Phys 131 2016

Lecture 23

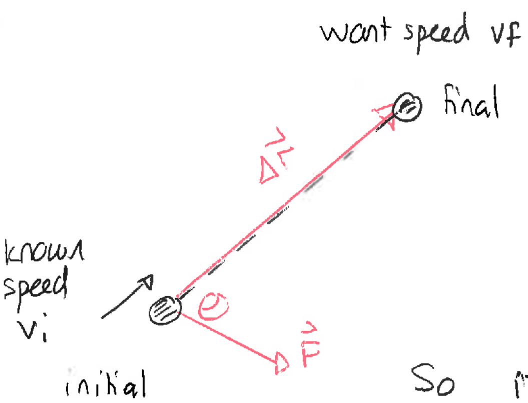
Thurs: Seminar 12:30pm WS 117

Fri: HW by 5pm
- Supp Ex 58
- Ch 9 Conc. Q 6, 8
- Ch 9 Prob 12, 14, 18, 43, 44

Mon: Warm Up 10

Work and Kinetic Energy

The concepts of work and kinetic energy offer alternative approaches to assessing the motion of objects. So far we have considered an object moving in a straight line while subject to various constant forces. The key result here is that:



> The kinetic energy satisfies:
>
> $$\Delta K = W_{net}$$
>
> where $W_{net} = W_1 + W_2 + \ldots$

So if we know the speed of the particle at the initial instant, we can determine the speed at a later instant using:

Determine initial kinetic energy $K_i = \frac{1}{2} m v_i^2$

Determine work done by each force $W = F \Delta r \cos\theta$

Determine net work $W_{net} = W_1 + \ldots$

Determine final kinetic energy $K_f = K_i + W_{net}$

Determine speed via $\frac{1}{2} m v_f^2 = K_f$

We stress that in the definition of work done:

1) both F and Δr are always positive

2) θ is the angle between the force and displacement

We can also observe that for any single force

$W > 0 \iff$ force tends to cause object to speed up

$W < 0 \iff$ " " " " " slow down.

Quiz 1 $80\% - 95\% \lessgtr 70\% - 100\%$

Vector dot product

Provided that the force acting on an object is constant and that it moves in a straight line, the work done only depends on two vectors:

1) displacement $\Delta \vec{r}$

2) force $\vec{F}$

We seek a multiplicative algebraic method for combining these vectors to give work. We can do this by defining the dot product of any two vectors

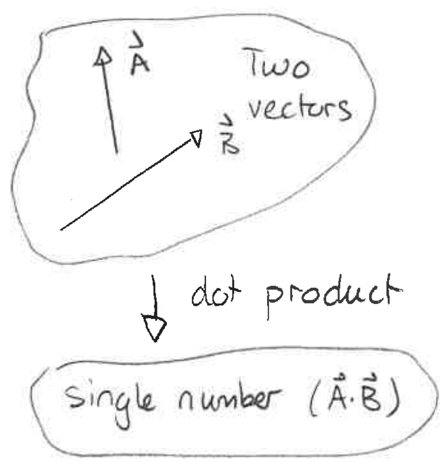
Let $\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$

$\vec{B} = B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k}$

along x   along y   along z

Then the dot product of $\vec{A}$ with $\vec{B}$ is a scalar:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$



The dot product has the following features:

1) $\vec{A}\cdot\vec{B}$ is a scalar – there are no $\hat{i}, \hat{j}$, etc,... left

2) $\vec{A}\cdot\vec{B}$ can be positive or negative

3) $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$ etc,...

