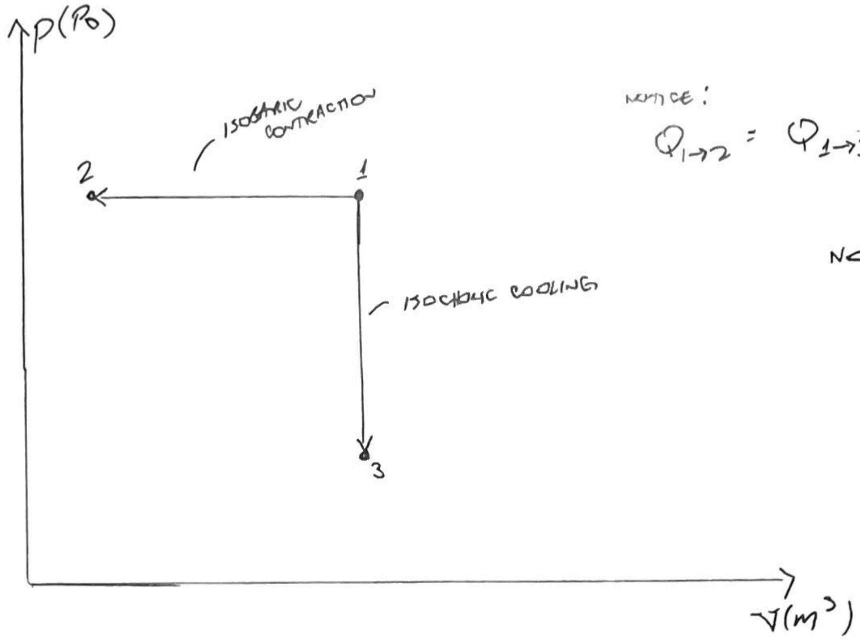
HOMEWORK SET 4 SOLUTIONS

CQ2



NOTICE: $Q_{1\to 2} = Q_{1\to 3} \equiv Q$

NOTE: $Q = -|Q|$ (HEAT WAS REMOVED)

FOR EITHER PROCESS,

$$\Delta E_{TH} = Q + W$$

FOR THE ISOCHORIC COOLING PROCESS...

$$W = 0 \quad \text{so}$$

$$\Delta E_{TH,1\to 3} = Q = -|Q| \qquad \text{NOTICE } \Delta E_{TH,1\to 3} < 0$$

FOR THE ISOBARIC CONTRACTION PROCESS...

$$W = +|W| \qquad (+ \text{ WORK WAS DONE ON THE GAS})$$

so

$$\Delta E_{TH,1\to 2} = -|Q| + |W| > -|Q| = \Delta E_{TH,1\to 3}$$

so

$$\Delta E_{TH,1\to 2} > \Delta E_{TH,1\to 3}$$

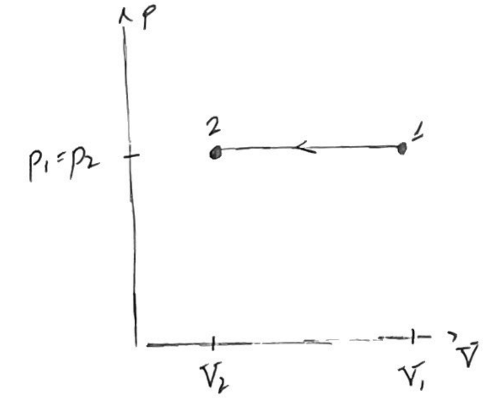
or

$$(T_{f,1\to 2} - T_i) > (T_{f,1\to 3} - T_i) \quad \text{SINCE } \Delta E_{TH} = mc\Delta T$$

so

$$\left[T_{f,1\to 2} > T_{f,1\to 3} \right]$$

Concept Question 9



This process corresponds to an <u>isobaric</u> contraction

Notice that

- $n_1 = n_2 \equiv n$
- $p_1 = p_2 \equiv p$

so

$pV_1 = nRT_1$ since $V_2 < V_1$

$pV_2 = nRT_2$ $\quad T_2 < T_1$

$\therefore \Delta E_{th,1\to2} < 0 \quad \therefore \Delta E_{th,1\to2} \equiv -|\Delta E_{th}|$

so $\boxed{E_{th,1} > E_{th,2}}$

Also, since the gas is being compressed,

$\boxed{W > 0} \quad \therefore W \equiv +|W|$

Now since..

