

Physics 230

Exam 2

1. A racer attempting to break the land speed record rockets by two markers spaced 100. m apart on the ground in a time of 4.0×10^{-7} s, as measured by an observer on the ground.
 - a) What are the two relevant events in this problem?
 - b) What is the speed of the racer, as measured by the ground observers? Write this speed as a fraction of c ?
 - c) How far apart are the two markers according to the racer?
 - d) What elapsed time does the racer measure?

2. A racer attempting to break the land speed record rockets by two markers spaced 100. m apart on the ground. He does this in a time of 4.0×10^{-7} s as measured by his watch. Calculate the speed of the racer, as measured by the ground observers? Write this speed as a fraction of c ?

3. A hungry lion chases a gazelle. According to observers on the ground, the lion moves to the right at $(4/5)c$ and the gazelle also moves to the right at a speed $(3/5)c$.
 - a) How fast, and in what direction, is the lion moving in the rest frame of the gazelle?
 - b) How fast, and in what direction, is the gazelle moving in the rest frame of lion?Write these speeds as fractions of c .

4. Consider two inertial reference frames S and S' , where S' moves in the $+x$ direction at a speed of $(12/13)c$ relative to the S frame. The origins of S and S' coincide at $t = t' = 0$. Later, an event occurs at $x = 75$ m at time $t = 2.0 \times 10^{-6}$ s, as measured by the S frame. What are the spacetime coordinates of this event according to the S' frame?

P1

a.)

E_1 +25/25

When the racer passes the first marker. ✓

+63/100

E_2

When the racer passes the second marker. ✓

b.)

Rest length of markers is 100 m

time measured by observers is 4.0×10^{-7} s

$D = 100 \text{ m}$

$t = 4.0 \times 10^{-7} \text{ s}$

$v = \frac{D}{t} = \frac{100 \text{ m}}{4.0 \times 10^{-7} \text{ s}} = 2.5 \times 10^8 \text{ m/s}$ $D = v \cdot t$

$v = 2.5 \times 10^8 \text{ m/s}$
 $c = 3.0 \times 10^8 \text{ m/s}$

$\frac{v}{c} = \frac{2.5 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = \frac{5}{6} c$

$v = \frac{5}{6} c$

c.)

since $\Delta x' = 0$

$d = D \sqrt{1 - (v/c)^2}$ ✓

$\sqrt{\frac{36-25}{36}} = \frac{\sqrt{11}}{6}$ ✓

$D = 100 \text{ m}$

$\therefore d = 100 \text{ m} \sqrt{1 - (\frac{5}{6})^2} = 100 \text{ m} (\frac{\sqrt{11}}{6}) = 55.2 \text{ m} \approx 55 \text{ m}$

$\frac{v}{c} = (\frac{5}{6}) c$

$d = 55 \text{ m}$

d.)

Racecar driver will measure shorter t since moving fast

$t' = \gamma(t)$

$t' = (4.0 \times 10^{-7} \text{ s}) (\frac{\sqrt{11}}{6}) = 2.2 \times 10^{-7} \text{ s} \approx 2.2 \times 10^{-7} \text{ s}$

$\gamma = (\frac{\sqrt{11}}{6})$

$t = 4.0 \times 10^{-7} \text{ s}$

$\Delta t' = 2.2 \times 10^{-7} \text{ s}$

P2

$$\Delta t' = 4.0 \times 10^{-7} \text{ s}$$

$$D_r = 100 \text{ m}$$

$$V = \frac{D}{t} \quad +4/25$$

racer will see a contracted distance of

$$d = D \sqrt{1 - v^2/c^2} = 100 \text{ m} \left(\frac{\sqrt{11}}{5} \right) = 55 \text{ m}$$

← ~~not~~ v \rightarrow V

he will see his velocity as

$$V = \frac{d}{t} = \frac{55 \text{ m}}{4.0 \times 10^{-7} \text{ s}} = 1.375 \times 10^8 \text{ m/s}$$

$$\frac{V}{c} = \frac{1.375 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = \left(\frac{11}{24} \right) c$$

