Taylor Larrechea
Dr. Gustafson
MATH 360
HW 9.9

9.9 # 5,9,11

9.9 #5

$\vec{v} = xyz[x,y,z]$

$\vec{v} = [x^2yz, xy^2z, xyz^2]$

Curl : $\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & xy^2z & xyz^2 \end{vmatrix} = \left[\frac{\partial}{\partial y}(xyz^2) - \frac{\partial}{\partial z}(xy^2z), \frac{\partial}{\partial z}(x^2yz) - \frac{\partial}{\partial x}(xyz^2), \frac{\partial}{\partial x}(xy^2z) - \frac{\partial}{\partial y}(x^2yz)\right]$

$= [xz^2 - xy^2, x^2y - yz^2, y^2z - x^2z]$

$\boxed{\vec{\nabla}\times\vec{v} = [x(z^2-y^2), y(x^2-z^2), z(y^2-x^2)]}$

9.9 #9

$\vec{v} = [0, 3z^2, 0]$

a.) Curl : $\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 3z^2 & 0 \end{vmatrix} = \left[\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(3z^2), \frac{\partial}{\partial z}(0) - \frac{\partial}{\partial x}(0), \frac{\partial}{\partial x}(3z^2) - \frac{\partial}{\partial y}(0)\right]$

$[-6z, 0, 0]$

$\vec{\nabla}\times\vec{v} = [-6z,0,0]$: $\vec{\nabla}\times\vec{v} \neq 0$ $\therefore$ the fluid is rotational

b.) Div : $\vec{\nabla}\cdot\vec{v} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]\cdot[0,3z^2,0] = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(3z^2) + \frac{\partial}{\partial z}(0) = 0$

$\vec{\nabla}\cdot\vec{v} = 0$, $\therefore$ the fluid is incompressible

c.) $\vec{v} = [0,3z^2,0] = [x'(t), y'(t), z'(t)]$

$y'(t) = 3z^2$

$x'(t) = 0$ $\quad x(t) = C_1$

$y'(t) = 3z^2$ $\quad y(t) = 3C_3^2 t + C_1$

$z'(t) = 0$ $\quad z(t) = C_3$

$\int dy = 3z^2$ $\quad z = C_3$

$y + c = 3z^2$

$y = 3z^2 - c$ $\quad +,-$

$y = 3C_3^2 + c$

a.) $\vec{\nabla}\times\vec{v} = [-6z,0,0]$ $\therefore$ the fluid is rotational
b.) $\vec{\nabla}\cdot\vec{v} = 0$, $\therefore$ the fluid is incompressible
c.) $\vec{v} = [0,3z^2,0] = [x'(t), y'(t), z'(t)]$
$x'(t) = 0$ $\quad x(t) = C_1$
$y'(t) = 3z^2$ $\quad y(t) = 3C_3^2 t + C_1$
$z'(t) = 0$ $\quad z(t) = C_3$

9.9 #11 $\vec{v} = [y, -2x, 0]$

a.) curl : $\vec{\nabla} \times \vec{v} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -2x & 0 \end{vmatrix} = \left[\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(-2x), \frac{\partial}{\partial z}(y) - \frac{\partial}{\partial x}(0), \frac{\partial}{\partial x}(-2x) - \frac{\partial}{\partial y}(y)\right]$
$= [0-0, 0-0, -2-1]$

$\vec{\nabla} \times \vec{v} = [0, 0, -3]$
$\vec{\nabla} \times \vec{v} \neq 0$ $\therefore$ the fluid is irrotational

b.) div : $\vec{\nabla} \cdot \vec{v} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right] \cdot [y, -2x, 0]$
$= \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(-2x) + \frac{\partial}{\partial z}(0)$
$= 0 + 0 + 0$
$\vec{\nabla} \cdot \vec{v} = 0$

$\vec{\nabla} \cdot \vec{v} = 0$, $\therefore$ the fluid is incompressible

c.) $\vec{v} = [y, -2x, 0] = [x'(t), y'(t), z'(t)]$
$x'(t) = y$
$y'(t) = -2x$ $\quad C = x^2 + \frac{1}{2}y^2$
$z'(t) = 0$ $\quad z(t) = c_3$

$\int y \frac{dy}{dt} \, dt = -2 \int x \frac{dx}{dt} \, dt$
$\int y \, dy = -2 \int x \, dx$
$\frac{1}{2}y^2 = -2\left(\frac{1}{2}x^2\right) + C$
$\frac{1}{2}y^2 = -x^2 + C$
$C = x^2 + \frac{1}{2}y^2$

a.) $\vec{\nabla} \times \vec{v} = [0, 0, -3]$
$\vec{\nabla} \times \vec{v} \neq 0$, $\therefore$ the fluid is irrotational
b.) $\vec{\nabla} \cdot \vec{v} = 0$, $\therefore$ the fluid is incompressible
c.) $\vec{v} = [y, -2x, 0] = [x'(t), y'(t), z'(t)]$
$x'(t) = y$ $\quad x(t) = 0$
$y'(t) = -2x$ $\quad C = x^2 + \frac{1}{2}y^2$
$z'(t) = 0$ $\quad z(t) = c_3$