Tues: Discussion / quiz

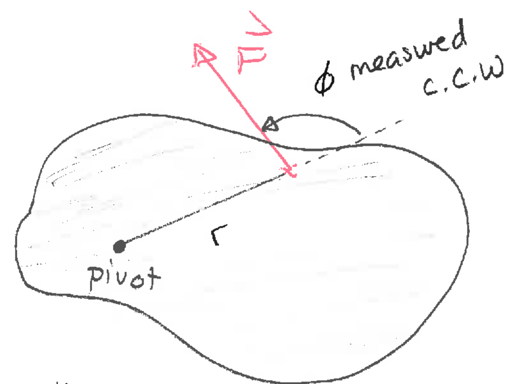
Ch12 Conc Q 1, 8

Prob 4, 7, 13, 20, 21

Torque:

The rotational effects of forces are determined via associated quantities called torque. With reference to the diagram, the torque produced by a force is:

$$\tau = rF\sin\phi$$



and this has units of N·m. In this formula:

1) r and F are magnitudes and are positive
2) ϕ can be such that torque is:

positive $\Rightarrow$ counterclockwise

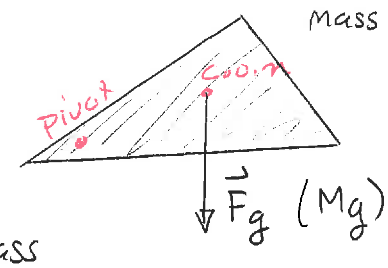
negative $\Rightarrow$ clockwise.

In general many forces might act on an object. Then the net torque is

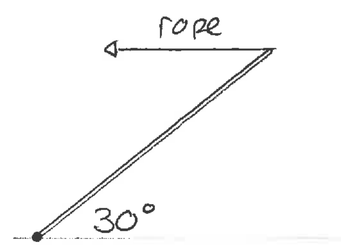
$$\boxed{\tau_{net} = \tau_1 + \tau_2 + \ldots = \sum_{\text{all forces}} \tau_i}$$

Additionally one can show that:

> The torque produced by the gravitational forces acting on various parts of an object is exactly the same as the torque produce by a downward force with magnitude Mg acting at the center of mass



<u>Example</u>: A 2.0m long 10kg rod can pivot about one end. A rope pulls horizontally with a 200N tension at the other end. Determine:

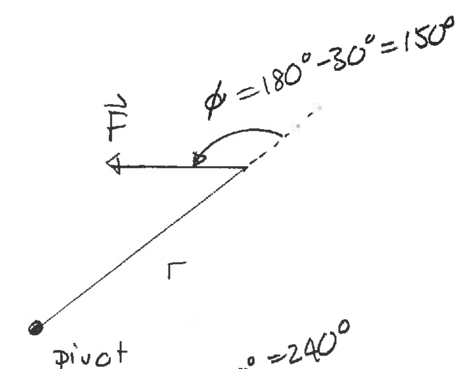


a) torque produced by rope

b) " " " gravity

c) " " " pivot

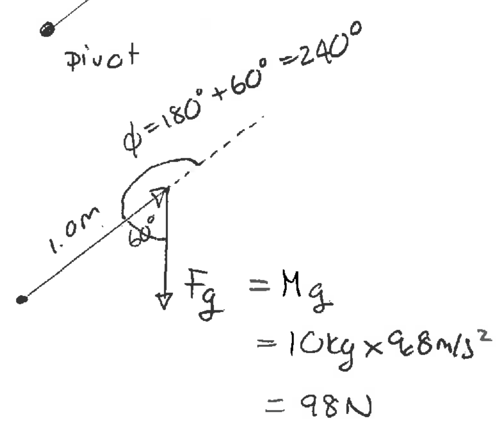
d) net torque



<u>Answer</u>: a) $\tau = rF\sin\phi$

$= 2.0m \times 200N \sin 150°$

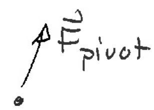
$\tau_{rope} = 200N\cdot m$



b) $\tau_g = rF\sin\phi$

$= 1.0m \times 98N \sin 240°$

$\tau_g = -85 N.m$



c) $\tau_{pivot} = rF\sin\phi$

But $r = 0 \Rightarrow \tau_{pivot} = 0 N\cdot m$

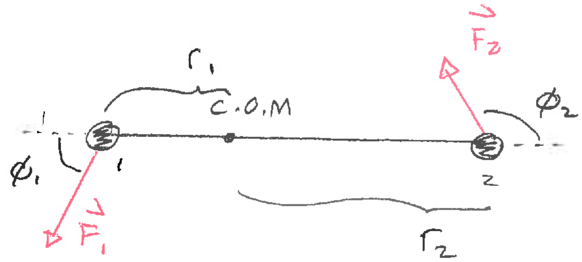
d) $\tau_{net} = \tau_{rope} + \tau_g + \tau_{pivot} = 200N\cdot m - 85N\cdot m + 0N\cdot m$

