

Supportive Information – Phonon-accurate machine-learning potentials from automated workflows

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ABSTRACT

The supportive information for the main article.

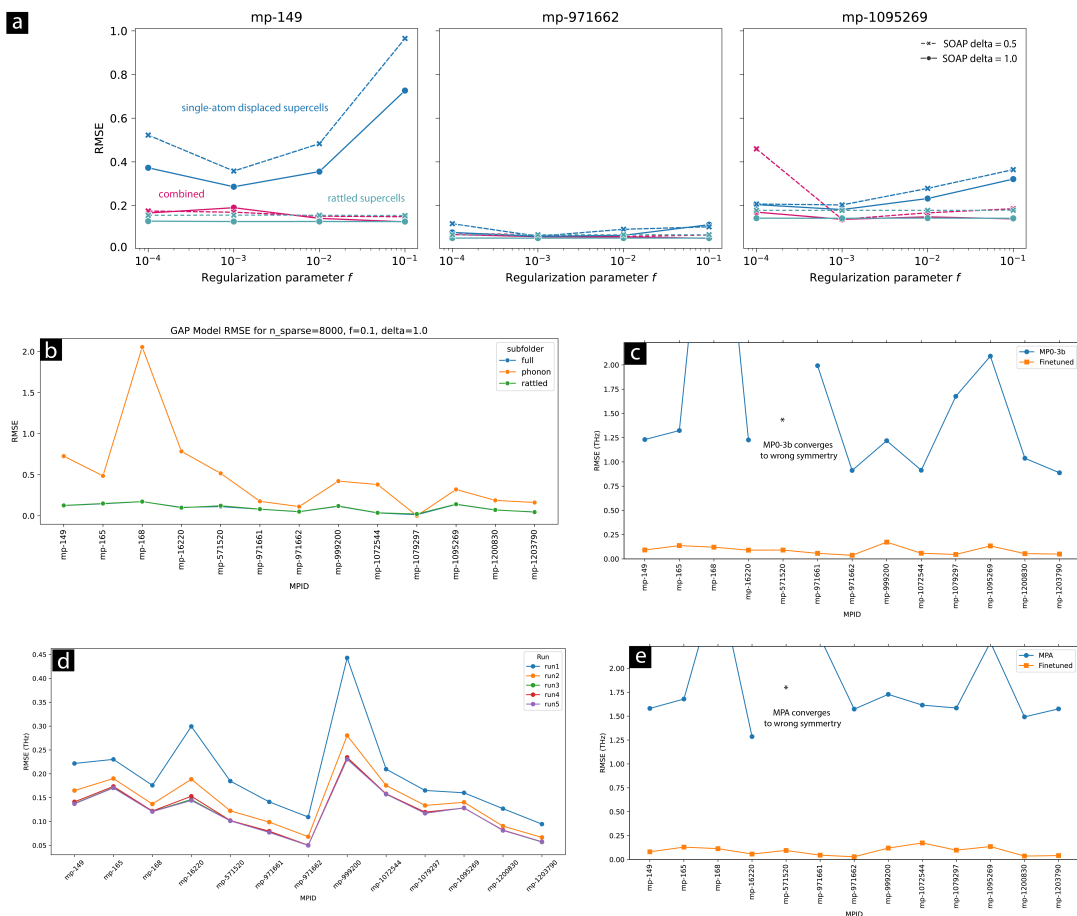


Figure S1. Si convergence tests. a) Regularization parameter f vs. RMSE for mp-149, mp-971662, mp-1095269. b) Si-MPIDs vs. RMSE for GAP with SOAP-delta = 1.0, $n_{\text{sparse}} = 8000$, $f = 0.1$. c) Si-MPIDs vs. MP0-3b and fine-tuned MP0-3b. d) Si-MPIDs vs. MACE from scratch for increasing run steps (run1 to run5) with different epoch times and training settings (see method section of main text). e) Si-MPIDs vs. MPA and fine-tuned MPA.

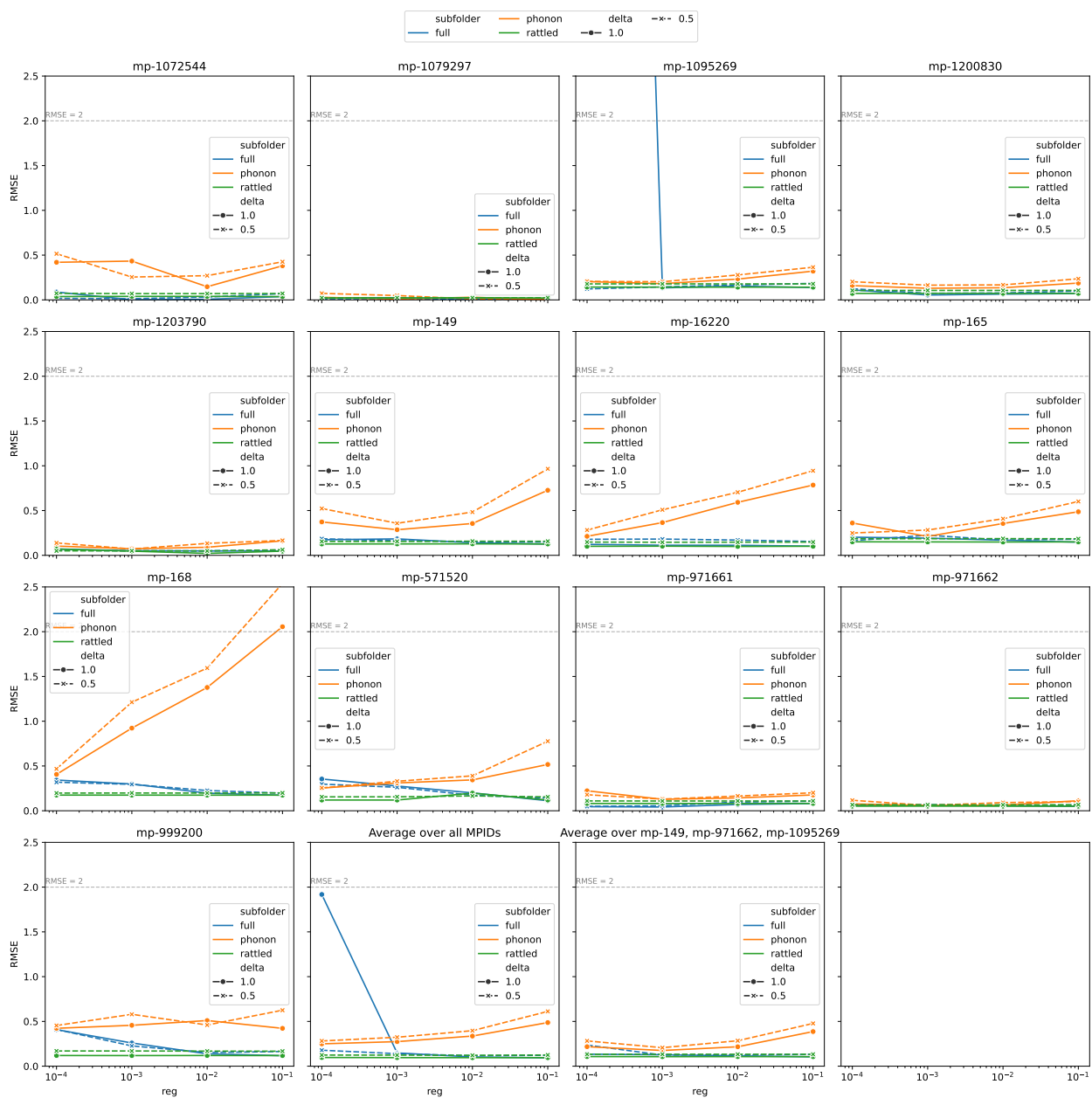


Figure S2. Si convergence tests. a) Regularization parameter (reg) vs. RMSE for $n_{\text{sparse}} = 7000$.

