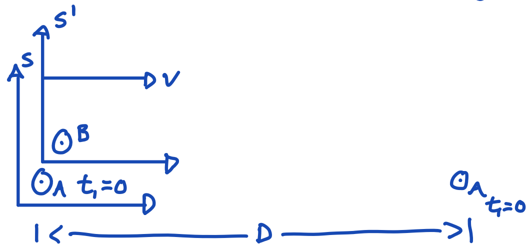
Paradoxes

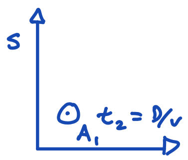
If your clock runs slow to me, how does my clock run slow to you?

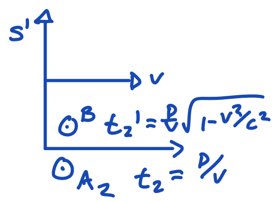
Clocks A_1 & A_2 in S : B in S', moving at speed V RT to S



Notice :

- D is the rest length Between A_1 : A_2
- A_1 : A_2 are synchronized in S
- Let B pass A_1 @ $t_1' = t_1 = 0$



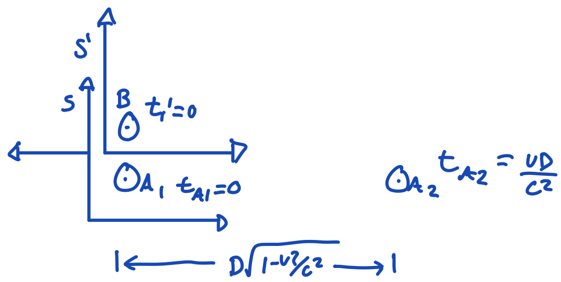


$$V = \frac{D}{\Delta t} \quad \therefore \quad \Delta t = \frac{D}{V}$$

$$\Delta t' = \Delta t\sqrt{1 - v^2/c^2} = \frac{D}{V}\sqrt{1-v^2/c^2}$$

Now, look @ the same experiment in S' frame...

- According to S', A_1-A_2 length must be contracted



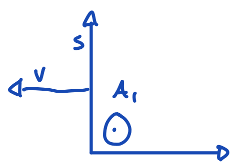
According to S'....

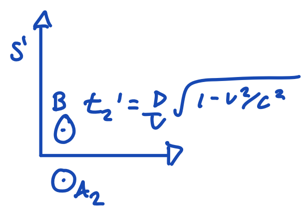
$$V = \frac{D\sqrt{1-v^2/c^2}}{\Delta t'} \quad \text{so} \quad \Delta t' = \frac{D\sqrt{1-v^2/c^2}}{V}$$

A_2 will read ?

$$t_{A2} = D/V$$

To agree with the 1st scenario





Now $t_{A_2} > t_B$ but moving clocks run slow

The way out of the paradox is due to clocks A_1 & A_2 are not synchronized to one another from S'.

$$t_{A2} = \frac{VD}{c^2}$$

$$\Delta t_{A_2} = \Delta t' \cdot \sqrt{1-v^2/c^2} = \frac{D}{V}\sqrt{1-v^2/c^2} \cdot \sqrt{1-v^2/c^2}$$

notice, here, $\Delta x = 0$

$$\Delta t_{A_2} = \frac{D}{V}(1 - v^2/c^2)$$

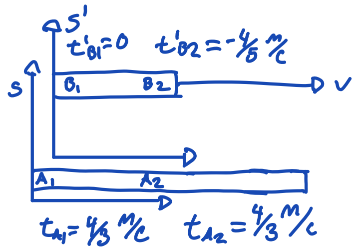
Time interval for A_1 & A_2 as seen by S'

The final reading on A_2's clock must be

$$t_{A_2f} = t_{A_2i} + \Delta t_{A_2} = \frac{VD}{c^2} + \frac{D}{V}(1 - v^2/c^2)$$

$$= \frac{VD^2}{c^2} + \frac{D}{V} - \frac{VD^2}{c^2} = \frac{D}{V}$$

$v = \frac{4}{5}c \quad D_B = 1\,m$



$$t'_{B2} = -\frac{VD}{c^2} = \frac{-(\frac{4}{5}c)(1m)}{c^2} = -\frac{4}{5}\,m/c$$

Notice:

From B's point of view, the measurement by A_2 was made too early, So of Course A concludes that B is too short.

Why does B think A's Stick is too short?

