Physics 231, Exam 3

Mainly Ch. 7

(Dated: May 3, 2018)

Abstract

NAME: Taylor Larrechea

DIRECTIONS: Show all work and make your reasoning clear. Make sure I can follow your steps and locate answers. Closed-book, only a calculator, single notecard, are permitted as resources. I will provide necessary integrals, polynomial tables, etc. on the board.

Mostly conceptual and short calculations

PACS numbers:

I. EXAM PROBLEMS TOTAL SCORE: 98

1.) If you wrote the Schrödinger equation for the hydrogen atom in Cartesian coordinates, excluding spin, how many quantum numbers would there be? **5 pts.**

There are 4 main quantum numbers: N - Principal: $n=0,1,2\ldots$
ℓ - Orbital Angular Momentum: $\ell=0,1,2,3,n-1$
m_ℓ - Magnetic: $-\ell, \ell+1, 0, \ell-1, \ell$
m_s - $\pm\frac{1}{2}$ for all m_ℓ

With spin there are 4
Without spin there are 3 ✓ 5

2.) Explain what is meant by the statement "an external applied field can *lift* or *split* the degeneracy of a quantum system." *HINT:* Think about the normal Zeeman effect as an example. **10 pts.**

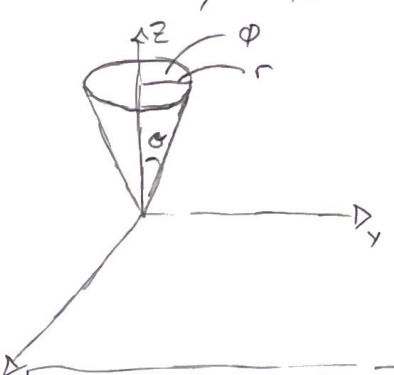
The normal Zeeman effect is when spectral lines are seperated due to an external magnetic field. What the phrase above is stating is the amount of energy is either increased distributed evenly ✓ amongst spectral lines that are split due to the normal Zeeman effect.

10

3.) Write an expression for the probability of finding the electron anywhere in a conical region $\pi/4$ radians wide centered on the z-axis for a hydrogen atom in a state n, ℓ, and m_ℓ in terms of radial and spherical eigenfunctions $R_{n,\ell}(r)$ and $Y_{\ell,m_\ell}(\theta,\phi)$. Do not evaluate anything, just set it up. Diagram helpful but not required. **25 pts.**

23

Probability: $P(r)\,dr = \int_{-\pi}^{r}\int_{-\phi}^{\phi}\int_{-\theta}^{\theta} R_{n,\ell}(r)\cdot Y_{\ell,m_\ell}(\theta,\phi)\, r^2\sin\theta\, d\theta\, d\phi\, dr$



θ: Angle off of z-axis: $\theta = \pi/4$
ϕ: Angle through x-y plane: $\phi = 2\pi$
r: Radial distance from origin of z-axis. This isn't stated so I am going to leave it going to ∞. It would be more correct to leave it in r, for any values of r.

$$P(r)\,dr = \int_0^{\infty}\int_0^{2\pi}\int_{-\pi/8}^{\pi/8} R_{n,\ell}(r)\cdot Y_{\ell,m_\ell}(\theta,\phi)\, r^2\sin\theta\, d\theta\, d\phi\, dr$$

-2

For the next 4 questions, consider a hydrogen atom in the state $3p$.

