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Chapter 12

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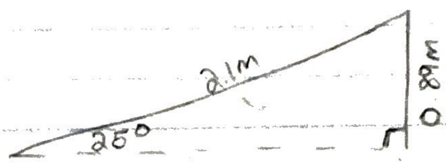
10:00-10:50 AM
PHYS 131

Problems

12.35 PE = KE

$r = 0.04m$
$m = 0.4\ kg$



$PE = mgh$
$KE = \frac{1}{2}I\omega^2$
$I = \frac{2}{5}mr^2$
$\omega = \frac{v}{R}$

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$
$mgh = \frac{1}{2}(\frac{2}{5}mr^2)\frac{v^2}{r^2} + \frac{1}{2}mv^2$
$mgh = \frac{1}{5}mv^2 + \frac{1}{2}mv^2$
$gh = \frac{7}{10}v^2$
$\frac{10}{7}gh = v^2$
$\sqrt{\frac{10}{7}gh} = v$

$V = \sqrt{\frac{10}{7}(9.8m/s^2)(0.89m)}$
$V = 3.52\ m/s$

$\omega = \frac{V}{R}$
$\omega = \frac{3.52m/s}{0.04m}$
$\omega = 88\ rad/s$

Rotational : $\frac{1}{2}I\omega^2$ $\quad I = \frac{2}{5}mr^2$ $\quad \omega = \frac{v^2}{r^2}$

$\frac{1}{2}(\frac{2}{5}mr^2)\frac{v^2}{r^2}$
$\frac{1}{5}mv^2 \longrightarrow \frac{1}{5}(0.4kg)(3.52m/s)^2$
$0.99\ J$

Total: $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $\quad I = \frac{2}{5}mr^2$
$\frac{1}{2}(mv^2) + \frac{1}{2}(\frac{2}{5}mr^2\frac{v^2}{r^2})$
$\frac{1}{2}(\frac{7}{5}mv^2)$
$\frac{1}{2}(\frac{7}{5}(0.4kg)(3.52m/s)^2) \longrightarrow 3.47J$

$\frac{K_R}{K_T} = \frac{0.99J}{3.47J} = 0.28 = \frac{2}{7}$

$\omega = 88\ rad/s$
$\frac{2}{7} \approx 0.28$

12.40 $\vec{F} = -10\hat{j}\ N$ $\quad \vec{r} = (5\hat{i} + 5\hat{j})$

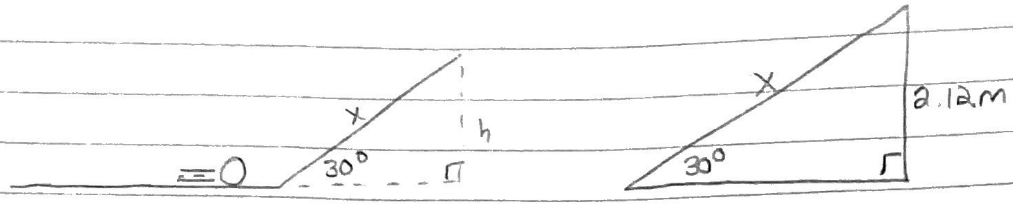
$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r} = \langle 5, 5, 0 \rangle$ $\quad r \times F$
$\vec{F} = \langle 0, -10, 0 \rangle$

$\langle 5(0)-0(-10), 0(0)-5(0), 5(-10)-5(0) \rangle$
$\langle 0, 0, -50 \rangle$

$\boxed{\tau = -50\hat{k}\ Nm}$

12.69



$V = 5.0\ m/s$

$PE = KE$ $\quad KE = K_R + K_T$ $\quad I = \frac{2}{3}mr^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $\quad \omega = \frac{v}{r}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\frac{v^2}{r^2}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{3}mv^2$ $\quad \sin 30 = \frac{2.12m}{x}$

3/3 $\quad mgh = \frac{5}{6}mv^2$ $\quad \frac{1}{2}x = 2.12m$

$gh = \frac{5}{6}v^2$ $\quad x = 4.24m$

$h = \frac{\frac{5}{6}v^2}{g}$ $\quad h = \frac{\frac{5}{6}(5.0m/s)^2}{9.8m/s^2} \Rightarrow h = 2.12m$

