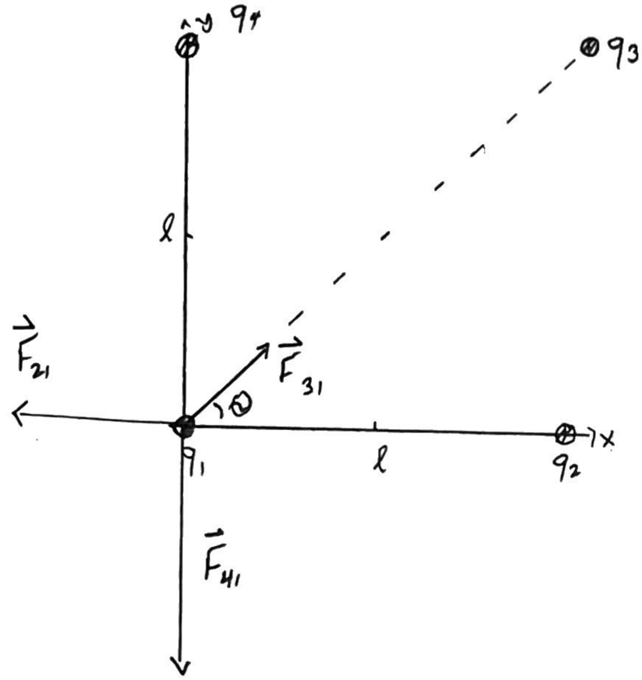
PHYS 311 HOMEWORK SET 4

1.



$q_1 = q$

$q_2 = 2q$

$q_3 = -q$

$q_4 = 3q$

NOTICE:

From trigonometry

$\tan\theta = \frac{l}{l} = 1 \quad \therefore \theta = 45°$

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{l^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{l^2} \quad \text{so} \quad \vec{F}_{21} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{l^2}\hat{x}$$

$$F_{31} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{2l^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2l^2}$$

NOW

$$F_{31,x} = F_{31}\cos(45°) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2l^2}\cdot\frac{1}{\sqrt{2}} \quad \text{so} \quad \vec{F}_{31} = F_{31,x}\hat{x} + F_{31,y}\hat{y}$$

$$F_{31,y} = F_{31}\sin(45°) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2l^2}\cdot\frac{1}{\sqrt{2}}$$

so

$$\vec{F}_{31} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2l^2}\cdot\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$$

LASTLY..

$$F_{41} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{l^2} = \frac{1}{4\pi\varepsilon_0}\frac{3q^2}{l^2} \quad \text{so} \quad \vec{F}_{41} = -\frac{1}{4\pi\varepsilon_0}\frac{3q^2}{l^2}\hat{y}$$

NOW

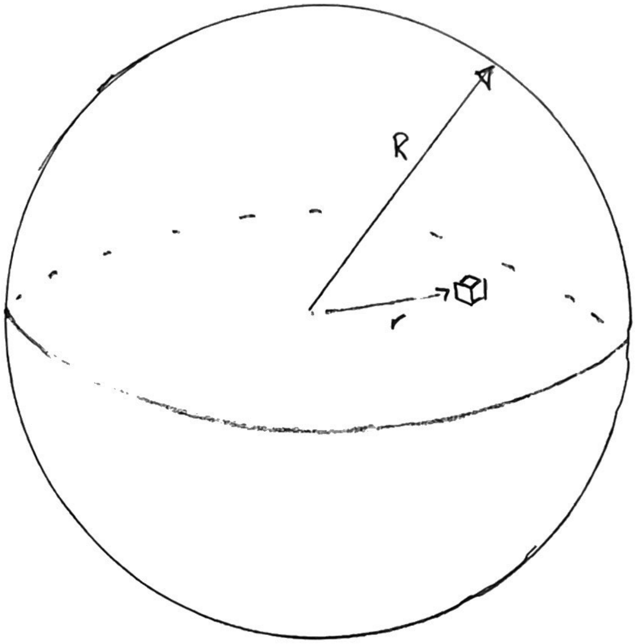
$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} = -\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{l^2}\hat{x} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2\sqrt{2}l^2}(\hat{x}+\hat{y}) - \frac{1}{4\pi\varepsilon_0}\frac{3q^2}{l^2}\hat{y}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\left[-2\hat{x} + \frac{1}{2\sqrt{2}}(\hat{x}+\hat{y}) - 3\hat{y}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\left[\left(\frac{1}{2\sqrt{2}} - 2\right)\hat{x} + \left(\frac{1}{2\sqrt{2}} - 3\right)\hat{y}\right]$$

2

a)



$\rho(\theta) = \rho_0 \cos^2\theta$

$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$

NOW, THE CHARGE DENSITY IS...