Since the observers are at rest

$$v_{x'} = \left(\frac{11}{24} \right) c$$

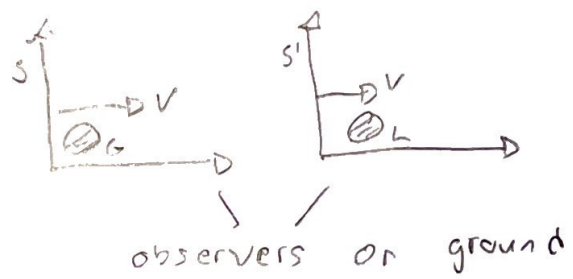
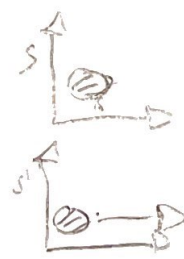
P3

$$V_L = (4/5)c \quad V_G = (3/5)c$$

a.) V_L relative to V_G

$$V = (3/5)c \quad \text{X-2}$$

$$V_{x'} = (4/5)c \quad \checkmark$$



$$V_x = \frac{V_{x'} + V}{1 + \frac{V \cdot V_{x'}}{c^2}} = \frac{((4/5)c + (3/5)c)}{1 + \frac{(4/5)c(3/5)c}{c^2}} = \frac{(7/5)c}{1 + \frac{12}{25}} = \frac{(7/5)c}{(32/25)} = \left(\frac{35}{32}\right)c$$

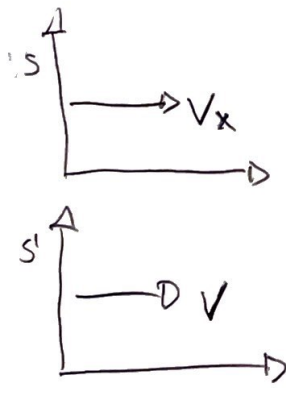
+13/25

$$V_x = \left(\frac{35/25}{32/25}\right) = \left(\frac{35}{32}\right)c$$

$$V_x = \left(\frac{35}{37}\right)c \quad \text{X-3}$$

To right

b.)



$$V = (4/5)c \quad \text{X-2}$$

$$V_x = (3/5)c \quad \checkmark$$

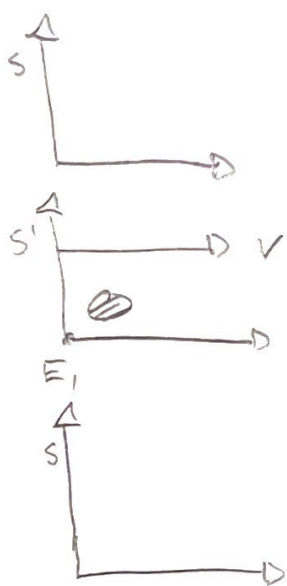
$$V_{x'} = \frac{V_x + V}{1 + \frac{V \cdot V_x}{c^2}}$$

$$V_x = \frac{(3/5)c + (4/5)c}{1 + \frac{(3/5)c(4/5)c}{c^2}} = \frac{(7/5)c}{1 + \frac{12}{25}} = \frac{(7/5)c}{(32/25)} = \left(\frac{35}{32}\right)c$$

$$V_x = \left(\frac{35}{37}\right)c \quad \text{X-3}$$

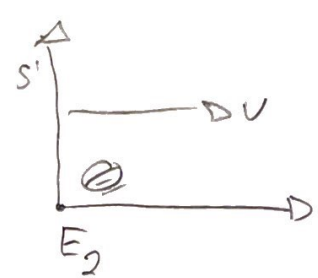
To right X-2

P4



$$v = (12/13)c$$

$$\frac{+19}{25}$$



s' sees both events happen at origin
 $\Delta x' = 0$

$$t = \gamma(t')$$

$$t' = \gamma(t)$$

$$x_2' \text{ \& } t_2' ?$$

$$v = (12/13)c \quad \gamma = 13/5 \quad \gamma^{-1} = (5/13)$$

$$x_2 = 75 \text{ m}$$

$$t_2 = 2.0 \times 10^{-6} \text{ s}$$

$$t' = \gamma(t - vx/c^2) \checkmark$$

$$x' = \gamma(x - vt)$$

$$\frac{900}{13} \text{ m/c} \Rightarrow 2.3 \times 10^{-7} \text{ s}$$

$$t_2' = \frac{13}{5} \left(2.0 \times 10^{-6} \text{ s} - \frac{(12/13)c(75) \text{ m}}{c^2} \right)$$

