (10.5)$$

Compute the denominator:

$$4\pi a_0 \ell_P = 4\pi (5.291\,772 \times 10^{-11})(1.616\,255 \times 10^{-35}) = 1.074\,783 \times 10^{-44}. \quad (10.6)$$

Then

$$R_\infty = \frac{1.179\,438 \times 10^{-37}}{1.074\,783 \times 10^{-44}} \quad (10.7)$$

$$= 1.097\,373 \times 10^7 \text{ m}^{-1}. \quad (10.8)$$

This is in excellent agreement with the CODATA 2022 value

$$R_\infty = 1.097\,373 \times 10^7 \text{ m}^{-1}. \quad (10.9)$$

For reference, the corresponding per-radian (angular) wavenumber is

$$k_\infty = 2\pi R_\infty = 6.895\,000 \times 10^7 \text{ m}^{-1}. \quad (10.10)$$

10.2 Bohr form of the Rydberg frequency

The Rydberg frequency is defined by

$$\nu_R = c R_\infty. \quad (10.11)$$

The canonical hydrogenic expression for R_∞ is

$$R_\infty = \frac{\alpha^2 m_e c}{2h}, \quad (10.12)$$

so that

$$\nu_R = c R_\infty = \alpha^2 \frac{m_e c^2}{2h}. \quad (10.13)$$

Using the Bohr identity

$$\alpha = \frac{v_B}{c}, \quad h = 2\pi\hbar, \quad (10.14)$$

Eq. (10.13) becomes

$$\nu_R = \frac{v_B^2 m_e}{4\pi\hbar}$$

(10.15)

This form contains no reference to charge, ϵ_0 , μ_0 , or the Coulomb potential. The Rydberg frequency emerges solely from:

- the Bohr orbital velocity v_B ,
- the electron mass m_e ,
- the reduced action quantum \hbar (the QLM rotational phase unit).

Numerical validation (CODATA 2022). Using

$$v_B = 2.187\,691 \times 10^6 \text{ m/s}, \quad m_e = 9.109\,384 \times 10^{-31} \text{ kg}, \quad \hbar = 1.054\,572 \times 10^{-34} \text{ J s}, \quad (10.16)$$

compute the numerator:

$$v_B^2 m_e = (2.187\,691 \times 10^6)^2 (9.109\,384 \times 10^{-31}) = 4.360\,084 \times 10^{-18}. \quad (10.17)$$

Compute the denominator:

$$4\pi\hbar = 4\pi (1.054\,572 \times 10^{-34}) = 1.325\,489 \times 10^{-33}. \quad (10.18)$$

Then

$$\nu_R = \frac{4.360\,084 \times 10^{-18}}{1.325\,489 \times 10^{-33}} \quad (10.19)$$

$$= 3.289\,842 \times 10^{15} \text{ Hz}, \quad (10.20)$$

in excellent agreement with the CODATA 2022 value

$$\nu_R = 3.289\,842 \times 10^{15} \text{ Hz.} \quad (10.21)$$

10.3 Planck Charge from Action and Impedance

In the QLM, electric charge is not a primitive postulate. It emerges algebraically from the reduced action quantum \hbar and the electromagnetic impedance of the vacuum.

From Maxwell electromagnetism,

$$q_P^2 = 4\pi\epsilon_0\hbar c, \quad (10.22)$$

and using

$$\epsilon_0 = \frac{1}{Z_0 c}, \quad (10.23)$$

we obtain

$$q_P^2 = \frac{4\pi\hbar}{Z_0}. \quad (10.24)$$

With the per-radian normalization

$$Z_P = \frac{Z_0}{4\pi}, \quad (10.25)$$

this becomes the compact QLM form

$$q_P^2 = \frac{\hbar}{Z_P} = \hbar Y_P, \quad (10.26)$$

where $Y_P = 1/Z_P$ is the per-radian admittance.

Therefore,

$$q_P = \sqrt{\frac{\hbar}{Z_P}} = \sqrt{\hbar Y_P}. \quad (10.27)$$

Thus Planck charge is determined entirely by the action-impedance pair (\hbar, Z_P) and is not an independent primitive of the model.

