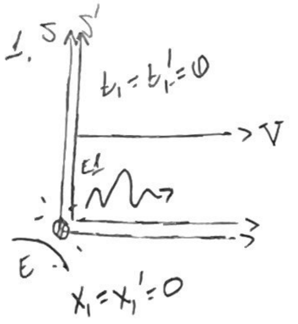
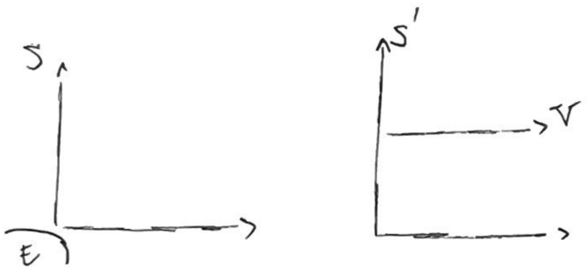
$V = \frac{5}{13}c \quad , D = 25.0\, c\cdot\text{years}$

E1: S' passes S, flash bulb explodes at origin

a)





b) Now

$x_2 = 25.0\, c\cdot\text{yrs}$

$t_2 = 25.0\ \text{yrs}$

Also

$$c = \frac{D}{\Delta t_{12}} \quad \therefore \quad \Delta t_{12} = \frac{D}{c} = \frac{25.0\, c\cdot\text{years}}{c} = 25.0\ \text{years} = t_2 - \cancel{t_1}$$

Using the Lorentz transformations...

$$x_2' = \gamma(x_2 - Vt_2) = \frac{1}{\sqrt{1-(5/13)^2}}\left(25.0\, c\cdot\text{yrs} - \left(\frac{5}{13}c\right)(25.0\ \text{yrs})\right)$$

$$= \frac{1}{12/13}(25.0\, c\cdot\text{yrs})\left(1 - \frac{5}{13}\right) = \frac{13}{12}(25.0\, c\cdot\text{yrs})\left(\frac{8}{13}\right) = \frac{2}{3}(25.0\, c\cdot\text{yrs})$$

