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PHYS 311
HW 2

Problem 1

$\vec{V}_a = x^4\vec{x} + 4x^2z^2\vec{y} - 2y^3z\vec{z}$
$\vec{V}_b = x^2y\vec{x} + 3x^2z\vec{y} + 2yz^2\vec{z}$
$\vec{V}_c = 4y^3\vec{x} + (2x^2z + z^3)\vec{y} + 2xy^2\vec{z}$

Divergence : $\vec{\nabla}\cdot\vec{v} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$

$\vec{V}_a$: $\vec{V}_a = x^4\vec{x} + 4x^2z^2\vec{y} - 2y^3z\vec{z}$

$V_1 = x^4$: $\frac{\partial V_1}{\partial x} = 4x^3$

$V_2 = 4x^2z^2$: $\frac{\partial V_2}{\partial y} = 0$

$V_3 = -2y^3z$ $\frac{\partial V_3}{\partial z} = -2y^3$

$\vec{\nabla}\cdot\vec{V}_a = 4x^3\hat{i} + 0\hat{j} - 2y^3\hat{k}$

V_b : $\vec{V}_b = x^2y\vec{x} + 3x^2z\vec{y} + 2yz^2\vec{z}$

$V_1 = x^2y$ $\frac{\partial V_1}{\partial x} = 2xy$

$V_2 = 3x^2z$ $\frac{\partial V_2}{\partial y} = 0$

$V_3 = 2yz^2$ $\frac{\partial V_3}{\partial z} = 4yz$

$\vec{\nabla}\cdot\vec{V}_b = 2xy\hat{i} + 0\hat{j} + 4yz\hat{k}$

$$\boxed{\begin{aligned}\vec{\nabla}\cdot\vec{V}_a &= 4x^3\hat{i} + 0\hat{j} - 2y^3\hat{k}\\ \vec{\nabla}\cdot\vec{V}_b &= 2xy\hat{i} + 0\hat{j} + 4yz\hat{k}\\ \vec{\nabla}\cdot\vec{V}_c &= 0\hat{i} + 0\hat{j} + 0\hat{k}\end{aligned}}$$

V_c : $\vec{V}_c = 4y^3\vec{x} + (2x^2z + z^3)\vec{y} + 2xy^2\vec{z}$

$V_1 = 4y^3$ $\frac{\partial V_1}{\partial x} = 0$

$V_2 = 2x^2z + z^3$ $\frac{\partial V_2}{\partial y} = 0$

$V_3 = 2xy^2$ $\frac{\partial V_3}{\partial z} = 0$

$\vec{\nabla}\cdot\vec{V}_c = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Problem 2

$$\vec{V}_a = x^4\hat{x} + 4x^2z^2\hat{y} - 2y^3z\hat{z}$$
$$\vec{V}_b = x^2y\hat{x} + 3x^2z\hat{y} + 2yz^2\hat{z}$$
$$\vec{V}_c = 4y^3\hat{x} + (2x^2z+z^3)\hat{y} + 2xy^2\hat{z}$$

Curl: $\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$

$\nabla \equiv \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

$\vec{V} \equiv \langle V_x, V_y, V_z \rangle$

$\vec{V}_a$: $\vec{V}_a = x^4\hat{x} + 4x^2z^2\hat{y} - 2y^3z\hat{z}$

$\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\imath}$: $\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\jmath}$: $\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$

$\frac{\partial V_z}{\partial y} = -6y^2z$ $\quad \frac{\partial V_x}{\partial z} = 0$ $\quad \frac{\partial V_y}{\partial x} = 8xz^2$

$\frac{\partial V_y}{\partial z} = 8x^2z$ $\quad \frac{\partial V_z}{\partial x} = 0$ $\quad \frac{\partial V_x}{\partial y} = 0$

$$\vec{\nabla}\times\vec{V}_a = (-6y^2z - 8x^2z)\hat{\imath} + 0\hat{\jmath} + (8xz^2)\hat{k}$$

