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Laboratory 4: Projectile Motion

A projectile is an object close to the Earth's surface which moves solely under the influence of the Earth's gravitational force. Newtonian physics predicts the exact trajectory of a projectile given knowledge of its initial state of motion. The aims of this laboratory are to verify these predictions and to familiarize you with the use of the equations of motion for projectile motion.

1 Projectile Motion with Horizontal Launch

Classical physics predicts that any projectile has a constant acceleration with components $a_x = 0 \text{ m/s}^2$ and $a_y = -g = -9.8 \text{ m/s}^2$. The trajectory depends on the initial location and velocity of the projectile and can be determined using

$$v_{fx} = v_{ix} + a_x \Delta t \quad (1)$$

$$x_f = x_i + v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \quad (2)$$

$$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x. \quad (3)$$

Similar equations apply for the vertical components. Simply replace x with y .

In this experiment a ball will be launched in a horizontal direction. The speed of the ball immediately prior to launch can be measured and is denoted v_i . The vertical height through which the ball drops can also be measured and is denoted h .

- Sketch the velocity vector, \vec{v}_i at the moment of launch. Use this to relate (using formulas) its components, v_{ix} and v_{iy} , to the speed, v_i , with which the ball is launched.
- Does the horizontal component of velocity change over time? Explain your answer.
- Does the time taken to drop to the floor depend on the initial speed? Explain your answer.
- Determine an expression for the time taken to fall to the floor, Δt , in terms of one or more of v_i , h , g and other relevant numbers or constants.
- Use your results to determine an expression for the horizontal distance traveled by the ball from the point of launch to the point where it hits the floor. Verify this with the instructor.

2 Experiment

The ball can be rolled along a groove in the track and launched so that it strikes a piece of paper taped to the floor. A piece of carbon paper placed above this will make a mark at the point where the ball strikes the floor.

- Tape a piece of paper to the floor in the approximate location where the ball will land. Place a sheet of carbon paper face down on top of this.
- Level the track and align one end with the table edge.
- Measure and record the vertical height through which the ball falls.
- The speed of the ball prior to launch can be measured by timing how long it takes to pass from one photogate to another. Start DataStudio and connect the two photogates separately. Configure these to read the time between photogates. Display the result by dragging "time between photogates" into Digits in the display menu. Describe how to use this to determine the speed of the ball prior to launch.
- Roll the ball along the track and measure the time taken to pass from one photogate to another. Use this to determine the speed with which the ball leaves the track. Use the formula you developed earlier to calculate the distance through which the ball will travel from the moment of launch off the track until it hits the floor and denote your result Δx_{calc} . Measure the horizontal distance through which the ball traveled from the moment of launch until it struck the floor and denote this Δx_{meas} .
- Determine the percentage difference between Δx_{meas} and Δx_{calc} via

$$\text{Percentage difference} = \frac{|\Delta x_{\text{meas}} - \Delta x_{\text{calc}}|}{\Delta x_{\text{calc}}} \times 100\%$$

This gives a rough method for comparing theory to experiment. A good comparison would be anything less than 5%.

- Repeat the procedure of parts (e) and (f) three more times.

3 Conclusion

- Do the equations of motion predict the horizontal distance covered correctly?
- What are possible sources of error in this experiment? Note that friction between the ball and track here is irrelevant.
- What is your main conclusion from this experiment?

4 Exercises

- One possible source of error is an inaccuracy in measuring Δx_{meas} . It is reasonable to assume that your measurement may be as much as 5 mm too small or too large. This will give a range of values for Δx_{meas} . Choose one run of your experiment and list the range of possible values for Δx_{meas} . Does Δx_{calc} lie within this range? Can the percentage error for this run be explained by such measurement discrepancies in Δx_{meas} ?

