Taylor Research

29 Oct 2019

Mixed state phase-shift estimation

Consider estimating phase shift parameter λ in

$$\hat{U}(\lambda) = e^{-i\lambda\hat{\sigma}_z/2}$$

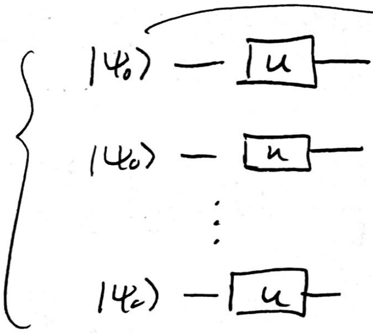
Exercise: Consider pure state inputs

1) single qubit $|\psi_0\rangle = a_0|0\rangle + a_1|1\rangle$

what is optimal a_0, a_1 and what is optimal QFI?

2) multiple qubits with independent channel use. use optimal from

n copies



get output state and QFI. Note that in the resulting calculations use:

$$\left[\langle\phi_1|\langle\phi_2|\ldots\langle\phi_n|\right]\left[|\psi_1\rangle|\psi_2\rangle\ldots|\psi_n\rangle\right] = \langle\phi_1|\psi_1\rangle\langle\phi_2|\psi_2\rangle\ldots\langle\phi_n|\psi_n\rangle$$

3) multiple entangled qubits. Try n qubit entangled state

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left\{|00\ldots0\rangle + |11\ldots1\rangle\right\}.$$

get QFI How does it compare to best unentangled with n independent uses?

Mixed state estimation

Now suppose initial state is mixed. Again want to estimate λ and determine QFI

$$\hat{\rho}_0 \longrightarrow \boxed{U(\lambda)} \longrightarrow \hat{\rho}_{final}(\lambda)$$

The final state will be mixed and we now have to use the general rule for determining QFI.

- get $\hat{L}$ via $\frac{\partial \rho}{\partial \lambda} = \frac{1}{2}\left[\hat{\rho}\hat{L} + \hat{L}\hat{\rho}\right]$
- then $H = \mathrm{Tr}\{\hat{\rho}\hat{L}^2\}$

  $H = \mathrm{Tr}\left[\frac{\partial \rho}{\partial \lambda}\hat{L}\right]$

- can get L via its eigenstates + eigenvalues of $\hat{\rho}$
  $|\phi_i\rangle$ p_i

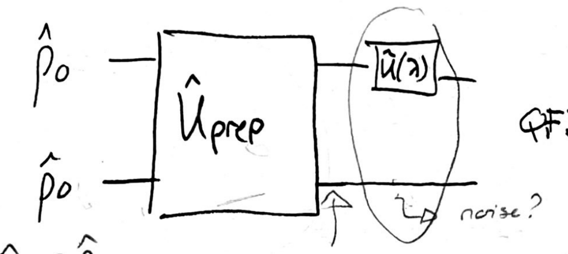
$$\hat{L} = 2\sum \frac{\langle \phi_j | \frac{\partial \rho}{\partial \lambda} | \phi_k\rangle}{p_j + p_k} |\phi_j\rangle\langle\phi_j|$$

- or can calculate $\hat{L}$ via general procedure for 2×2 matrices (see my article [20] PRA 87 032301 (2013))

Exercise: * Suppose $\hat{\rho}_0 = \frac{1}{2}\left(\hat{I} + r\hat{\sigma}_x\right)$ $0 \le r \le 1$

* Determine $\hat{\rho}_{final}(\lambda)$
* Determine QFI for

Now can we entangle / correlate these?



$\hat{\rho}_0 \otimes \hat{\rho}_0$

$\hat{U}(\lambda) \otimes \hat{I}$ $\quad \begin{pmatrix} e^{-i\lambda/2} & \\ & e^{+i\lambda/2} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

For two qubits:

$$\hat{U}_{prep} = \frac{1}{2}\left(\hat{I}\otimes\hat{I} + \hat{I}\otimes\hat{\sigma}_c + \hat{\sigma}_c\otimes\hat{I} - \hat{\sigma}_c\otimes\hat{\sigma}_c\right)$$

$\hat{U}_{prep}^{\dagger} = \hat{U}_{prep}$

for some unit vector $\vec{c}$ and

$$\hat{\sigma}_c = c_x\hat{\sigma}_x + c_y\hat{\sigma}_y + c_z\hat{\sigma}_z$$

<u>Exercise</u>: Use $\hat{\rho}_0 = \frac{1}{2}(\hat{I} + r\hat{\sigma}_x)$ and $\hat{c} = \hat{y}$

1) determine state after $\hat{U}_{prep}$

2) " " " unitary $\hat{U}(\lambda)$ acts on one.

3) " QFI after $\hat{U}(\lambda)$ has acted. Compare to QFI for