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PHYS 321
HW 6

3.9, 3.10, 3.11, 3.13, 3.17, 3.22, 3.27, 3.31, 3.37, 3.38

Problem 3.9

a.) ∞ Square well : $\boxed{\psi_n(x) = A\sin\left(\frac{n\pi x}{a}\right)}$

b.) Rectangular Barrier : $\boxed{\psi_n(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)}$

c.) $\boxed{[\hat{x},\hat{p}] = i}$

Problem 3.10

No, the ground state of the infinite Square well is not an eigenfunction of momentum. It is not an eigenfunction because momentum will not depend on the direction of the particle.

Problem 3.11

$\varphi(P,t)$: $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega}{2\hbar}x^2}$, $\varphi_0(t) = e^{\frac{-i\omega t}{2}}$

$$\psi_0(x,t) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega}{2\hbar}x^2} \cdot e^{\frac{-i\omega t}{2}}$$

$$\varphi_0(P,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} e^{-ipx/\hbar} \psi_0(x,t)\,dx = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} e^{\frac{-iPx}{\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} e^{\frac{-i\omega t}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-i\omega t}{2}} \int_{-\infty}^{+\infty} e^{-\left(\frac{m\omega}{2\hbar}x^2 + \frac{iPx}{\hbar}\right)} dx \qquad \alpha = \frac{m\omega}{2\hbar}, \quad \beta = \frac{iP}{\hbar}$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-i\omega t}{2}} \int_{-\infty}^{+\infty} e^{-(\alpha x^2 + \beta x)} dx \quad : \quad \int_{-\infty}^{+\infty} e^{-(\alpha x^2 + \beta x)} dx = \sqrt{\frac{\pi}{\alpha}}\, e^{\frac{\beta^2}{4\alpha}}$$

$$\int_{-\infty}^{+\infty} e^{-(\alpha x^2 + \beta x)} dx = \sqrt{\frac{\pi}{\alpha}}\, e^{\frac{\beta^2}{4\alpha}} \quad : \quad \beta^2 = -\frac{p^2}{\hbar^2} \quad : \quad 4\alpha = \frac{2m\omega}{\hbar} \quad : \quad \frac{\beta^2}{4\alpha} = \frac{\frac{-p^2}{\hbar^2}}{\frac{2m\omega}{\hbar}} = \frac{-p^2}{2m\omega\hbar}$$

$$= \sqrt{\frac{\pi}{\frac{1}{\frac{m\omega}{2\hbar}}}} = \sqrt{\frac{2\hbar\pi}{m\omega}}$$

$$\int_{-\infty}^{+\infty} e^{-(\alpha x^2 + \beta x)} dx = \sqrt{\frac{2\pi\hbar}{m\omega}}\, e^{\frac{-p^2}{2m\omega\hbar}}$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-i\omega t}{2}} \cdot \sqrt{\frac{2\pi\hbar}{m\omega}}\, e^{\frac{-p^2}{2m\omega\hbar}} = \frac{1}{\sqrt{m\omega}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-i\omega t}{2}} \cdot e^{\frac{-p^2}{2m\omega\hbar}}$$

$$\varphi_0(P,t) = \left(\frac{1}{m\omega\pi\hbar}\right)^{1/4} e^{\frac{-i\omega t}{2}} \cdot e^{\frac{-p^2}{2m\omega\hbar}} \quad : \quad p = \pm\sqrt{m\omega\hbar}$$

$$P = \int_{-\infty}^{-\sqrt{m\omega\hbar}} |\varphi|^2 dp + \int_{\sqrt{m\omega\hbar}}^{\infty} |\varphi|^2 dp \quad : \quad \int_{-\infty}^{+\infty} |\varphi|^2 dp - \int_{-\sqrt{m\omega\hbar}}^{\sqrt{m\omega\hbar}} |\varphi|^2 dp$$

$$P = \underbrace{\int_{-\infty}^{+\infty} |\varphi|^2 dp}_{(1)} - 2\underbrace{\int_{0}^{\sqrt{m\omega\hbar}} |\varphi|^2 dp}_{(2)} \quad : \quad |\varphi^2| = \left(\frac{1}{m\omega\pi\hbar}\right)^{1/2} e^{\frac{-p^2}{m\omega\hbar}}$$

$$(2): \quad 2\int_0^{\sqrt{m\omega\hbar}} |\varphi^2|\,dp = 2\left(\frac{1}{m\omega\pi\hbar}\right)^{1/2} \int_0^{\sqrt{m\omega\hbar}} e^{\frac{-p^2}{m\omega\hbar}} dp$$

$\alpha = \sqrt{\frac{2}{m\omega\hbar}}\,p$: $d\alpha = \sqrt{\frac{2}{m\omega\hbar}}\,dp$ $\quad p = 0, \ \alpha = 0$

$\quad p = \sqrt{m\omega\hbar}, \ \alpha = \sqrt{2}$

$$\frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{2}} e^{\frac{-\alpha^2}{2}} d\alpha = \frac{1}{\sqrt{2\pi}}(1.05617) = 0.42$$

$$P = \int_{-\infty}^{+\infty} |\varphi|^2 dp - 2\int_0^{\sqrt{m\omega\hbar}} |\varphi|^2 dp = 1 - 2(0.42) = 1 - 0.84 = 0.16 \qquad \boxed{P = 0.16}$$

