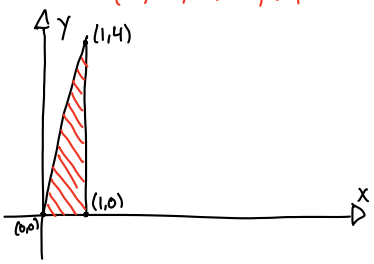
1.) $\oint xy\,dx + x^2y^3\,dy$

$(0,0), (1,0), (1,4)$



$P = xy \quad Q_x = 2xy^3$

$Q = x^2y^3 \quad P_y = x$

$\iint_D (2xy^3 - x)\,dA$

$\int_0^1 \int_0^{4x} x(2y^3 - 1)\,dy\,dx$

$\int_0^1 x(\frac{1}{2}y^4 - y)\Big|_0^{4x}\,dx$

$\int_0^1 x(128x^4 - 4x)\,dx$

$\int_0^1 128x^5 - 4x^2\,dx$

$\frac{64}{3}x^6 - \frac{4}{3}x^3\Big|_0^1$

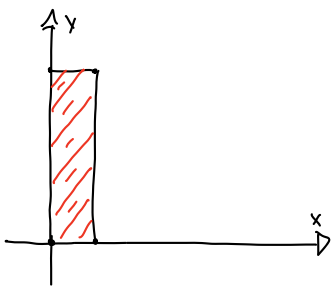
$\frac{64}{3} - \frac{4}{3} = \frac{60}{3}$

$\frac{60}{3}$

$4\overline{)256}$ 64, 24, 16, 16, 0

2.) $\int_C \cos y\,dx + x^2 \sin y\,dy$

$(0,0), (1,0), (1,4), (0,4)$



$P = \cos y \quad P_y = -\sin y$

$Q = x^2 \sin y \quad Q_x = 2x \sin y$

$\iint_D 2x\sin y + \sin y\,dA$

$\int_0^1 \int_0^4 2x\sin y + \sin y\,dy\,dx$

$\int_0^1 -2x\cos y - \cos y\Big|_0^4\,dx$

$\int_0^1 -2x\cos 4 - \cos 4 - (-2x\cos(0) - \cos(0))\,dx$

$\int_0^1 -2x\cos 4 - \cos 4 - (-2x - 1)\,dx$

$\int_0^1 -2x\cos 4 - \cos 4 + 2x + 1\,dx$

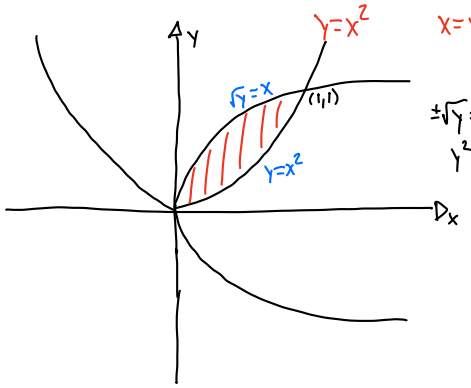
$-x^2\cos 4 - x\cos 4 + x^2 + x\Big|_0^1$

