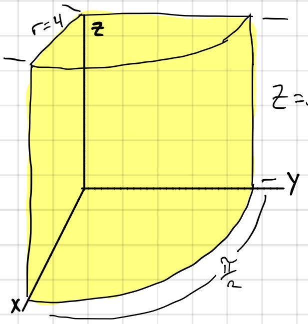
1.) $\iiint_E f(x,y,z)\, dv$

$$\int_0^{\frac{\pi}{2}} \int_0^{4} \int_0^{5} f \; r\, dz\, dr\, d\theta$$

(θ, r, z)

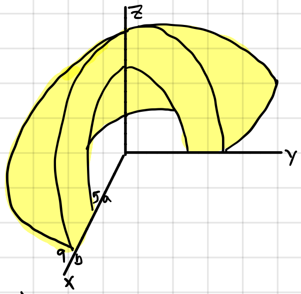
$f(x,y,z)$

$f(r\cos\theta, r\sin\theta, z$



2.) $\iiint_E f(x,y,z)\, dv$ $a=5$ $b=9$

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{2\pi} \int_5^{9}$$



$f(\rho\sin(\varphi)\cos(\theta), \rho\sin(\varphi)\sin\theta, \rho\cos(\varphi))$ $\rho^2\sin(\varphi)\, d\rho\, d\theta\, d\varphi$

3. $\iiint_B (x^2+y^2+z^2)^2\, dv$ origin radius 1 $\rho^2 = z^2+y^2+x^2$

$x=\rho\sin\varphi\cos\theta$ $y=\rho\sin\varphi\sin\theta$ $z=\rho\cos\varphi$

$$\iiint_B (\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta + \rho^2\cos^2\varphi)^2 \quad \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\iiint_B (\rho^2)^2 \quad \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\iiint_B \rho^6\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho^6\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\int_0^{\pi}\int_0^{2\pi} \frac{\rho^7}{7}\sin\varphi \Big|_0^1 \, d\theta\, d\varphi$$

$$\int_0^{\pi}\int_0^{2\pi} \frac{1}{7}\sin\varphi\, d\theta\, d\varphi$$

$$\int_0^{\pi} \frac{\theta}{7}\sin\varphi \Big|_0^{2\pi} d\varphi$$

$$\int_0^{\pi} \frac{2\pi}{7}\sin\varphi\, d\varphi$$

$$-\frac{2\pi}{7}\cos\varphi \Big|_0^{\pi}$$

$$-\frac{2\pi}{7}\cos\pi - \left(-\frac{2\pi}{7}\cos 0\right)$$

$$\frac{2\pi}{7} - \left(-\frac{2\pi}{7}\right)$$

$$\frac{2\pi}{7} + \frac{2\pi}{7} = \frac{4\pi}{7}$$

$\frac{4\pi}{7}$

4.) $\iiint_H (5-x^2-y^2)\, dv$ $x^2+y^2+z^2 \le 4$ $z \ge 0$

$\rho^2 \le 4$

$x=\rho\sin\varphi\cos\theta$ above xy plane
$y=\rho\sin\varphi\sin\theta$ $\varphi = \frac{\pi}{2}$
$z=\rho\cos\varphi$ $\theta = 2\pi$
$\rho = 2$

$f(x,y,z)$

$0 \le \rho \le 2$
$0 \le \theta \le 2\pi$
$0 \le \varphi \le \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{2} (5-\rho^2\sin^2\varphi\cos^2\theta - \rho^2\sin^2\varphi\sin^2\theta)\, \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{2} (5-\rho^2\sin^2\varphi(\cos^2\theta+\sin^2\theta))\, \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{2} (5-\rho^2\sin^2\varphi)\, \rho^2\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{2} 5\rho^2\sin\varphi - \rho^4\sin^3\varphi\, d\rho\, d\theta\, d\varphi$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{5}{3}\rho^3\sin\varphi - \frac{\rho^5}{5}\sin^3\varphi \Big|_0^2 \, d\theta\, d\varphi$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{40}{3}\sin\varphi - \frac{32}{5}\sin^3\varphi \, d\theta\, d\varphi$$

$$\int_0^{\frac{\pi}{2}} \frac{40}{3}(\theta)\sin\varphi - \frac{32}{5}(\theta)\sin^3\varphi \Big|_0^{2\pi} d\varphi$$

$$\int_0^{\frac{\pi}{2}} \frac{80\pi}{3}\sin\varphi - \frac{64\pi}{5}\sin^3\varphi \, d\varphi$$

$$\int_0^{\frac{\pi}{2}} \frac{80\pi}{3}\sin\varphi\, d\varphi - \int_0^{\frac{\pi}{2}} \frac{64\pi}{5}\sin^3\varphi\, d\varphi$$