$$x_2' = \frac{50.0}{3}\, c\cdot\text{yrs}$$

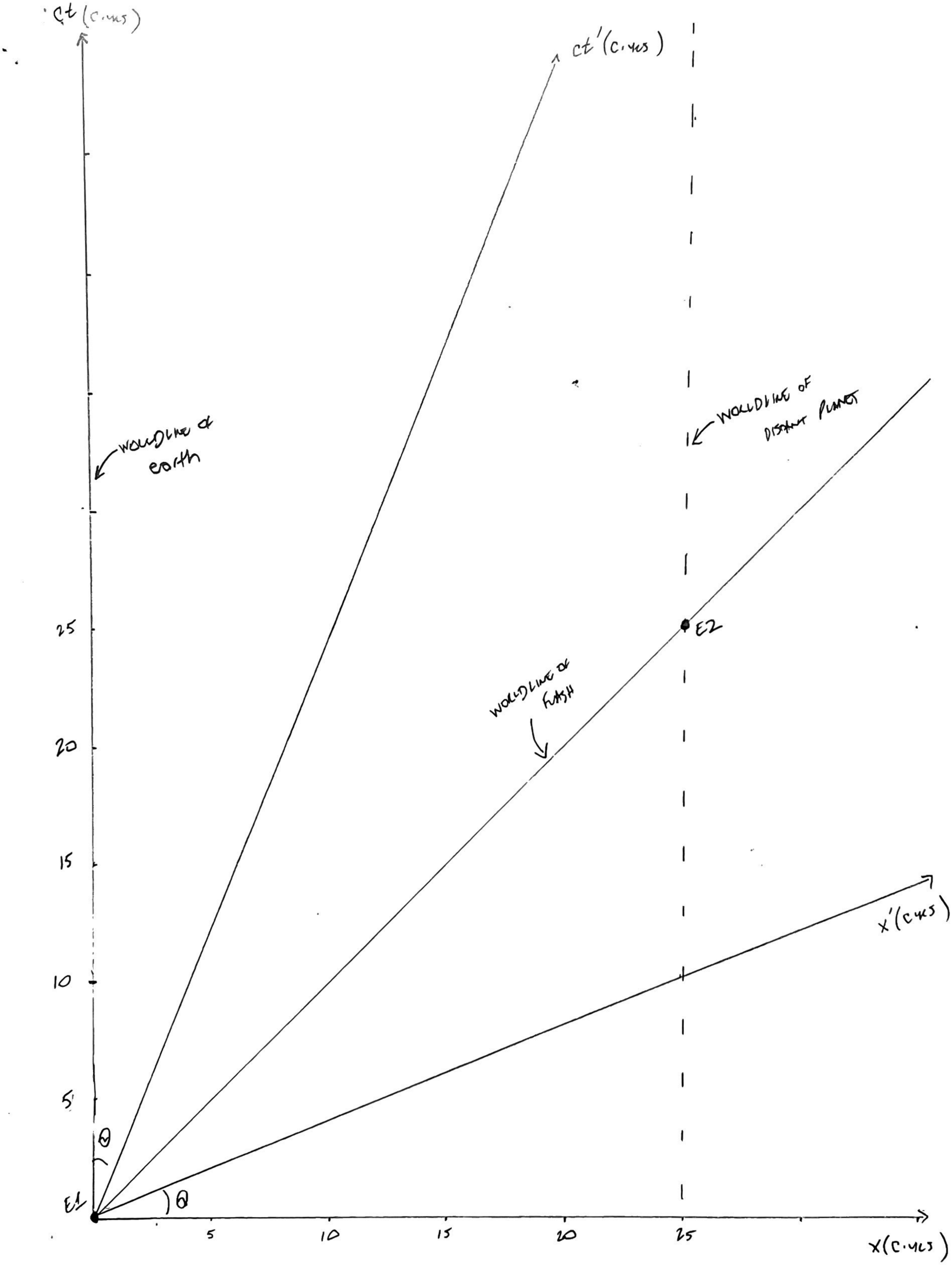
$$t_2' = \gamma\left(t_2 - \frac{Vx_2}{c^2}\right) = \frac{13}{12}\left(25.0\ \text{yrs} - \left(\frac{5}{13}c\right)\left(\frac{25.0\, c\cdot\text{yrs}}{c^2}\right)\right) = \frac{13}{12}(25.0\ \text{yrs})\left(1 - \frac{5}{13}\right) = \frac{50.0}{3}\ \text{years}$$

So, in summary...

$$\left[\begin{array}{ll} t_2 = 25.0\ \text{yrs} & t_2' = \frac{50}{3}\ \text{years} \\ x_2 = 25.0\, c\cdot\text{yrs} & x_2' = \frac{50}{3}\, c\cdot\text{yrs} \end{array}\right]$$

NOW THE ct' AXIS HAS A SLOPE OF $V/c = \frac{5}{13}$ WITH ct AXIS

SO $\tan\Theta = \frac{5}{13}$ $\therefore$ $\boxed{\Theta = 21^\circ}$



ct (c·yrs)
ct' (c·yrs)
WORLDLINE OF DISTANT PLANET
WORLDLINE OF EARTH
E2
WORLDLINE OF FLASH
x' (c·yrs)
θ
θ
E1
25
20
15
10
5
5
10
15
20
25
x (c·yrs)

7. USING THE LORENTZ TRANSFORMATIONS, THE SPATIAL : TEMPORAL SEPARATIONS BETWEEN TWO EVENTS ARE

$$\Delta x' = \gamma\left(\Delta x - \frac{V}{c}\Delta(ct)\right)$$

$$c\Delta t' = \gamma\left(c\Delta t - \frac{V}{c}\Delta x\right)$$

SO

$$\Delta s'^2 = -c^2\Delta t'^2 + \Delta x'^2$$

$$= -\gamma^2\left(c\Delta t - \frac{V}{c}\Delta x\right)^2 + \gamma^2\left(\Delta x - \frac{V}{c}c\Delta t\right)^2$$

$$= -\gamma^2\left(c^2\Delta t^2 - \cancel{2\frac{V}{c}\cdot c\Delta t\Delta x} + \frac{V^2}{c^2}\Delta x^2\right) + \gamma^2\left(\Delta x^2 - \cancel{2\frac{V}{c}\cdot c\Delta t\Delta x} + \frac{V^2}{c^2}c^2\Delta t^2\right)$$

NOW GROUPING LIKE TERMS..

$$\Delta s'^2 = \gamma^2\Delta x^2\left(1 - \frac{V^2}{c^2}\right) - \gamma^2 c^2\Delta t^2\left(1 - \frac{V^2}{c^2}\right)$$

NOW $\gamma^2\left(1 - \frac{V^2}{c^2}\right) = \frac{1}{1 - V^2/c^2}\cdot\left(1 - V^2/c^2\right) = 1$

$$= -c^2\Delta t^2 + \Delta x^2 = \Delta s^2 \checkmark$$

3. $V = 3/5$

Let S be the rest frame of B, S' be the rest frame of C.

