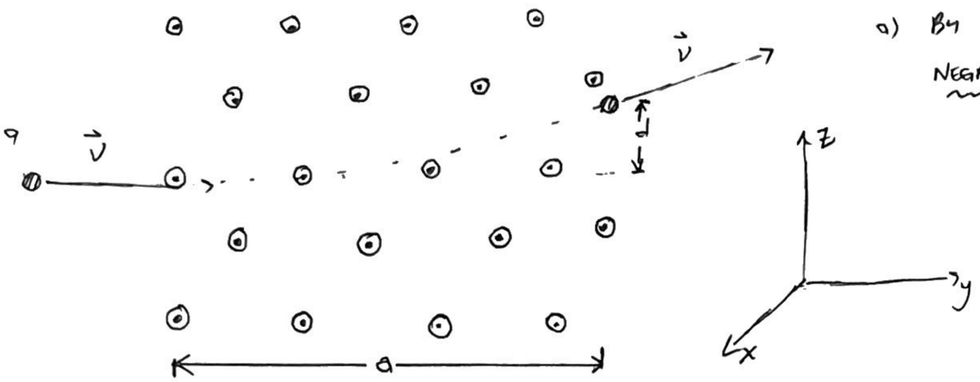
PHYS 311 HOMEWORK SET 9 SOLUTIONS

1.



a) BY THE RHR, THE CHARGE MUST BE NEGATIVE

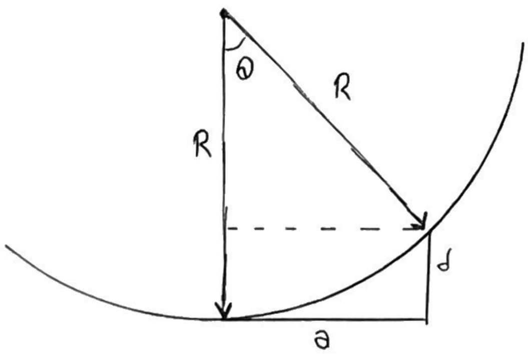
$\vec{B} = B\hat{x}$

$\vec{v}_i = v_i\,\hat{y}$

NOTICE THAT WHILE THE NEGATIVELY CHARGED PARTICLE IS IN THE UNIFORM $\vec{B}$-FIELD, IT EXPERIENCES A CENTRIPETAL FORCE

$$\vec{F}_{rot,c} = \frac{mv^2}{R}$$ WHERE R IS THE RADIUS OF THE CIRCULAR PATH.

$$qvB = \frac{mv^2}{R} \quad \therefore \quad v = \frac{qRB}{m}$$



NOTICE THAT...

$$a = R\sin\theta \qquad \therefore \qquad \sin\theta = \frac{a}{R}$$

$$d = R - R\cos\theta \qquad\qquad \cos\theta = 1 - \frac{d}{R}$$

NOW

$$1 = \sin^2\theta + \cos^2\theta = \frac{a^2}{R^2} + \left(1 - \frac{d}{R}\right)^2$$

OR

$$R^2 = a^2 + (R-d)^2 = a^2 + R^2 - 2dR + d^2$$

OR

$$0 = a^2 + d^2 - 2dR \quad \therefore \; R = \frac{1}{2d}(a^2 + d^2)$$

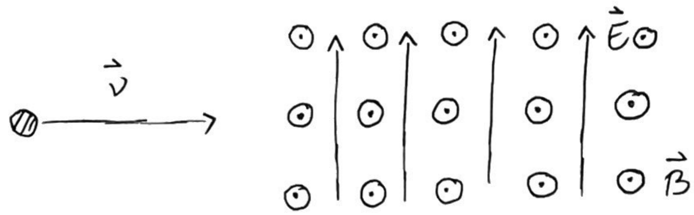
SO

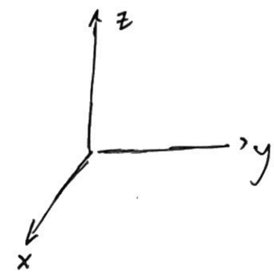
$$v = \frac{qB}{m} \cdot \frac{1}{2d}(a^2 + d^2)$$