ℓ = 0 1 2 3 4 5
Letter: S P d F g h

3P = n = 3, ℓ = 1

4.) What is the degeneracy (not including spin)? **5 pts.**

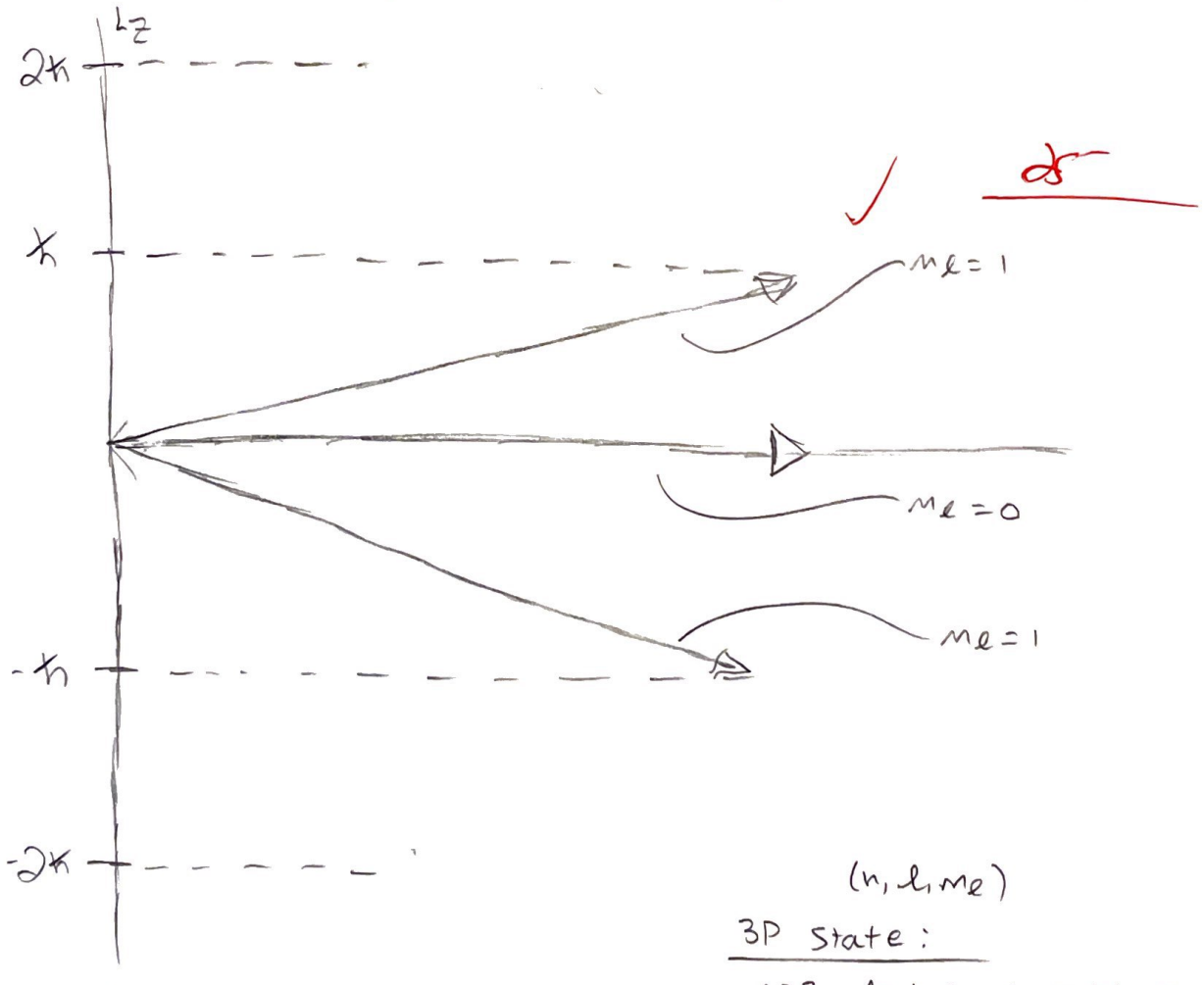
Degeneracy without spin, # of m_ℓ states = 3
Degeneracy with spin, 2(m_ℓ) states = 6

3P n=3, ℓ=1,0, m_ℓ = ±1, 0
(3,1,1), (3,0,0), (3,1,-1)
3 total m_ℓ states

5

This is my reasoning

5.) Draw a diagram representing the possible *orbital* angular momentum vectors. Indicate clearly the magnitude of the angular momenta and their z-components in units of $\hbar$.**25 pts.**



5

$l_z = m_\ell \hbar$

(n, ℓ, m_ℓ)

3P State:

n=3, ℓ=1,0, m_ℓ = ±1, 0

(3,1,1) (3,0,0)
(3,1,-1)

