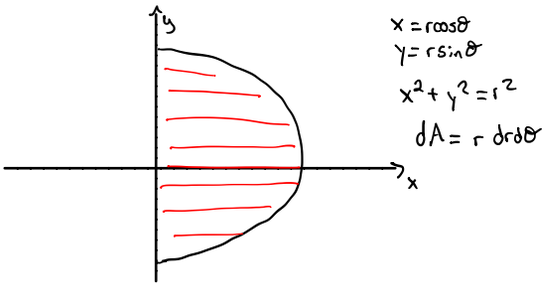
15.4.1 $\iint_D e^{-x^2-y^2}\,dA$ y-axis $x=\sqrt{25-y^2}$



$x=r\cos\theta$
$y=r\sin\theta$
$x^2+y^2=r^2$
$dA=r\,dr\,d\theta$

$\iint e^{-r^2}\,r\,dr\,d\theta$ $\quad 0\le r\le 5$, $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^5 re^{-r^2}\,dr\,d\theta$ $\quad u=-r^2$, $du=-2r\,dr$, $-\frac{1}{2}du=r\,dr$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\frac{1}{2}\int_0^5 e^u\,du\,d\theta$

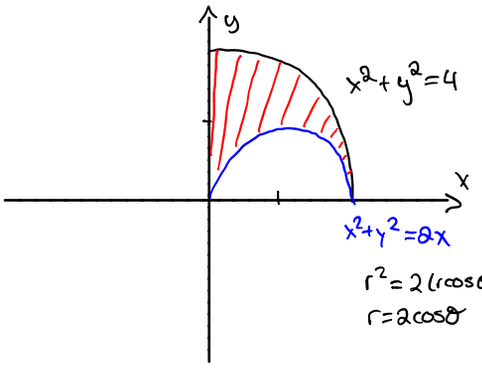
$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[-\frac{1}{2}e^{-r^2}\right]_0^5 d\theta$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\frac{1}{2}e^{-25}-\left(-\frac{1}{2}e^0\right)d\theta$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{2}\left(-e^{-25}+e^0\right)d\theta = \frac{\pi}{2}\left(1-e^{-25}\right)$

15.4.2 $\iint_D x\,dA$ Quadrant 1 Between $x^2+y^2=4$ $x^2+y^2=2x$

$x=r\cos\theta$
$y=r\sin\theta$



$r^2=2(r\cos\theta)$
$r=2\cos\theta$

$(x^2-2x+1)+y^2=0+1$

$(x-1)^2+y^2=1$

$\int_0^{\frac{\pi}{2}}\int_{2\cos\theta}^2 r\cos\theta\,r\,dr\,d\theta$

$\int_0^{\frac{\pi}{2}}\cos\theta\int_{2\cos\theta}^2 r^2\,dr\,d\theta$

$\int_0^{\frac{\pi}{2}}\cos\theta\left[\frac{1}{3}r^3\right]_{2\cos\theta}^2 d\theta$

$\int_0^{\frac{\pi}{2}}\cos\theta\left[\frac{8}{3}-\frac{8\cos^3\theta}{3}\right]d\theta$

$\int_0^{\frac{\pi}{2}}\frac{8\cos\theta}{3}-\frac{8\cos^4\theta}{3}\,d\theta$

$\frac{8}{3}\int_0^{\frac{\pi}{2}}\cos\theta-\cos^4\theta\,d\theta$

$\frac{8}{3}\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta-\frac{8}{3}\int_0^{\frac{\pi}{2}}\cos^4\theta\,d\theta$

$\frac{8}{3}\sin\theta\Big|_0^{\frac{\pi}{2}}-\frac{8}{3}\left[\frac{3}{8}\theta+\frac{1}{4}\sin(2\theta)+\frac{1}{32}\sin(4\theta)\right]_0^{\frac{\pi}{2}}$

$\int\cos^4\theta\,d\theta$

$\int(\cos^2\theta)^2\,d\theta$

$\int\left(\frac{1}{2}+\frac{1}{2}\cos(2\theta)\right)^2 d\theta$

$\int\frac{1}{4}+\cos(2\theta)+\frac{1}{4}\cos^2(2\theta)$ $\quad \frac{1}{4}\int\frac{1}{2}+\frac{1}{2}\cos 2\theta$

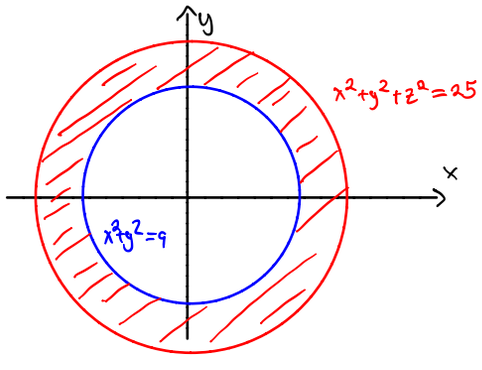
$\int\frac{1}{4}d\theta+\int\cos(2\theta)\,d\theta+\int\frac{1}{4}\cos^2(2\theta)$

$\frac{1}{4}\theta+\frac{1}{4}\sin 2\theta+\frac{1}{8}\theta+\frac{1}{32}\sin(4\theta)$

15.4.6 Find the volume

$x^2+y^2+z^2=25$ Sphere

$x^2+y^2=9$ Cylinder



$z^2=-x^2-y^2+25$ $\quad z=\pm\sqrt{-x^2-y^2+25}$

$\int_0^{2\pi}\int_3^5\sqrt{25-r^2}\,r\,dr\,d\theta$ $\quad u=25-r^2$, $du=-2r\,dr$, $-\frac{1}{2}du=r\,dr$

$\int_0^{2\pi}-\frac{1}{2}\int_3^5\sqrt{u}\,du\,d\theta$

$\int_0^{2\pi}-\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_3^5 d\theta$

$\int_0^{2\pi}-\frac{1}{2}\left[\left(\frac{2}{3}(0)^{\frac{3}{2}}\right)-\left(\frac{2}{3}(16)^{\frac{3}{2}}\right)\right]d\theta$

$\frac{64}{3}\int_0^{2\pi}1\,d\theta$

$\frac{64}{3}[\theta]_0^{2\pi} = \frac{128\pi}{3}\times 2$ For entire Volume

15.4.5 Find volume below $z = 32 - 2x^2 - 2y^2$ vertex $(0,0,32)$ above xy plane

$$V = \iint_D 32 - 2x^2 - 2y^2 \, dA$$

$0 = 32 - 2x^2 - 2y^2$

$x^2 + y^2 = 16$

$r = 4$

$0 \le r \le 4$

$$\int_0^{2\pi} \int_0^4 32 - 2r^2 \, r \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^4 32r - 2r^3 \, dr \, d\theta$$

$$\int_0^{2\pi} \left[16r^2 - \frac{r^4}{2} \right]_0^4 d\theta$$