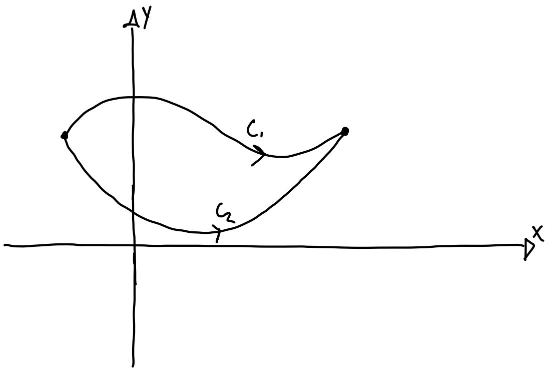
Fundamental Theorem of Line Integrals

$$\int_C \nabla f \cdot d\vec{r} = f(\text{End point}) - f(\text{Start point})$$

f is potential function of $\vec{F}$



Pf: $\int_C \nabla f \cdot d\vec{r}$ $\qquad \vec{r}(t) = \langle x(t), y(t) \rangle$

$$\int_{t=a}^{t=b} \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$$\int_a^b \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \cdot \langle x'(t), y'(t) \rangle$$

$$\int_a^b \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} \quad dt$$

$$\int_a^b \frac{d}{dt} f \, dt = f\Big|_{t=b} - f\Big|_{t=a}$$

When is a vectorfield $\vec{F}$ conservative?

$\vec{F} = \langle P(x,y), Q(x,y) \rangle \qquad P_y = Q_x$

$f_x = P \qquad f_y = Q$

$P_y = f_{xy} = f_{yx} = Q_x$

$P_y = Q_x$

Thm: $\vec{F} = \langle P, Q \rangle$

If $P_y = Q_x$, then $\vec{F}$ is conservative

Ex: $\vec{F}(x,y) = \langle 2x - 6y, -6x + 6y - 9 \rangle$

$P = 2x - 6y \qquad P_y = -6$

$Q = -6x + 6y - 9 \qquad Q_x = -6$

$x^2 - 6xy + g(y)$

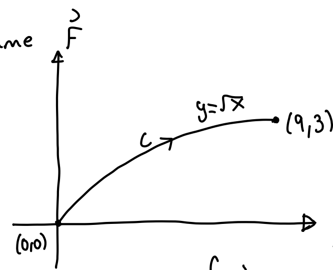
$-6x + g'(y) = -6x + 6y - 9$

$g'(y) = 6y - 9$

$g(y) = 3y^2 - 9y$

$f(x,y) = x^2 - 6xy + 3y^2 - 9y + K$

Same $\vec{F}$



$\vec{F} \langle 2x - 6y, -6x + 6y - 9 \rangle$

$f(x,y) = x^2 - 6xy + 3y^2 - 9y + C$

$\int_C \vec{F} \cdot d\vec{r} = f(9,3) - f(0,0)$

$9^2 - 6 \cdot 9 \cdot 3 + \cancel{3 \cdot 3^2} - \cancel{9 \cdot 3}$

$81 - 162 = -81$

-81

Ex: $\vec{F}(x,y,z)\ \langle yze^{xz}, e^{xz}, xye^{xz} \rangle = \nabla f(x,y,z)$

$f_x = yze^{xz} \qquad \int f_x \, dx = ye^{xz} + h(y,z)$

$e^{xz} + h_y(y,z) = e^{xz}$

$h_y(y,z) = 0$

$H(z)$

$ye^{xz} + g(z)$

$xye^{xz} + g'(z) = xye^{xz}$

$g'(z) = 0$

$g(z) = K$

$ye^{xz} + K$

$f(x,y,z) = ye^{xz} + K$

$\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$