PHYS 230 HOMEWORK SET 8 SOLUTIONS

1) $D_A = 150m$

$D_B = 450m$

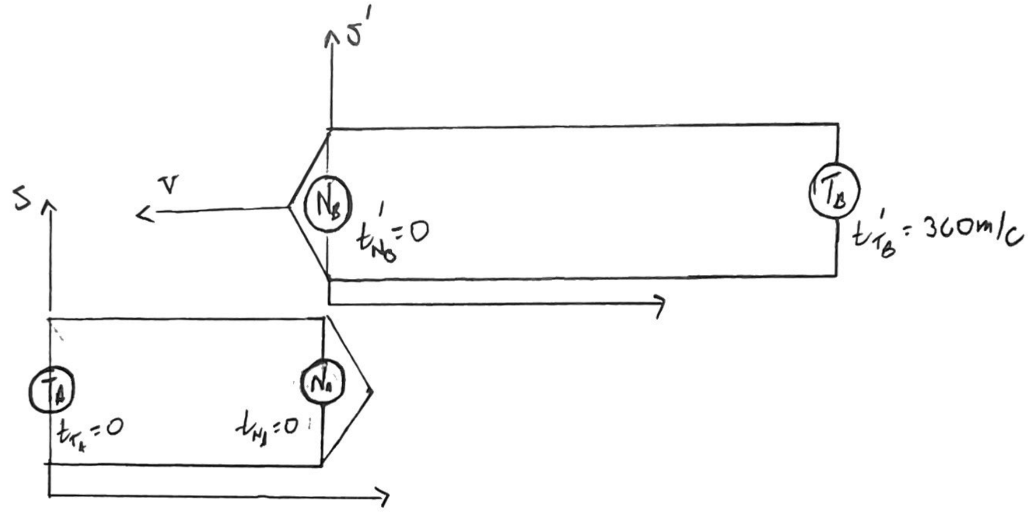
$V = \frac{4}{5}c$

let 1 INCH = 100m

ACCORDING TO SPACESHIP A, SPACESHIP B IS LENGTH CONTRACTED TO...

$$d_B = D_B\sqrt{1 - V^2/c^2} = 450m\sqrt{1-(4/5)^2} = 450m \cdot \frac{3}{5} = 270m$$

a)



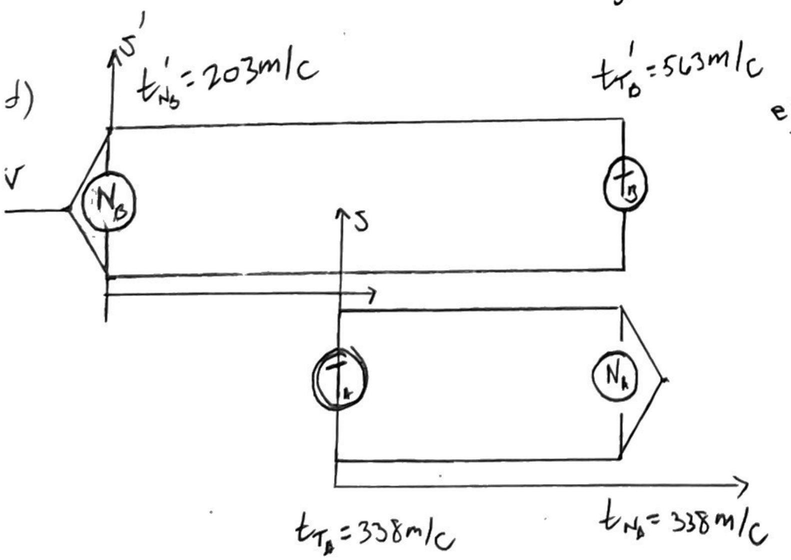
b) NOW, $t_{T_A} = t_{N_A} = 0$ AS BOTH CLOCKS ARE SYNCHRONIZED IN THE S FRAME

$$t'_{N_B} = 0$$

$$t'_{T_B} = \frac{VD_B}{c^2} = \frac{(\frac{4}{5}c)(450m)}{c^2} = 360m/c$$

c) NOW $V = \frac{d_B}{\Delta t}$ $\therefore$ $\Delta t = \frac{d_B}{V} = \frac{270m}{\frac{4}{5}c} = 337.5\,m/c = 338\,m/c$

d)



e) NOW A TIME INTERVAL OF $\Delta t = 338m/c$ TICKED BY ON THE A CLOCKS SO

$$t_{T_A} = t_{N_A} = 338\,m/c$$

MOVING CLOCKS TICK SLOW, SO...

