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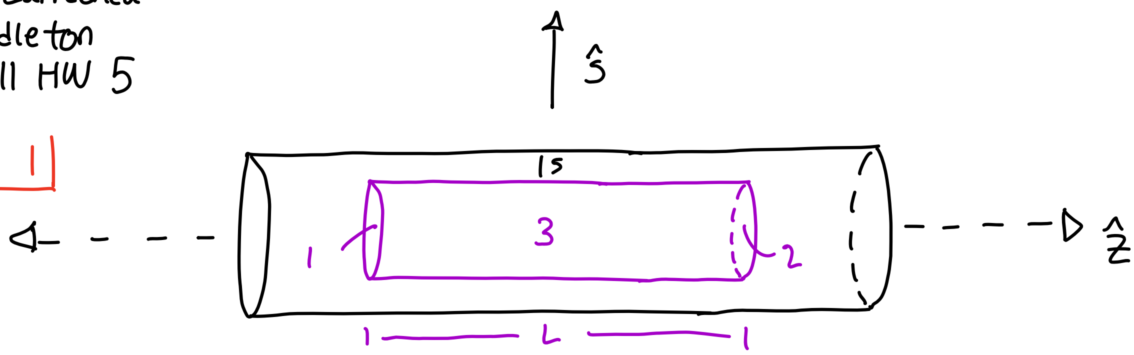
Dr. Middleton
PHYS 311 HW 5

Problem 1



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{Enc}}{\varepsilon_0} \qquad d\vec{a} = s\,d\phi\,dz\,\hat{s} \quad 0 \le z \le L$$

$$\lambda = \frac{\text{charge}}{\text{Length}},\; Q_{ENC} = \underline{\lambda \cdot L}$$

$$\int dq = \int \lambda\, dl$$

$$q_{enc} = \lambda L$$

$$\oint \vec{E}\cdot d\vec{a} = \int_0^L \int_0^{2\pi} \vec{E}\cdot d\vec{a}$$

Here we can pull E outside of the integral due to the symmetry of our wire. The electric field is also uniform for our wire. The electric field also points in the same direction as the area element of surface three for our Gaussian surface. We can discard calculating sides one and two due to their area elements being perpendicular to the electric field ($\vec{E}$ points radially away in the $\hat{s}$ direction) where as sides one and two point in the $\pm\hat{z}$ direction. Thus,

$$\vec{E} = E\hat{s}$$

$$d\vec{a} = s\,d\phi\,dz\,\hat{s}$$

$$ES\int_0^L \int_0^{2\pi} d\phi\, dz$$

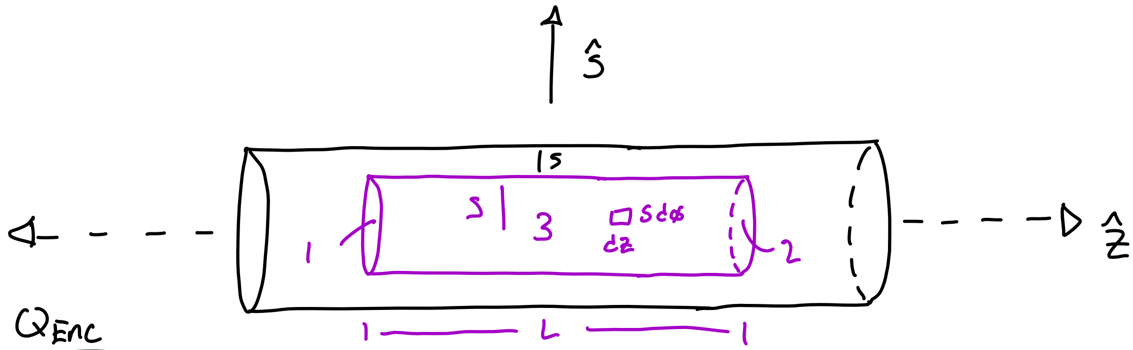
$$ES\int_0^L \phi\Big|_0^{2\pi} dz$$

$$2\pi ES\int_0^L dz = 2ES\pi \cdot z\Big|_0^L = 2\pi ESL$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{Enc}}{\varepsilon_0} \iff 2\pi ESL = \frac{\lambda L}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi S\varepsilon_0}\hat{s} \ \frac{N}{C}}$$

Problem 1



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{Enc}}{\varepsilon_0}$$

$\lambda = \frac{\text{Charge}}{\text{Length}}$ $\therefore$ Chager $= \lambda \cdot L$

$\int dq = \int \lambda\, dL \Rightarrow q = \lambda L$

$$Q_{Enc} = \lambda L$$

$$\oint \vec{E}\cdot d\vec{a} : \underbrace{\oint_1 \vec{E}\cdot d\vec{a} + \oint_2 \vec{E}\cdot d\vec{a}}_{\vec{E}\perp d\vec{a}} + \underbrace{\oint_3 \vec{E}\cdot d\vec{a}}_{\vec{E}\parallel d\vec{a}}$$

