Lecture 20

Mon: HW due by 5pm

Supp Ex 46, 52 → d) 4.8 m/s

Ch 7 CQ 13 → $2.3\,m/s^2$

Ch 7 Prob 38, 40 → $a = 0.48\,m/s^2$ $T = 21\,N$

Ch 8 CQ 4,

Ch 8 Prob 6, 13

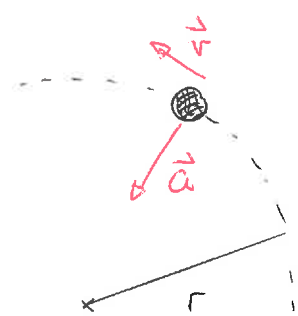
→ $6.8 \times 10^3\,N$

Tues: Review

Weds: Exam II Ch 5, 6, 7, 8

Circular motion

Any object which moves in a circle will have a non-zero acceleration since its velocity constantly changes direction. The simplest such case involves an object moving in a circle with constant speed - uniform circular motion.



In this case standard kinematics gives:

For uniform circular motion the acceleration points radially inward and has magnitude

$$a = v^2/r$$

where r is the radius of the circle

Standard geometrical reasoning relates this to angular velocity ω via:

$v = \omega r$ ⇒ uniform motion $\boxed{a = \omega^2 r}$

Dynamics of uniform circular motion

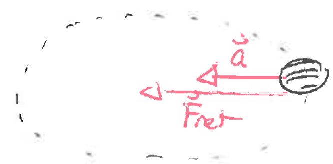
As there is a radially inward acceleration for an object undergoing uniform circular motion, there must be one or more forces acting on the object so as to produce the acceleration. We aim to use Newton's 2^{nd} and 3^{rd} law to assess the relationship between forces on objects moving in circles and the eventual motion. This has applications to:

1) orbital motion of celestial objects
2) motion of charged particles in magnetic fields
3) motion of objects in amusement park rides (rollercoaster loops)

In all uniform motion situations, Newton's 2^{nd} Law,

$$\vec{F}_{net} = m\vec{a}$$

implies that the net force points radially inward

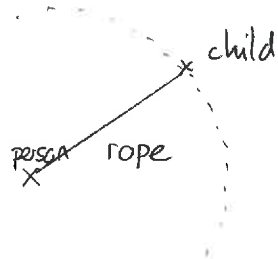


Warm Up 1

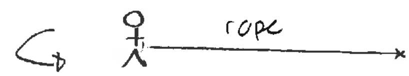
One common example of such circular motion is to objects that swing in circles on the end of a rope.

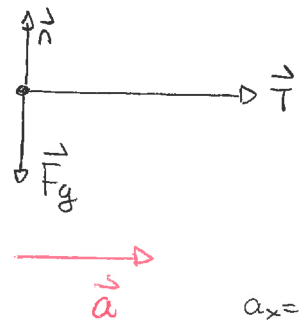
Example A person is fixed to a horizontal sheet of ice. The person swings a child, connected by a rope, in a circle of constant speed. There is no friction between the child + the surface and the rope is always horizontal. Determine an expression for the tension in the rope in terms of the mass of the child, the length of the rope and the child's angular velocity.

Answer: Top



Side (left





Using the left "side view" the FBD is as illustrated. The acceleration is radially inward (so it points right in this diagram). Then letting m be the mass of the child:

$a_x = a$

$a_y = 0$

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y = 0$$

Quiz 1

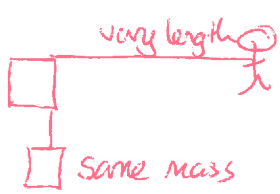
The horizontal components give:

$$T = ma$$

where T is the tension in the string If the length of the string is L and the angular velocity is ω, then

