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PHYS 231 - HW 3

Ch. 4 P. # 11, 12, 13, 14, 18, 23, 28, 34, 55

Problem 11

$r_{Al} = 3.6\ fm \quad r_{Au} = 7.0\ fm$
$r_p = 1.3\ fm \quad r_\alpha = 2.6\ fm$

a.) Potential Energy is Transformed into Kinetic Energy $\quad P.E \Rightarrow K.E$

$$U = \frac{kQ_1Q_2}{r} \Rightarrow \begin{array}{l} Q_1 \equiv \text{Particle 1} : Q_1 \Rightarrow \alpha \\ Q_2 \equiv \text{Particle 2} : Q_2 \Rightarrow Al \end{array}$$

$$U = k\frac{Z_1Z_2e^2}{r} \Rightarrow 8.99\times10^9 \frac{Nm^2}{c^2} \frac{2(13)(1.6\times10^{-19}c)^2}{((2.6+3.6)\times10^{-15}m)} = 9.651\times10^{-13}\ J$$

$Z_1 = \alpha = 2$
$Z_2 = A_l = 13$

$U = 9.65\times10^{-13}\ J$
$1\ eV = 1.6\times10^{-19}\ J$

$$U = \frac{9.65\times10^{-13}\ J}{1.6\times10^{-19}\ J} = 6.03\times10^6\ eV$$

$$U = \frac{kQ_1Q_2}{r} = k\frac{Z_1Z_2e^2}{r}$$

$Z_1 = 2 \Rightarrow \alpha$
$Z_2 = 47 \Rightarrow A_u$

$$U = 8.99\times10^9 \frac{Nm^2}{c^2} \frac{(2)(79)(1.6\times10^{-19}c)^2}{((7.0+2.6)\times10^{-15}m)} = 3.79\times10^{-12}\ J$$

$$U = \frac{3.79\times10^{-12}\ J}{1.6\times10^{-9}\ J} = 2.37\times10^7\ eV \Rightarrow 23.7\ MeV$$

$$\boxed{\begin{array}{l} U_{Al} = 6.03\ MeV \\ U_{Au} = 23.7\ MeV \end{array}}$$

b.) $U = \frac{kZ_1Z_2e^2}{r}$

$Z_1 = \text{Proton} = 1$
$Z_2 = Al = 13$

$$U = 8.99\times10^9 \frac{Nm^2}{c^2} \frac{1(13)(1.6\times10^{-19}c)^2}{((1.3+3.6)\times10^{-15}m)} = 6.11\times10^{-13}\ J$$

$$U = \frac{6.11\times10^{-13}\ J}{1.6\times10^{-19}\ J} = 3.82\times10^6\ eV = 3.82\ MeV$$

$$\boxed{\begin{array}{l} U_p = 3.82\ MeV \\ U_{Au} = 13.7\ MeV \end{array}}$$

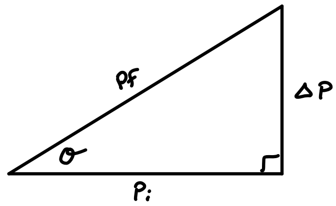
$Z_1 = \text{Proton} = 1$
$Z_2 = Au = 79$

$$U = 8.99\times10^9 \frac{Nm^2}{c^2} \frac{1(79)(1.6\times10^{-19}c)^2}{((7.0+1.3)\times10^{-15}m)} = 2.19\times10^{-12}\ J$$

$$U = \frac{2.19\times10^{-12}\ J}{1.6\times10^{-19}\ J} = 1.37\times10^7\ eV = 13.7\ MeV$$

Problem 12

a.) $\theta = \dfrac{2Z_2e^2}{4\pi\varepsilon_0 KR}$ $\quad \Delta t = \dfrac{2R}{v}$



$\tan\theta = \dfrac{\Delta P}{P}$ $\quad P = mv$

Small $\theta \sim \tan\theta = \theta$

Max Coulomb Force : $F = \dfrac{2Z_2e^2}{4\pi\varepsilon_0 R^2}$

$F = \dfrac{\Delta P}{\Delta t}$ $\quad \Delta t = \dfrac{2R}{v}$

$$\Delta P = F\cdot\Delta t = \frac{2Z_2e^2}{4\pi\varepsilon_0 R^2}\left(\frac{2R}{v}\right) = \frac{4Z_2e^2}{4\pi\varepsilon_0 Rv}$$

$$\theta = \frac{\Delta P}{P} = \frac{4Z_2e^2}{4\pi\varepsilon_0 Rv}\cdot\frac{1}{mv} = \frac{4Z_2e^2}{4\pi\varepsilon_0 Rmv^2} = \frac{4Z_2e^2}{2\cdot 4\pi\varepsilon_0 RK} = \frac{2Z_2e^2}{4\pi\varepsilon_0 RK} \checkmark$$

