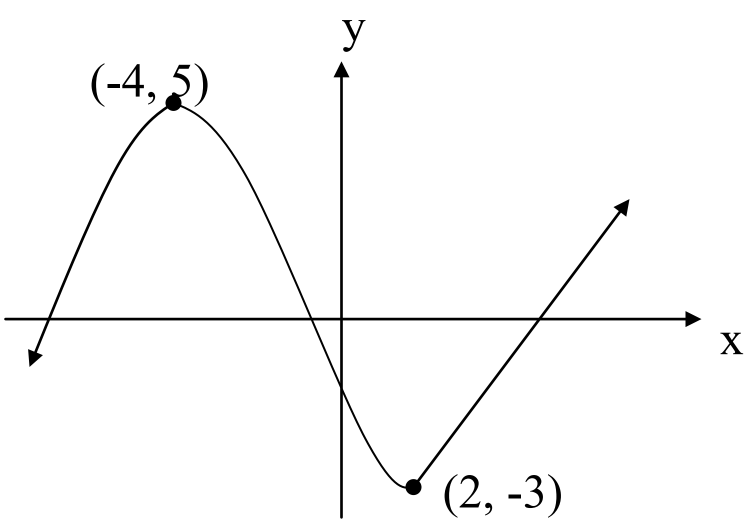
MAT 201
Larson/Edwards – Section 3.1
Extrema on an Interval

Consider the following graph:



What do we call the points (-4, 5) and (2, -3)?

<u>Definition of Extrema</u>:

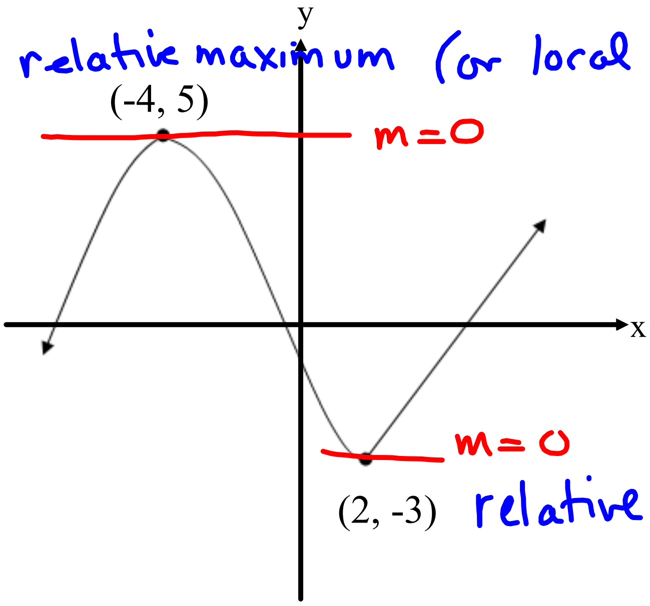
Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of *f* on I** if $f(c) \le f(x)$ for all x in I.
2. $f(c)$ is the **maximum of *f* on I** if $f(c) \ge f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the ***extreme values***, or ***extrema*** of the function on the interval. The minimum and maximum of a function on an interval are also called the ***absolute minimum*** and ***absolute maximum***, or the ***global minimum*** and ***global maximum***, on the interval.

Consider the following graph:

relative maximum (or local max)



m = 0

m = 0

relative minimum (or local min)

What do we call the points (-4, 5) and (2, -3)?

Extreme Value Theorem:
If *f* is continuous on a closed interval [a, b], then *f* has both a minimum and a maximum on the interval.

Why does the extreme value theorem require that a function *f* be continuous on a closed interval [a, b] in order for *f* to have both a minimum and maximum?

The "Official" Definition of Relative Extrema:

1. If there is an open interval containing *c* on which *f(c)* is a maximum, then *f(c)* is called a ***relative maximum*** of *f*, or you can say that *f* has a ***relative maximum at* (*c, f(c)*)**.
2. If there is an open interval containing *c* on which *f(c)* is a minimum, then *f(c)* is called a ***relative minimum*** of *f*, or you can say that *f* has a **relative *mimimum at* (*c, f(c)*)**.

