# QUANTUM LATTICE MODEL

Planck Energy Density Cap and Minimal Saturated-Core Completion

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**Abstract**

The Quantum Lattice Model (QLM) formulates physics as deterministic phase–action evolution on a discrete Planck-scale spacetime lattice. In the foundational QLM framework, a universal Planck energy density arises from the transport of one quantum of reduced action per lattice four-volume. In this work, that result is elevated to an explicit local constraint, interpreted as a local, action-limited Planck energy density cap. We show that this cap implies a maximum lattice mass density and enforces a minimal, non-singular interior completion for gravitational collapse. The resulting interior consists of a finite saturated core whose radius scales as the cube root of the total mass, while the exterior gravitational field remains exactly Schwarzschild. No exotic matter, additional parameters, or modifications of general relativity outside the core are introduced. Numerical examples spanning terrestrial, stellar, and supermassive mass scales confirm the analytic scaling relations. Singular collapse is thereby forbidden as a direct consequence of the QLM density bound.

# 1 Introduction

Classical general relativity predicts the formation of spacetime singularities under gravitational collapse, signaling a breakdown of the theory at high densities. While such singularities are widely interpreted as indicators of missing ultraviolet physics, many proposed resolutions introduce additional fields, exotic states of matter, or phenomenological modifications of the Einstein equations. A central open question is whether singular collapse can be avoided as a direct and unavoidable consequence of a more fundamental description of spacetime itself, without altering the exterior gravitational field or introducing free parameters.

The Quantum Lattice Model (QLM), developed in *Quantum Lattice Model: Reduced-Action Foundations & Planck-Unit Derivations* (December 4, 2025), formulates physics as deterministic phase–action evolution rules on a discrete Planck-scale spacetime lattice. In this framework, each lattice four-volume supports exactly one radian of coherent phase evolution and transports one quantum of reduced action. From this primitive structure, a universal Planck energy density emerges, expressed entirely in terms of the lattice parameters $\hbar$, $\ell_P$, and $t_P$. This density was derived in the foundational work as a fundamental property of the lattice, rather than as an imposed cutoff or phenomenological bound.

The present paper builds directly on that result. Its purpose is not to introduce new physics, but to make explicit a consequence that is already implicit in the QLM foundations: the Planck energy density functions as a local, action-limited density cap. When treated as a strict constraint, this cap implies a maximum lattice mass density and forbids unlimited compression. We show that gravitational collapse must therefore terminate in a finite, non-singular interior configuration corresponding to a fully saturated lattice state.

The resulting interior completion consists of a compact saturated core whose radius scales as the cube root of the total mass. Importantly, the exterior gravitational field depends only on the enclosed mass and remains exactly Schwarzschild. No exotic matter, additional parameters, or modifications of general relativity outside the core are required. The construction is fixed entirely by the QLM lattice primitives and their associated density bound.

This paper is organized as follows. Section 2 reviews the QLM primitives and gravitational impedance formulation. Section 3 formulates the Planck energy density cap and its interpretation as an action-limited constraint. Section 4 derives the corresponding maximal mass density and the resulting saturated-core interior completion. Section 5 examines the scaling of the core radius relative to the Schwarzschild radius. Section 6 provides numerical examples spanning terrestrial, stellar, and supermassive mass scales, confirming the analytic scaling relations. A numerical appendix lists the constants and reference values used throughout.

## 2 Definitions and QLM Primitives

We work in Quantum Lattice Model (QLM) primitives,

$$\ell_P \quad \text{(Planck cell edge length)}, \qquad t_P \quad \text{(Planck tick)}, \qquad \hbar \quad \text{(reduced action quantum)}. \tag{2.1}$$

The single–cell (Planck cell) volume is

$$\mathcal{V}_P = \ell_P^3. \tag{2.2}$$

QLM adopts the exact kinematic identity

$$c \equiv \frac{\ell_P}{t_P}. \tag{2.3}$$

The primitive per–radian Planck energy is

$$E_P \equiv \frac{\hbar}{t_P}. \tag{2.4}$$

The QLM Planck mass follows directly as

$$m_P \equiv \frac{E_P}{c^2} = \frac{\hbar\, t_P}{\ell_P^2}. \tag{2.5}$$

All subsequent quantities in this work are expressed in terms of the invariant QLM primitives $\{\hbar, \ell_P, t_P\}$ and derived combinations thereof.

## 2.1 Gravitational Impedance / Throttle Factor

In the Quantum Lattice Model, gravity manifests as a modulation of local phase–flow bandwidth rather than as a geometric deformation. Gravitational time dilation is interpreted as an impedance that throttles the rate of lattice updates relative to an asymptotic reference frame.

