2.3 (The Imaginary Unit and Complex Numbers)

Imaginary Unit

i.) imaginary unit $i = \sqrt{-1}$, $i^2 = -1$

ii.) $z = a+bi$ is a complex number

iii.) a is the real part, b is the imaginary

Equality of Complex Numbers

$a+bi$ & $c+di$

$a+bi = c+di$

iff $a=c$ & $b=d$

Magnitude of a Complex Number

$z = a+bi$

$|z| = \sqrt{a^2+b^2}$

Complex Conjugate

$z = a+bi$

$\bar{z} = a-bi$

$|z| = |\bar{z}|$

Complex Conjugate Magnitude

$z = a+bi$ $\quad \bar{z} = a-bi$

$|z| = \sqrt{a^2+b^2} = \sqrt{a^2+(-b)^2} = |\bar{z}|$

Addition and Subtraction of Complex Numbers

$(a+bi) \pm (c+di)$

$(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$

Conjugate Sum Property

$z_1, z_2, \ldots z_n$ are complex

$\overline{z_1 + z_2 \ldots z_n} = \bar{z}_1 + \bar{z}_2 + \ldots \bar{z}_n$

Euler's Formula

$e^{int} = \cos(nt) + i\sin(nt)$

Periodicity Identity

$e^{i(t+2\pi n)} = e^{it}$