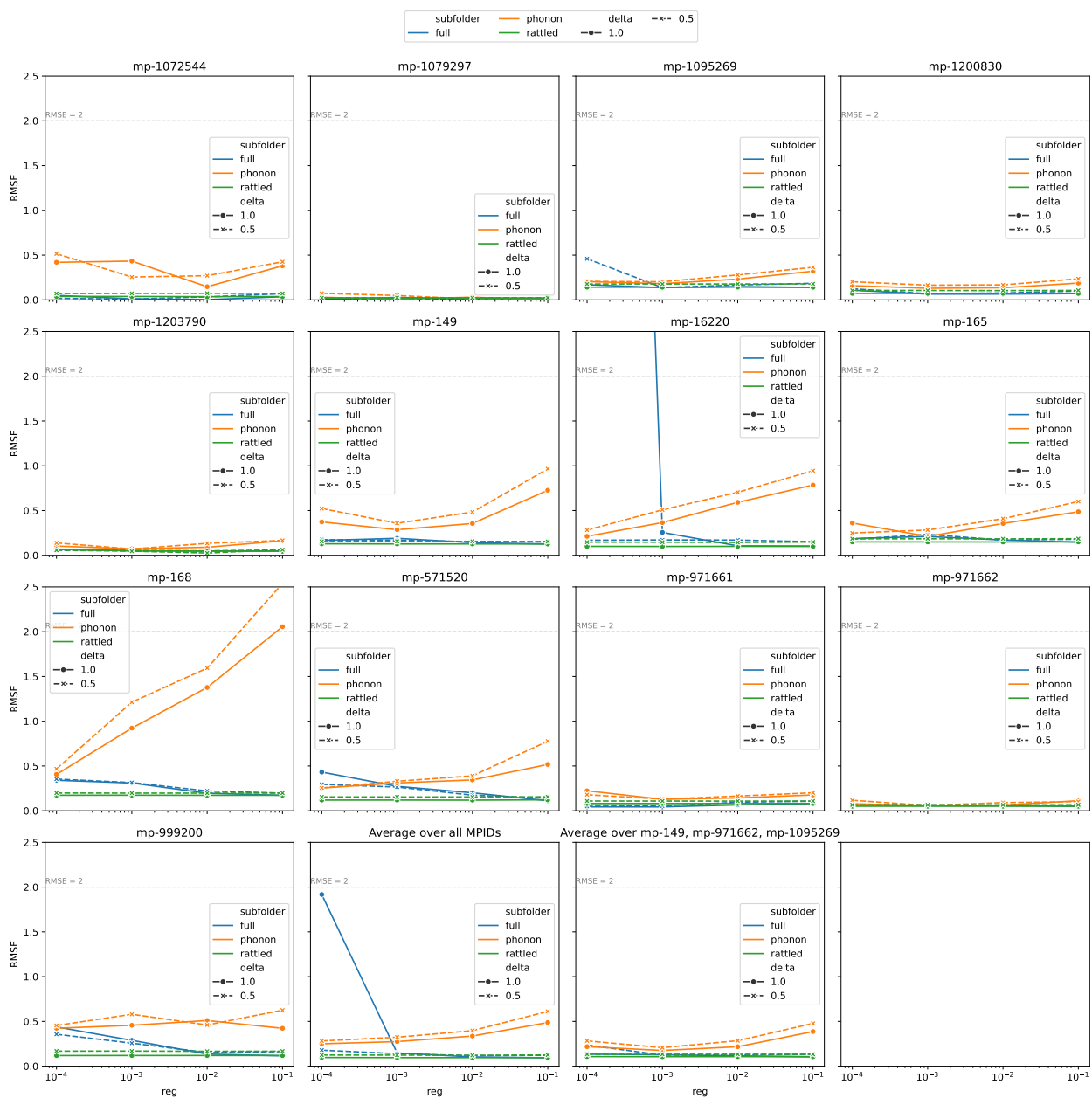


Figure S3. Si convergence tests. a) Regularization parameter (reg) vs. RMSE for $n_{\text{sparse}} = 8000$.