Problem 3.13

$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

a.) $[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B}$

$$[\hat{A}\hat{B},\hat{C}] = \hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B}$$

$$\hat{A}[\hat{B},\hat{C}] = \hat{A}\left(\hat{B}\hat{C} - \hat{C}\hat{B}\right) = \hat{A}\hat{B}\hat{C} - \hat{A}\hat{C}\hat{B} \longrightarrow (3)$$

$$[\hat{A},\hat{C}]\hat{B} = \left(\hat{A}\hat{C} - \hat{C}\hat{A}\right)\hat{B} = \hat{A}\hat{C}\hat{B} - \hat{C}\hat{A}\hat{B} \longrightarrow (4)$$

$$(3)+(4) = \hat{A}\hat{B}\hat{C} - \cancel{\hat{A}\hat{C}\hat{B}} + \cancel{\hat{A}\hat{C}\hat{B}} - \hat{C}\hat{A}\hat{B} = \hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B} = [\hat{A}\hat{B},\hat{C}]$$

$$\boxed{[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B}}$$

b.) $[x^n,\hat{p}]$: $\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$

$$[x^n,\hat{p}] = x^n\cdot\hat{p} - \hat{p}\cdot x^n = x^n\cdot\frac{\hbar}{i}\frac{d}{dx} - \frac{\hbar}{i}\frac{d}{dx}(x)^n$$

$$= \cancel{x^n\cdot\frac{\hbar}{i}\frac{d}{dx}}^{\,0} + i\hbar(n)x^{n-1} = i\hbar n x^{n-1}$$

$$\boxed{[x^n,p^n] = i\hbar n x^{n-1}}$$

c.) $[f(x),\hat{p}] = i\hbar\frac{df}{dx}$

$$[f(x),\hat{p}] = f(x)\cdot\hat{p} - \hat{p}f(x) = f(x)\cdot\frac{\hbar}{i}\frac{d}{dx} - \frac{\hbar}{i}\frac{d}{dx}f(x)$$

$$= \cancel{f(x)\cdot\frac{\hbar}{i}\frac{d}{dx}}^{\,0} + \hbar i\frac{df}{dx} = i\hbar\frac{df}{dx}$$

$$\boxed{[f(x),\hat{p}] = i\hbar\frac{df}{dx}}$$

Problem 3.17

Eq. (3.71) : $\frac{d}{dt}\langle\hat{Q}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$

a.) $Q=1$

$$\boxed{\frac{d}{dt}\langle 1\rangle = 0 \quad : \text{ Conservation of normalization, } \Psi \text{ is constant w/ time}}$$

b.) $Q=H$

$$\frac{d}{dt}\langle\hat{H}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{H}]\rangle + \left\langle\frac{\partial\hat{H}}{\partial t}\right\rangle$$

$$= \frac{i}{\hbar}\langle \cancelto{0}{H^2 - H^2}\rangle + \left\langle\frac{\partial\hat{H}}{\partial t}\right\rangle = \left\langle\frac{\partial\hat{H}}{\partial t}\right\rangle \quad : \quad \frac{\partial\hat{H}}{\partial t} = 0$$

$$= \frac{i}{\hbar}(0) + (0) = 0$$

$$\boxed{\frac{d\langle\hat{H}\rangle}{dt} = 0 \quad : \text{ Energy is conserved, } \Psi \text{ is constant w/ time}}$$

c.) $Q=x$

$$\frac{d}{dt}\langle\hat{x}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{x}]\rangle + \left\langle\frac{\partial\hat{x}}{\partial t}\right\rangle \qquad \hat{H} = \frac{P^2}{2m} + V$$

$$= \frac{i}{\hbar}\langle[P^2/2m + V, x]\rangle + \left\langle\frac{\partial x}{\partial t}\right\rangle$$

$$\langle[P^2/2m + V, x]\rangle = \left\langle \frac{1}{2m}[P^2,x] + \cancelto{0}{[V,x]}\right\rangle = \left\langle \frac{1}{2m}\cdot\hat{P}[\hat{P},\hat{x}] + \frac{1}{2m}[\hat{P},\hat{x}]\hat{P}\right\rangle$$

$$[A^2,B] = A[A,B] + [A,B]A$$

$$[\hat{x},\hat{P}] = i\hbar \quad \therefore \quad [\hat{P},\hat{x}] = -i\hbar$$

$$= \left\langle \tfrac{1}{2m}\cdot P\cdot(-i\hbar) + \tfrac{1}{2m}\cdot P\cdot(-i\hbar)\right\rangle = \frac{1}{2m}\langle -Pi\hbar - Pi\hbar\rangle = -\frac{2iP\hbar}{2m} = \left\langle -\frac{i\hbar P}{m}\right\rangle$$

$$= \frac{i}{\hbar}\left\langle\left(-\frac{i\hbar P}{m}\right)\right\rangle + \cancelto{0}{\left\langle\frac{\partial x}{\partial t}\right\rangle} = \frac{\langle P\rangle}{m}$$

$$\boxed{\frac{d\langle\hat{x}\rangle}{dt} = \frac{\langle P\rangle}{m} \quad : \text{ Energy is conserved, } \Psi \text{ is constant w/ time.}}$$

d.) $Q=P$

$$\frac{d}{dt}\langle\hat{P}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{P}]\rangle + \left\langle\frac{\partial\hat{P}}{\partial t}\right\rangle \qquad \hat{H} = \frac{P^2}{2m} + V$$

$$\frac{i}{\hbar}\langle[P^2/2m + V, P] + \left\langle\frac{\partial\hat{P}}{\partial t}\right\rangle$$

$$\langle [p^2/2m + V, p] \rangle = \langle \tfrac{1}{2m}[\hat{p}^2, \hat{p}] + [V, \hat{p}] \rangle \qquad \hat{H} = \frac{p^2}{2m} + V, \quad \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$= \langle \tfrac{1}{2m}(\cancel{\hat{p}^3} - \hat{p}^3)^{0} + [V, \hat{p}] \rangle$$

