Statistical and Thermal Physics: Homework 1

Due: 24 January 2020

Some homework problems and class exercises will use simulations developed by the text authors. These can all be run from a java "launcher." You should first install the launcher on the computer that you are using. To do this:

1. Go to the following webpage: http://www.compadre.org/STP/items/detail.cfm?ID=7308.

2. Download the file called "download 3239kb.jar." You may have to save it and then run it.

Running the file produces a "launcher" window (with the tab title Open Source Physics: osp_stp.xset) which lists a long collection of simulations for statistical and thermal physics these are sorted into folders in the directory on the left. Double click on any entry to run the relevant simulation.

General tips about running the simulations:

1. Clicking on a simulation in the "launcher" will produce documentation for the simulation in the launcher window.

2. The "launcher" usually opens a controller window but not necessarily the other windows associated with the simulation. The controller window lets you set parameters. Once you have done this clicking Initialize will open the simulation usually with some other graphical windows. Clicking Start will actually run the simulation.

3. Graphs and images can be save by following Tools → Snapshot . . . .

4. The data that is generated by the simulation can be accessed in tabular form by following Tools → Data Tool, selecting one of the starred objects and clicking OK. This contains a Statistics button that will computer averages and deviations. Data that goes into such a calculation can be selected by highlighting the relevant entries in the columns.

1 Particles in a box with two sections

Suppose that N particles are placed in a box with two sections ("left" and "right"). Suppose that initially all particles are in the left section and at $t = 0$ a partition between the sections is removed. Let $n(t)$ be the number of particles in the left section at time t.

a) How do you expect $n(t)$ will evolve as time passes?

This can be simulated using one of the text simulators. Run the "launcher" and select Approach to equilibrium → Particles in a box. This will open a controller window where the number of particles can be adjusted by setting N in the relevant box (and hitting Initialize). The simulation describes the number of particles on a single side of the box.

b) Starting with $N = 20$, run the simulation. Does n appear to approach a fixed value? If so what is the fixed value? How large are the typical fluctuations in n? Does n ever return to 20?

c) Starting with $N = 200$, run the simulation. Does n appear to approach a fixed value? If so what is the fixed value? How large are the typical fluctuations in n? Does n ever return to 200?

d) Starting with $N = 2000$, run the simulation. Does n appear to approach a fixed value? If so what is the fixed value? How large are the typical fluctuations in n? Does n ever return to 2000?

e) In which case, $N = 20$, $N = 200$ or $N = 2000$, is there are more distinct equilibrium state. What is the equilibrium state?

f) The simulation lets you record the average number of particles $\overline{n}$ and the standard deviation in the number of particles, σ, which quantifies the fluctuations in n. In each case, after the simulation has appeared to reach equilibrium, determine $\overline{n}$ and σ for a period during which the system is close to equilibrium. A measure of the typical size of fluctuations is $\sigma/\overline{n}$. Determine this for each case. How does it behave as N increases? What does this imply about the precision of the equilibrium state as N increases?

## 2 Interacting systems in equilibrium

Much of thermodynamics deals with systems that interact. The usual consideration here, is that there are two isolated systems that are in equilibrium states. These are brought into thermal contact and they than reach a new equilibrium state. The issue is to find the common property that describes this new equilibrium state. The simulation Identification of the Temperature explores this. Run the "launcher" and select Identification of the Temperature and, using the default parameters, click "Initialize."

The simulation shows two systems, which by default are thermally isolated. They can be brought into thermal contact by clicking the "Contact" button.

a) Run the simulation with the two systems isolated. The simulation will display fluctuating energies for both systems. By averaging over a reasonable duration, record the average kinetic and potential energies per particle for each system. Use these to determine the total energy for each subsystem. Are any of these quantities the same for each system?

b) Without resetting, click the "Contact" button. Which of the energies change?

c) After the systems have reached a common equilibrium record the average kinetic and potential energies per particle for each system. Use these to determine the total energy for each subsystem. Which way did energy flow?

d) After the two systems have reached a common thermal equilibrium, which quantity is the same for the two systems? Given that the temperature of two systems in thermal equilibrium must be the same, which quantity best represents temperature?

