1.) Find an equation of the tangent plane to the surface with the point. $z = \sqrt{xy}$ $P(3,3,3)$

Tangent Plane

$$z - z_0 = \frac{\partial z}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial z}{\partial y}(x_0, y_0)(y - y_0)$$

$$\frac{\partial z}{\partial x} = (xy)^{1/2}$$

$$\frac{1}{2}(xy)^{-1/2} \cdot y$$

$$= \frac{y}{2\sqrt{xy}}$$

$$(3,3) \quad \frac{3}{2\sqrt{9}}$$

$m_y = \frac{1}{2}$

$$\frac{3}{6} = \frac{1}{2}$$

$$\frac{\partial z}{\partial y} = (xy)^{\frac{1}{2}}$$

$$\frac{1}{2}(xy)^{-\frac{1}{2}} \cdot x$$

$$= \frac{x}{2\sqrt{xy}}$$

$$(3,3) = \frac{3}{2\sqrt{9}}$$

$m_y = \frac{1}{2}$

$$\frac{3}{6} = \frac{1}{2}$$

$$z - 3 = \frac{1}{2}(x - 3) + \frac{1}{2}(y - 3)$$

$$= \frac{1}{2}x - \frac{3}{2} + \frac{1}{2}y - \frac{3}{2}$$

$$z - 3 = \frac{1}{2}x + \frac{1}{2}y - 3$$

$$z = \frac{1}{2}x + \frac{1}{2}y$$

$$\boxed{z = \frac{1}{2}x + \frac{1}{2}y}$$

2.) $f(x,y) = \sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$ at $(0,0)$

Find the linear approximation!

$$f_x\left(\sqrt{y + \cos^2 x}\right)$$

$u = \cos x$

$u' = \sin x$

$$(y + \cos^2 x)^{\frac{1}{2}}$$

$$(y + u^2)^{\frac{1}{2}}$$

$$\frac{1}{2}(y + u^2)^{-\frac{1}{2}} \cdot 2u \cdot u'$$

$$\frac{1}{2\sqrt{y + \cos^2 x}} \cdot 2\cos x \cdot -\sin x$$

$$\frac{-2\cos x \sin x}{2\sqrt{y + \cos^2 x}}$$

$$\frac{-\cos x \sin x}{\sqrt{y + \cos^2 x}}$$

$$f_y\left(\sqrt{y + \cos^2 x}\right)$$

$$(y + \cos^2 x)^{\frac{1}{2}}$$

$$\frac{1}{2}(y + \cos^2 x)^{-\frac{1}{2}} \cdot 1$$

$$\frac{1}{2\sqrt{y + \cos^2 x}}$$

$$f_x(0,0) = \frac{-\cos(0)\sin(0)}{\sqrt{0 + \cos^2(0)}} = 0$$

$$f_y(0,0) = \frac{1}{2\sqrt{0 + \cos^2 0}} = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$f_x = \frac{-\cos x \sin x}{\sqrt{y + \cos^2 x}}$$

$$f_y = \frac{1}{2\sqrt{y + \cos^2 x}}$$

$$f_x(0,0) = \frac{-\cos(0)\sin(0)}{\sqrt{0 + \cos^2(0)}} = 0$$

2.) Find the linear approximation of the function $f(x,y,z)=\sqrt{x^2+y^2+z^2}$ $(8,8,4)$

$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}$ $\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}$

$\frac{\partial z}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\frac{\partial z}{\partial x}(8,8,4) = \frac{8}{\sqrt{64+64+16}} = \frac{8}{\sqrt{144}} = \frac{8}{12} = \frac{2}{3}$

$\frac{\partial z}{\partial y}(8,8,4) = \frac{8}{\sqrt{64+64+16}} = \frac{8}{\sqrt{144}} = \frac{8}{12} = \frac{2}{3}$

$\frac{\partial z}{\partial z}(8,8,4) = \frac{4}{\sqrt{64+64+16}} = \frac{4}{\sqrt{144}} = \frac{4}{12} = \frac{1}{3}$

$w = \frac{2}{3}(x-8) + \frac{2}{3}(y-8) + \frac{1}{3}(z-4)$

$w = \frac{2x}{3} - \frac{16}{3} + \frac{2y}{3} - \frac{16}{3} + \frac{z}{3} - \frac{4}{3} + 12$

$w = \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z - \frac{36}{3} + 12$

$w = \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z - 12 + 12$

$w(8.02, 7.99, 3.97) = \frac{2}{3}(8.02) + \frac{2}{3}(7.99) + \frac{1}{3}(3.97)$

11.9967

4.)

$(40,20)$

$f_v(v,t) \approx \lim_{h \to 0} \frac{(v+h,t) - f(v,t)}{h}$ $h=10$

$f_v = 1.15$

$f_t = 1/2$

$f(40,20) = 28$

$f_v(40,20)_{h=10} = \frac{f(40+10,20) - f(40,20)}{10}$

$f_v(40,20) = \frac{f(50,20) - f(40,20)}{10}$

$f_v(40,20) = \frac{40-28}{10} = \frac{12}{10} = 1.2$

$f_v(30,20)_{h=-10} = \frac{f(30,20) - f(40,20)}{-10}$ $f_v = 1.15$ $\frac{+}{2}$