$\vec{V}_b$: $\vec{V}_b = x^2y\hat{x} + 3x^2z\hat{y} + 2yz^2\hat{z}$

$\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\imath}$: $\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\jmath}$: $\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$

$\frac{\partial V_z}{\partial y} = 2z^2$ $\quad \frac{\partial V_x}{\partial z} = 0$ $\quad \frac{\partial V_y}{\partial x} = 6xz$

$\frac{\partial V_y}{\partial z} = 3x^2$ $\quad \frac{\partial V_z}{\partial x} = 0$ $\quad \frac{\partial V_x}{\partial y} = x^2$

$$\vec{\nabla}\times\vec{V}_b = (2z^2 - 3x^2)\hat{\imath} + (0)\hat{\jmath} + (6xz - x^2)\hat{k}$$

$\vec{V}_c$: $\vec{V}_c = 4y^3\hat{x} + (2x^2z+z^3)\hat{y} + 2xy^2\hat{z}$

$\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\imath}$: $\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\jmath}$: $\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$

$\frac{\partial V_z}{\partial y} = 4xy$ $\quad \frac{\partial V_x}{\partial z} = 0$ $\quad \frac{\partial V_y}{\partial x} = 4xz$

$\frac{\partial V_y}{\partial z} = (2x^2 + 3z^2)$ $\quad \frac{\partial V_z}{\partial x} = 2y^2$ $\quad \frac{\partial V_x}{\partial y} = 12y^2$

$$\vec{\nabla}\times\vec{V}_c = (4xy - 2x^2 - 3z^2)\hat{\imath} + (-2y^2)\hat{\jmath} + (4xz - 12y^2)\hat{k}$$

$$\boxed{\begin{aligned} \vec{\nabla}\times\vec{V}_a &= (-6y^2z - 8x^2z)\hat{\imath} + 0\hat{\jmath} + (8xz^2)\hat{k} \\ \vec{\nabla}\times\vec{V}_b &= (2z^2 - 3x^2)\hat{\imath} + (0)\hat{\jmath} + (6xz - x^2)\hat{k} \\ \vec{\nabla}\times\vec{V}_c &= (4xy - 2x^2 - 3z^2)\hat{\imath} + (-2y^2)\hat{\jmath} + (4xz - 12y^2)\hat{k} \end{aligned}}$$

Problem 3

Curl: $\vec{\nabla} \times \vec{v} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$

Divergence: $\vec{\nabla} \cdot \vec{v} = \underset{V_y \wedge V_z}{\frac{\partial V_x}{\partial x}} + \underset{V_x \wedge V_z}{\frac{\partial V_y}{\partial y}} + \underset{V_x \wedge V_y}{\frac{\partial V_z}{\partial z}}$

Curl: $\vec{\nabla} \times \vec{v} = \left(\underset{V_x \wedge V_z}{\frac{\partial V_z}{\partial y}} - \underset{V_x \wedge V_y}{\frac{\partial V_y}{\partial z}}\right)\hat{\imath} + \left(\underset{V_x \wedge V_y}{\frac{\partial V_x}{\partial z}} - \underset{V_y \wedge V_z}{\frac{\partial V_z}{\partial x}}\right)\hat{\jmath} + \left(\underset{V_y \wedge V_z}{\frac{\partial V_y}{\partial x}} - \underset{V_x \wedge V_z}{\frac{\partial V_x}{\partial y}}\right)\hat{k}$

$$\begin{aligned} \vec{V}_x &= zy \\ \vec{V}_y &= zx \\ \vec{V}_z &= yx \end{aligned}$$

$\vec{v} = \langle V_x, V_y, V_z \rangle$
$\vec{v} = \langle zy, zx, yx \rangle$
$\vec{v} = zy\hat{\imath} + zx\hat{\jmath} + yx\hat{k}$

Divergence: $\vec{\nabla} \cdot \vec{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

$= \frac{\partial}{\partial x}(zy) + \frac{\partial}{\partial y}(zx) + \frac{\partial}{\partial z}(yx)$

