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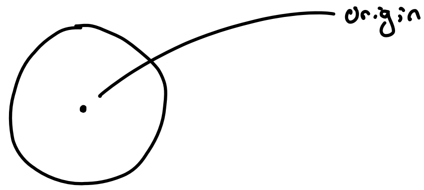
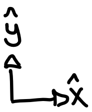
Dr. Workman
PHYS 342 HW 8

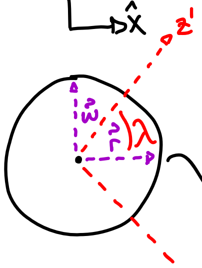


Problem 1

a.)



i.)



Centripetal accel $= \vec{\omega} \times (\vec{\omega} \times \vec{r})$
Centrifugal accel $= -\vec{\omega} \times (\vec{\omega} \times \vec{r})$

Equator : $\omega = \dfrac{2\pi}{24\text{hrs} \cdot \frac{60\text{min}}{1\text{hr}} \cdot \frac{60\text{s}}{1\text{min}}} = 7.272 \times 10^{-5} \ \frac{\text{rad}}{\text{s}}$: $\vec{\omega} = |\omega| \hat{y}$

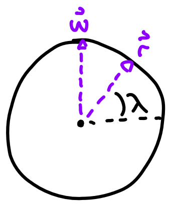
Radius of Earth : $R_E = 6.371 \times 10^{6}\,\text{m}$: $\vec{R} = |R_E| \hat{x}$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & |\omega| & 0 \\ |R_E| & 0 & 0 \end{vmatrix} = (0)\hat{x} + (0)\hat{y} - (\omega \cdot R_E)\hat{z}$$

$$-\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -|\omega| & 0 \\ 0 & 0 & -(\omega \cdot R_E) \end{vmatrix} = (\omega^2 \cdot R_E)\hat{x} + (0)\hat{y} + (0)\hat{z} \quad : \quad \omega^2 \cdot R_E = (7.272 \times 10^{-5} \tfrac{\text{rad}}{\text{s}})^2 \cdot 6.371 \times 10^{6}\,\text{m}$$

@ Equator $\boxed{\vec{A}_{CF} = 0.034 \ \text{m/s}^2}$

ii.)



$\lambda = 45^\circ$: $\vec{\omega} = |\omega| \hat{y}$: $\vec{R} = (R_E \cdot \cos(\lambda))\hat{x} + (R_E \cdot \sin(\lambda))\hat{y}$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & |\omega| & 0 \\ R_E\cos(\lambda) & R_E\sin(\lambda) & 0 \end{vmatrix} = (0)\hat{x} + (0)\hat{y} - (\omega \cdot R_E\cos(\lambda))\hat{z}$$

$$-\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -|\omega| & 0 \\ 0 & 0 & -\omega \cdot R_E\cos(\lambda) \end{vmatrix} = (\omega^2 \cdot R_E\cos(\lambda))\hat{x} + (0)\hat{y} + (0)\hat{z} :$$

$$\omega^2 \cdot R_E\cos(\lambda) = (7.272 \times 10^{-5} \tfrac{\text{rad}}{\text{s}})^2 \cdot 6.371 \times 10^{6}\,\text{m} \cdot \cos(45)$$

@ 45° $\boxed{\vec{A}_{CF} = 0.024 \ \text{m/s}^2 \ \hat{x}}$

iii.)

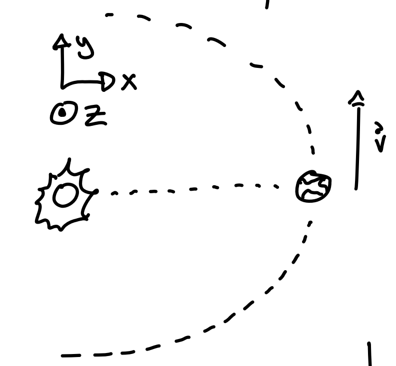
$\vec{\omega} = |\vec{\omega}| \hat{y}$: $\vec{r} = |R_E| \hat{y}$

here : $\vec{\omega} \times \vec{r} = 0$, so $-\vec{\omega} \times (\vec{\omega} \times \vec{r}) = 0$, so $\vec{A}_{CF} = 0$ at pole

@ pole $\boxed{\vec{A}_{CF} = 0 \ \text{m/s}^2}$

Problem 1) continued

b.) Same as part a.) but with different $\vec{\omega}$'s and R_E's.



