Phys 131 2016

Lecture 28

Mon: Pumpkin drop

Tues: Discussion / quiz

Supp Ex 65

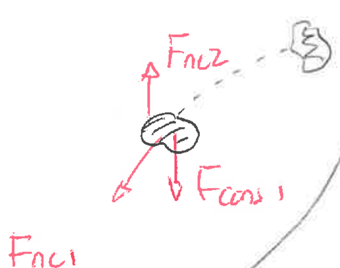
Ch 10 CQ 8, 13

Ch 10 Prob 35, 49, 54

Energy Conservation

The general scheme for using energy to assess motion is:

Separate forces into conservative and non-conservative



→ Construct mechanical energy

$$E = K + U_1 + U_2 + \dots$$

where U_i is potential energy for $F_{cons\,i}$

Construct work for each non-conservative force:

$$W_{noncons1} = F_{nc1} \Delta r \cos\theta$$

→ Work done by all non-conservative forces

$$W_{nc} = W_{noncons1} + W_{noncons2} + \dots$$

Then $\Delta E = W_{nc}$

Typical conservative forces are: gravity, springs The usual conservation of energy follows

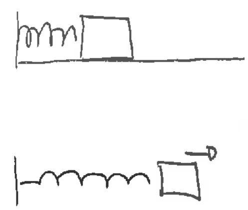
66 Bumper cart

A 200 kg bumper cart can move on a rough horizontal surface. The cart is pushed against a spring, with spring constant 6000 N/m, compressing it by 0.25 m. It is released and begins to move right. The coefficient of friction between the cart and the surface is 0.20.

a) Determine the speed of the cart at the moment that it leaves the spring.

b) Suppose that while the cart moves right it is also pulled with a rope that pulls with tension 400 N to the right. Determine the speed of the cart when it leaves the spring.

Answer: a) $\Delta E = W_{nc} \Rightarrow E_f = E_i + W_{nc}$



initial

$y_i = 0\,m$

$v_i = 0\,m/s$

$\Delta s = 0.25\,m$

$E = K + U_g + U_{spring}$

$y_f = 0\,m$

$v_f = ??$

$\Delta s_f = 0\,m$

So

$$K_f + \cancel{U_{gf}} + \cancel{U_{spf}} = \cancel{K_i} + \cancel{U_{gi}} + U_{spi} + W_{nc}$$

$$\Rightarrow \frac{1}{2}mv_f^2 + \cancel{mgy_f} + \cancel{\frac{1}{2}k\Delta s_f^2} = \cancel{\frac{1}{2}mv_i^2} + \cancel{mgy_i} + \frac{1}{2}k(\Delta s_i)^2 + W_{n.c.}$$

Now $W_{nc} = W_{friction} = f_k \Delta r \cos\theta$

$f_k \quad \Delta r$

$$= \mu_k mg\,\Delta s_i \cos(180^\circ)$$

$\Sigma F_y = 0$

$n = mg$

$\Rightarrow f_k = \mu_k mg$

$$= -0.20 \times 200\,kg \times 9.8\,m/s^2 \times 0.25\,m$$

$$= -98\,J$$

So $\frac{1}{2}mv_f^2 = \frac{1}{2}\,6000\,N/m\,(0.25\,m)^2 - 98\,J$

$$= 188\,J - 98\,J = 90\,J$$

$\Rightarrow \frac{1}{2}\,200\,kg\,v_f^2 = 90\,J \Rightarrow v_f^2 = 0.90\,m^2/s^2$

$\Rightarrow v_f = 0.95\,m/s$

b) Here $W_{nc} = W_{friction} + W_{rope}$

$$W_{rope} = F_{rope} \Delta s \cos\theta$$

$$= 400N \times 0.25m \cos 0° = 100J$$

So $\Delta E = W_{nc}$

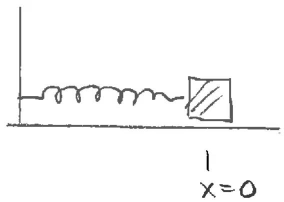
$$\Rightarrow \frac{1}{2} m v_f^2 = \frac{1}{2} k (\Delta s_i)^2 + 100J - 98J$$

$$\Rightarrow 100kg\, v_f^2 = 190J$$

$$\Rightarrow V_f = \sqrt{1.90 m^2/s^2} \Rightarrow V_f = 1.4 m/s$$

Graphing energy

Consider a block that can move along the x axis and is attached to a spring. We can choose the origin so that $x=0$ corresponds to the spring's relaxed position. Then the spring potential energy is

