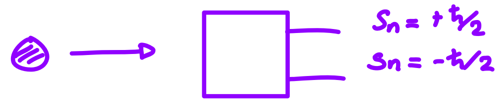
Exercise 1

$|\varphi\rangle = \cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle$:



Problem 1 | SG $\hat{z}$: $|+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\varphi}\sin(\theta/2)|-\hat{z}\rangle$: $|-\hat{n}\rangle = \sin(\theta/2)|+\hat{z}\rangle - e^{i\varphi}\cos(\theta/2)|-\hat{z}\rangle$

$\theta = 0,\ \varphi = 0$

$|+\hat{z}\rangle = \cos(0)|+\hat{z}\rangle + e^{0}\cdot\sin(0)|-\hat{z}\rangle = |+\hat{z}\rangle$: $\therefore$ $|+\hat{z}\rangle = |+\hat{z}\rangle$

$|-\hat{z}\rangle = \sin(0)|+\hat{z}\rangle - e^{0}\cdot\cos(0)|-\hat{z}\rangle = -|-\hat{z}\rangle$: $\therefore$ $|-\hat{z}\rangle = |-\hat{z}\rangle$

$\langle+\hat{z}|\varphi\rangle = (\langle+\hat{z}| + 0\cdot\langle-\hat{z}|)(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = \cos(\alpha/2)\cancelto{1}{\langle+\hat{z}|+\hat{z}\rangle} + 0\cdot\cancelto{0}{\sin(\alpha/2)}\cancelto{1}{\langle-\hat{z}|-\hat{z}\rangle}$

$\langle+\hat{z}|\varphi\rangle = \cos(\alpha/2)$: $|\langle+\hat{z}|\varphi\rangle|^2 = \cos^2(\alpha/2) = \frac{n_+}{n}$

$\langle-\hat{z}|\varphi\rangle = (0\cdot\langle+\hat{z}| - \langle-\hat{z}|)(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = 0\cdot\cancelto{0}{\cos(\alpha/2)}\cancelto{1}{\langle+\hat{z}|+\hat{z}\rangle} - \sin(\alpha/2)\cancelto{1}{\langle-\hat{z}|-\hat{z}\rangle}$

$\langle-\hat{z}|\varphi\rangle = -\sin(\alpha/2)$: $|\langle-\hat{z}|\varphi\rangle|^2 = \sin^2(\alpha/2) = \frac{n_-}{n}$

$|\langle+\hat{z}|\varphi\rangle|^2 + |\langle-\hat{z}|\varphi\rangle|^2 = 1$: $\cos^2(\alpha/2) + \sin^2(\alpha/2) = 1$ $\therefore$ α can be any value

The α value can be calculated via

$$\boxed{\cos^2(\alpha/2) = \frac{n_+}{n},\quad \sin^2(\alpha/2) = \frac{n_-}{n}}$$

Problem 2 | SG $\hat{x}$: $|+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\varphi}\sin(\theta/2)|-\hat{z}\rangle$: $|-\hat{n}\rangle = \sin(\theta/2)|+\hat{z}\rangle - e^{i\varphi}\cos(\theta/2)|-\hat{z}\rangle$

$\theta = \pi/2,\ \varphi = 0$

$|+\hat{x}\rangle = \cos(\pi/4)|+\hat{z}\rangle + e^{0}\sin(\pi/4)|-\hat{z}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{1}{\sqrt{2}}|-\hat{z}\rangle$ $\therefore$ $|+\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{1}{\sqrt{2}}|-\hat{z}\rangle$

$|-\hat{x}\rangle = \sin(\pi/4)|+\hat{z}\rangle - e^{0}\cos(\pi/4)|-\hat{z}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle - \frac{1}{\sqrt{2}}|-\hat{z}\rangle$ $\therefore$ $|-\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle - \frac{1}{\sqrt{2}}|-\hat{z}\rangle$

$\langle+\hat{x}|\varphi\rangle = \frac{1}{\sqrt{2}}(\langle+\hat{z}| + \langle-\hat{z}|)(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = \frac{1}{\sqrt{2}}(\cos(\alpha/2)\cancelto{1}{\langle+\hat{z}|+\hat{z}\rangle} + \sin(\alpha/2)\cancelto{1}{\langle-\hat{z}|-\hat{z}\rangle})$

