Taylor Larrechea
Dr. Collins
PHYS 362 HW 5

Problem 1

$F = PV^n$, $PV = NkT$ $\quad V = \left(\frac{NkT}{P}\right)^n \quad T = \frac{PV}{Nk}$

a.) $\left(\frac{\partial F}{\partial T}\right)_V$: $\longrightarrow$ V & T are independent , $F = NkTV^{n-1}$ $\therefore \left(\frac{\partial F}{\partial T}\right)_V = NkV^{n-1}$

$\left(\frac{\partial F}{\partial T}\right)_P$: $\longrightarrow$ P & T are independent , $F = P \cdot \frac{(Nk)^n T^n}{P^n} = P^{1-n}(Nk)^n T^n$

$$\left(\frac{\partial F}{\partial T}\right)_P = n P^{1-n}(Nk)^n T^{n-1} = n P^{1-n}(Nk)^n \left(\frac{PV}{Nk}\right)^{n-1} = n \cdot \cancel{P^{1-n}} \cdot \cancel{P^{n-1}} \frac{\cancel{N^n}}{\cancel{N^{n-1}}} \cdot \frac{\cancel{k^n}}{\cancel{k^{n-1}}} V^{n-1} = nNkV^{n-1}$$

$$\left(\frac{\partial F}{\partial T}\right)_P = nNkV^{n-1}$$

$$\boxed{\left(\frac{\partial F}{\partial T}\right)_V = NkV^{n-1}, \quad \left(\frac{\partial F}{\partial T}\right)_P = nNkV^{n-1}}$$

b.) $\left(\frac{\partial F}{\partial T}\right)_V = NkV^{n-1}$ $\quad \left(\frac{\partial F}{\partial T}\right)_P = nNkV^{n-1}$

$\boxed{\text{These are equal when } n=1}$

Problem 2

a.) $z = x^2 y$, $x = \frac{\sqrt{z}}{\sqrt{y}}$, $y = \frac{z}{x^2}$

$$\left(\frac{\partial z}{\partial y}\right)_x = x^2 \ , \ \left(\frac{\partial z}{\partial x}\right)_y = 2xy \ , \ \left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2} z^{1/2} y^{-3/2}$$

$$\left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2} z^{-1/2} y^{-1/2} \ , \ \left(\frac{\partial y}{\partial x}\right)_z = \frac{-2z}{x^3} \ , \ \left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{x^2}$$

$\left(\frac{\partial z}{\partial y}\right)_x = x^2$, $\left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{x^2}$ $\longrightarrow$ These are reciprocals

$\left(\frac{\partial z}{\partial x}\right)_y = 2xy$, $\left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2}\sqrt{\frac{1}{zy}} = \frac{1}{2}\sqrt{\frac{1}{x^2y^2}} = \frac{1}{2xy}$ $\longrightarrow$ These are reciprocals

$$\boxed{\left[\left(\frac{\partial z}{\partial y}\right)_x = x^2 \ , \ \left(\frac{\partial y}{\partial z}\right)_x\right], \left[\left(\frac{\partial z}{\partial x}\right)_y = 2xy \ , \ \left(\frac{\partial x}{\partial z}\right)_y\right] \text{ are reciprocals}}$$

b.)

i.) Prove $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = 1$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \ : \ dx = \left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial x}{\partial y}\right)_z dy$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left[\left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial x}{\partial y}\right)_z dy\right] + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dz - \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z dy - \left(\frac{\partial z}{\partial y}\right)_x dy = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz$$

Because y is held constant, $dy = 0$ $\therefore$

$$\cancel{dz} = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y \cancel{dz}$$

$$\boxed{\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = 1}$$

ii.) Prove $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \ : \ dx = \left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial x}{\partial y}\right)_z dy$$

