PHYS 230 HOMEWORK SET 7

1. $V = \frac{12}{13}c$

a) E1: SPACESHIP PASSES THE ORIGIN OF OUR REFERENCE FRAME.

E2: SPACESHIP IS SOMEWHERE WHEN THE SPACESHIP CLOCK MEASURES $\Delta t' = 1.0$HR

THE SPACESHIP MEASURES $\Delta x' = 0$

b)

c) NOW $\Delta t = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}}$ SINCE $\Delta x' = 0$

SO

$$\Delta t = \frac{1.0\text{HR}}{\sqrt{1 - (12/13)^2}} = \frac{1.0\text{HR}}{\sqrt{1 - 144/169}} = \frac{1.0\text{HR}}{\sqrt{25/169}} = \frac{1.0\text{HR}}{5/13} = \frac{13}{5}\text{HRS} = 2.6\text{HRS}$$

$$\boxed{\Delta t = 2.6\text{HRS}}$$

d) ALSO $V = \frac{D}{\Delta t}$ SO $D = V\Delta t = \left(\frac{12}{\cancel{13}}c\right)\left(\frac{\cancel{13}}{5}\text{HRS}\right) = \frac{12}{5}c\cdot\text{HRS}$

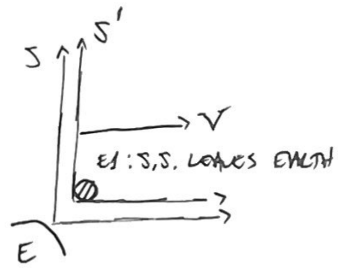
$$\boxed{D = \frac{12}{5}c\cdot\text{HRS}}$$

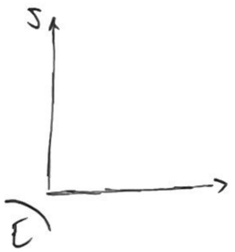
e) ACCORDING TO THE SPACESHIP, THE DISTANCE THEY TRAVEL IS LENGTH CONTRACTED TO...

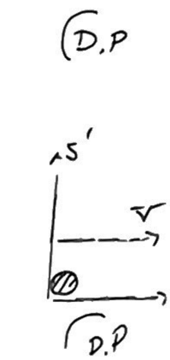
$$d = D\sqrt{1 - V^2/c^2} = \left(\frac{12}{5}c\cdot\text{HRS}\right)\left(\frac{5}{13}\right) = \frac{12}{13}c\cdot\text{HRS}$$

$$\boxed{d = \frac{12}{13}c\cdot\text{HRS}}$$

2. $V = \frac{5}{13} c$







E2: S.S. ARRIVES @ DISTANT PLANET.

NOTICE THAT $\Delta x' = 0$ AS BOTH EVENTS OCCUR AT THE SAME x' COORDINATE.

$$\Delta x = 25\ c\cdot yrs \equiv D \quad \text{(THIS IS THE REST LENGTH)}$$

a) $V = \frac{\Delta x}{\Delta t}$ SO $\Delta t = \frac{\Delta x}{V} = \frac{25\ c\cdot yrs}{\frac{5}{13} c} = 25\ yrs \cdot \frac{13}{5} = 65\ yrs$

$$\boxed{\Delta t = 65\ yrs}$$

b) NOW $\Delta t' = \Delta t\sqrt{1 - V^2/c^2}$ SINCE $\Delta x' = 0$ SO

$$\Delta t' = 65\ yrs \cdot \sqrt{1 - (5/13)^2} = 65\ yrs\sqrt{1 - \frac{25}{169}} = 65\ yrs \cdot \sqrt{\frac{144}{169}} = 65\ yrs \cdot \frac{12}{13} = 60\ yrs$$

$$\boxed{\Delta t' = 60\ yrs}$$

c) ACCORDING TO THE SPACESHIP, THE DISTANCE THEY HAVE TRAVELED IS LENGTH CONTRACTED TO...

