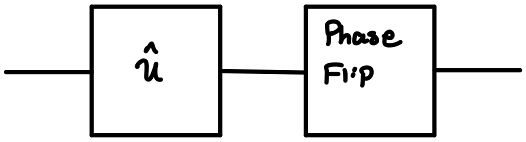
Exercise 1



$$\hat{U} = e^{-i\hat{\sigma}_z \lambda/2}$$

a.) determine $\hat{\rho}(\lambda)$

$$\hat{U}(\lambda) = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix} = \begin{pmatrix} e^{-i\lambda/2} & 0 \\ 0 & e^{i\lambda/2} \end{pmatrix}$$

Suppose $\hat{\rho}_0 = \frac{1}{2}(\hat{I} + r\hat{\sigma}_x) = \frac{1}{2}\begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}$

$$\hat{\rho}(\lambda) = \hat{U}(\lambda)\hat{\rho}_0\hat{U}^\dagger(\lambda) = \begin{pmatrix} e^{-i\lambda/2} & 0 \\ 0 & e^{i\lambda/2} \end{pmatrix} \frac{1}{2}\begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix} \begin{pmatrix} e^{i\lambda/2} & 0 \\ 0 & e^{-i\lambda/2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & re^{-i\lambda} \\ re^{i\lambda} & 1 \end{pmatrix}$$

$$\boxed{\hat{\rho}(\lambda) = \frac{1}{2}\begin{pmatrix} 1 & re^{-i\lambda} \\ re^{i\lambda} & 1 \end{pmatrix}}$$

b.) determine phase flip

$$\hat{\rho}(\lambda) = \frac{1}{2}\left[1\cdot|0\rangle\langle 0| + re^{-i\lambda}|0\rangle\langle 1| + re^{i\lambda}|1\rangle\langle 0| + 1\cdot|1\rangle\langle 1|\right]$$

$\hat{\rho}_i = |0\rangle\langle 0| \longrightarrow \hat{\rho}_f = |0\rangle\langle 0|$: $\hat{\rho}_i = |1\rangle\langle 1| \longrightarrow \hat{\rho}_f = |1\rangle\langle 1|$

$\hat{\rho}_i = |0\rangle\langle 1| \longrightarrow \hat{\rho}_f = (1-2\alpha)|0\rangle\langle 1|$: $\hat{\rho}_i = |1\rangle\langle 0| \longrightarrow \hat{\rho}_f = (1-2\alpha)|1\rangle\langle 0|$

$$\hat{\rho}(\lambda) = \frac{1}{2}\left[|0\rangle\langle 0| + (1-2\alpha)re^{-i\lambda}|0\rangle\langle 1| + (1-2\alpha)re^{i\lambda}|1\rangle\langle 0| + |1\rangle\langle 1|\right]$$

$$\boxed{\hat{\rho}(\lambda) = \frac{1}{2}\begin{pmatrix} 1 & (1-2\alpha)re^{-i\lambda} \\ (1-2\alpha)re^{i\lambda} & 1 \end{pmatrix}}$$

c.) determine QFI

i.) Calculate $\hat{L}$: $\hat{L} = \frac{1}{Tr(\hat{A})}\left[2\cdot\frac{\partial\hat{A}}{\partial\lambda} - \frac{\partial \ln(\alpha)}{\partial\lambda}\hat{A}\right] + \frac{\partial}{\partial\lambda}\left[\ln(\alpha) - \ln(Tr(\hat{A}))\right]\hat{I}$

$\alpha = Tr(\hat{A}^2) - Tr(\hat{A})^2$; $\hat{A} = \frac{1}{2}\begin{pmatrix} 1 & (1-2\alpha)re^{-i\lambda} \\ (1-2\alpha)re^{i\lambda} & 1 \end{pmatrix}$

$$\hat{A}^2 = \frac{1}{4}\begin{pmatrix} 1 & (1-2\alpha)re^{-i\lambda} \\ (1-2\alpha)re^{i\lambda} & 1 \end{pmatrix}\begin{pmatrix} 1 & (1-2\alpha)re^{-i\lambda} \\ (1-2\alpha)re^{i\lambda} & 1 \end{pmatrix}$$

$$\hat{A}^2 = \frac{1}{4}\begin{pmatrix} 1 + r^2(1-2\alpha)^2 & 2(1-2\alpha)re^{-i\lambda} \\ 2(1-2\alpha)re^{i\lambda} & 1 + r^2(1-2\alpha)^2 \end{pmatrix}$$

Exercise 1 Continued

$$Tr(\hat{A}^2) = \frac{1}{2}\left(1 + r^2(1-2\alpha)^2\right) \; : \; Tr(\hat{A}) = 1 \quad \therefore \quad Tr(\hat{A})^2 = 1$$

$$\alpha = \frac{1}{2}\left(1 + r^2(1-2\alpha)^2\right) - 1 \longrightarrow \alpha \text{ does not depend upon } \lambda \; !$$

$$\therefore \quad \hat{L} = \frac{1}{Tr(\hat{A})}\left[2 \cdot \frac{\partial \hat{A}}{\partial \lambda} - \cancelto{0}{\frac{\partial \ln(\alpha)}{\partial \lambda}} \hat{A}\right] + \cancelto{0}{\frac{\partial}{\partial \lambda}\left[\ln(\alpha) - \ln(Tr(\hat{A}))\right]} \hat{I} = \frac{1}{Tr(\hat{A})} \cdot 2 \cdot \frac{\partial \hat{A}}{\partial \lambda}$$

$$\frac{\partial \hat{A}}{\partial \lambda} = \frac{1}{2}\begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix} \quad \therefore \quad \hat{L} = \begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix}$$

$$\boxed{\hat{L} = \begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix}}$$

ii.) Calculate QFI: $H = Tr\left[\hat{L} \cdot \frac{\partial \hat{A}}{\partial \lambda}\right]$

