Handy Reference For Complex #'S :

$i = \sqrt{-1}$

$z = x + iy$ <= cartesian Form

$z^* = x - iy$ <= Complex conjugate

$z = re^{i\theta}$ <= Polar Form

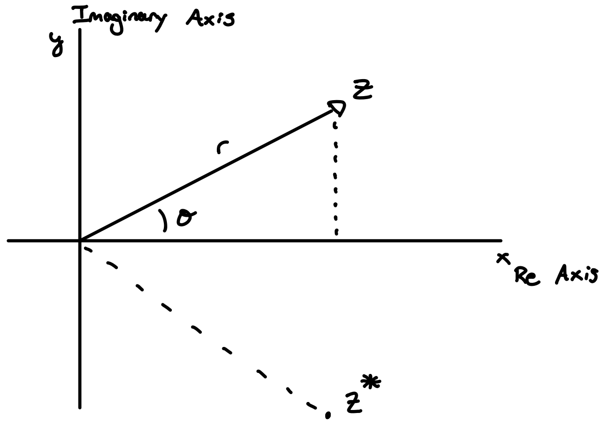
$x = Re(z)$ <-> Real

$y = Im(z)$ <-> Imaginary

$|z| \equiv$ Modulus

w/ $|z|^2 = z \cdot z = x^2 + y^2 = r^2$

$\theta \Rightarrow \tan\theta = y/x$



Summarizing up to 5.7

1) Probabilistic nature
2) Probability associated w/ a Function, (ϕ or ψ)

$$P \sim |\psi|^2$$

3) Normalization : If it exists, it must be some place w/ probabilities = 1

$$1 = \int \rho \, d\Omega = \int \psi^* \psi \, d^3\vec{r}$$

Central to copenhagen Interpretation :

1) uncertainty relations between canonically conjugate Variables (I.E x & P_x or y & P_y or E & t ...)

$$\Delta\vec{A}\Delta\vec{B} \geq \hbar/2$$

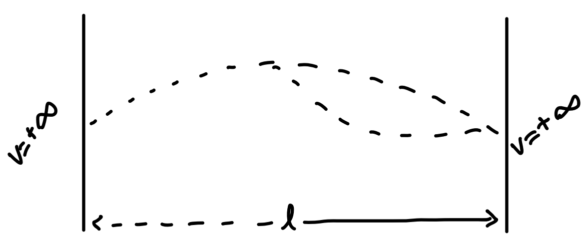
$$\Delta x P_x \geq \hbar/2$$

$$\Delta E \Delta t \geq \hbar/2$$

2) Bohr's Complementarity
3) The probabilistic interpretations

Consequences :

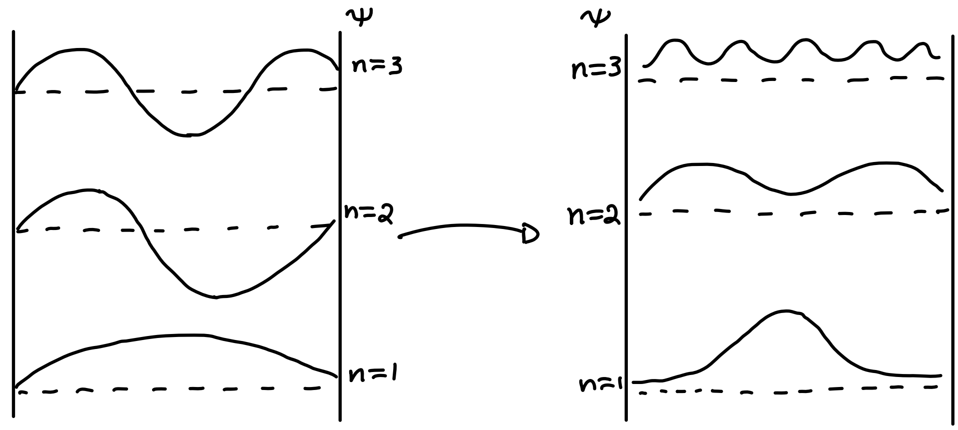
1) Not deterministic
2) Measurement itself plays a critical role
3) Non - Local
4) Is the wave Function "An element of physical reality ?"



