## The Classical (Planetary) model

## Bohrs model

$$E = K + U = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r} < 0$$

A bound system

$$F = -\frac{e^2}{4\pi\varepsilon_0 r^2} = -\frac{mv^2}{r}$$

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m r}} \Rightarrow v = \frac{e}{\sqrt{4\pi\varepsilon_0 m r}}$$

## Bohr's Assumptions:

1) Stationary States: Stable, non-classical States of definite energy E where in the electrons don't Continuously radiate E.M waves while "orbiting"

2) Emission or absorbtion of radiation corresponds to transition between the Stationary States. The Frequence is related to the energy difference.

$$\Delta E = h\nu = E_1 - E_2$$

3) The dynamic equilibrium in Stationary states is governed by classical mechanics. However transitions are Manifestly non-classical.

4) The average Kinetic energy (of $e^-$) is $K = \frac{1}{2} n h \nu$ → orbital frequency

$$L \sim n\frac{h}{2\pi} \qquad \frac{h}{2\pi} = \hbar$$

( De Broglie later (ch 5) associated this w/ matter waves )

## Bohr's Derivation :

$$\vec{L} = \vec{r} \times \vec{p} \qquad L = mvr = n\hbar$$

$$\frac{e^2}{4\pi\varepsilon_0 m r} = \frac{n^2\hbar^2}{m^2 r^2}$$

n → Principle quantum number

Ground State, $n=1$

$$\frac{r^2 e^2}{4\pi\varepsilon_0 m r} = \frac{n^2\hbar^2}{m^2}$$

$$r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{e^2 m} = n^2 a_0$$

The nuclear mass here is huge

$$M_{Nuc} \longrightarrow \infty$$

$$M >> M_e$$

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = 0.63 \times 10^{-10}\, m$$

## To Get Energies:

$$E_n = \frac{-e^2}{8\pi\varepsilon_0 r_n} = \frac{-e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv \frac{-E_0}{n^2}$$

$$-E_0 = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{(1.6\times10^{-19}C)}{8\pi(8.85\times10^{-12}F/m)(0.53\times10^{-10}m)} = -2.17\times10^{-18}J$$

$$= -13.6\,eV$$

## Transitions

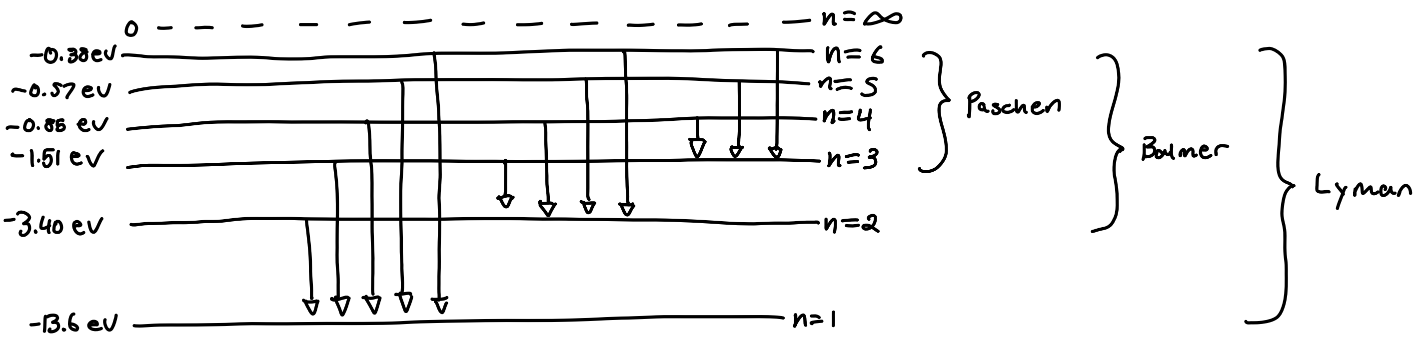
$$h\nu = E_u - E_\ell \qquad \nu = c/\lambda$$

$$\frac{hc}{\lambda} = E_u - E_\ell = -E_0\left(\frac{1}{n_u^2} - \frac{1}{n_\ell^2}\right)$$

$$\frac{1}{\lambda} = \frac{-E_0}{hc}\left(\frac{1}{n_u^2} - \frac{1}{n_\ell^2}\right)$$

