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11-16

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Phys 131L
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Laboratory 12: Equilibrium of a Rigid Body

The rotational version of Newton's second law states that for any object

$$\tau_{\text{net}} = I\alpha \tag{1}$$

where τ_{net} is the net torque on the object, I is its moment of inertia and α is the angular acceleration of the object. If the object is in equilibrium then $\alpha = 0$ implies that

$$\tau_{\text{net}} = 0. \tag{2}$$

The aim of this laboratory exercise is to verify this.

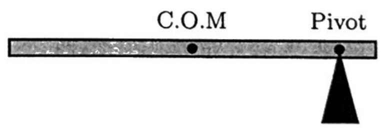
1 Center of mass of the meter stick

The Earth exerts gravitational forces on each segment of any rigid object. This multitude of gravitational forces acting on an object of total mass M can be replaced by a *single collective gravitational force*, with magnitude Mg, acting on the object at a location called the *center of mass* (C.O.M.) of the object. The center of mass depends on the mass distribution and *it is not necessarily at the center of the object.*

Knowing the location of the center of mass of any object is essential for assessing the rotational motion of the object. The aim of this part of the laboratory is to determine the mass and the center of mass of the meter stick.

a) Measure and record the mass of the meter stick. Measure and record the masses of the clamps.

b) The meter stick in this experiment will have an adjustable pivot point. Suppose that the pivot point is located right of the center of mass while the meter stick is horizontal.



Sketch the forces acting on the meter stick and determine whether the net torque on the meter stick about the pivot is zero or not. Will the meter stick be in equilibrium in this situation?

c) At which point should the pivot be located so that the net torque on the meter stick about the pivot is zero?

d) Attach a pivot clamp to the meter stick and either place the clamp in the stand or suspend it from a string. Adjust the location of the clamp until the meter stick stays at rest horizontally. Once you have adjusted the pivot position so that the meter stick is horizontal and in equilibrium, then the pivot will be located at the center of mass. Record the location of the center of mass of the meter stick.

2 Equilibrium: pivot at the center of mass

a) Balance the meter stick with its center of mass at the pivot point. Now add another clamp between 40 cm and 50 cm from the center of mass. Suspend 150 g (in addition to the clamp mass) from this clamp. Without anything else attached the meter stick will rotate. Attach another clamp, from which 250 g is suspended, to the meter stick so that the meter stick is at rest horizontally. Record the positions of the clamps and the masses suspended from them.

b) Determine the counterclockwise (positive) torque about the pivot point (don't forget about the clamp!). Determine the clockwise (negative) torque about the pivot point. If the meter stick is at rest horizontally, how should these compare to each other?

c) Determine the percentage difference between the magnitude of the clockwise torque and the counterclockwise torque.

d) Repeat parts 2 a) to c) starting with the 150 g mass suspended between 30 cm and 40 cm from the center of mass.

e) Repeat parts 2 a) to c) starting with a 100 g mass suspended between 40 cm and 50 cm from the center of mass and using a 250 g mass on the other side.

3 Equilibrium: pivot away from the center of mass

a) Arrange the meter stick so that the pivot is between 15 cm and 20 cm from the center of mass. Attach one other clamp and suspend 100 g from it. Adjust the position of the clamp so that the meter stick is at rest horizontally. Record the positions of the clamp and the mass suspended from it.

b) Determine the counterclockwise (positive) torque about the pivot point (don't forget about the clamp!). Determine the clockwise (negative) torque about the pivot point. If the meter stick is at rest horizontally, how should these compare to each other?

c) Determine the percentage difference between the magnitude of the clockwise torque and the counterclockwise torque.

d) Repeat parts 3 a) to c) starting with the the pivot is between 20 cm and 25 cm from the center of mass.

4 Conclusion

a) What was the main finding of these experiments?

b) Explain any discrepancies between your observations and the theoretical predictions. For your explanation you should consider:

   i) How accurately you were able to measure the quantities needed to carry out the calculations.

ii) Whether there are any torques present that were not included in the analysis or whether the calculation of the torques ignored any small details.