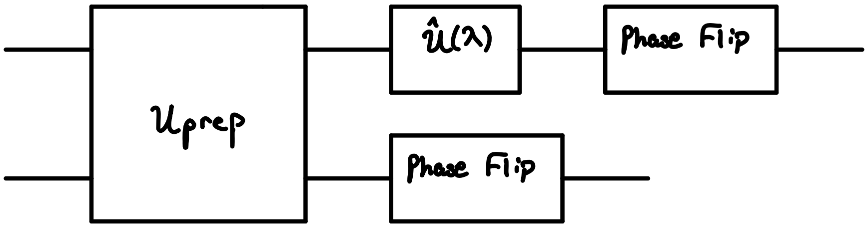
$$\frac{\partial \hat{A}}{\partial \lambda} = \frac{1}{2}\begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix}, \quad \hat{L} = \begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix}$$

$$\hat{L} \cdot \frac{\partial \hat{A}}{\partial \lambda} = \begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 0 & -i(1-2\alpha)re^{-i\lambda} \\ i(1-2\alpha)re^{i\lambda} & 0 \end{pmatrix}$$

$$\hat{L} \cdot \frac{\partial \hat{A}}{\partial \lambda} = \frac{1}{2}\begin{pmatrix} (1-2\alpha)^2 r^2 & 0 \\ 0 & (1-2\alpha)^2 r^2 \end{pmatrix} \quad \therefore \quad Tr\left[\hat{L} \cdot \frac{\partial \hat{A}}{\partial \lambda}\right] = (1-2\alpha)^2 r^2$$

$$\boxed{H = (1-2\alpha)^2 r^2}$$

Exercise 2



$$\hat{U}(\lambda) = e^{-i\lambda\hat{\sigma}_z/2}$$

a.) Write $\hat{\rho}_{prep} = \dots |0\rangle|0\rangle\langle 0|\langle 0| + |0\rangle|0\rangle\langle 0|\langle 1| \dots\dots$

$$\hat{\rho}_{prep} = \hat{U}_{prep}\,\hat{\rho}\,\hat{U}^{\dagger}_{prep} = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (-8ir) \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (8ir) & 0 & 0 & (4+4r^2) \end{pmatrix}$$

$$\hat{\rho}_{prep} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]$$

$$\boxed{\hat{\rho}_{prep} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]}$$

b.) Determine $\hat{U}(\lambda)$ on $\hat{\rho}_{prep}$

$$\hat{U}\hat{\rho}_{prep}\hat{U}^{\dagger} = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (-8ir)e^{-i\lambda} \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (8ir)e^{i\lambda} & 0 & 0 & (4+4r^2) \end{pmatrix}$$

$$\hat{U}\hat{\rho}_{prep}\hat{U}^{\dagger} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)e^{-i\lambda}|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)e^{i\lambda}|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]$$

$$\boxed{\hat{U}\hat{\rho}_{prep}\hat{U}^{\dagger} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)e^{-i\lambda}|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)e^{i\lambda}|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]}$$

Exercise 2 continued

c.) Determine phase flip on second particle

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (-8ir)e^{-i\lambda} \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (8ir)e^{i\lambda} & 0 & 0 & (4+4r^2) \end{pmatrix}$$

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes|0\rangle\langle 0| + (-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes|0\rangle\langle 1| + (4-4r^2)|0\rangle\langle 0|\otimes|1\rangle\langle 1| +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes|0\rangle\langle 0| + (8ir)e^{i\lambda}|1\rangle\langle 0|\otimes|1\rangle\langle 0| + (4+4r^2)|1\rangle\langle 1|\otimes|1\rangle\langle 1|\Big]$$

Phase flip acts on second part, i.e:

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes\underline{|0\rangle\langle 0|} + (-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes\underline{|0\rangle\langle 1|} + (4-4r^2)|0\rangle\langle 0|\otimes\underline{|1\rangle\langle 1|} +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes\underline{|0\rangle\langle 0|} + (8ir)e^{i\lambda}|1\rangle\langle 0|\otimes\underline{|1\rangle\langle 0|} + (4+4r^2)|1\rangle\langle 1|\otimes\underline{|1\rangle\langle 1|}\Big]$$

$\hat{\rho}_i = |0\rangle\langle 0| \rightarrow \hat{\rho}_f = |0\rangle\langle 0|$: $\hat{\rho}_i = |1\rangle\langle 1| \rightarrow \hat{\rho}_f = |1\rangle\langle 1|$:

$\hat{\rho}_i = |0\rangle\langle 1| \rightarrow \hat{\rho}_f = (1-2\alpha)|0\rangle\langle 1|$: $\hat{\rho}_i = |1\rangle\langle 0| \rightarrow \hat{\rho}_f = (1-2\alpha)|1\rangle\langle 0|$

These rules evolve $\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger$ to