$\boxed{4.24\ m\ \text{up ramp}}$

12.74

$P.E = K.E$

$PE = mgh$	$P.E = K.E$
$KE = \frac{1}{2}I\omega^2$	$mgh = \frac{1}{2}I\omega^2$
$I = \frac{1}{3}mL^2$	$mgh = \frac{1}{6}mL^2(\omega^2)$
$R = L$	$g(\frac{L}{2}) = \frac{1}{6}L^2\omega^2$
$h = \frac{L}{2}$	$6g(\frac{L}{2}) = L^2\omega^2$
$V = \omega R$	$3gL = L^2\omega^2$
$V = \omega L$	$\frac{3g}{L} = \omega^2$
$V = \sqrt{\frac{3g}{L}}\,(L)$	$\omega = \sqrt{\frac{3g}{L}}$

$$\boxed{\omega = \sqrt{\frac{3g}{L}} \qquad V_f = \sqrt{\frac{3g}{L}}\,(L)}$$

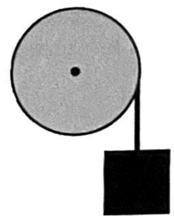
R - Radius

L - length of rod

Conceptual

12.C.7 Yes you can by rolling them down an inclined ramp. The Shelled out sphere will roll down the ramp slower due to it having a larger moment of inertia, the solid sphere will roll down the ramp faster due to it having a smaller moment of inertia.

77 Mass suspended from a rotating wheel



A 10 kg solid wheel with radius 0.40 m can rotate about a frictionless axle through its center. A 5.0 kg block is suspended from a string which is wrapped around the wheel. The system is held at rest with the block 2.0 m above the ground. The block is then released, causing the wheel to rotate (it's axle does not drop).

a) Determine the total energy of the system at the moment that the block is released. What type of energy is this?

b) What type of energy does the system have at the instant just before the block hits the ground? Determine an expression for the total energy of the system at this instant; the expression should include the speed of the block, v, and the angular velocity of the wheel ω.

c) If the string does not slip along the wheel the speed of the block v satisfies $v = \omega r$ where r is the radius of the wheel. Substitute this into the result of the previous part to show that the total energy just before the block hits the ground is

$$E = \frac{1}{2} m_{\text{block}} r^2 \omega^2 + \frac{1}{2} I_{\text{wheel}} \omega^2.$$

Use this to determine the angular velocity of the wheel just before the block hits the ground.

d) Determine the speed of the block just before it hits the ground.

a.) E_{total} Only potential energy $M = 5.0\,kg$ $h = 2.0\,m$

$mgh \Rightarrow 5.0kg(9.8m/s^2)(2.0m)$

$98\ m^2/s^2 \cdot kg$ 98 N

b.) E : $K_{rotational} + K_{translational}$ $I = \frac{1}{2}Mr^2$

$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$\frac{1}{4}Mv^2 + \frac{1}{2}mv^2$ $\boxed{\frac{1}{2}I_{wheel}\,\omega^2 + \frac{1}{2}m_{Block}v^2}$

C.)

$E = \frac{1}{4}m_W v^2 + \frac{1}{2}m_B v^2$ $\omega = \frac{V}{R}$ $\omega = \frac{4.4m/s}{0.4m}$

3/3

$98N = \frac{1}{4}(10kg)v^2 + \frac{1}{2}(5.0kg)v^2$ $\frac{98N}{5.0kg} = v^2$ $\omega = 11\ rad/s$

$98N = (2.5kg + 2.5kg)v^2$ $19.6\ m^2/s^2 = v^2$

$\omega = 11\ rad/s$ $V = 4.4\ m/s$

d.)

$V = 4.4\ m/s$

Cross products on back

78 Vectors: Cross Products

For each of the following, determine A_x, A_y, B_x, B_y and A_z and B_z (if applicable) and determine the cross product, $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$. *Note: If your answer for the cross product does not contain* $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ *or* $\hat{\mathbf{k}}$ *then it is very incorrect!*

a) $\boxed{0\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ $A_x = 2\hat{i}$ $A_y = 2\hat{j}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ $B_x = 2\hat{i}$ $B_y = 2\hat{j}$

b) $\boxed{-8\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ $A_x = 2\hat{i}$ $A_y = 2\hat{j}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ $B_x = 2\hat{i}$ $B_y = -2\hat{j}$

c) $\boxed{8\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ $A_x = 2\hat{i}$ $A_y = -2\hat{j}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ $B_x = 2\hat{i}$ $B_y = 2\hat{j}$

d) $\boxed{0\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ $A_x = 2\hat{i}$ $B_x = -2\hat{j}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ $B_x = 2\hat{i}$ $B_y = -2\hat{j}$

