MAT 201
Larson/Edwards – Section 4.1
Antiderivatives and Indefinite Integration

Ex: Find a possible function *f* in which the derivative is $f'(x) = 2x$?

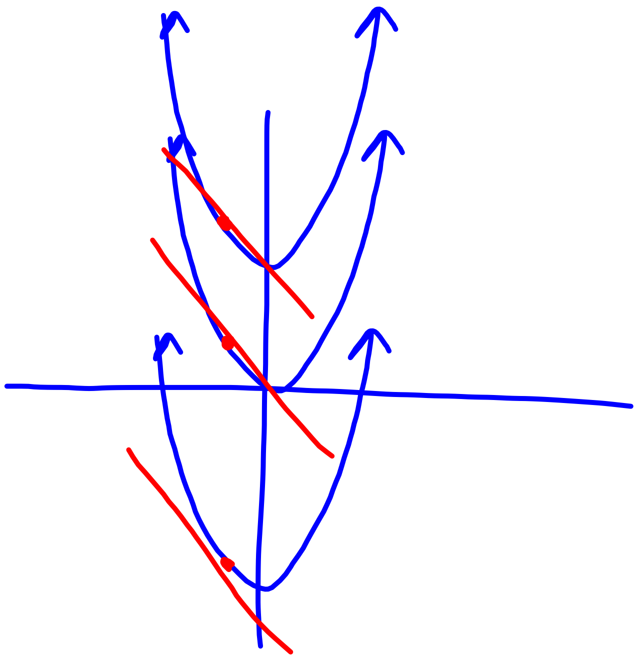
Up to this point in the book, we have been concerned primarily with this problem: ***given a function, find its derivative***. Many important applications of calculus involve the inverse problem as seen above. The operation of determining the original function from its derivative is the inverse operation of differentiation, so it seems fit that we call the process ***antidifferentiation***; that is, the process of finding the ***antiderivative***. Also known as ***integration***, we will focus on this process and the applications involving integrals in Chapter 7.

Definition of Antiderivative: A function F is an ***antiderivative*** of a function f on an interval I if $F'(x) = f(x)$. Note: If $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$, where C is a constant, is also an antiderivative of $f(x)$. WHY???

Ex: Find a possible function f in which the derivative is $f'(x) = 2x$?

$f(x) = x^2 + 1$

$f(x) = x^2 + C$ c is a constant



Ex: Use integral notation to write your answer for our 1st example.

$$f'(x) = 2x$$

$$\boxed{\int 2x \, dx = x^2 + C}$$

What about the notation???

$\int$ This is the ***integral sign***. I'll explain why this symbol later.

$\int f(x)dx$ This is an ***indefinite integral*** because there are no limits of integration (I'll explain what that means later). We read this as "the indefinite integral (or antiderivative) of *f(x)* with respect to *x*". *f(x)* is the ***integrand*** and *dx* is the ***differential***.

$\int f(x)dx = F(x) + C$ *F(x)* + *C* is the ***Antiderivative***.

<u>Putting it All Together</u>: The notation $\int f(x)dx = F(x) + C$ where *C* is an arbitrary constant, means that *F* is an antiderivative of *f*. That is, $F'(x) = f(x)$ for all *x* in the domain of *f*.

Ex: Use integral notation to write your answer for our 1st example.

How do we evaluate indefinite integrals? The inverse relationship between integration and differentiation allows us to develop integration formulas from differentiation formulas!

Basic Integration Rules:

1. $\int 0\,dx = C$
2. $\int k\,dx = kx + C$ k and C are constants.
3. $\int k \cdot f(x)dx = k\int f(x)dx$
4. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
5. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C, \quad n \neq -1$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sin x\,dx = -\cos x + C$
8. $\int \sec^2 x\,dx = \tan x + C$
9. $\int \sec x \tan x\,dx = \sec x + C$
10. $\int \csc^2 x\,dx = -\cot x + C$
11. $\int \csc x \cot x\,dx = -\csc x + C$

Ex: Find the following indefinite integrals:

a) $\int 5\,dx$

b) $\int -2t\,dt$

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a) $\int 5\,dx \;=\; 5x + C$

b) $\int -2t\,dt \;=\; -t^2 + C$

$-2\left[\int t^1\,dt\right]$

$= -2\cdot\left(\frac{t^2}{2} + C\right)$

$= -t^2 - 2C \qquad -t^2 + K$

c) $\int x \sqrt[3]{x}\, dx$

d) $\int \left(4r^3 - 5r + 2\right) dr$

e) $\int \frac{x^3 - 3x^2}{x^5}\, dx$

f) $\int \left(1 - \csc t \cot t\right) dt$

c) $\int x \sqrt[3]{x}\, dx$ $= \int x^1 \cdot x^{1/3}\, dx$

$= \int x^{4/3}\, dx$

$= \dfrac{x^{4/3+1}}{4/3+1} + C = \boxed{\dfrac{3x^{7/3}}{7} + C}$

d) $\int \left(4r^3 - 5r + 2\right) dr$

$\int 4r^3 dr - \int 5r^1 dr + \int 2\, dr$

$\dfrac{4r^{3+1}}{3+1} - \dfrac{5r^{1+1}}{1+1} + 2r + C$

$\boxed{r^4 - \dfrac{5r^2}{2} + 2r + C}$

e) $\int \frac{x^3 - 3x^2}{x^5}\,dx = \int \left(\frac{x^3}{x^5} - \frac{3x^2}{x^5}\right) dx$

$= \int (x^{-2} - 3x^{-3})\,dx$

$= \int x^{-2}\,dx - \int 3x^{-3}\,dx$

$= \frac{x^{-2+1}}{-2+1} - \frac{3x^{-3+1}}{-3+1} + C$

$= \boxed{-x^{-1} + \frac{3x^{-2}}{2} + C \underset{\text{OR}}{=} -\frac{1}{x} + \frac{3}{2x^2} + C}$

f) $\int (1 - \csc t \cot t)\,dt$

$= \int 1\,dt - \int \csc t \cot t\,dt$

$= t - (-\csc t) + C$

$= \boxed{t + \csc t + C}$

<u>Basics of Differential Equations</u>: A ***differential equation*** in x and y is an equation that involves x, y, and derivatives of y. The ***general solution*** of $\frac{dy}{dx}$ is its antiderivative. For instance, if $\frac{dy}{dx} = 3x$, then the general solution of the differential equation is $y = \frac{3}{2}x^2 + C$. Sometimes, if we are given information about the original function, that is, we're given ***initial conditions***, then we may be able to find the ***particular solution***.

Ex: a) Find the general solution of $f'(x) = 4x + 2$.

b) Find the particular solution of $f'(x) = 4x + 2$ that satisfies the initial condition of $f(1) = 8$.

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a) $\int 4x + 2 \, dx = 2x^2 + 2x + C$

General Solution: $\boxed{f(x) = 2x^2 + 2x + C}$

b) Given the initial condition of $f(1) = 8$, we can find C in our general solution by substitution.

$f(1) = 8 \Rightarrow 8 = 2(1)^2 + 2(1) + C$

$8 = 4 + C$

$4 = C$

Particular Solution: $\boxed{f(x) = 2x^2 + 2x + 4}$