$\rho = \dfrac{dq}{d\tau}$ so $dq = \rho \, d\tau$

SO THE TOTAL CHARGE IS...

$$q = \int \rho \, d\tau = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho_0 \cos^2\theta \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$$

so

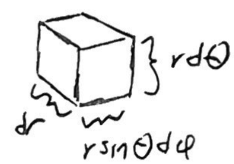
$$q = \rho_0 \int_0^{2\pi} d\phi \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta \int_0^{R} r^2 dr = \rho_0 \, \phi \Big|_0^{2\pi} \cdot -\int_1^{-1} u^2 du \cdot \frac{r^3}{3}\Big|_0^{R} = \rho_0 \, 2\pi \, \frac{u^3}{3}\Big|_{-1}^{1} \frac{R^3}{3}$$

let

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$= \rho_0 2\pi \left(\frac{1}{3} - \left(\frac{-1}{3}\right)\right)\frac{R^3}{3} = \left[\rho_0 \frac{4\pi R^3}{9} = q\right]$$

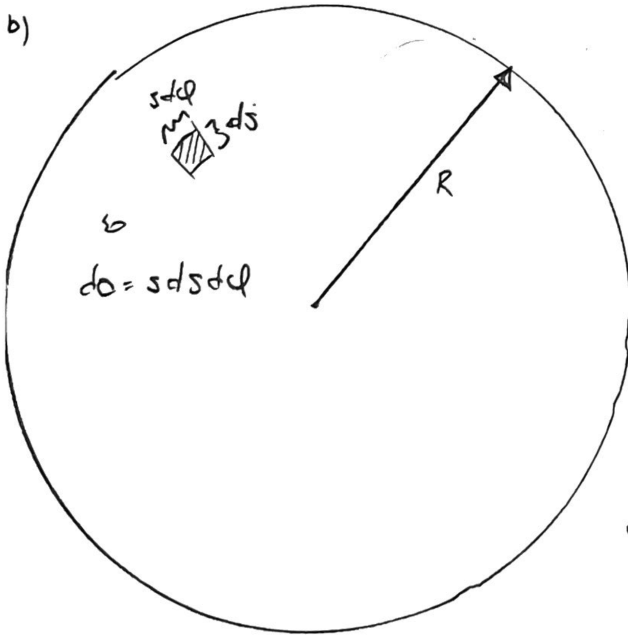


so

$d\tau = dr \cdot r\sin\theta \, d\phi \cdot r \, d\theta$

$= r^2 \sin\theta \, dr \, d\theta \, d\phi$

b)



so

$d\sigma = s \, ds \, d\phi$

$\sigma(s) = \sigma_0 \dfrac{R}{s}$

NOW, THE CHARGE DENSITY IS...

$\sigma = \dfrac{dq}{d\sigma}$ so $dq = \sigma \, d\sigma$

SO THE TOTAL CHARGE IS...

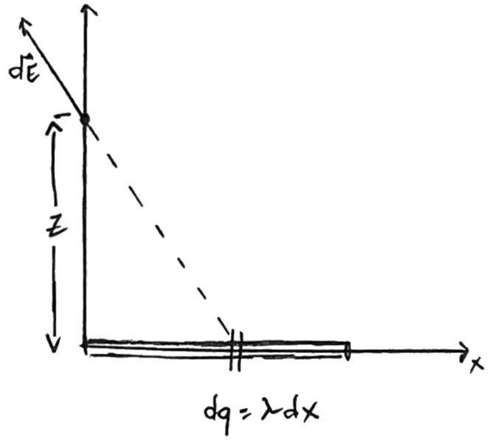
$$q = \int \sigma \, d\sigma = \int_0^{2\pi}\int_0^{R} \sigma_0 \frac{R}{s} \cdot s \, ds \, d\phi$$

so

$$q = \sigma_0 R \int_0^{R} ds \int_0^{2\pi} d\phi = \sigma_0 R \, s\Big|_0^{R} \phi \Big|_0^{2\pi}$$

$$\left[q = 2\pi \sigma_0 R^2\right]$$

3.



