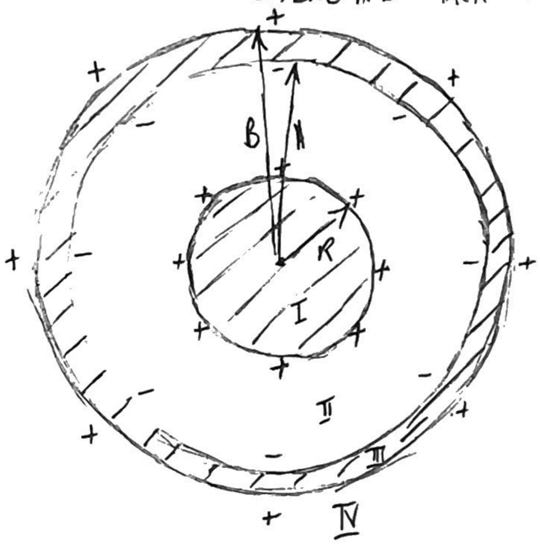
PHYS 311 HOMEWORK SET 7 SOLUTIONS

CROSS SECTION OF SPHERE AND THICK CONCENTRIC METAL SHELL...



let Region I : when $r < R$

Region II when $R < r < A$

Region III when $A < r < B$

Region IV when $B < r$

$\sigma = \frac{\text{CHARGE}}{\text{AREA}}$

NOTICE:

- REGIONS I & III MUST HAVE $\vec{E} = 0$, SINCE CONDUCTORS HAVE ZERO $\vec{E}$-FIELD INSIDE
- DRAWING A GAUSSIAN SURFACE CONCENTRIC W/ THE SPHERE AND THICK CONCENTRIC W/ $A < r < B$..

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon}$$ NOW, SINCE $\vec{E} = 0$ IN RIII, $\oint \vec{E} \cdot d\vec{a} = 0$ IN RIII $\therefore Q_{enc} = 0$ $\therefore$

$\therefore$ THE INNER SURFACE OF THE THICK CONCENTRIC METAL SHELL MUST HAVE $-Q$. NOW SINCE THE OUTER SHELL IS NEUTRAL, THE OUTER SURFACE OF THE THICK CONCENTRIC METAL SHELL MUST HAVE $+Q$

a) $\sigma(r = R) = +\frac{Q}{4\pi R^2}$

$\sigma(r = A) = -\frac{Q}{4\pi A^2}$

$\sigma(r = B) = +\frac{Q}{4\pi B^2}$

b) USE GAUSS' LAW IN EACH REGION TO GET THE $\vec{E}$-FIELD

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

IN RI: $\vec{E}_I = 0$ SINCE $Q_{enc} = 0$

IN RII:

$$E_{II} 4\pi r^2 = +\frac{Q}{\varepsilon_0} \quad \therefore \quad \vec{E}_{II} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

IN RIII: $\vec{E}_{III} = 0$ SINCE $Q_{enc} = 0$

IN RIV: $$E_{IV} 4\pi r^2 = +\frac{Q}{\varepsilon_0} \quad \therefore \quad \vec{E}_{IV} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

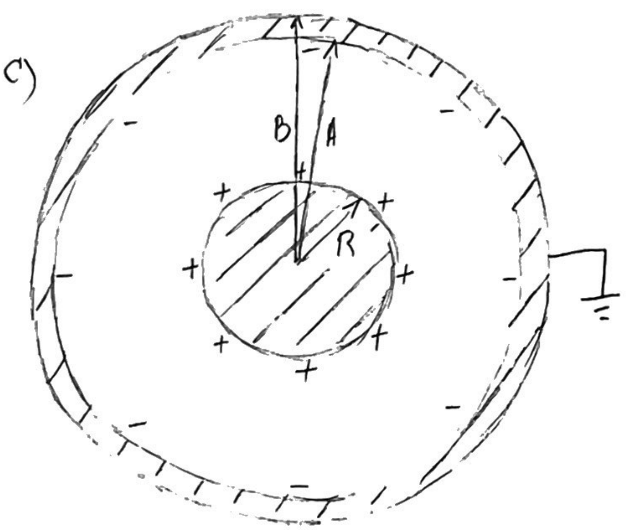
NOW

$$V(r=0) = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{B} \vec{E}_{IV}\cdot d\vec{l} - \int_{B}^{A} \cancel{\vec{E}_{III}}\cdot d\vec{l} - \int_{A}^{R} \vec{E}_{II}\cdot d\vec{l} - \int_{R}^{0} \cancel{\vec{E}_{I}}\cdot d\vec{l}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[\int_{\infty}^{B}\frac{dr}{r^2} + \int_{A}^{R}\frac{dr}{r^2}\right] = -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\Big|_{\infty}^{B} - \frac{1}{r}\Big|_{A}^{R}\right]$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{B} + \frac{1}{R} - \frac{1}{A}\right]$$

c)



NOTICE :

- TOUCHING THE OUTER SURFACE TO A GROUNDING WIRE REMOVES THE CHARGE $+Q$ (SINCE $V(r=B)=0$) BUT LEAVES THE CHARGE ON THE INNER SURFACE UNAFFECTED
- HOW DO WE KNOW THIS ?