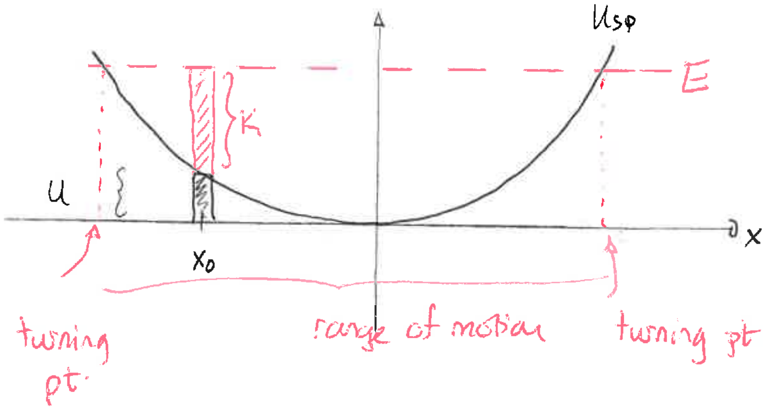


$$U_{sp} = \frac{1}{2} k (\Delta s)^2$$

where $\Delta s = x$ gives:

$$U_{sp} = \frac{1}{2} k x^2$$

We can plot this potential energy versus block location. With no other forces, the total energy is



$$E = K + U_{sp}.$$

Since $K \geq 0$ we have $E \geq U_{sp}$

Since the total energy is constant it is represented by a horizontal line.

At any location x_0, we can represent the potential and kinetic energies via bars. Since $K \geq 0$, this gives us :

1) range of motion $\rightarrow$ values of x s.t $E \geq U_{sp}$
2) turning points $\rightarrow$ locations where $v=0 \Rightarrow K=0$
3) location where speed is largest $\rightarrow$ lowest U_{sp} ($x=0$ here).

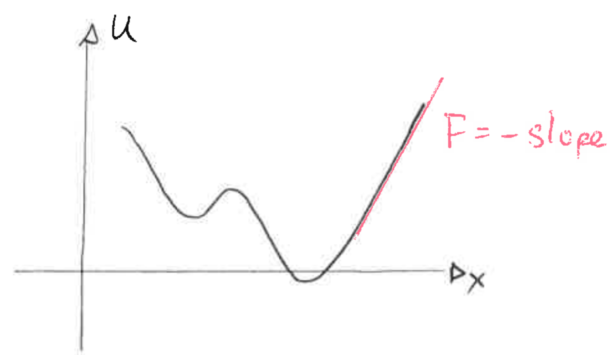
Warm Up!

Force and Potential Energy

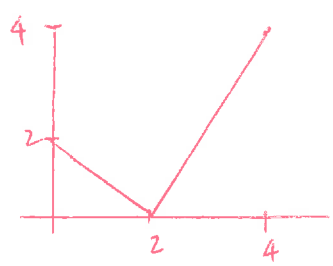
Potential energy is associated with a force. A mathematical theorem shows that:

> For any object moving along the x-axis only, the force associated with the potential energy has horizontal component
>
> $$F_x = -\frac{dU}{dx}$$

Graphically this is the negative of the slope of U w.r.t x



Warm Up 2



~~Warm~~ Quiz 1

Quiz 2

Here $U = x^3 \Rightarrow F_x = -\frac{d}{dx} x^3 = -3x^2 \Rightarrow F_x = -3x^2$

Then when $x = 1$ $\quad F_x \equiv F_1 = -3$

$x = 2$ $\quad F_x \equiv F_2 = -12$ $\quad$ 4 times.