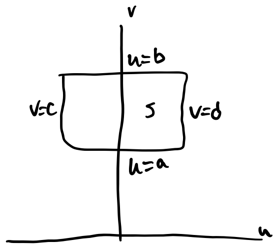
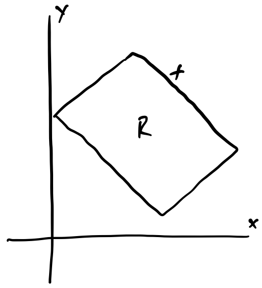
3.) $y=2x-3$, $y=2x+3$, $y=3-x$, $y=5-x$





$x+y=3$ $\quad x+y=5$ $\qquad 2x-y=3$ $\quad 2x-y=-3$

$3 \le u \le 5$ $\qquad -3 \le v \le 3$

$u=x+y$ $\qquad v=2x-y$

$u+v=x+y+2x-y$

$u+v=3x$

$\frac{u+v}{3}=x$

$x=\frac{u+v}{3}$ $\quad 3 \le u \le 5$

$y=\frac{2u-v}{3}$ $\quad -3 \le v \le 3$

$v=2\left(\frac{u+v}{3}\right)-y$

$v=\frac{2u+2v}{3}-y$

$-y=v-\frac{2u+2v}{3}$

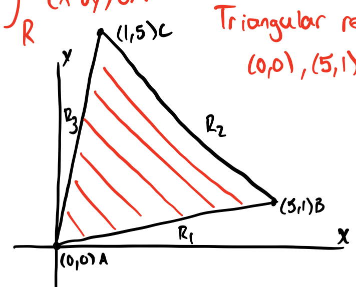
$y=-v+\frac{2u+2v}{3}$

$y=\frac{2u+2v}{3}-v$

$y=\frac{2u+2v-3v}{3}$

$y=\frac{2u-v}{3}$

4.) $\iint_R (x-6y)\,dA$ $\quad$ Triangular region $(0,0),(5,1),(1,5)$ $\quad x=5u+v$, $y=u+5v$



$x=5u+v$ $\qquad \frac{\partial x}{\partial u}=5$ $\quad \frac{\partial x}{\partial v}=1$

$y=u+5v$ $\qquad \frac{\partial y}{\partial u}=1$ $\quad \frac{\partial y}{\partial v}=5$

$\begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix}$ $\quad 25-1=24$

R_1 $\quad \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ $\quad$ ① $(0,0)$ ② $(5,1)$

$\frac{x-0}{5-0}=\frac{y-0}{1-0}$

$\frac{x}{5}=y$

$x=5y$

$5u+v=5(u+5v)$

$5u+v=5u+25v$

$0u=24v$

$u=0$

R_2 $\quad \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ $\quad$ ① $(5,1)$ ② $(1,5)$

$\frac{x-5}{1-5}=\frac{y-1}{5-1}$

$\frac{x-5}{-4}=\frac{y-1}{4}$

$-x+5=y-1$

$-x+6=y$

$y=6-x$

$y+x=6$

$(u+5v)+(5u+v)=6$

$6u+6v=6$

$u+v=1$

R_3 $\quad \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ $\quad$ ① $(1,5)$ ② $(0,0)$

$\frac{x-1}{0-1}=\frac{y-5}{0-5}$

$\frac{x-1}{-1}=\frac{y-5}{-5}$

$-x+1=\frac{y-5}{-5}$

$5x-5=y-5$

$5x=y$

$y=5x$

$u+5v=5(5u+v)$

$u+5v=25u+5v$

$0v=24u$

$v=0$

$0 \le u \le 1$ $\quad 24\int_0^1\int_0^{1-u} 5u+v-6(u+5v)\,dv\,du$

$0 \le v \le 1-u$ $\quad 24\int_0^1\int_0^{1-u} 5u+v-6u-30v\,dv\,du$

$24\int_0^1\int_0^{1-u} -u-29v\,dv\,du$ $\qquad (-u+1)(-u+1)$

$\qquad u^2-2u+1$

$24\int_0^1 -uv-\frac{29v^2}{2}\Big|_0^{1-u}\,du$

$24\int_0^1 -u(1-u)-\frac{29}{2}(u^2-2u+1)\,du$

$24\int_0^1 -u+u^2-\frac{29}{2}(u^2-2u+1)\,du$

$24\left[-\frac{u^2}{2}+\frac{u^3}{3}-\frac{29}{2}\left(\frac{u^3}{3}-u^2+u\right)\right]_0^1$

