

# Announcements

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## □ Homework for tomorrow...

Ch. 29: CQ 10, Probs. 20, 22, & 54

CQ4a)  $E_1 = E_2 = -(10 \text{ V/m}) \hat{i}$

b)  $E_1 = -(5 \text{ V/m}) \hat{i}$ ,  $E_2 = -(20 \text{ V/m}) \hat{i}$

CQ5: accelerates right

29.6:  $W = 1.6 \times 10^{-13} \text{ J}$

29.12:  $E_x(0\text{cm} < x < 2\text{cm}) = +2,500 \text{ V/m}$      $E_x(2\text{cm} < x < 3\text{cm}) = -10,000 \text{ V/m}$

29.44:  $E = 40 \text{ V/m}$ ,  $\theta = 27^\circ$  ccw from  $+x$ -axis

## □ Office hours...

MW 10-11 am

TR 9-10 am

F 12-1 pm

## □ Tutorial Learning Center (TLC) hours:

MTWR 8-6 pm

F 8-11 am, 2-5 pm

Su 1-5 pm

# Chapter 29

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## Potential & Field *(Combinations of Capacitors)*

# Review...

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## □ *Kirchoff's Loop Rule...*

$$\Delta V_{loop} = \sum_i (\Delta V)_i = 0$$

## □ *Conductors in Electrostatic Equilibrium...*

## □ *Capacitance...*

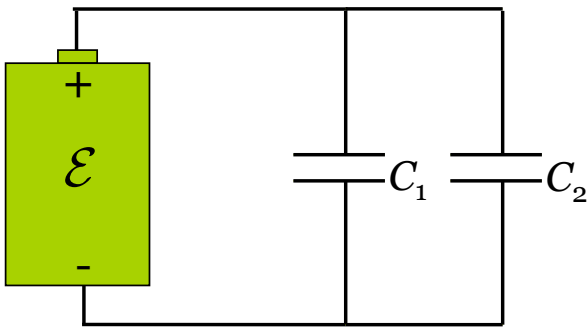
$$C \equiv \frac{Q}{\Delta V_C}$$

$$C = \frac{\epsilon_0 A}{d} \quad (\text{parallel-plate capacitor})$$

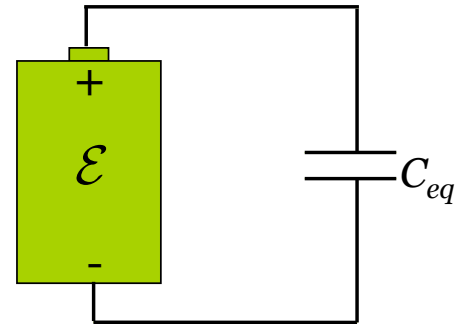
# Combinations of Capacitors

Consider two capacitors in *parallel*...

- Can we find an *equivalent capacitor*,  $C_{eq}$ , to the two capacitors,  $C_1$  &  $C_2$ ?



$\stackrel{?}{=}$



$$\Delta V_B = \Delta V_1 = \Delta V_2$$

$$Q = Q_1 + Q_2$$

$$C = \frac{Q}{\Delta V}$$

$$Q = C \Delta V$$

$$Q_1 = C_1 \Delta V$$

$$Q_2 = C_2 \Delta V$$

$$Q_{eq} = C_{eq} \Delta V_{eq}$$

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

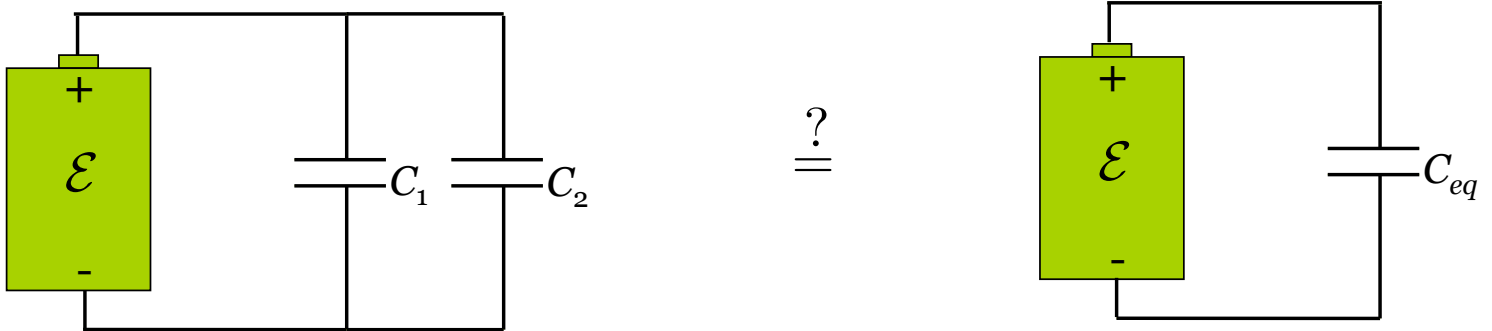
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# Combinations of Capacitors