OR

$$\boxed{v = \frac{qB}{2dm}(a^2 + d^2)}$$

2. a)

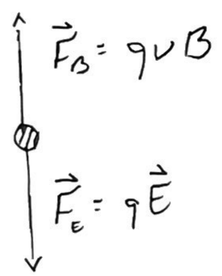




CONSIDER A UNIFORM $\vec{E}$- AND $\vec{B}$-FIELD w/

$$\vec{E} = E\hat{z}$$
$$\vec{B} = B\hat{x}$$

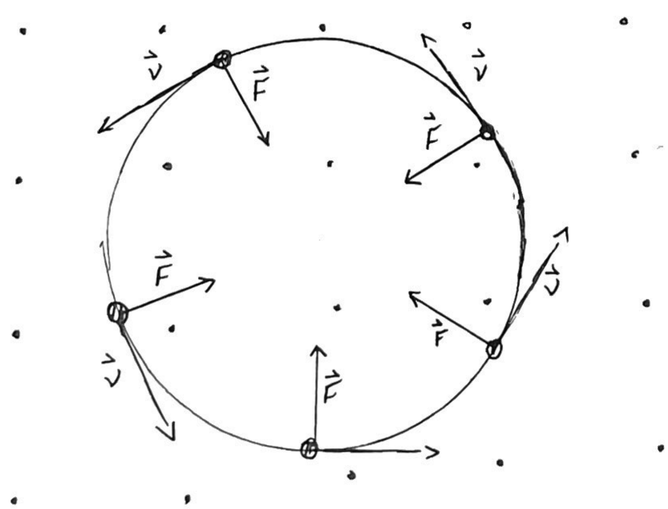
THE FREE-BODY DIAGRAM FOR THE NEGATIVELY CHARGED ELECTRON IS..



IF THE e^- ISN'T DEFLECTED, THEN WE HAVE...

$$\vec{F}_{NET,z} = 0$$
$$qvB - qE = 0 \qquad \therefore \left[v = \frac{E}{B} \right]$$

i) NOW w/ ZERO $\vec{E}$-FIELD...



NEWTON'S 2ND LAW FOR CENTRIPETAL ACCELERATION IS...

$$\vec{F}_{NET,C} = m\vec{a}_c$$
$$qvB = \frac{mv^2}{R}$$
$$\therefore \left[v = \frac{qRB}{m} \right]$$

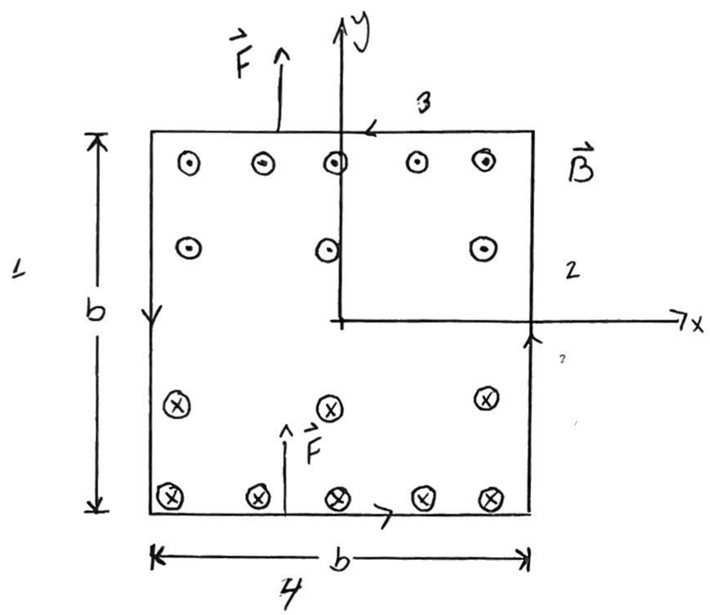
NOW EQUATING THE TWO YIELDS..

