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Laboratory 9: Conservation of Energy

The mechanical energy of a physical system obeys

$$\Delta E = W_{nc} \quad (1)$$

where $\Delta E = E_f - E_i$ is the change in mechanical energy from an initial instant to a final instant and W_{nc} is the work done by all the non-conservative forces between these instants. If $W_{nc} = 0$ then

$$\Delta E = 0. \quad (2)$$

This is the law of conservation of energy.

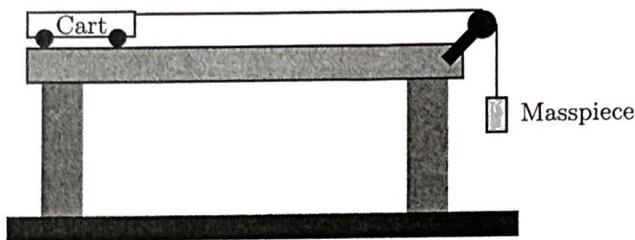


Figure 1: A cart, which can move along a horizontal track, is attached to a suspended masspiece. These objects will be released from rest.

In this experiment, you will consider energy conservation for the system illustrated in Fig. 1. It can be shown that $W_{nc} = 0$ and thus mechanical energy is conserved. Here, the mechanical energy of the system is

$$E_{\text{total}} = K_{\text{total}} + U_{\text{grav total}} \quad (3)$$

where K_{total} is the kinetic energy of *all parts of the system* and $U_{\text{grav total}}$ is the gravitational potential energy of *all parts of the system* at any single moment.

The cart and suspended masspiece will be released from rest. The task will be to compare the total energy at the moment of release to the total energy just before the suspended masspiece hits the floor. Denoting these the initial and final moments respectively, $\Delta E_{\text{total}} = 0$ and Eq. (3) imply

$$\Delta K_{\text{total}} = -\Delta U_{\text{grav total}} \quad (4)$$

where

$$\Delta K_{\text{total}} = K_{\text{total } f} - K_{\text{total } i} \quad (5)$$

$$\Delta U_{\text{total}} = U_{\text{total } f} - U_{\text{total } i}. \quad (6)$$

Various measurements enable calculation of the energies involved, and ultimately the conservation of mechanical energy, via Eq. (4) can be checked.

1 Qualitative Considerations

The cart and suspended masspiece will be released from rest. Consider the initial instant to be that at which the cart is released and the final instant to be that immediately before the suspended masspiece hits the floor.

- a) Is ΔU_{grav} for the cart positive, negative or zero?
- b) Is ΔU_{grav} for the suspended masspiece positive, negative or zero?
- c) Is ΔK for the cart positive, negative or zero?
- d) Is ΔK for the masspiece positive, negative or zero?
- e) Is ΔK_{total} positive, negative or zero?
- f) Is ΔU_{total} positive, negative or zero?

2 Theory and Experimental Design

- a) Describe how to determine a numerical value for ΔU_{total} . Your description should include a list of the quantities that you need to measure and a procedure for calculating ΔU_{total} from these measured numbers.
- b) Describe how to determine a numerical value for ΔK_{total} . Your description should include a list of the quantities that you need to measure and a procedure for calculating ΔK_{total} from these measured numbers.
- c) You will need to measure the velocity of the cart at the instant before the suspended masspiece hits the floor. Explain, using *physics*, why it is sufficient to measure the velocity of the cart *after* the masspieces have hit the floor.
- d) You will be supplied with a cart, suspended masspieces and an electronic timer which can measure the speed of the cart. Briefly describe how you could use these to build an experiment to verify that the mechanical energy is conserved in the process described above.

3 Experiment

- a) Level the track and adjust the pulley so that the string runs horizontally and freely.
- b) Attach the photogate trigger (plexiglas sheet with black stripes) to the cart and check, by running it through the photogate, that the detector is triggering on the 5 cm long black strip.
- c) Place the photogate so that it triggers shortly *after the moment at which the suspended masspiece has hit the floor*.

- d) Open DataStudio, click Create an Experiment and connect the photogate. Drag Time in Gate into digits. This will display the time that the black strip takes to pass through the photogate.
- e) Hold the cart at rest near the end of the track opposite the pulley. Measure the distance from the base of the masspiece hanger to the floor. Start the photogate timer, release the cart and record the duration for which the black strip interrupted the photogate. Repeat this three times with the same suspended mass, releasing the cart *from the same position each time* and recording the duration for which the black strip interrupted the photogate for each. Use the average of the three times to calculate the final speed of the cart.
- f) Calculate ΔK_{total} and ΔU_{total} . Describe whether the magnitudes of these are the same, approximately the same, or substantially different. To get an idea of the size of the discrepancy between them, determine the percentage difference via
- $$\frac{|\Delta K_{\text{total}}| - |\Delta U_{\text{total}}|}{|\Delta U_{\text{total}}|} \times 100\%.$$
- g) Repeat parts e) and f) for two additional values of m_{susp} .