$$\vec{A}_{CF} = -\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{\omega} = \frac{2\pi}{1\,\text{Year}} = \frac{2\pi}{365\,\text{days} \cdot \frac{24\,\text{hrs}}{1\,\text{day}} \cdot \frac{60\,\text{min}}{1\,\text{hr}} \cdot \frac{60\,\text{s}}{1\,\text{min}}} = 1.993 \times 10^{-7}\ \frac{\text{rad}}{s}\ \hat{y}$$

$$\vec{R} = |R_E|\,\hat{x} \quad : \quad R_E = 1.496 \times 10^{11}\ m$$

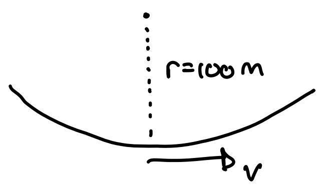
$$(\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & |\omega| & 0 \\ |R_E| & 0 & 0 \end{vmatrix} = (0)\hat{x} + (0)\hat{y} - (\omega \cdot R_E)\,\hat{z}$$

$$-\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -\omega & 0 \\ 0 & 0 & -\omega \cdot R_E \end{vmatrix} = (\omega^2 \cdot R_E)\,\hat{x} + (0)\,\hat{y} + (0)\,\hat{z} \ : \ \omega^2 R_E = (1.993 \times 10^{-7}\,\tfrac{\text{rad}}{s})^2 \cdot (1.496 \times 10^{11}\,m)\,\hat{x}$$

@ 1 AU : $\boxed{\vec{A}_{CF} = 0.006\ m/s^2\ (\hat{x})}$

The centrifugal acceleration on Earth is about 6 times higher than that of the centrifugal acceleration due to the Sun.

c.)



$$ma = \frac{mv^2}{r} \ : \ r\,a = v^2 \ : \ v = \sqrt{r \cdot a} = \sqrt{100\,m \cdot 9.81\,m/s^2} = 3.\sqrt{109}\ m/s$$

$31.3\ m/s \rightarrow 70.0\ \frac{mi}{hr}$: $\boxed{V = 70\ mi/hr}$

d.) Coriolis Force : $\vec{F} = 2m\vec{\omega} \times \vec{v}$, centripetal Force : $\vec{F} = \frac{mv^2}{r}$

$$\vec{\omega} \times \vec{v} \rightarrow \omega v \sin(\lambda) \Big|_{\lambda = 45^\circ}$$

$$2\cancel{m}\omega\cancel{v}\sin(\lambda) = \frac{\cancel{m}v^{\cancel{2}}}{r}$$

$$2\omega\sin(\lambda) = \frac{v}{r} \rightarrow v = 2 \cdot \omega r \cdot \sin(45)$$

$\omega = 7.272 \times 10^{-5}\ \frac{\text{rad}}{s}$

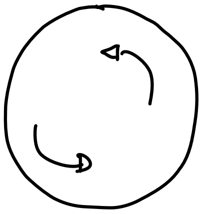
$r = 6.371 \times 10^6\ m$

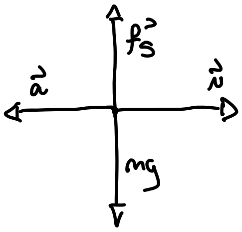
$$V = 2 \cdot (7.272 \times 10^{-5}\,\text{rad}/s) \cdot (6.371 \times 10^6\,m) \cdot \sin(45)$$

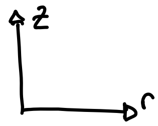
$$V = 655.2\ m/s \rightarrow V = 1465.2\ \frac{mi}{hr}$$

$$\boxed{V = 1465.2\ mi/hr}$$

Problem 2)







$$\vec{a} = (\ddot r - r\dot\phi^2)\hat r + (\cancelto{0}{r\ddot\phi + 2\dot r\dot\phi})\hat\phi + (\ddot z)\hat z$$

Force : $m\vec{a} = m(\ddot r - r\dot\phi^2)\hat r + m(\ddot z)\hat z$

Z Direction : $\mu_s N - mg = m\ddot z \rightarrow \mu_s N = m(g+\ddot z) \rightarrow N = \dfrac{m(g+\ddot z)}{\mu_s}$

