Last time...

- The *total change in thermal energy* for any process...

$$\boxed{\Delta E_{th} = nC_V \Delta T} \quad \text{(any ideal-gas process)}$$

- Quantity of *heat* needed to change the temp of n moles of gas by ΔT

Molar specific heat at constant *volume*

$$Q = nC_V \Delta T \quad \text{(temp change at constant volume)}$$

$$Q = nC_P \Delta T \quad \text{(temp change at constant pressure)}$$

Molar specific heat at constant *pressure*

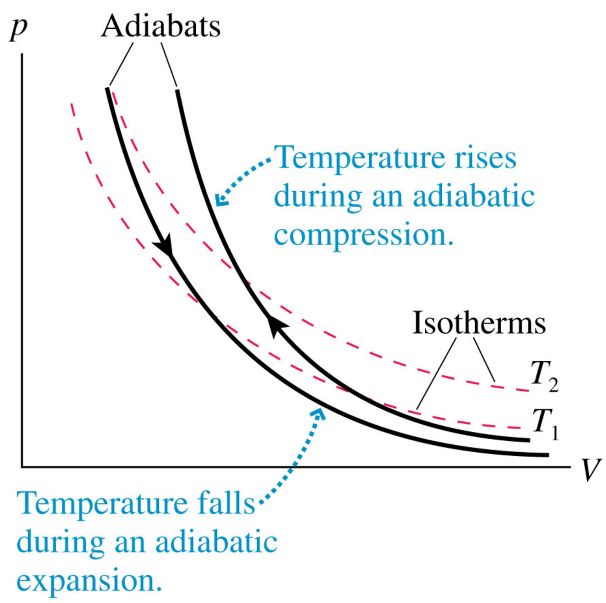
Last time…

- *Molar specific heat* relationship..

$$\boxed{C_P = C_V + R}$$

- For an *adiabatic process* (Q = 0)..

$$\boxed{pV^\gamma = \text{constant}}$$



where

$$\gamma = \frac{C_P}{C_V} = 1.67 \quad \text{monatomic gas}$$

$$= 1.40 \quad \text{diatomic gas}$$

i.e. 17.9: An adiabatic compression

Air containing gasoline vapor is admitted into the cylinder of an internal combustion engine at 1.00 atm pressure and 30° C. The piston rapidly compresses the gas from 500 cm³ to 50 cm³, a compression ratio of 10.

a. What are the final temp and pressure of the gas? $T = 758\ K$
b. Show the compression on a *pV* diagram.
c. How much work is done to compress the gas?

$PV = nRT$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$T_2 = \frac{T_1 P_2 V_2}{P_1 V_1} = 25\left(\frac{1}{10}\right)(303K) = 758K$

$P_1 = 1.01 \times 10^5 Pa$

$T_1 = 303 K$

$\frac{V_1}{V_2} = 10$

$P_2 = \frac{P_1 V_1^{\gamma}}{V_2^{\gamma}} = P_1 (10)^{\gamma} = (1.01 \times 10^5 Pa)(10)^{1.4} = 25.1 \times 10^5 Pa$

$\Delta E_{th} = nC_V \Delta T = 190\ J$

$W?$

$\Delta E_{th} = nC_V \Delta T$

$PV = nRT$

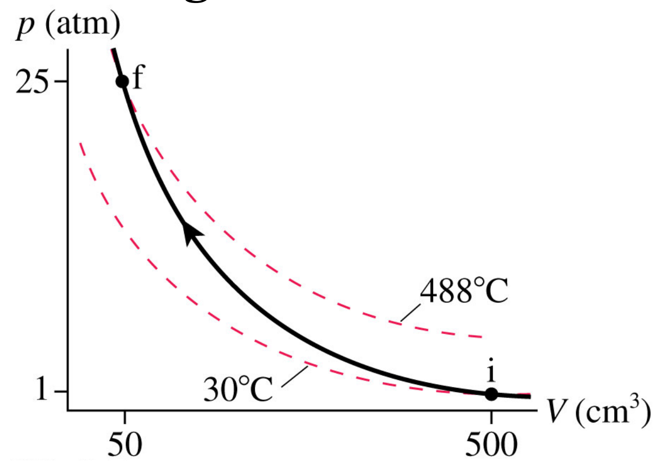
$n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 Pa)(5.0 \times 10^{-4} m^3)}{(8.31\ J/mol\,K)(303K)} = 0.020\ mol$

