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PHYS 342
HW 7

Problem 2

$$\theta(r) = \int \frac{(\ell/r^2)\,dr}{\sqrt{2\mu\left(E + \frac{k}{r} - \frac{\ell^2}{2\mu r^2}\right)}} - \frac{\pi}{2} \qquad \alpha = \frac{\ell^2}{2\mu},\ u = \frac{1}{r} \ : \ \frac{du}{dr} = \frac{-1}{r^2} \ \therefore\ dr = -r^2 du$$

$$\theta(r) = \int \frac{\ell \cdot u^2}{\sqrt{2\mu(E + k\cdot u - \alpha \cdot u^2)}}\,dr - \frac{\pi}{2} \Rightarrow \int \frac{\ell \cdot u^2}{\sqrt{2\mu(E + ku - \alpha u^2)}} \cdot -r^2 du - \frac{\pi}{2} = \frac{\ell}{\sqrt{2\mu}} \int \frac{du}{\sqrt{-\alpha u^2 + ku + E}} - \frac{\pi}{2}$$

$$\theta(r) = \frac{\ell^2}{2\mu} \int \frac{1}{\sqrt{-\alpha u^2 + ku + E}}\,du - \frac{\pi}{2} \ : \ \theta(r) = \int \frac{1}{\sqrt{-u^2 + \frac{k}{\alpha}u + \frac{E}{\alpha}}}\,du - \frac{\pi}{2} \qquad a = -1,\ b = \frac{k}{\alpha},\ c = \frac{E}{\alpha}$$

Using appendix E):
$$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = -\frac{1}{\sqrt{-a}} \sin^{-1}\left(\frac{2ax + b}{\sqrt{b^2 - 4ac}}\right)$$

$$\theta(r) = -\frac{1}{\sqrt{-(-1)}} \sin^{-1}\left(\frac{-2u + \frac{k}{\alpha}}{\sqrt{(\frac{k}{\alpha})^2 + 4(\frac{E}{\alpha})}}\right) - \frac{\pi}{2} \ : \ -\theta(r) - \frac{\pi}{2} = \sin^{-1}\left(\frac{\frac{k}{\alpha} - 2u}{\sqrt{(\frac{k}{\alpha})^2 + 4(\frac{E}{\alpha})}}\right)$$

$$\sin\left(-\theta(r) - \frac{\pi}{2}\right) = \frac{\frac{k}{\alpha} - 2u}{\sqrt{(\frac{k}{\alpha})^2 + 4(\frac{E}{\alpha})}} \ : \ -\cos(\theta(r)) = \frac{\frac{k}{\alpha} - 2u}{\sqrt{(\frac{k}{\alpha})^2 + 4(\frac{E}{\alpha})}}$$

$$\downarrow$$

$$-\sin\left(\theta(r) + \frac{\pi}{2}\right) = -\cos(\theta(r))$$

$$\cos(\theta(r)) = \frac{2u - \frac{k}{\alpha}}{\sqrt{(\frac{k}{\alpha})^2 + 4(\frac{E}{\alpha})}} \ : \ \frac{\frac{2}{r} - \frac{k \cdot 2\mu}{\ell^2}}{\sqrt{\frac{k^2 \cdot 4\mu^2}{\ell^4} + \frac{4E \cdot 2\mu}{\ell^2}}} \left(\frac{\frac{\ell^2}{2\mu k}}{\frac{\ell^2}{4\mu k}}\right) \qquad u = \frac{1}{r} \ : \ \alpha = \frac{\ell^2}{2\mu}$$

$$\cos(\theta(r)) = \frac{\frac{\ell^2}{\cancel{2}\mu k} \cdot \frac{\cancel{2}}{r} - 1}{\sqrt{1 + \frac{2E\ell^2}{\mu k^2}}} = \frac{\frac{\ell^2}{\mu k r} - 1}{\sqrt{1 + \frac{2E\ell^2}{\mu k^2}}}$$