10.4 Vacuum Impedance, Admittance, Permittivity, and Permeability

The electromagnetic constants of free space follow directly from the QLM lattice increments (ℓ_P, t_P) together with the vacuum impedance Z_0 . From

$$Z_0 = \mu_0 c = \frac{1}{\epsilon_0 c}, \quad c = \frac{\ell_P}{t_P}, \quad (10.28)$$

the permittivity and permeability take the geometric lattice forms

$$\mu_0 = Z_0 \frac{t_P}{\ell_P}, \quad \epsilon_0 = \frac{t_P}{Z_0 \ell_P}. \quad (10.29)$$

Consistency check. These expressions satisfy identically

$$\mu_0 \epsilon_0 c^2 = 1, \quad (10.30)$$

confirming that the Maxwell identity is a direct geometric consequence of the lattice relation $c = \ell_P/t_P$.

Impedance and Admittance Form

Using the per-radian normalization

$$Z_P = \frac{Z_0}{4\pi}, \quad Y_P = \frac{1}{Z_P}, \quad (10.31)$$

the Planck charge identity

$$q_P^2 = \frac{\hbar}{Z_P} = \hbar Y_P, \quad (10.32)$$

expresses the electromagnetic vacuum constants entirely in terms of the action-impedance pair (\hbar, Z_P) .

Equivalently,

$$Z_0 = 4\pi Z_P = \frac{4\pi\hbar}{q_P^2}, \quad (10.33)$$

$$\mu_0 = \frac{Z_0}{c}, \quad (10.34)$$

$$\epsilon_0 = \frac{1}{Z_0 c}. \quad (10.35)$$

Thus the electromagnetic triad

$$(\epsilon_0, \mu_0, Z_0) \quad (10.36)$$

is not an independent sector of physics but a derived consequence of the reduced action quantum \hbar , the lattice spacetime increments (ℓ_P, t_P) , and the impedance-admittance structure of the Planck cell.

10.5 The Planck Electromagnetic Quartet

Planck voltage and current.

$$V_P = \frac{E_P}{q_P} = \frac{\hbar}{t_P q_P}, \quad (10.37)$$

$$I_P = \frac{q_P}{t_P}. \quad (10.38)$$

Planck impedance (per–radian). The impedance follows directly from the ratio

$$Z_P = \frac{V_P}{I_P} = \frac{E_P t_P}{q_P^2} = \frac{\hbar}{q_P^2}. \quad (10.39)$$

Using the QLM charge identity

$$q_P^2 = \frac{\hbar}{Z_P}, \quad (10.40)$$

we obtain the per–radian Planck impedance

$$\boxed{Z_P = \frac{Z_0}{4\pi}} \quad (10.41)$$

For a full 2π phase loop,

$$\boxed{Z_P^{(\text{loop})} = 2\pi Z_P = \frac{Z_0}{2}} \quad (10.42)$$

Equivalently,

$$\boxed{Z_P = \frac{Z_P^{(\text{loop})}}{2\pi}} \quad (10.43)$$

Planck power (per–radian).

$$P_P = V_P I_P = \frac{\hbar}{t_P^2}. \quad (10.44)$$

Thus

$$\{V_P, I_P, Z_P, P_P\} \quad (10.45)$$

form the Planck electromagnetic quartet, defined per radian of coherent lattice phase advance.

Numeric values (CODATA 2022, exact defined constants). Using

$$Z_0 = 3.767\,303 \times 10^2 \Omega, \quad (10.46)$$

the loop impedance is

$$\begin{aligned} Z_P^{(\text{loop})} &= \frac{Z_0}{2} = \frac{3.767\,303 \times 10^2 \Omega}{2} \\ &= 1.883\,652 \times 10^2 \Omega. \end{aligned} \quad (10.47)$$

The per–radian impedance is

$$\begin{aligned} Z_P &= \frac{Z_0}{4\pi} = \frac{3.767\,303 \times 10^2 \Omega}{4\pi} \\ &= 2.997\,925 \times 10^1 \Omega. \end{aligned} \quad (10.48)$$

Verification:

$$\frac{Z_P^{(\text{loop})}}{2\pi} = \frac{1.883\,652 \times 10^2 \Omega}{2\pi} = 2.997\,925 \times 10^1 \Omega = Z_P. \quad (10.49)$$

