General Equations

$$\hat{H}\Psi = E\Psi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = E\Psi$$

$$\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$$

$$\eta = \int_{-\infty}^{\infty} |\Psi(x,0)^*\Psi(x,0)| \ dx = 1$$

$$\mathcal{O} = \int_{-\infty}^{\infty} \Psi(x,0)^*\Psi(x,0) \ dx$$

$$P = \int_a^b \rho(x) \ dx$$

$$P_n = |C_n|^2$$

$$< \hat{Q} >= \int \Psi^*\hat{Q}\Psi \ dx =< \Psi|\hat{Q}\Psi >$$

$$< x >= \int_{-\infty}^{\infty} x|\Psi(x,t)|^2 \ dx$$

$$< x^2 >= \int_{-\infty}^{\infty} x^2|\Psi(x,t)|^2 \ dx$$

$$< \hat{p} >= m\frac{d < x >}{dt} = -i\hbar\int \left(\Psi^*\frac{\partial\Psi}{\partial x}\right)\Psi \ dx$$

$$< \hat{p}^2 >= \int \Psi^*\left(-\hbar^2\frac{\partial^2}{\partial x^2}\right)\Psi \ dx$$

$$< \hat{Q}(x,p) >= \int \Psi^* Q\left(x, \frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi \ dx$$

$$< \hat{T} >= \left\langle \frac{\hat{p}^2}{2m} \right\rangle$$

$$< \hat{Q} >= \sum_n q_n|c_n|^2$$

$$< j >= \frac{\sum jN(j)}{N} = \sum_{j=0}^{\infty} jP(j)$$

$$< j^2 >= \sum_{j=0}^{\infty} j^2P(j)$$

$$< f(j) >= \sum_{j=0}^{\infty} f(j)P(j)$$

$$\frac{d}{dt} < \hat{Q} >= \frac{i}{\hbar} < [\hat{H},\hat{Q}] > + \left\langle \frac{\partial\hat{Q}}{\partial t} \right\rangle$$

$$< H >= E$$

$$< H^2 >= E^2$$

$$< H >= \sum_{n=1}^{\infty} |C_n|^2E_n$$

$$\sigma = \sqrt{< j^2 > - < j >^2}$$

$$\sigma^2 =< (\Delta x^2) >$$

$$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda}$$

$$\sigma_x\sigma_p \geq \frac{\hbar}{2}$$

$$\Delta t\Delta E \geq \frac{\hbar}{2}$$

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i} < [\hat{A},\hat{B}] >\right)^2$$

$$\hat{E} = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\hat{T} = \frac{\hat{p}^2}{2m}$$

$$\hat{H} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

$$\phi(t) = e^{-iE_nt/\hbar}$$

$$\Psi(x,t) = \Psi(x)\phi(t)$$

$$\Psi(x,t) = \Psi(x)e^{-iE_nt/\hbar}$$

$$\Psi_n(r,t) = \Psi_n(r)e^{-iE_nt/\hbar}$$

$$\Psi(r,t) = \sum C_n\Psi_n(r)e^{-iE_nt\hbar}$$

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n\Psi_n(x)e^{-iE_nt/\hbar}$$

$$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \Psi(x,0)e^{-ikx} \ dx$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \phi(k)e^{i\left(kx-\frac{\hbar k^2}{2m}t\right)} \ dk$$

$$\Psi(x,t) = \frac{\sqrt{m\alpha}}{\hbar}e^{-m\alpha|x|/\hbar^2}e^{-iEt/\hbar}$$

$$\phi(p,t) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{-ipx/\hbar}\Psi(x,t) \ dx$$

$$\phi(x,t) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{-ipx/\hbar}\Psi(p,t) \ dp$$

$$E_n = \frac{\hbar^2k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2} \ \text{(I.S.W)}$$

$$\Psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right) \ \text{(I.S.W)}$$