FOR MY INFINITESIMAL CHUNK OF CHARGE...

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathscr{r}^2}\hat{\mathscr{r}}$$

NOW

$\vec{r}' = x\hat{x}$ SO $\vec{\mathscr{r}} = -x\hat{x} + z\hat{z}$

$\vec{r} = z\hat{z}$

$\mathscr{r}^2 = x^2 + z^2$ SO $\mathscr{r} = \sqrt{x^2+z^2}$

$\therefore \hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}} = \frac{-x\hat{x}+z\hat{z}}{\sqrt{x^2+z^2}}$

PUTTING THIS ALL TOGETHER...

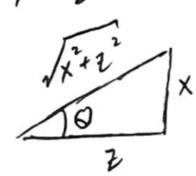
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dx}{x^2+z^2}\frac{(-x\hat{x}+z\hat{z})}{\sqrt{x^2+z^2}} = \frac{\lambda}{4\pi\varepsilon_0}\frac{(-x\hat{x}+z\hat{z})}{(x^2+z^2)^{3/2}}dx$$

SO THE TOTAL $\vec{E}$-FIELD IS FOUND BY INTEGRATING OVER THE LENGTH L..

$$\vec{E} = \int_0^L d\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2+z^2)^{3/2}}(-x\hat{x}+z\hat{z}) = \frac{\lambda}{4\pi\varepsilon_0}\left[-\hat{x}\int_0^L\frac{x\,dx}{(x^2+z^2)^{3/2}} + z\hat{z}\int_0^L\frac{dx}{(x^2+z^2)^{3/2}}\right]$$

let $u = x^2+z^2$
$du = 2x\,dx$

and
$x = z\tan\theta$
$dx = z\sec^2\theta\, d\theta$



$$= \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{\hat{x}}{2}\int_{z^2}^{L^2+z^2}\frac{du}{u^{3/2}} + z\hat{z}\int_0^{\tan^{-1}(L/z)}\frac{z\sec^2\theta\,d\theta}{z^3(1+\tan^2\theta)^{3/2}}\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{1}{2}\hat{x}\,\frac{u^{-1/2}}{-1/2}\Bigg|_{z^2}^{L^2+z^2} + \frac{\hat{z}}{z}\int_0^{\tan^{-1}(L/z)}\frac{\sec^2\theta\,d\theta}{\sec^3\theta\,d\theta}\right]$$

SINCE $\tan^2\theta + 1 = \sec^2\theta$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\hat{x}\left[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{z}\right] + \frac{\hat{z}}{z}\int_0^{\tan^{-1}(L/z)}\cos\theta\,d\theta\right] = \frac{\lambda}{4\pi\varepsilon_0}\left[\left[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{|z|}\right]\hat{x} + \frac{\hat{z}}{z}\sin\theta\Bigg|_0^{\tan^{-1}(L/z)}\right]$$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left[\left[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{z}\right]\hat{x} + \frac{L}{\sqrt{L^2+z^2}}\cdot\frac{\hat{z}}{z}\right] = \frac{\lambda}{4\pi\varepsilon_0}\left[\left[\frac{1}{z}\left(1+\frac{L^2}{z^2}\right)^{-1/2} - \frac{1}{z}\right]\hat{x} + \frac{L}{z^2}\left(1+\frac{L^2}{z^2}\right)^{-1/2}\hat{z}\right]$$

NOW

$$\lim_{z\gg L}\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{z}\left(\cancel{1} - \frac{1}{2}\frac{L^2}{z^2} + O\left(\frac{L^4}{z^4}\right) - \cancel{1}\right)\hat{x} + \frac{L}{z^2}\left(1 - \frac{1}{2}\frac{L^2}{z^2} + O\left(\frac{L^4}{z^4}\right)\right)\hat{z}\right]$$

SINCE $(1+x)^n \simeq 1 + nx + ...$ FOR SMALL x

SO $\lim_{z\gg L}\vec{E} \simeq \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{z^2}\hat{z} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\hat{z}$ – A POINT CHARGE!