$$\frac{E}{B} = \frac{q}{m} \cdot RB \qquad \text{so} \quad \left[\frac{q}{m} = \frac{E}{RB^2} \right]$$

3. $\vec{B} = ky\,\hat{z}$ - CONSIDER THIS NON UNIFORM $\vec{B}$-FIELD

NOW THE MAGNETIC FORCE ON A SEGMENT OF CURRENT-CARRYING WIRE IS

$$\vec{F}_{MAG} = \int I\,(d\vec{l} \times \vec{B})$$



WE NEED TO CALCULATE THE FORCE ON EACH SIDE OF THE SQUARE LOOP (LABELED 1 THROUGH 4)

FOR SIDE 1...

$$d\vec{l} = dy\,\hat{y} \quad \text{so} \quad \vec{F}_{MAG,1} = I\int_{b/2}^{-b/2} dy\,\hat{y} \times ky\,\hat{z} = Ik(-\hat{x})\int_{b/2}^{-b/2} y\,dy = Ik\hat{x}\,\frac{y^2}{2}\Bigg|_{-b/2}^{b/2}$$

$$= \frac{Ik}{2}\hat{x}\left(\left(\frac{b}{2}\right)^2 - \left(-\frac{b}{2}\right)^2\right) = 0$$

LIKEWISE FOR SIDE 2..

$$d\vec{l} = dy\,\hat{y} \quad \text{so} \quad \vec{F}_{MAG,2} = I\int_{-b/2}^{b/2} dy\,\hat{y} \times ky\,\hat{z} = Ik(\hat{x})\int_{-b/2}^{b/2} y\,dy = Ik\hat{x}\,\frac{y^2}{2}\Bigg|_{-b/2}^{b/2} = 0$$

AS BEFORE!

NOTICE!

- SIDES 1 : 2 YIELD NO FORCE!

FOR SIDE 3...

$$d\vec{l} = dx\,\hat{x} \quad \text{so} \quad \vec{F}_{MAG,3} = I\int dx\,\hat{x} \times ky\,\hat{z} = Iky\Big|_{y=b/2}(-\hat{y})\int_{b/2}^{-b/2} dx = Ik\frac{b}{2}(-\hat{y})\,x\Bigg|_{+b/2}^{-b/2}$$

$$= Ik\frac{b}{2}(-\hat{y})\left[-\frac{b}{2} - \frac{b}{2}\right] = \left(Ik\frac{b^2}{2}\right)\hat{y}$$

LIKEWISE FOR SIDE 4...

$$d\vec{l} = dx\,\hat{x} \quad \ldots\text{SO} \quad \vec{F}_{MAG,4} = I\int_{-b/2}^{b/2} dx\,\hat{x} \times ky\,\hat{z} = Iky\Big|_{y=-b/2}(-\hat{y})\int_{-b/2}^{b/2} dx = Ik\frac{b}{2}(\hat{y})\,x\Big|_{-b/2}^{b/2}$$

$$= Ik\frac{b}{2}(\hat{y})\left[\frac{b}{2} - \left(\frac{-b}{2}\right)\right] = \left(\frac{Ikb^2}{2}\right)\hat{y}$$

NOW, THE NET FORCE ACTING ON THE CURRENT LOOP IS..