$\langle+\hat{x}|\varphi\rangle = \frac{1}{\sqrt{2}}(\cos(\alpha/2) + \sin(\alpha/2))$: $|\langle+\hat{x}|\varphi\rangle|^2 = \frac{1}{2}(\cos^2(\alpha/2) + \sin^2(\alpha/2) + 2\cos(\alpha/2)\sin(\alpha/2))$

$= \frac{1}{2}(1 + \sin(\alpha))$: $|\langle+\hat{x}|\varphi\rangle|^2 = \frac{1}{2}(1+\sin(\alpha)) = \frac{n_+}{n}$

$\langle-\hat{x}|\varphi\rangle = \frac{1}{\sqrt{2}}(\langle+\hat{z}| - \langle-\hat{z}|)(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = \frac{1}{\sqrt{2}}(\cos(\alpha/2)\cancelto{1}{\langle+\hat{z}|+\hat{z}\rangle} - \sin(\alpha/2)\cancelto{1}{\langle-\hat{z}|-\hat{z}\rangle})$

$\langle-\hat{x}|\varphi\rangle = \frac{1}{\sqrt{2}}(\cos(\alpha/2) - \sin(\alpha/2))$: $|\langle-\hat{x}|\varphi\rangle|^2 = \frac{1}{2}(\cos^2(\alpha/2) + \sin^2(\alpha/2) - 2\cos(\alpha/2)\sin(\alpha/2))$

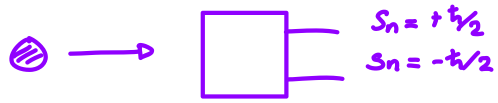
$= \frac{1}{2}(1 - \sin(\alpha))$: $|\langle-\hat{x}|\varphi\rangle|^2 = \frac{1}{2}(1 - \sin(\alpha)) = \frac{n_-}{n}$

The α value can be calculated via

$$\boxed{\frac{1}{2}(1+\sin(\alpha)) = \frac{n_+}{n},\quad \frac{1}{2}(1-\sin(\alpha)) = \frac{n_-}{n}}$$

Exercise 1 Continued

$|\varphi\rangle = \cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle$:



Problem 3 | SG $\hat{y}$: $|+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\varphi}\sin(\theta/2)|-\hat{z}\rangle$: $|-\hat{n}\rangle = \sin(\theta/2)|+\hat{z}\rangle - e^{i\varphi}\cos(\theta/2)|-\hat{z}\rangle$

$\theta = \pi/2,\ \varphi = \pi/2$

$$|+\hat{y}\rangle = \cos(\pi/4)|+\hat{z}\rangle + e^{i\cdot\frac{\pi}{2}}\sin(\pi/4)|-\hat{z}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{i}{\sqrt{2}}|-\hat{z}\rangle \quad \therefore \quad |+\hat{y}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{i}{\sqrt{2}}|-\hat{z}\rangle$$

$$|-\hat{y}\rangle = \sin(\pi/4)|+\hat{z}\rangle - e^{i\frac{\pi}{2}}\cos(\pi/4)|-\hat{z}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle - \frac{i}{\sqrt{2}}|-\hat{z}\rangle \quad \therefore \quad |-\hat{y}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle - \frac{i}{\sqrt{2}}|-\hat{z}\rangle$$

$$\langle +\hat{y}|\varphi\rangle = \frac{1}{\sqrt{2}}\left(\langle +\hat{z}| - i\langle -\hat{z}|\right)\left(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle\right) = \frac{1}{\sqrt{2}}\left(\cos(\alpha/2)\cancelto{1}{\langle +\hat{z}|+\hat{z}\rangle} - i\sin(\alpha/2)\cancelto{1}{\langle -\hat{z}|-\hat{z}\rangle}\right)$$

$$\langle +\hat{y}|\varphi\rangle = \frac{1}{\sqrt{2}}\left(\cos(\alpha/2) - i\sin(\alpha/2)\right)$$

$$|\langle +\hat{y}|\varphi\rangle|^2 = \frac{1}{2}\left(\cos(\alpha/2) - i\sin(\alpha/2)\right)\left(\cos(\alpha/2) + i\sin(\alpha/2)\right) = \frac{1}{2}\left(\cos^2(\alpha/2) + \sin^2(\alpha/2)\right)$$