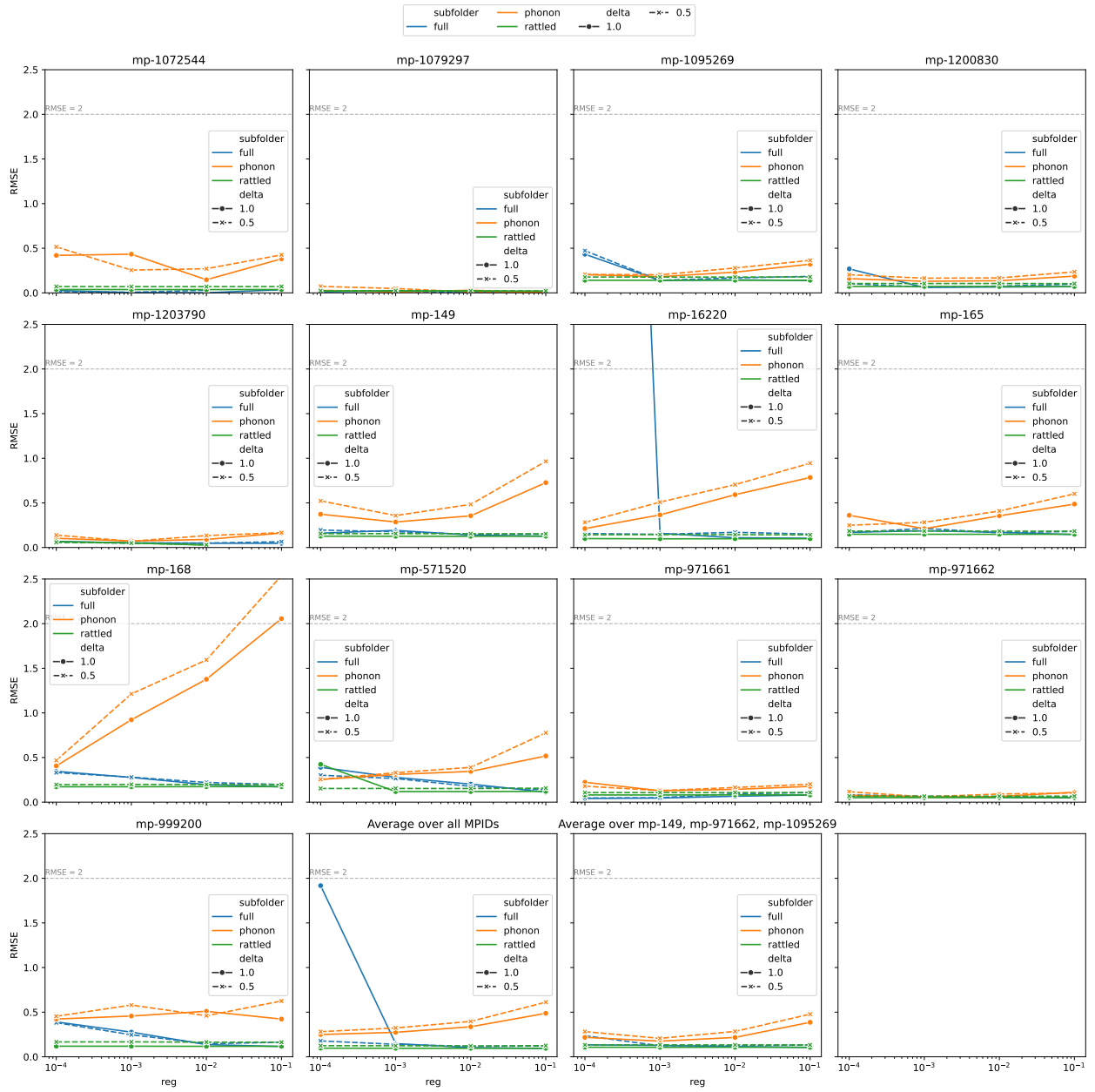


Figure S4. Si convergence tests. a) Regularization parameter (reg) vs. RMSE for $n_{\text{sparse}} = 9000$.

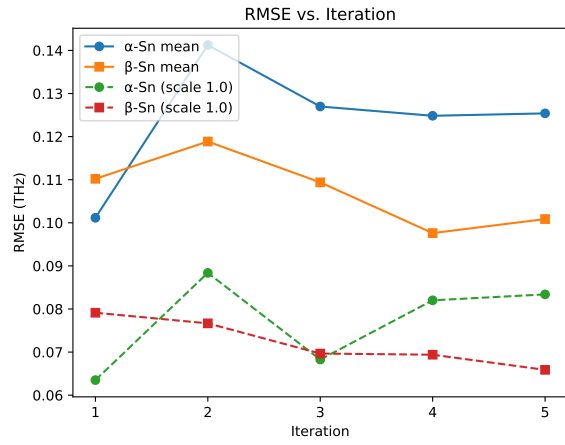
Advanced benchmark

Next, we test the robustness of the Si potentials that we acquired from `autoplex`. Si MPIDs have been filtered for suitable candidates (regarding supercell size) with a function[?] in `autoplex` from which five MPIDs have been tested, and three of them produced sensible results. Our test structures are mp-34, mp-644693, and mp-988210. We test the accuracy of the different potentials as a kind of "stress test".

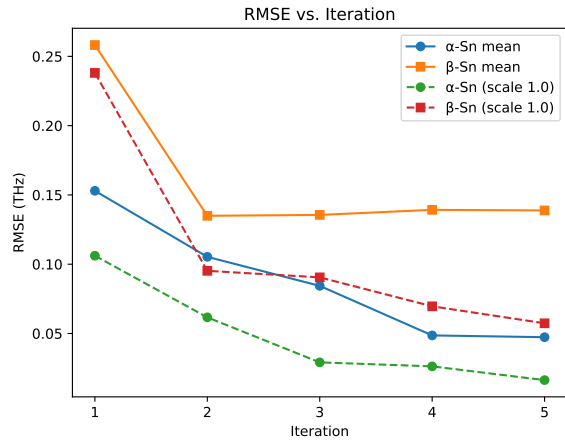
| Potential | MPID | RMSE (THz) |
|-------------------|-----------|------------|
| GAP | mp-34 | 8.80 |
| GAP | mp-644693 | 1.18 |
| GAP | mp-988210 | 0.61 |
| MACE finetuned | mp-34 | 1.98 |
| MACE finetuned | mp-644693 | 1.10 |
| MACE finetuned | mp-988210 | — |
| MP0-3b | mp-34 | 2.81 |
| MP0-3b | mp-644693 | — |
| MP0-3b | mp-988210 | 2.76 |
| MACE from scratch | mp-34 | 10.90 |
| MACE from scratch | mp-644693 | 1.39 |
| MACE from scratch | mp-644693 | — |

Table 1. RMSE values per MPID and potential type. There is no entry when the structure did not converge to the correct symmetry with the respective potential.

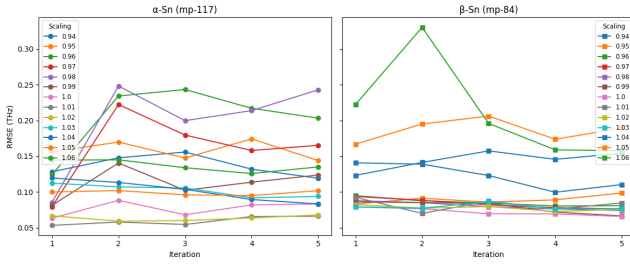
The overall outcome of this further benchmark is rather negative, as the (average) RMSE is relatively high, and the structures do not converge to the correct symmetry. Only mp-988210 yields acceptable results with the GAP potential. The reason for this outcome could be that the suitable MPIDs already went into the data generation, and mp-644693 and mp-988210 are polycrystalline/amorphous structures, which are not represented in the training data at all. Still, in the case of mp-34, it is evident that the foundation model MP0-3b performs better than the potentials trained from scratch and that the RMSE can be further reduced by fine-tuning.



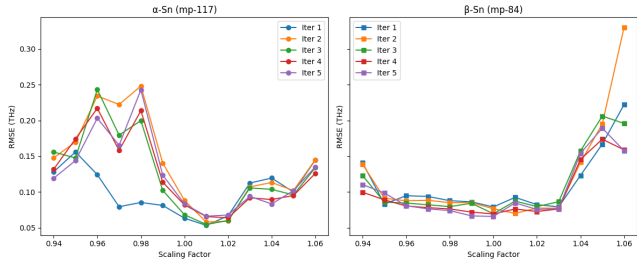
(a) Average RMSE vs. iteration GAP.



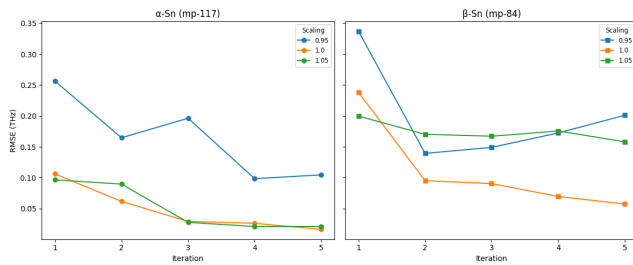
(b) Average RMSE vs. iteration MACE.



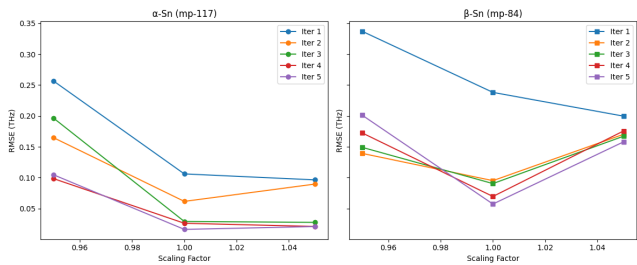
(c) RMSE vs. iteration GAP.