$$\lambda_n = \frac{2l}{n} \qquad p = h/\lambda$$

$$k_n = \frac{2\pi}{\lambda} = \frac{\pi n}{l}$$

$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m}\left(\frac{n^2}{4l^2}\right) = \frac{n^2 h^2}{8ml^2}$$



Schröedingers Equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi \qquad i\hbar \frac{\partial \psi}{\partial t} = \overbrace{\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]}^{=H}\psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$\psi, V \Rightarrow$ can be $V(x,t)$
$\psi(x,t)$

Classical Wave Eqn

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2} \qquad (1)$$

Diffusion (Heat) Equation

$$\frac{\partial \rho}{\partial t} = -D\frac{\partial^2 \rho}{\partial x^2} \qquad (2)$$

$t^* \to -it$

Solution To Wave Equations

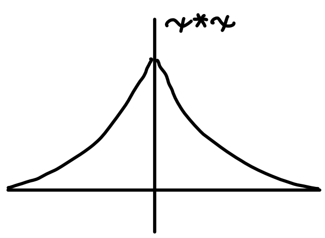
Traveling classical wave: $\psi = A\sin(Kx - \omega t + \phi)$
more general Form: $\psi = Ae^{i(Kx-\omega t+\phi)}$ $\Leftrightarrow$ Traveling plane waves
(A can be real or complex)

Normalization:

$$1 = \int_{-\infty}^{\infty} P\,dx = \int \psi^*\psi\,dx$$

$$P(\in\{x,x'\}) = \int_x^{x'} P\,dx$$

$\psi(x) = Ae^{-\alpha|x|}$



$$\begin{aligned}1 &= \int_{-\infty}^{\infty} \psi^*\psi\,dx \\ &= \int_{-\infty}^{\infty} Ae^{-\alpha|x|}\left(Ae^{-\alpha|x|}\right)dx \\ &= \int_{-\infty}^{\infty} A^2 e^{-2\alpha|x|}\,dx \\ &= -\frac{A^2}{\alpha}e^{-2\alpha|x|}\Big|_{-\infty}^{\infty} = \frac{A^2}{\alpha} = 1 \qquad A = \sqrt{\alpha}\end{aligned}$$

$$\psi = \sqrt{\alpha}\,e^{-\alpha|x|}$$

1) ψ Everywhere bounded (Finite)
2) ψ is single-valued
3) If V is bounded for all x
ψ & $\frac{\partial \psi}{\partial x}$ must be continuous
4.) Normalization requires that $\psi \xrightarrow[x \to \pm\infty]{} 0$
*Except for plane waves.

$\Psi(x,t) = \psi(x) f(t)$ <= For time-independent V
(& Special time-dep cases...)

Put into Schröedingers equation

$$i\hbar \psi \frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m} f \frac{\partial^2 \psi}{\partial x^2} + V\psi f$$

$$i\hbar \frac{1}{f}\frac{df}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V$$

$$i\hbar \frac{1}{f}\frac{df}{dt} = B \qquad B = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V$$

Solutions: $f(t) = e^{-iBt/\hbar} + C \qquad C = 0 \qquad B = E$

$$i\hbar \frac{1}{f}\frac{df}{dt} = E$$ Time dependent

$$-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V_{(x)} = E$$ Time independent

Ex: Free Particle:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$E \geq 0$ — Free Particle

$E \leq 0$ — Damped particle

Time indepent

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

$$\psi \sim e^{mx}$$
$$m^2 = -k^2$$
$$m = \pm ik$$

Solution

$$\psi = Ae^{ikx}$$

$$\Psi(x,t) = \psi(x) f(t) = Ae^{ikx} e^{-iEt/\hbar}$$
$$= Ae^{i(kx-\omega t)}$$

$$1 = \int_{-\infty}^{\infty} A^* e^{-i(kx-\omega t)} A e^{i(kx-\omega t)}\, dx = |A|^2 \int_{-\infty}^{\infty} dx$$

Expectation Values

Assume a normalized ψ
we want to know the expected average value for some dynamical variable upon measurement.