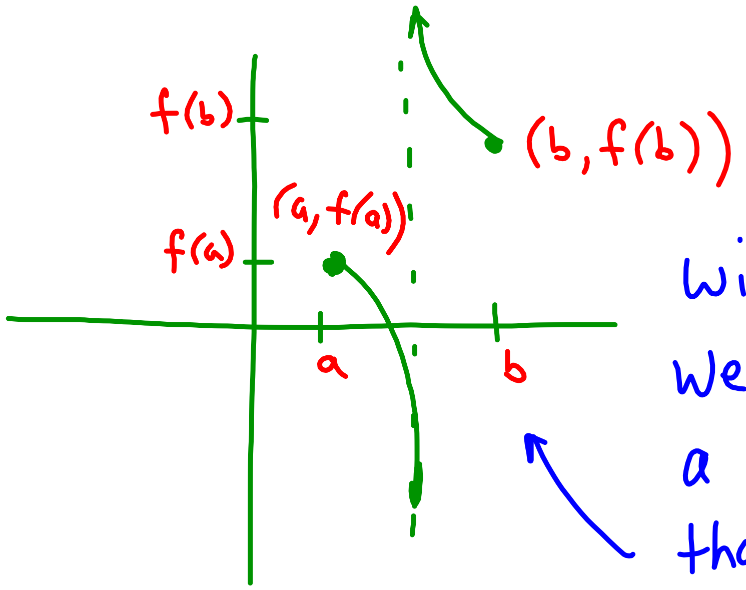
Relative Extremum (As I See It):

1. The point at which a function changes from an increasing status to decreasing (or constant) status is a relative maximum.
2. The point at which a function changes from a decreasing (or constant) status to an increasing status is a relative minimum.

What is the relationship between relative extrema, and the derivative?

Extreme Value Theorem:
If *f* is continuous on a closed interval [a, b], then *f* has both a minimum and a maximum on the interval.

Why does the extreme value theorem require that a function *f* be continuous on a closed interval [a, b] in order for *f* to have both a minimum and maximum?



without continuity we could have a case like this that doesn't have a clear max & min.

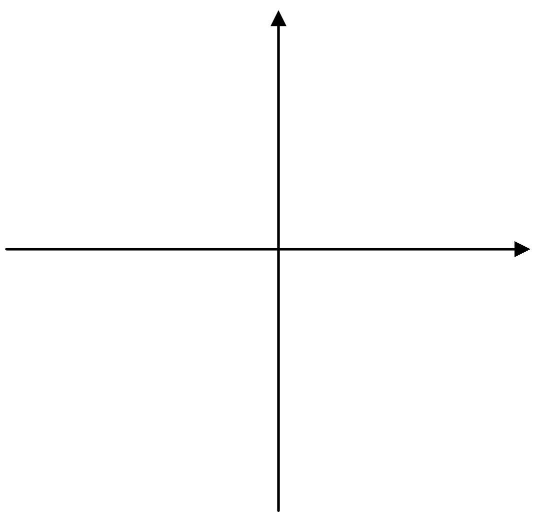
What is the relationship between relative extrema, and the derivative?

At extreme values (in most cases) the slope of the tangent line is defined to be zero.

