Electromagnetic Theory II: Homework 9

Due: 9 March 2021

1 Separable solutions to the wave equation

Consider a solution to the wave equation of the form

$$f(z,t) = \sin(\omega t)\, h(z).$$

where ω is a constant.

a) By substituting into the classical wave equation, determine the most general form $h(z)$ which solves the classical wave equation. *Hint: You will know when you have the most general solution, if your solution has two undetermined parameters.*

b) Suppose that you required your solution to satsify

$$\frac{\partial f}{\partial t} = 0$$

for all z at $t = 0$ but that in general $f \neq 0$. Would this be possible given the solution above? If not suggest a simple modification to the solution which would render it possible.

2 Sinusoidal waves and energy transport

A sinusoidal traveling wave on a string has frequency ω, wavenumber k and amplitude A. In each of the following, briefly explain your answer.

a) Suppose that the amplitude is increased to $3A$ without changing the other parameters. Which of the following is true regarding in rate at which energy is transported?

   i) It remains unchanged.
   ii) It increases to a factor of 3 of what it had been.
   iii) It increases to a factor of 9 of what it had been.

b) Suppose that the tension in the string is adjusted so that the wave speed increases to three times what it had been previously. The frequency and the amplitude remain unchanged. Which of the following is true regarding in rate at which energy is transported?

   i) It remains unchanged.
   ii) It increases to a factor of 3 of what it had been.
   iii) It increases to a factor of 9 of what it had been.

3 Energy transport on a string

The displacement of an infinitely long string with mass per unit length μ and tension T has the form

$$f(z,t) = Be^{-(z-vt)^2/a^2}$$

where $B > 0$ and $a > 0$ are constants, each with units of meters.

a) Determine the total energy stored in the string.

b) Determine the rate at which energy passes from left to right at $z = 0$. Describe how this will differ at another point $z_0 > 0$.

4 Energy of a normal mode of a string with both ends fixed

Consider a string spanning the range $0 \leqslant x \leqslant L$ with both ends fixed. The standing waves on this string, or normal modes, are of the form

$$y_n(x,t) = A_n \sin(k_n x) \cos(\omega_n t + \phi).$$

where A_n is a constant, $k_n = n\pi/L$, $\omega = vk_n$ with v being the wavespeed and $n = 1, 2, \ldots$

a) Determine an expression for the energy of the n^{th} normal mode.

b) Rewrite this in terms of the total mass of the string.

It is possible to create electromagnetic standing waves. Using the analogous quantity for the energy of these is the starting point for a quantum theory description of light.