R Direction : $\dfrac{mv^2}{r} - N = m\ddot r - mr\dot\phi^2$

$$\frac{mv^2}{r} - \frac{m(g+\ddot z)}{\mu_s} = m\ddot r - mr\dot\phi^2$$

$$\frac{v^2}{r} = \frac{(g+\ddot z)}{\mu_s} + \ddot r - r\dot\phi^2$$

$$v^2 = \frac{r(g+\ddot z)}{\mu_s} + \ddot r\cdot r - r^2\dot\phi^2$$

$v = \omega\cdot r \quad : \quad \omega = \dfrac{2\pi}{T}$

$v = \dfrac{2\pi}{T}\cdot r$

$$\frac{\mu_s(2\pi)^2}{T^2} = r(g+\ddot z) + \mu_s(\ddot r\cdot r - r^2\dot\phi^2)$$

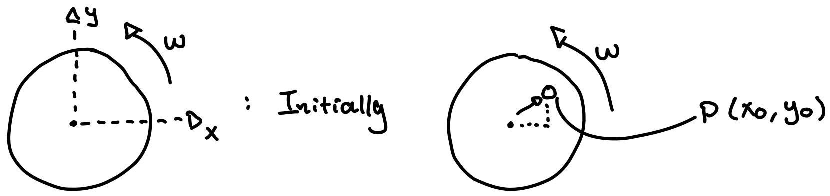
$$\mu_s\cdot(2\pi)^2 = T^2\left(r(g+\ddot z) + \mu_s(\ddot r\cdot r - r^2\dot\phi^2)\right)$$

$$T^2 = \frac{\mu_s\cdot(2\pi)^2}{\left(r(g+\ddot z) + \mu_s(\ddot r\cdot r - r^2\dot\phi^2)\right)} \rightarrow \boxed{T = \sqrt{\frac{\mu_s\cdot(2\pi)^2}{\left(r(g+\ddot z) + \mu_s(\ddot r\cdot r - r^2\dot\phi^2)\right)}}}$$

Speeds up : Time goes down

Slows down : Time goes up

Problem 3



Equation 10.30: $\vec{F}_{net} = \cancel{F} + \cancel{m\vec{g}_0} - \cancel{m\ddot{\vec{R}}_0} - \cancel{m\dot{\vec{\omega}}\times\vec{r}} - \cancel{m\vec{\omega}\times(\vec{\omega}\times\vec{r})} - 2m\vec{\omega}\times\vec{v}$

All zero except for coriolis force

$$m\vec{a} = -2m(\vec{\omega}\times\vec{v}) \quad : \quad \vec{\omega} = |\vec{\omega}|\hat{z} \ , \quad \vec{v} = V_{x0}(\hat{x}) + V_{y0}(\hat{y})$$

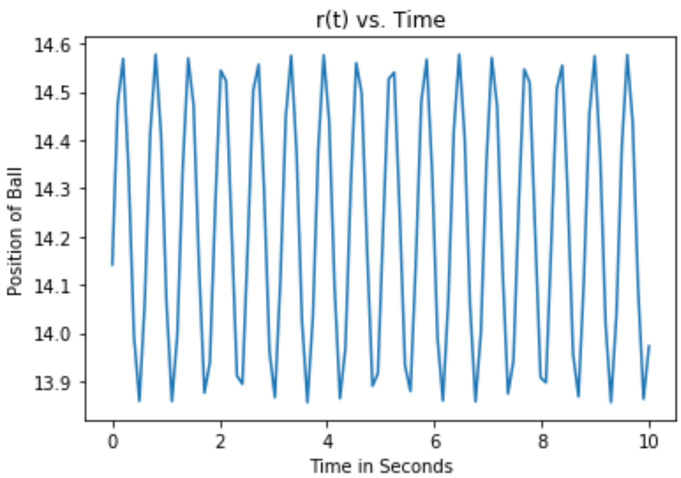
$$\vec{a} = -2(\vec{\omega}\times\vec{v})$$

$$\vec{\omega}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ V_{x0} & V_{y0} & 0 \end{vmatrix} = -\omega V_{y0}(\hat{x}) + \omega V_{x0}(\hat{y}) \quad : \quad -2\times\left(-\omega V_{y0}(\hat{x}) + \omega V_{x0}(\hat{y})\right) = 2\omega V_{y0}(\hat{x}) - 2\omega V_{x0}(\hat{y})$$

$$\vec{a} = 2\omega V_{y0}(\hat{x}) - 2\omega V_{x0}(\hat{y}) \quad : \quad \vec{a}_x = 2\omega V_{y0} \ , \quad \vec{a}_y = -2\omega V_{x0}$$

$$\ddot{x} = 2\omega\dot{y}_0 \ , \quad \ddot{y} = -2\omega\dot{x}_0 \quad : \quad r = \sqrt{x^2+y^2}$$

