Quiz on Thursday

-3.1 & 3.2

$uv - \int v\,du$

$$h_n(t) = a_0 + \sum_{k=0}^{n} a_k \cos\left(\frac{2\pi k}{T} t\right) + \sum_{k=1}^{n} b_k \sin\left(\frac{2\pi k}{T} t\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$
$$a_k = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi k}{T}t\right)dt$$
$$b_k = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi k}{T}t\right)dt$$

$T=1$
$$a_0 = \int_0^1 f(t)\,dt$$
$$a_k = 2\int_0^1 f(t)\cos(2\pi k t)\,dt$$
$$b_k = 2\int_0^1 f(t)\sin(2\pi k t)\,dt$$

$f(t) = 3 \qquad 0 \le t \le \frac{1}{2}$
$f(t) = 7 \qquad \frac{1}{2} \le t \le 1$

$$\int_0^{1/2} 3\cdot\sin(2\pi\cdot k\cdot t)\,dt + \int_{1/2}^1 7\cdot\sin(2\pi\cdot k\cdot t)\,dt$$

$\sin(u)$
$u = (2\pi\cdot k\cdot t)$
$du = 2\pi k\,dt$
$\frac{du}{2\pi k} = dt$

$$\int_0^{1/2} \frac{3\cdot\sin(u)\,du}{2\pi k}$$
$$\frac{3}{2\pi k}\int \sin(u)\,du$$
$$\frac{3}{2\pi k}\left[-\cos(2\pi\cdot k\cdot t)\right]_0^{1/2}$$
$$\frac{3}{2\pi k}(+1-1) = 0$$

$$\int_{1/2}^1 7\sin(2\pi\cdot k\cdot t)\,dt$$
$$\frac{7}{2\pi k}\int_{1/2}^1 \sin(u)\,du$$
$$\frac{7}{2\pi k}\left[-\cos(u)\right]_{1/2}^1$$
$$\frac{7}{2\pi k}\left[-\cos(2\pi\cdot k\cdot t)\right]_{1/2}^1$$
$$\frac{7}{2\pi k}\left(-\cos(2\pi\cdot k) + \cos(\pi\cdot k)\right)$$
$$\frac{7}{2\pi k}(-1+1) = 0$$

$b_k = 0$

$f(t) = t,\ [0,1]$

$$b_k = 2\int_0^1 t\sin(2\pi\cdot k\cdot t)\,dt$$

$u = t \qquad dv = \sin(2\pi\cdot k\cdot t)$
$du = 1 \qquad v = -\frac{1}{2\pi k}\cos(2\pi\cdot k\cdot t)$

$$2\left(-\frac{t}{2\pi k}\cos(2\pi\cdot k\cdot t)\right) + \frac{1}{2\pi k}\int \cos(2\pi\cdot k\cdot t)\,dt$$
$$2\left(-\frac{t}{2\pi k}\cos(2\pi\cdot k\cdot t)\right) + \frac{1}{(2\pi k)^2}\left(\sin(2\pi\cdot k\cdot t)\right)\Big|_0^1$$
$$2\left(\frac{-t}{2\pi k}\cos(2\pi\cdot k\cdot t)\Big|_0^1\right) = \frac{-2}{2\pi k}\cdot\cos(2\pi\cdot k) = \frac{2\cos(2\pi\cdot k)}{2\pi k} = \frac{-\cos(2\pi\cdot k)}{\pi k}$$

$$\frac{-\cos(2\pi\cdot k)}{(\pi k)} = \frac{-1}{\pi k}$$