$$\vec{F}_{MAG} = \sum_i \vec{F}_{MAG,i} = 0 + 0 + \left(\frac{Ikb^2}{2}\right)\hat{y} + \left(\frac{Ikb^2}{2}\right)\hat{y}$$

$$\therefore \boxed{\vec{F}_{MAG} = Ikb^2\,\hat{y}}$$

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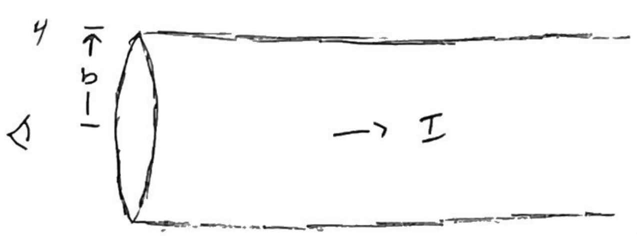
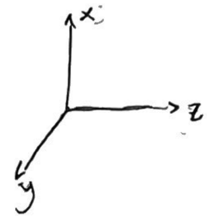




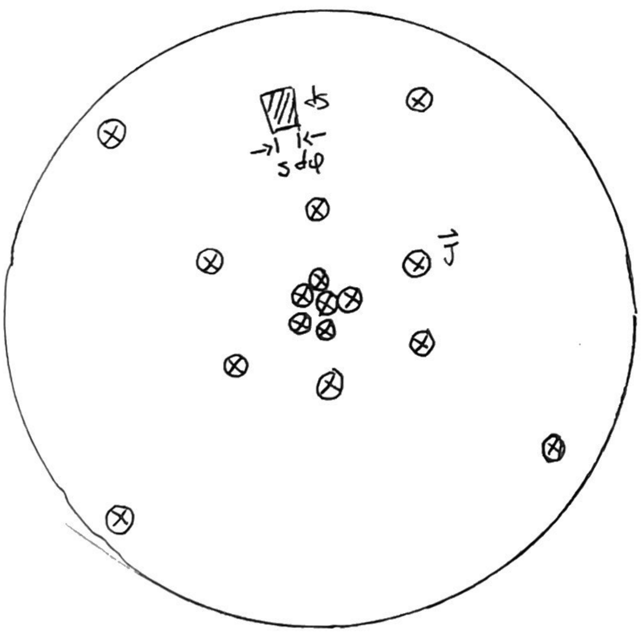
a) $K = \frac{dI}{dl_\perp}$

NOTICE THAT THE CURRENT IS "UNIFORMLY DISTRIBUTED OVER THE SURFACE"

so $K = \frac{I}{l_\perp} = \frac{I}{2\pi b}$ so $\left[\vec{K} = \frac{I}{2\pi b}\hat{y}\right]$

b) $J \propto \frac{1}{s}$ so

$J = \frac{k}{s}$ OR IN VECTOR FORM $\vec{J} = \frac{k}{s}\hat{z}$: $d\vec{a} = s\,d\phi\,ds\,\hat{z}$



LOOKING DOWN THE Z AXIS

so $\vec{J}\cdot d\vec{a} = \frac{k}{s}\hat{z}\cdot s\,d\phi\,ds\,\hat{z} = k\,d\phi\,ds$

so

$$I = \int \vec{J}\cdot d\vec{a} = \int_0^b\int_0^{2\pi} k\,d\phi\,ds = k\int_0^b ds\int_0^{2\pi} d\phi$$

