

2.) $f(t) = 2t + 1$

$$\begin{aligned} \langle f, B \rangle \int_0^{\frac{1}{4}} (2t+1) dt &= t^2 + t \Big|_0^{\frac{1}{4}} = 0.3125 \\ &= t^2 + t \Big|_{\frac{1}{4}}^{\frac{1}{2}} = 0.4375 \\ &= t^2 + t \Big|_{\frac{1}{2}}^{\frac{3}{4}} = 0.5625 \\ &= t^2 + t \Big|_{\frac{3}{4}}^1 = 0.6875 \end{aligned}$$

$$\begin{aligned} \langle B, B \rangle \int_0^1 (1)^2 dt &= t \Big|_0^1 = \frac{1}{4} \\ t \Big|_{\frac{1}{4}}^{\frac{1}{2}} &= (\frac{1}{2} - \frac{1}{4}) = \frac{1}{4} \\ t \Big|_{\frac{1}{2}}^{\frac{3}{4}} &= (\frac{3}{4} - \frac{1}{2}) = \frac{1}{4} \\ t \Big|_{\frac{3}{4}}^1 &= (1 - \frac{3}{4}) = \frac{1}{4} \end{aligned}$$

$c_1 = 1.25, c_2 = 1.75, c_3 = 2.25, c_4 = 2.75$

$$g(t) = 1.25 \cdot B_1(t) + 1.75 \cdot B_2(t) + 2.25 \cdot B_3(t) + 2.75 \cdot B_4(t)$$

10.) $f(t) = 2t + 1$

$$c_1 = \frac{\langle f, B \rangle}{\langle B, B \rangle} : \frac{\int_0^1 (2t+1) dt}{\int_0^1 (1)^2 dt} = \frac{t^2 + t \Big|_0^1}{1} = 2$$

$$c_2 = \frac{\langle f, w \rangle}{\langle w, w \rangle} : \frac{\int_0^{\frac{1}{2}} (2t+1) dt + \int_{\frac{1}{2}}^1 -(2t+1) dt}{\int_0^{\frac{1}{2}} (1)^2 dt + \int_{\frac{1}{2}}^1 (-1)^2 dt} = \frac{t^2 + t \Big|_0^{\frac{1}{2}} - (t^2 + t \Big|_{\frac{1}{2}}^1)}{t \Big|_0^{\frac{1}{2}} + t \Big|_{\frac{1}{2}}^1} = \frac{-\frac{1}{2}}{1} = -0.5$$

$$c_3 = \frac{\langle f, w_0' \rangle}{\langle w_0', w_0' \rangle} : \frac{\int_0^{\frac{1}{4}} (2t+1) dt + \int_{\frac{1}{4}}^{\frac{1}{2}} -(2t+1) dt}{\int_0^{\frac{1}{4}} (1)^2 dt + \int_{\frac{1}{4}}^{\frac{1}{2}} (-1)^2 dt} = \frac{t^2 + t \Big|_0^{\frac{1}{4}} - (t^2 + t \Big|_{\frac{1}{4}}^{\frac{1}{2}})}{t \Big|_0^{\frac{1}{4}} + t \Big|_{\frac{1}{4}}^{\frac{1}{2}}} = \frac{-\frac{1}{8}}{\frac{1}{2}} = -0.25$$

$$c_4 = \frac{\langle f, w_1' \rangle}{\langle w_1', w_1' \rangle} : \frac{\int_{\frac{1}{2}}^{\frac{3}{4}} (2t+1) dt - \int_{\frac{3}{4}}^1 (2t+1) dt}{\int_{\frac{1}{2}}^{\frac{3}{4}} (1)^2 dt + \int_{\frac{3}{4}}^1 (-1)^2 dt} = \frac{t^2 + t \Big|_{\frac{1}{2}}^{\frac{3}{4}} - (t^2 + t \Big|_{\frac{3}{4}}^1)}{t \Big|_{\frac{1}{2}}^{\frac{3}{4}} + t \Big|_{\frac{3}{4}}^1} = \frac{-\frac{1}{8}}{\frac{1}{2}} = -0.25$$

$c_1 = 2, c_2 = -0.5, c_3 = -0.25, c_4 = -0.25$

$$g(t) = 2 \cdot B(t) - 0.5 \cdot W(t) - 0.25 \cdot w_0'(t) - 0.25 \cdot w_1'(t)$$