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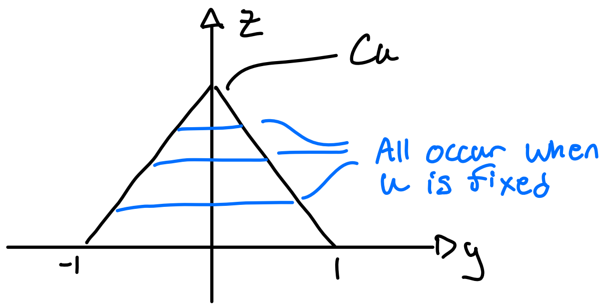
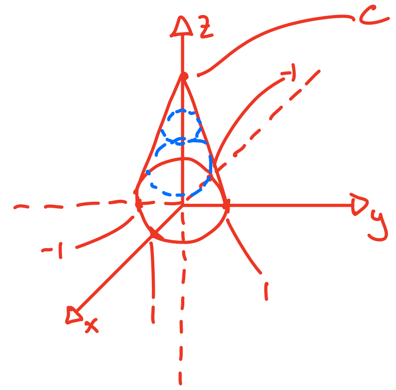
Dr. Gustafson
MATH 360
HW Ch. 10.5

#3-5, 14, 15

Problem 10.5.3 Cone $\vec{r}(u,v) = [u\cos(v), u\sin(v), cu]$

$0 \le u \le \infty$
$0 \le v \le 2\pi$

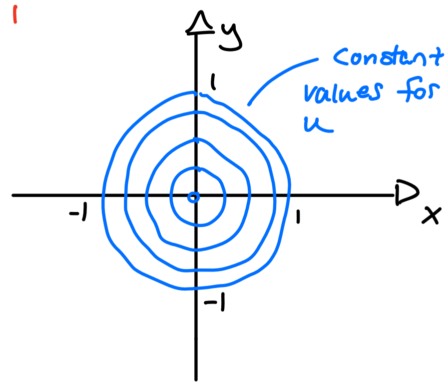




a.)

Parameter Curves

$u=1,\ v=0 \qquad \vec{r}(1,0) = [1, 0, c]$
$u=1,\ v=\pi/2 \qquad \vec{r}(1,\pi/2) = [0, 1, c] \implies$
$u=1,\ v=\pi \qquad \vec{r}(1,\pi) = [-1, 0, c]$
$u=1,\ v=3\pi/2 \qquad \vec{r}(1,3\pi/2) = [0, -1, c]$



we observe there is a circle in the x-y plane centered at the origin. The equation of a circle is $r^2 = x^2+y^2$. Since the circle is in the x-y plane and the cone will extend vertically in the z-direction.

In xy plane circle is $z = \sqrt{x^2+y^2}$

Since the cone extends in the z direction we have a multiplying factor of c, so the equation becomes

$$\boxed{z = c\sqrt{x^2+y^2}}$$

b.) $\vec{N} = \vec{r}_u \times \vec{r}_v$

$\vec{r}_u = [\cos(v), \sin(v), c]$
$\vec{r}_v = [-u\sin(v), u\cos(v), 0]$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos(v) & \sin(v) & c \\ -u\sin(v) & u\cos(v) & 0 \end{vmatrix} = \hat{i}[0\sin(v) - cu\cos(v)] + \hat{j}[-cu\sin(v) - \cos(v)(0)] + \hat{k}[u\cos^2(v) + u\sin^2(v)]$$

