Tues: Discussion / quiz.

Supp Ex 56, 59

Ch 9 CQ 12

Ch 9 Probs 28, 36 (const speed), 39, 59

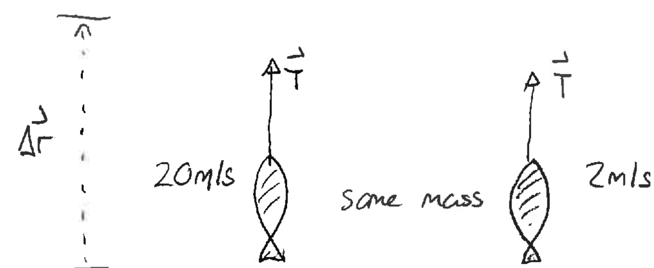
Power:

Neither work nor kinetic energy include information about the time taken for any physical process. Consider two objects that are lifted at constant speeds. Then

$$W_{net} = \Delta K = 0$$

$$\Rightarrow \quad W_{rope} + W_{grav} = 0$$

$$\Rightarrow \quad W_{rope} = - W_{grav}$$



Now if the objects move the same vertical distance then W_{grav} is the same. Thus the rope does the same work. But the work is done more rapidly for the object that is lifted faster. We need a physical quantity to describe this situation. Thus we define

> The power delivered by any force is
>
> $$P = \frac{W}{\Delta t}$$
>
> where W is the work done by the force and Δt the time taken for the object to move.

Units: Watt $W = J/s$

In the example the object lifted faster has smaller Δt for the same $\Delta \vec{r}$. This rope delivers a greater power. Power is also more generally related to energy via:

$$\boxed{P = \frac{\Delta E}{\Delta t}}$$

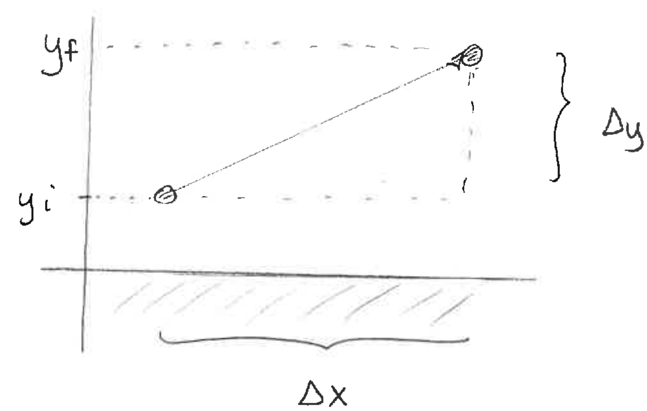
where ΔE is the energy delivered in time Δt.

Quiz 1

Work done by gravitational force

Consider any object that moves while Earth's gravity acts on it. If the object moves in any straight trajectory then the work done by gravity is

$$W_g = \vec{F}_{grav} \cdot \Delta \vec{r}$$



Here

$$\vec{F}_{grav} = 0\hat{\imath} - mg\hat{\jmath}$$

$$\Delta \vec{r} = \Delta x \hat{\imath} + \Delta y \hat{\jmath}$$

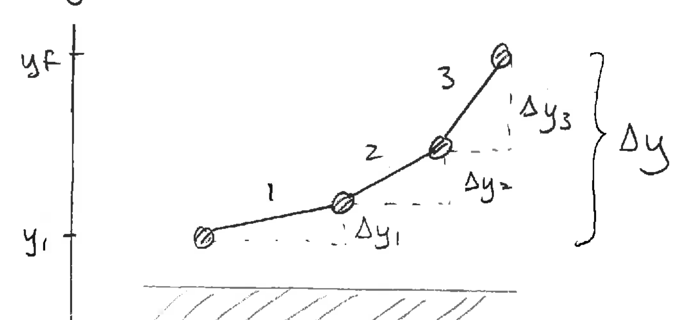
$$\Rightarrow \quad W_{grav} = 0\,\Delta x - mg\Delta y \qquad \Rightarrow \qquad W_{grav} = -mg\Delta y$$

