1.) $\iiint_E \sqrt{x^2+y^2}\, dV$ E lies inside cylinder $x^2+y^2=4$
$z=1 \quad z=3$

$x^2+y^2=4$ circle radius 2

$0 \le r \le 2$
$0 \le \theta \le 2\pi$
$1 \le z \le 3$

$$\iiint \sqrt{x^2+y^2}\, dV$$

$$\iiint \sqrt{r^2}\, dV$$

$$\int_0^{2\pi}\int_1^3\int_0^2 r \quad r\, dr\, dz\, d\theta$$

$$\int_0^{2\pi}\int_1^3\int_0^2 r^2\, dr\, dz\, d\theta$$

$$\int_0^{2\pi}\int_1^3 \frac{1}{3}r^3 \Big|_0^2 dz\, d\theta$$

$$\int_0^{2\pi}\int_1^3 \frac{8}{3}\, dz\, d\theta$$

$$\int_0^{2\pi} \frac{8}{3}z \Big|_1^3 d\theta$$

$$\int_0^{2\pi} \frac{8}{3}(3) - \frac{8}{3}(1)\, d\theta$$

$$\int_0^{2\pi} 8 - \frac{8}{3}\, d\theta$$

$$\frac{24}{3} - \frac{8}{13} = \frac{16}{3}$$

$$\int_0^{2\pi} \frac{16}{3}\, d\theta$$

$$\frac{16}{3}\theta \Big|_0^{2\pi}$$

$$\frac{16(2\pi)}{3} - \frac{16}{3}(0)$$

$$\frac{32\pi}{3}$$

$$\iiint_E \sqrt{x^2+y^2}\, dV = \frac{32\pi}{3}$$

2.) $\iiint_E x\, dV$ E enclosed by planes $z=0$, $z=x+y+9$ Cylinders $x^2+y^2=16$ $x^2+y^2=25$

$x=r\cos\theta$ $r=4$ $r=5$
$y=r\sin\theta$

$z=r\cos\theta+r\sin\theta+9$

$4 \le r \le 5$
$0 \le \theta \le 2\pi$
$0 \le z \le r\cos\theta + r\sin\theta + 9$

$$\iiint_E x\, dV$$

$$\int_0^{2\pi}\int_4^5\int_0^{r\cos\theta+r\sin\theta+9} r\cos\theta \quad r\, dz\, dr\, d\theta$$

$$\iiint x\, dV = \frac{369\pi}{4}$$

$$\int_0^{2\pi}\int_4^5 r^2\cos\theta[z]\Big|_0^{r\cos\theta+r\sin\theta+9} dr\, d\theta$$

$$\int_0^{2\pi}\int_4^5 r^2\cos\theta[r\cos\theta+r\sin\theta+9]\, dr\, d\theta$$

$\cos^2\theta = \frac{1}{2}+\frac{1}{2}\cos 2\theta$

$$\int_0^{2\pi}\int_4^5 r^3\cos^2\theta + r^3\cos\theta\sin\theta + 9r^2\cos\theta\, dr\, d\theta$$

$$\int_0^{2\pi} \frac{r^4}{4}\cos^2\theta + \frac{r^4}{4}\cos\theta\sin\theta + 3r^2\cos\theta \Big|_4^5 d\theta$$

$$\int_0^{2\pi} \frac{625}{4}\cos^2\theta + \frac{625}{4}\cos\theta\sin\theta + 75\cos\theta - (64\cos^2\theta + 64\cos\theta\sin\theta + 48\cos\theta)\, d\theta$$

$$\int_0^{2\pi} \frac{369}{4}\cos^2\theta + \frac{369}{4}\cos\theta\sin\theta + 27\cos\theta\, d\theta$$

$u=2\theta$
$du=2\,d\theta$
$\frac{1}{2}du=d\theta$
$\int \frac{1}{2}\cos 2\theta\, d\theta$

$$\int_0^{2\pi} \frac{369}{4}\cos^2\theta\, d\theta + \int_0^{2\pi}\frac{369}{4}\cos\theta\sin\theta\, d\theta + \int_0^{2\pi} 27\cos\theta\, d\theta$$

$\frac{1}{4}\sin 2\theta$
$\frac{1}{2}\cos 2\theta$

$$\frac{369}{4}\int_0^{2\pi} \frac{1}{2}+\frac{1}{2}\cos 2\theta\, d\theta + \frac{369}{4}\int_0^{2\pi}\cos\theta\sin\theta\, d\theta + 27\int_0^{2\pi}\cos\theta\, d\theta$$

$$\frac{369}{4}\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{2\pi} \quad \frac{369}{4\cos\theta}\int_0^{2\pi} u\, du \quad 27\sin\theta\Big|_0^{2\pi}$$

