1.) $f(x,y) = x^2+y^2+x^{-2}y^{-2}+1$

$\frac{\partial f}{\partial x} = 2x - 2x^{-3}y^{-2} + x^{-2}(0)$

$\frac{\partial f}{\partial x} = 2x - 2x^{-3}y^{-2}$

$\frac{\partial f}{\partial x} = 2x - \frac{2}{x^3y^2}$

$0 = 2x - \frac{2}{x^3y^2}$

$-2x = -\frac{2}{x^3y^2}$

$2x^4y^2 = 2$

$x^4y^2 = 1$

$y^2 = \frac{1}{x^4}$

$y = \pm \frac{1}{x^2}$

$D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2$

$\frac{\partial f}{\partial y} = 2y + (0)y^{-2} + x^{-2}(-2y^{-3})$

$\frac{\partial f}{\partial y} = 2y - 2x^{-2}y^{-3}$

$\frac{\partial f}{\partial x} = 2y - \frac{2}{x^2y^3}$

$0 = 2y - \frac{2}{x^2y^3}$

$-2y = -\frac{2}{x^2y^3}$

$2x^2y^4 = 2$

$x^2y^4 = 1$

$x^2 = \frac{1}{y^4}$

$x = \pm \frac{1}{y^2}$

(0, 0)
(1, -1) (-1, -1)
(1, 1) (-1, 1)

$y = \frac{1}{\frac{1}{y^2}}$

$y = y^2$

$y^2 - y = 0$

$y(y-1) = 0 \quad y=0 \quad y=\pm 1$

$\frac{\partial^2 f}{\partial x^2} = 2x - \frac{2}{x^3y^2}$

$2x - 2x^{-3}y^{-2}$

$2 + 6x^{-4}(y^{-2}) - 2x^{-3}(0)$

$2 + \frac{6}{x^4y^2}$

$\frac{\partial^2 f}{\partial y} = 2y - \frac{2}{x^2y^3}$

$2y - 2x^{-2}y^{-3}$

$2 - (0)y^{-3} - 2x^{-2}(-3y^{-4})$

$2 + 6x^{-2}y^{-4}$

$2 + \frac{6}{x^2y^4}$

$\frac{\partial^2 f}{\partial y \partial x} = 2x - 2x^{-3}y^{-2}$

$0 - (0)y^{-2} - 2x^{-3}(-2y^{-3})$

$4x^{-3}y^{-3}$

$\frac{4}{x^3y^3}$

$D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2$

$D(1,1) = \left(2+\frac{6}{1^4(1)^2}\right)\left(2+\frac{6}{1^2(1)^4}\right) - (4)^2$

$(8)(8) - 16 = 64 - 16 = 48 > 0$

$\therefore$ minimum (1,1)

$D(1,-1) = \left(2+\frac{6}{1^4(-1)^2}\right)\left(2+\frac{6}{1^2(-1)^4}\right) - (-4)^2$

$(8)(8) - (16) = 64 - 16 = 48 > 0$

$\therefore$ minimum (1,-1)

$D(-1,-1) = \left(2+\frac{6}{-1^4(-1)^2}\right)\left(2+\frac{6}{-1^2(-1)^4}\right) - (4)^2$

$(8)(8) - (16) = 64 - 16 = 48 > 0$

$\therefore$ minimum (-1,-1)

$D(-1,1) = \left(2+\frac{6}{-1^4(1)^2}\right)\left(2+\frac{6}{-1^2(1)^4}\right) - (-4)^2$

$(8)(8) - (16) = 64 - 16 = 48 > 0$

$\therefore$ minimum (-1,1)

(1,1)
(-1,1)
(-1,-1)
(1,-1)
(0,0)
} Are all critical points

$f_{xy}\left(\frac{4}{x^3y^3}\right)$: $(1,1) = \frac{4}{1^3 1^3} = 4$

(0,0): inconclusive $(1,-1) = \frac{4}{1^3 \cdot -1^3} = -4$

$\frac{4}{0} \neq$ und $(-1,-1) = \frac{4}{-1^3 \cdot -1^3} = 4$

$(-1,1) = \frac{4}{-1^3 \cdot 1^3} = -4$

$f_{xx} = \left(2+\frac{6}{x^4y^2}\right)$: $(1,1) = 2 + \frac{6}{1^4 1^2} = 2+6 = 8$

