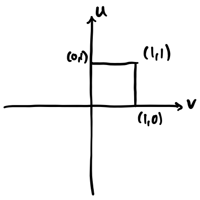
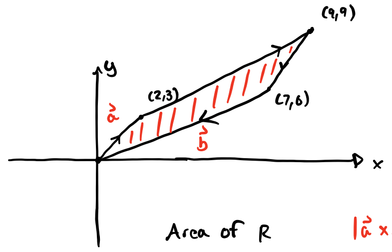
$x = 7u + 2v$ $\quad \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 7 & 2 \\ 6 & 3 \end{vmatrix} \quad 21 - 12 = 9$

$y = 6u + 3v$

$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$(u,v) \longmapsto (7u+2v, 6u+3v)$




$R = T(S)$

Area of R $\quad |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

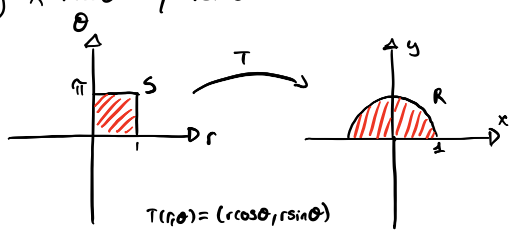
$(a_2b_3 - a_3b_2)$ $\quad \langle 2,3,0\rangle$

$(a_3b_1 - a_1b_3)$ $\quad \langle 7,6,0\rangle$

$(a_1b_2 - a_2b_1)$ $\quad \langle 3(0)-0(6), 0(7)-2(0), 2(6)-3(7)\rangle$

$|\langle 0,0,-9\rangle| = 9$

Ex 2: $x = r\cos\theta \quad y = r\sin\theta$




$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} \quad r(\cos^2\theta + \sin^2\theta) = r$

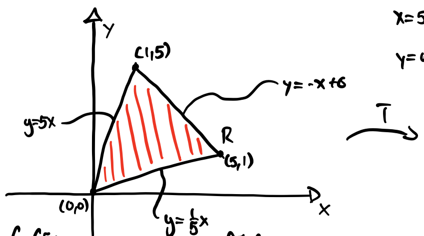
$$\iint_R f(x,y)\,dA = \iint_S f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dA$$

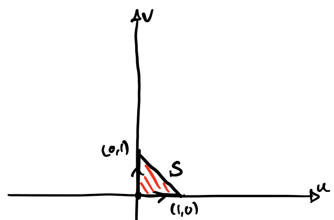
fn of u & v

15.10.4 $\iint_R (x - 6y)\, dA$

$x = 5u + v$

$y = u + 5v$




$$\int_0^1 \int_{\frac{1}{5}x}^{5x} (x-6y)\,dy\,dx + \int_1^5 \int_{\frac{1}{5}x}^{-x+6} (x-6y)\,dy\,dx$$

$$\int_0^1 \int_0^{-u+1} 5u+v-6(u+5v) \left|\begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{matrix}\right| dv\,du$$

$$\int_0^1 \int_0^{-u+1} -u-29v\, |24|\, dv\,du$$

$x = \rho\sin\varphi\cos\theta$

$y = \rho\sin\varphi\sin\theta$ $\quad$ Spherical

$z = \rho\cos\varphi$

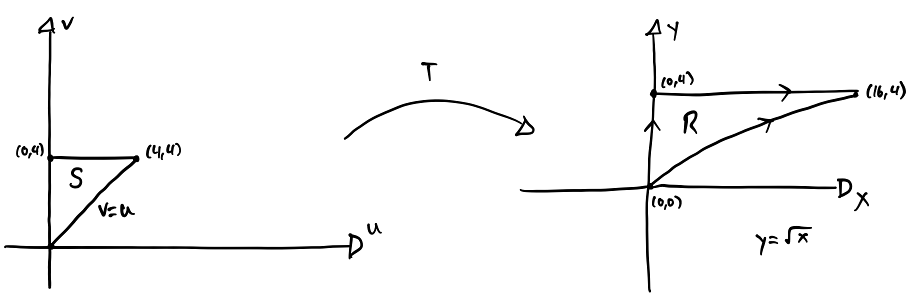
$dv = \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

