Ch. 3 Formalism

3-11-19

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$, Span, basis, dimension

$\hat{\imath}, \hat{\jmath}, \hat{k} \to$ basis
$3 \to$ dimension

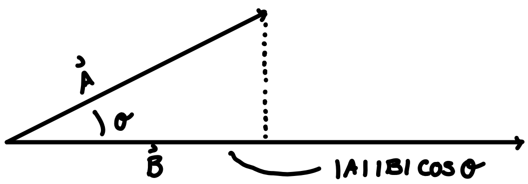
$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$
$|\hat{e}_1\rangle, |\hat{e}_2\rangle, |\hat{e}_3\rangle, |\hat{e}_k\rangle, \ldots |\hat{e}_n\rangle$
$|\alpha\rangle = \sum_i a_i|e_i\rangle \to f(x) = \sum c_n|\psi_n\rangle$

Inner Product $= \langle\alpha|\alpha\rangle \to$ Vectors as you know them

$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2$
$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$\langle\alpha|$ - Bra
└ conjugate
$|\alpha\rangle$ - ket

Properties
$\langle B|\alpha\rangle = \langle\alpha|B\rangle^*$
$\langle\alpha|\alpha\rangle \geq 0$
Only 0 if $|\alpha\rangle = 0$



$\langle\alpha|\Big[b|B\rangle + c|\gamma\rangle\Big] = b\langle\alpha|B\rangle + c\langle\alpha|\gamma\rangle$

$\psi(x,0) = \sum c_n|\hat{e}_n\rangle \to c_n = \langle e_n|\psi(x,0)\rangle$: $\langle\alpha|\alpha\rangle = 1$

$\langle\alpha_i|\alpha_j\rangle = \delta_{ij}$, $\langle\psi|\psi\rangle = 1$ $\quad |F\rangle = \sum c_n|\psi_n\rangle$

$f(x) \to$ Linear Algebra , $\hat{Q}(x,p) = \hat{Q}(x, -i\hbar\frac{\partial}{\partial x})$, $\hat{Q} \to$ Matrices
Functions

$\hat{T}_{ij} =$
row, column
$\begin{bmatrix} T_{ii} & T_{12} & T_{13} & \cdots & T_{ij} \\ T_{21} & \ddots & & & \vdots \\ T_{31} & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ T_{i1} & \cdots & \cdots & \cdots & T_{ij} \end{bmatrix}$

$\hat{Q} \to$ Represent measurements of physical variables

iff $\hat{Q}\psi_n = q_n\psi_n$, Then q_n is measured and stays q_n
$a_1|\psi_1\rangle + a_2|\psi_2\rangle + a_3|\psi_3\rangle$
$\sum |a_i|^2 = 1$

Goal of Q.M is to determine our complete set of counting observables

$[x, p_x] = i\hbar$

Einstein Summation : $A_{ij}B_{jk} = C_{ik}$

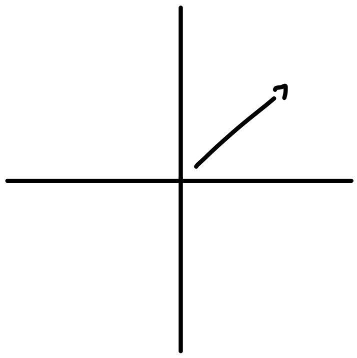
$C_{11} = (A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31})$
$C_{12} = (A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32})$
$C_{13} = (A_{11}B_{13} + A_{12}B_{23} + A_{13}B_{33})$

$\langle b|a\rangle$

$(1\ 2\ 3)\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = 32$
$B_{13}A_{31} = C_{11}$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}(4\ 5\ 6) = \begin{bmatrix} 4 & & \\ & 10 & \\ & & 18 \end{bmatrix}$
$A_{31}B_{13} = C_{33}$

$|\vec{A}\rangle = A_x \begin{pmatrix}1\\0\\0\end{pmatrix} + A_y \begin{pmatrix}0\\1\\0\end{pmatrix} + A_z \begin{pmatrix}0\\0\\1\end{pmatrix} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$



$= \sum C_i |e_i\rangle = |\alpha\rangle$

$= \sum C_n |\psi_n\rangle = |\psi(x,0)\rangle$

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$|a\rangle = \begin{pmatrix}1\\2\\3\end{pmatrix}, \quad \langle B| = 4\,5\,6$