$f(0,0) = \infty$

$(1,-1) = 2 + \frac{6}{1^4 \cdot -1^2} = 2+6 = 8$

$(-1,-1) = 2 + \frac{6}{-1^4 \cdot -1^2} = 2+6 = 8$

$(-1,1) = 2 + \frac{6}{-1^4 \cdot 1^2} = 2+6 = 8$

$f_{yy} = \left(2+\frac{6}{x^2y^4}\right)$: $(1,1) = 2 + \frac{6}{1^2 \cdot 1^4} = 2+6 = 8$

$f(0,0) = \infty$

$(1,-1) = 2 + \frac{6}{1^2 \cdot -1^4} = 2+6 = 8$

$(-1,-1) = 2 + \frac{6}{-1^2 \cdot -1^4} = 2+6 = 8$

$(-1,1) = 2 + \frac{6}{-1^2 \cdot 1^4} = 2+6 = 8$

$f(1,1) = x^2+y^2+x^{-2}y^{-2}+1$

$= 1^2 + 1^2 + \frac{1}{1^2}\left(\frac{1}{1^2}\right) + 1$

$1+1+1+1$

4

minimum values at (1,1) is 4

2.) $f(x,y) = 3xye^{-x^2-y^2}$

$f_x = 3y(e^{-x^2-y^2}) + 3xy(e^{-x^2-y^2}) \cdot 2x$

$f_x = 3y(e^{-x^2-y^2}) - 6x^2y(e^{-x^2-y^2})$

$f_x = e^{-x^2-y^2}(3y-6x^2y)$

$3y - 6x^2y = 0$ $\quad 3y = 0$

$3y(1-2x^2) = 0$ $\quad y = 0$

$\quad x = \pm \frac{1}{\sqrt{2}}$

$1 = 2x^2$

$\frac{1}{2} = x^2$

$x = \pm \frac{1}{\sqrt{2}}$

$f_y = 3xe^{-x^2-y^2} + 3xy(e^{-x^2-y^2}) \cdot 2y$

$3xe^{-x^2-y^2} - 6xy^2(e^{-x^2-y^2})$

$f_y = e^{-x^2-y^2}(3x - 6xy^2)$

$3x(1-2y^2) = 0$ $\quad 3x = 0$

$\quad x = 0$

$\quad y = \pm \frac{1}{\sqrt{2}}$

$1 = 2y^2$

$\frac{1}{2} = y^2$

$y = \pm \frac{1}{\sqrt{2}}$

$(0,0)\ (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$f_{xx} = e^{-x^2-y^2}(3y - 6x^2y)$

$(-2x)e^{-x^2-y^2}(3y-6x^2y) + e^{-x^2-y^2}(-12xy)$

$e^{-x^2-y^2}(-2x(3y-6x^2y) - 12xy)$

$e^{-x^2-y^2}(-6xy + 12x^3y - 12xy)$

$f_{xx} = e^{-x^2-y^2}(12x^3y - 18xy)$

$f_{yy} = e^{-x^2-y^2}(3x-6xy^2)$

$(-2y)e^{-x^2-y^2}(3x-6xy^2) + e^{-x^2-y^2}(-12xy)$

$e^{-x^2-y^2}(-2y(3x-6xy^2) - 12xy)$

$e^{-x^2-y^2}(-6xy + 12xy^3 - 12xy)$

$f_{yy} = e^{-x^2-y^2}(12xy^3 - 18xy)$

$f_{xy} = e^{-x^2-y^2}(3y - 6x^2y)$

$-2y(e^{-x^2-y^2})(3y-6x^2y) + e^{-x^2-y^2}(3-6x^2)$

$e^{-x^2-y^2}(-2y(3y-6x^2y) + (3-6x^2))$

$e^{-x^2-y^2}(-6y^2 + 12x^2y^2 + 3 - 6x^2)$

$f_{xy} = e^{-x^2-y^2}(3 - 6x^2 - 6y^2 + 12x^2y^2)$

$D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - (f_{xy}(a,b))^2$

$(0,0) = e^0(0)e^0(0) - e^0(3)$

$(0,0) = -3$

$-3 < 0$ ∴ Saddle at (0,0)

Saddle at (0,0)

$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$f_{xx}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -2.21 \sim$ max