$$= \langle [V, \hat{p}] \rangle = \langle V\hat{p} - \hat{p}V \rangle$$

$$= \langle V \cdot \cancel{\tfrac{\hbar}{i}\tfrac{\partial}{\partial x}}^{0} - \tfrac{\hbar}{i}\tfrac{\partial}{\partial x} V \rangle = \langle i\hbar\, \partial V/\partial x \rangle \qquad \frac{\partial \hat{p}}{\partial t} = 0$$

$$\frac{d\langle \hat{p} \rangle}{dt} = \frac{i}{\hbar} \langle i\hbar\, \partial V/\partial x \rangle + \langle 0 \rangle = \langle -\partial V/\partial x \rangle$$

$$\boxed{\frac{d\langle \hat{p} \rangle}{dt} = \langle -\partial V/\partial x \rangle : \text{Ehrenfests theorem, } \Psi \text{ is constant w/ time.}}$$

Problem 3.22

$$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle, \quad |\beta\rangle = i|1\rangle + 2|3\rangle$$

a.)

$\langle\alpha|$: $\langle\alpha| = (a_1^* a_2^* \dots a_n^*)$ $\qquad$ $\langle\beta|$: $\langle\beta| = (b_1^* b_2^* \dots b_n^*)$

$$\boxed{\langle\alpha| = -i\langle 1| - 2\langle 2| + i\langle 3|, \quad \langle\beta| = -i\langle 1| + 2\langle 3|}$$

b.)

$\langle\alpha|\beta\rangle$: $\langle\alpha| = -i\langle 1| - 2\langle 2| + i\langle 3|$, $\quad |\beta\rangle = i|1\rangle + 2|3\rangle$

$$\langle\alpha|\beta\rangle = -i^2\langle 1\cancel{|}1\rangle + 2i\langle 3\cancel{|}3\rangle = 1+2i$$

$$\boxed{\langle\alpha|\beta\rangle = 1+2i}$$

$\langle\beta|\alpha\rangle$: $\langle\beta| = -i\langle 1| + 2\langle 3|$, $\quad |\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle$

$$\langle\beta|\alpha\rangle = -i^2\langle 1\cancel{|}1\rangle - 2i\langle 3\cancel{|}3\rangle = 1-2i$$

$$\boxed{\langle\beta|\alpha\rangle = 1-2i}$$

$\langle\alpha|\beta\rangle = 1+2i$: $\langle\alpha|\beta\rangle^* = 1^* + (2i)^* = 1 - 2i = \langle\beta|\alpha\rangle$

$$\therefore \quad \boxed{\langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle}$$

c.) $\hat{A} \equiv |\alpha\rangle\langle\beta| \longrightarrow A_{mn} = \langle m|A|n\rangle$

$A_{11} = \langle 1|\hat{A}|1\rangle = \langle 1|\alpha\rangle\langle\beta|1\rangle = (i)(-i) = 1$: $A_{11} = 1$

$A_{12} = \langle 1|\hat{A}|2\rangle = \langle 1|\alpha\rangle\langle\beta|2\rangle = (i)(0) = 0$: $A_{12} = 0$

$A_{13} = \langle 1|\hat{A}|3\rangle = \langle 1|\alpha\rangle\langle\beta|3\rangle = (i)(2) = 2i$: $A_{13} = 2i$

$A_{21} = \langle 2|\hat{A}|1\rangle = \langle 2|\alpha\rangle\langle\beta|1\rangle = (-2)(-i) = 2i$: $A_{21} = 2i$

$A_{22} = \langle 2|\hat{A}|2\rangle = \langle 2|\alpha\rangle\langle\beta|2\rangle = (-2)(0) = 0$: $A_{22} = 0$

$A_{23} = \langle 2|\hat{A}|3\rangle = \langle 2|\alpha\rangle\langle\beta|3\rangle = (-2)(2) = -4$: $A_{23} = -4$

$A_{31} = \langle 3|\hat{A}|1\rangle = \langle 3|\alpha\rangle\langle\beta|1\rangle = (-i)(-i) = -1$: $A_{31} = -1$

$A_{32} = \langle 3|\hat{A}|2\rangle = \langle 3|\alpha\rangle\langle\beta|2\rangle = (-i)(0) = 0$: $A_{32} = 0$

$A_{33} = \langle 3|\hat{A}|3\rangle = \langle 3|\alpha\rangle\langle\beta|3\rangle = (-i)(2) = -2i$: $A_{33} = -2i$

$$\boxed{\hat{A} = \begin{pmatrix} 1 & 0 & 2i \\ 2i & 0 & -4 \\ -1 & 0 & -2i \end{pmatrix}, \quad \hat{A}^\dagger = \begin{pmatrix} 1 & -2i & -1 \\ 0 & 0 & 0 \\ -2i & -4 & 2i \end{pmatrix} \quad \text{Since } \hat{A} \neq \hat{A}^\dagger, \text{ this matrix is not Hermetian}}$$

Problem 3.27

$$\psi_1 = (3\varphi_1 + 4\varphi_2)/5 \quad , \quad \psi_2 = (4\varphi_1 - 3\varphi_2)/5$$

a.) $\hat{A}\psi = a\psi$

$$\hat{A}\psi_1 = a_1\psi_1$$
$$\hat{A}\psi_2 = a_2\psi_2$$

$$\hat{A}\left[(3\varphi_1 + 4\varphi_2)/5\right] = a_1\left[(3\varphi_1 + 4\varphi_2)/5\right]$$

After a_1 is measured, nothing happens to the wavefunction and we remain in ψ_1.

