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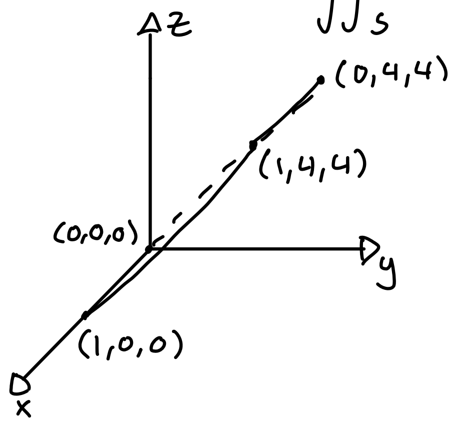
MATH 360
HW 5

10.9 #1,2,4,14

Problem 10.9.1

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\,dA = \oint_C \vec{F}\cdot\vec{r}\,'ds \qquad F=[z^2,-x^2,0]$$



$\vec{r}(u,v)=[u,v,4u]$
$0\le u\le 1$
$0\le v\le 4$
$\vec{r}_u=[1,0,4]$
$\vec{r}_v=[0,1,0]$
$\vec{r}_u\times\vec{r}_v=\vec{n}$

$F=[z^2,-x^2,0]$

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ z^2 & -x^2 & 0\end{vmatrix}$$

$$\hat{x}\left(\frac{\partial}{\partial y}(0)-\frac{\partial}{\partial z}(-x^2)\right)+\hat{y}\left(\frac{\partial}{\partial z}(z^2)-\frac{\partial}{\partial x}(0)\right)+\hat{z}\left(\frac{\partial}{\partial x}(-x^2)-\frac{\partial}{\partial y}(z^2)\right)$$

$$\vec{\nabla}\times\vec{F}=\left[\hat{x}(0)+\hat{y}(2z)+\hat{z}(-2x)\right]$$

$$\vec{r}_u\times\vec{r}_v=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 1 & 0 & 4\\ 0 & 1 & 0\end{vmatrix}=\hat{x}\left(0(0)-4(1)\right)+\hat{y}\left(4(0)-1(0)\right)+\hat{z}\left(1(1)-0(0)\right)$$

$$\vec{r}_u\times\vec{r}_v=\left[\hat{x}(-4)+\hat{y}(0)+\hat{z}(1)\right]=\vec{n}$$

$$(\vec{\nabla}\times\vec{F})\cdot(\vec{r}_u\times\vec{r}_v)=(0)(-4)+(2z)(0)-(2x)(1) = -2x \equiv 2u \quad \text{Since } x=u$$

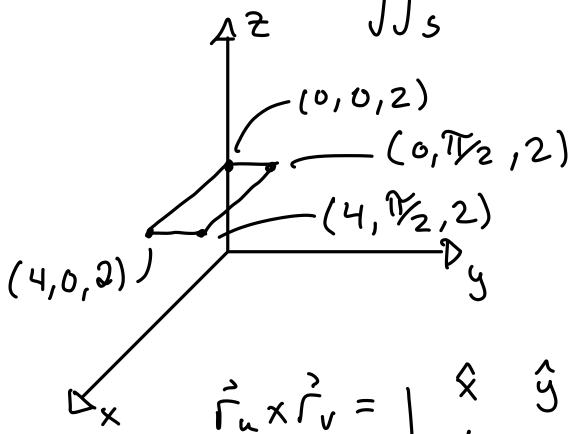
$$-2\int_0^4\int_0^1 u\,du\,dv = -\int_0^4\left[u^2\right]_0^1 dv = -\int_0^4 dv = -4$$

$$\boxed{\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\,dA = -4}$$

Problem 10.9.2

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\, dA = \oint_C \vec{F}\cdot\vec{r}\,' ds \qquad \vec{F} = [-13\sin(y), 3\sinh(z), x]$$

