

Physics 132

Exam 2

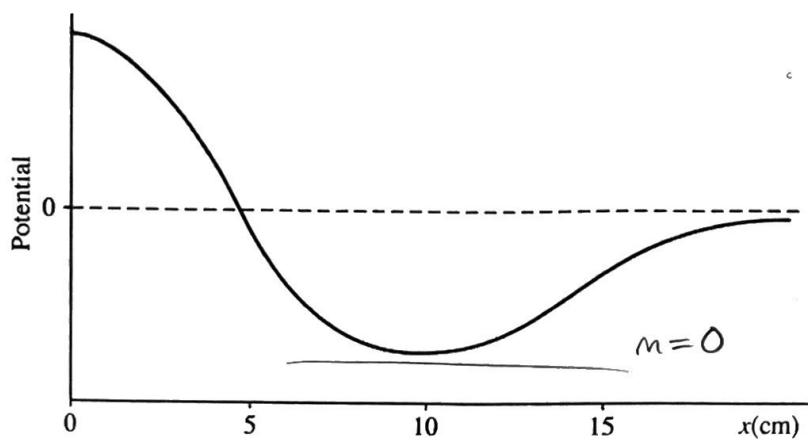
Instructions: This exam contains 5 multiple choice questions worth 4 points each and 4 problems worth 20 points each. Do not refer to any book or notes during the exam. The time limit for this test is 50 minutes.

(I)

Section I - Circle the letter that corresponds to the correct answer.

+03/100

1. The potential, $V(x)$, as a function of position, x , is shown graphically in the figure below. Which statement about the electric field is true?



$$E = -\frac{dV}{dx}$$

- (a) The electric field is zero at $x = 0$, its magnitude is at a maximum at $x = 5$ cm, and the field is directed to the right there.
- b) The electric field is zero at $x = 5$ cm, its magnitude is at a maximum at $x = 0$, and the field is directed to the right there.
- c) The electric field is zero at $x = 0$, its magnitude is at a maximum at $x = 15$ cm, and the field is directed to the left there.
- (d) The electric field is zero at $x = 10$ cm, its magnitude is at a maximum at $x = 5$ cm, and the field is directed to the left there.

2. A wire of resistivity ρ must be replaced in a circuit by a wire of the same material but 4 times as long. If, however, the resistance of the new wire is to be the same as the resistance of the original wire, the diameter of the new wire must be

- a) the same as the diameter of the original wire.
- b) $1/2$ the diameter of the original wire.
- c) $1/4$ the diameter of the original wire.
- (d) 2 times the diameter of the original wire.
- (e) 4 times the diameter of the original wire.

$$R = \frac{\rho L}{A} \quad A = \pi r^2$$

$$R = \frac{\rho 4L}{\pi (\frac{d}{2})^2} = \frac{\rho 4L}{\pi (\frac{d}{2})^2} = \frac{\rho 4L}{\pi d^2}$$

$$\frac{\rho 4L}{\pi (\frac{4d}{2})^2} = \frac{\rho 4L}{\pi 4d^2}$$

diameter in half
radius in half

3. A cylindrical wire has a resistance R and resistivity ρ . If its length and diameter are both cut in half, what will be its resistance?

- a) $4R$
- ☒ b) $2R$
- c) R
- ☒ d) $R/2$
- e) $R/4$

$$R = \frac{\rho L}{A} = \frac{\rho(\frac{1}{2}L)}{\pi(\frac{1}{2})^2} = \frac{\rho(\frac{1}{2}L)}{\pi(\frac{1}{2}r)^2} = \frac{\rho L}{2\pi(r^2)}$$

$d = 2R$
 $\frac{1}{2}d = R$
 $\frac{1}{2}d = R$
 $\frac{\rho L}{2\pi(r^2)}$

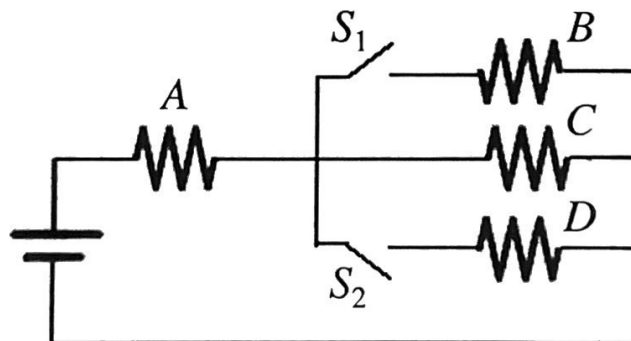
4. As more resistors are added in series across a constant voltage source, the power supplied by the source