A <= Some dynamical variable (Repressed by an operator)

$$\langle A \rangle = \frac{\sum_i A_i f(A_i)}{\sum_i f(A_i)} \qquad A \text{ is D.R.V}$$

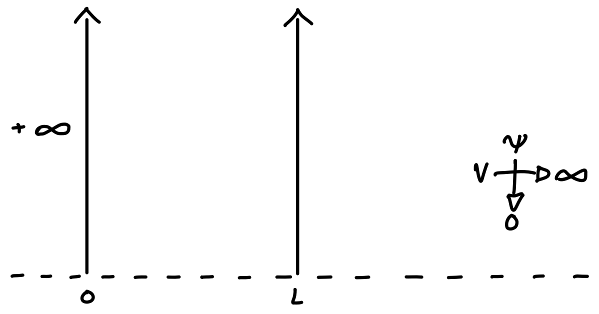
if A is a C.R.V

$$\langle A \rangle = \frac{\int A f(A)}{\int f(lh)\, dx}$$

Q.M Expectation Values:

$$\langle A \rangle = \frac{\int_{-\infty}^{\infty} \Psi^*(x,t) A \Psi(x,t)\, dx}{\int_{-\infty}^{\infty} \psi^* \psi \, dx} = \int_{-\infty}^{\infty} \psi^* A \psi \, dx$$

6.3 Infinite Square Well Problem



$$V(x) = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < L \\ \infty & x \geq L \end{cases}$$

$$\frac{d^2\psi}{dx^2} = -k^2\psi \quad \text{w/ } k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi \sim e^{imx} \longrightarrow \psi = Ae^{ikx} + Be^{-ikx}$$

$$\Updownarrow$$

$$\psi = A\sin kx + B\cos kx$$

B.C.'s

$$\psi(0) = 0 \Rightarrow A' + B' = 0 \Rightarrow A' = -B'$$

$$\psi(x) = A'(e^{ikx} - e^{-ikx})$$

– or –

$$\psi(x) = A\sin kx$$

$$\psi(L) = 0 = A\sin kL \longrightarrow kL = n\pi$$

$$kL = \sqrt{\frac{2mE}{\hbar^2}}\,L = n\pi \Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$1 = \int_{-\infty}^{\infty} \psi^*\psi \, dx = \int_0^L A^2 \sin^2(kx)\, dx$$

$$= \int_0^L A^2 \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$= A^2\, L/2 = 1$$

$$A = \sqrt{\frac{2}{L}}$$

Normalized Wave Functions

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

$$\psi(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) e^{-iE_nt/\hbar}$$

Now, what do we get when we measure:

1) Position: $\hat{x}$

2) Momentum: $\hat{p} = i\hbar \frac{\partial}{\partial x}$

3) Momentum²: $\hat{p}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$

Position

$$\hat{p}f(x) \neq f(x)\hat{p}$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x,t)\, x\, \psi(x,t)\, dx = \int_0^L \psi^*(x)\, x\, \psi(x)\, dx$$

$$\frac{2}{L}\int_0^L x\sin^2(Kx)\,dx = \frac{2}{L}\left(\frac{L}{\pi}\right)\left[\left(\frac{n\pi}{L}x\right)^2\frac{1}{4} - \frac{\left(\frac{n\pi}{L}x\right)\sin\left(\frac{2n\pi}{L}x\right)}{4} - \frac{\cos\left(\frac{2n\pi}{L}x\right)}{8}\right]_0^L = \frac{L}{2}$$

momentum

$$\langle P_x\rangle = \int_0^L \psi^*\left(-i\hbar\,\partial/\partial x\right)\psi\,dx = \frac{2}{L}\int_0^L \sin\left(\frac{n\pi}{L}x\right)(-i\hbar)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$= -i\hbar\frac{2}{L}\int_0^L \sin\left(\frac{n\pi}{L}x\right)\cos\left(\frac{n\pi}{L}x\right)\left(\frac{n\pi}{L}\right)dx$$