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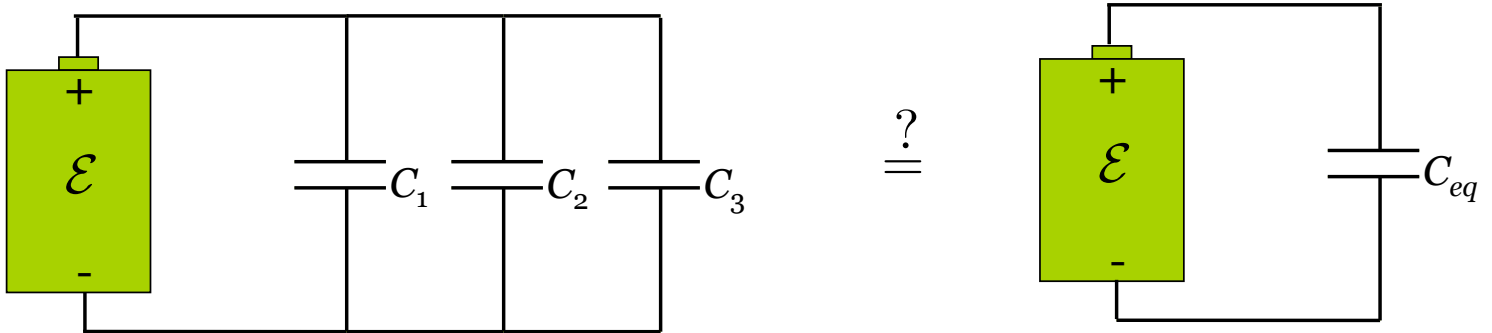
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# Combinations of Capacitors

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What about for *several* capacitors in *parallel*?

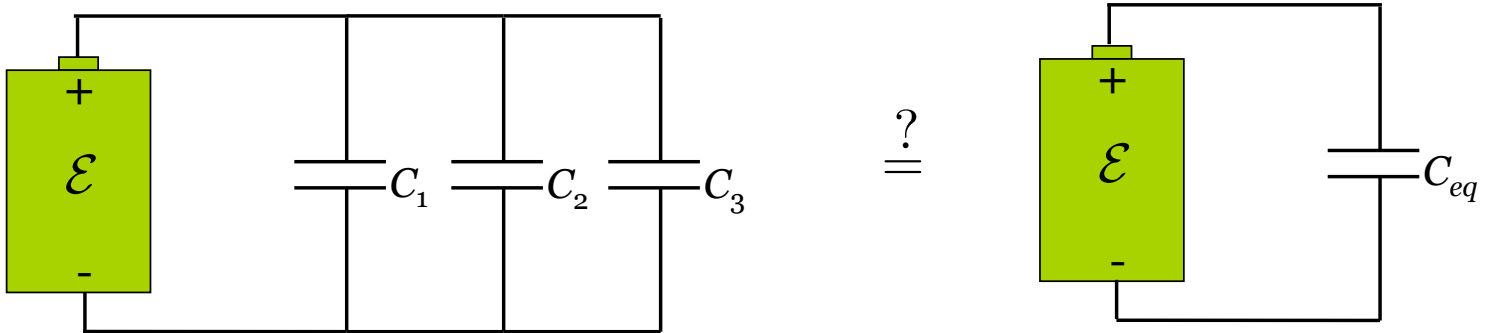
- Can we find an *equivalent capacitor*,  $C_{eq}$ , to the capacitors,  $C_1$ ,  $C_2$ , ... (all in parallel)?



