4.2 orthogonal Vector Expansions

Discrete and Inverse Discrete Transform Matrices

Let v be a vector of length n expanded in terms of n orthogonal vectors $g_1, g_2, \ldots, g_n$, each of length n

$$V = C_1 g_1 + C_2 g_2 + \ldots + C_n g_n$$

$$C_K = \frac{\langle V, g_K \rangle}{\langle g_K, g_K \rangle}, \quad K = 1, 2, \ldots n$$

The discrete transform matrix relative to $g_1, g_2, \ldots, g_n$ is the $n \times n$ matrix H

The K^{th} row consists of the orthonormalized vector h_K^T

$$h_K = \frac{g_K}{\langle g_K, g_K \rangle}$$

$\langle g_K, g_K \rangle = C, \quad K = 1, 2, \ldots, n,$
for some constant C, then H has the form

$$H = \frac{1}{C} \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{bmatrix}$$