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PHYS 362 HW 17

Problem 1

a.) $Z = \sum_s e^{-E_s \beta}$, $k = 8.617\times10^{-5} \frac{eV}{K}$, $T = 10^5 K$ $\therefore$ $\beta = \frac{1}{kT} = \frac{1}{8.617\times10^{-5} eV/K \cdot 10^5 K} = 0.11605 \ eV^{-1}$

System A: $E_1 = 0eV$, $E_2 = 10eV$ ⋮ System B: $E_1 = 0eV$, $E_{2,3} = 10eV$ ⋮ System C: $E_{1,2} = 0eV$, $E_{3,4} = 10eV$

$Z_A = 1 + e^{-10eV(0.11605 eV^{-1})} = 1.31333$ ⋮ $Z_B = 1 + 2\cdot e^{-10eV(0.11605)eV^{-1}} = 1.62666$ ⋮ $Z_C = 2 + 2\cdot e^{-10eV(0.11605)eV^{-1}} = 2.62666$

From this we can see $Z_C > Z_B > Z_A$. Therefore the correct answer is v) None of the partition functions are the same.

b.) $Z_A = 1 + e^{-10eV(0.11605)eV^{-1}} = 1.31333$, $Z_B = (Z_{\text{single particle}})^N = (1.31333)^2 = 1.72484$

$Z_A = Z_B$ is not true due to System B having two distinguishable particles. $Z_B > Z_A$.

Problem 2

$$Z = \sum e^{-E_s\beta} = e^0 + 3e^{-4.7\times10^{-4}eV\cdot\beta} = 1 + 3e^{-4.7\times10^{-4}eV\cdot\beta} \;:\; Z = 1 + 3e^{-4.7\times10^{-4}eV\cdot\beta}$$

$$P = \frac{e^{-4.7\times10^{-4}eV\cdot\beta}}{1+3e^{-4.7\times10^{-4}eV\cdot\beta}} = 0.1 \;:\; e^{-4.7\times10^{-4}eV\cdot\beta} = 0.1 + 0.3\,e^{-4.7\times10^{-4}eV\cdot\beta} \;:\; e^{-4.7\times10^{-4}eV\cdot\beta}(1-0.3) = 0.1$$

$$e^{-4.7\times10^{-4}eV\cdot\beta} = \frac{0.1}{0.7} \;:\; -4.7\times10^{-4}eV\cdot\beta = \ln(0.1/0.7) \;:\; 4.7\times10^{-4}eV\cdot\frac{1}{kT} = \ln(0.7/0.1) \;:\; k = 8.617\times10^{-5}\frac{eV}{K}$$

$$T = \frac{4.7\times10^{-4}eV}{k\cdot\ln(7)} = \frac{4.7\times10^{-4}eV}{8.617\times10^{-5}\frac{eV}{K}\cdot\ln(7)} = 2.80297\text{ K} \;:\; T \approx 2.80\text{ K}$$

$$\boxed{T = 2.80\text{ K}}$$

Problem 3

a.) $Z = \sum e^{-E_s\beta} = e^{\epsilon\beta} + e^{-\epsilon\beta}$

$$\boxed{Z = e^{\epsilon\beta} + e^{-\epsilon\beta}}$$

b.) $\bar{E} = -\frac{\partial}{\partial\beta}\ln(Z)$

$$\frac{\partial}{\partial\beta}\ln(e^{\epsilon\beta}+e^{-\epsilon\beta}) = \frac{\epsilon e^{\epsilon\beta} - \epsilon e^{-\epsilon\beta}}{e^{\epsilon\beta}+e^{-\epsilon\beta}} \quad : \quad -\frac{\partial}{\partial\beta} = \epsilon\left(\frac{e^{-\epsilon\beta} - e^{\epsilon\beta}}{e^{\epsilon\beta}+e^{-\epsilon\beta}}\right) = -\epsilon \operatorname{Tanh}(\epsilon\beta)$$

$$\boxed{\bar{E} = -\epsilon \operatorname{Tanh}(\epsilon\beta)}$$

c.) $C_V = \frac{\partial \bar{E}}{\partial T} \quad : \quad \frac{\partial \bar{E}}{\partial T} = \frac{\partial \bar{E}}{\partial \beta} \cdot \frac{\partial \beta}{\partial T}$

$$\frac{\partial \bar{E}}{\partial \beta} = \frac{-\epsilon^2}{(\cosh(\epsilon\beta))^2} \quad , \quad \frac{\partial\beta}{\partial T} = \frac{\partial}{\partial T}\left(\frac{1}{kT}\right) = -\frac{1}{kT^2}$$

$$C_V = \frac{-\epsilon^2}{(\cosh(\epsilon\beta))^2} \cdot -\frac{1}{kT^2} = \frac{\epsilon^2}{kT^2(\cosh(\epsilon\beta))^2}$$

$$\boxed{C_V = \frac{\epsilon^2}{kT^2(\cosh(\epsilon/kT))^2}}$$

d.)