if $mv^2 = 2K$

$$\boxed{\theta = \frac{2Z_2e^2}{4\pi\varepsilon_0 RK}}$$

b.) $K = 8.0\ MeV$
$r = 0.136\ nm$
$Z = 79$
$\varepsilon_0 = 8.85\times10^{-12}\ F/m$

$$\theta = \frac{2(79)(1.6\times10^{-19}c)^2}{4\pi(8.85\times10^{-12}F/m)(0.135\times10^{-9}m)(8.0\times10^6 eV)} = \frac{2(79)(1.44\ eV\ nm)}{(0.135\times10^{-9}m)(8.0\times10^6 eV)} = 2.11\times10^{-4}\ \text{rads}$$

$$\frac{(1.6\times10^{-19}c)^2}{4\pi\cdot 8.85\times10^{-12}F/m} = 2.30\times10^{-28}\ J = 1.44\times10^{-9}\ eV\ m$$
$$= 1.44\ eV\ nm$$

$$\theta = 2.11\times10^{-4}(\text{rads}) = 2.11\times10^{-4}(\text{rads})\left(\frac{180}{\pi}\right) = 0.012^\circ$$

rads : $\left(\frac{\pi}{180}\right)$

deg : $\left(\frac{180}{\pi}\right)$

$$\boxed{\begin{array}{l}\theta = 0.012^\circ \\ \theta = 2.11\times10^{-4}(\text{rads})\end{array}}$$

Problem 13

a.) $\theta = \dfrac{2Z_2e^2}{4\pi\varepsilon_0 KR}$

$$\theta = \frac{2(79)(1.6\times10^{-19}c)^2}{4\pi(8.85\times10^{-12}F/m)(10\times10^6 eV)(10^{-10}m)} = \frac{2(79)(1.44\times10^{-9})eV\ m}{(10\times10^6 eV)(10^{-10}m)} = 2.28\times10^{-4}\ \text{rads}$$

$\varepsilon_0 = 8.85\times10^{-12}\ F/m$
$Z_2 = 79$
$K = 10\times10^6\ eV$
$e = 1.6\times10^{-19}\ C$
$R = 10\times10^{-10}\ m$

$$\frac{(1.6\times10^{-19}c)^2}{4\pi(8.85\times10^{-12}F/m)} = 2.30\times10^{-28}J = 1.44\times10^{-9}\ eV\ m$$

$$\boxed{\begin{array}{l}\theta = 0.013^\circ \\ \theta = 2.28\times10^{-4}(\text{rads})\end{array}}$$

b.) The scattering angle is slightly greater for a proton compared to an electron. This scattering angle has an increase of (0.001°) from the electron scattering angle.

> The results are slightly larger for a proton

Problem 14 $r_H = 1.2$ Fm

a.)

$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 m r}}$$

$\varepsilon_0 = 8.85\times10^{-12}$ F/m $\quad e = 1.6\times10^{-19}$ C

$m_e = 9.1\times10^{-31}$ kg $\quad m_e = 0.51100$ MeV/c^2

$r_H = 1.2\times10^{-15}$ m

$$v = \frac{1.6\times10^{-19}\,C}{\sqrt{4\pi(8.85\times10^{-12}\,F/m)(9.1\times10^{-31}\,kg)(1.2\times10^{-15}\,m)}} = 4.59\times10^{8}\,m/s$$

$\boxed{v = 4.59\times10^{8}\,m/s}$

b.)

$$E = \frac{-e^2}{8\pi\varepsilon_0 r}$$

$e = 1.6\times10^{-19}$ C

$\varepsilon_0 = 8.85\times10^{-12}$ F/m

$r_H = 1.2\times10^{-15}$ m

$$E = \frac{-(1.6\times10^{-19}c)^2}{8\pi(8.85\times10^{-12}F/m)(1.2\times10^{-15}m)} = \frac{-9.59\times10^{-14}\,J}{1.6\times10^{-14}\,J} \Rightarrow -0.59\text{ MeV}$$

$\boxed{E = -0.6\text{ MeV}}$

c.)

These are not reasonable. The planetary model has a speed that is faster than the speed of light. The total mechanical Energy of the System is negative as well. These do not make sense.

