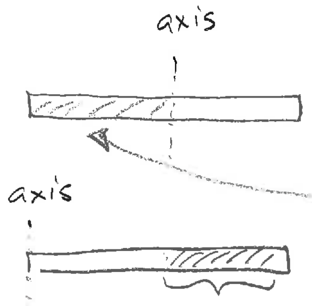
57, 10

Knight Ch 12

Conc Q 5



all of this mass is further from axis of rotation than this

Conc Q 6

$$I = \frac{2}{5} M R^2$$

Both M and R change here

$$M = \rho V = \rho \frac{4}{3} \pi R^3$$

ρ: density = const

V: volume

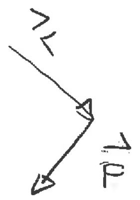
$$\Rightarrow \quad I = \frac{8}{15} \rho \pi R^5$$

Doubling R increases R^5 by $(2R)^5 = 32 R^5$

Factor of $\underline{32}$

Knight Ch 12

Conc Q 9

a) 

$\vec{r} \times \vec{F}$ into page

$\Rightarrow$ Torque negative

b), c) $\tau_{net} = 0 \Rightarrow \alpha = 0 \Rightarrow \omega =$ constant

Conc Q 10

$$\vec{\tau} = I\vec{\alpha} \Rightarrow |\tau| = I|\alpha|$$

all τ and all α are positive $\Rightarrow \tau = I\alpha$

$$\Rightarrow \alpha = \frac{\tau}{I}$$

a) $\tau = rF \qquad I = mr^2 \Rightarrow \alpha_a = \frac{F}{mr}$

b) $\tau = 2Fr \qquad I = 2mr^2 \Rightarrow \alpha_b = \frac{F}{mr}$

c) $\tau = 2rF \qquad I = m4r^2 \Rightarrow \alpha_c = \frac{F}{2mr}$

d) $\tau = 4rF \qquad I = 2m4r^2 \Rightarrow \alpha_d = \frac{F}{2mr}$

$$\alpha_a = \alpha_b > \alpha_c = \alpha_d$$

Prob 15

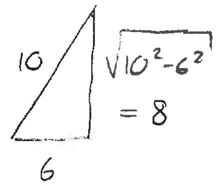
a)

$$X_{c.o.m} = \frac{M_A X_A + M_B X_B + M_C X_C}{M_A + M_B + M_C}$$

	x	y
A	0.06m	0.08m
B	0m	0m
C	0.12m	0m

$$= \frac{0.200kg \times 0.060m + 0.100kg \times 0.12m}{0.400kg}$$

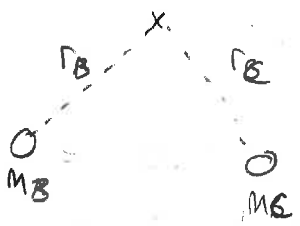
$$= \frac{0.024m\,kg}{0.400\,kg} = 0.06\ m$$



$$y_{c.o.m} = \frac{M_A y_A + M_B y_B + M_C y_C}{M_A + M_B + M_C}$$

$$= \frac{0.08m \times 0.200kg}{0.400kg} = 0.04m$$

b)

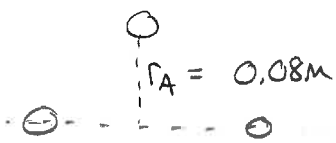


$$I = M_B r_B^2 + M_C r_C^2$$

$$= 2 M_B r_B^2 = 2 \times 0.100kg \times (0.10m)^2$$

$$= 0.0020\,kg\,m^2$$

c)



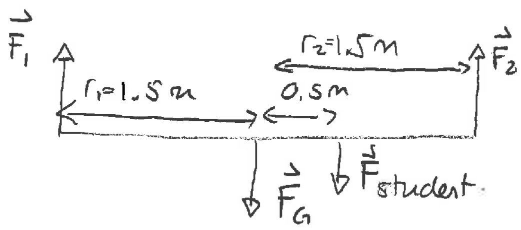
$$I = M_A r_A^2 = 0.200kg \times (0.08m)^2$$

$$= 0.0013kg\,m^2$$

Knight Ch 12

4ed Prob 31 ~~Prob 61~~

Consider the beam. The forces are as illustrated



Now $F_{student} = M_{student}\, g$

$F_G = M_{beam}\, g$

In equilibrium $\vec{F}_{net} = 0$

$\vec{\tau}_{net} = 0$

So $\vec{F}_{net} = 0 \Rightarrow F_1 + F_2 - F_G - F_{student} = 0$

$\Rightarrow F_1 + F_2 = (M_{student} + M_{beam})\, g \Rightarrow F_2 = (M_{student} + M_{beam})g - F_1$

