2016 Supp Ex 77

a) $E_i = K_i + U_{gi}$

$= \underbrace{K_{trans\,i}}_{\text{suspended}} + \underbrace{K_{rot\,i}}_{\text{wheel}} + \underbrace{U_{gi}}_{\text{suspended}}$

$= \cancel{\frac{1}{2} m_{susp} V_{susp\,i}^2} + \cancel{\frac{1}{2} I_{wheel} \omega_{wheel\,i}^2} + m_{susp}\, g y_i$

$= 5.0\text{kg} \times 9.8\,\text{m/s}^2 \times 2.0\text{m}$

$E_i = 98\text{J}$

b) Kinetic only $\rightsquigarrow$ rotational for wheel

$\rightsquigarrow$ translational for block

$E = K_{trans\,f} + K_{rot\,f}$

$= \frac{1}{2} M_{block} V_i^2 + \frac{1}{2} I_{wheel}\, \omega^2$

c) But the speed of the block is the same as the string, which is the same as the edge of the wheel, so

$v = \omega r$ — radius wheel.

$\Rightarrow \quad E = \frac{1}{2} M_{block} (\omega r)^2 + \frac{1}{2} I_{wheel}\, \omega^2$

$= \frac{1}{2} M_{block}\, r^2 \omega^2 + \frac{1}{2} I_{wheel}\, \omega^2$

*

2016 Supp 77

So

$$98\,J = \frac{1}{2}\,5.0kg\,(0.40m)^2\,\omega^2 + \frac{1}{2}\,I_{wheel}\,\omega^2$$

But $I_{wheel} = \frac{1}{2} M_{wheel}\, r_{wheel}^2$

$$= \frac{1}{2}\,10kg \times (0.40m)^2$$

$$= 0.80\,kg\,m^2$$

$$\Rightarrow \quad 98\,J = \frac{1}{2}\,5.0kg \times (0.40m)^2\,\omega^2 + \frac{1}{2}\,0.80kgm^2\,\omega^2$$

$$= 0.40\,kgm^2\,\omega^2 + 0.40kgm^2\,\omega^2$$

$$= 0.80\,kgm^2\,\omega^2$$

$$\Rightarrow \omega^2 = 120\,rad^2/s^2 \quad \Rightarrow \quad \omega = \sqrt{120\,rad^2/s^2} = 11\,rad/s.$$

d) $v = \omega r = 11\,rad/s \times 0.40m$

$v = 4.4\,m/s$

Phys 131 2016

Supp Ex 78

a) $\vec{A}, \vec{B}$ are same $\Rightarrow$ $\vec{A}\times\vec{B} = 0$

b) $$\vec{A}\times\vec{B} = (2\hat{\imath}+2\hat{\jmath})\times(2\hat{\imath}-2\hat{\jmath}) = \overset{0}{\cancel{4\hat{\imath}\times\hat{\imath}}} - 4\overbrace{\hat{\imath}\times\hat{\jmath}}^{\hat{k}} + 4\underbrace{\hat{\jmath}\times\hat{\imath}}_{-\hat{k}} - \underset{0}{\cancel{4\hat{\jmath}\times\hat{\jmath}}} = -8\hat{k}$$

c) Reverse of part c) $$(2\hat{\imath}-2\hat{\jmath})\times(2\hat{\imath}+2\hat{\jmath}) = \underset{0}{\cancel{2\hat{\imath}\times2\hat{\imath}}} + 2\underbrace{\hat{\imath}\times\hat{\jmath}}_{\hat{k}} - 2\underbrace{\hat{\jmath}\times\hat{\imath}}_{-\hat{k}}2 - \underset{0}{\cancel{4\hat{\jmath}\times\hat{\jmath}}} = 8\hat{k}$$

d) $\vec{A} = \vec{B}$ $\Rightarrow$ $\vec{A}\times\vec{B} = 0$

e) $$\vec{A}\times\vec{B} = (2\hat{\imath}+2\hat{\jmath}+3\hat{k})\times(2\hat{\imath}+2\hat{\jmath}) = \cancel{2\hat{\imath}\times2\hat{\imath}} + 2\hat{\imath}\times2\hat{\jmath}$$
$$+ 2\hat{\jmath}\times2\hat{\imath} + \cancel{2\hat{\jmath}\times2\hat{\jmath}}$$
$$+ 3\hat{k}\times2\hat{\imath} + 3\hat{k}\times2\hat{\jmath}$$
$$= \cancel{4\hat{k} - 4\hat{k}} + 6\hat{\jmath} - 6\hat{\imath} = -6\hat{\imath} + 6\hat{\jmath}$$