b.) $\hat{B}\psi = b\psi$

$$\hat{B}\psi_1 = b_1\psi_1 \qquad b_1 = 3/5 \quad , \quad b_2 = 4/5$$
$$\hat{B}\psi_2 = b_2\psi_2$$

$$\boxed{|b_1| = 9/25 \rightarrow 36\%}$$
$$\boxed{|b_2| = 16/25 \rightarrow 64\%}$$

c.) $\psi_1 = (3\varphi_1 + 4\varphi_2)/5 \quad , \quad \psi_2 = (4\varphi_1 - 3\varphi_2)/5$

$$\varphi_1 = \frac{5\psi_1 - 4\varphi_2}{3} \qquad \varphi_2 = \frac{4\varphi_1 - 5\psi_2}{3}$$

$$\varphi_1 = \frac{5}{3}\psi_1 - \frac{4}{3}\left(\frac{4\varphi_1 - 5\psi_2}{3}\right) = \frac{5}{3}\psi_1 - \frac{16\varphi_1}{9} + \frac{20\psi_2}{9} \quad : \quad \varphi_1 + \frac{16\varphi_1}{9} = \frac{5}{3}\psi_1 + \frac{20\psi_2}{9}$$

$$\frac{25\varphi_1}{9} = \frac{15}{9}\psi_1 + \frac{20}{9}\psi_2 \rightarrow \left[\varphi_1 = \frac{3}{5}\psi_1 + \frac{4}{5}\psi_2\right]^{(*)}$$

$$\varphi_2 = \frac{4}{3}\left(\frac{5\psi_1 - 4\varphi_2}{3}\right) - \frac{5}{3}\psi_2 = \frac{20}{9}\psi_1 - \frac{16\varphi_2}{9} - \frac{5}{3}\psi_2 \quad : \quad \varphi_2 + \frac{16\varphi_2}{9} = \frac{20}{9}\psi_1 - \frac{5}{3}\psi_2$$

$$\frac{25}{9}\varphi_2 = \frac{20}{9}\psi_1 - \frac{15}{9}\psi_2 \rightarrow \left[\varphi_2 = \frac{4}{5}\psi_1 - \frac{3}{5}\psi_2\right]^{(**)}$$

$$|\varphi_1| = \frac{9}{25}|\psi_1| + \frac{16}{25}|\psi_2| \quad : \quad |\psi_1| = 9/25 \quad , \quad |\psi_2| = 16/25$$

$$= \frac{9}{25}\left(\frac{9}{25}\right) + \frac{16}{25}\left(\frac{16}{25}\right) = \frac{81}{625} + \frac{256}{625} = \frac{337}{625}$$

$$\boxed{|a_1| \approx 53.9\%}$$

Problem 3.31

$$\frac{d}{dt}\langle Q\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$$

$Q = xP$: $\frac{d}{dt}\langle xP\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{x}\hat{P}]\rangle + \left\langle\frac{\partial\hat{x}\hat{P}}{\partial t}\right\rangle$

$$[\hat{H},\hat{x}\hat{P}] = [\hat{H},\hat{x}]\hat{P} + \hat{x}[\hat{H},\hat{P}]$$

From problem 3.17 : $[\hat{H},\hat{x}] = \frac{-i\hbar P}{m}$ $\quad [\hat{H},\hat{P}] = i\hbar\frac{\partial V}{\partial x}$

$[\hat{H},\hat{x}\hat{P}] = -\frac{i\hbar P}{m}(P) + (x)i\hbar\frac{\partial V}{\partial x}$: $\left(-\frac{i\hbar P}{m}(P) + (x)i\hbar\frac{\partial V}{\partial x}\right) = \frac{-i\hbar P^2}{m} + xi\hbar\frac{\partial V}{\partial x}$

$\frac{d\langle xP\rangle}{dt} = \frac{i}{\hbar}\left\langle\frac{-i\hbar P^2}{m} + xi\hbar\frac{\partial V}{\partial x}\right\rangle + \left\langle\frac{\partial\hat{x}\hat{P}}{\partial t}\right\rangle$: $\frac{\partial\hat{x}\hat{P}}{\partial t} = 0$ for stationary states

$$= \langle P^2/m - x\cdot\partial V/\partial x\rangle + \cancel{\left\langle\frac{\partial\hat{x}\hat{P}}{\partial t}\right\rangle}^{\,0}$$

$= 2\cdot\langle P^2\rangle/2m - \langle x\,\partial V/\partial x\rangle$: $\frac{\langle P^2\rangle}{2m} = \langle T\rangle$

$$\boxed{\frac{d\langle xP\rangle}{dt} = 2\langle T\rangle - \langle x\,\partial V/\partial x\rangle}$$

$\frac{d\langle xP\rangle}{dt}$ is zero since both $\langle x\rangle$ and $\langle P\rangle$ are independent of time when stationary.