$= 0 + 0 + 0$

$\vec{\nabla} \cdot \vec{v} = 0$ ✓

Curl: $\vec{\nabla} \times \vec{v} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$

$(x - x)\hat{\imath} + (y - y)\hat{\jmath} + (z - z)\hat{k}$

$\frac{\partial V_z}{\partial y} = x \qquad \frac{\partial V_x}{\partial z} = y \qquad \frac{\partial V_y}{\partial x} = z$

$\frac{\partial V_y}{\partial z} = x \qquad \frac{\partial V_z}{\partial x} = y \qquad \frac{\partial V_x}{\partial y} = z$

$\vec{\nabla} \times \vec{v} = (0)\hat{\imath} + (0)\hat{\jmath} + (0)\hat{k}$

$\vec{\nabla} \times \vec{v} = 0$ ✓

$$\boxed{\begin{aligned} \vec{v} &= \langle zy, zx, yx \rangle \\ \vec{v} &= (zy)\hat{\imath} + (zx)\hat{\jmath} + (yx)\hat{k} \end{aligned}}$$

Problem 4

$\vec{A} = 3y\hat{x} + z\hat{y} + 2x\hat{z}$
$\vec{B} = 2x\hat{x} - 3z\hat{y}$

Product Rule (vi): $\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{\nabla}\cdot\vec{B})\vec{A} - (\vec{\nabla}\cdot\vec{A})\vec{B}$

$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
$\vec{A} = 3y\hat{x} + z\hat{y} + 2x\hat{z}$
$\vec{B} = 2x\hat{x} - 3z\hat{y} + 0\hat{z}$

A B C

Cross Product : $(\vec{A}\times\vec{B}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$

$\vec{\nabla}\times(\vec{A}\times\vec{B})$:

$\vec{A}\times\vec{B} = (z(0) - 2x(-3z))\hat{\imath} + (2x(2x) - 3y(0))\hat{\jmath} + ((3y)(-3z) - z(2x))\hat{k}$
$= (0 + 6xz)\hat{\imath} + (4x^2 - 0)\hat{\jmath} + (-9yz - 2xz)\hat{k}$
$\vec{A}\times\vec{B} = (6xz)\hat{\imath} + (4x^2)\hat{\jmath} + (-9yz - 2xz)\hat{k}$

$\vec{\nabla}\times(\vec{A}\times\vec{B}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xz & 4x^2 & -9yz-2xz \end{vmatrix} = \left(\frac{\partial}{\partial y}(-9yz-2xz) - \frac{\partial}{\partial z}(4x^2)\right)\hat{\imath} + \left(\frac{\partial}{\partial z}(6xz) - \frac{\partial}{\partial x}(-9yz-2xz)\right)\hat{\jmath} + \left(\frac{\partial}{\partial x}(4x^2) - \frac{\partial}{\partial y}(6xz)\right)\hat{k}$
$= ((-9z - 0))\hat{\imath} + ((6x - (-2z))\hat{\jmath} + ((8x - 0))\hat{k}$
$= (-9z)\hat{\imath} + (6x + 2z)\hat{\jmath} + (8x)\hat{k} \quad (1^*)$

$\vec{\nabla}\times(\vec{A}\times\vec{B}) = (-9z)\hat{\imath} + (6x+2z)\hat{\jmath} + (8x)\hat{k}$
$\vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$

$(\vec{\nabla}\cdot\vec{B}) = \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(-3z) + \frac{\partial}{\partial z}(0)$
$= 2 + 0 + 0$
$(\vec{\nabla}\cdot\vec{B}) = 2$
$2(3y\hat{x} + z\hat{y} + 2x\hat{z})$
$6y\hat{x} + 2z\hat{y} + 2x\hat{z}$

$(\vec{\nabla}\cdot\vec{A}) = \frac{\partial}{\partial x}(3y) + \frac{\partial}{\partial y}(z) + \frac{\partial}{\partial z}(2x)$
$= 0 + 0 + 0$
$(\vec{\nabla}\cdot\vec{A}) = 0$