$$Z = \sum e^{-E_s\beta} \; : \; E_s = \epsilon + E_0 \, , \; E_s = E_0 - \epsilon \; : \; Z = e^{-(\epsilon+E_0)\beta} + e^{-(E_0-\epsilon)\beta} \longleftarrow \text{Different from part A}$$

$$\bar{E} = -\frac{\partial}{\partial\beta}\ln(Z) = -\frac{\partial}{\partial\beta}\ln\left(e^{-(\epsilon+E_0)\beta} + e^{-(E_0-\epsilon)\beta}\right) = -\left(\frac{-(\epsilon+E_0)e^{-(\epsilon+E_0)\beta} - (E_0-\epsilon)e^{-(E_0-\epsilon)\beta}}{e^{-(\epsilon+E_0)\beta} + e^{-(E_0-\epsilon)\beta}}\right)$$

$$\bar{E} = \frac{(\epsilon+E_0)e^{-(\epsilon+E_0)\beta} + (E_0-\epsilon)e^{-(E_0-\epsilon)\beta}}{e^{-(\epsilon+E_0)\beta} + e^{-(E_0-\epsilon)\beta}} \longleftarrow \text{Different from part B}$$

$$P_{E_s} = \frac{e^{-E_s\beta}}{e^{E_s\beta}+e^{-E_s\beta}} \quad , \quad P_{E_s+E_0} = \frac{e^{-(E_s+E_0)\beta}}{e^{(E_s+E_0)\beta}+e^{-(E_s+E_0)\beta}} \quad : \text{Lets say } \beta = E_s = 1 \, , \; E_0 = 0$$

$$P_{E_s} = \frac{e^{-1}}{e^{1}+e^{-1}} \quad , \quad P_{E_s+E_0} = \frac{e^{-1}}{e^{1}+e^{-1}} \quad \therefore \quad P_{E_s} = P_{E_s+E_0}$$

We can see that Z and $\bar{E}$ changes while the probabilities are different.

Problem 4

a.) $E=\hbar\omega(n+1/2)$: $\bar{E}=-\frac{\partial}{\partial\beta}\ln(Z)$

$Z=\sum_i e^{-E_i\beta}$: $Z_{Ensemble}=(Z_{single})^N$

$Z=\sum e^{-\hbar\omega n\beta}e^{-\hbar\omega\beta/2}=e^{-\hbar\omega\beta/2}\sum e^{-\hbar\omega n\beta}$: $\sum e^{-\hbar\omega n\beta}=\frac{1}{1-e^{-\hbar\omega\beta}}$

$Z=\left(\frac{e^{-\hbar\omega\beta/2}}{1-e^{-\hbar\omega\beta}}\right)^N$: $\bar{E}=-\frac{\partial}{\partial\beta}\ln\left[\left(\frac{e^{-\hbar\omega\beta/2}}{1-e^{-\hbar\omega\beta}}\right)^N\right]$

$\bar{E}=-\frac{\partial}{\partial\beta}\cdot N\left[\ln(e^{-\hbar\omega\beta/2})-\ln(1-e^{-\hbar\omega\beta})\right]=-N\left[-\frac{\hbar\omega}{2}\frac{\cancel{e^{-\hbar\omega\beta/2}}}{\cancel{e^{-\hbar\omega\beta/2}}}-\frac{-\hbar\omega e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right]$ $\frac{e^{-x}}{1-e^{-x}}=\frac{1}{e^x-1}$

$\bar{E}=N\left[\frac{\hbar\omega}{2}+\frac{\hbar\omega e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right]=N\hbar\omega\left[\frac{1}{2}+\frac{e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right]$: $\bar{E}=N\hbar\omega\left[\frac{1}{2}+\frac{e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right]$ $\therefore$

$$\boxed{\bar{E}=N\hbar\omega\left[\frac{1}{2}+\frac{e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right]=N\hbar\omega\left[\frac{1}{2}+\frac{1}{e^{\hbar\omega\beta}-1}\right]}$$

b.) $C_V=\frac{\partial\bar{E}}{\partial T}=\frac{\partial\bar{E}}{\partial\beta}\frac{\partial\beta}{\partial T}$