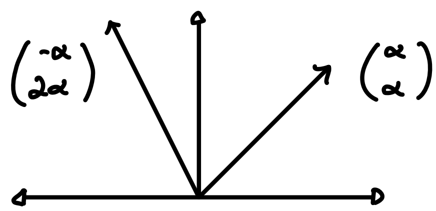
$\langle B|a\rangle = 4+10+18 = 32$

$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix} :$ $\quad C_{11} = A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31}$

$C_{23} = A_{21}B_{13} + A_{22}B_{23} + A_{23}B_{33}$

Ex: $\hat{A} = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}, \quad \vec{B} = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 7 & -4 \end{bmatrix}, \quad \hat{A}\cdot\vec{B} = \vec{C} = \begin{bmatrix} 8 & 34 & 4 \\ -1 & -8 & -13 \end{bmatrix}$

$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}(a_1^* \; a_2^* \; a_3^*) \rightarrow = |\alpha\rangle\langle\alpha|$



$\hat{Q} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{Q}_{22}|a_{21}\rangle = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix}\alpha\\\alpha\end{pmatrix} = \begin{pmatrix}-\alpha\\-\alpha\end{pmatrix}$

$\hat{R}_{22} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}, \quad \hat{R}|a\rangle = \begin{pmatrix}-\alpha\\2\alpha\end{pmatrix}$

$\hat{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad |\alpha\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$

$\hat{R}|\alpha\rangle = \begin{bmatrix} \cos\theta(\alpha) - \sin\theta(\beta) \\ \sin\theta(\alpha) + \cos\theta(\beta) \end{bmatrix}$ Matrices are linear transformations

$\langle a|b\rangle \rightarrow$ Inner product. $(a_1^* \; a_2^* \; a_3^*)\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} = a_1^* b_1 + a_2^* b_2 + a_3^* b_3$

$\langle C|D\rangle = \int C(x)^* D(x)\, dx$

$F(x) = \sum C_n \psi_n$

$C_n = \langle \psi | f(x)\rangle$, $a_i = \langle \hat{e}_i | \alpha \rangle$

Determinant $(\hat{A})$ - $|(\hat{A})|$

2×2 $\vec{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

3×3 $\vec{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a(ei - fh) + b(fg - di) + c(dh - eg)$

Ex: $\hat{E} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

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$|\alpha\rangle = \sum a_i e_i$

$|F\rangle = \sum C_n \psi_n$

$C_n = \langle \psi_n | f \rangle$

$a_1 = \langle e_i | \alpha \rangle$

$\hat{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\hat{A}^{-1} = \frac{1}{\det(\hat{A})} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\hat{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\hat{A}^{-1} = -\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$

$\hat{A} \cdot \hat{A}^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$ ✓

We need to know how to multiply, take determinants, Eigenvalues, Eigenvectors

Hermetian Matrix:

$\hat{A} = \hat{A}^+ = (A^*)^T$ IF Hermitian,

$\hat{A} = \hat{A}^+ \longrightarrow \langle \hat{T}^+ \alpha | \beta \rangle = \langle \alpha | \hat{T} \beta \rangle$: bra $\rightarrow$ Transpose conjugate of a ket.

$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = |\alpha\rangle$, $\langle \alpha | = a_1^* \, a_2^* \, a_3^*$

means $\langle \phi | \hat{A} | \psi \rangle = \langle \hat{A}\phi | \psi \rangle$: Ex $\langle \psi | \hat{Q} | \psi \rangle = \langle \hat{Q} \rangle$

Operators in QM represent physically observable quantities. $\longrightarrow$ on next test

If an operator is not Hermetian, it cannot be valid.