f) $$\vec{A}\times\vec{B} = (2\hat{\imath}+2\hat{\jmath}+3\hat{k})\times(2\hat{\imath}-2\hat{\jmath}) = \cancel{2\hat{\imath}\times2\hat{\imath}} - 2\hat{\imath}\times2\hat{\jmath} + 2\hat{\jmath}\times2\hat{\imath} - \cancel{2\hat{\jmath}\times2\hat{\jmath}} + 3\hat{k}\times2\hat{\imath} - 3\hat{k}\times2\hat{\jmath}$$
$$= \cancel{-4\hat{k}} \cancel{+4\hat{k}} + 6\hat{\jmath} + 6\hat{\imath} = 6\hat{\jmath} + 6\hat{\imath}$$

g) $$\vec{A}\times\vec{B} = (2\hat{\imath}-2\hat{\jmath}+3\hat{k})\times(2\hat{\imath}+2\hat{\jmath}+3\hat{k})$$
$$= 2\hat{\imath}\times2\hat{\jmath} + 6\hat{\imath}\times\hat{k} - 4\hat{\jmath}\times\hat{\imath} - 6\hat{\jmath}\times\hat{k} + 6\hat{k}\times\hat{\imath} + 6\hat{k}\times\hat{\jmath}$$
$$= 4\hat{k} - \cancel{6\hat{\jmath}} + 4\hat{k} - 6\hat{\imath} + \cancel{6\hat{\jmath}} - 6\hat{\imath} = 8\hat{k} - 12\hat{\imath}$$

h) $(\hat{i} - 2\hat{j} + 3\hat{k}) \times (2\hat{i} + \hat{j} - 3\hat{k})$

$= \hat{i}\times\hat{j} - 3\hat{i}\times\hat{k} - 4\hat{j}\times\hat{i} + 6\hat{j}\times\hat{k} + 6\hat{k}\times\hat{i} + 3\hat{k}\times\hat{j}$

$= \hat{k} + 3\hat{j} + 8\hat{k} + 6\hat{i} + 6\hat{j} - 3\hat{i}$

$= 3\hat{i} + 9\hat{j} + 9\hat{k}$

a) $\vec{A} = 4\hat{\imath} + \hat{\jmath}$

$\vec{B} = \hat{\imath} + 2\hat{\jmath}$

$\vec{A} \times \vec{B} = (4\hat{\imath} + \hat{\jmath}) \times (\hat{\imath} + 2\hat{\jmath})$

$= 8\,\hat{\imath} \times \hat{\jmath} + \hat{\jmath} \times \hat{\imath}$

$= 8\hat{k} - \hat{k} = 7\hat{k}$

$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = -7\hat{k}$

b) $\vec{A} = 4\hat{\imath} + \hat{\jmath}$

$\vec{B} = 2\hat{\imath} + \hat{\jmath}$

$\vec{A} \times \vec{B} = (4\hat{\imath} + \hat{\jmath}) \times (2\hat{\imath} + \hat{\jmath})$

$= 4\,\hat{\imath} \times \hat{\jmath} + 2\hat{\jmath} \times \hat{\imath} = 2\hat{k}$

$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = -2\hat{k}$

c) $\vec{B} = -2\hat{\imath} + \hat{\jmath}$

$\vec{A} \times \vec{B} = (4\hat{\imath} + \hat{\jmath}) \times (-2\hat{\imath} + \hat{\jmath})$

$= -2\,\hat{\jmath} \times \hat{\imath} + 4\hat{\imath} \times \hat{\jmath} = 6\,\hat{k}$

