Taylor Larrechea
Jacob (Max) Doose

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Laboratory 3: Vectors

Vectors and vector algebra are used throughout classical physics. This laboratory exercise will illustrate vector algebra using physical and graphical methods and compare the results to those that can be obtained with purely mathematical constructions.

1 Walking displacement vectors

Displacement vectors describe the location of an object at a later instant relative to its location at an earlier instant. These vectors provide a useful conceptual illustration of vector addition: two displacements can be combined in succession to provide an overall displacement in the same way that two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, can be added, giving the sum $\vec{\mathbf{A}} + \vec{\mathbf{B}}$. This exercise illustrates such vector addition via walking various displacements.

Consider the following sequence of displacements:

$$\vec{\mathbf{A}} = 5.0\,\text{m due east and}$$
$$\vec{\mathbf{B}} = 4.0\,\text{m at an angle of } 50^\circ \text{ north of west}$$

and let $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. This is illustrated schematically below.

The aim of this exercise will be to follow (by walking) the two displacements successively and to measure the resulting overall displacement vector $\vec{\mathbf{C}}$. The measured overall displacement will then be compared to a calculated overall displacement.

a) Take rope, a meter stick and a protractor outside. Stretch the rope in a straight line approximately east/west. This will establish what we regard as "east."

b) Stand at one end of the rope and leave a "marker" object at this location. This is the *initial* location.

c) Follow the displacement vector $\vec{\mathbf{A}}$ by walking 5.0 m along the rope. At the end of this you have reached an intermediate location.

d) Starting at this intermediate location, follow the displacement described by $\vec{\mathbf{B}}$. This takes you to a *final* location. Mark this location.

e) Determine the overall displacement from your initial to final locations. This should consist of *both* a *distance* and a *direction* (e.g. angle measured from "east" toward "north" using the "line of sight" from the initial to the final location). The "line of sight" is illustrated in a different example below.

Record the data for the displacement below.

4 meters 50° east toward north

The rest of this exercise should be conducted in the lab room.

f) You will now use components to calculate the magnitude and direction of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ *without using the results from the walking activity.* Let "east" correspond to the x axis and "north" the y axis when you do this. The obtain both components for $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Use addition to find the components of $\vec{\mathbf{C}}$. Then use these to sketch the vector $\vec{\mathbf{C}}$ and obtain its magnitude and angle. The table beneath might be useful.

Vector	x component	y component
$\vec{\mathbf{A}}$	5 m ✓	0 m ✓
$\vec{\mathbf{B}}$	-2.6 m ✓	3.1 m ✓
$\vec{\mathbf{C}}$	2.4 m ✓	3.1 m ✓

m = meters

g) Do the magnitude and direction of the calculated $\vec{\mathbf{C}}$ agree with those of the measured displacement as determined via walking?

Yes, they do. We calculated 3.9m and walked 4m.

h) Using the notation that A is the magnitude of $\vec{\mathbf{A}}$, B is the magnitude of $\vec{\mathbf{B}}$, etc, ..., is it true that $C = A + B$ in this exercise? Explain your answer.

A=5m
B=4m
C=4m

No, the magnitudes of $\vec{A}$ and $\vec{B}$ added together equal nine meters (A+B=5m+4m=9m). And $\vec{C}$'s magnitude is equal to four meters (4m). $4m \neq 9m$, $C \neq A+B$

i) Is it true that $C^2 = A^2 + B^2$ in this exercise? Explain your answer.

A=5m
B=4m
C=4m

$4m^2 = 5m^2 + 4m^2$

$16m^2 = 25m^2 + 16m^2$

$16m \neq 41m^2$

This figure is not a right triangle △. Pathagorean's theorem only applies to right triangles. So therefore $C^2 \neq a^2 + b^2$ in our triangles case

2 Vector addition: graphical methods versus algebraic methods

A more accurate comparison of graphical methods and algebraic methods of adding vectors can be illustrated by drawing and adding vectors accurately on a sheet of paper. Consider the three vectors:

$\vec{A} = 8.0\,\text{cm}$ along the x axis,

$\vec{B} = 6.0\,\text{cm}$ at an angle of 45° counterclockwise from the x axis and

$\vec{C} = 12.0\,\text{cm}$ at an angle of 120° counterclockwise from the x axis.

