2.1 1.5 # 24

2.1 # (19, 20, 27, 28, 37, 38) $y' + ay = b$ $y = ce^{-at} + \frac{b}{a}$

2.1.19 $y' + y = 2$

$\frac{dy}{dt} = 2 - y$

$\int \frac{dy}{2-y} = \int dt$

$-\ln(2-y) = t + C$

$\ln(2-y) = -t - C$

$(2-y) = ke^{-t}$

$-y = ke^{-t} - 2$

$y = 2 + ke^{-t}$

$y = 2 + ke^{-t}$

2.1.20 $y' - 0.08y = 100$

$y' = 100 + 0.08y$

$\frac{dy}{100 + 0.08y} = dt$

$u = 100 + 0.08y$

$du = 0.08\,dy$

$\frac{1}{0.08}du = dy$

$\frac{1}{0.08}\int \frac{1}{u}du = \int dt$

$\frac{1}{0.08}\ln|100 + 0.08y| = t + C$

$\ln|100 + 0.08y| = 0.08t + C$

$100 + 0.08y = ke^{0.08t}$

$0.08y = ke^{0.08t} - 100$

$y = \frac{ke^{0.08t} - 100}{0.08}$

$y = ke^{0.08t} - \frac{100}{0.08}$

$y = \frac{ke^{0.08t} - 100}{0.08}$

2.1.27 $y'' + 4y = 0$ $y_1 = \sin 2t$ $y_2 = \cos 2t$

$y'' = -4\sin 2t$ $y'' + 4y = 0$ $-4\sin 2t + 4\sin 2t = 0$

$y'' = -4\cos 2t$ $y'' + 4y = 0$ $-4\cos 2t + 4\cos t = 0$

y_1 ✓

y_2 ✓

any constant would be a solution

y_1 ✓
y_2 ✓
Any constant

2.1.28 $2y'' + y' - y = 0$ $y_1 = e^{t/2}$ $y_2 = e^{-t}$

$y_1 \quad e^{t/2}$

$y'' = \frac{1}{4}e^{t/2}$

$2y'' + y' - y = 0$

$\frac{1}{2}e^{t/2} + \frac{1}{2}e^{t/2} - e^{t/2} = 0$

$e^{t/2} - e^{t/2} = 0$

y_1 ✓

$y_2 = e^{-t}$

$y' = -e^{-t}$

$y'' = e^{-t}$

$2y'' + y' - y = 0$

$2e^{-t} - e^{-t} - e^{-t} = 0$

$2e^{-t} - 2e^{-t} = 0$

y_2 ✓

y_1 ✓
y_2 ✓
Any constant is a solution

2.1.37 $y' - y = e^t$

Homogeneous Solution

$y = y_h + y_p$

$y' - y = 0$

$y' = y$

$\frac{dy}{dt} = y$

$\frac{dy}{y} = dt$

$\ln y = t + c$

$y_h = Ke^t$

$y_p = te^t$

$y' = te^t + e^t$

$y = te^t$

$y_p = te^t$

$y' - y = e^t$

$te^t + e^t - te^t = e^t$

$e^t = e^t$

Homogeneous Solutions

$y = Ke^t + te^t$

2.1.38 $y'' - a^2 y = 0$

$y = e^t$

$y' = e^t$

$y'' = e^t$

$a = 1$

$e^t - a^2 e^t = 0$

$0 = 0$

Ke^t

Ke^t is a solution