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MATH 361 HW 1.2

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1, 6, 7, 9, 15, 19, 20, 21, 22, 25

Problem 1 absolute error $= |p - p^*|$, relative error $= \frac{|p - p^*|}{|p|}$

a.) $p = \pi$, $p^* = 22/7$

absolute error $= |p - p^*| = |\pi - 22/7| = |-0.001264| = 0.001264$

abs error = 0.001264

relative error $= \frac{|p - p^*|}{|p|} = \frac{|\pi - 22/7|}{|22/7|} = |-0.000402| = 0.000402 = 4.023 \times 10^{-4}$

relative error = 4.023×10^{-4}

b.) $p = \pi$, $p^* = 3.1416$

absolute error $= |p - p^*| = |\pi - 3.1416| = |-0.000007| = 0.000007 = 7.346 \times 10^{-6}$

abs error = 7.346×10^{-6}

relative error $= \frac{|p - p^*|}{|p|} = \frac{|\pi - 3.1416|}{|3.1416|} = |-0.000002| = 0.000002 = 2.338 \times 10^{-6}$

relative error = 2.338×10^{-6}

c.) $p = e$, $p^* = 2.718$

absolute error $= |p - p^*| = |e - 2.718| = |0.000282| = 0.000282 = 2.818 \times 10^{-4}$

abs error = 2.818×10^{-4}

relative error $= \frac{|p - p^*|}{|p|} = \frac{|e - 2.718|}{|e|} = |0.000104| = 0.000104 = 1.037 \times 10^{-4}$

relative error = 1.037×10^{-4}

d.) $p = \sqrt{2}$, $p^* = 1.414$

absolute error $= |p - p^*| = |\sqrt{2} - 1.414| = |0.000214| = 2.136 \times 10^{-4}$

abs error = 2.316×10^{-4}

relative error $= \frac{|p - p^*|}{|p|} = \frac{|\sqrt{2} - 1.414|}{|\sqrt{2}|} = \frac{|0.000214|}{|\sqrt{2}|} = |0.000151| = 1.510 \times 10^{-4}$

Relative error = 1.510×10^{-4}

Problem 6 absolute error $= |p - p^*|$: relative error $= \dfrac{|p - p^*|}{|p|}$

a.) $133 + 0.921 \approx 134$: abs error $= |133.921 - 134| = |-0.079| = 0.079$

absolute error $= 0.079$

relative error $= \dfrac{|133.921 - 134|}{|133.921|} = |-0.00059| = 5.90 \times 10^{-4}$

relative error $= 5.90 \times 10^{-4}$

b.) $133 - 0.499 \approx 133$: abs error $= |132.501 - 133| = |-0.499| = 0.499$

absolute error $= 0.499$

relative error $= \dfrac{|132.501 - 133|}{|132.501|} = |-0.0037661| = 3.77 \times 10^{-3}$

relative error $= 3.77 \times 10^{-3}$

C.) $(121 - 0.327) - 119 \approx 1.673$: abs error $= |1.673 - 1.671| = |0.0031| = 3.0 \times 10^{-3}$

absolute error $= 3.0 \times 10^{-3}$

relative error $= \dfrac{|1.673 - 1.671|}{|1.673|} = |0.0017931| = 1.79 \times 10^{-3}$

relative error $= 1.79 \times 10^{-3}$

d.) $(121 - 119) - 0.327 \approx 1.673$: abs error $= |1.673 - 1.671| = |0.0031| = 3.0 \times 10^{-3}$

absolute error $= 3.0 \times 10^{-3}$

relative error $= \dfrac{|1.673 - 1.671|}{|1.673|} = |0.0017931| = 1.79 \times 10^{-3}$

relative error $= 1.79 \times 10^{-3}$

Problem 7

absolute error $= |p - p^*|$: relative error $= \frac{|p - p^*|}{|p|}$

a.) $\frac{13/14 - 6/7}{2e - 5.4} = 1.80$

$13/14 = 0.929$
$6/7 = 0.857$
$2e = 5.44$

$p = \frac{13/14 - 6/7}{2e - 5.4}$
$p^* = 1.80$

absolute error $= |p - p^*| = 0.16354 = 0.164$

A.E = 0.164

relative error $= \frac{|p - p^*|}{|p|} = 0.078596 = 0.0786$

R.E = 0.0786

b.) $-10\pi + 6e - 3/62 = -15.1$

$-10\pi = -31.4$
$6e = 16.3$
$3/62 = 0.0484$

absolute error $= |p - p^*| = 0.054623 = 0.0546$

A.E = 0.0546

relative error $= \frac{|p - p^*|}{|p|} = 0.003604 = 0.00360$

R.E = 3.60×10^{-3}

c.) $(2/9) \cdot (9/7) = 0.286$?