Problem 18

$$P = \frac{2Q^2}{4\pi\varepsilon_0 3c^3}\left(\frac{d^2\vec{r}}{dt^2}\right)^2 = \frac{2Q^2a^2}{4\pi\varepsilon_0 3c^3} = \frac{2e^2a^2}{4\pi\varepsilon_0 3c^3} = \frac{e^2a^2}{6\pi\varepsilon_0 c^3} = \frac{e^2a^2}{6\varepsilon_0 c^3}$$

P ≡ Power

T ≡ Period

$$\Delta E_0 = -\frac{e^2}{8\pi\varepsilon_0 r}$$

$a = v^2/r$; $v = \omega r$: $a = \omega^2 r$

$$\Delta T = -\int_{r_H}^{0} \frac{dE_0}{dr}\,\frac{1}{P}\,dr$$

$$\Delta T = -\int_{r}^{0} \frac{dE_0}{dr}\,\frac{1}{P}\,dr \qquad \frac{dE_0}{dr} = \frac{e^2}{8\pi\varepsilon_0 r^2}$$

$$P = \frac{e^2a^2}{6\varepsilon_0 c^3} \qquad F_e = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad P = \frac{e^2(\omega^2 r)^2}{6\varepsilon_0 c^3}$$

$$\frac{8\pi\varepsilon_0 r(0) + e^2(8\pi\varepsilon_0)}{(8\pi)^2(\varepsilon_0)^2 r^2} = \frac{e^2}{8\pi\varepsilon_0 r^2}$$

$$ma = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$\omega^2 r = \frac{e^2}{4\pi\varepsilon_0 m r^2}$$

$$\omega^2 = \frac{e^2}{4\pi\varepsilon_0 m r^3}$$

$$\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m r^3}}$$

$$P = \frac{e^2\left(\frac{e^2 r}{4\pi\varepsilon_0 m r^3}\right)^2}{6\varepsilon_0 c^3}$$

$$P = \frac{e^2\left(\frac{e^4}{16\pi^2\varepsilon_0^2 m^2 r^4}\right)}{6\varepsilon_0 c^3}$$

$$P = \frac{e^6}{96\pi^2\varepsilon_0^3 c^3 m^2 r^4}$$

$$\frac{1}{P} = \frac{96\pi^2\varepsilon_0^3 c^3 m^2 r^4}{e^6}$$

$x = 10^{-10}$ m

$$\Delta T = -\int_{x}^{0} \frac{e^2}{8\pi\varepsilon_0 r^2}\,\frac{96\pi^2\varepsilon_0^3 c^3 m^2 r^4}{e^6}\,dr$$

$$\Delta T = \frac{12\pi\,\varepsilon_0^2 c^3 m^2}{e^4}\int_0^x r^2\,dr$$

$$\frac{12\pi\,\varepsilon_0^2 c^3 m^2}{e^4}\left(\frac{1}{3}r^3\Big|_0^x\right) = \frac{12\pi(8.85\times10^{-12}F/m)^2(3.0\times10^{8}m/s)^3(9.1\times10^{-31}kg)^2}{(1.6\times10^{-19}c)^4}\left(\frac{1}{3}(10^{-10}m)^3\right) = 3.35\times10^{-11}\,s$$

$\boxed{3.4\times10^{-11}\,s}$

Problem 23

$\lambda = 410 \times 10^{-9} m$

$E = \frac{hc}{\lambda}$ $\quad E = \frac{1239.8 \text{ eV nm}}{410 \text{ nm}} = 3.02 \text{ eV}$

$hc = 1239.8 \text{ eV}$

$\lambda = 410 \times 10^{-9} m$ $\quad \Delta E = 3.02 \text{ eV}$

$n=2 ; \; E = 3.40 \text{ eV}$ $\quad n=2 - n=6 : 3.40 \text{ eV} - 0.38 \text{ eV} = 3.02 \text{ eV}$ ✓

$n=6 ; \; E = 0.38 \text{ eV}$

The initial state was @ n=2
and the final state was @ n=6

Problem 28

a.) For hydrogen atoms, the energy state is typically at n=1, where the spectral lines are usually in the Lyman series. However the Balmer series is typically where the visible spectrum is for hydrogen.

$\frac{1}{\lambda} = R\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)$ $\quad n=2: 3,4,5,6$ $\quad R = 1.097 \times 10^7 m^{-1}$

n=3 $\quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) = 1.523 \times 10^6 m^{-1} : \lambda = 656.3 \text{ nm}$

n=4 $\quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = 2.056 \times 10^6 m^{-1} : \lambda = 486.2 \text{ nm}$

n=5 $\quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{25}\right) = 2.303 \times 10^6 m^{-1} : \lambda = 434.1 \text{ nm}$

n=6 $\quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{36}\right) = 2.438 \times 10^6 m^{-1} : \lambda = 410.2 \text{ nm}$

$\lambda = 656.3$ nm
$\lambda = 486.2$ nm
$\lambda = 434.1$ nm
$\lambda = 410.2$ nm

b.) The spectral lines of an ionized hydrogen atom are going from a lower energy state to a higher energy state. These reside in the higher n levels.

