$$\int_0^{\sqrt{6}}\int_{x^2}^{2} 6xy^2 \, dy\, dx$$
$$\int_0^{6}\int_0^{\sqrt{y}} 6xy^2 \, dx\, dy$$
} Equal

$\iint_D 9y^2 \, dA \quad D = \{(x,y) \mid -1 \le y \le 1, -y-2 \le x \le y\}$

$$\int_{-1}^{1}\int_{-y-2}^{y} 9y^2 \, dx\, dy$$

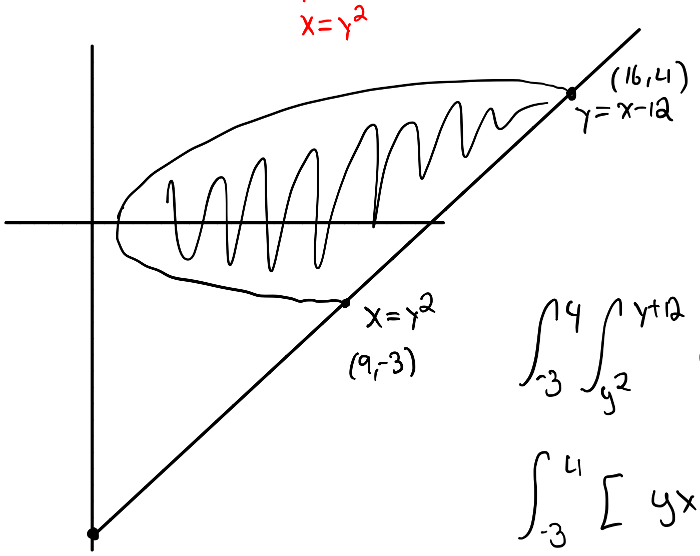
$$\int_{-1}^{1} \left[ 9y^2 x \right]_{-y-2}^{y} dy$$

$$\int_{-1}^{1} \left[ 9y^2(y) - 9y^2(-y-2) \right) dy$$

$$\int_{-1}^{1} \left[ 9y^3 + 9y^3 + 18y^2 \right] dy$$

$$\frac{18}{4}y^4 + 6y^3 \Big|_{-1}^{1}$$

15.3.4) $\iint_D y \, dA$ D is bounded by $y = x-12$, $x = y^2$




$D = \{(x,y) \mid y^2 \le x \le y+12, -3 \le y \le 4\}$

$x = y^2 \quad y = x-12$

$y = y^2 - 12$

$0 = y^2 - y - 12$

$(y-4)(y+3)$

$y = 4, -3$

$$\int_{-3}^{4}\int_{y^2}^{y+12} y \, dx\, dy$$

$$\int_{-3}^{4} \left[ yx \right]_{y^2}^{y+12} dy$$

$$\int_{-3}^{4} \left( y(y+12) - y(y^2) \right) dy$$

$$\int_{-3}^{4} \left( y^2 + 12y - y^3 \right) dy$$

$$\frac{1}{3}y^3 + 6y^2 - \frac{1}{4}y^4 \Big|_{-3}^{4}$$

15.3.5

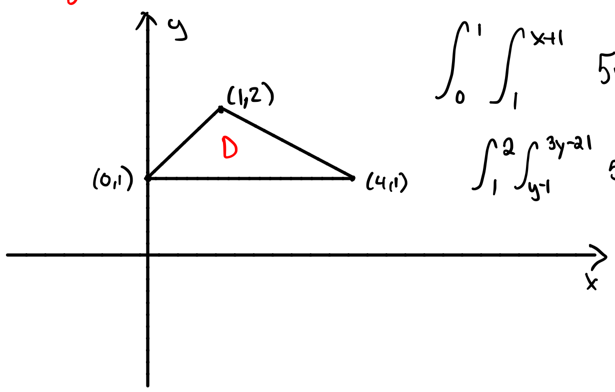
$\iint_D 5y^2 \, dA$ (0,1) (1,2) (4,1)

$y = x+1$

$x = y-1$

$y = \frac{1}{3}x + \frac{7}{3}$

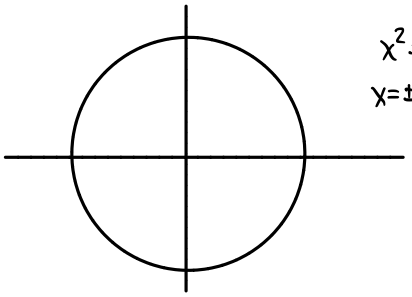
$y - \frac{7}{3} = \frac{1}{3}x$




$$\int_0^1 \int_1^{x+1} 5y^2 \, dy\,dx + \int_1^4 \int_1^{-\frac{1}{3}x+\frac{7}{3}} 5y^2 \, dy\,dx$$

