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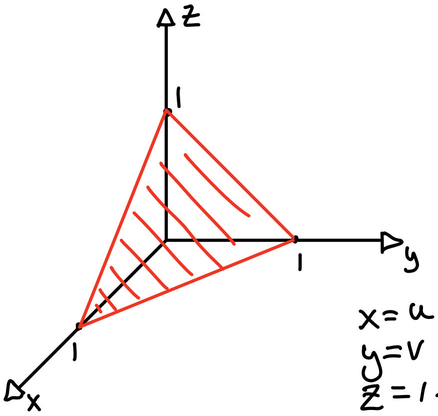
MATH 360
HW 4

\# 2, 4, 12, 15

Problem 10.6.2

$\vec{F} = [e^y, e^x, 1]$ S: $x + y + z = 1$, $x \geq 0$, $y \geq 0$, $z \geq 0$



$x = y = 0$: $z = 1$
$x = z = 0$: $y = 1$
$y = z = 0$: $x = 1$

S: $x + y + z = 1$

This is a plane with intercepts at positive one on each of the axes.

$x = u$
$y = v$
$z = 1 - u - v$

$\vec{r}(u,v) = [u, v, 1 - u - v]$

$x \rightarrow u$
$y \rightarrow v$

$0 \leq u \leq 1 - v$
$0 \leq v \leq 1$

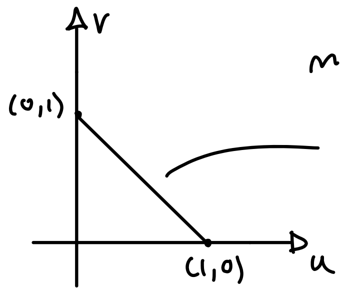
$\vec{r}_u = [1, 0, -1]$
$\vec{r}_v = [0, 1, -1]$

$$\iint_S \vec{F} \cdot \vec{n}\, dA = \iint_S \vec{F}(\vec{r}(u,v)) \cdot \vec{N}\, da$$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(0(-1) + 1(1)) + \hat{j}(-1(0) + 1(1)) + \hat{k}(1(1) - 0(0))$$
$$= \hat{i}(1) + \hat{j}(1) + \hat{k}(1)$$

$\vec{N} = [1, 1, 1]$

$\vec{F}(\vec{r}(u,v)) = [e^v, e^u, 1]$



$m = -1$, $b = 1$

$v = 1 - u$

$$\iint_S \vec{F} \cdot \vec{n}\, dA = \int_0^1 \int_0^{1-v} [e^v, e^u, 1] \cdot [1, 1, 1]\, du\, dv$$

$$\int_0^1 \int_0^{1-v} e^v + e^u + 1\, du\, dv$$

$$\int_0^1 [ue^v + e^u + u]_0^{1-v}\, dv$$

$$\int_0^1 [(1-v)e^v + e^{1-v} + (1-v)] - [0(e^v) + e^0 + 0]\, dv$$

$$\int_0^1 e^v - ve^v + e^{1-v} - v\, dv$$

$u = v$ $\quad dv = e^v$
$du = 1\, dv$ $\quad v = e^v$

$ve^v - \int e^v\, dv$
$ve^v - e^v$

$$e^v - ve^v + e^v - e^{1-v} - \frac{v^2}{2} \Big|_0^1$$

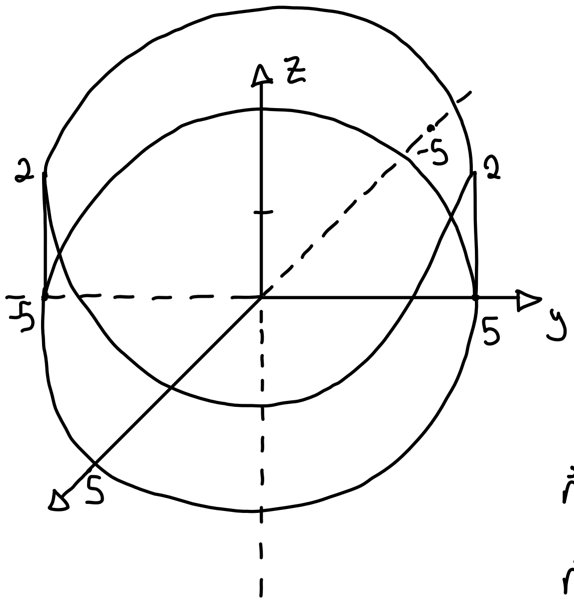
$\int ve^v = ve^v - e^v$

$$[e^1 - e^1 + e^1 - e^0 - 1/2] - [e^0 - 0 + e^0 - e^1 - 0/2]$$
$$(e - 3/2) - (2 - e)$$

$$\boxed{2e - 7/2}$$

Problem 10.6.4 $\vec{F} = [e^y, -e^z, e^x]$ $S: x^2+y^2=25$ $x \geq 0, y \geq 0, 0 \leq z \leq 2$

