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MATH 360
CP Ch. 10.5

Write Up Equation (2)

A parametric representation of a surface S in space is of the form

(2) $$r(u,v) = [x(u,v), y(u,v), z(u,v)] = x(u,v)\hat{\imath} + y(u,v)\hat{\jmath} + z(u,v)\hat{k}$$

Parametric Representations

Cylinder: A parametric representation is,

$$\vec{r}(u,v) = [a\cos(u), a\sin(u), v] = a\cos(u)\hat{\imath} + a\sin(u)\hat{\jmath} + (v)\hat{k}$$
$$0 \le u \le 2\pi, \quad -1 \le v \le 1, \quad 0 \le a \le \infty$$

Sphere: A sphere can be represented in the form,

$$\vec{r}(u,v) = a\cos(v)\cos(u)\hat{\imath} + a\cos(v)\sin(u)\hat{\jmath} + a\sin(v)\hat{k}$$
$$0 \le u \le 2\pi, \quad -\tfrac{\pi}{2} \le v \le \tfrac{\pi}{2}, \quad 0 \le a \le \infty$$

Cone: A circular cone can be represented by,

$$\vec{r}(u,v) = [u\cos(v), u\sin(v), u] = u\cos(v)\hat{\imath} + u\sin(v)\hat{\jmath} + (u)\hat{k}$$
$$0 \le u \le H, \quad 0 \le v \le 2\pi$$

Equations (4) & (5)

The normal vector $\vec{N}$ of S at P

(4) $$\vec{N} = r_u \times r_v \neq 0$$

The corresponding unit normal vector $\vec{n}$ of S at P is

$$\vec{n} = \frac{1}{|\vec{N}|}N = \frac{1}{|r_u \times r_v|} r_u \times r_v$$

Example 4 & 5

Ex 4 unit normal vector of a Sphere

From (5*) we find that the sphere $g(x,y,z) = x^2+y^2+z^2-a^2=0$ has the unit normal vector

$$\vec{n}(x,y,z) = \left[\frac{x}{a}, \frac{y}{a}, \frac{z}{a}\right] = \frac{x}{a}\hat{\imath} + \frac{y}{a}\hat{\jmath} + \frac{z}{a}\hat{k}$$

We see that n has the direction of the position vector $[x,y,z]$ of the corresponding point. Is it obvious that this must be the case?

Ex 5 unit normal vector of a cone

At the apex of the cone $g(x,y,z) = -z + \sqrt{x^2+y^2} = 0$ in Example 3, the unit normal vector n becomes undetermined because from (5*) we get

$$\vec{n} = \left[\frac{x}{\sqrt{2(x^2+y^2)}}, \frac{y}{\sqrt{2(x^2+y^2)}}, \frac{-1}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}}\left(\frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j} - \hat{k}\right)$$