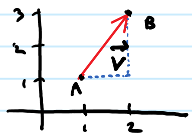
Math 360 Ch 9.1 – 9.3: Vectors, Inner Products, Cross Products

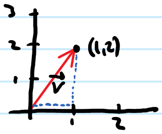
Scalars are quantities that are determined by **magnitude** only. (Temperature, speed, etc)

Vectors are quantities that are determined by **magnitude** and **direction** (Force, velocity, etc)

When graphing a vector, it can be convenient and useful to locate its initial point at the origin and its terminal point at a certain ordered pair (in $\mathbb{R}^2$) and ordered triple in $\mathbb{R}^3$, etc.

Example Consider the vector $\vec{v} = \overrightarrow{AB}$, where $A = (1,1)$, $B = (2,3)$.



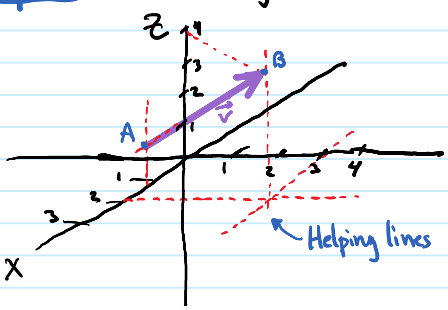


- In graph on left, we graph $\vec{v}$ by first plotting A & B.
- In graph on right, we graph the position vector for $\vec{v}$, given by $\vec{v} = [2-1, 3-1] = [1,2]$

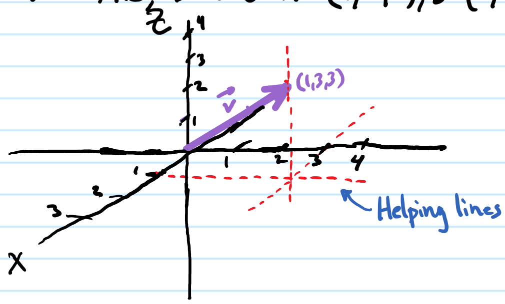
Note that the magnitude of $\vec{v}$ is given by
$$|\vec{v}| = \sqrt{1^2+2^2} = \sqrt{5}$$

The unit vector $\vec{u}$ in the direction of $\vec{v}$ is given by
$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{5}}[1,2] = \left[\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right] = \frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}$$

Example Let's try this in $\mathbb{R}^3$: $\vec{v} = \overrightarrow{AB}$, where $A = (1,0,1)$, $B = (2,3,4)$



Graph of $\vec{v}$ found by connecting points A and B



Graph of $\vec{v}$ using position vector: $\vec{v} = [2-1, 3-0, 4-1] = [1,3,3]$

Read:
- Vector addition definition on page 357 for position vectors, and review triangle law for vector addition in Figure 170, for general vectors
- Also read Ex 2 in Ch 9.1, and review $\vec{i}, \vec{j}, \vec{k}$ vectors (unit coord vectors)

DEFINITION

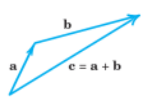


Fig. 170. Vector addition

Addition of Vectors

The **sum a + b** of two vectors $\mathbf{a} = [a_1, a_2, a_3]$ and $\mathbf{b} = [b_1, b_2, b_3]$ is obtained by adding the corresponding components,

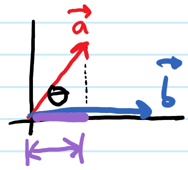
$$\mathbf{a} + \mathbf{b} = [a_1 + b_1, \quad a_2 + b_2, \quad a_3 + b_3]. \tag{3}$$

Geometrically, place the vectors as in Fig. 170 (the initial point of **b** at the terminal point of **a**); then **a + b** is the vector drawn from the initial point of **a** to the terminal point of **b**.

Two Ways of Multiplying Vectors

1) **Inner Product** $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$

Properties:
1) $\vec{a}\cdot\vec{b}$ is scalar valued
2) $\vec{a}\cdot\vec{b}$, roughly speaking, measures how much of $\vec{a}$ is in direction of $\vec{b}$
3) $\vec{a}\cdot\vec{b}$ measures amount of work done by force $\vec{a}$ in direction $\vec{b}$ (see p.364)



adj side $= |\vec{a}|\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ } projection of $\vec{a}$ onto $\vec{b}$

- If $\theta = \frac{\pi}{2}$ rads $= 90°$, then $\cos\theta = 0$ and $\vec{a}\cdot\vec{b} = 0$.

In this case $\vec{a}$ & $\vec{b}$ are said to be orthogonal.

Example $\vec{a} = [1,2,3]$, $\vec{b} = [-1,0,3]$, $\vec{c} = [0,-6,4]$

$\vec{a}\cdot\vec{b} = (1)(-1) + (2)(0) + (3)(3) = 8$ = work done by $\vec{a}$ along displacement $\vec{b}$

$\vec{a}\cdot\vec{c} = (1)(0) + (2)(-6) + (3)(4) = 0 \Rightarrow \vec{a}$ & $\vec{b}$ orthogonal

2) **Cross Product**

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - b_2a_3)\vec{i} - (a_1b_3 - b_1a_3)\vec{j} + (a_1b_2 - b_1a_2)\vec{k} = \text{vector valued}$$

Properties
1) $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$
2) $\vec{a}\times\vec{b}$ is orthogonal to $\vec{a}$ & $\vec{b}$
3) $\vec{a}\times\vec{b}$ is a normal vector to plane determined by $\vec{a}$ and $\vec{b}$.
4) $|\vec{a}\times\vec{b}|$ = area of parallelogram determined by $\vec{a}$ & $\vec{b}$
5) $\vec{a}\times\vec{b}$ yields moment vector whose direction is given by axis of rotation.

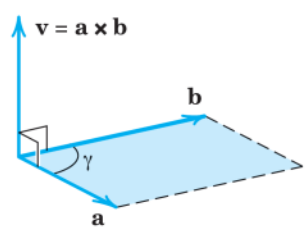


Fig. 185. Vector product

Read
1) Ex 1 on p.370 gives example of cross product calculations.
2) Ex 3 on p.371 gives example of moment of force
3) Ex 5 on p.372 discusses velocity of rotating body
4) Scalar Triple Product - p.373
   - ↳ volume of parallelepiped → see Ex 6
   - ↳ this gets used in Ch 10 derivations