$n = 0.02\ mol$

i.e. 17.9: An adiabatic compression

Air containing gasoline vapor is admitted into the cylinder of an internal combustion engine at 1.00 atm pressure and 30° C. The piston rapidly compresses the gas from 500 cm^3 to 50 cm^3, a compression ratio of 10.

a. What are the final temp and pressure of the gas?
b. Show the compression on a *pV* diagram.
c. How much work is done to compress the gas?



Quiz Question 1

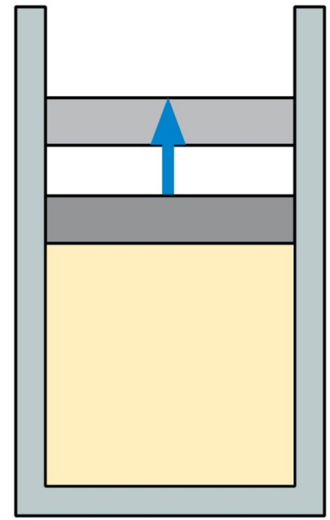
A gas in a container expands rapidly, pushing the piston out. The temperature of the gas

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Delta E_{th} = nC_V\Delta T = Q + W$$

$$\Delta T = \frac{W}{nC_V} \quad : \quad W = -1$$

$$\frac{T_1P_2V_2}{P_1V_1} = T_2$$

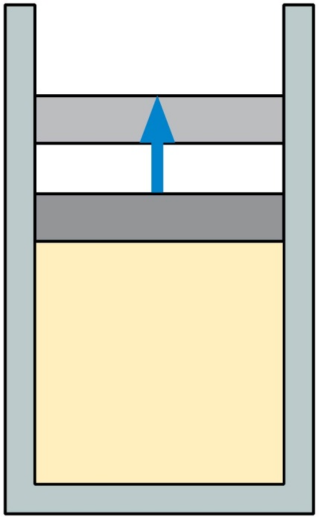


1. rises.
2. is unchanged.
3. falls.
4. can't say without knowing more.

Quiz Question 2

A gas in a container expands rapidly, pushing the piston out. The temperature of the gas falls.

This is because



1. the gas pressure falls.
2. the gas density falls.
3. heat energy is removed.
4. work is done.
5. both 3. and 4.

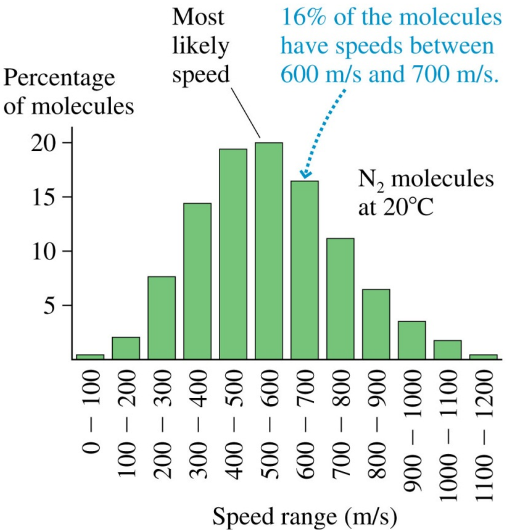
Knight: Chapter 18

The Micro/Macro Connection

(*Molecular Speeds and Collisions, Pressure in a Gas, & Temperature*)

Molecular Speeds and Collisions

Consider Fig 18.2 which displays the *distribution of molecular speeds* in a sample of gas…



Notice:

- Changing the *temp* or the *gas* changes the *most likely speed* but NOT the *shape* of the distribution.