$$= (13/5) \left(2.0 \times 10^{-6} \text{ s} - \frac{900}{13} \text{ m/c} \right)$$

$$t_2' = (13/5) (2.0 \times 10^{-6} \text{ s} - 2.3 \times 10^{-7} \text{ s}) = 4.6 \times 10^{-6} \text{ s} \checkmark \quad t_2' = 4.6 \times 10^{-6} \text{ s}$$

$$x_2' = \gamma(x_2 - vt_2) \checkmark$$

$$x_2 = 75 \text{ m}$$

$$v = (12/13)c$$

$$t_2 = 2.0 \times 10^{-6} \text{ s}$$

$$\gamma = (13/5)$$

$$x_2' = (13/5) (75 \text{ m} - (12/13)c(2.0 \times 10^{-6} \text{ s}))$$

$$= (13/5) (75)$$

$$x_2' = 195 \text{ m}$$

$$t_2' = 4.6 \times 10^{-6} \text{ s}$$

$$x_2' = 195 \text{ m} \text{ X-U}$$

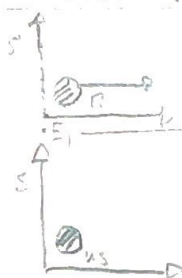
$$L = \gamma L_0 = \gamma (L_0 \sqrt{1 - v^2/c^2})$$

$$4.24 \times 10^{-7} \text{ s} =$$

$$6.0 \times 10^{-8} \text{ s}$$

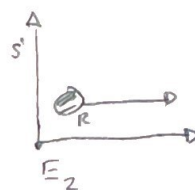
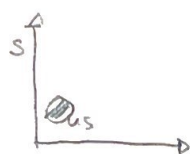
Exam 2

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$$+63 + \frac{3.1}{5} = 70.4\%$$

F0



100 m

a) when the racer (Ed) passes marker 1, when Ed passes marker 2.

b) $D = 100 \text{ m}$
 $\Delta t = 4.0 \times 10^{-7} \text{ s}$

$$v = \frac{D}{\Delta t} = \frac{100 \text{ m}}{4.0 \times 10^{-7} \text{ s}} = 2.5 \times 10^8 \text{ m/s}$$

$$c = 3.0 \times 10^8 \text{ m/s} \quad \frac{v}{c} = \frac{2.5 \times 10^8 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = \frac{5}{6}$$

$$v = \left(\frac{5}{6}\right)c$$

c) Since $\Delta x' = 0$

$$d = D \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} = 100 \text{ m} \sqrt{1 - \left(\frac{5}{6}\right)^2} = 100 \text{ m} \left(\frac{\sqrt{11}}{\sqrt{36}}\right) = 100 \text{ m} \left(\frac{\sqrt{11}}{6}\right) = 55 \text{ m}$$

$$D = 100 \text{ m}$$

$$d = 55 \text{ m}$$

d) Since $\Delta x' = 0$

$$t = \gamma(t') \therefore t' = t \gamma^{-1}$$

$$t = 4.0 \times 10^{-7} \text{ s}$$

$$\gamma = \frac{\sqrt{11}}{6}$$

$$\gamma = \frac{6\sqrt{11}}{11}$$

$$\Delta t' = \frac{\sqrt{11}}{6} (4.0 \times 10^{-7} \text{ s}) = 2.2 \times 10^{-7} \text{ s}$$

