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PHYS 362 HW 8

Problem 1

a.) $\Delta S = \int_{T_i}^{T_F} \frac{C_V}{T} dT = C_V(\ln(T_F) - \ln(T_i)) = C_V \ln(T_F/T_i)$

$P_iV_i = NkT_i \therefore T_i = \frac{P_iV_i}{Nk}$

$P_FV_F = NkT_F \therefore T_F = \frac{P_FV_F}{Nk}$

$V_i = V_F$, $P_F = \frac{1}{4}P_i$ $\therefore$ $\Delta S = C_V \ln\left(\frac{P_F\cancel{V_F}/\cancel{Nk}}{P_i\cancel{V_i}/\cancel{Nk}}\right) = C_V \ln\left(\frac{P_F}{P_i}\right) = -C_V\ln(4)$

$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{3}{2}Nk = \frac{3}{2}\frac{P_iV_i}{T_i}$

$$\boxed{\Delta V = -\frac{3}{2}\frac{P_iV_i}{T_i}\ln(4)}$$

b.) $\Delta S = \int_{T_i}^{T_F} \frac{C_P}{T} dT = C_P(\ln(T_F) - \ln(T_i)) = C_P\ln(T_F/T_i)$

$P_iV_i = NkT_i \therefore T_i = \frac{P_iV_i}{Nk}$

$P_FV_F = NkT_F \therefore T_F = \frac{P_FV_F}{Nk}$

$P_F = P_i$, $V_F = 4V_i$ $\therefore$ $\Delta S = C_P \ln\left(\frac{4\cancel{P_i}\cancel{V_i}/\cancel{Nk}}{\cancel{P_i}\cancel{V_i}/\cancel{Nk}}\right) = C_P\ln(4)$

$C_P = \frac{5}{2}Nk$

$$\boxed{\Delta S = \frac{5}{2}\frac{P_iV_i}{T_i}\ln(4)}$$

c.) $dE = \delta Q - PdV$: $dE = TdS - PdV + \mu \cancel{dN}^{0}$ $\therefore$ $\delta Q - \cancel{PdV} = TdS - \cancel{PdV}$

$\delta Q = TdS$: $\delta Q = \left[P + \frac{dE}{dV} + \frac{dP}{dV}\frac{dE}{dP}\right]dV$

$\frac{dE}{dV} = \frac{3}{2}P$: $\frac{dP}{dV} = \frac{-NkT}{V^2}$: $\frac{dE}{dP} = \frac{3}{2}V$: $\delta Q = \left[\frac{NkT}{V} + \frac{3}{2}P - \frac{NkT}{V^{\cancel{2}}}\cdot\frac{3}{2}\cancel{V}\right]dV$

$\delta Q = \left[\frac{NkT}{V} + \cancel{\frac{3}{2}\frac{NkT}{V}} - \cancel{\frac{3}{2}\frac{NkT}{V}}\right]dV \therefore \delta Q = \frac{NkT}{V}dV \longrightarrow dS \cdot T = \frac{NkT}{V}dV$

$$\boxed{dS = \frac{Nk}{V}dV}$$

$\int dS = Nk\int_{V_i}^{V_F} 1/V \, dV \therefore$ $\boxed{\Delta S = Nk \cdot \ln(V_F/V_i)}$

Problem 1 Continued

d.) $V_F = \frac{1}{4} V_i$ $\therefore$ $\boxed{\Delta S = -\frac{P_iV_i}{T_i} \ln(4)}$

$Nk = \frac{P_iV_i}{T_i}$

e.) $\Delta S_{Tot} = -\frac{3}{2}\frac{P_iV_i}{T_i}\ln(4) + \frac{5}{2}\frac{P_iV_i}{T_i} - \frac{P_iV_i}{T_i}\ln(4) = 0$

$$\boxed{\Delta S = 0}$$

f.) $W_{net}: -P\int_{V_i}^{4V_i} dV = -P(4V_i - V_i) = -P(3V_i) = -\frac{3}{4}P_iV_i$

