3.4 Determinants and Cramers Rule

3.4 # 4, 19, 29, 43

3.4.4

$$\begin{bmatrix} 1 & -4 & 2 & -2 \\ 4 & 7 & -3 & 5 \\ 3 & 0 & 8 & 0 \\ -5 & -1 & 6 & 9 \end{bmatrix} = x$$

$|x| = 1(-1)^{1+1}(M) - 4(-1)^{1+2}(N) + 2(-1)^{1+3}(O) - 2(-1)^{1+4}(P)$ (First Row)

$|X| = 1(544) + 4(659) + 2(-204) + 2(-365)$

$|X| = 2042$

$|x| = 2042$

$$M = \begin{bmatrix} 7 & -3 & 5 \\ 0 & 8 & 0 \\ -1 & 6 & 9 \end{bmatrix} \quad N = \begin{bmatrix} 4 & -3 & 5 \\ 3 & 8 & 0 \\ -5 & 6 & 9 \end{bmatrix}$$

$|M| = 544$ $\quad |N| = 659$

$$O = \begin{bmatrix} 4 & 7 & 5 \\ 3 & 0 & 0 \\ -5 & -1 & 9 \end{bmatrix} \quad P = \begin{bmatrix} 4 & 7 & -3 \\ 3 & 0 & 8 \\ -5 & -1 & 6 \end{bmatrix}$$

$|O| = -204$ $\quad |P| = -365$

3.4.19

$$\begin{bmatrix} 6 & 22 & 0 & -3 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = x$$

$$A = \begin{bmatrix} 6 & 22 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 13 \end{bmatrix} \quad |A| = -78$$

$6(-1)(13)(4) = -312$

$|X| = -312$

$= 4(-1)^{4+4}(|A|)$

$= 4(1)(-78)$

$= -312$

$13(-1)^{3+3}(-6)$

$13(-6)$

-78

3.4.29

$|A^{-1}| = \frac{1}{|A|}$

$A^{-1} = \frac{1}{A}$ $\quad |A| \neq 0$ $\quad$ i.e $\quad A\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$|A| = |A|$

$|A^{-1}| = \frac{|1|}{|A^{-1}|}$

$|A^{-1}| = \frac{1}{|A^{-1}|}$

$|A| = 1 \quad 1 \neq 0$

$|A^{-1}| = 4-(3) = 1 \quad |A^{-1}| = 1$

$|A^{-1}| = \frac{1}{|A|} \cdot 1 = \frac{1}{1} : 1 = 1$ ✓

3.4.39

$x + 2y = 2$

$2x + 5y = 0$

Cramers rule: $x_i = \frac{|A_i|}{|A|}$

$$\vec{b} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \vec{a} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

$$\vec{A}_1 = \begin{bmatrix} 2 & 2 \\ 0 & 5 \end{bmatrix} \quad \vec{A}_2 = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \quad \vec{A} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

$|A_1| = 10 \quad |A_2| = -4 \quad |A| = 1$

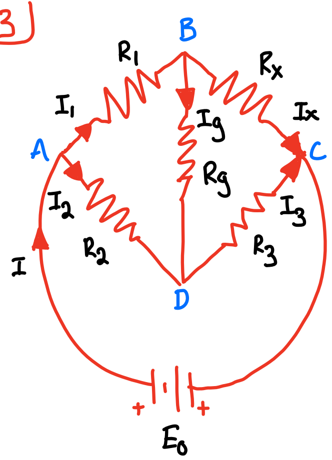
$x_1 = \frac{|A_1|}{|A|} : \frac{10}{1} = 10$ $\quad x_1 = 10 \quad x_1 \equiv x$

$x_2 = \frac{|A_2|}{|A|} : \frac{-4}{1} = -4$ $\quad x_2 = -4 \quad x_2 \equiv y$

$x = 10$

$y = -4$

3.4.43)



$I = \sum I$

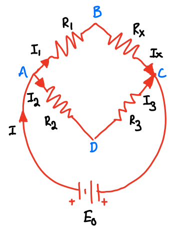
a.) $I_A = I_1 + I_2$ (current going in)

$I_B = I_g + I_x$

$I_C = I_x + I_3$

$I_D = I_2 + I_g$ (current going out)

C.)



Balanced $I_g = 0$

$R_x I_x - R_3 I_3 = 0$

$R_1 I_1 - R_2 I_2 = 0$

$R_2 R_x = R_1 R_3$

Voltage drop $= IR$ $\quad V = IR$

b.) Circuit BCD

$R_x I_x - R_3 I_3 - R_g I_g = 0$

Circuit BDA

$R_g I_g - R_2 I_2 + R_1 I_1 = 0$

Battery ABC

$I_1 R_1 + I_x R_x = E_0$

$I_g = 0 \qquad I_1 = I_x \ : \ I_3 = I_2$

$$\frac{R_x I_1}{R_1 I_1} = \frac{R_3 I_2}{R_2 I_2}$$

$$\frac{R_x}{R_1} = \frac{R_3}{R_2}$$