Taylor Larrechea
Shelby Cerise

Phys 132L
Spring 2017

Laboratory 2: Electric Field for a Continuous Charge Distribution: Numerical and Analytical Computations

It is often necessary to determine the electric field produced by a continuous charge distribution. This laboratory describes a general procedure for doing this analytically (i.e. just using calculus and algebra). Sometimes it is possible to carry out the resulting mathematical manipulations. However, in most situations there is no known way to do this analytically. Fortunately, it is always possible to compute the electric fields to a high degree of accuracy numerically (i.e. by having a computer carry out a vast number of summations). Such *numerical approximations* are important in many sciences.

This laboratory employs both techniques to compute the electric field in a simple situation: that of a uniformly charged rod.

1 Electric Field for a Uniformly Charged Rod

Consider a rod of length L that lies along the x axis with one end at the origin as illustrated in Fig. 1. The rod is uniformly charged with a total positive charge of Q. This charge distribution produces an electric field and the task is to compute electric field vectors at any point in the vicinity of the rod.

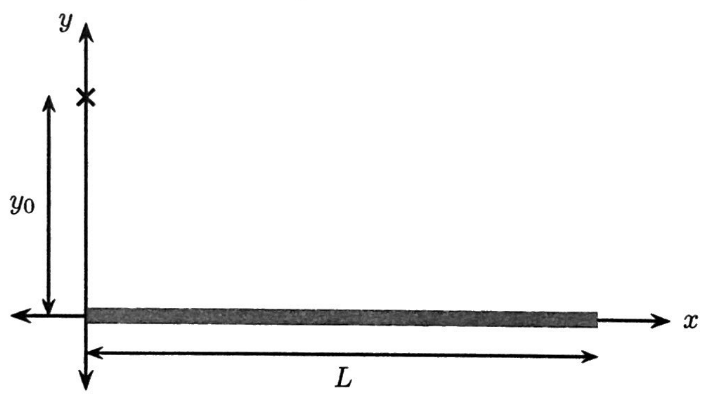


Figure 1: Charged rod and field point.

Specifically this exercise will focus on calculating the electric field vector at any point directly above the left end of the rod. The coordinates for this point will be denoted $(0, y_0)$.

The procedure for calculating the electric field in such cases involves splitting the rod in to small sections of equal length as illustrated in Fig. 2 and treating each as a point charge. The total electric field is computed by summing the electric fields produced by all the segments. The resulting electric field will be an approximation, whose accuracy improves when recalculated using smaller segments. The figure illustrates the segment whose left edge is located at x. The task is to determine the components of the electric field that this segment produces at the point $(0, y_0)$.

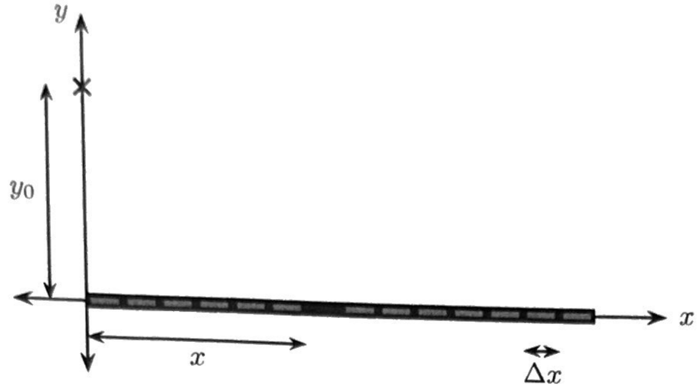


Figure 2: Decomposition of a continuous charge distribution into pointlike segments.

a) Determine an expression for the linear charge density, λ, in terms of Q and L.

b) Denote the charge contained in the shaded segment in Fig. 2 by Δq. Determine an expression, in terms of λ and Δx, for Δq.

c) Provide a diagram similar to that of Fig. 2 which shows the axes, the point at which the field is to be calculated, **only the shaded section of the charged rod** and the variables x, y_0 and Δx appropriately. Indicate the electric field vector produced by the shaded segment at the location $(0, y_0)$ on this diagram.

d) Determine an expression for the *magnitude of the electric field,* denoted ΔE, produced by the shaded segment illustrated in Fig. 2. For simplicity, assume that the segment is equivalent to a point charge located at $(x, 0)$.

e) Determine an expression for the x component of the electric field, denoted ΔE_x, produced by the shaded segment illustrated in Fig. 2.

f) Determine an expression for the y component of the electric field, denoted ΔE_y, produced by the shaded segment illustrated in Fig. 2.

g) **Verify with the instructor** that your expressions for ΔE_x and ΔE_y are correct.

h) This process must be carried out for each segment of the charged rod. However, you have obtained expressions for ΔE_x and ΔE_y which will work for every segment. Identify the terms in each of these expressions which are the same for all segments and those which change depending on the segments.

