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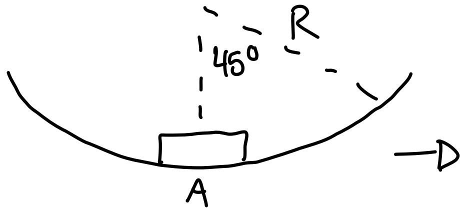
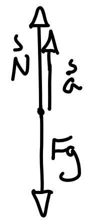
Dr. Workman
PHYS 342 HW 3

Problem 1

a.) $\vec{N}$ @ A ?





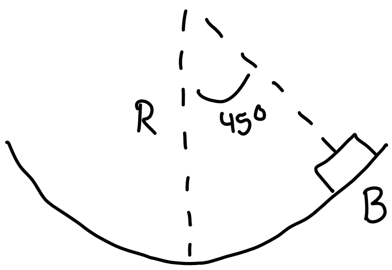
$\sum F_y$: $\vec{N} - mg = m\frac{\dot{y}_0^2}{r}$

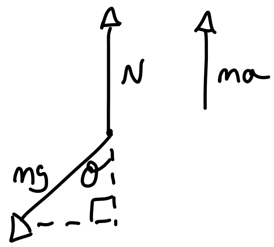
$\vec{N} = m\left(g + \frac{\dot{y}_0^2}{r}\right)$

$$\boxed{N = m\left(g + \frac{\dot{y}_0^2}{R}\right) \, N}$$

$m = 1.62$ kg
$g = 9.81$ m/s^2

b.)





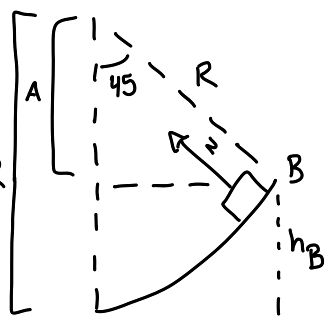
$\sum F_y$: $N - mg\cos\theta = ma$ $\quad a \leftrightarrow \frac{\dot{y}^2}{r}$

$N - mg\cos\theta = m\left(\frac{\dot{y}^2}{r}\right)$

$m = 1.62$ kg
$g = 9.81$ m/s^2

$$\boxed{N = m\left(g\cos\theta - \frac{\dot{y}^2}{R}\right) N}$$

c.)



$E_i = E_f$: $\cancel{T_i}^{0} + U_i = T_f + U_f$

$mgh = \frac{1}{2}mV_B^2 + mgh_B$

$gh - gh_B = \frac{1}{2}V_B^2$

$2g(h - h_B) = V_B^2$

$h \equiv$ starting height
$h_B \equiv$ Height off ground at location B

$R = R/\sqrt{2} + h_B$

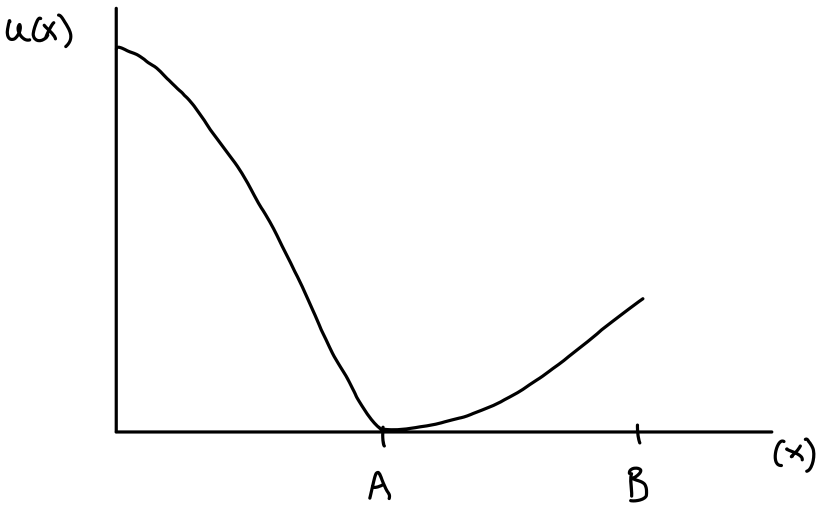
$h_B = R - R/\sqrt{2}$

$h_B = R(1 - 1/\sqrt{2})$: $V_B = \sqrt{2g(h - h_B)}$

$$\boxed{V_B = \sqrt{2g(h - R - R/\sqrt{2})}}$$

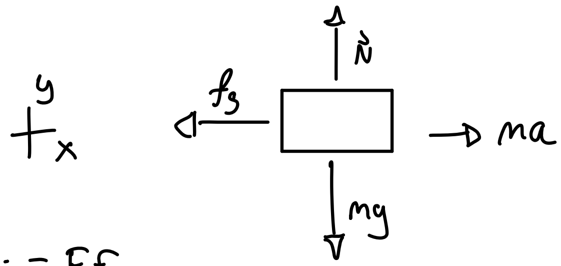
Problem 11 Continued

e.)