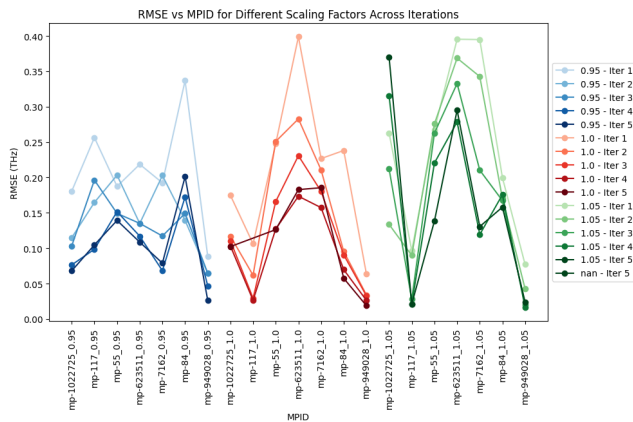
(d) RMSE vs. scaling GAP.



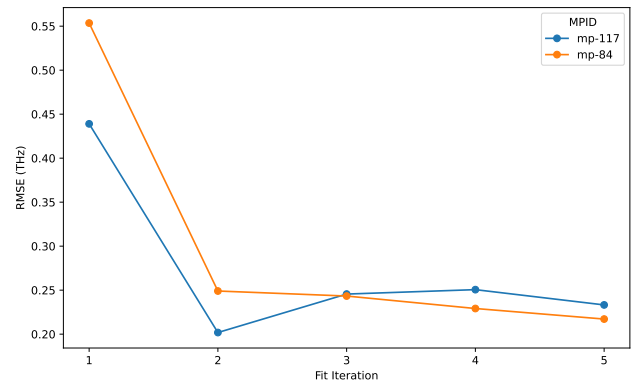
(e) RMSE vs. iteration finetuned MP0-3b.



(f) RMSE vs. scaling finetuned MP0-3b.

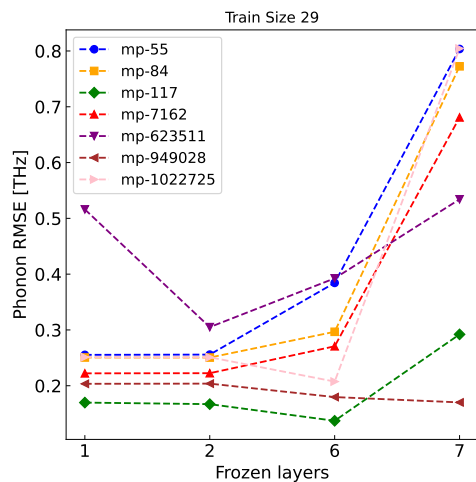
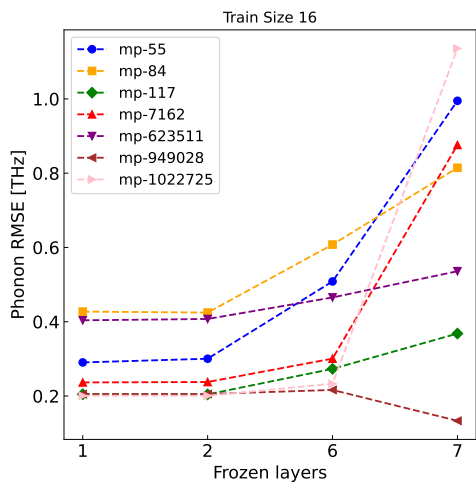


(g) RMSE vs. MPIDs finetuned MP0-3b.

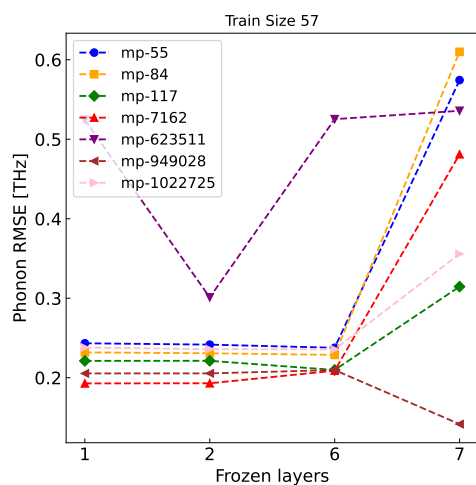
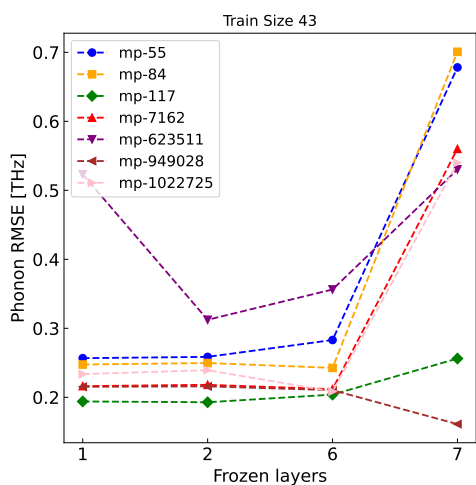


(h) RMSE vs. fit iteration for finetuned MPA.

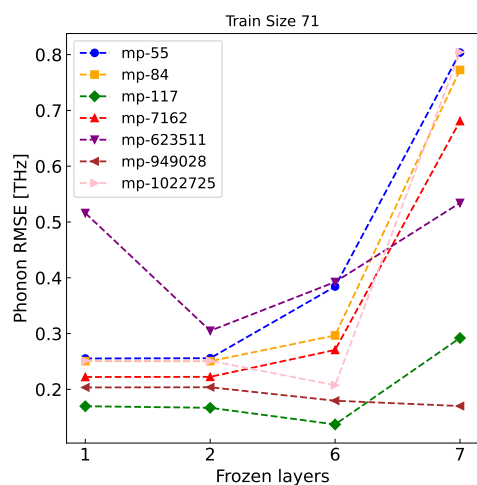
Figure S5. Sn plots.



(a) RMSE vs. frozen layers for training size 16. (b) RMSE vs. frozen layers for training size 29.

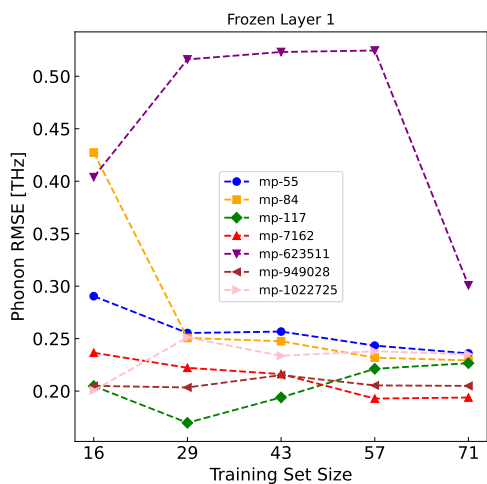


(c) RMSE vs. frozen layers for training size 43. (d) RMSE vs. frozen layers for training size 57.