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)$$

$$R_\infty \equiv \frac{E_0}{hc} = 1.097373\times10^{7}\,m^{-1}$$




$$v_n = \frac{n\hbar}{m r_n} = \frac{1}{n}\frac{\hbar}{m a_0}$$

$$v_1 \simeq 2.0\times10^{6}\,m/s$$

## The Correspondence Principle:

"In the limits where classical & Quantum theories should agree, the Quantum theory must reduce to the classical result"

Considering orbital frequency:

$$\nu_{cl} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{v}{r} \qquad \nu_{cl} = \frac{1}{2\pi}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m r^3}}$$

$$= \frac{m e^4}{4\pi\varepsilon_0^2 h^3 n^3}$$

$$\nu_{Bohr} = \frac{E_0}{h}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$= \frac{E_0}{h}\left(\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right) = \frac{E_0}{h}\frac{2n+1}{n^2(n+1)^2}$$

$$= \frac{E_0}{h} \frac{2 + 1/n}{n(n+1)^2}$$

$$\approx \frac{E_0}{h} \frac{2}{n^3} = \frac{me^4}{4\pi\varepsilon_0^2 h^3} \frac{1}{n^3}$$

## Successes & Failures, limitations

Bohr's model gets H-Spectrum right, predicts new series, ETC.

However: Only good for single-electron atoms/ions

Also doesn't allow for Nuclear motion $\Rightarrow$ use reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \Rightarrow R_{\substack{\text{Finite} \\ \text{mass}}} = \frac{\mu e}{m e} R_\infty$$

For Hydrogen:

$R_H = 1.09677 \times 10^7 \, m^{-1}$

## Fine Structure: Relativistic & magnetic Corrections

Result in single lines corresponding to closely spaced doublets, Etc.

## Angular Momentum!

Assumption #4 is kind of ad hoc

De Broglie showed this is a consequence of matter having wave-like properties.

## Main Points

Rutherford: 1) Demolished "Plum pudding" model
2) "Planetary" Model

Bohr: 1) Verified theoretically $a_0 \sim 10^{-10} m$
2) Demonstrated that a Quantum planetary model does in fact get the H-Spectrum right.
3) Predict new lines (For single atoms)
4) Qualitative pictures

## Series Limit

$Li^{++}$ $Z = 3$

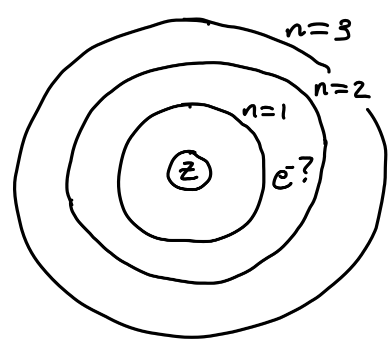
$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_e^2} - \frac{1}{n_u^2}\right)$$

