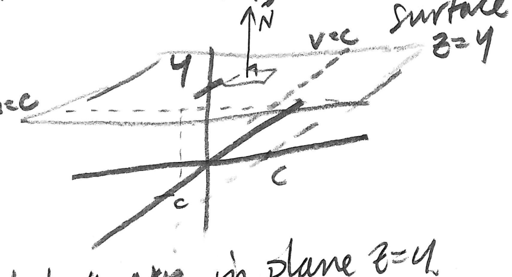
Ch 10.5 In-Class Examples

① Parameterization for $z=4$ plane, parallel to x-y plane.



Soln $\vec{r}(u,v) = [u, v, 4]$

$-\infty \le u \le \infty,\ -\infty \le v \le \infty$

$x = u =$ constant $\Rightarrow x = c \Rightarrow$ line parallel to y-axis in plane $z=4$.

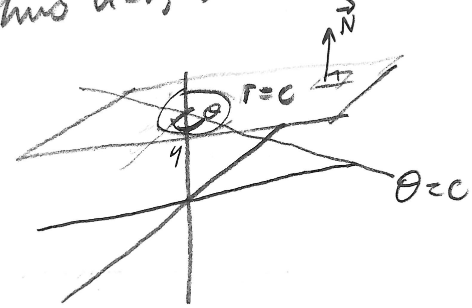
$y = v =$ constant $\Rightarrow y = c \Rightarrow$ line parallel to x-axis in $z=4$ plane

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1\vec{k}$$

② Parameterization for $z=4$ plane, parallel to xy plane, using polar coords.

Soln $\vec{r}(u,v) = [u\cos v,\ u\sin v,\ 4]$ (thus $u=r$, $v=\theta$)

$-\infty \le u \le \infty,\ 0 \le v \le 2\pi$



$u = r =$ constant $\Rightarrow r = c$
$\Rightarrow$ concentric circles centered at $(0,0)$ with radius $r=c$.

$v = \theta =$ constant $\Rightarrow \theta = c \Rightarrow$ line passing through $(0,0,4)$

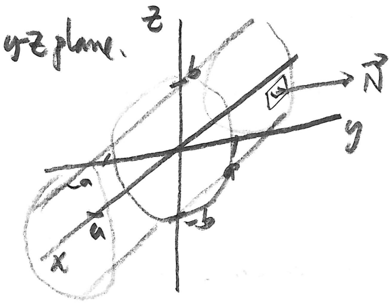
$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (u\cos^2 v + u\sin^2 v)\vec{k} = u\vec{k}$$

(3) Elliptic Cylinder $\vec{r}(u,v) = [u, a\cos v, b\sin v]$

$x = u$

$\left.\begin{array}{l} y = a\cos v \\ z = b\sin v \end{array}\right\} \quad \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \Rightarrow$ ellipse in yz plane $\Big\}$ linear in x, since $x=u$

$\therefore$ elliptical cylinder coming out along x-axis



$u = \text{const} \Rightarrow$ ellipse in $x=u$ plane, parallel to yz plane.

$v = \text{const} \Rightarrow \left.\begin{array}{l} x = u \\ y = k_1 a \\ z = k_2 b \end{array}\right\}$ line with point $(u, k_1 a, k_2 b)$

$\Rightarrow$ parallel to x-axis, along wall of cylinder, since $-\infty \le u \le \infty$

$\left.\begin{array}{l} \& \ y = k_1 a = \text{fixed} \\ z = k_2 b = \text{fixed} \end{array}\right\}$ y-z satisfy eqn of ellipse

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & -a\sin v & b\cos v \end{vmatrix} = -b\cos v\,\vec{j} - a\sin v\,\vec{k}$$

$$= [0, -b\cos v, -a\sin v]$$

↑ vector in yz plane, no direction in x

(4) Plane $2x + 3y + 5z = 4$

Soln $z = \frac{4 - 2x - 3y}{5} = \frac{4}{5} - \frac{2}{5}x - \frac{3}{5}y$

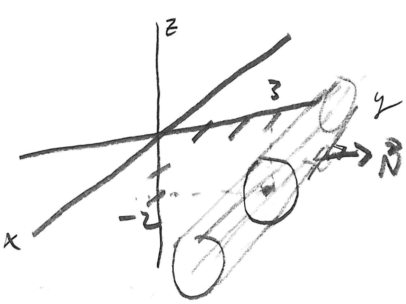
$u = x, \quad v = y, \quad z = 4/5 - 2/5 u - \frac{3}{5} v$

