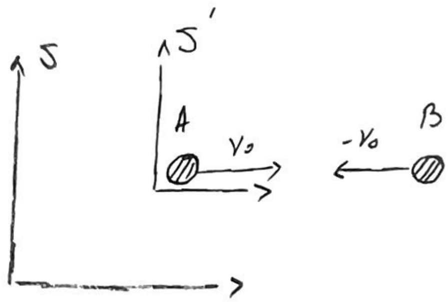
CHAPTERS 1-2 SPECIAL RELATIVITY

1.)



THE GALILEAN VELOCITY TRANSFORMATION ARE

$$v_x' = v_x - V$$
$$v_y' = v_y$$
$$v_z' = v_z$$

a) TO FIND B's VELOCITY IN A's REST FRAME, LABEL A's FRAME AS THE S' FRAME...

NOTICE:

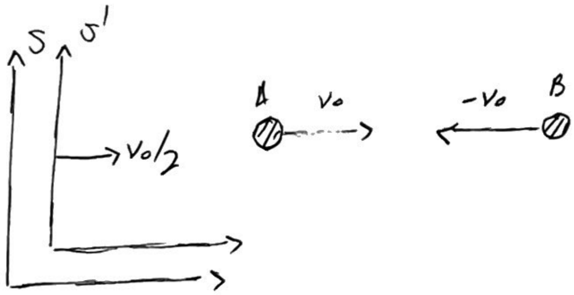
$v_x = -v_0$ (THE X-COMP OF VELOCITY OF B RELATIVE TO S)

$V = v_0$ (THE VELOCITY OF S' RELATIVE TO S IS A's VELOCITY)

SO

$$\left[v_x' = (-v_0) - v_0 = -2v_0 \right]$$

b)



HENCE

$v_x = v_0$

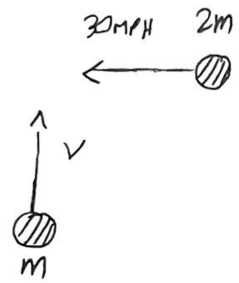
$V = v_0/2$

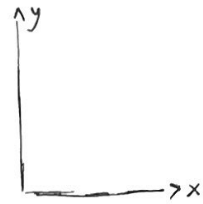
SO

$$\left[v_x' = v_0 - v_0/2 = v_0/2 \right]$$

1-7

INITIALLY...





AFTER COLLISION...

MOVING EXACTLY NORTHWEST MEANS THAT $\theta = 45°$

$\therefore \; v_{f,x} = v_{f,y}$

WHERE v IS OUR UNKNOWN SPEED...

CONSERVATION OF MOMENTUM, WHICH IS A VECTOR RELATIONSHIP, IMPLIES...

$$\vec{p}_{i,x} = \vec{p}_{f,x} \qquad\qquad \vec{p}_{i,y} = \vec{p}_{f,y}$$

1) $-2m(30\,MPH) = -3m\,v_{f,x}$ 2) $mv = 3m\,v_{f,y}$

AS BOTH RIGHTSIDES ARE EQUIVALENT, WE HAVE..

$$+2m(30\,MPH) = mv$$

SO $\boxed{v = 60\,\frac{\text{MILES}}{\text{HR}}}$

NOW

2) $v_{f,y} = \frac{1}{3}v = 20\,\text{MILES}/\text{HR}$

1) $v_{fx} = \frac{2}{3}\left(30\,\frac{\text{MILES}}{\text{HR}}\right) = 20\,\text{MILES}/\text{HR}$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(20\,MPH)^2 + (20\,MPH)^2}$$

$$\boxed{v_f = 28\,\frac{\text{MILES}}{\text{HR}}}$$

2-4 $V_a = 50 m/s$ (AIRPLANE SPEED RELATIVE TO THE AIR)

$\vec{V}_w = 40 m/s\ \hat{i}$ (WIND SPEED RELATIVE TO THE GROUND)

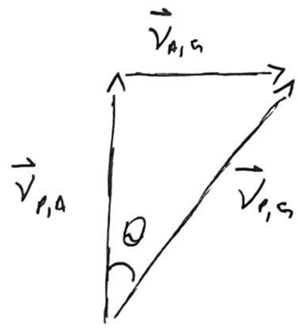
a) $\vec{V}_0 = 50 m/s\ \hat{j}$

THIS IS A VECTOR ADDITION PROBLEM..

$$\vec{V}_{PLANE,GROUND} = \vec{V}_{PLANE,AIR} + \vec{V}_{AIR,GROUND}$$

SO

$$\vec{V}_{PLANE,GROUND} = (50 m/s)\hat{j} + (40 m/s)\hat{i}$$



NOW

$$V_{P,G} = \sqrt{(50 m/s)^2 + (40 m/s)^2}$$

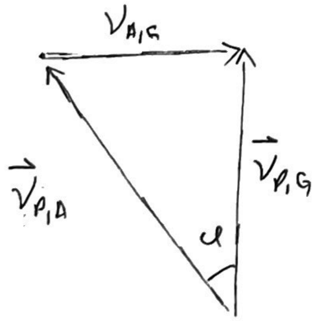
$$\boxed{V_{P,G} = 64 m/s}$$

ALSO

$$\tan\theta = \frac{40 m/s}{50 m/s} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{4}{5}\right)$$

$$\boxed{\theta = 39^\circ}$$

b) HERE, THE VECTORS LOOK LIKE...