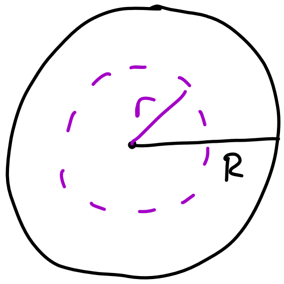
$\vec{E} \parallel d\vec{a}$ — E constant at given radius

$$\oint_3 \vec{E}\cdot d\vec{a} : d\vec{a} = s\, d\phi\, dz : E s \int_0^L \int_0^{2\pi} d\phi\, dz = E\cdot 2\pi s L$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{Enc}}{\varepsilon_0} : E\cdot 2\pi s L = \frac{\lambda L}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s}\hat{s}}$$

Problem 2



$$\rho(r') = k r'^{\pi}$$

$$\oint E \cdot da = \frac{Q_{ENC}}{\varepsilon_0}$$

$$Q_{ENC} = \int \rho(r')\, d\tau' \qquad d\tau = r'^2 \sin\theta\, dr'\, d\theta'\, d\phi'$$

$$Q_{ENC} = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} k r'^{\pi} \cdot r'^2 \sin\theta\, dr'\, d\theta'\, d\phi'$$

$$= k \int_0^{2\pi}\int_0^{\pi}\int_0^{r} r'^{(\pi+2)}\, dr'\, d\theta\, d\phi'$$

$$= k \int_0^{2\pi}\int_0^{\pi} \frac{r'^{(\pi+3)}}{(\pi+3)}\Big|_0^r \sin\theta\, d\theta'\, d\phi'$$

$$\int_0^{\pi} \sin\theta = -\cos\theta \Big|_0^{\pi} = 2$$

$$= \frac{k r^{(\pi+3)}}{(\pi+3)} \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta'\, d\phi'$$

$$= \frac{k r^{(\pi+3)}}{(\pi+3)} (4\pi) = \frac{4\pi k r^{(\pi+3)}}{(\pi+3)} \qquad Q_{net} = \frac{4\pi k r^{(\pi+3)}}{(\pi+3)}$$

$$\oint \vec{E} \cdot d\vec{a} \; : \; d\vec{a} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{r} \Rightarrow E\, r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi$$

Similar to problem 1, we can pull E outside of the integral due it being uniform in the sphere. The area element and Electric field point in the same direction (The $\hat{r}$ direction).

$$E \cdot r^2 \int_0^{2\pi} -\cos(\theta)\Big|_0^{\pi} d\phi \qquad -\cos(\pi) - (-\cos(0))$$

$$-(-1) - (-1)$$

$$1 + 1 = 2$$

$$2 E r^2 \int_0^{2\pi} d\phi$$

$$2 E r^2\, \phi \Big|_0^{2\pi} = 4\pi E r^2$$

$$\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{ENC}$$

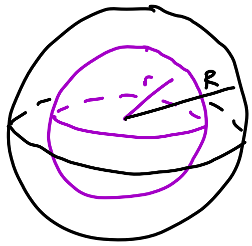
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \qquad \oint E \cdot da = E \cdot 4\pi r^2 \; : \; \frac{Q_{ENC}}{\varepsilon_0} = \frac{4\pi k r^{(\pi+3)}}{\varepsilon_0(\pi+3)}$$

$$E \cdot 4\pi r^2 = \frac{4\pi k r^{(\pi+3)}}{\varepsilon_0(\pi+3)}$$

$$\vec{E} = \frac{k r^{(\pi+1)}}{\varepsilon_0(\pi+3)}$$

$$\boxed{\vec{E} = \frac{k r^{(\pi+1)}}{\varepsilon_0(\pi+3)}}$$

Problem 2)

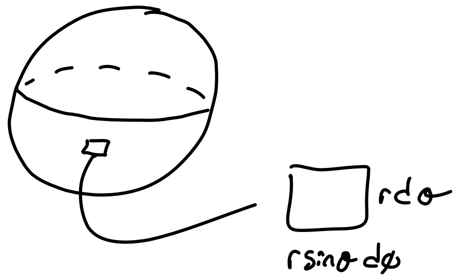


$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \qquad \rho(r) = k r^{\pi}$$

$$\rho \equiv \frac{\text{Charge}}{\text{Volume}} \quad \therefore \quad \text{Charge} = \rho \cdot V$$

$$\int dQ = \int_V \rho \, dv$$

Q : $dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$



$$k \int_0^{2\pi}\int_0^{\pi}\int_0^{r'} r^{(\pi+2)} \sin\theta \, dr \, d\theta \, d\phi$$

$$k \int_0^{2\pi}\int_0^{\pi} \left. \frac{r^{(\pi+3)}}{(\pi+3)} \right|_0^{r'} \sin\theta \, d\theta \, d\phi$$

$$\frac{k \cdot r'^{(\pi+3)}}{(\pi+3)} \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = \frac{k \cdot r'^{(\pi+3)}}{(\pi+3)} \cdot 2\pi \cdot 2$$

$$\therefore \quad Q_{ENC} = \frac{4\pi k \, r'^{(\pi+3)}}{(\pi+3)}$$

$\oint \vec{E}\cdot d\vec{a}$: E costant at given r, $d\vec{a} \leftrightarrow$ on surface of gaussian sphere

