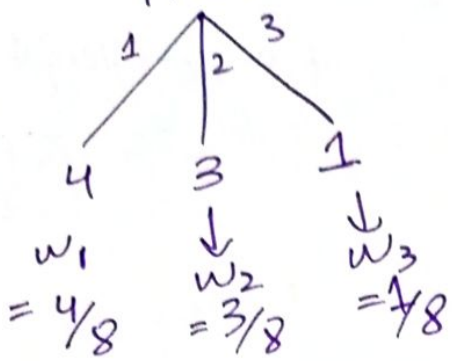
# Solution of Example #2 in Slides




$$H(Hair) = w_1 H(Hair=1) + w_2 H(Hair=2) + w_3 H(Hair=3)$$

$$H(Hair=1) = -\sum_{i=1}^{n} p_i \log_2 p_i$$

$$= -\left[\frac{2}{4}\log_2(2/4) + 2/4 \log_2(2/4)\right]$$

$$= -\left[1/2(-1) + 1/2(-1)\right]$$

$H(Hair=1) = 1$

| Hair=1 | result |
|---|---|
| 1 | 1 |
| 1 | 0 |
| 1 | 1 |
| 1 | 0 |

$P_1 = 2/4$

$P_0 = 2/4$

$H(Hair=2) = -[0 + 1 \log_2(1)]$

$= 0$

| | |
|---|---|
| 2 | 0 |
| 2 | 0 |
| 2 | 0 |

$P_1(0)$ $P_0 = 3/3 = 1$

$H(Hair=3) = 0$

| | |
|---|---|
| 3 | 1 |

$P_1 = 1/1 = 1$

$P_0 = 0$

$$H(Hair) = 4/8 \times 1 + 3/8 \times 0 + 1/8 \times 0$$

$$= 4/8$$

$$IG(Hair) = H(Parent) - H(Hair)$$

$H(Parent)$ ← result

$$IG(Hair) = 0.955 - 4/8 = 0.455$$

$$H(Result) = -(3/8 \log_2 3/8 + 5/8 \log_2 5/8) = 0.955$$

Height


$w_1 = 3/8$ $w_2 = 3/8$ $w_3 = 2/8$

$H(Height) = w_1 H(Height=1) + w_2 H(Height=2) + w_3 H(Height=3)$

=

Height = 1

| | |
|---|---|
| 1 | 0 |
| 1 | 1 |
| 1 | 0 |

$P_1 = 1/3$ $P_0 = 2/3$

$H(Height=1) = -(1/3 \log_2 1/3 + 2/3 \log_2 2/3)$

$= (-0.528 - 0.3899)$

$= 0.9179$

$H(Height=2) = 0.9179$

Height = 2

| | |
|---|---|
| 2 | 1 |
| 2 | 1 |
| 2 | 0 |

$P_1 = 2/3$ $P_0 = 1/3$

Height = 3

| | |
|---|---|
| 3 | 0 |
| 3 | 0 |

$P_1 = 0$ $P_0 = 2/2 = 1$

$H(Height=3) = -(0 + 1 \log_2(1))$

$H(Height=3) = 0$

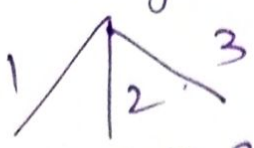
$H(Height) = 3/8 \times 0.9179 + 3/8 \times 0.9179 + 2/8 \times 0$

$= 0.6884$

$IG(Height) = H(Result) - H(Height)$

$IG(Height) = 0.955 - 0.6884 = 0.267$

Weight




$w_1 = 2/8$ $w_2 = 3/8$ $w_3 = 3/8$

weight = 1

1 1
1 0

$P_1 = 1/2$, $P_0 = 1/2$

weight = 2

2 0
2 0
2 1

$P_1 = 1/3$, $P_0 = 2/3$

weight = 3

3 1
3 0
3 0

$P_1 = 1/3$, $P_0 = 2/3$

$H(\text{weight}=3) = 0.918$

$H(\text{weight}=1) = 1$

$H(\text{weight}=2) = 0.918$

$H(\text{weight}) = 2/8 \times 1 + 3/8 \times 0.918 + 3/8 \times 0.918$

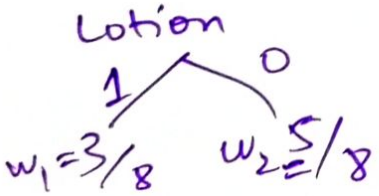
$\cong 0.939$

$IG(\text{weight}) = H(\text{Result}) - H(\text{weight})$

$= 0.955 - 0.939$

$\boxed{IG(\text{weight}) = 0.016}$

Lotion




$w_1 = 3/8$ $w_2 = 5/8$

Lotion = 1

1 0
1 0
1 0

$P_1 = 0$, $P_0 = 3/3 = 1$

Lotion = 0

0 1
0 1
0 1
0 0
0 0

$P_1 = 3/5$, $P_0 = 2/5$

$H(\text{Lotion}=1) = 0$

$H(\text{Lotion}=0) = -\left(\left(3/5 \log_2 3/5\right) + 2/5 \log_2 2/5\right)$

$H(\text{Lotion}=0) = 0.971$

$IG(\text{Lotion}) = H(\text{Result}) - H(\text{Lotion})$

$= 0.955 - \left[3/8 \times 0 + 5/8 \times 0.971\right]$

$= 0.955 - 0.607$

$\boxed{IG(\text{Lotion}) = 0.348}$