$\frac{1}{\lambda} = R\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)$ $\quad n=4: 5,6,7,8$ $\quad R_{\infty} = 4R$ $\quad$ 4R because ionized

n=5 $\quad \frac{1}{\lambda} = R_\infty\left(\frac{1}{16} - \frac{1}{25}\right) = 987{,}300 \; m^{-1} = 1012.9 \text{ nm}$

n=6 $\quad \frac{1}{\lambda} = R_\infty\left(\frac{1}{16} - \frac{1}{36}\right) = 1.523 \times 10^6 m^{-1} = 656.3 \text{ nm}$

n=7 $\quad \frac{1}{\lambda} = R_\infty\left(\frac{1}{16} - \frac{1}{49}\right) = 1.847 \times 10^6 = 541.4 \text{ nm}$

n=8 $\quad \frac{1}{\lambda} = R_\infty\left(\frac{1}{16} - \frac{1}{64}\right) = 2.220 \times 10^6 = 450.4 \text{ nm}$

$\lambda = 1012.9$ nm
$\lambda = 656.3$ nm
$\lambda = 541.4$ nm
$\lambda = 450.4$ nm

Problem 34

a.) Bohr radius between r_1 & r_2

$r_n = n^2 a_0$
$a_0 = 0.53 \times 10^{-10} m$

$r_2 = (2)^2(0.53 \times 10^{-10} m) = 2.12 \times 10^{-10} m$
$r_1 = (1)^2(0.53 \times 10^{-10} m) = 5.3 \times 10^{-11} m$

$r_2 - r_1 = 2.12 \times 10^{-10} m - 5.3 \times 10^{-11} m = 1.59 \times 10^{-10} m$

$\boxed{\Delta r_{12} = 1.59 \times 10^{-10} m}$

b.) Bohr radius between r_5 & r_6

$r_6 = (6)^2(0.53 \times 10^{-10} m) = 1.908 \times 10^{-9} m$
$r_5 = (5)^2(0.53 \times 10^{-10} m) = 1.325 \times 10^{-9} m$

$r_6 - r_5 = 1.908 \times 10^{-9} m - 1.325 \times 10^{-9} m = 5.83 \times 10^{-10} m$

$\boxed{\Delta r_{56} = 5.83 \times 10^{-10} m}$

c.) Bohr radius between r_{10} & r_{11}

$r_{11} = (11)^2(0.53 \times 10^{-10} m) = 6.413 \times 10^{-9} m$
$r_{10} = (10)^2(0.53 \times 10^{-10} m) = 5.3 \times 10^{-9} m$

$r_{11} - r_{10} = 6.413 \times 10^{-9} m - 5.3 \times 10^{-9} m = 1.113 \times 10^{-10} m$

$\boxed{\Delta r_{10-11} = 1.11 \times 10^{-10} m}$

d.) $r_{n+1} - r_n$:
$r_{n+1} = (n+1)^2 a_0$
$r_n = (n)^2 a_0$

$$r_{n+1} - r_n = (n^2 + 2n + 1)a_0 - (n)^2 a_0$$
$$= n^2 a_0 + 2n + 1(a_0) - n^2 a_0$$
$$r_{n+1} - r_n = (2n+1)(a_0)$$

At huge values of (n) (Rydberg Atoms) we can leave off the (+1) of (2n+1) because it is only making small contributions at this level. ∴

$$r_{n+1} - r_n = \Delta r_n = 2na_0$$

$\boxed{\Delta r_n = 2na_0}$

Problem 55

$L = n\hbar \qquad K = nhf_{orb}/2$

$f_{orb} = \dfrac{v}{2\pi r} \qquad v_n = \dfrac{1}{n}\dfrac{e^2}{4\pi\varepsilon_0\hbar} \qquad r_n = \dfrac{4\pi\varepsilon_0 n^2\hbar^2}{me^2} \qquad K = \dfrac{me^4}{2\hbar^2(4\pi\varepsilon_0)^2}$

$$f_{orb} = \frac{1}{2\pi} \cdot v \cdot \frac{1}{r} = \frac{1}{2\pi} \cdot \frac{e^2}{4\pi\varepsilon_0 n\hbar} \cdot \frac{me^2}{4\pi\varepsilon_0 n^2\hbar^2} = \frac{me^4}{32\pi^3\varepsilon_0^2 n^3\hbar^3} = \frac{me^4}{(4\pi\varepsilon_0)^2 2\hbar^2} \cdot \frac{1}{\pi n\hbar}$$
$$= \frac{me^4}{(4\pi\varepsilon_0)^2 2\hbar^2} \cdot \frac{1}{nh}$$
$$f = 2\left(\frac{me^4}{(4\pi\varepsilon_0)^2 2\hbar^2} \cdot \frac{1}{nh}\right)$$

$f_{orb} = \dfrac{2K}{nh}$ ∴ $\boxed{K = \dfrac{nhf_{orb}}{2}}$ ✓