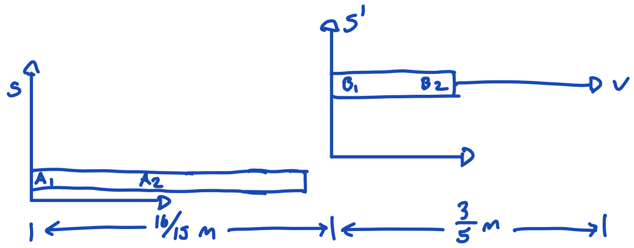
$$\Delta t_B' = \frac{4}{5}\,m/c$$

According to S, this corresponds to a time interval of ....

$$\Delta t_B = \frac{\Delta t_B'}{\sqrt{1 - v^2/c^2}} = \frac{\frac{4}{5}\,m/c}{\frac{3}{5}c} = \frac{4}{3}\,m/c$$

How far has the stick advanced?

$$D = V \cdot \Delta t = (\tfrac{4}{5}c)(\tfrac{4}{3}\,m/c) = \tfrac{16}{15}\,m$$

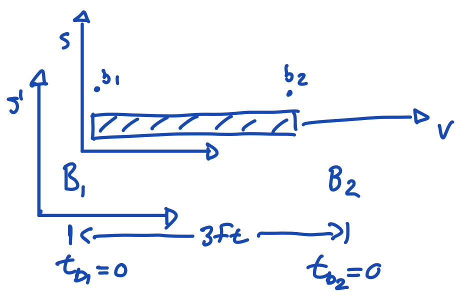


Both observers belive the other's Stick is Short

$x_{B_2} = \frac{5}{3}\,m$

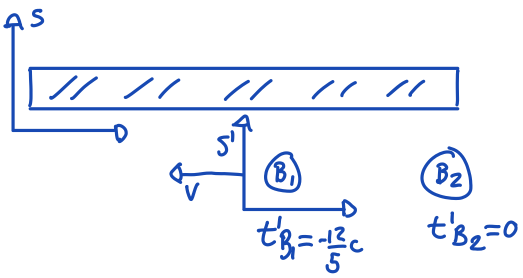
Magicians Assitant

$D_A = 5\,Ft$
$D_E = 3\,Ft$
$d_A = 3\,Ft$
$d_E = 1.8\,Ft$



$$d_A = (5)\sqrt{1 - (\tfrac{4}{5})^2} = 5(\tfrac{3}{5}) = 3\,Ft$$
$$d_E = (3)\sqrt{1 - (\tfrac{4}{5})^2} = 3(\tfrac{3}{5}) = 1.8\,Ft$$

Blade Chops are Simultaneous to Exe
Not simultaneous to Assistant



According to S, S's D is contracted
$d = 1.8\,Ft$ but $D = 3\,Ft$

$$\therefore\ t'_{B_1} = -\frac{VD}{c^2} = \frac{-\frac{4}{5}c\,(3Ft)}{c^2} = -\frac{12}{5}\,\frac{ft}{c}$$

Time must past by of
$$\Delta t' = \frac{12}{5}\,\frac{ft}{c}$$
according to S'

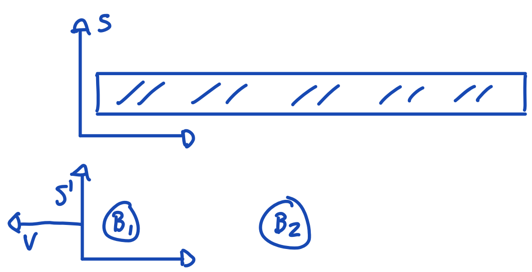
During this time interval, executioners will move

$$\Delta x = V \cdot \Delta t = \tfrac{4}{5}c \cdot 4\,Ft/c = 3.2\,Ft$$

S will measure

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \frac{\frac{12}{5}\,Ft/c}{\frac{3}{5}} = 4\,Ft/c$$

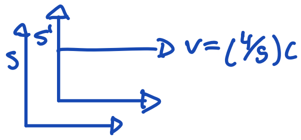
$$\Delta t = 4\,Ft/c$$



The blades chop at...
$3.2\,Ft + 1.8\,Ft = 5.0\,Ft$

That bitch lives!!

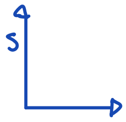
<u>Twin Paradox</u>

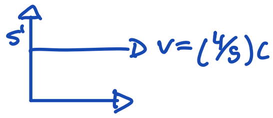


$$D = v\Delta t = \left(\tfrac{4}{5}c\right)(5\,\text{yrs})$$
$$= 4\ c\cdot\text{yrs}$$

AL - S

Bertha - S'





Bertha turns back

$$\Delta t' = 5\,\text{yrs} \cdot \sqrt{1 - v^2/c^2} = 5\,\text{yrs} \cdot \tfrac{3}{5} = 3\,\text{yrs}$$

$t_B = 6\,\text{yrs}$

$t_{Al} = 10\,\text{yrs}$