Problem 2 Continued

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left[\left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial x}{\partial y}\right)_z dy \right] + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz + \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dz - \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z dy - \left(\frac{\partial z}{\partial y}\right)_x dy = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y dz$$, z is held constant

$$\overset{0}{\cancel{dz}} - \left(\frac{\partial z}{\partial x}\right)_y \overset{0}{\cancel{\left(\frac{\partial x}{\partial z}\right)_y}} dz - \left(\frac{\partial z}{\partial y}\right)_x dy = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z dy$$

$$-\left(\frac{\partial z}{\partial y}\right)_x \cancel{dy} = \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z \cancel{dy}$$

$$\boxed{\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x}$$

c.) $z(x,y) = x^2 y$: $x(y,z) = \sqrt{z/y}$: $y(x,z) = z/x^2$ $\qquad z^{1/2} y^{-1/2} \rightarrow -\frac{1}{2} z^{1/2} y^{-3/2}$

i.) Show $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = 1$ is true :

$\left(\frac{\partial z}{\partial x}\right)_y = 2xy$: $\left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2}\sqrt{\frac{1}{zy}}$ $\qquad z = x^2 y$ $\therefore$ $\left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2}\sqrt{\frac{1}{x^2y^2}} = \frac{1}{2}\frac{1}{xy}$

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = \cancel{2xy} \cdot \frac{1}{\cancel{2}} \frac{1}{\cancel{xy}} = 1 \quad ✓$$

$$\boxed{\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial z}\right)_y = 1 \text{ is true}}$$

ii.) Show $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x$ is true $\qquad y^3 = \frac{z^3}{x^6}$

$\left(\frac{\partial z}{\partial y}\right)_x = x^2$: $\left(\frac{\partial z}{\partial x}\right)_y = 2xy$: $\left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\sqrt{\frac{z}{y^3}}$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\sqrt{\frac{z}{y^3}} = \frac{-1}{2}\sqrt{\frac{z}{z^3/x^6}} = \frac{-1}{2}\sqrt{\frac{x^6}{z^2}} = \frac{-1}{2}\sqrt{\frac{x^6}{x^4y^2}} = \frac{-1}{2}\sqrt{\frac{x^2}{y^2}} = \frac{-1}{2}\frac{x}{y}$$

$\left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\frac{x}{y}$ $\qquad \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial z}{\partial x}\right)_y = 2x\cancel{y} \cdot \frac{-1}{\cancel{2}}\frac{x}{\cancel{y}} = -x^2 = -\left(\frac{\partial z}{\partial y}\right)_x$

Problem 2 continued

$$\boxed{\left(\frac{\partial z}{\partial x}\right)_y\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x \quad \text{is true}}$$

d.)

i.) Prove $\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial x}\right)_z = 1$: $x = \sqrt{\frac{z}{y}}$: $y = \frac{z}{x^2}$: $z = x^2 y$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\sqrt{\frac{z}{y^3}} \quad : \quad \left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\sqrt{\frac{x^2}{y^2}} = \frac{-1}{2}\,\frac{x}{y}$$

$$\left(\frac{\partial y}{\partial x}\right)_z = -\frac{2z}{x^3} \quad : \quad \left(\frac{\partial y}{\partial x}\right)_z = -\frac{2y}{x}$$

$$\left(\frac{\partial x}{\partial y}\right)_z\cdot\left(\frac{\partial y}{\partial x}\right)_z = \frac{-1}{2}\,\frac{\cancel{x}}{\cancel{y}}\cdot\frac{-2\cancel{y}}{\cancel{x}} = 1 \quad \checkmark$$

$$\boxed{\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial x}\right)_z = 1}$$

ii.) Prove $\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x = -\left(\frac{\partial x}{\partial z}\right)_y$: $x = \sqrt{\frac{z}{y}}$: $y = \frac{z}{x^2}$: $z = x^2 y$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\sqrt{\frac{z}{y^3}} \quad : \quad \left(\frac{\partial x}{\partial y}\right)_z = \frac{-1}{2}\sqrt{\frac{x^2}{y^2}} = \frac{-1}{2}\,\frac{x}{y} \quad : \quad \left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{x^2}$$

