Phys 131 2016

Lecture 9

Mon: Warm Up 4

Tues: HW by 5pm

Supp Ex: 18, 20

Ch 4 Conc Q: 9, 13

Ch 4 Probs: 13, 15, 51, 56

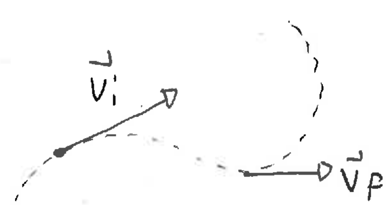
Acceleration and velocity change

Recall that velocity is a vector with

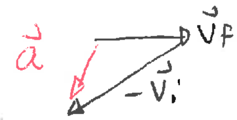
- direction tangent to trajectory of motion
- magnitude equal to speed.

Then acceleration is a vector which describes the rate of change of the velocity vector. The average acceleration is:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$



This can be extended to general cases by defining the instantaneous acceleration as:



$$\boxed{\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}}$$

This is again a vector and in component form is:

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

In general physics provides information about acceleration and one aims to infer information about velocity from this. For a constant acceleration situation:

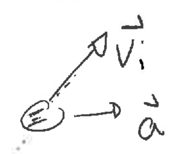
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} = \vec{a}\,\Delta t \Rightarrow \vec{v}_f - \vec{v}_i = \vec{a}\,\Delta t.$$

Thus

$$\boxed{\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t}$$

This result implies the following scheme:

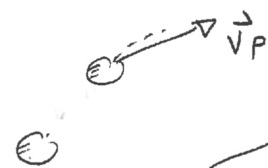
Knowing acceleration $\vec{a}$ and velocity $\vec{v}_i$ at one instant



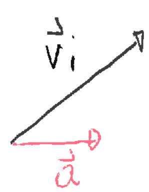
$\longrightarrow$

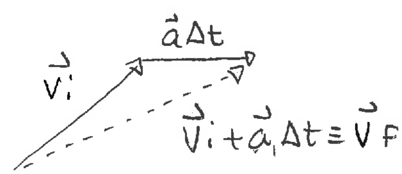
Determine velocity at a later instant

$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$



This requires a construction:





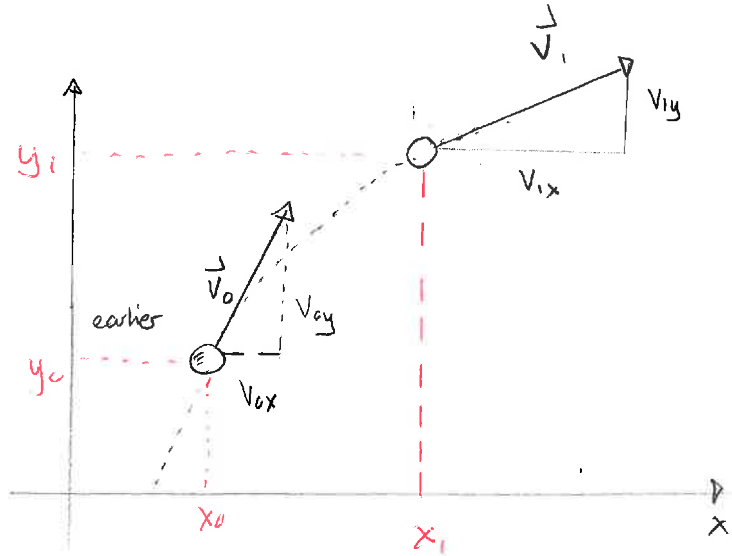
By comparing $\vec{v}_i$ and $\vec{v}_f$, we can describe changes in motion

Quiz 1 70% → 70% ~ 40% – 90%

Quiz 2 80% – 100% ~ 90%

Motion with constant acceleration

If the acceleration of an object is constant then the later velocity ($\vec{v}_1$) is related to earlier velocity, $\vec{v}_0$, by:



$$\vec{v}_1 = \vec{v}_0 + \vec{a}\,\Delta t$$

In terms of components:

$$v_{1x} = v_{0x} + a_x\,\Delta t$$

$$v_{1y} = v_{0y} + a_y\,\Delta t$$

This is an example of the mathematically proveable fact:

> If the acceleration is constant then the horizontal and vertical components of the motion can be treated independently. The kinematic equations for constant acceleration apply to each direction separately.