$24\left[-\frac{1}{2}+\frac{1}{3}-\frac{29}{2}\left(\frac{1}{3}-1+1\right)\right]$

$24[-5]$

-120 $\qquad -120$

5.) $\iint_R 7x^2\,dA$ $\quad$ Bounded by $16x^2+9y^2=144$ $\quad x=3u$, $y=4v$

$\frac{\partial x}{\partial u}=3$ $\quad \frac{\partial x}{\partial v}=0$

$\frac{\partial y}{\partial u}=0$ $\quad \frac{\partial y}{\partial v}=4$

$\begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix}$ $\quad 12-0=12$

$16(3u)^2+9(4v)^2=144$ $\qquad -1 \le v \le 1$ $\qquad u^2=1-v^2$ $\qquad x=3u$

$9(16)u^2+9(16)v^2=144$ $\qquad -\sqrt{1-v^2} \le u \le \sqrt{1-v^2}$ $\qquad u=\pm\sqrt{1-v^2}$ $\qquad y=4v$

$144u^2+144v^2=144$

$u^2+v^2=1$ $\qquad \iint_R 7x^2$

$189\int_0^{2\pi} \cos^2\theta\,d\theta$ $\qquad \frac{1}{2}+\frac{1}{2}\cos 2\theta$

$189\int_0^{2\pi} \frac{1}{2}+\frac{1}{2}\cos 2\theta\,d\theta$

$189\int_0^{2\pi} \frac{1}{2}\,d\theta + 189\int_0^{2\pi}\frac{1}{2}\cos 2\theta\,d\theta$ $\qquad u=2\theta$

$\qquad du=2\,d\theta$

$\frac{189}{2}[\theta]_0^{2\pi} + \frac{189}{2}\int_0^{2\pi}\cos 2\theta\,d\theta$ $\qquad \frac{1}{2}du=d\theta$

$\frac{189}{2}[2\pi] + \frac{189}{2}\int_0^{2\pi}\cos u\,du$ $\qquad \sin u=\sin 2\theta$

189π $\qquad \frac{189}{2}[\sin u]_0^{2\pi}$ $\qquad \sin 2\theta\,\big|_0^{2\pi}$

$\qquad \sin 4\pi-\sin 0$

$\qquad 0-0$

$189\pi+0$

$7\iint_R 9u^2 \, dudv$ $\quad u=x=r\cos\theta$

$0 \le r \le 1$
$0 \le \theta \le 2\pi$

189π

$63\int_{-1}^{1}\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} u^2 (12) \, dvdu$

$756\int_0^{2\pi}\int_0^1 r^2\cos^2\theta \, r \, dr d\theta$

$756\int_0^{2\pi}\int_0^1 r^3\cos^2\theta \, dr d\theta$

$756\int_0^{2\pi} \frac{r^4}{4}\cos^2\theta \Big|_0^1 \, d\theta$

$756\int_0^{2\pi} \frac{1}{4}\cos^2\theta \, d\theta$

$189\int_0^{2\pi} \cos^2\theta \, d\theta$

6.) $\iint_R 10\frac{x-4y}{7x-y} \, dA$

R is Enclosed by

$x-4y=0 \quad x-4y=5$
$7x-y=7 \quad 7x-y=8$

$u=x-4y$
$v=7x-y$
$0 \le u \le 5$
$7 \le v \le 8$

$$\begin{vmatrix} 1 & -4 \\ 7 & -1 \end{vmatrix}$$

$-1-(-28)$
$-1+28$
27

$\star = 27$
$J = \frac{1}{27}$

$\int_7^8\int_0^5 10\frac{u}{v} \, dudv$

$10\int_7^8\int_0^5 \frac{u}{v} \, dudv$

$10\int_7^8 \frac{1}{v}\left[\frac{u^2}{2}\right]_0^5 dv$

$\frac{25}{2}\cdot\frac{10}{1}\int_7^8 \frac{1}{v} \, dv$

$125\left[\ln v\right]_7^8$

$125[\ln 8 - \ln 7]$

$\frac{125}{27}[\ln 8 - \ln 7]$

$\frac{125}{27}[\ln 8 - \ln 7]$

7.) $\iint_R 7(x+y)e^{x^2-y^2} \, dA$

$x-y=0 \quad x+y=0$
$x-y=5 \quad x+y=6$

$x-y(x+y)$ $\quad x^2+xy-xy-y^2$
x^2-y^2

$u=x-y \quad v=x+y$
$0 \le u \le 5 \quad 0 \le v \le 6$

$\frac{\partial u}{\partial x}=1 \quad \frac{\partial u}{\partial y}=-1$
$\frac{\partial v}{\partial x}=1 \quad \frac{\partial v}{\partial y}=1$

