Phys 131 2016

Lecture 13

Thurs: Seminar

Fri: HW due by 5pm

Supp Ex 29, 31

Ch 6 Conc Q 12

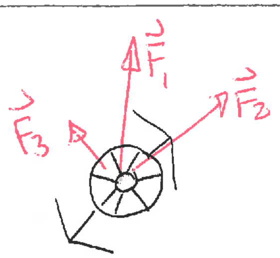
Ch 6 Prob 1, 12, 15, 39, 42

Newton's 2nd Law

The connection between forces + motion is given via Newton's 2nd Law. The key concept is:

Forces determine acceleration

Newton's 2nd Law specifies exactly how.



object mass m

1) Pick an object of interest and list all forces on the object: $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$

2) Construct net force (vector sum)

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = \underbrace{\sum \vec{F}_i}_{\text{all force vectors}}$$

3) Acceleration of object is given by

$$\vec{a} = \vec{F}_{net}/m \quad \text{or} \quad \vec{F}_{net} = m\vec{a}$$

Note that 1) force describes interactions; acceleration describes motion

2) force does not directly determine velocity.

Equilibrium

An object which is at rest or is moving with a constant velocity will have zero acceleration. This type of situation is called equilibrium. Examples include:

1) stationary structures (buildings...)

2) parts of human body (e.g. in traction...)

If $\vec{a} = 0$ then $\vec{F}_{net} = m\vec{a} = 0$. Thus we get

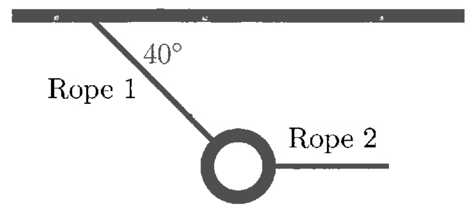
$$\boxed{\text{Equilibrium} \iff \vec{F}_{net} = 0}$$

This vector equation actually produces several algebraic equations - one for each component direction. Thus

$$\vec{F}_{net} = \sum \vec{F}_i = 0 \implies \begin{cases} \sum_i F_{ix} = 0 & \leftarrow \text{add all } x \text{ components and set } = 0 \\ \sum_i F_{iy} = 0 & \leftarrow \text{add all } y \text{ components and set } = 0 \end{cases}$$

33 Suspended object in equilibrium

A 2.50 kg ring is suspended from the ceiling and is held at rest by two ropes as illustrated. Rope 2 pulls horizontally. The aim of this exercise is to use Newton's 2nd Law to determine the tension in each rope. One piece of background information that you will need to answer this is that the magnitude of the gravitational force on an object of mass m is $F_g = mg$.



a) Draw a free body diagram for the ring. Label the tension forces $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$.

b) Write Newton's 2nd Law in its component form, i.e. write

$$F_{\text{net } x} = \Sigma F_{ix} = ma_x \tag{1}$$
$$F_{\text{net } y} = \Sigma F_{iy} = ma_y \tag{2}$$

Insert as much information as possible about the acceleration. You will return to these equations shortly; they will generate the algebra that eventually gives you the tensions.

c) These equations require all components of all forces, including the two unknown tension forces. In order to manage these, you should express the components of each tension force in terms of its magnitude. When doing this denote the magnitude of the tension in rope 1 by T_1 and for rope 2, by T_2.

d) List as much information as possible about each component for each force; each could be a number or an algebraic expression. Use one of the two formats below.

$$F_{gx} = \cdots$$
$$F_{gy} = \cdots$$
$$T_{1x} = \cdots$$
$$T_{1y} = \cdots$$
$$\vdots$$

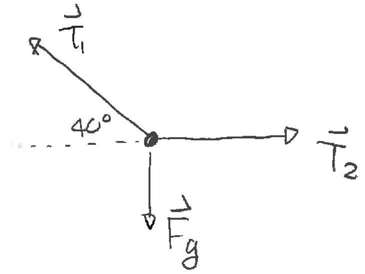
Force	x comp	y comp
$\vec{\mathbf{F}}_g$		
$\vec{\mathbf{T}}_1$		
$\vdots$		

e) Use Eq. (1) to obtain an equation relating various quantities that appear in this problem. Do the same with Eq. (2). You should get two expressions that contain the two unknowns T_1 and T_2. Solve them for the unknowns.

f) If you had one rope that is rated to break when the tension exceeds 30 N and another rated to break when the tension exceeds 40 N, which one would you use to suspend the object as illustrated above?

