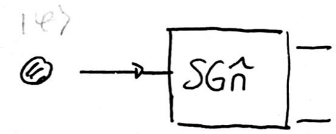
Taylor 10 Sept 2019

Lectures 8, 9

I Consider problem of estimation. Given a collection of spin-½ particles each in state

$$|\psi\rangle = \cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle$$

we want to estimate α. Suppose there are N such particles. Imagine the following process:

a) 

record outcome... $S_n = +\hbar/2$

$S_n = -\hbar/2$

b) Suppose we get n_+ outcomes $S_n = +\hbar/2$. How can we estimate α from this?

Exercise

Specific cases.

1) suppose measurement is $SG\hat{z}$. How can we estimate α?

2) " " " $SG\hat{x}$ " " " " α?

3) " " " $SG\hat{y}$ " " " " α?

II Entangled states.

Have two distinguishable particles: A, B. Suppose measure $|0\rangle, |1\rangle$ ($SG\hat{z}$) on each.

etc..

Possible outcomes + states:

$S_z A$	$S_z B$	
+	+	$\lvert 0\rangle_A \lvert 0\rangle_B$
+	−	$\lvert 0\rangle \lvert 1\rangle$
−	+	$\lvert 1\rangle \lvert 0\rangle$
−	−	$\lvert 1\rangle \lvert 1\rangle$.

product states

In general any state which can be written $\underbrace{|\varphi\rangle_A}_{\text{single particle state}} |\psi\rangle_B$ is a product state.

<u>Exercise</u> Let

$$|\phi_1\rangle = |0\rangle$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle.$$

express each of following as a superposition in $|0\rangle|0\rangle, |0\rangle|1\rangle, \ldots, |1\rangle|1\rangle$

$|\phi_1\rangle|\phi_2\rangle$

$|\phi_1\rangle|\phi_3\rangle$

$|\phi_2\rangle|\phi_3\rangle$

General state has form

$$|\Psi\rangle = a_0 |0\rangle|0\rangle + a_1|0\rangle|1\rangle + a_2|1\rangle|0\rangle + a_3|1\rangle|1\rangle$$

How to interpret measurements? Need bra/kets... If

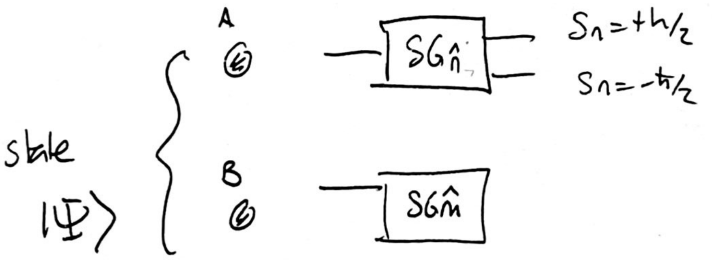
$$|\Psi\rangle = |\psi_A\rangle|\psi_B\rangle$$

$$|\Phi\rangle = |\phi_A\rangle|\phi_B\rangle$$

Then $\langle\Phi| = \langle\phi_A|\langle\phi_B|$ and

$$\langle\Phi|\Psi\rangle = \underbrace{\langle\phi_A|\psi_A\rangle}_{\text{number}}\underbrace{\langle\phi_B|\psi_B\rangle}_{\text{number}}.$$

So if we have



Then

S_n	S_m	Assoc state	Prob
$+\hbar/2$	$+\hbar/2$	$\langle +\hat{n}|\langle +\hat{m}|$	$|\langle +\hat{n}|\langle +\hat{m}|\Psi\rangle|^2$
$+\hbar/2$	$-\hbar/2$	$\langle +\hat{n}|\langle -\hat{m}|$	- - - ..

Exercise

a) Let state be

$$|\Psi\rangle = \frac{1}{2}\left[\overset{A}{|0\rangle}\overset{B}{|0\rangle} + |0\rangle|1\rangle - |1\rangle|0\rangle - |1\rangle|1\rangle\right]$$

and suppose we measure

1) SG$\hat{z}$ for A $\{|0\rangle, |1\rangle\}$ get probs of all four outcomes
SG$\hat{z}$ for B $\{|0\rangle, |1\rangle\}$

2) SG$\hat{x}$ for A $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$
SG$\hat{z}$ " B $\{|0\rangle, |1\rangle\}$

3) SG$\hat{x}$ for A
SG$\hat{x}$ for B

Is there one pair of measurements that always yield same outcome with certainty?

b) Repeat for

$$|\Psi\rangle = \frac{1}{2}\left[|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle\right]$$

c) For each state $|\Psi\rangle$ can you find $|\psi_A\rangle, |\psi_B\rangle$ s.t.

$$|\Psi\rangle = |\psi_A\rangle|\psi_B\rangle$$

Try $|\psi_A\rangle = a_0|0\rangle + a_1|1\rangle$

$|\psi_B\rangle = b_0|0\rangle + b_1|1\rangle$

and see...

1) Product state

$$|\Psi\rangle = |\psi_A\rangle |\psi_B\rangle$$

state for A

can find some

θ_A, ϕ_A s.t.

SG$\hat{n}$

gives $+\hbar/2$

certainty

state for B

can find some

θ_B, ϕ_B

SG$\hat{m}$

s.t. $+\hbar/2$ certainty

2) Entangled state.

- not possible to find some meas. for A and some meas for B, s.t. one pair of outcomes occurs with certainty.