Cylinder of radius 5 with a height of 2 in the xy-plane



$$\int_S \vec{F}\cdot\vec{n}\, da = \iint_S \vec{F}(\vec{r}(u,v))\cdot\vec{N}\, da$$

$x = 5\cos(u)$ $0 \leq u \leq 2\pi$, $0 \leq v \leq 2$

$y = 5\sin(u)$

$z = v$ $\vec{r}(u,v) = [5\cos(u), 5\sin(u), v]$

$\vec{F}(\vec{r}(u,v)) = [e^{5\sin(u)}, e^{-v}, e^{5\cos(u)}]$

$\vec{N} = \vec{r}_u \times \vec{r}_v$

$\vec{r}_u = [-5\sin(u), 5\cos(u), 0] = \vec{a}$

$\vec{r}_v = [0, 0, 1]$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5\sin(u) & 5\cos(u) & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{i}(5\cos(u) - 0) + \hat{j}(0 + 5\sin(u)) + \hat{k}(0-0)$$

$\vec{N} = [5\cos(u), 5\sin(u), 0]$

$$\int_0^2\int_0^{2\pi} [e^{5\sin(u)}, e^{-v}, e^{5\cos(u)}]\cdot[5\cos(u), 5\sin(u), 0]\, du\, dv$$

$$\int_0^2\int_0^{2\pi} 5\cos(u)e^{5\sin(u)} + 5\sin(u)e^{-v}\, du\, dv$$

$$\int_0^2 \left[e^{5\sin(u)} - 5\cos(u)e^{-v}\right]_0^{2\pi} dv$$

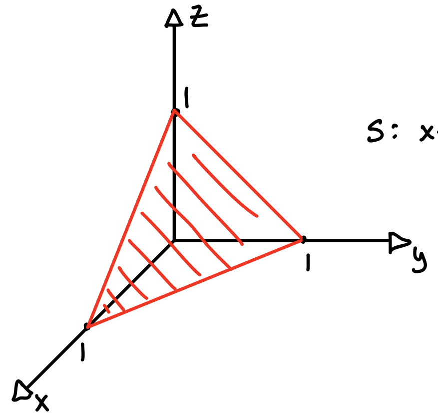
$$\int_0^2 [e^0 - 5e^{-v}] - [e^0 - 5e^{-v}]\, dv$$

$$\int_0^2 0\, dv = 0$$

$$\boxed{0}$$

Problem 10.6.12 $G = \cos(x) + \sin(x)$, $S: x+y+z=1$, First octant

$$\iint_S G(\vec{r}(u,v))\,|\vec{N}|\,dA$$



$S: x+y+z=1$ ← This surface is a plane with intercepts at 1 on each coordinate axis (x,y,z).

$x=u$, $y=v$, $z=1-u-v$ $\quad 0 \le u \le 1-v$, $0 \le v \le 1$

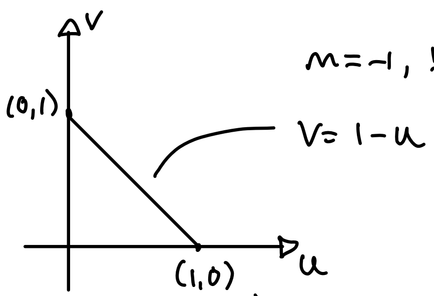
$\vec{r}(u,v) = [u, v, 1-u-v]$ $\quad G(\vec{r}(u,v)) = \cos(u) + \sin(v)$

$\vec{r}_u = [1, 0, -1]$

$\vec{r}_v = [0, 1, -1]$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(0(-1)+1(1)) + \hat{j}(-1(0)+1(1)) + \hat{k}(1(1)-0(0))$$
$$= (1)\hat{i} + (1)\hat{j} + (1)\hat{k}$$

$\vec{N} = [1,1,1]$ $\quad |N| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$