$$f_v(30,20) = \frac{17-28}{-10} = \frac{-11}{-10} = 1.1$$

$$\underset{h=10,-10}{f_t(40,20)} = \frac{f(v,t+h)-f(v,t)}{h}$$

$$\underset{h=10}{f_t(40,20)} = \frac{f(40,20+10)-f(40,20)}{10}$$

$$f_t(40,20) = \frac{f(40,30)-f(40,20)}{10}$$

$$f_t(40,20) = \frac{31-28}{10} = \frac{3}{10} = 0.3$$

$$\underset{h=-10}{f_t(40,20)} = \frac{f(v,t+h)-f(v,t)}{h}$$

$$f_t(40,20) = \frac{f(40,20-10)-f(40,20)}{-10}$$

$$f_t(40,20) = \frac{f(40,10)-f(40,20)}{-10}$$

$$f_t(40,20) = \frac{21-28}{-10} = \frac{-7}{-10} = \frac{7}{10} = 0.7$$

$$\frac{0.3+0.7}{2} = \frac{1}{2}$$

$$f(v,t) = 1.15v + 0.5t - 28$$

$$f(41,23) = 30.65 \text{ Ft}$$

5.) Find the differential of the function

$$m = p^5 q^4$$

$$\partial m / \partial p = 5p^4 q^4 \quad \partial p$$

$$\partial m / \partial q = 4q^3 p^5 \quad \partial q$$

6.) Find the total differential of the function

$$T = \frac{v}{5+uvw}$$

$$\partial T / \partial u = \frac{(5+uvw)(0) - v(0 + 1 \cdot vw)}{(5+uvw)^2}$$

$$= \frac{-v^2 w}{(5+uvw)^2}$$

$$\boxed{\partial T / \partial u = \frac{-v^2 w}{(5+uvw)^2}}$$

$$\partial T / \partial v = \frac{(5+uvw)(1) - v(0 + 1 \cdot uw)}{(5+uvw)^2}$$

$$= \frac{5 + uvw - uvw}{(5+uvw)^2}$$

$$= \frac{5}{(5+uvw)^2}$$

$$\boxed{\frac{\partial T}{\partial v} = \frac{5}{(5+uvw)^2}}$$

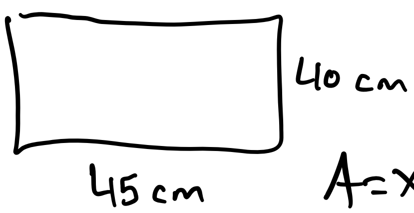
$$\frac{\partial T}{\partial w} = \frac{(5+uvw)(0) - v(0+1\cdot uv)}{(5+uvw)^2}$$

$$= \frac{0 - uv^2}{(5+uvw)^2}$$

$$= \frac{-uv^2}{(5+uvw)^2}$$

$$\boxed{\frac{\partial T}{\partial w} = \frac{-uv^2}{(5+uvw)^2}}$$

7.)



$A = xy$

$L = 45\text{cm}$

$w = 40\text{ cm}$

Error $\leq 0.1\text{cm}$ each

Maximum error?

$dx = 0.1$

$dy = 0.1$

$A = xy$

$$\partial A = \frac{\partial A}{\partial x}(xy)\,dx + \frac{\partial A}{\partial y}(xy)\,dy$$

$$\partial A = y(0.1) + x(0.1)$$

$$\partial A = 40(0.1) + 45(0.1)$$

$$\partial A = 4 + 4.5$$

$$\partial A = 8.5$$

$$\boxed{8.5\text{ cm}^2}$$

8.) $PV = 8.31T$ $\qquad \frac{\partial P}{\partial V} = 0.6L \qquad \frac{\partial P}{\partial T} = -10K$

$$P = \frac{8.31T}{V}$$

$$\frac{\partial P}{\partial V} = \frac{V(0) - 8.31T(1)}{V^2}$$

$$\frac{\partial P}{\partial V} = \frac{-8.31T}{V^2}$$

$$\frac{\partial P}{\partial T} = \frac{8.31T}{V}$$

$$\frac{\partial P}{\partial T} = \frac{V(8.31) - 8.31T(0)}{V^2}$$

$$\frac{\partial P}{\partial T} = \frac{8.31V}{V^2} \qquad \frac{\partial P}{\partial T} = \frac{8.31}{V}$$

$T_0(\overset{V}{10}, \overset{T}{325})$

$dV = 0.6L$

$dT = -10K$

$$dP = \frac{\partial P}{\partial V}(dV) + \frac{\partial P}{\partial T}(dt)$$

$$dP = \frac{-8.31T}{V^2}(dV) + \frac{8.31T}{T}(dt)$$

$$dP = \frac{-8.31(325)}{10^2}(0.6L) + \frac{8.31(325)}{325}(-10K)$$

$$dP = -16.2045 - 83.1$$

-99.3045 kilo pascals

9.) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ $\qquad R_1 = 10\,\Omega \quad R_2 = 80\,\Omega \quad R_3 = 20\,\Omega$

$E = 0.5\%$

$$\frac{-1}{R^2} = \frac{-1}{R_1^2} + \frac{-1}{R_2^2} + \frac{-1}{R_3^3}$$

$$\frac{0.5}{100} = 0.005$$

$\Delta R_{1,2,3} = 0.005$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{80} + \frac{1}{20}$$

$$\frac{1}{R} = \frac{13}{80}$$

$$R = \frac{80}{13} \times \Delta R$$

$$dr = 0.030769$$

$$\approx 0.0030769 \text{ ohms}$$