Problem 21

a.)



$m = 2\,kg$
$\dot{x}_0 = 4\ m/s$
$K = 6\ N/m$

$\dot{x}_1^2 = \dot{x}_0^2 + 2a\Delta x$

$E_i = E_f$

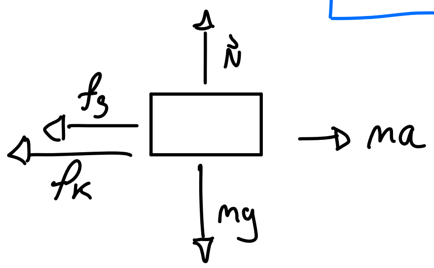
$\cancel{u_i} + K_i = u_f + \cancel{K_f}$

$$\frac{1}{2} m \dot{x}_0^2 = \frac{1}{2} k \Delta x^2$$

$$\Delta x = \sqrt{\frac{m \dot{x}_0^2}{K}} = \sqrt{\frac{(2kg)(4\,m/s)^2}{6\ N/m}} = 2.31\ m$$

$$\boxed{\Delta x = 2.31\ m}$$

b.)



$E_i = E_f$

$\cancel{u_i} + K_i = u_F + \cancel{K_f} - W_{nc}$

$$\frac{1}{2} m \dot{x}_0^2 = \frac{1}{2} k \Delta x^2 - \mu_K N d - \mu_K N \Delta x$$

$m = 2\,kg$
$\dot{x}_0 = 4\ m/s$
$\mu_K = 0.2$
$N = mg$
$K = 6\ N/m$
$d = 2\,m$

$$0 = \frac{1}{2} K \Delta x^2 - \mu_K N \Delta x - \mu_K N d - \frac{1}{2} m \dot{x}_0^2$$

$a = \frac{1}{2} k \quad b = -\mu_K m g \quad C = -\frac{1}{2} m \dot{x}_0^2 - \mu_K m g d$

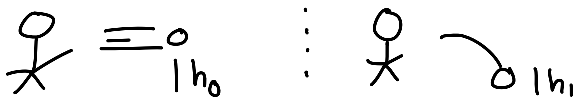
$$0 = \frac{1}{2}(6\,N/m)\Delta x^2 - 0.2(2kg \cdot 9.81\,m/s^2)\Delta x - \frac{1}{2}(2kg)(4\,m/s)^2 - 0.2(2kg)(9.81\,m/s^2)(2\,m)$$

Put in quadratic formula

$$\boxed{\Delta x = 1.1\ m}$$

Problem 3

(a) $E_i = E_f$

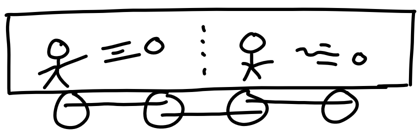


$T_i + U_i = T_f + U_f$ $\qquad$ $T_i = 0 \text{ J}$

$mgh_0 = T_f + mgh_1$

$$\boxed{\Delta T = \frac{1}{2}mv^2 \text{ J}}$$

(b) $E_i = E_f$ $\qquad$ work $= \Delta KE$



$\Delta T = T_f - T_i$

$= \frac{1}{2}m(v+u)^2 - \frac{1}{2}m(v)^2$

$= \frac{1}{2}m(v^2 + 2uv + u^2) - \frac{1}{2}m(v^2)$

$= \frac{1}{2}m(2uv + u^2)$

$$\boxed{\Delta T = \frac{1}{2}m(2uv + u^2)}$$

(c) $W = \Delta KE$

$T_i = 0$ $\qquad$ $T_f = \frac{1}{2}mv^2$

$\Delta T = T_f - T_i = \frac{1}{2}mv^2 - 0$

$$\boxed{W = \frac{1}{2}mv^2 \text{ J}}$$

(d) $W = \Delta KE$

$T_i = \frac{1}{2}mv^2$, $T_f = \frac{1}{2}mv^2$

$\Delta T = \frac{1}{2}mv^2 - \frac{1}{2}mv^2 = 0$

$$\boxed{\Delta T = 0}$$

Problem 4

$F = -kx + \frac{kx^3}{\alpha^2}$

$u(x) = k \cdot \int_{x_0}^{x_1} -x + \frac{x^3}{\alpha^2}\, dx$

$u(x) = k \cdot \left[\frac{-x^2}{2} + \frac{x^4}{4\alpha^2} \right]_{x_0}^{x_1}$

$u(x) = k \cdot \left[\left(-\frac{x_1^2}{2} + \frac{x_1^4}{4\alpha^2} \right) - \left(-\frac{x_0^2}{2} + \frac{x_0^4}{4\alpha^2} \right) \right]$