$f_{xx}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 2.21 \sim$ min

$f_{xx}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = -2.21 \sim$ max

$f_{xx}(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 2.21 \sim$ min

$f_{yy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = -2.21 \sim$ max

$f_{yy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 2.21 \sim$ min

$f_{yy}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = -2.21 \sim$ max

$f_{yy}(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 2.21 \sim$ min

$f_{xy}(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$

$f_{xy}(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$

$f_{xy}(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 0$

$f_{xy}(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 0$

$$\left.\begin{matrix} f(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \\ (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \end{matrix}\right\} 3xye^{-x^2-y^2} = \frac{3e^{(-1)}}{2} \Big\}\ \text{Maximum}$$

$$\left.\begin{matrix} f(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \\ (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \end{matrix}\right\} 3xye^{-x^2-y^2} = \frac{-3e^{(-1)}}{2} \Big\}\ \text{Minimum}$$

$$\left.\begin{matrix} f(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \\ f(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \end{matrix}\right\} = \frac{3e^{(-1)}}{2} \Leftrightarrow \text{maximum}$$

Saddle at (0,0,0)

$$\left.\begin{matrix} f(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \\ f(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \end{matrix}\right\} = \frac{-3e^{(-1)}}{2} \Leftrightarrow \text{minimum}$$

3.) Find absolute max and min values of f on the set D.