$\vec{E} = E\hat{r}$ $\qquad d\vec{a} = r'^2 \sin\theta \, d\theta \, d\phi$

$$\oint \vec{E}\cdot d\vec{a} = E \cdot r'^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = E \cdot r'^2 \cdot 2\pi \cdot 2 = E \cdot 4\pi r'^2$$

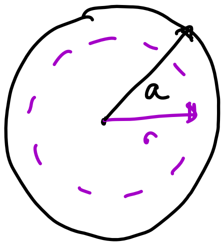
$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \quad \vdots \quad E \cdot 4\pi r'^2 = \frac{4\pi k \, r'^{(\pi+3)}}{(\pi+3)\,\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{k \, r'^{(\pi+1)}}{(\pi+3)\,\varepsilon_0} \hat{r}}$$

Problem 3

$\rho(r) = kr^{\pi}$

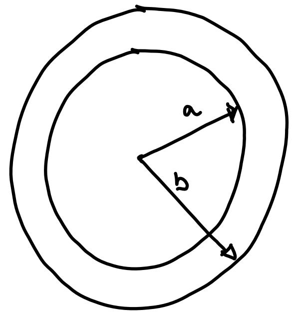
a.) $r < a$



Gauss' Law

$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$

$Q_{ENC} = 0$ For inside the shell. ∴



radius of "s"

$\oint_s \vec{E} \cdot d\vec{a} = 0$

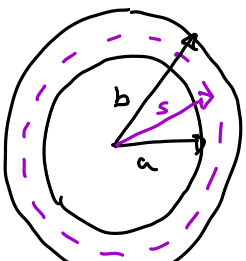
$d\vec{a} = r^2 \sin\theta \, d\theta d\phi \, \hat{r} \Rightarrow \int_0^{2\pi}\int_0^{\pi} E \cdot \boxed{r^2 \sin\theta \, d\theta d\phi} = 0$

$\neq 0$ ∴ $E = 0$

$\boxed{\vec{E} = 0, \text{ no charge inside shell.}}$

b.) $a < r < b$

$a \Rightarrow s$



$Q_{ENC} = \int_V \rho(r) \, d\tau \, , \quad \rho(r) = Kr^{\pi} \quad d\tau = r^2 \sin\theta \, dr d\theta d\phi$

$$Q_{ENC} = K \int_0^{2\pi}\int_0^{\pi}\int_a^{s} r^{(\pi+2)} \sin\theta \, dr d\theta d\phi$$

$$K \int_0^{2\pi}\int_0^{\pi} \frac{r^{(\pi+3)}}{(\pi+3)} \Bigg|_a^s \sin\theta \, d\theta d\phi \qquad \frac{1}{(\pi+3)}\left(s^{(\pi+3)} - a^{(\pi+3)}\right)$$

$$\frac{K(s^{(\pi+3)} - a^{(\pi+3)})}{(\pi+3)} \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta d\phi$$

$$\frac{K(s^{(\pi+3)} - a^{(\pi+3)})}{(\pi+3)} \phi \Big|_0^{2\pi} -\cos\theta \Big|_0^{\pi} = \frac{4\pi K(s^{(\pi+3)} - a^{(\pi+3)})}{(\pi+3)}$$

$$Q_{ENC} = \frac{4\pi K(s^{(\pi+3)} - a^{(\pi+3)})}{(\pi+3)}$$

$\oint \vec{E} \cdot d\vec{a}$: Electric field is constant in Gaussian surface ∴ E can be pulled out. The electric field and ($d\vec{a}$) point in the same direction (The $\hat{R}$ direction), so,

$d\vec{a} = s^2 \sin\theta \, d\theta d\phi \, \hat{r}$: $\vec{E} = E\hat{r}$ $\qquad r \equiv s$, in this part

$$\oint \vec{E} \cdot d\vec{a} \Rightarrow E \cdot s^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta d\phi$$

$$E \cdot s^2 \int_0^{2\pi} -\cos\theta \Big|_0^{2\pi} d\phi$$

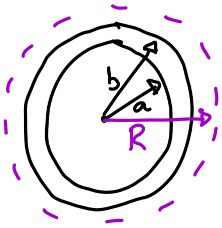
$$2Es^2 \int_0^{2\pi} d\phi = 2Es^2 \phi \Big|_0^{2\pi} = E \cdot 4\pi s^2 \qquad \oint \vec{E} \cdot d\vec{a} = E \cdot 4\pi s^2$$

Problem 3 | Continued

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \quad ; \quad E\cdot 4\pi s^2 = \frac{4\pi k}{\varepsilon_0(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right]$$

$$\boxed{\vec{E} = \frac{k}{\varepsilon_0 s^2(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right] \frac{N}{C}\ \hat{r}}$$

c.) $R > b$ Outer Radius $\equiv R$ $\rho(r) = kr^{\pi}$



$$Q_{ENC} = \int_v \rho(r)\, d\tau \qquad d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q_{ENC} = k\int_0^{2\pi}\int_0^{\pi}\int_a^b r^{(\pi+2)} \sin\theta\, dr\, d\theta\, d\phi$$

$$= k\int_0^{2\pi}\int_0^{\pi} \frac{r^{(\pi+3)}}{(\pi+3)}\Big|_a^b\, d\theta\, d\phi \qquad \frac{1}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]$$

$$= \frac{k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]\int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi$$

$$= \frac{k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]\int_0^{2\pi} -\cos\theta\Big|_0^{\pi} d\phi$$

$$= \frac{2k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]\int_0^{2\pi} d\phi$$

$$Q_{ENC} = \frac{4\pi k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]$$

$\oint \vec{E}\cdot d\vec{a}$: Electric field is constant in Gaussian surface $\therefore$ E can be pulled out. The electric field and ($d\vec{a}$) point in the same direction (The $\hat{R}$ direction). So,

$$d\vec{a} = R^2 \sin\theta\, d\theta\, d\phi\ \hat{r} \quad , \quad \vec{E} = E\hat{r}$$

$$\oint \vec{E}\cdot d\vec{a} : E\cdot R^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi$$