3 Constant volume gas thermometer

A constant volume gas thermometer consists of a gas inside a flask whose volume is fixed. The pressure of the gas is monitored. The container can be immersed inside any fluid and the pressure can be monitored. The flask is immersed into fluid A and the pressure is monitored. After the pressure has settled it is recorded, giving a pressure of $10\,\text{kPa}$. Then the flask is removed and immersed in fluid B, which is isolated from fluid A. After the pressure has settled it is recorded, giving a pressure of $20\,\text{kPa}$.
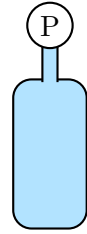
a) List the variables that can be used to describe the state of the gas inside the flask. For each variable, describe whether it is the same (after settling) for fluid A as for fluid B. Is the state of the gas the same for fluid A as for fluid B? Explain your answer.

b) Would fluid A and fluid B be in thermal equilibrium if they were placed into contact with each other? Do they have the same or different temperatures? Explain your answers.

4 Venus' atmosphere

The atmosphere of Venus consists of 96.5% carbon dioxide and 3.5% molecular nitrogen (N_2).

a) Determine the mass of one mole (6.02×10^{23} molecules) of Venus' "air."

b) The surface temperature of Venus is $740\,\text{K}$ and the surface pressure $9.2 \times 10^6\,\text{Pa}$. Determine the density of Venus' atmosphere.

a) Expect that after some time $n(t) = N/2$.

b) with $N=20$ there is a lot of fluctuation. between $n=4$ and $n=16$
   $\Rightarrow$ fluctuates around $n=10$. We observe

   $\bar{n} =$ 10.1

   $\sigma =$ 2.4

   Does not return to 20

c) There are still fluctuations from
   $n = 85 \rightarrow 115$
   Does not   return to 200

d) There are still fluctuations

   $n =$

   does not return to 2000

e) It is more distinct for larger N

f)   See table $\rightarrow$

   As N increases $\sigma/\bar{n}$ decreases

     $\Rightarrow$ equilibrium state more precise

N	$\bar{n}$	σ	$\sigma/\bar{n}$
20	10.1	2.4	0.24
200	102	13	0.13
2000	1003	16	0.016

unnamed

Measure | Analyze | Data Builder... | Help

max	5.630E2	1.600E1
min	5.000E1	4.000E0
mean	3.065E2	1.010E1
sd	1.485E2	2.353E0
se	6.551E0	1.038E-1
n	514	514

markers		✔
lines		☐
style		☐
axis	horiz	vert

row	x	y
541	541	9
542	542	8
543	543	9
544	544	10
545	545	9
546	546	10
547	547	11
548	548	12
549	549	13
550	550	14
551	551	13
552	552	12
553	553	13
554	554	14
555	555	13
556	556	14
557	557	13

Correlation coefficient (R-squared) = 0.091

non-editable

	unnamed	unnamed_1	

Measure	Analyze		Data Builder...	Help

max	3.803E3	2.000E2
min	7.990E2	7.600E1
mean	2.301E3	1.018E2
sd	8.676E2	1.329E1
se	1.583E1	2.153E-1
n	3005	3813
markers		☑
lines		☐
style		☐
axis	horiz	vert

row	x	y
3781	3,781	109
3782	3,782	108
3783	3,783	109
3784	3,784	108
3785	3,785	109
3786	3,786	108
3787	3,787	107
3788	3,788	108
3789	3,789	107
3790	3,790	108
3791	3,791	109
3792	3,792	108
3793	3,793	109
3794	3,794	108
3795	3,795	109
3796	3,796	110
3797	3,797	111