4 Conclusion

- a) What does this experiment establish concerning mechanical energy?
- b) Explain any discrepancies between your observations and the theoretical predictions. Specifically, consider what would happen if you were to repeat the experiment several times for exactly the same combination of suspended mass and drop height. Would you expect to get the same results for ΔK_{total} and ΔU_{total} ? Based on this what could you have done to your experiment to improve your results? Also consider the accuracy with which various measurements could be made, using the equipment available to you.

5 Exercises

- a) Suppose that the experiment is run twice with the suspended mass released from the same position in both runs. In run 2 the suspended mass has exactly 4 times the mass that of run 1. Which of the following is true for velocities of the carts at the instant before the suspended mass hits the ground?
- i) $v_2 = 2v_1$
 - ii) $v_2 = \frac{1}{2} v_1$
 - iii) $v_2 = v_1$
 - iv) $2v_1 > v_2 > v_1$
 - v) $v_2 > 2v_1$

Explain your answer.

- b) An experiment of the type done in this lab is carried out for a situation where $m_{\text{cart}} = 800 \text{ g}$, $m_{\text{susp}} = 200 \text{ g}$ and the masspiece is dropped through height 50 cm. The photogate records the duration of passage of the 5.0 cm black band as 0.040 s. Determine ΔK_{total} and ΔU_{total} . The experiment is repeated, this time giving the duration of passage of the black band as 0.038 s. Fill out the following table.

	Time	ΔK_{total}	ΔU_{total}	percentage difference
Exp 1	0.040 s	.78175 J	.98 J	20%
Exp 2	0.038 s	.87 J	.98 J	11%

Table 1: Question b).

For approximately how much of the percentage difference is the discrepancy in the timing responsible? How would this enter into the discussion of the discrepancies in your experiment?

55% is the difference due to timing.
 You could enter these into the discussion by showing how a .002 second difference can drastically change the results.

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1 Qualitative considerations

- A. The change in gravitational potential energy is zero because it never changes height.
- B. The change in gravitational potential energy is negative because the height goes from a higher height to a lower height.
 $\Delta U_G = mg(y_{\text{smaller}} - y_{\text{larger}})$
- C. ΔK for the cart would be positive because it begins at rest and ends while it is moving.
- D. ΔK for the hanging masspiece would be positive because it begins at rest and ends while it descending to the ground.
- E. ΔK_{total} has to be positive because all of initial potential energy is converted to kinetic in this scenario.
- F. ΔU_{total} has to be negative because all of the potential changes to kinetic in this scenario.

2 Theory and Experimental Design

$$A \quad \Delta U_{\text{total}} = U_{\text{Total f}} - U_{\text{Total i}} \quad U_g = mgY$$

f = final position of masspiece in terms of height

i = initial position of masspiece in terms of height

m_H = Hanging piece mass

$$g = 9.8 \text{ m/s}^2$$

y_f = Final height

y_i = Initial height

$$\Delta U_{\text{total}} = m_H y_f - m_H y_i$$

$$B \quad \Delta K_{\text{total}} = K_{\text{total f}} - K_{\text{total i}} \quad K = \frac{1}{2}mv^2$$

v_i = initial Velocity

M = mass of cart

v_f = Final Velocity

V = Velocity

m_H = mass of hanger

$$(\frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2) + (\frac{1}{2}m_H v_f^2 - \frac{1}{2}m_H v_i^2) = \Delta K$$

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$y = 602 \text{ m}$

$m_C = \text{mass of cart}$

$m_H = \text{mass of hanger}$

$m_C = 504 \text{ g} \quad m_C = 0.504 \text{ kg}$

$M_H = 50 \text{ g} \quad M_H = 0.05 \text{ kg}$

C. Since friction and drag are negated, when the masspiece hits the floor the cart coasts with no acceleration.

D. We can use the height of the mass piece measured with a meter stick to find the y_i value for the potential energy. We can then measure the black flags length and divide it by the time taken for it to pass in front of the electronic timer and that will give us velocity to be used for kinetic energy.