$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = -6\hat{k}$

d) $\vec{A} = -4\hat{\imath} + \hat{\jmath}$

$\vec{B} = -\hat{\imath} + 2\hat{\jmath}$

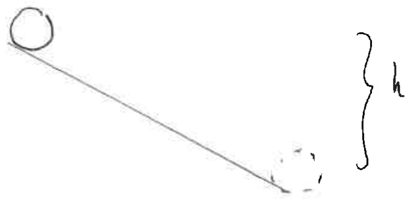
$\vec{A} \times \vec{B} = (-4\hat{\imath} + \hat{\jmath}) \times (-\hat{\imath} + 2\hat{\jmath})$

$= -8\,\hat{\imath} \times \hat{\jmath} - \hat{\jmath} \times \hat{\imath} = -7\hat{k}$

$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = 7\hat{k}$

Knight Ch 12
4ed Conc Q 7

Roll them down the same plane, starting from the same height. Then energy conservation implies:



$$E_f = E_i$$

$$K_{trans\,f} + K_{rot\,f} = U_{g\,i}$$

$$\frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 = M g y_i$$

$$\omega_f = v_f / R \quad \Rightarrow \quad \frac{1}{2}\Big(M + \underbrace{\frac{I}{R^2}}_{\text{larger}}\Big) \underbrace{v_f^2}_{\text{smaller}} = \underbrace{M g y_i}_{\text{same}}$$

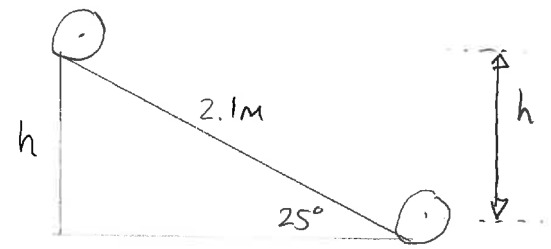
The object with the larger moment of inertia will travel slower. The hollow sphere has the larger inertia and moves more slowly – it will arrive later. Thus:

1) roll both from rest at the same height
2) the hollow sphere arrives later than the solid sphere.

Knight Ch12

4ed Prob 35

a)



$y_i = h \qquad y_f = 0$

$v_i = 0\,m/s \qquad v_f = ?$

$\omega_i = 0\,rad/s \qquad \omega_f = ?$

Energy conserved. $\quad E_f = E_i$

$$K_f + U_{gf} = K_i + U_{gi}$$

$$K_{trans\,f} + K_{rot\,f} + \underbrace{\cancel{U_{gf}}}_{0} = \underbrace{\cancel{K_{trans\,i}}}_{0} + \underbrace{\cancel{K_{rot}}\,i}_{0} + U_{gi}$$

$$\tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega_f^2 = m g y_i$$

But $I = \frac{2}{5} m r^2$ gives:

$$\tfrac{1}{2} \cancel{m} v_f^2 + \tfrac{1}{2}\,\tfrac{2}{5} \cancel{m} r^2 \omega_f^2 = \cancel{m} g h$$

and $\quad v_f = \omega_f r \quad$ gives:

$$\tfrac{1}{2} \omega_f^2 r^2 + \tfrac{1}{5} r^2 \omega_f^2 = gh$$

$$\Rightarrow \left(\tfrac{1}{2} + \tfrac{1}{5}\right) r^2 \omega_f^2 = gh$$

By trigonometry $\quad h/2.1m = \sin 25° \quad \Rightarrow h = 2.1m \sin 25° = 0.89m$

Thus $\quad \left(\frac{5+2}{10}\right)(0.040m)^2 \omega_f^2 = (9.8m/s^2) \times 0.89m$

$$\Rightarrow \omega_f^2 = 7\,800\,rad^2/s^2 \Rightarrow \omega_f = 88\,rad/s$$

b) From above $\quad K_{trans} = \frac{1}{2} m(\omega_f r)^2 \quad$ and $\quad K_{rot} = \frac{1}{2}\frac{2}{5} m r^2 \omega_f^2$

$$\Rightarrow K = \left(\frac{1}{2}+\frac{1}{5}\right) m \omega_f^2 r^2 . \quad \text{So} \quad \frac{K_{rot}}{K} = \frac{\frac{1}{5} \cancel{M} r^2 \omega_f^2}{\frac{7}{10} \cancel{M} \omega_f^2 \cancel{r^2}} = \frac{2}{7}$$