For a spherically symmetric mass $M$, the gravitational impedance (time–dilation throttle) factor is

$$Z_g(r) = \left(1 - \frac{r_s}{r}\right)^{-1/2}. \tag{2.6}$$

Expressed entirely in QLM Planck variables, this becomes

$$Z_g(r) = \left[1 - 2\left(\frac{M}{m_P}\right)\left(\frac{\ell_P}{r}\right)\right]^{-1/2}. \tag{2.7}$$

The Schwarzschild radius is therefore

$$r_s = 2\left(\frac{M}{m_P}\right)\ell_P. \tag{2.8}$$

The divergence of $Z_g(r)$ as $r \to r_s$ corresponds to complete throttling of outward action transport in the asymptotic frame. In QLM, this divergence reflects a bandwidth limit imposed by the lattice rather than the appearance of a physical membrane or singular behavior of spacetime itself.

Equations (2.6)–(2.8) reproduce the standard Schwarzschild time–dilation relations exactly, while admitting a direct interpretation in terms of phase–action transport on the Planck lattice.

# 3 Planck Energy Density Cap (Action-Limited)

## 3.1 Per–Cell Action Transport

Each Planck four–volume,

$$\mathcal{V}_4 = \ell_P^3 t_P, \tag{3.1}$$

supports exactly one radian of coherent phase evolution and therefore transports one quantum of reduced action,

$$\Delta S = \hbar \qquad \text{per tick } t_P. \tag{3.2}$$

Because energy is action per unit time, this immediately defines the fundamental per–radian Planck energy,

$$E_P = \frac{\hbar}{t_P}. \tag{3.3}$$

## 3.2 Planck Energy Density

The corresponding Planck energy density is

$$\boxed{u_P \equiv \frac{E_P}{\ell_P^3} = \frac{\hbar}{\ell_P^3 t_P}.} \tag{3.4}$$

This quantity is identical to the Planck energy density derived in Sec. 8.3 of *Quantum Lattice Model: Reduced–Action Foundations & Planck–Unit Derivations* (December 4, 2025).

Because energy is action per unit time, the Planck energy density already encodes the maximal action throughput permitted by the lattice. No additional physical assumptions or phenomenological cutoffs are introduced.

## 3.3 Local Energy Density Cap

The Quantum Lattice Model therefore enforces the universal local constraint

$$\boxed{u(r) \leq u_P.} \tag{3.5}$$

This bound is an intrinsic property of the lattice dynamics. It is not imposed externally and introduces no new density scale beyond the QLM primitives.

# 4 Gravitational Throttling of Energy Density

Gravitational time dilation relates the local proper time of a lattice cell to the time coordinate measured by an observer at infinity according to

$$Z_g(r) \equiv \frac{dt_\infty}{dt_{\text{local}}}. \tag{4.1}$$

Let $Q$ denote any physical quantity defined locally on the lattice, such as energy, action, or action density. Rates of change with respect to asymptotic time are related to local rates by the chain rule,

$$\frac{dQ}{dt_\infty} = \frac{dt_{\text{local}}}{dt_\infty}\frac{dQ}{dt_{\text{local}}} = \frac{1}{Z_g(r)}\frac{dQ}{dt_{\text{local}}}. \tag{4.2}$$

Thus, while local lattice update rates remain fixed by the Planck-scale action throughput, any rate expressed per unit asymptotic time is throttled by the gravitational impedance factor $Z_g(r)$.

Applying this general relation to the Planck energy density $u_P$, which is a local lattice quantity fixed by the action-limited throughput constraint, yields the energy density as observed from infinity,

$$\boxed{u_\infty(r) = \frac{u_P}{Z_g(r)}.} \tag{4.3}$$

Gravity therefore acts as a bandwidth limiter on energy transport: the local energy density cap remains fixed at $u_P$, while the rate at which energy can be communicated to infinity is progressively reduced by gravitational impedance.

No modification of the local density bound is required, and the exterior Schwarzschild geometry remains exact.

# 5 Minimal Interior Completion: Saturated Core

## 5.1 Maximum Lattice Mass Density

Using the relation $u = \rho c^2$ together with the kinematic identity $c = \ell_P / t_P$, the Planck energy density cap implies a maximum lattice mass density,

$$\boxed{\rho_{\max} = \frac{u_P}{c^2} = \frac{m_P}{\ell_P^3}.} \tag{5.1}$$

This quantity represents the maximum mass that can be supported per Planck cell volume without violating the action–throughput constraint of the lattice.