$\vec{\tau}_{net} = 0 \Rightarrow \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_G + \vec{\tau}_{student} = 0.$

Take torques about the center of mass So $\tau_G = 0.$

$\tau_1 = r_1 F_1 \sin\theta = 1.5\,m\, F_1 \sin 270° = -1.5\, F_1$

$\tau_2 = r_2 F_2 \sin\theta = 1.5\,m\, F_1 \sin 90° = 1.5\, F_2$

$\tau_{student} = r_{student}\, F_{student} \sin\theta$ $\quad \vec{r}_{student}$, $\vec{F}_{student}$

$= 0.5\, M_{student}\, g \sin 270°$

$= -0.5\, M_{student}\, g$

$\vec{\tau}_{net} = 0 \Rightarrow -1.5\, F_1 + 1.5\, F_2 - 0.5\, M_{student}\, g = 0$

substitute

$-1.5\, F_1 + 1.5\left((M_{student} + M_{beam})\, g - F_1\right) = 0.5\, M_{student}\, g.$

$\Rightarrow -3F_1 = -1.5(M_{student} + M_{beam})g + 0.5 M_{student} g$

Knight Ch 12

4ed Prob 31 ~~Prob 61~~ cont

So $F_1 = \left[\frac{1.5}{3}(M_{student} + M_{beam}) - \frac{0.5}{3} M_{student}\right] g$

$= \left[\frac{1.5}{3}(180\text{ kg}) - \frac{0.5}{3} 80\text{kg}\right] g$

$= 750\text{ N}$

Then $F_2 = (M_{student} + M_{beam})g - F_1$

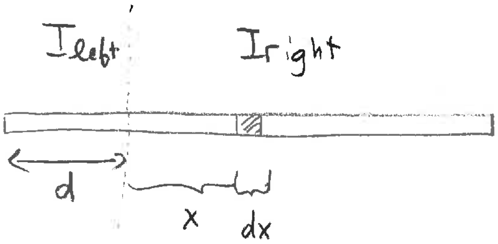
$= 180\text{kg} \times 9.8 - 750\text{N}$

$= 1000\text{ N}$

Knight Ch ~~16~~ 12

4ed Prob 53

~~Prob 55~~



The moment of inertia is $I_{left} + I_{right} = I$

For right the shaded region gives:

$$dI = dm\, x^2$$

But $\dfrac{dm}{M} = \dfrac{dx}{L} \Rightarrow dm = dx\,\dfrac{M}{L} \Rightarrow dI = \dfrac{M}{L} x^2 dx$

So $I_{right} = \dfrac{M}{L}\displaystyle\int_0^{L-d} x^2\,dx = \dfrac{M}{L}\,\dfrac{x^3}{3}\Big|_0^{L-d}$

$$= \frac{M}{L}\,\frac{(L-d)^3}{3}$$

Similarly $I_{left} = \dfrac{M}{L}\displaystyle\int_0^{d} x^2\,dx = \dfrac{M}{L}\,\dfrac{x^3}{3}\Big|_0^{d} = \dfrac{M}{L}\,\dfrac{d^3}{3}$

Thus

$$I = \frac{M}{3L}\left\{(L-d)^3 + d^3\right\}$$

$$= \frac{M}{3L}\left\{L^3 - 3L^2d + 3Ld^2 - \cancel{d^3} + \cancel{d^3}\right\}$$

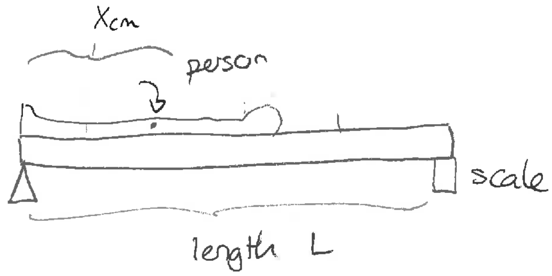
$$= \frac{M}{3}\left\{L^2 - 3Ld + 3d^2\right\}$$

If $d = 0$ $\quad \frac{M}{3}L^2$ ✓

$d = L$ $\quad \frac{M}{3}L^2$ ✓ $\qquad d = L/2 \quad I = \frac{M}{3}\left(L^2 - \frac{3}{2}L^2 + \frac{3}{4}L^2\right) = \frac{M}{12}L^2$ ✓

