

4.8 For spin-1/2 state evolves with time)

$$\vec{B} \rightarrow \uparrow \uparrow \vec{B} \rightarrow$$

for some time

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

↑  
Hamiltonian

If  $\hat{H}$  is time independent

$$|\psi(t)\rangle = \underbrace{e^{-i\hat{H}t/\hbar}}_{\hat{U}(t)} |\psi(0)\rangle$$

For spin-1/2 particle in  $\vec{B}$  - field what is  $\hat{H}$ ?

$$\hat{H} = - \underbrace{\frac{q\hbar}{2m}}_{\gamma} \vec{B} \cdot \vec{\sigma}$$

$$\hat{H} = \frac{\gamma\hbar}{2} [B_x \hat{\sigma}_x + B_y \hat{\sigma}_y + B_z \hat{\sigma}_z]$$

so

$$\hat{U}(t) = e^{-i \gamma [B_x \hat{\sigma}_x + B_y \hat{\sigma}_y + B_z \hat{\sigma}_z] t/2}$$

known describes how particle responds to  $\vec{B}$  field

known factor

$$\vec{B} \rightarrow \uparrow \uparrow \vec{B} \uparrow \uparrow \rightarrow$$

time t

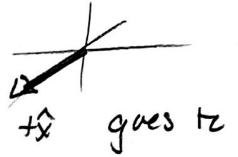
① Suppose  $\vec{B} = B \hat{z}$  with known  $B, t$

a) Determine  $\hat{u}(t)$

b) Suppose  $|\psi(0)\rangle = |\hat{z}\rangle$  get  $|\psi(t)\rangle = |\hat{n}\rangle$   
 $\xrightarrow{t=0}$  what direction



Suppose  $|\psi(0)\rangle = |\hat{x}\rangle$  get  $|\psi(t)\rangle = |\hat{n}\rangle$   
 $\xrightarrow{t=0}$  what direction



② Suppose  $\vec{B} = B \hat{y}$

a) Determine  $\hat{u}(t)$

b) Suppose  $|\psi(0)\rangle = |\hat{z}\rangle$  get  $|\psi(t)\rangle = |\hat{n}\rangle$   
 $\xrightarrow{t=0}$  what direction



③ How does this relate to estimating  $\lambda$  in

$$\hat{u}(\lambda) = e^{-i\lambda \hat{\sigma}_z / 2}$$

$\lambda$  corresponds to what?