Math 361 **Numerical Analysis** Taylor Larrechea
10/10/19 Quiz 6 Name

Answer the following questions in the space provided. **Show all work.** (40 pts. total.)

1. Use Newton's method to approximate the root of $f(x) = x^4 - 3x^2 - 2$ by finding p_1, p_2, p_3, given that $p_0 = 1.5$. Be sure to specify your iterating function $g(x)$, and use function notation with g to indicate how you obtain each iterate, as discussed in class. (10 pts)

$$g(x) = x - \frac{x^4 - 3x^2 - 2}{4x^3 - 6x}$$

$P_1 = g(P_0) = 2.319444444$

$P_2 = g(P_1) = 2.019328224$

$P_3 = g(P_2) = 1.904324485$

2. Consider the equation $x^4 - 3x^2 - 2 = 0$ for $x \in [1,2]$. Given $p_0 = 1$, $p_1 = 2$, use the Secant Method to find p_2, p_3, p_4. Be sure to specify your iterating function $g(x, y)$, and use function notation with g to indicate how you obtain each iterate, as discussed in class. (10 pts.)

$$g(x,y) = x - \frac{(x^4 - 3x^2 - 2)(x - y)}{(x^4 - 3x^2 - 2) - (y^4 - 3y^2 - 2)}$$

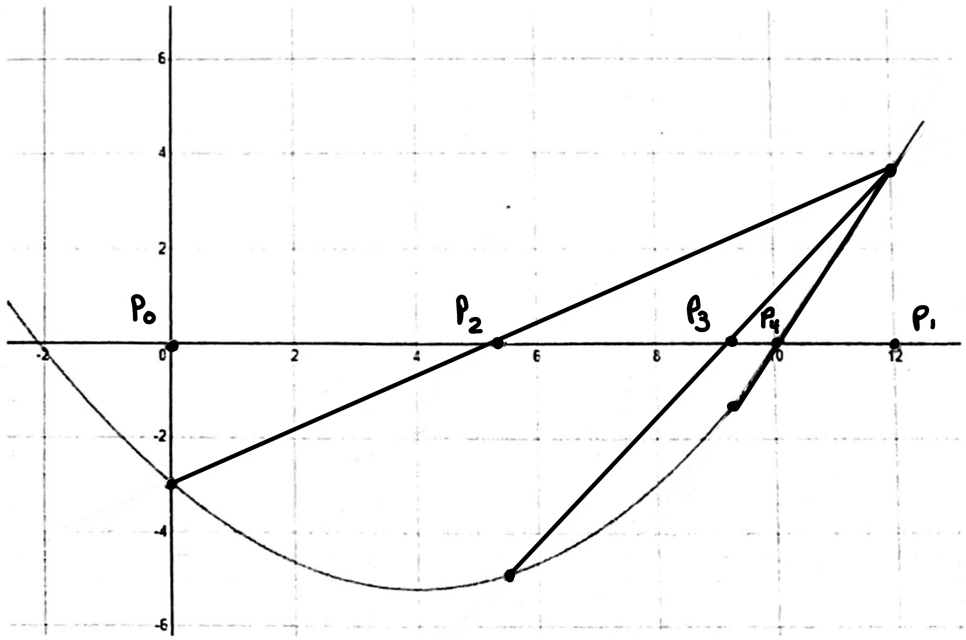
$P_2 = g(P_1, P_0) = 1.666666667$

$P_3 = g(P_2, P_1) = 1.855614973$

$P_4 = g(P_3, P_2) = 1.897354461$

3. The graph of a function f is given below for parts (a) and (b), where the solution to $f(x) = 0$ is sought using the numerical method indicated. (10 pts.)

Given $p_0 = 0$ and $p_1 = 12$, use the False Position method to label p_2, p_3, p_4 on figure below.



4. Show that the sequence $p_n = (1/3)^n$ converges linearly to $p = 0$; that is, show that the order of convergence is $\alpha = 1$. (10 pts)

$$\lim_{n \to \infty} \frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lim_{n \to \infty} \frac{|(1/3)^{n+1} - 0|}{|(1/3)^n - 0|^1} = \lim_{n \to \infty} |(1/3)| = \frac{1}{3}$$

Since $\lambda > 0$ and finite, $p_n \to p$ on the order of $\alpha = 1$

$\lambda = 1/3$