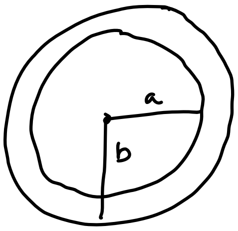
$$E\cdot R^2 \int_0^{2\pi}\left[-\cos\theta\right]_0^{\pi} d\phi \implies E\cdot 2R^2\int_0^{2\pi} d\phi = E\cdot 4\pi R^2$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \qquad \oint_s \vec{E}\cdot d\vec{a} = E\cdot 4\pi R^2$$

$$E\cdot 4\pi R^2 = \frac{4\pi k}{\varepsilon_0(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]$$

$$\boxed{\vec{E} = \frac{k}{\varepsilon_0 R^2(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right] \frac{N}{C}\ \hat{r}}$$

Problem 3



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}, \quad \rho(r) = k r^{\pi}$$

$$\rho \equiv \frac{\text{Charge}}{\text{Volume}} \quad \therefore \quad \text{Charge} = \rho \cdot \text{Volume}$$

$$\int dQ = \int_{v} \rho \, dv$$

a.) $S < a$

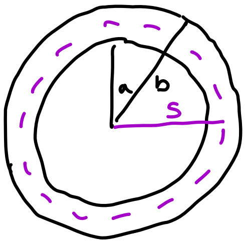
When we are on the inside, there is no charge.

$$\therefore \quad Q_{ENC} = 0 \quad \therefore \quad \oint \vec{E}\cdot d\vec{a} = 0 \quad \therefore \quad E = 0$$

$$\boxed{\vec{E} = 0}$$

$$dv = r^2 \sin\theta \, dr\, d\theta\, d\phi$$

b.) $a < s < b$



$$Q_{ENC}: \quad k \int_0^{2\pi}\int_0^{\pi}\int_a^{s} r^{(\pi+2)} \sin\theta \, dr\, d\theta\, d\phi$$

$$k \int_0^{2\pi}\int_0^{\pi} \frac{r^{(\pi+3)}}{(\pi+3)}\Bigg|_a^s \sin\theta \, d\theta\, d\phi$$

$$k \int_0^{2\pi}\int_0^{\pi} \frac{1}{(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right] \sin\theta \, d\theta\, d\phi$$

$$\frac{k}{(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right] \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta\, d\phi = \frac{k}{(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right]\cdot 2\pi \cdot 2$$

$$Q_{ENC} = \frac{4\pi k}{(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right]$$

$\oint \vec{E}\cdot d\vec{a}$: E constant at given radius

$d\vec{a} \leftrightarrow$ Exists on gaussian sphere

$$\vec{E} = E\hat{r}$$

$$d\vec{a} = s^2 \sin\theta \, d\theta\, d\phi$$

$$\oint \vec{E}\cdot d\vec{a} = E \cdot s^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta\, d\phi = E\cdot s^2 \cdot 2\pi \cdot 2 = E \cdot 4\pi s^2$$

$$\oint \vec{E}\cdot d\vec{a} = E\cdot 4\pi s^2$$

$$Q_{ENC} = \frac{4\pi k}{(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right]$$

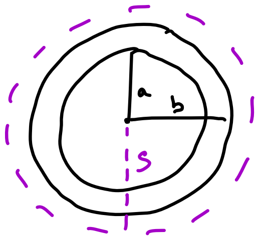
$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$$

$$E\cdot 4\pi s^2 = \frac{4\pi k}{\varepsilon_0(\pi+3)}\left[s^{(\pi+3)} - a^{(\pi+3)}\right]$$

$$\boxed{\vec{E} = \frac{k}{\varepsilon_0(\pi+3)s^2}\left[s^{(\pi+3)} - a^{(\pi+3)}\right]\hat{r}}$$

Problem 3 | Continued

c.) $s > b$



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \quad , \quad \rho(r) = kr^{\pi}$$

$$\rho = \frac{\text{Charge}}{\text{Volume}} \quad \therefore \quad \text{Chare} = \rho \cdot \text{volume} \qquad d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\int dQ = \int \rho\, dv \quad , \quad Q = \rho \cdot V$$

$$Q_{ENC} = k \int_0^{2\pi}\int_0^{\pi}\int_a^b r^{(\pi+2)} \sin\theta\, dr\, d\theta\, d\phi$$

$$= k \int_0^{2\pi}\int_0^{\pi} \left. \frac{r^{(\pi+3)}}{(\pi+3)} \right|_a^b \sin\theta\, d\theta\, d\phi$$

$$= \frac{k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right] \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = \frac{k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]\cdot 2\pi \cdot 2$$

$$Q_{ENC} = \frac{4\pi k}{(\pi+3)}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]$$