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes|0\rangle\langle 0| + (1-2\alpha)(-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes|0\rangle\langle 1| + (4-4r^2)|0\rangle\langle 0|\otimes|1\rangle\langle 1| +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes|0\rangle\langle 0| + (1-2\alpha)(8ir)e^{i\lambda}|1\rangle\langle 0|\otimes|1\rangle\langle 0| + (4+4r^2)|1\rangle\langle 1|\otimes|1\rangle\langle 1|\Big]$$

$$\boxed{\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (1-2\alpha)(-8ir)e^{-i\lambda} \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (1-2\alpha)(8ir)e^{i\lambda} & 0 & 0 & (4+4r^2) \end{pmatrix}}$$

d.) Determine phase flip on first particle

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes|0\rangle\langle 0| + (1-2\alpha)(-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes|0\rangle\langle 1| + (4-4r^2)|0\rangle\langle 0|\otimes|1\rangle\langle 1| +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes|0\rangle\langle 0| + (1-2\alpha)(8ir)e^{i\lambda}|1\rangle\langle 0|\otimes|1\rangle\langle 0| + (4+4r^2)|1\rangle\langle 1|\otimes|1\rangle\langle 1|\Big]$$

Phase flip acts on first particle

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)\underline{|0\rangle\langle 0|}\otimes|0\rangle\langle 0| + (1-2\alpha)(-8ir)e^{-i\lambda}\underline{|0\rangle\langle 1|}\otimes|0\rangle\langle 1| + (4-4r^2)\underline{|0\rangle\langle 0|}\otimes|1\rangle\langle 1| +$$
$$(4-4r^2)\underline{|1\rangle\langle 1|}\otimes|0\rangle\langle 0| + (1-2\alpha)(8ir)e^{i\lambda}\underline{|1\rangle\langle 0|}\otimes|1\rangle\langle 0| + (4+4r^2)\underline{|1\rangle\langle 1|}\otimes|1\rangle\langle 1|\Big]$$

Exercise 2 Continued

$\hat{\rho}_i = |0\rangle\langle 0| \longrightarrow \hat{\rho}_f = |0\rangle\langle 0|$: $\hat{\rho}_i = |1\rangle\langle 1| \longrightarrow \hat{\rho}_f = |1\rangle\langle 1|$:

$\hat{\rho}_i = |0\rangle\langle 1| \longrightarrow \hat{\rho}_f = (1-2\beta)|0\rangle\langle 1|$: $\hat{\rho}_i = |1\rangle\langle 0| \longrightarrow \hat{\rho}_f = (1-2\beta)|1\rangle\langle 0|$

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes|0\rangle\langle 0| + (1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes|0\rangle\langle 1| + (4-4r^2)|0\rangle\langle 0|\otimes|1\rangle\langle 1| +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes|0\rangle\langle 0| + (1-2\beta)(1-2\alpha)(8ir)e^{i\lambda}|1\rangle\langle 0|\otimes|1\rangle\langle 0| + (4-4r^2)|1\rangle\langle 1|\otimes|1\rangle\langle 1|\Big]$$

These rules evolve $\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger$ to

$$\boxed{\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda} \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (1-2\alpha)(1-2\beta)(8ir)e^{i\lambda} & 0 & 0 & (4+4r^2) \end{pmatrix}}$$

e.) Calculate QFI

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda} \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (1-2\alpha)(1-2\beta)(8ir)e^{i\lambda} & 0 & 0 & (4+4r^2) \end{pmatrix}$$

$$\hat{A}_1 = \frac{1}{16}\begin{pmatrix} 4-4r^2 & 0 \\ 0 & 4-4r^2 \end{pmatrix}, \quad \hat{A}_2 = \frac{1}{16}\begin{pmatrix} (4+4r^2) & (1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda} \\ (1-2\alpha)(1-2\beta)(8ir)e^{i\lambda} & (4+4r^2) \end{pmatrix}$$

$$\hat{L} = \frac{1}{Tr(\hat{A})}\left[2\cdot\frac{\partial\hat{A}}{\partial\lambda} - \frac{\partial \ln(\alpha)}{\partial\lambda}\hat{A}\right] + \frac{\partial}{\partial\lambda}\left[\ln(\alpha) - \ln(Tr(\hat{A}))\right]\hat{I}, \quad H_i = Tr\left[L_i\frac{\partial A_i}{\partial\lambda}\right]$$

i.) Calculate $\hat{L}_1$: $\alpha = Tr(\hat{A}_1^2) - Tr(\hat{A}_1)^2$

$$\hat{A}_1^2 = \frac{1}{16^2}\begin{pmatrix} 4-4r^2 & 0 \\ 0 & 4-4r^2 \end{pmatrix}\begin{pmatrix} 4-4r^2 & 0 \\ 0 & 4-4r^2 \end{pmatrix} = \frac{1}{16^2}\begin{pmatrix} (4-4r^2)^2 & 0 \\ 0 & (4-4r^2)^2 \end{pmatrix}$$

$$Tr(\hat{A}_1^2) = \frac{1}{16^2}\left[(4-4r^2)^2 + (4-4r^2)^2\right] = \frac{2}{16^2}\left[16-32r^2+16r^4\right]$$

$$Tr(\hat{A}_1) = \frac{1}{16}\left[4-4r^2+4-4r^2\right] = \frac{1}{16}\left[8-8r^2\right] = \frac{1}{2}\left[1-r^2\right] : Tr(\hat{A}_1)^2 = \frac{1}{4}\left[1-2r^2+r^4\right]$$

$$\alpha = Tr(\hat{A}_1^2) - Tr(\hat{A}_1) = \frac{1}{128}\left[16-32r^2+16r^4\right] - \frac{32}{128}\left[1-2r^2+r^4\right]$$
$$= \frac{16}{128} - \frac{32r^2}{128} + \frac{16r^4}{128} - \frac{32}{128} + \frac{64r^2}{128} - \frac{32r^4}{128} = -\frac{16}{128} + \frac{32r^2}{128} - \frac{16r^4}{128} = -\frac{1}{8} + \frac{r^2}{4} - \frac{r^4}{8}$$

