

## Laboratory 11: Rotational Motion

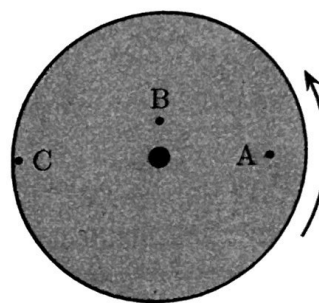
The behavior of physical systems in general can be described in terms of standard quantities such as velocity, acceleration, force, energy and momentum. Although these apply to all classical physical systems, modified versions of the standard quantities are useful for describing orbital or rotational motion.

The goal of this laboratory is to introduce you to quantities suitable for describing the kinematics of rotational motion: angular position, angular velocity and angular acceleration.

### 1 Angular velocity and angular acceleration: qualitative ideas

Conventional mechanics describes motion in terms of position, speed, velocity, etc.,... We shall consider this in the context of a solid wheel which spins at a constant rate about an axle through its center. The aims of this exercise are to investigate whether conventional descriptions of motion are appropriate for describing such motion or whether there are alternative descriptions appropriate for rotational motion. *Much of this exercise is based on a similar exercise in Tutorials in Introductory Physics by McDermott and Shaffer.*

- a) A view of the wheel from above is illustrated and indicates counterclockwise rotation. Three points are marked on the wheel and labeled A, B and C. Rank the times taken by each point to complete one complete rotation in order of increasing time. Indicate whether any of these times are equal to any others.



Shortest time to longest

$$A = B = C$$

Rank the speeds of points A, B and C in order of increasing speed, indicating ties whenever possible. **Explain your answer.**

$$v = \omega^2 r$$

$$B < A < C$$

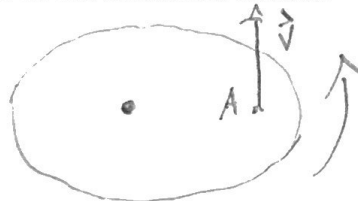
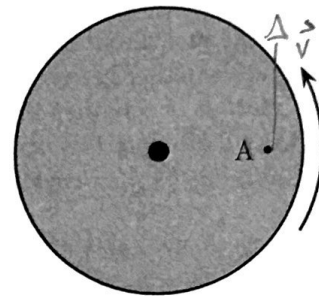
C has the largest radius so it has the greatest velocity. A, B, and C have the same,  $\omega$ . A follows C and then A.

- b) Zog and Geraldine argue about the following claim made by Geraldine: "The wheel rotates at a constant rate and so I can describe its motion by saying that the whole wheel travels with the same constant speed (e.g. 35 m/s). One value of speed is all that I need to describe the wheel's motion!" Is this true or not? Explain your answer.

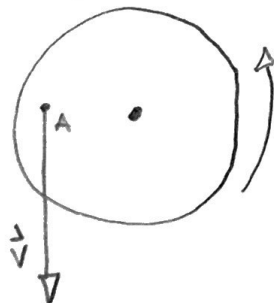
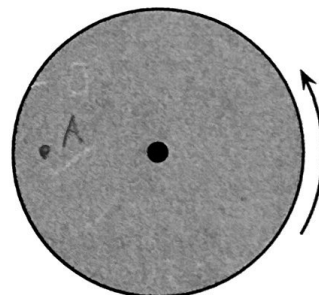
$v \propto r$

Since the radii of the points are different, the smaller the radius the slower the velocity. Faster farther out, slower closer in.

- c) Clearly various points on the wheel move with different speeds. Perhaps the motion of the wheel could be described by one speed if one observed *only one point on the wheel*. For example, one could focus on point A. However, motion must be described in terms of velocity and not speed. Indicate the velocity vector for point A at the illustrated instant.



- d) Suppose that the wheel has completed *one half turn* after the moment indicated above. Indicate the new position of the point labeled A in the previous diagram. Sketch the velocity vector at this point at the new instant. How does the velocity vector at point A compare (in magnitude and direction) to the velocity vector of point A at the earlier instant illustrated in the previous part?



It is equal in magnitude, but opposite in direction.

- c) Is it possible to describe the wheel's motion by a single velocity vector that is constant as time passes? **Explain your answer.**

No because the velocity vector is determined from magnitude and direction, if it has the same magnitude and direction this would be correct. But direction is constantly changing. So no.

For this example we would like a single quantity that describes the rate of rotation. This must not vary from one location to another and must remain constant as time passes. Clearly, even for a simple case such as rotation at a constant rate, conventional linear motion quantities such as speed and velocity do not satisfy this requirement. We seek alternatives for rotational motion.

- f) Suppose that the wheel completes a single revolution in exactly 4 s. Determine the total change in angle for *each of the marked points* during this period.

$2\pi$  radians ✓

Determine the rate of change of this angle for *each of the marked points*.

