Phys 131 2016

Lecture 16

Fri: HW due by 5pm

Supp Ex 34, 35, 39

Ch 6 CQ 14

Probs 29, 49, 57, 58

Thurs: Seminar

Mon: Warm Up 7

Objects on Inclined Surfaces

In many physical situations an object moves along an inclined surface while Earth's gravity and friction act on it. Examples include:

1) skier sliding down a slope
2) truck moving up a runaway truck ramp.

The same basic process, using Newton's 2nd Law, allows us to determine accelerations of objects moving in this way. First consider the frictionless case:

Quiz 1 20% − 40% ? 50% − 70%

Quiz 2 50% − 80% ? 75% − 95%

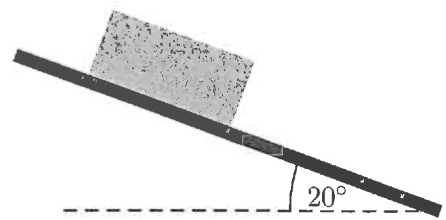
Demo: PhET Ramp: − Ice
− push object on frictionless horiz surface
− observe forces

Now suppose that friction is present

Warm Up 2 from prev class.

41 Box on a ramp with friction

A 10 kg box can move along a 4.0 m long rough ramp angled 20° from the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.25 and the coefficient of static friction is 0.30. The box is released from rest at the top of the ramp and moves down the ramp. The aim of this exercise is to determine the speed of the box at the bottom of the ramp.



a) Draw a free body diagram for the box.

b) Describe your choice of x and y axes.

c) Write Newton's Second Law in vector form and also in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_x = \cdots \tag{9}$$
$$F_{\text{net } y} = \Sigma F_y = \cdots \tag{10}$$

where x and y refer to your specially chosen axes. Insert as much information as possible about the components of acceleration at this stage. The resulting equations will generate much of the algebra that follows.

d) Determine expressions for the *magnitudes* of the gravitational and the friction forces.

e) List all the components of all the forces, using one of the two formats below.

$$\begin{aligned} F_{Gx} &= \cdots \\ F_{Gy} &= \cdots \\ n_x &= \cdots \\ n_y &= \cdots \\ &\vdots \end{aligned}$$

Force	x comp	y comp
$\vec{\mathbf{F}}_G$		
$\vec{\mathbf{n}}$		
$\vdots$		

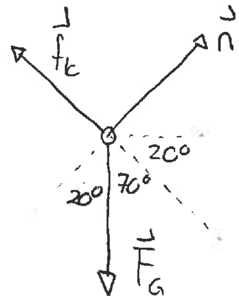
f) Use Eq. (9) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (10). Use the resulting equations to find the acceleration of the box.

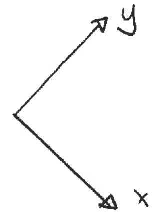
g) Determine the speed of the box when it reaches the bottom of the ramp.

h) Do these results depend on the mass of the box?

i) What would be the minimum force, pushing parallel to the ramp, required to keep the box at rest?

Answer: a)





b) Use x parallel to ramp
y perpendicular to ramp.

c) $\sum F_x = ma_x$

$\sum F_y = \underbrace{ma_y}_{=0}$ since object does not leave surface

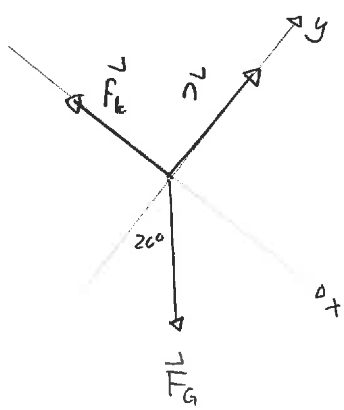
$\Longrightarrow$ $\sum F_x = ma_x$

$\sum F_y = 0$

d) $F_G = mg = 10\,kg \times 9.8\,m/s^2 \Rightarrow F_G = 98N$

$f_k = \mu_k n \Rightarrow f_k = 0.25n$

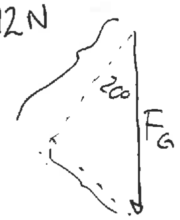
e)