$$a = \omega^2 L \quad \Rightarrow \quad T = m\omega^2 L$$

Demo

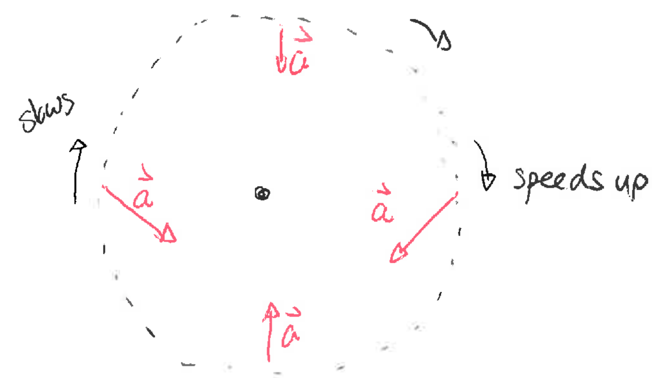


In terms of speed $a = v^2/L \Rightarrow T = \dfrac{mv^2}{L}$.

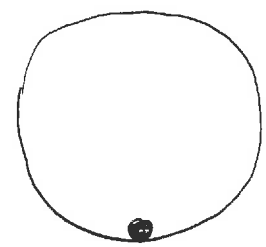
Warm Up 2

Quiz 2

Motion in a vertical circle in a gravitational field can be complicated by the fact that the object might speed up or slow down and at many stages the acceleration will not be radially inward. However, at the top and bottom the acceleration will be radially inward and will have magnitude $a = v^2/r = \omega^2 r$



<u>Example:</u> A blob of tar is stuck to the inside of a drum. The drum rotates in a vertical circle. When the tar is at the bottom how does the normal force compare to the gravitational force on the tar?



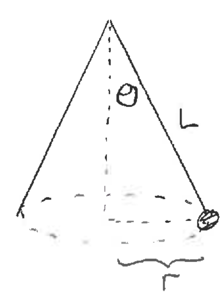
<u>Answer:</u>

Acceleration is inward (i.e. up here) $\Rightarrow$ $\vec{F}_{net}$ is up.

$\uparrow \vec{F}_{net} \quad \Rightarrow \quad n > F_g \quad \Rightarrow \quad n > mg$ ▪

<u>Example:</u> A ball on the end of a string of length L traces out a horizontal circle. The string sweeps out a cone at the illustrated angle. Determine expressions for:

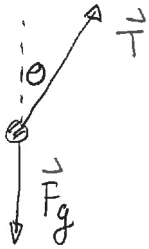
1) tension in the string (in terms of mass + angle)
2) speed of the ball



<u>Answer:</u> The ball moves in a circle of radius r. The acceleration is radially inward. Note that

$$\frac{r}{L} = \sin\theta \quad \Rightarrow \quad r = L\sin\theta$$

a) 1) FBD (when ball is at left)



$a_x = a = V^2/r$

$a_y = 0$

2) Newton's 2^{nd} Law

$\sum F_x = ma_x$

$\sum F_y = ma_y = 0$

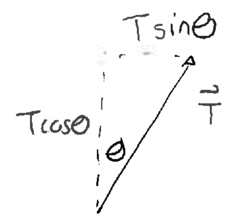
	x	y
$\vec{T}$	$T\sin\theta$	$T\cos\theta$
$\vec{F_g}$	0	$-mg$

where m is mass of ball. Thus

$\sum F_x = ma_x$

$\Rightarrow T\sin\theta = ma = mv^2/r$

$\Rightarrow \boxed{T\sin\theta = \frac{mv^2}{r}}$



$\sum F_y = 0 \Rightarrow T\cos\theta - mg = 0$

$\Rightarrow T\cos\theta = mg \Rightarrow \boxed{T = \frac{mg}{\cos\theta}}$

b) Combining gives

$$\frac{\cancel{m}g}{\cos\theta}\sin\theta = \frac{\cancel{m}v^2}{r} \Rightarrow v^2 = g\frac{\sin\theta}{\cos\theta}r$$

$$\Rightarrow v^2 = g\frac{\sin\theta}{\cos\theta}L\sin\theta \Rightarrow \boxed{v = \sqrt{gL\frac{\sin^2\theta}{\cos\theta}}}$$

Optional challenge: Show that

a) $\omega = \sqrt{\frac{g}{L\cos\theta}}$

b) The time to complete one orbit is $T = m\omega^2 L$