4.4 Undetermined Coefficients

4.4 # 22, 24, 44, 36, 42

Consider the linear ODE

$L(y) = f$

The general solution is

$y = y_h + y_p$

y_h Solution of $L(y) = 0$

y_p Solution of $L(y) = f$

Ex: 1 $y'' - y' - 2y = 2t + 1 - 2e^t$

$L(y) = y'' - y' - 2y$

$f_1 = 2t + 1$

$f_2 = -2e^t$

$f = f_1 + f_2$

f_1: $y'' - y' - 2y = 2t + 1$

f_1: $y = -t$

f_2: $y'' - y' - 2y = -2e^t$

f_2: $y = e^t$

$f = f_1 + f_2$

$f = -t + e^t$

$f = e^t - t$

$y_p = e^t - t$

y_h: $y'' - y' - 2y = 0$ $\quad r = 2, -1$

$r^2 - r - 2$

$(r-2)(r+1)$

$y_h = c_1 e^{2t} + c_2 e^{-t}$

$y = y_h + y_p$

$y = c_1 e^{2t} + c_2 e^{-t} + e^t - t$

$y'' - y' - 2y = t + \frac{1}{2} + 8e^t$

$y_p = -\frac{t}{2} - 4e^t$

$y = c_1 e^{2t} + c_2 e^{-t} - t/2 - 4e^t$

$ay'' + by' + cy = d$

$y_p = d/c$

$c \neq 0$

$y'' + y' = 1$

$t = 0$

$y'' + p(t)y' + q(t)y = f(t)$

$y = y_h + y_p$

- $y'' + p(t)y' + q(t)y = 0$

$y_h = c_1 y_1(t) + c_2 y_2(t)$

- $y'' + p(t)y' + q(t)y = f(t)$

$y_p = v_1(t) y_1(t) + v_2(t) y_2(t)$

$y_p' = v_1' y_1 + v_1 y_1' + v_2' y_2 + v_2 y_2'$

$v_1' y_1 + v_2' y_2 = 0 \Leftarrow$ Auxilary condition

$y_p' = v_1 y_1' + v_2 y_2'$

$y_p'' = v_1' y_1' + v_1 y_1'' + v_2' y_2' + v_2 y_2''$

$v_1' y_1' + v_1 y_1'' + v_2' y_2' + v_2 y_2'' + p(t)(v_1 y_1' + v_2 y_2') + q(t)(v_1 y_1 + v_2 y_2) = f(t)$

$y'' \quad + p \quad y' \quad + q \quad y \quad = f(t)$

$v_1(y_1'' + p y_1' + q y_1) + v_2(y_2'' + p y_2' + q y_2) + (v_1' y_1' + v_2' y_2') = f(t)$

$v_1' y_1' + v_2' y_2' = f(t)$

$v_1' y_1 + v_2' y_2 = 0$

$$\begin{bmatrix} y_1' & y_2' \\ y_1 & y_2 \end{bmatrix} \begin{bmatrix} v_2' \\ v_1' \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

$$v_1' = \frac{\begin{vmatrix} 0 & y_2' \\ f & y_2 \end{vmatrix}}{\begin{vmatrix} y_1' & y_2' \\ y_1 & y_2 \end{vmatrix}} = \frac{-y_2' f}{W(y_1, y_2)} \qquad v_2' = \frac{-y_1' f}{W(y_1, y_2)}$$

Ex:

$y'' - y' - 2y = 5e^{2t}$

$(r^2 - r - 2) = 0$

$(r-2)(r+1)$

$r_1 = 2, r_2 = -1$

$y_h = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$y_h = c_1 e^{2t} + c_2 e^{-t}$

$y_p = \frac{5}{3} t e^{2t}$

$y_p = Ate^{2t} \Leftarrow$ (must be linearly independent)

$y_p' = Ae^{2t} + 2Ate^{2t}$

$y_p'' = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t}$

$4Ae^{2t} + 4Ate^{2t} - (Ae^{2t} + 2Ate^{2t}) - 2Ate^{2t} = 5e^{2t}$

$4A + 4At - A - 2At - 2At = 5$

$3A = 5$

$A = \frac{5}{3}$

$y_G = c_1 e^{2t} + c_2 e^{-t} + \frac{5}{3} t e^{2t}$