Phys 131 2016

Lecture 36

Tues: Discussion / quiz

Ch 12 Conc Q ~~6~~ ~~7,8~~ 3, 11

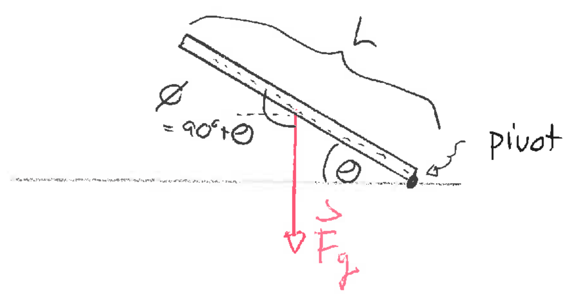
Prob 10, 27, 66

Rotational Energy

While rotational dynamics can describe the rotational motion of any object in principle, in practise it can be difficult to implement. One example that illustrates this is a pivoting rod / tree / chimney

Demo: Falling chimney page (Physics Central + video links)

Consider a pivoting rod. The net torque is entirely due to gravity. Thus



$$\tau_{net} = I\alpha$$

$$\Rightarrow \quad \tau_g = I\alpha$$

But $\tau_g = r F_g \sin\phi$ implies

$$\tau_{net} = mg \, L/2 \, \sin(90° + \theta)$$

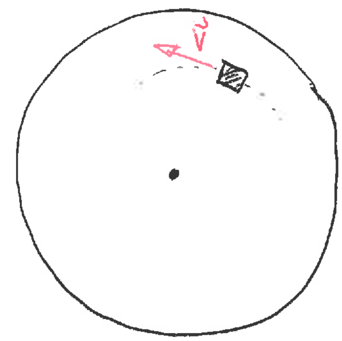
$$\Rightarrow \quad \tau_{net} = \frac{mgL}{2} \cos\theta$$

So the angular acceleration changes as the rod pivots. Ordinary constant angular acceleration kinematics is insufficient here.

But we can get limited information by using energy.

Rotational kinetic energy.

Any moving mass has non-zero kinetic energy. Consider a disk that rotates about its center. The illustrated shaded section, with mass m has kinetic energy



$$K = \frac{1}{2} m v^2$$

But $v = \omega r$ implies:

$$K = \frac{1}{2} m (\omega r)^2 = \frac{1}{2}(m r^2)\,\omega^2$$

Then $m r^2$ is just the moment of inertia of the shaded section. Adding the contributions for all shaded sections (angular velocity is the same) gives $K = \frac{1}{2} I \omega^2$. This is an example of a general rule:

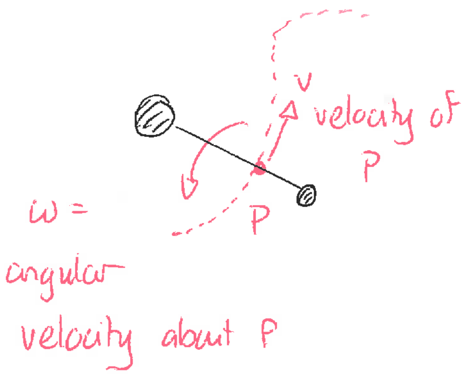
> A rotating object has rotational kinetic energy
>
> $$K_{rot} = \frac{1}{2} I \omega^2$$
>
> where I is the moment of inertia of the object about the axis of rotation and ω is the angular velocity about that point.

Warm Up 1

Quiz 1 90% $\}$ 90%

Energy Conservation.