3 Experiment

E

$$m_C = 0.504 \text{ kg} \quad m_H = 0.05 \text{ kg}$$

Run 1

Run 2

Run 3

$$D = 0.05 \text{ m}$$

$$D = 0.05 \text{ m}$$

$$D = 0.05 \text{ m}$$

$$T = 0.04765 \text{ s}$$

$$T = 0.04775 \text{ s}$$

$$T = 0.0473 \text{ s}$$

$$V = \frac{D}{T}$$

$$V = \frac{D}{T}$$

$$V = \frac{D}{T}$$

$$V_1 = 1.05 \text{ m/s} \quad V_2 = 1.048 \text{ m/s} \quad V_3 = 1.057 \text{ m/s}$$

$$V_1 + V_2 + V_3 = 1.05 \text{ m/s} + 1.048 \text{ m/s} + 1.057 \text{ m/s}$$

(A) Average = 1.05 m/s ✓

E. I. $m_C = 0.504 \text{ kg} \quad m_H = 0.1 \text{ kg}$

Run 1

Run 2

Run 3

$$D = 0.05 \text{ m}$$

$$D = 0.05 \text{ m}$$

$$D = 0.05 \text{ m}$$

$$T = 0.0365 \text{ s}$$

$$T = 0.03465 \text{ s}$$

$$T = 0.0355 \text{ s}$$

$$V_1 = \frac{D}{T}$$

$$V_2 = \frac{D}{T}$$

$$V_3 = \frac{D}{T}$$

$$V_1 = 1.43 \text{ m/s} \quad V_2 = 1.45 \text{ m/s} \quad V_3 = 1.43 \text{ m/s}$$

(B) Average = 1.43 m/s ✓

E.2 $m_c = 0.504 \text{ kg}$ $M_H = 0.15 \text{ kg}$

Run 1	Run 2	Run 3
$D = 0.05 \text{ m}$	$D = 0.05 \text{ m}$	$D = 0.05 \text{ m}$
$T = 0.02985$	$T = 0.02975$	$T = 0.02985$
$V_i = 0$	$V_f = 0$	$V_f = 0$
$V_1 = 1.68 \text{ m/s}$	$V_2 = 1.68 \text{ m/s}$	$V_3 = 1.68 \text{ m/s}$

③ Average Value = 1.68 m/s Hanger

$$\Delta K = \frac{1}{2} M_H V_f^2 - \frac{1}{2} m_c V_i^2$$

$$\Delta K = \frac{1}{2} (0.05 \text{ kg}) (1.05 \text{ m/s})^2$$

$$\Delta K_{\text{total}} = K_{\text{total}} f - K_{\text{total}}$$

$$= 0.028$$

Ⓐ $V_f = 1.05 \text{ m/s}$ $\Delta K = \frac{1}{2} m_c V_f^2 - \frac{1}{2} m_c V_i^2$

$m_c = 0.504 \text{ kg}$ $\Delta K_f = \frac{1}{2} (0.504 \text{ kg}) (1.05 \text{ m/s})^2$

$V_i = 0 \text{ m/s}$ $\Delta K_i = 0.28 \text{ J}$ + $= 0.308 \text{ J}$

$$V_f = 1.05 \text{ m/s}$$

$$M_H = 0.05 \text{ kg}$$

$$V_i = 0 \text{ m/s}$$

$M_H = 0.05 \text{ kg}$ $\Delta u_g = m_H g y_f - m_H g y_i$

$$Y_f = 0.602 \text{ m} \quad \Delta u_g = 0.05 \text{ kg} (9.8 \text{ m/s}^2) (0.602 \text{ m})$$

$$Y_i = 0 \text{ m} \quad \Delta u_g = -0.89 \text{ J}$$

$$\left| \frac{0.285 + 0.008 \text{ J} - 0.89 \text{ J}}{-0.89 \text{ J}} \right| \times 100 \% = 6.2 \% \quad \checkmark$$

Ⓑ $V_f = 1.43 \text{ m/s}$ $V_f = 1.43 \text{ m/s}$

$m_c = 0.504 \text{ kg}$ $M_H = 0.1 \text{ kg}$ $\Delta R = \frac{1}{2} m_c V_f^2 - \frac{1}{2} m_c V_i^2$