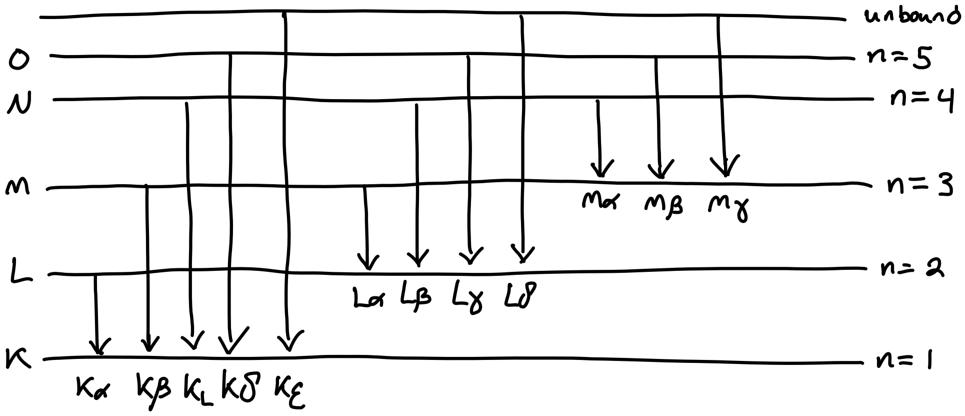
$n_e = 1$ $\quad n \to \infty$

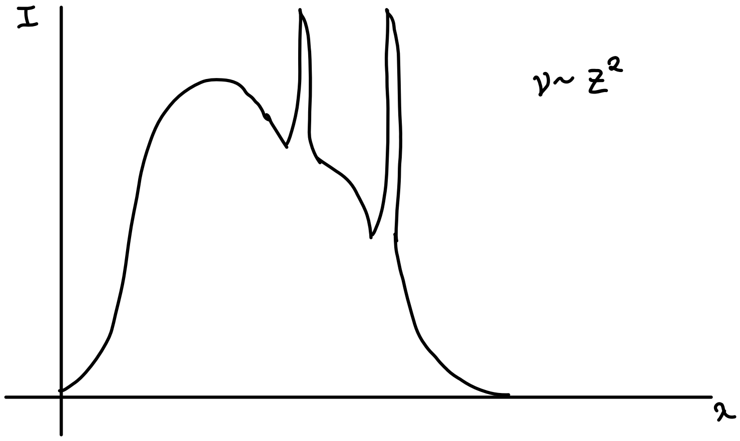
$$\frac{1}{\lambda} = 9R(1)$$

$$\lambda = \frac{1}{9R} = 10.1 \text{ nm}$$

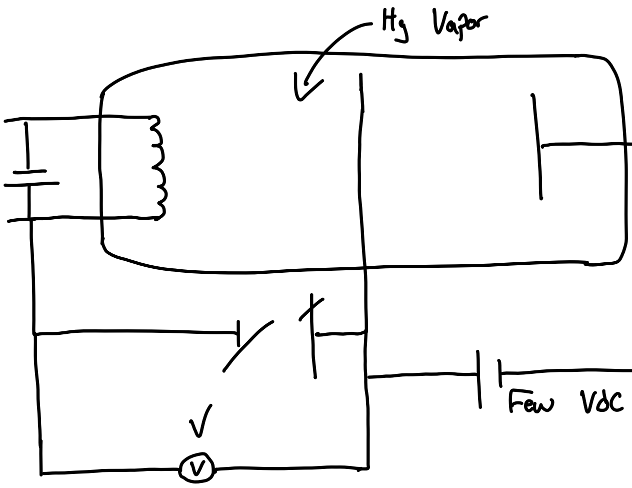
Bohr Model

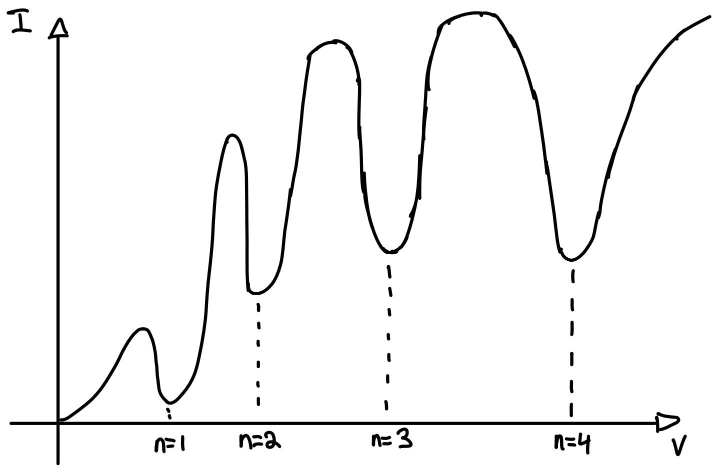



unbound
O
n=5
N
n=4
M
n=3
L
n=2
K
n=1
Kα
Kβ
KL
Kδ
Kε
Lα
Lβ
Lγ
Lδ
Mα
Mβ
Mγ


I
λ

$\nu \sim Z^2$


Hg Vapor
A
V
V
Few VdC


I
n=1
n=2
n=3
n=4
V