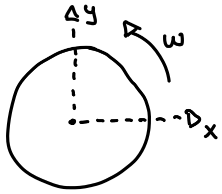
Need to solve for $x(t)$ & $y(t)$ $\qquad r(t) = \sqrt{x(t)^2 + y(t)^2}$



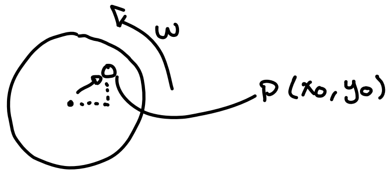
$x_0 = 10$, $y_0 = 10$ $\quad \omega = 5$

$V_{x0} = 2$, $V_{y0} = 3$

Problem 4



: Initially



Equation 10.30: $\vec{F}_{net} = \cancel{\vec{F}} + \cancel{m\vec{g}_0} - \cancel{m\ddot{\vec{R}}_0} - \cancel{m\dot{\vec{\omega}}\times\vec{r}} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v}$

$$\vec{F}_{net} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m(\vec{\omega}\times\vec{v})$$
$$m\vec{a} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m(\vec{\omega}\times\vec{v})$$
$$\vec{a} = -\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m(\vec{\omega}\times\vec{v})$$

$\vec{\omega} = |\vec{\omega}|\hat{z}$, $\vec{r} = x_0(\hat{x}) + y_0(\hat{y})$

$\vec{v} = V_{x_0}(\hat{x}) + V_{y_0}(\hat{y})$

$$\vec{\omega}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ V_{x_0} & V_{y_0} & 0 \end{vmatrix} = -\omega V_{y_0}(\hat{x}) + \omega V_{x_0}(\hat{y}) \quad : -2\times\left(-\omega V_{y_0}(\hat{x}) + \omega V_{x_0}(\hat{y})\right) = 2\omega V_{y_0}(\hat{x}) - 2\omega V_{x_0}(\hat{y})$$

$$\vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ x_0 & y_0 & 0 \end{vmatrix} = (-\omega\cdot y_0)(\hat{x}) + (\omega\cdot x_0)(\hat{y}) + 0(\hat{z}) = (-\omega\cdot y_0)(\hat{x}) + (\omega\cdot x_0)(\hat{y})$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ -\omega y_0 & \omega x_0 & 0 \end{vmatrix} = (-\omega^2\cdot x_0)(\hat{x}) - (\omega^2 y_0)(\hat{y}) + (0)\hat{z} = (-\omega^2\cdot x_0)(\hat{x}) - (\omega^2\cdot y_0)(\hat{y})$$

$$-\vec{\omega}\times(\vec{\omega}\times\vec{r}) = (\omega^2\cdot x_0)(\hat{x}) + (\omega^2 y_0)(\hat{y})$$

$$-\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2(\vec{\omega}\times\vec{v}) = (\omega^2\cdot x_0)(\hat{x}) + (\omega^2 y_0)(\hat{y}) + (2\omega V_{y_0})(\hat{x}) - (2\omega V_{x_0})(\hat{y})$$

$\vec{a}_x = \omega^2\cdot x_0 + 2\omega\cdot V_{y_0}$, $\vec{a}_y = \omega^2\cdot y_0 - 2\omega\cdot V_{x_0}$

$\ddot{x} = \omega^2\cdot x_0 + 2\omega\cdot\dot{y}_0$, $\ddot{y} = \omega^2\cdot y_0 - 2\omega\cdot\dot{x}_0$

$\eta = x + iy$

$\ddot{\eta} = \ddot{x} + i\ddot{y}$

$\ddot{x} - \omega^2 x = 2\omega\cdot\dot{y}$ $\qquad$ $\ddot{y} - \omega^2 y = -2\omega\cdot\dot{x}$

$$\begin{array}{rl} & \ddot{x} - \omega^2 x = 2\omega\cdot\dot{y} \\ + & i\ddot{y} - i\omega^2 y = -2i\omega\cdot\dot{x} \\ \hline \end{array}$$

$$\ddot{\eta} - \omega^2\eta = 2\omega(\dot{y} - i\dot{x}) = 2\omega i\left(\frac{\dot{y}}{i} - \dot{x}\right) = -2\omega i\left(\dot{x} - \frac{\dot{y}}{i}\cdot\frac{i}{i}\right) = -2\omega i(\dot{x} + i\dot{y}) = -2\omega i\,\dot{\eta}$$