$\frac{272\pi}{15}$

$1-\cos^2\varphi = \sin^2\varphi$

$$\frac{64\pi}{5}\int_0^{\frac{\pi}{2}} \sin\varphi(\sin\varphi)^2\, d\varphi$$

$u=\cos\varphi$
$du = -\sin\varphi\, d\varphi$
$-du = \sin\varphi\, d\varphi$

$$\frac{64\pi}{5}\int_0^{\frac{\pi}{2}} \sin\varphi(1-\cos^2\varphi)\, d\varphi$$

$$\frac{64\pi}{5}\int_0^{\frac{\pi}{2}} (1-u^2)\, d\varphi$$

$$\frac{64\pi}{5}\left[u - \frac{u^3}{3}\right]_0^{\frac{\pi}{2}}$$

$$\frac{64\pi}{5}\cos\left(\frac{\pi}{2}\right) - \frac{64\pi}{5}\frac{\cos^3\left(\frac{\pi}{2}\right)}{3}$$

5.) $\rho \le 7$ $\quad \phi = \frac{\pi}{6}$, $\phi = \frac{\pi}{3}$ } Between

$$z^2 = x^2 + y^2$$
$$\rho^2 = x^2 + y^2 + z^2 \qquad \rho = 7$$

$x = \rho \sin\varphi \cos\theta$
$y = \rho \sin\varphi \sin\theta$
$z = \rho \cos\varphi$

$0 \le \rho \le 7$
$0 \le \theta \le 2\pi$
$\frac{\pi}{6} \le \varphi \le \frac{\pi}{3}$

$$\int_0^{2\pi} \int_{\pi/6}^{\pi/3} \int_0^{7} \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$$
$$\int_0^{2\pi} \int_{\pi/6}^{\pi/3} \left[\frac{\rho^3}{3} \sin\varphi \right]_0^7 d\varphi\, d\theta$$
$$\int_0^{2\pi} \int_{\pi/6}^{\pi/3} \frac{343}{3} \sin\varphi \; d\varphi\, d\theta$$
$$\frac{343}{3} \int_0^{2\pi} \int_{\pi/6}^{\pi/3} \sin\varphi \; d\varphi\, d\theta$$
$$\frac{343}{3} \int_0^{2\pi} -\cos\varphi \Big|_{\pi/6}^{\pi/3} d\theta$$
$$\frac{343}{3} \int_0^{2\pi} -\frac{1}{2} - \left(-\frac{\sqrt{3}}{2}\right) d\theta$$
$$\frac{343}{3} \int_0^{2\pi} \frac{\sqrt{3}}{2} - \frac{1}{2} \; d\theta$$
$$\frac{343}{3} \left[\frac{\sqrt{3}}{2}\theta - \frac{1}{2}\theta \right]_0^{2\pi}$$
$$\frac{343}{3} \left[\sqrt{3}\pi - \pi \right]$$
$$\frac{343\sqrt{3}\pi}{3} - \frac{343\pi}{3}$$

$\frac{343}{3}(\sqrt{3}\pi - \pi)$

6.) radius $= 4$ $\quad$ two planes $\quad$ diameter $\frac{\pi}{2}$

$\rho = 4$

$0 \le \rho \le 4$
$0 \le \theta \le \frac{\pi}{2}$
$0 \le \varphi \le \pi$

$x = \rho \sin\varphi \cos\theta$
$y = \rho \sin\varphi \sin\theta$
$z = \rho \cos\varphi$

$$\int_0^{\pi/2} \int_0^{\pi} \int_0^{4} \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$$
$$\int_0^{\pi/2} \int_0^{\pi} \frac{\rho^3}{3} \sin\varphi \Big|_0^4 d\varphi\, d\theta$$
$$\int_0^{\pi/2} \int_0^{\pi} \frac{64}{3} \sin\varphi \; d\varphi\, d\theta$$
$$\frac{64}{3} \int_0^{\pi/2} -\cos\varphi \Big|_0^{\pi} d\theta$$