$$\Delta t' = \Delta t\sqrt{1 - V^2/c^2} = 338\frac{m}{c} \cdot \frac{3}{5}$$

$$\Delta t' = 203\,m/c$$

SO

$$t'_{N_B} = 0 + 203\,m/c = 203\,m/c$$

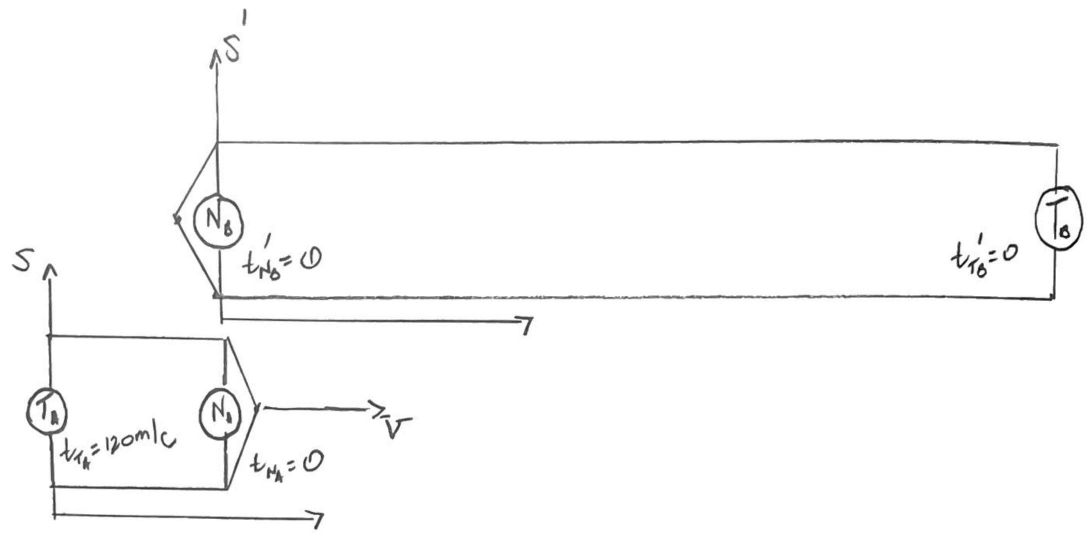
$$t'_{T_B} = 360\,m/c + 203\,m/c = 563\,m/c$$

ACCORDING TO SPACESHIP B, SPACESHIP A IS LENGTH CONTRACTED TO...

2.

$$d_A = D_A\sqrt{1-V^2/c^2} = 150m \cdot \sqrt{1-(4/5)^2} = 150m \cdot \frac{3}{5} = 90m$$

a)



b) NOW, $t'_{N_B} = t'_{T_B} = 0$ AS BOTH CLOCKS ARE SYNCHRONIZED IN THE S' FRAME

$$t_{N_A} = 0$$

$$t_{T_A} = \frac{VD_A}{c^2} = \frac{\frac{4}{5}c(150m)}{c^2} = 120m/c$$

) NOW $V = \frac{D_B}{\Delta t'}$, $\therefore \Delta t' = \frac{D_B}{V} = \frac{450m}{\frac{4}{5}c} = 563m/c$ $\quad \therefore \left[\Delta t' = 563m/c\right]$

MOVING CLOCKS TICK SLOW, SO..