## 5.2 Equivalent Action and Mass Forms

Substituting the QLM Planck mass,

$$m_P = \frac{\hbar t_P}{\ell_P^2}, \tag{5.2}$$

into Eq. (5.1) yields

$$\rho_{\max} = \frac{\hbar t_P}{\ell_P^5}. \tag{5.3}$$

An equivalent result follows directly from the Planck energy density,

$$\rho_{\max} = \frac{u_P}{c^2} = \frac{\hbar}{\ell_P^3 t_P}\left(\frac{t_P}{\ell_P}\right)^2 = \frac{\hbar t_P}{\ell_P^5}. \tag{5.4}$$

Thus the action–based, energy–based, and mass–based formulations all reduce to the same invariant lattice density, fixed entirely by the QLM primitives.

## 5.3 Interior Density Profile

Once the Planck energy density cap is reached, further compression is forbidden. The Quantum Lattice Model therefore enforces a saturated interior configuration,

$$\rho(r) = \rho_{\max}, \qquad (r \le R_c), \tag{5.5}$$

corresponding to a maximally occupied Planck lattice state.

This saturated region represents the minimal non–singular interior completion of gravitational collapse.

## 5.4 Interior Mass Function

For a spherically symmetric saturated core, the enclosed mass for $r \le R_c$ is

$$M(r) = \frac{4\pi}{3}\,\rho_{\max}\, r^3. \tag{5.6}$$

Imposing the condition $M(R_c) = M$ fixes the core radius uniquely,

$$\boxed{R_c(M) = \left(\frac{3}{4\pi}\frac{M}{m_P}\right)^{1/3} \ell_P.} \tag{5.7}$$

The saturated–core radius therefore scales as $M^{1/3}$, in contrast to the linear scaling of the Schwarzschild radius.

## 5.5 Exterior Matching

For radii exterior to the saturated core,

$$M(r) = M, \qquad (r \geq R_c), \tag{5.8}$$

and by spherical symmetry the exterior gravitational field is exactly Schwarzschild,

$$r_s = 2\left(\frac{M}{m_P}\right)\ell_P. \tag{5.9}$$

No modification of the exterior geometry is required. The saturated–core completion alters only the interior structure, while all standard exterior predictions of general relativity remain intact.

# 6 Numerical Validation of Scaling Relations

All numerical values in this section use CODATA 2022 constants identical to those employed in the December 4, 2025 QLM foundations paper. The purpose of these examples is purely confirmatory: to illustrate the magnitude and scaling behavior of the saturated–core radius relative to the Schwarzschild radius across a wide range of masses.

## 6.1 Earth–Mass Example

For a mass equal to that of the Earth,

$$M = M_\oplus = 5.9722 \times 10^{24}\ \text{kg}, \tag{6.1}$$

the Schwarzschild radius is

$$r_s(M_\oplus) \approx 8.87 \times 10^{-3}\ \text{m}, \tag{6.2}$$

while the corresponding saturated–core radius is

$$R_c(M_\oplus) \approx 6.5 \times 10^{-25}\ \text{m}. \tag{6.3}$$

The ratio of the core radius to the Schwarzschild radius is therefore

$$\frac{R_c}{r_s} \sim 10^{-22}, \tag{6.4}$$

demonstrating that even for terrestrial masses the saturated core lies far below the horizon scale.

## 6.2 Solar–Mass Example

For a solar–mass object,

$$M = M_\odot = 1.98847 \times 10^{30} \text{ kg}, \tag{6.5}$$

the Schwarzschild radius is

$$r_s(M_\odot) \approx 2.95 \times 10^3 \text{ m}, \tag{6.6}$$

while the saturated–core radius is

$$R_c(M_\odot) \approx 4.5 \times 10^{-23} \text{ m}. \tag{6.7}$$

This yields

$$\frac{R_c}{r_s} \sim 10^{-26}, \tag{6.8}$$

indicating that for stellar masses the saturated core is even more deeply hidden within the horizon.

## 6.3 Supermassive Example ($10^6 M_\odot$)

For a supermassive black hole of mass

$$M = 10^6 M_\odot = 1.98847 \times 10^{36} \text{ kg}, \tag{6.9}$$

the Schwarzschild radius is

$$r_s(10^6 M_\odot) \approx 2.95 \times 10^9 \text{ m}, \tag{6.10}$$

while the saturated–core radius is

$$R_c(10^6 M_\odot) \approx 4.5 \times 10^{-21} \text{ m}. \tag{6.11}$$

The ratio becomes

$$\frac{R_c}{r_s} \sim 10^{-30}, \tag{6.12}$$

showing that for astrophysical black holes the saturated core is effectively invisible to all exterior classical observables.