$\langle \varphi | \hat{A} | \psi \rangle = \langle \hat{A}\varphi | \psi \rangle$: Hermetian , $\langle \alpha | c\beta \rangle = c\langle \alpha | \beta \rangle$, $\overline{123} \cdot ② \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

$\langle c\alpha | \beta \rangle = c^{*}\langle \alpha | \beta \rangle$

Properties of Hermetian operators

$\hat{Q}|\psi_n\rangle = q|\psi_n\rangle$, $|\psi_n\rangle \rightarrow$ Orthonormal complete set

$\langle \psi_m | \psi_n \rangle = \delta_{mn}$

Ex : $\overset{\hat{i}}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}, \overset{\hat{j}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}, \overset{\hat{k}}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$

$\lambda \rightarrow$ Eigenvalue of $\hat{Q}$: These values must be real

$\langle \hat{Q}\psi | \psi \rangle = \langle \lambda\psi | \psi \rangle = \lambda^{*}\underbrace{\langle \psi | \psi \rangle}_{1} = \lambda^{*}$

$\langle \psi | \hat{Q}\psi \rangle = \langle \psi | \lambda\psi \rangle = \lambda\langle \psi | \psi \rangle = \lambda$

$\lambda^{*} = \lambda$

If $\hat{Q}$ is Hermetian, $\langle \psi_m | \psi_n \rangle = \delta_{mn} \longrightarrow$ Shows (ψ_m, ψ_n) are orthogonal

If $\hat{Q}$ is Hermetian, Eigenvalues are real

$\hat{Q}|\psi_n\rangle = \lambda_n|\psi_n\rangle$, $\hat{Q}|\psi_m\rangle = \lambda_m|\psi_m\rangle$

$\langle \psi_m | \hat{Q}\psi_n \rangle = \lambda_n\langle \psi_m | \psi_n \rangle$

$\langle \hat{Q}\psi_m | \psi_n \rangle = \lambda_m\langle \psi_m | \psi_n \rangle$

$\lambda_n\underbrace{\langle \psi_m | \psi_n \rangle}_{0} = \lambda_m\underbrace{\langle \psi_m | \psi_n \rangle}_{0} \longrightarrow$ case 1: $m \neq n$

$\lambda_n\underbrace{\langle \psi_m | \psi_n \rangle}_{1} = \lambda_n\underbrace{\langle \psi_m | \psi_n \rangle}_{1} \longrightarrow$ case 2: $m = n$

$\hat{Q}$ Hermetian operator : $\int (\hat{Q}\psi)^{*}\psi = \int \psi^{*}\hat{Q}\psi$

Set such that $\langle \psi_m | \psi_n \rangle = \delta_{mn}$

$|F\rangle = \sum a_i \hat{e}_i$: Set of eigenvalues $\rightarrow$ called Spectrum of Eigenvalues

Eigenvalues solve Schrodinger equation

$\uparrow$

Matrix

3-25-19

$\hat{H}\psi_n = E_n\psi_n$: $\alpha(\hat{S})\psi_n = S_n\psi_n$

$\hat{Q}|\psi_n\rangle = \lambda_n|\psi_n\rangle$

$$\left[\hat{Q} - \hat{I}(\lambda)\right]|\psi_n\rangle = 0$$

$\hat{Q}|\psi_n\rangle = \lambda_n|\psi_n\rangle$, $\hat{Q} = \begin{pmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{pmatrix} \longrightarrow$ not hermetion $\hat{Q}^{+}$ — $+ \equiv$ Transpose Conjugate

$$\hat{Q} - \hat{I}(\lambda) = \begin{vmatrix} 2-\lambda & 0 & -2 \\ -2i & i-\lambda & 2i \\ 1 & 0 & -1-\lambda \end{vmatrix} = 2-\lambda\left((i-\lambda)(-1-\lambda)\right) - 2(\lambda - i) = 0$$

$$= -\lambda^3 + (1+i)\lambda^2 - i\lambda = 0 \rightarrow \lambda\left[(\lambda-i)(\lambda-1)\right] = 0$$

$\lambda = 0$:
$$\begin{pmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 0\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} 2a_1 + 0a_2 - 2a_3 &= 0 \\ -2ia_1 + ia_2 + 2ia_3 &= 0 \\ a_1 + 0a_2 - a_3 &= 0 \end{aligned} \qquad \begin{aligned} a_1 &= a_3 \\ a_2 &= 0 \end{aligned} \longrightarrow a_1\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \longrightarrow \langle\psi_0|\psi_0\rangle = 1$$