$|\langle +\hat{y}|\varphi\rangle|^2 = \frac{1}{2}\left(\cos^2(\alpha/2) + \sin^2(\alpha/2)\right) \longrightarrow$ This means α can be anything

> We cannot learn anything about α by measuring along the Y direction.

Exercise 2

$$|\varphi_1\rangle = |0\rangle \;,\; |\varphi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \;,\; |\varphi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

Problem 1 | $|\varphi_1\rangle|\varphi_2\rangle$

$$|\varphi_1\rangle|\varphi_2\rangle = (|0\rangle)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|1\rangle$$

$$\boxed{|\varphi_1\rangle|\varphi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |0\rangle|1\rangle\right)}$$

Problem 2 | $|\varphi_1\rangle|\varphi_3\rangle$

$$|\varphi_1\rangle|\varphi_3\rangle = (|0\rangle)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle|0\rangle - \frac{i}{\sqrt{2}}|0\rangle|1\rangle$$

$$\boxed{|\varphi_1\rangle|\varphi_3\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle - i|0\rangle|1\rangle\right)}$$

Problem 3 | $|\varphi_2\rangle|\varphi_3\rangle$

$$|\varphi_2\rangle|\varphi_3\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \cdot \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right) = \frac{1}{2}\left(|0\rangle|0\rangle - i|0\rangle|1\rangle + |1\rangle|0\rangle - i|1\rangle|1\rangle\right)$$

$$\boxed{|\varphi_2\rangle|\varphi_3\rangle = \frac{1}{2}\left(|0\rangle|0\rangle - i|0\rangle|1\rangle + |1\rangle|0\rangle - i|1\rangle|1\rangle\right)}$$

Exercise 3

Problem 1

$$|\psi\rangle = \frac{1}{2}\left[|0\rangle|0\rangle + |0\rangle|1\rangle - |1\rangle|0\rangle - |1\rangle|1\rangle\right]$$

a.) SG$\hat{z}$ for A, SG$\hat{z}$ for B $\{|0\rangle, |1\rangle\}$

SG$\hat{z}$(A)	SG$\hat{z}$(B)	State
+	+	$\|0\rangle\|0\rangle$
+	-	$\|0\rangle\|1\rangle$
-	+	$\|1\rangle\|0\rangle$
-	-	$\|1\rangle\|1\rangle$

$$\langle 0|\langle 0|\psi\rangle = \frac{1}{2}\left[\langle 0|\overset{1}{\cancel{\langle 0|0\rangle}}|0\rangle + \langle 0|\overset{0}{\cancel{\langle 0|0\rangle}}|1\rangle - \langle 0|\overset{0}{\cancel{\langle 0|1\rangle}}|0\rangle - \langle 0|\overset{0}{\cancel{\langle 0|1\rangle}}|1\rangle\right]$$

$$\langle 0|\langle 0|\psi\rangle = \frac{1}{2} \quad \therefore \quad \boxed{|\langle 0|\langle 0|\psi\rangle|^2 = 1/4}$$

$$\langle 0|\langle 1|\psi\rangle = \frac{1}{2}\left[\langle 0|\overset{0}{\cancel{\langle 1|0\rangle}}|0\rangle + \langle 0|\overset{1}{\cancel{\langle 1|0\rangle}}|1\rangle - \langle 0|\overset{0}{\cancel{\langle 1|1\rangle}}|0\rangle - \langle 0|\overset{0}{\cancel{\langle 1|1\rangle}}|1\rangle\right]$$

$$\langle 0|\langle 1|\psi\rangle = \frac{1}{2} \quad \therefore \quad \boxed{|\langle 0|\langle 1|\psi\rangle|^2 = 1/4}$$