$\left|\frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)}\right| \longrightarrow \det \begin{vmatrix} x_\rho & x_\theta & x_\varphi \\ y_\rho & y_\theta & y_\varphi \\ z_\rho & z_\theta & z_\varphi \end{vmatrix} = \det \begin{vmatrix} {}^{+}\sin\varphi\cos\theta & {}^{-}\rho\sin\varphi\sin\theta & {}^{+}\rho\cos\varphi\cos\theta \\ {}^{-}\sin\varphi\sin\theta & {}^{+}\rho\sin\varphi\cos\theta & {}^{-}\rho\cos\varphi\sin\theta \\ {}^{+}\cos\varphi & {}^{-}0 & {}^{+}-\rho\sin\varphi \end{vmatrix}$ ← Expand along row 3

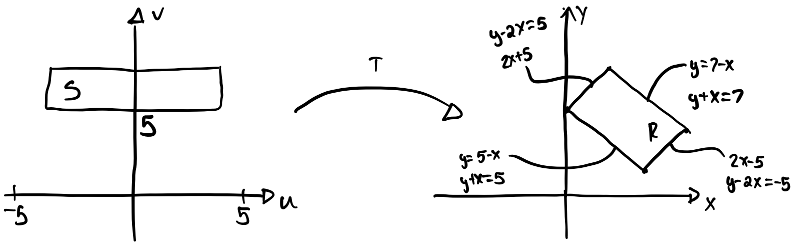
$$\cos\varphi \begin{pmatrix} -\rho\sin\varphi\sin\theta & \rho\cos\varphi\cos\theta \\ \rho\sin\varphi\cos\theta & \rho\cos\varphi\sin\theta \end{pmatrix} - 0 \cdot \begin{pmatrix} \sin\varphi\cos\theta & \rho\cos\varphi\cos\theta \\ \sin\varphi\sin\theta & \rho\cos\varphi\sin\theta \end{pmatrix} - \rho\sin\varphi \begin{pmatrix} \sin\varphi\cos\theta & -\rho\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & \rho\sin\varphi\cos\theta \end{pmatrix}$$

$$= \rho^2 \sin\varphi$$

15.10.2 $x = u^2, \quad y = v$



## 15.10.3




$y=2x-5$ $\quad y=5-x$

$y=2x+5$ $\quad y=7-x$

① $v=y+x$ $\qquad 1-2$

② $u=y-2x$ $\qquad v-u=(y+x)-(y-2x)$ $\qquad 2v+u=2y+2x+y-2x$

$v-u=3x$ $\qquad 2v+u=3y$

$\frac{v-u}{3}=x$ $\qquad \frac{2v+u}{3}=y$

## 15.10.6

$\iint_R 5\left(\frac{x-5y}{7x-y}\right)\,dA$

R is enclosed by

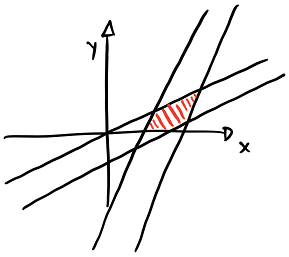
$x-5y=0$ $\quad 7x-y=6$ $\quad u=x-5y$

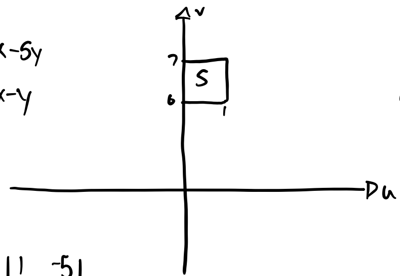
$x-5y=1$ $\quad 7x-y=7$ $\quad v=7x-y$

$y=\frac{1}{5}x$ $\quad y=7x-6$

$y=\frac{1}{5}x-1$ $\quad y=7x-7$

$x=\frac{5v-u}{34}$ $\quad y=\frac{v-7u}{34}$




$0\le u\le 1$

$6\le v\le 7$

$\begin{vmatrix} \frac{-1}{34} & \frac{5}{34} \\ \frac{-7}{34} & \frac{1}{34} \end{vmatrix} = \frac{1}{34}$