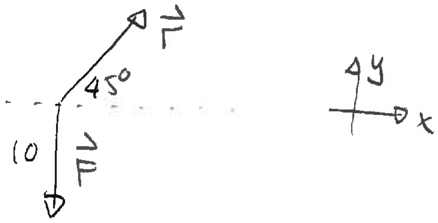
$$= \frac{7}{10} m \omega_f^2 r^2$$

Knight Ch 12

4^{ed} Prob 40

~~Prob 42~~ $\vec{\tau} = \vec{r} \times \vec{F}$

direction into page $\Rightarrow$ along $-\hat{z}$



$\tau = rF\sin\phi$

$= 5\sqrt{2}\ \text{m} \times 10\ \text{N} \sin 135°$

$= 50\ \text{N}\cdot\text{m}$

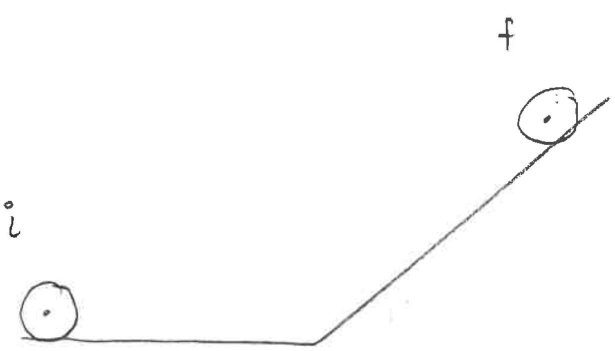
$r = \sqrt{5^2+5^2} = \sqrt{2}\,5$

$\Rightarrow \vec{\tau} = -50\ \text{N}\cdot\text{m}\ \hat{k}$

Knight Ch 12*

~~Prob 33~~

4ed Prob 69



$\omega_i = V_i R$

$V_i = 5.0 \, m/s$

$y_i = 0$

$\omega f = 0$

$V f = 0$

$y f = ?$

Energy conserved

$$E_i = E_f$$

$$K_{rot\,i} + K_{trans\,i} + \cancel{U_{gi}}^{0} = \cancel{K_{rot\,f}} + \cancel{K_{trans\,f}} + U_{gf}$$

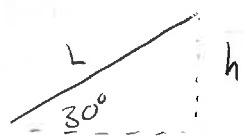
$$\frac{1}{2} M V_i^2 + \frac{1}{2} I \omega_i^2 = m g y_f$$

But $I = \frac{2}{3} m R^2$

$$\Rightarrow \frac{1}{2} M V_i^2 + \frac{1}{3} m \underbrace{R^2 \omega_i^2}_{V_i^2} = m g y_f$$

$$\Rightarrow \frac{1}{2} \cancel{M} V_i^2 + \frac{1}{3} \cancel{M} V_i^2 = \cancel{M} g y_f \Rightarrow \frac{5}{6} V_i^2 = g y_f$$

$$\Rightarrow y_f = \frac{5 V_i^2}{6 g} = 2.13 \, m$$



$h/L = \sin 30°$

h

$L = \frac{}{\sin 30°}$

$= 4.3 m$

Knight Ch 12

~~Prob 76~~ 4^{ed} Prob 74

a) Energy is conserved

initial

$\omega_i = 0$ rad/s

$y_{cmi} = L/2$

final

$\omega_f = ?$

$y_{cmf} = 0$

$$E_i = E_f \Rightarrow \cancel{K_{rot\,i}}^{0} + U_{gi} = K_{rot\,f} + \cancel{U_{gf}}^{0}$$

$$Mg\,y_{cmi} = \frac{1}{2} I \omega_f$$

But $I = \frac{1}{3} M L^2$

$$\Rightarrow \quad Mg\,y_{cmi} = \frac{1}{6} ML^2 \omega_f^2$$

$$\Rightarrow \quad \cancel{M}g\,\cancel{L}/2 = \frac{1}{6} \cancel{M} L^{\cancel{2}} \omega_f^2$$

$$\Rightarrow \quad \omega_f = \sqrt{3g/L}$$

b) $v = r\omega \Rightarrow \quad V_{tip} = L\omega_f = \sqrt{3gL}$