- a) increases.
- ☒ b) decreases.
- c) does not change.

$$P = I^2 R = \frac{(\Delta V_R)^2}{R} \rightarrow \text{Bigger}$$

$$= I(\Delta V_R)$$

5. In the circuit shown in the figure, four identical resistors labeled A to D are connected to a battery as shown. S_1 and S_2 are switches. Which of the following actions would result in the *greatest* amount of current through resistor A ?



- ☒ a) closing both switches
- b) closing S_1 only
- c) closing S_2 only
- ☒ d) leaving both switches open as shown.

+4/20

$$\mu = 10 \times 10^{-6}$$

$$n = 10 \times 10^{-9}$$

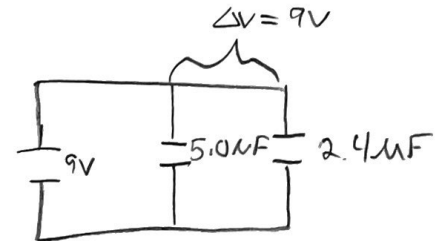
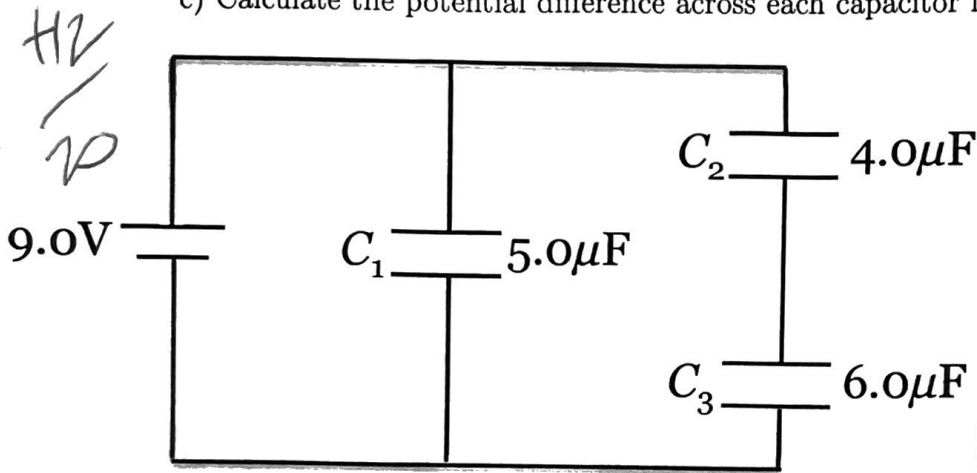
$$C = \frac{Q}{\Delta V_C}$$

$$Q = 6.66 \times 10^{-5} \quad Q = C \Delta V_C$$



Section II - For credit, show all work!

- Calculate the equivalent capacitance of the circuit below.
- Calculate the charge on each capacitor in the figure below.
- Calculate the potential difference across each capacitor in the figure below.



$$C = \frac{Q}{\Delta V_C} = \frac{\epsilon_0 A}{d}$$

$$\Delta V_C = \frac{Q}{C}$$

Series $\frac{1}{C}$

$$a.) C_2 + C_3 = \frac{1}{C_{23}} = \left(\frac{1}{4 \mu F} + \frac{1}{6 \mu F} \right) \quad C_{23} = \frac{12}{5} \mu F$$

$$C_1 + C_{23} = 7.4 \mu F$$

$$C_{eq} = 7.4 \mu F$$

$$b.) Q = \Delta V C$$

$$C_1: Q_1 = \Delta V_1 C_1$$

$$\Delta V_1 = 9V + 1V$$

$$C_1 = 5.0 \mu F$$

$$Q_1 = 4.5 \times 10^{-5} C + 1$$

$$Q_2 (4 \mu F) + Q_3$$

$$c.) C_1: \Delta V_{C_1} = \frac{Q_1}{C_1} + 1$$

$$\Delta V_{C_1} = \frac{4.5 \times 10^{-5} C}{5 \times 10^{-6} F}$$

2. The terminals of a 1.5 V battery are connected by a 120 m long gold wire, which has a diameter of 0.15 mm. Gold has a resistivity of $2.4 \times 10^{-8} \Omega \cdot \text{m}$.

