Physics 311

Homework Set 3

1. Check the *fundamental theorem for gradients* using $T(x, y, z) = 3z^2 + 4xz + yz^3$, the points $\vec{a} = (0, 0, 0)$, $\vec{b} = (1, 1, 1)$, and the three paths in Fig. 1.28:

   a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$;

   b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$;

   c) the parabolic path $z = x^2$; $y = x$.

2. Test the *Divergence theorem* for the function

$$\vec{v} = (y^2 z^2)\hat{x} + (3z^2 x^2)\hat{y} + (4x^2 y^2)\hat{z}. \tag{1}$$

   Take as your volume the cube shown in Fig. 1.30, with sides of length 2.

3. Test *Stokes' theorem* for the function

$$\vec{v} = (y^2 z^2)\hat{x} + (3z^2 x^2)\hat{y} + (4x^2 y^2)\hat{z}. \tag{2}$$

   using the triangular shaded area of Fig. 1.34.

4. Compute the divergence of the function

$$\vec{v} = (r^2 \sin\theta)\hat{r} + (r^2 \cos\theta)\hat{\theta} + (r^2 \sin\phi \cos\theta)\hat{\phi}. \tag{3}$$

   Check the *Divergence theorem* for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (see Fig. 1.40).

5. a) Find the divergence of the function

$$\vec{v} = s^2(3 + \sin^2\phi)\hat{s} + s^2 \sin\phi \cos\phi\ \hat{\phi} + 5z^2\hat{z} \tag{4}$$

   b) Test the divergence theorem for this function, using the quarter-cylinder (radius 2, height 5), shown in Fig. 1.43.

   c) Find the curl of $\vec{v}$.

6. Let

$$\begin{aligned}
\vec{F}_1 &= y^2\hat{x} + z^2\hat{y} + x^2\hat{z} \\
\vec{F}_2 &= x^2\hat{x} + y^2\hat{y} + z^2\hat{z}
\end{aligned} \tag{5}$$

a) Calculate the divergence and the curl of $\vec{F}_1$ and $\vec{F}_2$. Which one can be written as the gradient of a scalar? Find a scalar potential that does the job. Which one can be written as the curl of a vector? Find a suitable vector potential.

b) Show that $\vec{F}_3 = (2yz)\hat{x} + (2zx)\hat{y} + (2xy)\hat{z}$ can be written both as the gradient of a scalar and as the curl of a vector. Find scalar and vector potentials for this function.