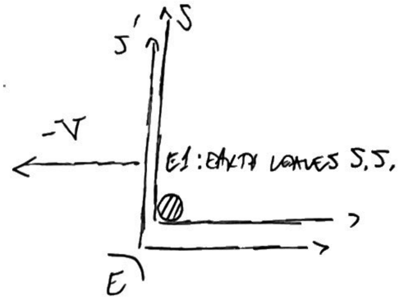
$$d = D\sqrt{1 - v^2/c^2} = (25\ c\cdot yrs)\sqrt{1 - (5/13)^2} = (25\ c\cdot yrs)\cdot\frac{12}{13} = \frac{300}{13}\ c\cdot yrs$$

$$\simeq 23.1\ c\cdot yrs$$

3. ACCORDING TO THE SPACESHIP... $V = \frac{5}{13}c$

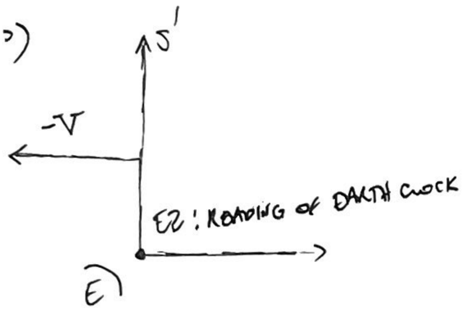
E1: THE EARTH LEAVES THE SPACESHIP

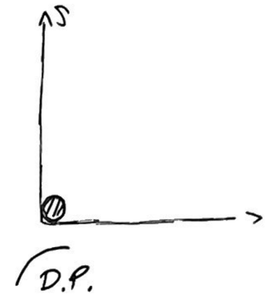
E2: OBSERVER IN S.S. FRAME READS THE TIME ON THE EARTH CLOCK.



a) THE EARTH OBSERVER SEES BOTH EVENTS OCCUR AT THE SAME SPATIAL COORDINATE...
LET S' BE THE EARTH-DISTANT PLANET FRAME, SO $\Delta x' = 0$

b)





c) THE REST LENGTH DISTANCE HERE WAS FOUND IN PART c) OF THE PREVIOUS PROBLEM

$$\Delta X = \frac{300}{13}\ c\cdot yrs \equiv D$$

THE SPACESHIP OBSERVER MEASURES THIS DISTANCE.

d) AS THE EARTH FRAME IS MOVING RELATIVE TO THIS REST LENGTH, IT MEASURES A CONTRACTED LENGTH OF..

$$d = D\sqrt{1 - V^2/c^2} = \frac{300}{13}\ c\cdot yrs\sqrt{1 - \left(5/13\right)^2} = \left(\frac{300}{13}\ c\cdot yrs\right)\cdot\frac{12}{13}$$