$m=-1,\ b=1$

$\sin(u) - v\sin(v)$

$u = v \quad dv = \sin(v)$

$du = 1\,dv \quad v = -\cos(v)$

$-v\cos(v) + \int \cos(v)\,dv$

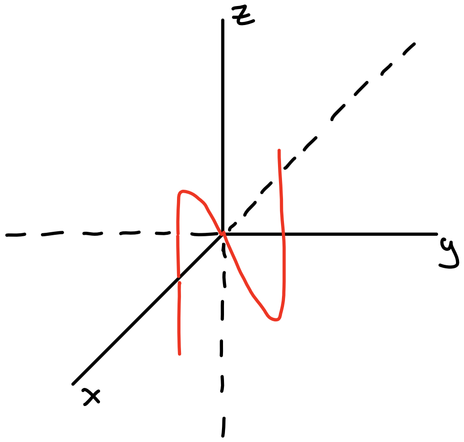
$-v\cos(v) + \sin(v)$

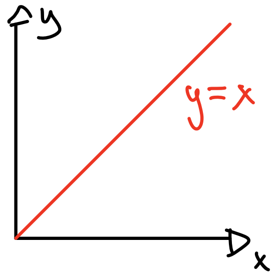
$\int v\sin(v) = -v\cos(v) + \sin(v)$

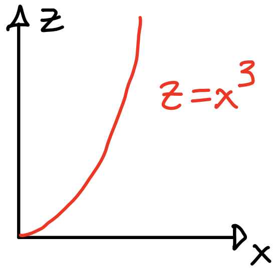
$$\sqrt{3}\int_0^1\int_0^{1-v} \cos(u) + \sin(v)\,du\,dv$$
$$\sqrt{3}\int_0^1 \Big[\sin(u) + u\sin(v)\Big]_0^{1-v} dv$$
$$\sqrt{3}\int_0^1 \sin(1-v) + (1-v)\sin(v)\,dv$$
$$\sqrt{3}\int_0^1 \sin(1-v) + \sin(v) - v\sin(v)\,dv$$
$$\sqrt{3}\Big[-\cos(1-v) - \cos(v) + v\cos(v) + \sin(v)\Big]_0^1$$
$$\sqrt{3}\Big[(-\cos(0) - \cos(1) + \cos(1) + \sin(1)) - (-\cos(1) - \cos(0) + 0 + 0)\Big]$$
$$\sqrt{3}\Big[(\sin(1) - 1) - (1 - \cos(1))\Big]$$

$$\boxed{\sqrt{3}\,[\sin(1) + \cos(1) - 2]}$$

Problem 10.6.15 $G=(1+9xz)^{3/2}$ $S: \vec{r}=[u, v, u^3]$ $0 \le u \le 1$, $-2 \le v \le 2$







$S: \vec{r}=[u, v, u^3]$

This surface is cubic function extending in the z-direction while expanding linearly in the xy plane

$$\iint_S G(\vec{r}(u,v))\,|\vec{N}|\,dA \qquad G(\vec{r}(u,v)) = (1+9u^4)^{3/2}$$

$$\vec{N} = r_u \times r_v \; : \; \vec{r}_u = [1, 0, 3u^2] \qquad \vec{r}_v = [0, 1, 0]$$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 3u^2 \\ 0 & 1 & 0 \end{vmatrix} = \hat{i}(0-3u^2) - \hat{j}(0-0) + \hat{k}(1-0) = (-3u^2)\hat{i} - (0)\hat{j} + (1)\hat{k}$$

$$|\vec{N}| = \sqrt{(-3u^2)^2 + 1^2} = \sqrt{9u^4+1}$$

$$\int_{-2}^{2}\int_0^1 (9u^4+1)^{3/2}(9u^4+1)^{1/2}\,du\,dv$$

$$\int_{-2}^{2}\int_0^1 (9u^4+1)^2\,du\,dv$$

$$\int_{-2}^{2}\int_0^1 81u^8 + 18u^4 + 1\,du\,dv$$

$$\int_{-2}^{2}\left[9u^9 + 18/5\,u^5 + u\right]_0^1 dv \qquad \left(9\cdot 1^9 + 18/5\,(1)^5 + 1\right) - (0+0+0)$$

$$10 + 18/5 = 68/5$$

$$\int_{-2}^{2} 68/5\,dv$$

$$\frac{68}{5}\left[v\right]_{-2}^{2} \qquad \frac{68}{5}\left(2-(-2)\right) = \frac{272}{5}$$

$$\boxed{\frac{272}{5}}$$