1.5 # 1-5

a.) Does Picards apply?

b.) What is the largest rectangle Picards conditions could hold?

C.) If (a) is no, there are other initial conditions $y(t_0)=y_0$ for which the answer would be yes. Describe these points.

1.5.1) $y'+ty=1 \quad y(0)=0$

$t_0=0 \quad y_0=0$

$y'=1-ty \quad y(t_0)=y_0$

Continuous

$\frac{\partial y'}{\partial y}=-t$

Continuous

a.) y' & $\frac{\partial y'}{\partial y}$ are continous so therefore Picards theorem applies.

b.) Since both functions are continuous, the solution holds true in the entire t-y plane.

1.5.2) $ty'+y=2 \quad y(0)=1$

$ty'=2-y$

$y'=\frac{2-y}{t}$

$y'=\frac{2}{t}-\frac{y}{t}$

$\frac{\partial y'}{\partial y}=-\frac{1}{t}$

a.) y' is not continous at t_0 $\therefore$ Picards theorem does not apply.

C.) Uniqueness and existence hold true over the entire rectangle

1.5.3) $y'=y^{4/3} \quad y(0)=0$

$\frac{\partial y'}{\partial y}=\frac{4}{3}y^{1/3}$

a.) Both y' & $\frac{\partial y'}{\partial y}$ are continuous so therefore Picard's theorem does apply.

b.) Since both functions are continous, Picard's theorem holds true in the entire ty plane.

1.5.4) $y'=\frac{t-y}{t+y} \quad y(0)=-1$

a.) Both y' and $\partial y'$ are continous everywhere except $y=-t$. Picards theorem does apply. The rectangle cannot intersect the line $y=t$.

b.) Uniqueness and existence hold true everywhere except the line $y=-t$.

1.5.5 $y' = \frac{1}{t^2+y^2}$ $y(0)=0$

$$\frac{\partial y'}{\partial y} = \frac{(t^2+y^2)(0) - 1(2y)}{(t^2+y^2)^2}$$
$$= \frac{-2y}{(t^2+y^2)^2}$$

a.) Both functions are continuous therefore Picard's theorem does apply. But since (0,0) is the IV, and (0,0) is the only point on y' where it is discontinuous, Picards theorem does not apply.

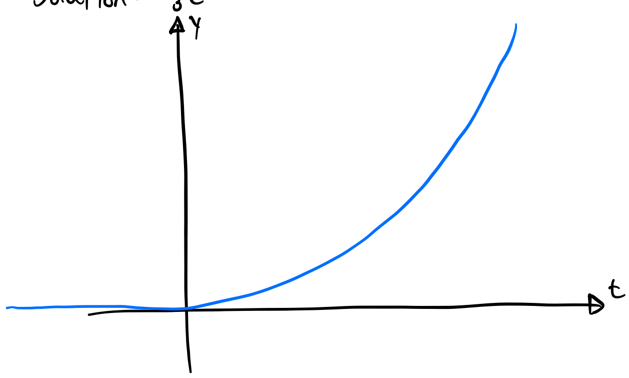
C. Uniqueness and existence describe the solution set everywhere except (0,0).

1.5.20 $y' = y^{\frac{1}{3}}$ $y(0)=0$

$f_y = \frac{1}{3} y^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{y^2}} = y \neq 0$ IV $=0$ $\therefore$ Picards does not apply

$y(t)=0$

Solution $= \frac{2}{3}t^{\frac{3}{2}}$



$\frac{dy}{dt} = y^{\frac{1}{3}}$

$y^{-\frac{1}{3}}\,dy = dt$ $y = \left(\frac{2}{3}t + c\right)^{\frac{3}{2}}$

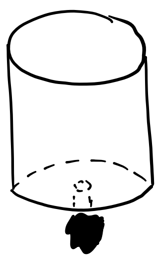
$\frac{3}{2} y^{\frac{2}{3}} = t + C$ $y(0)=0$

$y^{\frac{2}{3}} = \frac{2}{3}t + \frac{2}{3}C$ $0 = \left(\frac{2}{3}(0) + c\right)^{\frac{3}{2}}$

$y = \left(\frac{2}{3}t\right)^{\frac{3}{2}} + C^{\frac{3}{2}}$ $0 = C$

$y = \left(\frac{2}{3}t\right)^{\frac{3}{2}}$

1.5.24



$\frac{dh}{dt} = -k\sqrt{h}$

$h(0)=0$

a.) Because the partial derivative in respect to "h" isn't continuous

b.) $\frac{dh}{dt} = -K\sqrt{h}$

$dh = -K\sqrt{h}\,dt$

$\frac{dh}{-K\sqrt{h}} = dt$

$\int\left(-K^{-1}(h)^{-\frac{1}{2}}\right) dh = \int dt$

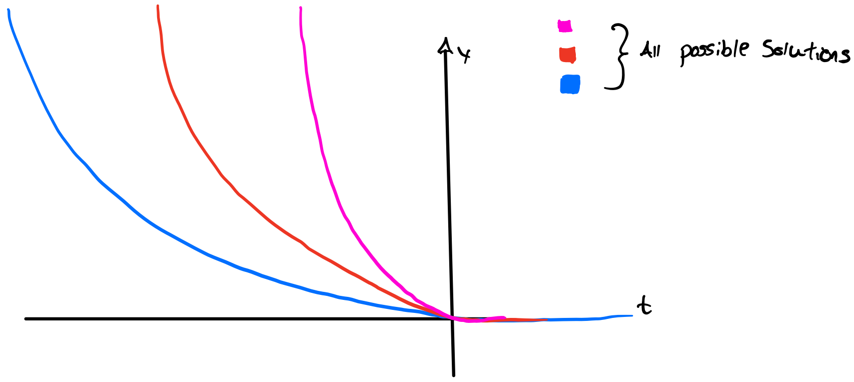
$-2K(h)^{\frac{1}{2}} = t$

$K(h)^{\frac{1}{2}} = t/_{-2}$

$(h)^{\frac{1}{2}} = t/_{-2K}$

$h = \left(t/_{-2K}\right)^2$

It is impossible to tell how much time has ellapsed because the bucket could have been sitting empty for quite some time



c.)