3.5 Vector Spaces and Subspaces

3.5 #'s 15, 16, 18, 22, 38, 39, 45, 46, 57

3.5.15) The set of all polynomials of even degree

Ex: x^2, x^4, x^6 Commutativity

$P = x^6 + x^3 + x + 1$

$Q = -x^6 + x^3 + x + 1$

$P+Q = (x^6+x^3+x+1) + (-x^6+x^3+x+1)$

$= 2x^3 + 2x + 2$

$(P+Q)$ is of odd degree $\therefore$ no vector space

3.15.16) The set of all diagonal 2x2 matrices

$A = \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix}$ $B = \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix}$

Since (A & B) are both compatible, there are no properties that do not apply $\therefore$ yes this is a vector space.

3.5.18) The set of all invertible 2x2 matrices

$A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$ $\begin{bmatrix} \frac{z}{wz-xy} & \frac{-x}{wz-xy} \\ \frac{-y}{wz-xy} & \frac{w}{wz-xy} \end{bmatrix} = A^{-1}$

Invertible matrix $\equiv |A| \neq 0$

$A_1 \begin{bmatrix} w & x \\ y & z \end{bmatrix}$ $A_2 \begin{bmatrix} -w & -x \\ -y & -z \end{bmatrix}$ $A_1 + A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A_1 + A_2 \equiv a$ $|a| = 0$

There exists vectors such that create an invertible matrix $\therefore$ not a vector space.

3.5.22) The set of all differentiable functions on $(-\infty, \infty)$

$x(t) = \cos(t)$

$\dot{x}(t) = -\sin(t)$

$y(t) = \sin(t)$

$\dot{y}(t) = \cos(t)$

$\dot{x} - \dot{x} = -\sin(t) + \sin(t) = 0$

$\dot{x} + 0 = -\sin(t)$

$\dot{y} + \dot{x} = \dot{x} + \dot{y}$: $\cos(t) - \sin(t) = -\sin(t) + \cos(t)$

All properties are met this is a vector space

3.5.38 $\mathbb{V} = \mathbb{R}^2$ $\mathbb{W} = \{(x,y) \mid x^2+y^2=1\}$

$x = \cos t$ $\cos^2 t + \sin^2 t = 1$ $\vec{u} = \langle \cos t, \sin t \rangle$ $c\vec{u} \notin W$

$y = \sin t$ $5(x^2+y^2) = 1$ $5\vec{u} \neq 1$

$5 \neq 1$

This is not a subspace due to $c\vec{u} \notin W$

3.5.39 $\mathbb{V} = \mathbb{R}^3$ $\mathbb{W} = \{(x_1, x_2, x_3) \mid x_3 = 0\}$

$\vec{u} + \vec{v} \in W$

$\vec{u} = \langle 1,1,0 \rangle$ $\vec{u} + \vec{v} \in W$ ✓

$\vec{v} = \langle 2,2,0 \rangle$

$c\vec{u} \in W$ $5\vec{u} \in W$ ✓ since x_3 is always zero, x_1 & x_2 can be anything

$c = 5$

This is a subspace

3.5.45 $\mathbb{V} = C^2[0,1]$ $\mathbb{W} = \{f(t) \mid f'' + f = 0\}$

$f(t) = \cos(t)$ $-\cos(t) + \cos(t) = 0$

$f'(t) = -\sin(t)$ $0 = 0$

$f''(t) = -\cos(t)$ $5(-\cos(t) + \cos(t)) = 0$

$0 = 0$

This is a subspace because no theorem is violated

3.5.46 $\mathbb{V} = C^2[0,1]$ $\mathbb{W} = \{f(t) \mid f'' + f = 1\}$

There is not a condition in which $\mathbb{W}$ is satisified $\therefore$ no subspace

3.5.57 closed under vector addition but not under scalar multiplication

$\vec{u} = \cos^2 t$ $\vec{u} + \vec{v} = 1$ ✓

$\vec{v} = \sin^2 t$ $5(\vec{u} + \vec{v}) \neq 1$

Subset: $\mathbb{W} \in \vec{u} + \vec{v} = 1$

3.5.11 The set of vectors in the First quadrant of the plane.

No because the closure property is violated.

$c\vec{x} \in V$

Also no Additive inverse

3.5.12 The set of vectors in the first octent (x,y,z) space

No, closure property is violated

Closure proprty

3.5.13 The set of all pairs of real numbers (x,y) such that $x \geq y$

No, the negative does not exist i.e $[2,1] \Rightarrow [-2,-1]$

$x \ngeq y$

Negative of the set violates $x \geq y$

3.5.14 The set of all polynomials degree 2

$f(t): \; x^2+x+1 = A$

$-(x^2+2x+4) = B$

$f(t): \; A\vec{0} = 0 \neq \mathbb{R}$

$\vec{0}$

$A+B = 0-x-3$

$A+B = -x-3$

$\neq \mathbb{V}$

Not a vector space

3.5.15 The set of all polynomials of even degree

0 vector and negative property do not hold true.

Not a vector space

3.5.16 The set of all diagonal 2x2 matrices

No properties are violated

This is a vector space

3.5.17 The set of all 2x2 matrices with determinant equal to 0

No this is not a vector space, not closed under addition

not a vector space.

3.5.18 The set of all invertible 2x2 matrices.

NO, not closed under negative addition

Not a vector space

3.5.19 The set of all 3x3 upper triangular matrices

No properties violated

Vector space

3.5.20 All continuous functions f on the interval $[0,1]$ such that $f(0)=1$

$f(t)=e^t$

The $\vec{0}$ vector identity is violated

Not a vector space

3.5.21 All continuous functions f defined on the interval $[0,1]$ such that $f(t)>0$

Not a vector space, $\vec{0}$ is violated, Also Addition

Not a vector space

3.5.22 The set of all differentiable functions on $(-\infty,\infty)$

No properties violated

This is a vector space

3.5.23) The Set of all Functions integrable On the interval [0,1]

No properties violated

This is a vector space