Substitutions : NEW!!! $\alpha = \frac{\ell^2}{\mu k}$

$$\varepsilon = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}}$$

$$\cos(\theta(r)) = \frac{\frac{\alpha}{r} - 1}{\varepsilon} \ : \ r\varepsilon\cos(\theta(r)) = \alpha - r \ : \ r\varepsilon\cos(\theta(r)) + r = \alpha \ : \ \varepsilon\cos(\theta(r)) + 1 = \frac{\alpha}{r}$$

$$\therefore \boxed{\frac{\alpha}{r} = 1 + \varepsilon\cos(\theta(r))}$$

Problem 3

a.) $\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{\ell^2}\cdot\frac{1}{u^2}F(1/u)$ $\qquad F(r) = \frac{-k}{r^3}$ $\qquad \ell^2 = \mu k,\ \ell^2 > \mu k,\ \ell^2 < \mu k$

$F(1/u) = -k\cdot u^3$

$$\frac{d^2u}{d\theta^2} + u = \cancel{-}\frac{\mu}{\ell^2}\cdot\frac{1}{\cancel{u^2}}\cancel{-}k\cdot u^{\cancel{3}} = \frac{\mu k}{\ell^2}\cdot u$$

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu k}{\ell^2}u \quad : \quad u'' + u - \frac{\mu k}{\ell^2}u = 0 \quad : \quad u'' + u\left(1 - \frac{\mu k}{\ell^2}\right) = 0$$

i.) $\ell^2 = \mu k$

$$\frac{d^2u}{d\theta^2} + u = -\frac{\cancel{\mu}}{\cancel{\mu k}}\cdot\frac{1}{\cancel{u^2}}\cdot(\cancel{-k})u^{\cancel{3}} \quad : \quad \frac{d^2u}{d\theta^2} + u = u \quad \therefore \quad \frac{d^2u}{d\theta^2} = 0$$

$\boxed{u(\theta) = c_1\theta + c_2}$ Lets take $C_1 = 2,\ C_2 = 5$ through out

ii.) $\ell^2 > \mu k$: $\nu = \frac{\mu k}{\ell^2}$: $u'' + u(1-\nu) = 0$: $r^2 + (1-\nu) = 0$: $r^2 = \nu - 1$

$r = \pm\sqrt{\nu - 1}$

$$u(\theta) = c_1 e^{\sqrt{\nu-1}\,\theta} + c_2 e^{-\sqrt{\nu-1}\,\theta} \longrightarrow \boxed{u(\theta) = c_1 e^{\sqrt{\mu k/\ell^2 - 1}\,\theta} + c_2 e^{-\sqrt{\mu k/\ell^2 - 1}\,\theta}}$$

$c_1 = 2,\ c_2 = 5,\ \frac{\mu k}{\ell^2} = 10$

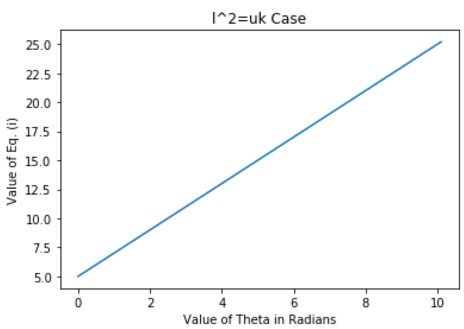
iii) $\ell^2 < \mu k$: $\nu = -\frac{\mu k}{\ell^2}$: $u'' + u(1+\nu) = 0$: $r^2 + (1+\nu) = 0$: $r^2 = -(\nu+1)$

