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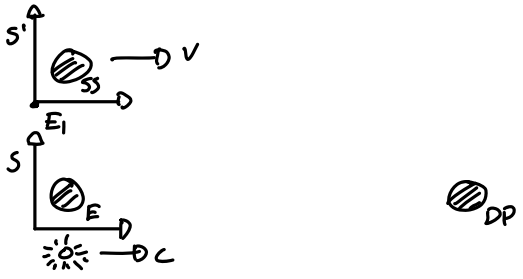
Dr. Middleton
PHYS 230 HW 10

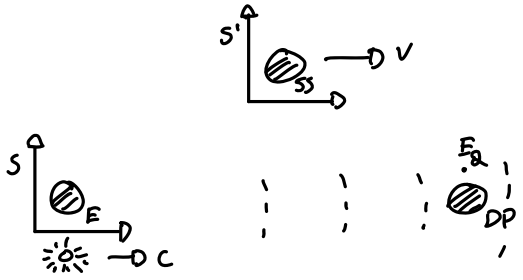
Problem 1

a.) $v = (3/13)c$

E_1



E_2



b.) $x_2 = 25$ c·yrs $\quad x_2' =$

$t_2 = 25$ yrs $\quad t_2' =$

$$\Delta t = \frac{D}{V} = \frac{25 \text{ c·yrs}}{c} = 25 \text{ yrs}$$

$x' = \gamma(x - vt) \quad t' = \gamma(t - vx/c^2)$

$\gamma = (13/12)$

$x = 25$ c·yrs

$v = (5/13)c$

$t = 25$ yrs

$x' = (13/12)(25 \text{ c·yrs} - (5/13)c(25)\text{yrs})$

$x' = \frac{50}{3}$ c·yrs

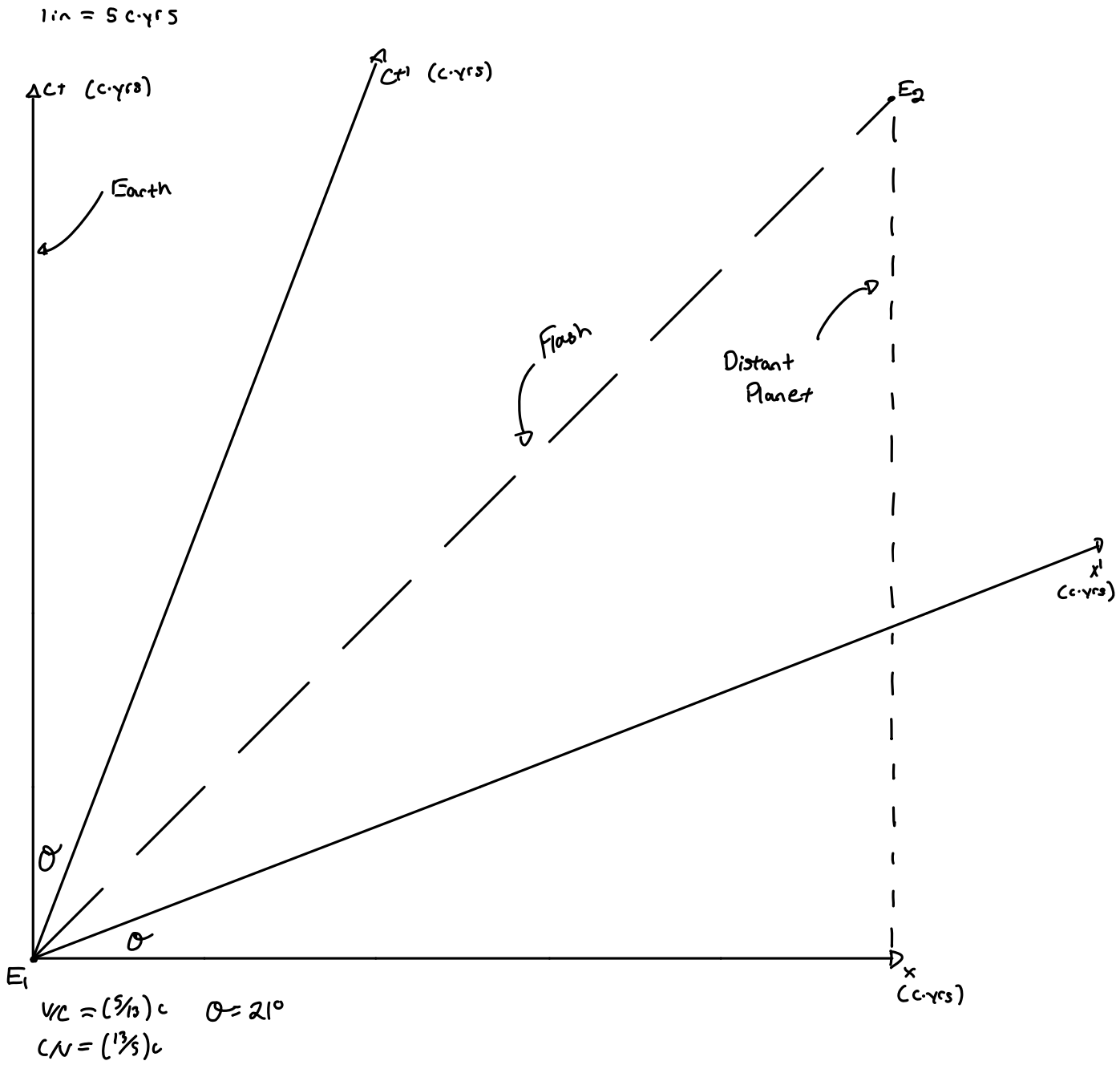
$t' = (13/12)(25 \text{ yrs} - (5/13)c(25)\text{c·yrs})$