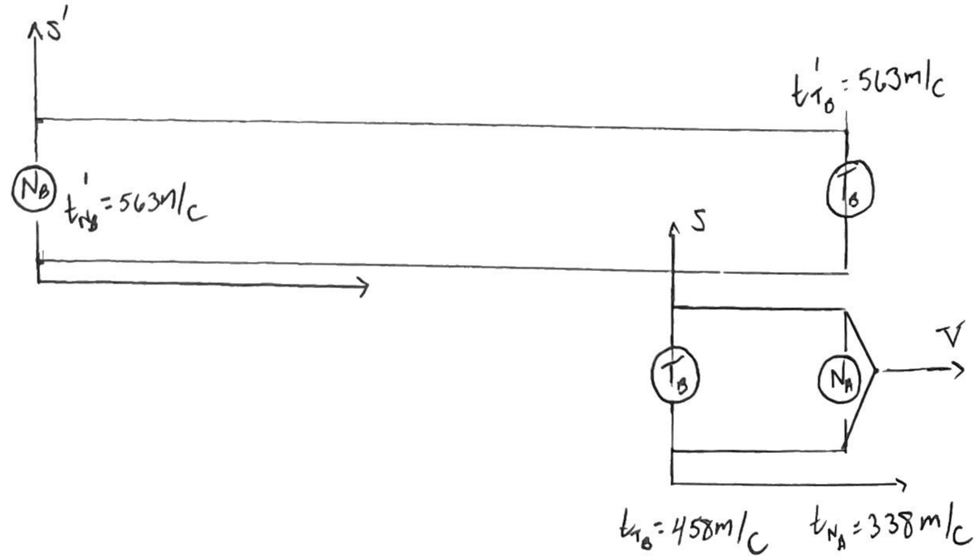
so

$$\Delta t = \Delta t'\sqrt{1-V^2/c^2} = (563m/c)\sqrt{1-(4/5)^2} = (563m/c)\cdot\frac{3}{5} = 338m/c$$

so

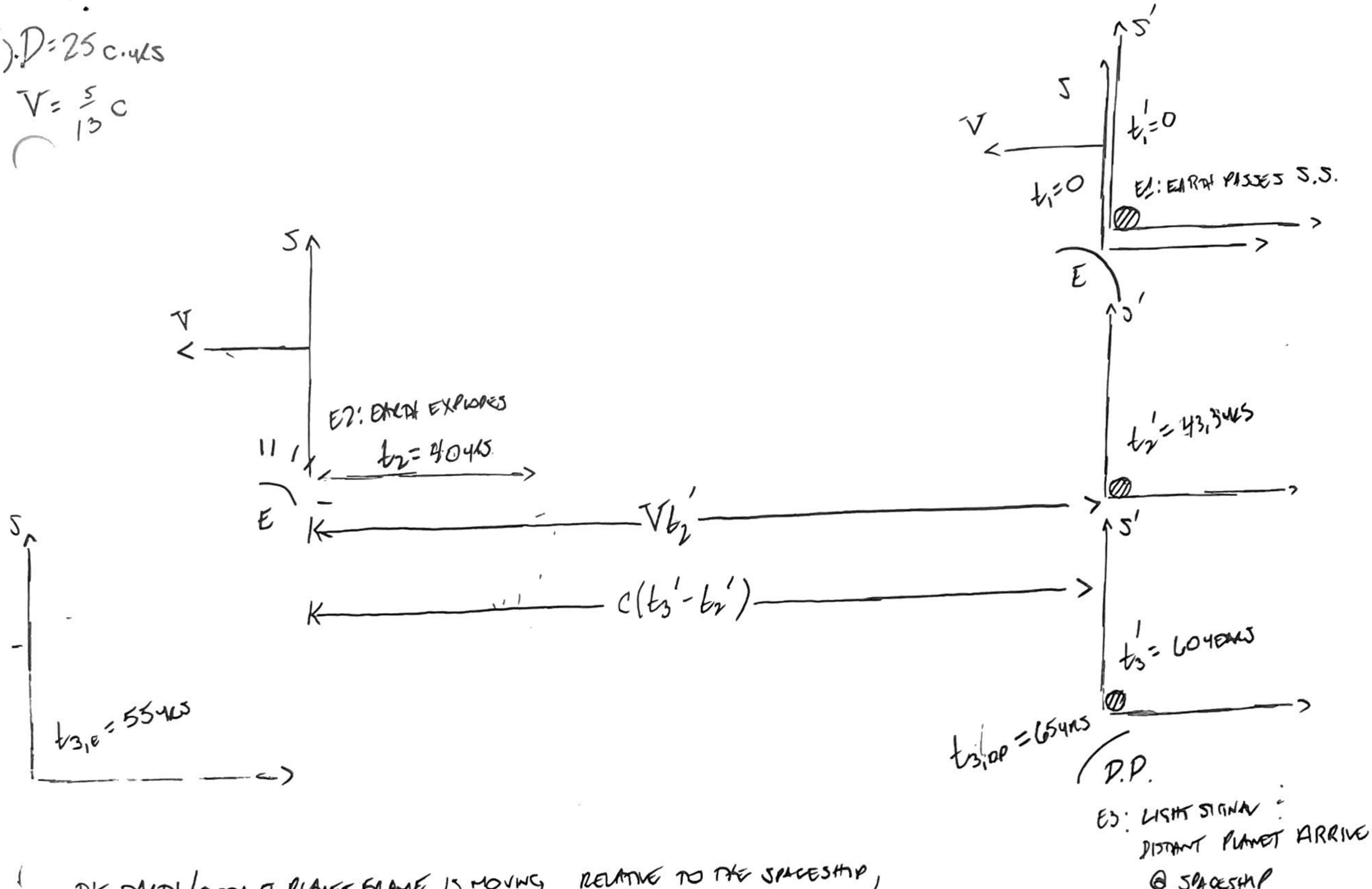
$t_{N_A} = 0 + 338m/c = 338m/c$

$t_{T_A} = 120m/c + 338m/c = 458m/c$



). $D = 25\ c\cdot yrs$

$V = \frac{5}{13}c$



) ... THE EARTH / DISTANT PLANET FRAME IS MOVING RELATIVE TO THE SPACESHIP, THE SPACESHIP OBSERVERS SEE A CONTRACTED LENGTH