The Planck power is

$$\begin{aligned} P_P &= \frac{\hbar}{t_P^2} \\ &= 3.628\,254 \times 10^{52} \text{ W}. \end{aligned} \quad (10.50)$$

From $V_P = \sqrt{P_P Z_P}$ and $I_P = \sqrt{P_P / Z_P}$,

$$V_P = 2.614\,262 \times 10^{27} \text{ V}, \quad I_P = 1.387\,869 \times 10^{25} \text{ A}. \quad (10.51)$$

These satisfy identically

$$V_P I_P = P_P, \quad \frac{V_P}{I_P} = Z_P, \quad I_P = \frac{q_P}{t_P}, \quad V_P = \frac{E_P}{q_P}. \quad (10.52)$$

10.6 Planck Electromagnetic Field Strengths

The fundamental (per–radian) Planck energy of the QLM lattice is

$$E_P = \frac{\hbar}{t_P}, \quad (10.53)$$

so the associated Planck energy density is

$$u_P = \frac{E_P}{\ell_P^3} = \frac{\hbar}{\ell_P^3 t_P} = \hbar \Phi_P, \quad (10.54)$$

representing the maximal coherent electromagnetic energy storables in a single Planck four–volume per radian of phase advance.

Electric and magnetic field amplitudes. Using the conventional electromagnetic energy–density relations,

$$u = \frac{1}{2} \epsilon_0 \mathcal{E}^2 = \frac{\mathcal{B}^2}{2\mu_0}, \quad (10.55)$$

the per–radian QLM Planck electromagnetic fields are

$$\mathcal{E}_P = \sqrt{\frac{2u_P}{\epsilon_0}} = \sqrt{\frac{2\hbar}{\epsilon_0 \ell_P^3 t_P}}, \quad (10.56)$$

$$\mathcal{B}_P = \sqrt{2\mu_0 u_P} = \sqrt{\frac{2\mu_0 \hbar}{\ell_P^3 t_P}}. \quad (10.57)$$

Wave-speed and impedance relations. Their ratio satisfies the plane-wave identity

$$\frac{\mathcal{E}_P}{\mathcal{B}_P} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c = \frac{\ell_P}{t_P}, \quad (10.58)$$

which is a wave-speed relation arising from the lattice kinematic identity.

Impedance enters through the magnetic field intensity $\mathcal{H}_P = \mathcal{B}_P/\mu_0$, giving

$$\frac{\mathcal{E}_P}{\mathcal{H}_P} = \sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0, \quad (10.59)$$

so the standard vacuum impedance relation is recovered exactly at the Planck scale.

Numeric values (CODATA 2022). Using

$$\hbar = 1.054571817 \times 10^{-34} \text{ J s}, \quad \ell_P = 1.616255 \times 10^{-35} \text{ m}, \quad t_P = 5.391247 \times 10^{-44} \text{ s}, \quad (10.60)$$

$$\epsilon_0 = 8.8541878128 \times 10^{-12} \text{ F/m}, \quad \mu_0 = 1.25663706212 \times 10^{-6} \text{ N/A}^2, \quad (10.61)$$

the energy density is

$$u_P = \frac{\hbar}{\ell_P^3 t_P} = 4.63294706 \times 10^{113} \text{ J/m}^3. \quad (10.62)$$

Thus the fundamental Planck electromagnetic fields are

$$\mathcal{E}_P = \sqrt{\frac{2u_P}{\epsilon_0}} = 3.23496288 \times 10^{62} \text{ V/m}, \quad (10.63)$$

$$\mathcal{B}_P = \sqrt{2\mu_0 u_P} = 1.07906746 \times 10^{53} \text{ T}. \quad (10.64)$$

Both satisfy $\mathcal{E}_P/\mathcal{B}_P = c$ and $\mathcal{E}_P/\mathcal{H}_P = Z_0$ to machine precision.