$V_i = 0 \text{ m/s}$ $V_i = 0 \text{ m/s}$ $\Delta K = \frac{1}{2} (0.504 \text{ kg}) (1.43 \text{ m/s})^2$

$$\Delta K = 0.515 \text{ J}$$

$$\Delta K = \frac{1}{2} M_H V_f^2 - \frac{1}{2} M_H V_i^2$$

$$\Delta K = \frac{1}{2} (0.1 \text{ kg}) (1.43 \text{ m/s})^2$$

$$\Delta K = 0.1 \text{ J}$$

$$M_H = 0.1 \text{ kg}$$

$$Y_i = 0.602 \text{ m}$$

$$Y_f = 0 \text{ m}$$

$$\Delta u_g = 0.1 \text{ kg} (9.8 \text{ m/s}^2) (0.602 \text{ m})$$

$$\Delta u_g = -0.59 \text{ J}$$

$$\left| \frac{0.515 \text{ J} + 0.1 \text{ J} - 0.59 \text{ J}}{-0.59 \text{ J}} \right| \times 100 \% = 4.23 \% \quad \checkmark$$

④ $V_f = 1.68 \text{ m/s}$, $V_f = 1.68 \text{ m/s}$

$$m_c = 0.504 \text{ kg}$$
 $M_H = 0.15 \text{ kg}$ $\Delta K = \frac{1}{2} m_c V_f^2 - \frac{1}{2} m_c V_i^2$

$$V_i = 0 \text{ m/s}$$
 $V_i = 0 \text{ m/s}$ $\frac{1}{2} (0.504) (1.68)^2$

$$\Delta K = 0.711 \text{ J}$$

$$\Delta K = \frac{1}{2} m_H v F_H^2 - \frac{1}{2} m_H v_i^2$$

$$\Delta K = \frac{1}{2} (1.15)(1.63)^2 \quad m_H = .15 \text{ kg}$$

$$Y_i = .602 \text{ m}$$

$$Y_f = 0 \text{ m}$$

$$\Delta K = .1993 \text{ J}$$

$$\Delta U_g = m_H g Y_f - m_H g Y_i$$

$$= .15(9.8)(1.602)$$

$$\Delta U_g = -.885 \text{ J}$$

$$\left| \frac{.711 + .2 - .885}{-.885} \right| \times 100 = \boxed{2.94\%} \quad \checkmark$$

4. a) This experiment establishes mechanical energy is the sum of Kinetic and Potential energy.

b) The discrepancies could be explained by slight friction and air resistance. If the experiment was repeated several times we should get the same results. To improve accuracy the experiment should be in a closed environment with no air and friction. Using a larger distance to get velocity would give a better reading as well.

$$5. a) \Delta K = \frac{1}{2} m v^2 = \frac{1}{2} \frac{\Delta K}{m} = v^2$$

$$\Delta K = \frac{1}{2} 4m v^2 = 2 \frac{\Delta K}{m} = v^2$$

$$\text{II: } v_2 = \frac{1}{2} v_1 \quad X-1$$

$$b) T_C = 1.75 \text{ ms} V_{fH} = 1.75 \text{ m/s}$$

$$T_C = .8 \text{ kg m}_H = .8 \text{ kg}$$

$$\therefore 0 = 0 \text{ m/s}$$

$$\Delta E = \frac{1}{2} m_H V_{fH}^2 - \frac{1}{2} m_H V_i^2$$
$$(.8)(1.75)^2$$

$$\Delta E = .625 \text{ J}$$

$$\Delta E = \frac{1}{2} m_H (V_{fH}^2 - \frac{1}{2} m_E V_i^2)$$
$$= \frac{1}{2} (.8)(1.75)^2$$

$$\Delta E = .15625 \text{ J}$$

$$m_F = .2 \text{ kg} \quad \Delta u_g = m_F g Y_f - m_H g Y_i$$

$$Y_i = .50 \text{ m} \quad = .2(9.8)(.5)$$

$$Y_F = 0 \text{ m} \quad \Delta U_G = .98 \text{ J} \quad \checkmark$$

$$\left| \frac{.625 + .15625 - .98}{-.98} \right| \times 100 = 20\% \quad \checkmark$$

$$V_{fC} = 1.32 \text{ m/s} \quad V_{fH} = 1.32 \text{ m/s}$$

$$m_C = .8 \text{ kg} \quad m_H = .2 \text{ kg}$$

$$U_i = 0 \text{ m/s} \quad U_f = 0 \text{ m/s}$$

$$\Delta E = \frac{1}{2} (.8)(1.32)^2$$

$$\Delta E = .7 \text{ J}$$

$$\Delta E = \frac{1}{2} (.8)(1.32)^2$$

$$\Delta E = .17 \text{ J}$$

$$m_H = .2 \text{ kg} \quad \Delta U_G = .2(9.8)(.5)$$

$$Y_i = .5 \text{ m} \quad = .98 \text{ J} \quad \checkmark$$

$$Y_F = 0 \text{ m}$$

$$\left| \frac{.7 + .17 - .98}{.98} \right| \times 100 = 11\% \quad \checkmark$$