$f(x,y) = x^2 + y^2 + x^2y + 4$ $\quad -1 \le x \le 1$

$D = \{(x,y) \mid |x| \le 1, |y| \le 1\}$ $\quad -1 \le y \le 1$

$f_x = 2x + 2xy$

$2x(1+y) = 0$

$2x = 0 \quad x = 0$

$x = 0 \quad y = -1$

$1 + y = 0$

$y = -1$

$f_y = 2y + x^2$

$-x^2 = 2y$

$y = \frac{-x^2}{2}$

$2x + 2x\left(\frac{-x^2}{2}\right)$

$2x - \frac{2x^3}{2} = 0$

$2x\left(1 - \frac{x^2}{2}\right) = 0$

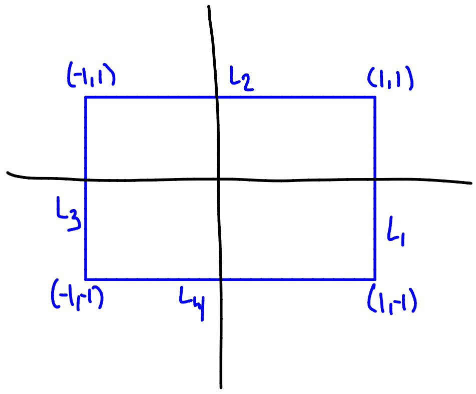
$x = 0$

$(0,0)$

$\frac{-x^2}{2} = -1$

$x^2 = 2$

$x = \pm\sqrt{2}$



$f(0,0) = 0^2 + 0^2 + 0^2(0) + 4$

$f(0,0) = 4$

$L_1(1,y) = 1 + y^2 + y + 4$

$f(1,1)$ $\quad y^2 + y + 5 \quad -1 \le y \le 1$

$g(y) = y^2 + y + 5$

$g'(y) = 2y + 1$ $\quad 2y + 1 = 0$

$y = -\frac{1}{2}$ $\quad 2y = 1$, $y = \frac{1}{2}$

$g(-1) = -1^2 - 1 + 5$

$1 - 1 + 5$

$g(-1) = 5$

max @ $(1,1) = 7$

min @ $(1,-0.5) = 4.75$

$g(1) = 1^2 + 1 + 5$

$g(1) = 7$

$g(-0.5) = 0.25 - 0.5 + 5$

$-0.25 + 5$

4.75

$L_2(x,1) = x^2 + 1 + x^2 + 4$

$f(1,1)$ $\quad g(x) = 2x^2 + 5 \quad -1 \le x \le 1$

$g(-1) = 2(-1)^2 + 5$

$g(-1) = 7$

$g(1) = 2(1)^2 + 5$

7

max @ $(-1,1) = 7$, $(1,1) = 7$

min @ $(0,5) = 5$

$g'(x) = 4x$ $\quad g(0) = 2(0)^2 + 5$

$4x = 0$ $\quad g(0) = 5$

$x = 0$

$L_3(-1,y) = 1 + y^2 + y + 4$

$(-1,1)$ $\quad g(y) = y^2 + y + 5 \quad -1 \le y \le 1$

$g(-1) = -1^2 - 1 + 5$

$1 - 1 + 5$

$g(-1) = 5$

$g(1) = 1^2 + 1 + 5$

$g(1) = 7$

$(-1,1)$ is max 7

$(-1,-\frac{1}{2})$ is min 4.75

$g(-\frac{1}{2}) = 4.75$

$2y + 1 = 0$

$y = -\frac{1}{2}$

$L_4(x,-1) = x^2 + 1 - x^2 + 4$

$= 5$

constant

Maximum critical points

$(1,1), (-1,1), (0,0)$

$f(1,1) = 7$

$f(-1,1) = 7$

$f(0,0) = 0$

Minimum critical points

$(-1,-0.5), (0,5), (0,0)$

$f(-1,-0.5) = 4.75$

$f(0,5) = 29$

$f(0,0) = 4$

Maximum $= 7$

Minimum $= 4$

4.) Find the absolute max and minimum values of f on D

$f(x,y) = x^2y+4 \quad D=\{(x,y) \mid x\ge 0, y\ge 0, x^2+y^2\le 3\}$

$f_x = 2xy$

$x=0$

$y=0$

$f_y = x^2$

$X=0$

$x^2+y^2 \le 3$

$0^2+y^2 \le 3$

$y \le \sqrt{3}$

$y^2 \le 3-x^2$

$x^2+0^2 \le 3$

$x \le \sqrt{3}$

$0 \le x \le \sqrt{3}$

$f(x,x) = x^2(3-x^2)+4$

$f(x,y) = 3x^2-x^4+4$

$g(x) = 3x^2-x^4+4$

$g'(x) = 6x-4x^3$

$g'(x)=0: \quad 6x-4x^3=0$

$x(6-4x^2)=0$

$x=0 \quad -4x^2=-6$

$x^2=\frac{3}{2}$

$x=\pm\sqrt{3/2}$

$x=\sqrt{3/2}$

$0 \le x \le \sqrt{3}$

$y^2=3-x^2$

$y^2=3-\frac{3}{2}$

$y^2=\frac{3}{2}$

$y=\pm\sqrt{3/2}$

$(\sqrt{3/2}, \sqrt{3/2}), (\sqrt{3/2}, -\sqrt{3/2})$

$(x_1, y_1) \quad (x_2, y_2)$

$f(x_1,y_1) = \frac{3\sqrt{3}}{8}+4 = 8.6$

$f(x_2,y_2) = \frac{-3\sqrt{3}}{8}+4 = 7.4$

$(0,0) \ (\sqrt{3},0) \ (0,\sqrt{3})$

$f(0,0) = 0^2(0)+4=4$

$f(\sqrt{3},0) = (\sqrt{3})^2(0)+4=4$

$f(0,\sqrt{3}) = 0^2(\sqrt{3})+4=4$

$0 \le x \le \sqrt{3}$

$0 \le y \le \sqrt{3}$

$(0,0) \ (\sqrt{3},0) \ (0,\sqrt{3})$

$f(x,y) = x,y^2+4$

$f(x) = x(9-x^2)+4$