AGAIN

$$\vec{V}_{P,G} = \vec{V}_{P,A} + \vec{V}_{A,G}$$

WHERE

$$\vec{V}_{A,G} = (40 m/s)\hat{i}$$

$$\vec{V}_{P,G} = V_{PG}\ \hat{j}$$

$$|\vec{V}_{P,A}| = 50 m/s$$

USING THE PYTHAGOREAN THEOREM..

$$(50 m/s)^2 = (40 m/s)^2 + V_{PG}^2 \qquad \text{so} \qquad V_{PG} = \sqrt{(50 m/s)^2 - (40 m/s)^2}$$

$$\boxed{V_{AG} = 30\,m/s}$$

AGAIN, FROM TRIGONOMETRY..

$$\tan\varphi = \frac{V_{A,G}}{V_{I,G}} = \frac{40\,m/s}{30\,m/s} \quad \text{so} \quad \varphi = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\boxed{\varphi = 53^\circ}$$

2-6

WHERE

$V_{A,G} = 25 \frac{\text{MILES}}{\text{HR}}$

VECTOR DECOMPOSITION GIVES THE COMPONENTS OF THE WIND..

$$V_{A,Gx} = V_{A,G}\cos(60^\circ) = (25\,\text{MPH})\cos(60^\circ) = 12.5\,\text{MPH}$$

$$V_{A,Gy} = V_{A,G}\sin(60^\circ) = (25\,\text{MPH})\sin(60^\circ) = 21.7\,\text{MPH}$$

SO

$$\vec{V}_{A,G} = (12.5\,\text{MPH})\,\hat{i} + (21.7\,\text{MPH})\,\hat{j}$$

THIS IS ANOTHER VELOCITY ADDITION PROBLEM

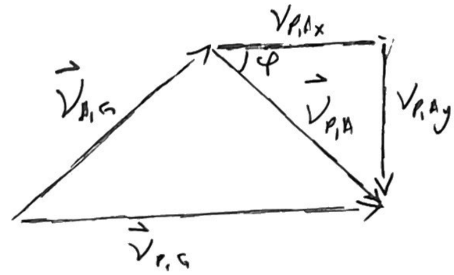
$$\vec{V}_{\text{PLANE, GROUND}} = \vec{V}_{\text{PLANE, AIR}} + \vec{V}_{\text{AIR, GROUND}}$$

OR

(*) $$\vec{V}_{P,G} = \vec{V}_{P,A} + \vec{V}_{A,G}$$

WHERE

$V_{P,A} = 25\,\text{MPH}$



WE NEED

$$\vec{V}_{P,G} = V_{P,G}\,\hat{i}$$

SO, THE (*) EQUATION IS OF THE FORM..

$$V_{P,G}\,\hat{i} = \underbrace{\left[(25\,\text{MPH})\cos\varphi\,\hat{i} - (25\,\text{MPH})\sin\varphi\,\hat{j}\right]}_{\vec{V}_{P,A}} + \underbrace{\left[(12.5\,\text{MPH})\,\hat{i} + (21.7\,\text{MPH})\,\hat{j}\right]}_{\vec{V}_{A,G}}$$

EQUATING COMPONENTS YIELDS..

1. $0 = (-25\,\text{MPH})\sin\varphi + (21.7\,\text{MPH})$ (y-COMP)

2. $V_{P,G} = (25\,\text{MPH})\cos\varphi + (12.5\,\text{MPH})$ (x-COMP)

THE 1^{st} EQN YIELDS..

$$\sin\varphi = \frac{21.7\,\text{MPH}}{25\,\text{MPH}} \quad \therefore \quad \varphi = \sin^{-1}\left(\frac{21.7}{25}\right) \quad \therefore \quad \boxed{\varphi = 60^\circ}$$

PLUGGING THIS INTO EQN. 2 YIELDS..