$\Delta E_{th} = W + Q$

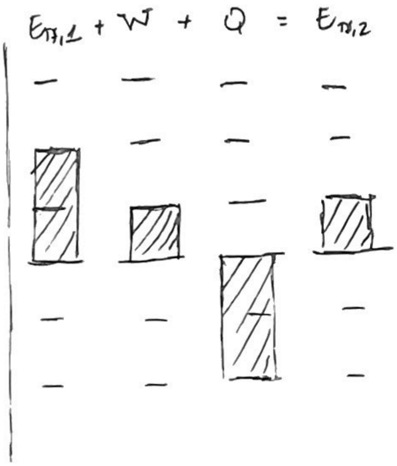
$-|\Delta E_{th}| = +|W| + Q$

so

$Q = -|\Delta E_{th}| - |W| < 0$

$\equiv -|Q|$

So, a 1st law bar chart describing this process is of the form..



so...

- Place cylinder on ice to remove heat
- Remove cylinder from ice when gas reaches desired volume

Concept Question 11

a) $Q < 0$ since heat is removed

$W > 0$ since the gas is compressed

$\Delta E_{th} < 0$ since $T_f < T_i$ (since $V_f < V_i$; $p_f = p_i$)

b) See image above!

P1. $V_i = 3.60\times10^3 cm^3 \times \left(\frac{1m}{100cm}\right)^3 = 3.60\times10^{-3} m^3$

$n = 0.100 mol$

$T_i = 120.^\circ C + 273 = 393K$

$V_f = 5.40\times10^3 cm^3 = 5.40\times10^{-3} m^3$

NOW

$$p_i = \frac{nRT_i}{V_i} = \frac{(0.100mol)(8.31 J/mol\cdot K)(393K)}{(3.60\times10^{-3}m^3)} = 9.07\times10^4 Pa$$

a) FOR CONSTANT PRESSURE...

$$W = -p\Delta V = -(9.07\times10^4 Pa)(5.40\times10^{-3}m^3 - 3.60\times10^{-3}m^3) = -163J$$

$$\boxed{W = -163J}$$

b) FOR CONSTANT TEMPERATURE...

$$p_f = \frac{p_iV_i}{V_f} = \frac{(9.07\times10^4 Pa)(3.60\times10^{-3}m^3)}{(5.40\times10^{-3}m^3)} = 6.05\times10^4 Pa$$

$$W = -nRT\ln\left(\frac{V_f}{V_i}\right) = -(0.100mol)\left(8.31\frac{J}{mol\cdot K}\right)(393K)\ln\left(\frac{5.40\times10^{-3}m^3}{3.60\times10^{-3}m^3}\right)$$

$$\boxed{W = -132J}$$

P2 a) FOR THE ISOBARIC PROCESS...

$$p_i = p_f = 9.07 \times 10^4 Pa$$

NOW

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f} \therefore T_f = T_i \frac{V_f}{V_i} = \frac{(393K)(5.40 \times 10^{-3} m^3)}{(3.60 \times 10^{-3} m^3)} = 590K$$

NOW

i) $\Delta E_{TH} = n C_V \Delta T = (0.100 mol)(20.9 \frac{J}{mol \cdot K})(590K - 393K)$

$$\left[\Delta E_{TH} = 412 J\right]$$

20.8 to two sig figs

ii) $W = -163K$ FROM ISOBARIC AND $\Delta E_{TH} = Q + W$ so $Q = \Delta E_{TH} - W$

$$= 412J - (-163K)$$

$$\left[Q = 575J\right]$$

b) FOR THE ISOTHERMAL PROCESS...