$\alpha = -\frac{1}{8}(1-2r^2+r^4)$, Since α does not depend upon λ, $\hat{L}$ changes to

Exercise 2 continued

$$\hat{L} = \frac{1}{Tr(\hat{A})}\left[2\cdot\frac{\partial \hat{A}}{\partial \lambda} - \overset{0}{\frac{\partial \cancel{\ln(\alpha)}}{\cancel{\partial\lambda}}}\hat{A}\right] + \overset{0}{\frac{\cancel{\partial}}{\cancel{\partial\lambda}}}\left[\cancel{\ln(\alpha)} - \ln(\cancel{Tr(\hat{A})})\right]\hat{I} = \frac{2}{Tr(\hat{A}_1)}\cdot\frac{\partial \hat{A}}{\partial\lambda}$$

$\hat{A}_1$ does not depend on λ $\therefore$ $\hat{L}_1 = 0$

$$\boxed{\hat{L}_1 = 0}$$

ii.) Calculate $\hat{L}_2$: $\alpha = Tr(\hat{A}_2^2) - Tr(\hat{A}_2)^2$

$$\hat{A}_2^2 = \frac{1}{16^2}\begin{pmatrix} (4+4r^2) & (1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda} \\ (1-2\alpha)(1-2\beta)(8ir)e^{i\lambda} & (4+4r^2)\end{pmatrix}\begin{pmatrix} (4+4r^2) & (1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda} \\ (1-2\alpha)(1-2\beta)(8ir)e^{i\lambda} & (4+4r^2)\end{pmatrix}$$

$$\hat{A}_2^2 = \frac{1}{16^2}\begin{pmatrix} (4+4r^2)^2 + 64r^2(1-2\alpha)^2(1-2\beta)^2 & 2(4+4r^2)(1-2\alpha)(1-2\beta)(-8ir)e^{-i\lambda} \\ 2(4+4r^2)(1-2\alpha)(1-2\beta)(8ir)e^{i\lambda} & (4+4r^2)^2 + 64r^2(1-2\alpha)^2(1-2\beta)^2\end{pmatrix}$$

$$Tr(\hat{A}_2^2) = \frac{1}{16^2}\left(2(4+4r^2) + 128r^2(1-2\alpha)^2(1-2\beta)^2\right) : Tr(\hat{A}_2)^2 = \frac{1}{64}(4+4r^2)^2$$

$$\therefore \quad \alpha = \frac{1}{16^2}\left(2(4+4r^2) + 128r^2(1-2\alpha)^2(1-2\beta)^2\right) - \frac{1}{8^2}(4+4r^2)^2$$

Since α does not depend upon λ, $\hat{L}$ reduces to

$$\hat{L} = \frac{1}{Tr(\hat{A})}\left[2\cdot\frac{\partial \hat{A}}{\partial \lambda} - \overset{0}{\frac{\partial \cancel{\ln(\alpha)}}{\cancel{\partial\lambda}}}\hat{A}\right] + \overset{0}{\frac{\cancel{\partial}}{\cancel{\partial\lambda}}}\left[\cancel{\ln(\alpha)} - \ln(\cancel{Tr(\hat{A})})\right]\hat{I} = \frac{2}{Tr(\hat{A}_2)}\cdot\frac{\partial \hat{A}_2}{\partial\lambda}$$

$$\hat{L}_2 = \frac{2}{Tr(\hat{A}_2)}\cdot\frac{\partial \hat{A}_2}{\partial\lambda} : \frac{\partial \hat{A}_2}{\partial\lambda} = \frac{1}{16}\begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0\end{pmatrix}$$

$$\hat{L}_2 = \frac{16}{(4+4r^2)}\cdot\frac{1}{16}\begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0\end{pmatrix}$$

$$\boxed{\hat{L}_2 = \frac{1}{(4+4r^2)}\begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0\end{pmatrix}}$$

iii.) Calculate QFI

$$H = H_1 + H_2 : H_1 = 0, \quad H_i = Tr\left[L_i \frac{\partial A_i}{\partial\lambda}\right] : H_2 = Tr\left[\hat{L}_2\cdot\frac{\partial \hat{A}_2}{\partial\lambda}\right]$$

$$\hat{A}_2 = Tr\left[\hat{L}_2\cdot\frac{\partial \hat{A}_2}{\partial\lambda}\right]$$

$$\frac{\partial \hat{A}_2}{\partial\lambda} = \frac{1}{16}\begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0\end{pmatrix}$$

$$\hat{L}_2 = \frac{1}{(4+4r^2)}\cdot\begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0\end{pmatrix}$$