$$d = D\sqrt{1 - V^2/c^2} = 25c\cdot yrs\sqrt{1 - (5/13)^2} = (25c\cdot yrs)\frac{12}{13} = 23.1\ c\ yrs$$

$$\left[d = 23.1\ c\cdot yrs\right]$$

:) NOW

$$V = \frac{d}{\Delta t_{31}'} \quad \therefore \quad \Delta t_{31}' = \frac{d}{V} = \frac{23.1\ c\cdot yrs}{\frac{5}{13}c} = 60\ \text{YEARS} = t_3' - \cancel{t_1'} \quad \text{so}$$

$$\left[t_3' = 60\ \text{YEARS}\right]$$

d) NOW THE MOVING EARTH CLOCK RUNS AT A SLOWER RATE, SO

$$t_3 = t_3'\sqrt{1 - V^2/c^2} = (60\ \text{YEARS})\sqrt{1 - (5/13)^2} = (60\ \text{years})\cdot\frac{12}{13} = 55.4\ \text{YEARS}$$

so

$$\left[t_3 = 55.4\ \text{YEARS}\right] = \Delta t_{31}$$

e) ACCORDING TO THE SHIP FRAME, THE DISTANT PLANET CLOCK IS NOT SYNCHRONIZED W/ THE EARTH CLOCK, SO

$$t_{1,DP} = \frac{VD}{c^2} = \frac{\left(\frac{5}{13}c\right)(25\,c\,\text{yrs})}{c^2} = 9.62\text{ yrs}$$

A TIME INTERVAL $\Delta t_{31} = 55.4\text{ yrs}$ TICKS BY, SO

$$t_{3,DP} = t_{1,DP} + \Delta t_{31} = 55.4\text{ yrs} + 9.62\text{ yrs} = 65\text{ YEARS}$$

$$\boxed{t_{3,DP} = 65\text{ YEARS}}$$

f) NOTICE THAT

$$c(t_3' - t_2') = V t_2' \quad \text{so} \quad (c+V)t_2' = c t_3' \quad \therefore \quad t_3' = \left(1 + \frac{V}{c}\right)t_2' \quad \therefore \quad t_2' = \frac{t_3'}{(1+V/c)}$$

$$\text{so} \quad t_2' = \frac{60\text{ yrs}}{1 + 5/13} = \frac{60\text{ yrs}}{18/13} = \frac{13}{18}\cdot(60\text{ yrs})$$

$$\boxed{t_2' = 43.3\text{ yrs}}$$

g) NOW THE MOVING EARTH CLOCK TICKS SLOW, SO ...

$$t_2 = t_2'\sqrt{1 - V^2/c^2} = (43.3\text{ yrs})\sqrt{1 - (5/13)^2} = (43.3\text{ yrs})\frac{12}{13} = 40\text{ yrs}$$

