Physics 311

Homework Set 2

1. Calculate the *divergence* of the following vector functions:

$$\begin{aligned}
\vec{v}_a &= x^4\ \hat{x} + 4x^2z^2\ \hat{y} - 2y^3z\ \hat{z} \\
\vec{v}_b &= x^2y\ \hat{x} + 3x^2z\ \hat{y} + 2yz^2\ \hat{z} \\
\vec{v}_c &= 4y^3\ \hat{x} + (2x^2z + z^3)\ \hat{y} + 2xy^2\ \hat{z} \qquad (1)
\end{aligned}$$

2. Calculate the *curl* of the vector functions in the previous problem.

3. Construct a vector function that has *zero* divergence and *zero* curl everywhere, a constant vector function is too trivial!

4. Check product rule (vi) (by calculating each term separately) for the functions

$$\begin{aligned}
\vec{A} &= 3y\ \hat{x} + z\ \hat{y} + 2x\ \hat{z} \\
\vec{B} &= 2x\ \hat{x} - 3z\ \hat{y} \qquad (2)
\end{aligned}$$

5. Calculate the Laplacian of the following functions:

   a) $T_a = z^2 + 2zx + 3y + 5$

   b) $T_b = \sin x \cos y \sin z$

   c) $T_c = e^{-2z} \sin 3x \cos 4y$

   d) $\vec{v} = z^2\ \hat{x} + 2zy^2\ \hat{y} - zy\ \hat{z}$

6. Consider the generic vector function

$$\vec{B} = B_x\ \hat{x} + B_y\ \hat{y} + B_z\ \hat{z} \qquad (3)$$

   where B_x, B_y, B_z are the components.

   a) Show that the divergence of a curl of this generic vector function is always zero.

   b) Show that the curl of a gradient of the scalar function, $g = g(x, y, z)$, is always zero.

7. Calculate the line integral of the function $\vec{v} = z^2\,\hat{x} + 2zy^2\,\hat{y} - zy\,\hat{z}$ from the origin to the point $(1,1,1)$ by three different routes:

   a) $(0,0,0) \;\rightarrow\; (1,0,0) \;\rightarrow\; (1,1,0) \;\rightarrow (1,1,1)$;

   b) $(0,0,0) \;\rightarrow\; (0,0,1) \;\rightarrow\; (0,1,1) \;\rightarrow (1,1,1)$;

   c) The direct straight line.