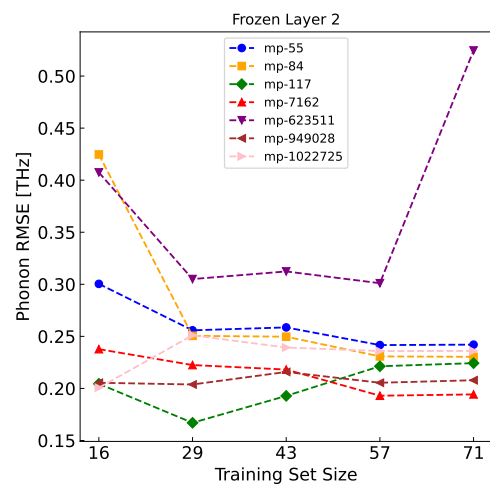


(e) RMSE vs. frozen layers for training size 71.

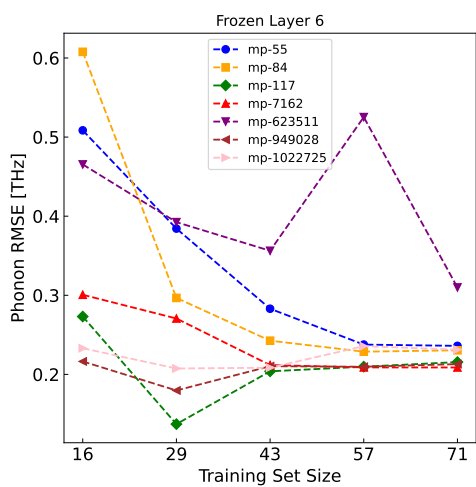
Figure S6. Sn plots MACE frozen layer parameter.



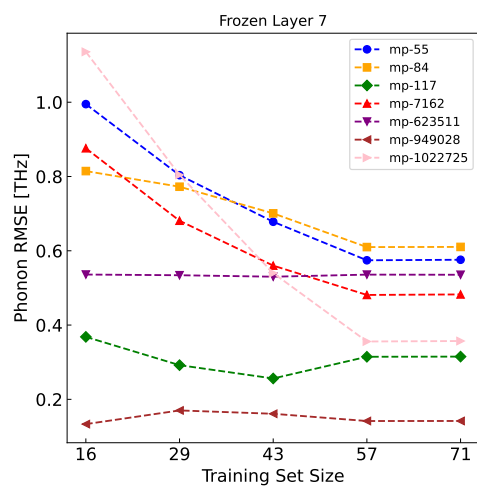
(a) RMSE vs. frozen layers for frozen layer 1.



(b) RMSE vs. frozen layers for frozen layer 2.



(c) RMSE vs. frozen layers for frozen layer 6.



(d) RMSE vs. frozen layers for frozen layer 7.

Figure S7. Sn plots MACE frozen layer parameter.

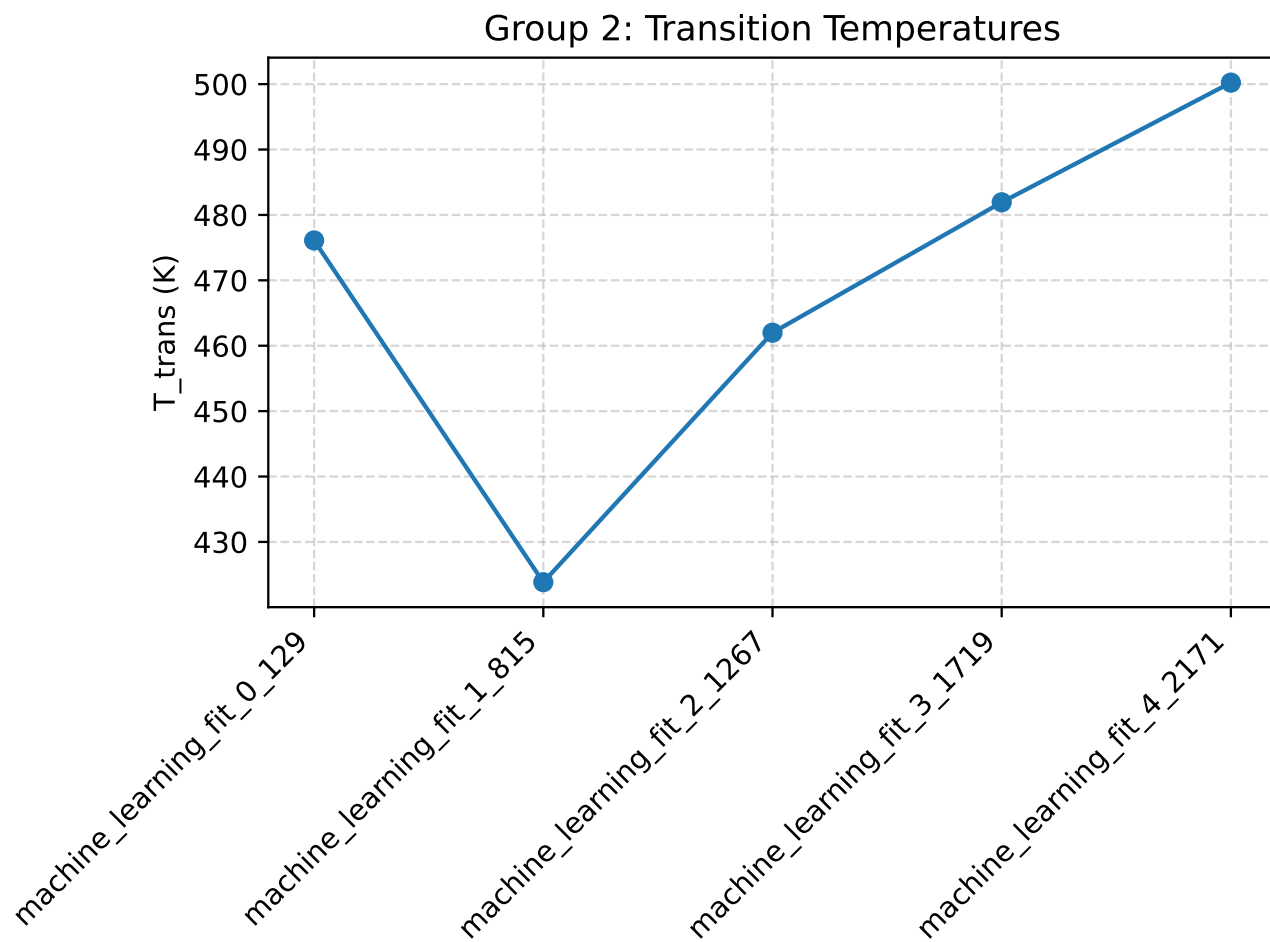


Figure S8. Sn transition temperature in dependence on iterations.

