1.) $\iiint_E y\,dv$ $E = \{(x,y,z) \mid 0 \le x \le 5,\ 0 \le y \le x,\ x-y \le z \le x+y\}$

$0 \le x \le 5 \quad 0 \le y \le x \quad x-y \le z \le x+y$

$$\int_0^5 \int_0^x \int_{x-y}^{x+y} y \, dz\,dy\,dx$$

$$\int_0^5 \int_0^x [yz]_{x-y}^{x+y} \, dy\,dx$$

$$\int_0^5 \int_0^x y(x+y) - y(x-y) \, dy\,dx$$

$$\int_0^5 \int_0^x yx + y^2 - yx + y^2 \, dy\,dx$$

$$\int_0^5 \int_0^x 2y^2 \, dy\,dx$$

$$\int_0^5 \frac{2}{3}y^3 \Big|_0^x \, dx$$

$$\int_0^5 \frac{2}{3}x^3 \, dx$$

$$\frac{2}{12}x^4 \Big|_0^5 = \frac{1}{6}x^4 \Big|_0^5 = \frac{625}{6}$$

$\frac{625}{6}$

2.) $\iiint_E 4xy \, dV$ under the plane $z = 1+x+y$ above xy plane bounded by the curves $y = \sqrt{x}$, $y=0$, $x=1$

$$\int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 4xy \, dz\,dy\,dx$$

$\frac{65}{42}$

$$\int_0^1 \int_0^{\sqrt{x}} 4xyz \Big|_0^{1+x+y} \, dy\,dx$$

$4xy + 4x^2y + 4xy^2$

$$\int_0^1 \int_0^{\sqrt{x}} 4xy(1+x+y) \, dy\,dx$$

$$\int_0^1 \int_0^{\sqrt{x}} 4xy + 4x^2y + 4xy^2 \, dy\,dx$$

$$\int_0^1 2xy^2 + 2x^2y^2 + \frac{4xy^3}{3} \Big|_0^{\sqrt{x}} \, dx$$

$$\int_0^1 2x^2 + 2x^3 + \frac{4x(\sqrt{x})^3}{3} \, dx$$

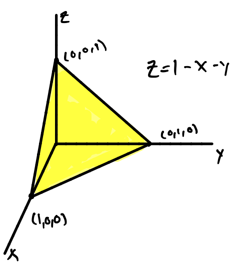
$$\int_0^1 2x^2 + 2x^3 + \frac{4x^{\frac{5}{2}}}{3} \, dx$$

$$\frac{2}{3}x^3 + \frac{x^4}{2} + \frac{8x^{\frac{7}{2}}}{21} \Big|_0^1$$

$$\frac{2}{3} + \frac{1}{2} + \frac{8}{21}$$

$$\frac{65}{42}$$

3.) $\iiint_T 3x^2 \, dV$ T tetrahedron $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$



$z = 1-x-y$

z: $z=0$, $\int_0^{x+y-1}$ $\quad 0 = 1-x-y$, $-1 = -x-y$, $1 = x+y$, $0 = x+y-1$

y: $0 = 1-x-y$, $y = 1-x$, $\int_0^{1-x}$

x: $0 = x-1-0$, $1 = x$, $\int_0^1$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3x^2 \, dz\,dy\,dx$$

$$\int_0^1 \int_0^{1-x} 3x^2z \Big|_0^{1-x-y} \, dy\,dx$$

$$\int_0^1 \int_0^{1-x} 3x^2(1-x-y) \, dy\,dx$$

$$\int_0^1 \int_0^{1-x} 3x^2 - 3x^3 - 3x^2y \, dy\,dx$$

$$\int_0^1 3x^2y - 3x^3y - \frac{3x^2y^2}{2} \Big|_0^{1-x} \, dx$$

$$\int_0^1 3x^2(1-x) - 3x^3(1-x) - \frac{3x^2(1-x)^2}{2} \, dx$$

$$\int_0^1 3x^2 - 3x^3 - \cancel{3x^3} + 3x^4 - \frac{3x^4}{2} + \cancel{3x^3} - \frac{3x^2}{2} \, dx$$

$$\int_0^1 \frac{3}{2}x^4 - 3x^3 + \frac{3}{2}x^2 \, dx$$

$$\frac{3}{10}x^5 - \frac{3x^4}{4} + \frac{1}{2}x^3 \Big|_0^1$$

$$\frac{3}{10} - \frac{3}{4} + \frac{1}{2}$$

$$\frac{12}{40} - \frac{30}{40} + \frac{20}{40}$$

$$\frac{12}{40} - \frac{10}{40}$$

$$\frac{2}{40} = \frac{1}{20}$$

4.) $\iiint_E 10x\,dv$ E bounded by $x=7y^2+7z^2$ plane $x=7$

$7=7y^2+7z^2$

$1=y^2+z^2$ ← circle

$1=r^2$

$r=\pm1$

$0\le r\le 1$

$0\le\theta\le 2\pi$

$7r^2\le x\le 7$

$x=7(r^2)$

$x=7r^2 \to 7$

$\frac{490\pi}{3}$

$$\int_0^{2\pi}\int_0^1\int_{7r^2}^{7} 10xr\;dx\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 \left[5x^2\right]_{7r^2}^{7} r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 (245-245r^4)r\;dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 245r-245r^5\;dr\,d\theta$$