i) The x and y-components of the electric field produced by the rod is

$$E_x = \sum_{\text{all segments}} \Delta E_x \tag{1}$$

$$E_y = \sum_{\text{all segments}} \Delta E_y. \tag{2}$$

Use these to verify that

$$E_x = -\frac{kQ}{L} \sum_{\text{all segments}} \frac{x}{(x^2 + y_0^2)^{3/2}} \Delta x \tag{3}$$

$$E_y = \frac{kQy_0}{L} \sum_{\text{all segments}} \frac{1}{(x^2 + y_0^2)^{3/2}} \Delta x. \tag{4}$$

Eqs. (3) and (4) are the main equations which will guide the computation of the electric field. It should be clear that the crux of this problem is to calculate the summations. The terms preceding these are constants that can be inserted later. Thus the main calculations are:

$$\sum_{\text{all segments}} \frac{x}{(x^2 + y_0^2)^{3/2}} \Delta x \tag{5}$$

$$\sum_{\text{all segments}} \frac{1}{(x^2 + y_0^2)^{3/2}} \Delta x. \tag{6}$$

2 Numerical Computation

In the following consider a rod of length 1.0 m.

a) In order to calculate the electric field components by hand consider the case where $y_0 = 0.50\,\text{m}$ and there are four segments. Number the segments, $1, 2, 3, \ldots$ from left to right. Use a table of the following form to carry out, with a calculator, the summations of Eqs. (5) and (6).

Segment number	x	Δx	$x\Delta x/(x^2 + y_0^2)^{3/2}$	$\Delta x/(x^2 + y_0^2)^{3/2}$
1	0	0.25	0	2
2	$\ldots$	$\ldots$	$\ldots$	$\ldots$
3	$\ldots$	$\ldots$	$\ldots$	$\ldots$
4	$\ldots$	$\ldots$	$\ldots$	$\ldots$
Sum			$\ldots$	$\ldots$

Use the results to obtain an expressions for the x and y-components of the electric field.

b) This method uses approximations which become more accurate as the segment size decreases and the number of segments increases. The resulting summations can be computed efficiently by having a computer. Set up an Excel spreadsheet, formatted like the table above but which is configured to manage a large number of segments. This should be as automated as possible and there should be cells into which the number of segments and the value of y_0 are entered. All calculations should refer to these cells when necessary. Verify that your spreadsheet reproduces the values that you computed in the previous part.

c) Carry out the calculation for 10, 50 and 100 segments. For each case use the results to obtain an expressions for the x and y-components of the electric field.

d) To what values do the x and y-components of the electric field appear to converge as the number of segments increases?

3 Analytical Computation

In the limit as the number of segments approaches infinity, $\Delta x \to 0$ and the summations reduce to the following integrals.

$$\sum_{\text{all segments}} \frac{x}{(x^2+y_0^2)^{3/2}} \Delta x \to \int_0^L \frac{x}{(x^2+y_0^2)^{3/2}}\, dx \tag{7}$$

$$\sum_{\text{all segments}} \frac{1}{(x^2+y_0^2)^{3/2}} \Delta x \to \int_0^L \frac{1}{(x^2+y_0^2)^{3/2}}\, dx. \tag{8}$$

Standard techniques for evaluating these integrals give:

Σ_x

$$\int_0^L \frac{x}{(x^2+y_0^2)^{3/2}}\, dx = -\left.\frac{1}{\sqrt{x^2+y_0^2}}\right|_0^L$$

Σ_y

$$\int_0^L \frac{1}{(x^2+y_0^2)^{3/2}}\, dx = \left.\frac{x}{y_0^2\sqrt{x^2+y_0^2}}\right|_0^L$$

a) Use the exact integrals to determine an exact expressions for the x and y-components of the electric field. Compare these to the results of your numerical simulation with 100 segments and $L = 1.0\,\text{m}$ and $y_0 = 0.50\,\text{m}$.

b) Use the exact integrals to determine an exact expressions for the x and y-components of the electric field. Compare these to the results of your numerical simulation with 100 segments and $L = 1.0\,\text{m}$ and $y_0 = 0.25\,\text{m}$.

c) Suppose that $y_0 \gg L$ (i.e. y_0 is very much larger than L). Use the exact integrals to determine an approximate expressions for the x and y-components of the electric field and show that these resemble the results at a distance y_0 from a point charge of magnitude Q. Discuss why you may have expected this.

d) This entire exercise has provided a rule for computing the electric field at any location above a rod of any length which is uniformly charged. Explain (without actually doing the calculation) how this rule could be used to compute the electric field at a point *anywhere above or below* the rod.