$\oint \vec{E}\cdot d\vec{a}$: E Constant at given radius
$d\vec{a} \longrightarrow$ exists on Gaussian Surface

$$d\vec{a} = s^2 \sin\theta\, d\theta\, d\phi\, \hat{s} \quad , \quad \vec{E} = E\,\hat{s}$$

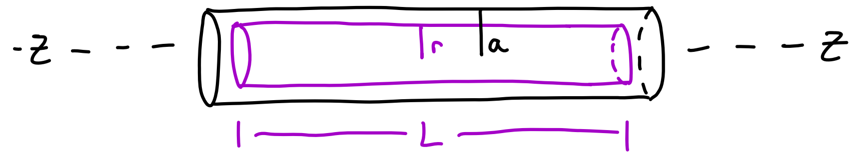
$$\oint \vec{E}\cdot d\vec{a} = E\cdot s^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = E\cdot s^2 \cdot 2\pi \cdot 2 = E \cdot 4\pi s^2$$

$$\oint \vec{E}\cdot d\vec{a} = E\cdot 4\pi s^2 \quad \vdots \quad \oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$$

$$E \cdot 4\pi s^2 = \frac{4\pi k}{(\pi+3)\varepsilon_0}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]$$

$$\boxed{\vec{E} = \frac{k}{s^2(\pi+3)\varepsilon_0}\left[b^{(\pi+3)} - a^{(\pi+3)}\right]\hat{s}}$$

Problem 4



a.) $s < a$

$\rho(s) = \rho_0 \frac{s}{a}$

inner cylinder radius $\equiv r$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$$

$Q_{ENC} = \int_V \rho(s)\, d\tau$, $d\tau = s\, ds\, d\phi\, dz$

$$Q_{ENC} : \frac{\rho_0}{a} \int_0^L \int_0^{2\pi} \int_0^r s^2\, ds\, d\phi\, dz$$

$$\frac{\rho_0}{a} \int_0^L \int_0^{2\pi} \frac{s^3}{3}\Big|_0^r d\phi\, dz$$

$$\frac{r^3 \rho_0}{3a} \int_0^L \int_0^{2\pi} d\phi\, dz = 2\pi L \cdot \frac{r^3 \rho_0}{3a} \qquad Q_{ENC} = \frac{2\pi L r^3 \rho_0}{3a}$$

Because of cylindrical symmetry of our gaussian surface we can say that the $\vec{E}$ field is constant at a given radius away. We can also discard finding the $\vec{E}$ field at the ends of the wire due to the $\vec{E}$ field being orthogonal to $d\vec{a}$. Therefore the long surface of Gaussian surface is the only side of interest.

$\underbrace{\vec{E} = E\,\hat{s} \qquad d\vec{a} = r\, d\phi\, dz\, \hat{s}}$ Both in $\hat{s}$ direction!

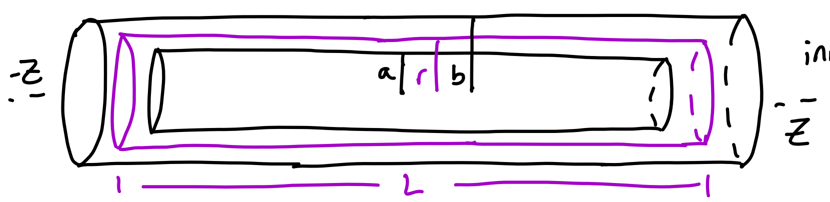
$$\oint \vec{E} \cdot d\vec{a} : E \cdot r \int_0^L \int_0^{2\pi} d\phi\, dz = E \cdot 2\pi r L \qquad \oint \vec{E} \cdot d\vec{a} = E \cdot 2\pi r L$$

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \Leftrightarrow E \cdot 2\pi r L = \frac{2\pi L r^3 \rho_0}{3a\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{r^2 \rho_0}{3a\varepsilon_0} \ \frac{N}{C} \ \hat{s}}$$

b.) $0 < s < a$

$\rho(s) = \rho_0 \frac{s}{a}$



inner cylinder radius $\equiv r$

$Q_{ENC} = \int_V \rho(s)\, d\tau \qquad d\tau = s\, ds\, d\phi\, dz\, \hat{s}$

$$= \frac{\rho_0}{a} \int_0^L \int_0^{2\pi} \int_0^a s^2\, ds\, d\phi\, dz$$

$$= \frac{\rho_0}{a} \int_0^L \int_0^{2\pi} \frac{s^3}{3}\Big|_0^a d\phi\, dz \Leftrightarrow \frac{\rho_0 a^3}{3a} \int_0^L \int_0^{2\pi} d\phi\, dz = \frac{2\pi L \rho_0 a^2}{3}$$

$$Q_{ENC} = \frac{2\pi L \rho_0 a^2}{3}$$

Problem 4 | Continued

Because of cylindrical symmetry of our gaussian surface we can say that the $\vec{E}$ field is constant at a given radius away. We can also discard finding the $\vec{E}$ field at the ends of the wire due to the $\vec{E}$ field being orthogonal to $d\vec{a}$. Therefore the long surface of Gaussian surface is the only side of interest.

