Physics 311

Homework Set 8

1a) Consider Laplace's equation in spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0. \tag{1}$$

Find the general solution to Laplace's equation in spherical coordinates, for the case where the electric potential depends only on r.

b) Consider Laplace's equation in cylindrical coordinates:

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2} = 0. \tag{2}$$

Find the general solution to Laplace's equation in cylindrical coordinates, for the case where the electric potential depends only on s.

2. Find the potential in the infinite slot of Ex. 3.3 if the boundary at $x = 0$ consists of two metal strips: one, from $y = 0$ to $y = a/4$, is held at a constant potential V_0, and the other, from $y = a/4$ to $y = a$, is at potential $-V_0/4$.

3. A rectangular pipe, running parallel to the z-axis (from $-\infty$ to ∞), has three grounded metal sides at $y = 0$, $y = b$, and $x = 0$. The fourth side, at $x = a$, is maintained at a specific potential $V_0(y)$.

   a) Develop a general formula for the potential within the pipe.

   b) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).

4. A cubical box (sides of length a) consists of five metal plates, which are welded together and grounded. The plate at $y = a$ is made of a separate sheet of metal, insulated from the others, and held at a constant potential V_0. Find the potential inside the box.