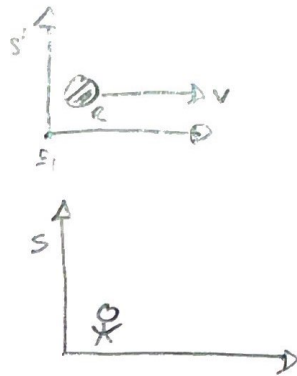
$$\Delta t' = 2.2 \times 10^{-7} \text{ s}$$

Exam 2

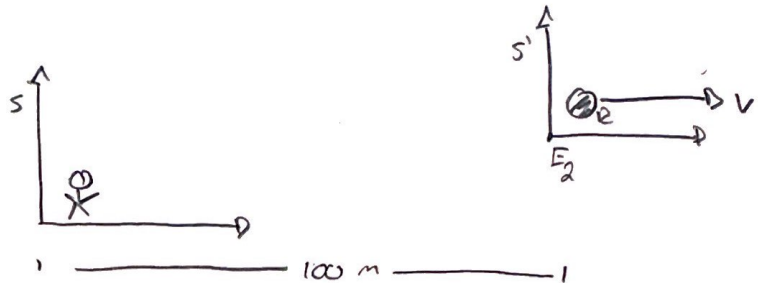
Problem 6

E1

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E2



$$\Delta x = 100 \text{ m}$$

$$\Delta t' = 4.0 \times 10^{-7} \text{ s}$$

$$\Delta x' = 0$$

∞ observers in S' $\therefore \Delta t' = 0$ $\therefore x = \gamma x'$ $\therefore x' = \gamma^{-1}(x)$

$$x = \gamma(vt')$$

$$\Delta x = \frac{v(\Delta t')}{\sqrt{1-v^2/c^2}}$$

$$\Delta x(\sqrt{1-v^2/c^2}) = v(\Delta t') \quad \checkmark$$

$$(\Delta x)^2(1-v^2/c^2) = v^2(\Delta t')^2 \quad \checkmark$$

$$\Delta x^2 - \Delta x^2(v^2/c^2) = v^2(\Delta t')^2 \quad \checkmark$$

$$\Delta x^2 = v^2(\Delta t')^2 + \Delta x^2(v^2/c^2) \quad \checkmark$$

$$\frac{\Delta x^2}{c^2} = \frac{v^2}{c^2} \Delta t'^2 + \Delta x^2 \frac{v^2}{c^4} \quad \checkmark$$

$$\frac{\Delta x^2}{c^2} = \frac{v^2}{c^2} \left(\Delta t'^2 + \frac{\Delta x^2}{c^2} \right) \quad \checkmark$$

$$\frac{v^2}{c^2} = \frac{\frac{\Delta x^2}{c^2}}{\Delta t'^2 + \frac{\Delta x^2}{c^2}} \quad \checkmark$$

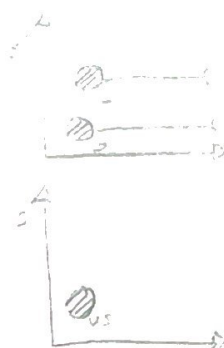
$$\frac{v}{c} = \sqrt{\frac{\Delta x^2}{c^2 \Delta t'^2 + \Delta x^2}} \quad \checkmark$$

$$\frac{v}{c} = \sqrt{\frac{(100 \text{ m})^2}{(3.00 \times 10^8 \text{ m/s})^2 (4.0 \times 10^{-7} \text{ s})^2 + (100 \text{ m})^2}}$$

$$\frac{v}{c} = 0.64 \quad \checkmark$$

$$V_x = \frac{V_x' + V}{1 + \frac{V V_x'}{c^2}}$$

$$V_x' = \frac{V_x - V}{1 - \frac{V V_x}{c^2}}$$



$$V_L = \left(\frac{4}{5}\right)c \quad V_B = \left(\frac{3}{5}\right)c$$

a.) RF of Gazelle...