$$\int_1^2 \int_{y-1}^{3y-21} 5y^2 \, dx\,dy$$

$\iint_D (6x - 5y) \, dA$



$x^2 + y^2 = 1$

$x^2 = 1 - y^2$

$x = \pm\sqrt{1-y^2}$

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (6x - 5y) \, dx\,dy$$

$$\int_{-1}^1 \left[ 3x^2 - 5yx \right]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy$$

$$\int_{-1}^1 \left[ (3\sqrt{1-y^2})^2 - 5y(\sqrt{1-y^2}) \right) - \left( 3(-\sqrt{1-y^2})^2 - 5y(-\sqrt{1-y^2}) \right)$$

$$\int_{-1}^1 -10y\sqrt{1-y^2} \, dy$$

$u = 1 - y^2$

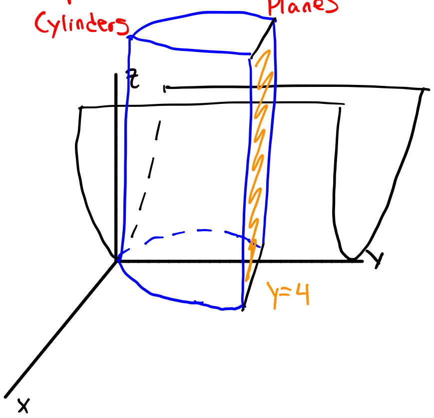
$du = 2y\,dy$

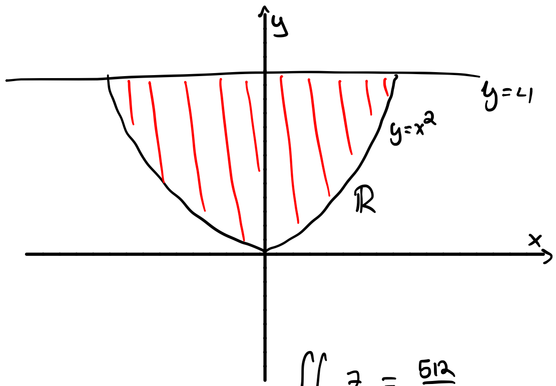
$\frac{1}{2} du = y\,dy$

$$\int_{-1}^1 -5\sqrt{u} \, du$$

$$\int_{-1}^1 -5(u)^{\frac{1}{2}} \, du$$

$\underbrace{z = 4x^2, \; y = x^2}_{\text{Cylinders}}, \; \underbrace{z = 0, \; y = 4}_{\text{Planes}}$




$\iint_R z = \frac{512}{15}$

$$\iint_R z \, dA$$

$$\int_{-2}^2 \int_{x^2}^4 4x^2 \, dy\,dx$$

$$\int_{-2}^2 \left[ 4x^2 y \right]_{x^2}^4 dx$$

$$\int_{-2}^2 \left[ 4x^2(4) - 4x^2(x^2) \right] dx$$

$$\int_{-2}^2 \left[ 16x^2 - 4x^4 \right] dx$$

$$\left[ \frac{16}{3}x^3 - \frac{4x^5}{5} \right]_{-2}^2$$

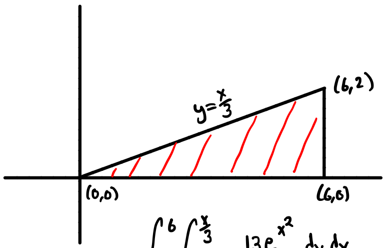
$$\frac{16}{3}(8) - \frac{4(32)}{5} - \left( \frac{16}{3}(-2)^3 - \frac{4(-2)^5}{5} \right)$$

$$\frac{128}{3} - \frac{128}{5} - \left( \frac{-128}{3} + \frac{128}{5} \right)$$