$\vec{r}_u \times \vec{r}_v = \vec{N} = [-cu\cos(v), -cu\sin(v), u]$

$$\boxed{\vec{N} = [-cu\cos(v), -cu\sin(v), u]}$$

Problem 10.5.41

Elliptic Cylinder $\vec{r}(u,v) = [a\cos(v), b\sin(v), u]$

$x = a\cos(v)$

$y = b\sin(v)$

$z = u$

Elliptic cylinder : $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $\boxed{z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}}$

a.) $\therefore$ This cylinder is coming out of the z-axis

$u =$ Constant, ellipse in $z=u$ plane parallel to xy plane

$v =$ constant, $z = u$

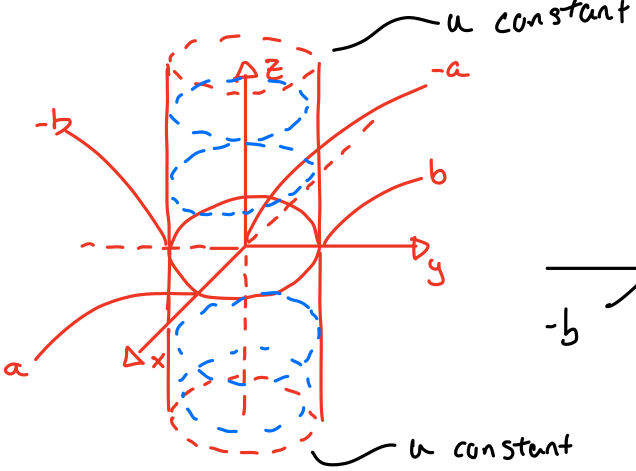
$x = k_1 a$ $\Rightarrow (k_1 a, k_2 b, u)$ — Line parallel to z-axis, this shows that

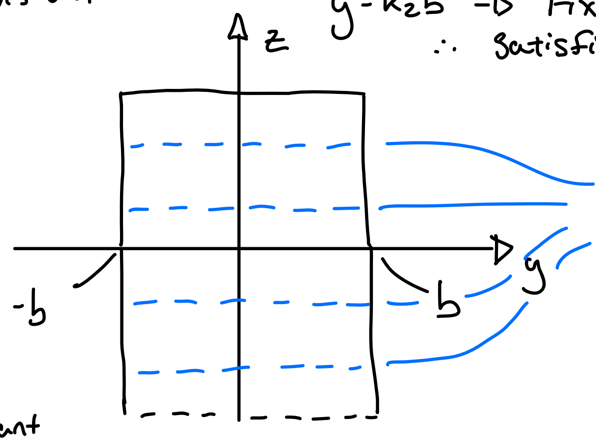
$y = k_2 b$

$x = k_1 a \rightarrow$ Fixed

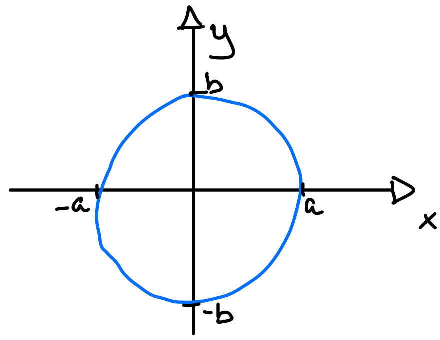
$y = k_2 b \rightarrow$ Fixed

$\therefore$ Satisfies ellipse equation





All occur when v is a constant



b.) $\vec{N} = \vec{r}_u \times \vec{r}_v$

$\vec{r}_u = [0, 0, 1]$

$\vec{r}_v = [-a\sin(v), b\cos(v), 0]$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -a\sin(v) & b\cos(v) & 0 \end{vmatrix}$$

$$\vec{r}_u \times \vec{r}_v = \hat{i}\,[0(0) - 1\cdot b\cos(v)] + \hat{j}\,[-a\sin(v) - 0(0)] + \hat{k}\,[0\cdot b\cos(v) + 0\cdot a\sin(v)]$$