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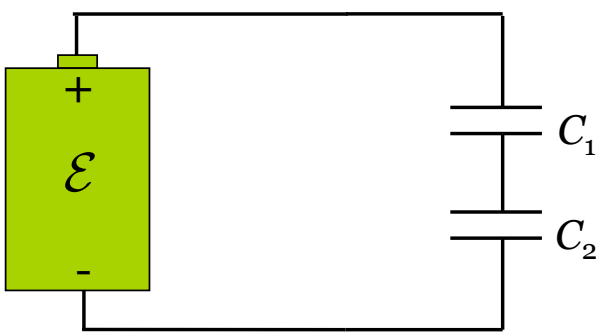


$$C_{eq} = C_1 + C_2 + \dots$$

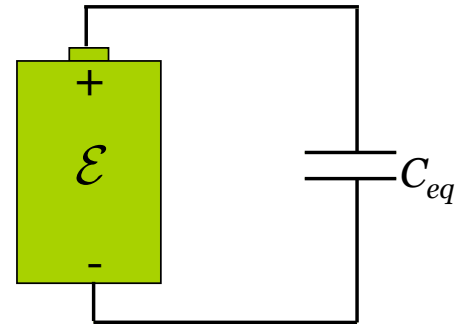
# Combinations of Capacitors

Consider two capacitors in *series*...

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$\stackrel{?}{=}$



$$1.) \Delta \bar{V}_{\text{Bat}} = \Delta V_1 + \Delta V_2$$

$$2.) Q = Q_1 = Q_2$$

$$C = \frac{Q}{\Delta V} \therefore \Delta V = \frac{Q}{C}$$

$$\Delta V_1 = \frac{Q_1}{C_1}$$

$$\Delta V_2 = \frac{Q_2}{C_2}$$

$$\Delta V_{eq} = \frac{Q}{C_{eq}}$$

Plugging into 2)

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

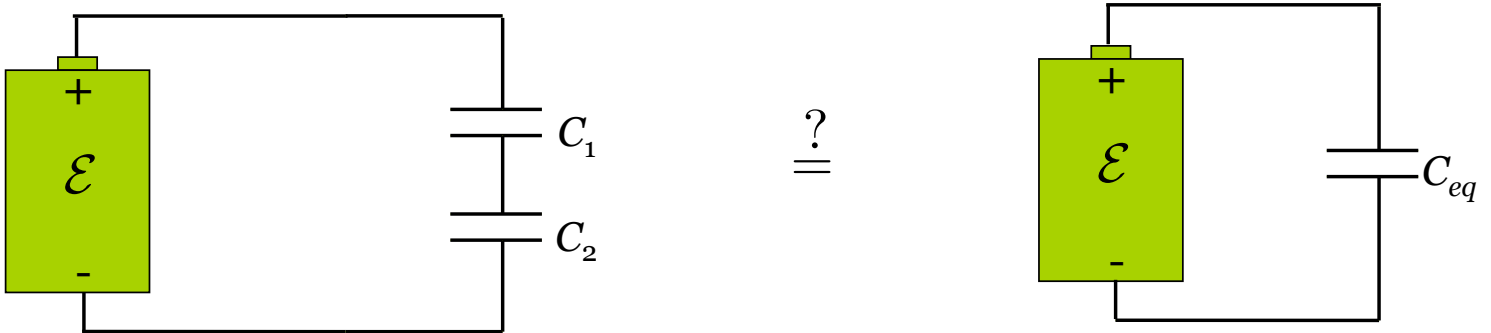
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



# Combinations of Capacitors

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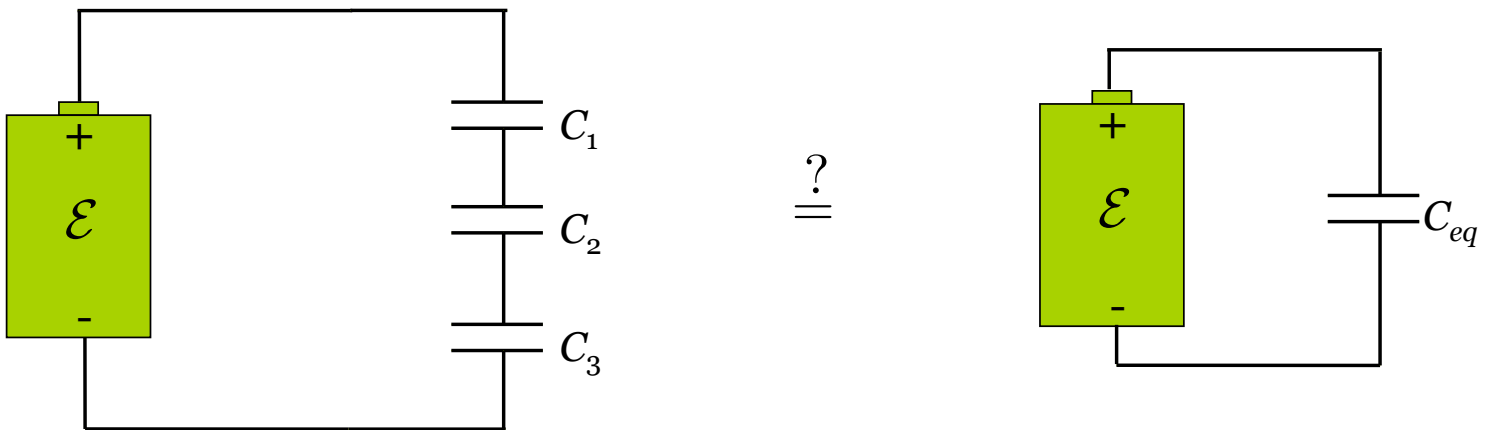
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# Combinations of Capacitors