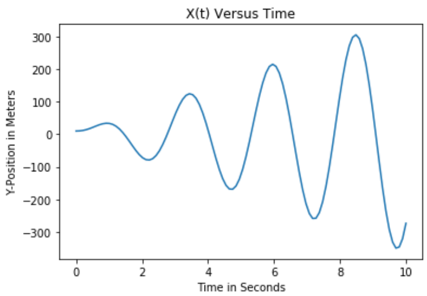
$$\therefore \quad \boxed{\ddot{\eta} + 2\omega i\,\dot{\eta} - \omega^2\eta = 0}$$

Problem 4) Continued

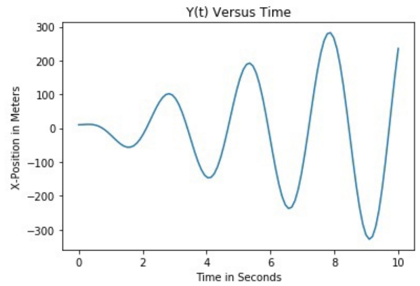
IC : $x_0=10,\ y_0=10,\ v_{x0}=0,\ v_{y0}=1$

$w=2.5$

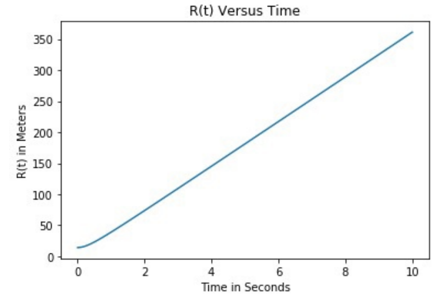
Solution in x



Solution in Y



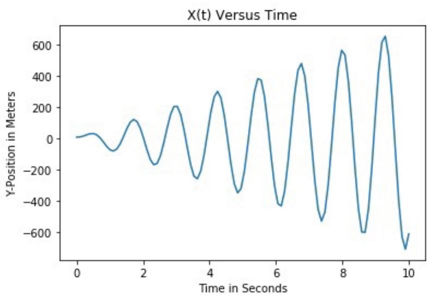
Solution for r(t)



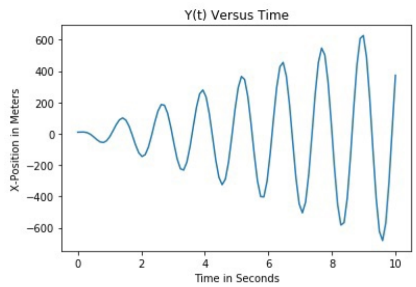
IC : $x_0=10,\ y_0=10,\ v_{x0}=0,\ v_{y0}=1$

$w=5.0$

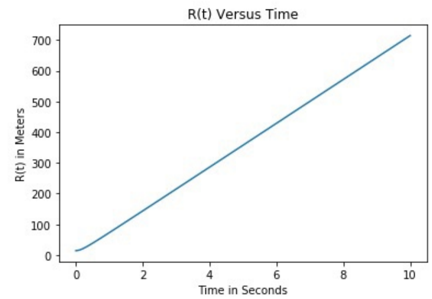
Solution in x



Solution in Y



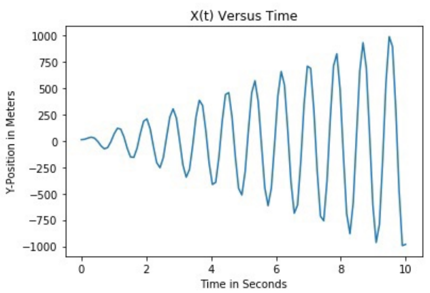
Solution for r(t)



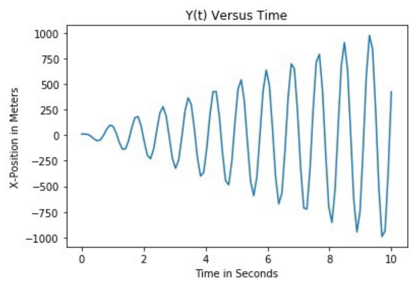
IC : $x_0=10,\ y_0=10,\ v_{x0}=0,\ v_{y0}=1$