$2\langle T\rangle = \left\langle x\frac{dV}{dx}\right\rangle$: $V(x) = \frac{1}{2}m\omega^2x^2$

$\frac{dV}{dx} = m\omega^2 x$: $x\cdot\frac{dV}{dx} = m\omega^2x^2 = 2\cdot V$

$$\therefore \cancel{2}\langle T\rangle = \cancel{2}\langle V\rangle$$

$$\boxed{\langle T\rangle = \langle V\rangle}$$

Problem 3.37

$$H = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}$$

a.) $|\mathcal{S}(0)\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\qquad \hat{H} - \hat{I}(\lambda) = \hat{G}$

$$\hat{G} = \begin{pmatrix} a-\lambda & 0 & b \\ 0 & c-\lambda & 0 \\ b & 0 & a-\lambda \end{pmatrix} \qquad \det(\hat{G}) = (c-\lambda)(A^2 - 2A\lambda + \lambda^2 - B^2) = 0$$
$$(c-\lambda)((\lambda - A)^2 - B^2) = 0$$

$c - \lambda = 0$ $\quad$: $\quad (\lambda - A)^2 - B^2 = 0$
$\lambda = c$ $\qquad\qquad (\lambda - A)^2 = B^2 \qquad \lambda = A+B \,, \quad \lambda = A-B$
$\lambda - A = \pm B$
$\lambda = A \pm B$

$\lambda = c$

$$\begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = c\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{aligned} a(\alpha) + b(\gamma) &= c(\alpha) \\ c(\beta) &= c(\beta) \\ b(\alpha) + a(\gamma) &= c(\gamma) \end{aligned} \longrightarrow \beta = 1$$

$a(\alpha) + b(\gamma) = c(\alpha)$: $b(\gamma) = \alpha(c - a) \longrightarrow$ (5)
$b(\alpha) + a(\gamma) = c(\gamma)$: $b(\alpha) = \gamma(c - a) \longrightarrow$ (6)

(5) + (6) : $b(\gamma) + b(\alpha) = c(\alpha) - a(\alpha) + c(\gamma) - a(\gamma)$

$$0 = (c - b - a)\alpha + (c - b - \gamma)\gamma \longrightarrow (7)$$

(7) yields $\alpha = \gamma = 0$ $\quad \therefore \quad \left[|\lambda_1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right]^{(*)}$

$\lambda = a + b$

$$\begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = a+b\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{aligned} a(\alpha) + b(\gamma) &= a(\alpha) + b(\alpha) \\ c(\beta) &= a(\beta) + b(\beta) \\ b(\alpha) + a(\gamma) &= a(\gamma) + b(\gamma) \end{aligned} \longrightarrow \beta = 0$$

$a(\alpha) + b(\gamma) = a(\alpha) + b(\alpha)$: $b(\gamma) = b(\alpha) \longrightarrow$ (8)
$b(\alpha) + a(\gamma) = a(\gamma) + b(\gamma)$: $b(\alpha) = b(\gamma) \longrightarrow$ (9)

(8) + (9) :: $b(\gamma) + b(\alpha) = b(\alpha) + b(\gamma)$

$$\gamma + \alpha = \alpha + \gamma \; : \; \alpha = \gamma = 1 \longrightarrow (10)$$

(10) yields $\alpha = \gamma = 1$ $\quad \therefore \quad \left[|\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right]^{(**)}$

$A^2(1+1) = 1$
$A^2 = 1/2$
$A = \pm\sqrt{1/2}$

$\lambda = a - b$

$$\begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = a - b\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \to \begin{matrix} a(\alpha) + b(\gamma) = a(\alpha) - b(\alpha) \\ c(\beta) = a(\beta) - b(\beta) \\ b(\alpha) + a(\gamma) = a(\gamma) - b(\gamma) \end{matrix} \longrightarrow \beta = 0$$

$a(\alpha) + b(\gamma) = a(\alpha) - b(\alpha) \;:\; b(\gamma) = -b(\alpha) \longrightarrow (11)$

$b(\alpha) + a(\gamma) = a(\gamma) - b(\gamma) \;:\; b(\alpha) = -b(\gamma) \longrightarrow (12)$

$(11) + (12) :: \; b(\gamma) + b(\alpha) = -b(\alpha) - b(\gamma)$

$$(\gamma) + (\alpha) = -(\alpha + \gamma) \longrightarrow (13)$$

(13) yields $\alpha = -\gamma \;:\; \alpha = 1 \to \gamma = -1$

$$\left[|\lambda_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right]^{(***)}$$

$$|\mathcal{S}(0)\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \;:\; |\mathcal{S}(t)\rangle = e^{\frac{-iE_n t}{\hbar}} |\lambda_1\rangle$$

$$= e^{\frac{-iE_n t}{\hbar}} |\mathcal{S}(0)\rangle$$

$$= e^{\frac{-iE_n t}{\hbar}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\therefore \boxed{|\mathcal{S}(t)\rangle = e^{\frac{-iE_1 t}{\hbar}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$$

b.) $|\mathcal{S}(0)\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \;:\; |\mathcal{S}(t)\rangle = e^{\frac{-iE_n t}{\hbar}} (\mathcal{S}(0))$

$$|\mathcal{S}(0)\rangle = \frac{1}{2}|\lambda_2\rangle + \frac{1}{2}|\lambda_3\rangle$$

$$|\mathcal{S}(t)\rangle = \frac{1}{2}\left(e^{\frac{-iE_2 t}{\hbar}} |\lambda_2\rangle + e^{\frac{-iE_3 t}{\hbar}} |\lambda_3\rangle \right)$$

$e^{\frac{-iat - ibt}{\hbar}}$

$e^{\frac{-iat + ibt}{\hbar}}$

$$= \frac{1}{2}\left(e^{\frac{-i(a+b)t}{\hbar}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + e^{\frac{-i(a-b)t}{\hbar}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)$$

$$= \frac{1}{2}\left[e^{\frac{-iat}{\hbar}} \left(e^{ibt/\hbar} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + e^{-ibt/\hbar} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right) \right] \longrightarrow (14)$$