$$\frac{512}{5}$$

$\int_0^2 \int_{3y}^6 13e^{x^2}\,dx\,dy$

$x=6$
$x=3y$




$\int_0^6 \int_0^{\frac{x}{3}} 13e^{x^2}\,dy\,dx$

$\int_0^6 \left[13e^{x^2}(y)\right]_0^{\frac{x}{3}}\,dx$

$\int_0^6 \left[13e^{x^2}\left(\frac{x}{3}\right)\right]dx$

$u=x^2$
$du=2x$
$\frac{1}{2}du=x\,dx$

$\int_0^6 \left[\frac{13}{3}xe^{x^2}\right]dx$

$\frac{13}{3}\cdot\frac{1}{2}\int_0^6 e^u\,du$

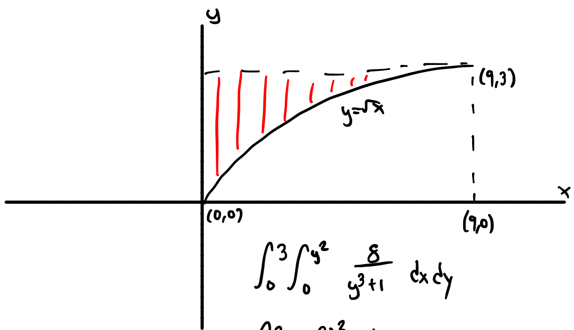
$\frac{13}{6}(e^{36}-e^0)$

$\frac{13}{6}\left[e^{x^2}\right]_0^6$

$\frac{13}{6}(e^{36}-e^0)$

$\int_0^9 \int_{\sqrt{x}}^3 \frac{8}{y^3+1}\,dy\,dx$

$y=\sqrt{x}$ $x=0$
$y=3$ $x=9$




$\int_0^3 \int_0^{y^2} \frac{8}{y^3+1}\,dx\,dy$

$\frac{8}{3}\ln(28)$

$\int_0^3 8\int_0^{y^2} \frac{1}{y^3+1}\,dx\,dy$

$\int_0^3 8\left[\frac{x}{y^3+1}\right]_0^{y^2}\,dy$

$u=y^3+1$
$du=3y^2\,dy$
$\frac{1}{3}du=y^2\,dy$

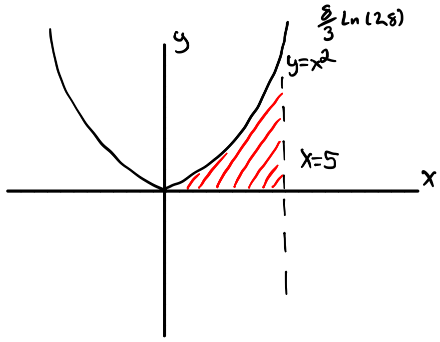
$\int_0^3 \frac{8y^2}{y^3+1}\,dy$

$\frac{8}{3}\int_0^3 \frac{1}{u}\,du$

$\frac{8}{3}\ln(y^3+1)\Big|_0^3$

$\frac{8}{3}\ln(28)$

$f(x,y)=3x\sin(y)$
$y=0$, $y=x^2$, $x=5$




$f_{avg}=\frac{1}{\Delta A}\iint_D f(x,y)\,dA$

$A=\iint_D dA$

$A=\int_0^5\int_0^{x^2} 1\,dy\,dx$

$A=\int_0^5 [y]_0^{x^2}\,dx$

$A=\int_0^5 x^2\,dx$

$A=\left[\frac{1}{3}x^3\right]_0^5$

$A=\frac{125}{3}$

$\frac{1}{A}\iint_D f(x,y)\,dy\,dx$

$\frac{3}{125}\int_0^5\int_0^{x^2} 3x\sin(y)\,dy\,dx$

$\frac{3}{125}\int_0^5 3x\int_0^{x^2}\sin(y)\,dy\,dx$

$\frac{3}{125}\int_0^5 3x\left[-\cos(y)\right]_0^{x^2}\,dx$

$\frac{3}{125}\int_0^5 3x\left(-\cos(x^2)+\cos(0)\right)\,dx$

$\frac{3}{125}\int_0^5 -3x\cos(x^2)+3x\,dx$

$u=x^2$
$du=2x\,dx$
$\frac{1}{2}du=x\,dx$

$\frac{9(-\sin(25)+25)}{250}$

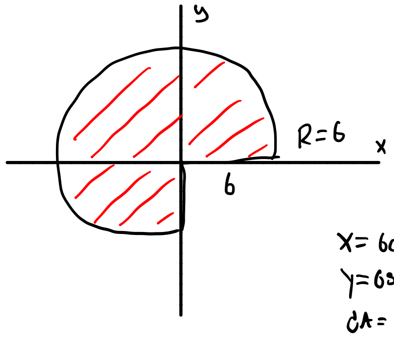
$\frac{3}{125}\int_0^5 3x(-\cos(x^2)+1)\,dx$

$\frac{3}{125}\cdot\frac{3}{2}\int_0^5 -\cos(u)+1\,du$

$-\cos(x^2)+x^2$

$\frac{9}{250}\left[-\sin(u)+u\right]_0^5$

$\frac{9}{250}\left[(-\sin(25)+25)-(-\sin(0)+(0))\right]$




$0 \le \theta \le \frac{3\pi}{2}$

$0 \le R \le 6$

$$\iint_R f(x,y)\, dA$$

$x = 6\cos\theta$

$y = 6\sin\theta$

$dA = r\,dr\,d\theta$

$$\int_{\theta=0}^{\frac{3\pi}{2}}\int_{r=0}^{6} f(6\cos\theta, 6\sin\theta)\, r\,dr\,d\theta$$

$$\int_{-6}^{6}\int_{0}^{\sqrt{36-x^2}} f(x,y)\,dy\,dx + \int_{-6}^{0}\int_{0}^{-\sqrt{36-x^2}} f(x,y)\,dy\,dx$$