Phys 131 2016

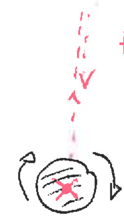
Lecture 32

Weds: Review III Thurs Physics seminar

Fri: Exam III Ch 9,10,11

Center of Mass Motion

How can we separate the rotational and translational motion of an object? Consider a ball which is thrown vertically and also spins. We know that one can represent the translational motion by tracking the position of the center. It seems reasonable to describe the rotational motion in terms of rotation about the center. In these terms, the center is a special point on the object that enters into the description of both types of motion



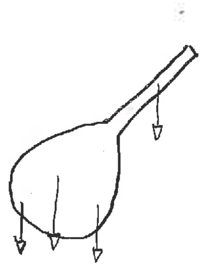
Locating such a special point is more complicated when the object is not symmetrical or has a non-uniform mass distribution

Demo: MIT Co.m video

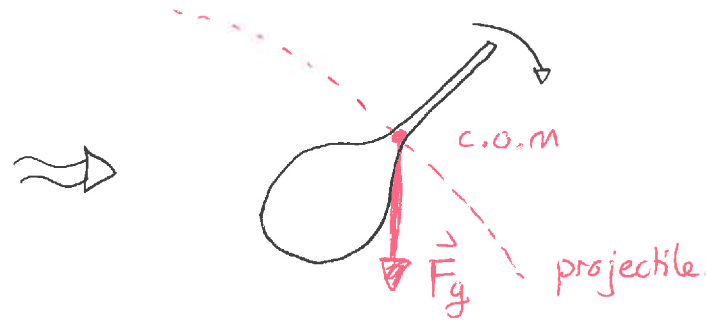
The video illustrates:

> For any object there is a special point, called the center of mass, such that when various forces act on the object at various locations, the motion of the center of mass is exactly that of a point particle with all the mass and forces concentrated at this point.

In this video the forces are all gravitational and the center of mass undergoes projectile motion.



small gravitational forces act at all locations



Note that this does not describe the rotational motion of the object. So

$\boxed{\text{The translational motion is described by the center of mass motion}}$

The center of mass is formally defined using:

> The center of mass is a location with co-ordinates
>
> $$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum m_i x_i}{\sum m_j}$$
>
> $$Y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum m_i y_i}{\sum m_j}$$
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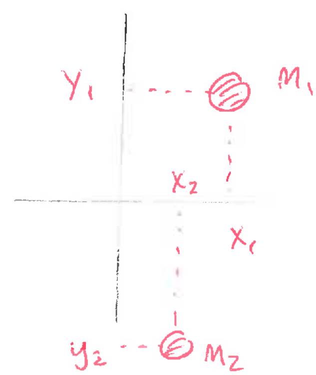
> 
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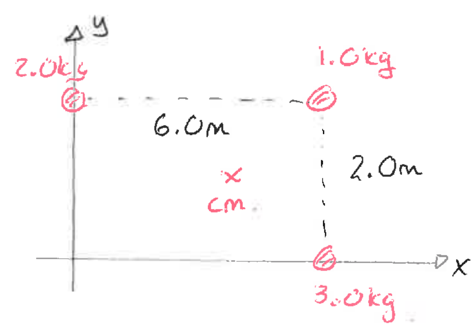


Example: Determine the center of mass of the illustrated objects.

Answer:

Object	Mass	x	y
1	$m_1 = 2.0kg$	$x_1 = 0m$	$y_1 = 2.0m$
2	$m_2 = 1.0kg$	$x_2 = 6.0m$	$y_2 = 2.0m$
3	$m_3 = 3.0kg$	$x_3 = 6.0m$	$y_3 = 0m.$

Here $\sum m_j = 2.0kg + 1.0kg + 3.0kg = 6.0kg$. So

$$X_{c.m} = \sum x_i m_i / \sum m_j = [0m \times 2.0kg + 6.0m \times 1.0kg + 6.0m \times 3.0kg] / 6.0kg = 4.0m$$

$$Y_{cm} = \sum y_i m_i / \sum m_j = [2.0m \times 2.0kg + 2.0m \times 1.0kg + 0m \times 3.0kg] / 6.0kg = 1.0m$$

Note that the center of mass does not need to be located on the actual objects.

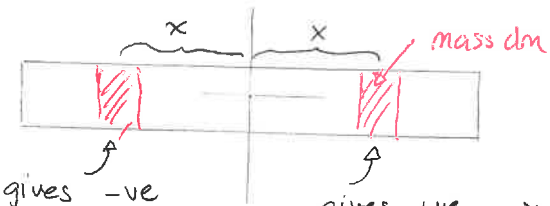
For extended objects with a continuous mass distribution, we break the object into infinitesimal parts, treat each as a point particle and calculate the c.o.m. in the usual fashion. So

$$x_{cm} = \frac{\sum x\, dm}{M} \qquad y_{cm} = \frac{\sum y\, dm}{M}$$

total mass

For uniform symmetrical objects the center of mass is at the geometric center.