4)
$\hat{i}\cdot\hat{i} = 1$ $\quad$ $\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{j} = 1$ $\quad$ $\hat{i}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{k} = 1$ $\quad$ $\hat{j}\cdot\hat{k} = 0$

An additional useful geometric result is that

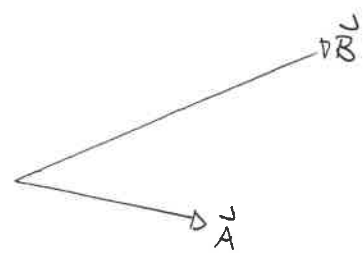
For any vectors $\vec{A}, \vec{B}$

$$\vec{A}\cdot\vec{B} = A\,B\cos\alpha$$

where $A \geq$ is magnitude of $\vec{A}$

$B \geq$ " " " $\vec{B}$

α is angle between $\vec{A}$ and $\vec{B}$



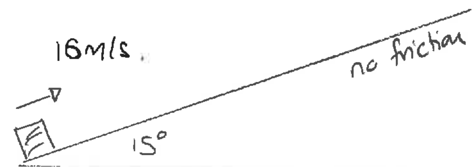
Quiz 2 $\quad$ 50% → 90% $\quad$ & $\quad$ 80% → 100%

Quiz 3 $\quad$ 70% → 95% $\quad$ & $\quad$ 70% → 90%

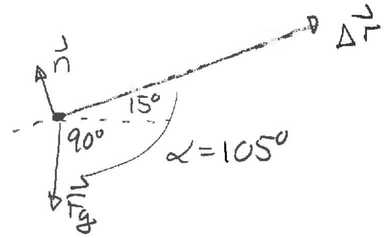
With this definition we can see that

> If $\vec{F}$ is a constant force acting on an object which moves in a straight line with displacement $\Delta\vec{r}$ then the work done by the force is
>
> $$W = \vec{F}\cdot\Delta\vec{r}$$

Quiz 4



<u>Example:</u> A crate is launched up a ramp with speed 16m/s at the base of the ramp. Determine how far along the ramp it slides before coming to a halt.



<u>Answer:</u> $W_{net} = \Delta K$

$$= K_f - K_i$$

$$W_{net} = \cancel{\tfrac{1}{2}mv_f^2} - \tfrac{1}{2}mv_i^2$$

Now $W_{net} = \underbrace{W_{normal}}_{\substack{=0 \\ \text{since } \vec{n},\Delta\vec{r} \\ \text{perp.}}} + W_{grav}$

Thus $W_{grav} = -\tfrac{1}{2}mv_i^2$.

Then $W_{grav} = \vec{F_g}\cdot\Delta\vec{r} = F_g\,\Delta r\cos\alpha = mg\,\Delta r\cos 105°$

$\Rightarrow \cancel{m}g\,\Delta r\cos 105° = -\tfrac{1}{2}\cancel{m}v_i^2$

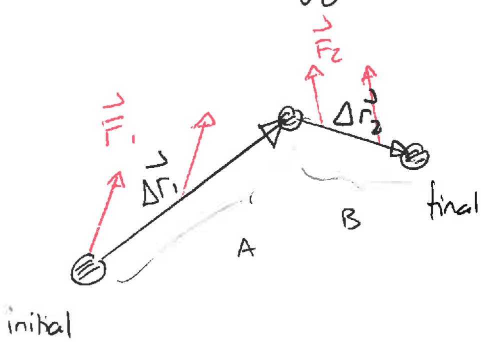
$\Rightarrow \Delta r = -\dfrac{v_i^2}{2g\cos 105°} = -\dfrac{(16\,m/s)^2}{2\times 9.8\,m/s^2\cos 105°} \Rightarrow \boxed{\Delta r = 50m}$

Work in more general cases

What if the object does not move in a straight line or the force is not constant? We will encounter examples of these:

- springs
- objects moving in curved trajectories.

In all cases the definition of work can be generalized so that the work kinetic energy theorem is still true. Consider the illustrated example:



Here we can form the work for the entire path

$$W_{entire} = W_A + W_B$$
$$= \vec{F}_1 \cdot \Delta\vec{r}_1 + \vec{F}_2 \cdot \Delta\vec{r}_2$$

We can show that the work kinetic energy theorem is still true in this case. More generally we can break the trajectory into infintesimally short sections. Then

$$\boxed{W = \sum \vec{F}_i \cdot \Delta\vec{r}_i}$$

If the object moves along the x-axis then:

$$W = \sum_i F_{ix} \Delta x_i = \int F_x \, dx$$

In all these cases, the work kinetic energy theorem is still true.

$$\boxed{W_{net} = \Delta K}$$