## 6.4 Scaling Confirmation

These examples confirm the analytic scaling relations derived earlier. Increasing the mass by a factor of $10^6$ increases the Schwarzschild radius by the same factor, while the saturated–core radius increases only by a factor of $10^2$. Consequently, the ratio of the two scales obeys

$$\frac{R_c}{r_s} \propto M^{-2/3}, \tag{6.13}$$

and rapidly decreases with increasing mass.

The numerical results therefore verify the fundamental QLM predictions

$$r_s \propto M, \tag{6.14}$$

$$R_c \propto M^{1/3}, \tag{6.15}$$

and demonstrate that the saturated–core completion remains deeply hidden inside the horizon for all astrophysical black holes.

# 7 QLM Interpretation of Black-Hole Thermodynamics

Classical black-hole thermodynamics assigns a temperature and entropy to Schwarzschild black holes, despite their ostensibly vacuum exterior. In standard treatments, these quantities are introduced through semiclassical quantum-field calculations on curved spacetime. In contrast, the Quantum Lattice Model (QLM) reproduces the same scaling laws associated with black-hole thermodynamics, but reinterprets what those quantities mean physically.

## 7.1 Horizon Size as Impedance Divergence

The Schwarzschild radius,

$$r_s = 2\left(\frac{M}{m_P}\right)\ell_P, \tag{7.1}$$

corresponds in QLM to the radius at which the gravitational impedance factor,

$$Z_g(r) = \left(1 - \frac{r_s}{r}\right)^{-1/2}, \tag{7.2}$$

diverges in the asymptotic frame.

At this surface, outward action transport toward infinity is fully throttled. The event horizon therefore represents not a physical membrane, but a bandwidth limit imposed by the lattice on phase–action propagation.

## 7.2 Hawking Temperature from Action Throughput

The standard Hawking temperature,

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}, \tag{7.3}$$

may be expressed entirely in QLM primitives by using the kinematic identity $c = \ell_P/t_P$ together with the reduced gravitational coupling,

$$\frac{G}{c^2} = \frac{\ell_P^3}{\hbar t_P}, \tag{7.4}$$

which follows directly from the fundamental QLM definition $G = \ell_P^5/(t_P^3\hbar)$.

Substitution yields

$$\boxed{T_H = \frac{1}{8\pi k_B}\frac{\hbar}{t_P}\frac{1}{M/m_P}.} \tag{7.5}$$

The quantity $\hbar/t_P$ represents the per–tick Planck action energy transported by a single lattice four–volume, while the inverse factor $(M/m_P)^{-1}$ reflects the number of Planck mass quanta sourcing the gravitational impedance.

In QLM, Hawking temperature therefore characterizes the residual action leakage rate from a maximally throttled lattice configuration, rather than the thermodynamic state of interior degrees of freedom or microscopic states.

### 7.3 Entropy as Frozen Lattice Capacity

The Bekenstein–Hawking entropy,

$$S_{\rm BH} = \frac{k_B c^3 A}{4\hbar G}, \tag{7.6}$$

with horizon area $A = 4\pi r_s^2$, reduces in Quantum Lattice Model (QLM) variables to

$$\boxed{S_{\rm BH} = 4\pi k_B \left(\frac{M}{m_P}\right)^2.} \tag{7.7}$$

Entropy is therefore proportional to the square of the dimensionless mass ratio $M/m_P$. In the Quantum Lattice Model, this quantity counts the total number of lattice action-transport channels rendered inaccessible to outward propagation by gravitational impedance.

Entropy is thus not a measure of interior microstates. Instead, it represents *frozen lattice action capacity* stored at the impedance boundary, arising as a direct consequence of the Planck energy density cap and the resulting throttling of phase–action transport.

### 7.4 Unified QLM Scaling

All Schwarzschild black-hole properties are controlled by the single dimensionless mass ratio

$$\frac{M}{m_P}. \tag{7.8}$$

They scale as

$$r_s \propto \frac{M}{m_P}, \tag{7.9}$$

$$T_H \propto \left(\frac{M}{m_P}\right)^{-1}, \tag{7.10}$$

$$S_{\rm BH} \propto \left(\frac{M}{m_P}\right)^2. \tag{7.11}$$

These relations follow directly from the Planck energy density cap and the gravitational impedance formulation already established in this work. Black-hole thermodynamics is therefore an emergent consequence of action-limited lattice dynamics, not an independent physical principle.