4. THIS PROBLEM IS VERY SIMILAR TO THE PREVIOUS ONE, WITH ONLY THE LINEAR CHARGE DENSITY DIFFERING FROM A CONSTANT.

FROM PROBLEM 3

$$d\vec{E} = \frac{\lambda}{4\pi\epsilon_0} \frac{(-x\hat{x} + z\hat{z})}{(x^2+z^2)^{3/2}} dx \qquad \text{WHERE, HERE, } \lambda(x) = \lambda_0 \frac{x^2}{L^2}$$

SO, THE TOTAL $\vec{E}$-FIELD IS OF THE FORM..

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_0^L \lambda_0 \frac{x^2}{L^2} \frac{(-x\hat{x} + z\hat{z})}{(x^2+z^2)^{3/2}} dx$$

$$= \frac{\lambda_0}{4\pi\epsilon_0 L^2} \left[-\hat{x} \int_0^L \frac{x^3\, dx}{(x^2+z^2)^{3/2}} + z\hat{z} \int_0^L \frac{x^2 dx}{(x^2+z^2)^{3/2}} \right] \checkmark$$

NOW $\cosh^2\theta - \sinh^2\theta = 1$

NOW

$$\int_0^L \frac{x^2 dx}{(x^2+z^2)^{3/2}} = \int_0^L x \cdot \frac{x\,dx}{(x^2+z^2)^{3/2}} = \left.\frac{-x}{\sqrt{x^2+z^2}}\right|_0^L + \int_0^L \frac{dx}{\sqrt{x^2+z^2}} = \frac{-L}{\sqrt{L^2+z^2}} + \int_0^{\sinh^{-1}(L/z)} \frac{z\cosh\theta\, d\theta}{z\sqrt{1+\sinh^2\theta}}$$

LET $u = x \qquad dv = \frac{x\,dx}{(x^2+z^2)^{3/2}}$

LET $x = z\sinh\theta$

$dx = z\cosh\theta\, d\theta$

$$du = dx \qquad v = \int \frac{x\,dx}{(x^2+z^2)^{3/2}} = \frac{1}{2}\int \frac{dw}{w^{3/2}} = \frac{1}{2}\frac{w^{-1/2}}{-1/2} = -\frac{1}{\sqrt{x^2+z^2}}$$

LET $w = x^2+z^2$

$dw = 2x\,dx$

SO

$$\int_0^L \frac{x^2 dx}{(x^2+z^2)^{3/2}} = \frac{-L}{\sqrt{L^2+z^2}} + \int_0^{\sinh^{-1}(L/z)} d\theta$$

$$= \frac{-L}{\sqrt{L^2+z^2}} + \sinh^{-1}(L/z) \checkmark$$

WHEREAS

$$\int_0^L \frac{x^3 dx}{(x^2+z^2)^{3/2}} = \int_0^L x \cdot \frac{x^2 dx}{(x^2+z^2)^{3/2}} = \left. x\left(\frac{-x}{\sqrt{x^2+z^2}} + \sinh^{-1}\left(\frac{x}{z}\right)\right)\right|_0^L - \int_0^L \left[\frac{-x}{\sqrt{x^2+z^2}} + \sinh^{-1}\left(\frac{x}{z}\right)\right] dx$$

$u = x \qquad dv = \frac{x^2 dx}{(x^2+z^2)^{3/2}}$

$du = dx \qquad v = \frac{-x}{\sqrt{x^2+z^2}} + \sinh^{-1}\left(\frac{x}{z}\right)$

