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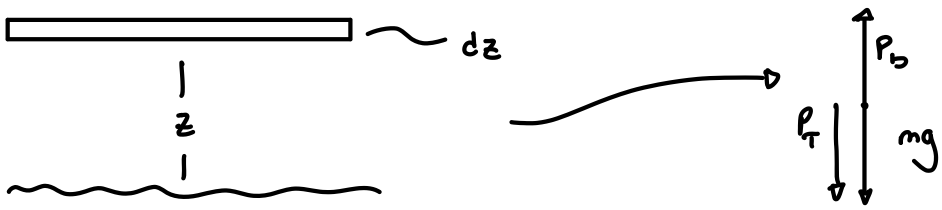
Problem 1

a.)



$PA = mg$, $m = \rho V$

$P_b A - P_T A = mg$ $\quad V = dz \cdot A$ $\quad P_T - P_b = dP$

$A(P_b - P_T) = mg$ $\quad (P_b - P_T)\cancel{A} = \rho \cdot \cancel{A} \cdot g \cdot dz$ $\quad \boxed{\frac{dP}{dz} = -\rho g}$

$A(P_b - P_T) = \rho V g$ $\quad P_T - P_b = -\rho g dz$

b.) $PV = NKT$, $P\frac{m}{\rho} = NKT$ $\therefore$ $\rho = \frac{Pm}{NKT}$

$$\frac{dP}{dz} = -g \cdot \frac{Pm}{NKT} \quad \therefore \quad \int \frac{1}{P} dP = \int \frac{-gm}{NKT} dz$$

$$\ln(P) = \frac{-gm}{NKT} z + C \longrightarrow C = 1.0 \text{ atm}$$

$$\boxed{P(z) = Ce^{-gmz/NKT} , \quad \rho(z) = \frac{Ce^{-gmz/NKT} m}{NKT}}$$

c.)

N_2 : $m_{N_2} = 28 \frac{g}{mol}$, O_2 : $m_{O_2} = 32 \frac{g}{mol}$, Ar : $m_{Ar} = 40 \frac{g}{mol}$

1 mol = 6.02×10^{23} molecules

$m_{N_2} = 4.65 \times 10^{-26} \frac{kg}{mole}$, $m_{O_2} = 5.32 \times 10^{-26} \frac{kg}{mole}$, $m_{Ar} = 6.64 \times 10^{-26} \frac{kg}{mole}$

$$\boxed{m = 4.81 \times 10^{-26} \frac{kg}{molecule}}$$

d.) 11,000 ft = 3353.7 m

$$P(z) = 1.01 \times 10^5 Pa \cdot e^{-9.8 m/s^2 \times 4.81 \cdot 10^{-26} kg/molecule \times 3353.7 m / 1 \times 1.38 \times 10^{-23} J/K \times 293 K} = 68.3 \text{ kPa}$$

$$P = 68.3 \text{ kPa}$$

Problem 1 Continued

4,583 Ft = 1397.3 m

$$P(z) = 1.01\times10^5 Pa \cdot e^{-9.8 m/s^2 \times 4.81\cdot10^{-26} kg/molecule \times 1397.3m / 1\times1.38\times10^{-23} J/K \times 293 K} = 85.8 \text{ kPa}$$

$$P = 85.8 \text{ kPa}$$

$$\boxed{P = 68.3 \text{ kPa on mesa}, \; P = 85.8 \text{ kPa in GJ}}$$

e.) $$\rho(z=0) = \frac{1.01\times10^5 Pa \cdot 4.81\times10^{-26} kg/molecule}{1.38\times10^{-23} J/K \cdot 293 K} = 1.2 \frac{kg}{m^3}$$

$$\frac{\rho \cdot NKT}{m} = ce^{-gmz/NKT} \quad : \quad \frac{\rho NKT}{cm} = e^{-gmz/NKT} \quad \ln\left(\frac{\rho NKT}{cm}\right) = \frac{-gmz}{NKT}$$

$$z = -\ln\left(\frac{\rho N KT}{cm}\right) \cdot \frac{NKT}{gm} = 19,761.7 \text{ m}$$

$$\boxed{z = 19,762 \text{ m}}$$

Problem 2

Constant volume, pressure doubled

$$\left(P + \frac{N^2}{V^2}a\right)\left(V - Nb\right) = NkT \quad : \quad \frac{N^2}{V^2}a \to \alpha \quad V - nb \to \beta \quad NK \longrightarrow \gamma$$

$$P_f = 2P_i$$

$$(P_i + \alpha)(\beta) = \gamma \cdot T_i \quad : \quad T_i = \frac{(P_i + \alpha)(\beta)}{\gamma} \quad : \quad (2P_i + \alpha)(\beta) = \gamma \cdot T_f$$

$$P_i = \frac{\gamma \cdot T_i}{\beta} - \alpha \quad : \quad \left(2 \cdot \left(\frac{\gamma \cdot T_i}{\beta} - \alpha\right) + \alpha\right)(\beta) = \gamma T_f$$

$$\left(\frac{2\gamma T_i}{\beta} - 2\alpha + \alpha\right)(\beta) = \gamma T_f$$

$$\left(\frac{2\gamma T_i}{\beta} - \alpha\right)(\beta) = \gamma T_f$$

$$\left(2\gamma T_i - \alpha\beta\right) = \gamma T_f \quad \therefore \quad T_f = 2T_i - \frac{\alpha\beta}{\gamma}$$

$\boxed{T_f = 2T_i - \frac{\alpha\beta}{\gamma}}$, $\therefore$ The temperature increases by less than a factor of two. iii.)

Problem 3

$$\left(P + \frac{N^2}{V^2}a\right)\left(V - Nb\right) = NkT \quad , \quad N_A = N_B \quad , \quad T_A = T_B \quad , \quad V_A = V_B$$

a.) The only thing that could cause gas A to have a greater pressure than gas B or vice versa are the coefficients : a and b in the Van der Waals gas equation. More precisely, if attractive forces of the molecules in gas A are greater than that of gas B, then we should expect the pressure of B to be greater than that of gas A.

Therefore we shouldn't expect the pressures to be the same

b.) The ideal gas law predicts the pressure of these gases to be the same. This could be correct if the coefficient of a and b were the same for each gas.

c.) The van der Waals equation of state could predict a difference in pressure of the gases if and only if the "a" coefficient were different for each gas. For instance if b were the same for each gas, then

if $a_a > a_b$ then $P_b > P_a$
if $a_a < a_b$ then $P_b < P_a$

and thus van der Waals equation state will predict different pressures of these gases.