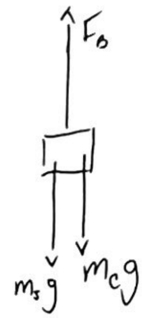
PHYS 230 HOMEWORK SET 2

TRIVIAL

P1. $l = 75.0\,m$
$w = 30.0\,m$
$h = 30.0\,m$
$m_s = 3.75\times10^{6}\,kg$

$\therefore V_{ship} = lwh$

FREE-BODY DIAGRAM



THE MAXIMUM CARGO LOAD WOULD CORRESPOND TO THE SHIP DISPLACING ITS VOLUME OF WATER, WITH THE WATER LEVEL BEING AT THE TOP OF THE SHIP

$$V_{\substack{fluid\\displaced}} = V_{ship}$$

NOW, NEWTON'S 2ND LAW TAKES THE FORM...

$$\sum \vec{F}_y = 0 \quad \alpha$$

$$F_B - m_s g - m_c g = 0 \qquad \text{with } F_B = \rho_{fluid} V_{fluid}\, g = \rho_F\, lwh\, g$$

SOLVINGING FOR m_c ...

$$m_c = \rho_F\, lwh - m_s = \left(1000\,\tfrac{kg}{m^3}\right)(75.0\,m)(30.0\,m)(30.0\,m) - 3.75\times10^{6}\,kg$$

$$= 6.75\times10^{7}\,kg - 3.75\times10^{6}\,kg$$

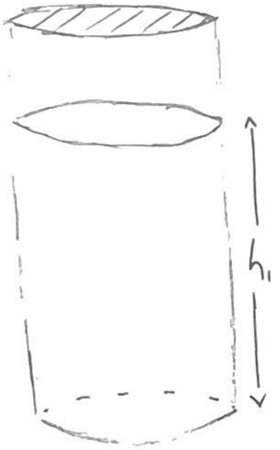
$$\left[\, m_c = 6.38\times10^{7}\,kg \,\right]$$

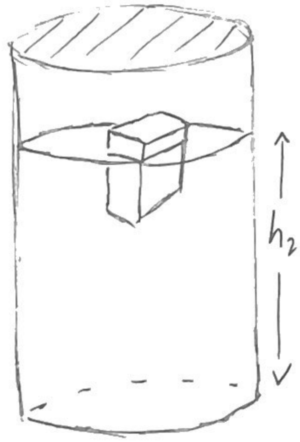
P2

FIRST GLASS..

SECOND GLASS...

Let A be the cross-sectional area of end of the glass





$m_{H_2O,1} = 0.200\,kg$

$\rho_{H_2O} = 1000\,kg/m^3$

now $V_{H_2O,1} = \dfrac{m_{H_2O,1}}{\rho_{H_2O}} = \dfrac{0.200\,kg}{1000\,kg/m^3}$

$= 2.00\times10^{-4}\,m^3 = Ah_1$

$M_{H_2O,2} = 0.175\,kg$

$\rho_{H_2O} = 1000\,kg/m^3$

In the second glass, the volume of the just the water is...

$$V'_{H_2O,2} = \frac{M_{H_2O,2}}{\rho_{H_2O}} = \frac{0.185\,kg}{1000\,kg/m^3} = 1.85\times10^{-4}\,m^3$$

Now, the floating ice displaces H_2O, which raises the height of the H_2O.

Consider a free body diagram for the ice..

now

$\sum F_y = 0$

$F_B - m_{ICE}\,g = 0$ where $F_B = \rho_{FL} V_{FL}\, g$

so

$\rho_{FL} V_{FL}\, g = m_{ICE}\, g$ so $V_{FL} = \dfrac{m_{ICE}}{\rho_{FL}} = \dfrac{2.5\times10^{-2}\,kg}{1000\,kg/m^3}$

$= 2.5\times10^{-5}\,m^3$

THIS DISPLACED FLUID ADDS TO THE VOLUME OF THE GLASS, SO THE TOTAL VOLUME IS..

$$V_{H_2O,2} = V'_{H_2O,2} + V_{fl} = 2.10 \times 10^{-4} m^3 = Ah_2$$

SO, TAKING THE RATIO OF THE TOTAL VOLUMES..

$$\frac{V_{H_2O,1}}{V_{H_2O,2}} = \frac{Ah_1}{Ah_2} = \frac{h_1}{h_2} = \frac{2.00 \times 10^{-4} m^3}{2.10 \times 10^{-4} m^3}$$

OR

$$\boxed{\frac{h_2}{h_1} = 1.05}$$

P3

$\theta_1 = 47^\circ$

$\Delta y = 8.0\,m$

THE KINEMATIC EQUATIONS FOR FREE FALL TAKE THE FORM..

$$v_{2y} = v_{1y} - g\Delta t \qquad v_{2x} = v_{1x}$$

$$\Delta y = v_{1y}\Delta t - \frac{1}{2} g \Delta t^2 \qquad \Delta x = v_{1x}\Delta t$$

$$v_{2y}^2 = v_{1y}^2 - 2g\Delta y$$

AS THE H_2O HITS THE BUILDING HORIZONTALLY, $v_{2y} = 0$, AND THE 3rd EQUATION TAKES THE FORM..

$$0 = v_{1y}^2 - 2g\Delta y \quad \text{so} \quad v_{1y} = \sqrt{2g\Delta y} = v_1 \sin\theta_1$$

so

$$v_1 = \frac{\sqrt{2g\Delta y}}{\sin\theta_1} = \frac{\sqrt{2(9.8\,m/s^2)(8.0\,m)}}{\sin(47^\circ)}$$

$$\left[v_1 = 17\,m/s \right]$$

THE CROSS-SECTIONAL AREA OF THE GARDEN HOSE IS..