$\frac{\partial\bar{E}}{\partial\beta}=\frac{\partial}{\partial\beta}\left(\frac{N\hbar\omega}{2}+N\hbar\omega(e^{\hbar\omega\beta}-1)^{-1}\right)=-N\hbar\omega(e^{\hbar\omega\beta}-1)^{-2}\cdot\hbar\omega e^{\hbar\omega\beta}$: $\frac{\partial\beta}{\partial T}=\frac{-1}{kT^2}$

$C_V=\frac{-N\hbar^2\omega^2e^{\hbar\omega\beta}}{(e^{\hbar\omega\beta}-1)^2}\cdot-\frac{1}{kT^2}=\frac{N\hbar^2\omega^2e^{\hbar\omega\beta}}{kT^2(e^{\hbar\omega\beta}-1)^2}$

$$\boxed{C_V=\frac{N\hbar^2\omega^2e^{\hbar\omega\beta}}{kT^2(e^{\hbar\omega\beta}-1)^2}=\frac{N\hbar^2\omega^2e^{-\hbar\omega\beta}}{kT^2(1-e^{-\hbar\omega\beta})^2}}$$

c.) $kT\gg\hbar\omega$: $\bar{E}=N\hbar\omega\left[\frac{1}{2}+\frac{e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right]$: $e^{-\hbar\omega/kT}=1-\frac{\hbar\omega}{kT}=1-\hbar\omega\beta$

$\frac{\hbar\omega}{kT}\ll1$

$\bar{E}=N\hbar\omega\left[\frac{1}{2}+\frac{1-\hbar\omega\beta}{1-1+\hbar\omega\beta}\right]=N\hbar\omega\left[\frac{1}{2}+\frac{1-\hbar\omega\beta}{\hbar\omega\beta}\right]=N\hbar\omega\left[\frac{\hbar\omega\beta+2-2\hbar\omega\beta}{2\hbar\omega\beta}\right]$

$\bar{E}=N\cancel{\hbar\omega}\left[\frac{2-\hbar\omega\beta}{2\cancel{\hbar\omega}\beta}\right]=\frac{N(2-\hbar\omega\beta)}{2\beta}=\frac{\cancel{2}N}{\cancel{2}\beta}-\frac{N\hbar\omega\cancel{\beta}}{2\cancel{\beta}}=NkT-\frac{N\hbar\omega}{2}$

$\lim_{kT\gg\hbar\omega}\left(NkT-\frac{N\hbar\omega}{2}\right)=NkT$ $\qquad$ $\bar{E}=NkT$

$C_V=\frac{N\hbar^2\omega^2}{kT^2}\cdot\frac{e^{-\hbar\omega\beta}}{(1-e^{-\hbar\omega\beta})^2}=\frac{N\hbar^2\omega^2}{kT^2}\cdot\frac{(1-\hbar\omega\beta)}{(1-1+\hbar\omega\beta)^2}=\frac{N\cancel{\hbar^2\omega^2}}{kT^2}\cdot\frac{(1-\hbar\omega\beta)}{\cancel{\hbar^2\omega^2}\beta^2}=\frac{N}{kT^2}\cdot\frac{(1-\hbar\omega\beta)}{\beta^2}$

Problem 4 continued

$$C_V = \frac{Nk^{\cancel{2}}\cancel{T^2}}{\cancel{kT^2}}(1-\hbar\omega\beta) = Nk(1-\hbar\omega\beta) = Nk - \frac{Nk\hbar\omega}{kT}$$

$$\lim_{kT \gg \hbar\omega}\left(Nk - \frac{Nk\hbar\omega}{kT}\right) = Nk \qquad C_V = Nk$$

$$\boxed{\bar{E} = NkT,\ C_V = Nk}$$

d.) $$\bar{E} = N\hbar\omega\left[\frac{1}{2} + \frac{e^{-\hbar\omega\beta}}{1-e^{-\hbar\omega\beta}}\right] = N\hbar\omega\left[\frac{1}{2} + \frac{1}{e^{\hbar\omega\beta}-1}\right] \qquad \frac{\hbar\omega}{kT} \gg 1 \rightarrow \hbar\omega \gg kT$$

$$\hbar\omega \gg kT \quad (1-e^{-\hbar\omega\beta}) = (1+e^{-\hbar\omega\beta})$$