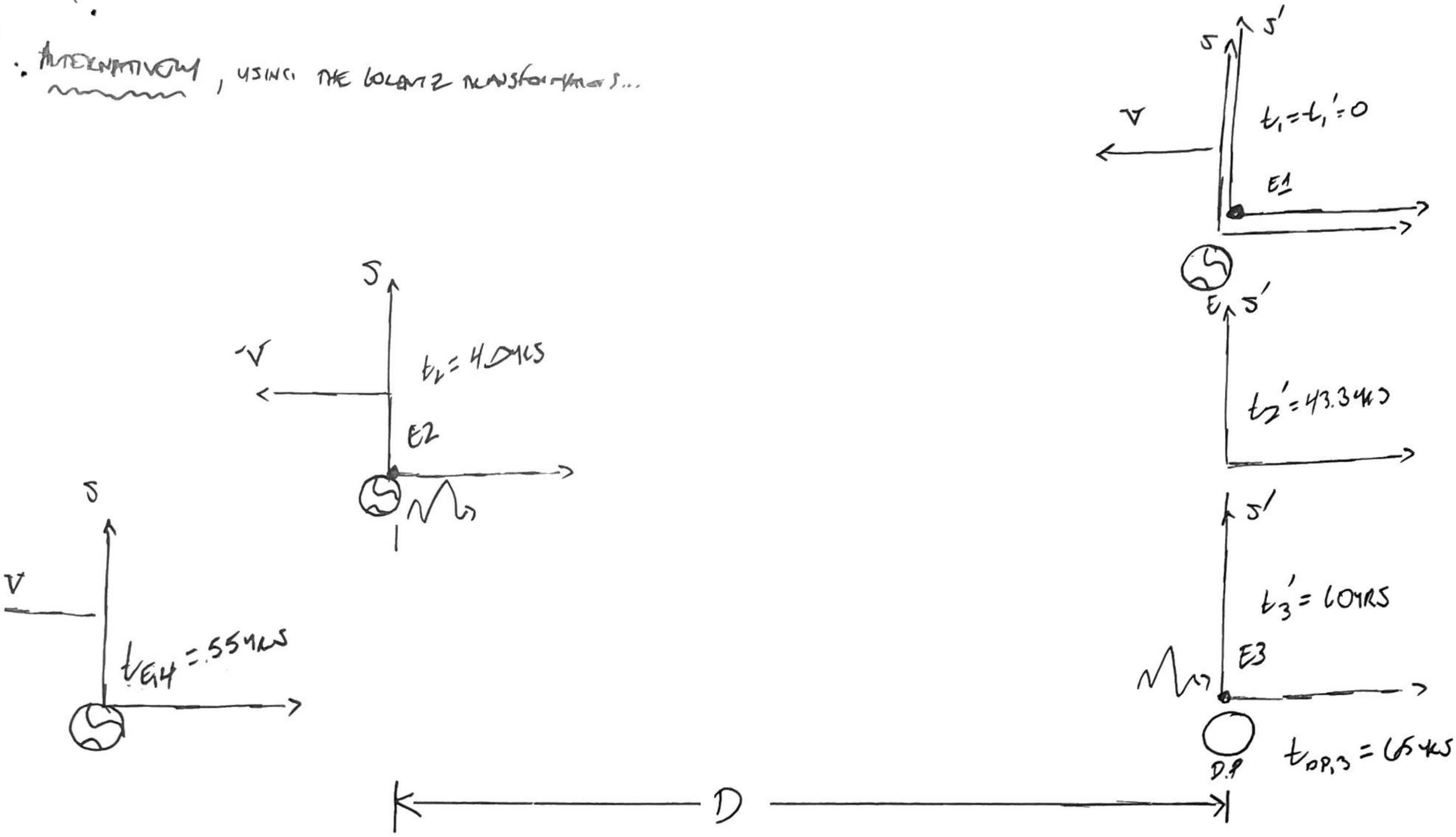
$$\boxed{t_2 = 40\text{ yrs}} = \Delta t_{21}$$

h) THE DISTANT PLANET CLOCK READS ...

$$t_{2,DP} = t_{1,DP} + \Delta t_{21} = 9.62\text{ yrs} + 40\text{ yrs}$$

$$\boxed{t_{2,DP} = 50\text{ yrs}}$$

∴ ALTERNATIVELY, USING THE LORENTZ TRANSFORMATIONS...



NOW

$$D = c(t_3' - t_2') = V t_2' \quad \text{so} \quad c t_3' = (V + c) t_2' \quad \therefore \; t_3' = \left(1 + \frac{V}{c}\right) t_2' \quad (*)$$

NOTICE:

$\Delta X_{12} = 0$ ∴ $\Delta X_{13}' = 0$

$$\text{LT 1)} \quad \Delta t' = \gamma\left(\Delta t - \frac{V}{c^2}\Delta X\right)$$

$$\text{LT 2)} \quad \Delta X' = \gamma(\Delta X - V\Delta t)$$

$$\text{ILT 1)} \quad \Delta t = \gamma\left(\Delta t' + \frac{V}{c^2}\Delta X'\right)$$

$$\text{ILT 2)} \quad \Delta X = \gamma(\Delta X' + V\Delta t')$$

COMPARING E1 ∴ E3 AND USING $\Delta X_{13}' = 0$...

$$\text{ILT 2)} \quad \Delta X_{13} = V\Delta t_{13} = D \quad \text{so}$$

$$\Delta t_{13} = \frac{D}{V} = \frac{25c \text{ YRS}}{\frac{5}{13}c}$$

$$\Delta t_{13} = \frac{13}{5} \cdot 25 \text{ YRS} = 65 \text{ YRS} = t_3 - \cancel{t_1}^{\,0} \quad \therefore \; \boxed{t_{DP,3} = 65 \text{ YRS}}$$

