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MATH 361 HW 1.1

1a ✓, 2a ✓, 3a ×, 4a ×, 5a ✓, 6a ✓, 7 ×, 9 ✓, 11 ×

Problem 1a

$f(x) \equiv x\cos(x) - 2x^2 + 3x - 1 = 0$ on $[0.2, 0.3]$

- There are no discontinuities for f on $[0.2, 0.3]$, $f \in [0.2, 0.3]$ ✓

- $f(0.2) = 0.2\cos(0.2) - 2(0.2)^2 + 3(0.2) - 1 = -0.283987$
  $f(0.3) = 0.3\cos(0.3) - 2(0.3)^2 + 3(0.3) - 1 = 0.006601$

  $f(0.2) \cdot f(0.3) = -0.001875 < 0$ ✓

$\boxed{\therefore \exists\, c \in [0.2, 0.3] \text{ such that } f(c) = 0}$

$f(x) \equiv x\cos(x) - 2x^2 + 3x - 1 = 0$ on $[1.2, 1.3]$

- There are no discontinuities for f on $[1.2, 1.3]$, $f \in [1.2, 1.3]$ ✓

- $f(1.2) = 1.2\cos(1.2) - 2(1.2)^2 + 3(1.2) - 1 = 0.154829$
  $f(1.3) = 1.3\cos(1.3) - 2(1.3)^2 + 3(1.3) - 1 = -0.132252$

  $f(1.2) \cdot f(1.3) = 0.154829 \cdot -0.132252 = -0.020476 < 0$ ✓

$\boxed{\therefore \exists\, c \in [1.2, 1.3] \text{ such that } f(c) = 0}$

Problem 2a

$f(x) = \sqrt{x} - \cos(x) = 0$ on $[0,1]$

- There are no discontinuities for f on $[0,1]$, $f \in [0,1]$ ✓

- $f(0) = \sqrt{0} - \cos(0) = 0 - 1 = -1$
  $f(1) = \sqrt{1} - \cos(1) = 1 - 0.540302 = 0.459698$

  $f(0) \cdot f(1) = -1 \cdot 0.459698 = -0.459698 < 0$ ✓

$$\boxed{\therefore \ \exists\, c \in [0,1] \text{ such that } f(c) = 0}$$

Problem 5a)

$f(x) = \frac{1}{3}(2 - e^{x} + 2x) \qquad [0,1]$

$f'(x) = \frac{1}{3} \cdot (0 - e^{x} + 2) = \frac{2}{3} - \frac{e^{x}}{3} \quad : \quad f'(x) = \frac{1}{3}(2 - e^{x})$

$0 = \frac{1}{3}(2 - e^{x}) \quad : \quad 0 = 2 - e^{x} \quad : \quad 2 = e^{x} \quad x = \ln(2)$

$\boxed{x = \ln(2)}$

Problem 6a)

$f(x) = \frac{2x}{(x^2+1)}$ $[0,2]$: $f(x) = 2x\,(x^2+1)^{-1}$

$f'(x) = 2\cdot(x^2+1)^{-1} - 2x\,(x^2+1)^{-2}\cdot 2x = \frac{2}{(x^2+1)} - \frac{4x^2}{(x^2+1)^2}$

$0 = \frac{2}{(x^2+1)} - \frac{4x^2}{(x^2+1)^2}$: $\frac{4x^2}{(x^2+1)^2} = \frac{2}{(x^2+1)}$: $\frac{1}{(x^2+1)} = \frac{1}{2x^2}$: $2x^2 = x^2+1$

$x^2 = 1 \quad \therefore \quad x = \pm 1$

$\boxed{x=1}$

Problem 7)

a.) $f(x) = 1 - e^x + (e-1)\sin\left(\frac{\pi}{2}x\right)$: $f'(x) = -e^x + \frac{\pi}{2}(e-1)\cos\left(\frac{\pi}{2}x\right)$

$0 = -e^x + \frac{\pi}{2}(e-1)\cos\left(\frac{\pi}{2}x\right)$ $\therefore$ $e^x = \frac{\pi}{2}(e-1)\cos\left(\frac{\pi}{2}x\right)$

Problem 9

$f(x) = x^3$ @ $x_0 = 0$ $\quad P_n(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^{(n)}}{n!}$: $R_n(x) = \frac{f^{(n)}\xi(x)}{(n+1)!}(x-a)^{(n)}$

a.) $f(0) = 0$

$f'(x) = 3x^2$: $f'(0) = 0$

$f''(x) = 6x$: $f''(0) = 0$

$\boxed{P_2(x) = 0}$

b.) $R_2(x) = \frac{6x \cdot \xi(x)}{3!}(x-0)^2 = x \cdot \xi(x) \cdot x^2$: $R_2(x) = \xi(x) \cdot x^3 = \xi(0.5) \cdot 0.5^3 = 0.125$

$\boxed{R_2(0.5) = 0.125}$

c.) $f(1) = 1$

$f'(x) = 3x^2$: $f'(1) = 3$

$f''(x) = 6x$: $f''(1) = 6$: $f'''(x) = 6$

$\boxed{P_2(x) = 1 + \frac{3 \cdot (x-1)}{1!} + \frac{6 \cdot (x-1)^2}{2!}}$

d.) $R_2(x) = \frac{6 \cdot \xi(x)}{3!}(x-1)^2$ $\therefore$ $R_2(0.5) = \xi(0.5) \cdot (-0.5)^2 = 0.5 \cdot 0.25 = 0.125$

$\boxed{R_2(0.5) = 0.125}$

Problem 11

$$f(x) = e^x \cos(x) \;@\; x=0 \;:\; P_n(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!} \;:\; R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-a)^{(n+1)}$$

a.) $f(0) = 1$

$$f'(x) = e^x\cos(x) - e^x\sin(x) \;:\; f'(0) = 1$$

$$f''(x) = e^x\cancel{\cos}(x) - e^x\sin(x) - e^x\sin(x) - e^x\cancel{\cos}(x)$$

$$f''(x) = -2e^x\sin(x) \;:\; f''(0) = 0$$

$$f'''(x) = -2e^x\sin(x) - 2e^x\cos(x) = -2e^x(\sin(x) + \cos(x))$$

$$f'''(\xi(x)) = -2e^{\xi(x)}(\sin(\xi(x)) + \cos(\xi(x)))$$

$$P_2(x) = 1 + x \;:\; P_2(0.5) = 1.5$$

$$f(0.5) = e^{0.5}\cdot\cos(0.5) = 1.446889 \;:\; |f(0.5) - P_2(0.5)| = 0.0531109$$

$$R_2(0.5) = \frac{f^{(3)}(\xi(0.5))}{3!}(0.5-0)^3 = \frac{-2e^{\xi(0.5)}(\sin(\xi(0.5)) + \cos(\xi(0.5)))\cdot(0.125)}{6}$$

$$\boxed{R_2(0.5) = -(0.125)e^{\xi(0.5)}(\sin(\xi(0.5)) + \cos(\xi(0.5))) \;,\; |f(0.5) - P_2(0.5)| = 0.053119}$$