Taylor Research 3 Sept

* Read / do class exercises from Lecture 5, 6

* Ket / bra algebra

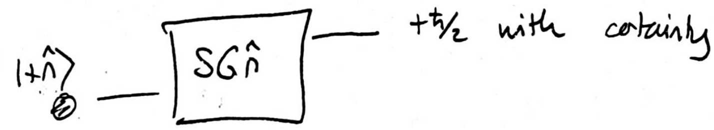
The general expression for a spin-½ system ket.

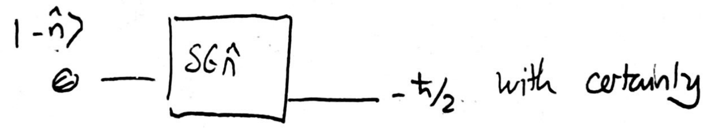
Given ordinary vector direction $\hat{n}$ ~ angles, θ, ϕ

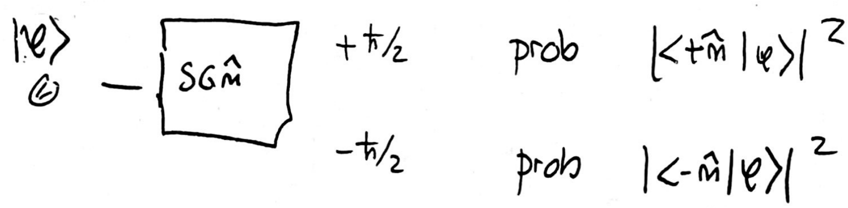
$$|+\hat{n}\rangle = \cos\theta/2 \, |+\hat{z}\rangle + e^{i\phi} \sin\theta/2 \, |-\hat{z}\rangle$$

$$|-\hat{n}\rangle = \sin\theta/2 \, |+\hat{z}\rangle - e^{i\phi} \cos\theta/2 \, |-\hat{z}\rangle$$

Connects to measurements via







Exercise: Consider the following states

i) $|\psi\rangle = \frac{3}{5}|+\hat{z}\rangle + \frac{4}{5}|-\hat{z}\rangle$

ii) $|\psi\rangle = \frac{3}{5}|+\hat{z}\rangle + \frac{4i}{5}|-\hat{z}\rangle$

iii) $|\psi\rangle = \frac{12}{13}|+\hat{z}\rangle - \frac{i5}{13}|-\hat{z}\rangle$

a) Determine the direction along which SG apparatus must be oriented s.t. get $+\hbar/2$ with certainty.

b) Determine prob of outcomes for
- SG $\hat{z}$ meas
- SG $\hat{x}$ "

In general for any state $|\psi\rangle$ one can find θ, ϕ, α s.t.

$$|\psi\rangle = \underbrace{e^{i\alpha}}_{\text{irrelevant phase}} \Big\{ \underbrace{\cos\tfrac{\theta}{2}\,|+\hat{z}\rangle + e^{i\phi}\sin\tfrac{\theta}{2}\,|-\hat{z}\rangle}_{\text{corresponds to } |+\hat{n}\rangle \text{ for some } \hat{n}} \Big\}$$

So there is some direction $\hat{n}$ s.t. $|\psi\rangle \to \boxed{SG\hat{n}}$ — $+\hbar/2$ certainty

any state

$\hat{n}$ depends on

Exercise: Repeated ~~ex~~ measurements

Suppose one has a particle in state

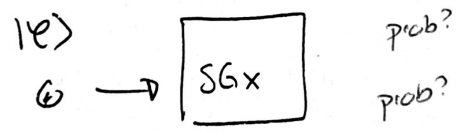
$$|\psi\rangle = \cos\theta/2\, |+\hat{z}\rangle + \sin\theta/2\, |-\hat{z}\rangle$$

and we want to learn θ. by measurements. We can try.