Correlation coefficient (R-squared) = 0.024

non-editable

Measure | Analyze   Data Builder... | Help

max	6.998E3	1.072E3
min	4.211E3	1.004E3
mean	5.604E3	1.033E3
sd	8.050E2	1.630E1
se	1.525E1	3.086E-1
n	2788	2788
markers		☑
lines		☐
style		▫
axis	horiz	vert

row	x	y
6976	6,976	1,014
6977	6,977	1,015
6978	6,978	1,014
6979	6,979	1,013
6980	6,980	1,014
6981	6,981	1,013
6982	6,982	1,014
6983	6,983	1,015
6984	6,984	1,016
6985	6,985	1,017
6986	6,986	1,016
6987	6,987	1,015
6988	6,988	1,014
6989	6,989	1,015
6990	6,990	1,014
6991	6,991	1,013
6992	6,992	1,012

Correlation coefficient (R-squared) = 0.000   non-editable

a)

per particle

	$\overline{KE}$	$\overline{PE}$
Yellow	0.0927	−3.057
Blue	0.1407	−4.402

number yellow = 81

"   blue = 64

Total energy = number particles × (average per particle),

Yellow $E = 81 \times [0.0927 \quad -3.057] = -240$ $\Big\}$ not the same

Blue $E = 64 \times [0.1407 - 4.402] = -273$

Total energy = −240 −273 = −513

b) The kinetic energy per particle changes. Potential energy per particle changes a little

c)

	$\overline{KE}$	$\overline{PE}$
Yellow	0.1614	−2.931
Blue	0.1565	−4.471

Yellow $E = 81 \times [0.1614 - 2.931] = -224$ $\Big\}$ total = −500

Blue $E = 64 \times [0.1565 - 4.471] = -276$

Energy flowed from   Blue to Yellow (from higher KE per particle to lower KE/particle)

d) Approximately   KE/particle ← best represents temp.

a)   P, V, T, N

pressure ↓ | ↓ temp ↘ number particles
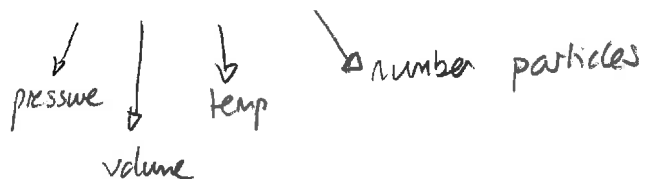
↓ volume

V stays   same since volume is same

N "   "   "   no gas molecules can escape.

P changes   - problem states this.

T changes   - e.g. for ideal   gas   PV=NkT.

b) NO * If they were then each would be in equilib with

flask of   gas

* But the state of flask changes when going from A to B

so it is not in thermal equilibrium with both

a) One mole contains

$$6.02 \times 10^{23} \text{ particles.}$$

$\Rightarrow$ # CO_2 molecules = $0.965 \times 6.02 \times 10^{23}$

$= 5.81 \times 10^{23}$ molecules of CO_2

Then there will be $(6.02 - 5.81) \times 10^{23}$ molecules of N_2

$\Rightarrow$ 0.21×10^{23} molecules of N_2.

One mole of CO_2 has mass $\quad 12g + 2 \times 16g = 44g$

" $\quad$ " $\quad$ " $\quad N_2$ " $\quad$ " $\quad 2 \times 14g = 28g$

There are 0.965 mol CO_2 and 0.035mol N_2. So mass is:

$$0.965 \times 44g + 0.035 \times 28g = 43.4g = 0.0434 kg$$

b) Suppose volume V is occupied by n moles with mass M

Then density is

$$\rho = \frac{M}{V} = \frac{n \times \text{mass one mole}}{V}$$

But $PV = nRT \Rightarrow \dfrac{n}{V} = \dfrac{P}{RT} \Rightarrow \rho = \dfrac{P}{RT} \times 0.0434 kg$

$$\Rightarrow \rho = \frac{9.2 \times 10^6 \text{ N/m}^2 \times 0.0434 kg}{8.314 \text{ J/K mol} \times 740 K}$$

$$= 65 kg/m^3$$