Tues: Discussion/quiz

Ch 9 Conc Q 3, 4, 5

Ch 9 Prob 2, 5, 9, 10

Weds: Lecture

Energy in Physics

Newton's system of laws provides a framework for understanding motion of any classical system. However, we will see that the method it prescribes can result in apparently insurmountable mathematical difficulties. Consider a skater on a frictionless surface

Demo Energy Skate Park Basics

→ Intro → W skate track

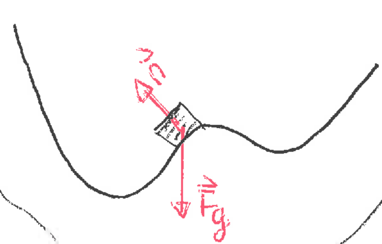
→ Release from high point

→ Questions: What is max speed of skater?

How long to return to high point?

Newton's mechanics says:

Specify forces



→ Find acceleration via $\vec{a} = \vec{F}_{net}/m$

Specify initial velocity

→ Use $\vec{a} = \frac{d\vec{v}}{dt}$ to get velocity $\vec{v}$ at all times

Specify initial position

→ Use $\vec{v} = \frac{d\vec{x}}{dt}$ to get positions

The difficulty here is that the normal force changes with time, so acceleration is time dependent,... Eventually the mathematics becomes intractible.

However, we can still answer certain questions about this system by using a concept associated with Newton's laws called energy. We shall show that, with appropriate definitions, Newton's laws implies certain general rules that energy must satisfy.

Demo: PhET skater → show bar graphs.

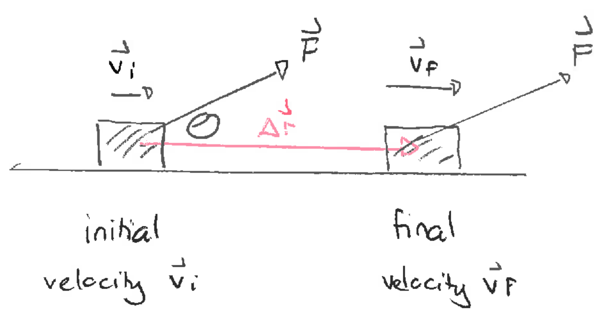
The concept of energy occurs throughout physics + other physical sciences and is used in:

- quantum theory
- thermodynamics
- relativity
- chemistry
- biology
- climate science, ...

Work + Energy: Single Constant Force

Consider an object moving along a horizontal frictionless surface. During this period a single constant force acts on the object. The object is displaced horizontally by distance Δr

The angle between the force + this displacement is θ. We aim to determine how the speed of the object changes.

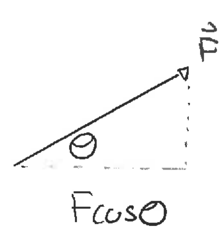


The force produces an acceleration along the horizontal direction. Using

$$\sum F_x = ma_x$$

gives

$$F\cos\theta = ma_x$$



and thus the acceleration is constant.

Now constant acceleration kinematics implies:

$$v_f^2 = v_i^2 + 2a_x \Delta x$$

$$\Rightarrow \quad mv_f^2 = mv_i^2 + 2\underbrace{(ma_x)}_{F\cos\theta}\overbrace{\Delta x}^{\Delta r}$$

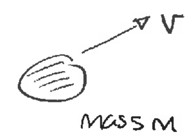
$$\Rightarrow \quad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + (F\cos\theta)\,\Delta r$$

$$\Rightarrow \quad \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = F\,\Delta r\cos\theta$$

This motivates the following definitions:

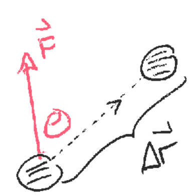
The kinetic energy of an object with mass m moving with speed v is

$$K = \frac{1}{2}mv^2$$



Units: Joules $\quad 1\,J = 1\,kg\,m^2/s^2 = 1\,Nm$

If a constant force acts on an object that moves in a straight line between an initial and a final location, then the work done by the force between these instants is



$$W = F\,\Delta r\cos\theta$$

where F = magnitude of force

Δr = distance traveled

θ = angle between displacement vector and force

This also has units of Joules

We have seen that for a single force then

$$K_f - K_i = W \quad \Rightarrow \quad \Delta K = W$$

where $K_F = \frac{1}{2} m v_f^2$ is the kinetic energy of the object at the final instant.

This is an example of a more general result. At the moment we can extend this to:

> If an object moves in a straight line while various constant forces act on it then the change in its kinetic energy is
>
> $$\Delta K = W_{net}$$
>
> where the net work done is
>
> $$W_{net} = W_{force\,1} + W_{force\,2} + \cdots$$

This is a limited version of the work-kinetic energy theorem.

Quiz 1

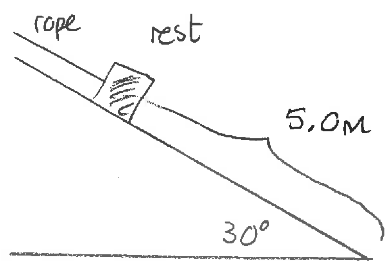
Note that $W_{net} = 0$ in this case since $\Delta K = 0$. Thus $W_{grav} = -W_{rope}$

Warm Up 1

Warm Up 2

We can observe that the sign of work done by a force depends only on the relative orientation of the force + displacement vectors.

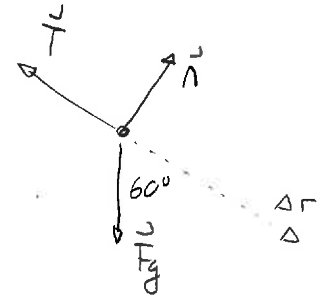
Example:



A 20kg box is lowered down a frictionless incline. The rope that lowers it is parallel to the incline and exerts a constant 70N force

Determine the speed of the block when it reaches the bottom of the incline

Answer:



$$\Delta K = W_{net}$$
$$= W_n + W_{grav} + W_{rope}$$

$$K_f - K_i = W_n + W_{grav} + W_{rope}.$$

Then

$$V_i = 0 \Rightarrow K_i = 0$$

$$\Rightarrow \frac{1}{2} M V_f^2 = W_n + W_{grav} + W_{rope}.$$

We need individual works.

normal: $W_n = n \Delta r \cos\theta$ ← 90° $\Rightarrow$ $W_n = 0J$

gravity $W_g = F_g \Delta r \cos\theta = mg \Delta r \cos 60°$

$= 20kg \times 9.8 m/s^2 \times 5.0m \times \cos 60°$

$\Rightarrow W_g = 490J$

rope $W_{rope} = F_{rope} \Delta r \cos\theta$ ← 180° $\Rightarrow W_{rope} = 70N \times 5.0m \cos 180°$

$W_{rope} = -350J$

$$\Rightarrow \frac{1}{2} M V_f^2 = 0J + 490J - 350J$$

$$\Rightarrow \frac{1}{2} 20kg\, V_f^2 = 140J \Rightarrow V_f^2 = 14\, m^2/s^2 \Rightarrow V_f = \sqrt{14 m^2/s^2} = 3.7 m/s$$