Taylor Larrechea   Ch. 5 # 7, 18, 23, 24, 27, 31, 33, 34, 67

Dr. Schiffbauer

PHYS 231 HW 4

Problem 7

Limited by lens ∴ relativistic condition

$$p_{rel} = \frac{\sqrt{KE^2 + 2KEmc^2}}{c} \qquad \lambda = \frac{h}{p} \qquad KE = e \cdot \Delta V$$

$$\lambda = \frac{h}{\frac{\sqrt{KE^2+2KEmc^2}}{c}} = \frac{hc}{\sqrt{KE^2+2KEmc^2}} \quad : \quad hc = 1240 \text{ eV}\cdot\text{nm}, \quad mc^2 = 0.511 \frac{\text{MeV}}{c^2}$$

$\Delta V = 40$ kV

$KE = (1.6\times10^{-19} C)(40{,}000 V) = 6.4\times10^{-15} J$

$KE = 40$ keV

$\Delta V = 100$ keV

$KE = (1.6\times10^{-19} C)(100{,}000 V) = 1.6\times10^{-14} J$

$KE = 100{,}000$ keV

a.) $\Delta V = 40$ kV

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{(40\text{keV})^2 + 2(40\text{keV})(0.511\frac{\text{MeV}}{c^2})}} = 0.00602 \text{ nm} = 6.02 \text{ pm}$$

b.) $\Delta V = 100$ kV

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{(100\text{keV})^2 + 2(100\text{keV})(0.511\frac{\text{MeV}}{c^2})}} = 0.00370 \text{ nm} = 3.7 \text{ pm}$$

$$\boxed{\begin{aligned}\lambda_{40\,kV} &= 6.02 \text{ pm} \\ \lambda_{100\,kV} &= 3.7 \text{ pm}\end{aligned}}$$

Problem 18

Relativistic Momentum

$$\lambda = \frac{h}{p} \qquad p = \frac{\sqrt{KE^2+2KEmc^2}}{c}$$

$$\lambda = \frac{hc}{\sqrt{KE^2+2KEmc^2}} \quad : \quad hc = 1240 \text{ eV}\cdot\text{nm}, \quad mc^2 = 938.27 \frac{\text{MeV}}{c^2}$$

a.) $KE = 1.0\times10^{12}$ eV

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{(1.0\times10^{12}\text{eV})^2 + 2(1.0\times10^{12}\text{eV})(938.27\frac{\text{MeV}}{c^2})}} = 1.24\times10^{-9} m = 1.24 \text{ nm}$$

b.) $KE = 7.0\times10^{12}$ eV

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{(7.0\times10^{12}\text{eV})^2 + 2(7.0\times10^{12}\text{eV})(938.27\frac{\text{MeV}}{c^2})}} = 1.77\times10^{-10} m = 0.177 \text{ nm}$$

$$\boxed{\begin{aligned}\lambda_{KE=1.0\,TeV} &= 1.24 \text{ nm} \\ \lambda_{KE=7.0\,TeV} &= 0.177 \text{ nm}\end{aligned}}$$

Problem 23

$\Delta\omega\Delta t = 2\pi$ $\qquad\qquad\Delta\omega = 2\pi f$

$2\pi f\Delta t = 2\pi$

$f\Delta t = 1$

Time between measurements is half of the bandwidth relationship

$f\Delta t = \frac{1}{2}$

$\Delta t = \frac{1}{2f}$ $\qquad f = 0.3\ Hz$

$\Delta t = \frac{1}{2(0.3Hz)} = \frac{1}{0.6Hz} = 1.67s$

$\boxed{\Delta t = 1.67\ s}$

Problem 24

a.) 6.0 ev

$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240\ ev\cdot nm}{\sqrt{2(0.511\times10^6 ev)(6.0ev)}} = 0.501nm$