$t' = \frac{50}{3}$ c·yrs

$x_2 = 25$ c·yrs $\quad x_2' = 50/3$ c·yrs

$t_2 = 25$ yrs $\quad t_2' = 50/3$ yrs

C.)



$v/c = (5/13)c$ $\theta = 21°$

$c/v = (13/5)c$

Problem 2

$x' = \gamma(x - v/c\, ct)$

$ct' = \gamma(ct - v/c\, x)$

$\gamma^{-1} ct' = ct - v/c\, x$ $\qquad$ $x'\gamma^{-1} = x - v/c\, ct$

$\gamma^{-1} ct' + v/c\, \Delta x = c\Delta t$ $\qquad$ $\Delta x'\gamma^{-1} + v/c\, c\Delta t = \Delta x$

$c\Delta t = \gamma^{-1} c\Delta t' + v/c\, \Delta x$

$\Delta x = \Delta x'\gamma^{-1} + v/c\, c\Delta t$

$\Delta s'^2 = \Delta s^2$

$\Delta s^2 = -c^2\Delta t^2 + \Delta x^2$

$c^2\Delta t^2$: $(\gamma^{-1} c\Delta t' + (v/c)\Delta x)(\gamma^{-1} c\Delta t' + (v/c)\Delta x)$

$\gamma^{-2} c^2 \Delta t'^2 + 2\gamma^{-1} c\Delta t' (v/c)\Delta x + (v/c)^2 \Delta x^2$

Δx^2 : $(\Delta x'\gamma^{-1} + (v/c) c\Delta t)(\Delta x'\gamma^{-1} + (v/c) c\Delta t)$

$\Delta x'^2 \gamma^{-2} + 2\Delta x'\gamma^{-1}(v/c) c\Delta t + (v/c)^2 c^2 \Delta t^2$