THIS IS THE TIME ON THE DISTANT PLANET CLOCK! (CLOCK WHERE E3 OCCURS)

LT 1) YIELDS..

$$\Delta t_{13} = \gamma \Delta t_{13}' \quad \text{so} \quad \Delta t_{13}' = \frac{1}{\gamma}\Delta t_{13} = \sqrt{1 - V^2/c^2}\,\Delta t_{13} = \sqrt{1 - (5/13)^2}\,(65 \text{ YRS})$$

$$= \frac{12}{13} \cdot 65 \text{ YRS} = 60 \text{ YRS} = t_3' - \cancel{t_1'}$$

SO

$$\boxed{t_3' = 60 \text{ YRS}}$$

NOW (*) YIELDS t_2' . .

$$t_2' = \frac{t_3'}{1+V/c} = \frac{60\,\mu s}{1+5/13} = \frac{60\,\mu s}{18/13} = \frac{13}{18}(60\,\mu s) = 43.3\,\mu s$$

$$\therefore \boxed{t_2' = 43.3\,\mu s}$$

NOW, COMPARING E1 & E2 AND USING $\Delta x_{12} = 0$ . . .

LT1) YIELDS $\Delta t_{12}' = \gamma \Delta t_{12} = t_2' - \cancel{t_1'}^{\,0}$

$$\therefore \Delta t_{12} = \frac{1}{\gamma} t_2' = \sqrt{1 - V^2/c^2}\, t_2' = \sqrt{1 - (5/13)^2}\,(43.3\,\mu s)$$

$$= \frac{12}{13}(43.3\,\mu s) = 40\,\mu s = t_2 - \cancel{t_1}^{\,0}$$

SO $\boxed{t_2 = 40\,\mu s}$

HOW DO I GET $t_{E,13}$? LET E4 BE THE READING OF THE EARTH CLOCK

SO $\Delta x_{14} = 0$ SO

$$\Delta t_{14}' = \gamma \Delta t_{14} \quad \therefore \quad \Delta t_{14} = \frac{1}{\gamma} \Delta t_{14}'$$

$$= \sqrt{1 - V^2/c^2}\, \Delta t_{14}'$$

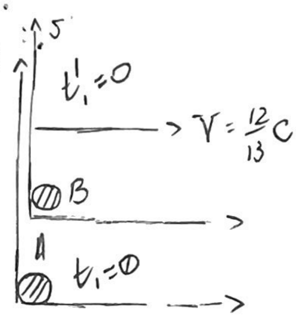
NOTICE THAT $t_4' = t_3'$ AS THESE EVENTS ARE SIMULTANEOUS FOR THE PRIMED OBSERVER $\therefore \Delta t_{14}' = \Delta t_{13}'$

SO $$\Delta t_{14} = \sqrt{1 - (5/13)^2}\; 60\,\mu s = \frac{12}{13} \cdot 60\,\mu s$$

$$\boxed{\therefore t_{E,4} = 55.4\,\mu s}$$

i...

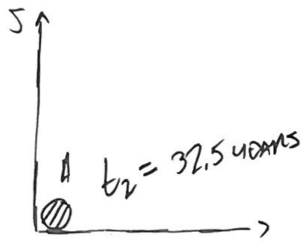
a)

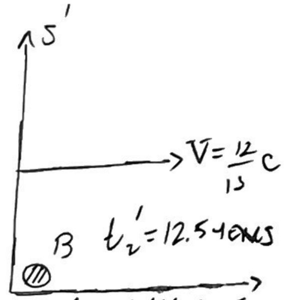


E1: BILLY DEPARTS FROM A

$t_1 = 0$

DISTANT PLANET





E2: BILLY ARRIVES AT DISTANT PLANET

) $\Delta t' = 12.5$ YRS

$\Delta x' = 0$ so $\Delta t = \dfrac{\Delta t'}{\sqrt{1 - V^2/c^2}} = \dfrac{12.5\text{ YRS}}{\sqrt{1 - (12/13)^2}} = \dfrac{12.5\text{ YRS}}{5/13} = 32.5$ YRS

