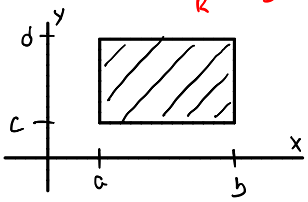
Fubinis Theorem : $\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy$



15.2.5) $\iint \frac{3(1+x^2)}{1+y^2}\,dA$ $\quad R = \{(x,y) \mid 0 \le x \le 4,\ 0 \le y \le 1\}$

$$\int_{y=0}^{y=1}\int_{x=0}^{x=4} \frac{3(1+x^2)}{1+y^2}\,dx\,dy$$

$$\int_0^1 \left[\frac{3}{1+y^2}\int_0^4 (1+x^2)\,dx\right] dy$$

$$\int_0^1 \frac{3}{1+y^2}\left[x+\frac{1}{3}x^3\right]_0^4 dy$$

$$\int_0^1 \frac{3}{1+y^2}\left[76/3\right] dy$$

$$3\cdot\frac{76}{3}\int_0^1 \frac{1}{1+y^2}\,dy$$

$$76 \arctan \Big|_0^1 \qquad 76\arctan(1) - 76\arctan(0) = 19\pi$$

$\iint_R \frac{6x}{1+xy}\,dA$ $\quad R = [0,6]\times[0,1]$

$$\int_0^6\int_0^1 \frac{6x}{1+xy}\,dy\,dx$$

$$\int_0^6 \left[6x\int_0^1 \frac{1}{1+xy}\,dy\right] dx$$

$u = 1+xy$

$du = x\,dy$

$\frac{du}{x} = dy$

$$\int_0^6 6x\int_{u=1}^{1+x} \frac{1}{u}\,\frac{du}{x}$$

$$\int_0^6 6x\cdot\frac{1}{x}\int_{u=1}^{1+x} \frac{1}{u}\,du \quad dx$$

$$\int_0^6 6\left[\ln u\right]\Big|_{u=1}^{u=1+x} dx$$

$$6\int_0^6 \ln(1+x) - \ln(1)\,dx$$

$$6\int_0^6 \underbrace{\ln(1+x)\,dx}_{\text{Integration by parts}} \qquad 6\big((1+x)(\ln(1+x)-1)\big)\Big|_0^6$$

$$6\,(7\ln(7)-1)$$

$uv - \int v\,du$

$u = \ln t \quad dv = dt$

$du = \frac{1}{t}dt \quad v = t$

$t = (x+1)$

$$\int \ln(1+x)\,dx =$$

$$t\ln(t) - \int t\cdot\frac{1}{t}\,dt$$

$$t\ln t - t + C$$

$$(1+x)(\ln(1+x)-1) + C$$

15.2.7) $\int_0^1\int_0^1 7-x-2y\,dx\,dy$

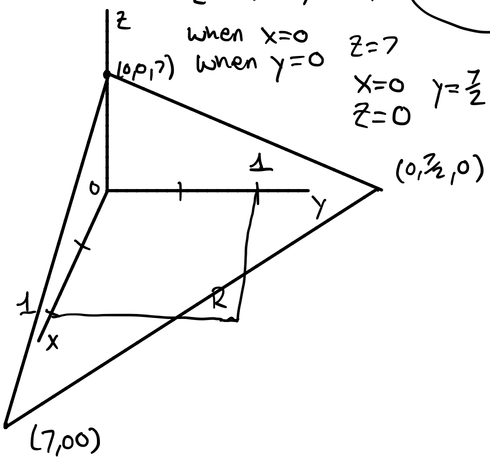
$z=7-x-2y\,dx\,dy$

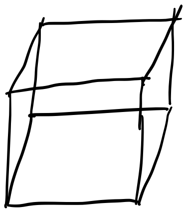
Likes this problem a lot

when $x=0$ $z=7$

when $y=0$ $x=0$ $y=\frac{7}{2}$

$z=0$





15.2.10) Find the volume enclosed by

$z=3+x^2+(y-2)^2$

planes $z=1$, $x=-2$, $x=2$, $y=0$, $y=2$

$$\iint_R z(x,y)\,dA$$

$$\int_0^2\int_{-2}^2 3+x^2+(y-2)^2-1\,dx\,dy$$

$$\int_0^2\left[3x+\frac{1}{3}x^3+x(y-2)^2-x\right]_{-2}^{2}dy$$

$$\int_0^2\left[2x+\frac{1}{3}x^3+x(y-2)^2\right]dy$$