$$= \frac{3600}{169}\ c\cdot yrs \simeq 21.3\ c\cdot yrs$$

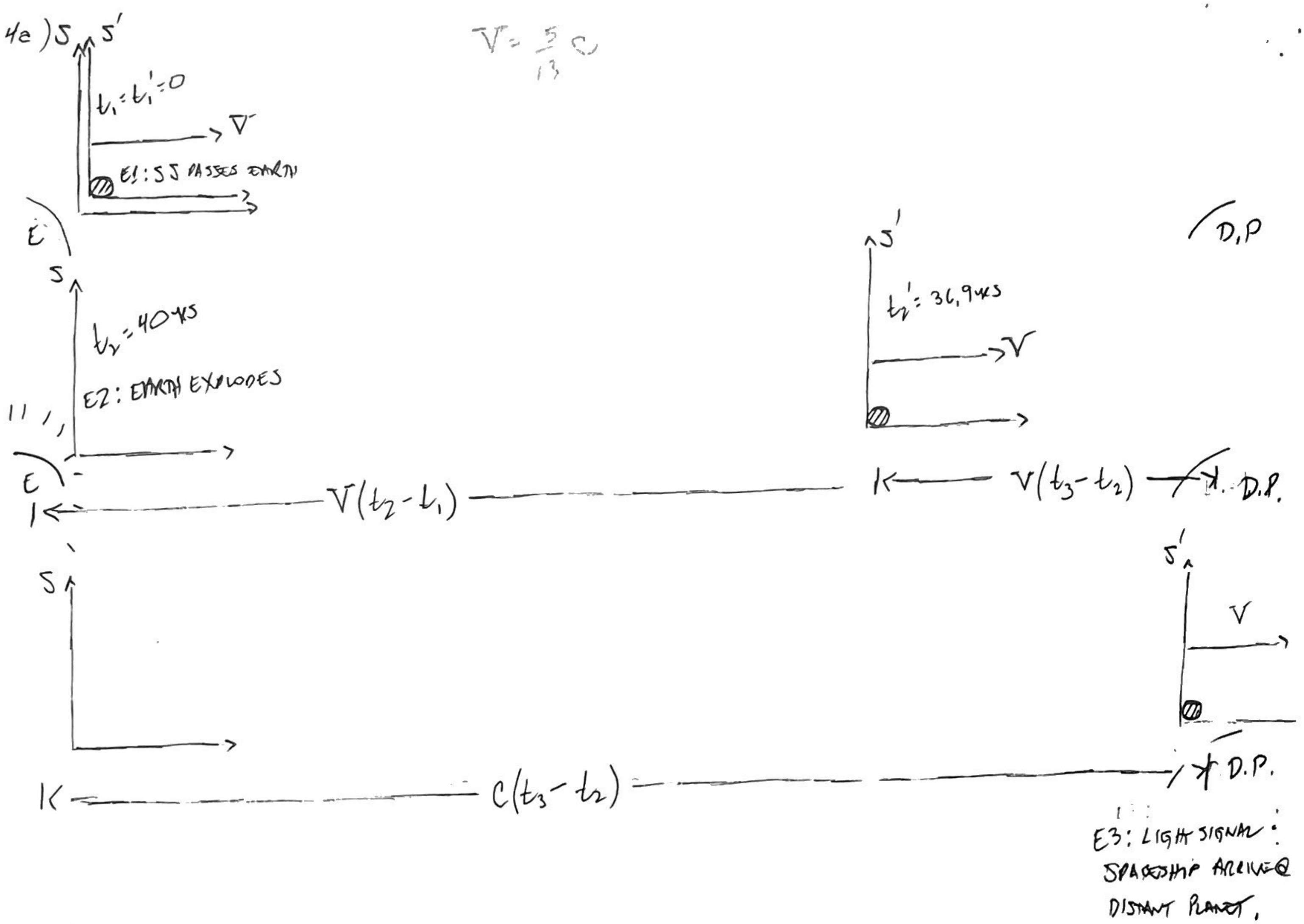
so

$$V = \frac{d}{\Delta t'}, \quad \therefore \quad \Delta t' = \frac{d}{V} = \frac{3600}{169}\ c\cdot yrs \cdot \frac{13}{5c} = \frac{720}{13}\ yrs$$

$$\boxed{\Delta t' = \frac{720}{13}\ yrs \simeq 55.4\ yrs}$$

4e)

$V = \frac{5}{13}c$



1) t_2?

NOTICE THAT EVENTS 1 & 2 OCCUR AT THE SAME X COORDINATE FOR THE S OBSERVER...

NOTICE THAT...

$V = \frac{D}{\Delta t}$ WHERE $\Delta t = t_3 - t_1 = t_3$ AS $t_1 = 0$

SO

$$t_3 = \frac{D}{V} = \frac{25c\cdot\text{yrs}}{\frac{5}{13}c} = 65\text{ yrs}$$

ALSO NOTICE (FROM THE ABOVE PICTURES) THAT..

$$c(t_3 - t_2) = V(t_2 - \underbrace{t_1}_{0}) + V(t_3 - t_2) = Vt_3$$

NOW SOLVE FOR t_2...

$$(c - V)t_3 = ct_2 \quad \text{so} \quad t_2 = \left(1 - \frac{V}{c}\right)t_3 = \left(1 - \frac{5}{13}\right)(65\text{ yrs}) = \frac{8}{13}\cdot 65\text{ yrs}$$

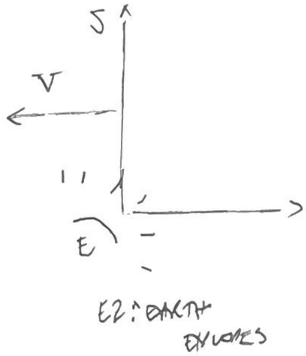
[t_2 = 40 yrs]

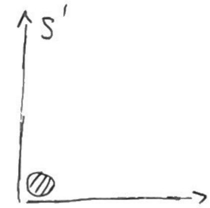
$\therefore$ NOW MOVING CLOCKS RUN SLOW SO

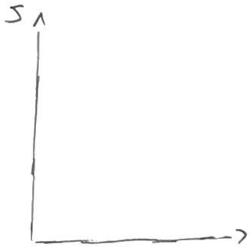
$$t_2' = t_2\sqrt{1 - v^2/c^2} = 40\,\text{yrs}\sqrt{1 - (5/13)^2} = 40\,\text{yrs}\cdot\frac{12}{13}$$