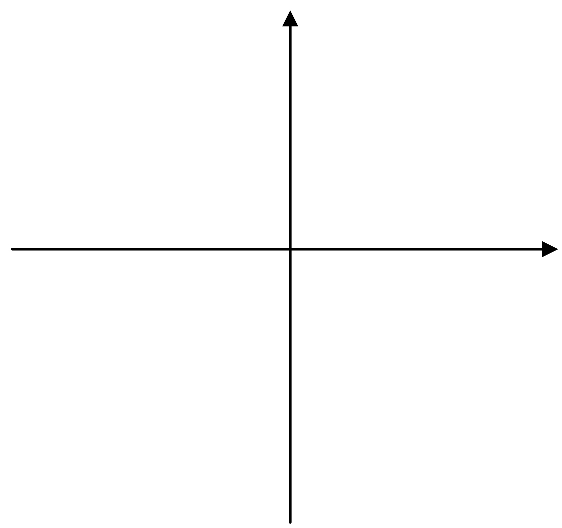
<u>Critical Numbers</u>:
Let f be defined at c. If $f'(c)=0$ or if $f'(c)$ is not differentiable at c, then c is a critical number of f.

Give an example (graphically) of each type of function f, yielding critical numbers c.

$f'(c)=0$



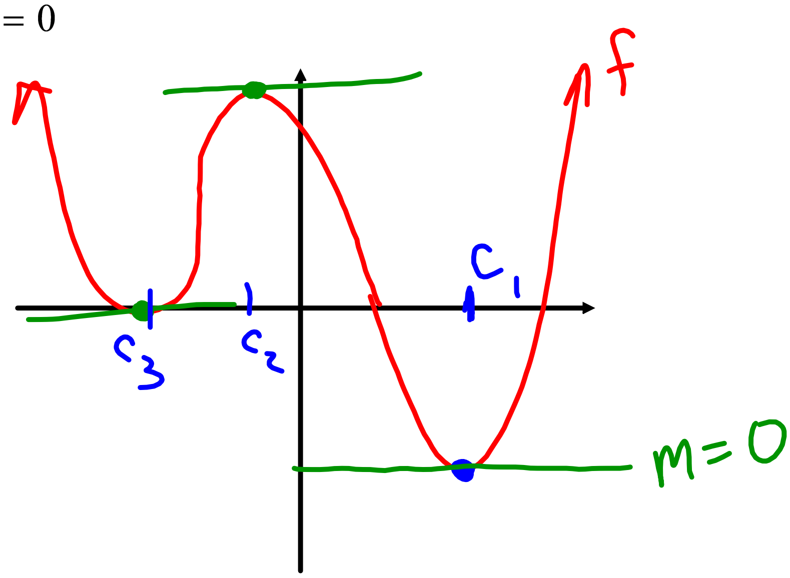
$f'(c)$ is not differentiable at c



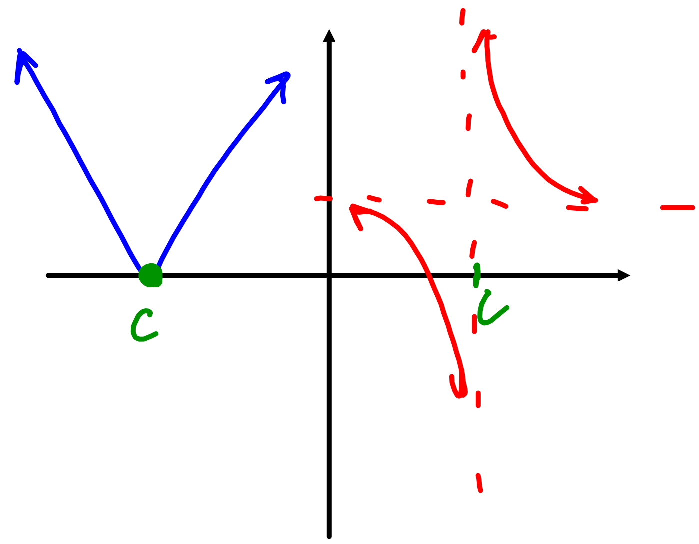
<u>Relative Extrema Occur Only At Critical Numbers</u>: If f has a relative minimum or relative maximum at $x=c$, then c is a critical number of f.

Give an example (graphically) of each type of function f, yielding critical numbers c.