Warm Up !



gives $-$ve $x\,dm$ since x negative $\Rightarrow$ add to zero.

gives $+$ve $x\,dm$

$\Rightarrow x_{cm} = 0$

In general one can show that:

> The acceleration of the center of mass is determined via:
>
> $$\vec{F}_{net} = M\vec{a}_{cm}$$
>
> where M is the total mass of the object and
>
> $\vec{F}_{net}$ = sum of all forces on all parts

If the net force is zero the the c.o.m stays at rest or moves with constant velocity.

Quiz 1 ~ 60% $\quad \xi$ 20%

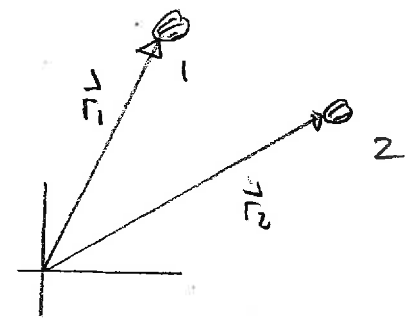
Quiz3 50% → 60% $\lesssim$ 30% − 60%

Proof of Center of Mass Dynamics:

Suppose there are N particles at locations $\vec{r}_1, \vec{r}_2, \ldots$

Then for particle i

$$\vec{F}_{net\,i} = m_i \vec{a}_i$$
$$= m_i \frac{d^2\vec{r}_i}{dt^2}$$



But $\vec{F}_{net\,i} = \underbrace{\vec{F}_{ext\,i}}_{\text{by external objects.}} + \underbrace{\vec{F}_{int\,i}}_{\text{exerted by all particles } j=1,\ldots,N \;\; j\neq i}$

Then $\vec{F}_{int\,i} = \sum_{j\neq i} \vec{F}_{j\,on\,i}$ gives:

$$\vec{F}_{ext\,i} + \sum_{j\neq i} \vec{F}_{j\,on\,i} = m_i \frac{d^2\vec{r}_i}{dt^2}$$

Adding gives:

$$\vec{F}_{ext\,1} + \vec{F}_{2\,on\,1} + \vec{F}_{3\,on\,1} + \cdots$$
$$+ \vec{F}_{ext\,2} + \vec{F}_{1\,on\,2} + F_{3\,on\,2} + $$
$$+ \cdots \qquad\qquad + \cdots = \sum_i m_i \frac{d^2\vec{r}_i}{dt^2}$$

cancel via Newton's Third Law. All internal forces cancel. Thus

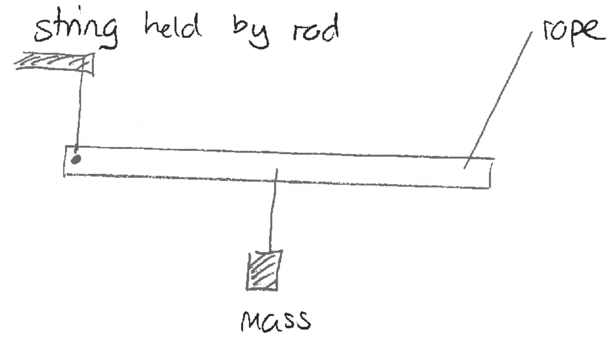
$$\underbrace{\sum_i F_{ext\,i}} = \frac{d^2}{dt^2} \sum m_i \vec{r}_i$$

$$\Rightarrow \vec{F}_{net} = M \frac{d\vec{R}}{dt^2} \quad \text{where } \vec{R} = \frac{\sum m_i \vec{r}_i}{M} \text{ is the center of mass} \quad \boxtimes$$

Forces, Torques + Rotational Motion

How do forces affect the rotational motion of an object?
Consider an extended object such as a meter stick and which can pivot about a single point.

Demo →



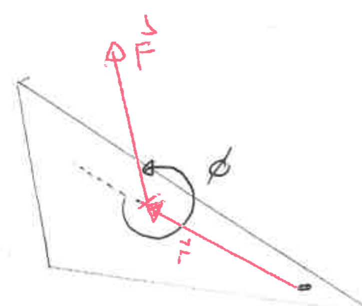
Clearly the rotational effects of the force are only partly determined by the magnitude and direction of the force.

We see that the location at which it acts also matters. There are three important features that affect the rotational motion.

1) the magnitude of the force
2) the angle at which the force acts.
3) the location " " " acts.

These are all built in to the definition of torque

1) Choose a reference point O (often pivot)
2) Draw a vector from the ref. point to where the force acts. Extend this line
3) Measure the angle ϕ c.c.w from the extension to the force vector



4) The "torque produced by $\vec{F}$ about O" is

$$\tau = rF\sin\phi$$

Units N·m

Note: 1) In the definition, F, r are positive.

2) $\sin\phi$ can be positive or negative. We can see that

$0^\circ \le \phi \le 180^\circ \Rightarrow \tau > 0 \Rightarrow$ promotes c.c.w rotation

$180^\circ \le \phi \le 360^\circ \Rightarrow \tau < 0 \Rightarrow$ " c.w "