$t_{2,DP} = 32.5$ YEARS

so $\boxed{\Delta t_{TOT} = 2 \cdot \Delta t = 65 \text{ YEARS}}$

) $V = \dfrac{D}{\Delta t}$ $\therefore$ $D = V \Delta t = \left(\frac{12}{13}c\right)(32.5\text{ YRS})$ $\therefore$ $\boxed{D = 30\text{ c} \cdot \text{YRS}}$

) $d = D\sqrt{1 - V^2/c^2} = (30\text{ cYRS})\sqrt{1 - (12/13)^2} = (30\text{ cYRS}) \cdot \frac{5}{13} = 11.5$ c YRS

$\boxed{d = 11.5 \text{ c YRS}}$

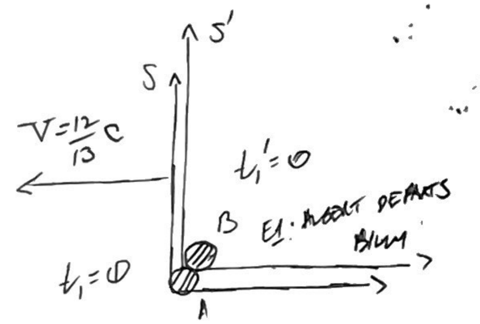
) NOW $\Delta t' = 12.5$ YEARS so $t'_2 = 12.5$ YEARS

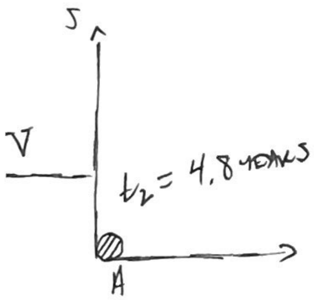
$\Delta t = 32.5$ YEARS so $t_{2,A} = 32.5$ YEARS

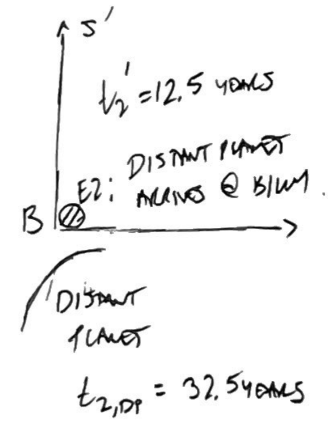
$t_{2,DP} = 32.5$ YEARS

5)

a)







b) AS ALBERT DEPARTS BLUM, $t_1 = t_1' = 0$

$$t_{1,DP} = \frac{VD}{c^2} = \frac{\left(\frac{12}{13}c\right)(30\, c \cdot yrs)}{c^2} = 27.7 \text{ YEARS}$$

$$\boxed{t_{1,DP} = 27.7 \text{ YEARS}}$$

c) ACCORDING TO BLUM...

$V = \frac{d}{\Delta t'}$, SO $\Delta t' = \frac{d}{V} = \frac{11.5 \text{ C YEARS}}{\frac{12}{13}c} = 12.5 \text{ YEARS}$

$$\boxed{\Delta t' = 12.5 \text{ YEARS}} = t_2'$$

MOVING CLOCKS TICK SLOW SO

$$\Delta t = \Delta t' \sqrt{1 - V^2/c^2} = (12.5 \text{ YEARS})\sqrt{1 - (12/13)^2} = 12.5 \text{ YEARS} \cdot \frac{5}{13} = 4.8 \text{ YEARS}$$

