2.2

2.2.5

$\frac{dy}{dt} + y = \frac{1}{1+e^t}$

$y' + p(t)y = f(t)$

$y(t) = y_n + y_p \qquad y_p = v(t)e^{-\int p(t)\,dt}$

$v(t) = \int f(t) e^{\int p(t)\,dt}\,dt$

$\frac{dy}{dt} + y = \frac{1}{1+e^t}$

$\mu(t) = e^{\int (1)\,dt} = e^t$

Integration factor method

$\left(\frac{dy}{dt} + y\right)e^t = \frac{e^t}{1+e^t}$

$\frac{d}{dt} y e^t = \frac{e^t}{1+e^t}$

$ye^t = \frac{e^t}{(1+e^t)}\,dt$

$ye^t = \ln(1+e^t) + C$

$y = \ln(1+e^t)(e^{-t}) + Ke^{-t}$

$y = \ln(1+e^t)(e^{-t}) + Ke^{-t}$

2.2.6

$\frac{dy}{dt} + 2ty = t$

$e^{t^2}(y' + 2ty) = te^{t^2}$

$\frac{d}{dt} e^{t^2} y = te^{t^2}$

$e^{t^2} y = te^{t^2}\,dt$

$e^{t^2} y = \frac{1}{2}\int e^u\,du$

$e^{t^2} y = \frac{1}{2} e^{t^2} + C$

$y = \frac{1}{2} e^{t^2}(e^{-t^2}) + Ke^{-t^2}$

$y = \frac{1}{2} + Ke^{-t^2}$

$e^{\int 2t\,dt} = e^{t^2}$

$u = t^2$

$du = 2t\,dt$

$\frac{1}{2}du = t\,dt$

$y = \frac{1}{2} + Ke^{-t^2}$

2.2.10

$\cos t \frac{dy}{dt} + y\sin t = 1$

$\frac{dy}{dt} + \tan t\, y = \cos^{-1}(t)$

$e^{\int p(t)}$

$e^{\int \tan t\,dt}$

$e^{\ln(\sec t)}$

$\sec(t)$

$\frac{d}{dt} \sec t\, y = \sec^2 t$

$\tan t = \frac{\sin t}{\cos t}$

$\frac{\sin t}{\frac{1}{\cos t}}$

$\frac{\sin t(\cos t)}{\cos t}$

$\sin(t)$

$\sec t\,(y) = \int \sec^2 t\,dt$

$\sec t\, y = \tan t + C$

$y = \frac{\tan t}{\frac{1}{\cos t}} + \frac{C}{\sec t}$

$y = \sin t + \frac{K}{\sec t}$

$y = \sin t + \frac{K}{\sec t}$

2.2.24

$y' - y = e^{3t}$

Integrating factor method

$y' + p(t)y = f(t)$

$\mu(t) = e^{\int p(t)\,dt}$

$P(t) = -1$

$\mu = e^{\int -1\,dt}$

$\mu = e^{-t}$

$$y(t) = \frac{1}{\mu(t)} \int \mu(t) f(t)\, dt + C\frac{1}{\mu(t)}$$

$$y(t) = \frac{1}{e^{-t}} \int e^{-t} e^{3t}\, dt + \frac{K}{e^{-t}}$$

$$= e^{t} \int e^{2t}\, dt + Ke^{t}$$

$$= e^{t}\left(\tfrac{1}{2}e^{2t}\right) + Ke^{t}$$

$$= \tfrac{1}{2}e^{3t} + Ke^{t}$$

$\int e^{2t}$

$u = 2t$

$du = 2\,dt$

$\frac{1}{2}du = dt$

$\frac{1}{2}\int e^{u}$

$\frac{1}{2}e^{2t} + C$

$$\boxed{y = \tfrac{1}{2}e^{3t} + Ke^{t}}$$

2.2.25 $y' + y = \sin t$

$$y(t) = \frac{1}{\mu(t)} \int \mu(t) f(t)\, dt + C\frac{1}{\mu(t)}$$

$\mu(t) = e^{\int P(t)\, dt}$

$y' + P(t)y = f(t)$

$P(t) = 1$

$\mu(t) = e^{t}$

$$y(t) = \frac{1}{e^{t}} \int e^{t} \sin t\, dt + K/e^{t}$$

$$= \frac{1}{e^{t}}\left(\frac{1}{2}e^{t}\sin t - \frac{1}{2}e^{t}\cos t\right) + Ke^{-t}$$

$$= \frac{1}{2}\sin t - \frac{1}{2}\cos t + Ke^{-t}$$

$$= \tfrac{1}{2}(\sin t - \cos t) + Ke^{-t}$$

$$\boxed{y(t) = \tfrac{1}{2}(\sin t - \cos t) + Ke^{-t}}$$

$\int e^{t}\sin t\, dt$

$u = e^{t}$ $du = e^{t}$

$dv = \sin t$ $v = -\cos t$

$-e^{t}\cos t - \int -e^{t}\cos t\, dt$

$u = -e^{t}$ $dv = \cos t\, dt$

$du = -e^{t}$ $v = \sin t\, dt$

$-e^{t}\cos t - (-e^{t}\sin t - \int -e^{t}\sin t\, dt)$

$-e^{t}\cos t + e^{t}\sin t - \int e^{t}\sin t\, dt = \int e^{t}\sin t\, dt$

$e^{t}\sin t - e^{t}\cos t = 2\int e^{t}\sin t\, dt$

$\frac{1}{2}e^{t}\sin t - \frac{1}{2}e^{t}\cos t = \int e^{t}\sin t\, dt$