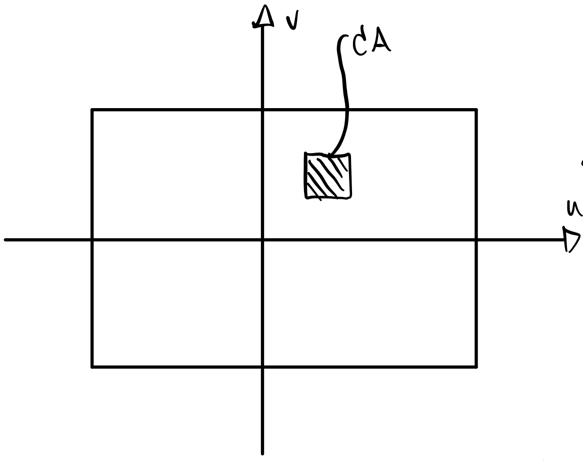
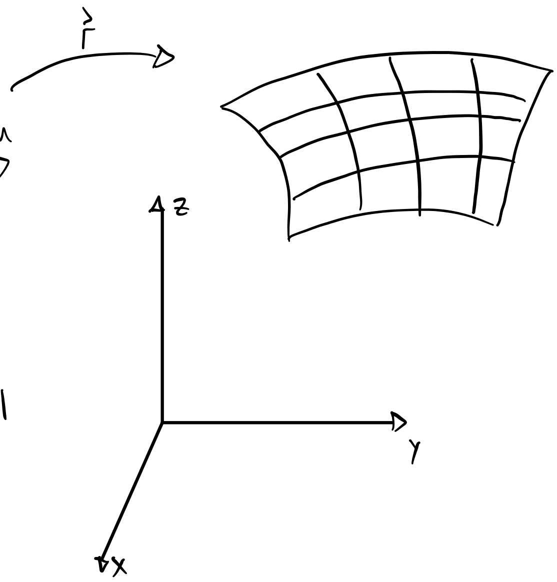


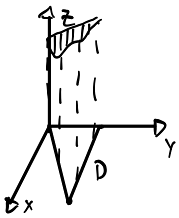


$ds = |\vec{r}_u \times \vec{r}_v|$

$\text{Area}(S) = \iint_S ds$

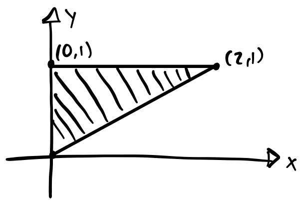
16.6.9] S is the portion of $z = 5 + 3x + 2y^2$

Above ; $(0,0), (0,1), (2,1)$



Parametrize with x,y

$\vec{r}(x,y) = \langle x, y, 5 + 3x + 2y^2 \rangle$



$\iint_S ds = \iint_D |\vec{r}_u \times \vec{r}_v|$ dxdy

$= \int_0^2 \int_{\frac{1}{2}x}^{1} |\langle 1,0,3 \rangle \times \langle 0,1,4y \rangle|$

$\begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & 4y \end{vmatrix}$

$= \int_0^1 \int_0^{2y} \sqrt{10 + 16y^2} \, dx\, dy$

$\langle 0(4y) - 3(1), 3(0) - 1(4y), 1(1) - 0(0) \rangle$

$\langle -3, -4y, 1 \rangle$

$\sqrt{9 + 16y^2 + 1} = \sqrt{10 + 16y^2}$

$u = 10 + 16y^2$

$du = 32y\,dy$

$\frac{1}{16} du = 2y\,dy$

$= \int_0^1 x\sqrt{10 + 16y^2} \Big|_0^{2y} dy$

$= \int_0^1 2y\sqrt{10 + 16y^2}\, dy$

$= \int_0^1 \sqrt{u}\, du$

$= \frac{2}{3}(10 + 16y^2)^{\frac{3}{2}} \Big|_0^1$

$= \frac{2}{3}(26)^{\frac{3}{2}} - \frac{2}{3}(10)^{\frac{3}{2}}$

$= \frac{2}{3}(26^{\frac{3}{2}} - 10^{\frac{3}{2}})$

≈ 67.3

$$\iint_S f\, ds = \lim \sum f(x_i^*) \Delta S$$

16.7.1) $x^2 + y^2 + z^2 = 26$, $z \geq 0$

$f(3,4,1) = 13$

$f(3,-4,1) = 13$

$f(-3,4,1) = 14$

$f(-3,-4,1) = 15$

$\frac{1}{4} \times 2\pi \times (\sqrt{26})^2$

$\frac{\pi}{2} \times 26$

$13\pi\,(f(x_1^*) + f(x_2^*) + f(x_3^*) + f(x_4^*))$

$13\pi\,(13 + 13 + 14 + 15)$

13π (55)

115π

115π

16.7.2 $(x+y+z)\,ds$ $\quad x=u+v \quad y=u-v \quad z=1+2u+v$

$0 \le u \le 4$

$0 \le v \le 1$

$\vec{r}_u = \langle 1, 1, 2 \rangle$

$\vec{r}_v = \langle 1, -1, 1 \rangle$

$\begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$ $\langle 1(1)-2(-1), 2(1)-1(1), 1(-1)-1(1) \rangle$

$\langle 1+2, 2-1, -1-1 \rangle$

$\langle 3, 1, -2 \rangle$

$\sqrt{9+1+4}$

$\sqrt{14}$

$\iint_S \, ds$

$= \iint (u+v)+(u-v)+(1+2u+v)\ \sqrt{14}$

$= \iint (4u+v+1)(\sqrt{14})$

$38\sqrt{14}$

$= \sqrt{14} \int_0^1 \int_0^4 4u+v+1 \ du\,dv$

$= \sqrt{14} \int_0^1 [2u^2+vu+u]_0^4 \, dv$

$= \sqrt{14} \int_0^1 [32+4v+4] \, dv$

$= \sqrt{14} \int_0^1 36+4v \ dv$

$= \sqrt{14} \, [36v+2v^2]_0^1$

$= \sqrt{14} \, [36+2]$

$= 38\sqrt{14}$

16.7.3 $\iint_S y \, ds$ $\quad y = x^2+z^2 \quad x^2+z^2=1$

$\sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1} \Rightarrow \sqrt{(2x)^2+(2z)^2+1}$

$= \sqrt{4x^2+4z^2+1}$

$= \sqrt{4(x^2+z^2)+1}$

$= \sqrt{4r^2+1}$

$\int_0^{2\pi} \int_0^1 r^2\left(\sqrt{4r^2+1}\right) r \, dr\,d\theta$

$\int_0^{2\pi} \int_0^1 r^3 \sqrt{4r^2+1} \ dr\,d\theta$