2.3 # 6, 10, 20

2.3.6 Proportional to, 1 gram reduced to 0.80 grams in one week

a.) $y = Ke^{t}$ $\qquad y(0) = 1$

$y = K(e^{0})$

$1 = K$

$0.80 = e^{?(1)}$

$\ln 0.80 = K$

$K = -0.23$

$$\boxed{\begin{array}{l} \text{a.)}\ y = e^{-0.23t} \\ \text{b.)}\ y = 3.11 \text{ weeks} \\ \text{c.)}\ y = 0.1 \text{ grams} \end{array}}$$

b.) $0.50 = e^{-0.23t}$

$\ln(0.50) = -0.23t$

$-0.69 = -0.23t$

$t = 3.11$

c.) $y(10)$: $y = e^{-0.23(t)}$

$y = e^{-0.23(10)}$

$y = 0.10$

2.3.10 $y = e^{Kt}$

$Q_1 = Q_0 e^{Kt_1}$

$Q_2 = Q_0 e^{Kt_2}$

$Q_0 = \dfrac{Q_1}{e^{Kt}}$

$Q_2 = \dfrac{Q_1}{e^{Kt_1}} e^{Kt_2}$

$Q_2 = Q_1 (e^{Kt_2})(e^{-Kt_1})$

$\dfrac{Q_2}{Q_1} = e^{Kt_2 - Kt_1}$

$\ln(Q_2/Q_1) = Kt_2 - Kt_1$

$K = \dfrac{\ln(Q_2/Q_1)}{(t_2 - t_1)}$

$e^{Kt} = \frac{1}{2}$

$Kt = \ln(1/2)$

$t = \dfrac{\ln(1/2)}{K}$

$t = -\dfrac{\ln 2}{K}$

$t = \dfrac{-\ln 2}{\dfrac{\ln(Q_2/Q_1)}{(t_2 - t_1)}}$

$t = \dfrac{-\ln 2 (t_2 - t_1)}{\ln(Q_2/Q_1)}$

$t = \dfrac{\ln 2 (t_2 - t_1)}{\ln(Q_1/Q_2)}$

$t_h = \dfrac{\ln 2 (t_2 - t_1)}{\ln(Q_1/Q_2)}$

2.3.20 $y = e^{Kt}$

Doubles every 12 hours how long till 5x as big?

$Q_1 = Q_0 e^{Kt}$

$y(t) = Q_0 e^{12K}$

$2 = e^{12K}$

$\ln 2 = 12K$

$\dfrac{\ln 2}{12} = K$

$K = 0.057762$

$y = e^{0.057762t}$

$5 = e^{0.057762t}$

$\ln 5 = 0.057762t$

$t = 27.9$

$t = 27.9$ hrs