$\begin{vmatrix} 1 & -5 \\ 7 & -1 \end{vmatrix}$ $\quad -1-(-35)$

$-1+35$

$\frac{1}{34}$ $\quad 34$

$\frac{5}{68}\left[\ln(7)-\ln(6)\right]$

$\iint_R 5\left(\frac{x-5y}{7x-y}\right)\,dA$

$\iint_S 5\left(\frac{u}{v}\right)\left|\frac{\partial(x,v)}{\partial(u,v)}\right|\,dA$

$\int_6^7\int_0^1 5\left(\frac{u}{v}\right)\left|\frac{1}{34}\right|\,du\,dv$

$\frac{5}{34}\int_6^7\int_0^1 \frac{u}{v}\,du\,dv$

$\frac{5}{34}\int_6^7 \frac{1}{v}\left[\frac{u^2}{2}\right]_0^1\,dv$

$\frac{5}{34}\int_6^7 \frac{1}{v}\left(\frac{1}{2}\right)\,dv$

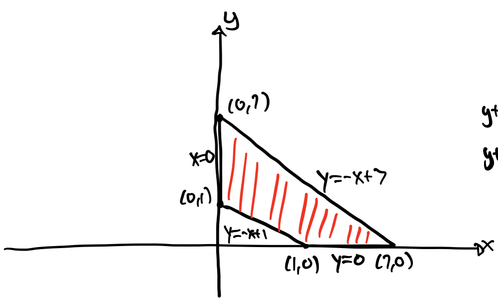
$\frac{5}{68}\int_6^7 \frac{1}{v}\,dv$

$\frac{5}{68}\left[\ln v\right]_6^7$

$\frac{5}{68}\left[\ln(7)-\ln(6)\right]$

## 15.10.8

$\iint_R 7\cos\left(5\left(\frac{y-x}{y+x}\right)\right)\,dA$ $\qquad$ R is

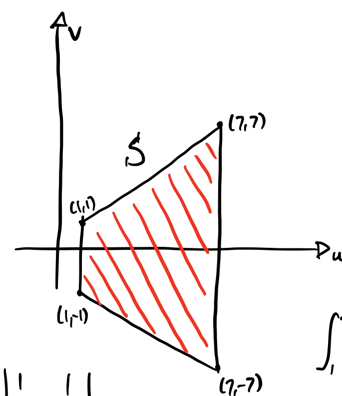



$y+x=7$

$y+x=1$

$u=y+x$ $\quad v=y-x$

$1\le u\le 7$




$u=y+x$ $\quad v=y-x$

$\frac{\partial u}{\partial x}=1$ $\quad \frac{\partial v}{\partial x}=-1$

$\frac{\partial u}{\partial y}=1$ $\quad \frac{\partial v}{\partial y}=1$

$\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$ $\quad 1-(-1)=2$

$\frac{1}{2}$

$w=\frac{5v}{u}$

$dw=\frac{5}{u}\,dv$

$dw\,\frac{u}{5}=dv$

$\frac{336}{10}\sin(5)$

$\int_1^7\int_{-u}^{u} 7\cos\left(5\frac{v}{u}\right)\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,dA$

$7\int_1^7\int_{-u}^{u} \cos\left(5\frac{v}{u}\right)\left|\frac{1}{2}\right|\,dv\,du$

$\frac{7}{2}\int_1^7\int_{-u}^{u} \cos\left(5\frac{v}{u}\right)\,dv\,du$

$\frac{7}{2}\int_1^7\int_{-u}^{u} \cos(w)\frac{u}{5}\,dw\,du$

$\frac{7}{2}\int_1^7 \frac{u}{5}\left(\sin\left(\frac{5v}{u}\right)\right)\Big|_{-u}^{u}\,du$

$\frac{7}{2}\int_1^7 \frac{u}{5}\left(\sin(5)-\sin(-5)\right)\,du$

$\frac{7}{2}\int_1^7 \frac{u}{5}\left(2\sin(5)\right)\,du$

$\frac{7}{2}\left[\frac{u^2}{10}2\sin(5)\right]_1^7$

$\frac{7}{2}\left[\frac{49}{10}2\sin(5)-\frac{1}{10}2\sin(5)\right]$

$\frac{7}{2}\left[\frac{48}{10}2\sin(5)\right]$

$\frac{7}{2}\left(\frac{48}{5}\right)$

$\frac{336}{10}\sin(5)$