e) $\boxed{-6\hat{j} + 6\hat{j} + 0\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ $A_x = 2\hat{i}$ $A_y = 2\hat{j}$ $A_z = 3\hat{k}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ $B_x = 2\hat{i}$ $B_y = 2\hat{j}$

f) $\boxed{6\hat{i} + 6\hat{j} - 8\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ $A_x = 2\hat{i}$ $A_y = 2\hat{j}$ $A_z = 3\hat{k}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ $B_x = 2\hat{i}$ $B_y = -2\hat{j}$

g) $\boxed{-12\hat{i} + 0\hat{j} + 8\hat{k}}$

$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ $A_x = 2\hat{i}$ $A_y = 2\hat{j}$ $A_z = 3\hat{k}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ $B_x = 2\hat{i}$ $B_y = 2\hat{j}$ $B_z = 3\hat{k}$

h) $\boxed{3\hat{i} + 9\hat{j} + 5\hat{k}}$

$\vec{\mathbf{A}} = 1\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ $A_x = \hat{i}$ $A_y = -2\hat{j}$ $A_z = 3\hat{k}$

$\vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 1\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ $B_x = 2\hat{i}$ $B_y = 1\hat{j}$ $B_z = -3\hat{k}$

a.) $\begin{vmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \end{vmatrix}$ $\langle 2(0)-0(2),\ 0(2)-2(0),\ 2(2)-2(2)\rangle$

$\langle 0, 0, 0\rangle$

b.) $\begin{vmatrix} 2 & 2 & 0 \\ 2 & -2 & 0 \end{vmatrix}$ $\langle 2(0)-0(-2),\ 0(2)-2(0),\ 2(-2)-2(2)\rangle$

$\langle 0, 0, -8\rangle$

c.) $\begin{vmatrix} 2 & -2 & 0 \\ 2 & 2 & 0 \end{vmatrix}$ $\langle -2(0)-0(2),\ 0(2)-2(0),\ 2(2)+2(2)\rangle$

$\langle 0, 0, 8\rangle$

d.) $\begin{vmatrix} 2 & -2 & 0 \\ 2 & -2 & 0 \end{vmatrix}$ $\langle -2(0)-0(-2),\ 0(2)-2(0),\ 2(-2)+2(2)\rangle$

$\langle 0, 0, 0\rangle$

e.) $\begin{vmatrix} 2 & 2 & 3 \\ 2 & 2 & 0 \end{vmatrix}$ $\langle 2(0)-3(2),\ 3(2)-2(0),\ 2(2)-2(2)\rangle$

$\langle -6, 6, 0\rangle$

f.) $\begin{vmatrix} 2 & 2 & 3 \\ 2 & -2 & 0 \end{vmatrix}$ $\langle 2(0)-3(-2),\ 3(2)-2(0),\ 2(-2)-2(2)\rangle$

$\langle 6, 6, -8\rangle$

g.) $\begin{vmatrix} 2 & -2 & 3 \\ 2 & 2 & 3 \end{vmatrix}$ $\langle -2(3)-3(2),\ 3(2)-2(3),\ 2(2)+2(2)\rangle$

$\langle -12, 0, 8\rangle$

h.) $\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -3 \end{vmatrix}$ $\langle -2(-3)-3(1),\ 3(2)-1(-3),\ 1(1)+2(2)\rangle$

$\langle 3, 9, 5\rangle$

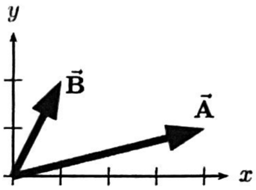
Cross products on back

79 Vectors: Cross Products

For each of the following, express $\vec{\mathbf{A}}$ in component form $\vec{\mathbf{B}}$ using unit vectors and determine both cross products, $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ and $\vec{\mathbf{B}} \times \vec{\mathbf{A}}$. *Note: If your answer for the cross product does not contain* $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ *or* $\hat{\mathbf{k}}$ *then it is very incorrect!*