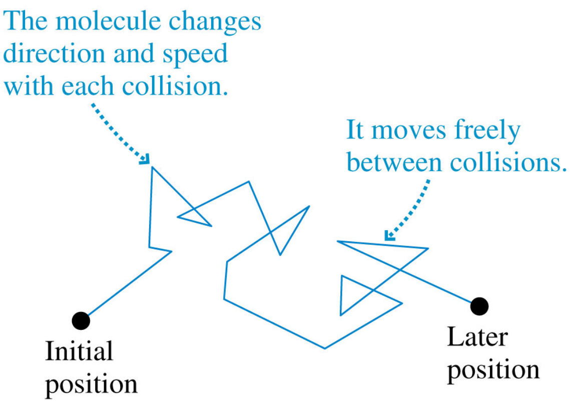
The micro/macro connection is built on the idea that:

- the *macroscopic properties* of a system (i.e. temp or pressure) are related to the *average* behavior of the *atoms and molecules*.

Mean free path..

- is the *average distance between the collisions* given by…

$$\boxed{\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}}$$



- N/V is the *number density* of the gas.
- r is the *radius of the molecules* modeled as hard spheres.
- Rule of thumb:

$$r \approx 0.5 \times 10^{-10}\ \mathrm{m} \quad \text{(monatomic gas)}$$
$$r \approx 1.0 \times 10^{-10}\ \mathrm{m} \quad \text{(diatomic gas)}$$

i.e. 18.1: The mean free path at room temp

What is the mean free path of a nitrogen molecule at 1.0 atm pressure and room temperature (20° C)?

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

$r = 1.0\times10^{-10}\,m$
$P = 1.01\times10^{5}\,Pa$
$T = 293\,K$
$k_B = 1.38\times10^{-23}\,J/K$

$$PV = Nk_BT$$

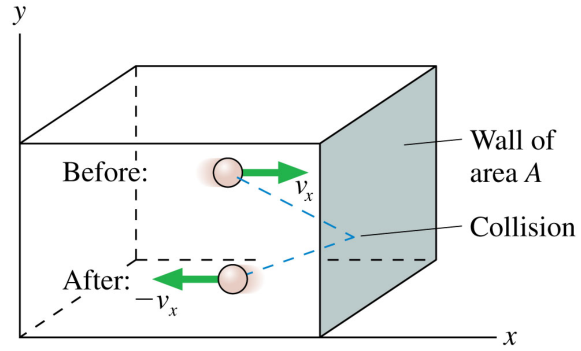
$$\frac{P}{k_BT} = \frac{N}{V} = \frac{1.01\times10^{5}\,Pa}{1.38\times10^{-23}\,J/K\,(293\,K)} = 2.5\times10^{25}\,m^{-3}$$

$$\frac{N}{V} = 2.5\times10^{25}\,m^{-3}$$

Pressure in a *Gas*…

- is due to *collisions* of the molecules with the walls of the container.
- is the *average force* on the walls *per unit area*.

$$p = \frac{F}{A} = \frac{1}{3}\frac{N}{V}mv_{rms}^2$$



Notice:

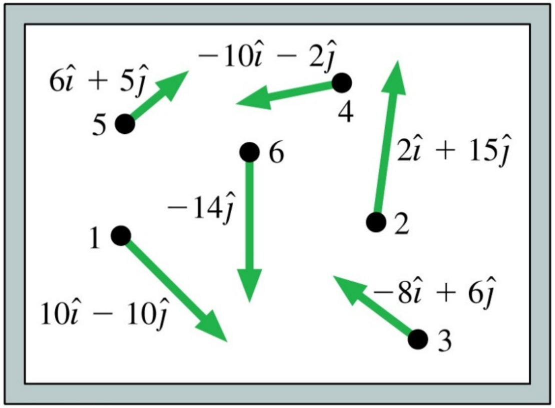
- the *average velocity* of many molecules traveling in random directions is *zero*.
- v_{rms} is the *root-mean-square speed* of the molecules:

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

i.e. 18.2: Calculating the root-mean-square speed

Figure 18.8 shows the velocities of all the molecules in a six molecule, 2D gas.