Exercise 2 continued

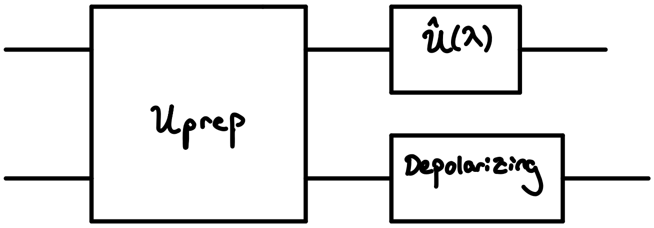
$$\hat{L}_2 \cdot \frac{\partial \hat{A}_2}{\partial \lambda} = \frac{1}{16(4+4r^2)} \begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0 \end{pmatrix} \times \begin{pmatrix} 0 & -(1-2\alpha)(1-2\beta)(8r)e^{-i\lambda} \\ -(1-2\alpha)(1-2\beta)(8r)e^{i\lambda} & 0 \end{pmatrix}$$

$$\hat{L}_2 \cdot \frac{\partial \hat{A}_2}{\partial \lambda} = \frac{1}{16(4+4r^2)} \begin{pmatrix} (1-2\alpha)^2(1-2\beta)^2(8r)^2 & 0 \\ 0 & (1-2\alpha)^2(1-2\beta)^2(8r)^2 \end{pmatrix}$$

$$\mathrm{Tr}\left[\hat{L}_2 \cdot \frac{\partial \hat{A}_2}{\partial \lambda}\right] = \frac{2 \cdot \cancel{64} r^2 (1-2\alpha)^2 (1-2\beta)^2}{\cancel{64}(1+r^2)} = \frac{2r^2(1-2\alpha)^2(1-2\beta)^2}{(1+r^2)}$$

$$\boxed{H = \frac{2r^2(1-2\alpha)^2(1-2\beta)^2}{(1+r^2)}}$$

Exercise 3



a.) Write $\hat{\rho}_{prep} = \dots |0\rangle|0\rangle\langle 0|\langle 0| + |0\rangle|0\rangle\langle 0|\langle 1| \dots\dots$

$$\hat{\rho}_{prep} = \hat{\mathcal{U}}_{prep}\,\hat{\rho}\,\hat{\mathcal{U}}_{prep}^{\dagger} = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (-8ir) \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (8ir) & 0 & 0 & (4+4r^2) \end{pmatrix}$$

$$\hat{\rho}_{prep} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]$$

$$\boxed{\hat{\rho}_{prep} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]}$$

b.) Determine $\hat{\mathcal{U}}(\lambda)$ on $\hat{\rho}_{prep}$

$$\hat{\mathcal{U}}\hat{\rho}_{prep}\hat{\mathcal{U}}^{\dagger} = \frac{1}{16}\begin{pmatrix} (4+4r^2) & 0 & 0 & (-8ir)e^{-i\lambda} \\ 0 & (4-4r^2) & 0 & 0 \\ 0 & 0 & (4-4r^2) & 0 \\ (8ir)e^{i\lambda} & 0 & 0 & (4+4r^2) \end{pmatrix}$$

$$\hat{\mathcal{U}}\hat{\rho}_{prep}\hat{\mathcal{U}}^{\dagger} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)e^{-i\lambda}|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)e^{i\lambda}|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]$$

$$\boxed{\hat{\mathcal{U}}\hat{\rho}_{prep}\hat{\mathcal{U}}^{\dagger} = \frac{1}{16}\Big[(4+4r^2)|0\rangle|0\rangle\langle 0|\langle 0| + (-8ir)e^{-i\lambda}|0\rangle|0\rangle\langle 1|\langle 1| + (4-4r^2)|0\rangle|1\rangle\langle 0|\langle 1| + (4-4r^2)|1\rangle|0\rangle\langle 1|\langle 0| + (8ir)e^{i\lambda}|1\rangle|1\rangle\langle 0|\langle 0| + (4+4r^2)|1\rangle|1\rangle\langle 1|\langle 1|\Big]}$$

Exercise 3

c.) Calculate depolarizing channel : $\hat{\rho} = \frac{1-\alpha}{2} Tr[\rho]\hat{I} + \hat{\rho}$

$$\hat{u}\hat{\rho}_{prep}\hat{u}^{+} = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes|0\rangle\langle 0| + (-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes|0\rangle\langle 1| + (4-4r^2)|0\rangle\langle 0|\otimes|1\rangle\langle 1| +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes|0\rangle\langle 0| + (8ir)e^{i\lambda}|1\rangle\langle 0|\otimes|1\rangle\langle 0| + (4+4r^2)|1\rangle\langle 1|\otimes|1\rangle\langle 1|\Big]$$

Depolarizing acts on second particle

$$\hat{u}\hat{\rho}_{prep}\hat{u}^{+} = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0|\otimes\underline{\underline{|0\rangle\langle 0|}} + (-8ir)e^{-i\lambda}|0\rangle\langle 1|\otimes\underline{\underline{|0\rangle\langle 1|}} + (4-4r^2)|0\rangle\langle 0|\otimes\underline{\underline{|1\rangle\langle 1|}} +$$
$$(4-4r^2)|1\rangle\langle 1|\otimes\underline{\underline{|0\rangle\langle 0|}} + (8ir)e^{i\lambda}|1\rangle\langle 0|\otimes\underline{\underline{|1\rangle\langle 0|}} + (4+4r^2)|1\rangle\langle 1|\otimes\underline{\underline{|1\rangle\langle 1|}}\Big]$$

i.) $\hat{\rho}_i = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = |0\rangle\langle 0|$

$$\hat{\rho}_f = \frac{1-\alpha}{2} Tr[\hat{\rho}_i]\hat{I} + \alpha\hat{\rho}_i = \frac{1-\alpha}{2}\hat{I} + \alpha\hat{\rho}_i = \begin{pmatrix} \frac{1-\alpha}{2} & 0 \\ 0 & \frac{1-\alpha}{2} \end{pmatrix} + \alpha\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} + \begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix}$$

