Ch. 4 Quantum Mechanics in Three Dimensions

$p_x \to \frac{\hbar}{i}\frac{\partial}{\partial x}$, $p_y \to \frac{\hbar}{i}\frac{\partial}{\partial y}$, $p_z \to \frac{\hbar}{i}\frac{\partial}{\partial z}$: $p \to \frac{\hbar}{i}\nabla$: $i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$: $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$

$\Psi_n(r,t) = \psi_n(r)e^{-iE_nt/\hbar}$: $\Psi(r,t) = \sum c_n\psi_n(r)e^{-iE_nt/\hbar}$: $P_\ell^m(x) \equiv (1-x^2)^{|m|/2}\left(d/dx\right)^{|m|}P_\ell(x)$

$P_\ell(x) \equiv \frac{1}{2^\ell \ell!}\left(\frac{d}{dx}\right)^\ell (x^2-1)^\ell$: $Y_\ell^m(\theta,\varphi) = \epsilon\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-|m|)!}{(\ell+|m|)!}}\,e^{im\varphi}P_\ell^m(\cos\theta)$: $j_\ell(x) = (-x)^\ell\left(\frac{1}{x}\frac{d}{dx}\right)^\ell\frac{\sin x}{x}$

$n_\ell(x) = -(-x)^\ell\left(\frac{1}{x}\frac{d}{dx}\right)^\ell\frac{\cos x}{x}$: $E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}$: $E_1 = -13.6\ \text{eV}$: $\psi_{100}(r,\theta,\varphi) = \frac{e^{-r/a}}{\sqrt{\pi a^3}}$

$\psi_{n\ell m} = \sqrt{\left(\frac{2}{na}\right)^3\frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}}\,e^{-r/na}\left(\frac{2r}{na}\right)^\ell\left[L_{n-\ell-1}^{2\ell+1}(2r/na)\right]Y_\ell^m(\theta,\varphi)$: $\Delta E = E_1\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$: $R = 1.097\times10^7\,m^{-1}$: $P = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}(f(r))^2 r^2\sin^2\theta\, dr\,d\theta\,d\varphi$: $L = r\times p$

$L_x = yp_z - zp_y$, $L_y = zp_x - xp_z$, $L_z = xp_y - yp_x$: $[L_x,L_y] = i\hbar L_z$, $[L_y,L_z] = i\hbar L_x$, $[L_z,L_x] = i\hbar L_y$

$[L^2,L] = 0$: $L_\pm = L_x \pm iL_y$: $L^2 f_\ell^m = \hbar^2\ell(\ell+1)f_\ell^m$, $L_z f_\ell^m = \hbar m f_\ell^m$: $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}$

$L_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi}$: $L_\pm = \pm\hbar e^{\pm i\varphi}\left(\partial/\partial\theta \pm i\cot\theta\,\partial/\partial\varphi\right)$: $L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$

$[S_x,S_y] = i\hbar S_z$, $[S_y,S_z] = i\hbar S_x$, $[S_z,S_x] = i\hbar S_y$: $\chi = \begin{pmatrix}a\\b\end{pmatrix} = a\chi_+ + b\chi_-$

$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$: $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$: $S^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1&0\\0&1\end{pmatrix}$: $S_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$: $S_y = \frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$: $S_z = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$, $\lambda = \frac{\hbar}{2}$: $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$, $\lambda = -\frac{\hbar}{2}$: $\chi_+^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $\lambda = \frac{\hbar}{2}$: $\chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, $\lambda = -\frac{\hbar}{2}$

$\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$, $\lambda = \frac{\hbar}{2}$: $\chi_-^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$, $\lambda = -\frac{\hbar}{2}$: $\chi_+^{(z)} = \begin{pmatrix}1\\0\end{pmatrix}$, $\lambda = \frac{\hbar}{2}$: $\chi_-^{(z)} = \begin{pmatrix}0\\1\end{pmatrix}$, $\lambda = -\frac{\hbar}{2}$

$\chi = \left(\frac{a+b}{\sqrt{2}}\right)\chi_+^{(i)} + \left(\frac{a-b}{\sqrt{2}}\right)\chi_-^{(i)}$: $\langle S\rangle = \chi^* S\chi$: $H = -\mu\cdot B = -\gamma B\cdot S$: $\chi(t) = \begin{pmatrix}\cos(\alpha/2)e^{i\gamma B_0 t/2}\\ \sin(\alpha/2)e^{-i\gamma B_0 t/2}\end{pmatrix}$

$P_\pm^{(i)} = |c_\pm^{(i)}|^2$: $c_\pm^{(i)} = \chi_\pm^{(i)*}\chi_\pm$: $i\hbar\frac{\partial\chi}{\partial t} = H\chi$