## 8 Derived Near–Horizon Implications of the Planck Energy Density Cap

The Planck energy density cap introduced in Secs. 3–5 has unavoidable *dynamical implications* near the Schwarzschild radius. Once the local action–throughput limit of the lattice is enforced, the gravitational impedance necessarily diverges as this bound is approached. This divergence constrains how gravitational perturbations and outward action flux may propagate, without introducing new boundary conditions or modifying the exterior Schwarzschild geometry.

Two closely related consequences follow directly from this constraint: (i) inward–propagating gravitational perturbations encounter a high–impedance region and are partially reflected, leading to delayed ringdown echoes; and (ii) outward action leakage becomes intrinsically bandwidth–limited, implying a late–time breakdown of semiclassical evaporation and a natural endpoint. Both effects arise from the same lattice throughput limit and require no additional postulates.

## 8.1 Impedance Divergence and Reflection of Gravitational Perturbations

In classical general relativity, the interior of a Schwarzschild black hole is treated as a perfect absorber: inward–propagating perturbations are lost to a singularity and no reflection occurs. In the Quantum Lattice Model, this assumption fails once the Planck energy density cap is enforced.

As shown in Secs. 4 and 5, saturation of the lattice forces the effective impedance to diverge. Additional phase–action transport then becomes impossible, and a fully saturated region behaves as a high–impedance termination for propagating impedance disturbances. Inward–propagating gravitational waves therefore encounter partial or full reflection once the required local update rate exceeds the lattice bandwidth, prior to horizon crossing in the asymptotic frame.

Together with the exterior Schwarzschild perturbation potential, this reflection creates a finite near–horizon cavity that gives rise to delayed secondary signals ("echoes") in the gravitational–wave ringdown.

## 8.2 Radial Propagation and Echo Delay

Impedance perturbations propagate according to the causal wave equation derived in Appendix B,

$$\frac{\partial^2(\delta Z)}{\partial t^2} = c^2\nabla^2(\delta Z), \tag{8.1}$$

which governs the transport of phase–action disturbances on the lattice.

In a Schwarzschild exterior, it is convenient to introduce the standard *tortoise coordinate*,

$$\boxed{r_* \equiv r + r_s \ln\left(\frac{r}{r_s} - 1\right),} \qquad (r > r_s), \tag{8.2}$$

which restores the characteristic propagation speed $c$ in the $(t, r_*)$ variables. This coordinate is employed here purely as a mathematical reparameterization that linearizes radial propagation in the asymptotic frame; it does not represent a physical radial distance or a distinct lattice scale.

Echoes arise from repeated propagation between two effective regions:

- the exterior perturbation barrier associated with the Schwarzschild potential, located near the photon sphere,

$$\boxed{r_{\text{peak}} \approx \frac{3}{2}\, r_s,} \tag{8.3}$$

- and an inner region where inward propagation becomes dynamically forbidden because the required local lattice update rate would exceed the maximum permitted action throughput.

A single echo corresponds to a round trip of an impedance disturbance between these regions, giving an echo delay

$$\boxed{\Delta t_{\text{echo}} \approx \frac{2}{c}\left[r_*(r_{\text{peak}}) - r_*(r_{\text{ref}})\right],} \tag{8.4}$$

where $r_{\text{ref}}$ denotes the radius at which inward propagation saturates the lattice bandwidth.

Evaluating the tortoise coordinate at the exterior barrier gives

$$r_*(r_{\text{peak}}) = \left(\frac{3}{2} - \ln 2\right) r_s \approx 0.8069\, r_s. \tag{8.5}$$

## 8.3 Cap–Limited Bandwidth and Determination of the Reflection Radius

A disturbance of asymptotic frequency $f_\infty$ experiences gravitational blueshift according to the impedance throttle factor,

$$f_{\text{local}}(r) = Z_g(r)\, f_\infty, \tag{8.6}$$

with

$$Z_g(r) = \left(1 - \frac{r_s}{r}\right)^{-1/2}. \tag{8.7}$$

The Planck lattice admits a maximum local update rate,

$$f_P \equiv \frac{1}{t_P}, \tag{8.8}$$

fixed by the per–radian action quantum.

Inward propagation ceases once the locally required update rate equals this maximum,

$$f_{\text{local}}(r_{\text{ref}}) = f_P. \tag{8.9}$$

This condition does not represent an imposed boundary or a physical surface. Rather, it marks the radius at which further inward transport would require super–Planckian action throughput and is therefore dynamically forbidden by the lattice.