$\tau_{net} = 115 N\cdot m$

Quiz 1

Torque and dynamics

How do torques determine the dynamics of any object? As an example, consider two masses connected by a rigid rod with negligible mass. The rod can rotate about an axle through its center of mass.



We compute the angular acceleration about the c.o.m, via the tangential acceleration. For 1

$$F_{1t} = m_1 a_{1t}$$

where F_{1t} is the tangential component of the force: $F_{1t} = F_1 \sin\phi_1$. Then $a_{1t} = r_1 \alpha$ gives:

$$F_1 \sin\phi_1 = m_1 r_1 \alpha \quad \Rightarrow \quad r_1 F_1 \sin\phi_1 = m_1 r_1^2 \alpha \quad \Rightarrow \quad \tau_1 = m_1 r_1^2 \alpha$$

Similarly $\tau_2 = m_2 r_2^2 \alpha$ and combining gives:

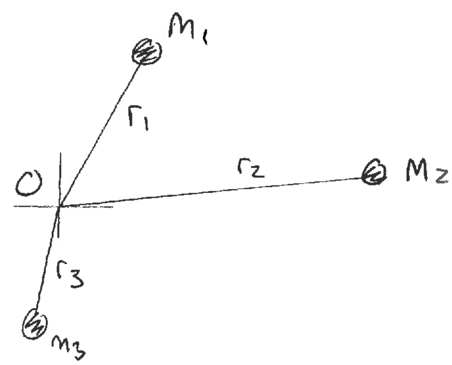
$$\tau_{net} = \tau_1 + \tau_2 = \left[m_1 r_1^2 + m_2 r_2^2\right] \alpha$$

This shows that not only mass, but also the location of mass determines angular acceleration. This motivates the definition of the moment of inertia:

The moment of inertia of a collection of point like objects about a point O is

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots$$

$$I = \sum m_i r_i^2$$



Units $kg \cdot m^2$

Thus the moment of inertia is determined by:

1) mass present
2) location of the mass.

Warm Up 1

The derivation presented is an example of a general rule:

> The net torque on an object determines its angular acceleration via
>
> $$\tau_{net} = I\alpha$$

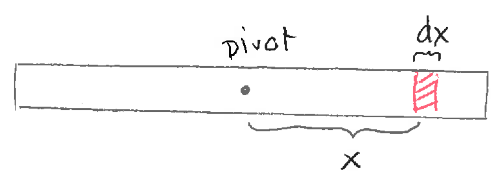
Warm Up 2

Demo: Red / blue sticks

Moment of inertia: continuous mass distributions

The previous derivation only considered point masses. But the calculation can be extended to continuous mass distributions. The following example illustrates this:

Example: Determine the moment of inertia of a uniform rod with length L and total mass M. (about its center)



Answer: 1) Decompose object into small pieces. Consider the piece of width dx at location x. The mass of this piece is dm. Ratios give

$$\frac{dm}{M} = \frac{dx}{L} \Rightarrow dm = \frac{M}{L}\,dx$$

2) Determine moment of inertia of each piece:

$$dI = r^2\,dm = x^2\,\frac{M}{L}\,dx$$

3) Add all such moments for all segments

For the rod, this means adding all contributions from $x = -L/2$ to $x = L/2$.

$$I = \sum_{\text{all segments}} dI = \int dI$$

$$\Rightarrow \quad I = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx$$

$$= \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx = \frac{M}{L} \frac{1}{3} x^3 \Big|_{-L/2}^{L/2}$$

$$= \frac{M}{L} \frac{1}{3} \left\{ \left(\frac{L}{2}\right)^3 - \left(\frac{-L}{2}\right)^3 \right\}$$

$$\Rightarrow \quad I = \frac{M}{3L} \left\{ \frac{L^3}{8} + \frac{L^3}{8} \right\} = \frac{ML^2}{12}$$

$$\Rightarrow \quad I = \frac{1}{12} ML^2 \qquad \blacksquare$$

Similar calculations can be done for various symmetrical extended objects