$w=7.5$

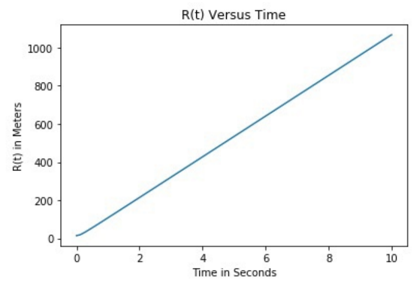
Solution in x



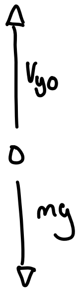
Solution in Y



Solution for r(t)



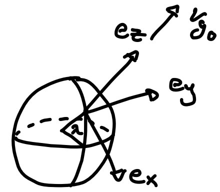
Problem 5



Coriolis force : $C_F = -2m\cdot(\vec{\omega}\times\vec{v}_r)$

$$\vec{a} = g - 2\cdot(\vec{\omega}\times\vec{v}_r)$$

$$\vec{\omega} = -\omega\cdot\cos(\lambda)(\hat{x}) + \omega\cdot\sin(\lambda)(\hat{y})$$
$$\vec{V}_r = V_{y0} - gt$$



$$(\vec{\omega}\times\vec{r}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\omega\cos(\lambda) & 0 & \omega\sin(\lambda) \\ 0 & 0 & V_{y0}-gt \end{vmatrix} = (0)(\hat{x}) + (\omega\cos(\lambda)(V_{y0}-gt))(\hat{y}) + (0)(\hat{z})$$

$$(\vec{\omega}\times\vec{r}) = \omega\cos(\lambda)(V_{y0}-gt)(\hat{y}) \quad \therefore \quad 2(\vec{\omega}\times\vec{r}) = 2\omega\cos(\lambda)(V_{y0}-gt)(\hat{y})$$

$$\vec{a}_x = 0$$
$$\vec{a}_y = 2\omega\cos(\lambda)(V_{y0}-gt)$$
$$\vec{a}_z = -g$$

$$\ddot{y} = 2\omega\cos(\lambda)V_{y0} - 2\omega\cos(\lambda)gt \;,\; \dot{y} = 2\omega\cos(\lambda)V_{y0}t - \omega\cos(\lambda)gt^2 ,\; y = \omega\cos(\lambda)V_{y0}t^2 - \omega\cos(\lambda)g\frac{t^3}{3}$$

$$\ddot{z} = -g \;:\; \dot{z} = -gt + \underbrace{\cancel{V_{z0}}}_{=0} \;:\; z = -g\frac{t^2}{2} + \cancel{V_{z0}} + \underbrace{\cancel{x_{z0}}}_{=0} \quad \therefore \quad z = -g\frac{t^2}{2}$$

let $z \to h$

$$t = \sqrt{\frac{2h}{|g|}}$$

$$y = \omega\cos(\lambda)V_{0y}t^2 - \omega\cos(\lambda)g\frac{t^3}{3} \;,\; t = \sqrt{\frac{2h}{|g|}}$$

$$\therefore \quad y(t) = \omega\cos(\lambda)V_0\left(\frac{2h}{|g|}\right) - \frac{\omega\cos(\lambda)}{3}\cdot g\cdot\left(\frac{2h}{|g|}\right)^{3/2}$$

$$\boxed{d = \omega\cos(\lambda)V_0\left(\frac{2h}{|g|}\right) - \frac{\omega\cos(\lambda)}{3}\cdot g\cdot\left(\frac{2h}{|g|}\right)^{3/2}} \qquad (1)$$

Ball:

$\lambda \cong 39^o$
$\omega = 7.272\times10^{-5}$ rad/s $\longrightarrow$ put into (1) above
$h = 10{,}000$ m/s
$g = 9.81$ m/s^2
$V_{0y} = 45$ m/s

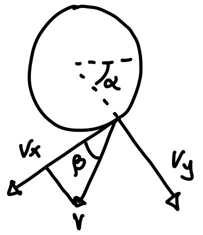
$$\boxed{d = 12.0 \text{ m}}$$

Missile:

$\lambda \cong 39^o$
$\omega = 7.272\times10^{-5}$ rad/s $\longrightarrow$ put into (1) above
$h = 10{,}000$ m/s
$g = 9.81$ m/s^2
$V_{0y} = 5{,}000$ m/s

$$\boxed{d = 559 \text{ m}}$$

Problem 6



$$m\frac{dv}{dt} = 2m(\vec{w}\times\vec{v})$$

$$\frac{dv}{dt} = 2(\vec{w}\times\vec{v})$$

$$\vec{w} = (-w\cos(\alpha))\hat{i} + (-w\sin(\alpha))\hat{j}$$
$$\vec{v} = (V_0\cos(\beta))\hat{i} + (V_0\sin(\beta) - gt)\hat{j}$$

$$2\cdot\vec{w}\times\vec{v} = 2\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -w\cos(\alpha) & -w\sin(\alpha) & 0 \\ V_0\cos(\beta) & V_0\sin(\beta)-gt & 0 \end{vmatrix} = 2\left[(0)\hat{i} + (0)\hat{j} + \left(w\cos(\alpha)(gt - V_0\sin(\beta)) + wV_0\sin(\alpha)\cos(\beta)\right)\hat{k}\right]$$

$$\cdot\vec{w}\times\vec{v} = \left(w\cos(\alpha)gt - wV_0\cos(\alpha)\sin(\beta) + wV_0\sin(\alpha)\cos(\beta)\right)\hat{k}$$
$$= \left(w\cos(\alpha)gt + wV_0(\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta))\right)\hat{k}$$

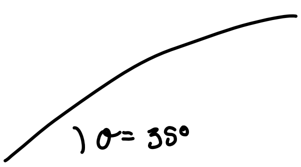
note: $\sin(A\pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$

$\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \equiv \sin(A-B)$: $\gamma = \alpha - \beta$: $\alpha = 40°$, $\beta = 35°$, $\gamma = 5°$

$$= (w\cos(\alpha)gt + wV_0\sin(\gamma))$$
$$= w(\cos(\alpha)gt + V_0\sin(\gamma)) \quad : \quad 2\vec{w}\times\vec{v} = 2w(\cos(\alpha)gt + V_0\sin(\gamma))$$

$$\frac{dv}{dt} = 2w((\cos(\alpha)gt + V_0\sin(\gamma))$$
$$\int dv = 2w\int \cos(\alpha)gt + V_0\sin(\gamma)\,dt$$
$$V = 2w\left[\cos(\alpha)\frac{gt^2}{2} + V_0 t\sin(\gamma) + C\right] \quad : \quad C = 0$$
$$V = w\left[\cos(\alpha)gt^2 + 2V_0 t\sin(\gamma)\right]$$

Deflection Velocity: $V(t) = w\cdot\left[\cos(\alpha)gt^2 + 2V_0 t\sin(\gamma)\right]$



$V_1 = V_0 + a\Delta t$
$\Delta x = x_0 + V_{x0}\Delta t + \frac{1}{2}a_x\Delta t^2$
$V_1^2 = V_0^2 + 2a\Delta x$

: In y $V_1 = 0$ @ top
$-V_0 = a\Delta t$
$\Delta t = \frac{-V_0}{a} \rightarrow a = -9.81\ m/s^2$
$V_0 = 1000\ m/s \cdot \sin(35°)$

$$\Delta t = \frac{-1000\cdot\sin(85°)\ m/s}{(-9.81\ m/s^2)} = 58.47\ s$$

Time to hit ground will be twice Δt ∴ $\Delta t_{tot} = 116.9 \approx 117\ s$

Deflection is then $d = V(t)\cdot\Delta t$

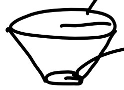
$5° = \frac{\pi}{36}$, $V_0 = 1{,}000\ m/s$
$w = 7.272\times10^{-5}\ \frac{rad}{s}$
$\alpha = 40°$
$\gamma = 5°$
$t = 117\ s$

$$d = (7.272\times10^{-5}\ rad/s)\cdot\left[\cos(40°)\cdot(9.81\ m/s^2)\cdot(117\ s)^2 + 2\cdot1000\ m/s\cdot(117\ s)\cdot\sin(5°)\right]\cdot 117\ s$$

$$\boxed{d = 1049\ m}$$

Problem 7)

a.)