$\therefore \vec{r}(u,v) = [u, v, 4/5 - 2/5 u - 3/5 v]$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2/5 \\ 0 & 1 & -3/5 \end{vmatrix} = 2/5 \vec{i} + 3/5 \vec{j} + \vec{k}$$

$$= \frac{1}{5}[2, 3, 5]$$

(5) Cylinder of revolution $(y-3)^2 + (z+2)^2 = 1$



$x = u$
$y = 3 + \cos v$
$z = -2 + \sin v$

$\therefore \vec{r}(u,v) = [u, 3 + \cos v, -2 + \sin v]$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & -\sin v & \cos v \end{vmatrix} = -\cos v \vec{j} - \sin v \vec{k}$$

$$= [0, -\cos v, -\sin v]$$

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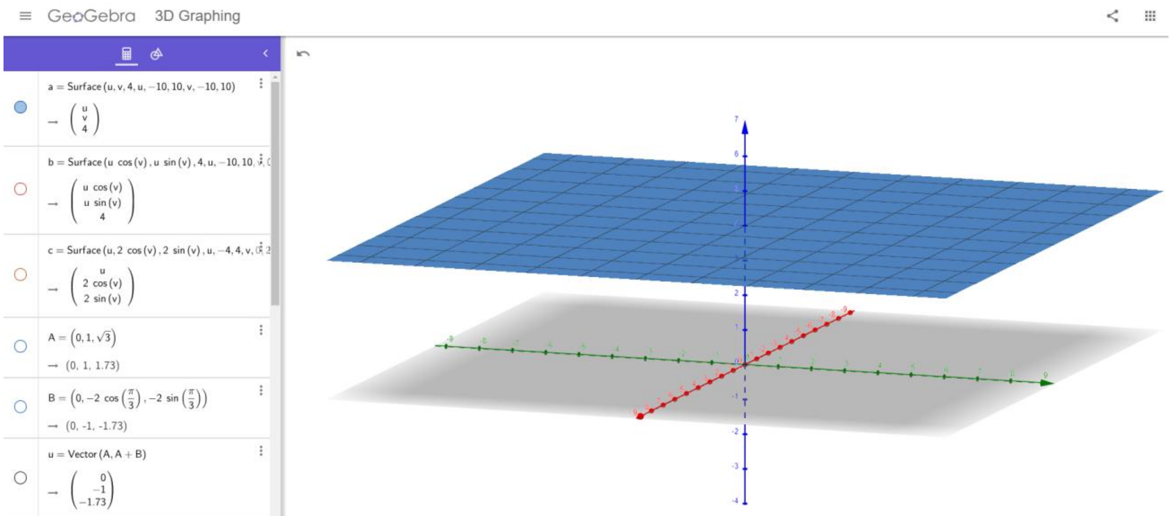
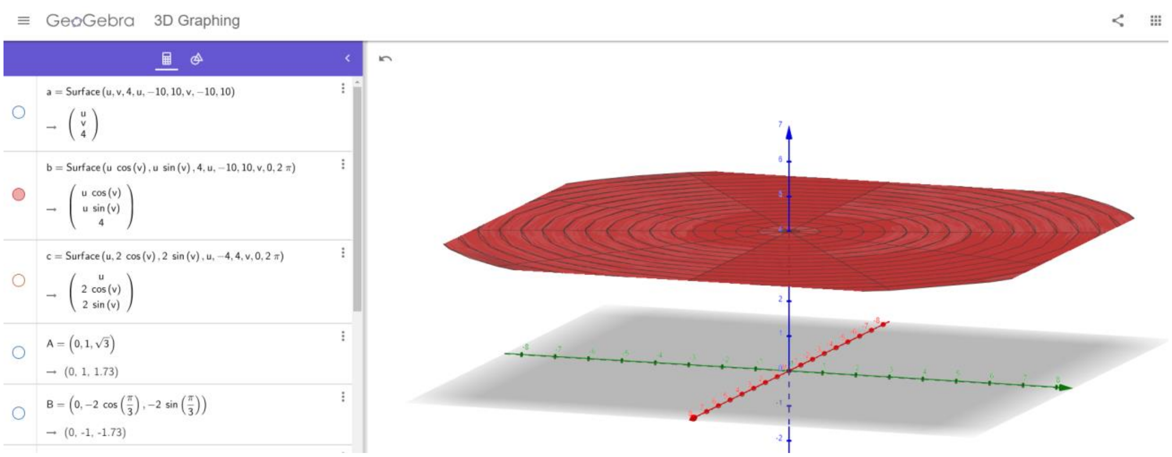
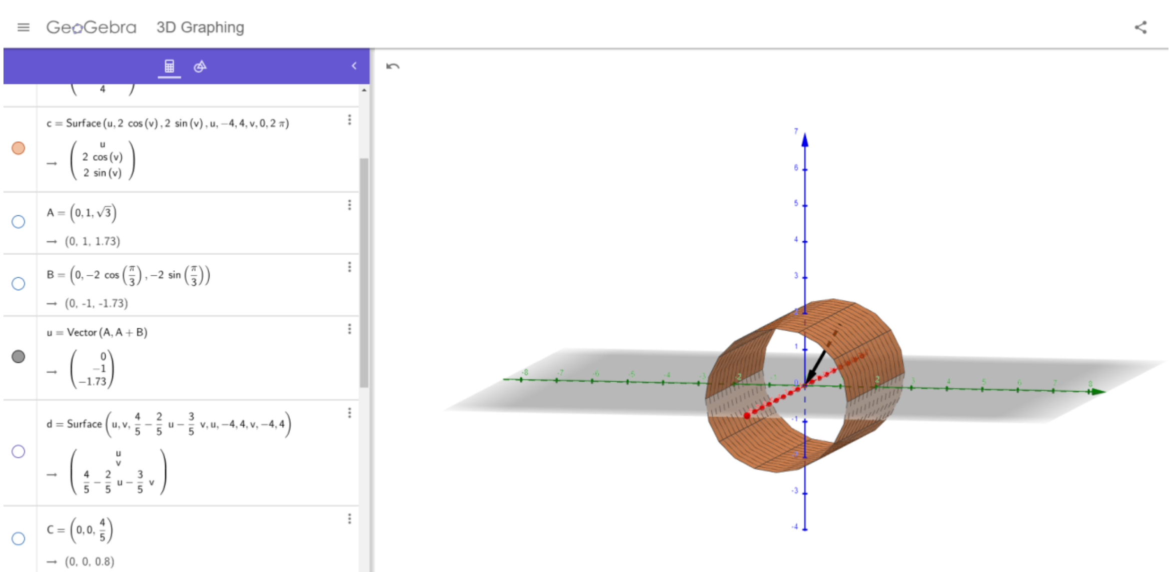
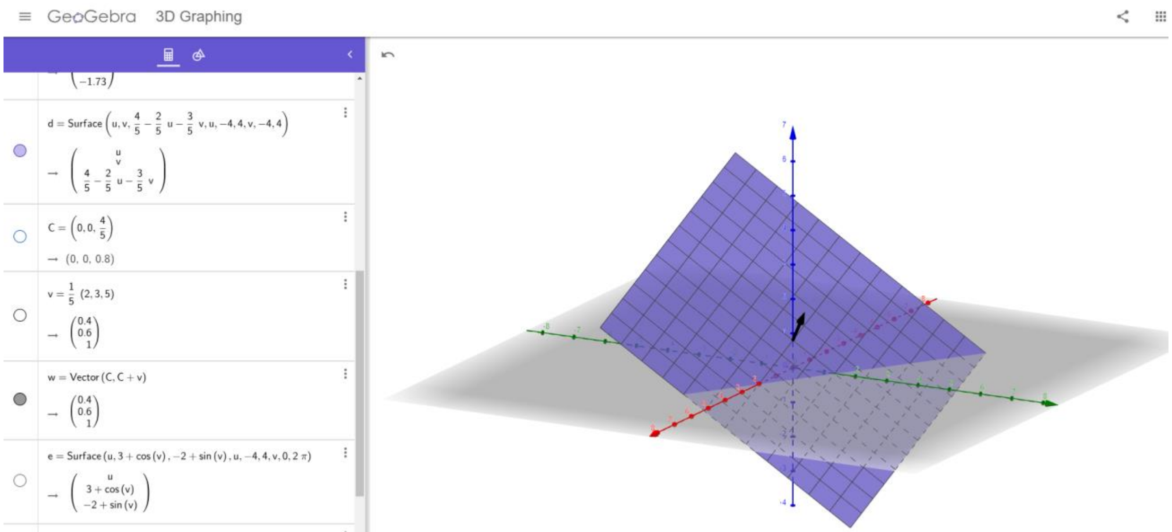
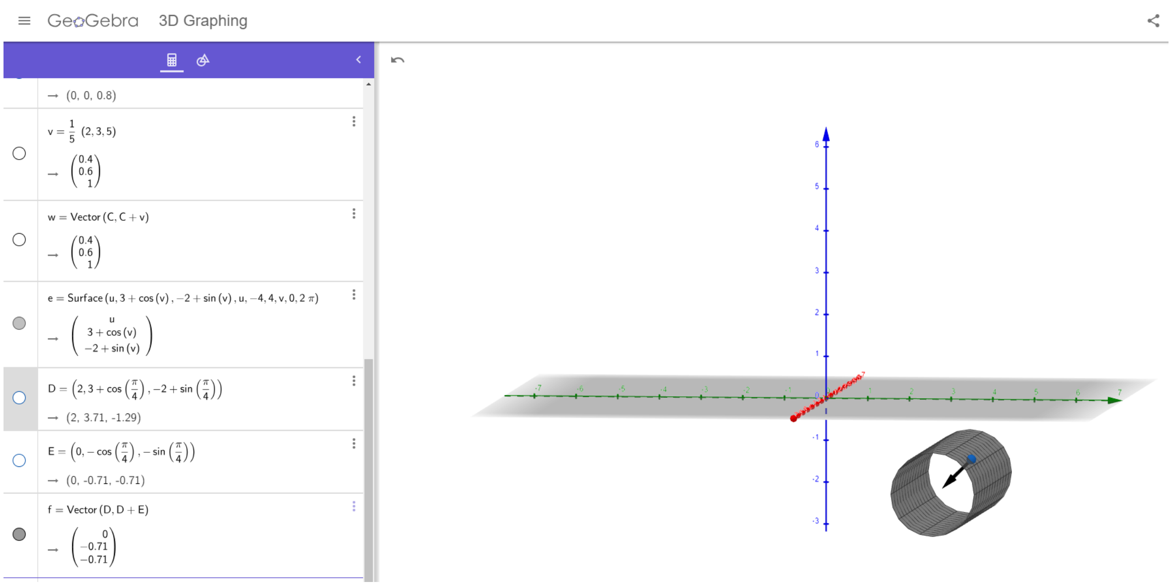