Thus:

$$\boxed{\begin{aligned} v_{1x} &= v_{0x} + a_x\,\Delta t \\ x_1 &= x_0 + v_{0x}\,\Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \\ v_{1x}^2 &= v_{0x}^2 + 2a_x\,\Delta x \end{aligned}}$$

similarly with $x \to y$

Projectile motion

A projectile is an object that moves only under the influence of Earth's gravity. Experiments show that the components of motion are independent and

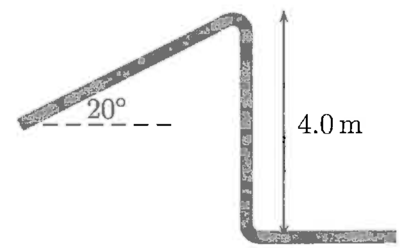
> The acceleration of a projectile is constant regardless of the projectile with
>
> $a_x = 0\,m/s^2 \qquad a_y = -g$

Demo: Ball launched / dropped

Demo: Track / cart with launcher

24 Launching off a ski ramp

A ski ramp is arranged as illustrated. A skier launches off the ramp with a speed of 15 m/s. Initially the aim of this exercise is to determine the maximum height reached by the skier and the velocity at this point. A later goal is to determine the distance at which the skier lands from the bottom of the ramp.



a) Sketch the situation with the "earlier" instant being that at which the skier launches and the "later" instant being the moment when she reaches its highest point. List as many of the variables as possible. Use the format

$$
\begin{array}{ll}
t_0 = & t_1 = \\
x_0 = & x_1 = \\
y_0 = & y_1 = \\
v_{0x} = & v_{1x} = \\
v_{0y} = & v_{1y} = \\
a_x = & a_y =
\end{array}
$$

b) Draw the velocity vector at the earlier instant and use this to determine the components of $\vec{\mathbf{v}}_0$. Enter these in the list above.

c) Draw the velocity vector at the later instant. Describe whether the components are positive, negative or zero and enter as much information about these in the list above.

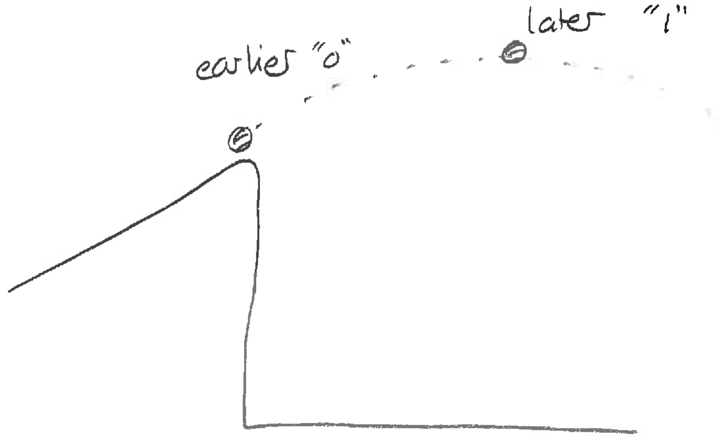
d) Determine the time taken to reach the maximum height and then the horizontal distance traveled by the skier to reach her maximum height. Determine the velocity at this point.

You will now consider the motion from the highest point back to the ground.

e) Repeat the problem set-up with the"earlier" instant being that at which the skier is at maximum height and the "later" instant being the moment *just before* she reaches hits the ground. Determine the time taken for this portion of the motion and use it to determine the horizontal distance from the base of the ramp to the skier's landing point.