$$= ks\Big|_0^b \phi\Big|_0^{2\pi} = kb\,2\pi$$

so

$$\left[I = 2\pi b k\right]$$

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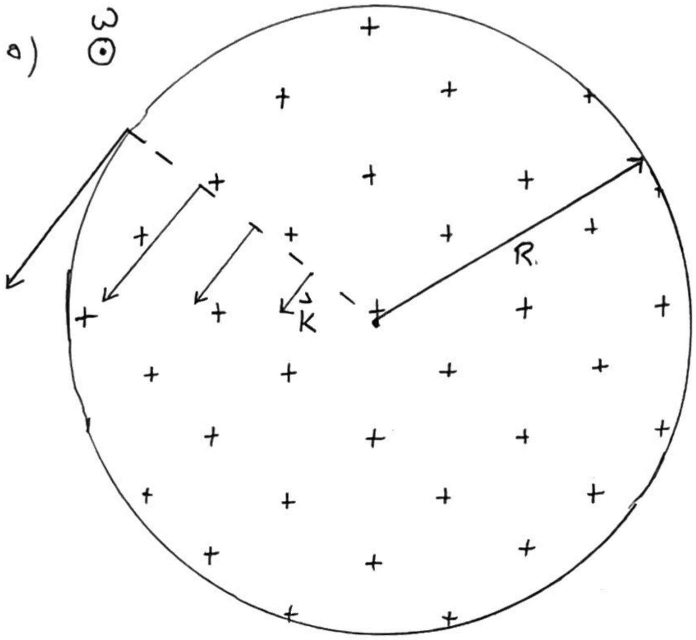
a) 

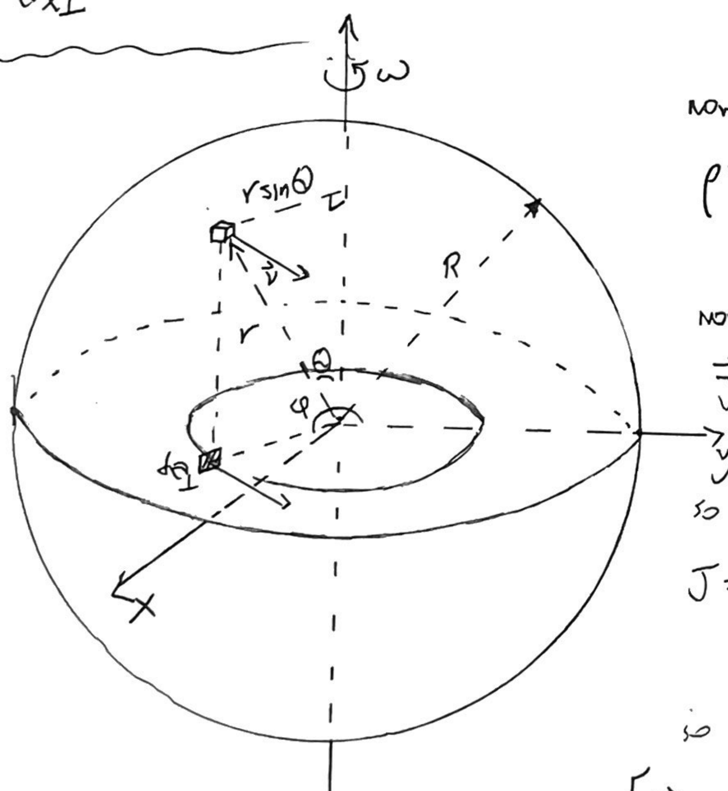
SINCE A UNIFORM SURFACE CHARGE DENSITY

$$\sigma = \frac{dq}{da} = \frac{Q}{\pi R^2}$$

NOW

$$\vec{K} = \frac{d\vec{I}}{dl_\perp} = \sigma \vec{v}$$

NOW $v = r\omega$ SO $\left[K = \sigma r \omega \right]$ OR $\left[\vec{K}(r) = \sigma \omega r \, \hat{\varphi} \right]$

b) 

NOW

SINCE A UNIFORM VOLUME CHARGE DENSITY

$$\rho = \frac{dq}{d\tau} = \frac{+Q}{\frac{4}{3}\pi R^3}$$

NOW

$$\vec{J} = \frac{d\vec{I}}{da_\perp} = \rho \vec{v}$$

SO

$$J = \frac{Q}{\frac{4}{3}\pi R^3} \cdot r \sin\theta \, \omega = \frac{3Q\omega}{4\pi R^3} r \sin\theta$$

SO

$$\left[\vec{J} = \frac{3Q\omega}{4\pi R^3} r \sin\theta \, \hat{\varphi} \right]$$

NOW

$$v = r \sin\theta \, \omega$$