$-\int_{4V_F}^{V_F} \frac{NkT}{V} dV = -NkT(\ln(V_F) - \ln(4V_F)) = NkT\ln(4) = P_iV_i\ln(4)$

$\boxed{W_{net} > 0 \text{, work is done on gas}}$

H_{net}: $\Delta E = Q$ $\quad Q = \frac{3}{2}\cancel{Nk}\left(\frac{P_iV_i}{4\cancel{Nk}}\right) - \frac{3}{2}\cancel{Nk}\left(\frac{P_iV_i}{\cancel{Nk}}\right) = \frac{3}{8}P_iV_i - \frac{3}{2}P_iV_i = -\frac{9}{8}P_iV_i$

$Q = \frac{3}{2}P(4V_i - V_i) + \frac{3}{4}P_iV_i = \frac{3}{2}\cdot\frac{P_i}{4}(3V_i) + \frac{3}{4}P_iV_i = \frac{9P_iV_i}{8} + \frac{6}{8}P_iV_i = \frac{15}{8}P_iV_i$

$Q = -P_iV_i\ln(4)$ $\quad Q_{net} = \frac{6}{8}P_iV_i - \ln(4)P_iV_i < 0$

$\boxed{Q_{net} < 0 \text{, heat is taken away from gas}}$

Since the heat is negative some of it is lost to the surroundings and therefore the change in entropy cannot be zero

Problem 2

$m_z = 0.750\ kg \quad T_{zi} = 393\ K \quad : \quad m_w = 1.25\ kg \quad T_{wi} = 293\ K$

$C_z = 387\ \frac{J}{kg\,K} \qquad C_w = 4.186 \times 10^3\ \frac{J}{kg\,K}$

a.) $m_z C_z (T_f - T_{zi}) + m_w C_w (T_f - T_{wi}) = 0$

$m_z C_z T_f - m_z C_z T_{zi} + m_w C_w T_f - m_w C_w T_{wi} = 0$

$T_f (m_z C_z + m_w C_w) = m_z C_z T_{zi} + m_w C_w T_{wi} \quad \therefore \quad T_f = \dfrac{m_z C_z T_{zi} + m_w C_w T_{wi}}{m_z C_z + m_w C_w}$

$$T_f = \frac{(0.750\ kg)(387\ J/kg\cdot K)(393\ K) + (1.25\ kg)(4.186 \times 10^3\ J/kg\,K)(293\ K)}{0.750\ kg(387\ J/kg\,K) + (1.25\ kg)(4.186 \times 10^3\ J/kg\,K)} = 298\ K$$

$$\Delta S = \int_{T_i}^{T_f} \frac{1}{T} C_v\, dT = C_v \int_{393\ K}^{298\ K} \frac{1}{T}\, dT = C_v \cdot \ln(298\ K / 393\ K) \cdot 0.750\ kg = -80.3\ \frac{J}{K}$$

$$\boxed{\Delta S = -80.3\ \frac{J}{K}}$$

b.)

$$\Delta S = \int_{293\ K}^{298\ K} \frac{1}{T} C_v\, dT = C_v \int_{293\ K}^{298\ K} \frac{1}{T}\, dT = C_v \cdot \ln(298\ K / 293\ K) \cdot 1.25\ kg = 88.5\ \frac{J}{K}$$

$$\boxed{\Delta S = 88.5\ \frac{J}{K}}$$

c.) $\Delta S_{Tot} = -80.3\ \frac{J}{K} + 88.5\ \frac{J}{K} = 8.2\ \frac{J}{K}$

$$\boxed{\Delta S_{Tot} = 8.2\ \frac{J}{K}}$$

Problem 3

a.)

