# Line Integrals

$\int_c F\, ds$ $\quad \vec{r}(t) = \langle x(t), y(t) \rangle$

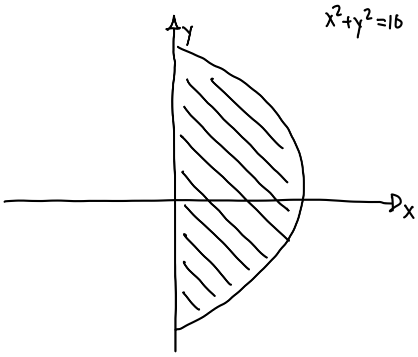
$ds = |\vec{r}\,'(t)|\, dt = \sqrt{(x')^2 + (y')^2}\, dt$

$\int_c F\, ds = \int_{t=a}^{t=b} F(x(t), y(t))\, |\vec{r}\,'(t)|\, dt$

**16.2.1** $\int_c xy^4\, ds$

C is right half of circle

$x^2+y^2=16$




$x = 4\cos t$ $\quad \vec{r}(t) = \langle 4\cos t, 4\sin t \rangle \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$y = 4\sin t$ $\quad \vec{r}\,'(t) = \langle -4\sin t, 4\cos t \rangle$

$\sqrt{(4\sin t)^2 + (4\cos t)^2} = \sqrt{16\sin^2 t + 16\cos^2 t} = \sqrt{16} = 4$

$4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos t\,(4^4 \sin^4 t)\, dt$

$4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4^5 \cos t \sin^4 t\, dt$ $\quad u = \sin t\, dt$

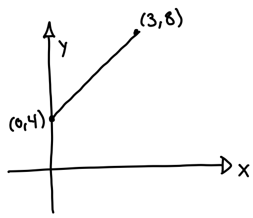
$du = \cos t\, dt$

$4^6\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \sin^4 t\, dt$

$4^6\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u^4\, du$ $\quad \frac{4^6}{5}(2)$

$4^6\left[\frac{1}{5}\sin^5 t\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

**16.2.2** $\int_c x\sin y\, ds$ $\quad (0,4) \to (3,8)$




$\langle 3,4 \rangle$

$\langle 0,4 \rangle + t\langle 3,4 \rangle$

$\langle 3t, 4+4t \rangle$

$\int_0^1 3t\sin(4+4t)\sqrt{(3)^2+(4)^2}\, dt$ $\quad u = 3t \quad dv = \sin(4+4t)$

$5\int_0^1 3t\sin(4+4t)\, dt$ $\quad du = 3\, dt \quad v = -\frac{1}{4}\cos(4+4t)$

$3\left(-\frac{1}{4}\cos(4+4t)\right) + \frac{1}{4}\int_0^1 \cos(4+4t)\, 3\, dt$

$-\frac{3}{4}\cos(4+4t) + \frac{3}{4}\int_0^1 \cos(4+4t)$

$-\frac{3}{4}\cos(4+4t) + \frac{3}{16}\sin(4+4t) \Big|_0^1$

**16.2.5** $\int_c z^2 dx + x^2 dy + y^2 dz$ $\quad (1,0,0)$ to $(5,1,2)$

$\langle 4,1,2 \rangle$

$\int_0^1 (2t)^2 4\, dt + (1+4t)^2 (1)\, dt + (t^2)2\, dt$ $\quad \langle 1,0,0 \rangle + t\langle 4,1,2 \rangle$

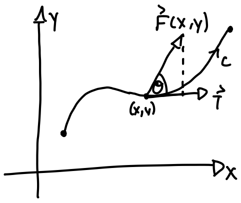
$\int_0^1 16t^2 + (1+4t)^2 + 2t^2\, dt$

$x = 1+4t \quad 0 \le t \le 1$

$y = t$

$z = 2t$

**Defn** The integral of vector field $\vec{F}$ along curve $C$ is defined to be $\int_C \vec{F}\cdot\vec{T}\,ds$




$$\vec{F}\cdot\vec{T} = |\vec{F}||\vec{T}|\cos\theta$$
$$= |\vec{F}|\cos\theta$$

Let $\vec{r}(t)$, $a \le t \le b$, be a parametrization of $C$

$$\vec{T} = \frac{r'(t)}{|\vec{r}'|} \qquad ds = |\vec{r}'|\,dt$$

$$\int_{t=a}^{t=b} \vec{F}(x(t),y(t)) \cdot \frac{\vec{r}'(t)}{|\vec{r}'(t)|} \cdot |\vec{r}'(t)|\,dt = \int_a^b \vec{F}\cdot\vec{r}'(t) \quad c$$