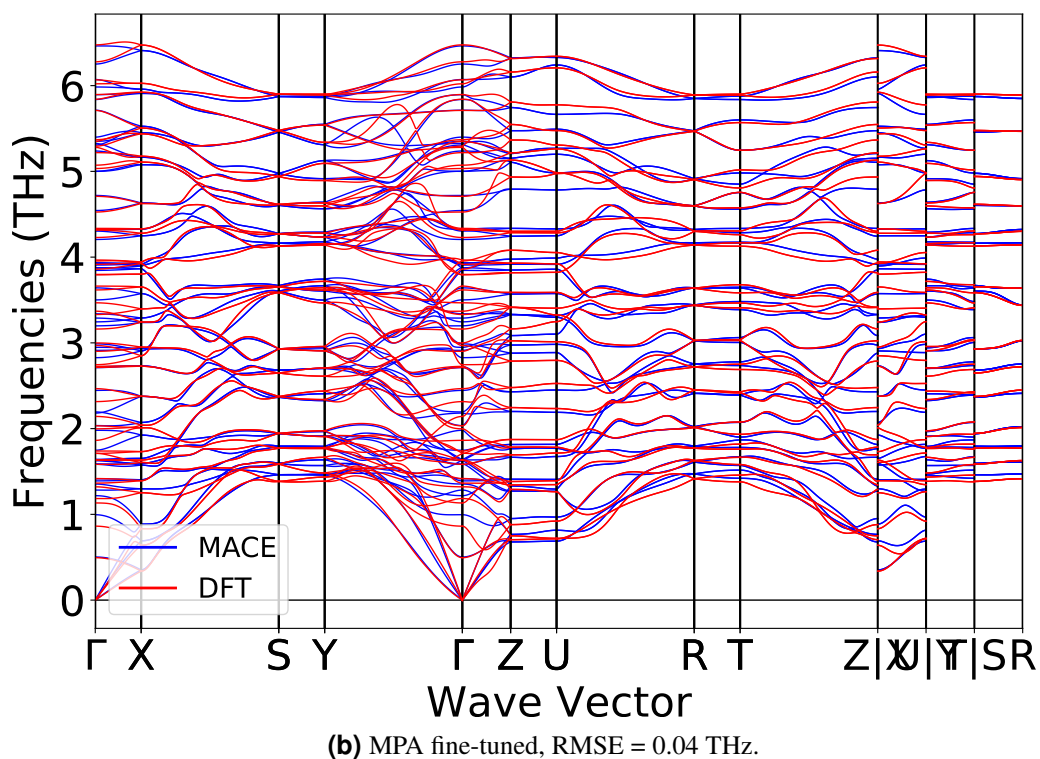
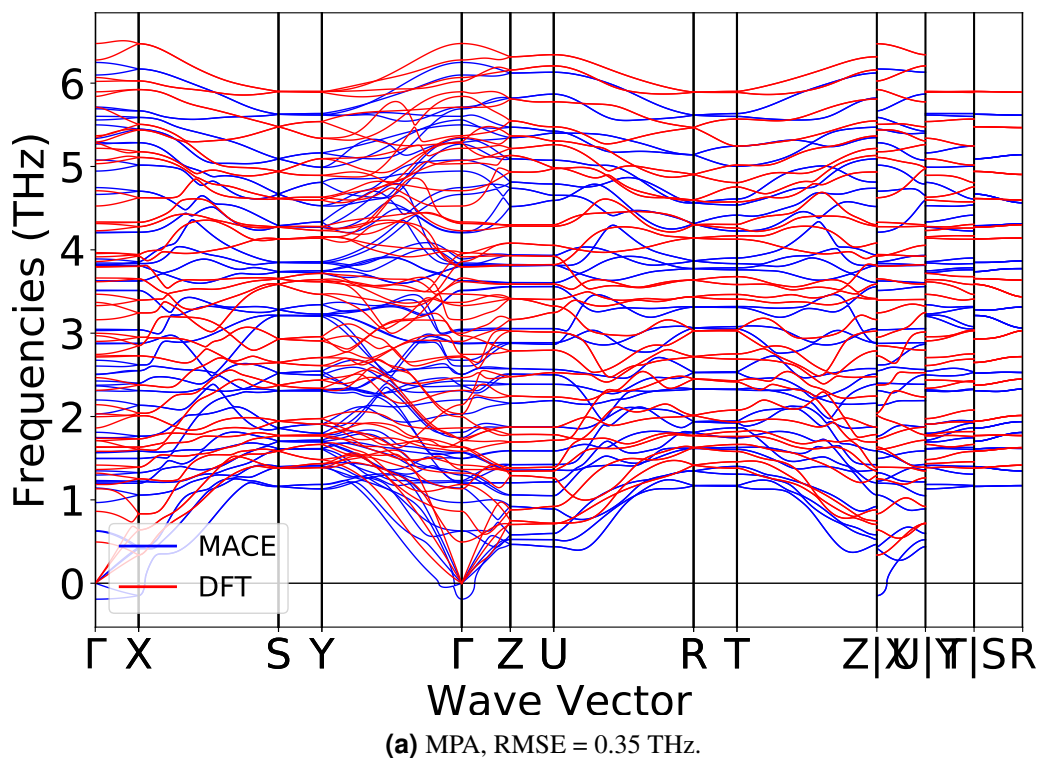
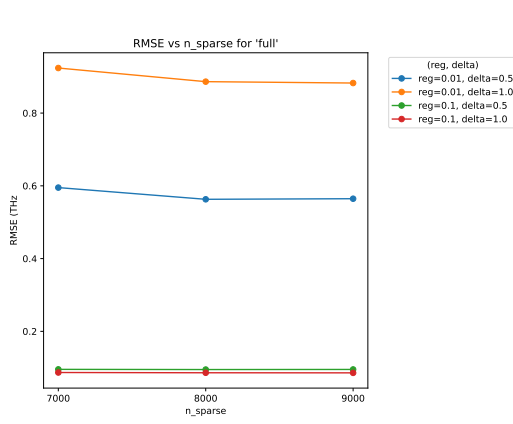
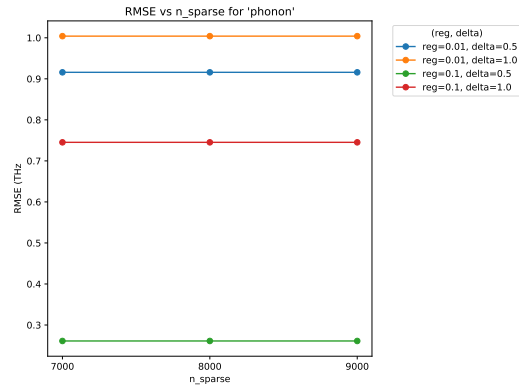


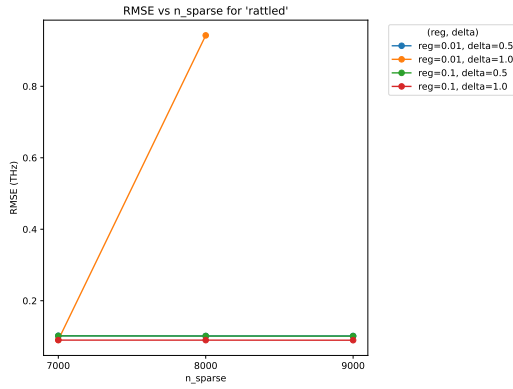
Figure S9. Sb_2Se_3 MPA and MPA fine-tuned based phonon structures. Note that the MPA is at DFT PBE level, while the DFT and MPA fine-tuned are at DFT PBE-sol level.



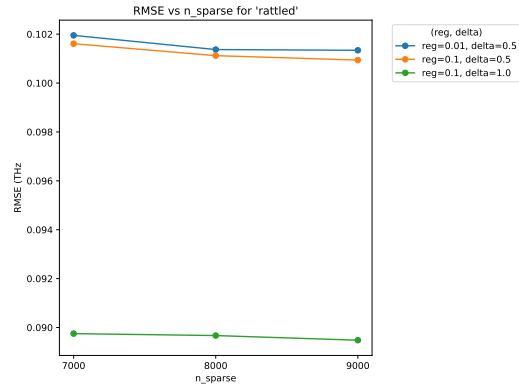
(a) RMSE vs. n_{sparse} for the full dataset.