Problem 3.37 Continued

(14) $\longrightarrow$ $= \frac{e^{-iat/\hbar}}{2}\left[\begin{pmatrix} e^{ibt/\hbar} + e^{-ibt/\hbar} \\ e^{ibt/\hbar} - e^{ibt/\hbar} \end{pmatrix}\right]$ $\longrightarrow$ (15)

$$e^{ix} = \cos(x) + i\sin(x) \quad : \quad e^{-ix} = \cos(x) - i\sin(x)$$

$$e^{ix} + e^{-ix} = 2\cdot\cos(x) \quad : \quad e^{ix} - e^{-ix} = 2i\sin(x)$$

(15) becomes $\frac{e^{-iat/\hbar}}{2}\begin{pmatrix} 2\cos(bt/\hbar) \\ 2i\sin(bt/\hbar) \end{pmatrix} = e^{-iat/\hbar}\begin{pmatrix} \cos(bt/\hbar) \\ i\sin(bt/\hbar) \end{pmatrix}$

$$\boxed{|\mathcal{S}(t)\rangle = e^{-iat/\hbar}\begin{pmatrix} \cos(bt/\hbar) \\ i\sin(bt/\hbar) \end{pmatrix}}$$

Problem 3.38

$$\hat{H} = \hbar\omega\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{A} = \xi\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{B} = \mu\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

a.)

i.) $\hat{H}$: $\hat{H} - \hat{I}(\lambda) = \begin{pmatrix} \hbar\omega-\lambda & 0 & 0 \\ 0 & 2\hbar\omega-\lambda & 0 \\ 0 & 0 & 2\hbar\omega-\lambda \end{pmatrix} = \hat{H}^*$

$$|\hat{H}^*| = (\hbar\omega-\lambda)(2\hbar\omega-\lambda)^2 = 0$$

$$\lambda = \hbar\omega, \quad (2\hbar\omega-\lambda)^2 = 0 \rightarrow \lambda = 2\hbar\omega$$

$$\lambda = \hbar\omega, \quad \lambda = 2\hbar\omega$$

$\lambda_1 = \hbar\omega$

$$\begin{pmatrix} \hbar\omega & 0 & 0 \\ 0 & 2\hbar\omega & 0 \\ 0 & 0 & 2\hbar\omega \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \hbar\omega\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{aligned} \hbar\omega(\alpha) &= \hbar\omega(\alpha) \rightarrow \alpha = 1 \\ 2\hbar\omega(\beta) &= \hbar\omega(\beta) \rightarrow \beta = 0 \\ 2\hbar\omega(\gamma) &= \hbar\omega(\gamma) \rightarrow \gamma = 0 \end{aligned}$$

$$\left[\lambda_1 = \hbar\omega, \quad |\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]^{(*)}$$

$\lambda_2 = 2\hbar\omega$

$$\begin{pmatrix} \hbar\omega & 0 & 0 \\ 0 & 2\hbar\omega & 0 \\ 0 & 0 & 2\hbar\omega \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 2\hbar\omega\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{aligned} \hbar\omega(\alpha) &= 2\hbar\omega(\alpha) \rightarrow \alpha = 0 \\ 2\hbar\omega(\beta) &= 2\hbar\omega(\beta) \rightarrow \beta = 1 \\ 2\hbar\omega(\gamma) &= 2\hbar\omega(\gamma) \rightarrow \beta = 1 \end{aligned}$$

$$\left[\lambda_2 = 2\hbar\omega, \quad |\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right]^{(**)}$$

Same result for $\lambda_3 = 2\hbar\omega$ as for λ_2

$\therefore$ For $\hat{H}$:

$$\boxed{\lambda_1 = \hbar\omega, \ |\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} : \lambda_2 = 2\hbar\omega, \ |\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} : \lambda_3 = 2\hbar\omega, \ |\lambda_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}$$

Linear Combo : $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ii.) $\hat{A}$: $\hat{A} - \hat{I}(\lambda) = \begin{pmatrix} -\lambda & \xi & 0 \\ \xi & -\lambda & 0 \\ 0 & 0 & 2\xi-\lambda \end{pmatrix} = \hat{A}^*$

$$|\hat{A}^*| = (2\xi-\lambda)(\lambda^2-\xi^2) = 0$$

$$\lambda = 2\xi \ , \quad \lambda = \pm\xi$$

$\lambda_1 = 2\xi$

$$\begin{pmatrix} 0 & \xi & 0 \\ \xi & 0 & 0 \\ 0 & 0 & 2\xi \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 2\xi\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{array}{l} \xi(\beta) = 2\xi(\alpha) \rightarrow \beta = \alpha = 0 \\ \xi(\alpha) = 2\xi(\beta) \\ 2\xi(\gamma) = 2\xi(\gamma) \rightarrow \gamma = 1 \end{array}$$

$$\left[\lambda_1 = 2\xi \ , \ |\lambda_1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]^{(*)}$$

$\lambda_2 = \xi$

$$\begin{pmatrix} 0 & \xi & 0 \\ \xi & 0 & 0 \\ 0 & 0 & 2\xi \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \xi\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{array}{l} \xi(\beta) = \xi(\alpha) \rightarrow \alpha = \beta = 1 \\ \xi(\alpha) = \xi(\beta) \\ 2\xi(\gamma) = \xi(\gamma) \rightarrow \gamma = 0 \end{array}$$

$$\left[\lambda_2 = \xi \ , \ |\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right]^{(**)}$$

$\lambda_3 = -\xi$

$$\begin{pmatrix} 0 & \xi & 0 \\ \xi & 0 & 0 \\ 0 & 0 & 2\xi \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = -\xi\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{array}{l} \xi(\beta) = -\xi(\alpha) \rightarrow \alpha = -\beta \\ \xi(\alpha) = -\xi(\beta) \\ 2\xi(\gamma) = -\xi(\gamma) \rightarrow \gamma = 0 \end{array}$$

$$\left[\lambda_3 = -\xi \ , \ |\lambda_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right]^{(***)}$$

