Before 1895
- Classical Mechanics
- E & M optics
- Thermodynamics

Questions Asked
1.) What is matter made of?
2.) How does it interact with other matter & energy?
3.) What kind of medium transmits E&M waves?

How classical Physicists Saw The World:
1.) Matter may or may not be "lumpy" it always has a definite position & momentum
2.) Energy is continuous
3.) Energy & Matter are distinct & conserved
4.) Time & space are distinct

Thornton
Ch. 1, 3-8, selected topics from 9-13

Taylor Expansion of A Function

Defs: (1) $f(x)$ is a suitable well-behaved function. (Real or complex)
(2) Regular point : x_0 in some interval such that $|f(x_0)| < \infty$
(3) Singular point : x_0 such that $\lim_{x \to x_0} f(x) \to \infty$
Ex $\frac{1}{x-1}$ has $x=1$ as a singularity

(4) Taylor Exp.
About a regular point x_0,

$$f(x) = f(x_0) + f'(x_0)x + \frac{f''(x_0)x^2}{2!} + \frac{f'''(x_0)x^3}{3!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{\partial f}{\partial x^n}\Big|_{x=x_0} x^N \frac{1}{n!}$$

$|x| \ll 1$ "order of"

$$f(x) \approx f(x_0) + f'(x_0)x + \mathcal{O}(x^2)$$

Ex: e^x for $x \ll 1$

$$e^x = \sum_n \frac{1}{n!}\frac{\partial^n}{\partial x^n}(e^x)\Big|_{x=0} x^n = \frac{e^0 x^0}{0!} + \frac{e^0 x^1}{1!} + \frac{e^0 x^2}{2!} + \dots$$

$$= 1 + x + \frac{x^2}{2} + \dots$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{x^N}{N!}}$$

Consider $x = 0.1$

$e^{0.1} = 1.105170918$ ("Exact") - From calculator

"$\varnothing$-Th" order $\quad e^{0.1} \approx 1$

1^{st} order $\quad e^{0.1} \approx 1 + 0.1 = 1.1$

2^{nd} order $\quad e^{0.1} \approx 1 + 0.1 + \frac{(0.1)^2}{2} = 1.105$

3^{rd} order $\quad e^{0.1} \approx 1 + 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3!} = 1.105167$

Ex: $F \sim 1/r^2$

Consider small perturbations or about Equilibrium

$$F(r_0+\delta r) \sim \frac{1}{(r_0+\delta r)^2} = F(r_0) + F'(r_0)\,\delta r + \mathcal{O}(\delta r^2)$$
$$= \underbrace{1/r_0^2}_{\text{"EQ" Value}} \underbrace{- \frac{2}{r_0}\delta r}_{\text{linear Perturbation}} + \mathcal{O}(\delta r^2)$$

Behavior near a Singular point:

$F(x) = \ln|x-x_0|$

$$= \sum_n \frac{\partial^n f}{\partial x^n}\Big|_{x=x_0} \frac{x^n}{n!} = \ln|x-x_0| + \frac{1}{(x-x_0)}\Big|_{x=x_0}^{x} - \frac{1}{2(x-x_0)^2}x^2 + \cdots$$

Complex Representation of Periodic Functions

$$\sin ax = \sum_n \frac{\partial^n}{\partial x^n}\sin(ax)\frac{x^n}{n!} = \emptyset + ax\,\cancelto{1}{\cos(0)} - \frac{a^2}{2!}\cancelto{0}{\sin(0)}x - \frac{a^3}{3!}x^3 + \cancelto{0}{\frac{a^4(0)}{4!}} + \frac{a^5}{5!}x^5 + \cdots$$
$$= ax - \frac{(ax)^3}{3!} + \frac{(ax)^5}{5!} - \frac{(ax)^6}{6!} + \cdots\cdots = \sum_{n\text{-odd}} (-1)^{n+1}\frac{(ax)^n}{n!}$$

$$\cos ax = \sum_{n\text{-even}} (-1)^{n/2}\frac{(ax)^n}{n!}$$

$$e^{ax} = \sum_n \frac{(ax)^n}{n!} \overset{?}{=} \sum_m \frac{(-1)^n (ax)^{2m+1}}{(2m+1)!}$$

Consider Sin: n-odd $\rightarrow m = ?$ $\qquad \sin(ax) = \sum_m \frac{(-1)^n (ax)^{2m+1}}{(2m+1)!}$

what about $m+1 = n$?

Consider cos:

$$\cos(ax) = \sum_m (-1)^m \frac{(ax)^{2m}}{(2m)!}$$

Now Consider the expansion of $e^{\pm ix}$ $\quad (i = \sqrt{-1})$

$$e^{ix} = \sum_m \frac{(ix)^m}{m!} \qquad e^{ix} + e^{-ix} = \sum_m \left(\frac{x^m}{m!}\right)(i^m + (-i)^m) \;=\; \frac{1}{2}(e^{ix}+e^{-ix}) = \cos(x)$$

$$e^{-ix} = \sum_m \frac{(-ix)^m}{m!}$$

if $m = 0, 2, 4 \ldots \Rightarrow m=0 \Rightarrow 2$ $\qquad$ for Sin: $\frac{1}{2i}(e^{ix}-e^{-ix}) = \sin(x)$

$m=2 \Rightarrow -2$

$m=4 \Rightarrow 2$

if $m = 1, 3, 5 \ldots, \Rightarrow m=1 \Rightarrow 0$ $\qquad e^{ix} = \cos(x) + i\sin(x)$

$m=3 \Rightarrow 0$

$m=5 \Rightarrow 0$

Consider ..... $re^{i\theta} = r\cos\theta + ir\sin\theta$

$x + iy$