$\sin(\,) \equiv u \qquad (\,)\cos(\,) = du$

$$= \frac{-i\hbar^2}{L}\,\frac{1}{2}u^2\Big|_0^L = \frac{-i\hbar}{L}\sin^2\left(\frac{n\pi}{L}x\right)\Big|_0^L = 0$$

momentum^2

$$\langle P_x^2\rangle = \int_0^L \psi^*(x)\left(-\hbar\frac{\partial^2}{\partial x^2}\right)\psi(x)\,dx$$

$$= -\hbar^2\int_0^L \psi^*\left(\frac{d}{dx}\frac{n\pi}{L}\sqrt{\frac{2}{L}}\cos\left(\frac{n\pi}{L}x\right)\right)dx$$

$$= \hbar^2\int_0^L \frac{2}{L}\sin^2\left(\frac{n\pi}{L}x\right)\left(\frac{n\pi}{L}\right)^2dx$$

$$= \frac{2n^2\pi^2\hbar^2}{L^3}\int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx = \frac{2n^2\pi^2\hbar^2}{L^3}\left[\frac{1}{2}\left(\frac{n\pi}{L}x\right) - \frac{1}{4}\sin\left(\frac{n\pi}{L}x\right)\right]_0^L = \frac{n^2\pi^2\hbar^2}{L^2} > 0$$

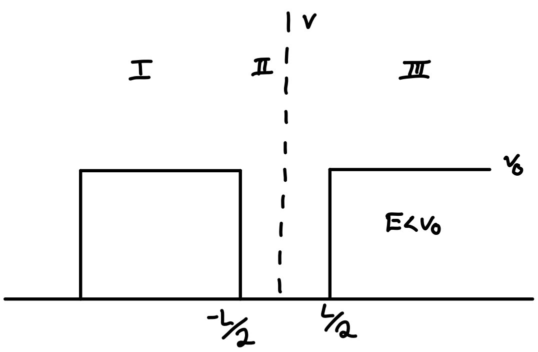
Suppose

$$\psi(x,t) = e^{-iE_1t/\hbar}\psi_1 + e^{-iE_2t/\hbar}\psi_2$$

$$\psi^*\psi = \psi_1^*\psi_1 + \psi_2^*\psi_2 + \left(e^{iE_1t/\hbar}\psi_1^*\right)\left(e^{-iE_2t/\hbar}\psi_2\right) + \left(e^{iE_2t/\hbar}\psi_2^*\right)\left(e^{-iE_1t/\hbar}\psi_1\right)$$

$$= |\psi_1|^2 + |\psi_2|^2 + \psi_1^*\psi_2e^{-i(E_2-E_1)t/\hbar} + \psi_2^*\psi_1e^{-i(E_1-E_2)t/\hbar} \;:\; \Delta E = \frac{-i(E_1-E_2)t}{\hbar} \;\Big\|\; \frac{-i(E_2-E_1)t}{\hbar}$$

Finite well



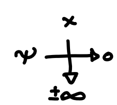
Region I & III

$$\frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} = E - V_0$$

$$\alpha^2 = \frac{2m(V_0-E)}{\hbar} > 0$$

Region II

$$\frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}\;E$$



$\psi \sim e^{\pm\alpha x}$

$\psi_I(x) = Ae^{\alpha x}$

$\psi_{III}(x) = Be^{-\alpha x}$

Region II

$$\frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} = E \equiv -k^2 \implies \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -k^2\psi$$

$\psi(x) \sim e^{\pm ikx}$

$\psi_{II} = Ce^{ikx} + De^{-ikx}$

Based on symmetry, ground state looks like

$\psi_{II}(x) = C'\cos(Kx)$

To Solve Such A System

$\psi_I(-L/2) = \psi_{II}(-L/2)$

$\psi_{II}(L/2) = \psi_{III}(L/2)$

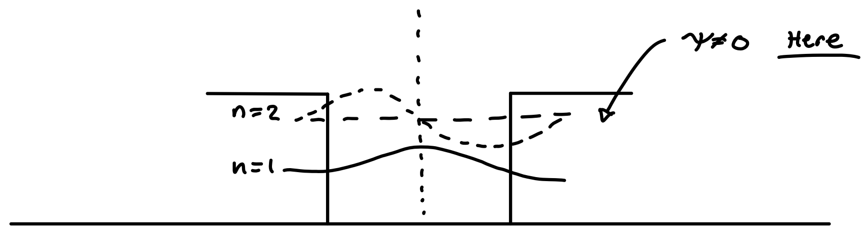
$\psi_I'(-L/2) = \psi_{II}'(-L/2)$

$\psi_{II}'(L/2) = \psi_{III}''(L/6)$

We match with Solutions in I & III:

