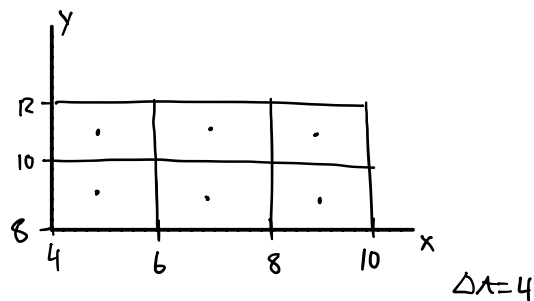1.) Estimate the volume of the solid that lies below the surface $z = xy$ and above the following rectangle

$R = \{(x,y) \mid 4 \leq x \leq 10, \ 8 \leq y \leq 12\}$

$M = 3$
$n = 2$

$\Delta A = 4$

$V = \sum_{i=1}^{3} \sum_{j=1}^{2} f(x_i, y_j) \Delta A$

$= f(6,10) \Delta A + f(6,12) \Delta A + f(8,10) \Delta A + f(8,12) \Delta A + f(10,10) \Delta A + f(10,12) \Delta A$

$= 60(4) + 72(4) + 80(4) + 96(4) + 100(4) + 120(4)$

$= 2112$

$\boxed{2112}$

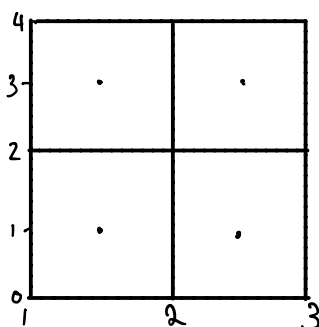$V = \sum_{i=1}^{3} \sum_{j=1}^{2} f(\bar{x}_i, \bar{y}_j) \Delta A \qquad \Delta A = 4$

$= f(5,9) \Delta A + f(5,11) \Delta A + f(7,9) \Delta A + f(7,11) \Delta A + f(9,9) \Delta A + f(9,11) \Delta A$

$= 45(4) + 55(4) + 63(4) + 77(4) + 81(4) + 99(4)$

$= 1680$

$\boxed{1680}$

4.) $f(x,y) \qquad R = [1,3] \times [0,4]$
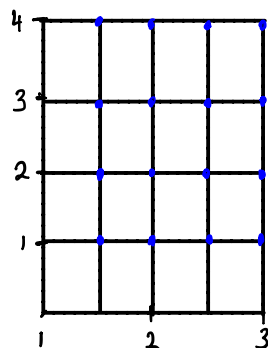

$\Delta x = 1$
$\Delta y = 2$
$M = 2 = n$

$\Delta A = 2$

$f(1.5,1) \Delta A + f(1.5,3) \Delta A + f(2.5,1) \Delta A + f(2.5,3) \Delta A$

$1(2) - 7(2) + 6(2) - 1(2)$

$-14 + 2 + 12 - 2$

$-16 + 14$

-2

$\boxed{\iint_R f(x,y) \, dA = -2}$

$f(1.5,1) \Delta A + f(1.5,2) \Delta A + f(1.5,3) \Delta A + f(1.5,4) \Delta A$
$f(2,1) \Delta A + f(2,2) \Delta A + f(2,3) \Delta A + f(2,4) \Delta A$
$f(2.5,1) \Delta A + f(2.5,2) \Delta A + f(2.5,3) \Delta A + f(2.5,4) \Delta A$
$f(3,1) \Delta A + f(3,2) \Delta A + f(3,3) \Delta A + f(3,4) \Delta A$

$\Delta x = 0.5$
$\Delta y = 1$
$\Delta A = 0.5$
$= -2.5$