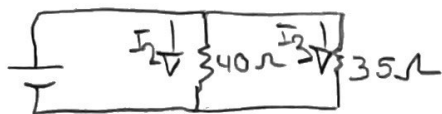
- a) Calculate the resistance of the gold wire.
 b) Calculate the current in the wire.
 c) Calculate the current density in the wire.
 d) Calculate the electric field in the wire.

A.) $R = \frac{\rho L}{A}$ ✓ $A = \pi r^2$
 $L = 120 \text{ m}$ $A = \pi \left(\frac{d}{2}\right)^2$ ✓
 $\rho = 2.4 \times 10^{-8} \Omega \cdot \text{m}$ $A = \pi (7.5 \times 10^{-5} \text{ m})^2$
 $R = \frac{2.4 \times 10^{-8} \Omega \cdot \text{m} (120 \text{ m})}{\pi (7.5 \times 10^{-5} \text{ m})^2}$
 $R = 162.975 \Omega$ ✓

B.) $I = \frac{\Delta V}{R}$ ✓ $I = \frac{1.5 \text{ V}}{162.975 \Omega}$
 $\Delta V = 1.5 \text{ V}$
 $R = 162.975 \Omega$
 $I = 0.009204 \text{ A}$ ✓

C.) $J = \frac{I}{A}$ ✓ $J = \frac{0.009204 \text{ A}}{\pi (7.5 \times 10^{-5} \text{ m})^2}$
 $I = 0.009204 \text{ A}$
 $A = \pi (7.5 \times 10^{-5} \text{ m})^2$
 $J = 520,833.33 \text{ A/m}^2$ ✓

D.) $J = \sigma E$ ✓ $E = \frac{J}{\sigma}$
 $\rho = \frac{1}{\sigma}$ ✓
 $\sigma = \frac{1}{\rho}$
 $\sigma = 4.16 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1}$
 $J = 520,833.33 \text{ A/m}^2$
 $E = \frac{520,833.33 \text{ A/m}^2}{4.16 \times 10^7 \Omega^{-1} \cdot \text{m}^{-1}}$
 $E = 1.25 \times 10^{-2} \text{ V/m}$ ✓



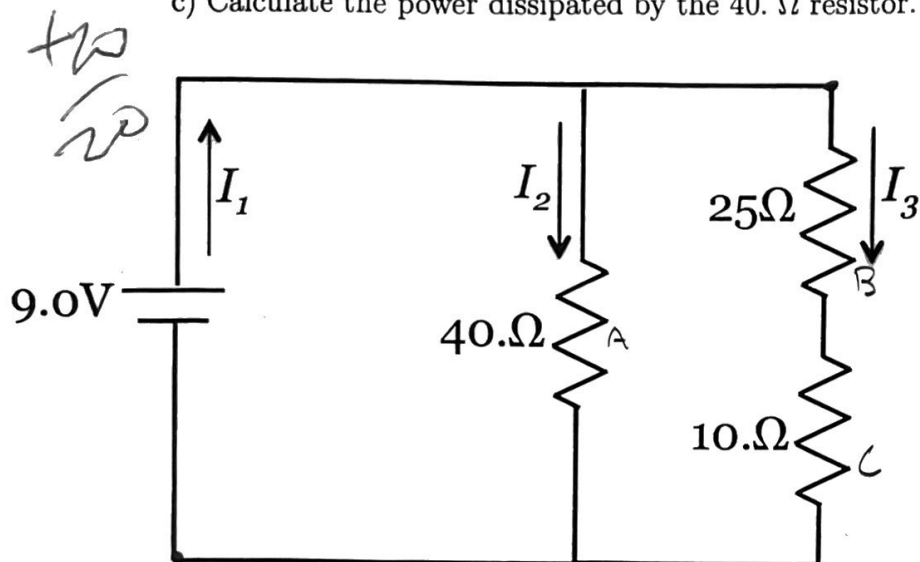
3. a) Calculate the equivalent resistance of the circuit below.

b) Calculate the current through each resistor in the figure below.

c) Calculate the power dissipated by the 40. Ω resistor.