$$\psi_I(-L/2) = \psi_{II}(-L/2) \Rightarrow A e^{-\alpha(L/2)} = C\cos(-k(L/2))$$
$$\psi_I'(-L/2) = \psi_{II}'(L/2) \Rightarrow \alpha A e^{-\alpha(L/2)} = -kC\sin(-kL/2)$$
$$\cos(-k(L/2)) = -k/4\,\sin(-k(L/2))$$
$$\Rightarrow \quad -\frac{k}{4} = \tan(-k(L/2))$$
$$\tan\left[\left(\frac{mL}{2\hbar}E\right)^{\frac{1}{2}}\right] = \left[\frac{V_0 - E}{E}\right]^{\frac{1}{2}}$$

Tunneling



Quick Note on 3D well:

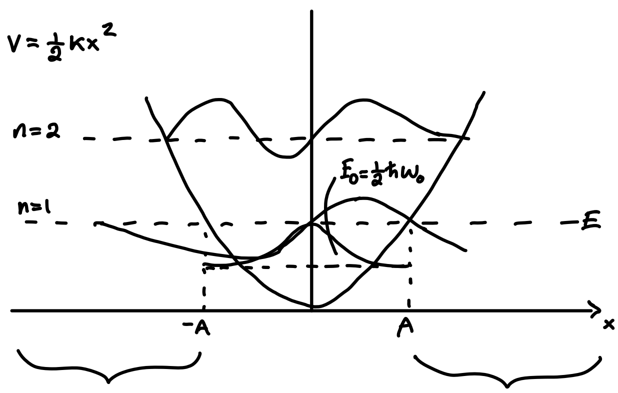
Degeneracy: Different States w/ Same E

$$E = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

Ground State: $(n=1)$
$n_x = n_y = n_z = 1$ } Non degenerate

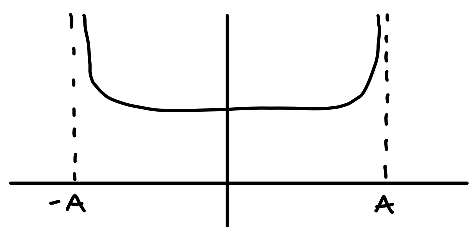
1^{st} Excited State:

Triple Degeneracy {
$n_x = 2 : n_y = n_z = 1$
$n_y = 2 : n_x = n_z = 1$
$n_z = 2 : n_x = n_y = 1$



Classically forbidden regions beyond turning points (-A, A)

All bound $V \xrightarrow[\pm\infty]{x} \infty \Rightarrow \psi \xrightarrow[\pm\infty]{x} 0$



$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}\left(E - \frac{kx^2}{2}\right)\psi$$

$$\alpha^2 = \frac{mk}{\hbar^2} \qquad \beta = \frac{2mE}{\hbar^2}$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = (\alpha^2x^2 - \beta)\psi$$

1 Step more $\Rightarrow$ new Coordinate: $\xi^2 = \frac{\sqrt{mk}}{\hbar}x^2 \qquad \lambda = \frac{2E}{\hbar}\sqrt{m/k} = \frac{2E}{\hbar\omega_0}$

$$\frac{d^2\psi}{d\xi^2} = (\xi^2 - \lambda)\psi$$

Guess: $\psi_0 \sim e^{-\xi^2/2}$

Trial: $\frac{d\psi}{d\xi} = e^{-\xi^2/2}(-\xi)$; $\frac{d^2\psi}{d\xi^2} = \left(\xi^2 e^{\xi^2/2}\right) - e^{-\xi^2/2}$
$= (\xi^2 - 1)e^{-\xi^2/2}$