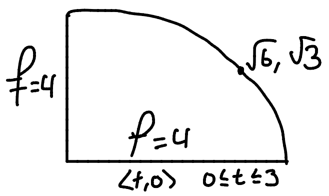
$9x-x^3+4$

$0=9-3x^2$

$-9=-3x^2$

$3=x^2$

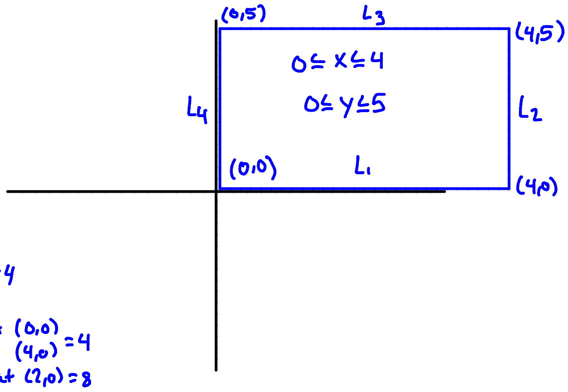
$\pm\sqrt{3}$



5.) Find the absolute max and min

$f(x,y) = 4x+6y-x^2-y^2+4$

$D=\{(x,y)\mid 0\le x\le 4, 0\le y\le 5\}$



$f_x = 4-2x$ $\quad f_y = 6-2y$

$0=4-2x \quad 0=6-2y$

$-4=-2x \quad -6=-2y$

$4=2x \quad 6=2y$

$x=2 \quad 3=y$

$(2,3)$

$f(2,3)=4(2)+6(3)-(2)^2-(3)^2+4$

$8+18-4-9+4$

$26-13+4$

$26-9$

17

Absolute max at 17
Absolute min at 4

$f(2,3)=8+18-4-9+4$

$4x+6y-x^2-y^2+4$

$L_1 \quad f(x,0) = 4x-x^2+4 \quad 0\le x\le 4$

$g(x)=-x^2+4x+4$

$g'(x)=-2x+4$

$0=-2x+4$

$x=(2)$

min at (0,0) (4,0) = 4

Max at (2,0) = 8

$f(2,0)=8-4+4 \quad f(0,0)=4$

$f(2,0)=8 \quad f(4,0)=4$

$L_2 = f(4,y) = 16+6y-16-y^2+4 \quad 0\le y\le 5$

$g(y)=-y^2+6y+4 \quad f(4,0)=4$

$g'(y)=-2y+6 \quad f(4,3)=13$

$0=-2y+6 \quad f(4,5)=9$

$y=3$

Min @ $f(4,0)=4$

Max @ $f(4,3)=13$

$L_3=(x,5) \quad f(x,5) = 4x+30-x^2-25+4 \quad 0\le x\le 4$

$g(x)=-x^2+4x+9 \quad f(0,5)=9$

$0=-2x+4 \quad f(2,5)=-4+8+9$

$x=2 \quad 13$

$f(4,5)=-16+16+9$

9

min @ (0,5) (4,5) = 9

Max @ (2,5) = 13

$L_4 = f(0,y) = 6y-y^2+4 \quad 0\le y\le 5$

$g(y)=6y-y^2+4 \quad f(0,0)=4$

$g'(y)=6-2y \quad f(0,3)=13$

$y=3 \quad f(0,5)=9$

Min @ $f(0,0)=4$

Max @ $f(0,3)=13$

6.) Find the absolute max and min

$f(x,y) = 2x^3+y^4+7 \quad D=\{(x,y)\mid x^2+y^2\le 1\}$

$f_x=6x^2 \quad f_y=4y^3$

$x=0 \quad y=0$

$(0,0)$

$y^2=1-x^2 \quad 0=1-x^2 \quad -1\le x\le 1$

$-1=-x^2$

$x=\pm 1$

$f(x,y)=2x^3+(1-x^2)^2+7$

$f(x,y)=2x^3+(x^4-2x^2+1)+7 \quad -1\le x\le 1$

$f(x,y)=x^4+2x^3-2x^2+8$

$f_x=4x^3+6x^2-4x$

$2x(2x^2+3x-2)$

$x=0$

$x=0.5 \quad -1\le x\le 1 \therefore x=0, x=0.5$

$x=-2$

$(0,1)\ (0,-1)$

$y^2=1-x^2 \quad y^2=1-x^2$

$y^2=1-(0)^2 \quad y^2=1-(0.5)^2$

$y^2=1 \quad y^2=1-0.25$

$y=\pm 1 \quad y^2=\frac{3}{4}$

$y=\pm\frac{\sqrt{3}}{2}$

$f(0,1)=2x^3+y^4+7 \quad f(0,-1)=2x^3+y^4+7$

$f(0,1)=2(0)^3+(1)^4+7 \quad f(0,-1)=2(0)^3+(-1)^4+7$

$1+7 \quad 1+7$

$8 \quad 8$

Absolute min @ (-1,0) = 5
Absolute max @(1,0) = 9

$(\frac{1}{2},\frac{\sqrt{3}}{2}),(\frac{1}{2},-\frac{\sqrt{3}}{2})$

$f(\frac{1}{2},\frac{\sqrt{3}}{2})=2(\frac{1}{2})^3+(\frac{\sqrt{3}}{2})^4+7$

$\frac{1}{4}+\frac{9}{16}=\frac{13}{16}$

$f(\frac{1}{2},\frac{\sqrt{3}}{2})=2(\frac{1}{2})^3+(-\frac{\sqrt{3}}{2})^4+7$

$\frac{1}{4}+\frac{9}{16}=\frac{13}{16}$

$-1\le x\le 1$

$f(-1,0)=2(-1)^3+(0)^4+7$

$-2+7$

5

$f(1,0)=2(1)^3+(0)^4+7$

$2+7$

9

7.) Find the points on the Surface $y^2 = 36 + xz$ $(0,0,0)$

$d^2 = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

$d^2 = x^2 + y^2 + z^2$ $\qquad y^2 = 36 + xz$

$f(x,y,z) = x^2 + y^2 + z^2$

$0 = x + 2z$

$x = -2z$

$f(x,z) = x^2 + 36 + xz + z^2$

$f_x = 2x + z$ $\qquad f_z = x + 2z$ $\qquad x - 4x$ $\qquad f_x = 2x + z$ $\qquad f_z = x + 2z$