a) $\vec{A} = \langle +4\hat{i},\ 1\hat{j} \rangle$

$\vec{B} = \langle +1\hat{i},\ 2\hat{j} \rangle$

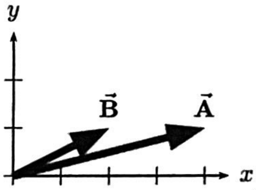


$\vec{A} \times \vec{B} = 0\hat{i} + 0\hat{j} + 7\hat{k}$

$\vec{B} \times \vec{A} = 0\hat{i} + 0\hat{j} - 7\hat{k}$

b) $\vec{A} = \langle +4\hat{i},\ 1\hat{j} \rangle$

$\vec{B} = \langle 2\hat{i},\ 1\hat{j} \rangle$

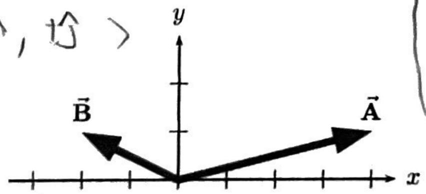


$\vec{A} \times \vec{B} = 0\hat{i} + 0\hat{j} + 2\hat{k}$

$\vec{B} \times \vec{A} = 0\hat{i} + 0\hat{j} - 2\hat{k}$

c) $\vec{A} = \langle 4\hat{i},\ 1\hat{j} \rangle$

$\vec{B} = \langle -2\hat{i},\ 1\hat{j} \rangle$

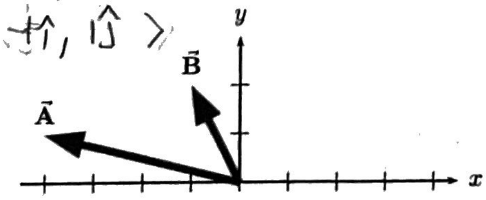


$\vec{A} \times \vec{B} = 0\hat{i} + 0\hat{j} + 6\hat{k}$

$\vec{B} \times \vec{A} = 0\hat{i} + 0\hat{j} - 6\hat{k}$

d) $\vec{A} = \langle -4\hat{i},\ 1\hat{j} \rangle$

$\vec{B} = \langle -1\hat{i},\ 1\hat{j} \rangle$



$\vec{A} \times \vec{B} = 0\hat{i} + 0\hat{j} - 7\hat{k}$

$\vec{B} \times \vec{A} = 0\hat{i} + 0\hat{j} + 7\hat{k}$

1) $\vec{A} \times \vec{B}$ $\begin{vmatrix} 4 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$ $\langle 1(0)-0(2), 0(1)-4(0), 4(2)-1(1) \rangle$

$\langle 0, 0, 8-1 \rangle$

$\langle 0,0,7 \rangle$

$\vec{B} \times \vec{A}$ $\begin{vmatrix} 1 & 2 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ $\langle 2(0)-0(1), 0(4)-1(0), 1(1)-2(4) \rangle$

$\langle 0,0,1-8 \rangle$

$\langle 0,0,-7 \rangle$

2.) $\vec{A} \times \vec{B}$ $\begin{vmatrix} 4 & 1 & 0 \\ 2 & 1 & 0 \end{vmatrix}$ $\langle 1(0)-0(1), 0(2)-4(0), 4(1)-1(2) \rangle$

$\langle 0,0,4-2 \rangle$

$\langle 0,0,2 \rangle$

$\vec{B} \times \vec{A}$ $\begin{vmatrix} 2 & 1 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ $\langle 1(0)-0(1), 0(4)-2(0), 2(1)-1(4) \rangle$

$\langle 0,0, 2-4 \rangle$

$\langle 0,0,-2 \rangle$

3.) $\vec{A} \times \vec{B}$ $\begin{vmatrix} 4 & 1 & 0 \\ 2 & 1 & 0 \end{vmatrix}$ $\langle 1(0)-0(1), 0(-2)-4(0), 4(1)-1(-2) \rangle$

$\langle 0,0,6 \rangle$

$\vec{B} \times \vec{A}$ $\begin{vmatrix} -2 & 1 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ $\langle 1(0)-0(1), 0(4)+2(0), -2(1)-1(4) \rangle$

$\langle 0,0,-2-4 \rangle$

$\langle 0,0,-6 \rangle$

4) $\vec{A} \times \vec{B}$ $\begin{vmatrix} -4 & 1 & 0 \\ -1 & 2 & 0 \end{vmatrix}$ $\langle 1(0)-0(1), 0(-1)+4(0), -4(2)-1(-1) \rangle$

$\langle 0,0,-8+1 \rangle$

$\langle 0,0,-7 \rangle$

$\vec{B} \times \vec{A}$ $\begin{vmatrix} -1 & 2 & 0 \\ -4 & 1 & 0 \end{vmatrix}$ $\langle 2(0)-0(1), 0(-4)-1(0), -1(1)-2(-4) \rangle$

$\langle 0,0,7 \rangle$

$$\vec{A} \times \vec{B} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$