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MATH 360
HW 5

10.7 #9,12

Problem 10.7.9

$$\iint_S \vec{F}\cdot\hat{n}\, dA = \iiint \vec{\nabla}\cdot\vec{F}\, dv$$

$\vec{F}=[x^2, 0, z^2]$ $\quad -1 \le x \le 1$, $-3 \le y \le 3$, $0 \le z \le 2$

$$\vec{\nabla}\cdot\vec{F} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]\cdot\left[x^2, 0, z^2\right] = \frac{\partial}{\partial x}x^2 + \frac{\partial}{\partial y}0 + \frac{\partial}{\partial z}z^2$$

$\vec{\nabla}\cdot\vec{F} = 2x + 2z$ $\quad \therefore$

$$\int_0^2\int_{-3}^3\int_{-1}^1 2x+2z \; dx\,dy\,dz$$

$$\int_0^2\int_{-3}^3 \left[x^2 + 2zx\right]_{-1}^1 dy\,dz$$

$(1^2+2z)-(1^2-2z)$

$4z$

$$\int_0^2\int_{-3}^3 4z \; dy\,dz$$

$$\int_0^2 \left[4zy\right]_{-3}^3 dz$$

$(4z(3))-(4(z)(-3))$

$24z$

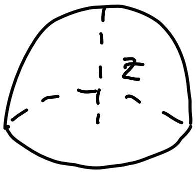
$$\int_0^2 24z\; dz$$

$$12z^2\Big|_0^2 = 12(4)-12(0) = 48$$

$$\boxed{\mathrm{div}(\vec{F}) = 2x+2z \;:\; 48}$$

Problem 10.7.12

$$\iint_S \vec{F}\cdot\vec{n}\,dA = \iiint \vec{\nabla}\cdot\vec{F}\,dV$$



$$\vec{F} = [x^3 - y^3,\ y^3 - z^3,\ z^3 - x^3]$$

Spherical : $r = 5$

$$\operatorname{div}(\vec{F}) = \vec{\nabla}\cdot\vec{F} = \left[\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z\right]\cdot\left[x^3-y^3,\ y^3-z^3,\ z^3-x^3\right]$$

$$\operatorname{div}(\vec{F}) = \frac{\partial}{\partial x}(x^3-y^3) + \frac{\partial}{\partial y}(y^3-z^3) + \frac{\partial}{\partial z}(z^3-x^3) = 3x^2+3y^2+3z^2$$

$\theta \in [0, \pi/2]$ $\quad \phi \in [0, 2\pi]$

$$3(x^2+y^2+z^2)$$

$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$$3(r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta)\ d\tau$$

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^5 3(r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta)\ r^2\sin\theta\ dr\,d\theta\,d\phi$$

$$3\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^5 r^4\sin^3\theta\cos^2\phi + r^4\sin^3\theta\sin^2\phi + r^4\sin\theta\cos^2\theta\ dr\,d\theta\,d\phi$$

$$r^4\sin^3\theta(\cos^2\phi + \sin^2\phi) + r^4\sin\theta\cos^2\theta$$
$$r^4\sin^3\theta(1) + r^4\sin\theta\cos^2\theta$$
$$r^4\sin\theta(\sin^2\theta + \cos^2\theta)$$
$$r^4\sin\theta\,(1)$$

$$3\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^5 r^4\sin\theta\ dr\,d\theta\,d\phi$$

$$3\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{r^5}{5}\sin\theta\,\Big|_0^5\ d\theta\,d\phi$$

$$3\cdot 625\int_0^{2\pi} -\cos(\theta)\,\Big|_0^{\frac{\pi}{2}}\ d\phi$$

$$\int_0^{2\pi} 1875\ d\phi = 1875\,\phi\,\Big|_0^{2\pi} = 3750\pi$$

$$\boxed{\operatorname{div}(\vec{F}) = [3x^2, 3y^2, 3z^2] : 3750\pi}$$