i.) $\Delta S = C_v \int_{273\,K}^{363\,K} \frac{1}{T}\,dT = C_v \cdot \ln(363K/273K) = 4.184 \times 10^3 \frac{J}{K \cdot kg} \cdot \ln(363/273) \cdot 1\,kg$

$$\boxed{\Delta S = 1{,}192 \ \frac{J}{K}}$$

ii.) $\Delta S = \frac{-Q}{T} = \frac{-mC\Delta T}{T} = \frac{-1\cdot kg \cdot 4.184 \times 10^3\,J/K\cdot kg \cdot (90\,K)}{363\,K} = -1{,}037 \ \frac{J}{K}$

$$\boxed{\Delta S = -1{,}037 \ \frac{J}{K}}$$

iii.) $\Delta S_w + \Delta S_{HB} = 1{,}192 \ \frac{J}{K} - 1{,}037 \ \frac{J}{K} = 155 \ \frac{J}{K}$

$$\boxed{\Delta S_T = 155 \ \frac{J}{K}}$$

b.)

i.) $\Delta S = C_v \int_{273\,K}^{318\,K} \frac{1}{T}\,dT = C_v \cdot \ln(318K/273K) = 4.184 \times 10^3 \frac{J}{kg \cdot K} \ln(318K/273K) \cdot 1\,kg = 638 \ \frac{J}{K}$

$\Delta S = \frac{-Q}{T} = \frac{-mC\Delta T}{T} = \frac{-1\cdot kg\,(4{,}184\,J/kg\,K)(45\,K)}{318\,K} = -592 \ \frac{J}{K}$

$$\boxed{\Delta S = 46 \ \frac{J}{K}}$$

ii.) $\Delta S = C_v \int_{318\,K}^{363\,K} \frac{1}{T}\,dT = C_v \cdot \ln(363K/318K) = 4.184 \times 10^3 \frac{J}{kg \cdot K} \ln(363/318) \cdot 1\,kg = 554 \ \frac{J}{K}$

$\Delta S = \frac{-Q}{T} = \frac{-mC\Delta T}{T} = \frac{-1\cdot kg \cdot (4{,}184\,J/kg\cdot K)(45\,K)}{363\,K} = -519 \ \frac{J}{K}$

$$\boxed{\Delta S = 35 \ \frac{J}{K}}$$

iii.) $\Delta S_{Tot} = 46 \ \frac{J}{K} + 35 \ \frac{J}{K} = 81 \ \frac{J}{K}$

$$\boxed{\Delta S_T = 81 \ \frac{J}{K}}$$

c.) There would have to be an increase in temperature of infinitely many steps for the change in entropy of the entire system to be 0.

Problem 4

a.) $\Delta S = C\int_{T_i}^{T_f} \frac{1}{T}\, dT \quad \therefore \quad \Delta S = C \ln(T_f/T_i) = C \ln(T_B/T_{sys}) \quad : \quad \Delta S_{sys} = C \cdot \ln(T_B/T_{sys})$

$$\Delta S_B = -\frac{Q}{T_B} = -\frac{\Delta E + P\cancel{dV}^{0}}{T_B} = -\frac{\Delta E}{T_B} = -\frac{C\Delta T}{T_B} = -C\left(\frac{T_B}{T_B} - \frac{T_{sys}}{T_B}\right) = C\left(\frac{T_{sys}}{T_B} - 1\right)$$

$\Delta S = \Delta S_{sys} + \Delta S_B$

$$\boxed{\Delta S = C\left(\ln(T_B/T_{sys}) + \frac{T_{sys}}{T_B} - 1\right)}$$

b.) $f'(x) = \frac{1}{x} - \frac{1}{x^2} \quad : \quad 0 = \frac{1}{x} - \frac{1}{x^2} \quad : \quad \frac{1}{x^2} = \frac{1}{x} \quad \frac{1}{x} = 1 \quad \therefore \quad x = 1$

$$\boxed{\text{When } T_{sys} = T_B \quad \Delta S = 0}$$

c.) $\lim_{x \to 0} \ln(x) + \frac{1}{x} - 1 = \infty$

$$\boxed{\text{When } T_B < T_{sys} \quad \Delta S > 0 \quad \text{Always}}$$

d.) $\lim_{x \to \infty} \ln(x) + \frac{1}{x} - 1 = \infty$

$$\boxed{\text{When } T_B > T_{sys} \quad \Delta S > 0 \quad \text{Always}}$$