Phys 131 2016

Lecture 30

Mon : HW due by 5pm

Supp 64

Ch 11 Conc Q 6, 13

Ch 11 Prob 15, 18, 28, 29, 49

Conservation of momentum

For any system of objects:

> If the net external force (sum of all forces exerted by objects outside the system) is zero then the total momentum of the system:
>
> $$\vec{P}_{tot} = \vec{p}_1 + \vec{p}_2 + \cdots$$
>
> remains constant.

This is the conservation of momentum. This idea is routinely applied to understand collisions.

Example: A 0.20kg cart is at rest. A 0.050kg ball approaches it with speed 50m/s, collides and rebounds in the opposite direction with speed 20m/s. Determine the velocity of the cart after the collision.

Answer: Initial Final

Ball Cart

$V_{bi} = 50\,m/s$ $V_{ci} = 0\,m/s$ $V_{bf} = -20\,m/s$ $V_{cf} = ??$

velocity of ball initially velocity of cart initially

As the net external force is zero, the total momentum is constant

$$P_{tot\,f} = P_{tot\,i}$$

$$\Rightarrow \quad p_{bf} + p_{cf} = p_{bi} + p_{ci}$$

$$\Rightarrow \quad M_b V_{bf} + M_c V_{cf} = M_b V_{bi} + \cancel{M_c V_{ci}}^{\,0}$$

$$\Rightarrow \quad 0.050\,kg\,(-20\,m/s) + 0.20\,kg\,V_{cf} = 0.050\,kg\,(50\,m/s)$$

$$\Rightarrow \quad -1.0\,kg\,m/s + 0.20\,kg\,V_{cf} = 2.5\,kg\,m/s$$

$$\Rightarrow \quad 0.20\,kg\,V_{cf} = 3.5\,kg\,m/s$$

$$\Rightarrow \quad V_{cf} = 17.5\,m/s \quad \boxtimes$$

Quiz 1

Demo: Happy sad / ball

Explosions

In an "explosion" two objects that are initially together separate. Again, if the net external force is zero, then

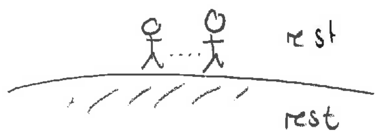
Quiz 3

Demo: PhET My Solar System ⇝ 2 planets

Example: One hundred 70kg people each simultaneously jump vertically. The speed with which they leave Earth's surface is 5.0m/s. Determine the speed with which Earth recoils.

Answer:

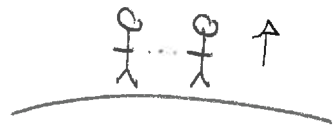
Initial



$V_{pi} = 0 m/s$

$V_{Ei} = 0 m/s$

Final (just after leaving...)



$V_{pf} = 5.0 m/s$

$V_{Ef} = ??$

If Earth + people are system then there are no external forces. So momentum is conserved.

$$P_{tot\,f} = P_{tot\,i}$$

$$M_p V_{pf} + M_E V_{Ef} = M_{pi} \cancel{V_{pi}} + M_E \cancel{V_{Ei}} = 0 kgm/s$$

mass of all people

$$\Rightarrow M_E V_{Ef} = -M_p V_{pf}$$

$$\Rightarrow V_{Ef} = -\frac{M_p}{M_E} V_{pf}$$

$$= -\frac{100 \times 70kg}{5.98 \times 10^{24} kg}(5.0 m/s) = 5.8 \times 10^{-21} m/s.$$

Challenge: a) Determine the maximum displacement of the people Ans 1.3m

b) " " " " " " Earth Ans: 1.5×10^{-21}m

Momentum + Energy in Collisions

Energy is not necessarily conserved in collisions. For example, with two identical objects that approach at the same speeds + stick.

$p_{tot\,i} = 0$ $\qquad$ $p_{tot\,f} = 0 \Rightarrow v_f = 0$

$K_i \neq 0$ $\qquad$ $K_f = 0.$

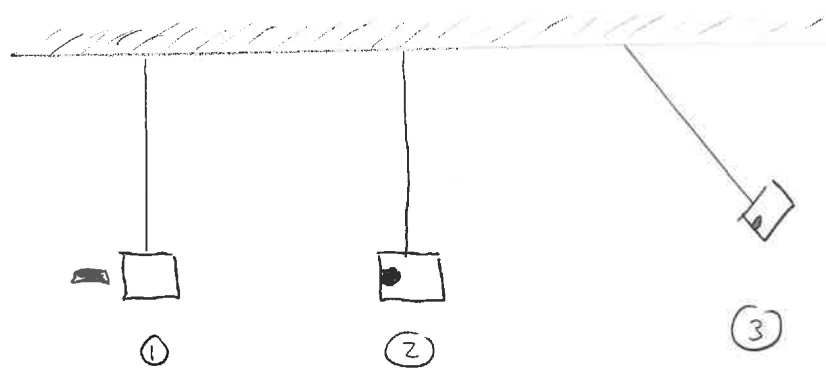
So the total mechanical energy is not conserved. But combinations of energy + momentum conservation can be applicable.

Example: A 0.010kg bullet is fired into a 0.200kg wooden block suspended from a string. The bullet sticks in the block and the pair swing upwards reaching a maximum height h. Determine

a) an expression for the speed of the bullet in terms of h

b)

Answer:



From ① → ② net external force zero ⇒ momentum conserved

From ② → ③ work done by non-cons = 0 ⇒ energy conserved

First consider ① → ②. Let v_{bi} = velocity of bullet

v_{wi} = " " wood

before

Let v_2 = velocity of pair after.

$$P_{tot\,2} = P_{tot\,1}$$

$$\Rightarrow \quad m_w v_2 + m_b v_2 = m_w \cancel{v_{wi}}^{0} + m_b v_{bi}$$

$$\Rightarrow \quad (m_w + m_b) v_2 = m_b v_{bi} \qquad -(1)$$

Now consider ② → ③. Arrange the vertical axis so $y=0$ corresponds to position of block when suspended vertically. Then

$$E_3 = E_2$$

$$\Rightarrow \quad \cancel{K_3}_{0} + U_{g3} = K_2 + \cancel{U_{g2}}_{0}$$

$$\Rightarrow \quad \cancel{(m_w+m_b)}\, g h = \frac{1}{2} \cancel{(m_w + m_b)}\, v_2^2$$

$$\Rightarrow \quad g h = \frac{1}{2} v_2^2 \qquad -(2)$$

Now (1) $\Rightarrow$ $v_2 = \left(\frac{m_b}{m_w+m_b}\right) v_{bi}$ and thus.

$$g h = \frac{1}{2}\left(\frac{m_b}{m_w+m_b}\right)^2 v_{bi}^2 \quad \Rightarrow \quad (v_{bi})^2 = 2gh \Big/ \left(\frac{m_b}{m_w+m_b}\right)^2$$

$$\Rightarrow \quad v_{bi} = \frac{m_w+m_b}{m_b}\sqrt{2gh}$$