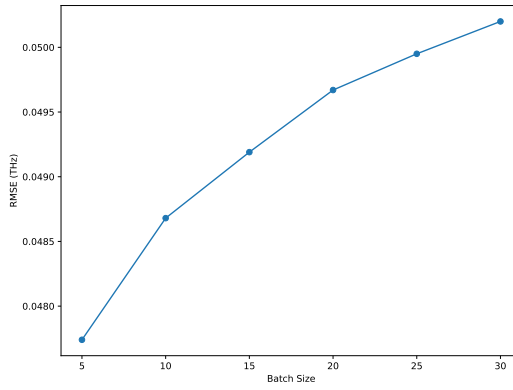
(b) RMSE vs. n_{sparse} for the single-atom displaced sub-dataset.



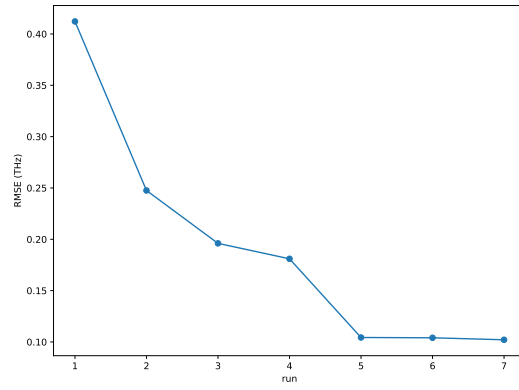
(c) RMSE vs. n_{sparse} for the rattled sub-dataset.



(d) RMSE vs. n_{sparse} for the rattled sub-dataset excluding extreme outliers.

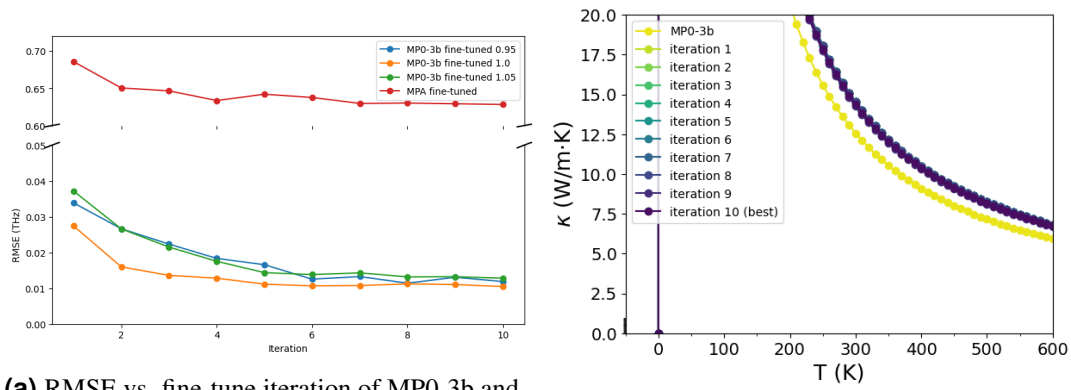


(e) RMSE vs. batch size of the MACE fine-tuning.



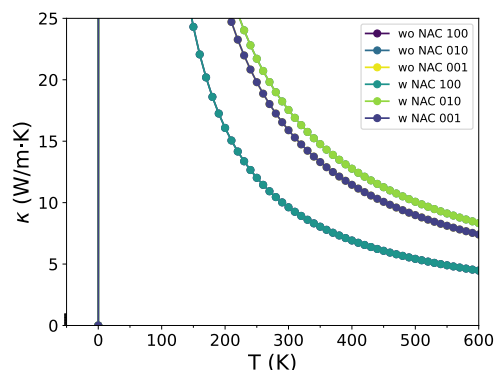
(f) RMSE vs. run iteration of the MACE from scratch generation.

Figure S10. Sb_2Se_3 convergence tests.



(a) RMSE vs. fine-tune iteration of MP0-3b and MPA

(b) Total κ (RTA) in dependence of fit iteration.



(c) Anisotropic κ (RTA, with NAC = w NAC, without NAC = wo NAC).

Figure S11. Ga₂O₃ convergence tests.

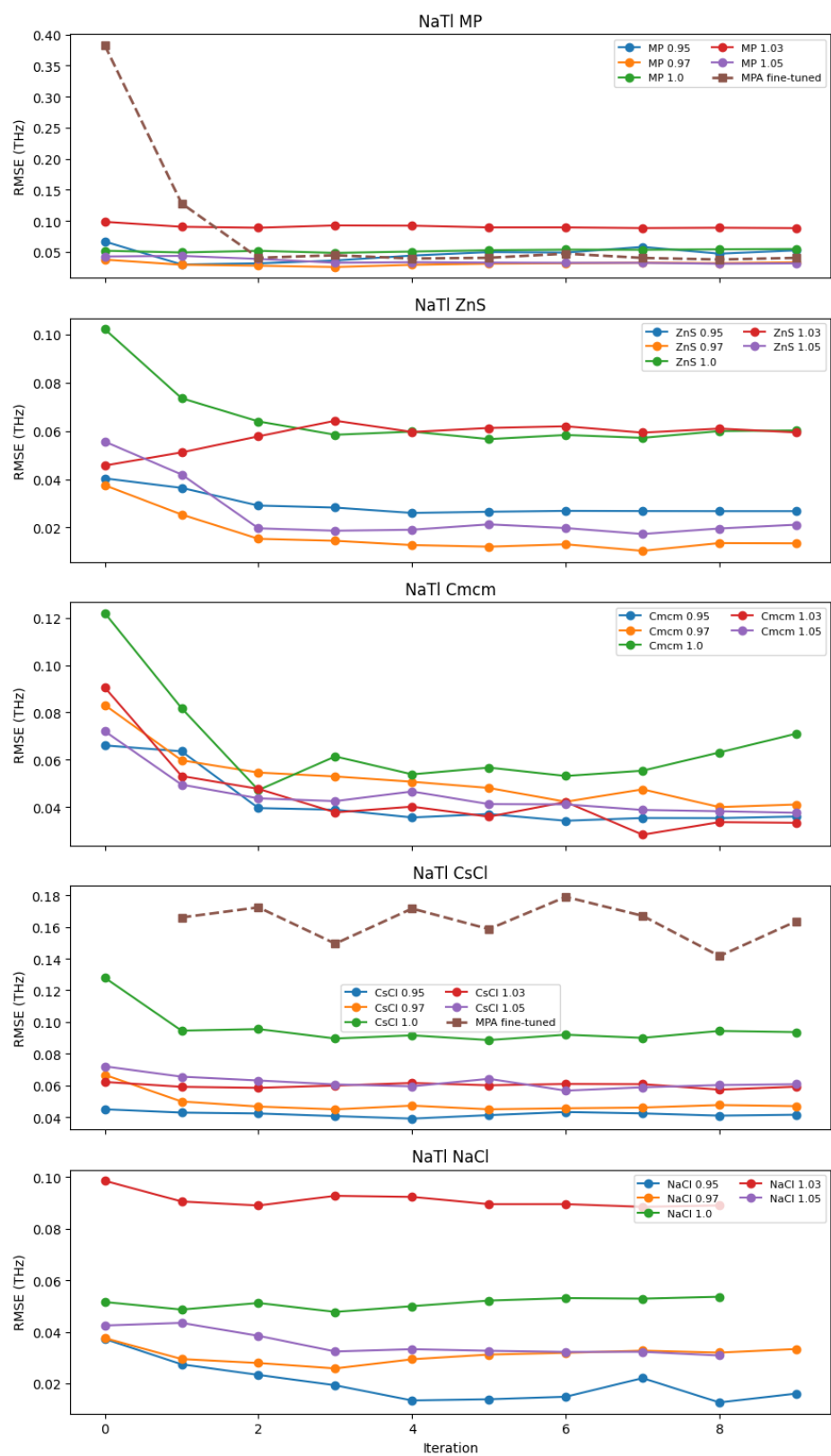


Figure S12. NaI convergence tests.