Solving Eq. (8.9) gives

$$1 - \frac{r_s}{r_{\text{ref}}} = (f_\infty t_P)^2, \tag{8.10}$$

and substitution into Eq. (8.4) yields the parameter–free echo delay,

$$\boxed{\Delta t_{\text{echo}} \approx \frac{2r_s}{c}\left[2\ln\left(\frac{1}{f_\infty t_P}\right) - 0.1931\right].} \tag{8.11}$$

### 8.4 Frequency–Selective Leakage and Evaporation Endpoint

The same action–throughput constraint governs outward propagation. Consistency with the Planck energy density cap requires that escaping modes satisfy

$$f_\infty \leq \frac{f_P}{Z_g(r)}. \tag{8.12}$$

As $r \to r_s$, the impedance factor $Z_g$ diverges and the allowed escaping frequency range collapses toward zero. Near–horizon leakage is therefore intrinsically low–frequency, while higher–frequency emission can originate only at larger radii where the lattice bandwidth is less severely throttled.

From Sec. 5, gravitational collapse terminates in a finite saturated core of radius

$$R_c(M) = \left(\frac{3}{4\pi}\frac{M}{m_P}\right)^{1/3} \ell_P. \tag{8.13}$$

Requiring the saturated core to comprise a finite number of Planck cells implies

$$\boxed{R_c \geq \ell_P,} \tag{8.14}$$

which yields a minimum mass

$$\boxed{M_{\min} = \frac{4\pi}{3}\, m_P.} \tag{8.15}$$

As this scale is approached, further evaporation would require super–Planckian action throughput or violation of the lattice density cap, both of which are forbidden. The semi-classical scaling $T_H \propto M^{-1}$ must therefore break down at late times, leading to evaporation stall or a long–lived Planck–scale remnant.

# Unified Interpretation

Gravitational–wave echoes and the evaporation endpoint arise from the same underlying mechanism: divergence of gravitational impedance enforced by the Planck energy density cap. Inward perturbations are halted once local lattice bandwidth is exhausted, while outward leakage is progressively filtered to ultra–low–frequency modes and ultimately terminated. Both effects follow directly from the QLM action–throughput constraint and require no additional postulates, exotic matter, or modification of the exterior Schwarzschild geometry.

- The Quantum Lattice Model enforces a universal Planck energy density cap,

$$u_P = \frac{\hbar}{\ell_P^3 t_P}, \tag{8.16}$$

  arising directly from the transport of one quantum of reduced action per lattice four–volume per lattice tick $t_P$.

- This cap implies a maximum lattice mass density admitting equivalent action–based, energy–based, and mass–based formulations.
- Gravitational time dilation acts as an impedance that throttles energy and action transport without modifying the local lattice bound.
- Gravitational collapse halts into a finite saturated core of radius $R_c(M) \propto M^{1/3}$, providing a minimal non–singular interior completion.
- The exterior gravitational field remains exactly Schwarzschild; no exotic matter, additional parameters, or modifications of general relativity are introduced.
- Near–horizon consequences of the density cap include partial reflection of gravitational perturbations (producing echoes) and intrinsic frequency filtering of outward action leakage, yielding a natural evaporation endpoint.
- Black–hole temperature and entropy arise as *derived, emergent* measures of throttled and frozen lattice action capacity.

The propagating impedance perturbations responsible for these near-horizon effects also govern gravitational-wave radiation in the QLM; a detailed lattice-level derivation is provided in Appendix B.

# A Numerical Appendix: Constants and Reference Values

This appendix lists the numerical constants and reference values used throughout the paper. Values follow CODATA 2022 and are presented in the same rounded format employed in the December 4, 2025 QLM foundations paper.

## A.1 Fundamental Constants (CODATA 2022)

$$\begin{aligned} c &= 2.997\,925 \times 10^{8}\ \mathrm{m\,s^{-1}}, \\ \hbar &= 1.054\,572 \times 10^{-34}\ \mathrm{J\,s}, \\ G &= 6.674\,300 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}} \quad (\text{derived from } G = \ell_P^5/(t_P^3\hbar)), \\ t_P &= 5.391\,247 \times 10^{-44}\ \mathrm{s}, \\ \ell_P &= 1.616\,255 \times 10^{-35}\ \mathrm{m}. \end{aligned} \tag{A.1}$$

## A.2 Derived QLM Planck Quantities

Per–radian Planck energy:

$$E_P = \frac{\hbar}{t_P} \approx 1.956\,081 \times 10^{9}\ \mathrm{J}. \tag{A.2}$$