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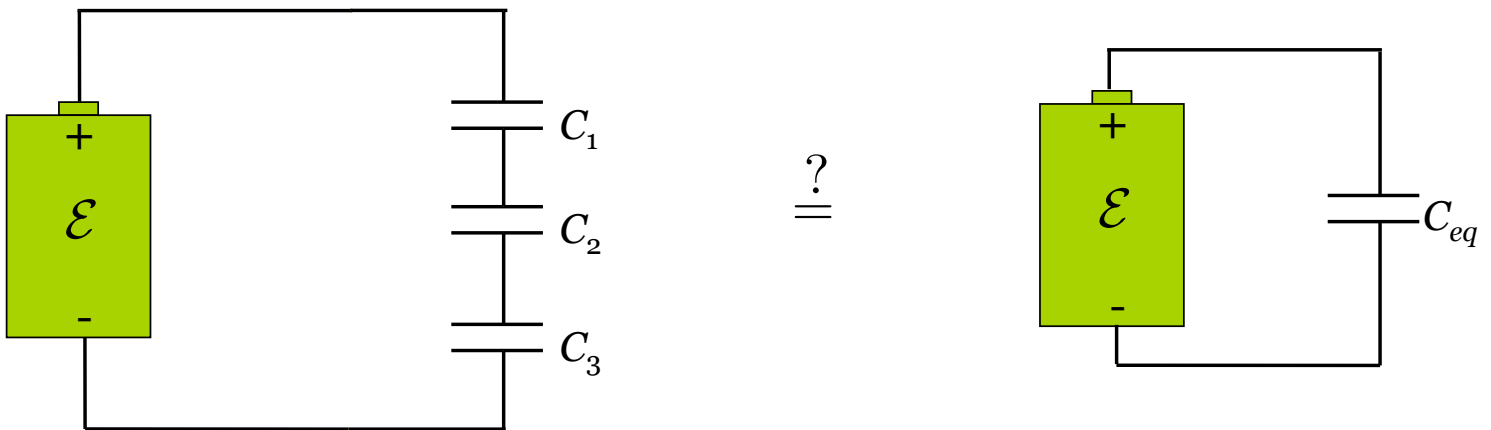
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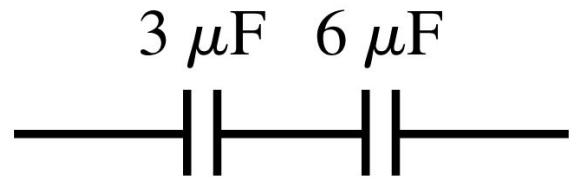


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

## Quiz Question 1

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The equivalent capacitance is



1.  $9\ \mu\text{F}$ .

2.  $6\ \mu\text{F}$ .

3.  $3\ \mu\text{F}$ .

④.  $2\ \mu\text{F}$ .