$$\left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2}\sqrt{\frac{1}{zy}} \quad : \quad \left(\frac{\partial x}{\partial z}\right)_y = \frac{1}{2}\sqrt{\frac{1}{x^2y^2}} = \frac{1}{2}\cdot\frac{1}{xy}$$

$$\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x = \left(\frac{-1}{2}\,\frac{x}{y}\right)\left(\frac{1}{x^2}\right) = \frac{-1}{2}\,\frac{1}{xy} = -\left(\frac{\partial x}{\partial z}\right)_y \quad \checkmark$$

$$\boxed{\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_x = -\left(\frac{\partial x}{\partial z}\right)_y}$$

e.)

i.) Prove $\left(\frac{\partial z}{\partial x}\right)_y\left(\frac{\partial x}{\partial z}\right)_y = 1 \longrightarrow \left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial P}\right)_V = 1$: $P = \frac{NkT}{V}$, $T = \frac{PV}{Nk}$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{Nk}{V} \quad : \quad \left(\frac{\partial T}{\partial P}\right)_V = \frac{V}{Nk} \quad \therefore \quad \left(\frac{\partial P}{\partial T}\right)_V\cdot\left(\frac{\partial T}{\partial P}\right)_V = \frac{Nk}{V}\cdot\frac{V}{Nk} = 1 \quad \checkmark$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial P}\right)_V = 1}$$

Problem 2 Continued

ii.) Prove $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x \longrightarrow \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P = -\left(\frac{\partial P}{\partial V}\right)_T$ $P = \frac{NkT}{V}$, $T = \frac{PV}{Nk}$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{Nk}{V} \quad : \quad \left(\frac{\partial T}{\partial V}\right)_P = \frac{P}{Nk} \quad : \quad \left(\frac{\partial P}{\partial V}\right)_T = \frac{-NkT}{V^2}$$

$$\left(\frac{\partial P}{\partial T}\right)_V \cdot \left(\frac{\partial T}{\partial V}\right)_P = \frac{\cancel{Nk}}{V} \times \frac{P}{\cancel{Nk}} = \frac{P}{V} = \frac{NkT}{V^2} = -\left(\frac{\partial P}{\partial V}\right)_T \quad \checkmark$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P = -\left(\frac{\partial P}{\partial V}\right)_T}$$

Problem 3

$$E = \frac{3}{2} NkT \quad , \quad PV = NkT$$

a.)

i.) $$C_p = \left(\frac{\partial E}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial E}{\partial T}\right)_P = \frac{3}{2}Nk = \frac{3}{2}\frac{PV}{T} \quad , \quad \left(\frac{\partial V}{\partial T}\right)_P = \frac{Nk}{P}$$

$$C_p = \frac{3}{2}Nk + P \cdot \frac{Nk}{P} = \frac{3}{2}Nk + Nk = \frac{5}{2}Nk \quad \therefore \quad C_p = \frac{5}{2}Nk$$

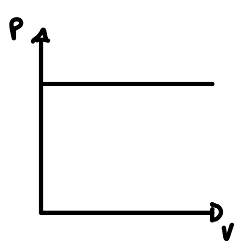
$$\boxed{C_p = \frac{5}{2}Nk}$$

ii.) $$C_v = \left(\frac{\partial E}{\partial T}\right)_V = \frac{3}{2}Nk$$

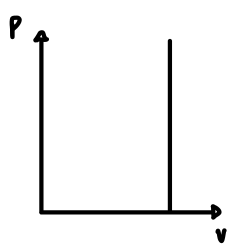
$$\boxed{C_v = \frac{3}{2}Nk}$$

b.) At constant volume all heat goes to changing E
At constant pressure some heat goes into work done by the gas

CP



Cv



$\Delta E = Q + W \quad \therefore \quad Q = \Delta E - W$

$W = \pm$

$V_i < V_f$: $W > 0$

$V_f < V_i$:

$W = 0$

So a smaller portion of heat is devoted to changing E.
The change in E is proportional to change in T.
So at constant volume change in T is greater per heat