10.7 Summary of Electromagnetic Closure

$$\boxed{\alpha = \frac{v_B}{c}} \quad \boxed{q_P^2 = \frac{\hbar}{Z_P} = \hbar Y_P} \quad \boxed{Z_P = \frac{Z_0}{4\pi}} \quad (10.65)$$

$$\boxed{V_P = \frac{E_P}{q_P}, \quad I_P = \frac{q_P}{t_P}, \quad P_P = \frac{\hbar}{t_P^2}} \quad (10.66)$$

$$\boxed{\mathcal{E}_P = \sqrt{\frac{2\hbar}{\epsilon_0 \ell_P^3 t_P}}, \quad \mathcal{B}_P = \sqrt{\frac{2\mu_0 \hbar}{\ell_P^3 t_P}}} \quad (10.67)$$

Electromagnetism in the QLM is therefore a fully geometric, phase-coherent sector arising directly from the lattice primitives (ℓ_P, t_P, \hbar, c) together with the vacuum impedance Z_0 , with loop-aggregated relations obtained by replacing Z_P by $Z_P^{(\text{loop})} = 2\pi Z_P = Z_0/2$ when full 2π phase cycles are considered.

11 Bohr Radius (Consistency and QLM–Equivalent Forms)

Canonical and QLM–consistent forms. The canonical definition of the Bohr radius with the Bohr Momentum being $p_B = m_e \cdot v_B$ is

$$a_0 = \frac{\hbar}{m_e c \alpha} = \frac{\hbar}{m_e v_B} = \frac{\hbar}{p_B} \quad (11.1)$$

Using the QLM action definition

$$E_P t_P = \hbar \quad E_P = \frac{\hbar}{t_P} \quad (11.2)$$

we may rewrite the canonical \hbar as $E_P t_P$, giving the equivalent QLM form

$$a_0 = \frac{E_P t_P}{m_e c \alpha}. \quad (11.3)$$

Applying the lattice kinematic identity $c = \ell_P / t_P$,

$$a_0 = \frac{E_P t_P^2}{m_e \alpha \ell_P}. \quad (11.4)$$

Equations (11.1), (11.3), and (11.4) are algebraically identical and differ only by QLM substitutions ($E_P = \hbar / t_P$, $c = \ell_P / t_P$).

Bohr identity $\Rightarrow \alpha = v_B / c$ (complete cancellation). Insert the Bohr identity $\hbar = m_e v_B a_0$ into Eq. (11.1):

$$a_0 = \frac{m_e v_B a_0}{m_e c \alpha} = a_0 \frac{v_B}{c \alpha} \Rightarrow 1 = \frac{v_B}{c \alpha} \Rightarrow \boxed{\alpha = \frac{v_B}{c}}. \quad (11.5)$$

The same cancellation occurs if we start from the QLM form $a_0 = E_P t_P / (m_e c \alpha)$ with $E_P t_P = \hbar$.

Thus, in the QLM, the Bohr radius directly encodes the kinematic identity $\alpha = v_B / c$ and contains no independent charge dependence.

Units checks. From (11.1):

$$[\hbar] = \text{kg m}^2 \text{s}^{-1}, \quad [m_e c \alpha] = \text{kg m s}^{-1}, \quad \Rightarrow \quad [a_0] = \text{m}. \quad (11.6)$$

From (11.4):

$$[E_P t_P^2] = (\text{J})(\text{s}^2) = \text{kg m}^2, \quad [m_e \alpha \ell_P] = \text{kg m}, \quad (11.7)$$

again giving $[a_0] = \text{m}$.

Numerical validation (CODATA 2022). Canonical form (\hbar, m_e, c, α):

$$a_0 = \frac{1.054\,572 \times 10^{-34} \text{ J s}}{(9.109\,384 \times 10^{-31} \text{ kg})(2.997\,925 \times 10^8 \text{ m s}^{-1})(7.297\,353 \times 10^{-3})} = 5.291\,772 \times 10^{-11} \text{ m.} \quad (11.8)$$

QLM per-radian form ($E_P = \hbar/t_P$):

$$a_0 = \frac{1.956\,081 \times 10^9 \text{ J} (5.391\,247 \times 10^{-44} \text{ s})}{(9.109\,384 \times 10^{-31} \text{ kg})(2.997\,925 \times 10^8 \text{ m s}^{-1})(7.297\,353 \times 10^{-3})} = 5.291\,772 \times 10^{-11} \text{ m.} \quad (11.9)$$