$2/9 = 0.222$
$9/7 = 1.29$

absolute error $= |p - p^*| = 0.0002857 = 0.000286$

A.E = 2.86×10^{-4}

relative error $= \frac{|p - p^*|}{|p|} = 0.0009999 = 0.001$

R.E = 1.0×10^{-3}

d.) $\frac{\sqrt{13} + \sqrt{11}}{\sqrt{13} - \sqrt{11}} = 23.9$

$\sqrt{13} = 3.61$
$\sqrt{11} = 3.32$

absolute error $= |p - p^*| = 0.058261 = 0.0583$

A.E = 0.0058

relative error $= \frac{|p - p^*|}{|p|} = 0.0024317 = 0.00243$

R.E = 2.43×10^{-3}

Problem 9

a.) $\frac{13/14 - 6/7}{2e - 5.4}$: $13/14 \approx 0.928$, $6/7 \approx 0.857$, $2e \approx 5.43$, $5.4 \approx 5.4$?

$P = \frac{13/14 - 6/7}{2e - 5.4}$, $P^* \approx 2.36$: absolute error $= |P - P^*| = 0.406$

A.E = 0.406

relative error $= |P - P^*| / |P| = 0.208$

R.E = 0.208

b.) $-10\pi + 6e - 3/62$: $-10\pi \approx -31.4$, $6e \approx 16.3$, $3/62 \approx 0.0483$?

$P = -10\pi + 6e - 3/62$, $P^* \approx -15.1$: absolute error $= |P - P^*| = 0.0546$

A.E = 0.0546

relative error $= |P - P^*| / |P| = 0.00360$

R.E = 0.00360

c.) $\left(\frac{2}{9}\right) \cdot \left(\frac{9}{7}\right)$: $\left(\frac{2}{9}\right) \approx 0.222$, $\left(\frac{9}{7}\right) \approx 1.28$

$P = \left(\frac{2}{9}\right) \cdot \left(\frac{9}{7}\right)$, $P^* \approx 0.284$: absolute error $= |P - P^*| = 0.00171$

A.E = 0.00171

relative error $= |P - P^*| / |P| = 0.006$

R.E = 0.006

d.) $\frac{\sqrt{13} + \sqrt{11}}{\sqrt{13} - \sqrt{11}}$: $\sqrt{13} \approx 3.60$, $\sqrt{11} \approx 3.31$?

$P = \frac{\sqrt{13} + \sqrt{11}}{\sqrt{13} - \sqrt{11}}$, $P^* \approx 23.8$: absolute error $= |P - P^*| = 0.158261$

A.E = 0.158

relative error $= |P - P^*| / |P| = 0.00660$

R.E = 0.00660

Problem 19

a.) 0 1000000 1010 1001001100 . . . . . 0 : S=0 : $(-1)^S 2^{C-1023}(1+f)$

$C = 1000000\,1010$: $0\cdot 2^0 + 1\cdot 2^1 + 0\cdot 2^2 + 1\cdot 2^3 + 0 + 1\cdot 2^{10} = 2 + 8 + 1024 = 1034$

$C = 1034$

$f = 1001001100$: $1\cdot\left(\frac{1}{2}\right)^1 + 0\cdot\left(\frac{1}{2}\right)^2 + 0\cdot\left(\frac{1}{2}\right)^3 + 1\cdot\left(\frac{1}{2}\right)^4 + 0\cdot\left(\frac{1}{2}\right)^5 + 0\cdot\left(\frac{1}{2}\right)^6 + 1\cdot\left(\frac{1}{2}\right)^7 + 1\cdot\left(\frac{1}{2}\right)^8$

$f = \frac{1}{2} + \frac{1}{16} + \frac{1}{128} + \frac{1}{256} = \frac{147}{256}$: $f = \frac{147}{256}$

$\# = (-1)^0 2^{11}\left(1 + {}^{147}/_{256}\right) = 3224$

3224

b.) 1 1000000 1010 10010011

$C = 1000000\,1010$: $0\cdot 2^0 + 1\cdot 2^1 + 0\cdot 2^2 + 1\cdot 2^3 + 0 + 1\cdot 2^{10} = 2 + 8 + 1024 = 1034$

$C = 1034$

$f = 10010011$: $1\cdot\left(\frac{1}{2}\right)^1 + 0\cdot\left(\frac{1}{2}\right)^2 + 0\cdot\left(\frac{1}{2}\right)^3 + 1\cdot\left(\frac{1}{2}\right)^4 + 0\cdot\left(\frac{1}{2}\right)^5 + 0\cdot\left(\frac{1}{2}\right)^6 + 1\cdot\left(\frac{1}{2}\right)^7 + 1\cdot\left(\frac{1}{2}\right)^8$

$f = \frac{1}{2} + \frac{1}{16} + \frac{1}{128} + \frac{1}{256} = \frac{147}{256}$: $f = \frac{147}{256}$

$\# = (-1)^1 2^{11}\left(1 + {}^{147}/_{256}\right) = -3224$

-3224

c.) 0 01111111111 01010011

$C = 01111111111$: $1\cdot 2^0 + 1\cdot 2^1 + 1\cdot 2^2 + 1\cdot 2^3 + 1\cdot 2^4 + 1\cdot 2^5 + 1\cdot 2^6 + 1\cdot 2^7 + 1\cdot 2^8 + 1\cdot 2^9 + 0\cdot 2^{10}$