$f'(c) = 0$

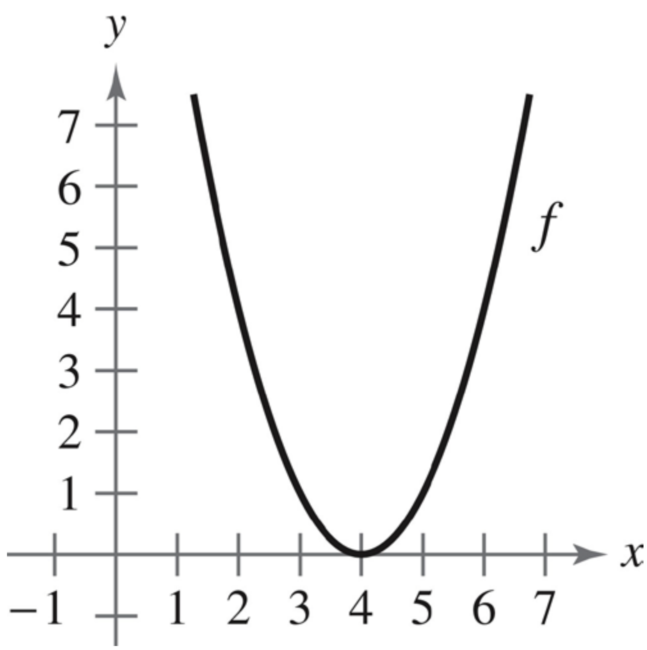


$f'(c)$ is not differentiable at c

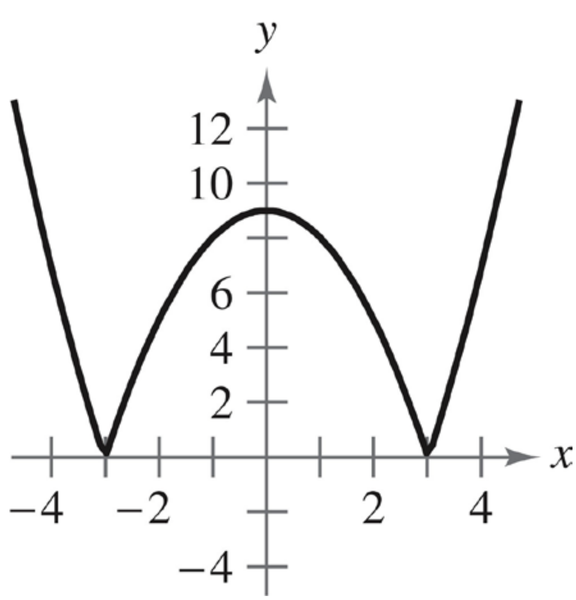


Ex: Approximate the critical numbers of the function shown in the graph. Determine whether the function has a relative maximum, relative minimum, an absolute maximum, absolute minimum or none of these at each critical number on the interval shown.

a)

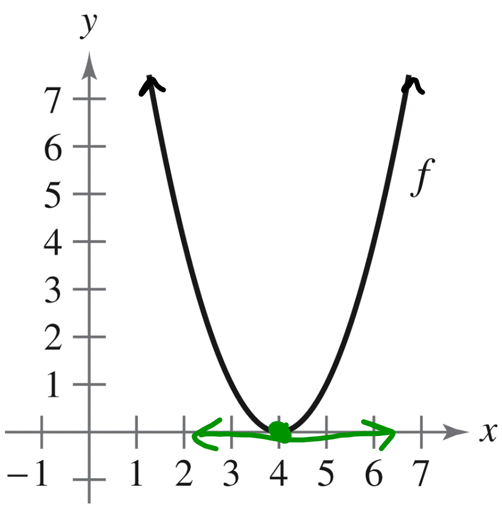


b)



Ex: Approximate the critical numbers of the function shown in the graph. Determine whether the function has a relative maximum, relative minimum, an absolute maximum, absolute minimum or none of these at each critical number on the interval shown.