$$T_f = T_i = 393K$$

$$p_f = 6.05 \times 10^4 Pa$$

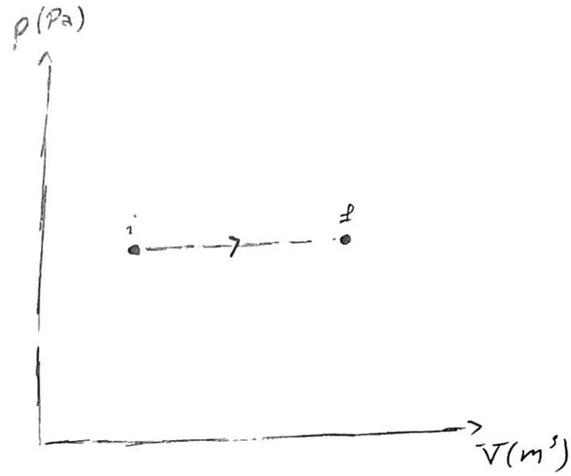
$\Delta E_{TH} = n C_V \Delta T = 0 = Q + W$ so $Q = -W = 132J$

$$\left[\Delta E_{TH} = 0\right]$$

$$\left[Q = 13.2 J\right]$$

FOR AN

• P3 ISOBARIC EXPANSION..



a) $W = -p\Delta V$

$\Delta E_{th} = nC_V \Delta T = Q + W$

ALSO

$Q = nC_p \Delta T$

NOTICE:

• $W < 0$

• $T_f > T_i$ SINCE $V_f > V_i$ $\therefore$ $p_f = p_i$

SO $\Delta E_{th} > 0$

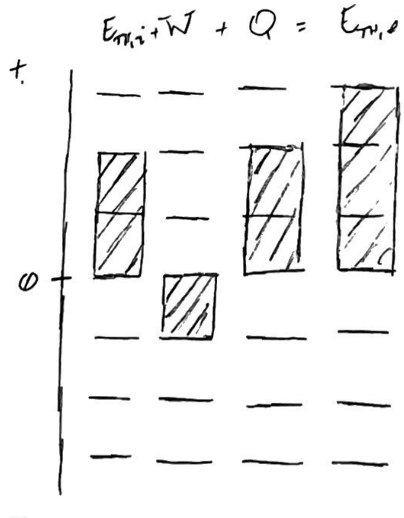
$Q > 0$

b) • W/ A CONSTANT PRESSURE ESTABLISHED W/ A FIXED AMOUNT OF MASS SITTING ON THE PISTON, HEAT THE GAS TO EXPAND THE GAS.

• WHEN DESIRED VOLUME IS REACHED, REMOVE HEAT SOURCE.

• INSULATE THE BOTTOM OF THE CYLINDER.

c)



P4 $M_{H_2O} = 0.220\,kg$

$M_{ICE} = .055\,kg = 5.5\times10^{-2}\,kg$

$C_{ICE} = 2{,}090\,\frac{J}{kg\,K}$

$T_{i,ICE} = 259K$ $\quad T_{f,ICE} = T_{f,H_2O}$

$C_{H_2O} = 4{,}190\,\frac{J}{kg\cdot K}$

$T_{i,H_2O} = 299K$

$L_{f,H_2O} = 3.33\times10^{5}\,J/kg$

$Q_{ICE} + Q_{H_2O} = 0$

$$2M_{ICE}C_{ICE}(14K) + 2M_{ICE}L_{f,H_2O} + 2M_{ICE}C_{H_2O}(T_f - 273K) + M_{H_2O}C_{H_2O}(T_f - 299K) = 0$$

NOW SOLVING FOR T_f...

$$(2M_{ICE}C_{H_2O} + M_{H_2O}C_{H_2O})T_f = M_{H_2O}C_{H_2O}(299K) + 2M_{ICE}C_{H_2O}(273K) - 2M_{ICE}C_{ICE}(14K) - 2M_{ICE}L_{f,H_2O}$$

NOW

$$2M_{ICE}C_{H_2O} + M_{H_2O}C_{H_2O} = 2(5.5\times10^{-2}kg)(4{,}190\,J/kg\cdot K) + (0.220\,kg)(4{,}190\,J/kg\cdot K)$$

$$= 1{,}382.7\,J/K$$

$$M_{H_2O}C_{H_2O}(299K) + 2M_{ICE}C_{H_2O}(273K) - 2M_{ICE}C_{ICE}(14K) - 2M_{ICE}L_{f,H_2O}$$

$$= \left[(0.220\,kg)(299K) + 2(0.055\,kg)(273K)\right]\left(4{,}190\,\frac{J}{kg\cdot K}\right) - 2(\underbrace{0.55\,kg}_{(0.055\,kg)})\left[\left(2{,}090\,\frac{J}{kg\cdot K}\right)(14K) + \left(3.33\times10^{5}\,\frac{J}{kg}\right)\right]$$

$$= 401{,}443.9\,J - 398{,}486\,J = 2{,}957.9\,J$$

SO

$$T_f = \frac{M_{H_2O}C_{H_2O}(299K) + 2M_{ICE}C_{H_2O}(273K) - 2M_{ICE}C_{ICE}(14K) - 2M_{ICE}L_{f,H_2O}}{(2M_{ICE}C_{H_2O} + M_{H_2O}C_{H_2O})}$$

$$= \frac{2{,}957.9\,J}{1{,}382.7\,J/K} = 2.1K\,?$$

$\therefore$ NOT ALL OF THE ICE MELTED!