Let $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. The aim of this exercise is to determine $\vec{D}$ graphically and algebraically

a) A graphical method for determining $\vec{D}$ would first add $\vec{A}+\vec{B}+\vec{C}$ using the "head-to-tail" method. Draw these three vectors "head-to-tail" as accurately as possible (using a ruler and a protractor) on a separate sheet. Use this to draw $\vec{D}$.

b) Determine the magnitude of $\vec{D}$ graphically (i.e. by measuring from your drawing). Record this below.

Graphically : $\boxed{\vec{D} = 15.5\text{ cm}}$

c) Draw the x and y components of $\vec{D}$ on your diagram. Write the results below.

The components of $\vec{D}$; $x = 5.8$ cm $y = 14.4$ cm

d) Without using your drawing, calculate the components and magnitude of $\vec{D}$ mathematically from the components of $\vec{A}$, $\vec{B}$, and $\vec{C}$ using algebra and trigonometry (this is the "algebraic" method). How do these compare to result that you obtained graphically?

A = 8.0 cm
B = 6.0 cm
C = 12.0 cm

$\vec{A}$ = x-component $\boxed{8.0\text{cm}}$
y-component $\boxed{0.0\text{cm}}$

$\vec{B}$ = x-comp
$H\cos\theta$ H=6.0cm $\theta = 45°$
$6\cos 45$
$\boxed{x = 4.2\text{cm}}$
y-comp
$H\sin\theta$ H=6.0cm $\theta = 45°$
$6\sin 45$
$\boxed{y = 4.2\text{cm}}$

$\vec{C}$ = x-comp
$H\cos\theta$ H=12.0cm $\theta = 120°$
$12\cos 120 = -6 = 6\text{cm}$
$\boxed{x = -6\text{cm}}$ = Because C points NW
y-comp H=12.0cm
$H\sin\theta$ $\theta = 120°$
$12\sin 120° = 10.4\text{cm}$
$\boxed{y = 10.4\text{cm}}$

$\vec{D}$ = x-comp
$H\cos\theta$ H=15.5 $\theta = 68°$
$15.5\cos 68 = 5.8\text{cm}$
$\boxed{x = 5.8\text{cm}}$
y-comp
$H\sin\theta$ H=15.5 $\theta = 68°$
$15.5\sin 68 = 14.4\text{cm}$
$\boxed{y = 14.4\text{cm}}$

$\vec{A}$ = (8.0cm) ✓
x comp
$\vec{B}$ = (4.2cm) + (4.2 cm) ✓
x-comp ✓ y-comp ✓
$\vec{C}$ = (-6cm) + (10.4 cm)
x-comp ✓ y-comp ✓
$\vec{D}$ = (5.8 cm) + (14.4 cm)
x-comp y-comp

They Differ in about ±2 cm from what we graphed

3 Vector subtraction: graphical methods versus algebraic methods

Vector subtraction is a variant of vector addition. Consider any two vectors $\vec{v}_1$ and $\vec{v}_2$ and define the difference between these as

$$\Delta\vec{v} := \vec{v}_2 - \vec{v}_1 \tag{1}$$

One way of constructing the difference involves addition of a negative vector. Thus

$$\begin{aligned} \Delta\vec{v} &= \vec{v}_2 - \vec{v}_1 \\ &= \vec{v}_2 + (-\vec{v}_1) \end{aligned} \tag{2}$$

Thus one can perform the subtraction by adding the vectors $\vec{v}_2$ and $-\vec{v}_1$. Alternatively, the difference between the vectors must satisfy

$$\vec{v}_2 = \vec{v}_1 + \Delta\vec{v} \tag{3}$$

and, given the two vectors $\vec{v}_1$ and $\vec{v}_2$, one can try to find $\Delta\vec{v}$ that satisfies Eq. (3).