QLM Planck mass:

$$m_P = \frac{\hbar t_P}{\ell_P^2} \approx 2.176\,434 \times 10^{-8}\ \mathrm{kg}. \tag{A.3}$$

Planck energy density:

$$u_P = \frac{\hbar}{\ell_P^3 t_P} \approx 4.632\,947 \times 10^{113}\ \mathrm{J\,m^{-3}}. \tag{A.4}$$

Maximum lattice mass density:

$$\rho_{\max} = \frac{m_P}{\ell_P^3} = \frac{\hbar t_P}{\ell_P^5} \approx 5.154\,849 \times 10^{96}\ \mathrm{kg\,m^{-3}}. \tag{A.5}$$

### A.3 Reference Masses

$$\begin{aligned} M_\oplus &= 5.9722 \times 10^{24}\ \mathrm{kg}, \\ M_\odot &= 1.98847 \times 10^{30}\ \mathrm{kg}, \\ 10^6 M_\odot &= 1.98847 \times 10^{36}\ \mathrm{kg}. \end{aligned} \tag{A.6}$$

### A.4 Reference Schwarzschild Radii

Using $r_s = 2GM/c^2$,

$$\begin{aligned} r_s(M_\oplus) &\approx 8.87 \times 10^{-3}\ \mathrm{m}, \\ r_s(M_\odot) &\approx 2.95 \times 10^{3}\ \mathrm{m}, \\ r_s(10^6 M_\odot) &\approx 2.95 \times 10^{9}\ \mathrm{m}. \end{aligned} \tag{A.7}$$

### A.5 Reference Saturated–Core Radii

Using $R_c(M) = \left(\frac{3}{4\pi}\frac{M}{m_P}\right)^{1/3} \ell_P$,

$$\begin{aligned} R_c(M_\oplus) &\approx 6.5 \times 10^{-25}\ \mathrm{m}, \\ R_c(M_\odot) &\approx 4.5 \times 10^{-23}\ \mathrm{m}, \\ R_c(10^6 M_\odot) &\approx 4.5 \times 10^{-21}\ \mathrm{m}. \end{aligned} \tag{A.8}$$

## B Gravitational Waves as Vacuum–Impedance Collapse

This appendix provides a detailed derivation of the dynamical impedance perturbations used in the main text. The results establish that gravitational radiation in the Quantum Lattice Model (QLM) corresponds to propagating modulations of lattice phase–flow impedance, rather than to geometric curvature waves. This derivation is included here for completeness and does not alter the exterior Schwarzschild solutions employed in the main body of the paper.

## B.1 Impedance Decomposition

In QLM, gravity is a modulation of lattice phase–flow bandwidth. We decompose the total lattice impedance as

$$Z(\mathbf{x}, t) = Z_0 + Z_g(\mathbf{x}) + \delta Z(\mathbf{x}, t), \tag{B.1}$$

where $Z_0$ is the vacuum impedance corresponding to minimal phase–volume, $Z_g(\mathbf{x})$ is the static gravitational impedance sourced by a stationary mass distribution, and $\delta Z(\mathbf{x}, t)$ represents time–dependent impedance perturbations.

Only $\delta Z(\mathbf{x}, t)$ contributes to gravitational radiation. A static mass distribution produces no propagating disturbance.

## B.2 Action Conservation and Causality

The QLM vacuum does not store energy. Instead, it supports a conserved action density $a(\mathbf{x}, t)$ whose transport obeys a local continuity relation,

$$\frac{\partial a}{\partial t} + \nabla \cdot \mathbf{J}_a = 0, \tag{B.2}$$

Here $a(\mathbf{x}, t)$ denotes the local action density, and $\mathbf{J}_a$ the corresponding action flux density, representing the transport of reduced action per unit area per unit time across the lattice.

Because each Planck cell transports a fixed quantum of reduced action $\hbar$ per lattice tick, local accumulation of excess action is forbidden. Instantaneous nonlocal redistribution is also prohibited by causality. Therefore, time–dependent impedance variations cannot remain localized and must propagate outward as a causal lattice response.

## B.3 Vacuum Collapse as the Restoring Mechanism

When a moving mass transiently dilates the lattice, the local impedance increases. Once the supporting mass distribution shifts, the lattice is no longer sustained in the dilated configuration.

The vacuum lattice exhibits a universal tendency to return to its minimal phase–volume state. This enforced return acts as a restoring mechanism:

- impedance increase corresponds to lattice displacement,
- vacuum phase minimization supplies the restoring drive,
- causal lattice updates enforce outward propagation.