E1: B departs C

E2: B fires at C

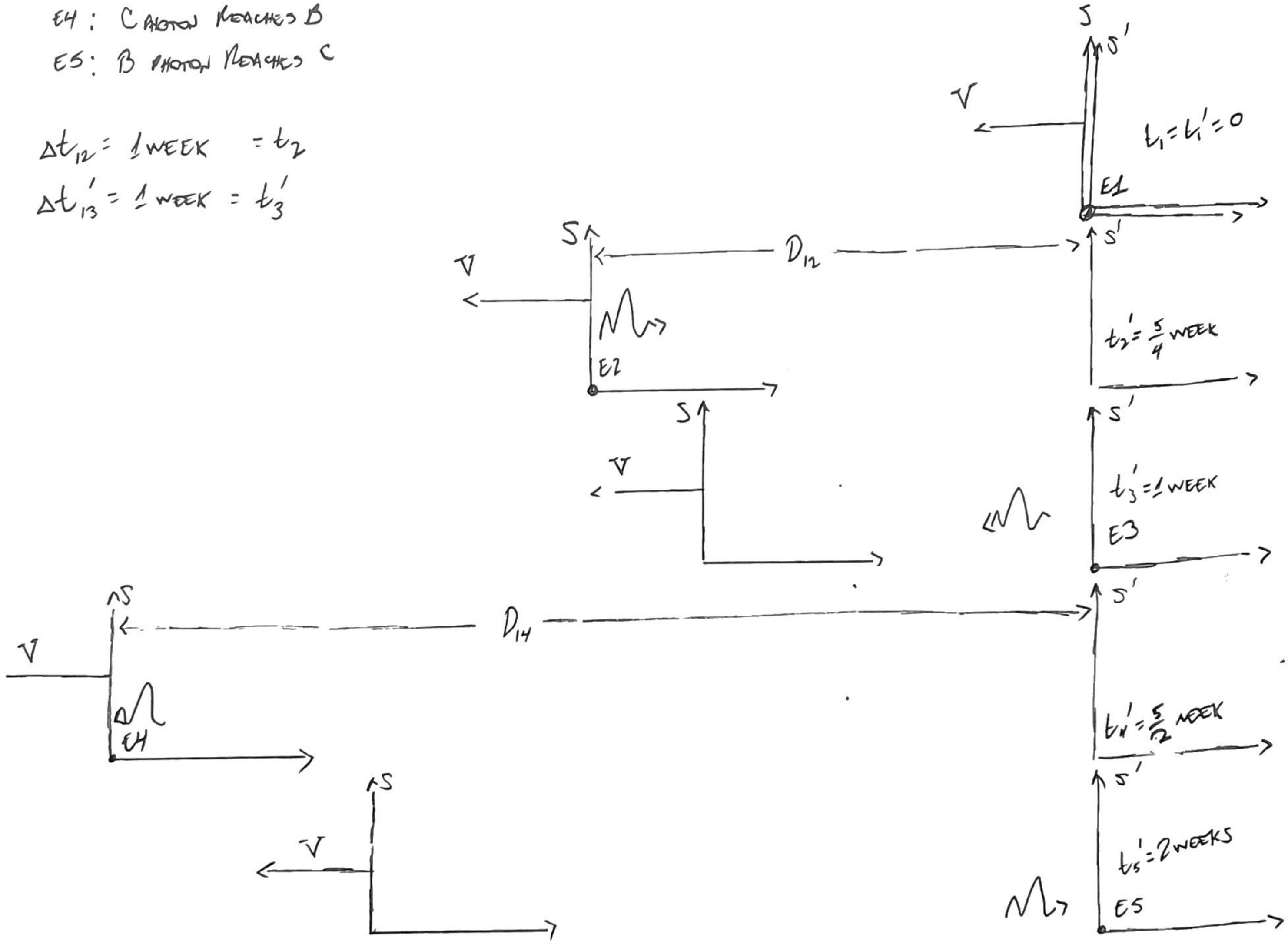
E3: C fires at B

E4: C photon reaches B

E5: B photon reaches C

$\Delta t_{12} = 1 \text{ WEEK} = t_2$

$\Delta t'_{13} = 1 \text{ WEEK} = t'_3$



Notice that the spaceship B observer, in the S frame, witnesses both E1 & E2 at the same x-coordinate

