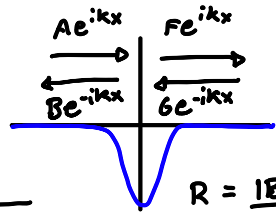
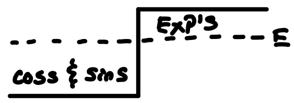
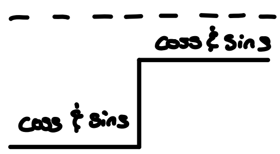
## 2.4 Free Particle

$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \varphi(k) e^{i(kx - \frac{\hbar k^2}{2m}t)} dk \; : \; \varphi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0) e^{-ikx} dx$

## 2.5 The Delta Function Potential



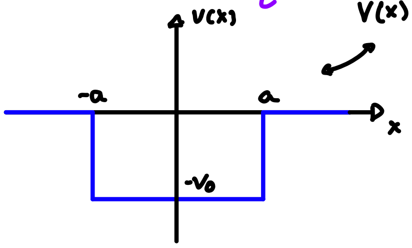

$\int_b^c f(x)\delta(x-a)\,dx = f(a)$ a $\in [b,c]$ :

$R = \frac{1}{1 + (2\hbar^2 E / m\alpha^2)}$ , $T = \frac{1}{1 + (m\alpha^2 / 2\hbar^2 E)}$

$R = \frac{|B|^2}{|A|^2}$, $T = \frac{|F|^2}{|A|^2}$, $R + T = 1$

$x \to \infty \; (ce^{-\alpha x} + \cancel{De^{\alpha x}})$ ($D = 0$)
$x \to -\infty \; (\cancel{ce^{-\alpha x}} + De^{\alpha x})$ ($c = 0$)

## 2.6 The Finite Square Well




$V(x) = \begin{cases} -V_0, & \text{for } -a < x < a \\ 0, & \text{for } |x| > a \end{cases}$

$\ell = \sqrt{2m(E+V_0)}/\hbar$
$k = \sqrt{-2mE}/\hbar$

: $\Psi(x) = Ce^{-i\ell x} + De^{i\ell x}$, for $-a < x < a$
: $\Psi(x) = Ae^{-\kappa x} + Be^{\kappa x}$, for $x < -a$
: $\Psi(x) = Ee^{-\kappa x} + Fe^{\kappa x}$, for $x > a$

## 3.1 Hilbert Space

$|\alpha\rangle \to a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\langle\alpha| \to a^* = (a_1^* \, a_2^* \, a_3^*)$, wave functions live in Hilbert Space, $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$

$\langle\alpha|\alpha\rangle = \int_a^b |f(x)|^2 dx$, $\langle f_m | f_n \rangle = \delta_{mn}$ : Finite inner product $\to$ In Hilbert Space

## 3.2 Observables

$\langle Q \rangle = \int \Psi^* \hat{Q} \Psi \, dx = \langle \Psi | \hat{Q}\Psi \rangle$ : $Q(x,p)$ : $\langle f | \hat{Q} g \rangle = \langle \hat{Q} f | g \rangle$ :

Observables are represented by hermitian operators : operators in QM represent physically observable quantities

Determinate States : Every measurement of $\hat{Q}$ returns the same value $q$

: Determinate states are eigenfunctions of $\hat{Q}$

## 3.4 Generalized Statistical Interpretation

$|c_n|^2$ where $c_n = \langle f_n | \Psi \rangle$ : $|c(z)|^2 dz = \langle f_z | \Psi \rangle$ : $c_n = \langle f_n | \Psi \rangle = \int f_n(x)^* \Psi(x,t)\,dx$ : $\langle \hat{Q} \rangle = \sum_n q_n |c_n|^2$

$\varphi(p,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} e^{-ipx/\hbar} \cdot \Psi(x,t)\,dx$ : $\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} e^{ipx/\hbar} \varphi(p,t)\,dp$

$\Psi(x,t) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2} e^{-iEt/\hbar}$ : $E = -\frac{m\alpha^2}{2\hbar^2}$ : $\langle \hat{A} \rangle = \langle \mathcal{S}(0) | \hat{A} | \mathcal{S}(0) \rangle$ : $P = |\langle \lambda_n | \mathcal{S}(t) \rangle|^2$

$|\mathcal{S}(t)\rangle = \varphi(t) \cdot |\mathcal{S}(0)\rangle$ : $|\mathcal{S}(0)\rangle = |\lambda_n\rangle$ : $\varphi(t) = e^{\frac{-iE_n t}{\hbar}}$

## 3.5 The Uncertainty Principle

$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ : $\sigma_A^2 \sigma_B^2 \geq \left( \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$ : $[\hat{x}, \hat{p}] = i\hbar$ : $\Delta t \Delta E \geq \frac{\hbar}{2}$ : $\frac{d}{dt}\langle Q \rangle = \frac{i}{\hbar} \langle [\hat{H}, \hat{Q}] \rangle + \left\langle \frac{\partial \hat{Q}}{\partial t} \right\rangle$

## 3.6 Dirac Notation

$\Psi(x,t) = \langle x | \mathcal{S}(t) \rangle$ : $\varphi(p,t) = \langle p | \mathcal{S}(t) \rangle$ : $c_n(t) = \langle n | \mathcal{S}(t) \rangle$ : $\langle e_m | \hat{Q} | e_n \rangle = Q_{mn}$ : $i\hbar \frac{d}{dt} |\mathcal{S}\rangle = H |\mathcal{S}\rangle$

$H|\mathcal{S}\rangle = E|\mathcal{S}\rangle$ : $\hat{P} = |\alpha\rangle\langle\alpha|$ : $\hat{P}|\beta\rangle = \langle\alpha|\beta\rangle|\alpha\rangle$ : $\langle e_m | e_n \rangle = \delta_{mn}$

## Other

Discrete Values :

$c_n = \langle \Psi_n | \Psi(x,0) \rangle$
$\Psi(x,t) = \sum c_n e^{-iE_n t/\hbar}$
$\langle \Psi_m | \Psi_n \rangle = \delta_{mn}$

Continuous :

$\varphi(k) = \langle F_{p'} | \Psi(x,t) \rangle$
$\Psi(x,t) = \langle \varphi(k) | F_p e^{-iE_n t/\hbar} \rangle$
$\langle F_p | F_p \rangle = \delta(p - p')$

$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$ : $\hat{H}\Psi = \hat{E}\Psi$

$\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$ : $\hat{E} = i\hbar \frac{\partial \Psi}{\partial t}$ : $\hat{P} = \frac{\hbar}{i} \frac{\partial}{\partial x}$ : $\hat{T} = \frac{\hat{P}^2}{2m}$

$\int_0^\infty x^n e^{-x/a} dx = n!\, a^{n+1}$ : $\int_0^\infty x^{2n} e^{-x^2/a^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left( \frac{a}{2} \right)^{2n+1}$ : $\int_0^\infty x^{2n+1} e^{-x^2/a^2} dx = \frac{n!}{2} a^{2n+2}$

$[x^2 + y, z] = [x^2, z] + [y, z]$ : $[x^2, z] = x[x, z] + [x, z]x$