$$\int \Psi_m^*(x)\Psi_n(x) \ dx = \delta_{mn}$$

$$C_n = \int \Psi_n^*(x)\Psi(x,0) \ dx$$

$$C_n =< f_n|\Psi >= \int f_n(x)^*\Psi(x,0) \ dx$$

$$V(x) = \frac{1}{2}mw^2x^2 \ \text{(H.O)}$$

$$[x,p] = i\hbar$$

$$[\hat{x},\hat{p}] = i\hbar$$

$$E_n = (n+1/2)\hbar w \ \text{(H.O)}$$

$$a_\pm = \frac{1}{\sqrt{2\hbar mw}}(\mp ip + mwx)$$

$$a_+\Psi_n = \sqrt{n+1}\Psi_{n+1}$$

$$a_-\Psi_n = \sqrt{n}\Psi_{n-1}$$

$$(a_+)^2\Psi_n \ \propto \ \Psi_{n+2}$$

$$(a_-)^2\Psi_n \ \propto \ \Psi_{n-2}$$

$$a_+a_-\Psi_n = n\Psi_n$$

$$a_-a_+\Psi_n = (n+1)\Psi_n$$

$$a_-\Psi_0 = 0$$

$$\Psi_0(x) = \left(\frac{mw}{\pi\hbar}\right)^{\frac{1}{4}} e^{\frac{-mw}{2\hbar}x^2}$$

$$\Psi_n(x) = A_n(a_+)^n\Psi_0(x)$$

$$\Psi_n = \frac{(a_+)^n}{\sqrt{n!}}\Psi_0$$

$$x = \sqrt{\frac{\hbar}{2mw}}(a_+ + a_-)$$

$$\hat{p} = i\sqrt{\frac{\hbar mw}{2}}(a_+ - a_-)$$

$$R = (1 + (2\hbar^2E/m\alpha^2))^{-1}$$

$$T = (1 + (m\alpha^2/2\hbar^2E))^{-1}$$

$$R = \frac{|B|^2}{|A|^2}, \ T = \frac{|F|^2}{|A|^2}$$

$$R + T = 1$$

$$|\alpha >\rightarrow a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$< \alpha| \rightarrow a^* = \begin{pmatrix} a_1^* & a_2^* & a_3^* \end{pmatrix}$$

$$\text{H.S iff} < \beta|\alpha >=< \alpha|\beta >^*$$

$$< \alpha|\alpha >= \int_a^b |f(x)|^2 \ dx$$

$$< f_m|f_n >= \delta_{mn}$$

$$< \hat{A} >=< \delta(0)|\hat{A}|\delta(0) >$$

$$P = | < \lambda_n|\delta(t) > |^2$$

$$|\delta(t) >= \phi(t)|\delta(0) >$$

$$|\delta(0) >= |\lambda_n >$$

$$\Psi(x,t) =< x|\delta(t) >$$

$$\phi(p,t) =< p|\delta(t) >$$
$$C_n(t) =< n|\delta(t) >$$
$$\hat{Q}_{mn} =< e_m|\hat{Q}|e_n >$$
$$i\hbar\frac{d}{dt}|\delta >= H|\delta >$$
$$\hat{H}|\delta >= E|\delta >$$
$$\hat{P} = |\alpha >< \alpha|$$
$$\hat{P}|\delta >=< \alpha|\beta|\alpha >$$
$$< e_m|e_n >= \delta_{mn}$$
$$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x},\ p_y = \frac{\hbar}{i}\frac{\partial}{\partial x},\ p_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$$
$$p = \frac{\hbar}{i}\nabla$$
$$P_l(x) = \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2-1)^l$$
$$P_l^m(x) = (1-x^2)^{|m|/2}\left(\frac{d}{dx}\right)^{|m|} P_l(x)$$
$$Y_l^m(\theta,\phi) = \left[\epsilon\sqrt{\frac{(2l+1)}{(4\pi)}\frac{(l-|m|)!}{(l+|m|)!}}\right.$$
$$\left.\times\, e^{im\phi}\ P_l^m(\cos\theta)\right],\ \epsilon = (-1)^m$$
$$j_l(x) = (-x)^l\left(\frac{1}{x}\frac{d}{dx}\right)\frac{\sin(x)}{x}$$
$$n_l(x) = -(-x)^l\left(\frac{1}{x}\frac{d}{dx}\right)^l\frac{\cos(x)}{x}$$
$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}$$
$$\Psi_{100}(r,\theta,\phi) = \frac{e^{-r/a}}{\sqrt{\pi a^3}}$$
$$\Psi_{nlm} = \left[\sqrt{\left(\frac{2}{na}\right)^3\frac{(n-l-1)!}{2n[(n+1)!]^3}}\ e^{-r/na}\right.$$
$$\left.\times\left(\frac{2r}{na}\right)^l\left[L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)\right]Y_l^m(\theta,\phi)\right]$$