Calculate and compare the *average velocity*, the *average speed*, and the *rms speed*.



i.e. 18.2: Calculating the root-mean-square speed

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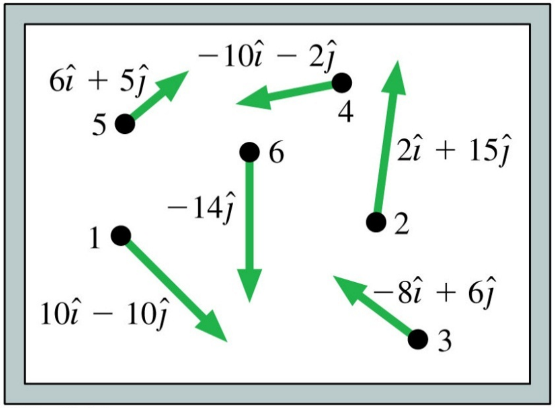

Molecule	v_x	v_y	v_x^2	v_y^2	v^2	v
1	10	−10	100	100	200	14.1
2	2	15	4	225	229	15.1
3	−8	6	64	36	100	10.0
4	−10	−2	100	4	104	10.2
5	6	5	36	25	61	7.8
6	0	−14	0	196	196	14.0
Average	0	0			148.3	11.9

Temperature

- Define ϵ_{ave} to be the *average translational* KE of molecules in a gas.

$$\boxed{\epsilon_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{1}{2}mv_{rms}^2}$$

Using our expression for pressure and the ideal gas law…

$$\mathcal{E}_{Avg} = (\overline{KE}) = (\overline{\tfrac{1}{2}mv^2}) = \tfrac{1}{2}m(v^2)_{AVG} = \tfrac{1}{2}mv_{Rms}^2$$

$$P = \tfrac{1}{3}(N/V)\,mV_{rms}^2 \qquad \frac{PV}{N} = k_BT$$

$$\frac{3PV}{N} = mV_{rms}^2$$

$$3k_BT = mv_{rms}^2$$

$$\mathcal{E}_{avg} = \tfrac{3}{2}k_BT$$

Temperature

- Define ϵ_{ave} to be the *average translational* KE of molecules in a gas.

$$\boxed{\epsilon_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{1}{2}mv_{rms}^2}$$

Using our expression for pressure and the ideal gas law…

$$\boxed{\epsilon_{avg} = \frac{3}{2}k_B T}$$

Notice:

- Temp is proportional to the average *translational* KE of molecules in a gas.
  - A *higher temp* corresponds to a *larger* ϵ_{avg} and thus to *higher molecular speeds*.
- *Absolute zero* is the temp where $\epsilon_{avg} = 0$, therefore all molecular motion ceases.

Temperature

- Define ϵ_{ave} to be the *average translational* KE of molecules in a gas.

$$\boxed{\epsilon_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{1}{2}mv_{rms}^2}$$

Using our expression for pressure and the ideal gas law…

$$\boxed{\epsilon_{avg} = \frac{3}{2}k_B T}$$

Equating these expressions and solving for rms speed…

Temperature

- Define ε_{ave} to be the *average translational* KE of molecules in a gas.

$$\epsilon_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{1}{2}mv_{rms}^2$$

Using our expression for pressure and the ideal gas law…

$$\epsilon_{avg} = \frac{3}{2}k_B T$$

Equating these expressions and solving for rms speed…

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

i.e. 18.4: Total microscopic KE
i.e. 18.5: Calculating a rms speed

What is the total translational KE of the molecules in 1.0 mol of gas at standard pressure and room temperature (20° C)?

What is the rms speed of nitrogen molecules at room temperature?