$$\vec{E} = \vec{E}\,\hat{s} \quad , \quad d\vec{a} = d\phi dz$$

$$\oint \vec{E} \cdot d\vec{a} \; : \; E \int_0^L \int_0^{2\pi} d\phi dz = E \cdot 2\pi r L \qquad \oint \vec{E} \cdot d\vec{a} = E \cdot 2\pi r L$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \; : \; E \cdot 2\pi r L = \frac{2\pi L \rho_0 a^2}{3\varepsilon_0}$$

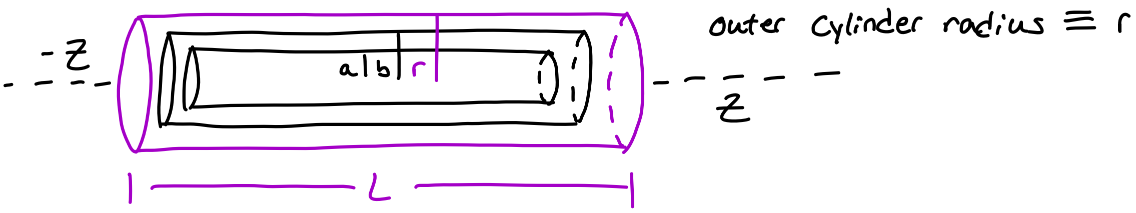
$$\boxed{\vec{E} = \frac{\rho_0 a^2}{3\varepsilon_0 r}\,\hat{s}}$$

c.) $s > b$

$\sigma(s) = \sigma\,\hat{s}$

$Q_{ENC} = \int_S \sigma\, da$

$da = b\, d\phi dz\, \hat{s}$



$$Q_{ENC} = \sigma b \int_0^L \int_0^{2\pi} d\phi dz = \sigma 2\pi L b$$

$$Q_{Tot} = Q_{Vol} + Q_{Sur} = \frac{2\pi}{3} \cdot \rho_0 L a^2 + 2\pi \cdot \sigma L b$$

Because of cylindrical symmetry of our gaussian surface we can say that the $\vec{E}$ field is constant at a given radius away. We can also discard finding the $\vec{E}$ field at the ends of the wire due to the $\vec{E}$ field being orthogonal to $d\vec{a}$. Therefore the long surface of Gaussian surface is the only side of interest.

$$\vec{E} = E\,\hat{s} \qquad d\vec{a} = r\, d\phi dz\, \hat{s}$$

Both in $\hat{s}$ direction

$$\oint \vec{E} \cdot d\vec{a} \; : r E \int_0^L \int_0^{2\pi} d\phi dz = E \cdot 2\pi r L \qquad \oint \vec{E} \cdot d\vec{a} = E \cdot 2\pi r L$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \qquad E \cdot 2\pi r L = \frac{\left(\frac{2\pi}{3} \cdot \rho_0 l a^2 + 2\pi \cdot \sigma l b\right)}{\varepsilon_0}\,\hat{s}$$

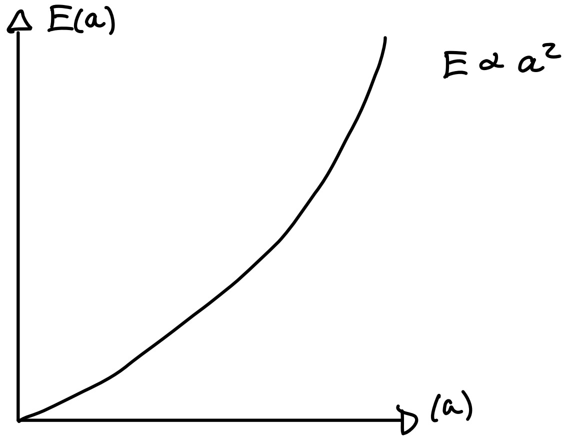
$$E \cdot 2\pi r l = \frac{2\pi l}{\varepsilon_0}\left(\frac{\rho_0}{3} a^2 + \sigma b\right)\hat{s}$$

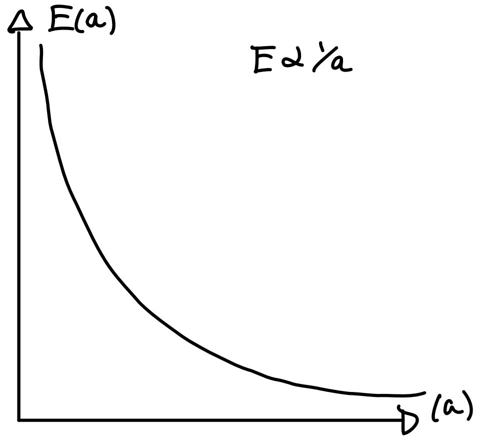
$$\boxed{\vec{E} = \frac{1}{\varepsilon_0 r}\left(\frac{\rho_0}{3} a^2 + \sigma b\right)\hat{s}}$$

Problem 4 | Continued

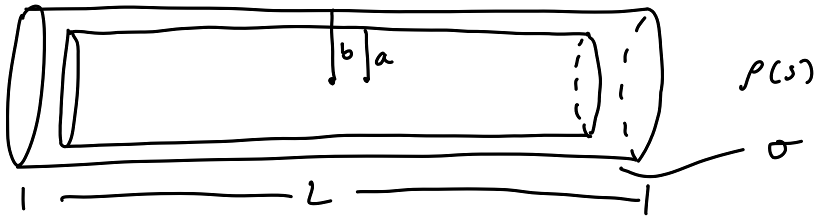
d.)