$$V = \left(\frac{3}{5}\right)c$$

$$V_x = \left(\frac{4}{5}\right)c$$

$$V_x' = \frac{\left(\frac{4}{5}\right)c - \left(\frac{3}{5}\right)c}{1 - \frac{\left(\frac{4}{5}\right)c \left(\frac{3}{5}\right)c}{c^2}} = \frac{\left(\frac{1}{5}\right)c}{\frac{13}{25}} = \frac{\frac{5}{25}}{\frac{13}{25}} = \frac{5}{13}$$

$$V_x' = \left(\frac{5}{13}\right)c$$

Rightward

b.) RF of Lion...

$$V = \left(\frac{4}{5}\right)c$$

$$V_x = \left(\frac{3}{5}\right)c$$

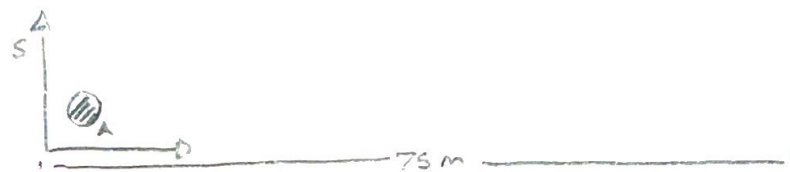
$$V_x' = \frac{\left(\frac{3}{5}\right)c - \left(\frac{4}{5}\right)c}{1 - \frac{\left(\frac{3}{5}\right)c \left(\frac{4}{5}\right)c}{c^2}} = \frac{-\left(\frac{1}{5}\right)c}{\left(\frac{13}{25}\right)} = \frac{-\left(\frac{5}{25}\right)c}{\left(\frac{13}{25}\right)} = \left(-\frac{5}{13}\right)c$$

$$V_x' = \left(-\frac{5}{13}\right)c$$

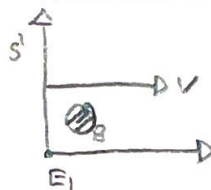
Leftward

Exam 2 Problem 4

E₁



$$V = \left(\frac{12}{13}\right)c$$



E₂



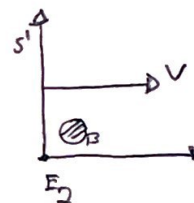
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$$\begin{array}{llll} x_1 = 0 & t_1 = 0 & x_2 = 75 \text{ m} & t_2 = 2.0 \times 10^{-6} \text{ s} \\ x'_1 = 0 & t'_1 = 0 & x'_2 = & t'_2 = 4.6 \times 10^{-6} \text{ s} \end{array}$$

$$V = \left(\frac{12}{13}\right)c \quad \gamma = \left(\frac{13}{5}\right)$$

$$t' = \gamma(t - Vx/c^2) \quad t = \gamma(t' + Vx'/c^2)$$

$$x' = \gamma(x - Vt) \quad x = \gamma(x' + Vt')$$



$$\begin{aligned} t'_2 &= \left(\frac{13}{5}\right) \left(2.0 \times 10^{-6} \text{ s} - \frac{\left(\frac{12}{13}\right)c(75 \text{ m})}{c^2} \right) & x'_2 &= \left(\frac{13}{5}\right) \left(75 \text{ m} - \left(\frac{12}{13}\right)c(2.0 \times 10^{-6} \text{ s}) \right) \\ &= \left(\frac{13}{5}\right) \left(2.0 \times 10^{-6} \text{ s} - \frac{900}{13} \text{ m/c} \right) & &= \left(\frac{13}{5}\right) (75 \text{ m} - 1.84 \times 10^{-6} \text{ c s}) \\ &= \left(\frac{13}{5}\right) (1.769 \times 10^{-6} \text{ s}) & &= \left(\frac{13}{5}\right) (75 \text{ m} - 554 \text{ m}) \\ t'_2 &= 4.6 \times 10^{-6} \text{ s} & x'_2 &= -1245.4 \text{ m} \end{aligned}$$

$$\boxed{\begin{array}{l} t'_2 = 4.6 \times 10^{-6} \text{ s} \\ x'_2 = -1200 \text{ m} \end{array}}$$

✓