IFF $\lambda = 1 \Rightarrow \psi \sim e^{-\xi^2/2}$

$\equiv \lambda_0 \qquad E_0 = \frac{1}{2}\hbar\omega_0$

Normalized Ground State Wave Function

$$\psi_0(x) = \left(\frac{mk}{\pi^2\hbar^2}\right)^{1/8} e^{-\sqrt{mk}\cdot x^2/2\hbar}$$

w/ $\Psi_0(x,t) = \psi_0(x) e^{-iE_0 t/\hbar}$

Assume this Solution

$$\psi \sim H(\xi) e^{-\xi^2/2}$$

$$\psi' = \frac{dH}{d\xi} e^{-\xi^2/2} - H\xi e^{-\xi^2/2}$$

$$\psi'' = \frac{d^2H}{d\xi^2} e^{-\xi^2/2} - 2\frac{dH}{d\xi}\xi e^{-\xi^2/2} - He^{-\xi^2/2} + H\xi^2 e^{-\xi^2/2}$$

$$\frac{d^2\psi}{d\xi^2} = (\xi^2 - \lambda)\psi$$

$$\left[\frac{d^2H}{d\xi^2} - 2\xi\frac{dH}{d\xi} - H + H\xi^2\right] = (\xi^2 - \lambda) H e^{-\xi^2/2}$$

$$\frac{d^2H}{d\xi^2} - 2\xi\frac{dH}{d\xi} + (\lambda - 1)H = 0 \longrightarrow \lambda_n = 2n+1$$

$$H_n(\xi) = \begin{cases} \sum_{k\text{-even}=0}^{n} a_k \xi^k & n\text{-even} \\ \sum_{k\text{-odd}=1}^{n} a_k \xi^k & n\text{-odd} \end{cases}$$

N-Even (Because We already have $H_0(\xi) = 1$)

$$\frac{d^2H}{d\xi^2} - 2\xi\frac{dH}{d\xi} + (\lambda - 1)H = 0$$

$$H_n = a_0 + a_2\xi^2 + a_4\xi^4 + \dots + a_n\xi^n$$

$$H_n' = 2a_2\xi + 4a_4\xi^3 + \dots + n a_n \xi^{n-1}$$

$$H_n'' = 2a_2 + 4\cdot 3 a_4 \xi^2 + 6\cdot 5\cdot a_6\xi^4 + \dots n(n-1)a_n\xi^{n-2}$$

$$[2a_2 + 3(4)a_4\xi^2 + \dots + n(n-1)a_n\xi^{n-2}] - 2\xi[2a_2\xi + \dots + na_n\xi^{n-1}] + (\lambda-1)[a_0 + a_2\xi^2 + \dots + a_n\xi^n] = 0$$

Goal: (1) Find λ_n

(2) Find a relation for the a_k's

ξ^0: $2a_2(\lambda - 1)a_0 = 0$

ξ^2: $3(4)a_4 - 2(2)a_2 + (\lambda - 1)a_2 = 0$

ξ^4: $5(6)a_6 - 2(4)a_4 + (\lambda - 1)a_4 = 0$

$\vdots$

$$(k+2)(k+1)a_{k+2} - 2(k)a_k + (\lambda - 1)a_k$$

(To find λ: Look at ξ^n terms)

$$(k+2)(k+1)a_{k+2} - 2ka_k + (2n)a_k = 0$$

$$a_{k+2} = \frac{2ka_k - (2n)a_k}{(k+2)(k+1)} \Rightarrow a_{k+2} = \frac{2(k-n)}{(k+2)(k+1)} a_k$$

$$H_n = (-1)^n e^{\xi^2}\frac{\partial^n}{\partial\xi^n} e^{-\xi^2}$$

$H_0(\xi) = 1$

$H_1(\xi) = 2\xi$

$H_2(\xi) = 4\xi^2 - 2$

$H_3(\xi) = 8\xi^3 - 12\xi$

Example: Orthogonality of Eigenstates:

(a) Infinite Square well

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$\int_{-\infty}^{\infty} \psi_{n'}\psi_n \, dx \Rightarrow \frac{2}{L}\int_{-\infty}^{\infty} \sin\left(\frac{n'\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \left[\frac{\sin[(n'-n)x\pi/L]}{2(n'-n)} - \frac{\sin[(n'+n)x\pi/L]}{2(n'+n)}\right]_{-\infty}^{\infty}$$

$$= \left[\frac{\sin[(n'-n)\pi x/L]}{2(n'-n)} - \frac{\sin[(n'+n)\pi x/L]}{2(n'+n)}\right]^{\infty} + \left[\frac{\sin[(n'+n)\pi x/L]}{2(n'+n)} - \frac{\sin[(n'-n)\pi x/L]}{2(n'-n)}\right]^{\infty}$$