$\therefore$ For $\hat{A}$

$$\boxed{\lambda_1 = 2\xi \ , \ |\lambda_1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} : \ \lambda_2 = \xi \ , \ |\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} : \ \lambda_3 = -\xi \ , \ |\lambda_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}$$

iii.) $\hat{B}$: $\hat{B} - \hat{I}(\lambda) = \begin{pmatrix} 2\mu-\lambda & 0 & 0 \\ 0 & -\lambda & \mu \\ 0 & \mu & -\lambda \end{pmatrix} = \hat{B}^*$

$$|\hat{B}^*| = (2\mu-\lambda)(\lambda^2-\mu^2) = 0$$

$$\lambda = 2\mu \ , \quad \lambda = \pm\mu$$

$\lambda_1 = 2\mu$

$$\begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 0 & \mu \\ 0 & \mu & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 2\mu\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{array}{l} 2\mu(\alpha) = 2\mu(\alpha) \rightarrow \alpha = 1 \\ \mu(\gamma) = 2\mu(\beta) \\ \mu(\beta) = 2\mu(\gamma) \rightarrow \beta = \gamma = 0 \end{array}$$

$$\left[\lambda_1 = 2\mu \ , \ |\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]^{(*)}$$

$\lambda_2 = \mu$

$$\begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 0 & \mu \\ 0 & \mu & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \mu\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{array}{l} 2\mu(\alpha) = \mu(\alpha) \rightarrow \alpha = 0 \\ \mu(\gamma) = \mu(\beta) \\ \mu(\beta) = \mu(\gamma) \rightarrow \beta = \gamma = 1 \end{array}$$

$$\left[\lambda_2 = \mu \ , \ |\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right]^{(**)}$$

$\lambda_3 = -\mu$

$$\begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 0 & \mu \\ 0 & \mu & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = -\mu\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \rightarrow \begin{array}{l} 2\mu(\alpha) = -\mu(\alpha) \rightarrow \alpha = 0 \\ \mu(\gamma) = -\mu(\beta) \\ \mu(\beta) = -\mu(\gamma) \rightarrow \beta = -\gamma \end{array}$$

$$\left[\lambda_3 = -\mu \ , \ |\lambda_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right]^{(***)}$$

$\therefore$ For $\hat{A}$

$$\boxed{\lambda_1 = 2\mu, \ |\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ : \ \lambda_2 = \mu \ , \ |\lambda_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \ : \ \lambda_3 = -\mu, \ |\lambda_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}}$$

b.) $\langle \hat{Y} \rangle = \langle \mathcal{S}(0) | \hat{Y} | \mathcal{S}(0) \rangle$

i.) $\langle \hat{H} \rangle = \langle \mathcal{S}(0) | \hat{H} | \mathcal{S}(0) \rangle = (c_1^* c_2^* c_3^*) \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

$$= \hbar\omega \, (c_1^* c_2^* c_3^*)\begin{pmatrix} c_1 \\ 2c_2 \\ 2c_3 \end{pmatrix} = \hbar\omega\left(|c_1|^2 + 2|c_2|^2 + 2|c_3|^2\right)$$

$$\boxed{\langle \hat{H} \rangle = \hbar\omega\left(|c_1|^2 + 2|c_2|^2 + 2|c_3|^2\right)}$$

ii.) $\langle \hat{A} \rangle = \langle \mathcal{S}(0) | \hat{A} | \mathcal{S}(0) \rangle = (c_1^* c_2^* c_3^*) \lambda \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

$$= \lambda (c_1^* c_2^* c_3^*) \begin{pmatrix} c_2 \\ c_1 \\ 2c_3 \end{pmatrix} = \lambda \left(c_1^* c_2 + c_2^* c_1 + 2|c_3|^2 \right)$$

$$\boxed{\langle \hat{A} \rangle = \lambda \left(c_1^* c_2 + c_2^* c_1 + 2|c_3|^2 \right)}$$

iii.) $\langle \hat{B} \rangle = \langle \mathcal{S}(0) | \hat{B} | \mathcal{S}(0) \rangle = (c_1^* c_2^* c_3^*) \mu \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$

$$= \mu (c_1^* c_2^* c_3^*) \begin{pmatrix} 2c_1 \\ c_3 \\ c_2 \end{pmatrix} = \mu \left(2|c_1|^2 + c_2^* c_3 + c_3^* c_2 \right)$$

$$\boxed{\langle \hat{B} \rangle = \mu \left(2|c_1|^2 + c_2^* c_3 + c_3^* c_2 \right)}$$

c.)

i.) $|\mathcal{S}(0)\rangle = c_1 |\lambda_1\rangle + c_2 |\lambda_2\rangle + c_3 |\lambda_3\rangle \;:\; |\mathcal{S}(t)\rangle = e^{\frac{-iE_n t}{\hbar}} \cdot |\mathcal{S}(0)\rangle$

$$|\mathcal{S}(t)\rangle = c_1 e^{\frac{-iE_1 t}{\hbar}} |\lambda_1\rangle + c_2 \cdot e^{\frac{-iE_2 t}{\hbar}} |\lambda_2\rangle + c_3 \cdot e^{\frac{-iE_3 t}{\hbar}} |\lambda_3\rangle$$

$$= c_1 e^{\frac{-iE_1 t}{\hbar}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{\frac{-iE_2 t}{\hbar}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{\frac{-iE_3 t}{\hbar}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1 e^{-i\omega t} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 e^{-2i\omega t} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c_3 e^{-2i\omega t} \end{pmatrix}$$