$hc = 1240\ ev\cdot nm$

$mc^2 = 0.511\times10^6 ev$

$K = 6.0ev$

600 kev $\quad$ Higher energy ∴ relativistic

$\lambda = \frac{h}{p'} = \frac{hc}{\sqrt{K^2+2mc^2K}} = \frac{1240\ ev\cdot nm}{\sqrt{(600\times10^3ev)^2+2(0.511\times10^6ev)(600\times10^3ev)}}$

$\lambda = 0.00126\ nm$

$\boxed{\begin{array}{l}\lambda_{6ev} = 501\ pm\\ \lambda_{600ev} = 1.26\ pm\end{array}}$

b.) 6.0 eV

$KE = \frac{1}{2}mv^2 = \frac{h^2}{2m\lambda^2}$

$mv^2 = \frac{h^2}{m\lambda^2}$: $v^2 = \frac{h^2}{m^2\lambda^2}$: $v = \frac{h}{m\lambda}$

$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}J\cdot S}{(9.11\times10^{-31}kg)(501\times10^{-12}m)} = 1.45\times10^6\ m/s$

$\lambda = 501\ pm$

$m = 9.11\times10^{-31}\ kg$

600 keV

$E_e = 600\ kev$ $\qquad E_\alpha = \left(\frac{E_e + E_0}{E_0}\right) \cong 2.17$

$v' = \frac{h}{E_\alpha m\lambda'}$

$m = 9.11\times10^{-31}\ kg$

$\lambda' = 1.26\ pm$

$v' = \frac{6.626\times10^{-34}J\cdot S}{E_\alpha(9.11\times10^{-31}kg)(1.26\times10^{-12}m)} = 2.66\times10^8\ m/s$

$\boxed{\begin{array}{l}V_{6eV} = 1.45\times10^6\ m/s\\ V_{600keV} = 2.66\times10^8\ m/s\end{array}}$

c.) 6.0 eV $\quad V_{ph} = \frac{E_0}{P} = \frac{E_0}{mV}$

$E_0 = 0.511\times10^6\ eV$

$m = 9.11\times10^{-31}\ kg$

$V = 1.45\times10^6\ m/s$

$V' = 2.66\times10^8\ m/s$

$V_{ph} = \frac{(0.511\times10^6 eV)\cdot(1.6\times10^{-19} J/eV)}{(9.11\times10^{-31}kg)(1.45\times10^6\ m/s)} = 6.19\times10^{10}\ m/s$

600 keV $\quad V_{ph} = \frac{E_0}{p'} = \frac{E_0}{E_\alpha mV'}$

$V_{ph} = \frac{0.511\times10^6 eV\ (1.6\times10^{-19}\ J/eV)}{(E_\alpha)(9.11\times10^{-31}kg)(2.66\times10^8 m/s)} = 1.55\times10^8\ m/s$

$\boxed{\begin{array}{l}V_{6eV} = 6.19\times10^{10}\ m/s\\ V_{600keV} = 1.55\times10^8\ m/s\end{array}}$

d.) 6.0 ev $\qquad$ 600 kev

$u_{gr} = v = 1.45\times10^6\ m/s$ $\qquad u_{gr} = v = 2.66\times10^8\ m/s$

$\boxed{\begin{array}{l}u_{6eV} = 1.45\times10^6\ m/s\\ u_{600keV} = 2.66\times10^8\ m/s\end{array}}$

Problem 27

$\Psi(x,t) = A\sin(kx - \omega t)$

a.) $\Psi(x,t)_1 = 0.0030\sin(6x - 300t)$ $\quad$ $\Psi(x,t)_2 = 0.0030\sin(7x - 250t)$

$$\Psi(x,t) = \Psi_1 + \Psi_2 = A\sin(k_1x - \omega_1 t) + A\sin(k_2x - \omega_2 t)$$

$$\Psi(x,t) = 2A\sin\left(\frac{\Delta k}{2}x + \frac{\Delta\omega}{2}t\right)\cos(k_{Av}x - \omega_{Av}t)$$