$$= 0 \qquad n' \neq n$$

$$\int_{-\infty}^{\infty} H_0(\xi)H_1(\xi)e^{-\xi^2}\,d\xi$$

$$= \int_{-\infty}^{\infty} 2\xi e^{-\xi^2}d\xi = -\int_{-\infty}^{\infty} \cancel{2\xi e^{-\xi^2} d\xi} \; 0$$

$$= -e^{-\xi^2}\Big|_{-\infty}^{\infty} = 0$$

$$\int_{-\infty}^{\infty} H_1(\xi)e^{-\xi^2/2} H_3(\xi)e^{-\xi^2/2}\,d\xi$$

$$= \int_{-\infty}^{\infty} (2\xi)(4\xi^2-2)e^{-\xi^2}d\xi = \int_{-\infty}^{\infty}\left[8\xi^3 e^{-\xi^2} - 4\xi e^{-\xi^2}\right]d\xi = 0$$

For any eigenfunctions (Distinct)

$$\boxed{\int_{-\infty}^{\infty} \psi_{n'}\psi_n \, dx = 0} \leftarrow \text{orthogonality}$$

$$\boxed{\int_{-\infty}^{\infty} \psi_{n'}\psi_n \, dx = \delta_{nn'}}$$

orthonormal

$$\delta_{nn} \equiv \text{Kronnecker} = \begin{cases} 1 & n'=n \\ 0 & \text{else} \end{cases}$$

The Charged SHO: Radiative dipole transitions

$$|\vec{P}| = e\langle x\rangle = e\int_{-\infty}^{\infty} \psi_n^*(x,t)\, x\, \psi_n(x,t)\,dx$$

$$\hat{H} = \frac{\hat{P}^2}{2m} + \hat{V}_0(x) + \hat{V}_P(t) \; : \; \hat{V}_0(x) = \tfrac{1}{2}kx^2$$

$$\psi(x,t) = c\psi_n e^{-iE_nt/\hbar} + c'\psi_{n'}e^{-iE_{n'}t/\hbar}$$

$$|\vec{P}| = e\langle x\rangle = e\left\{\int_{-\infty}^{\infty} |c\psi_n|^2 + |c'\psi_n|^2 + c^*c'\psi_n^*\psi_n e^{i\Delta E_{nn'}t/\hbar} + c'^*c\,\psi_n'\psi_n e^{-i\Delta E_{nn'}t/\hbar}\,dx\right\}$$

$$\Delta E_{nn'} = h\nu \Rightarrow \mathcal{E} = h\nu$$

Focus on amplitudes from $n \to n'$

$$\int_{-\infty}^{\infty} \psi_{n'}^* x \psi_n \, dx \to \int_{-\infty}^{\infty} H_{n'}(\xi)e^{-\xi^2/2} H_n(\xi)e^{-\xi^2/2}\,d\xi$$

$$H_{n+1} - 2\xi H_n + 2nH_{n-1} = 0$$

$$H_n = \frac{H_{n+1} + 2nH_{n-1}}{2\xi}$$

$$= \frac{1}{2}\left\{\int_{-\infty}^{\infty} H_{n'}(\xi)e^{-\xi^2/2}H_{n+1}(\xi)^{-\xi^2/2}d\xi + 2n\int_{-\infty}^{\infty} H_n(\xi)e^{-\xi^2/2}H_{n-1}(\xi)e^{-\xi^2/2}\,d\xi\right\}$$

Bound vs. Unbound States

1) Main Condition:

a) $E = K + U < 0 \Rightarrow$ Bound States

(Corresponding classically to closed orbits in phase space, $\{\dot{q}, \dot{p}\}$)

b) $E = K + U > 0 \Rightarrow$ unbound States

Hyperbolic orbits, escape, open orbits (Flux is conserved)

