Taylor 24 Sept.

Estimation

For previous estimation strategies

1) α in $|\psi\rangle = \cos(\alpha/2)|0\rangle + \sin(\alpha/2)|1\rangle$

2) ϕ in $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\phi}|1\rangle\right)$

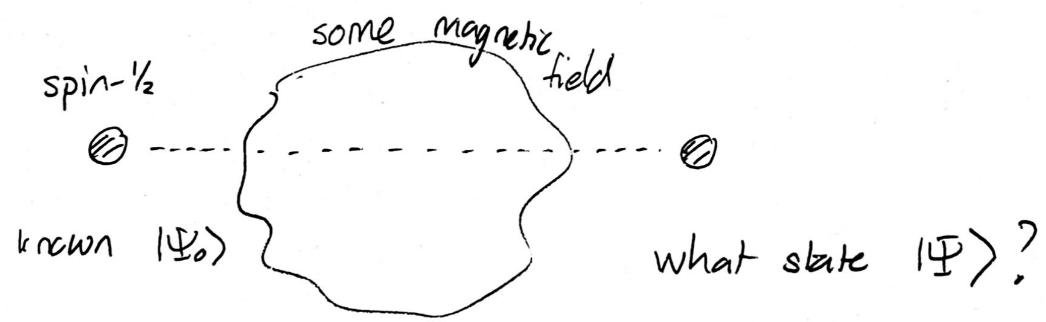
and the measurements that you considered:

a) determine Fisher info

b) how will the measurements compare in terms of least uncertainty?

State Evolution

Situation



General rule for evolution of state:

1) evolution is linear.

IF $|\Psi_{01}\rangle \to |\Psi_1\rangle$

and $|\Psi_{02}\rangle \to |\Psi_2\rangle$

then $\alpha_1|\Psi_{01}\rangle + \alpha_2|\Psi_{02}\rangle \to \alpha_1|\Psi_1\rangle + \alpha_2|\Psi_2\rangle$

2) evolution can be described via a matrix or operator

$$|\Psi\rangle = \hat{u}\,|\Psi_0\rangle$$

state after — state before

where $\hat{u}$ depends on physical situation (e.g. magnetic field, $\vec{B}$, duration of exposure to $\vec{B}$, ...)

3) the evolution must be unitary (for a closed system - will be generalized later)

$$\hat{u}^{+}\hat{u} = \hat{I}$$

$\hookrightarrow$ c.c. transpose

Exercise: Let

$$\hat{u} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show

a) $\hat{u}$ is unitary

b) determine effect of $\hat{u}$ on $|\Psi_0\rangle$ =

i) $|0\rangle$

ii) $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

iii) $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

c) Describe vector representing each state before $|\hat{n}_0\rangle \equiv |\Psi_0\rangle$ this and state after $|\hat{n}\rangle = |\Psi\rangle$ this

How can one describe evolution in terms of these vectors? Rotation?

Exercise. Repeat previous for

$$\hat{U} = \begin{pmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix}$$

and

$$|\Psi_0\rangle = |0\rangle$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

Exercise: Repeat for

$$\hat{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & +i \\ +i & 1 \end{pmatrix}$$

and same states.

Exercise Repeat for

$$\hat{U} = \begin{pmatrix} \cos\alpha/2 & +i\sin\alpha/2 \\ i\sin\alpha/2 & \cos\alpha/2 \end{pmatrix}$$

Special unitaries.

Pauli operators

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Hadamard

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Exercise: Check that these are unitary.

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state $$|\psi\rangle = e^{i\alpha_0/2} \underset{\text{real}}{a_0} |0\rangle + e^{i\alpha_1/2} \underset{\text{real}}{a_1} |1\rangle$$

$$\cos\theta/2\, |0\rangle + e^{i\phi} \sin\theta/2\, |1\rangle$$

$$= \underbrace{\cancel{e^{i\alpha_0/2}}}_{\text{global phase}} \left[a_0 |0\rangle + e^{i(\alpha_1 - \alpha_0)/2} a_1 |1\rangle \right]$$

$$v = \frac{p(1-p)}{N}$$

separately

$$F = \frac{N}{p(1-p)}$$

$$v \geq \frac{1}{F} = \frac{p(1-p)}{N}$$

$$v = \overline{p_{est}^2} - \overline{p_{est}}^2$$

$$\sigma = \sqrt{v} = \frac{\sqrt{p(1-p)}}{\sqrt{N}}$$