Taylor Larrechea
Dr. Collins
PHYS 362 HW 3

Problem 1

a.) $\alpha_1 = 1.65 \times 10^{-5}\ K^{-1}$, $L_0 = 10\,m$, $T_0 = 263\,K$, $T_1 = 303\,K$

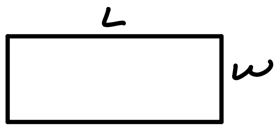
$$\frac{1}{L} \cdot \frac{\partial L}{\partial T} = \alpha_1 \longrightarrow \frac{\partial L}{L} = \alpha_1 \partial T \longrightarrow \int \frac{\partial L}{L} = \alpha_1 \int \partial T$$

$$\ln(L) = \alpha_1 T + C \longrightarrow L = k e^{\alpha_1 T} \quad \therefore \quad k = \frac{L}{e^{\alpha_1 T_1}} \quad : \quad k = 9.96\,m$$

$$L = (10.0\,m)\, e^{1.65 \times 10^{-5} K^{-1} (40\,K)} = 10.0066\,m$$

$$\boxed{\Delta L = 0.0066\,m}$$

b.)



$$\gamma = \frac{1}{A} \frac{\partial A}{\partial T} \quad : \quad A = L \cdot W \quad \therefore \quad L = \frac{A}{W} \quad : \quad L = \frac{1}{\alpha_1} \frac{\partial L}{\partial T}$$

$$\frac{dA}{dT} = \frac{dL}{dT} w + L \cdot \frac{dw}{dT} = \alpha_1 \cdot L w + L w \cdot \alpha_1 = 2\alpha_1 L w = 2\alpha_1 A$$

$$\frac{dA}{dT} = 2\alpha_1 A \quad : \quad \gamma \cancel{A} = 2\alpha_1 \cancel{A} \quad \therefore \quad \gamma = 2\alpha_1$$

$$2\alpha_1 = \frac{1}{A} \frac{\partial A}{\partial T} \longrightarrow \frac{\partial A}{A} = 2\alpha_1 \cdot \partial T \longrightarrow \int \frac{\partial A}{A} = 2\alpha_1 \int \partial T \longrightarrow \ln(A) = 2\alpha_1 T + C$$

$$A = k e^{2\alpha_1 T} \quad \therefore \quad k = \frac{A}{e^{2\alpha_1 T}} \longrightarrow k = 40.0\,m$$

$$A = (40.0\,m)\, e^{2(1.3 \times 10^{-5} k^{-1})(40\,k)} = 40.048\,m^2$$

$$\boxed{\Delta A = 0.05\,m^2}$$

Problem 2

$$\left[P + \frac{N^2}{V^2}a\right]\left[V - Nb\right] = NkT \quad : \quad \alpha = \frac{1}{V}\frac{\partial V}{\partial T} \qquad \frac{\partial V}{\partial T} = \frac{1}{\frac{\partial T}{\partial V}}$$

$$PV - PNb + \frac{N^2a}{V} - \frac{N^3ab}{V^2} = NkT \quad : \quad T = \frac{PV}{Nk} - \frac{PNb}{Nk} + \frac{Na}{kV} - \frac{N^2ab}{kV^2}$$

$$\frac{\partial T}{\partial V} = \frac{P}{Nk} - \frac{Na}{kV^2} + \frac{2N^2ab}{kV^3} = \frac{PV^3}{NkV^3} - \frac{N^2aV}{NkV^3} + \frac{2N^3ab}{NkV^3} = \frac{PV^3 - N^2aV + 2N^3ab}{NkV^3}$$

$$\frac{\partial T}{\partial V} = \frac{PV^3 - N^2aV + 2N^3ab}{NkV^3} \quad \therefore \quad \frac{\partial V}{\partial T} = \frac{NkV^3}{PV^3 - N^2aV + 2N^3ab}$$

$$\alpha = \frac{1}{V}\frac{\partial V}{\partial T} = \frac{1}{V}\cdot\frac{NkV^3}{PV^3 - N^2aV + 2N^3ab} = \frac{NkV^2}{PV^3 - N^2aV + 2N^3ab} \quad : \quad \alpha = \frac{NkV^2}{PV^3 - N^2aV + 2N^3ab}$$

$$\boxed{\alpha = \frac{NkV^2}{PV^3 - N^2aV + 2N^3ab}}$$

Problem 3

$PV = NkT$, $K = -\frac{1}{V}\frac{\partial V}{\partial P}$

$V = \frac{NkT}{P}$: $\frac{\partial V}{\partial P} = -\frac{NkT}{P^2}$: $K = -\frac{1}{V} \cdot -\frac{NkT}{P^2} = \frac{1}{V}\frac{NkT}{P^2}$: $K = \frac{1}{V}\frac{NkT}{P^2}$

$K = \frac{1}{\cancel{V}}\frac{\cancel{P}\cancel{V}}{P^{\cancel{2}}} = \frac{1}{P}$: $K = \frac{1}{P}$

$$\boxed{K = \frac{1}{P}}$$

This quantity for isothermal compressibility is >0 beacause $P \geq 0$. Thus $K > 0$

Problem 4

$$V = V_0(1 + aT - bP)$$

a.)

i.) $K = -\frac{1}{V}\frac{\partial V}{\partial P}$ Isothermal compressibility

$$\frac{\partial V}{\partial P} = -V_0 b \quad \therefore \quad K = -\frac{1}{V}(-V_0 b) = \frac{V_0 b}{V} = \frac{\cancel{V_0} b}{\cancel{V_0}(1 + aT - bP)}$$

$$\boxed{K = \frac{b}{(1 + aT - bP)}}$$

ii.) $\alpha = \frac{1}{V}\frac{\partial V}{\partial T}$ Isobaric expansion

$$\frac{\partial V}{\partial T} = V_0 a \quad \therefore \quad \alpha = \frac{1}{V}(V_0 a) = \frac{V_0 a}{V} = \frac{\cancel{V_0} a}{\cancel{V_0}(1 + aT - bP)}$$

$$\boxed{\alpha = \frac{a}{(1 + aT - bP)}}$$

b.) $aT \ll 1$, $bP \ll 1$

i.) $K = -\frac{1}{V}\frac{\partial V}{\partial P}$ Isothermal compressibility $\longrightarrow aT \ll 1$ & $bP \ll 1$

$$K = \frac{b}{(1 + \underbrace{\cancel{aT}}_{0} - \underbrace{\cancel{bP}}_{0})} = b : \qquad \boxed{K = b}$$

Practically

ii.) $\alpha = \frac{1}{V}\frac{\partial V}{\partial T}$ Isobaric expansion $\longrightarrow aT \ll 1$ & $bP \ll 1$

$$\alpha = \frac{\alpha}{(1 + \underbrace{\cancel{aT}}_{0} - \underbrace{\cancel{bP}}_{0})} = \alpha : \qquad \boxed{\alpha = a}$$

Practically