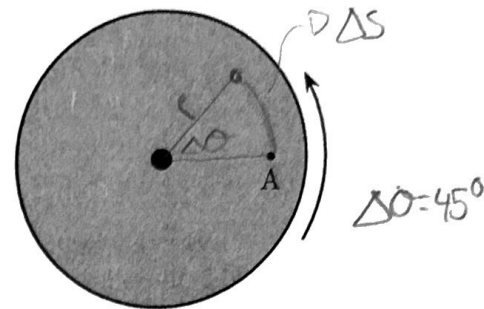
$\frac{\pi}{2}$  rad/sec ✓

- g) Is it possible to associate a single angular rate of change for all points on this wheel? Does this change as time passes in the situation described above? **Explain your answer.**

Yes because all points on the circle complete one revolution at the same rate. This does not change as time passes.

The angular rate of change is called the angular velocity and is denoted  $\omega$  and is measured in radians per second. We aim to relate this to linear speed.

- h) During a short interval of duration  $\Delta t$  the wheel rotates through angle  $\Delta\theta$ . Indicate the path taken by point A during this interval (your diagram will be reasonably clear if you draw  $\Delta\theta$  anywhere between about  $30^\circ$  and  $45^\circ$ ). Denote the arc length traveled by A by  $\Delta s$  and the distance from point A to the axle by  $r$ . Label  $\Delta\theta$ ,  $r$  and  $\Delta s$  on the diagram. Provide a general expression relating  $\Delta s$  to  $\Delta\theta$  and  $r$ , i.e.



$\Delta s =$  equation involving  $\Delta\theta$  and  $r$ .

$$\Delta s = r \Delta\theta$$

$$r \Delta\theta = \Delta s$$

- i) Starting with the definition of speed,  $v = \Delta s / \Delta t$ , determine an expression for the speed of point A in terms of  $\Delta\theta$ ,  $r$  and  $\Delta t$ . Use this to derive a relationship between  $v$  and  $\omega$ .

$$v = \frac{\Delta s}{\Delta t}$$

$$\Delta s = r \Delta\theta \quad \omega = \frac{\Delta\theta}{\Delta t}$$

$$v = \frac{r \Delta\theta}{\Delta t}$$

$$v = \omega r$$

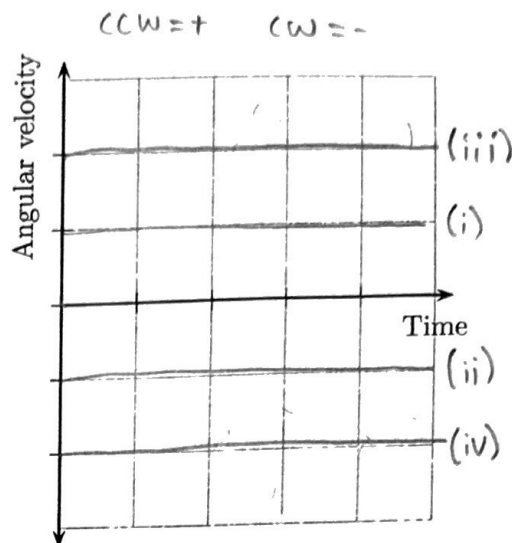
## 2 Angular velocity and angular acceleration: experiment.

The laboratory is equipped with a PASCO rotary motion sensor with an attached rotating arm. The rotary motion sensor can measure angular position and velocity. There is also a PASCO smart pulley which can be used to measure linear (ordinary) velocity.

- Connect the PASCO rotary motion sensor and configure it to record angular velocity, measured in rad/s.
- Configure DataStudio to display graphs of angular velocity vs. time.

- c) Sketch and label, using the illustrated axes, **predicted** graphs of angular velocity vs. time for the following motions of the arm, each at a constant rate:

- i) slower counterclockwise rotation,
- ii) slower clockwise rotation,
- iii) faster counterclockwise rotation, and
- iv) faster clockwise rotation.



- d) For each of the four situations above, carry out the motion by rotating the arm. Run the motion sensor, display the four **observed** graphs of angular velocity vs. time on the same set of axes, print this and attach it to the worksheet.
- e) Do your predictions and observations agree? Resolve any major discrepancies that you may have noticed.

Yes our predictions agree, discrepancies could include friction and drag.

- f) What is the main distinction between a graph for counterclockwise rotation and one for clockwise rotation?

Counterclockwise = (+) ✓

clockwise = (-)

### 3 Relationship between angular and linear speeds.

For an object that rotates in a circle at a distance  $r$  from the axis of rotation, the speed is given by

$$v = \omega r \quad (1)$$

where  $\omega$  is the angular speed. In this experiment you will check this relationship.