SO

$$t_2 = 4.8 \text{ YEARS}$$

$$t_{2,DP} = t_{1,DP} + \Delta t = 27.7 \text{ YEARS} + 4.8 \text{ YEARS}$$

$$\boxed{t_{2,DP} = 32.5 \text{ YEARS}}$$

2a $R = \frac{450m}{90m} = 5$

If $4.5 < R < 5.5$ +2

or $4 < R < 6$ +1

b) $t'_{NB} = t'_{TB} = 0$ } +1

$t_{N_B} = 0$

$t_{T_B} = 120 m/c$ +1

c) $\Delta t' = 563 m/c$ +1 $= 1.88\times10^{-6}$ s

d) If $4.5 < R < 5.5$ +2

$4 < R < 6$ +1

e) $t'_{NB} = t'_{TB} = 563 m/c$ +1 $= 1.88\times10^{-6}$ s

$t_{T_A} = 458 m/c$ +1 $= 1.53\times10^{-6}$ s

$t_{N_A} = 338 m/c$ +1 $= 1.13\times10^{-6}$ s

3a 3 correct pictures, +3

3 events event labeled

b) $d = 23.1$ c·yrs +1

c) $t'_3 = 60$ yrs +1

d) $t_3 = 55$ yrs +1

e) $t_{3,DP} = 65$ yrs +1

f) $t'_2 = 31.9$ yrs +1

g) $t_2 = 34.1$ yrs +1

h) $t_{2,DP} = 43.7$ yrs +1

4a) Two correct pictures +2

Dium measures $\Delta x' = 0$ +1

b) 65 yrs +1

c) $D = 30$ c·yrs +1

d) 11.5 cyrs +1

e) $t'_2 = 12.5$ yrs +1

$t_{2,Az} = 32.5$ yrs +1

$t_{2,DP} = 32.5$ yrs +1

5 a) Two correct pictures +2

b) $t'_{1,B} = 0$ +1

$t_{1,A} = 0$ +1

$t_{1,DP} = 27.7$ yrs +1

c) $t'_{2,B} = 12.5$ yrs +1

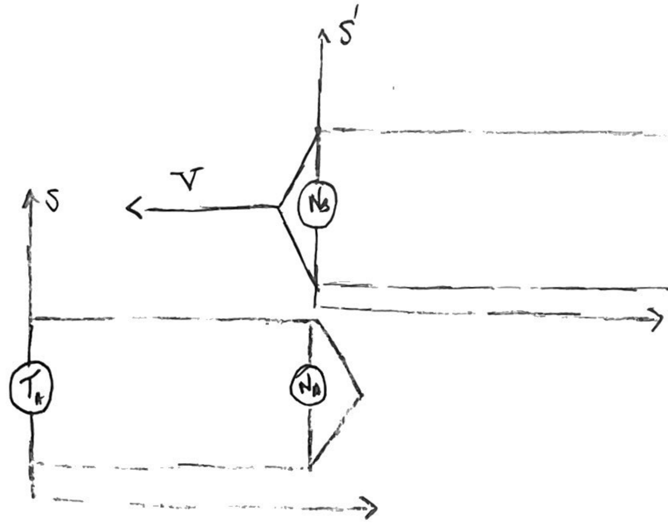
$t_{2,A} = 4.8$ yrs +1

$t_{2,DP} = 37.5$ yrs +1

X c·yrs $= X \times 3\times10^8 \frac{m}{s} \times 1\,yr \times \frac{365\,days}{1\,yr} \times \frac{24\,hrs}{1\,day} \times \frac{3600\,s}{1\,hr}$

$= 9.46\times10^{15} X$

3)



$R = \frac{270}{150} = 1.8$

IF $1.6 < R < 2.0$ +2

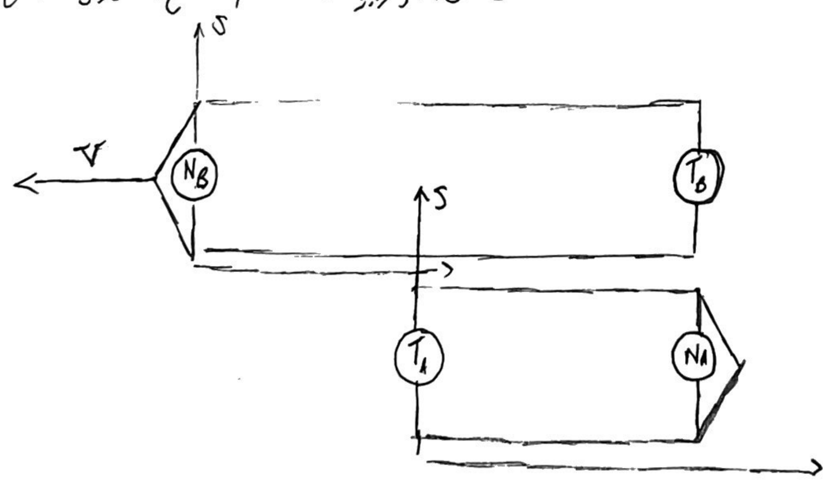
or $1.4 < R < 2.2$ +1

) $t_{N_A} = t_{T_A} = 0$
$t_{N_B}' = 0$ } +1

$t_{T_B}' = 360 \frac{m}{c}$ +1

) $\Delta t = 338\, m/c$, +1 $= 1.13 \times 10^{-6}$ s

d)



IF $1.6 < R < 2.0$ +2

or $1.4 < R < 2.2$ +1

e.) $t_{T_A} = t_{N_A} = 338\, m/c$ +1 $= 1.13 \times 10^{-6}$ s

$t_{N_B}' = 203\, m/c$ +1 $= 6.77 \times 10^{-7}$ s

$t_{T_B}' = 563\, m/c$ +1 $= 1.88 \times 10^{-6}$ s