$$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$1-(-1(1))$
$1+1=2$

$\iint 7ve^{uv}\left(\frac{1}{2}\right) dudv$

$x=uv$
$dx = v\,du$
$\frac{1}{v}dx = du$

$\frac{7}{2}\iint ve^{uv} \, dudv$

$\frac{7}{2}\int_0^6\int_0^5 ve^{uv} \, dudv$

$\frac{7}{2}\int_0^6 \frac{ve^{uv}}{v}\Big|_0^5 dv$

$\frac{7}{2}\int_0^6 e^{5v}-1 \, dv$

$\frac{7}{10}(e^{30}-31)$

$\frac{7}{2}\left[\frac{1}{5}e^{5v}-v\right]_0^6$

$\frac{7}{2}\left[\frac{1}{5}e^{30}-6-\left(\frac{1}{5}e^0-0\right)\right]$ $\quad -\frac{30}{5}-\frac{1}{5}$

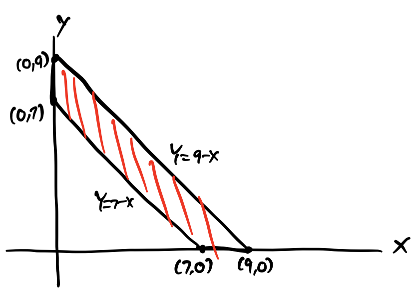
$\frac{7}{2}\left[\frac{1}{5}e^{30}-6-\frac{1}{5}\right]$

$\frac{7}{2}\left[\frac{e^{30}}{5}-\frac{31}{5}\right]$

$\frac{7}{10}[e^{30}-31]$

8.) $\iint_R 5\cos\left(7\left(\frac{y-x}{y+x}\right)\right) dA$

$(7,0),(9,0),(0,9),(0,7)$

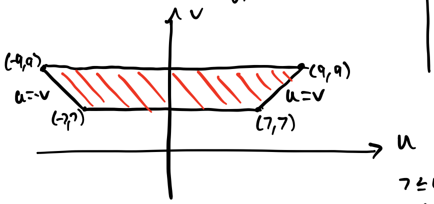


$u=y+x$
$v=y-x$

$\frac{\partial u}{\partial x}=1 \quad \frac{\partial u}{\partial y}=1$
$\frac{\partial v}{\partial x}=-1 \quad \frac{\partial v}{\partial x}=1$

∂x

(x, y)	(u, v)
(7, 0)	(-7, 7)
(9, 0)	(-9, 9)
(0, 9)	(9, 9)
(0, 7)	(7, 7)



$\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$ $\quad 1+1=2$

$\frac{1}{2} = J$

$7 \le u \le 9$

$-u \le v \le u$

$w = 7\frac{v}{u}$

$dw = \frac{7}{u}\,dv$

$u\,dw = 7\,dv$

$\frac{u}{7}dw = dv$

$$\int_7^9 \int_{-u}^{u} 5\cos\left(7\frac{v}{u}\right) dv\,du$$

$$\frac{5}{2}\int_7^9 \int_{-u}^{u} \cos 7\frac{v}{u}\, dv\,du$$

$$\frac{5}{2}\int_7^9 \int_{-u}^{u} \frac{u}{7}\cos w\, dw\,du$$

$$\frac{5}{2}\int_7^9 \frac{u}{7}\sin\left(7\frac{v}{u}\right)\Big|_{-u}^{u} du$$

$$\frac{5}{2}\int_7^9 \frac{u}{7}\sin(7) - \frac{u}{7}\sin(-7)$$

$$\frac{5}{2}\int_7^9 \frac{u}{7}(2\sin(7))\, du$$

$$5\sin(7)\int_7^9 \frac{u}{7}\, du$$

$$5\sin(7)\left[\frac{u^2}{14}\right]_7^9$$

$$5\sin(7)\left[\frac{81}{14} - \frac{49}{14}\right]$$

$$5\sin(7)\left[\frac{32}{14}\right]$$

$$\frac{80}{7}\sin(7)$$

$\frac{80}{7}\sin(7)$