3.1.5 $f(t)=\sin(2\pi\cdot 3\cdot t)$, $g(t)=\cos(2\pi\cdot 3\cdot t)$ $[0,1]$

$u=\sin(2\pi\cdot 3\cdot t)$

$du=6\pi\cos(2\pi\cdot 3t)\,dt$

$\frac{1}{6\pi}du=\cos(2\pi\cdot 3t)\,dt$

$$\int_0^1 \sin(2\pi\cdot 3\cdot t)\cdot\cos(2\pi\cdot 3\cdot t)\,dt$$
$$\frac{1}{6\pi}\int_0^1 u\,du$$
$$\frac{1}{6\pi}\left[\frac{1}{2}u^2\right]_0^1$$
$$\frac{1}{12\pi}\left[\sin(2\pi\cdot 3\cdot t)^2\right]_0^1$$
$$\frac{1}{12\pi}(\sin(6\pi)-\sin(0))$$
$$\frac{1}{12\pi}(0)$$
$$0$$

$\langle f,g\rangle=0$ ✓
$\therefore \langle f,g\rangle$ are orthogonal

3.1.6 $f(t)=\sin(2\pi\cdot 3\cdot t)$, $g(t)=\sin(2\pi\cdot 3\cdot t)$ $[0,1]$

$\langle f,g\rangle=\int_a^b f(t)g(t)\,dt$

$$\int_0^1 \sin(2\pi\cdot 3\cdot t)\sin(2\pi\cdot 3\cdot t)\,dt$$
$$\int_0^1 \sin^2(2\pi\cdot 3\cdot t)\,dt$$
$$\int_0^1 \frac{1-\cos(2\cdot 2\pi\cdot 3\cdot t)}{2}\,dt$$
$$\frac{1}{2}\int_0^1 1-\cos(12\pi\cdot t)\,dt$$
$$\frac{1}{2}\left[t-\frac{1}{12\pi}\sin(12\pi\cdot t)\right]_0^1$$
$$\frac{1}{2}[(1-0)-(0-0)]$$
$$\frac{1}{2}(1)=\frac{1}{2}$$

$\cos(2at)=1-2\sin^2(at)$

$\sin^2(at)=\dfrac{1-\cos(2at)}{2}$

$\cos(at)-1=-2\sin^2(at)$

$\dfrac{1-\cos(2at)}{2}=\sin^2(at)$

$\langle f,g\rangle\neq 0$, $\langle f,g\rangle=\frac{1}{2}$
$\therefore$ $f(t)$ & $g(t)$ are not orthogonal