Gravitational radiation therefore arises from repeated cycles of lattice dilation, release, and vacuum collapse. Radiation is not caused by energy emission from the mass itself, but by the lattice's inability to sustain time–dependent impedance configurations.

## B.4 Impedance Wave Equation

We relate perturbations of the local action density to impedance variations through a linear constitutive relation,

$$\delta a = \chi\,\delta Z, \tag{B.3}$$

where $\chi$ is the *lattice action susceptibility*, characterizing the change in locally supported action density produced by a given impedance variation. This coefficient plays a role analogous to a susceptibility in standard linear–response theory and does not represent energy storage or dissipation.

The corresponding action–flux response is taken to be purely reactive (lossless),

$$\frac{\partial \mathbf{J}_a}{\partial t} = -\gamma\,\nabla(\delta Z), \tag{B.4}$$

where $\gamma$ is a *reactive lattice stiffness* coefficient relating spatial gradients of impedance to the time evolution of the action flux. The symbol $\gamma$ here does not denote a Lorentz factor, decay rate, or thermodynamic index; it is a lattice transport coefficient characterizing the restoring response of the vacuum impedance field.

Combining Eqs. (B.2)–(B.4) yields the impedance wave equation,

$$\boxed{\frac{\partial^2(\delta Z)}{\partial t^2} = v_g^2\,\nabla^2(\delta Z),} \tag{B.5}$$

with propagation speed

$$v_g^2 = \frac{\gamma}{\chi}. \tag{B.6}$$

Causality of the Planck lattice enforces

$$v_g = c = \frac{\ell_P}{t_P}, \tag{B.7}$$

thereby fixing the ratio $\gamma/\chi$ uniquely in terms of the lattice primitives. Neither $\chi$ nor $\gamma$ constitutes a free phenomenological parameter.

Thus, gravitational impedance perturbations propagate as causal lattice waves at the universal phase speed, identical to electromagnetic disturbances, while remaining entirely distinct from geometric curvature perturbations.

## B.5 Maxwell–Like First–Order Formulation

The impedance dynamics may equivalently be expressed in a first–order, Maxwell–like system. Defining $\delta Z(\mathbf{x},t)$ and $\mathbf{J}_a(\mathbf{x},t)$ as conjugate lattice variables, we write

$$\boxed{\frac{\partial(\chi\,\delta Z)}{\partial t} + \nabla\cdot\mathbf{J}_a = 0} \qquad \text{(action continuity)}, \tag{B.8}$$

$$\boxed{\frac{\partial \mathbf{J}_a}{\partial t} + \gamma\,\nabla(\delta Z) = 0} \qquad \text{(reactive lattice response)}. \tag{B.9}$$

These equations reproduce the wave equation (B.5) and clarify the causal structure of gravitational radiation in QLM.

## B.6 Relation to Observables

Interferometric detectors respond to the integrated phase delay induced by impedance perturbations along their arms. The measured strain therefore scales as

$$\frac{\Delta L}{L} \propto \int_{\text{path}} \frac{\delta Z(\mathbf{x},t)}{Z_0}\, d\lambda. \tag{B.10}$$

This relation connects lattice impedance dynamics directly to observable gravitational–wave signals without invoking spacetime curvature perturbations.

## B.7 Notation and Symbols

The following symbols are used throughout the paper. All quantities are defined at their first occurrence; this table is provided for reference.

- $\ell_P$ — Planck lattice cell edge length.
- $t_P$ — Planck lattice tick (fundamental time step).
- $\hbar$ — Reduced quantum of action per lattice tick.
- $c$ — Lattice phase speed, $c = \ell_P/t_P$.
- $\mathcal{V}_P$ — Single Planck cell volume, $\ell_P^3$.
- $E_P$ — Per–radian Planck energy, $\hbar/t_P$.
- $m_P$ — QLM Planck mass, $\hbar t_P/\ell_P^2$.
- $u_P$ — Planck energy density cap, $\hbar/(\ell_P^3 t_P)$.
- $\rho_{\max}$ — Maximum lattice mass density, $m_P/\ell_P^3$.
- $M$ — Total gravitational mass sourcing the lattice.
- $r_s$ — Schwarzschild radius, $2(M/m_P)\ell_P$.
- $R_c$ — Saturated–core radius.
- $Z_g(r)$ — Gravitational impedance (time–dilation) factor.
- $a(\mathbf{x},t)$ — Local action density.
- $\mathbf{J}_a$ — Action–flux density.
- $\chi$ — Lattice action susceptibility.
- $\gamma$ — Reactive lattice stiffness coefficient.
- $\delta Z$ — Impedance perturbation.

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