So

$$\therefore \int_0^L \frac{x^3\,dx}{(x^2+z^2)^{3/2}} = \frac{-L^2}{\sqrt{L^2+z^2}} + L\sinh^{-1}\left(\frac{L}{z}\right) + \int_0^L \frac{x}{\sqrt{x^2+z^2}}\,dx - \int_0^L \sinh^{-1}\left(\frac{x}{z}\right)dx$$

u

$u = x^2+z^2$

$du = 2x\,dx$

$w = \frac{x}{z}$

$dw = \frac{dx}{z}$

$$= \frac{-L^2}{\sqrt{L^2+z^2}} + L\sinh^{-1}(L/z) + \frac{1}{2}\int_{z^2}^{L^2+z^2} \frac{du}{u^{1/2}} - z\int_0^{L/z} \sinh^{-1}(w)\,dw$$

Now

$$\int \sinh^{-1}(w)\,dw = w\sinh^{-1}(w) - \sqrt{w^2+1}$$ (from WolframAlpha)

So

$$\int_0^L \frac{x^3\,dx}{(x^2+z^2)} = \frac{-L^2}{\sqrt{L^2+z^2}} + L\sinh^{-1}(L/z) + \frac{1}{2}\left.\frac{u^{1/2}}{1/2}\right|_{z^2}^{L^2+z^2} - z\Big[w\sinh^{-1}(w) - \sqrt{w^2+1}\Big]_0^{L/z}$$

$$= \frac{-L^2}{\sqrt{L^2+z^2}} + \cancel{L\sinh^{-1}(L/z)} + \left[\sqrt{L^2+z^2} - z\right] - z\left[\cancel{\frac{L}{z}\sinh^{-1}\left(\frac{L}{z}\right)} - \sqrt{\frac{L^2}{z^2}+1}\right]$$

$$= \frac{-L^2}{\sqrt{L^2+z^2}} + \cancel{\sqrt{L^2+z^2}} - z - \cancel{\sqrt{L^2+z^2}}$$

$$= \frac{-L^2}{\sqrt{L^2+z^2}} - z$$

So

$$\vec{E} = \frac{\lambda_0}{4\pi\varepsilon_0 L^2}\left[+\left[\frac{L^2}{\sqrt{L^2+z^2}} + z\right]\hat{x} + z\left[-\frac{L}{\sqrt{L^2+z^2}} + \sinh^{-1}(L/z)\right]\hat{z}\right]$$

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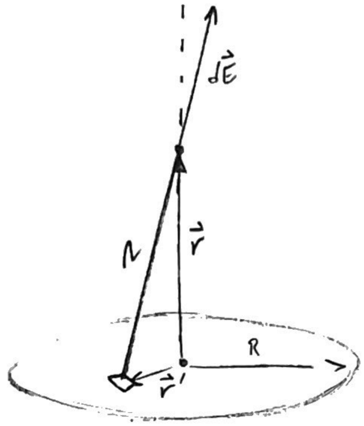

FOR AN INFINITESIMAL AREA OF CHARGE...

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{\eta^2}\hat{\eta}$$

NOW

$$\vec{r}' = s\hat{s} \qquad \text{so} \qquad \vec{\eta} = -s\hat{s} + z\hat{z}$$

$$\vec{r} = z\hat{z} \qquad \eta^2 = s^2 + z^2 \quad \text{so} \quad \eta = \sqrt{s^2+z^2}$$

$$\text{so} \quad \hat{\eta} = \frac{\vec{\eta}}{\eta} = \frac{-s\hat{s} + z\hat{z}}{\sqrt{s^2+z^2}}$$

ALSO

$$dq = \sigma\, da = \sigma s\, ds\, d\varphi$$

NOW PUTTING THIS ALL TOGETHER...

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{\sigma s\, ds\, d\varphi}{s^2+z^2}\cdot\frac{(-s\hat{s}+z\hat{z})}{\sqrt{s^2+z^2}} = \frac{\sigma}{4\pi\epsilon_0}\frac{s(-s\hat{s}+z\hat{z})\,ds\,d\varphi}{(s^2+z^2)^{3/2}}$$

NOW THE TOTAL $\vec{E}$-FIELD IS FOUND BY INTEGRATING OVER THE DISK...

$$\vec{E} = \frac{\sigma}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^R \frac{s(-s\hat{s}+z\hat{z})\,ds\,d\varphi}{(s^2+z^2)^{3/2}} = \frac{\sigma}{4\pi\epsilon_0}\left[-\hat{s}\int_0^{2\pi} d\varphi\int_0^R \frac{s^2\,ds}{(s^2+z^2)^{3/2}} + z\hat{z}\int_0^{2\pi} d\varphi\int_0^R \frac{s\,ds}{(s^2+z^2)^{3/2}}\right]$$

NOTICE!