$(\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$

$(\vec{B}\cdot\vec{\nabla}) : \langle 2x, -3z, 0\rangle \cdot \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle = \left(2x\cdot\left(\frac{\partial}{\partial x}\right) - 3z\left(\frac{\partial}{\partial y}\right) + 0\left(\frac{\partial}{\partial z}\right)\right)$
$(\vec{\nabla}\cdot\vec{B}) : \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle \cdot \langle 2x, -3z, 0\rangle = \left(\frac{\partial}{\partial x}(2x) - \frac{\partial}{\partial y}(3z) + \frac{\partial}{\partial z}0\right) = 2$
$(\vec{A}\cdot\vec{\nabla}) : \langle 3y, z, 2x\rangle \cdot \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle = \left(3y\left(\frac{\partial}{\partial x}\right) + z\left(\frac{\partial}{\partial y}\right) + 2x\left(\frac{\partial}{\partial z}\right)\right)$
$(\vec{\nabla}\cdot\vec{A}) : \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\rangle \cdot \langle 3y, z, 2x\rangle = \left(\frac{\partial}{\partial x}(3y) + \frac{\partial}{\partial y}(z) + \frac{\partial}{\partial z}(2x)\right) = 0$

$(\vec{B}\cdot\vec{\nabla})\vec{A} : \left(2x\cdot\left(\frac{\partial}{\partial x}\right) - 3z\left(\frac{\partial}{\partial y}\right) + 0\left(\frac{\partial}{\partial z}\right)\right)\langle 3y, z, 2x\rangle$
$\langle 2x\cdot\frac{\partial}{\partial x}(3y) - 3z\cdot\frac{\partial}{\partial y}(3y) + 0\cdot\frac{\partial}{\partial z}(3y),\ 2x\cdot\frac{\partial}{\partial x}(z) - 3z\cdot\frac{\partial}{\partial y}(z) + 0\cdot\frac{\partial}{\partial z}(z),\ 2x\cdot\frac{\partial}{\partial x}(2x) - 3z\cdot\frac{\partial}{\partial y}(2x) + 0\cdot\frac{\partial}{\partial z}(2x)\rangle$
$\langle 2x\cdot 0 - 3z(3) + 0(0),\ 2x(0) - 3z(0) + 0(1),\ 2x(2) - 3z(0) + 0(0)\rangle$
$\langle -9z, 0, 4x\rangle$

$(\vec{A}\cdot\vec{\nabla})\vec{B} : \left(3y\left(\frac{\partial}{\partial x}\right) + z\left(\frac{\partial}{\partial y}\right) + 2x\left(\frac{\partial}{\partial z}\right)\right)\langle 2x, -3z, 0\rangle$
$\langle 3y\cdot\frac{\partial}{\partial x}(2x) + z\cdot\frac{\partial}{\partial y}(2x) + 2x\cdot\frac{\partial}{\partial z}(2x),\ 3y\cdot\frac{\partial}{\partial x}(-3z) + z\cdot\frac{\partial}{\partial y}(-3z) + 2x\cdot\frac{\partial}{\partial z}(-3z),\ 3y\cdot\frac{\partial}{\partial x}(0) + z\cdot\frac{\partial}{\partial y}(0) + 2x\cdot\frac{\partial}{\partial z}(0)\rangle$
$\langle 3y(2) + z(0) + 2x(0),\ 3y(0) + z(0) + 2x(-3),\ 3y(0) + z(0) + 2x(0)\rangle$
$\langle 6y, -6x, 0\rangle$

$\vec{A}(\vec{\nabla}\cdot\vec{B}) : \langle 3y, z, 2x\rangle \circ 2 = \langle 6y, 2z, 4x\rangle$
$\vec{B}(\vec{\nabla}\cdot\vec{A}) : \langle 2x, -3z, 0\rangle \circ 0 = \langle 0, 0, 0\rangle$