Consider an object that undergoes rotational and translational motion. We can track one particular point, say P. Then the total kinetic energy can be divided into two parts:



- translational kinetic energy associated with motion of P

$$\boxed{K_{trans} = \frac{1}{2} M v^2}$$

total mass — speed of P

- rotational kinetic energy about P

$$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$$

moment of inertia about P

The total kinetic energy is

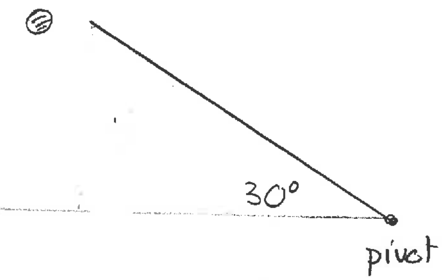
$$K = K_{trans} + K_{rot}$$

and

If non-conservative forces do zero work, then the energy $E = K + U_g + U_{sp} + \ldots$ ($K \to K_{trans} + K_{rot}$) is constant (conserved)

Warm Up 2

Example: A 2.0 m long 0.200 kg rod is held at rest as illustrated. The rod pivots about one end and is released. Determine:

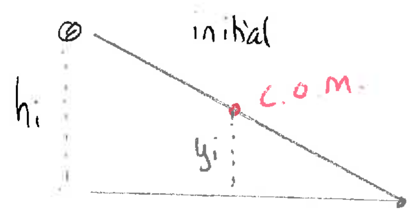


a) the angular velocity as the tip reaches the ground

b) " speed of the tip as it reaches the ground.

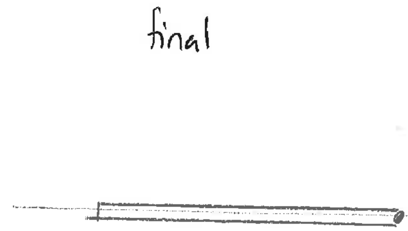
c) " speed of a ball dropped from the same height, as it reaches the ground

Answer:



$\omega_i = 0$

$y_i =$



$\omega_f = ??$

$y_f = 0$

a) Energy is conserved $\Rightarrow$ $E_f = E_i$

Then $E = K + U_g = K_{trans} + K_{rot} + U_g$ gives:

K_{trans}: 0 for point P.

$$K_{rotf} + \cancel{U_{gf}} = \cancel{K_{roti}} + U_{gi}$$

$$\Rightarrow \frac{1}{2} I \omega_f^2 = mgy_i$$

y_i: c.o.m initially

where I is the moment of inertia of the rod about one end:

$$I = \frac{1}{3} m L^2$$

where L is the length of the rod.

Thus:

$$\frac{1}{2}\left(\frac{1}{3} mL^2\right)\omega_f^2 = mgy_i$$

$$\frac{1}{6} \cancel{m}L^2\omega_f^2 = \cancel{m}gy_i$$

$$\omega_f^2 = 6gy_i/L^2 \quad \Rightarrow \quad \omega_f = \frac{\sqrt{6gy_i}}{L}$$

In this case:

$$y_i/(L/2) = \sin 30^\circ \Rightarrow y_i = \frac{L}{2}\sin 30^\circ$$

$$= 1.0\text{m} \sin 30^\circ = 0.50\text{m}$$

$$\text{Thus} \quad \omega_f = \frac{\sqrt{6 \times 9.8\text{m/s}^2 \times 0.50\text{m}}}{(2.0\text{m})} \Rightarrow \omega_f = 2.7\text{rad/s}.$$

b) Use $v_f = r\omega_f$ and here r is the distance from P to the tip. So.

$$V_f = L\omega_f = \sqrt{6gy_i} \Rightarrow V_f = \sqrt{6gy_i}$$

Here $v_f = \sqrt{6 \times 9.8\text{m/s}^2 \times 0.50\text{m}}$

$$\Rightarrow v_f = 5.4\text{m/s}$$

c) For the ball energy conservation implies $K_f = U_{gi}$

$$\Rightarrow \frac{1}{2}\cancel{M_{ball}}\, V_{f\,ball}^2 = \cancel{M}_{ball}\, g h_i \Rightarrow V_{f\,ball} = \sqrt{2gh_i}$$

But $h_i = 2y_i$ by geometry. So $V_{f\,ball} = \sqrt{4gy_i}$

The ball drops more slowly.

Demo: U Iowa Falling Chimney.

Challenge: 1) Show that there is one point on the rod that hits the ground at the same speed as ~~the~~ a freely falling ball dropped from the same point.

Show it is $\frac{2}{3}L$ from the pivot.

2) Show that when the rod is at angle θ, the acceleration of the tip is

$$a_t = \frac{3}{2} g \cos\theta .$$