Length-time substitution form ($c = \ell_P/t_P$):

$$a_0 = \frac{1.956\,081 \times 10^9 \text{ J} (5.391\,247 \times 10^{-44} \text{ s})^2}{(9.109\,384 \times 10^{-31} \text{ kg})(7.297\,353 \times 10^{-3}) 1.616\,255 \times 10^{-35} \text{ m}} = 5.291\,772 \times 10^{-11} \text{ m.} \quad (11.10)$$

Velocity confirmation (consistency loop). Using $\alpha = v_B/c$:

$$v_B = \alpha c = (7.297\,353 \times 10^{-3})(2.997\,925 \times 10^8 \text{ m s}^{-1}) = 2.187\,691 \times 10^6 \text{ m s}^{-1}. \quad (11.11)$$

This confirms perfect consistency between canonical quantum mechanics and the QLM substitutions ($E_P t_P = \hbar, c = \ell_P/t_P$).

12 Elementary Charge (Closed Loop)

Planck charge from action and impedance. Using the QLM substitutions

$$\epsilon_0 = \frac{t_P}{Z_0 \ell_P}, \quad c = \frac{\ell_P}{t_P}, \quad (12.1)$$

the Planck charge becomes

$$\begin{aligned} q_P^2 &= 4\pi \epsilon_0 \hbar c \\ &= 4\pi \hbar \frac{t_P}{Z_0 \ell_P} \frac{\ell_P}{t_P} = \boxed{\frac{4\pi \hbar}{Z_0}}. \end{aligned} \quad (12.2)$$

Introducing the per-radian Planck impedance

$$Z_P = \frac{Z_0}{4\pi}, \quad (12.3)$$

gives the compact QLM identity

$$\boxed{q_P^2 = \frac{\hbar}{Z_P} = \hbar Y_P}, \quad Y_P \equiv \frac{1}{Z_P}. \quad (12.4)$$

For a full 2π phase loop,

$$Z_P^{(\text{loop})} = 2\pi Z_P = \frac{Z_0}{2}. \quad (12.5)$$

Elementary charge from fine-structure scaling. Since

$$\alpha = \frac{e^2}{q_P^2}, \quad (12.6)$$

we immediately obtain

$$e^2 = \alpha q_P^2 = \alpha \frac{\hbar}{Z_P} = \alpha \hbar Y_P. \quad (12.7)$$

Using $q_P^2 = 4\pi\epsilon_0\hbar c$, this is equivalently

$$e^2 = 4\pi \epsilon_0 \hbar c \alpha, \quad (12.8)$$

which is the standard SI definition.

Thus the elementary charge is

$$e = q_P \sqrt{\alpha}. \quad (12.9)$$

Impedance form of the fine-structure constant. Using $q_P^2 = \hbar/Z_P$, the fine-structure constant takes the canonical QLM impedance form

$$\alpha = \frac{e^2 Z_P}{\hbar} = \frac{e^2}{\hbar Y_P}. \quad (12.10)$$

In loop-aggregated form,

$$\alpha = \frac{e^2 Z_P^{(\text{loop})}}{2\pi \hbar}. \quad (12.11)$$

Units check.

$$[\epsilon_0] = \text{C}/(\text{V m}), \quad [\hbar] = \text{V C s}, \quad [c] = \text{m/s}, \quad (12.12)$$

$$[\epsilon_0 \hbar c] = \text{C}^2, \quad [e^2] = \text{C}^2. \quad (12.13)$$

Numerical validation (CODATA 2022). *QLM route:*

$$\begin{aligned} e^2 &= \alpha \frac{\hbar}{Z_P} \\ &= (7.2973525643 \times 10^{-3})(1.054571817 \times 10^{-34}) \left(\frac{4\pi}{376.730313412} \right) \text{C}^2 \end{aligned} \quad (12.14)$$

$$= 2.5669699665078037 \times 10^{-38} \text{ C}^2. \quad (12.15)$$

Conventional route:

$$\begin{aligned} e^2 &= 4\pi \epsilon_0 \hbar c \alpha \\ &= 4\pi (8.8541878128 \times 10^{-12})(1.054571817 \times 10^{-34})(2.99792458 \times 10^8)(7.2973525643 \times 10^{-3}) \text{C}^2 \end{aligned} \quad (12.16)$$

$$= 2.5669699665078037 \times 10^{-38} \text{ C}^2. \quad (12.17)$$

Exact SI value:

$$e_{\text{exact}}^2 = (1.602176634 \times 10^{-19})^2 = 2.566969966535569956 \times 10^{-38} \text{ C}^2. \quad (12.18)$$

Both expressions agree with the exact SI value to machine precision.