$C = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023$: $C = 1023$

$f = 01010011$: $0\cdot\left(\frac{1}{2}\right)^1 + 1\cdot\left(\frac{1}{2}\right)^2 + 0\cdot\left(\frac{1}{2}\right)^3 + 1\cdot\left(\frac{1}{2}\right)^4 + 0\cdot\left(\frac{1}{2}\right)^5 + 0\cdot\left(\frac{1}{2}\right)^6 + 1\cdot\left(\frac{1}{2}\right)^7 + 1\cdot\left(\frac{1}{2}\right)^8$

$f = \frac{1}{4} + \frac{1}{16} + \frac{1}{128} + \frac{1}{256} = \frac{83}{256}$: $f = \frac{83}{256}$

$\# = (-1)^0 2^0\left(1 + {}^{83}/_{256}\right) = 1.32421875$

1.32421875

Problem 19 | Continued

$$\# = (-1)^s 2^{c-1023}(1+f)$$

0 01111111111 0101001101.....1

d.) $C = \cancel{0111111111}$: $1\cdot2^0 + 1\cdot2^1 + 1\cdot2^2 + 1\cdot2^3 + 1\cdot2^4 + 1\cdot2^5 + 1\cdot2^6 + 1\cdot2^7 + 1\cdot2^8 + 1\cdot2^9 + \cancel{0\cdot2^{10}}$

$C = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023$: $C = 1023$

$f = \cancel{0101001101}$: $\cancel{0\cdot\left(\frac{1}{2}\right)^1} + 1\cdot\left(\frac{1}{2}\right)^2 + \cancel{0\cdot\left(\frac{1}{2}\right)^3} + 1\cdot\left(\frac{1}{2}\right)^4 + \cancel{0\cdot\left(\frac{1}{2}\right)^5} + \cancel{0\cdot\left(\frac{1}{2}\right)^6} + 1\cdot\left(\frac{1}{2}\right)^7 + 1\cdot\left(\frac{1}{2}\right)^8 + 1\cdot\left(\frac{1}{2}\right)^{52}$

$f = \frac{1}{4} + \frac{1}{16} + \frac{1}{128} + \frac{1}{256} + \frac{1}{2^{52}} = 0.3242187499999999$

$$\# = (-1)^0 2^0 (1 + 0.3242187499999999) = 1.32422$$

1.32421875

Problem 20

a.) 0 10000001010 10010011100.....0

Smallest = 0 10000001010 10010011111.....1

Biggest = 0 10000001010 10010011100.....1

b.) 1 10000001010 10010011.....0

Smallest = 1 10000001010 10010011111 ...1

Biggest = 1 10000001010 10010011100 ... 1

c.) 0 01111111111 01010011.....0

Smallest = 0 01111111111 01010011111 1

Biggest = 0 01111111111 01010011100 ... 1

Problem 25

$$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99$$

a.)

$$\boxed{f(x) = (((1.01e^{x} - 4.62)e^{x} - 3.11)e^{x} + 12.2)e^{x} - 1.99}$$

b.) $f(1.53) = 1.01e^{4\cdot1.53} - 4.62e^{3\cdot1.53} - 3.11e^{2\cdot1.53} + 12.2e^{1.53} - 1.99$

$e^{x} = 4.62$

$e^{2x} = (4.62)(4.62) = 21.344 \approx 21.3$

$e^{3x} = (21.3)(4.62) = 98.406 \approx 98.4$

$e^{4x} = (98.4)(4.62) = 454.608 \approx 455$

$1.01(455) = 459.55 \approx 460$ (↳ e^{4x})

$4.62(98.4) = 454.608 \approx 455$ (↳ e^{3x})

$3.11(21.3) = 66.243 \approx 66.2$ (↳ e^{2x})

$12.2(4.62) = 56.364 \approx 56.4$ (↳ e^{x})

$460 - 455 - 66.2 + 56.4 - 1.99 = 5 - 66.2 + 56.4 - 1.99 = -61.2 + 56.4 - 1.99 = -4.8 - 1.99 = -6.79$

$$\boxed{f(1.53) = -6.79}$$

c.)

$1.01 \cdot e^{x} = 4.664358591 \approx 4.67$

$(1.01e^{x} - 4.62) = 4.67 - 4.62 = 0.05$

$(1.01e^{x} - 4.62)e^{x} = (0.05)(4.62) = 0.231$

$((1.01e^{x} - 4.62)e^{x} - 3.11) = 0.231 - 3.11 = -2.88$

$((1.01e^{x} - 4.62)e^{x} - 3.11)e^{x} = (-2.88)(4.62) = -13.3056 \approx -13.3$

$(((1.01e^{x} - 4.62)e^{x} - 3.11)e^{x} + 12.2) = -13.3 + 12.2 = -1.1$

$(((1.01e^{x} - 4.62)e^{x} - 3.11)e^{x} + 12.2)e^{x} = (-1.1)(4.62) = -5.082 \approx -5.08$

$(((1.01e^{x} - 4.62)e^{x} - 3.11)e^{x} + 12.2)e^{x} - 1.99 = -5.08 - 1.99 \approx -7.07$

$$\boxed{f(1.53) = -7.07}$$