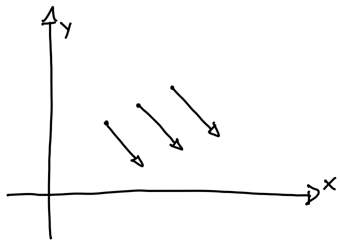
A vector field is a function that assigns to each point in R^2, a vector in R_3

Ex: $F: \mathbb{R}^2 \to \mathbb{R}^2$

$f(x,y) = 0.4\hat{i} - 0.3\hat{j}$



Ex: $f(x,y) = \dfrac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$

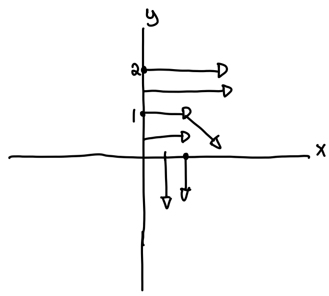
$F(0,0)$ undefined

$F(1,1) \quad \dfrac{i-j}{\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle$

$F(0,2) \quad \dfrac{2i-0j}{2} = \langle 1, 0 \rangle$

$F(1,0) \quad \dfrac{-j}{1} = \langle 0, -1 \rangle$

$F(\tfrac{1}{2},0) \quad \dfrac{-\frac{1}{2}j}{\sqrt{(\frac{1}{2})^2}} = \langle 0, -1 \rangle$



Gradient vector field $\nabla f(x,y)$

$f(x,y) = xe^{9xy}$

$f_x = e^{9xy} + x \cdot 9y e^{9xy}$

$f_y = 9x^2 e^{9xy}$

$\nabla f = e^{9xy} \langle 1 + 9xy, 9x^2 \rangle$

$f(x,y,z) = 3\sqrt{x^2+y^2+z^2}$

$f_x = \frac{3}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2x = \dfrac{3x}{\sqrt{x^2+y^2+z^2}}$

$f_y = \frac{3}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2y = \dfrac{3y}{\sqrt{x^2+y^2+z^2}}$

$f_z = \frac{3}{2}(x^2+y^2+z^2)^{-\frac{1}{2}} \cdot 2z = \dfrac{3z}{\sqrt{x^2+y^2+z^2}}$

$\dfrac{3}{\sqrt{x^2+y^2+z^2}} \langle x, y, z \rangle$