$f_x = 2(-2z) + z$ $\qquad 0 = x + 2z$ $\qquad -3x = 0$ $\qquad f_{xx} = 2$ $\qquad f_{zz} = 2$

$0 = -4z + z$ $\qquad -x = 2z$ $\qquad x = 0$

$0 = -3z$ $\qquad x = -2z$ $\qquad f_{xz} = 1$

$z = 0$ $\qquad (0,0)$ C.P

$D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - (f_{xy}(a,b))^2$

$= 2(2) - 1$

$= 4 - 1$

$= 3 \therefore$ min

$y^2 = 36 - xz$

$y^2 = 36 - (0)(0)$

$y^2 = 36$

$y = \pm 6$

$(0,-6,0), (0,6,0)$

Smaller y-value $= (0,-6,0)$

Bigger y-value $= (0,6,0)$

8.) Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r.

$V = L \cdot W \cdot H$ $\qquad x^2 + y^2 + z^2 = r^2$

$L = 2x$ $\qquad 2x(2y)2z = V$ $\qquad g(x,y,z) = x^2 + y^2 + z^2 - r^2$

$W = 2y$

$H = 2z$ $\qquad 8xyz = V$ $\qquad \nabla g = \langle 2x, 2y, 2z \rangle$

Constraint

$f(x,y,z) = 8xyz$ $\qquad x^2 + y^2 + z^2 - r^2 = 0$

$\nabla f = \langle 8yz, 8xz, 8xy \rangle$ $\qquad \nabla g = \langle 2x, 2y, 2z \rangle$

$\langle 8yz, 8xz, 8xy \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$8yz = \lambda(2x)$ $\qquad 8xz = \lambda(2y)$ $\qquad 8xy = \lambda(2z)$

$\frac{4yz}{x} = \lambda$ ① $\qquad \frac{4xz}{y} = \lambda$ ② $\qquad \frac{4xy}{z} = \lambda$ ③

1 & 2 $\qquad$ 1 & 3

$\frac{4yz}{x} = \frac{4xz}{y}$ $\qquad \frac{4yz}{x} = \frac{4xy}{z}$

$\frac{4y^2 z}{4z} = \frac{4x^2 z}{4z}$ $\qquad \frac{4yz^2}{4y} = \frac{4x^2 y}{4y}$

$y^2 = x^2$ $\qquad z^2 = x^2$

Constraint $\qquad x^2 = y^2 = z^2 : x = y = z$

$x^2 + y^2 + z^2 - r^2 = 0$

$x = y = z : x^2 + x^2 + x^2 - r^2 = 0$

$3x^2 = r^2$

$x^2 = \frac{r^2}{3}$

$x = \frac{r}{\sqrt{3}}$ $\qquad x > 0 \therefore (x = y = z) = \frac{r}{\sqrt{3}}$

$V = 8xyz$

$L = 2x$

$W = 2y$ $\qquad V = 8\left(\frac{r}{\sqrt{3}}\right)\left(\frac{r}{\sqrt{3}}\right)\left(\frac{r}{\sqrt{3}}\right)$

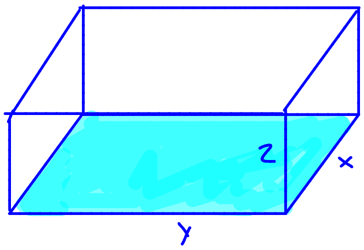
$H = 2z$

$V = 8 \frac{r^3}{3\sqrt{3}}$

$V = \frac{8r^3}{3\sqrt{3}}$

$\boxed{V = \frac{8r^3}{3\sqrt{3}}}$

9.) Minimize the cost of materials for an aquarium base.