$$a_1 a_1^* \underbrace{1 \ 0 \ 1}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 2|a|^2 = 1 : a = \frac{1}{\sqrt{2}} \quad : \quad \lambda_0 = 0 \quad \psi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda = 1$
$$\begin{pmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = 1\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\begin{aligned} 2a_1 - 2a_3 &= a_1 \\ -2ia_1 + ia_2 + 2ia_3 &= a_2 \\ a_1 - a_3 &= a_3 \end{aligned} \quad : a_1 = 2a_3 \rightarrow a_3 = \frac{a_1}{2}$$

$$\begin{aligned} -2ia_1 + ia_2 + ia_1 &= a_2 \\ -ia_1 &= a_2 - ia_2 \\ a_1 &= \frac{a_2 i - a_2}{i} \end{aligned} \quad : \quad a_1 = -(a_2 i^2 - a_2 i) = -a_2(-1-i) = a_2(1+i)$$

$$\langle\psi_1|\psi_1\rangle = (a_1^* a_1)\ \overbrace{\underbrace{1 \quad \tfrac{1+i}{2} \quad \tfrac{1}{2}}}\begin{pmatrix} 1 \\ \frac{1-i}{2} \\ 1/2 \end{pmatrix} = 1$$

$$|a_1|^2 = 1 + \frac{(1+i)}{2}\frac{(1-i)}{2} + \frac{1}{4} = 1 \quad \ldots\ldots \quad \lambda_1 = 1 \quad \psi_1 = \frac{2}{3}\begin{pmatrix} 1 \\ \frac{1+i}{2} \\ 1/2 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 2\lambda \end{pmatrix} \quad \lambda,\ |1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad : \quad \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 2\lambda \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \lambda\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad a_1 = a_2\ ,\ a_3 = 0$$

$$\lambda a_2 = \lambda a_1\ ,\ \lambda a_1 = \lambda a_2$$

$-\lambda,\ |2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$: $2\lambda,\ |3\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\quad 1 = \langle 1|1\rangle = \langle 2|2\rangle = \langle 3|3\rangle$

$$|\Psi(0)\rangle = \frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} = c_1|1\rangle + c_2|2\rangle + c_3|3\rangle$$

$$c_1 = \langle 1|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(1\ 1\ 0)\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} = \frac{3}{\sqrt{28}}$$

$$c_2 = \langle 2|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(1\ {-1}\ 0)\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} = \frac{-1}{\sqrt{28}}$$

$$c_3 = \langle 3|\Psi(0)\rangle = (0\ 0\ 1)\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix} = \frac{3}{\sqrt{14}}$$

$|c_1| + |c_2| + |c_3| = 1$

$$|\Psi(t)\rangle = \frac{3}{\sqrt{28}}e^{-i\lambda t/\hbar}|1\rangle - \frac{1}{\sqrt{28}}e^{i\lambda t/\hbar}|2\rangle + \frac{3}{\sqrt{14}}e^{-i2\lambda t/\hbar}|3\rangle$$

$\langle \hat{Q}\rangle$: $\hat{A} = \begin{pmatrix}0 & \lambda & 0\\ \lambda & 0 & 0\\ 0 & 0 & 2\lambda\end{pmatrix}$

$\langle\Psi|A|\Psi\rangle$: $(c_1^*\ c_2^*\ c_3^*)\begin{pmatrix}0 & \lambda & 0\\ \lambda & 0 & 0\\ 0 & 0 & 2\lambda\end{pmatrix}\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = (c_1^*\ c_2^*\ c_3^*)\begin{pmatrix}\lambda c_1\\ \lambda c_2\\ 2\lambda c_3\end{pmatrix}$

$$\hat{Q} = \hat{Q}^+ = (\hat{Q}^*)^T$$

$\hat{P} = -\hbar i\frac{d}{dx} \longrightarrow \langle\Psi|\hat{P}|\Psi\rangle = \int\Psi^*(-\hbar i\frac{d}{dx}\Psi)dx \quad u = \Psi^* \quad dv = \frac{d\Psi}{dx}dx$

$$= -\hbar i\Psi^*\Psi\Big|_{-\infty}^{\infty} - \int(-\hbar i)\frac{d\Psi^*}{dx}\Psi\,dx = \int\left(-\hbar i\frac{d\Psi}{dx}\right)^*\Psi\,dx = \langle\hat{P}\Psi|\Psi\rangle$$