For this exercise consider the vectors

$\vec{v}_1 = 5.0\,\text{cm}$ at an angle of 30° counterclockwise from the x axis and
$\vec{v}_2 = 5.0\,\text{cm}$ at an angle of 150° counterclockwise from the x axis.

a) On a separate sheet of paper, draw a set of x and y axes and draw $\vec{v}_1$ and $\vec{v}_2$ as accurately as possible.

b) The vectors $\vec{v}_1$ and $\vec{v}_2$ have the same magnitude. Does this mean that $\vec{v}_2 - \vec{v}_1 = 0$? Explain your answer.

No, $\vec{v}_2 - \vec{v}_1 \neq 0$. $\vec{v}_2 - \vec{v}_1$ yields a displacement vector.
If the statement were; the magnitude of $\vec{v}_2$ (v_2) minus the magnitude of $\vec{v}_1$ (v_1), then it would be true.
5cm-5cm=0 ($v_2 - v_1 = 0$) ($\vec{v}_2 - \vec{v}_1 \neq 0$)

c) Use the scheme of Eq (2) to draw $\Delta\vec{v}$ as accurately as possible on the same sheet of paper. Use your drawing to describe the direction of $\Delta\vec{v}$ and to determine its magnitude. Record these below.

$\Delta\vec{v}$ has a direction of 180° counter clockwise from the x-axis, with a magnitude of 8.5 cm.

d) Calculate the components of $\Delta\vec{v}$ using trigonometry and algebra. Use the result to describe the magnitude and direction of $\Delta\vec{v}$. How do these calculated results compare to your measured results?

$\Delta\vec{v}$ x-comp

$5\cos 30 = 4.3\text{cm} \cdot 2$

$\boxed{x = -8.6\text{ cm}}$

$|\Delta v| = 8.6\text{cm}$

$\vec{v}_2$ x-comp

$5\cos 30 = 4.3\text{cm}$

$\boxed{x = -4.3\text{cm}}$

y-comp

$5\sin 30 = 2.5\text{cm}$

$\boxed{y = 2.5\text{cm}}$

$\vec{v}_1$ x-comp

$5\cos 30 = 4.3\text{cm}$

$\boxed{x = -4.3\text{cm}}$

y-comp

$5\sin 30 = 2.5\text{cm}$

$\boxed{y = 2.5\text{cm}}$

These components differ graphically with a margin of about ± 1 cm

e) Use the "head-to-tail" method of addition and accurate drawings of $\vec{v}_1$ and $\Delta\vec{v}$ to construct $\vec{v}_1 + \Delta\vec{v}$ graphically. How does the resulting vector compare to $\vec{v}_2$? What does this imply (correct, incorrect, maybe correct) about Eq. (3)?

It is $-v_2$ when drawn out graphically. It implies equation 3 is correct

4 Unit vectors and vector components

Any vector $\vec{\mathbf{A}}$ can be expressed in terms of unit vectors as

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \tag{4}$$

where A_x and A_y are the components of $\vec{\mathbf{A}}$. This exercise will illustrate why this is possible and how this decomposition can be used to add vectors. Consider the two vectors:

$\vec{\mathbf{A}} = 10\,\text{cm}$ at an angle of $53°$ counterclockwise from the x axis and

$\vec{\mathbf{B}} = 5.0\,\text{cm}$ at an angle of $37°$ clockwise from the x axis.

a) On a separate sheet of paper draw a set of x and y axes. Draw the two standard unit vectors (each with length 1 cm on this sheet). Draw the vector $\vec{\mathbf{A}}$ as accurately as possible (using a ruler and a protractor).

b) Show, by drawing, how to construct $\vec{\mathbf{A}}$ as accurately as possible by repeatedly adding $\hat{\mathbf{i}}$ to itself followed by repeatedly adding $\hat{\mathbf{j}}$. An example for a different vector $\vec{\mathbf{D}}$ is illustrated below.