$-\cos(\pi) - (-\cos(0))$
$-(-1) - (-1)$
$1 + 1$

$$\frac{64}{3} \int_0^{\pi/2} 2 \; d\theta$$
$$\frac{64}{3} \left[2\theta \right]_0^{\pi/2}$$
$$\frac{64}{3} \left[2\left(\frac{\pi}{2}\right) \right]$$
$$\frac{64\pi}{3}$$

$\frac{64\pi}{3}$

7.) Torus $\rho = 7\sin\varphi$

$0 \le \rho \le 7\sin\varphi$
$0 \le \theta \le 2\pi$
$0 \le \varphi \le \pi$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{7\sin\varphi} \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$$
$$\int_0^{2\pi} \int_0^{\pi} \frac{\rho^3}{3} \sin\varphi \Big|_0^{7\sin\varphi} d\varphi\, d\theta$$
$$\int_0^{2\pi} \int_0^{\pi} \frac{343\sin^3\varphi}{3} \sin\varphi \; d\varphi\, d\theta$$
$$\frac{343}{3} \int_0^{2\pi} \int_0^{\pi} \sin^4\varphi \; d\varphi\, d\theta$$
$$\frac{343}{3} \int_0^{2\pi} \frac{3\pi}{8} \; d\theta$$
$$\frac{343}{3} \left[\frac{3\pi}{8}\theta \right]_0^{2\pi}$$
$$\frac{343}{3} \left[\frac{3\pi^2}{4} \right]$$
$$\frac{343\pi^2}{4}$$

$\frac{343\pi^2}{4}$

$$\int \sin^m(x)\,dx = -\frac{\cos(x)\sin^{m-1}(x)}{m} + \frac{m-1}{m} \int \sin^{-2+m}(x)\,dx$$

$$m=4 \quad = -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \sin^2(x)\,dx$$

$u = 2x$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$$-\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \frac{1}{2} - \frac{1}{2}\cos 2x \,dx$$
$$-\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{8}\int 1\,dx - \frac{3}{8}\int \cos 2x\,dx$$

$u = 2x$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$$-\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{8}x - \frac{3}{16}\sin 2x \Big|_0^{\pi}$$

8.) $\int_0^{10}\int_0^{\sqrt{100-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{200-x^2-y^2}} xy \quad dz\,dy\,dx$

$x=\rho\sin\phi\cos\theta$
$y=\rho\sin\phi\sin\theta$
$z=\rho\cos\phi$

$z=\sqrt{200-x^2-y^2}$
$z=\sqrt{x^2+y^2}$

$\sqrt{200-x^2-y^2}=\sqrt{x^2+y^2}$
$200-x^2-y^2=x^2+y^2$
$200=2(x^2+y^2)$
$100=x^2+y^2$

$\rho\cos\phi=\sqrt{(\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2}$
$=\sqrt{\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta}$
$=\sqrt{\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)}$
$=\sqrt{\rho^2\sin^2\phi}$
$\rho\cos\phi=\rho\sin\phi$
$1=\tan\phi$
$\tan^{-1}(1)=\phi$

$\theta=\frac{\pi}{2}$
$\phi=\frac{\pi}{4}$
$\rho=10\sqrt{2}$

$\int_0^{10}\int_0^{\sqrt{100-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{200-x^2-y^2}} xy\ dz\,dy\,dx=\frac{80{,}000\sqrt{2}}{3}-\frac{160000}{3}$

$z=\sqrt{200-x^2-y^2}$
$\rho\cos\phi=\sqrt{200-(\rho\sin\phi\cos\theta)^2-(\rho\sin\phi\sin\theta)^2}$
$\rho\cos\phi=\sqrt{200-\rho^2\sin^2\phi\cos^2\theta-\rho^2\sin^2\phi\sin^2\theta}$
$\rho\cos\phi=\sqrt{200-\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)}$
$\rho\cos\phi=\sqrt{200-\rho^2\sin^2\phi}$
$\rho^2\cos^2\phi=200-\rho^2\sin^2\phi$
$0=200-\rho^2\sin^2\phi-\rho^2\cos^2\phi$
$0=200-\rho^2(\sin^2\phi+\cos^2\phi)$
$0=200-\rho^2$
$200=\rho^2$
$\rho=\pm\sqrt{200}$
$\rho=10\sqrt{2}$