$$\Delta E = E_1\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$
$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$
$$P = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}(f(r))^2\, r^2\sin^2(\theta)\, dr d\theta d\phi$$
$$\eta = \int_0^{2\pi}\int_0^{\pi} f(r,\theta,\phi)^* f(r,\theta,\phi)\ \sin(\theta)\, d\theta d\phi$$
$$\mathcal{O} = \int_0^{2\pi}\int_0^{\pi} f(r,\theta,\phi)^* f(r,\theta,\phi)\sin(\theta)\, d\theta d\phi$$
$$L = r\ \mathbf{x}\ p$$
$$L_x = yP_z - zP_y$$
$$L_y = zP_x - xP_z$$
$$L_z = xP_y - yP_x$$
$$[L_x, L_y] = i\hbar L_z$$
$$[L_y, L_z] = i\hbar L_x$$
$$[L_z, L_x] = i\hbar L_y$$
$$[L^2, L] = 0$$
$$L_\pm = L_x \pm iL_y$$
$$L^2 f_l^m = \hbar^2 l(l+1) f_l^m$$
$$L_z f_l^m = \hbar m f_l^m$$
$$L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$
$$L_\pm = \pm\hbar e^{\pm i\phi}\left(\frac{\partial}{\partial\theta} \pm i\cot\theta\frac{\partial}{\partial\phi}\right)$$
$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$
$$[s_x, s_y] = i\hbar s_z$$
$$[s_y, s_z] = i\hbar s_x$$
$$[s_z, s_x] = i\hbar s_y$$
$$S^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$
$$S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$
$$S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$
$$\chi = \begin{pmatrix}a\\ b\end{pmatrix} = a\chi_+ + b\chi_-$$
$$\chi_+ = \begin{pmatrix}1\\ 0\end{pmatrix},\ \lambda = \frac{\hbar}{2}$$
$$\chi_- = \begin{pmatrix}0\\ 1\end{pmatrix},\ \lambda = \frac{-\hbar}{2}$$
$$\chi_+^{(x)} = \frac{1}{\sqrt{2}} = \begin{pmatrix}1\\ 1\end{pmatrix},\ \lambda = \frac{\hbar}{2}$$
$$\chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix},\ \lambda = \frac{-\hbar}{2}$$
$$\chi_+^{(y)} = \frac{1}{\sqrt{2}} = \begin{pmatrix}1\\ i\end{pmatrix},\ \lambda = \frac{\hbar}{2}$$
$$\chi_-^{(y)} = \frac{1}{\sqrt{2}} = \begin{pmatrix}1\\ -i\end{pmatrix},\ \lambda = \frac{-\hbar}{2}$$
$$\chi_+^{(z)} = \frac{1}{\sqrt{2}} = \begin{pmatrix}1\\ 0\end{pmatrix},\ \lambda = \frac{\hbar}{2}$$
$$\chi_-^{(z)} = \frac{1}{\sqrt{2}} = \begin{pmatrix}0\\ 1\end{pmatrix},\ \lambda = \frac{-\hbar}{2}$$
$$\chi = \left(\frac{a+b}{\sqrt{2}}\right)\chi_+^{(i)} + \left(\frac{a-b}{\sqrt{2}}\right)\chi_-^{(i)}$$
$$\chi(t) = \begin{pmatrix}\cos(\alpha/2)e^{i\gamma B_0 t/2}\\ \sin(\alpha/2)e^{-i\gamma B_0 t/2}\end{pmatrix}$$
$$< S >= \chi^* S\chi$$
$$\eta = \chi^*\chi = 1$$
$$\mathbf{H} = -\mu\cdot\mathbf{B} = -\gamma\mathbf{B}\cdot\mathbf{S}$$
$$P_\pm^{(i)} = |C_\pm^{(i)}|^2$$
$$C_\pm^{(i)} = \chi_\pm^{(i)^*}\chi_\pm$$
$$i\hbar\frac{\partial\chi}{\partial t} = H\chi$$

