MATH Preliminaries

1-25-19

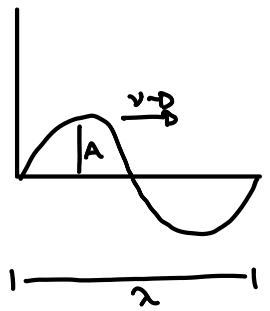
$\hat{H}\Psi = i\hbar \frac{\partial \Psi}{\partial T}$, $\hat{H}\Psi = E\Psi$

$\hat{H} = \hat{K}E + \hat{P}E$

$\Psi(r,t) - \Psi\bar{\Psi} \rightarrow$ Probabilities

QM $\rightarrow$ Statistical repeated measurements on identical systems, will yield expectation values

$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x-vt) + \delta\right)$: δ is an angular shift from the origin



ν = Frequency , $2\pi\nu = 2\pi\frac{v}{\lambda} = \omega$ = angular Frequency

$\left|\frac{2\pi}{\lambda}\right| = |\vec{k}|$: wavenumber, wavevector

$Ae^{i(kx-\omega t+\delta)} = Ae^{i\delta}e^{i(kx-\omega t)} = \tilde{A}e^{i(kx-\omega t)}$: $\tilde{A}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$, $\vec{r}$ = position vector , $\hat{k} = \hat{i}$

Complex Numbers

$Z = x + iy = re^{i\theta}$

$\bar{Z} = x - iy = re^{-i\theta}$

$r = (Z\cdot\bar{Z})^{\frac{1}{2}} = x^2 + y^2$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$, $re^{\pm i\theta} = r\left[\cos(\theta) \pm i\sin(\theta)\right]$

$Ae^{i(kx-\omega t)}$ $\longrightarrow$ Rightward

$Ae^{i(kx+\omega t)}$ $\longleftarrow$ Leftward

: HW Help $\rightarrow$

$\cos(-x) = \cos(x)$

$\sin(-x) = -\sin(x)$

$\sin(x \pm y) = \sin(x)\cos(y) \pm \sin(y)\cos(x)$

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Gaussian Integrals

$$I_{Even} = \int_{-\infty}^{\infty} x^{2n} e^{-ax^2}dx = 2\int_0^{\infty} x^{2n}e^{-ax^2}dx \quad : \quad \int_{-\infty}^{\infty} e^{-ax^2}dx = I_0 \xrightarrow{n=0} \int_{-\infty}^{\infty} e^{-ax^2}dx \int_{-\infty}^{\infty} e^{-ay^2}dy = (I_0)^2$$

$n = 0, 1, 2, 3$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(x^2+y^2)}dxdy = \int_0^{2\pi}\int_0^{\infty} e^{-ar^2}r\,dr\,d\theta = 2\pi\int_0^{\infty} re^{-ar^2}dr$$

Let $u = -ar^2$, $du = -2ar\,dr$

$r = 0$, $u = 0$

$r = \infty$, $u = -\infty$

$$-\frac{2\pi}{2a}\int_0^{-\infty} e^u du = \frac{\pi}{a}\int_{-\infty}^{0} e^u du = e^u\Big|_{\infty}^{0} = \frac{\pi}{a}[1-0] = \frac{\pi}{a} = I_0^2$$

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$$I_0 = \int e^{-ax^2} dx = \sqrt{\pi}\, a^{-1/2}$$

$$\frac{dI_0}{da} = \int \frac{d}{da} e^{-ax^2} dx = -\int x^2 e^{-ax^2} dx = \frac{d}{da}(\sqrt{\pi}\, a^{-1/2}) \quad : \quad +\int x^2 e^{-ax^2} dx = +I_1 = +\frac{1}{2}\sqrt{\pi}\, a^{-3/2}$$

$$I_{Even} = \int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = \frac{1\cdot 3\cdot 5\cdot (n+1)\pi^{1/2}}{2^{n/2} a^{\frac{n+1}{2}}}$$

$$\int_0^{\infty} x e^{-ax^2} dx \rightarrow \quad u = -ax^2 \quad du = -2ax\,dx \qquad = \frac{1}{2}a^{-1} = I_1 \qquad + \int x^3 e^{-ax^2} dx = I_3 = \frac{1}{2}a^{-2}$$

$$x = 0,\ u = 0 \qquad x = \infty,\ u = -\infty$$

$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)} dx \rightarrow e^{-a(x^2 + \frac{b}{a}x + \frac{c}{a})} \rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = (x+D)^2 + F = x^2 + 2Dx + D^2 + F$$

$$2D = \frac{b}{a} \rightarrow D = \frac{b}{2a} \qquad \frac{c}{a} = D^2 + F \rightarrow F = \frac{c}{a} - \frac{b^2}{4a^2}$$

$$\int_{-\infty}^{\infty} e^{-a\left[(x+\frac{b}{2a})^2 + (\frac{c}{a} - \frac{b^2}{4a^2})\right]} dx \rightarrow e^{-(c - \frac{b^2}{4a})} \int e^{-a(x+\frac{b}{2a})^2} dx \qquad u = x + \frac{b}{2u} \quad du = dx$$