$$\langle 1|\langle 0|\psi\rangle = \frac{1}{2}\left[\langle 1|\overset{0}{\cancel{\langle 0|0\rangle}}|0\rangle + \langle 1|\overset{0}{\cancel{\langle 0|0\rangle}}|1\rangle - \langle 1|\overset{1}{\cancel{\langle 0|1\rangle}}|0\rangle - \langle 1|\overset{0}{\cancel{\langle 0|1\rangle}}|1\rangle\right]$$

$$\langle 1|\langle 0|\psi\rangle = -\frac{1}{2} \quad \therefore \quad \boxed{|\langle 1|\langle 0|\psi\rangle|^2 = 1/4}$$

$$\langle 1|\langle 1|\psi\rangle = \frac{1}{2}\left[\langle 1|\overset{0}{\cancel{\langle 1|0\rangle}}|0\rangle + \langle 1|\overset{0}{\cancel{\langle 1|0\rangle}}|1\rangle - \langle 1|\overset{0}{\cancel{\langle 1|1\rangle}}|0\rangle - \langle 1|\overset{1}{\cancel{\langle 1|1\rangle}}|1\rangle\right]$$

$$\langle 1|\langle 1|\psi\rangle = -\frac{1}{2} \quad \therefore \quad \boxed{|\langle 1|\langle 1|\psi\rangle|^2 = 1/4}$$

b.) SG$\hat{x}$ for A, SG$\hat{z}$ for B $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$, $\{|0\rangle, |1\rangle\}$

SG$\hat{x}$(A)	SG$\hat{z}$(B)	State
+	+	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}, \|0\rangle = \frac{\|0\rangle\|0\rangle + \|1\rangle\|0\rangle}{\sqrt{2}}$
+	-	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}, \|1\rangle = \frac{\|0\rangle\|1\rangle + \|1\rangle\|1\rangle}{\sqrt{2}}$
-	+	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}, \|0\rangle = \frac{\|0\rangle\|0\rangle - \|1\rangle\|0\rangle}{\sqrt{2}}$
-	-	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}, \|1\rangle = \frac{\|0\rangle\|1\rangle - \|1\rangle\|1\rangle}{\sqrt{2}}$

$$\frac{\langle 0|\langle 0| + \langle 1|\langle 0|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\langle 0|\overset{1}{\cancel{\langle 0|0\rangle}}|0\rangle + \langle 1|\overset{0}{\cancel{\langle 0|0\rangle}}|0\rangle + \langle 0|\overset{0}{\cancel{\langle 0|0\rangle}}|1\rangle + \langle 1|\overset{0}{\cancel{\langle 0|0\rangle}}|1\rangle -$$

$$\langle 0|\overset{0}{\cancel{\langle 0|1\rangle}}|0\rangle - \langle 1|\overset{1}{\cancel{\langle 0|1\rangle}}|0\rangle - \langle 0|\overset{0}{\cancel{\langle 0|1\rangle}}|1\rangle - \langle 1|\overset{0}{\cancel{\langle 0|1\rangle}}|1\rangle\Big]$$

$$= \frac{1}{2\sqrt{2}}(1-1) = 0 \quad \therefore \quad \boxed{\left|\frac{\langle 0|\langle 0 + \langle 1|\langle 0|\psi\rangle}{\sqrt{2}}\right|^2 = 0}$$

Exercise 3 Continued

$$\frac{\langle 0|\langle 1| + \langle 1|\langle 1|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\overset{0}{\cancel{\langle 0|\langle 1|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|0\rangle}} + \overset{1}{\cancel{\langle 0|\langle 1|0\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|1\rangle}} -$$

$$\overset{0}{\cancel{\langle 0|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|1\rangle}} - \overset{1}{\cancel{\langle 1|\langle 1|1\rangle|1\rangle}}\Big]$$

$$= \frac{1}{2\sqrt{2}}(1-1) = 0 \quad \therefore \quad \boxed{\left|\frac{\langle 0|\langle 1| + \langle 1|\langle 1|\psi\rangle}{\sqrt{2}}\right|^2 = 0}$$