$$\int_0^{2\pi} \frac{245r^2}{2}-\frac{245r^6}{6}\Big|_0^1\,d\theta$$

$$\int_0^{2\pi}\frac{245}{2}-\frac{245}{6}\;d\theta$$

$$\frac{245}{2}\theta-\frac{245}{6}\theta\Big|_0^{2\pi}$$

$$245\pi-\frac{245\pi}{3}=\frac{490\pi}{3}$$

5.) $\iiint_E f(x,y,z)\,dv$

$y=x^2$ $z=0$ $y+3z=9$

dxdydz $x=\pm\sqrt{y}$

$3z=9-y$ $y=9-3z$

$z=3-\frac{y}{3}$ $y=0$

$0=3-\frac{y}{3}$ $z=0$

$-3=-\frac{y}{3}$ $z=3$

$-9=-y$

$y=9$

$9-3z=0$

$-3z=-9$

$z=3$

$$\int_0^3\int_0^{9-3z}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy\,dz$$

dxdzdy $x=\int_{-\sqrt{y}}^{\sqrt{y}}$ $z=\int_0^{3-\frac{y}{3}}$ $y=\int_0^9$

$$\int_0^9\int_0^{3-\frac{y}{3}}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$$

dydxdz

$9-3z=x^2$

$\pm\sqrt{9-3z}=x$

$9-3z=0$

$-3z=-9$

$z=3$

$y:\int_{x^2}^{9-3z}$ $x:\int_{-\sqrt{9-3z}}^{\sqrt{9-3z}}$ $z:\int_0^3$

$$\int_0^3\int_{-\sqrt{9-3z}}^{\sqrt{9-3z}}\int_{x^2}^{9-3z} dy\,dx\,dz$$

dydzdx

$y=x^2$

$z=3-\frac{y}{3}$

$3-\frac{x^2}{3}$

$y:\int_{x^2}^{9-3z}$ $z:\int_0^{3-\frac{x^2}{3}}$ $x:\int_{-3}^{3}$

$$\int_{-3}^3\int_0^{3-\frac{x^2}{3}}\int_{x^2}^{9-3z} dy\,dz\,dx$$

dzdydx $z=3-\frac{y}{3}$

$z:\int_0^{3-\frac{y}{3}}$ $y:\int_{x^2}^{9}$ $x:\int_{-3}^{3}$

$$\int_{-3}^3\int_{x^2}^9\int_0^{3-\frac{y}{3}} dz\,dy\,dx$$

dzdxdy

$z:\int_0^{3-\frac{y}{3}}$ $x:\int_{-\sqrt{y}}^{\sqrt{y}}$ $y:\int_0^9$

$$\int_0^9\int_{-\sqrt{y}}^{\sqrt{y}}\int_0^{3-\frac{y}{3}} dz\,dx\,dy$$

6.) $\int_0^{25}\int_{\sqrt{x}}^{5}\int_0^{5-y} f(x,y,z)\,dz\,dy\,dx$

$z=5-y$ $\quad -y=z-5$ $\quad y=5-z$ $\quad z=5-\sqrt{x}$

$dz\,dx\,dy$ $\quad y=\sqrt{x}$ $\quad x=y^2$

$z=\int_0^{5-y}$ $\quad x=\int_0^{y^2}$ $\quad y=\int_0^{5}$

$\int_0^5\int_0^{y^2}\int_0^{5-y} dz\,dx\,dy$

$dy\,dz\,dx$

$y=\int_{\sqrt{x}}^{5-z}$ $\quad z=\int_0^{5-\sqrt{x}}$ $\quad x=\int_0^{25}$

$\int_0^{25}\int_0^{5-\sqrt{x}}\int_{\sqrt{x}}^{5-z} dy\,dz\,dx$

$dx\,dy\,dz$

$x=\int_0^{y^2}$ $\quad y=\int_0^{5-z}$ $\quad z=\int_0^{5}$

$\int_0^5\int_0^{5-z}\int_0^{y^2} dx\,dy\,dz$

$dy\,dx\,dz$

$y=\int_{\sqrt{x}}^{5-z}$ $\quad x=\int_0^{(5-z)^2}$ $\quad z=\int_{-5}^{5}$

$\int_0^5\int_0^{(5-z)^2}\int_{\sqrt{x}}^{5-z} dy\,dx\,dz$

$z=5-\sqrt{x}$

$z-5=-\sqrt{x}$

$-z+5=\sqrt{x}$

$(5-z)^2=x$

$z^2-10z+25$

$x=\pm 5$

$dx\,dz\,dy$

$x=\int_0^{y^2}$ $\quad z=\int_0^{5-y}$ $\quad y=\int_0^{5}$

$\int_0^5\int_0^{5-y}\int_0^{y^2} dx\,dz\,dy$