$A = 0.0030$

$\Delta k = k_1 - k_2 = 6 - 7 = -1 \quad |\Delta k| = 1$ $\qquad$ $k_{Av} = \frac{k_1+k_2}{2} = \frac{6+7}{2} = 6.5$

$\Delta\omega = \omega_1 - \omega_2 = 300 - 250 = 50$ $\qquad$ $\omega_{Av} = \frac{\omega_1+\omega_2}{2} = \frac{300+250}{2} = 275$

$$\boxed{\Psi(x,t) = 0.0060\sin\left(\frac{1}{2}x + 25t\right)\cos\left(6.5x - 275t\right)}$$

b.) V_{ph} & V_{Gr}

$V_{ph} = \frac{\omega_{Av}}{k_{Av}} = \frac{275\ 1/s}{6.5\ 1/m} = 42.3\ m/s$ $\qquad$ $V_{Gr} = \frac{\Delta\omega}{\Delta k} = \frac{50\ 1/s}{1\ 1/m} = 50\ m/s$

$$\boxed{\begin{array}{l} V_{ph} = 42.3\ m/s \\ V_{Gr} = 50\ m/s \end{array}}$$

c.) $\Delta k \Delta x = 2\pi$

$\Delta k = 1 \quad \therefore \quad \Delta x = 2\pi\ m$

$$\boxed{\Delta x = 2\pi\ m}$$

d.) $\Delta k \Delta x = 2\pi$

$$\boxed{\Delta k \Delta x = 2\pi}$$

Problem 31

$$\Psi(x,t) = \int \tilde{A}(k)\cos(kx-\omega t)\,dk$$

$$\tilde{A}(k) = A_0$$

a.)

$$\Psi(x,0) = \int A_0 \cos(kx)\,dk \qquad u = kx \qquad \frac{du}{dk} = x \qquad du = x\,dk$$

$$\int_{k_0-\frac{\Delta k}{2}}^{k_0+\frac{\Delta k}{2}} A_0\cos(kx)\,dk$$

$$\frac{A_0}{x}\sin(kx)\Bigg|_{k_0-\frac{\Delta x}{2}}^{k_0+\frac{\Delta k}{2}}$$

$$\frac{A_0}{x}\sin\left(\left(k_0+\tfrac{\Delta k}{2}\right)x\right) - \frac{A_0}{x}\sin\left(\left(k_0-\tfrac{\Delta x}{2}\right)x\right)$$

$$\sin(a) - \sin(b) = \sin\left(\tfrac{a-b}{2}\right)\cos\left(\tfrac{a+b}{2}\right)$$

$$a = \left(k_0+\tfrac{\Delta k}{2}\right)x \qquad b = \left(k_0 - \tfrac{\Delta k}{2}\right)x$$

$$\frac{2A_0}{x}\left(\sin(a) - \sin(b)\right)$$

$$a-b: \left(\cancel{k_0x}+\tfrac{\Delta k}{2}x\right) - \left(\cancel{k_0x} - \tfrac{\Delta x}{2}x\right) = \Delta k x$$

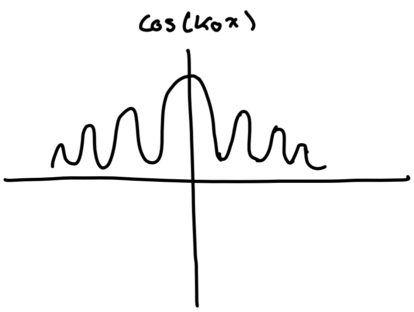
$$\frac{2A_0}{x}\left(\sin\left(\tfrac{a-b}{2}\right)\cos\left(\tfrac{a+b}{2}\right)\right)$$

$$a+b: \left(k_0x + \cancel{\tfrac{\Delta k}{2}x}\right) + \left(k_0x - \cancel{\tfrac{\Delta k}{2}x}\right) = 2k_0x$$