$$\frac{\langle 0|\langle 0| - \langle 1|\langle 0|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\overset{1}{\cancel{\langle 0|\langle 0|0\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 0|\langle 0|0\rangle|1\rangle}} - \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|1\rangle}} -$$

$$\overset{0}{\cancel{\langle 0|\langle 0|1\rangle|0\rangle}} + \overset{1}{\cancel{\langle 1|\langle 0|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 0|1\rangle|1\rangle}}\Big]$$

$$= \frac{1}{2\sqrt{2}}(1+1) = \frac{1}{\sqrt{2}} \quad \therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| - \langle 1|\langle 0|\psi\rangle}{\sqrt{2}}\right|^2 = \frac{1}{2}}$$

$$\frac{\langle 0|\langle 1| - \langle 1|\langle 1|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\overset{0}{\cancel{\langle 0|\langle 1|0\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|0\rangle}} + \overset{1}{\cancel{\langle 0|\langle 1|0\rangle|1\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|1\rangle}}$$

$$- \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|1\rangle}} + \overset{1}{\cancel{\langle 1|\langle 1|1\rangle|1\rangle}}\Big]$$

$$= \frac{1}{2\sqrt{2}}(1+1) = \frac{1}{\sqrt{2}} \quad \therefore \quad \boxed{\left|\frac{\langle 0|\langle 1| - \langle 1|\langle 1|\psi\rangle}{\sqrt{2}}\right|^2 = \frac{1}{2}}$$

c.) SG $\hat{x}$ for A, SG $\hat{x}$ for B $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$

SG $\hat{x}$ (A)	SG $\hat{x}$ (B)	State
+	+	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}, \frac{\|0\rangle+\|1\rangle}{\sqrt{2}} = \frac{1}{2}(\|0\rangle\|0\rangle + \|0\rangle\|1\rangle + \|1\rangle\|0\rangle + \|1\rangle\|1\rangle)$
+	−	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}, \frac{\|0\rangle-\|1\rangle}{\sqrt{2}} = \frac{1}{2}(\|0\rangle\|0\rangle - \|0\rangle\|1\rangle + \|1\rangle\|0\rangle - \|1\rangle\|1\rangle)$
−	+	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}, \frac{\|0\rangle+\|1\rangle}{\sqrt{2}} = \frac{1}{2}(\|0\rangle\|0\rangle + \|0\rangle\|1\rangle - \|1\rangle\|0\rangle - \|1\rangle\|1\rangle)$
−	−	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}, \frac{\|0\rangle-\|1\rangle}{\sqrt{2}} = \frac{1}{2}(\|0\rangle\|0\rangle - \|0\rangle\|1\rangle - \|1\rangle\|0\rangle + \|1\rangle\|1\rangle)$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[\overset{1}{\cancel{\langle 0|\langle 0|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 0|\langle 0|0\rangle|1\rangle}} - \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|1\rangle}}$$

$$+ \overset{0}{\cancel{\langle 0|\langle 1|0\rangle|0\rangle}} + \overset{1}{\cancel{\langle 0|\langle 1|0\rangle|1\rangle}} + \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|0\rangle}} + \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|1\rangle}}$$

$$- \overset{1}{\cancel{\langle 1|\langle 0|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 0|1\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|1\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|1\rangle|0\rangle}} - \overset{1}{\cancel{\langle 1|\langle 1|1\rangle|1\rangle}}\Big]$$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1+1-1-1) = 0$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = 0}$$

Exercise 3 Continued

$$\frac{\langle 0|\langle 0| - \langle 0|\langle 1| + \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[\langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle$$

$$- \langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle -$$

$$\langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle\Big]$$

$$\frac{\langle 0|\langle 0| - \langle 0|\langle 1| + \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1-1-1+1) = 0$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| - \langle 0|\langle 1| + \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = 0}$$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| - \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[\langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle$$

$$+ \langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle - \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle +$$

$$\langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle\Big]$$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| - \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1+1+1+1) = 1$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| + \langle 0|\langle 1| - \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = 1}$$

$$\frac{\langle 0|\langle 0| - \langle 0|\langle 1| - \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[\langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle -$$

$$\langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle +$$

$$\langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle - \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle\Big]$$

$$\frac{\langle 0|\langle 0| - \langle 0|\langle 1| - \langle 1|\langle 0| + \langle 1|\langle 1|}{2} = \frac{1}{4}(1-1+1-1) = 0$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| - \langle 0|\langle 1| - \langle 1|\langle 0| + \langle 1|\langle 1|}{2}\right|^2 = 0}$$