$-(\gamma^{-2} c^2\Delta t'^2 + 2\gamma^{-1} c\Delta t'(v/c)\Delta x + (v/c)^2\Delta x^2) + (\Delta x'^2\gamma^{-2} + 2\Delta x'\gamma^{-1}(v/c) c\Delta t + (v/c)^2 c^2\Delta t^2) = \Delta s^2$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\gamma^{-1}(v/c)(-c\Delta t'\Delta x + c\Delta t\Delta x') + (v/c)^2(-\Delta x^2 + c^2\Delta t^2) = -c^2\Delta t^2 + \Delta x^2$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\gamma^{-1}(v/c)(-c\Delta t'\Delta x + c\Delta t\Delta x') = -c^2\Delta t^2 + \Delta x^2 - (v/c)^2(-\Delta x^2 + c^2\Delta t^2) \qquad +(v/c)^2(\Delta x^2) - (v/c)^2 c^2\Delta t^2$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\gamma^{-1}(v/c)(-c\Delta t'\Delta x + c\Delta t\Delta x') = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\gamma^{-1}(v/c)\left(-(\gamma(c\Delta t - (v/c)\Delta x))\Delta x + c\Delta t(\gamma(\Delta x - (v/c)c\Delta t))\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + \left(-2\gamma^{-1}(v/c)(\gamma(c\Delta t - (v/c)\Delta x))\Delta x + 2\gamma^{-1}(v/c)(c\Delta t)(\gamma(\Delta x - (v/c)c\Delta t))\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + \left(-2(v/c)(c\Delta t - (v/c)\Delta x))\Delta x + 2(v/c)(c\Delta t)(\Delta x - (v/c)c\Delta t))\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\left(-(v/c)(c\Delta t - (v/c)\Delta x))\Delta x + (v/c)(c\Delta t)(\Delta x - (v/c)c\Delta t))\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\left(-(v/c)(c\Delta t\Delta x - (v/c)\Delta x^2) + (v/c)(c\Delta t\Delta x - (v/c)c^2\Delta t^2))\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\left((-c\Delta t\Delta x(v/c) + (v/c)^2\Delta x^2) + ((v/c)c\Delta t\Delta x - (v/c)^2 c^2\Delta t^2)\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2\left((v/c)^2\Delta x^2 - (v/c)^2 c^2\Delta t^2\right) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) + 2(v/c)^2(-c^2\Delta t^2 + \Delta x^2) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) = -c^2\Delta t^2 + \Delta x^2\left(1 + (v/c)^2\right) - 2(v/c)^2(-c^2\Delta t^2 + \Delta x^2)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) = -c^2\Delta t^2 + \Delta x^2 - c^2\Delta t^2 + \Delta x^2(v/c)^2 + 2(v/c)^2 c^2\Delta t^2 - 2(v/c)^2\Delta x^2$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) = -c^2\Delta t^2 + \Delta x^2 + (v/c)^2(-c^2\Delta t^2 + \Delta x^2 + 2c^2\Delta t^2 - 2\Delta x^2)$

$\gamma^{-2}(-c\Delta t'^2 + \Delta x'^2) = -c^2\Delta t^2 + \Delta x^2 + (v/c)^2(c^2\Delta t^2 - \Delta x^2)$

$\gamma = \dfrac{1}{\sqrt{1 - (v/c)^2}}$

$\gamma^{-1} = \sqrt{1 - (v/c)^2}$

$\gamma^{-2} = (1 - (v/c)^2)$

$(1 - (v/c)^2)(-c\Delta t'^2 + \Delta x'^2) = -c^2\Delta t^2 + \Delta x^2(1 - (v/c)^2) \qquad -c^2\Delta t^2 + \Delta x^2(1 - (v/c)^2)$

$-c\Delta t'^2 + \Delta x'^2 = -c^2\Delta t^2 + \Delta x^2$

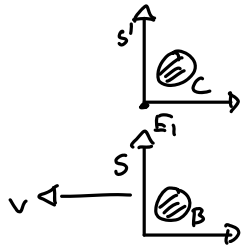
$\Delta s'^2 = \Delta s^2$

$\Delta s'^2 = \Delta s^2$

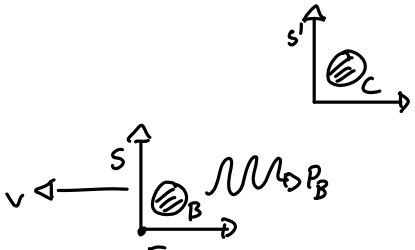
Problem 3

a.) $v=(3/5)c \quad \gamma=(5/4)$

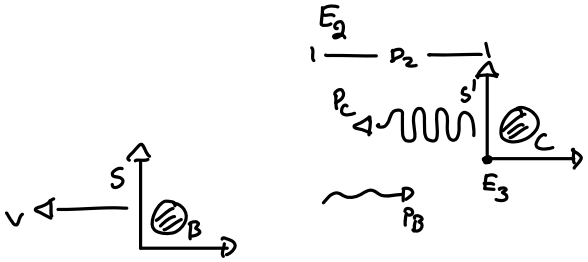
E_1
B departs C $\quad \Delta x'=0$
$t_1=0$
$t_1'=0$



