Physics 230

Homework Set 10

1. Reconsider Problem 1 of Homework Set 9 that featured a spaceship that passed earth traveling at a speed of $V = (5/13)c$ and was heading for a distant planet 25.0 c·years away. Both the earth and the distant planet were at rest relative to one another and their clocks were previously synchronized, both reading zero when the spaceship passed the earth. Just as the spaceship passed earth, the spaceship observer set his clock to also read zero and exploded a flash bulb at that point. Later, the flash was seen by an observer at rest on the distant planet.

   a) Draw a set of two pictures in the rest frame of the earth, with the earth located at the origin of its own coordinate system, illustrating the two important events. Indicate the locations of the two important events on these two pictures.

   b) Using the Lorentz transformations, calculate the spatial and temporal coordinates of $E2$, namely, t_2, x_2, t'_2, x'_2.

   c) Draw a spacetime diagram from the perspective of earth's frame. This spacetime diagram should have a scale of 5 c·yrs : 1 inch. Be sure to label each axis and include a proper scale.

   d) Calculate the angle that the (x', ct') axes make with the (x, ct) axes on this spacetime diagram.

   e) Draw the (x', ct') axes on this spacetime diagram, labeling them as such.

   f) Draw the world lines of the earth on this spacetime diagram and label it on the diagram as such.

   g) Indicate the locations of the two important events on this spacetime diagram with a dot, labeling them as $E1$ and $E2$.

   e) Draw the world line of the distant planet as a dashed line.

2. Using the Lorentz transformations

$$\begin{aligned} x' &= \gamma(x - \frac{V}{c}ct) \\ ct' &= \gamma(ct - \frac{V}{c}x), \end{aligned} \tag{1}$$

show that

$$\Delta s'^2 = \Delta s^2 \tag{2}$$

where

$$\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 \tag{3}$$

is the spacetime interval between two events.

3. Spaceships B and C, starting at the same location when each of their clocks reads zero, depart from one another with relative velocity $(3/5)c$. Relative to our frame of reference, spaceship B moves to the left and spaceship C moves to the right. Let S be the rest frame of spaceship B and S' be the rest frame of spaceship C.

One week later according to C's clocks, C's captain goes berserk and fires a photon torpedo at B. Similarly, when clocks on B read one week, B's captain goes crazy and fires a photon torpedo at C.

a) Draw a set of five pictures in the rest frame of spaceship C, illustrating the five important events. For the sake of uniformity, and to stay consistent with this example worked out in class from B's perspective, label the events as such: $E1$: B departs C, $E2$: B fires at C, $E3$: C fires at B, $E4$: C photons reach B, and $E5$: B photons reach C. Indicate the locations of the five important events on these five pictures.

Answer the following questions from the point of view of the observer at rest in spaceship C's frame at the origin of the S' coordinate system.

b) Calculate the five times corresponding to each of the five relevant events.

c) Draw a spacetime diagram according to spaceship C's frame. This spacetime diagram should take up most of a full sheet of paper with a scale of 1 c·week : 4 in. Be sure to label each axis and include a proper scale.

d) Draw the (ct, x) axes on this spacetime diagram.

e) Draw the world lines of spaceship B and spaceship C on this spacetime diagram.

f) Draw the world lines of the two photon trajectories as *dashed* lines.

g) Indicate the locations of the five important events on this spacetime diagram with dots, labeling them as $E1,\ E2,\ E3,\ E4,\ E5$, respectively.

h) According to the spaceship C frame, which spaceship fires their photon torpedo first? Which spaceship gets hit by the enemy photon torpedo first?

4. Reconsider the two spaceships of the previous problem, still from the perspective of spaceship C. Using the Lorentz transformations and the inverse Lorentz transformations, calculate the spatial and temporal coordinates for events $E2,\ E3$, $E4,\ E5$.
For example: for event 2, find $t_2',\ x_2',\ t_2,$ and x_2. Notice that these events should have *negative* (or zero) x' coordinates and *positive* (or zero) x coordinates. Use your spacetime diagram from Problem 4 to check that your answers make sense.