There is certainty with the SG $\hat{x}$ for A, SG $\hat{x}$ for B, pair of measurements. Precisely the $(-+)$ pair for A and B specifically.

Exercise 3 Continued

Problem 2 $\quad |\psi\rangle = \frac{1}{2}\Big[|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle\Big]$

a.) SG $\hat{z}$ for A, SG $\hat{z}$ for B $\quad \{|0\rangle, |1\rangle\}$

SG $\hat{z}$ (A)	SG $\hat{z}$ (B)	State
+	+	$\|0\rangle\|0\rangle$
+	−	$\|0\rangle\|1\rangle$
−	+	$\|1\rangle\|0\rangle$
−	−	$\|1\rangle\|1\rangle$

$$\langle 0|\langle 0|\psi\rangle = \frac{1}{2}\Big[\langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle\Big]$$

$$\langle 0|\langle 0|\psi\rangle = \frac{1}{2} \qquad \therefore \qquad \boxed{|\langle 0|\langle 0|\psi\rangle|^2 = 1/4}$$

$$\langle 0|\langle 1|\psi\rangle = \frac{1}{2}\Big[\langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle\Big]$$

$$\langle 0|\langle 1|\psi\rangle = \frac{1}{2} \qquad \therefore \qquad \boxed{|\langle 0|\langle 1|\psi\rangle|^2 = 1/4}$$

$$\langle 1|\langle 0|\psi\rangle = \frac{1}{2}\Big[\langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle\Big]$$

$$\langle 1|\langle 0|\psi\rangle = \frac{1}{2} \qquad \therefore \qquad \boxed{|\langle 1|\langle 0|\psi\rangle|^2 = 1/4}$$

$$\langle 1|\langle 1|\psi\rangle = \frac{1}{2}\Big[\langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle - \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle\Big]$$

$$\langle 1|\langle 1|\psi\rangle = \frac{-1}{2} \qquad \therefore \qquad \boxed{|\langle 1|\langle 1|\psi\rangle = 1/4}$$

b.) SG $\hat{x}$ for A, SG $\hat{z}$ for B $\quad \left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}, \{|0\rangle, |1\rangle\}$

SG $\hat{x}$ (A)	SG $\hat{z}$ (B)	State
+	+	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}, \|0\rangle = \frac{\|0\rangle\|0\rangle + \|1\rangle\|0\rangle}{\sqrt{2}}$
+	−	$\frac{\|0\rangle+\|1\rangle}{\sqrt{2}}, \|1\rangle = \frac{\|0\rangle\|1\rangle + \|1\rangle\|1\rangle}{\sqrt{2}}$
−	+	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}, \|0\rangle = \frac{\|0\rangle\|0\rangle - \|1\rangle\|0\rangle}{\sqrt{2}}$
−	−	$\frac{\|0\rangle-\|1\rangle}{\sqrt{2}}, \|1\rangle = \frac{\|0\rangle\|1\rangle - \|1\rangle\|1\rangle}{\sqrt{2}}$

$$\frac{\langle 0|\langle 0| + \langle 1|\langle 0|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle$$
$$+ \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle + \langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle\Big]$$

$$\frac{\langle 0|\langle 0| + \langle 1|\langle 0|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}(1+1) = \frac{1}{\sqrt{2}} \quad \therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| + \langle 1|\langle 0|\psi\rangle}{\sqrt{2}}\right|^2 = 1/2}$$

