MAT 201
Larson/Edwards – Section 2.2
Basic Differentiation Rules and Rates of Change

Recall the notation that we may use for the derivative:

$$f'(x) = \frac{dy}{dx} = y' = \frac{d}{dx}[f(x)] = D_x[y]$$

$f'(x) = \frac{d}{dx}[f(x)]$ (read: "*d*, *dx* of *f* of *x*") means "the derivative of *f* with respect to *x* at *x*".

<u>Basic Rules for Differentiation</u>: Proofs provided in text.
If *f* and *g* are differentiable functions;

1. ***Derivative of a Constant***: $\frac{d}{dx}[c] = 0, \quad c \in R$
2. ***The Power Rule***: $\frac{d}{dx}[x^n] = n \cdot x^{n-1} \quad n \in Rational\ \ Number$
3. ***The Constant Multiple Rule***: $\frac{d}{dx}[cf(x)] = cf'(x) \quad c \in R$
4. ***The Sum/Difference Rule***: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
5. ***Derivative of Sine and Cosine Functions***:
   $\frac{d}{dx}[\sin x] = \cos x \quad and \quad \frac{d}{dx}[\cos x] = -\sin x$

Ex: Find the derivative for each of the following functions:
a) *f(x) = 5*

b) $y = x^4$

c) $y = \frac{3}{x^2}$

d) $s(t) = \sqrt{t} + \cos t$

e) $f(x) = 6x^3 - 2x^2 + \frac{1}{x}$

Ex: Find the derivative for each of the following functions:

a) $f(x) = 5$

$$\boxed{f'(x) = 0}$$

b) $y = x^4$

$$y' = 4x^{4-1}$$

$$\boxed{y' = 4x^3}$$

c) $y = \dfrac{3}{x^2}$

$$y = 3x^{-2}$$

$$y' = 3 \cdot -2x^{-2-1}$$

$$\boxed{y' = -6x^{-3} \text{ or } y' = \frac{-6}{x^3}}$$

d) $s(t) = \sqrt{t} + \cos t$

$$s(t) = t^{\frac{1}{2}} + \cos t$$

$$s'(t) = \frac{1}{2}t^{\frac{1}{2}-1} + -\sin t$$

$$s'(t) = \frac{1}{2}t^{-1/2} - \sin t$$

or

$$s'(t) = \frac{1}{2\sqrt{t}} - \sin t$$

or

$$s'(t) = \frac{\sqrt{t}}{2t} - \sin t$$

e) $f(x) = 6x^3 - 2x^2 + \frac{1}{x}$

$\hookrightarrow x^{-1}$

$f'(x) = 6 \cdot 3x^{3-1} - 2 \cdot 2x^{2-1} + -1 \cdot x^{-1-1}$

$f'(x) = 18x^2 - 4x - x^{-2}$

or

$f'(x) = 18x^2 - 4x - \frac{1}{x^2}$

Ex: a) Find the slope and equation of the tangent line to $f(x)=\frac{1}{4}x^2 - 2$ at the point (-4, 2).

b) At what point is the tangent line horizontal?

<u>Rates of Change</u>:
Since the derivative is used to find the slope of a tangent line to a curve at a point, and slope can be thought of as a rate of change, we can use the derivative to determine the rate of change of one variable with respect to another in a wide variety of applications such as population growth rates, production rates, water flow rates, velocity, and acceleration.

1. **Average Rate of Change:** The average rate of change is the change in y, divided by the change in x when given two points. $\frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$
2. **Instantaneous Rate of Change:** The rate of change at any given point, c, on a function f. We can find the instantaneous rate of change by finding $f'(c)$.

Ex: Given $f(t) = t^2 - 7$.

a) Find the average rate of change over the interval [3, 4].
b) Find the instantaneous rate of change at t = 3.

Rates of Change Relating to Velocity:

Since velocity is a rate of change, we can find the ***average velocity*** by dividing the change in distance, *s*, by the change in time, *t*:

$$Average\ Velocity = \frac{\text{Change in } s}{\text{Change in } t} = \frac{\Delta s}{\Delta t}$$

We can find the ***instantaneous velocity*** (or simply ***velocity***) of an object at time *t* by evaluating the derivative of *f* at *t*:

$$Velocity = f'(t)$$

Note: A negative velocity indicates that the object is moving downward. The ***speed*** of an object is the absolute value of its velocity, that is, speed cannot be negative.

Position Function:

The position of a free-falling object (neglecting air resistance) under the influence of gravity can be represented by the equation:

$$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$$

where s_0 is the initial height of the object, v_0 is the initial velocity of the object, and g is the acceleration due to gravity. On earth, g = -32 feet/s^2 (U.S. customary) or g = -9.8 m/s^2 (metric system).

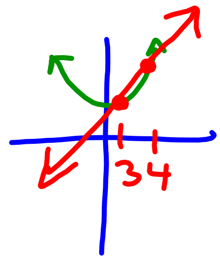
$\frac{f(b)-f(a)}{b-a}$

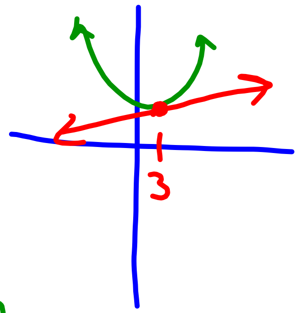
Ex: Given $f(t) = t^2 - 7$.

a b

a) Find the average rate of change over the interval [3, 4].

b) Find the instantaneous rate of change at t = 3.





a) $t=3 \rightarrow f(3)=9-7=2$

$(3,2)$

$t=4 \rightarrow f(4)=16-7=9$

$(4,9)$

slope of line containing these points.

$$\frac{9-2}{4-3} = \boxed{7}$$

b) $f'(t) = 2t$

$f'(3) = 2(3)$

$$\boxed{f'(3) = 6}$$