$\hat{\rho}_f = \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix}$: $\hat{\rho}_i \longrightarrow \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix}$

ii.) $\hat{\rho}_i = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = |0\rangle\langle 1|$

$$\hat{\rho}_f = \frac{1-\alpha}{2} Tr[\hat{\rho}_i]\hat{I} + \alpha\hat{\rho}_i = \frac{1-\alpha}{2}\cdot 0\cdot\hat{I} + \alpha\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}$$

$\hat{\rho}_f = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}$: $\hat{\rho}_i \longrightarrow \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}$

iii.) $\hat{\rho}_i = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = |1\rangle\langle 0|$

$$\hat{\rho}_f = \frac{1-\alpha}{2} Tr[\hat{\rho}_i]\hat{I} + \alpha\hat{\rho}_i = \frac{1-\alpha}{2}\cdot 0\cdot\hat{I} + \alpha\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix}$$

$\hat{\rho}_f = \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix}$: $\hat{\rho}_i \longrightarrow \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix}$

iv.) $\hat{\rho}_i = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = |1\rangle\langle 1|$

$$\hat{\rho}_f = \frac{1-\alpha}{2} Tr[\hat{\rho}_i]\hat{I} + \alpha\hat{\rho}_i = \frac{1-\alpha}{2}\cdot\hat{I} + \alpha\hat{\rho}_i = \begin{pmatrix} \frac{1-\alpha}{2} & 0 \\ 0 & \frac{1-\alpha}{2} \end{pmatrix} + \alpha\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \alpha \end{pmatrix}$$

$\hat{\rho}_f = \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix}$: $\hat{\rho}_i \longrightarrow \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix}$

$$\boxed{\begin{array}{l} \hat{\rho}_i = |0\rangle\langle 0| \longrightarrow \hat{\rho}_f = \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} \; : \; \hat{\rho}_i = |0\rangle\langle 1| \longrightarrow \hat{\rho}_f = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} \\ \hat{\rho}_i = |1\rangle\langle 0| \longrightarrow \hat{\rho}_f = \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix} \; : \; \hat{\rho}_i = |1\rangle\langle 1| \longrightarrow \hat{\rho}_f = \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix} \end{array}}$$

Exercise 3 Continued

v.) $\hat{\rho}_i = |0\rangle\langle 0| \longrightarrow \hat{\rho}_F = \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix}$: $\hat{\rho}_i = |0\rangle\langle 1| \longrightarrow \hat{\rho}_F = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}$

$\hat{\rho}_i = |1\rangle\langle 0| \longrightarrow \hat{\rho}_F = \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix}$: $\hat{\rho}_i = |1\rangle\langle 1| \longrightarrow \hat{\rho}_f = \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix}$

These rules evolve $\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger$ to

$$\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\Big[(4+4r^2)|0\rangle\langle 0| \overset{1}{\otimes} \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} + (-8ir)e^{-i\lambda}|0\rangle\langle 1| \overset{2}{\otimes} \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} + (4-4r^2)|0\rangle\langle 0| \overset{3}{\otimes} \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix}$$
$$(4-4r^2)|1\rangle\langle 1| \overset{4}{\otimes} \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} + (8ir)e^{i\lambda}|1\rangle\langle 0| \overset{5}{\otimes} \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix} + (4+4r^2)|1\rangle\langle 1| \overset{6}{\otimes} \frac{1}{2}\begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix}\Big]$$

1: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} = \begin{pmatrix} 1+\alpha & 0 & 0 & 0 \\ 0 & 1-\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 2: $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

3: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix} = \begin{pmatrix} 1-\alpha & 0 & 0 & 0 \\ 0 & 1+\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 4: $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\alpha & 0 \\ 0 & 0 & 0 & 1-\alpha \end{pmatrix}$

5: $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ \alpha & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 0 \end{pmatrix}$ 6: $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1-\alpha & 0 \\ 0 & 1+\alpha \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\alpha & 0 \\ 0 & 0 & 0 & 1+\alpha \end{pmatrix}$

① $= \begin{pmatrix} (2+2r^2)(1+\alpha) & 0 & 0 & 0 \\ 0 & (2+2r^2)(1-\alpha) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ ② $= \begin{pmatrix} 0 & 0 & 0 & (-8ir)e^{-i\lambda}(\alpha) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

③ $= \begin{pmatrix} (2-2r^2)(1-\alpha) & 0 & 0 & 0 \\ 0 & (2-2r^2)(1+\alpha) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ ④ $= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (2-2r^2)(1+\alpha) & 0 \\ 0 & 0 & 0 & (2-2r^2)(1-\alpha) \end{pmatrix}$

⑤ $= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (8ir)e^{i\lambda}(\alpha) & 0 & 0 & 0 \end{pmatrix}$ ⑥ $= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (2+2r^2)(1-\alpha) & 0 \\ 0 & 0 & 0 & (2+2r^2)(1+\alpha) \end{pmatrix}$