$$\bar{E} = N\hbar\omega\left[\frac{1}{2} + \frac{e^{-\hbar\omega\beta}}{1+e^{-\hbar\omega\beta}}\right] \quad \lim_{x\to\infty} e^{-x} = 0 \rightarrow \lim_{\hbar\omega \gg kT} e^{-\hbar\omega/kT} = 0 \ \therefore$$

$$\bar{E} = N\hbar\omega\left[\frac{1}{2} + \frac{e^{-\hbar\omega\beta}}{1+0}\right] = N\hbar\omega\left[\frac{1}{2} + e^{-\hbar\omega\beta}\right]$$

$$\boxed{\bar{E} = N\hbar\omega\left[\frac{1}{2} + e^{-\hbar\omega/kT}\right]}$$

e.) $$C_V = \frac{N\hbar^2\omega^2}{kT^2}\cdot\frac{e^{-\hbar\omega\beta}}{(1-e^{-\hbar\omega\beta})^2} \quad : kT \ll \hbar\omega$$

$$\lim_{kT \ll \hbar\omega}(1-e^{-\hbar\omega/kT})^2 = 1 \quad \therefore \quad \lim_{kT \ll \hbar\omega}\frac{N\hbar^2\omega^2}{kT^2}\frac{e^{-\hbar\omega\beta}}{(1-e^{-\hbar\omega/kT})^2} = \frac{N\hbar^2\omega^2}{kT^2}e^{-\hbar\omega\beta}$$

$$C_V = \frac{N\hbar^2\omega^2}{kT^2}e^{-\hbar\omega/kT} \quad : \quad \lim_{T\to 0}\frac{N\hbar^2\omega^2}{kT^2}e^{-\hbar\omega/kT} = \infty \cdot e^{-\infty} = \infty \cdot 0 = 0$$

$$\boxed{C_V = \frac{N\hbar^2\omega^2}{kT^2}e^{-\hbar\omega/kT},\ C\to 0,\ T\to 0}$$

Problem 5

a.) $\frac{P_2}{P_1} = \frac{P_3}{P_1} = \frac{P_4}{P_1} = \ldots = 0$, $E_1 < E_2 < E_3 < \ldots$, $s = 1,2,3,\ldots$

$$P = \frac{e^{-E_s\beta}}{Z} \therefore \frac{P_2}{P_1} = \frac{e^{-E_2\beta}/\cancel{Z}}{e^{-E_1\beta}/\cancel{Z}} = \frac{e^{-E_2\beta}}{e^{-E_1\beta}} = 0 \,\vdots\, e^{(E_1-E_2)\beta} = 0 \,,\, e^{\frac{(E_1-E_2)}{kT}} = 0$$

$$e^{-\frac{(E_2-E_1)}{kT}} = 0 \longrightarrow e^{-\infty} = 0 \longrightarrow \text{This is only possible if } T=0$$

$$\boxed{T=0}$$

b.) $e^{-\Delta E/kT}$

$$\lim_{T\to 0} e^{-\Delta E/kT} , \; e^{-\Delta E/kT} \text{ goes to } e^{-\infty} , \; e^{-\infty} = 0$$

$$\boxed{\text{As } T\to 0 , \; \frac{P_{n+1}}{P_n} = 0}$$

c.) $S = k\ln(Z) + \bar{E}/T$

$$Z = \sum e^{-E_s\beta} = e^{-E_1\beta} + e^{-E_2\beta} = e^{-E_1\beta}\left[1 + e^{(E_1-E_2)\beta}\right] = e^{-E_1\beta}$$

$$S = k\ln(Z) + \bar{E}/T \; : \; \bar{E} = -\frac{\partial}{\partial\beta}\ln(Z) \,,\, \bar{E} = -\frac{\partial}{\partial\beta}\ln(e^{-E_1\beta}) = -\frac{\partial}{\partial\beta}(-E_1\beta) = E_1$$

$$S = k\ln(e^{-E_1\beta}) + \frac{E_1}{T} = \frac{\cancel{k}\cdot(-E_1)}{\cancel{k}T} + \frac{E_1}{T} = -\frac{E_1}{T} + \frac{E_1}{T} = 0$$

$$\boxed{\lim_{T\to 0}(S) = 0}$$

d.) $\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial G}{\partial T}\right)_P$: $dG = -SdT + VdP$: $-\left(\frac{\partial S}{\partial P}\right)_V = \left(\frac{\partial G}{\partial P}\right)_V$

$$\left(\frac{\partial}{\partial P}\right)_V\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial^2 G}{\partial P\partial T}\right) , \; -\left(\frac{\partial}{\partial T}\right)_P\left(\frac{\partial S}{\partial P}\right)_V = \left(\frac{\partial^2 G}{\partial T\partial P}\right) \longrightarrow \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\frac{\partial}{\partial P}\left[\lim_{T\to 0} S\right] = -\frac{\partial}{\partial P}(0) = 0$$