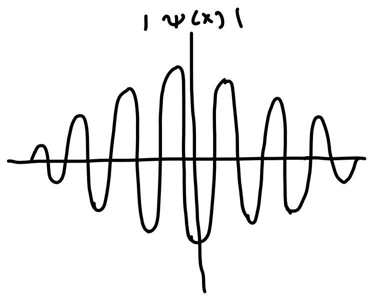
$$\boxed{\Psi(x,0) = \frac{2A_0}{x}\left(\sin\left(\frac{\Delta k}{2}x\right)\cos(k_0x)\right)}$$

b.)

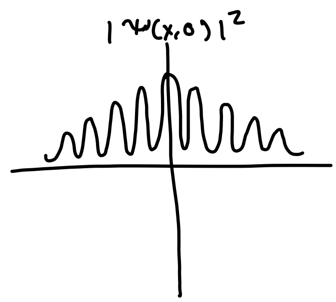
Oscillating term



Envelope Term



$|\Psi(x,0)|^2$



c.)

$$|\Psi(x,0)|^2 = \left(\frac{2A_0}{x}\right)^2 \sin^2\left(\frac{\Delta k}{2}x\right)\cos^2(k_0x)$$

$$\frac{\Delta k x}{2} = \pi \qquad \frac{x}{2} = \Delta x$$

now at half maximum

$$\frac{\Delta k x}{2} = \frac{\pi}{2}$$

$$\Delta k x = \pi$$

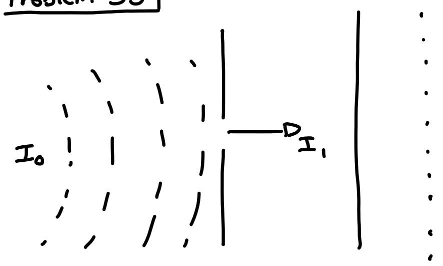
$$x = \frac{\pi}{\Delta k}$$

$$\boxed{\Delta x = \frac{\pi}{\Delta k}}$$

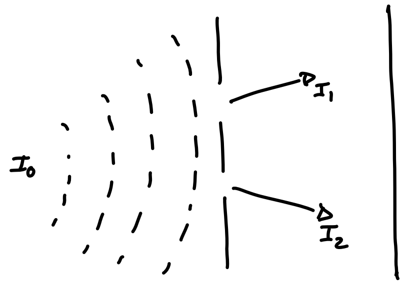
d.) $\Delta k \Delta x = ?$

$$\boxed{\Delta k \Delta x = \pi}$$

Problem 33



One Slit



Double Slit

$P = P_1 + P_2 + 2\sqrt{P_1 P_2}\cos\delta$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$

Intensity: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$

$= I_o + I_o + 2\sqrt{I_o I_o}\cos\theta$

$= 2I_o + 2I_o\cos(0)$

$I = 4I_o$

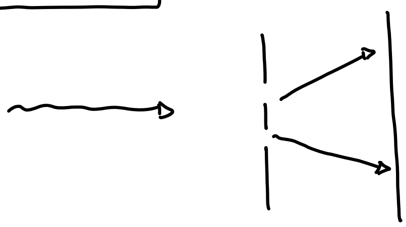
$I_1 = I_2 = I_o$

$\theta = 0$

When the light goes through the double slit, the intensity hitting the back screen increases by 4.