a)

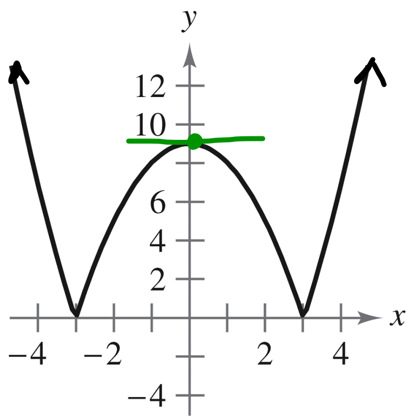


Critical #: $x = 4$

No relative max.
No absolute max.

Absolute min: (4, 0)
Relative min: (4, 0)

b)



Crit. #'s:

$x = 0$

(0,9) Relative max

No absolute max

$x = -3$

(-3,0) Absolute Min.

& Relative Min.

$x = 3$

(3,0) Absolute Min.

& Relative Min.

Ex: Locate the absolute extrema of the function on the closed interval.

a) $f(x) = 5x^2 - 3x + 1, \quad [-1, 1]$

b) $h(s) = \dfrac{1}{s - 2}, \quad [0, 1]$

Ex: Locate the absolute extrema of the function on the closed interval.

a) $f(x) = 5x^2 - 3x + 1, \quad [-1, 1]$

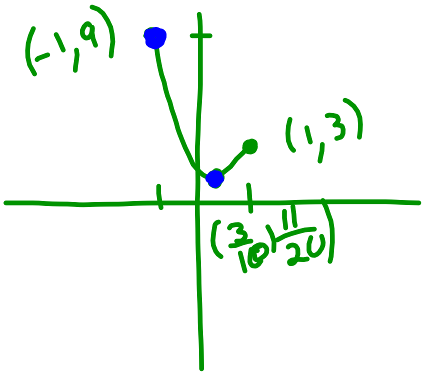
Find critical #'s that exist in $[-1, 1]$

$f'(x) = 10x - 3$

$0 = 10x - 3$

$3/10 = x$

$\frac{3}{10} \in [-1, 1]$



$f\left(\frac{3}{10}\right) = 5\left(\frac{3}{10}\right)^2 - 3\left(\frac{3}{10}\right) + 1 = \frac{11}{20} \quad \left(\frac{3}{10}, \frac{11}{20}\right)$

Compare Endpoints w/ all points at our critical #'s. The highest point is the absolute max & the lowest point is the absolute min.

$f(-1) = 5(-1)^2 - 3(-1) + 1 = 9 \quad (-1, 9)$

$f(1) = 5(1)^2 - 3(1) + 1 = 3 \quad (1, 3)$

$(-1, 9)$ is the absolute max.

$\left(\frac{3}{10}, \frac{11}{20}\right)$ is the absolute min.

b) $h(s) = \frac{1}{s-2}$, $[0, 1]$

$h(s) = (s-2)^{-1}$

$s \neq 2$

Critical value: $s = 2 \notin [0, 1]$

$h'(s) = -(s-2)^{-2} = \frac{-1}{(s-2)^2}$

$(s-2)^2 \left(0 = -\frac{1}{(s-2)^2} \right)$

$0 = -1$

$h(0) = \frac{1}{0-2} = -\frac{1}{2}$ $\left(0, -\frac{1}{2}\right)$

Abs. Max

$h(1) = \frac{1}{1-2} = \frac{1}{-1} = -1$ $(1, -1)$

abs. Min.

Tools for finding Absolute Extrema on a Closed Interval:
To find the extrema of a continuous function f on a closed interval $[a, b]$,

1. Evaluate f at each of its critical numbers in (a, b).
2. Evaluate f at each endpoint, a and b.
3. The least of these values is the absolute minimum, and the greatest is the absolute maximum.