$r_1 = 1\text{ ft} = 0.304878\text{ m}$

$r_2 = \frac{1}{4} r_1 = 0.0762\text{ m}$

From google

radius of drain $\simeq$ 3 in $\simeq$ 7.62 cm = 0.0762 m

water in toilet initially $\simeq$ 1.6 gal $\simeq$ 6 kg

From google

$$\int \rho v A\, dt = m \rightarrow \int \rho g t A = m \rightarrow m = \frac{\rho g t^2}{2} A \quad \therefore \quad t = \sqrt{\frac{2m}{\rho g A}}$$

$m = 6\text{ kg}$

$\rho = 1000\ \frac{\text{kg}}{\text{m}^3}$

$g = 9.81\ \text{m/s}^2$

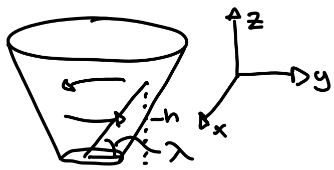
$A = (0.0762\text{m})^2 \pi$

$$t = \sqrt{\frac{2(6\text{kg})}{(1000\frac{\text{kg}}{\text{m}^3})(9.81\text{ m/s}^2)(0.0762\text{m})^2\pi}} = 0.259\text{ s}$$

$$\boxed{\Delta t = 0.259\text{ s}}$$

This is not accurate since not all of the water is traversing through the drain at once. Instead, some water is still flowing around the bowl in a circumferential direction. This is analgous to planets since planets travel in a similar direction.

b.)



Coriolis force: $2m(\vec{\omega} \times \vec{v})$: $\vec{v} = -gt\,(\hat{z})$

$\vec{w} = -w\cos(\lambda)(\hat{x}) - w\sin(\lambda)(\hat{y})$

@ Equator :

$$\vec{\omega} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ w & w & 0 \\ 0 & 0 & -gt \end{vmatrix} = (-wgt)\hat{x} + (wgt)\hat{y} + (0)\hat{z}$$

$$2(\vec{\omega} \times \vec{v}) = -2wgt\,(\hat{x}) + 2wgt\,(\hat{y}) \quad \therefore \quad \vec{F_c} = 2mwgt\,(\hat{y} - \hat{x})$$

$$\vec{a} = 2wgt\,(\hat{y} - \hat{x})$$
$$\vec{v} = wgt^2\,(\hat{y} - \hat{x})$$
$$\vec{x} = wg\frac{t^3}{3}\,(\hat{y} - \hat{x})$$

$\vec{x} = -\frac{1}{3} wgt^3 \cdot \hat{x} + \frac{1}{3} wgt^3 \cdot \hat{y}$

$\hat{x}$ - East / West

$\hat{y}$ - North / South

East/West Deflection: $w = 7.272 \times 10^{-5}$ rad/s , $g = 9.81$ m/s², $t = 0.259$ s

$$d = -\frac{1}{3}(7.272 \times 10^{-5}\text{ rad/s})(9.81\text{ m/s}^2)(0.259\text{ s})^3$$

$$\boxed{d = 4.13 \times 10^{-6}\text{ m}}$$

my $\hat{y}$ is your $\hat{z}$,

$\therefore$

$$\boxed{\vec{F}/m = \frac{2mwgt}{m}\,(\hat{y})}$$

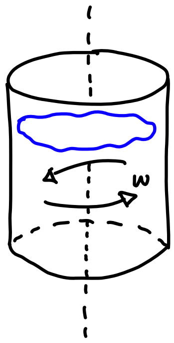
Problem 7 | Continued

c.)

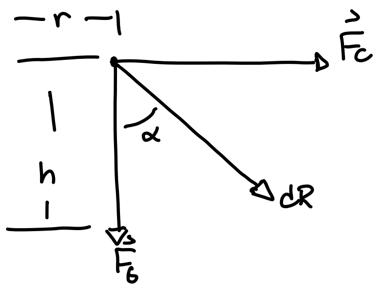
The coriolis force is significantly greater than the normal force on one particle of water. This is due to due to particle falling through the toilet as well as flying around the rim of a toilet. The combination of these two are greater than just gravity acting on the particle of water.

If the toilet were replaced with a tank from seaworld but through the same drain, the coriolis effect would increase due to there being greater circulation (ω) within the tank.

Problem 8



$$\vec{F_p} + m\vec{g} + \vec{F_c} = 0$$



$$\tan(\alpha) = \frac{dh}{dr} = \frac{F_c}{F_G} = \frac{m w^2 r}{mg} = \frac{w^2 r}{g}$$

$$\frac{w^2 r}{g} = \frac{dh}{dr} \rightarrow w^2 r\, dr = g\, dh$$

$$\int w^2 r\, dr = \int g\, dh$$

$$\frac{w^2 r^2}{2} = gh + h_0 \quad \therefore \quad h = \frac{w^2 r^2}{2g} - \frac{h_0}{g}$$

$$\boxed{Z(h) = \frac{h}{8} - \frac{w^2 r^2}{2g}}$$

This is the shape of a paraboloid.