Exercise 3 Continued

$$\frac{\langle 0|\langle 1| + \langle 1|\langle 1|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\overset{0}{\cancel{\langle 0|\langle 1|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|0\rangle}} + \overset{1}{\cancel{\langle 0|\langle 1|0\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|1\rangle}}$$
$$+ \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|1\rangle}} - \overset{1}{\cancel{\langle 1|\langle 1|1\rangle|1\rangle}}\Big]$$

$$\frac{\langle 0|\langle 1| + \langle 1|\langle 1|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}(1-1) = 0 \quad : \quad \boxed{\left|\frac{\langle 0|\langle 1| + \langle 1|\langle 1|\psi\rangle}{\sqrt{2}}\right|^2 = 0}$$

$$\frac{\langle 0|\langle 0| - \langle 1|\langle 0|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\overset{1}{\cancel{\langle 0|\langle 0|0\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 0|\langle 0|0\rangle|1\rangle}} - \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|1\rangle}}$$
$$+ \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|0\rangle}} - \overset{1}{\cancel{\langle 1|\langle 0|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 0|1\rangle|1\rangle}}\Big]$$

$$\frac{\langle 0|\langle 0| - \langle 1|\langle 0|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}(1-1) = 0 \quad : \quad \boxed{\left|\frac{\langle 0|\langle 0| - \langle 1|\langle 0|\psi\rangle}{\sqrt{2}}\right|^2 = 0}$$

$$\frac{\langle 0|\langle 1| - \langle 1|\langle 1|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\Big[\overset{0}{\cancel{\langle 0|\langle 1|0\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|0\rangle}} + \overset{1}{\cancel{\langle 0|\langle 1|0\rangle|1\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|1\rangle}}$$
$$+ \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|1\rangle}} + \overset{1}{\cancel{\langle 1|\langle 1|1\rangle|1\rangle}}\Big]$$

$$\frac{\langle 0|\langle 1| - \langle 1|\langle 1|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}}(1+1) = \frac{1}{\sqrt{2}} \quad : \quad \boxed{\left|\frac{\langle 0|\langle 1| - \langle 1|\langle 1|\psi\rangle}{\sqrt{2}}\right|^2 = 1/2}$$

c.) SG$\hat{x}$ for A, SG $\hat{x}$ for B $\left\{\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right\}$

SG $\hat{x}$ (A)	SG $\hat{x}$ (B)	State
+	+	$\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle+|1\rangle}{\sqrt{2}} = \frac{1}{2}(|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle + |1\rangle|1\rangle)$
+	−	$\frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{1}{2}(|0\rangle|0\rangle - |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle)$
−	+	$\frac{|0\rangle-|1\rangle}{\sqrt{2}}, \frac{|0\rangle+|1\rangle}{\sqrt{2}} = \frac{1}{2}(|0\rangle|0\rangle + |0\rangle|1\rangle - |1\rangle|0\rangle - |1\rangle|1\rangle)$
−	−	$\frac{|0\rangle-|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{1}{2}(|0\rangle|0\rangle - |0\rangle|1\rangle - |1\rangle|0\rangle + |1\rangle|1\rangle)$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[\overset{1}{\cancel{\langle 0|\langle 0|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 0|\langle 0|0\rangle|1\rangle}} + \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 0|1\rangle|1\rangle}} +$$
$$\overset{0}{\cancel{\langle 0|\langle 1|0\rangle|0\rangle}} + \overset{1}{\cancel{\langle 0|\langle 1|0\rangle|1\rangle}} + \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 0|\langle 1|1\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 0|0\rangle|1\rangle}} +$$
$$\overset{1}{\cancel{\langle 1|\langle 0|1\rangle|0\rangle}} - \overset{0}{\cancel{\langle 1|\langle 0|1\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|0\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|0\rangle|1\rangle}} + \overset{0}{\cancel{\langle 1|\langle 1|1\rangle|0\rangle}} - \overset{1}{\cancel{\langle 1|\langle 1|1\rangle|1\rangle}}\Big]$$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1+1+1-1) = \frac{1}{2}$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = 1/4}$$

