MAT 201
Larson/Edwards – Section 7.5
Work

In this section we will discuss how to calculate the amount of work that is required to move an object. We can define work in two ways; (1) if the force applied to the object is constant, and (2) if the force applied to the object is a variable force (changing).

Work Done By a Constant Force:
If an object is moved a distance D in the direction of an applied constant force F, then the work W done by the force is defined as $W = FD$.

A ***force*** can be thought of as a push or a pull; a force changes the state of rest or state of motion of a body. For gravitational forces on Earth, it is common to use units of measure corresponding to the weight of an object.

Units of Work: Since work is force times distance, units for work are given in the form "distance-weight" or "weight-distance". For instance, if the force is given in pounds and the distance is given in feet, the units for work will be given in ***foot-pounds***. In the metric system, work is measured in ***newton-meters*** (or ***joules***).

Ex: Determine the work done lifting a 75 lb object 7 feet.

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Constant Force: $W = F \cdot D$

$W = 75 \cdot 7$

$W = \boxed{525 \text{ ft.-lb.}}$

If a variable force is applied to an object, calculus is needed to determine the work done, because the amount of force changes as the object changes position.

Definition of Work Done by a Variable Force:
If an object is moving along a straight line (horizontally) by a continuously varying force *F(x)*, then the work *W* done by the force as the object is moved from $x = a$ to $x = b$ is:

$$W = \int_a^b F(x)\, dx$$

If the object is moving along a straight line (vertically) by a continuously varying force *F(y)*, then the work *W* done by the force as the object is moved from $x = c$ to $x = d$ is:

$$W = \int_c^d F(y)\, dy$$

We will discuss work done by a variable force in two ways:

1. $\Delta W = (\text{force})(\text{distance increment}) = (F)(\Delta x).$
2. $\Delta W = (\text{force increment})(\text{distance}) = (\Delta F)(x).$

Compressing a spring is an example of (1), the force times the distance increment.

We will need to use ***Hooke's Law*** to determine the force of compressing (or stretching) as spring:

The force F required to compress or stretch a spring (within its elastic limits) is proportional to the distance d that the spring is compressed or stretched from its original length. That is,

$$F = kd$$

where the constant of proportionality k (the spring constant) depends on the specific nature of the spring.

Ex: A force of 5 pounds compresses a 15-inch spring a total of 3 inches. How much work is done in compressing the spring from a length of 10 inches to a length of 6 inches?

Pumping water or lifting a chain is an example of (2), the force increment times the distance.

Ex: A rectangular tank with a base 10 feet by 10 feet and a height of 6 feet is full of water. The water weighs 62.4 pounds per cubic foot. How much work is done in pumping water out over the top edge in order to empty the tank?

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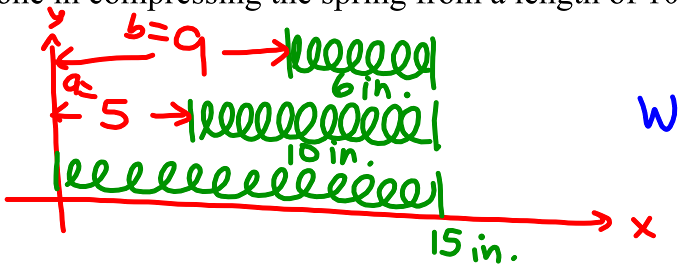