$$\therefore \quad \boxed{|\mathcal{S}(t)\rangle = \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix}}$$

ii.) $P = |\langle \lambda | \mathcal{S}(t) \rangle|^2$

$\hat{A}: \;\; \lambda = 2\xi \;:\; P_{2\xi} = |\langle \lambda_1 | \mathcal{S}(t) \rangle|^2 = \left| (0\,0\,1) \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix} \right|^2 = |c_3 e^{-2i\omega t}|^2 = |c_3|^2$

$$\left[P_{2\xi} = |c_3|^2 \right]^{(*)}$$

$$\lambda = \xi : \quad P_{\xi} = |\langle \lambda_2 | \mathcal{S}(t) \rangle|^2 = \left| \tfrac{1}{\sqrt{2}} (1\ 1\ 0) \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix} \right|^2 = \left| \tfrac{1}{\sqrt{2}} \left(c_1 e^{-i\omega t} + c_2 e^{-2i\omega t} \right) \right|^2$$

$t=0$

$$= \frac{1}{2} \left(c_1^* e^{i\omega t} + c_2^* e^{2i\omega t} \right) \left(c_1 e^{-i\omega t} + c_2 e^{-2i\omega t} \right)$$

$$\left[P_{\xi} = \frac{1}{2} \left[|c_1|^2 + c_1^* c_2 e^{-i\omega t} + c_2^* c_1 e^{i\omega t} + |c_2|^2 \right] \right]^{(**)}$$

$$\lambda = -\xi : \quad P_{-\xi} = |\langle \lambda_3 | \mathcal{S}(t) \rangle|^2 = \left| \tfrac{1}{\sqrt{2}} (1\ {-1}\ 0) \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix} \right|^2 = \left| \tfrac{1}{\sqrt{2}} \left(c_1 e^{-i\omega t} - c_2 e^{-2i\omega t} \right) \right|^2$$

$$= \frac{1}{2} \left(c_1^* e^{i\omega t} - c_2^* e^{2i\omega t} \right) \left(c_1 e^{-i\omega t} - c_2 e^{-2i\omega t} \right)$$

$$\left[P_{-\xi} = \frac{1}{2} \left[|c_1|^2 - c_1^* c_2 e^{-i\omega t} - c_2^* c_1 e^{i\omega t} + |c_2|^2 \right] \right]^{(***)}$$

So for $\hat{A}$:

$$\boxed{\begin{gathered} P_{\xi} = \tfrac{1}{2} \left[|c_1|^2 + c_1^* c_2 e^{-i\omega t} + c_2^* c_1 e^{i\omega t} + |c_2|^2 \right] \\ P_{-\xi} = \tfrac{1}{2} \left[|c_1|^2 - c_1^* c_2 e^{-i\omega t} - c_2^* c_1 e^{i\omega t} + |c_2|^2 \right] \\ P_{2\xi} = |c_3|^2 \end{gathered}}$$

iii.) $\hat{B}$: $\lambda = 2\mu$: $P_{2\mu} = |\langle \lambda_1 | \mathcal{S}(t) \rangle|^2 = \left| (1\ 0\ 0) \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix} \right|^2 = \left| c_1 e^{-i\omega t} \right|^2 = |c_1|^2$

$$\left[P_{2\mu} = |c_1|^2 \right]^{(*)}$$

$$\lambda = \mu : \quad P_{\mu} = |\langle \lambda_2 | \mathcal{S}(t) \rangle|^2 = \left| \tfrac{1}{\sqrt{2}} (0\ 1\ 1) \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix} \right|^2 = \left| \tfrac{1}{\sqrt{2}} \left(c_2 e^{-2i\omega t} + c_3 e^{-2i\omega t} \right) \right|^2$$

$$= \tfrac{1}{2} \left(c_2^* e^{2i\omega t} + c_3^* e^{2i\omega t} \right) \left(c_2 e^{-2i\omega t} + c_3 e^{-2i\omega t} \right)$$

$$\left[P_{\mu} = \tfrac{1}{2} \left[|c_2|^2 + c_2^* c_3 + c_3^* c_2 + |c_3|^2 \right] \right]^{(**)}$$

$$\lambda = -\mu : \quad P_{-\mu} = \langle \lambda_3 | \mathcal{S}(t) \rangle|^2 = \left| \tfrac{1}{\sqrt{2}} (0\ 1\ {-1}) \begin{pmatrix} c_1 e^{-i\omega t} \\ c_2 e^{-2i\omega t} \\ c_3 e^{-2i\omega t} \end{pmatrix} \right|^2 = \left| \tfrac{1}{\sqrt{2}} \left(c_2 e^{-2i\omega t} - c_3 e^{-2i\omega t} \right) \right|^2$$

$$= \tfrac{1}{2} \left(c_2^* e^{2i\omega t} - c_3^* e^{2i\omega t} \right) \left(c_2 e^{-2i\omega t} - c_3 e^{-2i\omega t} \right)$$

Problem 3.38 | continued

$$\left[P_{-\mu} = \frac{1}{2}\left[|c_2|^2 - c_2^* c_3 - c_3^* c_2 + |c_3|^2 \right] \right]^{(***)}$$

$$\boxed{\begin{aligned} P_{\mu} &= \frac{1}{2}\left[|c_2|^2 + c_2^* c_3 + c_3^* c_2 + |c_3|^2 \right] \\ P_{-\mu} &= \frac{1}{2}\left[|c_2|^2 - c_2^* c_3 - c_3^* c_2 + |c_3|^2 \right] \\ P_{2\mu} &= |c_1|^2 \end{aligned}}$$