① + ② + ③ + ④ + ⑤ + ⑥

$$\boxed{\hat{u}\hat{\rho}_{prep}\hat{u}^\dagger = \frac{1}{16}\begin{pmatrix} (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) & 0 & 0 & (-8ir)e^{-i\lambda}(\alpha) \\ 0 & (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) & 0 & 0 \\ 0 & 0 & (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) & 0 \\ (8ir)e^{i\lambda}(\alpha) & 0 & 0 & (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) \end{pmatrix}}$$

Exercise 3 Continued

d.) Calculate QFI

$$\hat{A}_1 = \frac{1}{16}\begin{pmatrix} (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) & 0 \\ 0 & (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) \end{pmatrix}$$

$$\hat{A}_2 = \frac{1}{16}\begin{pmatrix} (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) & (-8ir)e^{-i\lambda}(\alpha) \\ (8ir)e^{i\lambda}(\alpha) & (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) \end{pmatrix}$$

$$\hat{L} = \frac{1}{Tr(\hat{A})}\left[2\cdot\frac{\partial\hat{A}}{\partial\lambda} - \frac{\partial \ln(\beta)}{\partial\lambda}\hat{A}\right] + \frac{\partial}{\partial\lambda}\left[\ln(\beta) - \ln(Tr(\hat{A}))\right]\hat{I} \quad , \quad H_i = Tr\left[L_i\frac{\partial A_i}{\partial\lambda}\right]$$

i.) Calculate $\hat{L}_1$: $\alpha = Tr(\hat{A}_1^2) - Tr(\hat{A}_1)^2$

$$\hat{A}_1^2 = \frac{1}{16^2}\begin{pmatrix} (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) & 0 \\ 0 & (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) \end{pmatrix}$$

$$\times \begin{pmatrix} (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) & 0 \\ 0 & (2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha) \end{pmatrix}$$

$$\hat{A}_1^2 = \frac{1}{16^2}\begin{pmatrix} [(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)]^2 & 0 \\ 0 & [(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)]^2 \end{pmatrix}$$

$$Tr(\hat{A}_1^2) = \frac{1}{16^2}\cdot 2\left[(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)\right]^2$$

$$Tr(\hat{A}_1) = \frac{1}{8}\left[(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)\right] \ : \ Tr(\hat{A}_1)^2 = \frac{1}{8^2}\left[(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)\right]^2$$

$$\alpha = \frac{1}{16^2}\cdot 2\left[(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)\right]^2 - \frac{1}{8^2}\left[(2+2r^2)(1-\alpha)+(2-2r^2)(1+\alpha)\right]^2$$

Since α does not depend upon λ, $\hat{L}_1$ reduces to

$$\hat{L}_1 = \frac{1}{Tr(\hat{A})}\left[2\cdot\frac{\partial\hat{A}_1}{\partial\lambda} - \overset{0}{\cancel{\frac{\partial \ln(\alpha)}{\partial\lambda}}}\hat{A}_1\right] + \overset{0}{\cancel{\frac{\partial}{\partial\lambda}}}\left[\ln(\alpha) - \ln(Tr(\hat{A}_1))\right]\hat{I}$$

$$\hat{L}_1 = \frac{1}{Tr(\hat{A}_1)}\cdot 2\cdot\frac{\partial\hat{A}_1}{\partial\lambda}$$ and since $\hat{A}_1$ does not depend upon λ, $\hat{L}_1 = 0$

$$\boxed{\hat{L}_1 = 0}$$

And thus $H_1 = 0$

ii.) Calculate $\hat{L}_2$: $\alpha = Tr(\hat{A}_2^2) - Tr(\hat{A}_2)^2$

Exercise 3 continued

$$\hat{A}_2^2 = \frac{1}{16^2}\begin{pmatrix} (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) & (-8ir)e^{-i\lambda}(\alpha) \\ (8ir)e^{i\lambda}(\alpha) & (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) \end{pmatrix}$$

$$\times \begin{pmatrix} (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) & (-8ir)e^{-i\lambda}(\alpha) \\ (8ir)e^{i\lambda}(\alpha) & (2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha) \end{pmatrix}$$

$$\hat{A}_2^2 = \frac{1}{16^2}\begin{pmatrix} [(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]^2+64r^2(\alpha)^2 & 2(-8ir)e^{-i\lambda}(\alpha)[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)] \\ 2(8ir)e^{i\lambda}(\alpha)[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)] & [(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]^2+64r^2(\alpha)^2 \end{pmatrix}$$

$$Tr(\hat{A}_2^2) = \frac{1}{16^2}\left(2\left([(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]^2+64r^2(\alpha)^2\right)\right)$$

$$Tr(\hat{A}_2) = \frac{1}{8}[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)] \quad : \quad Tr(\hat{A}_2)^2 = \frac{1}{8^2}[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]^2$$

$$\alpha = \frac{1}{16^2}\left(2\left([(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]^2+64r^2(\alpha)^2\right)\right) - \frac{1}{8^2}[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]^2$$