$(\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$:
$\langle -9z, 0, 4x\rangle - \langle 6y, -6x, 0\rangle + \langle 6y, 2z, 4x\rangle + \langle 0, 0, 0\rangle$
$\langle -9z - 6y + 6y,\ 0 + 6x + 2z,\ 4x + 0 + 4x\rangle$
$\langle -9z, 6x + 2z, 8x\rangle$
$(-9z)\hat{\imath} + (6x + 2z)\hat{\jmath} + (8x)\hat{k} \quad (2^*)$

$(1^*) = (2^*)$
$\vec{\nabla}\times(\vec{A}\times\vec{B})$:
$(-9z)\hat{\imath} + (6x + 2z)\hat{\jmath} + (8x)\hat{k}$
$(\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$:
$(-9z)\hat{\imath} + (6x + 2z)\hat{\jmath} + (8x)\hat{k}$

Problem 5

a) $T_a = z^2 + 2zx + 3y + 5$

$$\nabla^2 T_a = \frac{\partial^2 T_a}{\partial x^2} + \frac{\partial^2 T_a}{\partial y^2} + \frac{\partial^2 T_a}{\partial z^2} = 0 + 0 + 2 = 2$$

$$\frac{\partial T_a}{\partial x} = 2z \qquad \frac{\partial T_a}{\partial y} = 3 \qquad \frac{\partial T_a}{\partial z} = 2z + 2x \qquad \nabla^2 T_a = 2$$

$$\frac{\partial^2 T_a}{\partial x^2} = 0 \qquad \frac{\partial^2 T_a}{\partial y^2} = 0 \qquad \frac{\partial^2 T_a}{\partial z^2} = 2$$

b.) $T_b = \sin(x)\cos(y)\sin(z)$

$$\nabla^2 T_b = \frac{\partial^2 T_b}{\partial x^2} + \frac{\partial^2 T_b}{\partial y^2} + \frac{\partial^2 T_b}{\partial z^2} = -\sin(x)\cos(y)\sin(z) - \cos(x)\sin(y)\sin(z) - \sin(x)\cos(y)\sin(z)$$

$$\frac{\partial T_b}{\partial x} = \cos(x)\cos(y)\sin(z) \qquad \frac{\partial T_b}{\partial y} = -\cos(x)\sin(y)\sin(z) \qquad \frac{\partial T_b}{\partial z} = \sin(x)\cos(y)\cos(z)$$

$$\frac{\partial^2 T_b}{\partial x^2} = -\sin(x)\cos(y)\sin(z) \qquad \frac{\partial^2 T_b}{\partial y^2} = -\cos(x)\sin(y)\sin(z) \qquad \frac{\partial^2 T_b}{\partial z^2} = -\sin(x)\cos(y)\sin(z)$$

$$\nabla^2 T_b = \sin(z)\left(-2\sin(x)\cos(y) - \cos(x)\sin(y)\right)$$

c.) $T_c = e^{-2z}\sin(3x)\cdot\cos(4y)$

$$\nabla^2 T_c = \frac{\partial^2 T_c}{\partial x^2} + \frac{\partial^2 T_c}{\partial y^2} + \frac{\partial^2 T_c}{\partial z^2} = -9e^{-2z}\sin(3x)\cos(4y) - 16e^{-2z}\sin(3x)\cos(4y) + 4e^{-2z}\sin(3x)\cos(4y)$$

$$\frac{\partial T_c}{\partial x} = 3e^{-2z}\cos(3x)\cos(4y) \qquad \frac{\partial T_c}{\partial y} = -4e^{-2z}\sin(3x)\sin(4y) \qquad \frac{\partial T_c}{\partial z} = -2e^{-2z}\sin(3x)\cos(4y)$$

$$\frac{\partial^2 T_c}{\partial x^2} = -9e^{-2z}\sin(3x)\cos(4y) \qquad \frac{\partial^2 T_c}{\partial y^2} = -16e^{-2z}\sin(3x)\cos(4y) \qquad \frac{\partial^2 T_c}{\partial z^2} = 4e^{-2z}\sin(3x)\cos(4y)$$