x = times

$V = xyz$

Slate $= 7x$

Glass $= 1x$

$$C = 2yz + 2xz + 7xy$$

$$z = \frac{V}{xy}$$

$$C\left(x, y, \frac{V}{xy}\right) = 2y\left(\frac{V}{xy}\right) + 2x\left(\frac{V}{xy}\right) + 7xy$$

$$C\left(x, y, \frac{V}{xy}\right) = \frac{2yV}{xy} + \frac{2xV}{xy} + 7xy$$

$$C\left(x, y, \frac{V}{xy}\right) = \frac{2V}{x} + \frac{2V}{y} + 7xy$$

$$C_x = \frac{x(0) - 2V(1)}{x^2} + 7y \qquad C_y = \frac{(y)(0) - (2V)(1)}{y^2} + 7x$$

$$C_x = \frac{-2V}{x^2} + 7y \qquad C_y = \frac{-2V}{y^2} + 7x$$

$$7y - \frac{2V}{x^2} = 7x - \frac{2V}{y^2}$$

$$y = x$$

$$7x - \frac{2V}{x^2} = 0$$

$$\frac{2V}{x^2} = 7x$$

$$2V = 7x^3$$

$$\frac{2V}{7} = x^3$$

$$\sqrt[3]{\frac{2V}{7}} = x$$

$$\sqrt[3]{\frac{2V}{7}} = y$$

$$z = \frac{V}{xy}$$

$x = y$

$$z = \frac{V}{\sqrt[3]{\frac{2V}{7}} \cdot \sqrt[3]{\frac{2V}{7}}}$$

$$x \cdot y = \frac{4V^2}{49}$$

$$z = \frac{V}{\frac{4V^2}{49}} \qquad \frac{\frac{V}{1}}{\frac{4v^2}{49}}$$

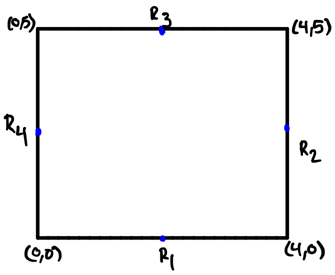
$$\frac{v}{4v^2(49)}$$

$$\frac{1}{4v(49)}$$

$$\frac{1}{196v}$$

$f(x,y) = 4x+6y-x^2-y^2+4$

$D = \{(x,y) \mid 0 \le x \le 4, 0 \le y \le 5\}$



R_1

$(0,0) + T(4,0)$ $\quad 4x+6y-x^2-y^2+4$

$\langle 4T, 0 \rangle \quad 0 \le t \le 1$ $\quad t = \frac{1}{2}$ $\quad (2,0)$

$4(4T)+6(0)-(4T)^2-(0)^2+4$

$16T-(16T)^2+4$

$-16T^2+16T+4$

$dt \; -32T+16$

$0=-32T+16$

$-16=-32T$

$T=\frac{1}{2}$

$-16T^2+16T+4$

$t=0, \frac{1}{2}, 1$

$t(0) = -16(0)^2+16(0)+4$
$= 4$

$t(\frac{1}{2}) = -16(\frac{1}{2})^2+16(\frac{1}{2})+4$
$= 8$

$t(1) = -16(1)^2+16(1)+4$
$= 4$

R_2 $\quad 4x+6y-x^2-y^2+4$

$\langle 4, 5T \rangle \quad 0 \le t \le 1$ $\quad t = \frac{3}{5}$ $\quad (4,3)$