Mathematical Equations and Constants

$$\int_0^\infty x^n e^{-x/a}dx = n!a^{n+1},\quad \int_0^\infty x^{2n}e^{-x^2/a^x}dx = \sqrt{\pi}\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1},\quad \int_0^\infty x^{2n+1}e^{-x^2/a^2}dx = \frac{n!}{2}a^{2n+2}$$
$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}\quad :\quad a\ddot{x} + b\dot{x} + x = 0,\quad \Delta = b^2 - 4ac$$
$$\text{iff } \Delta > 0,\ x(t) = Ae^{r_1 t} + Be^{r_2 t} :\quad \text{iff } \Delta = 0,\ x(t) = Ae^{rt} + Bte^{rt} :\quad \text{iff } \Delta < 0,\ x(t) = Ae^{irt} + Be^{-irt}$$
$$E_1 = -13.6\text{ ev},\ R = 1.097\cdot 10^7\ \text{m}^{-1},\ \hbar = 1.05457\cdot 10^{-34}\ \text{Js},\ c = 2.99792\cdot 10^8\ \text{m/s},\ m_e = 9.10938\cdot 10^{-31}\ \text{Kg}$$
$$m_p = 1.67262\cdot 10^{-27}\ \text{Kg},\ e_{+,-} = \pm 1.67262\cdot 10^{-19}\ \text{C},\ \epsilon_0 = 8.85419\cdot 10^{-12}\ \text{C}^2/\text{Jm},\ k_B = 1.38065\cdot 10^{-23}\ \text{J/K}$$
$$[x^2+y, z] = [x^2, z] + [y, z],\ [x^2, z] = x[x,z] + [x,z]x,\ [A,B] = AB - BA,\ [\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

Helpful Hints

Discrete Values:

$$C_n = < \Psi_n | \Psi(x,0)| >$$

$$\Psi(x,t) = \sum C_n e^{-iE_n t/\hbar}$$

$$< \Psi_m | \Psi_n >= \delta_{mn}$$

Continuous Values:

$$\phi(k) = < F_P^i | \Psi(x,t) >$$

$$\Psi(x,t) = < \phi(k) | F_P e^{-iE_n t/\hbar} >$$

$$< F_p | F_p >= \delta(P - P')$$

- Schrödingers equation solves for a spinless, nonrelativistic, quantum mechanical system.
- Discrete values are finite and can be known. i.e Energies.
- Unbound energies have continuous spectrum of Eigenvalues.
- Continus values are probabilities. i.e Position and Momentum.
- Wave functions live in Hilbert Space.
- Finite inner products indicate that the wave function given lives in Hilbert space.
- Observables are represented by hermitian operators.
- Operators in Quantum Mechanics represent physically observable quantities.
- Determinate States: Every measurement of $\hat{Q}$ returns the same value q.
- Determinate States are eigenfunctions of $\hat{Q}$.