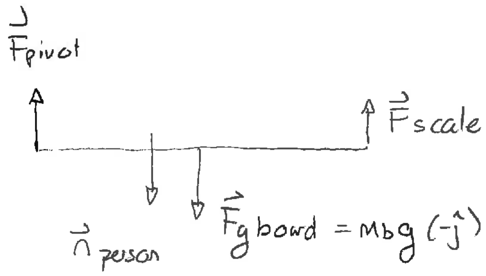
Knight Ch12

4ed Prob 57



$\vec{\tau}_{net} = 0$

$\vec{F}_{net} = 0$



Forces: $\vec{F}_{net} = 0$

$\Rightarrow F_{scale} + F_{pivot} - M_b g - M_p g = 0$

will give F_{pivot} (not useful)

Note $\quad \vec{n}_{person} = -\vec{F}_{g\,person}$

$n_{person} = M_p g$

Also $\quad F_{scale} = 25kg \times g$

Torques: $\quad \vec{\tau}_{net} = 0 \quad$ (about pivot)

$\Rightarrow \tau_{scale} + \tau_{board} + \tau_{person} + \tau_{pivot} = 0$

Individually $\quad \tau = rF\sin\phi$

pivot: $\quad r = 0 \Rightarrow \quad \tau_{pivot} = 0$

scale: $\quad \tau_{scale} = L\, F_{scale} \sin 90°$

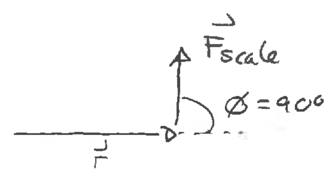
$= Lg\, 25kg$

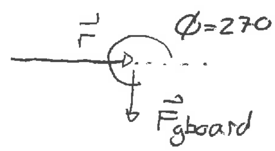
board: $\quad \tau_{board} = L/2\, M_b g \sin 270°$

$= -LM_b g/2$

person $\quad \tau_{person} = x_{cm} M_p g \sin 270°$

$= -x_{cm} M_p g$





$\tau_{net} = 0 \Rightarrow \quad Lg\,25kg - LM_b g/2 - x_{cm} M_b g \ldots$

$\Rightarrow \quad (25kg - M_b/2)\,L = x_{cm} M_b \quad \Rightarrow \quad x_{cm} = \dfrac{(25kg - 6.1kg/2)\,2.5m}{60kg}$

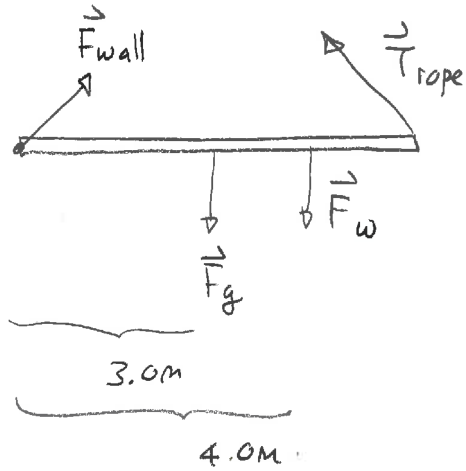
$\Rightarrow \boxed{x_{cm} = 0.91m}$

Knight Ch 12

~~Prob 62~~ 4ed 59

We require $\tau_{net} = 0$

$\vec{F}_{net} = 0$



In equilibrium

$F_w = M_w g$

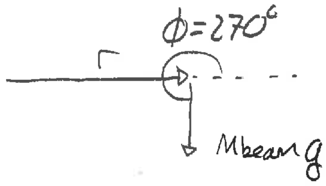
for the worker.

We work with torques about the wall pivot.

$\tau_{net} = 0 \Rightarrow \underbrace{\cancel{\tau_{wall}}}_{=0 \text{ since } r=0} + \tau_g + \tau_w + \tau_{rope} = 0$

gravity $\quad \tau_g = rF\sin\phi = rM_{beam}g\sin 270^\circ$

$= -3.0m \times 1450kg \times 9.8m/s^2 = -42630\,Nm$



similar for worker

worker $\quad \tau_w = rF\sin\phi = 4.0m \times 80kg \times 9.8m/s^2 \sin 270^\circ$

$= -3136\,Nm$

rope $\quad \tau_{rope} = rF\sin\phi = rT_{rope}\sin 150^\circ$

$= 6.0m \times T_{rope} \times \sin 150^\circ = 3.0m\,T_{rope}$

Thus $\quad \tau_{net} = 0 \Rightarrow \quad -42630Nm - 3136Nm + 3.0m\,T_{rope} = 0$

$\Rightarrow \quad T_{rope} = \dfrac{1}{3.0m}(45766\,Nm) = 15300\,N$

Calamity! Cable breaks.