$$I_1 = I_2 + I_3$$

$$I = I_1 + I_2$$



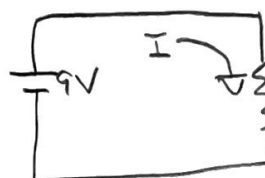
$$B+C = 25\Omega + 10\Omega \checkmark$$

$$BC = 35\Omega \checkmark$$

$$A+(BC) = \frac{1}{R_{ABC}} = \left(\frac{1}{40\Omega} + \frac{1}{35\Omega} \right) \checkmark$$

$$R_{ABC} = \frac{56}{3}\Omega \checkmark$$

$$9V - I\left(\frac{56}{3}\Omega\right) = 0$$



$$I = 27/56A$$

A.) $R_{eq} = \frac{56}{3}\Omega \checkmark$

B.) A: $I = \frac{\Delta V}{R} \quad I_2 = 0.225A \checkmark$

$$\Delta V = 9V$$

$$R = 40\Omega$$

$$I_1 = 0.482A \checkmark$$

$$I_2 = 0.225A \checkmark$$

$$I_3 = 0.257A \checkmark$$

$$9V = I\left(\frac{56}{3}\Omega\right)$$

$$I = 27/56A$$

$$-9V - I_2(40\Omega) = 0 \quad I_2 \checkmark$$

$$I_2 = 0.225A \checkmark$$

$$-I_3(35\Omega) + I_2(40\Omega) = 0 \checkmark$$

$$I_3(35\Omega) = 9$$

$$I_3 = 0.257A \checkmark$$

C.) $P_R = I^2 R$

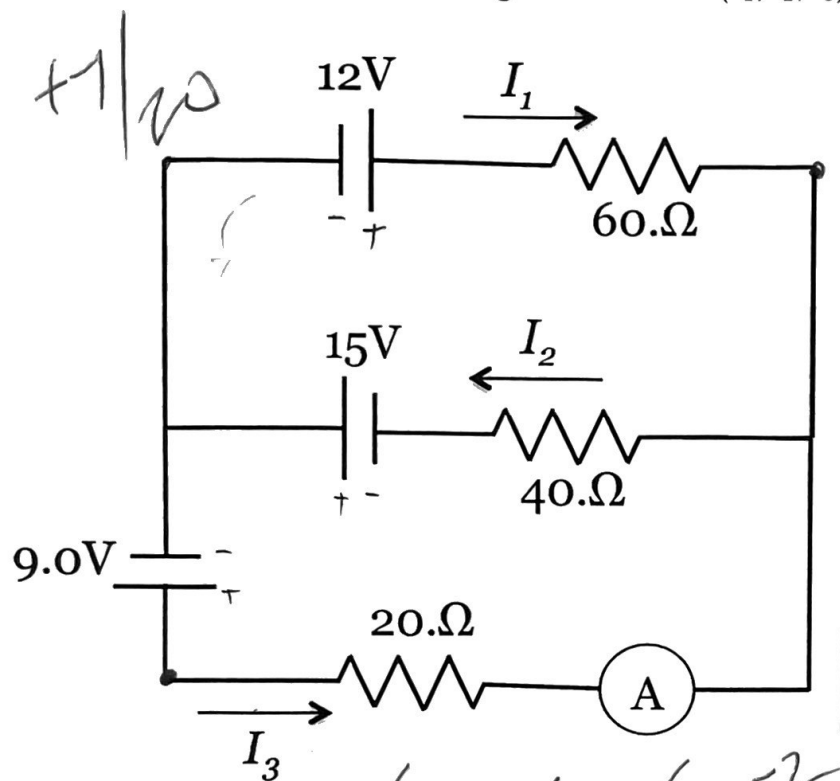
$$P_R = 2.025W \checkmark$$

$$I = 0.225A \checkmark$$

$$R = 40\Omega$$

4. Consider the circuit shown in the figure below where the ammeter reads 0.300 A.

Find the current through each resistor (I_1, I_2, I_3).