Exercise 3 Continued

$$
\begin{aligned}
\frac{\langle 0|\langle 0| - \langle 0|\langle 1| + \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[& \langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle \\
& - \langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle - \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \\
& \langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle \Big]
\end{aligned}
$$

$$\frac{\langle 0|\langle 0| - \langle 0|\langle 1| + \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1-1+1+1) = \frac{1}{2}$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| - \langle 0|\langle 1| + \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = \frac{1}{4}}$$

$$
\begin{aligned}
\frac{\langle 0|\langle 0| + \langle 0|\langle 1| - \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}\Big[& \langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle \\
& + \langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle \\
& - \langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 1|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle \Big]
\end{aligned}
$$

$$\frac{\langle 0|\langle 0| + \langle 0|\langle 1| - \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1+1-1+1) = \frac{1}{2}$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| + \langle 0|\langle 1| - \langle 1|\langle 0| - \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = \frac{1}{4}}$$

$$
\begin{aligned}
\frac{\langle 0|\langle 0| - \langle 0|\langle 1| - \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \Big[& \langle 0|\cancelto{1}{\langle 0|0\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 0|0\rangle}|1\rangle + \langle 0|\cancelto{0}{\langle 0|1\rangle}|0\rangle - \langle 0|\cancelto{0}{\langle 0|1\rangle}|1\rangle - \\
& \langle 0|\cancelto{0}{\langle 1|0\rangle}|0\rangle - \langle 0|\cancelto{1}{\langle 1|0\rangle}|1\rangle - \langle 0|\cancelto{0}{\langle 1|1\rangle}|0\rangle + \langle 0|\cancelto{0}{\langle 1|1\rangle}|1\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|0\rangle - \langle 1|\cancelto{0}{\langle 0|0\rangle}|1\rangle - \\
& \langle 1|\cancelto{1}{\langle 0|1\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 0|1\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 1|0\rangle}|0\rangle + \langle 1|\cancelto{0}{\langle 1|0\rangle}|1\rangle + \langle 1|\cancelto{0}{\langle 1|1\rangle}|0\rangle - \langle 1|\cancelto{1}{\langle 1|1\rangle}|1\rangle \Big]
\end{aligned}
$$

$$\frac{\langle 0|\langle 0| - \langle 0|\langle 1| - \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2} = \frac{1}{4}(1-1-1-1) = \frac{-1}{2}$$

$$\therefore \quad \boxed{\left|\frac{\langle 0|\langle 0| - \langle 0|\langle 1| - \langle 1|\langle 0| + \langle 1|\langle 1|\psi\rangle}{2}\right|^2 = \frac{1}{4}}$$

Exercise 3 Continued

Problem 3 Find $|\psi_A\rangle, |\psi_B\rangle$ s.t $|\psi\rangle = |\psi_A\rangle|\psi_B\rangle$

$$|\psi_A\rangle = a_0|0\rangle + a_1|1\rangle \quad , \quad |\psi_B\rangle = b_0|0\rangle + b_1|1\rangle$$

$$|\psi_A\rangle|\psi_B\rangle = a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle$$

for, $|\psi\rangle = \frac{1}{2}\left[|0\rangle|0\rangle + |0\rangle|1\rangle - |1\rangle|0\rangle - |1\rangle|1\rangle\right]$

$a_0 = \frac{-1}{\sqrt{2}}$, $b_0 = \frac{-1}{\sqrt{2}}$, $a_1 = \frac{1}{\sqrt{2}}$, $b_1 = \frac{-1}{\sqrt{2}}$ $\longrightarrow$ These Satisfy $|\psi\rangle$ $\therefore$

$$\boxed{|\psi_A\rangle = \frac{-1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad , \quad |\psi_B\rangle = \frac{-1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle}$$

$$|\psi_A\rangle = a_0|0\rangle + a_1|1\rangle \quad , \quad |\psi_B\rangle = b_0|0\rangle + b_1|1\rangle$$

$$|\psi_A\rangle|\psi_B\rangle = a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle$$

for, $|\psi\rangle = \frac{1}{2}\left[|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle\right]$

There is no combination of a_0, b_0, a_1, b_1 that will satisfy $|\psi\rangle$.

$\boxed{\text{This is an entangled state}}$