$$V_{P,G} = (25\,MPH)\cos(60^\circ) + (12.5\,MPH)$$

$$V_{P,G} = 25\,MPH =$$

b) NOW

$$V_{P,G} = \frac{d}{t} \qquad \text{WHERE} \quad d = 5\,\text{MILES}$$

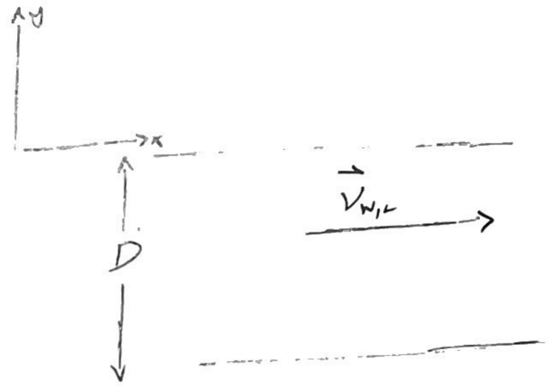
$$\text{SO} \quad t = \frac{d}{V} = \frac{5\,\text{MILES}}{25\,\text{MILES}/hr} = 0.20\,hr$$

2-8

$\vec{V}_{WATER, LAND} = 0.50 m/s \ \hat{i}$

$D = 35 m$

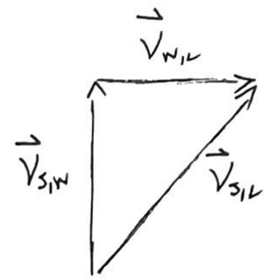
$V_{S, WATER} = 1.00 m/s$



a) $\vec{V}_{S,W} = (1.00 m/s)\hat{j}$

ANOTHER VECTOR ADDITION PROBLEM...

(*)
$$\vec{V}_{S,LAND} = \vec{V}_{S,W} + \vec{V}_{W,L}$$



SO

$$\vec{V}_{S,L} = (1.00 m/s)\hat{j} + (0.50 m/s)\hat{i}$$

TO ANSWER THESE QUESTIONS, WE CAN ANALYZE THE X- AND y-COMPONENTS SEPARATELY...

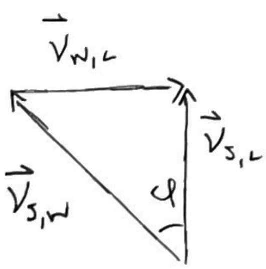
$$V_y = \frac{D}{\Delta t} \quad \text{SO} \quad \Delta t = \frac{D}{V_y} = \frac{35 m}{1.00 m/s} = 35 s$$

SO $\boxed{\Delta t = 35 s}$ - TIME TO GET ACROSS

$$\Delta x = V_x \Delta t = (0.50 m/s)(35 s)$$

$\therefore \boxed{\Delta x = 18 m}$ - DISTANCE SWEPT DOWNSTREAM

b)



HERE WE NEED

$$\vec{V}_{S,L} = V_{S,L}\hat{j}$$

THE (*) EQN, IN COMPONENT FORM, IS...

$$V_{S,L}\hat{j} = \left[+(1.00 m/s)\cos\varphi\,\hat{j} - (1.00 m/s)\sin\varphi\,\hat{i}\right] + (0.50 \tfrac{m}{s})\hat{i}$$

EQUATING COMPONENTS YIELDS..

$$0 = -(1.00\,m/s)\sin\varphi + (0.50\,m/s)$$

$$V_{S,L} = (1.00\,m/s)\cos\varphi$$

THE 1ST EQN YIELDS..

$$\sin\varphi = \frac{0.50\,m/s}{1.00\,m/s} \quad \therefore \quad \varphi = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

$$\boxed{\varphi = 30°}$$

THE 2ND EQN YIELDS..

$$V_{S,L} = (1.00\,m/s)\cos(30°)$$

$$V_{S,L} = 0.87\,m/s = \frac{D}{\Delta t} \quad \text{so} \quad \Delta t = \frac{D}{V_{S,L}} = \frac{35\,m}{0.87\,m/s}$$

$$\boxed{\Delta t = 40\,s}$$

1-2 a) $\bar{V} = v_0$ +1

$v_x = -v_0$ +1

$v_x' = v_x - \bar{V} = -2v_0$ +2

b) $\bar{V} = v_0/2$ +1

$v_x = v_0$ +1

$v_x' = v_x - \bar{V} = v_0 - v_0/2 = v_0/2$ +2

2-8 $\Delta t = 35s$ +2

$\Delta x = 17.5m$ +2

$\varphi = 30°$ +2

$\Delta t : 40.2s$ +2

1-7 $\vec{P}_{i,x} = \vec{P}_{f,x}$ + 1

$2m(-30\frac{miles}{hr}) = -3mv_{fx} = -3mv\cos(45°)$ +2

$\vec{P}_{iy} = \vec{P}_{f,y}$ +1

$mv = 3m\,v_{f,y} = 3mv\sin 45°$ +2

$v = 60MPH$ +2

$v_f = 28MPH$ +2

2-40) $v_{P,G} = 64 m/s$ +2

$\Theta = 39°$ +2

b) $v_{P,G} = 30 m/s$ +2

$\varphi = 53°$ +2

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a) $\varphi = 60°$ +2

b) $v_{P,G} = 25 mPH$ +2

$t = \frac{d}{v}$ +2

$t = 0.20 hr$ +2