  $\vec{E}_{III}$ MUST BE ZERO $\therefore$ $Q_{enc} = 0$

SO

$\vec{E}_{I} = 0$ (AS BEFORE)

$\vec{E}_{II} = \frac{1}{4\pi\epsilon_0}\frac{+Q}{r^2}\hat{r}$ (AS BEFORE)

$\vec{E}_{III} = 0$ (AS BEFORE)

$\vec{E}_{IV} = 0$ SINCE $Q_{enc} = 0$

SO

$\sigma(r=R) = +\frac{Q}{4\pi R^2}$

$\sigma(r=A) = -\frac{Q}{4\pi R^2}$

$\sigma(r=B) = 0$

NOW

$$V(r=0) = -\int_{A}^{R} \vec{E}_{II}\cdot d\vec{l} = -\frac{Q}{4\pi\epsilon_0}\int_{A}^{R}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\cdot\frac{1}{r}\Big|_{A}^{R} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{A}\right)$$

THE POTENTIAL AT $r=0$ DECREASED

THE SURFACE CHARGE DENSITY @ $r=B$ VANISHED !

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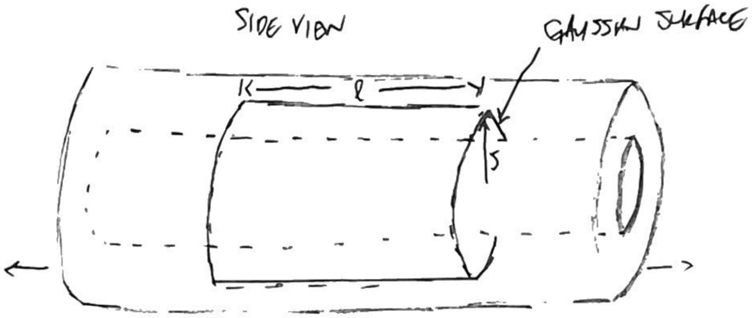

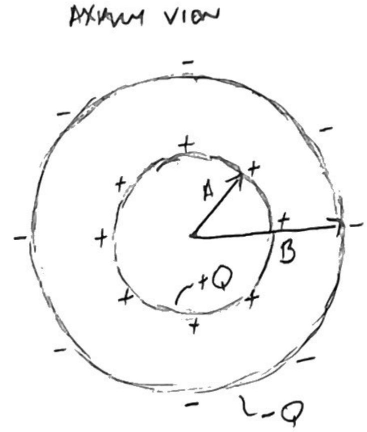

- PLACE $+\sigma_0$ ON INNER CYLINDER : $-\sigma_0$ ON OUTER CYLINDER
- CONSIDER GAUSSIAN SURFACES COAXIAL W/ THE TWO METAL CYLINDRICAL TUBES W/

RI: $s < A$

RII: $A < s < B$

$RIII$: $B < s$

NOW, USE GAUSS' LAW IN EACH OF THE 3 REGIONS...

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

IN RI:

$$\vec{E}_I = 0 \quad \text{SINCE} \quad Q_{enc} = 0$$

IN RII, GAUSS' LAW BECOMES...

$$E\, 2\pi s l = \frac{+\sigma_0}{\varepsilon_0} 2\pi A l \qquad \text{SO} \qquad E = \frac{\sigma_0 A}{\varepsilon_0} \frac{1}{s} \qquad \text{SO} \qquad \boxed{\vec{E} = \frac{\sigma_0 A}{\varepsilon_0} \frac{1}{s} \hat{s}}$$

IN $RIII$:

$$\vec{E}_3 = 0 \quad \text{SINCE} \quad Q_{enc} = 0$$

NOW, THE POTENTIAL DIFFERENCE BETWEEN THE TWO COAXIAL CYLINDRICAL TUBES IS..

$$\Delta V = V(A) - V(B) = -\int_B^A \vec{E} \cdot d\vec{l} = -\frac{\sigma_0 A}{\varepsilon_0} \int_B^A \frac{ds}{s} = -\frac{\sigma_0 A}{\varepsilon_0} \ln s \Big|_B^A = \frac{\sigma_0 A}{\varepsilon_0}\left[\ln B - \ln(A)\right]$$

$$d\vec{l} = ds\, \hat{s}$$

$$= \frac{\sigma_0 A}{\varepsilon_0} \ln(B/A)$$

NOW $\sigma = \dfrac{Q}{2\pi A l}$ SO $\Delta V = \dfrac{Q}{2\pi l \varepsilon_0} \ln(B/A)$

$$\text{SO} \quad \frac{Q}{\Delta V} = \frac{2\pi l \varepsilon_0}{\ln(B/A)} = C$$

SO $\boxed{\dfrac{C}{l} = \dfrac{2\pi\varepsilon_0}{\ln(B/A)}}$