$\Delta x_{12} = 0$ so

$$\Delta t'_{12} = \gamma\left(\Delta t_{12} - \frac{V}{c^2}\cancelto{0}{\Delta x_{12}}\right) = \frac{1}{\sqrt{1 - V^2/c^2}}\Delta t_{12} = \frac{1}{\sqrt{1-(3/5)^2}}(1 \text{ WEEK}) = \frac{1}{4/5}(1 \text{ WEEK})$$

$$= \frac{5}{4} \text{ WEEK} = t_2' - \cancelto{0}{t_1'} = t_2'$$

NOW, THE DISTANCE TRAVELED BY B, WHEN THE PHOTON TORPEDO IS FIRED FROM B IS..

$$D_{12} = V\Delta t_{12}'$$

THIS IS ALSO THE DISTANCE TRAVELED BY THE PHOTONS TO GET TO OBSERVER C IN THE TIME INTERVAL $\Delta t_{25}'$

SO

$$D_{12} = V\Delta t_{12}' = c\Delta t_{25}' \quad \text{WHERE} \quad \Delta t_{25}' = t_5' - t_2'$$
$$\Delta t_{12}' = t_2' - \cancel{t_1'}_{0} = t_2'$$

SO

$$V t_2' = c(t_5' - t_2') \quad \text{SO} \quad c t_5' = (V + c)t_2'$$
$$t_5' = \left(1 + \frac{V}{c}\right)t_2' = \left(1 + \frac{3}{5}\right)t_2' = \frac{8}{5}t_2'$$
$$\therefore \quad t_5' = \frac{8}{5} \cdot \frac{5}{4} \text{ WEEKS} = 2 \text{ WEEKS}$$
$$\left[t_5' = 2 \text{ WEEKS}\right]$$

NOW, CALCULATE THE TIME FOR C'S PHOTON TO GET TO B...

THE DISTANCE TRAVELED BY B, WHEN THE PHOTON TORPEDO REACHES B IS...

$$D_{14} = V\Delta t_{14}'$$

THIS IS ALSO THE DISTANCE TRAVELED BY THE PHOTONS TO REACH B IN THE TIME INTERVAL $\Delta t_{34}'$

SO

$$D_{14} = c\Delta t_{34}' = V\Delta t_{14}' \quad \text{WHERE} \quad \Delta t_{34}' = t_4' - t_3' = t_4' - 1 \text{ WEEK}$$
$$\Delta t_{14}' = t_4' - \cancel{t_1'}_{0} = t_4'$$