$$A_1 = \pi R_1^2 = \pi(2.5\times10^{-2}\,m)^2 = 2.0\times10^{-3}\,m^2$$

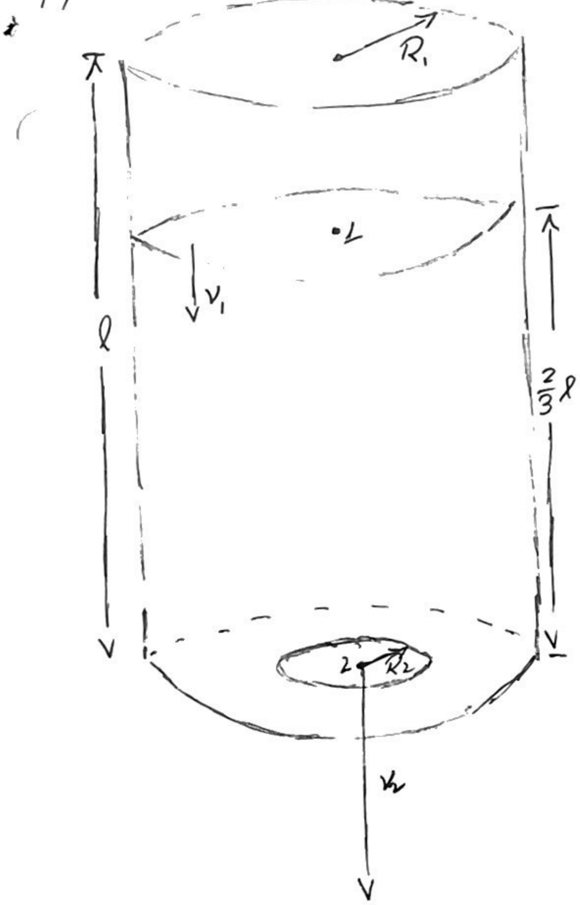
Diameter Dimension?

SO THE FLOW RATE IS.

$$Q = v_1 A_1 = (17\,m/s)(2.0\times10^{-3}\,m^2) = 3.3\times10^{-2}\,\frac{m^3}{s}$$

$$(17\,m/s)(4.9\times10^{-4}\,m^3) = 8.4\times10^{-3}\,m^3/s$$

P4



$l = 3.0m$

$R_1 = 0.25m$

$R_2 : 0.11m$

CONSIDERS PTS 1 & 2, WHICH CORRESPOND TO THE TOP OF THE WATER LEVEL AND A POINT IN THE HOLE WHERE THE WATER DRAINS OUT

NOTICE: $P_1 = P_2 = P_{ATM}$

SO BERNOULLI'S EQN TAKES THE FORM

$$\cancel{P_1} + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \cancel{P_2} + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

NOW THE EQUATION OF CONTINUITY IS ...

$$v_1 A_1 = v_2 A_2 \quad \text{so} \quad v_1 \pi R_1^2 = v_2 \pi R_2^2$$

$$\text{so} \quad \boxed{v_2 = v_1 \frac{R_1^2}{R_2^2}}$$

PLUGGING THIS EXPRESSION INTO BERNOULLI'S EQN..

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \frac{1}{2}\rho v_1^2 \frac{R_1^4}{R_2^4} + \rho g y_2 \qquad \text{NOW SOLVE FOR } v_1 ..$$

$$\rho g (y_1 - y_2) = \frac{1}{2}\rho\left(\frac{R_1^4}{R_2^4} - 1\right) v_1^2 \qquad \text{BUT } y_1 - y_2 = \frac{2}{3} l$$

so

$$g \cdot \frac{2}{3} l = \frac{1}{2}\left(\frac{R_1^4}{R_2^4} - 1\right) v_1^2 \quad \therefore \quad v_1^2 = \frac{\frac{4}{3} g l}{\left(R_1^4/R_2^4 - 1\right)} \quad \text{so} \quad v_1 = 2\left[\frac{g l}{3\left(R_1^4/R_2^4 - 1\right)}\right]^{1/2}$$

$$\therefore v_1 = 2\left[\frac{(9.8 m/s^2)(3.0m)}{3\left(\frac{(0.25m)^4}{(0.11m)^4} - 1\right)}\right]^{1/2} = 1.2 \ m/s$$

SO THE SPEED OF THE WATER EXITING THE HOLE IS..

$$v_2 = v_1 \frac{R_1^2}{R_2^2} = (1.2 m/s)\frac{(0.25m)^2}{(0.11m)^2} = 6.4 m/s$$

$$\boxed{v_2 = 6.4 m/s}$$

b) THE FLOW RATE IS GIVEN BY...

$$Q = v_2 A_2 = v_2 \pi R_2^2 = (3.2\,m/s)\pi(0.11\,m)^2 = 0.24\,\frac{m^3}{s}$$

$$\boxed{Q = 0.24\,m^3/s}$$

c) $\boxed{v_1 = 1.2\,m/s}$

d) NOW, LOOK AT AN OBJECT IN FREE FALL...

$$v_1 = -0.62\,m/s$$

$$\Delta y = -\tfrac{2}{3}\ell = -2.0\,m$$

$$v_2^2 = v_1^2 - 2g\Delta y$$

so

$$v_2 = -\sqrt{v_1^2 - 2g\Delta y} = -\sqrt{(-1.2\,m/s)^2 - 2(9.8\,m/s^2)(-2.0\,m)}$$

$$= -6.4\,m/s$$

SO THE SPEED OF THE FREELY FALLING OBJECT IS 6.4 m/s WHEREAS THE SPEED OF THE H_2O EXITING THE HOLE IS 6.4 m/s.