GeoGebra 3D Graphing
−1.73
d = Surface(u, v, 4/5 − 2/5 u − 3/5 v, u, −4, 4, v, −4, 4)
→ (u, v, 4/5 − 2/5 u − 3/5 v)
C = (0, 0, 4/5)
→ (0, 0, 0.8)
v = 1/5 (2, 3, 5)
→ (0.4, 0.6, 1)
w = Vector(C, C + v)
→ (0.4, 0.6, 1)
e = Surface(u, 3 + cos(v), −2 + sin(v), u, −4, 4, v, 0, 2π)
→ (u, 3 + cos(v), −2 + sin(v))


GeoGebra 3D Graphing
→ (0, 0, 0.8)
v = 1/5 (2, 3, 5)
→ (0.4, 0.6, 1)
w = Vector(C, C + v)
→ (0.4, 0.6, 1)
e = Surface(u, 3 + cos(v), −2 + sin(v), u, −4, 4, v, 0, 2π)
→ (u, 3 + cos(v), −2 + sin(v))
D = (2, 3 + cos(π/4), −2 + sin(π/4))
→ (2, 3.71, -1.29)
E = (0, − cos(π/4), − sin(π/4))
→ (0, -0.71, -0.71)
f = Vector(D, D + E)
→ (0, −0.71, −0.71)