$$I_3 = 0.300 \text{ A}$$

$$I_1 = I_2 + I_3 \quad \times$$

$$-I_3(20\Omega) - I_2(40\Omega) + 15V - 9.0V = 0$$

$$I_3(20\Omega) + I_2(40\Omega) = 6V$$

$$I_2(40\Omega) + I_3(60\Omega) = 27V$$

$$I_2(40\Omega) + I_3(20\Omega) = 6V$$

$$-I_2(40\Omega) + 15V + 12V - I_1(60\Omega) = 0$$

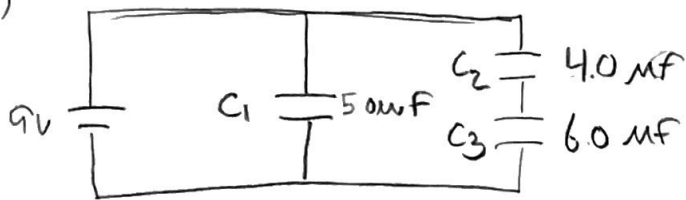
$$I_1(60\Omega) + I_2(40\Omega) = 27V$$

$$(I_2 + I_3)(60\Omega) + I_2(40\Omega) = 27V$$

$$I_2(100\Omega) + I_3(60\Omega) = 27V$$

$$\text{ref}\left(\begin{bmatrix} 100 & 60 & 27 \\ 40 & 20 & 6 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -9/20 \\ 0 & 1 & 6/5 \end{bmatrix} \quad \begin{matrix} I_2 = 9/20 \text{ A} \\ I_3 = 6/5 \text{ A} \end{matrix}$$

$$\begin{matrix} I_1 = 33/20 \text{ A} & \times & -3 \\ I_2 = 9/20 \text{ A} & \times & -3 \\ I_3 = 6/5 \text{ A} & \times & -2 \end{matrix}$$



Series: $\frac{1}{C}$

Parallel: $C_1 + C_2 + C_3$

a.) $\frac{1}{4 \mu F} + \frac{1}{6 \mu F} = C_2 + C_3 = 2.4 \mu F$
 $5.0 \mu F + 2.4 \mu F$
 $C_1 + (C_2 + C_3) = 7.4 \mu F$

$C_{eq} = 7.4 \mu F$

b.) $C = \frac{Q}{\Delta V}$

$Q = 7.4 \mu F (9V) = 66.6 \mu C$

$C_1 = 5.0 \mu F$

$C_4 = 7.4 \mu F$

$C_1 = \frac{Q}{\Delta V}$

$C_4 = \frac{Q}{\Delta V} \quad C_4 = 2.4 \mu F$

$Q_1 = 45 \mu C$

$Q_{eq} = 22 \mu C$

$Q = 45 \mu C$

$Q = 21.6 \mu C$

c.) $C_2 = \frac{Q}{\Delta V} \quad Q = 21.6 \mu C \quad C_3 = \frac{Q}{\Delta V} \quad Q = 21.6 \mu C$
 $\Delta V = \frac{Q}{C_2} \quad C_2 = 4.0 \mu F \quad \Delta V = \frac{Q}{C_3} \quad C_3 = 6.0 \mu F$
 $\Delta V = 5.4 \Delta V \quad \Delta V = 3.6 \Delta V$

$C_1: C_1 = \frac{Q}{\Delta V} \quad Q = 45 \mu C$

$\Delta V = \frac{Q}{C_1} \quad C_1 = 5 \mu C$

$\Delta V = 9V$

$C_1 = 9V$

$C_2 = 5.4V$

$C_3 = 3.6V$

2.) $\Delta V = 15 \text{ V}$ $L = 120 \text{ m}$ $\rho = 2.4 \times 10^{-8} \Omega \text{ m}$ $d = 0.15 \text{ mm}$

a.) $R = \frac{\rho L}{A}$ $R = 162.975 \Omega$ $\frac{\Omega \text{ m (m)}}{\text{m}^2} = \Omega$

$$A = \pi \left(\frac{1.5 \times 10^{-3}}{2} \right)^2$$

[a.] $A = (5.625 \times 10^{-7} \pi)$

b.) $I = \frac{\Delta V}{R}$ $\Delta V = 1.5$
 $R = 162.975 \Omega$

$I = 0.009204 \text{ A}$

c.) $J = \sigma E$ $\sigma = \frac{1}{\rho}$ $\sigma = 4.16 \times 10^7 (\Omega \text{ m})^{-1}$