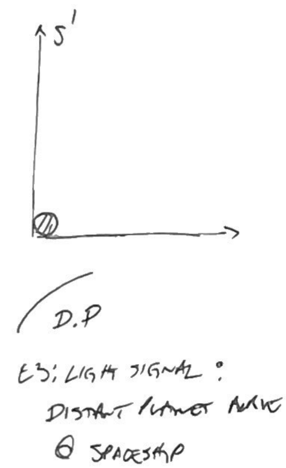
$$\left[t_2' = 36.9\,\text{yrs}\right]$$

5. $V = \frac{5}{13} c$









) THE EARTH/DISTANT PLANET IS MOVING RELATIVE TO THE SPACESHIP, SO THE SPATIAL SEPARATION IS LENGTH CONTRACTED...

$$d = D\sqrt{1 - V^2/c^2} = 25c\cdot yrs\sqrt{1-\left(\frac{5}{13}\right)^2} = (25c\cdot yrs)\cdot\frac{12}{13} = \frac{300}{13}\, c\, yrs \simeq 23.1\, c\, yrs$$

) $V = \frac{d}{\Delta t'}$, so $\Delta t' = \frac{d}{V} = \frac{\frac{300}{13}\, c\, yrs}{\frac{5}{13}\, c} = 60\ yrs$ ✓ so $\Delta t' = 60\ yrs$

) NOW, THE EARTH / DISTANT PLANET IS MOVING RELATIVE TO THE S' FRAME, SO THE TIME INTERVAL IS DILATED.

$$\Delta t = \Delta t'\sqrt{1 - V^2/c^2} = 60\ yrs \cdot \frac{12}{13} = 55\ yrs$$

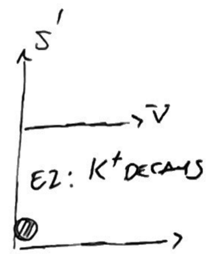
6.

$\Delta t' = 1.24 \times 10^{-8}$ s

Let S be the lab frame, S' be the K+ frame...

E1: creation of K^+ meson

E2: decay of K^+ meson



$$\Delta x = 10.0\,\text{m} = D$$

Now, according to the lab frame...

(*)
$$V = \frac{\Delta x}{\Delta t} = \frac{D}{\Delta t}$$

Since both events occur at the same x' coordinate,

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}} \quad \text{since } \Delta x' = 0$$

so (*) becomes..

$$V = \frac{D}{\Delta t'}\sqrt{1 - V^2/c^2} \quad \therefore \quad \frac{V}{c} = \frac{D}{c\Delta t'}\sqrt{1 - V^2/c^2}$$

Now squaring and solving for V/c...

$$\frac{V^2}{c^2} = \left(\frac{D}{c\Delta t'}\right)^2\left(1 - \frac{V^2}{c^2}\right)$$

$$\therefore \quad \frac{V^2}{c^2}\left(1 + \frac{D^2}{c^2\Delta t'^2}\right) = \left(\frac{D}{c\Delta t'}\right)^2$$

Now

$$\frac{D}{c\Delta t'} = \frac{10.0\,\text{m}}{(3.00 \times 10^8 \frac{\text{m}}{\text{s}})(\frac{5}{4} \times 10^{-8}\,\text{s})} = \frac{10.0\,\text{m}}{\frac{15}{4}\,\text{m}}$$

$$= \frac{40.0}{15} = 8/3$$

so

$$\frac{V}{c} = \frac{D/c\Delta t'}{\sqrt{1 + D^2/c^2\Delta t'^2}} = \frac{8/3}{\sqrt{1 + (8/3)^2}} = \frac{8/3}{\sqrt{1 + \frac{64}{9}}} = \frac{8/3}{\sqrt{73/9}} = \frac{8}{\sqrt{73}} \simeq 0.936$$

so $\boxed{V = 0.936\,c}$

b) Now, according to the LHC scientists...

$$V = \frac{D}{\Delta t}$$

$$\therefore \Delta t = \frac{D}{V} = \frac{10.0\,\text{m}}{0.936\,c} = 10.7\,\frac{\text{m}}{c}$$

$$\boxed{\Delta t = 3.56 \times 10^{-8}\,\text{s}}$$