$$\boxed{T_f = 273K = 0.0^\circ C}$$

∴ Now for 1 ice cube...

$Q_{Ice} + Q_{H_2O} = 0$ yields

$$T_f = \frac{m_{H_2O} C_{H_2O}(299K) + m_{Ice} C_{H_2O}(273K) - m_{Ice} C_{Ice}(14K) - m_{Ice} L_{f,H_2O}}{m_{Ice} C_{H_2O} + m_{H_2O} C_{H_2O}} \equiv \frac{N}{D}$$

so

$$N = (0.220kg)\left(4{,}190\frac{J}{kg \cdot K}\right)(299K) + (0.055kg)\left(4{,}190\frac{J}{kg \cdot K}\right)(273K) - (0.055kg)\left[\left(2090\frac{J}{kg \cdot K}\right)(14K) + 3.33\times10^5\frac{J}{kg}\right]$$

$= 3.186\times10^5 J$ ✓

$$D = (0.055kg + 0.220kg)\left(4{,}190\frac{J}{kg \cdot K}\right) = 1.152\times10^3 J/K$$ ✓

so

$\boxed{T_f = 277K \simeq 3°C}$

P5. $M_{ice} = 0.210\,kg$ $L_{v,H_2O} = 22.6 \times 10^5\ J/kg$

$T_{i,ice} = 0°C = 273K$ $L_{f,H_2O} = 3.33 \times 10^5\ J/kg$

$T_{i,steam} = 100°C = 373K$ $C_{H_2O} = 4{,}190\ \frac{J}{kg \cdot K}$

$T_f = 55°C = 328K$

M_{ST}?

now

$$0 = Q_{steam} + Q_{ice}$$

$$= -M_{ST} L_{v,H_2O} + M_{ST} C_{H_2O}(\underbrace{328K - 373K}_{-45K}) + M_{ice} L_{f,H_2O} + M_{ice} C_{H_2O}(\underbrace{328K - 273K}_{55K})$$

now solve for M_{ST} ...

$$M_{ST}\left(L_{v,H_2O} + C_{H_2O}(45K)\right) = M_{ice}\left(L_{f,H_2O} + C_{H_2O}(55K)\right)$$

so

$$M_{ST} = M_{ice}\frac{\left(L_{f,H_2O} + C_{H_2O}(55K)\right)}{\left(L_{v,H_2O} + C_{H_2O}(45K)\right)}$$

$$= (0.210\,kg)\frac{\left(3.33 \times 10^5\ J/kg + (4{,}190\ \frac{J}{kg \cdot K})(55K)\right)}{\left(22.6 \times 10^5\ J/kg + (4{,}190\ \frac{J}{kg \cdot K})(45K)\right)} = (0.210\,kg)\frac{(5.635 \times 10^5\ J/kg)}{(2.449 \times 10^6\ J/kg)}$$

$$\boxed{M_{ST} = 0.048\,kg}$$

P6. O_2 GAS : DIATOMIC

$M = 1.88 \times 10^{-3}\,kg$

NOW $n = \frac{M}{M_{mol}} = \frac{(1.88\times10^{-3}\,kg)}{(0.032\,kg/mol)} = 0.0588\,mol$

FROM THE ATTACHED IMAGE...

