Taylor Larrechea
Dr. Gustafson
MATH 360
HW 9.8

9.8 # 2, 10c, 10f, 15

9.8 | #2 $\vec{v} = [0, \cos(xyz), \sin(xyz)]$, $P: (2, \frac{1}{2}\pi, 0)$

$\text{div}(\vec{v}) = \vec{\nabla} \cdot \vec{v}$

$\vec{\nabla} \cdot \vec{v} = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}\cos(xyz) + \frac{\partial}{\partial z}\sin(xyz)$

$\frac{\partial}{\partial x}(0) = 0$ $\frac{\partial}{\partial y}(\cos(xyz)) = -xz\sin(xyz)$ $\frac{\partial}{\partial z}(\sin(xyz)) = xy\cos(xyz)$

$\vec{\nabla} \cdot \vec{v} = -xz\sin(xyz) + xy\cos(xyz)$

$\vec{\nabla} \cdot \vec{v}(2, \frac{\pi}{2}, 0) = -2(0)\sin(2 \cdot \frac{\pi}{2} \cdot 0) + 2 \cdot \frac{\pi}{2} \cdot \cos(2 \cdot \frac{\pi}{2} \cdot 0)$

$= 0 + \pi(1)$

$\vec{\nabla} \cdot \vec{v}(2, \frac{\pi}{2}, 0) = \pi$

$$\boxed{\begin{gathered} \vec{\nabla} \cdot \vec{v} = -xz\sin(xyz) + xy\cos(xyz) \\ \vec{\nabla} \cdot \vec{v}\Big|_{(2, \frac{\pi}{2}, 0)} = \pi \end{gathered}}$$

9.8 #10 $\vec{v} = x\hat{\imath} - y\hat{\jmath}$

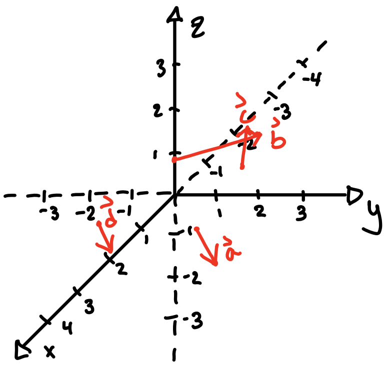
c.)

By inspection we can see that $div(\vec{v}) = 0$

$$div(\vec{v}) : \vec{\nabla}\cdot\vec{v} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]\cdot\left[x, -y, 0\right]$$
$$= \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(-y) + \frac{\partial}{\partial z}(0)$$

$$div(\vec{v}) = 1 - 1 = 0$$

$0 = 0 \therefore \quad div(\vec{v}) = 0$



$(1,1) \to \vec{v} = \hat{\imath} - \hat{\jmath}$
$(-1,-1) \to \vec{v} = -\hat{\imath} + \hat{\jmath}$
$(-1,1) \to \vec{v} = -\hat{\imath} - \hat{\jmath}$
$(1,-1) \to \vec{v} = \hat{\imath} + \hat{\jmath}$

$\boxed{div(\vec{v}) = 0}$

f.) $\vec{v} = (x^2+y^2)^{-1}(-y\hat{\imath} + x\hat{\jmath}) \longrightarrow$ must be Zero

$$\vec{v} = \frac{-y}{x^2+y^2}\hat{\imath} + \frac{x}{x^2+y^2}\hat{\jmath} + 0\hat{z}$$

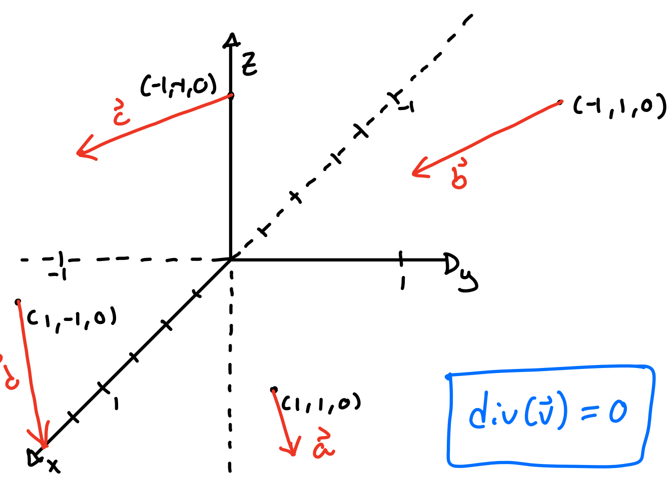
$$div(\vec{v}) = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]\cdot\left[\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right]$$
$$div(\vec{v}) = \frac{\partial}{\partial x}\left(\frac{-y}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) + \frac{\partial}{\partial z}(0)$$

$$\frac{\partial}{\partial x}\left(-\frac{y}{x^2+y^2}\right) \qquad \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)$$

$$\frac{(x^2+y^2)(0) + y(2x)}{(x^2+y^2)} = \frac{2xy}{(x^2+y^2)} \qquad \frac{(x^2+y^2)(0) - x(2y)}{(x^2+y^2)} = \frac{-2xy}{(x^2+y^2)}$$

$$div(\vec{v}) = \frac{2xy}{x^2+y^2} - \frac{2xy}{x^2+y^2} = 0$$

$(1,1) \to \vec{v} = \left[-\tfrac{1}{2}, \tfrac{1}{2}, 0\right] (\vec{a})$
$(-1,1) \to \vec{v} = \left[-\tfrac{1}{2}, -\tfrac{1}{2}, 0\right] (\vec{b})$
$(-1,-1) \to \vec{v} = \left[\tfrac{1}{2}, -\tfrac{1}{2}, 0\right] (\vec{c})$
$(1,-1) \to \vec{v} = \left[\tfrac{1}{2}, \tfrac{1}{2}, 0\right] (\vec{d})$



$\boxed{div(\vec{v}) = 0}$

9.8 #15 $f(x,y) = \cos^2(x) + \sin^2(y)$

$$\nabla^2 f = \text{div}(\text{grad}(f))$$

$$\text{grad}(f): \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\text{div}(\text{grad}(f)) = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right] \cdot \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\text{div}(\text{grad}(f)) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial f}{\partial x} = -2\sin(x)\cos(x) = -\sin(2x) \qquad \frac{\partial f}{\partial y} = 2\sin(y)\cos(y)$$

$$\frac{\partial^2 f}{\partial x^2} = -2\cos(2x) \qquad \frac{\partial^2 f}{\partial y^2} = 2\cos(2y)$$

$$\boxed{\nabla^2 f = 2(\cos(2y) - \cos(2x))}$$