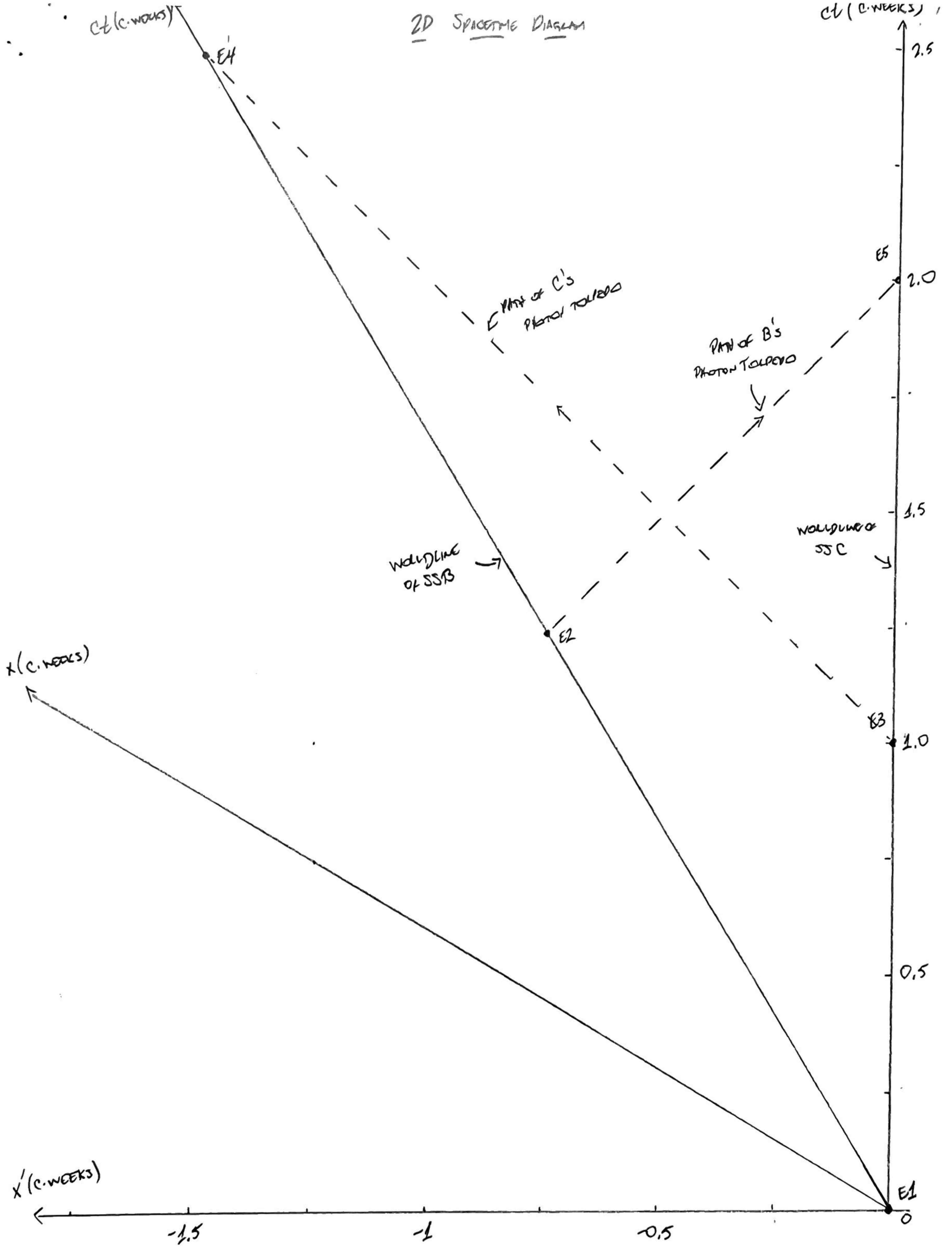
SO

$$c(t_4' - 1 \text{ WEEK}) = V t_4'$$
$$(c - V)t_4' = 1c \cdot \text{WEEK}$$
$$\left(1 - \frac{V}{c}\right)t_4' = 1 \text{ WEEK} = \left(1 - \frac{3}{5}\right)t_4' = \frac{2}{5}t_4' \quad \therefore \quad \left[t_4' = \frac{5}{2} \text{ WEEK}\right]$$

h) ACCORDING TO SPACESHIP C...

- C FIRES THEIR PHOTON TORPEDO FIRST.
- C RECEIVES THE ENEMY PHOTON TORPEDO FIRST.

2D Spacetime Diagram



4. NOTICE THAT E2 OCCURS AT THE ORIGIN OF S SO...

$x_2 = 0$

$t_2' = \frac{5}{4}$ WEEK

$x' = \gamma(x - vt)$ $\quad$ $x = \gamma(x' + vt')$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$ $\quad$ $t = \gamma\left(t' + \frac{v}{c^2}x'\right)$

SO

$x_2 = 0 = \gamma(x_2' + vt_2')$ $\quad \therefore \quad x_2' = -vt_2' = -\left(\frac{3}{5}c\right)\left(\frac{5}{4}\text{ WEEK}\right) = -\frac{3}{4}c\text{ WEEKS}$

$$t_2 = \gamma\left(t_2' + \frac{v}{c^2}x_2'\right) = \frac{1}{\sqrt{1-(3/5)^2}}\left(\frac{5}{4}\text{ WEEK} + \frac{\left(\frac{3}{5}c\right)\left(-\frac{3}{4}c\text{ WEEKS}\right)}{c^2}\right)$$

$$= \frac{1}{4/5}\left(\frac{5}{4}\text{ WEEKS} - \frac{9}{20}\text{ WEEKS}\right) = \frac{5}{4}\left(\frac{25-9}{20}\right)\text{WEEKS}$$

$$= \frac{5}{4}\cdot\frac{16}{20_4} = 1\text{ WEEK}$$

SO, IN SUMMARY..