Therefore since $\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$ and $-\left(\frac{\partial S}{\partial P}\right)_T = 0$, $\left(\frac{\partial V}{\partial T}\right)_P = 0$ as $T \to 0$

Problem 6

a.) $F = -NkT\left[\ln(v) + \frac{3}{2}\ln\left(\frac{mkT}{2\pi\hbar^2}\right)\right]$, $S = -\left(\frac{\partial F}{\partial T}\right)_{v,N}$

$$-\frac{\partial F}{\partial T} = +NkT\left[\frac{3}{2}\,\frac{\cancel{m}k/\cancel{2\pi\hbar^2}}{\cancel{m}kT/\cancel{2\pi\hbar^2}}\right] = \frac{3}{2}Nk + Nk\left[\ln(v) + \frac{3}{2}\ln\left(\frac{mkT}{2\pi\hbar^2}\right)\right]$$

$$\boxed{S = Nk\left[\frac{3}{2} + \ln(v) + \ln\left(\frac{mkT}{2\pi\hbar^2}\right)\right]}$$

b.) $\mu = \left(\frac{\partial F}{\partial N}\right)_{v,T}$

$$\frac{\partial F}{\partial N} = -kT\left[\ln(v) + \frac{3}{2}\ln\left(\frac{mkT}{2\pi\hbar^2}\right)\right]$$

$$\boxed{\mu = -kT\left[\ln(v) + \frac{3}{2}\ln\left(\frac{mkT}{2\pi\hbar^2}\right)\right]}$$

c.) $\mu = -kT\left[\ln(v) + \frac{3}{2}\left(\frac{mkT}{2\pi\hbar^2}\right)\right]$

Gas will flow from the higher energy per particle to the lower energy per particle.

$\mu_A = -kT\left[\ln(v) + \frac{3}{2}\left(\frac{m_A kT}{2\pi\hbar^2}\right)\right]$, $\mu_B = -kT\left[\ln(v) + \frac{3}{2}\left(\frac{m_B kT}{2\pi\hbar^2}\right)\right]$

Because $m_A < m_B$, $\mu_A > \mu_B$, energy will flow from A to B.

Problem 7

a.) $E = \frac{p^2}{2m} + u(x)$: $Z = \sum e^{-E_s\beta} = \sum \text{Probabilities} = A \int dx \int dp \, e^{\frac{-p^2\beta}{2m}} e^{-u(x)\beta}$

$$Z = e^{-\left(\frac{p^2}{2m} + u(x)\right)\beta} = e^{-\frac{p^2\beta}{2m} - u(x)\beta} = e^{\frac{-p^2\beta}{2m}} e^{-u(x)\beta} = \alpha_p e^{\frac{-p^2\beta}{2m}} \alpha_x e^{-u(x)\beta}$$

$$Z_x = \alpha_x \int e^{-u(x)\beta} dx \quad , \quad Z_p = \alpha_p \int e^{\frac{-p^2\beta}{2m}} dp \quad \therefore \quad Z = Z_x Z_p$$

$$\boxed{Z = Z_x Z_p}$$

b.) $P(x) = \frac{1}{Z} \int e^{-E\beta} dp$: $e^{-E\beta} = e^{\frac{-p^2\beta}{2m} - u(x)\beta} = e^{\frac{-p^2\beta}{2m}} e^{-u(x)\beta}$, $Z = Z_x Z_p$

$$P(x) = \frac{1}{Z_x Z_p} \underbrace{\int e^{-u(x)\beta} e^{\frac{-p^2\beta}{2m}} dp}_{Z_p} = \frac{1}{Z_x \cancel{Z_p}} e^{-u(x)\beta} \cdot \cancel{Z_p} = \frac{1}{Z_x} e^{-u(x)\beta}$$

$$\boxed{P(x) = \frac{1}{Z_x} e^{-u(x)\beta}}$$

c.) $\bar{x} = \int x \, p(x) \, dx$: $p(x) = \frac{1}{Z_x} e^{-u(x)\beta}$

$$\bar{x} = \int x \cdot \frac{1}{Z_x} e^{-u(x)\beta} dx = \frac{1}{Z_x} \int x e^{-u(x)\beta} dx$$

