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Dr. Gustafson
MATH 366
CP. Ch. 19.5

1)

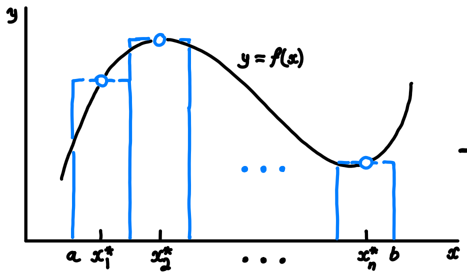
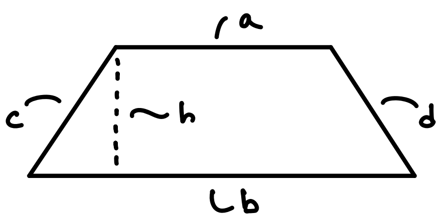


Fig. 441. Rectangular rule

If f was a constant Function, there wouldn't be any error.

2) a & b are bases
c & d are side lengths
h is the height



$$\boxed{A = \left(\frac{a+b}{2}\right)h}$$

3)

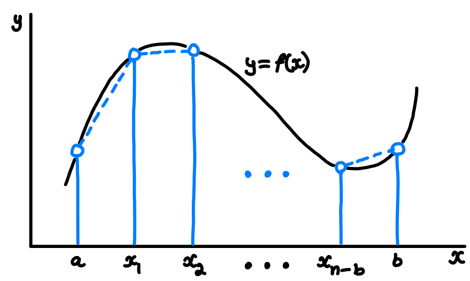


Fig. 442. Trapezoidal rule

$A = \frac{1}{2}[f(a) + f(x_1)]h$, $A = \frac{1}{2}[f(x_1) + f(x_2)]h$, ..., $A = \frac{1}{2}[f(x_{n-1}) + f(b)]h$.

If f were a linear Function, there wouldn't be any error.

4) The second derivative of f is used for M and M^*.

5)

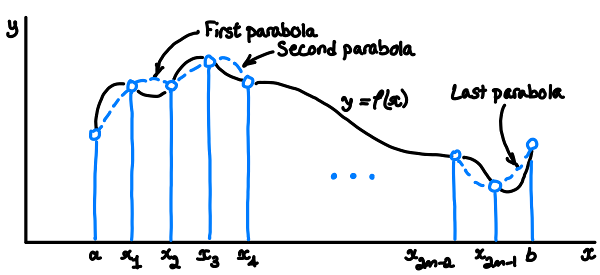


Fig. 443. Simpson's rule

A quadratic function was used for the simpsons rule.

6) The Fourth derivative is used for f to find M_4 and M_4^*.

7) Trapezoidal Rule : DP=1 , Simpsons Rule : DP=3 , Midpoint Rule : DP=0

8) $f^{(4)}$ is identically zero for a cubic polynomial. This makes Simpson's rule sufficiently accurate for most practical problems and accounts for its popularity.

9) Simpson's Rule is numerically stable.

11) The idea is to adapt step h to the variability of $f(x)$. That is, where f varies but little, we can proceed in large steps without causing a substantial error in the integral, but where f varies rapidly, we have to take small steps in order to stay everywhere close enough to the curve of f.

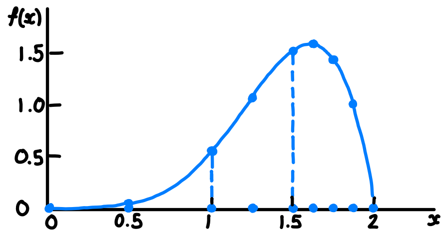


Fig. 444. Adaptive integration in Example 6

12)

$$\int_{-1}^{1} f(t)\, dt \approx \sum_{j=1}^{n} A_j f_j$$

13)

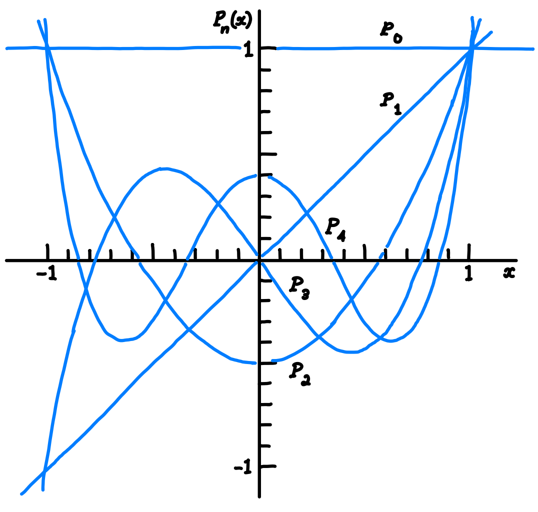


Fig. 107. Legendre polynomials

14)

(6) $$P_2(x) = \frac{x^2 - x \cdot x_2 - x \cdot x_1 + x_1 x_2}{h^2} f_0 + \frac{x^2 - x \cdot x_2 - x x_0 + x_0 x_2}{h^2} f_1 + \frac{x^2 - x x_1 - x x_0 + x_0 x_1}{h^2} f_2$$

$$P_2'(x) = \frac{2x - x_2 - x_1}{h^2} f_0 + \frac{2x - x_2 - x_0}{h^2} f_1 + \frac{2x - x_1 - x_0}{h^2} f_2$$