$\hat{Q} = i\frac{d}{d\varphi}$: $\langle\Psi|\hat{Q}|\Psi\rangle = \int\Psi^* i\frac{d\Psi}{d\sigma}d\sigma$: $\Psi(\sigma+2\pi) = \Psi(\sigma)$

$$= i(\Psi\Psi^*)\Big|_0^{2\pi} - i\int\frac{d\Psi^*}{d\sigma}\Psi\,d\sigma = +\int\left(i\frac{d\Psi}{d\sigma}\right)^*\Psi\,d\sigma$$

$$\hat{Q}|\Psi_n\rangle = q_n|\Psi_n\rangle$$

$i\frac{d\Psi}{d\sigma} = q\Psi$: $\frac{d\Psi}{\Psi} = -iq\,d\sigma$: $\ln(\Psi) = -iq\sigma + c$: $\Psi = Ae^{-iq\sigma} = Ae^{-iq(\sigma+2\pi)}$

$= A\cos(2\pi q) - i\sin(2\pi q)$

$q = \pm n, 0$

$$\Psi = Ae^{iq\sigma},\ q = 0, \pm 1, \pm 2, \pm 3$$

Unbound Energies, Continuous Spectrum of Eigenvalues

$\hat{P}_x = -\hbar i \frac{d}{dx}$, $\hat{P} F_p = p F_p$, $F_p = A e^{iP_x/\hbar} \longrightarrow \underset{\delta(P-P')}{\overset{\shortparallel}{\langle F_{p'} | F_p \rangle}} \longrightarrow$ dirac orthogonality

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discrete $\longrightarrow$ Continuous

$$\boxed{\begin{aligned} &\langle F_{p'} | F_p \rangle = \delta(p-p') \\ &|\langle F_{p'} | \psi \rangle|^2 dp = P(p) \end{aligned}}^{(*)}$$

$$-\hbar i \frac{d}{dx} = \hat{P}_x \longrightarrow \hat{P}_x \psi = p\psi$$

$$F_p = A e^{iP_x x/\hbar}$$

$$\langle F_p^* | F_p \rangle = \int_{-\infty}^{\infty} F_p^* F_p \, dx = |A|^2 \int_{-\infty}^{\infty} e^{i(P_x - P_x')x/\hbar} dx$$

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk \longrightarrow \delta\left(\frac{k}{\hbar}\right) = \hbar \delta(k)$$

$$\langle F_p | F_p \rangle = A^2 2\pi\hbar \cdot \delta(p-p')$$

$$\longrightarrow \varphi(P) \longrightarrow \psi(x)$$

Let $A = \frac{1}{\sqrt{2\pi\hbar}}$ guess $\int_{-\infty}^{+\infty} 2\pi\hbar |A|^2 \delta(p-p') \, dp = 2\pi\hbar |A|^2$

$$\hat{x} = g_y(x) = y \cdot g_y(x)$$

$$g_y(x) = \delta(x-y) \longrightarrow \int_{-\infty}^{+\infty} \delta(x-y')\delta(x-y) \, dx$$

$$\langle g_{y'} | g_y \rangle = \delta(x-y')$$

$$F(x) = \int_{-\infty}^{+\infty} c(y) g_y(x) \, dx = \int_{-\infty}^{+\infty} c(y) \delta(x-y) \, dy \longrightarrow c(y) = F(x)$$

$$g_y(x) = \delta(x-y)$$

$$c(y) = \langle g_y | \psi \rangle \longrightarrow \psi(y, T)$$

$$P(y) = |c_y|^2 dy = |\psi(y)|^2 dy$$

$$c(P) = \langle F_p | \psi \rangle \; : \; P(p) = |c(p)|^2 dp$$

Ex : $V(x) = -\alpha\delta(x)$

$P(p > m\alpha/\hbar)$: $\psi(x,t) = \frac{\sqrt{m\alpha}}{\hbar} e^{\frac{-m\alpha|x|}{\hbar^2}} e^{\frac{-iEt}{\hbar}}$ where $E = \frac{-m\alpha^2}{2\hbar^2}$

$$\varphi(P, T) = c(P) = c(\hbar k) \longrightarrow \langle F_p | \psi \rangle$$

$$\frac{1}{(2\pi\hbar)^{1/2}} \frac{\sqrt{m\alpha}}{\hbar} e^{-iEt/\hbar} \int_{-\infty}^{+\infty} e^{-ipx/\hbar} e^{-m\alpha|x|/\hbar^2} dx = c(p)$$