5.  $1/2\ \mu\text{F}$ .

$$\frac{1}{3\mu\text{F}} + \frac{1}{6\mu\text{F}} = \frac{3}{6\mu\text{F}} = \left(\frac{1}{2\mu\text{F}}\right)^{-1} = C_{eq}$$

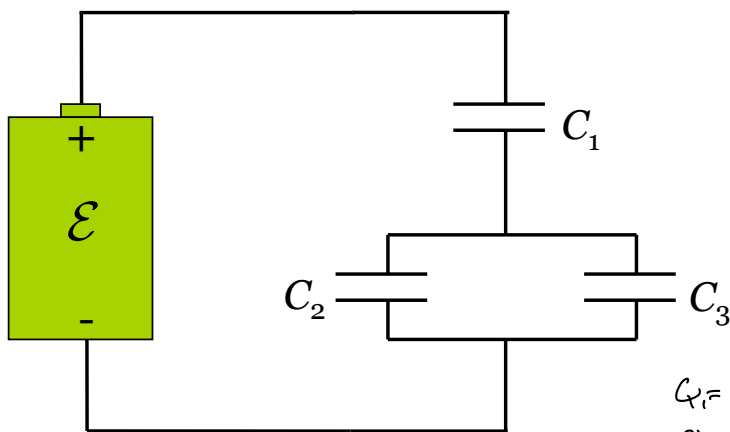
$$C_{eq} = 2\mu\text{F}$$

i.e. 29.7:

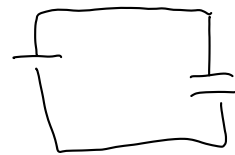
## A Capacitor Circuit

Find the charge on and the potential difference across each of the three capacitors in the figure below.

Given:  $\mathcal{E} = 12 \text{ V}$ ,  $C_1 = 3 \mu\text{F}$ ,  $C_2 = 5 \mu\text{F}$ ,  $C_3 = 1 \mu\text{F}$



$$C_2 + C_3 = 6 \mu\text{F} = C_4$$
$$\frac{1}{3 \mu\text{F}} + \frac{1}{6 \mu\text{F}} = \frac{1}{2 \mu\text{F}} = 2 \mu\text{F}$$



$$Q_1 = C_1 \Delta V_1$$
$$Q_2 = C_2 \Delta V_2$$
$$Q_3 = C_3 \Delta V_3$$
$$Q_1 = Q_4 = 24 \mu\text{C}$$
$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{24 \mu\text{C}}{3 \mu\text{F}} = 8 \text{ V}$$

$$Q_{\text{eq}} = C_{\text{eq}} \Delta V_{\text{eq}}$$
$$= 2 \mu\text{F} (12 \text{ V})$$
$$Q = 2.4 \times 10^{-5} \text{ C}$$

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{24 \mu\text{C}}{6 \mu\text{F}} = 4 \text{ V}$$

$$\Delta V_2 = \Delta V_3 = \Delta V_4 = 4 \text{ V}$$

$$Q_2 = C_2 \Delta V_2$$
$$= 5 \mu\text{F} (4 \text{ V})$$

$$Q_2 = 20 \mu\text{C}$$

$$Q_3 = C_3 \Delta V_3$$
$$= (1 \mu\text{F}) (4 \text{ V})$$

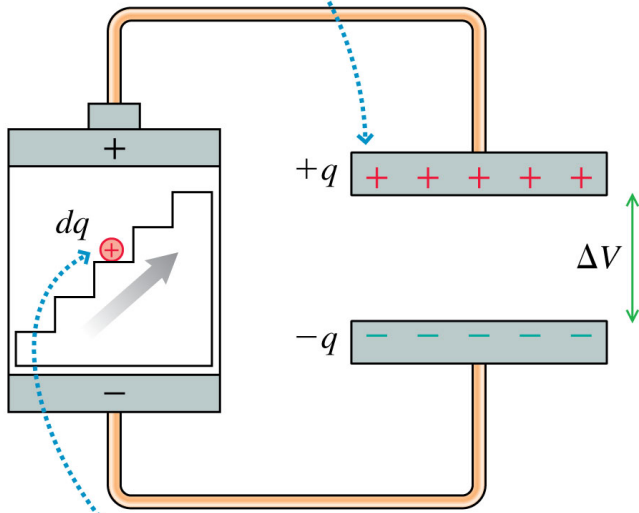
$$Q_3 = 4 \mu\text{C}$$

## 29.6:

# The Energy Stored in a Capacitor

How much *energy* is transferred from the battery to the capacitor?

The instantaneous charge on the plates is  $\pm q$ .



The charge escalator does work  $dq \Delta V$  to move charge  $dq$  from the negative plate to the positive plate.