Now suppose that there are several straight line sections. In the illustration there are three sections and

$$W_{grav} = W_{grav\,1} + W_{grav\,2} + W_{grav\,3}$$

$$= -mg\Delta y_1 - mg\Delta y_2 - mg\Delta y_3$$

$$= -mg\,\underbrace{(\Delta y_1 + \Delta y_2 + \Delta y_3)}_{\Delta y}$$

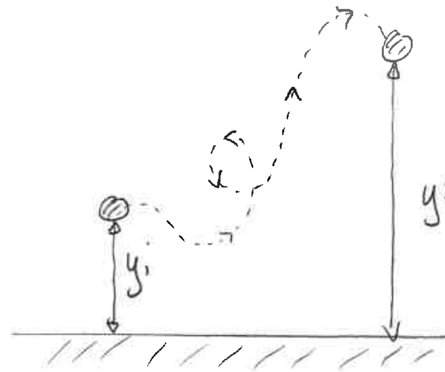


$$\Rightarrow \quad W_{grav} = -mg\Delta y$$

This extends to arbitrary collections of straight line segment paths and by the limiting procedure to any curved path. Thus we find

The work done by gravity when an object moves from one location to another is:

$$W_{grav} = -mg\Delta y$$



where $\Delta y = y_f - y_i$ and

y_i = initial vertical location of the object

y_f = final " " " "

We see a striking feature:

The work done by gravity does not depend on the path taken between initial + final locations. It only depends on the vertical position of the object

<u>Gravitational Potential Energy</u>

In many situations, the only force that does non-zero work is gravity. Examples include:

a) projectiles

b) objects moving along frictionless surfaces.

In such cases the work kinetic energy theorem gives:

$$\Delta K = W_{net}$$

$$\Rightarrow \Delta K = W_{grav}$$

$$\Rightarrow \Delta K = -mg\Delta y$$

$$\Rightarrow \Delta K = -\Delta(mgy). \quad \Rightarrow \quad \Delta K + \Delta(mgy) = 0$$

Thus we define

> The gravitational potential energy of an object of mass m at vertical position y is:
>
> $$U_g = mgy$$

Units: J

Warm Up 1

It follows that if gravity is the only force that does non-zero work then:

$$\Delta K + \Delta U_g = 0 \quad \Rightarrow \quad \Delta (K + U_g) = 0$$

We can define the mechanical energy of the system as:

$$\boxed{E_{mech} = K + U_g}$$

This gives a form of the conservation of energy:

> If gravity is the only force that does non-zero work on an object then the total mechanical energy is constant during the motion. For any two instants
>
> $$\Delta E_{mech} = 0 \iff \Delta K + \Delta U_g = 0$$

Warm Up 2

~~Quiz 1~~ Quiz 2

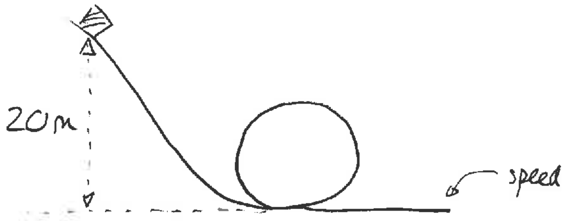
Demo PhET - tracks -> double well rollercoaster

-> bar graph

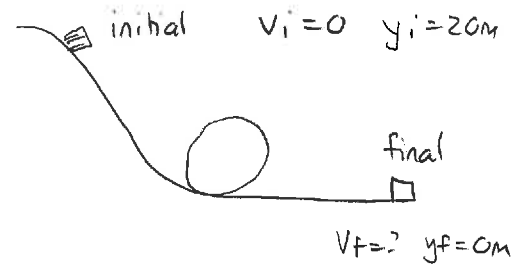
-> energy versus ~~position~~, time

-> scroll + find pt of agreement

Example: A 1000kg rollercoaster slides illustrated frictionless track and it completes the loop. It starts from rest at the illustrated point and completes the loop. Determine its speed at the end of the track



Answer: The mechanical energy is conserved



$$\Rightarrow \quad E_{mech f} = E_{mech i}$$

$$\Rightarrow \quad K_f + U_{gf} = K_i + U_{gi}$$

$$\Rightarrow \quad \frac{1}{2} m v_f^2 + \cancel{m g y_f}_{0} = \cancel{\frac{1}{2} m v_i^2}_{0} + m g y_i$$

$$\Rightarrow \quad \frac{1}{2} m v_f^2 = 1000kg \times 9.8m/s^2 \times 20m = 1.96 \times 10^5 J$$

$$\Rightarrow \quad \frac{1}{2} 1000kg \; v_f^2 = 1.96 \times 10^5 J$$

$$\Rightarrow \quad V_f^2 = \frac{1.96 \times 10^5 J}{500kg} = 392 \, m^2/s^2$$

$$\Rightarrow \quad V_f = \sqrt{392 m^2/s^2} \quad \Rightarrow \quad V_f = 20 m/s$$