Use "tilted" axes

	x	y
$\vec{F}_G$	34N	−92N
$\vec{n}$	0	n
$\vec{f}_k$	$-\mu_k n$	0

$F_{Gy} = F_G \cos 20°$
$= 98N \cos 20°$
$= 92N$



$F_{Gx} = F_G \sin 20°$
$= 98N \sin 20°$
$=$

f) $\sum F_x = ma_x \Rightarrow 34N - 0.25n = ma_x = 10kg\, a_x$

$\Rightarrow 34N - 0.25n = 10kg\, a_x$

$\sum F_y = 0 \Rightarrow -92N + n = 0 \Rightarrow n = 92N$

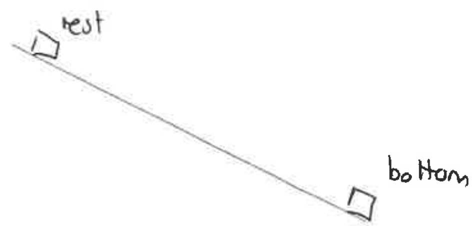
So $34N - 0.25 \times 92N = 10kg\, a_x$

$\Rightarrow 11N = 10kg\, a_x \Rightarrow a_x = 1.1m/s^2$

g) $x_0 = 0m$ $\quad x_1 = 4.0m$

$v_{0x} = 0m/s$ $\quad v_{1x} = ?$

$a_x = 1.1m/s^2$



$$v_{1x}^2 = v_{0x}^2 + 2a_x \Delta x$$

$\Rightarrow v_{1x}^2 = (0m/s)^2 + 2 \times 1.1m/s^2 \times 4.0m = 8.4m^2/s^2$

$\Rightarrow v_{1x} = \sqrt{8.4m^2/s^2} \Rightarrow v_{1x} = 2.9m/s.$

h) Working with a general mass.

	x	y
$\vec{F}_G$	$mg\sin 20°$	$-mg\cos 20°$
$\vec{n}$	0	n
$\vec{f}_k$	$-\mu_k n$	0

$\sum F_y = 0 \Rightarrow n = mg\cos 20°$

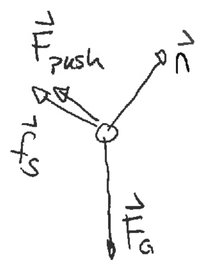
$\sum F_x = ma_x \Rightarrow mg\sin 20° - \mu_k n = ma_x$

$\Rightarrow \cancel{m}g\sin 20° - \mu_k \cancel{m}g\cos 20° = \cancel{m}a_x$

$\Rightarrow g(\sin 20° - \mu_k \cos 20°) = a_x$

Does not depend on mass

i) Static friction:



Again at rest

$$\Sigma F_x = ma_x \Rightarrow \quad \Sigma F_x = 0$$

$$\Sigma F_y = ma_y \Rightarrow \quad \Sigma F_y = 0$$

Thus $\Sigma F_x = 0 \Rightarrow \quad F_G \sin 20° - f_s - F_{push} = 0$

$$\Sigma F_y = 0 \Rightarrow \underbrace{-F_G \cos 20° + n = 0}$$

$$\Rightarrow n = mg\cos 20°$$

Minimum push occurs for max friction: $f_s = \mu_s n$

$$= \mu_s mg \cos 20°$$

Thus: $\quad mg\sin 20° - \mu_s mg\cos 20° = F_{push}$

$$\Rightarrow mg\left[\sin 20° - \mu_s \cos 20°\right] = F_{push}$$

$$\Rightarrow 10\,kg \times 9.8\,m/s^2 \left[\sin 20° - 0.30\cos 20°\right] = F_{push}$$

$$\Rightarrow F_{push} = 5.9\,N$$

Normally the coefficient of static friction is larger than the coefficient of kinetic friction. So a larger force is needed to keep the object at rest than moving with a constant force.

Demo: Forces + Motion

→ ~~Friction tab~~ Force Graphs

→ Control applied force with slider until moves.

→ Observe v vs t.

Quiz 3