Phys 131 2016

Lecture 39

Tues: Discussion / quiz

Supp 80, 81

Ch 12 Prob 46, 79

Ch 13 Conc Q 5

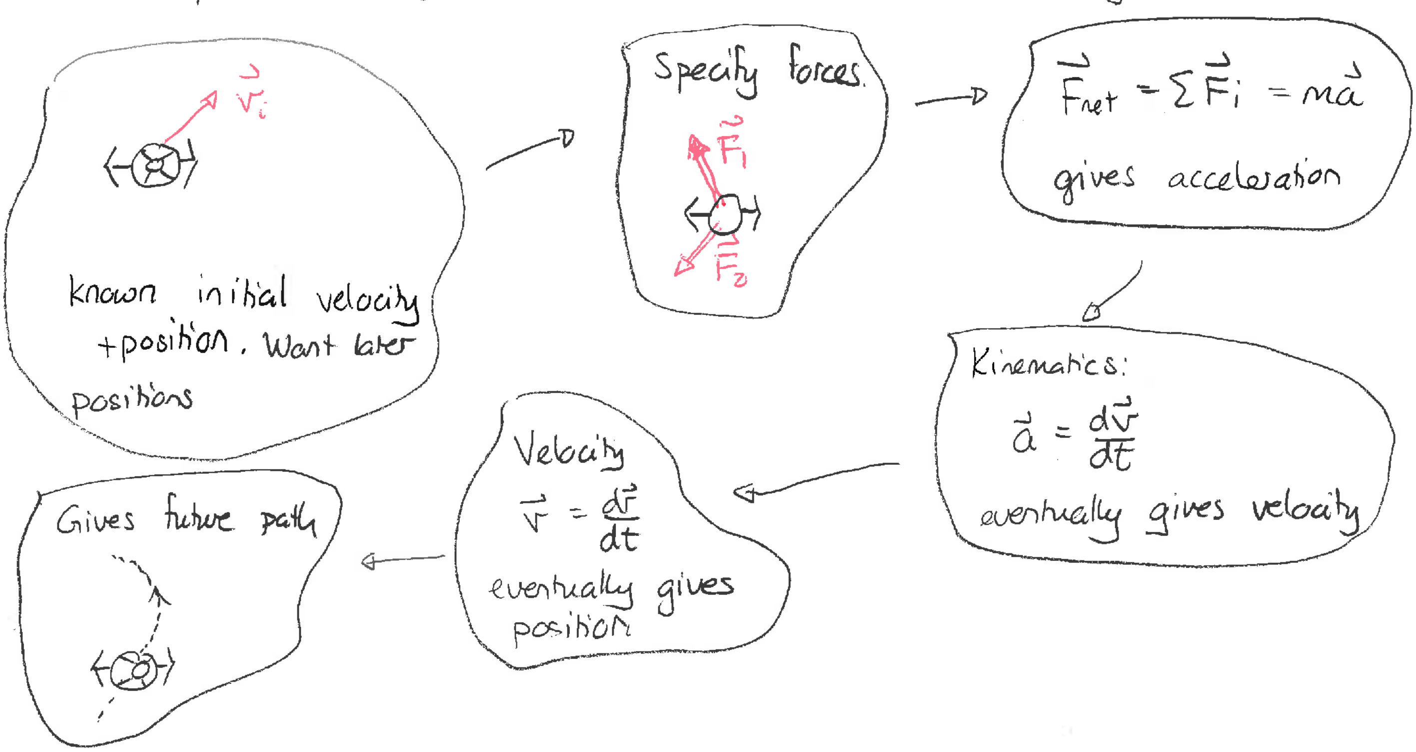
Prob 29, 35

Weds: Redo – Diagnostic test

– Up to 5pts extra credit.

Newton's system of mechanics

Newton's mechanics provides a method for describing the motion of any object given complete knowledge about its interactions with surroundings.



What is absent from this is a set of rules for computing forces. We have some rules such as for gravity near Earth's surface or springs but these are not fundamental in nature – we have to do specific measurements on specific springs, etc,... to determine the forces

In classical physics there are fundamental rules that allow for calculation of forces from first principles. These are:

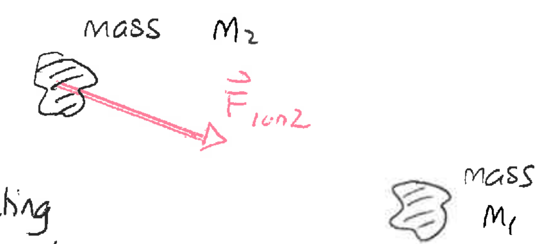
1) gravitational forces

2) electric + magnetic forces Phys 132

Gravitational forces

Gravitational forces are exerted by objects solely because these objects have mass. This realization plus assessment of the orbital motion of objects allowed Newton to provide a universal law of gravitation: Newton's Law of gravity states:

> Every object with non-zero mass exerts a force on any other object with non-zero mass. The force is attractive, directed along the line connecting the centers of mass of the two objects. The force has magnitude
>
> $$F_{1 on 2} = G \frac{M_1 M_2}{r^2}$$
>
> where r is the distance between the objects.