BECAUSE OF THE CYLINDRICAL-SYMMETRY OF THE PROBLEM, THE $\vec{E}$-FIELD MUST POINT ONLY IN THE $\hat{z}$-DIRECTION.

$\therefore$ THE $\hat{s}$ INTEGRAL MUST BE ZERO, I'LL SHOW THIS NOW...

$$\int_0^R \frac{s^2\,ds}{(s^2+z^2)^{3/2}} = \int_0^R s\cdot\frac{s\,ds}{(s^2+z^2)^{3/2}} = -\frac{s}{\sqrt{s^2+z^2}}\Bigg|_0^R + \int_0^R \frac{ds}{\sqrt{s^2+z^2}}$$

let $u = s$ $\qquad dv = \dfrac{s\,ds}{(s^2+z^2)^{3/2}}$

$du = ds$

$$v = \int \frac{s\,ds}{(s^2+z^2)^{3/2}} = \frac{1}{2}\int w^{-3/2}\,dw = \frac{1}{2}\frac{w^{-1/2}}{-1/2} = -\frac{1}{\sqrt{s^2+z^2}}$$

let $w = s^2+z^2$

$dw = 2s\,ds$

so

$$\int_0^R \frac{s^2\,ds}{(s^2+z^2)^{3/2}} = -\frac{R}{\sqrt{R^2+z^2}} + \int_0^R \frac{ds}{\sqrt{s^2+z^2}} = -\frac{R}{\sqrt{R^2+z^2}} + \int_0^{\sinh^{-1}(R/z)} \frac{z\cosh\theta\,d\theta}{z\sqrt{1+\sinh^2\theta}} = -\frac{R}{\sqrt{R^2+z^2}} + \int_0^{\sinh^{-1}(R/z)} d\theta$$

let $s = z\sinh\theta$ $\qquad$ so $\theta = \sinh^{-1}\left(\frac{s}{z}\right)$

$ds = z\cosh\theta\,d\theta$

so $\therefore$

$$\int_0^R \frac{s^2\,ds}{(s^2+z^2)^{3/2}} = \frac{-R}{\sqrt{R^2+z^2}} + \sinh^{-1}\left(\frac{R}{z}\right)$$

so

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0} z\hat{z}\int_0^{2\pi} d\varphi \int_0^R \frac{s\,ds}{(s^2+z^2)^{3/2}} = \frac{\sigma}{4\pi\varepsilon_0} z\hat{z}\cdot 2\pi \cdot \frac{1}{2}\int_{z^2}^{R^2+z^2} \frac{du}{u^{3/2}} = \frac{\sigma}{4\varepsilon_0} z\hat{z}\,\frac{u^{-1/2}}{-1/2}\Bigg|_{z^2}^{R^2+z^2}$$

let $u = s^2 + z^2$

$du = 2s\,ds$

$$= -\frac{\sigma}{2\varepsilon_0} z\hat{z}\left[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{z}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]\hat{z}$$

Now

$\lim_{R\to\infty} \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$: THE $\vec{E}$-FIELD OF AN ∞ PLANE!

6. THIS PROBLEM IS VERY SIMILAR TO THE PREVIOUS ONE, WITH ONLY THE SURFACE CHARGE DENSITY DIFFERING FROM THAT OF A CONSTANT.

