

Lecture 5

Thurs 12:30 Physics Seminar WS 117

Fri: HW by 5pm

Supp Ex 10, 12 ← pdf has live links

Ch 2 Conc Q 13

Ch 2 Prob ~~13~~ 18, 21, 49a, 55, 68

two parts - while engines on
- after engines shut off

two parts:

- while racing
- while braking

Mon: Warm Up 3

Motion with constant acceleration

Recall that:

acceleration = rate of change of velocity

conceptual idea

average acceleration from time t_i to t_f

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

time t_f



time t_i



(preliminary)
mathematical
definition

The instantaneous acceleration, denoted "a" and called acceleration is determined by taking the limit as $\Delta t \rightarrow 0$ in the previous definition.

(note text has subscript a_s ~~acc~~. Omit subscript).

Quiz 1 10% → 80% \approx 30% → 70%

Quiz 2 90% \approx 90%

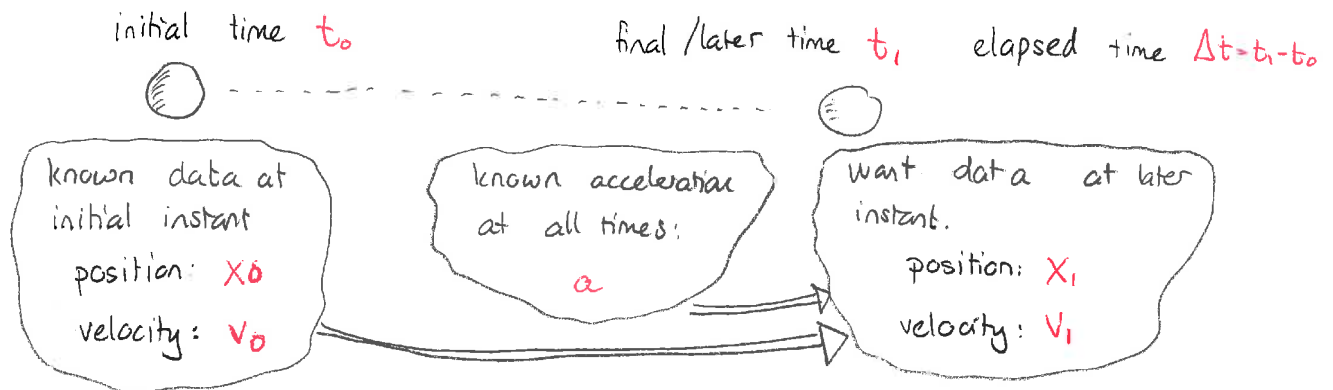
If the acceleration is constant then it is exactly true that

$$a = \frac{\Delta v}{\Delta t}$$

regardless of the time interval. This is not true if acceleration varies with time. We will consider such constant acceleration motion in detail because:

- 1) it occurs in many situations - free fall, ramps
- 2) it illustrates the conceptual scheme of kinematics and shows how one can make it work using algebra.

The scheme is:



It emerges that there are algebraic relationships between these. These are the kinematic equations:

If acceleration is constant then

$$v_1 = v_0 + a \Delta t$$
$$x_1 = x_0 + v_0 \Delta t + \frac{1}{2} a (\Delta t)^2$$
$$v_1^2 = v_0^2 + 2 a \Delta x$$

from $a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta v = a \Delta t$
 $\Rightarrow v_1 - v_0 = a \Delta t \Rightarrow v_1 = v_0 + a \Delta t$

→ from area under v vs t graph

→ combine the two algebraically to eliminate Δt

Here $\Delta x = x_1 - x_0$

Example: In the moving man animation, the man is initially moving with speed 6.0 m/s to the right and is at the -8.0 m location. What is the (minimum) constant acceleration needed so that he does not hit the wall at 10.0 m ?