Graph of $|E|$



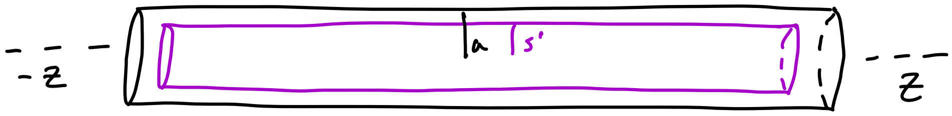


Problem 4)



$\rho(s) = \rho_0 \frac{s}{a}$

a.) $s < a$



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \;\vdots\; Q_{ENC} = \int_v \rho \, d\tau \qquad d\tau = s\,ds\,d\phi\,dz$$

$$Q_{ENC} = \frac{\rho_0}{a}\int_0^L\int_0^{2\pi}\int_0^{s'} s^2\,ds\,d\phi\,dz$$

$$= \frac{\rho_0}{a}\int_0^L\int_0^{2\pi} \frac{s^3}{3}\Big|_0^{s'} d\phi\,dz = \frac{\rho_0 s'^3}{a\,3}\int_0^L\int_0^{2\pi} d\phi\,dz = \frac{\rho_0 s'^3}{a\,3}\cdot 2\pi\cdot L$$

$$Q_{ENC} = \frac{2\pi}{3}\rho_0 \frac{l s'^3}{a}$$

$\oint \vec{E}\cdot d\vec{a}$ ⋮ E constant @ given radius, $d\vec{a} \rightarrow$ exists on sphere

$d\vec{a} = s'\,d\phi\,dz$

$$\oint \vec{E}\cdot d\vec{a} = E\cdot s'\int_0^L\int_0^{2\pi} d\phi\,dz = E\cdot s'\cdot 2\pi\cdot L$$

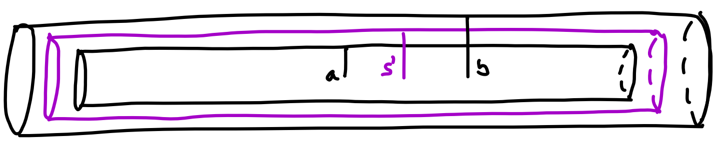
$$\oint \vec{E}\cdot d\vec{a} = E\cdot 2\pi L s'$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$$

$$E\cdot 2\pi l s' = \frac{2\pi\rho_0 l s'^3}{3a\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho_0 s'^2}{3a\varepsilon_0}\hat{s}}$$

b.) $a < s < b$



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \qquad d\tau = s\,ds\,d\phi\,dz, \quad \rho = \rho_0\frac{s}{a}$$

$$Q_{ENC} = \int_v \rho\,d\tau \;\vdots\; \frac{\rho_0}{a}\int_0^L\int_0^{2\pi}\int_0^a s^2\,ds\,d\phi\,dz$$

$$\frac{\rho_0}{a}\int_0^L\int_0^{2\pi}\frac{s^3}{3}\Big|_0^a d\phi\,dz = \frac{\rho_0 a^2}{3}\int_0^L\int_0^{2\pi} d\phi\,dz = \frac{\rho_0 a^2}{3}\cdot 2\pi\cdot L$$

$$Q_{ENC} = \frac{2\pi}{3}\cdot\rho_0 a^2 L$$

$\oint \vec{E}\cdot d\vec{a}$: E constant @ given radius $\qquad \vec{E} = E\hat{s}$, $d\vec{a} = s'\,d\phi dz$
$d\vec{a}$ lives on gaussian surface

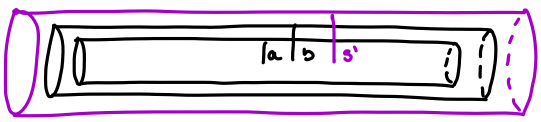
$\oint \vec{E}\cdot d\vec{a} = E\cdot s'\int_0^{2\pi}\int_0^L d\phi dz = E\cdot 2\pi\cdot L\cdot s' \qquad \oint \vec{E}\cdot d\vec{a} = E\cdot 2\pi L s'$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$$

$$E\cdot 2\pi L s' = \frac{2\pi}{\varepsilon_0 3}\rho_0 a^2 L$$

$$\boxed{\vec{E} = \frac{\rho_0}{3\varepsilon_0}\frac{a^2}{s'}\hat{s}}$$

c.) $s > b$



$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0} \qquad d\tau = s\,ds\,d\phi dz$, $\rho = \rho_0 \frac{s}{a}$ $\qquad d\vec{a}$ lives on Gaussian
$d\vec{a} = b\, d\phi dz$

$$Q_{ENC} = Q_{vol} + Q_{sur}$$

$$Q_{sur} = \int_s \sigma\, da = \sigma b\int_0^L\int_0^{2\pi} d\phi dz = \sigma b\cdot 2\pi\cdot L$$

$$Q_{ENC} = \frac{2\pi}{3}\rho_0 a^2 L + \sigma b 2\pi L \qquad 2\pi L\left(\frac{\rho_0}{3}a^2 + \sigma b\right)$$