$$dA = du\,dv$$



$$\vec{r}(u,v) = [u, v, 2] \qquad 0 \le u \le 4, \quad 0 \le v \le \pi/2$$

$$\vec{r}_u = [1,0,0]$$

$$\vec{r}_u \times \vec{r}_v \equiv \vec{n}$$

$$\vec{r}_v = [0,1,0]$$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{x}(0(0)-0(1)) + \hat{y}(0(0)-1(0)) + \hat{z}(1(1)-0(0))$$

$$\vec{r}_u \times \vec{r}_v = [0,0,1] \equiv \vec{n}$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -13\sin(y) & 3\sinh(z) & x \end{vmatrix}$$

$$= \hat{x}\left[\frac{\partial}{\partial y}(x) - \frac{\partial}{\partial z}(3\sinh(z))\right] + \hat{y}\left[\frac{\partial}{\partial z}(-13\sin(y)) - \frac{\partial}{\partial x}(x)\right] + \hat{z}\left[\frac{\partial}{\partial x}(3\sinh(z)) - \frac{\partial}{\partial y}(-13\sin(y))\right]$$

$$\vec{\nabla}\times\vec{F} = [1, -1, 13\cos(y)] \Rightarrow [1, -1, 13\cos(v)] \quad \text{since } y = v$$

$$\int_0^{\pi/2}\int_0^4 [1,-1,13\cos(v)]\cdot[0,0,1]\, du\,dv$$

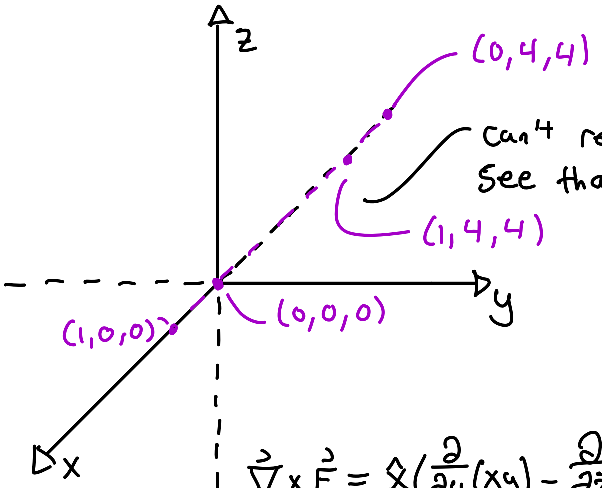
$$\int_0^{\pi/2}\int_0^4 13\cos(v)\, du\,dv \Rightarrow 13\int_0^{\pi/2} u\cos(v)\Big|_0^4 dv \Rightarrow 52\int_0^{\pi/2}\cos(v)\,dv$$

$$52\left[\sin(v)\right]_0^{\pi/2} = 52\left[\sin(\pi/2) - \sin(0)\right] = 52$$

$$\boxed{\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\, dA = 52}$$

Problem 10.9.4

$\vec{F} = [z^2, -x^2, xy]$ $\quad z = xy \quad 0 \le x \le 1,\ 0 \le y \le 4$ $\qquad \iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dA = \oint_C \vec{F} \cdot \vec{r}\,' ds$



$x = u$

$y = v$

$\vec{r}(u,v) = [u, v, 4u]$

$\vec{r}_u = [1, 0, 4]$, $\vec{r}_v = [0, 1, 0]$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & -x^2 & xy \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \hat{x}\left(\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(-x^2)\right) + \hat{y}\left(\frac{\partial}{\partial z}(z^2) - \frac{\partial}{\partial x}(xy)\right) + \hat{z}\left(\frac{\partial}{\partial x}(-x^2) - \frac{\partial}{\partial y}(z^2)\right)$$

$$\vec{\nabla} \times \vec{F} = (x - 0)\hat{x} + (2z - y)\hat{y} + (-2x + 0)\hat{z}$$

$$\vec{\nabla} \times \vec{F} = [x, 2z - y, -2x]$$

$$\vec{n} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 4 \\ 0 & 1 & 0 \end{vmatrix} = \hat{x}(0(0) - 4(1)) + \hat{y}(4(0) - 1(0)) + \hat{z}(1(1) - 0(0))$$