E_2
B fires at C $\quad \Delta x=0 \quad t'=\gamma t$
$t_2=1\,wk \quad D_2=v(t_2'-t_1')$
$t_2'=3/4\,wk \quad D_2=c(t_3'-t_2')$



E_3
C fires at B $\quad \Delta x'=0 \quad t=\gamma(t')$
$t_3=3/4\,wk \quad D_3=v(t_3'-t_2')$
$t_3'=1\,wk$



E_4
C's photon reaches B $\quad \Delta x=0 \quad t'=\gamma(t)$
$t_4=2\,wk \quad D_4=v(t_4'-t_1')$
$t_4'=3/2\,wk \quad D_4=c(t_4'-t_3')$

E_5
$\Delta x'=0 \quad t=\gamma(t')$
B's photon reaches C $\quad D_5=v(t_5'-t_1')$
$t_5=5/2\,wk$
$t_5'=2\,wk$

b.)

E_2
$t'=\gamma(t) \quad t_2'=3/4\,wk$
$\gamma=(5/4)$
$t=1\,wk$

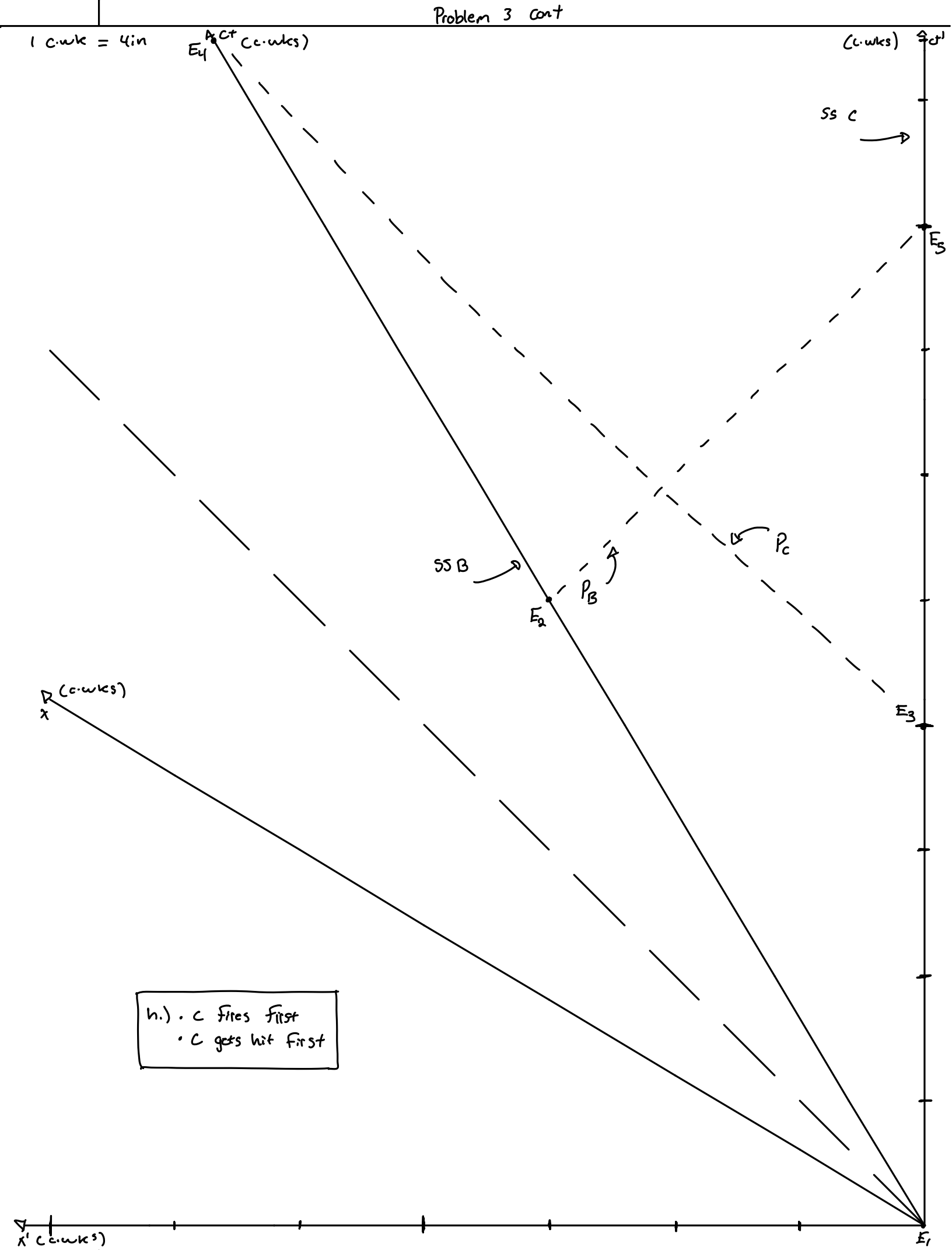
E_3
$t_3'=1\,wk$

E_4
$v(t_4')=c(t_4'-t_2') \quad t_4'=3/2\,wk$
$t_4'(v-c)=-ct_3'$
$t_4'=\dfrac{ct_3'}{c-v}=\dfrac{1\,wk}{(3/5)}=\dfrac{5}{2}\,wk$
$c-v=(2/5)$

E_5
$v(t_2')=c(t_5'-t_2') \quad t_5'=\dfrac{(5/4)wk\,(3/5)c}{c}=2\,wk$
$ct_5'=t_2'(v+c)$
$t_5'=\dfrac{t_2'(v+c)}{c}$
$v+c=8/5$
$t_2'=5/4\,wk$

$t_1'=0\,wk \quad t_4'=5/2\,wk$
$t_2'=3/4\,wk \quad t_5'=2\,wk$
$t_3'=1\,wk$

Problem 3 cont



Problem 4

E_1	E_2	E_3	E_4	E_5
$t_1 = t_1' = 0$	$t_2 = 1\,wk$	$t_3 = 3/4\,wk$	$t_4 = 2\,wk$	$t_5 = 5/2\,wk$