1) measure $SG\hat{z}$

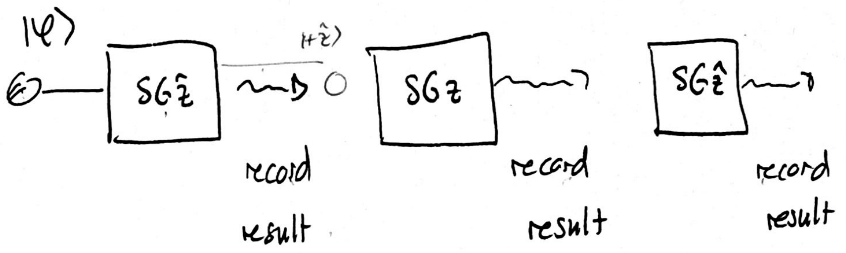
From outcome can we learn θ?

2) measure $SG\hat{x}$



can we learn θ?

3) repeated measurements



Does sequence of results give any more information than that after first measurement?

4) Repeated measurements:

(e) $$\text{SG}\hat{z} \xrightarrow[-\hbar/2]{+\hbar/2} \underbrace{\text{SG}\hat{x} \rightarrow \text{SG}\hat{x}}_{}$$

do these results depend on outcome of 1st measurement?

Bra/ket calculations.

Given $$|\varphi\rangle = \frac{3}{5}|+\hat{z}\rangle + \frac{4i}{5}|-\hat{z}\rangle$$

$$|\psi\rangle = \frac{1}{13}|+\hat{z}\rangle - \frac{12}{13}|-\hat{z}\rangle$$

determine $\langle\psi|\varphi\rangle$ by just doing algebra operations (see Ex 1 class 6)

Qubit states

Can relabel
$$|0\rangle \equiv |+\hat{z}\rangle \sim \begin{pmatrix}1\\0\end{pmatrix}$$
$$|1\rangle \equiv |-\hat{z}\rangle \sim \begin{pmatrix}0\\1\end{pmatrix}$$

and write states in the form

$$|\varphi\rangle = a_0|0\rangle + a_1|1\rangle. \sim \begin{pmatrix}a_0\\a_1\end{pmatrix} \leftarrow \text{qubit state}$$

The associated bra vectors is

$$\langle\varphi| = a_0^* \langle 0| + a_1^* \langle 1| \sim (a_0^* \; a_1^*)$$

Then

$$\langle 0|0\rangle = 1 \qquad \langle 0|1\rangle = 0$$
$$\langle 1|1\rangle = 1 \qquad \langle 1|0\rangle = 0$$

Exercise: Given
$$|\varphi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|\psi\rangle = \frac{4}{5}|0\rangle + \frac{3i}{5}|1\rangle$$

determine $\langle\varphi|\psi\rangle$

Qubit measurements

Describe measurements via

$\rightarrow$ two orthogonal states (basis) $\{|\chi_1\rangle, |\chi_2\rangle\}$

outcome m_1 $\quad m_2$

~~Prob~~

state $\quad$ meas basis

$|\psi\rangle \rightarrow$ $|\chi_1\rangle \rightsquigarrow$ outcome m_1 $\rightarrow$ $\text{prob}(m_1) = |\langle \chi_1 | \psi \rangle|^2$

$|\chi_2\rangle \rightsquigarrow$ outcome m_2 $\quad$ $\text{prob}(m_2) = |\langle \chi_2 | \psi \rangle|^2$

<u>Exercise</u>: For spin-½ particle, describe basis states & outcomes ($\rightarrow$ using $|0\rangle, |1\rangle$) for

* SG $\hat{z}$
* SG $\hat{x}$

measurements

<u>Exercise</u>: Which of the following are measurement bases?

1) $\left\{ \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \right\}$

2) $\left\{ \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right\}$.

3) $\left\{ \left(\frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle\right), \frac{4}{5}|0\rangle - \frac{3i}{5}|1\rangle \right\}$

4 $\left\{ \left(\frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle\right), \left(\frac{4}{5}|0\rangle + \frac{3i}{5}|1\rangle\right) \right\}$