Info = < basis function | ψ >   P(info in range) = $|\text{info}|^2$ d(variable)

$$c(p,t) = \sqrt{\frac{2}{\pi}} \frac{p_0^{3/2} e^{-iEt/\hbar}}{p^2+p_0^2}$$

$$\int_{p_0}^{\infty} |c(p,t)|^2 dp = \frac{2}{\pi} p_0^3 \int_{p_0}^{\infty} \frac{1}{p^2+p_0^2} dp = \frac{1}{\pi}\left[\frac{pp_0}{p^2+p_0^2} + \tan^{-1}\left(\frac{p}{p_0}\right)\Bigg|_{p_0}^{\infty}\right]$$

Solve SE → Normalize → use orthogonality to determine $c(p)$ or $\varphi(p)$ or c_n weights
Probabilities $|c_n|^2$ or $(\varphi(p))^2 dp$
ψ → Sum or integral over weights

discrete

$P(\underset{\substack{\uparrow \\ \text{Eigenvalue}}}{q_n}) = |c_n|^2$: $\underset{\text{Continuous}}{P(q) = |c[p]|^2 dp}$

look, measure ex $\psi(x,t)$ measure E_n

$\psi(x,t) \longrightarrow \psi_n$

measure $\psi(p,t)$ measure $p \rightarrow e^{ipx/\hbar}$

$\langle \hat{Q} \rangle \overset{\text{discrete}}{=} \sum q_n |c_n|^2$

4-1-19

$\hat{A}$, $\psi_1, \psi_2 \longrightarrow a_1, a_2$ $\psi = \sum c_n \psi_n$ or $\int \varphi(k) e^{ikx} dk$
$\langle f_p | \psi \rangle = \varphi(k)$, $\langle \psi_n | \psi \rangle = c_n$

$\hat{B}$, $\varphi_1, \varphi_2 \longrightarrow b_1, b_2$

$\psi_1 = \frac{3\varphi_1 + 4\varphi_2}{5}$, $\psi_2 = \frac{4\varphi_1 - 3\varphi_2}{5}$

$\hat{A}$ measured find a_1

$\hat{B}$ is now measured $\frac{9}{25} b_1$ and $\frac{16}{25} b_2$ $\hat{A} \rightarrow \psi_1 \rightarrow \hat{B} \rightarrow ?$ $\nearrow a_1$ $\searrow A_1$

$\hat{A}$ measured again $\longrightarrow$ $\varphi_1 = \frac{3\psi_1 + 4\psi_2}{5}$, $\varphi_2 = \frac{4\psi_1 - 3\psi_2}{5}$

$$\overset{b_1}{\left(\frac{9}{25}\right)}\overset{a_1}{\left(\frac{9}{25}\right)} + \overset{b_2}{\left(\frac{16}{25}\right)}\overset{a_2}{\left(\frac{16}{25}\right)} = 0.5392$$

$\sigma_x \sigma_p \geq \frac{\hbar}{2} \longrightarrow$ Minimum shift with a Gaussian

Stationary State. Ex: $\sqrt{\frac{2}{a}} \sin\left(\frac{3000\pi x}{a}\right)$

$$\frac{d}{dT}\langle \hat{Q} \rangle = \frac{d}{dT}\langle \Psi | \hat{Q} | \Psi \rangle = \langle \partial\Psi/\partial t | \hat{Q} | \Psi \rangle + \langle \Psi | \partial\hat{Q}/\partial T | \Psi \rangle + \langle \Psi | \hat{Q} | \partial\Psi/\partial T \rangle$$

$$i\hbar \frac{\partial \Psi}{\partial T} = \hat{H}\Psi \longrightarrow \frac{\partial \Psi}{\partial T} = -\frac{1}{i\hbar}\hat{H}\Psi$$