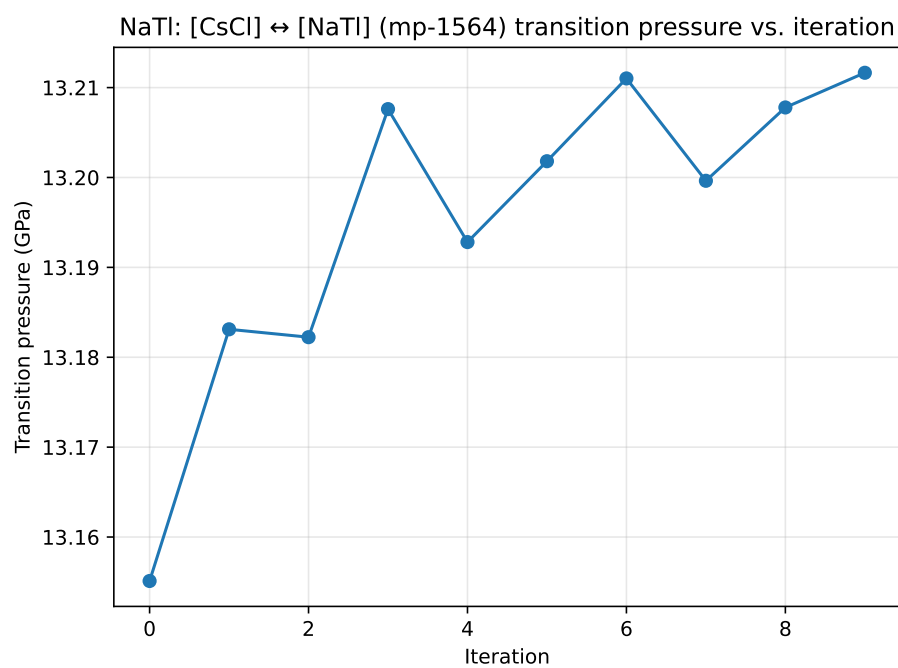


Figure S13. NaTl convergence test for MP0-3b fine-tuning all iterations.

The MACE hyperparameters varied during the MACE from scratch potential generation for the convergence tests for Si and Sb₂Se₃ are listed below in Tab.3.

| Run | Max Epochs | Energy Weight | Forces Weight | Stress Weight |
|------|------------|---------------|---------------|---------------|
| run1 | 5000 | 1000.0 | 1000.0 | 1.0 |
| run2 | 1000 | 1000.0 | 1200.0 | 1.0 |
| run3 | 1000 | 1000.0 | 1500.0 | 1.0 |
| run4 | 1000 | 1500.0 | 1000.0 | 1.0 |
| run5 | 1000 | 1000.0 | 2000.0 | 1.0 |

Table 2. Overview of MACE from scratch run parameters for Si.

| Run | Max Epochs | Energy Weight | Forces Weight | Stress Weight |
|------|------------|---------------|---------------|---------------|
| run1 | 5000 | 1000.0 | 1500.0 | 1.0 |
| run2 | 5000 | 1000.0 | 1200.0 | 1.0 |
| run3 | 5000 | 1000.0 | 1300.0 | 1.0 |
| run4 | 2000 | 1500.0 | 1000.0 | 1.0 |
| run5 | 2000 | 1000.0 | 2000.0 | 1.0 |
| run6 | 1000 | 1000.0 | 1500.0 | 1.0 |
| run7 | 2000 | 1000.0 | 1200.0 | 1.0 |

Table 3. Overview of MACE from scratch run parameters for Sb₂Se₃.

| Fit | Train | Test | % Train Data |
|--|-------|------|--------------|
| Sn (GAP) | | | |
| machine_learning_fit_0_81 | 44 | 22 | 38% |
| machine_learning_fit_1_869 | 61 | 31 | 54% |
| machine_learning_fit_2_1410 | 79 | 39 | 69% |
| machine_learning_fit_3_1951 | 96 | 48 | 84% |
| machine_learning_fit_4_2492 | 114 | 56 | 100% |
| Sn (fine-tuned MACE, α/β -Sn) | | | |
| machine_learning_fit_0_15 | 5 | 2 | 38% |
| machine_learning_fit_1_209 | 9 | 4 | 69% |
| machine_learning_fit_2_340 | 13 | 6 | 100% |
| Sn (fine-tuned MACE, all Sn) | | | |
| machine_learning_fit_0_129 | 15 | 7 | 21% |
| machine_learning_fit_1_815 | 29 | 14 | 41% |
| machine_learning_fit_2_1267 | 43 | 21 | 61% |
| machine_learning_fit_3_1719 | 57 | 28 | 80% |
| machine_learning_fit_4_2171 | 71 | 35 | 100% |
| NaTi (fine-tuned MACE) | | | |
| machine_learning_fit_0_153 | 18 | 9 | 8% |
| machine_learning_fit_1_1034 | 35 | 17 | 16% |
| machine_learning_fit_2_1604 | 99 | 49 | 46% |
| machine_learning_fit_3_2174 | 116 | 57 | 54% |
| machine_learning_fit_4_2744 | 133 | 65 | 61% |
| machine_learning_fit_5_3314 | 150 | 73 | 69% |
| machine_learning_fit_6_3884 | 166 | 82 | 77% |
| machine_learning_fit_7_4454 | 183 | 90 | 84% |
| machine_learning_fit_8_5024 | 200 | 98 | 92% |
| machine_learning_fit_9_5594 | 217 | 106 | 100% |
| Ga ₂ O ₃ (fine-tuned MACE) | | | |
| machine_learning_fit_0_21 | 21 | 10 | 5% |
| machine_learning_fit_1_261 | 41 | 20 | 10% |
| machine_learning_fit_2_414 | 61 | 30 | 15% |
| machine_learning_fit_3_567 | 81 | 40 | 20% |
| machine_learning_fit_4_720 | 101 | 49 | 25% |
| machine_learning_fit_5_873 | 121 | 59 | 30% |
| machine_learning_fit_6_1026 | 282 | 138 | 71% |
| machine_learning_fit_7_1179 | 320 | 158 | 80% |
| machine_learning_fit_8_1332 | 360 | 178 | 90% |
| machine_learning_fit_9_1485 | 400 | 198 | 100% |

Table 4. Overview of data per iteration for Sn, NaTi, and Ga₂O₃, including relative train set sizes.