$-\cos 4 - \cos 4 + 1 + 1$

$2 - 2\cos 4$

$2 - 2\cos 4$

3.) $\int_C (3y + 7e^{\sqrt{x}})\,dx + (8x + 9\cos y^2)\,dy$



$\pm\sqrt{y} = x$

$y^2 = x$

$P = 3y + 7e^{\sqrt{x}} \quad P_y = 3$

$Q = 8x + 9\cos y^2 \quad Q_x = 8$

$0 \le y \le 1$

$y^2 \le x \le \sqrt{y}$

$\int_0^1 \int_{y^2}^{\sqrt{y}} 5\,dx\,dy$

$\int_0^1 5x\Big|_{y^2}^{\sqrt{y}}\,dy$

$\int_0^1 5\sqrt{y} - 5y^2\,dy$

$\frac{10}{3}y^{\frac{3}{2}} - \frac{5}{3}y^3\Big|_0^1$

$\frac{10}{3} - \frac{5}{3} = \frac{5}{3}$

$\frac{5}{3}$

4.) $\int_C 8y^3\,dx - 8x^3\,dy$

$x^2 + y^2 = 4$

$x = 2\cos\theta$

$y = 2\sin\theta$

$r = 2$

$P = 8y^3 \quad P_y = 24y^2$

$Q = -8x^3 \quad Q_x = -24x^2$

$\iint_D -24x^2 - 24y^2\,dA$

$\iint_D -24(x^2 + y^2)\,dA$

$-24\iint_D r^2 \cdot r\,dr\,d\theta$

$-24\int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta$

$-24\int_0^{2\pi} \frac{1}{4}r^4\Big|_0^2\,d\theta$

$-24\int_0^{2\pi} 4\,d\theta$

$-24[4\theta]_0^{2\pi}$

$-24[8\pi]$

-192π

-192π

5.) $\int_C F \cdot dr$

$F(x,y) = \langle y - \cos y, x \sin y \rangle$ circle $(x-6)^2 + (y+5)^2 = 16$

$P = y - \cos y \quad P_y = 1 + \sin y$

$Q = x \sin y \quad Q_x = \sin y$

$\iint_D \sin y - (1 + \sin y)\, dA$

$\iint_D -1 \, dA$

$0 \le r \le 4$

$0 \le \theta \le 2\pi$

16π

$\int_0^{2\pi} \int_0^4 r \, dr \, d\theta$

$\int_0^{2\pi} \frac{1}{2} r^2 \Big|_0^4 \, d\theta$

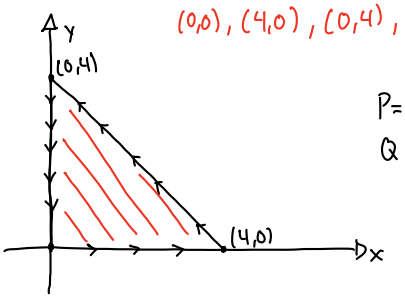
$\int_0^{2\pi} 8 \, d\theta$

$8\theta \Big|_0^{2\pi}$

16π

6.) $F(x,y) = x(x+y)\hat{i} + xy^2\hat{j}$

$(0,0), (4,0), (0,4), (0,0)$



$P = x^2 + xy \quad P_y = x$

$Q = xy^2 \quad Q_x = y^2$

$\frac{32}{3}$

$\iint_D (y^2 - x)\, dA$

$\int_0^4 \int_0^{4-x} y^2 - x \, dy \, dx$

$\int_0^4 \frac{1}{3} y^3 - yx \Big|_0^{4-x} \, dx$

$\int_0^4 \frac{1}{3}(4-x)^3 - (4-x)x \, dx$

$\int_0^4 \frac{1}{3}(4-x)^3 - 4x + x^2 \, dx$

$-\frac{1}{12}(4-x)^4 - 2x^2 + \frac{1}{3}x^3 \Big|_0^4$

$\frac{32}{3}$

$u = 4 - x$

$du = -1 \, dx$

7.) $x = t - \sin t \quad y = 1 - \cos t$

$\frac{1}{2} \oint_C x\,dy - y\,dx \qquad dy = \sin t$

$\oint_C x\,dy$

$\int_0^{2\pi} (t - \sin t)(\sin t)\, dt \qquad u = t \quad dv = \sin t \, dt$

$\int_0^{2\pi} t \sin t - \sin^2 t \, dt \qquad du = 1 \, dt \quad v = -\cos t$

$\int_0^{2\pi} t \sin t \, dt - \int_0^{2\pi} \sin^2 t \, dt$

$-t\cos t + \int_0^{2\pi} \cos t \, dt - \int_0^{2\pi} \frac{1}{2} - \frac{1}{2}\cos 2t \, dt$

$-t\cos t + \sin t \Big|_0^{2\pi} - \left(\frac{1}{2}t - \frac{1}{4}\sin 2t\right)\Big|_0^{2\pi}$

$-2\pi(1) + 0 - (0+0) - \left(\pi - \frac{1}{4}\sin 4\pi - \left(0 - \frac{1}{4}\sin 0\right)\right)$

$-2\pi \quad - \quad \pi$

$-(-3\pi)$

3π

3π

8.) $x = 10\cos t - \cos 10t \quad , \quad y = 10\sin t - \sin 10t$

$A = \frac{1}{2}\int x\,dy - y\,dx$

$dx = -10\sin t + 10\sin 10t$

$dy = 10\cos t - 10\cos 10t$

110π

$\frac{1}{2}\int_0^{2\pi} [(10\cos t - \cos 10t)(10\cos t - 10\cos 10t) - (10\sin t - \sin 10t)(-10\sin t + 10\sin 10t)]\, dt$

$\frac{1}{2}[220\pi] = 110\pi$