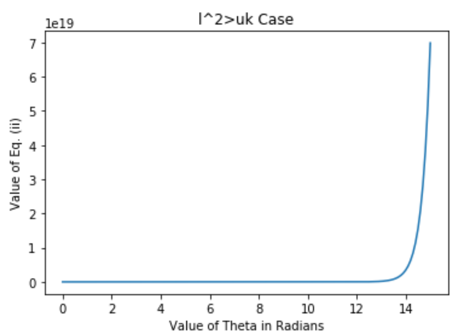
$r = \pm i\sqrt{\nu + 1}$

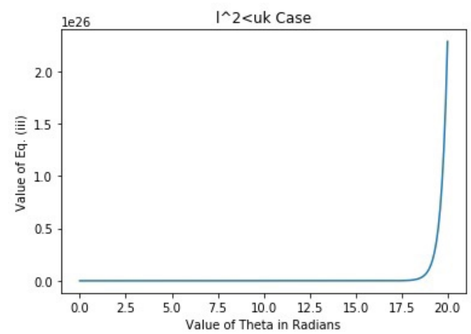
$$u(\theta) = c_1 e^{i\sqrt{\nu+1}\,\theta} + c_2 e^{-i\sqrt{\nu+1}\,\theta} \longrightarrow \boxed{u(\theta) = c_1 e^{i\sqrt{\mu k/\ell^2 + 1}\,\theta} + c_2 e^{-i\sqrt{\mu k/\ell^2 + 1}\,\theta}}$$

$c_1 = 2,\ c_2 = 5,\ \frac{\mu k}{\ell^2} = 8$

b.)



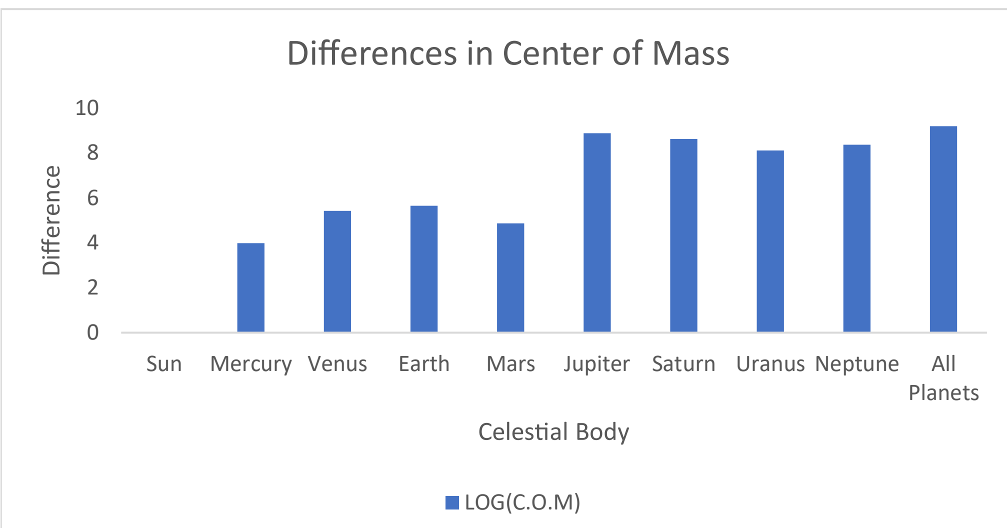




Problem 4

Celestial Body	Mass Of Celestial Body	Semi-Major Axis	Big M	Center of Mass
Sun	1.9885E+33			
Mercury	3.3011E+26	57909050000	1.99E+33	9613.454022
Venus	4.8675E+27	1.08208E+11	1.99E+33	264873.5986
Earth	5.97237E+27	1.49598E+11	1.99E+33	449309.5606
Mars	6.471E+26	2.27939E+11	1.99E+33	74176.21741
Jupiter	1.8982E+30	7.7857E+11	1.99E+33	742505481.6
Saturn	5.6834E+29	1.43353E+12	1.99E+33	409605051.7
Uranus	8.681E+28	2.87504E+12	1.99E+33	125507330.7
Neptune	1.02413E+29	4.50439E+12	1.99E+33	231976127.2
All Planets				1509210639