$$\nabla^2 T_c = -21e^{-2z}\sin(3x)\cos(4y)$$

d.) $\vec{v} = z^2\hat{x} + 2zy^2\hat{y} - zy\hat{z}$

$$\nabla^2 \cdot \vec{v} = \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_z}{\partial z^2} = 0 + 4z + 0 = 4z$$

$$\frac{\partial V_x}{\partial x} = 0 \qquad \frac{\partial V_y}{\partial y} = 4zy \qquad \frac{\partial V_z}{\partial z} = -y \qquad \nabla^2 \cdot \vec{v} = 4z$$

$$\frac{\partial^2 V_x}{\partial x^2} = 0 \qquad \frac{\partial^2 V_y}{\partial y^2} = 4z \qquad \frac{\partial^2 V_z}{\partial z^2} = 0$$

$$\boxed{\nabla^2 T_a = 2,\ \nabla^2 T_b = \sin(z)\left(-2\sin(x)\cos(y) - \cos(x)\sin(y)\right),\ \nabla^2 T_c = -21e^{-2z}\sin(3x)\cos(4y),\ \nabla^2 \cdot \vec{v} = 4z}$$

Problem 6 $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$

a.) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})$

$\vec{\nabla}\times\vec{B}$: $\vec{\nabla}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \left(\frac{\partial}{\partial y}B_z - \frac{\partial}{\partial z}B_y\right)\hat{\imath} + \left(\frac{\partial}{\partial z}B_x - \frac{\partial}{\partial x}B_z\right)\hat{\jmath} + \left(\frac{\partial}{\partial x}B_y - \frac{\partial}{\partial y}B_x\right)\hat{k}$

$f(x,y,z) = 3xyz^2$

$f_x = 3yz^2 \quad f_y = 3xz^2$

$f_{xy} = 3z^2 \quad f_{yx} = 3z^2$

$f_{xy} \equiv f_{yx}$

$\frac{\partial^2}{\partial xy} \equiv \frac{\partial^2}{\partial yx}$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}B_z - \frac{\partial}{\partial z}B_y\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}B_x - \frac{\partial}{\partial x}B_z\right) + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}B_y - \frac{\partial}{\partial y}B_x\right)$$

$$= \frac{\partial^2}{\partial xy}B_z - \frac{\partial^2}{\partial xz}B_y + \frac{\partial^2}{\partial yz}B_x - \frac{\partial^2}{\partial yx}B_z + \frac{\partial^2}{\partial zx}B_y - \frac{\partial^2}{\partial zy}B_x$$

$$= \left(\frac{\partial^2}{\partial yz}B_x - \frac{\partial^2 B_x}{\partial zy}\right) + \left(\frac{\partial^2}{\partial zx}B_y - \frac{\partial^2}{\partial xz}B_y\right) + \left(\frac{\partial^2}{\partial xy}B_z - \frac{\partial^2}{\partial yx}B_z\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0 + 0 + 0$$

$$\boxed{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0}$$

b.) $\vec{\nabla}\times(\vec{\nabla}g)$

$\vec{\nabla}g$: $g\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) = \left(\frac{\partial g}{\partial x}\hat{x} + \frac{\partial g}{\partial y}\hat{y} + \frac{\partial g}{\partial z}\hat{z}\right)$

$g = g(x,y,z)$

$\vec{\nabla}\times(\vec{\nabla}g)$: $\vec{\nabla}\times(\vec{\nabla}g) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \end{vmatrix} = \left(\frac{\partial^2 g}{\partial yz} - \frac{\partial^2 g}{\partial yz}\right)\hat{\imath} + \left(\frac{\partial^2 g}{\partial xz} - \frac{\partial^2 g}{\partial xz}\right)\hat{\jmath} + \left(\frac{\partial^2 g}{\partial xy} - \frac{\partial^2 g}{\partial xy}\right)\hat{k}$