Constructing $\vec{\mathbf{D}}$ required 4 copies of $\hat{\mathbf{i}}$ and 2 copies of $\hat{\mathbf{j}}$. Thus $\vec{\mathbf{D}} = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$. How many copies of each of these (reversed versions of the unit vectors count negatively, e.g. -1 copy) were required to construct $\vec{\mathbf{A}}$? Use the answer to this question to write

$$\vec{\mathbf{A}} = (??)\,\hat{\mathbf{i}} + (??)\,\hat{\mathbf{j}}$$

where the parentheses should be replaced by numbers.

(6,8) $\vec{A} = 6\hat{i} + 8\hat{j}$ ✓

c) Calculate the components of $\vec{\mathbf{A}}$ using trigonometry and algebra. Substitute these into Eq. (4). Does the resulting expression agree with that which you obtained in part (b)?

$x = H\cos\theta$ $H = 10$ cm $\theta = 53°$

$x = 10\cos 53$

$x = 6.0$ cm

$y = H\sin\theta$ $H = 10$ cm $\theta = 53°$

$10\sin 53$

$y = 8.0$ cm

X-Component = 6cm
Y-Component = 8cm
These compare exactly to what I found in part B. $\vec{A} = (6cm) + (8cm)$

d) Repeat parts (a) to (c) for $\vec{\mathbf{B}}$.

$\vec{B} = 4\hat{i}\ 3\hat{i}$
cm cm

x-component
$5\cos 37 = 4cm$
$\boxed{x = 4cm}$
y-component
$5\sin 37 = 3cm$
$\boxed{y = -3cm}$

x-component = 4cm
y-component = -3cm
$\vec{B} = (4cm) - (3cm)$
x-comp y-comp

e) Construct $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ graphically by drawing a succession of unit vectors, first those for $\vec{\mathbf{A}}$ and then for $\vec{\mathbf{B}}$. Verify that the result would be the same by first following all the copies of $\hat{\mathbf{i}}$ for both vectors and then all the copies of $\hat{\mathbf{j}}$ for both vectors (you will have to be careful about directions) Write the resulting vector in the form

$$\vec{\mathbf{C}} = (??)\,\hat{\mathbf{i}} + (??)\,\hat{\mathbf{j}}.$$

How does this compare to what you would calculate using the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$?

$\vec{A} = 6cm + 8cm$
$\vec{B} = 4cm - 3cm$
$\vec{C} = 10cm + 5cm$

This compares in that it is the same ratio as the calculations of $\vec{A}$ and $\vec{B}$ put together to get $\vec{C}$

$\boxed{\vec{C} = 2\hat{i} + 0\hat{j}}$

$\Downarrow$

Algebraically

$\boxed{\vec{C} = 10\hat{i} + 5\hat{j}}$
Units ($\hat{i}$) and ($\hat{j}$) are in cm

Taylor
Jacob

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

Part 2. A)

Components of $\vec{D}$ $\begin{cases} X = 15.5\cos 68 & x = 5.8\text{ cm} \\ Y = 15.5\sin 68 & y = 14.4\text{ cm} \end{cases}$

(c.)

y
10cm
8cm
6cm
4cm
2cm
$-\vec{U}_1$
$\vec{U}_2$
0cm
8cm
6cm
$\Delta\vec{U}$
4cm
2cm
x

e)

$X = 10\cos 53$ $\quad X =$

$Y = 10\sin 53$ $\quad Y = 8$

4. A.)

$X = 5\cos 37$

$Y = 5\sin 37$

$X = 4$

$Y = 3$

Part 4e