Answer: 1) Sketch situation



2) list variables

$$t_0 =$$

$$t_1 =$$

3) write kinematic

$$x_0 = -8.0\text{ m}$$

$$x_1 = 10.0\text{ m}$$

eqn + use it

$$v_0 = 6.0\text{ m/s}$$

$$v_1 = 0\text{ m/s}$$

$$v_1^2 = v_0^2 + 2a \Delta x$$

$$\Rightarrow (0\text{ m/s})^2 = (6.0\text{ m/s})^2 + 2a \Delta x$$

$$\Delta x = x_1 - x_0$$

$$= 10\text{ m} - (-8\text{ m})$$

$$\Rightarrow 0\text{ m}^2/\text{s}^2 = 36\text{ m}^2/\text{s}^2 + 2a \times 18\text{ m}$$

$$= 18\text{ m}$$

$$\Rightarrow -36\text{ m}^2/\text{s}^2 = 36\text{ m } a$$

$$\Rightarrow a = \frac{-36\text{ m}^2/\text{s}^2}{36\text{ m}}$$

$$\Rightarrow a = -1.0\text{ m/s}^2$$

□

Note that one cannot find time via $v = \frac{\Delta x}{\Delta t} \Rightarrow 6.0\text{ m/s} = \frac{18\text{ m}}{\Delta t}$ since the velocity is not constant.

Vertical motion: free fall

We can apply one dimensional kinematics to vertical motion with the following conventions:

- 1) position variable = y
- 2) upward motion \Rightarrow v positive
downward " \Rightarrow v negative.

If an object moves solely under the influence of Earth's gravity then it is said to be in free fall (even if it moves upwards)

Quiz 3 \rightarrow try to invent numerical values to answer

not done
today

Observations and experiments on freely falling objects show:

- 1) the acceleration due to gravity near Earth is the same for any object regardless of its mass or velocity
- 2) the acceleration due to gravity near Earth is constant and is

$$a = -g$$

where the magnitude of the acceleration due to Earth's gravity is

$$g = 9.80 \text{ m/s}^2$$

Demo: Guinea + feather.

The kinematic equations apply with "x" replaced by "y"

16 Ball launched vertically

A ball is launched vertically from Earth's surface with speed 10 m/s. The aim of the first part of this exercise is to determine the maximum height reached by the ball and time taken to reach the maximum height.

- a) Sketch the situation, illustrating the ball at two key instants.

"1" Ⓢ max height

"0" Ⓢ launch

List all relevant variables, including the acceleration, for the two instants:

$$t_0 = 0 \text{ s}$$

$$t_1 =$$

$$a = -9.8 \text{ m/s}^2$$

$$y_0 = 0 \text{ m}$$

$$y_1 =$$

$$v_0 = 10 \text{ m/s}$$

$$v_1 = 0 \text{ m/s} \quad (\text{minimum speed at max height})$$

- b) Determine the maximum height reached by the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.

$$v_1^2 = v_0^2 + 2a \Delta y$$

$$(0 \text{ m/s})^2 = (10 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2) \Delta y$$

$$0 \text{ m}^2/\text{s}^2 = 100 \text{ m}^2/\text{s}^2 - 19.6 \text{ m/s}^2 \Delta y$$

$$\Rightarrow -100 \text{ m}^2/\text{s}^2 = -19.6 \text{ m/s}^2 \Delta y$$

$$\Rightarrow \Delta y = \frac{-100 \text{ m}^2/\text{s}^2}{-19.6 \text{ m/s}^2}$$

$$\Rightarrow \Delta y = 5.1 \text{ m}$$

Answer: 5.1 m

- c) Determine the time taken to reach the maximum height by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the maximum height variable.

$$v_1 = v_0 + a \Delta t$$

$$0 \text{ m/s} = 10 \text{ m/s} + (-9.8 \text{ m/s}^2) \Delta t$$

$$\Rightarrow -10 \text{ m/s} = -9.8 \text{ m/s}^2 \Delta t \quad \Rightarrow \Delta t = 1.0 \text{ s}$$

Answer: 1.0 s

The second part of this exercise aims to find the speed of the ball just before returning to the ground using the fall from its maximum height.

d) Sketch the situation for the falling ball, illustrating the ball at two key instants.

max height ○ initial

○ final

List all relevant variables, including the acceleration, for the two instants:

$$t_0 = 0 \text{ s}$$

$$t_1 =$$

$$a = -9.8 \text{ m/s}^2$$

$$y_0 = 5.1 \text{ m}$$

$$y_1 = 0 \text{ m}$$

$$v_0 = 0 \text{ m/s}$$

$$v_1 = ??$$

e) Determine the speed of the ball just before hitting the ground the ball by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for the velocity variable.

