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MATH 360
HW 9.7

9.7 # 3,4,21,25

9.7 #3 $f(x,y) = y/x$

a.)

$\vec{\nabla} f$: $\vec{\nabla} = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

$\frac{\partial f}{\partial x}$: $f(x,y) = yx^{-1}$

$\frac{\partial f}{\partial x} = -yx^{-2} = \frac{-y}{x^2}$

$\frac{\partial f}{\partial y}$: $f(x,y) = yx^{-1}$

$\frac{\partial f}{\partial y} = x^{-1} = \frac{1}{x}$

$\vec{\nabla} f = \left[-y/x^2, 1/x\right]$

$$\boxed{\vec{\nabla} f = \left[-y/x^2, 1/x\right]}$$

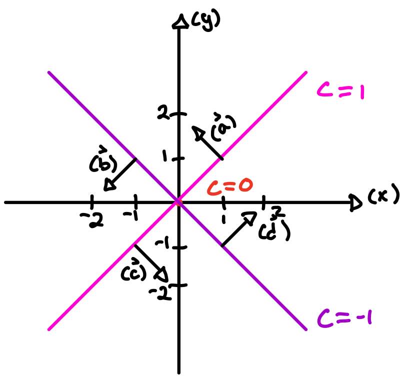
b.)

$c = y/x$

$c = 0$: $0 = \frac{y}{x}$, $y = 0$

$c = 1$: $1 = \frac{y}{x}$, $y = x$

$c = -1$: $-1 = \frac{y}{x}$, $y = -x$



$\vec{\nabla} f(1,1) = [-1, 1] \equiv (\vec{a})$

$\vec{\nabla} f(-1,1) = [-1, -1] \equiv (\vec{b})$

$\vec{\nabla} f(-1,-1) = [1, -1] \equiv (\vec{c})$

$\vec{\nabla} f(1,-1) = [1, 1] \equiv (\vec{d})$

9.7 #4 $f(x,y) = (y+6)^2 + (x-4)^2$

a.) $\vec{\nabla} f$: $\vec{\nabla} = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

$\frac{\partial f}{\partial x}$: $f(x,y) = (y^2 + 12y + 36) + (x^2 - 8x + 16)$

$f(x,y) = (y^2 + 12y) + (x^2 - 8x) + 52$

$\frac{\partial f}{\partial x} = 2x - 8$

$\frac{\partial f}{\partial y}$: $f(x,y) = (y^2 + 12y) + (x^2 - 8x) + 52$

$\frac{\partial f}{\partial y} = 2y + 12$

$\boxed{\vec{\nabla} f = [2x-8, 2y+12]}$

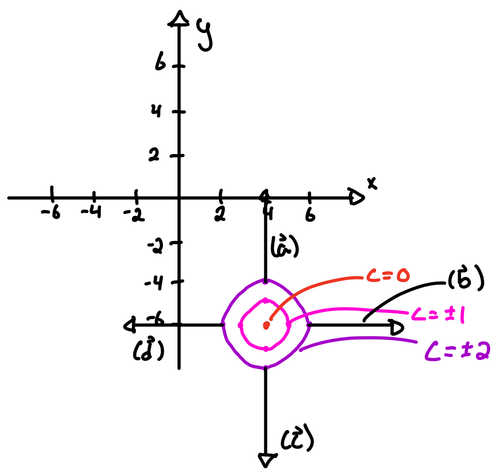
b.) $f(x,y) = (y+6)^2 + (x-4)^2$

Eq of Circle : $r^2 = (x-h)^2 + (y-k)^2$

$h = 4$ Centered @ $(4,-6)$

$k = -6$

$C = \pm 1 \therefore r = 1$ ⋮ $C = \pm 2 \therefore r = 4$ ⋮



$\vec{\nabla} f(4,-4) = [0, 4] \equiv (\vec{a})$

$\vec{\nabla} f(6,-6) = [4, 0] \equiv (\vec{b})$

$\vec{\nabla} f(4,-8) = [0, -4] \equiv (\vec{c})$

$\vec{\nabla} f(2,-6) = [-4, 0] \equiv (\vec{d})$

9.7 #21 $f(x,y) = e^x\cos(y)$ $P: (1, \frac{1}{2}\pi)$

$\vec{v} = \vec{\nabla} f$:

$$\frac{\partial f}{\partial x} = e^x\cos(y) \qquad \frac{\partial f}{\partial y} = -e^x\sin(y)$$

$$\vec{v} = [e^x\cos(y), -e^x\sin(y)]$$

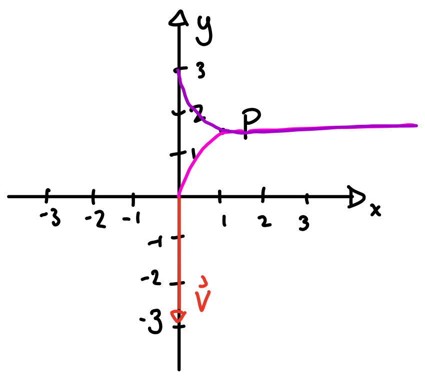
$$\vec{v}(1, \tfrac{1}{2}\pi) = [0, -e] \qquad e \simeq 2.718$$

$$\vec{v}(1, \tfrac{1}{2}\pi) \simeq [0, -2.718]$$

$C=0$: $0 = e^x\cos(y)$
$x=0, y=0$

$C=1$: $1 = e^x\cos(y)$
$\cos(y) = e^{-x}$
$y = \cos^{-1}(e^{-x})$

$C=-1$ $\cos(y) = -e^{-x}$
$y = \cos^{-1}(-e^{-x})$



$c=1$, $c=-1$

$$\boxed{\begin{aligned}&\vec{v} = [e^x\cos(y), -e^x\sin(y)]\\ &\vec{v}(1, \tfrac{1}{2}\pi) = [0, -e]\end{aligned}}$$

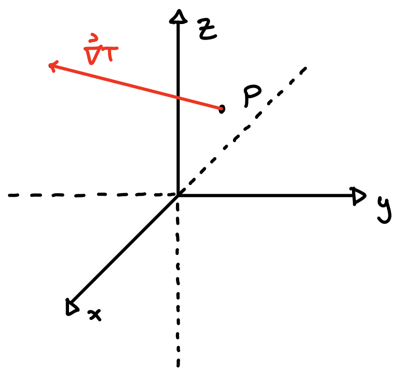
9.7 #25 $T = \dfrac{z}{x^2+y^2}$ $P: (0,1,2)$

$$\frac{\partial T}{\partial x}: \frac{(x^2+y^2)(0) - z(2x)}{(x^2+y^2)^2} = \frac{-2xz}{(x^2+y^2)^2}$$

$$\frac{\partial T}{\partial y}: \frac{(x^2+y^2)(0) - z(2y)}{(x^2+y^2)^2} = \frac{-2yz}{(x^2+y^2)^2}$$

$$\frac{\partial T}{\partial z}: \frac{(x^2+y^2)(1) - z(0)}{(x^2+y^2)^2} = \frac{(x^2+y^2)}{(x^2+y^2)^2} = \frac{1}{x^2+y^2}$$

$$\boxed{\begin{aligned}&\vec{\nabla} T = \left[\frac{-2xz}{(x^2+y^2)^2}, \frac{-2yz}{(x^2+y^2)^2}, \frac{1}{x^2+y^2}\right]\\ &\vec{\nabla} T(0,1,2) = [0, -4, 1]\end{aligned}}$$



9.7 # 30

$4x^2 + 9y^2 = 72$ $\quad P: (2, \sqrt{56}/3)$ $\quad n = \frac{1}{|\nabla f(P)|} \cdot \nabla f(P)$

$\nabla f = [8x, 18y]$

$\nabla f(P) = [16, 12 \cdot \sqrt{14}]$

$|\nabla f(P)| = \sqrt{16^2 + (12)^2 \cdot 14} = 4 \cdot \sqrt{142}$ $\quad \therefore$

$$\boxed{\vec{n} = \frac{1}{4 \cdot \sqrt{142}} \cdot [16, 12 \cdot \sqrt{14}]}$$