$\boxed{I = 4I_o}$

Problem 34



orders for maximum

$n\lambda = d\sin\theta$

Low energy ∴ nonrelativistic

$KE = \frac{p^2}{2m}$ ∴ $p = \sqrt{2mKE}$

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}}$

$n\lambda = d\sin\theta$ $\qquad \lambda = \frac{h}{\sqrt{2mK}}$

$\frac{nh}{\sqrt{2mK}} = d\sin\theta$

$\frac{nh}{\sin\theta\sqrt{2mK}} = d$

$h = 6.626\times10^{-34}\,J\cdot s$ $\quad m = 9.11\times10^{-31}\,kg$

$\theta = 1°$ $\quad K = 1.6\times10^{-16}\,J$

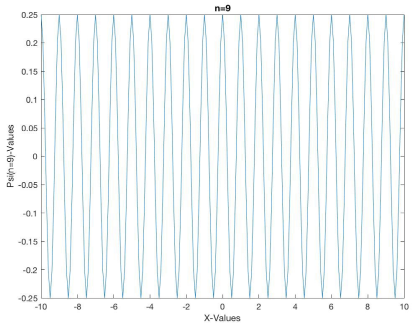
$d = \frac{(1)(6.626\times10^{-34}\,J\cdot s)}{\sin(1°)\sqrt{2(9.11\times10^{-31}\,kg)(1.6\times10^{-16}\,J)}} = 2.22\times10^{-9}\,m$

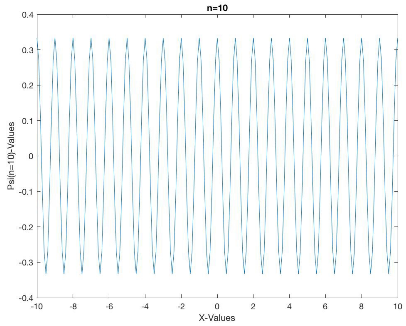
$\boxed{d = 2.22\text{ nm}}$

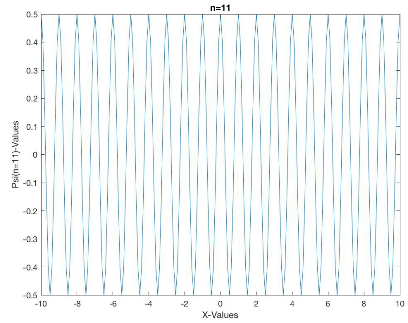
Problem 67

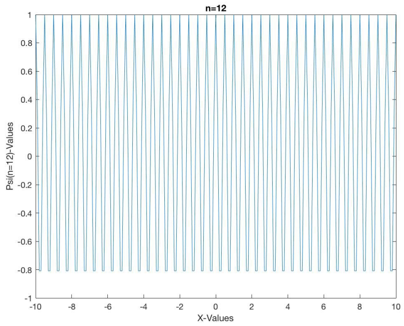
$$\Psi = A_n \cos(2\pi n x)$$

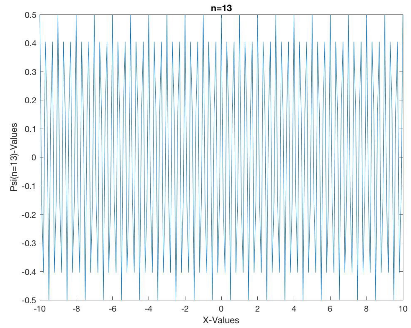
a.)

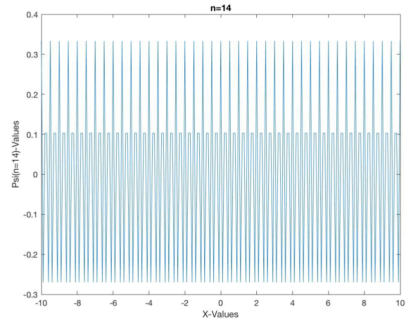


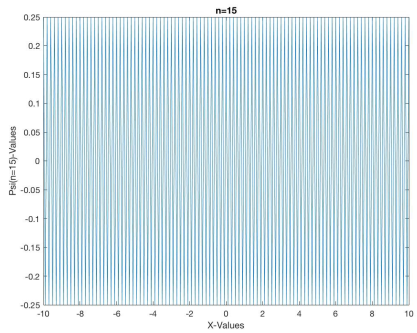












b.) The wave packets are all centered at zero. They repeat for every 1 x-value.