FROM PROBLEM 5...

$$d\vec{E} = \frac{\sigma}{4\pi\varepsilon_0} \frac{s(-s\hat{s}+z\hat{z})}{(s^2+z^2)^{3/2}} \, ds \, d\varphi \quad \text{WHERE } \sigma(s) = \sigma_0 \frac{s}{R}$$

SO

$$d\vec{E} = \frac{\sigma_0}{4\pi\varepsilon_0 R} \frac{s^2(-s\hat{s}+z\hat{z})}{(s^2+z^2)^{3/2}} \, ds \, d\varphi$$

NOW THE TOTAL $\vec{E}$-FIELD IS FOUND BE INTEGRATING OVER THE DISK..

$$\vec{E} = \frac{\sigma_0}{4\pi\varepsilon_0 R} \int_0^{2\pi}\int_0^R \frac{s^2(-s\hat{s}+z\hat{z})}{(s^2+z^2)^{3/2}} \, ds \, d\varphi = \frac{\sigma_0}{4\pi\varepsilon_0 R}\left[\cancel{-\hat{s}\int_0^{2\pi} d\varphi \int_0^R \frac{s^3 \, ds}{(s^2+z^2)^{3/2}}}^{\,0} + z\hat{z}\int_0^{2\pi} d\varphi \int_0^R \frac{s^2 \, ds}{(s^2+z^2)^{3/2}} \right]$$

NOW, IN THE LAST PROBLEM I SHOWED THAT...

$$\int_0^R \frac{s^2 \, ds}{(s^2+z^2)^{3/2}} = \frac{-R}{\sqrt{R^2+z^2}} + \sinh^{-1}(R/z)$$

SO

$$\vec{E} = \frac{\sigma_0}{4\pi\varepsilon_0 R} \cdot z\hat{z} \cdot 2\pi \cdot \left[-\frac{R}{\sqrt{R^2+z^2}} + \sinh^{-1}(R/z) \right]$$

$$= \frac{\sigma_0}{2\varepsilon_0} \cdot \frac{z}{R}\left[-\frac{R}{\sqrt{R^2+z^2}} + \sinh^{-1}(R/z) \right]\hat{z}$$

SO

$$\vec{E} = \frac{\sigma_0}{2\varepsilon_0}\left[-\frac{z}{\sqrt{R^2+z^2}} + \frac{z}{R}\sinh^{-1}(R/z) \right]\hat{z}$$

7. $\vec{E} = kr^5 \hat{r} = E_r \hat{r}$

NOW

a) $\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \cdot kr^5) = \frac{k}{r^2} \frac{\partial}{\partial r}(r^7) = \frac{k}{r^2} \cdot 7r^6 = 7kr^4$

BUT WE ALSO KNOW THAT

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ SO $\rho = \varepsilon_0 (\vec{\nabla} \cdot \vec{E})$ SO $\boxed{\rho(r) = 7k\varepsilon_0 r^4}$

b) NOW

$\rho = \frac{dq}{d\tau}$ SO $dq = \rho d\tau$ $\therefore$ $q = \int \rho d\tau$

SO

$$q = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} 7k\varepsilon_0 r^4 \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi = 7k\varepsilon_0 \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{R} r^6 dr$$

$$= 7k\varepsilon_0 \cdot 2\pi \cdot \cos\theta \Big|_{\pi}^{0} \frac{r^7}{7}\Big|_0^R = 7k\varepsilon_0 \cdot 2\pi \cdot 2 \cdot \frac{R^7}{7} = 4\pi\varepsilon_0 k R^7$$

WE COULD ALSO USE GAUSS' LAW...

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \text{ SO } Q_{enc} = \varepsilon_0 \oint \vec{E} \cdot d\vec{a} = \varepsilon_0 \int_0^{2\pi}\int_0^{\pi} kR^7 \sin\theta \, d\theta \, d\phi = \varepsilon_0 k R^7 \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta$$

NOW $\vec{E} = kr^5 \hat{r} \Big|_{r=R}$ SO $\vec{E} \cdot d\vec{a} = kR^7 \sin\theta \, d\theta \, d\phi$

$d\vec{a} = R^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$

SO

$Q_{enc} = \varepsilon_0 k R^7 \cdot 2\pi \cdot 2$ $\therefore$ $\boxed{Q_{enc} = 4\pi\varepsilon_0 k R^7}$

NOTICE THAT WE MADE OUR GAUSSIAN SURFACE TO BE A SPHERE OF RADIUS R,

THE EXACT SAME SIZE AS OUR SPHERE OF CHARGE.