2 In QM:

a) Bound States

(i) Normalizable energy eigenstates localized ψ_n

(ii) Discrete eigenvalue Spectrum

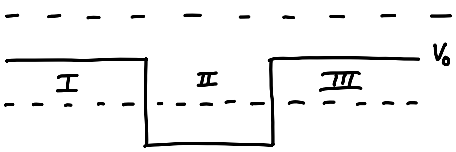
(iii) BC's $\Rightarrow E_n$

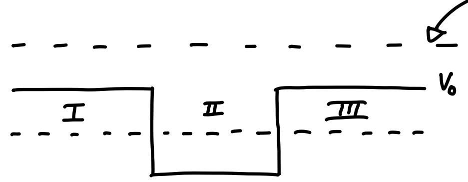
b) Unbound States

(i) Eigenfunctions are, $\sim e^{\pm ikx} \Rightarrow$ Not normalizable

(ii) Eigenvalues are continuous

(iii) Continuity imposed @ Boundaries & initial amplitude known





Allow $E > V_0$

$\alpha^2 = \dfrac{2m(V_0 - E)}{\hbar^2} < 0$

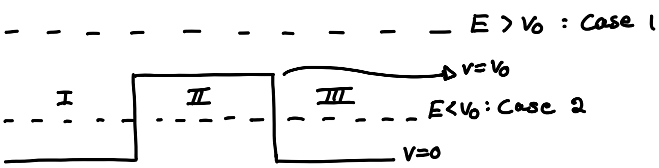
$\alpha = \pm ik \sim e^{\pm ikx}$

Case w/ well ($E < V_0$)

$\psi_{II} \sim Ce^{ikx} + De^{-ikx}$

& $\psi_{III} \sim e^{\pm \alpha x}$ $\qquad \alpha^2 = 2m(V_0 - E)/\hbar^2 > 0$

if $E < V_0$



Case 1:

$$\left.\begin{aligned} &\text{I.} \quad \frac{d^2\psi_I}{dx^2} + \frac{2m}{\hbar} E\psi_I = 0 \\ &\text{III.} \quad \frac{d^2\psi_{III}}{dx^2} + \frac{2m}{\hbar} E\psi_{III} = 0 \\ &\text{II.} \quad \frac{d^2\psi_{II}}{dx^2} + \frac{2m}{\hbar} E\psi_{II} = 0 \end{aligned}\right\}$$

$$\psi_I = \overbrace{Ae^{ik_I x}}^{\text{Incident}} + \overbrace{Be^{-ik_I x}}^{\text{Reflected}}$$

$$\psi_{III} = \underbrace{Fe^{ik_{III} x}}_{\text{Transmitted}} + Ge^{-ik_{III} x}$$

$$\psi_{II} = Ce^{ik_{II} x} + De^{-ik_{II} x}$$

For These

1) Choose the incident function

2) Impose continuity on ψ and $\frac{d\psi}{dx}$ at the boundaries

3) Do a bunch of algebra

Wavenumbers

$$k_I = k_{III} = \frac{\sqrt{2mE}}{\hbar}$$

$$k_{II} = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

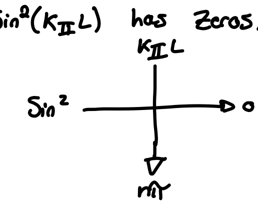
$R + T = 1$

$$R \equiv \frac{\text{Reflected intensity}}{\text{Incident intensity}} = \frac{B^*B}{A^*A} = \frac{|B|^2}{|A|^2}$$

$$T \equiv \frac{\text{Transmitted intensity}}{\text{Incident intensity}} = \frac{F^*F}{A^*A} = \frac{|F|^2}{|A|^2}$$

$$T = \left[1 + \frac{V_0^2 \sin^2(k_{II} L)}{4E(E - V_0)}\right]^{-1}$$

$\sin^2(k_{II} L)$ has Zeros:



$k_{II} L = n\pi \Rightarrow$ we get no reflected wave

$\psi_{II} = Ce^{\kappa x} + De^{-\kappa x}$ $\quad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$ $\Leftarrow$ Real

$T = \left[1 + \frac{V_0^2 \sinh^2(\kappa L)}{4E(V_0 - E)}\right]^{-1}$

κ^{-1} is like a wavelength (imaginary)

$[\kappa] = m^{-1}$

$\rightarrow$ Thick Barrier Limit $\longrightarrow$ $T = 16 \frac{E}{V_0}\left[1 - \frac{E}{V_0}\right] e^{-2\kappa L}$