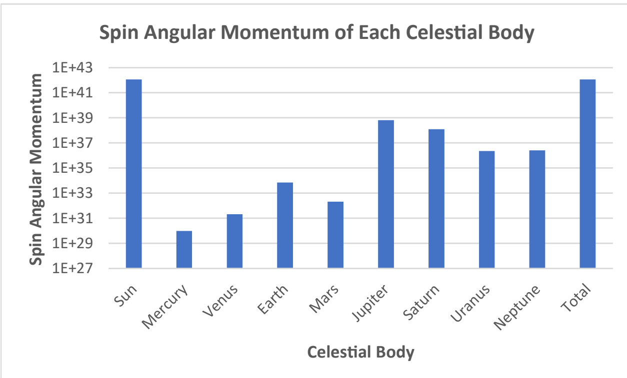
Planet	LOG(C.O.M)
Sun	0
Mercury	3.982879454
Venus	5.423038672
Earth	5.65254566
Mars	4.870264683
Jupiter	8.870699664
Saturn	8.612365304
Uranus	8.098669093
Neptune	8.365443294
All Planets	9.178749858

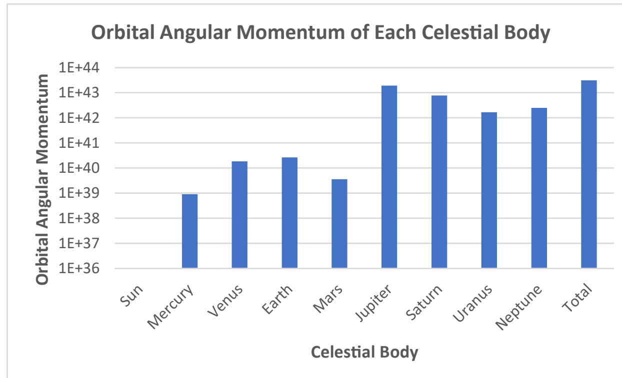


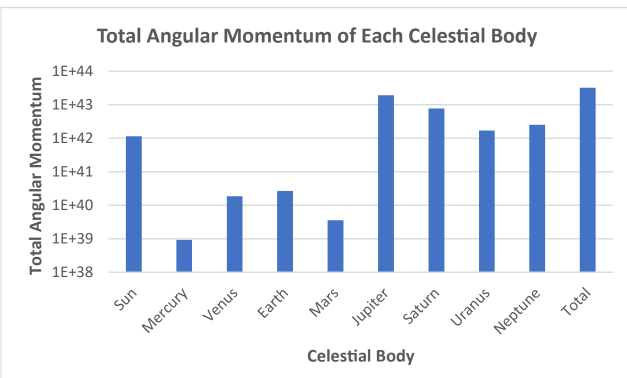
Problem 5

a.)

Celestial Body	Mass of Body	Radius of Body	Distance From Sun	I of Body	W Rotation	W Orbit	Spin L	Orbital L	Spin + Orbital L
Sun	1.9885E+30	695508000	0	3.8476E+47	2.9719E-06	0	1.1435E+42	0	1.14346E+42
Mercury	3.3011E+23	2440000	57909050000	7.86137E+35	1.2396E-06	8.264E-07	9.7448E+29	9.1482E+38	9.14819E+38
Venus	4.8675E+24	6052000	1.08208E+11	7.13122E+37	2.9927E-07	3.232E-07	2.1341E+31	1.8421E+40	1.84208E+40
Earth	5.97237E+24	6371000	1.49598E+11	9.69665E+37	7.2722E-05	1.992E-07	7.0516E+33	2.663E+40	2.663E+40
Mars	6.471E+23	3389000	2.27939E+11	2.97286E+36	6.9813E-05	1.059E-07	2.0754E+32	3.5589E+39	3.55893E+39
Jupiter	1.8982E+27	69911000	7.7857E+11	3.71102E+42	0.00017453	1.66E-08	6.4769E+38	1.9104E+43	1.91049E+43
Saturn	5.6834E+26	58232000	1.43353E+12	7.70889E+41	0.00015867	6.611E-09	1.2231E+38	7.7214E+42	7.72151E+42
Uranus	8.681E+25	25362000	2.87504E+12	2.23356E+40	0.00010267	2.346E-09	2.2931E+36	1.6833E+42	1.6833E+42
Neptune	1.02413E+26	24622000	4.50439E+12	2.48349E+40	0.00010267	1.208E-09	2.5497E+36	2.5101E+42	2.51014E+42
Total							1.1442E+42	3.1069E+43	3.22128E+43







b.)

