Physics 230

Homework Set 12

1. Reconsider the simple harmonic oscillator studied extensively in Homework Set 11 involving the mass $m = 0.550$ kg, which is attached to the spring and hung vertically. This time, however, the oscillator is submerged in a resistive medium. After oscillating for 6.00s, the maximum amplitude decreases to one-fourth of the initial value. Calculate
   a) the damping parameter, γ, and the damping constant, b,
   b) the angular frequency of oscillation, ω, and
   c) the quality factor, Q.

2. Reconsider the oscillator of the previous problem. At time $t = 0$, the mass is displaced from the equilibrium position by 5.50 cm in the *downward* direction and is given an initial shove, imparting a speed of 0.450 m/s on the mass in the *downward* direction toward the floor.
   a) Obtain an expression for the displacement of the mass in the form $x(t) = Ae^{-\gamma t/2}\cos(\omega t + \phi)$, giving numerical values for A, ω, γ, and ϕ. Notice that the oscillator is experiencing *very* light damping and should be treated as such.
   b) Use a spreadsheet program (i.e. Excel) to plot this function over the time period $t = 0$ to 4.00s, with a time step no greater than 0.0200s. Your columns of input data must be labeled with the correct SI units. Your plot needs to include a title and the axes should be labeled and include units.

3. Again reconsider the oscillator studied extensively in Homework Set 11 involving the mass $m = 0.550$ kg, which is attached to the spring and hung vertically. This time, the oscillator is placed in a resistive medium where the oscillator is found to be *critically damped*.
   a) Determine the value of the damping constant b, and therefore the damping parameter γ, that is required to produce this critical damping.

   At time $t = 0$, the mass is displaced from the equilibrium position by 5.50 cm in the *downward* direction and is given an initial shove, imparting a speed of 0.450 m/s on the mass in the *downward* direction toward the floor.

b) Obtain an expression for the displacement of the mass in the form
$x(t) = (A + Bt)e^{-\gamma t/2}$, giving numerical values for A, B, and γ.

c) Find the *maximum displacement* from equilibrium and the time, t', when this occurs.

4. Reconsider the oscillator of the previous problem. Using a spreadsheet program (i.e. Excel), plot this function over the time period $t = 0$ to 0.500s, with a time step no greater than 0.0100s. Your columns of input data must be labeled with the correct SI units. Your plot needs to include a title and the axes should be labeled and include units. From the plot, determine the *maximum displacement* from equilibrium and the time, t', when this occurs. How does this compare to the result from problem 3?

5. Again reconsider the oscillator studied extensively in Homework Set 11 involving the mass $m = 0.550$ kg, which is attached to the spring and hung vertically. This time, the oscillator is placed in a resistive medium where the oscillator is found to be *heavily damped.* The resistive medium is described by a damping parameter γ that is *triple* the value found in problem 3.

   At time $t = 0$, the mass is displaced from the equilibrium position by 5.50 cm in the *downward* direction and is given an initial shove, imparting a speed of 0.450 m/s on the mass in the *downward* direction toward the floor.

   a) Obtain an expression for the displacement of the mass in the form
   $x(t) = e^{-\gamma t/2}\left[Ae^{+\alpha t} + Be^{-\alpha t}\right]$, where $\alpha \equiv \sqrt{(\gamma/2)^2 - \omega_0^2}$, giving numerical values for A, B, γ, and α.

   b) Using a spreadsheet program (i.e. Excel), plot this function over the time period $t = 0$ to 0.500s, with a time step no greater than 0.0100s. Your columns of input data must be labeled with the correct SI units. Your plot needs to include a title and the axes should be labeled and include units. Include the plot generated in problem 4 for comparison.