13 Conclusion

The Quantum Lattice Model (QLM) reconstructs Planck-scale physics from a single deterministic phase-action postulate:

$$E_P t_P = \hbar, \quad c = \frac{\ell_P}{t_P}. \quad (13.1)$$

With (\hbar, ℓ_P, t_P) taken as primitive lattice increments, all canonical Planck quantities follow algebraically. Mass, momentum, energy density, force, electromagnetic charge, impedance, field amplitudes, and power reduce to minimal expressions containing no additional dimensional parameters.

Hydrogen provides the empirical calibration of this structure. The Bohr identities,

$$\hbar = m_e v_B a_0, \quad \alpha = \frac{v_B}{c}, \quad (13.2)$$

demonstrate that the fine-structure constant is a geometric phase-velocity ratio linking rotational atomic motion to linear lattice propagation. Atomic observables therefore anchor the Planck lattice without introducing free constants.

Electromagnetism emerges from the same phase-action primitive. The Planck charge is fixed by the vacuum impedance, and the electromagnetic quartet

$$(Z_P, V_P, I_P, P_P) \quad (13.3)$$

is uniquely determined by the per-radian identity $E_P t_P = \hbar$. Vacuum permittivity, permeability, and impedance follow directly from the lattice spacetime increments.

Across mechanical, electromagnetic, and hydrogenic sectors, all quantities collapse to a single internally consistent algebraic framework based on (\hbar, ℓ_P, t_P) and CODATA 2022 values. No additional curvature assumptions, supplementary fields, or independent coupling parameters are required at the level of Planck-unit construction.

The present work establishes the minimal canonical foundation of the Quantum Lattice Model. Subsequent developments apply this deterministic lattice structure to gravitational impedance, atomic structure, and cosmological dynamics.

Fundamental Constants and QLM Planck Units

Quantity	Symbol	Definition	Value / Status
<i>Exact SI-Defined Constants (Post-2019)</i>			
Speed of light	c	defined	2.99792458×10^8 m/s (exact)
Planck constant	h	defined	$6.62607015 \times 10^{-34}$ J · s (exact)
Reduced Planck const.	\hbar	$h/(2\pi)$	exact (derived from h)
Elementary charge	e	defined	$1.602176634 \times 10^{-19}$ C (exact)
<i>CODATA 2022 Measured Constants</i>			
Gravitational constant	G	measured	6.67430×10^{-11} m ³ kg ⁻¹ s ⁻²
Fine-structure const.	α	measured	$7.2973525643 \times 10^{-3}$
Vacuum impedance	Z_0	$\mu_0 c$	$3.76730313412 \times 10^2$ Ω
Vacuum permittivity	ϵ_0	$1/(Z_0 c)$	$8.8541878128 \times 10^{-12}$ F/m
Vacuum permeability	μ_0	Z_0/c	$1.25663706212 \times 10^{-6}$ N/A ²
<i>Derived Planck / QLM Quantities (Computed)</i>			
Planck time	t_P	$\sqrt{\hbar G/c^5}$	5.391247×10^{-44} s
Planck length	ℓ_P	ct_P	1.616255×10^{-35} m
Planck energy	E_P	\hbar/t_P	$1.95608143653 \times 10^9$ J
Planck mass	m_P	$\hbar t_P/\ell_P^2$	$2.17643412 \times 10^{-8}$ kg
Planck impedance (per-rad.)	Z_P	$Z_0/(4\pi)$	$2.99792458008 \times 10^1$ Ω
Planck admittance	Y_P	$1/Z_P$	$3.33564095198 \times 10^{-2}$ S
Planck charge	q_P	$\sqrt{\hbar/Z_P}$	$1.875545956 \times 10^{-18}$ C

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