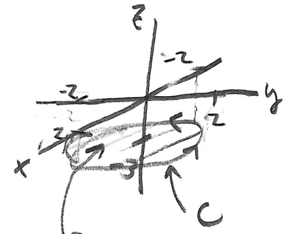
# Ch10.9 In-Class Examples: Stokes' Theorem

① (Ex 3 on p. 467), part 2.




$\vec{F} = [y, xz^3, -zy^3]$ , $C: x^2+y^2=4, \ z=-3$

(a) $\iint_S \text{curl}(\vec{F})\cdot\vec{n}\,dA = \iint_S (-28)\,dA = -28\pi(2)^2$

$= -112\pi$ (see text)

(b) $\iint_S \text{curl}(\vec{F})\cdot\vec{n}\,dA = \oint_C \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt = -\int_0^{2\pi}(56+52\cos 2t)\,dt$

$= -56t\Big|_0^{2\pi} - \cancelto{0}{\dfrac{52\sin 2t}{2}}\Big|_0^{2\pi}$

$= -112\pi$

$C: \vec{r}(t) = [2\cos t, 2\sin t, -3], \ 0 \le t \le 2\pi$

$\vec{r}\,'(t) = [-2\sin t, 2\cos t, 0]$

$\vec{F}(\vec{r}(t)) = [2\sin t, -54\cos t, 24\sin^3 t]$

$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = -4\sin^2 t - 108\cos^2 t$

$= -2(1-\cos 2t) - 54(1+\cos 2t)$

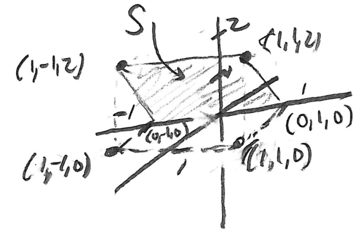
$= -56 + 2\cos 2t - 54\cos 2t$

$= -56 - 52\cos 2t$

# Ch 10.9 Stokes' Thm In-Class Examples (Continued)

(2) $\vec{F} = [x^2, 0, y^2]$ $S$ = rectangle (tilted) given by vertices $(0,-1,0)$, $(0,1,0)$, $(1,-1,2)$, $(1,1,2)$

Evaluate $\iint_S curl(\vec{F})\cdot\vec{n}\,dA$ directly.




Soln Need two cross products:

(1) $curl(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & 0 & y^2 \end{vmatrix} = [2y, 0, 0]$

Parameterize S

$\vec{r}(u,v) = [u, v, 2u]$, $0 \le u \le 1$, $-1 \le v \le 1$

(For each y between -1 & 1, $z = 2x$)

$curl(\vec{F})$ on $\vec{r}(u,v) = [2v, 0, 0]$

(2) $\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{vmatrix} = [-2, 0, 1]$

Next, need dot product:

$curl(\vec{F})\big|_{\vec{r}(u,v)} \cdot \vec{r}_u \times \vec{r}_v = [2v, 0, 0]\cdot[-2, 0, 1] = -4v$

Then

$$\iint_S curl(\vec{F})\cdot\vec{n}\,dA \overset{\text{p. 443 (Ch 10.6)}}{=} \iint_S curl(\vec{F})\big|_{\vec{r}(u,v)} \cdot (\vec{r}_u \times \vec{r}_v)\,du\,dv$$

$$= \int_{-1}^{1}\int_0^1 -4v\,du\,dv$$

$$= \int_{-1}^{1} -4uv\Big|_0^1\,dv$$

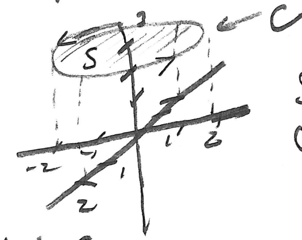
$$= \int_{-1}^{1} -4v\,dv$$

$$= -2v^2\Big|_{-1}^{1}$$

$$= 0$$

③ Evaluate $\oint_C \vec{F}\cdot\vec{r}'(s)\,ds$ using Stokes' Thm

$\vec{F} = [-y, x, z]$ , C: circle $x^2+y^2=4$, $z=3$.




S = disk
C = boundary (circumference)

Soln $\oint_C \vec{F}\cdot\vec{r}'(s)\,ds$

$= \iint_S \text{curl}(\vec{F})\Big|_{\vec{r}(u,v)}\cdot(\vec{r}_u\times\vec{r}_v)\,du\,dv$

$= \int_0^{2\pi}\int_0^2 2u\,du\,dv$

$= \int_0^{2\pi} u^2\Big|_0^2\,dv$

$= \int_0^{2\pi} 4\,dv$

$= 4v\Big|_0^{2\pi}$

$= 8\pi$

Parameterize S:

$\vec{r}(u,v) = [u\cos v, u\sin v, 3]$

$0\le u\le 2,\ 0\le v\le 2\pi$

$\vec{r}_u\times\vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = [0,0,u]$

Curl($\vec{F}$)

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & z \end{vmatrix} = [0,0,2]$