$$\frac{d\langle \hat{Q} \rangle}{dT} = \frac{-1}{i\hbar}\langle \hat{H}\Psi | \hat{Q}\Psi \rangle + \frac{1}{i\hbar}\langle \Psi | \hat{Q}\hat{H}\Psi \rangle + \langle \partial\hat{Q}/\partial T \rangle$$

$$\frac{d\langle \hat{Q} \rangle}{dT} = \frac{i}{\hbar}\langle \Psi | \hat{H}\hat{Q}\Psi \rangle - \frac{i}{\hbar}\langle \Psi | \hat{Q}\hat{H}\Psi \rangle + \langle \partial\hat{Q}/\partial T \rangle$$

$$\frac{d}{dT}\langle \hat{Q} \rangle = \frac{i}{\hbar}\langle [\hat{H}, \hat{Q}] \rangle + \langle \partial\hat{Q}/\partial T \rangle :$$

#1 $[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$

$$ABC - CAB = ABC - \cancel{ACB} + \cancel{ACB} - CAB$$

#2 $[A, B+C] = [A, B] + [A, C]$

$$[x, \hat{H}] = [x, p^2/2m + V] = \frac{1}{2m}\underbrace{[x, p^2]}_{a} + \underbrace{[x, V]}_{b}$$

b: $xVf - Vxf = 0$

$$a = \frac{1}{2m}[x, p^2] = \frac{1}{2m}(\hat{x}\hat{p}^2 - \hat{p}\hat{x}\hat{p} + \hat{p}\hat{x}\hat{p} + \hat{p}^2\hat{x})$$

$$= \frac{1}{2m}([\hat{x}, \hat{p}]\hat{p} - \hat{p}[\hat{x}, \hat{p}])$$

$$= \frac{1}{2m}(i\hbar\hat{p} + \hat{p}i\hbar) = \frac{i\hbar\hat{p}}{m} \quad : \quad \text{So } [\hat{H}, x] = \frac{-i\hbar P}{m}$$

$$\frac{d\langle x \rangle}{dT} = \frac{i}{\hbar}\langle [\hat{H}, x] \rangle = \langle i/\hbar \cdot (-i\hbar) \cdot P/m \rangle = \left\{ \frac{\langle P \rangle}{m} \right\}$$

$\vec{F} = -\frac{dV}{dx}$: $\langle \vec{F} \rangle = \langle d\vec{P}/dT \rangle \longrightarrow$ Homework Problem Hint

$$\sigma_a^2 \sigma_b^2 \geq \left(\tfrac{1}{2i}\langle [\hat{A}, \hat{B}] \rangle\right)^2 \quad : \quad \sigma_x^2 \sigma_H^2 \geq \left(\tfrac{1}{2i}\tfrac{i\hbar}{m}\langle P \rangle\right)^2 = \left(\tfrac{\hbar}{2m}\langle P \rangle\right)^2$$

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$$\sigma_a^2 \sigma_b^2 \geq \left(\frac{1}{2i}\langle [\hat{A}, \hat{B}] \rangle\right)$$

Ex: $[x,p] = i\hbar$

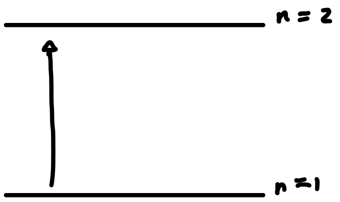
$$\sigma_x^2 \sigma_p^2 \geq \left(\frac{1}{2i} i\hbar\right)^2 = \frac{\hbar^2}{4} \longrightarrow \sigma_x \sigma_p \geq \frac{\hbar}{2}$$

$$\sigma_x \sigma_{p_x} = \sigma_x m \sigma_{v_x} \geq \frac{\hbar}{2} \longrightarrow \sigma_{v_x} \geq \frac{\hbar}{2m} \cdot \frac{1}{\sigma_x} = \frac{5.76 \times 10^{-5}}{\sigma_x} = \frac{6.626 \times 10^{-34}}{4\pi \cdot 9.13 \times 10^{-31}}$$

$\sigma_x = 1m \qquad \sigma_{vx} = 5.71 \times 10^{-5}\, m/s \quad : \quad (1\, m/s) \approx 2.2\, mi/hr$

$$T = \frac{d}{u} = \frac{1}{5.76} = 0.174 \quad : \quad 1.74 \times 10^{-1} \cdot 10^5 = 1.74 \cdot 10^4\, s$$

$$\sigma_Q = \left|\frac{d\langle\hat{Q}\rangle}{dt}\right| \Delta T \quad : \quad \frac{d\langle\hat{Q}\rangle}{dt} = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$$