$$\boxed{\vec{N} = [-b\cos(v), -a\sin(v), 0]}$$

Problem 10.5.5

Paraboloid of revolution $\vec{r}(u,v) = [u\cos(v), u\sin(v), u^2]$

$x = u\cos(v)$

$y = u\sin(v)$

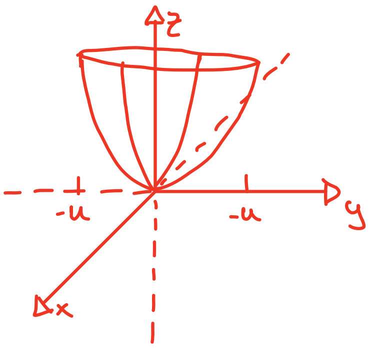
$z = u^2$

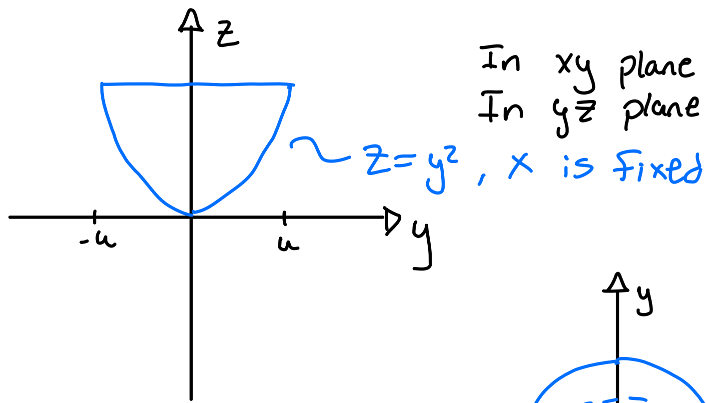
a.) Take for instance

$$x^2 = u^2\cos^2(v)$$
$$y^2 = u^2\sin^2(v)$$

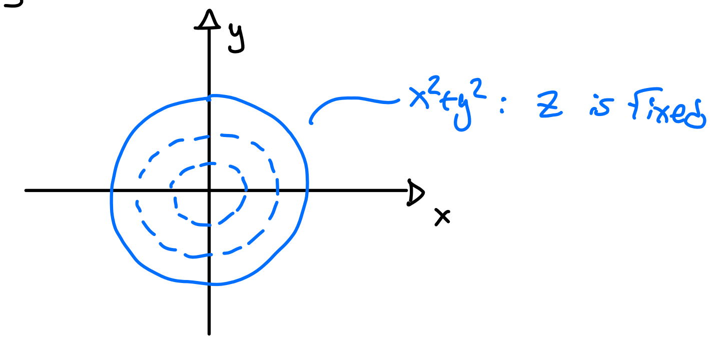
$$x^2+y^2 = u^2\cos^2(v) + u^2\sin^2(v) = u^2(\cos^2(v)+\sin^2(v)) = u^2 = z$$

$\therefore$ we can say that $\boxed{z = x^2+y^2}$





In xy plane there are circles.
In yz plane there are parabolas.



b.) $\vec{N} = \vec{r}_u \times \vec{r}_v$

$$\vec{r}_u = [\cos(v), \sin(v), 2u]$$
$$\vec{r}_v = [-u\sin(v), u\cos(v), 0]$$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos(v) & \sin(v) & 2u \\ -u\sin(v) & u\cos(v) & 0 \end{vmatrix}$$

$$= \hat{i}(0\cdot\sin(v) - 2u^2\cos(v)) + \hat{j}(-2u^2\sin(v) - 0\cos(v)) + \hat{k}(u\cos^2(v) + u\sin^2(v))$$