$$\left[\begin{array}{ll} t_2' = \frac{5}{4}\text{ WEEKS} & t_2 = 1\text{ WEEK} \\ x_2' = -\frac{3}{4}c\text{ WEEKS} & x_2 = 0 \end{array}\right]$$

FOR EVENT 3...

$t_3' = 1$ WEEK

$x_3' = 0$

SO $\quad x_3 = \gamma(x_3' + vt_3') = \frac{1}{\sqrt{1-(3/5)^2}}\left(0 + \frac{3}{5}\cdot 1\text{ WEEK}\right)$

$$= \frac{1}{4/5}\cdot\frac{3}{5}\text{ WEEK} = \frac{3}{4}\text{ WEEK}$$

$$t_3 = \gamma\left(t_3' + \frac{v}{c^2}x_3'\right) = \frac{1}{\sqrt{1-(3/5)^2}}\left(1\text{ WEEK} + 0\right) = \frac{1}{4/5}\cdot 1\text{ WEEK}$$

$$t_3 = \frac{5}{4}\text{ WEEK}$$

SO, IN SUMMARY..

$$\left[\begin{array}{ll} t_3' = 1\text{ WEEK} & t_3 = \frac{5}{4}\text{ WEEK} \\ x_3' = 0 & x_3 = \frac{3}{4}\text{ WEEK} \end{array}\right]$$

∴ FOR EVENT 4...

$t_4' = \frac{5}{2}$ WEEK

$x_4 = 0$

so $x_4 = 0 = \gamma(x_4' + v t_4')$

$\therefore x_4' = -v t_4' = -\frac{3}{5}c \cdot \frac{5}{2}$ WEEK $= -\frac{3}{2}c \cdot$WEEK

$$t_4 = \gamma\left(t_4' + \frac{v}{c^2} x_4'\right)$$

$$= \frac{1}{\sqrt{1-(3/5)^2}}\left(\frac{5}{2}\text{ WEEK} + \frac{\left(\frac{3}{5}c\right)\left(-\frac{3}{2}c\cdot\text{WEEK}\right)}{c^2}\right)$$

$$= \frac{5}{4}\left(\frac{5}{2}\text{ WEEK} - \frac{9}{10}\text{ WEEKS}\right) = \frac{5}{4}\left(\frac{25-9}{10}\right)\text{WEEKS} = \frac{\cancel{5}}{\cancel{4}_1}\cdot\frac{\cancel{16}^4}{\cancel{10}_2}\text{ WEEKS}$$

$t_4 = 2$ WEEK

SO, IN SUMMARY..

$$\left[\begin{array}{ll} t_4' = \frac{5}{2}\text{ WEEKS} & t_4 = 2\text{ WEEKS} \\ x_4' = -\frac{3}{2}c\cdot\text{WEEKS} & x_4 = 0 \end{array}\right]$$

FOR EVENT 5...

$x_5' = 0$

$t_5' = 2$ WEEKS

so $x_5 = \gamma(x_5' + v t_5') = \frac{1}{\sqrt{1-(3/5)^2}}\left(0 + \frac{3}{5}c \cdot 2\text{ WEEKS}\right)$

$$= \frac{5}{4}\cdot\frac{6}{5}\,c\cdot\text{WEEKS} = \frac{3}{2}\,c\cdot\text{WEEKS}$$

$$t_5 = \gamma\left(t_5' + \frac{v x_5'}{c^2}\right) = \frac{5}{4}\cdot 2\text{ WEEKS} = \frac{5}{2}\text{ WEEKS}$$

SO IN SUMMARY...

$$\left[\begin{array}{ll} t_5' = 2\text{ WEEKS} & t_5 = \frac{5}{2}\text{ WEEK} \\ x_5' = 0 & x_5 = \frac{3}{2}c\cdot\text{WEEKS} \end{array}\right]$$