$\oint \vec{E}\cdot d\vec{a}$: E constant at given radius
$d\vec{a}$ lives on gaussian $\qquad d\vec{a} = s'\,d\phi\, dz$

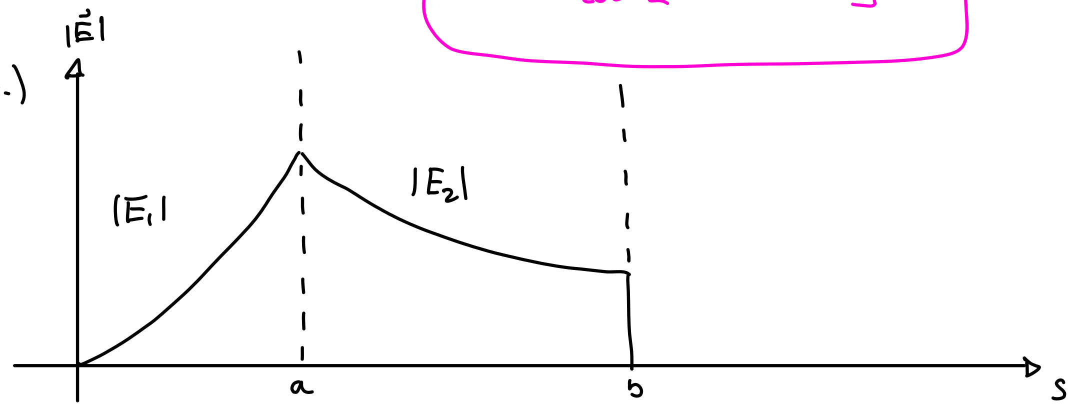
$$E\cdot s'\int_0^L\int_0^{2\pi} d\phi dz = E\cdot s'\cdot 2\pi\cdot L \quad : \quad E\cdot 2\pi L s'$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$$

$$E\cdot 2\pi L s' = \frac{2\pi L}{\varepsilon_0}\left(\frac{\rho_0}{3}a^2 + \sigma b\right)$$

$$\boxed{\vec{E} = \frac{1}{\varepsilon_0 s'}\left[\frac{1}{3}\rho_0 a^2 + \sigma b\right]\hat{s}}$$

d.)



Problem 5

Electrostatics $\rightarrow$ $(\vec{\nabla}\times\vec{E}) = 0$

a.) $\vec{E} = K[x^2y\,\hat{x} + 3yz^2\hat{y} + 2xz^3\hat{z}]$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 3yz^2 & 2xz^3 \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}(2xz^3) - \frac{\partial}{\partial z}(3yz^2)\right) + \hat{y}\left(\frac{\partial}{\partial z}(x^2y) - \frac{\partial}{\partial x}(2xz^3)\right) + \hat{z}\left(\frac{\partial}{\partial x}(3yz^2) - \frac{\partial}{\partial y}(x^2y)\right)$$

$$\vec{\nabla}\times\vec{E} = K[(-6yz)\hat{x} + (-2z^3)\hat{y} + (-x^2)\hat{z}]$$

b.) $\vec{E} = K[3y^2\hat{x} + (6xy+3z^2)\hat{y} + (6yz)\hat{z}]$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y^2 & (6xy+3z^2) & 6yz \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}(6yz) - \frac{\partial}{\partial z}(6xy+3z^2)\right) + \hat{y}\left(\frac{\partial}{\partial z}(3y^2) - \frac{\partial}{\partial x}(6yz)\right) + \hat{z}\left(\frac{\partial}{\partial x}(6xy+3z^2) - \frac{\partial}{\partial y}(3y^2)\right)$$

$$\vec{\nabla}\times\vec{E} = K[(0)\hat{x} + (0)\hat{y} + (0)\hat{z}] \quad \therefore$$

$\boxed{\vec{E} = K[x^2y\,\hat{x} + 3yz^2\hat{y} + 2xz^3\hat{z}] \text{ is an impossible electrostatic field}}$

$E_x = \frac{-dV}{dx}$

$\int -dV = \int E_x\,dx$

$-V = \int 3y^2\,dx$

$-V = 3y^2x + f(y,z)$

$V = -3y^2x + f(y,z)$

$E_y = \frac{-dV}{dy}$

$\int E_y\,dy = -\int dV$

$\int 6xy + 3z^2\,dy = -\int dV$

$3xy^2 + 3yz^2 + f(x,z) = -V$

$V = -3xy^2 - 3yz^2 + f(x,z)$

$E_z = \frac{-dV}{dz}$

$\int E_z\,dz = -\int dV$

$\int 6yz\,dz = -\int dV$

$3yz^2 + f(x,y) = -V$

$V = -3yz^2$

$$\boxed{V = -3xy^2 - 3yz^2}$$