$4(4)+6(5T)-(4)^2-(5T)^2+4$

$16+30T-16-25T^2+4$

$-25T^2+30T+4$

$dt = -50t+30$

$-30=-50t$

$\frac{3}{5}=t$

$-25t^2+30t+4$

$t=0, \frac{3}{5}, 1$

$t(0) = -25(0)^2+30(0)+4$
$= 4$

$t(\frac{3}{5}) = -25(\frac{3}{5})^2+30(\frac{3}{5})+4$
$= 13$

$t(1) = -25(1)^2+30(1)+4$
$= 9$

R_3 $\quad (0,5) \rightarrow (4,5)$

$\langle 4T, 5 \rangle$ $\quad 4x+6y-x^2-y^2+4$ $\quad t=\frac{1}{2}$

$0 \le t \le 1$

$4(4T)+6(5)-(4T)^2-(5)^2+4$ $\quad (2,5)$

$16T+30-16T^2-25+4$

$-16T^2+16T+9$

$dt = -32T+16$

$-16=-32t$

$t=\frac{1}{2}$

$-16t^2+16t+9$

$t=0$
$t=\frac{1}{2}$
$t=1$

$t(0) = -16(0)^2+16(0)+9$
$= 9$

$t(\frac{1}{2}) = -16(\frac{1}{2})^2+16(\frac{1}{2})+9$
$= 13$

$t(1) = -16(1)^2+16(1)+9$
$= 9$

R_4 $\quad (0,0) \rightarrow (0,5)$ $\quad 4x+6y-x^2-y^2+4$

$\langle 0, 5T \rangle \quad 0 \le t \le 1$ $\quad t=\frac{3}{5}$ $\quad (0,3)$

$4(0)+6(5T)-(0)^2-(5T)^2+4$

$30T-25T^2+4$

$-25T^2+30T+4$

$dt = -50T+30$

$-30=-50T$

$T=\frac{3}{5}$

$-25T^2+30T+4$

$t=0$
$t=\frac{3}{5}$
$t=1$

$t(0) = -25(0)^2+30(0)+4$
$= 4$

$t(\frac{3}{5}) = -25(\frac{3}{5})^2+30(\frac{3}{5})+4$
$= 13$

$t(1) = -25(1)^2+30(1)+4$
$= 9$

$4x+6y-x^2-y^2+4$

$f_x = 4-2x$

$0=4-2x$

$-4=-2x$

$x=2$ $\quad (2,3)$

$f_y = 6-2y$

$0=6-2y$

$-6=-2y$

$y=3$

(2,0)
(2,5)
(4,3)
(0,3)
(2,3)

$4x+6y-x^2-y^2+4$

$f(2,0) = 4(2)+6(0)-(2)^2-(0)^2+4$
$8-4+4$
8

$f(2,5) = 4(2)+6(5)-(2)^2-(5)^2+4$
$8+30-4-25+4$
13

$f(4,3) = 4(4)+6(3)-(4)^2-(3)^2+4$
$16+18-16-9+4$
13

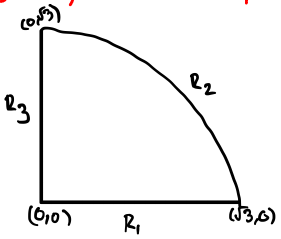
$f(0,3) = 4(0)+6(3)-(0)^2-(3)^2+4$
$18-9+4$
13

$f(2,3) = 4(2)+6(3)-(2)^2-(3)^2+4$
$8+18-4-9+4$
17

Maximum value is 17

Minimum value is 4

$f(x,y) = x^2y+4 \quad D = \{(x,y) \mid x \ge 0, y \ge 0, x^2+y^2 \le 3\}$



$x = \sqrt{3} \qquad 0 \le x \le \sqrt{3}$

$y = \sqrt{3} \qquad 0 \le y \le \sqrt{3}$

$x = \sqrt{3}\cos t$

$y = \sqrt{3}\sin t$

$\vec{r}(t) = \langle \sqrt{3}\cos t, \sqrt{3}\sin t \rangle$

$0 \le t \le \frac{\pi}{2}$

$r(0) = \langle \sqrt{3}, 0 \rangle$

$r(\frac{\pi}{2}) = \langle 0, \sqrt{3} \rangle$

$f(t) = (\sqrt{3}\cos(t))^2(\sqrt{3}\sin(t)) + 4$

$3\cos^2(t)\sqrt{3}\sin(t) + 4$

$3\sqrt{3}\cos^2(t)\sin(t) + 4$

$u = \cos(t)$

$u^2 : 2u \cdot u'$

$u' = -\sin(t)$

$2(\cos(t))\sin(t)$

$f'(t) = 6\sqrt{3}\cos(t)\sin(t)\sin(t) + 3\sqrt{3}\cos^2(t)\cos(t)$

$6\sqrt{3}\cos(t)\sin^2(t) + 3\sqrt{3}\cos^3(t)$

$3\sqrt{3}\cos(t)(2\sin^2(t) + \cos^2(t))$

$3\sqrt{3}\cos(t) = 0$

$\cos(t) = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin^2(t) + \cos^2(t) = 0$

$\cos^2(t) = -2\sin^2(t)$

$0 = \frac{-2\sin^2(t)}{\cos^2(t)}$

$0 = -2\tan^2(t)$

$0 = \tan^2(t)$

$0 = \pm\tan(t)$

$t = 0$