$10=10\sqrt{2}\sin(\frac{\pi}{4})\cos\theta$
$\frac{10}{10\sqrt{2}\sin(\frac{\pi}{4})}=\cos\theta$
$\theta=\cos^{-1}\left(\frac{10}{10\sqrt{2}\sin(\frac{\pi}{4})}\right)$
$\frac{10}{10}=1$
$\theta=\cos^{-1}(1)$
$\theta=0,\frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{4}}\int_0^{10\sqrt{2}}(\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)\ \rho^2\sin\phi\ d\rho\,d\phi\,d\theta$
$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{4}}\int_0^{10\sqrt{2}}\rho^4\sin^3\phi\cos\theta\sin\theta\ d\rho\,d\phi\,d\theta$
$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{4}}\frac{\rho^5}{5}\sin^3\phi\cos\theta\sin\theta\Big|_0^{10\sqrt{2}}\ d\phi\,d\theta$
$\frac{1}{2}(80{,}000\sqrt{2})\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{4}}\sin\phi\sin\phi^2\cdot2\cos\theta\sin\theta\ d\phi\,d\theta$
$40{,}000\sqrt{2}\int_0^{\frac{\pi}{2}}\sin2\theta\int_0^{\frac{\pi}{4}}\sin^3\phi\ d\phi\,d\theta$
$40{,}000\sqrt{2}\int_0^{\frac{\pi}{2}}\sin2\theta\left(\frac{2}{3}-\frac{5\sqrt{2}}{12}\right)d\theta$
$40{,}000\sqrt{2}\left(\frac{2}{3}-\frac{5\sqrt{2}}{12}\right)\int_0^{\frac{\pi}{2}}\sin2\theta$
$40{,}000\sqrt{2}\left(\frac{2}{3}-\frac{5\sqrt{2}}{12}\right)(1)=\frac{80{,}000\sqrt{2}}{3}-\frac{160000}{3}$

9.) $\int_{-5}^{5}\int_{-\sqrt{25-y^2}}^{\sqrt{25-y^2}}\int_{-\sqrt{25-x^2-y^2}}^{\sqrt{25-x^2-y^2}}(x^2z+y^2z+z^3)\ dz\,dx\,dy$

$x=\rho\sin\phi\cos\theta$
$y=\rho\sin\phi\sin\theta$
$z=\rho\cos\phi$

$0\le\phi\le\pi$
$0\le\theta\le2\pi$
$0\le\rho\le5$

$x^2+y^2+z^2=\rho^2$
$z(x^2+y^2+z^2)$
$\rho\cos\phi(\rho^2)$

0

$\int_0^{2\pi}\int_0^{\pi}\int_0^5\rho^3\cos\phi\cdot\rho^2\sin\phi\ d\rho\,d\phi\,d\theta$
$\int_0^{2\pi}\int_0^{\pi}\int_0^5\rho^5\cos\phi\sin\phi\ d\rho\,d\phi\,d\theta$
$\frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_0^5\rho^5 2\cos\phi\sin\phi\ d\rho\,d\phi\,d\theta$
$\frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_0^5\rho^5\sin2\phi\ d\rho\,d\phi\,d\theta$
$\frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\frac{\rho^6}{6}\sin2\phi\Big|_0^5\ d\phi\,d\theta$
$\frac{1}{2}\left(\frac{5^6}{6}\right)\int_0^{2\pi}\int_0^{\pi}\sin2\phi\ d\phi\,d\theta$

$u=2\phi$
$du=2d\phi$
$\frac{1}{2}du=d\phi$

$\frac{1}{4}\left(\frac{5^6}{6}\right)\int_0^{2\pi}\int_0^{\pi}\sin u\ du\,d\theta$
$\frac{5^6}{24}\int_0^{2\pi}-\cos2\phi\Big|_0^{\pi}\ d\theta$
$\frac{5^6}{24}\int_0^{2\pi}-\cos(2\pi)-(-\cos(0))\ d\theta$
$\frac{5^6}{24}\int_0^{2\pi}-(1)-(-1)\ d\theta$
$-1+1$
$\frac{5^6}{24}\int_0^{2\pi}0\ d\theta$
0