$$\boxed{\bar{x} = \frac{1}{Z_x} \int x e^{-u(x)\beta} dx}$$

d.) $u(x) = \frac{1}{2} k x^2$: $k = m\omega^2$: $u(x) = \frac{1}{2} m \omega^2 x^2$

$$\bar{x} = \frac{1}{Z_x} \int_{-\infty}^{\infty} x e^{-\frac{1}{2} m \omega^2 \beta x^2} dx \quad , \quad Z_x = \int_{-\infty}^{\infty} e^{-\frac{1}{2} m \omega^2 \beta x^2} dx$$

$$\bar{x} = \frac{1}{\int_{-\infty}^{\infty} e^{-\frac{1}{2} m \omega^2 \beta x^2} dx} \int_{-\infty}^{\infty} x e^{\frac{-1}{2} m \omega^2 \beta x^2} dx \quad : \quad \int_{-\infty}^{\infty} x e^{\frac{-1}{2} m \omega^2 \beta x^2} dx$$

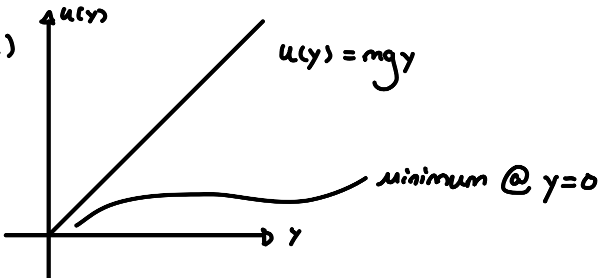
↳ odd Function

↳ even integral

odd Function over even integral = 0 $\therefore$

$$\bar{x} = \frac{1}{\int_{-\infty}^{\infty} e^{-\frac{1}{2} m \omega^2 \beta x^2} dx} \cdot 0 = 0 \quad \therefore \quad \boxed{\bar{x} = 0}$$

e.)



If a particle was released at $y > 0$, it would eventually end up at $y = 0$.

Problem 7 continued

$$\bar{y} = \frac{1}{Z_y}\int_0^\infty y e^{-u(y)\beta} dy = \frac{1}{\int_0^\infty e^{-u(y)\beta} dy}\int_0^\infty y e^{-u(y)\beta} dy = \frac{1}{\int_0^\infty e^{-mgy\beta} dy}\int_0^\infty y e^{-mgy\beta} dy$$

$$\int_0^\infty e^{-mgy\beta} dy = \frac{1}{-mg\beta} e^{-mgy\beta}\Big|_0^\infty = \frac{-1}{mg\beta}\left(\cancel{e^{-\infty}}^{0} - \cancel{e^{0}}^{1}\right) = \frac{1}{mg\beta}$$

$$\int_0^\infty y e^{-mgy\beta} dy \;:\; uv - \int v\,du \;:\; dv = e^{-mgy\beta},\; u = y,\; du = 1\,dy$$

$$v = \frac{-1}{mg\beta} e^{-mgy\beta}$$

$$\cancel{\frac{-y}{mg\beta} e^{-mgy\beta}}^{0}\Big|_0^\infty + \frac{1}{mg\beta}\int_0^\infty e^{-mgy\beta} dy = 0 + \frac{1}{mg\beta}\int_0^\infty e^{-mgy\beta} dy$$

$$\frac{1}{mg\beta}\cdot\frac{-1}{mg\beta}\cdot e^{-mgy\beta}\Big|_0^\infty = -\frac{1}{(mg\beta)^2}\left(\cancel{e^{-\infty}}^{0} - \cancel{e^{0}}^{1}\right) = \frac{1}{(mg\beta)^2}$$

$$\bar{y} = \cancel{mg\beta}\cdot\frac{1}{(mg\beta)^{\cancel{2}}} = \frac{1}{mg\beta} = \frac{kT}{mg}$$

$$\boxed{\begin{array}{l} \bar{y} = \dfrac{kT}{mg} \;:\; \text{As } T \to 0 \quad \lim\limits_{T\to 0} \dfrac{kT}{mg} = 0,\; \bar{y} \to 0 \\ \qquad\qquad\quad \text{As } T \to \infty \quad \lim\limits_{T\to\infty} \dfrac{kT}{mg} = \infty,\; \bar{y} \to \infty \end{array}}$$