Answer:

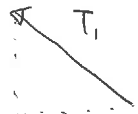
a)



b) $\sum F_{ix} = ma_x = 0 \qquad \sum F_{ix} = 0 \qquad -(1)$

$\sum F_{iy} = ma_y = 0 \qquad \sum F_{iy} = 0 \qquad -(2)$

c) For $\vec{T}$



$T_{1x} = -T_1 \cos 40°$

$T_{1y} = T_1 \sin 40°$

$T_{2x} = T_2$

$T_{2y} = 0$

d) $F_g = mg = 2.50\,kg \times 9.8\,m/s^2 = 24.5N$

$F_{gx} = 0$

$F_{gy} = -24.5N$

	x	y
$\vec{F}_g$	0	−24.5 N
$\vec{T}_1$	$-T_1 \cos 40°$	$T_1 \sin 40°$
$\vec{T}_2$	T_2	0

e) Eq (1) gives $\sum F_{ix} = 0 \Rightarrow -T_1 \cos 40° + T_2 = 0$

Eq (2) " $\sum F_{iy} = 0 \Rightarrow -24.5N + T_1 \sin 40° = 0$

$\Rightarrow T_1 \sin 40° = 24.5N \Rightarrow T_1 = \dfrac{24.5N}{\sin 40°}$

$\Rightarrow T_1 = 38.1N$

Then $-T_1 \cos 40° + T_2 = 0$

$\Rightarrow T_2 = T_1 \cos 40° = 38.1N \cos 40° = 29.2N$

f) Rope 1 would be the stronger rope (40N breaking)...

* Quiz 1

Non- equilibrium dynamics

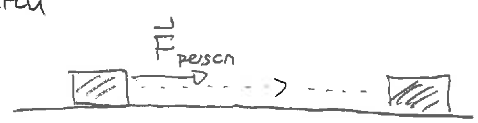
In generally $\vec{a} \neq 0$. Then in terms of components Newton's 2^{nd} Law gives:

$$\sum F_{ix} = ma_x \qquad \sum F_{iy} = ma_y$$

add all x components of forces
set equal to x ma_x

Example: A 50kg box is initially at rest on a horizontal surface. A person pushes with a constant force and the box moves 15m in 10s. During this time a 150N friction force opposes the motion. Determine the force exerted by the person.

Answer: ① Sketch



$t_0 = 0s$ $\quad$ $t_1 = 10s$

$x_0 = 0m$ $\quad$ $x_1 = 15m$

$v_0 = 0m/s$ $\quad$ $v_1 = ??$

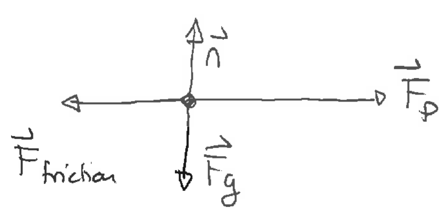
$a_x = ??$

We will use

$$\vec{F}_{net} = m\vec{a} \Rightarrow \sum F_{ix} = ma_x \qquad \sum F_{iy} = ma_y = 0$$

(no vertical motion.

② Forces



③ Components: only horizontal matter $\qquad \sum F_{ix} = ma_x$

magnitudes

$$\sum F_{ix} = F_p - F_{friction} = ma_x$$

$$\Rightarrow F_p - 150N = ma_x \Rightarrow \boxed{F_p = 150N + 50kg\, a_x}$$

④ Need acceleration from kinematics:

$$x_f = x_i + v_{ox}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$\Rightarrow 15m = \cancel{0m} + \cancel{0m/s \cdot 10s} + \frac{1}{2}a_x(10s)^2$$

$$\Rightarrow 15m = 50s^2 a_x \Rightarrow \boxed{a_x = 0.30m/s^2}$$

⑤ Combine: $F_p = 150N + 50kg \times 0.30m/s^2 = \boxed{165N}$ ☒