$p_A = p_B = 2.0\times10^5\,Pa$

$V_A = 1.0\times10^{-3}\,m^3$

$V_B = V_C = 3.0\times10^{-3}\,m^3$

$p_C = 4.0\times10^5\,Pa$

NOW

$p_A V_A = nRT_A$ SO $T_A = \frac{p_A V_A}{nR} = \frac{(2.0\times10^5\,Pa)(1.0\times10^{-3}\,m^3)}{(0.0588\,mol)(8.31\,\frac{J}{mol\cdot K})}$

$T_A = 409\,K$

a) $\frac{p_A V_A}{T_A} = \frac{p_B V_B}{T_B}$ SINCE $p_A = p_B$, WE HAVE $\frac{V_A}{T_A} = \frac{V_B}{T_B}$ SO $T_B = T_A \frac{V_B}{V_A} = (409\,K)\frac{(3.0\times10^{-3}\,m^3)}{(1.0\times10^{-3}\,m^3)}$

$= 1,230\,K$

LASTLY

$\frac{p_B V_B}{T_B} = \frac{p_C V_C}{T_C}$ SINCE $V_B = V_C$, WE HAVE $\frac{p_B}{T_B} = \frac{p_C}{T_C}$ SO $T_C = T_B \frac{p_C}{p_B} = (1,230\,K)\frac{(4.0\times10^5\,Pa)}{(2.0\times10^5\,Pa)}$

$= 2,460\,K$

SO

$T_A = 409\,K$

$T_B = 1,230\,K$

$T_C = 2,460\,K$

b) FOR PROCESS $A \rightarrow B$...

$\Delta E_{th} = nC_V\Delta T = (0.0588\,mol)(20.9\,\frac{J}{mol\cdot K})(1,230\,K - 409\,K) = 1,010\,J$

$W = -p\Delta V = -(2.0\times10^5\,Pa)(3.0\times10^{-3}\,m^3 - 1.0\times10^{-3}\,m^3) = -400\,J$

ALSO

$\Delta E_{th} = Q + W \therefore Q = \Delta E_{th} - W = 1,010\,J + 400\,J = 1,410\,J$

ALTERNATIVELY..

$Q = nC_p\Delta T = (0.0588\,mol)(29.2\,\frac{J}{mol\cdot K})(1,230\,K - 409\,K) = 1,410\,J$ ✓

For process $B \to C$...

$$\Delta E_{th} = nC_V \Delta T = (0.0588\,\text{mol})\left(20.9\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(2{,}460\,\text{K} - 1{,}230\,\text{K}) = 1{,}510\,\text{J}$$

For an isochoric process, $W = 0$

so

$$\Delta E_{th} = Q + W = Q \quad \text{so} \quad \boxed{Q = 1{,}510\,\text{J}}$$

For process $C \to A$...

$$\Delta E_{th} = nC_V \Delta T = (0.0588\,\text{mol})\left(20.9\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(409\,\text{K} - 2{,}460\,\text{K}) = -2{,}520\,\text{J}$$

$$W = -\int p\,dV$$

now

$$p = 1.0\times10^5\,\text{Pa} + \left(1.0\times10^8\,\frac{\text{Pa}}{\text{m}^3}\right)V = mV + b \quad \text{where} \quad m = 1.0\times10^8\,\text{Pa/m}^3, \quad b = 1.0\times10^5\,\text{Pa}$$

so

$$W = -\int_{V_C}^{V_A} (mV + b)\,dV = -\left[\frac{1}{2}mV^2 + bV\right]_{V_C}^{V_A} = -\frac{1}{2}m(V_A^2 - V_C^2) - b(V_A - V_C)$$

$$= \frac{1}{2}m(V_C^2 - V_A^2) + b(V_C - V_A)$$

$$= \frac{1}{2}\left(1.0\times10^8\,\frac{\text{Pa}}{\text{m}^3}\right)\left[(3.0\times10^{-3}\,\text{m}^3)^2 - (1.0\times10^{-3}\,\text{m}^3)^2\right] + (1.0\times10^5\,\text{Pa})\left[3.0\times10^{-3}\,\text{m}^3 - 1.0\times10^{-3}\,\text{m}^3\right]$$

$$= 4.0\times10^2\,\text{Pa}\,\text{m}^3 + 2.0\times10^2\,\text{Pa}\,\text{m}^3 = 600\,\text{J}$$

now

$$\Delta E_{th} = W + Q \quad \text{so} \quad Q = \Delta E_{th} - W = -2{,}520\,\text{J} - 600\,\text{J} = -3{,}120\,\text{J}$$

∴ SO IN SUMMARY

	A → B	B → C	C → A
ΔE_{th}	1,010J	1,510J	-2,520J
W	-400J	Ø	600J
Q	1,410J	1,510J	-3,120J