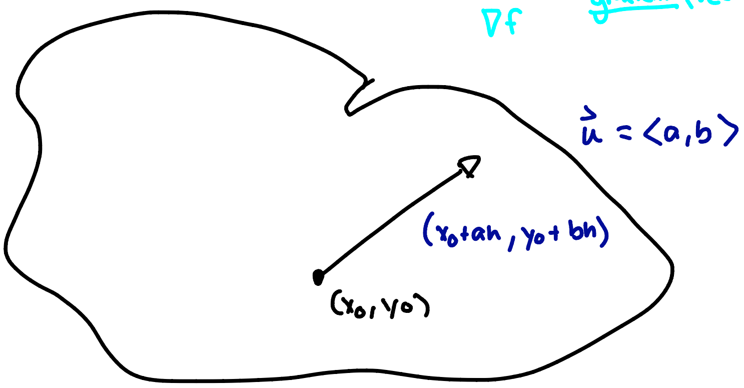
$$D_{\vec{u}} f(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+ah,\, y_0+bh) - f(x_0,y_0)}{h}$$

$$D_{\vec{u}} f(x_0,y_0) = \underbrace{\langle f_x, f_y \rangle_{(x_0,y_0)}}_{\nabla f} \cdot \vec{u}$$

gradient (vector) of f



Pf: Let $g(h) = f(x_0+ah,\, y_0+bh)$

$$D_{\vec{u}} f(x_0,y_0) = g'(0) = \lim_{h\to 0} \frac{g(h)-g(0)}{h}$$

$$\frac{dG}{dh} = \frac{\partial G}{\partial x}\frac{dx}{dh} + \frac{\partial G}{\partial y}\frac{dy}{dh}$$

$$= \frac{\partial f}{\partial x} a + \frac{\partial f}{\partial y} b$$

Proof

$$= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \cdot \langle a, b \rangle$$

14.6.4

$f(x,y,z) = xe^y + ye^z + ze^x$ $\qquad \vec{v} = \langle 5,3,-2 \rangle$

$P_{x_0,y_0,z_0}(0,0,0)$

$D_{\vec{u}} f(0,0,0)$

$\nabla f \langle e^y + ze^x,\ xe^y + e^z,\ ye^z + e^x \rangle$

$\nabla f(0,0,0) = \langle e^0 + 0e^0,\ 0e^0 + e^0,\ 0e^0 + e^0 \rangle$

$\nabla f(0,0,0) = \langle 1,1,1 \rangle$

$\vec{v} = \langle 5,3,-2 \rangle \quad \sqrt{25+9+4} = \sqrt{38}$

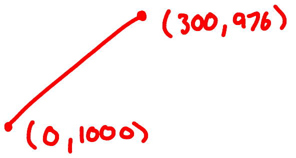
$\vec{u} = \left\langle \frac{5}{\sqrt{38}}, \frac{3}{\sqrt{38}}, \frac{-2}{\sqrt{38}} \right\rangle$

$\langle 1,1,1 \rangle \cdot \left\langle \frac{5}{\sqrt{38}}, \frac{3}{\sqrt{38}}, \frac{-2}{\sqrt{38}} \right\rangle$

$\frac{5}{\sqrt{38}} + \frac{3}{\sqrt{38}} - \frac{2}{\sqrt{38}}$

$\frac{6}{\sqrt{38}}$

$$\boxed{\frac{6}{\sqrt{38}}}$$

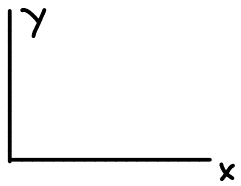


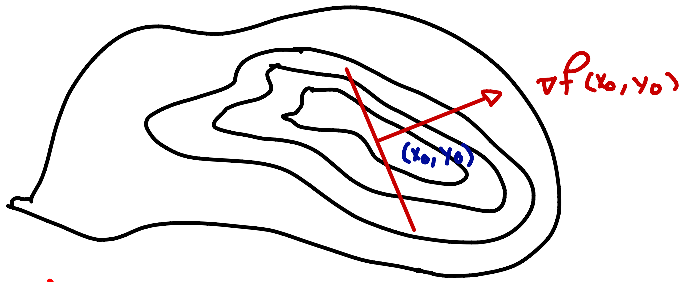
$$\frac{976-1000}{300-0} = \frac{-24}{300} = \frac{-2}{25}$$

Maximize: $D_{\vec{u}} f(x_0,y_0) = \nabla f(x_0,y_0) \cdot \vec{u} = |\nabla f(x_0,y_0)|\ |\vec{u}| \cos\theta$

where θ is the angle between $\vec{u} = \nabla f(x_0,y_0)$

- Max directional derivative at (x_0,y_0) is $|\nabla f(x_0,y_0)|$ and it occurs when $\vec{u}$ is in the direction $\nabla f(x_0,y_0)$
- Min is $-|\nabla f(x_0,y_0)|$ and it occurs in direction $-\nabla f(x_0,y_0)$
- No change in f in the direction that is $\perp$ to $\nabla f(x_0,y_0)$





$F(x,y,z) = 2(x-7)^2 + (y-2)^2 + (z-9)^2 \quad P(8,4,11)$

Write eqn. of the plane tangent to the surface

$$2(x-7)^2 + (y-2)^2 + (z-9)^2 = 10$$

Think of this as the level 10 surface of F.

∇f is orthogonal to the level surface

$$\nabla f = \langle 4(x-7),\ 2(y-2),\ 2(z-9) \rangle$$

$$\nabla f(8,4,11) = \langle 4, 4, 4 \rangle$$

$$4(x-8) + 4(y-4) + 4(z-11) = 0$$

$$\boxed{4(x-8) + 4(y-4) + 4(z-11) = 0}$$

$Z = 900 - 0.005x^2 - 0.01y^2 \quad (60, 40, 866) \quad \langle -1, -2.4 \rangle$

$$\frac{\partial z}{\partial x} = -0.01x \qquad \frac{\partial z}{\partial y} = -0.02y$$

a.) Ascend (0.8 m/s)

b.) Descend (-0.98 m/s)

$-0.01(60) = -0.6 \qquad -0.02(40) = -0.8$

$\langle -0.6, -0.8 \rangle \cdot \langle 0, -1 \rangle$

$(-0.6)(0) - 0.8(-1)$

$0.8m$

$NW = \langle -1, 1 \rangle$

$u = \sqrt{1+1}$

$u = \frac{1}{\sqrt{2}} \langle -1, 1 \rangle$

$u = \langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\nabla f \cdot \vec{u} = \langle -1, -2.4 \rangle \cdot \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$$\frac{1}{\sqrt{2}} - \frac{2.4}{\sqrt{2}} = -0.98 \text{ m/s}$$