$$\boxed{\vec{N} = [-2u^2\cos(v), -2u^2\sin(v), u]}$$

Problem 10.5.14 Plane $4x+3y+2z=12$

a.) $x = u$

$y = v$

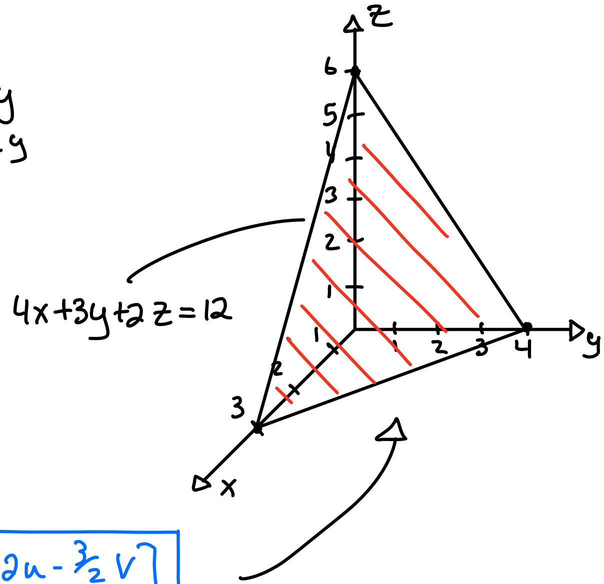
$z = 6 - 2u - \frac{3}{2}v$

$2z = 12 - 4x - 3y$

$z = 6 - 2x - \frac{3}{2}y$

$\vec{r}(u,v) = [u, v, 6 - 2u - \frac{3}{2}v]$

$4x + 3y + 2z = 12$

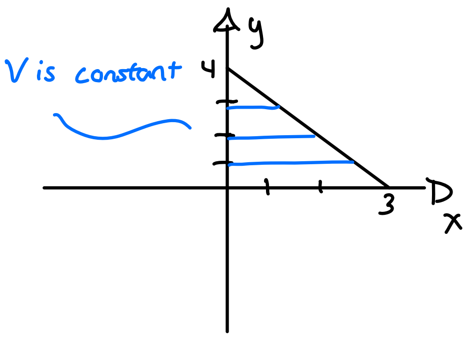


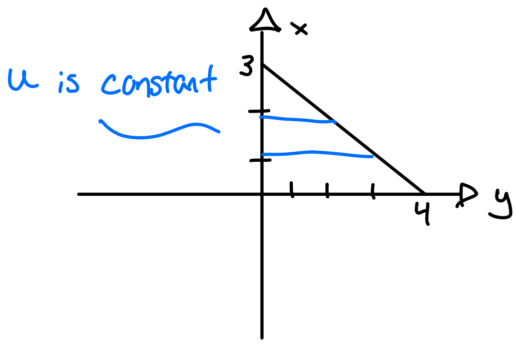
$(x,y) = 0$: $2z = 12$ $\quad z = 6$: $(0,0,6)$

$(x,z) = 0$: $3y = 12$ $\quad y = 4$: $(0,4,0)$

$(y,z) = 0$: $4x = 12$ $\quad x = 3$: $(3,0,0)$

$$\boxed{\vec{r}(u,v) = [u, v, 6 - 2u - \tfrac{3}{2}v]}$$





b.) $\vec{N} = \vec{r}_u \times \vec{r}_v$

$\vec{r}_u = [1, 0, -2]$

$\vec{r}_v = [0, 1, -\frac{3}{2}]$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & -2 \\ 0 & 1 & -\frac{3}{2} \end{vmatrix} = \hat{\imath}\left(0(-\tfrac{3}{2}) + 2(1)\right) + \hat{\jmath}\left(-2(0) - 1(-\tfrac{3}{2})\right) + \hat{k}\left(1(1) - 0(0)\right)$$