$$E_{binding} = \frac{3}{5} \cdot \frac{GM^2}{r} \quad , \quad T = L^2/(2I)$$

$E_{Binding}$ = Kinetic Energy :

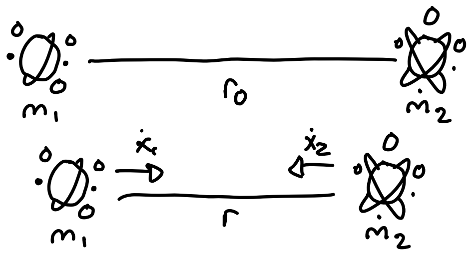
$$\frac{3}{5} \cdot \frac{GM^2}{r} = \frac{L^2}{2I}$$

$$L^2 = \frac{6}{5} \cdot \frac{GM^2 I}{r} \quad : \quad L = \sqrt{\frac{6}{5} \frac{GM^2 I}{r}}$$

$$\boxed{L = \sqrt{\frac{6}{5} \frac{GM^2 I}{r}}}$$

The Sun is not in danger of spinning itself out. Even if all the mass was put into the sun, it would not be problematic.

Problem 6)



$$E = T + U \; : \; T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 \; , \quad U = -\frac{Gm_1m_2}{r}$$

$$E_i = E_f \; : \; E_i = -\frac{Gm_1m_2}{r_0} \; , \quad E_f = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - \frac{Gm_1m_2}{r}$$

$$p = m\dot{x} \; : \; m_1\dot{x}_1 + m_2\dot{x}_2 = 0 \; : \; m_1\dot{x}_1 = -m_2\dot{x}_2 \; : \; \dot{x}_1 = -\frac{m_2}{m_1}\dot{x}_2 \; , \quad \dot{x}_2 = -\frac{m_1}{m_2}\dot{x}_1$$

$M = m_1 + m_2$

$\underline{\dot{x}_1}$

$$-\frac{Gm_1m_2}{r_0} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - \frac{Gm_1m_2}{r} \; : \; \frac{Gm_1m_2}{r} - \frac{Gm_1m_2}{r_0} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$

$$Gm_1m_2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}\cancel{m_2}\left(\frac{m_1^2}{m_2^{\cancel{2}}}\right)\dot{x}_1^2 \; : \; Gm_1m_2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}m_1\dot{x}_1^2\left[1 + \frac{m_1}{m_2}\right]$$

$$Gm_1m_2^2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}m_1\dot{x}_1^2\left[m_2 + m_1\right] \; : \; Gm_2^2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}\dot{x}_1^2 M \; , \quad \text{let } M = m_1 + m_2$$

$$\dot{x}_1^2 = \frac{2Gm_2^2}{M}\left[\frac{1}{r} - \frac{1}{r_0}\right] \quad \therefore \quad \dot{x}_1 = m_2\sqrt{\frac{2G}{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

$\underline{\dot{x}_2}$

$$Gm_1m_2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}\cancel{m_1}\left(\frac{m_2^2}{m_1^{\cancel{2}}}\right)\dot{x}_2^2 + \frac{1}{2}m_2\dot{x}_2^2 \; : \; Gm_1m_2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}\left(\frac{m_2^2}{m_1}\right)\dot{x}_2^2 + \frac{1}{2}m_2\dot{x}_2^2$$

$$Gm_1^2m_2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}(m_2^2)\dot{x}_2^2 + \frac{1}{2}m_2m_1\dot{x}_2^2 \; : \; Gm_1^2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_1\dot{x}^2$$

$$Gm_1^2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}\dot{x}_2^2\left[m_2 + m_1\right] \; : \; Gm_1^2\left[\frac{1}{r} - \frac{1}{r_0}\right] = \frac{1}{2}\dot{x}_2^2 M \; : \; \dot{x}_2^2 = \frac{2G}{M}m_1^2\left[\frac{1}{r} - \frac{1}{r_0}\right]$$

$$\boxed{\dot{x}_1 = m_2\sqrt{\frac{2G}{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)} \; , \quad \dot{x}_2 = m_1\sqrt{\frac{2G}{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)}}$$