- b) Suppose that the track was tilted slightly upwards at the launch end and that this had a negligible affect on the horizontal component of the initial velocity. However, it provides an non-zero positive vertical component of initial velocity. As a result, will Δx_{calc} (as calculated assuming a perfect horizontal launch) be too large or too small? Explain your answer.

$$\begin{aligned}
 v_{1x} &= v_{0x} + a_x \Delta t \\
 x_1 &= x_0 + v_{0x} \Delta t + \frac{1}{2} a_x (\Delta t)^2 \\
 v_{1x}^2 &= v_{0x}^2 + 2 a_x \Delta x
 \end{aligned}$$

$$\begin{aligned}
 v_{1y} &= v_{0y} + a_y \Delta t \\
 y_1 &= y_0 + v_{0y} \Delta t + \frac{1}{2} a_y (\Delta t)^2 \\
 v_{1y}^2 &= v_{0y}^2 + 2 a_y \Delta y
 \end{aligned}$$

30 $\frac{1}{8}$

10.1 cm

1.)

a) 

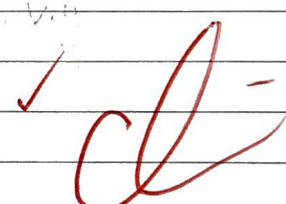
b) No, the horizontal component of velocity does not change over time because it is constant in projectile motion.

c) Yes, it is dependent on the initial vertical component of velocity because the smaller the initial velocity, the faster it hits the ground due to acceleration of gravity.

d) $x: y_1 - y_0 = \frac{1}{2} a_y (\Delta t)^2$ $\Delta t^2 = \frac{2(y_1 - y_0)}{a_y}$

$2(y_1 - y_0) = a_y (\Delta t)^2$ $\Delta t = \frac{\sqrt{2(y_1 - y_0)}}{a_y}$

e) $x_1 = x_0 + v_{0x} \left(\sqrt{\frac{2H}{g}} \right)$

$\Delta x = v_{1x} \sqrt{\frac{2h}{g}}$ ✓ 

2.)

c) Vertical distance of (0.80 m)

d) The time between the photogates with the distance between the photogates can give velocity at the initial instant of launch.

$$v_0 = \frac{x_1 - x_0}{t_1 - t_0}$$

$$v_0 = \frac{00.13 \text{ m}}{0.096 \text{ s}}$$

$$v_0 = 1.34 \text{ m/s}$$

$$h = 0.05 \text{ m}$$

$$g = -9.8 \text{ m/s}^2$$

0.78m height

0.01m Distance Between Gates

c) $\Delta x = v_0 x \sqrt{\frac{2h}{g}}$

$$\Delta x = 1.64 \text{ m/s} \sqrt{\frac{2(0.78 \text{ m})}{9.8 \text{ m/s}^2}}$$

$$\Delta x = 0.415 \text{ m}$$

Test # Distance from table % error

1) 1.433m 4.34% ✓

2) 1.537m 3.47% ✓

3) 1.375m 6.95% ✗

4) 1.638m 1.92% ✓

2)

$$\Delta x = 1.3 \text{ m/s} \sqrt{\frac{1.56 \text{ m}}{9.8 \text{ m/s}^2}}$$

$$\Delta x = 0.519 \text{ m}$$

3)

$$\Delta x = 1.01 \text{ m/s} \sqrt{\frac{1.56 \text{ m}}{9.8 \text{ m/s}^2}}$$

$$\Delta x = .403 \text{ m}$$

4)

$$\Delta x = 1.57 \text{ m/s} \sqrt{\frac{1.56 \text{ m}}{9.8 \text{ m/s}^2}}$$

$$\Delta x = .626 \text{ m}$$

4. a) Trial 4

$$x_{\text{meas}} = .638 \text{ m} \pm .5 \text{ m} = .643 \text{ m}, .633 \text{ m}$$

The x_{calc} for trial 4 was $\Delta x = .626 \text{ m}$ which does not lie within the range. This error could have been caused by not using proper tools, like levels, to measure out from the table.

b) If the track was tilted upwards the x_{calc} would be larger since there would be an initial positive non-zero vertical component allowing the ball to travel farther.