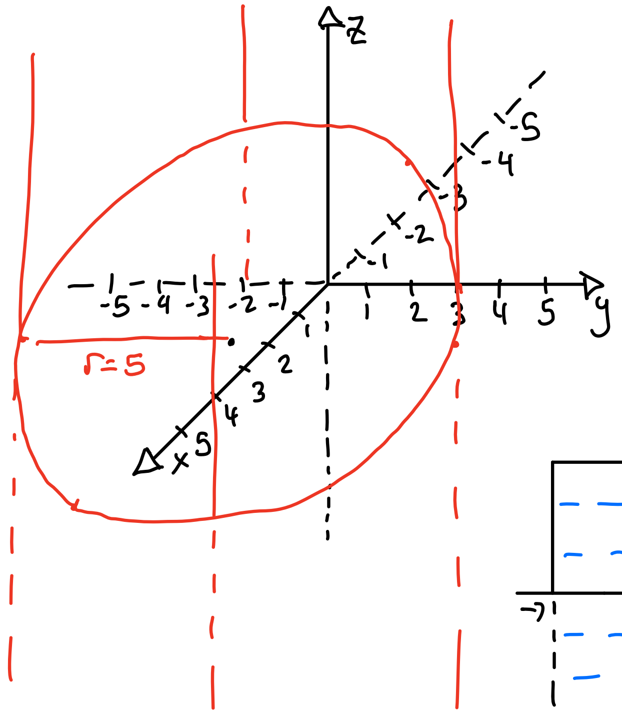
$$\boxed{\vec{N} = [2, \tfrac{3}{2}, 1]}$$

Problem 10.5.15 | Cylinder of revolution $(x-2)^2+(y+1)^2=25$

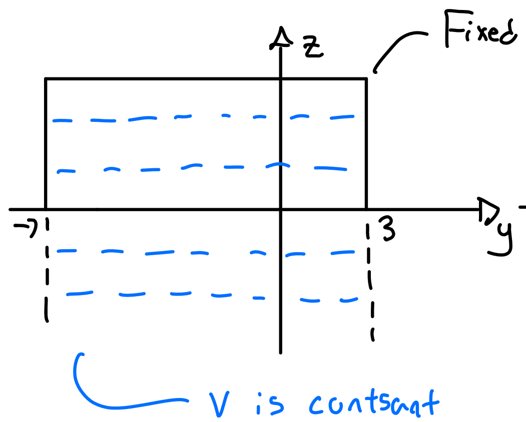
a.)

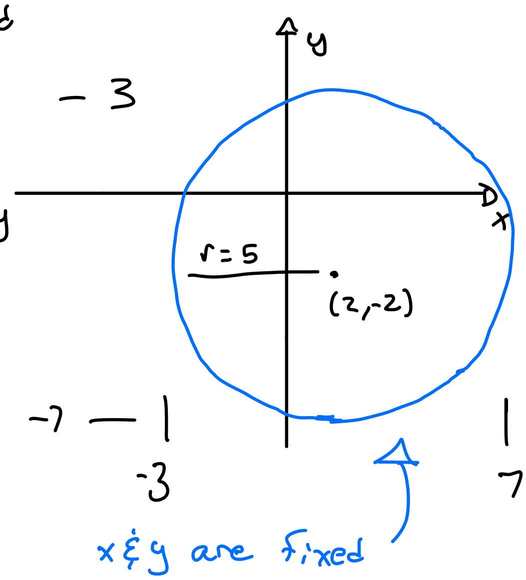
Circle centered at $(2,-1)$ w/ radius of 5.
The cylinder will extend in the z-direction

$$x=2+5\cos(u) \qquad -\infty \le u \le \infty$$
$$y=-1+5\sin(u) \qquad 0 \le v \le 2\pi$$
$$z=v$$



$$\boxed{\vec{r}(u,v)=[2+5\cos(u),\,-1+5\sin(u),\,v]}$$





b.) $\vec{N}=\vec{r}_u \times \vec{r}_v$

$$\vec{r}_u=[-5\sin(u),\,5\cos(u),\,0]$$
$$\vec{r}_v=[0,0,1]$$

$$\vec{r}_u\times\vec{r}_v=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -5\sin(u) & 5\cos(u) & 0\\ 0 & 0 & 1\end{vmatrix}=\hat{i}(5\cos(u)(1)-0(0))+\hat{j}(0(0)+5\sin(u)(1))+\hat{k}(-5\sin(u)(0)-5\cos(u)(0))$$

$$\boxed{\vec{N}=[5\cos(u),\,5\sin(u),\,0]}$$