$$W = \int_a^b F(x)\,dx$$

Hooke's Law: $F = kx$

$$5 = k \cdot 3$$

$$\frac{5}{3} = k$$

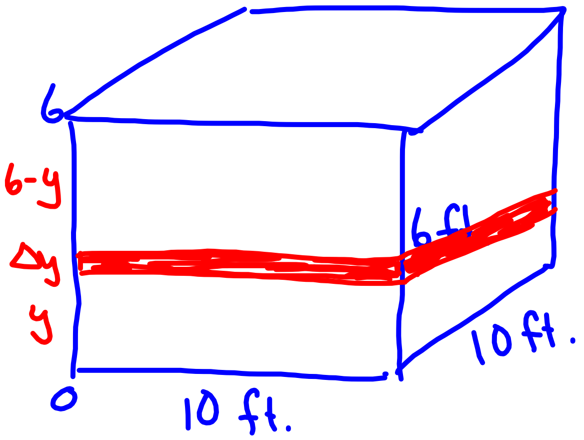
$$F(x) = \frac{5}{3}x$$

$$W = \int_5^9 \frac{5}{3}x\,dx$$

$$\boxed{W = 46.\overline{6}\ \text{in.-lb.}}$$

Ex: A rectangular tank with a base 10 feet by 10 feet and a height of 6 feet is full of water. The water weighs $\boxed{\text{62.4 pounds per cubic foot.}}$ How much work is done in pumping water out over the top edge in order to empty the tank?

$$W = \int_c^d F(y)\, dy$$



Force: $\underbrace{10 \cdot 10 \cdot \Delta y\, \cancel{ft.^3}}_{\text{Volume of shaded region.}} \cdot \dfrac{62.4\,lbs}{\cancel{ft.^3}}$

$$\text{Force} = 6240\, \Delta y \text{ lbs.}$$

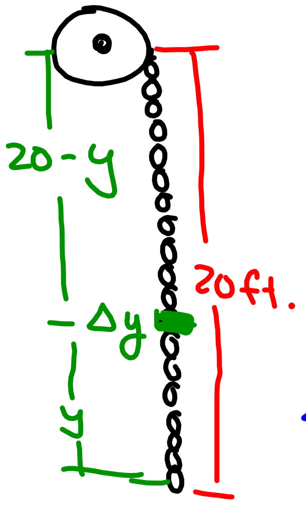
$$\Delta W = 6240\, \Delta y \cdot (6-y)$$

$$W = \int_0^6 6240(6-y)\, dy$$

$$\boxed{W = 112{,}320 \text{ ft-lbs.}}$$

Ex: Consider a 20-foot chain that weighs 3 pounds per foot hanging from a winch 20 feet above ground level. Find the work done by the winch in winding up one-third of the chain.

Ex: Consider a 20-foot chain that weighs 3 pounds per foot hanging from a winch 20 feet above ground level. Find the work done by the winch in winding up the entire chain.



$$W = \int_c^d F(y)\,dy$$

Force: $3 \cdot \Delta y$

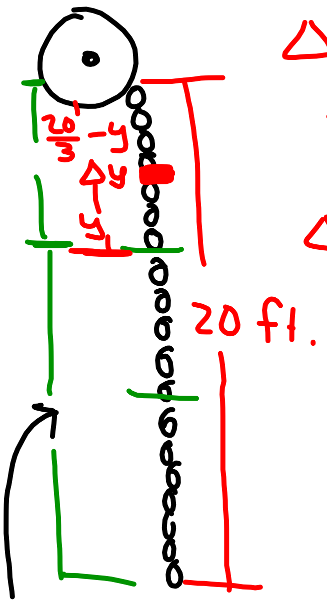
$$\Delta W = \Delta F \cdot \text{dist.}$$

$$\Delta W = 3\Delta y(20-y)$$

$$W = \int_0^{20} 3(20-y)\,dy$$

$$\boxed{W = 600 \text{ ft.-lbs.}}$$

Ex: Consider a 20-foot chain that weighs 3 pounds per foot hanging from a winch 20 feet above ground level. Find the work done by the winch in winding up one-third of the chain.



$$\Delta W = \Delta F \cdot \text{dist.}$$

$$\boxed{W_1 + W_2 = 333\tfrac{1}{3} \text{ ft.lb}}$$

$$F = 3\,\Delta y$$

$$\Delta W = 3\,\Delta y \cdot \frac{20}{3} - y$$

$$W_2 = \int_0^{20/3} 3\left(\frac{20}{3} - y\right) dy = \frac{400}{6} \text{ ft.lb.}$$

$$+$$

$$W_1 = \underbrace{\frac{\cancel{3}\,\text{lbs.}}{\cancel{\text{ft.}}} \cdot \frac{40\,\cancel{\text{ft.}}}{\cancel{3}}}_{\text{Force}} \cdot \underbrace{\frac{20}{3}\text{ ft}}_{\text{dist.}} = \frac{800}{3} \text{ ft.-lbs.}$$