$$\vec{n} = (0-4)\hat{x} + (4-0)\hat{y} + (1-0)\hat{z}$$

$$\vec{n} = [-4, 0, 1]$$

$$(\vec{\nabla} \times \vec{F}) \cdot (\vec{n}) = [x, 2z - y, -2x] \cdot [-4, 0, 1] = -4x - 2x = -6x \Big|_{x=u} = -6u$$

$$-6 \int_0^4 \int_0^1 u \, du\, dv$$

$$-3 \int_0^4 u^2 \Big|_0^1 dv = -3 \int_0^4 dv = -3(v)\Big|_0^4 = -12$$

$$\boxed{\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dA = -12}$$

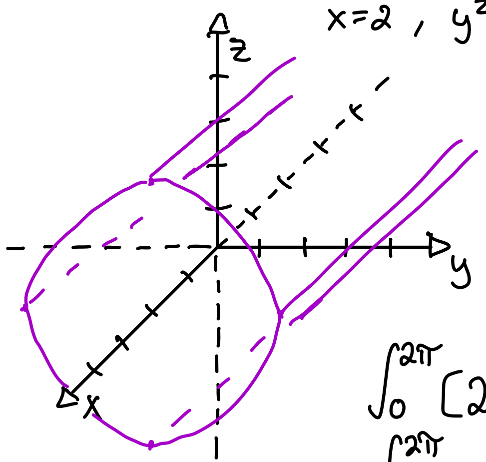
Problem 10.9.14

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\, dA = \oint_C \vec{F}\cdot\vec{r}\,' ds$$

$\vec{F} = [z^3, x^3, y^3]$

Circle of radius 3

$x=2$, $y^2+z^2=9$



$$\vec{r}(t) = [2, 3\cos(t), 3\sin(t)] \quad 0 \le t \le 2\pi$$

$$\vec{r}\,'(t) = [0, -3\sin(t), 3\cos(t)]$$

$$\vec{F}(\vec{r}(t)) = [27\sin^3(t), 8, 27\cos^3(t)]$$

$$\int_0^{2\pi} [27\sin^3(t), 8, 27\cos^3(t)][0, -3\sin(t), 3\cos(t)]\, dt$$

$$\int_0^{2\pi} -24\sin(t) + 81\cos^4(t)\, dt$$

$$-24\int_0^{2\pi} \sin(t)\, dt + 81\int_0^{2\pi} \cos(t)\cos^3(t)\, dt \quad \begin{matrix} u = \cos^3(t) & dv = \cos(t) \\ du = -3\cdot\cos^2(t)\cdot\sin(t)\, dt & v = \sin(t) \end{matrix}$$

$$-24\left[-\cos(t)\right]_0^{2\pi} + 81\left[\cos^3(t)\sin(t) + 3\int_0^{2\pi}\cos^2(t)\sin^2(t)\, dt\right]$$

$$-24\left[-\cos(2\pi) - (-\cos(0))\right] + 81\left[\cos^3(t)\sin(t) + \frac{3}{8}\int_0^{2\pi} 1 - \cos(4t)\, dt\right]$$

$$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta)) \qquad 81\left[\cos^3(t)\sin(t) + \frac{3}{8}\left[t - \frac{\sin(4t)}{4}\right]\right]_0^{2\pi}$$

$$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$$

$$\cos^2\theta\sin^2\theta = \frac{1}{8}(1-\cos(4\theta))$$

$$-24[-1+1] = 0$$

$$81\left[\cos^3(2\pi)\sin(2\pi) + \frac{3}{8}\left(2\pi - \frac{\sin(8\pi)}{4}\right) - \left(\cos^3(0)\sin(0) + \frac{3}{8}\left[0 - \frac{\sin(0)}{4}\right]\right)\right]$$

$$81\left[\left(0 + \frac{3}{8}(2\pi - 0)\right) - \left(0 + \frac{3}{8}(0-0)\right)\right]$$

$$81\left[\frac{3\pi}{4}\right] = \frac{243\pi}{4}$$

$$\boxed{\oint_C \vec{F}\cdot\vec{r}\,' ds = \frac{243\pi}{4}}$$