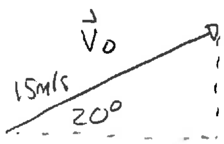
Answer:

a)



$t_0 = 0\,s$	$t_1 = \boxed{0.52s}$	$a_x = 0\,m/s^2$
$x_0 = 0\,m$	$x_1 = \boxed{7.3m}$	$a_y = -9.8\,m/s^2$
$y_0 = 4.0\,m$	$y_1 =$	
$V_{0x} = \boxed{14m/s}$	$V_{1x} = \boxed{14m/s}$	
$V_{0y} = \boxed{5.1m/s}$	$V_{1y} = \boxed{0m/s}$	

b)



$V_{0x} = V_0 \cos 20°$

$= 15m/s \cos 20° = 14m/s$

$V_{0y} = V_0 \sin 20°$

$= 15m/s \sin 20° = 5.1m/s$

c) $\xrightarrow{\vec{V}_1}$ horizontal. So $V_{1x} = ?$

$V_{1y} = 0m/s$

d) $V_{1y} = V_{0y} + a_y \Delta t$

$\Rightarrow 0m/s = 5.1m/s + (-9.8m/s^2)\Delta t$

$\Rightarrow -5.1m/s = -9.8m/s^2 \Delta t \Rightarrow \Delta t = \frac{5.1m/s^2}{9.8m/s^2} = 0.52s$

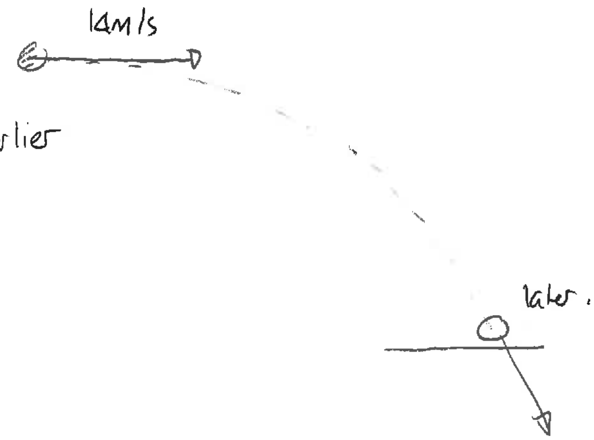
$X_1 = X_0 + V_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$

$= 0m + 14m/s \times 0.52s + \cancel{\frac{1}{2}\, 0m/s^2 (0.52s)^2}$

$= 7.3m$

$V_{1x} = V_{0x} + \cancel{a_x}\Delta t \Rightarrow V_{1x} = V_{0x} = 14m/s$ $\quad \xrightarrow{\vec{V}_1 \quad 14m/s}$

e)



$t_0 = 0s$ $\quad$ $t_1 =$

$x_0 = 7.3m$ $\quad$ $x_1 =$

$y_0 = 5.3m$ $\quad$ $y_1 = 0m$

$V_{0x} = 14m/s$ $\quad$ $V_{1x} = ?$

$V_{0y} = 0m/s$ $\quad$ $V_{1y} = ?$

We know $\quad y_1 = y_0 + V_{0y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$$0m = y_0 + \cancel{0m/s\,\Delta t} + \frac{1}{2}(-9.8\,m/s^2)\Delta t^2$$

We need y_0. From previous part $\quad$ part a,b,c,d $\quad y_1 = y_0 + V_{0y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$$y_1 = 4.0m + 5.1m/s^2 \times 0.52s + \frac{1}{2}(-9.8\,m/s^2)(0.52s)^2$$

$$y_1 = 5.3m$$

This is our new y_0

Then $\quad 0m = 5.3m - 4.9m/s^2(\Delta t)^2 \Rightarrow \frac{5.3m}{4.9m/s^2} = \Delta t^2$

$$\Rightarrow \Delta t = \sqrt{\frac{5.3m}{4.9m/s^2}} = 1.1s.$$

Now $\quad x_1 = x_0 + V_{0x}\Delta t + \frac{1}{2}\cancel{a_x(\Delta t)^2}$

$$\Rightarrow x_1 = 7.3m + 14m/s \times 1.1s \Rightarrow x_1 = 23m$$

Note that horizontal motion proceeds with constant velocity