$n=2 \rightarrow n=1$

$$\Delta E \Delta T \geq \frac{\hbar}{2} \rightarrow \Delta E \geq \frac{\hbar}{2\Delta T} \geq \frac{6.626 \times 10^{-34}}{4\pi \times 10^{-8}} \quad : \quad \Delta E \sim 2.2 \times 10^{-8}\, eV$$

<u>Test Problem</u>

$c_n = \langle \psi_n | A \rangle$

$\varphi(k) = \langle F_p | \psi \rangle$

$\langle \psi_m | \psi_n \rangle = \delta_{mn}$

$\langle F_{p'} | F_p \rangle = \delta(p - p')$

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|A\rangle = a|1\rangle + b|2\rangle \rightarrow a^2 + b^2 = 1$$

$$\hat{\psi} = \begin{pmatrix} h & g \\ g & h \end{pmatrix} \text{ with } |A(T=0)\rangle = |1\rangle$$

#1. Solve Schrödinger Equation

$$\hat{H}|\psi\rangle = E|\psi\rangle \rightarrow \left|\begin{pmatrix} h-E & g \\ g & h-E \end{pmatrix}\right| = 0 \rightarrow (h-E)^2 - g^2 = 0$$

$$(h-E)^2 = g^2 : h - E = \mp g \quad : \quad E = h \pm g$$

$E = h + g$

$$\begin{pmatrix} h & g \\ g & h \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = h+g\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \begin{matrix} h(\alpha) + g(\beta) = h(\alpha) + g(\alpha) \rightarrow \alpha = \beta = 1 \\ g(\alpha) + h(\beta) = h(\beta) + g(\beta) \rightarrow \end{matrix}$$

$$\left[|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right]^{(*)}$$

$E = h - g$

$$\begin{pmatrix} h & g \\ g & h \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = h-g\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow \begin{matrix} h(\alpha) + g(\beta) = h(\alpha) - g(\alpha) \rightarrow \alpha = -\beta = 1 \\ g(\alpha) + h(\beta) = h(\beta) - g(\beta) \rightarrow \end{matrix}$$

$$\left[|\psi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \right]^{(**)}$$

$$c_n = \langle \psi_n | A \rangle$$

$$c_1 = \langle \psi_1 | 1 \rangle = \frac{1}{\sqrt{2}}(1\ 1)\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}} = c_{A_+}$$

$$c_2 = \langle \psi_2 | 2 \rangle = \frac{1}{\sqrt{2}}(1\ -1)\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}} = c_{A_-}$$

$$|A(0)\rangle = \frac{1}{\sqrt{2}}(|A_+\rangle + |A_-\rangle) = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)\left[\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}1\\-1\end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|A(t)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}e^{\frac{-i(h+g)t}{\hbar}} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}e^{\frac{-i(h-g)t}{\hbar}}\right]$$

$$= \frac{1}{2}e^{\frac{-iht}{\hbar}}\begin{bmatrix} e^{\frac{-igt}{\hbar}} + e^{\frac{igt}{\hbar}} \\ e^{\frac{-igt}{\hbar}} - e^{\frac{igt}{\hbar}} \end{bmatrix} = e^{\frac{-iht}{\hbar}}\begin{pmatrix}\cos(gt/\hbar)\\ -i\sin(gt/\hbar)\end{pmatrix}$$

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$$c_n = \langle \psi_n | \psi(x,0) \rangle$$

$\psi(x,t) = \sum c_n \psi_n e^{-iE_nt/\hbar}$ ← Discrete

$$\langle \psi_m | \psi_n \rangle = \delta_{mn}$$

$$\langle F_p | F_p \rangle = \delta(p-p')$$

$\varphi(k) = \langle F_{p'} | \psi(x,t) \rangle$ ← Continuous

$$\psi(x,t) = \langle \varphi(k) | F_p e^{-iE_nt/\hbar} \rangle$$

Energy → c_n
Position → $\psi(x,t)$
Momentum → $\varphi(p,t)$