$$v_1^2 = v_0^2 + 2a \Delta y \quad \hookrightarrow y_1 - y_0$$

$$\Rightarrow v_1^2 = (0 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(0 \text{ m} - 5.1 \text{ m})$$

$$= 100 \text{ m}^2/\text{s}^2 \quad \Rightarrow v_1 = -\sqrt{100 \text{ m}^2/\text{s}^2} = -10 \text{ m/s}$$

$$\text{speed} = 10 \text{ m/s}$$

Answer: velocity: -10 m/s , speed: 10 m/s .

17 Parachuting package

A package is released from rest at a height of 100m above Earth's surface. It falls freely until it is 40m above Earth's surface. At that instant it deploys a parachute and after this it falls with a constant speed. The aim of this exercise is to determine the time taken to reach Earth.

To do this we will calculate the time taken for the free fall motion, Δt_A , and separately that time taken to fall with the parachute open, Δt_B .

The aim of the first part of this exercise is to determine Δt_A .

- a) Sketch the situation, illustrating the ball at two key instants that will allow one to determine the time for the free fall portion of the motion.

drop ☺

para.
opens ☹

List all relevant variables for the two instants and the acceleration.

$$t_0 = 0 \text{ s}$$

$$t_1$$

$$y_0 = 100 \text{ m}$$

$$y_1 = 40 \text{ m}$$

$$a = -9.8 \text{ m/s}^2$$

$$v_0 = 0 \text{ m/s}$$

$$v_1 = ??$$

- b) Determine Δt_A by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for Δt_A .

$$y_1 = y_0 + v_0 \Delta t_A + \frac{1}{2} a (\Delta t_A)^2$$

$$40 \text{ m} = 100 \text{ m} + 0 \text{ m/s} \Delta t_A + \frac{1}{2} (-9.8 \text{ m/s}^2) (\Delta t_A)^2$$

$$\Rightarrow -60 \text{ m} = -4.9 \text{ m/s}^2 (\Delta t_A)^2$$

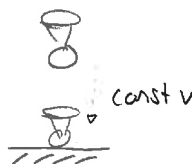
$$\Rightarrow \frac{60 \text{ m}}{4.9 \text{ m/s}^2} = (\Delta t_A)^2$$

$$\Rightarrow (\Delta t_A)^2 = 12.2 \text{ s}^2 \quad \Rightarrow \Delta t_A = 3.5 \text{ s}$$

Answer: 3.5 s.

The second part of this exercise aims to find Δt_B .

- c) Sketch the situation for the falling ball, illustrating the ball at two key instants that will allow one to determine the time for the parachuting portion of the motion.



List all relevant variables for the two instants and the acceleration.

$$t_0 = 3.5s$$

$$t_1 =$$

$$y_0 = 40m$$

$$y_1 = 0m$$

$$a = 0m/s^2$$

$$v_0 =$$

$$v_1 = -34m/s$$

- d) There is one quantity that one can obtain from the free fall part of the motion that will be needed to analyze the parachuting portion of the motion. Identify and compute this and insert it in to the list of variables from the previous part.

need v_0 which is v_1 from free fall part. Doing

$$v_1 = v_0 + a\Delta t_A \Rightarrow v_1 = 0m/s + (-9.8m/s^2)(3.5s)$$

$$\Rightarrow v_1 = -34m/s$$

- e) Now determine Δt_B by using one of the kinematic equations. Write down the equation, substitute from your list of variables and solve for Δt_B .

$$y_1 = y_0 + v_0 \Delta t_B + \frac{1}{2}a(\Delta t_B)^2$$

$$0m = 40m - 34m/s \Delta t_B + \frac{1}{2}0(\Delta t_B)^2$$

$$\Rightarrow -40m = -34m/s \Delta t_B$$

$$\Rightarrow \Delta t_B = 1.2s$$

Answer: 1.2s.

- f) Determine the total time taken to fall.

Answer: 4.7s.

$$\Delta t = \Delta t_A + \Delta t_B = 3.5s + 1.2s$$

$$= 4.7s$$