5 Exercises

a) Suppose that the pivot is at the center of mass and that a 200 g mass is suspended 40 cm to the left of the center of mass. An unknown mass is suspended 25 cm to the right of the center of mass. Determine the mass that is required to keep the meter stick in horizontal equilibrium.

b) Repeat the above part assuming that the meter stick is in equilibrium but not horizontal and makes an angle of 5.0° with respect to the horizontal. Is the mass required to balance the meter stick any different?

c) Again referring to the previous part suppose that the meter stick is in equilibrium but not horizontal and makes an angle of θ with respect to the horizontal. Is the mass required to balance the meter stick any different to that when the meter stick is horizontal? Will any deviation from horizontal affect the results of your experiment?

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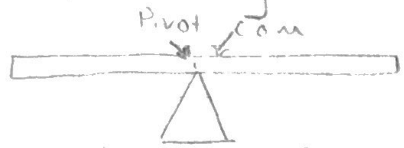
Part 1

A Meter stick - 0.087 kg
Clamp - 0.021 kg

B



The meter stick will be in equilibrium because its not moving.

C For the net torque to be zero, the pivot would have to be at the center of mass.

D The center of mass is at 51.1 cm on the meter stick.

Part 2

A Clamp mass$_A$ - 0.02 kg
clamp Mass$_B$ - 0.023 kg

0.273 kg 0.17 kg
78.8 cm ⟷ 0.788 m 5 cm ⟷ 0.05 m

measurements off of right end of meter stick

B Positive torque

$r = 0.287\,m$ $\phi = 90°$
$F = 0.273\,kg(9.8\,m/s^2)$
$F = 2.675\,N$
$\tau = r\vec{F}\sin\phi$
$\tau = 0.287m(2.675N)\sin(90)$
$\tau = 0.77\,Nm$ ✓

Negative Torque

$r = 0.451\,m$ $\phi = 270°$
$F = 0.17\,kg(9.8\,m/s^2)$
$F = 1.67\,N$
$\tau = r\vec{F}\sin\phi$
$\tau = 0.451m(1.67N)\sin 270$
$\tau = -0.76\,NM$ ✓

If the meter stick is at rest, The magnitudes should be equal but opposite.

% error is the difference over the value!