$= (0)\hat{\imath} + (0)\hat{\jmath} + (0)\hat{k}$

$= 0$

$$\boxed{\vec{\nabla}\times(\vec{\nabla}g) = 0}$$

Problem 7 $\vec{v} = z^2\hat{x} + 2zy^2\hat{y} - xy\hat{z}$

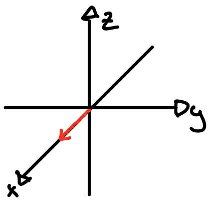
a) $(0,0,0) \to (1,0,0,) \to (1,1,0) \to (1,1,1)$

$\int_a^b \vec{v}\cdot d\vec{l}$: $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$\int_a^b z^2dx + 2zy^2dy - xz\,dz$

i.) $(0,0,0) \to (1,0,0)$ $dy = dz = 0$

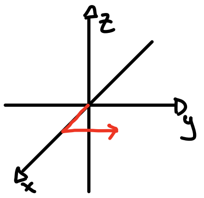
$z = 0$



$$\int_0^1 z^2dx = \int_0^1 0\,dx = 0$$

ii.) $(1,0,0) \to (1,1,0)$ $dx = dz = 0$

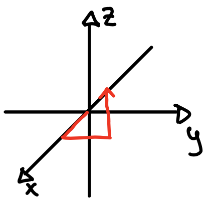
$x = 1$ $z = 0$



$$\int_0^1 2zy^2dy = \int_0^1 2(0)y^2dy = \int_0^1 0\,dy = 0$$

iii.) $(1,1,0) \to (1,1,1)$ $dx = dy = 0$

$x = 1$ $y = 1$



$$-\int_0^1 xz\,dz = -\int_0^1 z\,dz = -\tfrac{1}{2}z^2\Big|_0^1 = -\tfrac{1}{2}(1)^2 - \tfrac{1}{2}(0)^2 = -\tfrac{1}{2}$$

$\boxed{-\frac{1}{2}}$

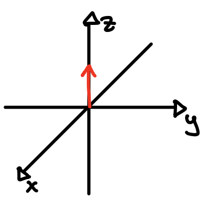
b.) $(0,0,0) \to (0,0,1) \to (0,1,1) \to (1,1,1)$

$\int_a^b \vec{v}\cdot d\vec{l}$: $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$\int_a^b z^2dx + 2zy^2dy - xz\,dz$

i.) $(0,0,0) \to (0,0,1)$ $dx = dy = 0$

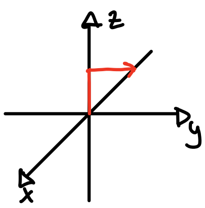
$x = 0$ $y = 0$



$$\int_0^1 -xz\,dz = -\int_0^1 (0)z\,dz = -\int_0^1 0\,dz = 0$$

ii.) $(0,0,1) \to (0,1,1)$ $dx = dz = 0$

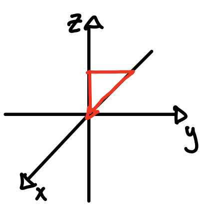
$x = 0$ $z = 1$



$$\int_0^1 2zy^2dy = 2\int_0^1 y^2dy = 2\cdot\tfrac{1}{3}y^3\Big|_0^1 = \tfrac{2}{3} - 0 = \tfrac{2}{3}$$

iii.) $(0,1,1) \to (1,1,1)$ $dy = dz = 0$

$y = 1$ $z = 1$

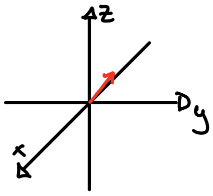


$$\int_0^1 z^2dx = \int_0^1 1\,dx = x\Big|_0^1 = 1$$

$\boxed{\frac{5}{3}}$

c.) $(0,0,0) \rightarrow (1,1,1)$ $dx = dy = dz = 1$

$x = y = z$



$\int_a^b \vec{v} \cdot d\vec{l}$: $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$\int_a^b z^2 dx + 2zy^2 dy - xz\,dz$

$$\boxed{\tfrac{1}{2}}$$

$$\int_0^1 x^2 + 2x^3 - x^2\,dx = \int_0^1 2x^3\,dx = \frac{x^4}{2}\Big|_0^1 = \tfrac{1}{2} - 0 = \tfrac{1}{2}$$