Since α does not depend upon λ, $\hat{L}_{2_0}$ reduces to

$$\hat{L}_2 = \frac{1}{Tr(\hat{A}_2)}\left[2\cdot\frac{\partial \hat{A}_2}{\partial\lambda} - \cancel{\frac{\partial \ln(\alpha)}{\partial\lambda}}\hat{A}_2\right] + \cancel{\frac{\partial}{\partial\lambda}}\left[\cancel{\ln(\alpha)} - \ln(\cancel{Tr}(\hat{A}_2))\right]\hat{I}$$

$$\hat{L}_2 = \frac{1}{Tr(\hat{A}_2)}\cdot 2\frac{\partial \hat{A}_2}{\partial\lambda} \quad : \quad \frac{\partial \hat{A}_2}{\partial\lambda} = \frac{1}{16}\begin{pmatrix} 0 & -(8r)e^{-i\lambda}(\alpha) \\ -(8r)e^{i\lambda}(\alpha) & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -(r)e^{-i\lambda}(\alpha) \\ -(r)e^{i\lambda}(\alpha) & 0 \end{pmatrix}$$

$$\hat{L}_2 = \frac{8}{[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]}\begin{pmatrix} 0 & -(r)e^{-i\lambda}(\alpha) \\ -(r)e^{i\lambda}(\alpha) & 0 \end{pmatrix}$$

$$\boxed{\hat{L}_2 = \frac{8}{[(2+2r^2)(1+\alpha)+(2-2r^2)(1-\alpha)]}\begin{pmatrix} 0 & -re^{-i\lambda}(\alpha) \\ -re^{i\lambda}(\alpha) & 0 \end{pmatrix}}$$

iii.) Calculate H : $H = Tr[\hat{L}_2 \cdot \frac{\partial \hat{A}_2}{\partial\lambda}]$

$$\hat{L}_2 = \frac{4}{[(1+r^2)(1+\alpha)+(1-r^2)(1-\alpha)]}\begin{pmatrix} 0 & -re^{-i\lambda}(\alpha) \\ -re^{i\lambda}(\alpha) & 0 \end{pmatrix} \quad \frac{\partial \hat{A}_2}{\partial\lambda} = \frac{1}{2}\begin{pmatrix} 0 & -re^{-i\lambda}(\alpha) \\ -re^{i\lambda}(\alpha) & 0 \end{pmatrix}$$

$$\hat{L}_2 \cdot \frac{\partial \hat{A}_2}{\partial\lambda} = \frac{2}{[(1+r^2)(1+\alpha)+(1-r^2)(1-\alpha)]}\begin{pmatrix} r^2\alpha^2 & 0 \\ 0 & r^2\alpha^2 \end{pmatrix}$$

$$Tr[\hat{L}_2 \cdot \frac{\partial \hat{A}_2}{\partial\lambda}] = \frac{4r^2\alpha^2}{[(1+r^2)(1+\alpha)+(1-r^2)(1-\alpha)]}$$

$$\boxed{H = \frac{4r^2\alpha^2}{[(1+r^2)(1+\alpha)+(1-r^2)(1-\alpha)]}}$$

Exercise 4

Simplify when $r \ll 1$

a.) $H = r^2$: $r \ll 1$ $\therefore$ $H = 0$

b.) $H = \dfrac{2r^2}{(1+r^2)}$: $H/r^2 = \dfrac{2}{(1+r^2)}$: $r \ll 1$ $\therefore$ $H = 2$

c.) $H = \dfrac{2r^2(1-2\alpha)^2}{(1+r^2)}$: $H/r^2 = \dfrac{2(1-2\alpha)^2}{(1+r^2)}$: $r \ll 1$ $\therefore$ $H = 2(1-2\alpha)^2$

d.) $H = \dfrac{2r^2\alpha^2}{((2\alpha-1)r^2+1)} + \dfrac{2r^2(1-\alpha)^2}{(1-(2\alpha-1)r^2)}$: $H/r^2 = \dfrac{2\alpha^2}{((2\alpha-1)r^2+1)} + \dfrac{2(1-\alpha)^2}{(1-(2\alpha-1)r^2)}$

: $r \ll 1$ $\therefore$ $H = 2\alpha^2 + 2(1-\alpha)^2$

e.) $H = (1-2\alpha)^2 r^2$: $r \ll 1$ $\therefore$ $H = (1-2\alpha)^2$

f.) $H = \dfrac{2r^2(1-2\alpha)^2(1-2\beta)^2}{(1+r^2)}$: $H/r^2 = \dfrac{2(1-2\beta)^2}{(1+r^2)}$: $r \ll 1$ $\therefore$ $H = 2(1-2\beta)^2$

g.) $H = \dfrac{4r^2\alpha^2}{[(1+r^2)(1+\alpha)+(1-r^2)(1-\alpha)]}$: $H/r^2 = \dfrac{4\alpha^2}{[(1+r^2)(1+\alpha)+(1-r^2)(1-\alpha)]}$

: $r \ll 1$ $\therefore$ $H = 2\alpha^2$

Exercise 5

a.) $H = r^2 \longrightarrow H \propto r^2$

b.) $H = \dfrac{2r^2}{(1+r^2)} \longrightarrow H \propto \dfrac{r^2}{(1+r^2)}$

c.) $H = \dfrac{2r^2(1-2\alpha)^2}{(1+r^2)} \longrightarrow H \propto \dfrac{r^2}{(1+r^2)} \longrightarrow H \propto (1-2\alpha)^2$

d.) $H = \dfrac{2r^2\alpha^2}{((2\alpha-1)r^2+1)} + \dfrac{2r^2(1-\alpha)^2}{(1-(2\alpha-1)r^2)}$