The quantity G is called the universal gravitational constant, and is the same for any pair of objects. Newton never determined the value of this – it was first measured about 100 years later by Cavendish. The value is:

$$G = 6.67 \times 10^{-11} \ Nm^2/kg^2$$

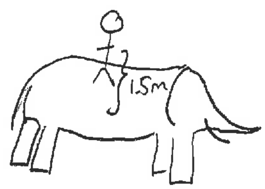
Quiz 1 90% & 80%

All objects with mass exert gravitational forces.

<u>Example:</u> A 100kg person sits on an elephant with mass 4000kg. Their centers of mass are 1.5m apart. Determine the gravitational force exerted by the person on the elephant.

<u>Answer:</u>

$r = 1.5\,m$



Here $F_{\text{person one}} = G \dfrac{M_p M_e}{r^2}$

$$= 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}\;\frac{100kg \times 4000kg}{(1.5m)^2}$$

$$= 1.2 \times 10^{-5}\,N$$

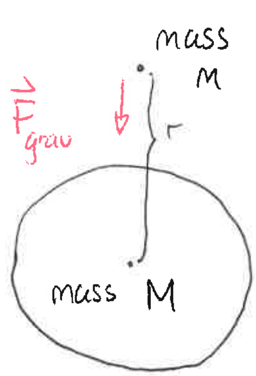
By comparison the gravitational force exerted by Earth on the elephant is $mg = 4000kg \times 9.8m/s^2 \approx 4.0\times10^{4}\,N$

With sufficiently precise equipment such forces can be detected. The experiment done by Cavendish did this

<u>Demo:</u> Cavendish expt

<u>Gravitational forces and acceleration near planets.</u>

Consider an object with mass m near a planet with mass M. Then for the object



$$\vec{F} = m\vec{a}$$

$$\Rightarrow \vec{F}_{grav} = m\vec{a}$$

$$\Rightarrow G\frac{\cancel{m}M}{r^2} = \cancel{m}a \quad \Rightarrow \quad a = G\frac{M}{r^2}$$

So the gravitational acceleration provided by the planet depends only on the mass of the planet + distance to the center of the planet

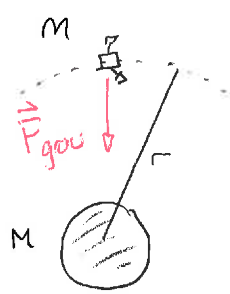
Warm Up 1

Also note that if the radius of the planet and gravity near to its surface are known then the mass of the planet is easily calculated.

Gravity and orbital motion

Demo: Newton's Cannon

Newton's system of mechanics plus the law of gravitation gives a complete description of orbital motion. Consider a satellite or moon with mass m in a circular orbit around a planet with mass M



Let r be the radius of orbit. Then Newton's second law gives:

$$\vec{F}_{net} = m\vec{a}$$

But the only force is gravity. Thus

$$F_{grav} = ma$$

But for circular motion $a = v^2/r$. Thus

$$F_{grav} = mv^2/r$$

Warm Up 2

Now $F_{grav} = G\dfrac{mM}{r^2}$ $\Rightarrow$ $G\dfrac{\cancel{m}M}{r^{\cancel{2}}} = \dfrac{\cancel{m}v^2}{\cancel{r}}$

$$\Rightarrow \quad v^2 = \frac{GM}{r}$$

This relates the speed of orbit to the radius and mass of the planet. Let T be the time for one orbit. Then $v = \dfrac{\Delta s}{\Delta t} = \dfrac{2\pi r}{T}$ gives:

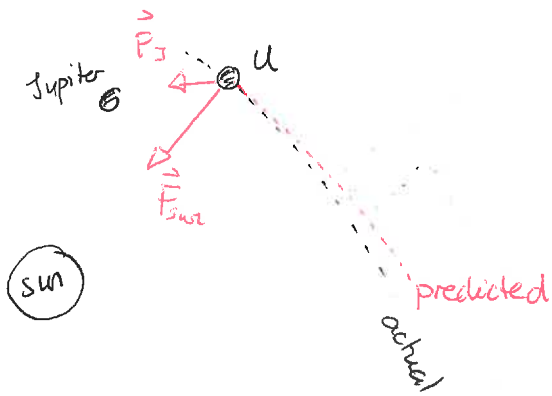
$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \Rightarrow \frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \Rightarrow GMT^2 = 4\pi r^3$$

$$\Rightarrow \boxed{T^2 = \frac{4\pi}{GM} r^3}$$

This behavior can be checked for the planets orbiting the sun. The T^2 vs r^3 behavior had been observed prior to Newton. His theory explained why.

Discovery of Neptune.

By the 1780s astronomical observations had succeeded in identifying Uranus as a planet. It's orbit was then tracked over the following years. Separately Newton's mechanics + gravitational law could be used to predict its orbit based on gravitational forces exerted by Sun and other plants. Even when all forces from the known planets Jupiter, Saturn, Earth, ... were included in the analysis, the predicted orbit did not match the actual orbit.



Demo: PhET Solar System

A) Single planet (2 body)

	m	x	y	v_x	v_y
1	200	0	0	0	0
2	5	120	0	0	140

B) Third planet

3	1	220	0	0	100

The one possible resolution was that there must be another unknown planet exerting a force on Uranus. Based on Uranus' actual orbit one could calculate where this planet should be. The new planet, Neptune, was observed at these locations in the 1840s

Demo: Planet Nine