-1/2

C $\frac{0.76\ NM}{0.77\ NM} = 98.7\%$ ✗ c.o.m = 50.1 cm

D 0.25 kg + 0.023 kg   0.15 kg + 0.02 kg   Positive Torque

72.4 cm   15.0 cm $r = 22.3\ cm \rightarrow 0.223\ m$ $\emptyset = 90^0$

0.724 m   0.15 m $F = 0.273\ kg (9.8\ m/s^2)$

off of right end of meter stick

$\vec{F} = 2.6754\ NM$

$\tau = r\vec{F}\sin\emptyset$

$\tau = 0.223\ m (2.6754\ NM) \sin 90$

$\tau = 0.597\ NM$ ✓

Negative Torque

$r = 35.1\ cm \rightarrow 0.351\ m$ $\emptyset = 270$

$\vec{F} = 0.17\ kg (9.8\ m/s^2)$

$\vec{F} = 1.666\ NM$

$\tau = r\vec{F}\sin\emptyset$

$\tau = 0.351\ m (1.666\ NM) \sin 270$

$\tau = -0.585\ NM$ ✓

$\frac{0.585}{0.597} = 97.9\%$ ✗

E   Hanger   Hanger

0.25 kg + 0.023 kg   0.10 kg + 0.02 kg

70.1 cm   6 cm

0.701 m   0.06 m

off of right end of meter stick

Positive Torque

$r = 20\ cm \rightarrow 0.2\ m$ $\emptyset = 90^0$

$\vec{F} = 0.273\ kg (9.8\ m/s^2)$

$\vec{F} = 2.6754\ NM$

$\tau = r\vec{F}\sin\emptyset$

$\tau = 0.2\ m (2.6754\ NM) \sin 90$

$\tau = 0.535\ NM$ ✓

Negative Torque

$r = 44.1\ cm \rightarrow 0.441\ m$ $\emptyset = 270^0$

$F = 0.12\ kg (9.8\ m/s^2)$

$\vec{F} = 1.176\ NM$

$\tau = r\vec{F}\sin\emptyset$

$\tau = 0.441\ m (1.176\ NM) \sin 270$

$\tau = -0.519\ NM$ ✓

$\frac{0.519\ NM}{0.535\ NM} = \boxed{97\%}$

Part 3

A New Pivot

33 cm off of right end of stick

0.33m

$0.10\ kg + 0.02\ kg$

$0.12\ kg$

21 cm off of right end

0.21 m

0.21 m off of right end of meter stick

0.12 kg total mass with hanger

B Positive Torque

$r = 0.33m$ $\emptyset = 90°$

$\vec{F} = mg$ $\vec{F} = m(9.8 m/s^2)$

$M = 0.044kg$ $\vec{F} = 0.044kg (9.8 M/s^2)$

$\vec{F} = 0.4312$ NM

$\tau = r\vec{F}\sin\emptyset$

$\tau = 0.33m (0.4312\ Nm) \sin 90$

$\tau = 0.142$ NM

Because meter stick isn't rotating

$Net_{\tau} = 0$

Negative Torque

$r = 0.12\ m$ $\emptyset = 270°$ $\vec{F} = mg$

$\vec{F} = 0.12kg (9.8 m/s^2)$

$\vec{F} = +1.176$ NM

$\tau = r\vec{F}\sin\emptyset$

$\tau = 0.12m (1.176\ Nm) \sin 270$

$\tau = 0.141$ NM

Positive and negative should balance out to = 0

C $\frac{0.141}{0.142} = 0.992$ $0.992 \times 100 = 99.2\%$

D Pivot at 30 cm from right end of stick

15.7 cm

$30^{cm} - 15.7^{cm} = 14.3cm$

Negative Torque

$r = 0.143\ m$ $\emptyset = 270°$ $M = 0.12kg$

$\vec{F} = mg$ $\vec{F} = 0.12kg (9.8\ m/s^2)$

$\vec{F} = 1.176$ Nm $\tau = r\vec{F}\sin\emptyset$

$\tau = 0.143m (1.176\ Nm) \sin 270$

$\tau = -0.168$ Nm

Positive Torque $\emptyset = 90$

$r = 0.35m$ $M = 0.049\ kg$

$\vec{F} = mg$ $\vec{F} = 0.049\ kg (9.8 m/s^2)$

$\vec{F} = 0.4802\ N$ $\tau = r\vec{F}\sin\emptyset$

$\tau = 0.35m (0.4802N) \sin 90$

$\tau = 0.169$ NM

$\frac{0.168\ NM}{0.169\ NM} = .994$ $\times 100 = 99.4\%$

99.4%

Part 4

a. Net Torque is 0 when in equilibrium, the magnitudes of the torques are equal but opposite.

B. The discrepancies in this experiment would be air resistance, friction, uneven surface that the pivot thing rests on. We were within 3 % error on our torques. In the instant that weights are on both sides there could be a torque of the meter stick that is being neglected

Part 5 In equilibrium

A $\tau_1 = \tau_2$ ✓

$r_1 \vec{F}_1 \sin\theta = r_2 F_2 \sin\theta$

$0.4m(0.2kg \cdot 9.8m/s^2)\sin 90 = 0.25m(m \cdot 9.8m/s^2)\sin 270$

$0.784\ kg \cdot m^2/s^2 = 2.45\ m^2/s^2 \cdot m$

$\boxed{m = 0.32\ kg}$ ✓

✓

B No the position would have to change so that it is getting closer to the center of mass. The radius from the pivot point would have to decrease causing the inertia to be less. So therefore the mass is not required since the radius can.

$I = mr^2$

C changing the mass is not required to keep an object in equilibrium since radius from pivot point can also affect the inertia of the object.