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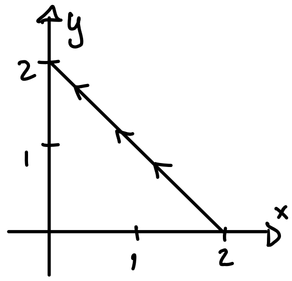
MATH 360
HW 10.1

#4,5,10

Problem 10.1.4 $F = [xy, x^2y^2]$ C from $(2,0)$ to $(0,2)$ $\int_C \vec{F}(r)\cdot d\vec{r}$



$0 \le t \le 1$

$\vec{r}(t) = [2-2t, 2t]$

$\frac{dr}{dt} = [-2, 2]$

$4t^2 - 8t + 4$

$\vec{F}(r) = [(2-2t)(2t), (2-2t)^2(4t^2)]$

$= [4t-4t^2, (4t^2-8t+4)(4t^2)]$

$\vec{F}(\vec{r}) = [4t-4t^2, (16t^4-32t^3+16t^2)]$

$\int_C \vec{F}(r)\cdot d\vec{r}$

$\int_0^1 [4t-4t^2, 16t^4-32t^3+16t^2]\cdot[-2, 2]\, dt$

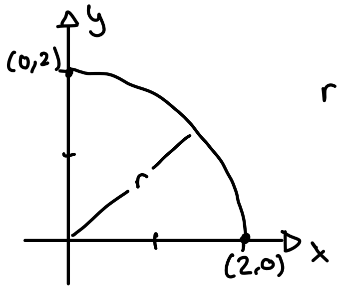
$\int_0^1 -8t+8t^2+32t^4-64t^3+32t^2\, dt$

$\int_0^1 32t^4-64t^3+40t^2-8t\, dt$

$\frac{32t^5}{5} - 16t^4 + \frac{40}{3}t^3 - 4t^2 \Big|_0^1 \left[\frac{32}{5} - 16 + \frac{40}{3} - 4\right] - 0 = \boxed{-\frac{4}{15}}$

$$\boxed{\int_C \vec{F}(r)\cdot d\vec{r} = -4/15}$$

Problem 10.1.5 $F=[xy, x^2y^2]$ C from 1/4 circle $(2,0) \to (0,2)$ with $(0,0)$ as center
radius = 2



$r=2$

$0 \le t \le \pi/2$

$\vec{r}(t) = [2\cos(t), 2\sin(t)]$

$\vec{r}\,'(t) = [-2\sin(t), 2\cos(t)]$

$r^2 = x^2+y^2$

$x = r\cos\underset{t}{\theta}, \quad y = r\sin\underset{t}{\theta}$

$\vec{F}(r) = [4\cos(t)\sin(t), 4\cos^2(t)\cdot 4\sin^2(t)]$

$$\int_C \vec{F}(r)\cdot d\vec{r} = \int_0^{\pi/2} [4\cos(t)\sin(t), 16\cos^2(t)\sin^2(t)]\cdot[-2\sin(t), 2\cos(t)]\, dt$$

$$\int_0^{\pi/2} -8\cos(t)\sin^2(t) + 32\cos^3(t)\sin^2(t)\, dt$$

$u = \sin(t)$
$du = \cos(t)\,dt$

$$-8\int_0^{\pi/2} \cos(t)\sin^2(t)\,dt + 32\int_0^{\pi/2} \cos^3(t)\sin^2(t)\,dt$$

$$-8\int_0^{\pi/2} u^2\,du + 32\int_0^{\pi/2} \cos(t)\cdot\cos^2(t)\cdot\sin^2(t)\,dt$$

$\cos^2(t) = (1-\sin^2(t))$

$$-8\left[\frac{u^3}{3}\right]\Big|_{u=\sin(t)}\Big|_0^{\pi/2} + 32\int_0^{\pi/2} \cos(t)(1-\sin^2(t))\cdot\sin^2(t)\,dt$$

$v = \sin(t)$
$dv = \cos(t)\,dt$

$$-8\left[\frac{\sin^3(t)}{3}\right]_0^{\pi/2} + 32\int_0^{\pi/2} \cos(t)(\sin^2(t) - \sin^4(t))\,dt$$

$$-8\left[\tfrac{1}{3} - 0\right] + 32\int_0^{\pi/2} v^2 - v^4\,dv$$

$$-\frac{8}{3} + 32\left[\frac{v^3}{3} - \frac{v^5}{5}\right]\Big|_{v=\sin(t)}$$

$$-\frac{8}{3} + 32\left[\frac{\sin^3(t)}{3} - \frac{\sin^5(t)}{5}\right]\Big|_0^{\pi/2}$$

$$-\frac{8}{3} + 32\left[\left(\frac{1}{3} - \frac{1}{5}\right) - 0\right] = -\frac{8}{3} + 32\left[\frac{2}{15}\right] = \frac{8}{5}$$

$$\boxed{\int_C \vec{F}(r)\cdot d\vec{r} = 8/5}$$

Problem 10.1.10 $F = [x, -z, 2y]$ $(0,0,0) \to (1,1,0) \to (1,1,1) \to (0,0,0)$

$\int_C \vec{F}(r) \cdot d\vec{r}$

(a) $(0,0,0) \to (1,1,0)$ $\qquad 0 \le t \le 1$

$[0,0,0] + t[1,1,0]$ $\qquad \vec{r}(t) = [t,t,0]$ $\qquad \vec{F}(r) = [t,0,2t]$

$[t,t,0]$ $\qquad \vec{r}\,'(t) = [1,1,0]$

$$\int_C \vec{F}(r) \cdot d\vec{r} = \int_0^1 [t,0,2t] \cdot [1,1,0]\, dt$$

$$= \int_0^1 t\, dt = \tfrac{1}{2} t^2 \Big|_0^1 = \tfrac{1}{2} \qquad \boxed{\tfrac{1}{2}}$$

(b) $(1,1,0) \to (1,1,1)$ $\qquad 0 \le t \le 1$

$[1,1,0] + t[0,0,1]$ $\qquad \vec{r}(t) = [1,1,t]$ $\qquad \vec{F}(r) = [1,-t,2]$

$[1,1,t]$ $\qquad r'(t) = [0,0,1]$

$$\int_C \vec{F}(r) \cdot d\vec{r} = \int_0^1 [1,-t,2] \cdot [0,0,1]\, dt$$

$$\int_0^1 2\, dt = 2t \Big|_0^1 = 2 \qquad \boxed{2}$$

(c) $(1,1,1) \to (0,0,0)$ $\qquad 0 \le t \le 1$

$[1,1,1] + t[-1,-1,-1]$ $\qquad \vec{r}(t) = [1-t, 1-t, 1-t]$ $\qquad \vec{F}(r) = [1-t, t-1, 2-2t]$

$[1-t, 1-t, 1-t]$ $\qquad \vec{r}\,'(t) = [-1,-1,-1]$

$$\int_C \vec{F}(r) \cdot d\vec{r} = \int_0^1 [1-t, t-1, 2-2t] \cdot [-1,-1,-1]\, dt$$

$$\int_0^1 (t-1) + (1-t) + (2t-2)\, dt$$

$$\int_0^1 2t-2\, dt = t^2 - 2t \Big|_0^1 = \boxed{-1}$$

$$(a) + (b) + (c) = \tfrac{1}{2} + 2 - 1 = \frac{3}{2} \qquad \boxed{\frac{3}{2}}$$