- Attach a string to the largest pulley on the rotary motion sensor. Run the string over the directional pulley and over the smart pulley. Suspend a 50 g mass from the string.
- Configure DataStudio to read smart pulley and display the graph of velocity vs. time in the same window as the graph of angular velocity vs. time. The smart pulley measures the speed with which the string moves.
- Which point on the rotary motion sensor and arm arrangement moves with the same speed as the string (e.g. Does the end of the arm move with the same speed as the string? Does the midpoint along the arm move with the same speed as the string? Some point on the pulley?). Measure  $r$  for this point.

$$r = 25.85 \text{ mm}$$

The point at which the string is tangent to the wheel. ~~It~~ it does not move along at the same speed at the center or on a different point of the wheel.

$$r = 0.0259 \text{ m}$$

$$r = 0.0259 \text{ m}$$

- Wind the string and release the suspended mass, after clicking Start.
- Observe the graphs of velocity (speed) vs. time and angular velocity vs. time. Choose one instant in time. Determine the magnitude of the angular velocity at this instant and use this with Eq. (1) to determine the speed of the point on the pulley and arm arrangement identified in part c). Determine the speed as measured by the smart pulley at the same instant. Record your results for these speeds. Are they similar or not? Repeat for two other instants.

$$\text{at } t = 3 \text{ s, } \omega = 7.07 \text{ rad/s, } v = 0.17 \text{ m/s } v = \omega r \quad r = 0.0259 \text{ m}$$

$$7.07 \text{ rad/s} (0.0259 \text{ m}) = 0.18 \text{ m/s } \checkmark$$

$$t = 4 \text{ s, } \omega = 9.16 \text{ rad/s, } v = 0.22 \text{ m/s } \checkmark$$

$$+ 9.16 \text{ rad/s} (0.0259 \text{ m}) = 0.23 \text{ m/s } \checkmark$$

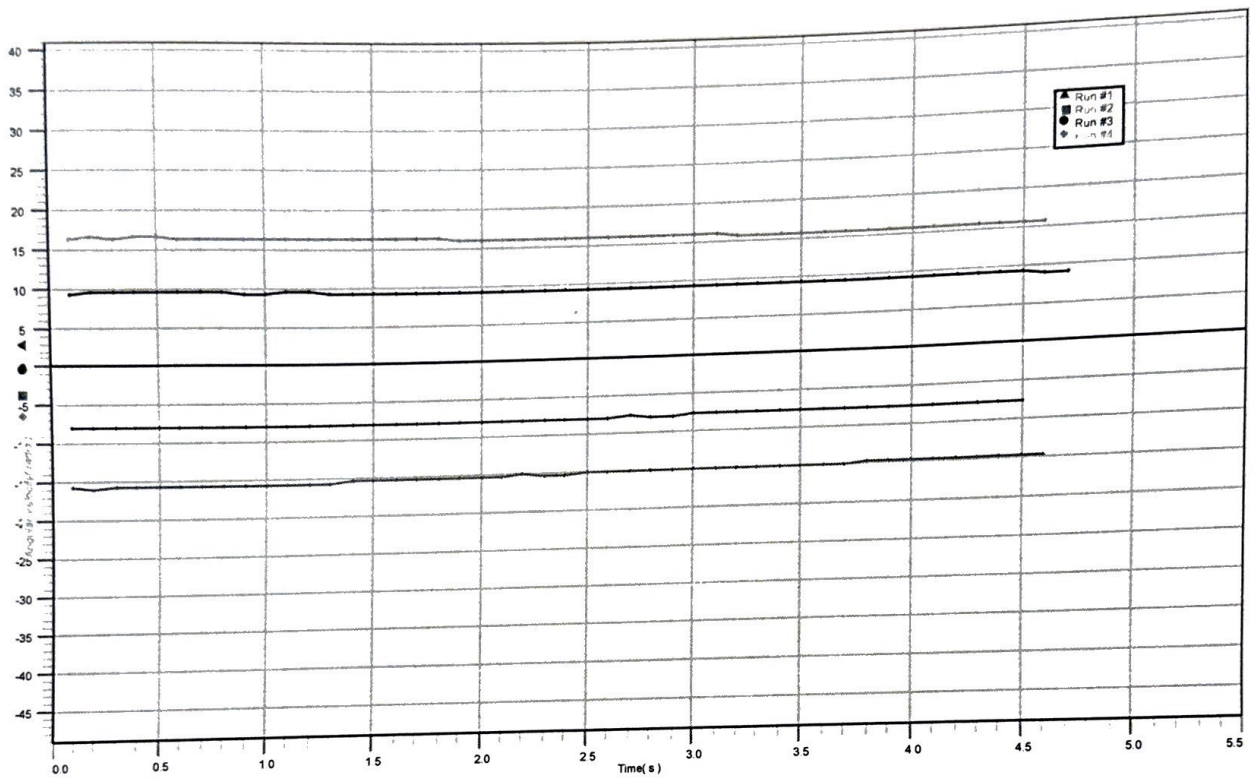
$$t = 6 \text{ s, } \omega = 9.85 \text{ rad/s, } v = 0.22 \text{ m/s } -$$

$$9.86 \text{ rad/s} (0.0259 \text{ m}) = 0.25 \text{ m/s } \checkmark$$

What can you conclude regarding the validity of Eq. (1)?

Equation 1 is correct

Graph 1



Graph 1

