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MATH 360
CP 9.7

Example 1

Find the directional derivative of $f(x,y,z) = 2x^2+3y^2+z^2$ at $P:(2,1,3)$ in the direction of $\vec{a}=[1,0,-2]$

Solution. grad $f = [4x, 6y, 2z]$ gives at P the vector grad $f(P) = [8,6,6]$. From this and (5*) we obtain,
Since $|a| = \sqrt{5}$

$$D_a f(P) = \frac{1}{\sqrt{5}}[1,0,-2]\cdot[8,6,6] = \frac{1}{\sqrt{5}}(8+0-12) = \frac{-4}{\sqrt{5}} = -1.789$$

The minus sign indicates that at P the function f is decreasing in the direction of $\vec{a}$.

Differential Operator

Remarks: For a definition of the gradient in curvilienear coordinates, see App. 3.4.
As a quick example, if $f(x,y,z) = 2y^3+4xz+3x$, then grad $f = [4z+3, 6y^2, 4x]$
Furthermore, we will show later in this section that (1) actually does define a vector.
The notation ∇f is suggested by the differential operator ∇ (read nabla) defined by

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Figure 216

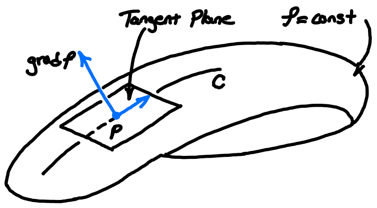

Fig. 216

The Laplacian

Laplaces equation is as follows,

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

The Laplace Operator

The differential operator,

$$\nabla^2 = \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (11)$$

where equation (11) is also known as the Laplacian.