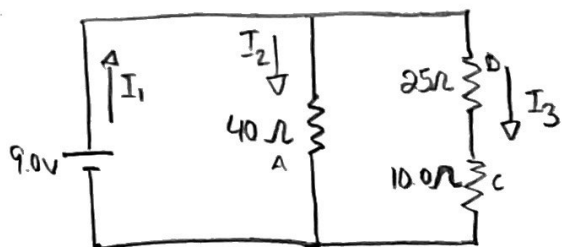
$$J = \frac{I}{A} = \frac{0.009204 \text{ A}}{\pi (0.075 \text{ mm})^2} = \boxed{J = 520,833 \text{ A/m}^2}$$

d.) $J = \sigma E$

$$E = \frac{J}{\sigma} = 1.25 \times 10^{-2} \text{ V/m}$$

$E = 1.25 \times 10^{-2} \text{ V/m}$

3.)



$$I_1 = I_2 + I_3$$

a) $B+C = 35\Omega$

$$A + (B+C)$$

$$\frac{1}{40\Omega} + \frac{1}{35\Omega} = \left(\frac{3}{56\Omega}\right)^{-1} = 18.67\Omega$$

$$R = 18.67\Omega$$

b) $9.0V - I_1(18.67\Omega) = 0$

$$I_1 = 0.482A$$

$$\therefore 9.0V - I_2(40\Omega) = 0$$

$$I_2 = 0.225A$$

$$\therefore 9.0V - I_3(35\Omega) = 0$$

$$I_3 = 0.257A$$

$$I_1 = 0.482A$$

$$I_2 = 0.225A$$

$$I_3 = 0.257A$$

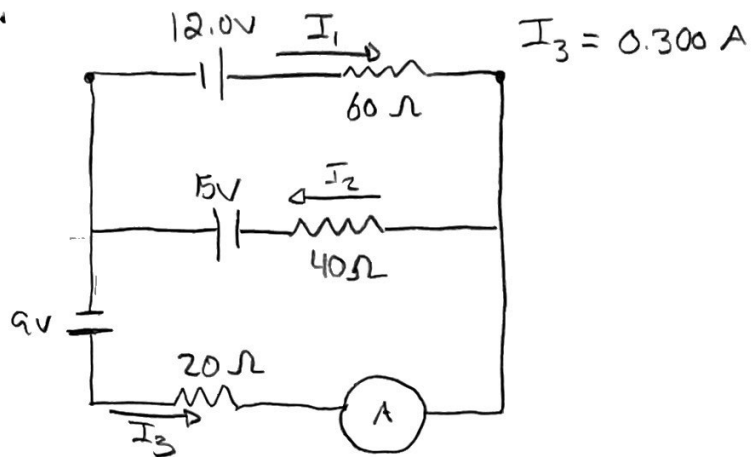
c.)

$$P = I^2 R$$

$$I = 0.225A$$

$$R = 40\Omega$$

$$P = 2.025W$$



loop 1

$$12\text{V} - I_1(60\Omega) + I_3(20\Omega) - 9\text{V} = 0$$

$$-I_1(60\Omega) + 9\text{V} = 0$$

$$I_1 = 0.15 \text{ A}$$

loop 2

$$12\text{V} - I_1(60\Omega) - I_2(40\Omega) + 15\text{V} = 0$$

$$-I_2(40\Omega) + 18\text{V} = 0$$

$$I_2 = 0.45 \text{ A}$$

$$I_1 = 0.15 \text{ A}$$

$$I_2 = 0.45 \text{ A}$$

$$I_3 = 0.300 \text{ A}$$

Conceptual

1.) (A)

$$2) R = \frac{\rho L}{A} \times 4 = 4R = \frac{\rho 4L}{\pi r^2} = \frac{4\rho L}{(2r)^2 \pi} = \frac{4\rho L}{4r^2 \pi} = \frac{\rho L}{\pi r^2}$$

$r = \frac{1}{2}d$
 $2r = d$ $2 \times r = 2 \times d$ (d)

$$3) R = \frac{\rho L}{A} \times \frac{1}{2} = \frac{\rho \frac{1}{2}}{A(\frac{1}{2})^2} = \frac{\rho \frac{1}{2}}{A(\frac{1}{4})} = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{4}{2} = 2r$$

$d = 2r$ $\frac{1}{2}d$ or (b)

$$4) P = I^2 R$$
$$= \frac{(\Delta V)^2}{R}$$

R increases $\therefore P$ decrease

(b)

5.) (A)