Initially :  $q = 0, U = 0$

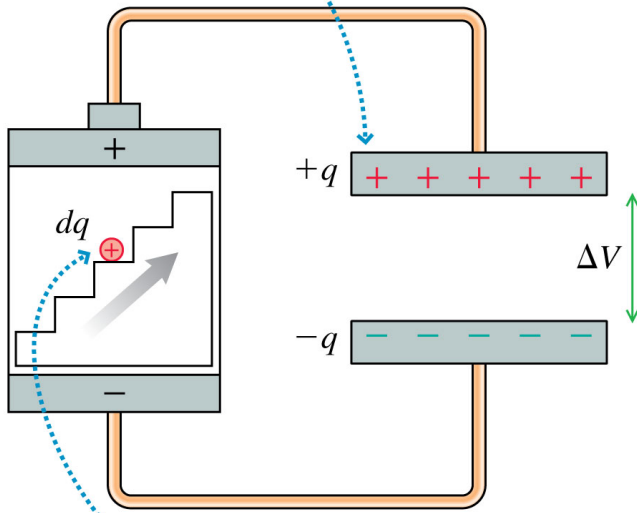
Finally :  $q = Q, U = U_C$

## 29.6:

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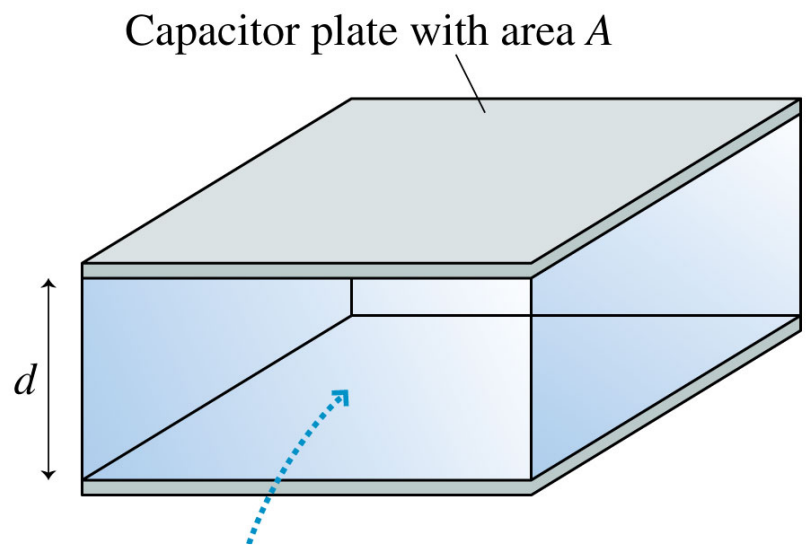
$$U_C = \frac{Q^2}{2C}$$

or

$$U_C = \frac{1}{2} C (\Delta V_C)^2$$

# The Energy in the Electric Field

Q: If a capacitor is analogous to a stretched spring, *where* is the stored energy?





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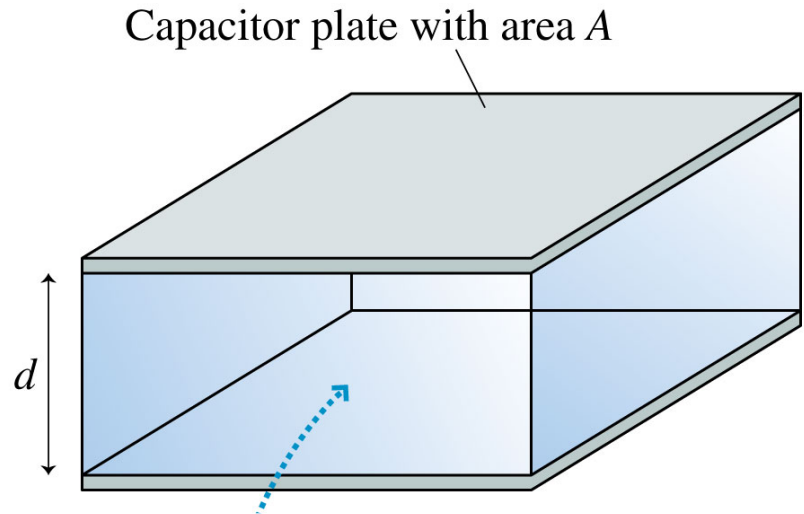
□ A: In the  $E$ -field!

$$u_E = \frac{1}{2} \epsilon_0 E^2$$

energy stored

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volume in which it is stored



When the capacitor is discharged, the energy is released as the  $E$ -field *collapses*.