$x_1 = x_1' = 0$	$t_2' = 5/4\,wk$	$t_3' = 1\,wk$	$t_4' = 3/2\,wk$	$t_5' = 2\,wk$

E_2 $\Delta x = 0$ $\quad \Delta x' = \gamma(-v\Delta t)$

$x_2 = 0$ $\quad x_2' = (5/4)(-(3/5)c(1wk))$

$x_2' = -3/4\ c\cdot wk$

$x_2 = 0\ c\cdot wk$

$x_2' = -3/4\ c\cdot wk$

E_3 $\Delta x' = 0$ $\quad \Delta x = \gamma(v\Delta t')$

$x_3' = 0\ c\cdot wk$ $\quad x_3 = (5/4)(3/5)c(1wk)$

$x_3 = (3/4)\ c\cdot wk$

$x_3' = 0\ c\cdot wk$

$x_3 = (3/4)c\cdot wk$

E_4 $\Delta x = 0$ $\quad \Delta x' = \gamma(-v\Delta t)$

$x_4 = 0$ $\quad x_4' = (5/4)(-3/5)c(2wk)$

$x_4' = -(3/2)\ c\cdot wk$

$x_4 = 0\ c\cdot wk$

$x_4' = -(3/2)c\cdot wk$

E_5 $\Delta x' = 0$ $\quad \Delta x = \gamma(v\Delta t)$

$x_5' = 0$ $\quad x_5 = (5/4)(3/5)c(2)wk$

$x_5 = (3/2)c\cdot wk$

$x_5 = (3/2)\ c\cdot wk$

$x_5' = 0\ c\cdot wk$

$t_1 = 0\,wk$	$t_1' = 0\,wk$	$x_1 = 0\ c\cdot wk$	$x_1' = 0\ c\cdot wk$
$t_2 = 1\,wk$	$t_2' = 5/4\,wk$	$x_2 = 0\ c\cdot wk$	$x_2' = -3/4\ c\cdot wk$
$t_3 = 5/4\,wk$	$t_3' = 1\,wk$	$x_3 = 3/4\ c\cdot wk$	$x_3' = 0\ c\cdot wk$
$t_4 = 2\,wk$	$t_4' = 3/2\,wk$	$x_4 = 0\ c\cdot wk$	$x_4' = -3/2\ c\cdot wk$
$t_5 = 5/2\,wk$	$t_5' = 2\,wk$	$x_5 = 3/2\ c\cdot wk$	$x_5' = 0\ c\cdot wk$