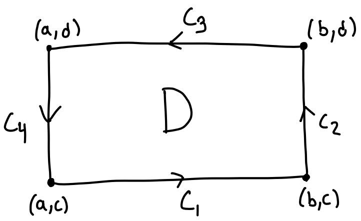
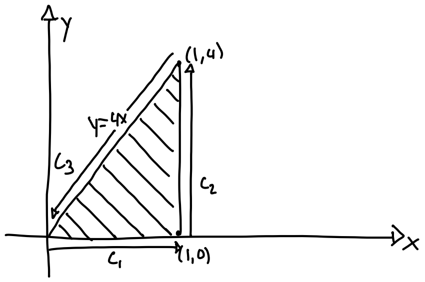
Greens Theorem

$$\iint_c (Q_x - P_y)\, dA$$



$$\int_c^d \int_a^b (Q_x - P_y)\, dx\,dy$$

Ex:



$$\int_c xy\, dx + x^2 y^3\, dy$$

$$\int_c P\,dx + Q\,dy$$

$$\int_{C_1} + \int_{C_2} + \int_{C_3}$$

$\vec{r}(t) = \langle t, 0\rangle$ $\quad$ $\vec{r}(t) = \langle 1, 4t\rangle$ $\quad$ $\vec{r}(t) = \langle 1-t, 4-4t\rangle$

$0 \le t \le 1$ $\quad$ $0 \le t \le 1$ $\quad$ $0 \le t \le 1$

$$\int_{C_1} 0 \quad 256\int_0^1 t^3\, dt = 64$$

$$\int_{C_2} 4t(0) + 64t^3(4)\, dt = 64$$

$$\int_0^1 (1-t)(4-4t)(-1)\, dt + (1-t)^2(4-4t)^3(-4)\, dt = -44$$

$64 + 0 - 44 = 20$

$P = xy \qquad Q = x^2y^3$

$P_y = x \qquad Q_x = 2xy^3$

$$\iint_D (Q_x - P_y)\, dA$$

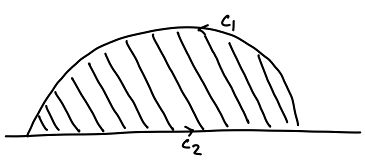
$$\int_0^1\int_0^{4x} 2xy^3 - x\;dy\,dx = 20$$

Ex:

$x = t - \sin t$

$y = 1 - \cos t$

$\vec{r}(t) = \langle t - \sin t, 1 - \cos t\rangle$



π

$$\int_{C_1} \vec{F}\cdot dr = \int_{C_1} P\,dx + Q\,dy$$

$$-\int_{-C_1} P\,dx + Q_y = -\int_0^{2\pi} 0\,dx + x\,dy$$

$$-\int_0^{2\pi} (t - \sin t)(\sin t)\, dt$$

$u = t \qquad v = -\cos t$

$dv = \sin t \qquad du = 1$

$$-\int_0^{2\pi} t\sin t - \sin^2 t\; dt$$

$\frac{1}{2} - \frac{1}{2}\cos 2\theta$

$$+\int_0^{2\pi} \sin^2 t\; dt$$

$$-\left(-t\cos t + \int_0^{2\pi} \cos t\; dt\right)$$

$$t\cos t - \int_0^{2\pi} \cos t\; dt$$

$$t\cos t - \sin t \Big|_0^{2\pi} \qquad \Big|_0^{2\pi}$$

$$2\pi\cos(2\pi) - (0\cos 0)$$

$$2\pi - 0$$

$$\int_0^{2\pi} \frac{1}{2} - \frac{1}{2}\cos 2t\; dt$$

$$\frac{1}{2}t - \frac{1}{4}\sin 2t \Big|_0^{2\pi}$$

$$\frac{1}{2}(2\pi) - \frac{1}{2}(0)$$

π $\qquad$ π