$u=\sin\theta$
$du=\cos\theta\, d\theta$
$\frac{1}{\cos\theta}du=d\theta$

$$\frac{369}{4}\left[\pi + \frac{1}{4}\sin(2\pi)\right] \quad \frac{369}{4\cos\theta}\left[\frac{1}{2}\sin\theta^2\right]_0^{2\pi} \quad 0$$

$$\frac{369}{4}[\pi] \quad 0$$

3.) $\iiint_E x^2\,dv$ cylinder $x^2+y^2=9$ plane $z=0$ cone $z^2=4x^2+4y^2$

$0\le r\le 3$
$0\le\theta\le 2\pi$

$x=r\cos\theta$
$y=r\sin\theta$

$4(r^2\cos^2\theta+r^2\sin^2\theta)$
$z^2=4(r^2)$
$z=2r$

$$\int_0^{2\pi}\int_0^3\int_0^{2r} x^2\,dv$$
$$\int_0^{2\pi}\int_0^3\int_0^{2r} r^2\cos^2\theta\ r\,dz\,dr\,d\theta$$
$$\int_0^{2\pi}\int_0^3 r^3\cos^2\theta\,[z]\Big|_0^{2r}\,dr\,d\theta$$
$$\int_0^{2\pi}\int_0^3 r^3\cos^2\theta\,[2r]\,dr\,d\theta$$
$$\int_0^{2\pi}\int_0^3 2r^4\cos^2\theta\,dr\,d\theta$$
$$\int_0^{2\pi}\frac{2r^5}{5}\cos^2\theta\Big|_0^3\,d\theta \qquad \cos^2\theta=\tfrac12+\tfrac12\cos2\theta$$
$$\int_0^{2\pi}\frac{486}{5}\cos^2\theta\,d\theta$$
$$\frac{486}{5}\int_0^{2\pi}\cos^2\theta\,d\theta$$
$$\frac{486}{5}\int_0^{2\pi}\tfrac12+\tfrac12\cos2\theta\,d\theta$$
$$\frac{486}{5}\left[\tfrac12\theta+\tfrac14\sin2\theta\right]_0^{2\pi}$$
$$\frac{486}{5}[\pi+0]$$
$$\frac{486\pi}{5}$$

$$\iiint_E x^2\,dv=\frac{486\pi}{5}$$

4.) $\displaystyle\int_{-7}^{7}\int_{-\sqrt{49-y^2}}^{\sqrt{49-y^2}}\int_{\sqrt{x^2+y^2}}^{8} xz\ dz\,dx\,dy$

$-7\le y\le 7$
$-\sqrt{49-y^2}\le x\le\sqrt{49-y^2}$
$\sqrt{x^2+y^2}\le z\le 8$

$x^2+y^2=64$
$r=8$

$x=r\cos\theta$
$y=r\sin\theta$

$\displaystyle\int_{\sqrt{x^2+y^2}}^{8}$

$r\le z\le 8$
$0\le r\le 8$
$0\le\theta\le 2\pi$

$$\int_0^{2\pi}\int_0^8\int_r^8 xz\ dz\,dx\,dy$$
$$\int_0^{2\pi}\int_0^8\int_r^8 r\cos\theta(z)\ r\ dz\,dr\,d\theta$$
$$\int_0^{2\pi}\int_0^8\int_r^8 r^2\cos\theta(z)\ dz\,dr\,d\theta$$
$$\int_0^{2\pi}\int_0^8 r^2\cos\theta\ \frac{z^2}{2}\Big|_r^8$$
$$\int_0^{2\pi}\int_0^8 r^2\cos\theta\,32-r^2\cos\theta\,\frac{r^2}{2}\ dr\,d\theta$$
$$\int_0^{2\pi}\int_0^8 32r^2\cos\theta-\frac{r^4}{2}\cos\theta\ dr\,d\theta$$
$$\int_0^{2\pi}\left[\frac{32r^3}{3}\cos\theta-\frac{r^5}{10}\cos\theta\right]_0^8 d\theta$$
$$\int_0^{2\pi}\left[\frac{16384}{3}\cos\theta-\frac{16384}{3}\cos\theta\right]d\theta$$
$$\int_0^{2\pi}0\ d\theta=0$$

$$\int_{-7}^{7}\int_{-\sqrt{49-y^2}}^{\sqrt{49-y^2}}\int_{\sqrt{x^2+y^2}}^{8} xz\ dz\,dx\,dy=0$$

5.) $\displaystyle\int_{-4}^{4}\int_0^{\sqrt{16-x^2}}\int_0^{16-x^2-y^2}\sqrt{x^2+y^2}\ dz\,dy\,dx$

$x^2+y^2=r^2$
$\sqrt{x^2+y^2}=r$
$0\le r\le 4$
$0\le\theta\le 2\pi$
$0\le z\le 16-r^2$

$z=0$
$z=16-x^2-y^2$
$0=16-x^2-y^2$
$16=x^2+y^2$
$r=4$

$$\int_0^{\pi}\int_0^4\int_0^{16-r^2} r\ r\ dz\,dr\,d\theta$$
$$\int_0^{\pi}\int_0^4\int_0^{16-r^2} r^2\ dz\,dr\,d\theta$$
$$\int_0^{\pi}\int_0^4 r^2z\Big|_0^{16-r^2}dr\,d\theta$$
$$\int_0^{\pi}\int_0^4 r^2(16-r^2)\ dr\,d\theta$$
$$\int_0^{\pi}\int_0^4 -r^4+16r^2\ dr\,d\theta$$
$$\int_0^{\pi} -\frac{r^5}{5}+\frac{16r^3}{3}\Big|_0^4 d\theta$$
$$\int_0^{\pi}\frac{2048}{15}\ d\theta$$
$$\frac{2048}{15}[\theta]_0^{\pi}$$
$$\frac{2048}{15}\pi$$

$\frac{2048\pi}{15}$