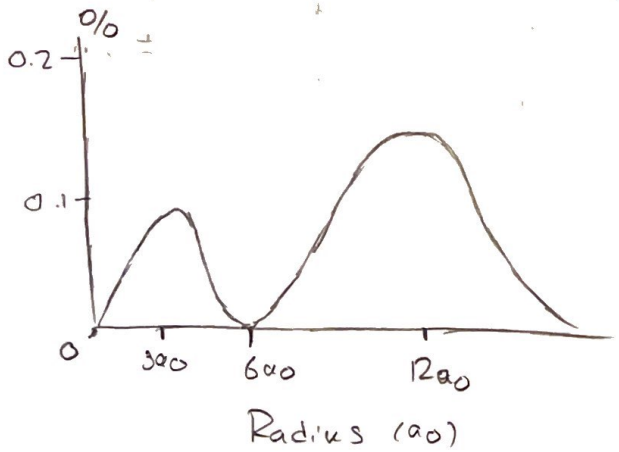
3,1

3P = $n=3$, $\ell=1,0$, $m_\ell=\pm1,0$

6.) Find the radii at which the probability density has local maxima. **25 pts.**

3P : (3, 1, ±1)

work on scratch paper



R = ($0a_0$, $3a_0$, $6a_0$, $12a_0$)

25

Maxima occur at ($3a_0$ & $12a_0$)

7.) Which one of these represents the most likely location of the electron? **5 pts.**

Since @ $12a_0$ the probability is at its highest, this is most likely where the electron is

5

Problem 6

$3P : (3, 1, \pm 1)$ $\quad P(r) = r^2 |R_{nl}(R)|^2$

$$R_{31} = \frac{1}{a_0^{3/2}} \frac{4}{81\sqrt{6}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$$

$$R_{31}^2 = \frac{1}{a_0^3} \frac{16}{81^2(6)} \left(6 - \frac{r}{a_0}\right)^2 \frac{r^2}{a_0^2} e^{-2r/3a_0}$$

$$R_{31}^2 r^2 = \frac{1}{a_0^3} \frac{16}{81^2(6)} (r^2) \left(6 - \frac{r}{a_0}\right)^2 \frac{r^2}{a_0^2} e^{-2r/3a_0}$$

$$P(r) = \frac{1}{a_0^3} \frac{16}{81^2(6)} \left(36 - \frac{12r}{a_0} + \frac{r^2}{a_0^2}\right) \frac{r^4}{a_0^2} e^{-2r/3a_0}$$

$$P(r) = \frac{1}{a_0^3} \frac{16}{81^2(6)} \left(\frac{36r^4}{a_0^2} - \frac{12r^5}{a_0^3} + \frac{r^6}{a_0^4}\right) e^{-2r/3a_0}$$

$$\frac{dP}{dr} = (B)\left(\frac{-2}{3a_0}\right)\left(36\frac{r^4}{a_0^2} - \frac{12r^5}{a_0^3} + \frac{r^6}{a_0^4}\right)e^{-2r/3a_0} + (B)\, e^{-2r/3a_0}\left(144\frac{r^3}{a_0^2} - \frac{60r^4}{a_0^3} + \frac{6r^5}{a_0^4}\right)$$

$$0 = e^{-2r/3a_0} B\left(-\frac{2}{3a_0}\left(36\frac{r^4}{a_0^2} - \frac{12r^5}{a_0^3} + \frac{r^6}{a_0^4}\right) + \left(144\frac{r^3}{a_0^2} - \frac{60r^4}{a_0^3} + \frac{6r^5}{a_0^4}\right)\right)$$

$$0 = -\frac{72r^4}{3a_0^3} + \frac{24r^5}{3a_0^4} - \frac{2r^6}{3a_0^5} + 144\frac{r^3}{a_0^2} - \frac{60r^4}{a_0^3} + \frac{6r^5}{a_0^4}$$

$$0 = 144\frac{r^3}{a_0^2} - \frac{84r^4}{a_0^3} + \frac{14r^5}{a_0^4} - \frac{2r^6}{3a_0^5}$$

$$0 = 144\frac{r^2}{a_0^2} - \frac{84r^3}{a_0^3} + \frac{14r^4}{a_0^4} - \frac{2r^5}{3a_0^5} \qquad x \equiv \frac{r}{a_0}$$

$$0 = 144x^2 - 84x^3 + 14x^4 - \frac{2}{3}x^5$$

using root tool on calculator

$x = 0, 3, 6, 12$

$r = x a_0$ $\quad \boxed{r = (0a_0, 3a_0, 6a_0, 12a_0)}$

$x e^{-x}$

$(1)e^{-x} + x$

$e^{-x} - x e^{-x}$