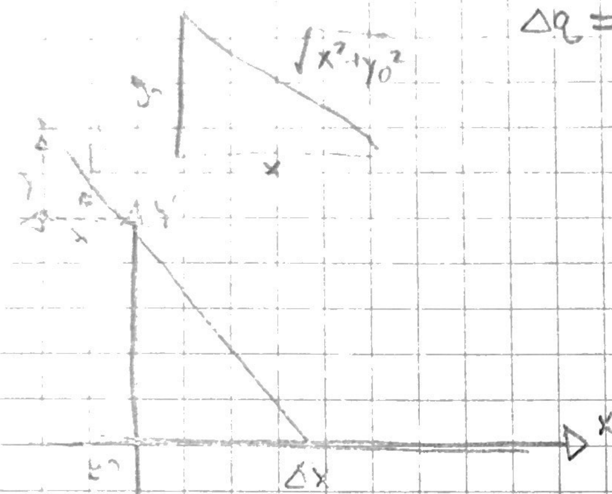
Taylor Larrechea
Shelby Cerise
132 L
2-3-17

$$E = \frac{KQ}{r^2}$$

$\Delta x \lambda - \Delta q$ $\quad r = \sqrt{x^2 + y_0^2}$

$$\vec{E} = \frac{K\lambda\Delta x}{x^2+y_0^2}$$

$$\Delta q = \Delta x \lambda$$



$$E = \frac{K\lambda\Delta x}{x^2+y_0^2}$$

$\vec{E}$: $E_x = E\cos\theta$
$E_y = E\sin\theta$

$$\vec{E} = -E\cos\theta\,\hat{i} + E\sin\theta\,\hat{j}$$

D.)

$E = \frac{KQ}{r^2}$ $\quad r = \sqrt{x^2+y_0^2}$ $\quad r^2 = x^2+y_0^2$

$$E = \frac{K\lambda\Delta x}{x^2+y_0^2}$$

E.) $\Delta E_x = \frac{K\lambda\Delta x}{x^2+y_0^2}\left(\frac{x}{\sqrt{x^2+y_0^2}}\right) = E\cos\theta$ ✓

F.) $\Delta E_y = \frac{K\lambda\Delta x}{x^2+y_0^2}\left(\frac{y_0}{\sqrt{x^2+y_0^2}}\right) = E\sin\theta$

H.) Constant : $\lambda, y_0, K, \Delta x$

Variable : x

$$\Delta E_x = \frac{K\lambda\Delta x(x)}{(x^2+y_0^2)^{3/2}} \qquad \Delta E_y = \frac{K\lambda\Delta x(y_0)}{(x^2+y_0)^{3/2}}$$

Part 3

(a.) $L = 1.0m$

$y_0 = 0.50m$

$$\Sigma_y \int_0^L \frac{x}{(x^2+y_0^2)^{3/2}} dx = -\frac{1}{\sqrt{x^2+y_0^2}}\Bigg|_0^L$$

$$\Sigma_y = \frac{1.106}{m} \Rightarrow \text{computer } \left(\frac{1.102}{m}\right)$$

$$\Sigma'_y \int_0^L \frac{1}{(x^2+y_0^2)^{3/2}} dx = \frac{x}{y_0^2\sqrt{x^2+y_0^2}}\Bigg|_0^L$$

$$\Sigma_y = \frac{3.58}{m} \Rightarrow \text{computer } \left(\frac{3.61}{m}\right)$$

(b.) $L = 1.0m$

$y_0 = 0.25m$

$$\Sigma_x \int_0^L \frac{x}{(x^2+y^2)^{3/2}} dx = \frac{-1}{\sqrt{x^2+y_0^2}}\Bigg|_0^L$$

$$\Sigma_x = \frac{3.03}{m} \Rightarrow \text{computer } \left(\frac{3.025}{m}\right)$$

$$\Sigma_y \int_0^L \frac{1}{(x^2+y^2)^{3/2}} dx = \frac{x}{y_0^2\sqrt{x^2+y_0^2}}\Bigg|_0^L$$

$$\Sigma'_y = \frac{15.522}{m} \Rightarrow \text{computer } \left(\frac{15.838}{m}\right)$$

(c.) $y_0 >> L$ the rod will appear to be a point charge and the rod will start to look like a point.

$$\Sigma_x \int_0^L \frac{x}{(x^2+y^2)^{3/2}} dx \quad x \to 0 \quad \int_0^L \frac{0}{(0^2+y^2)^{3/2}} dx = 0$$

$$\Sigma_y \int_0^L \frac{1}{(x^2+y^2)^{3/2}} dx \quad x \to 0 \quad \int_0^L \frac{1}{(0^2+y^2)^{3/2}} dx = \frac{1}{y^3}$$

$$E_x = -\frac{KQ}{L}\left[0\right] = 0 \qquad E_y = \frac{KQy_0}{L}\left[\frac{1}{y_0^3}\right] = \frac{KQ}{y_0^2}$$

d.) If $y_0 << L$, you can treat the x & y components of the $\vec{E}$ Field as a point charge yielding the equation $E = \frac{KQ}{y_0^2}$

$$\boxed{E = \frac{KQ}{y_0^2}}$$