$u(x) = k \left[\frac{-x_1^2}{2} + \frac{x_1^4}{4\alpha^2} + \frac{x_0^2}{2} - \frac{x_0^4}{4\alpha^2} \right]$

$\Delta x = x_1 - x_0$

$\boxed{u(x) = k \left[\frac{1}{2} (\Delta x)^2 - \frac{1}{4\alpha^2} (\Delta x)^4 \right] J}$

$k = 1, \alpha = 1$

$u = \frac{1}{2} x^2 - \frac{1}{4} x^4$

$u'(x) = \Delta x - \Delta x^3$

$x = [-1, 0, 1]$

Maxima

$0 = x - x^3$

$0 = x(1 - x^2)$

$x = 0, \quad x^2 - 1 = 0$

$x = \pm 1$

$u''(x) = 1 - 3x^2$

Inflection

$0 = 1 - 3x^2$

$3x^2 = 1$

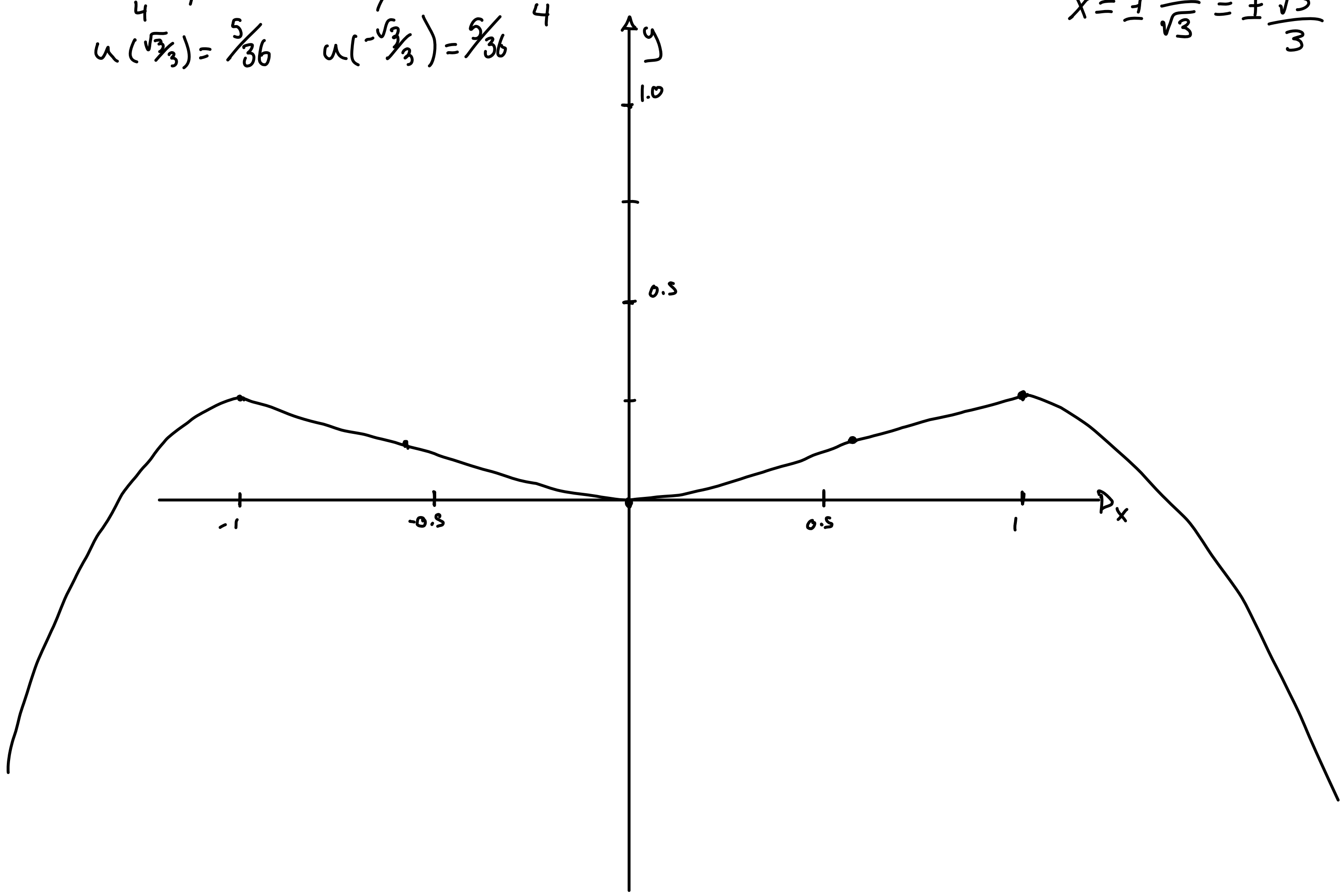
$x^2 = \frac{1}{3}$

$x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$

max, min, max

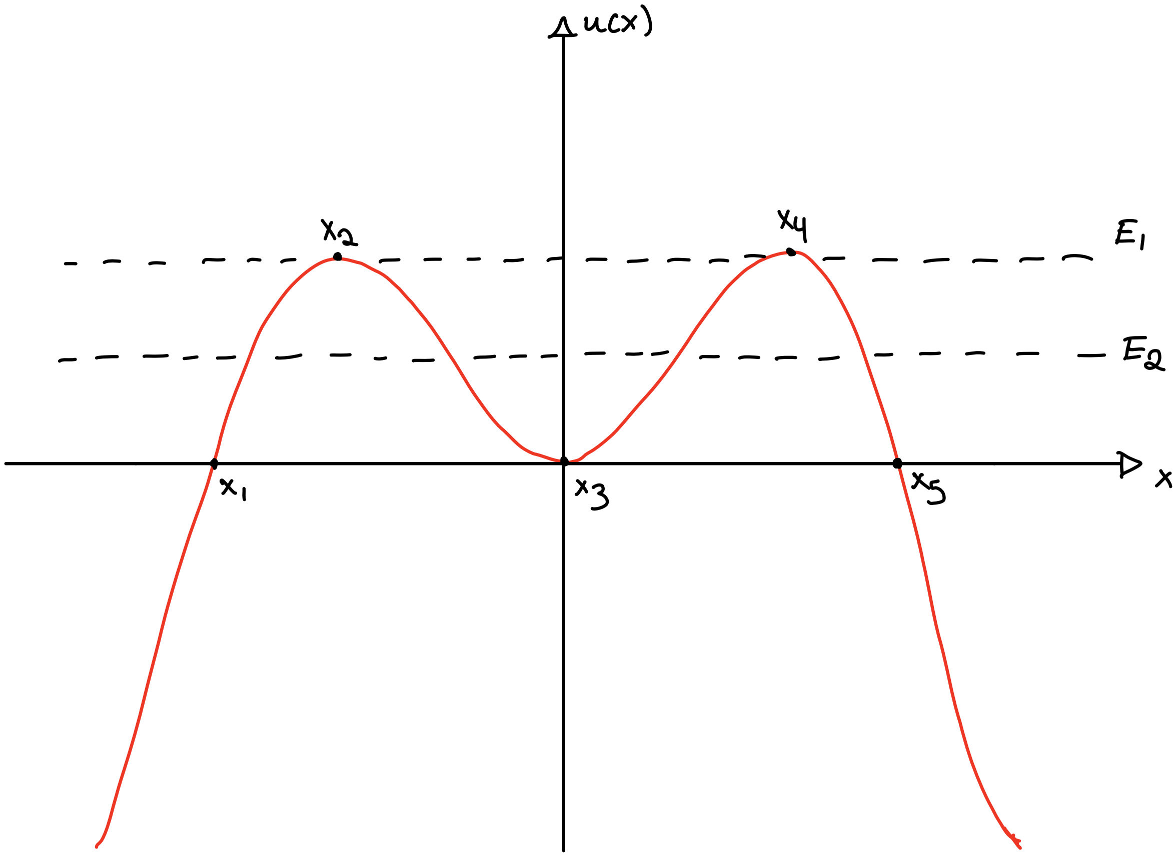
$u(-1) = \frac{1}{4}, \quad u(0) = 0, \quad u(1) = \frac{1}{4}$

$u(\sqrt{3}/3) = 5/36 \quad u(-\sqrt{3}/3) = 5/36$



x_3 - Stable equilibrium
x_2, x_4 - Unstable equilibrium

x_2 - Perturbed to left, goes to $-\infty$
x_2 - Perturbed to right, oscillates between x_2 & x_4
x_4 - Perturbed to right, goes to $-\infty$
x_4 - Perturbed to left, oscillates between x_2 & x_4
x_3 - Stable equilibrium, will oscillate between x_2 & x_4



Problem 5

$u(x) = u_0\left(\frac{a}{x} + \frac{x}{a}\right)$ $\qquad u(x) = \underbrace{\frac{2}{x} + \frac{x}{2}}_{f}$ $\qquad 2x^{-1} + \frac{1}{2}x$

$u_0 = 1J$

$a = 2m$

$u'(x) = \frac{-2}{x^2} + \frac{1}{2}$ $\therefore$ $0 = \frac{-2}{x^2} + \frac{1}{2}$ $\therefore$ $\frac{-1}{2} = \frac{-2}{x^2}$

$u'(x) = -2x^{-2} + \frac{1}{2}$

$\frac{x^2}{2} = 2 \quad \therefore \quad x^2 = 4$

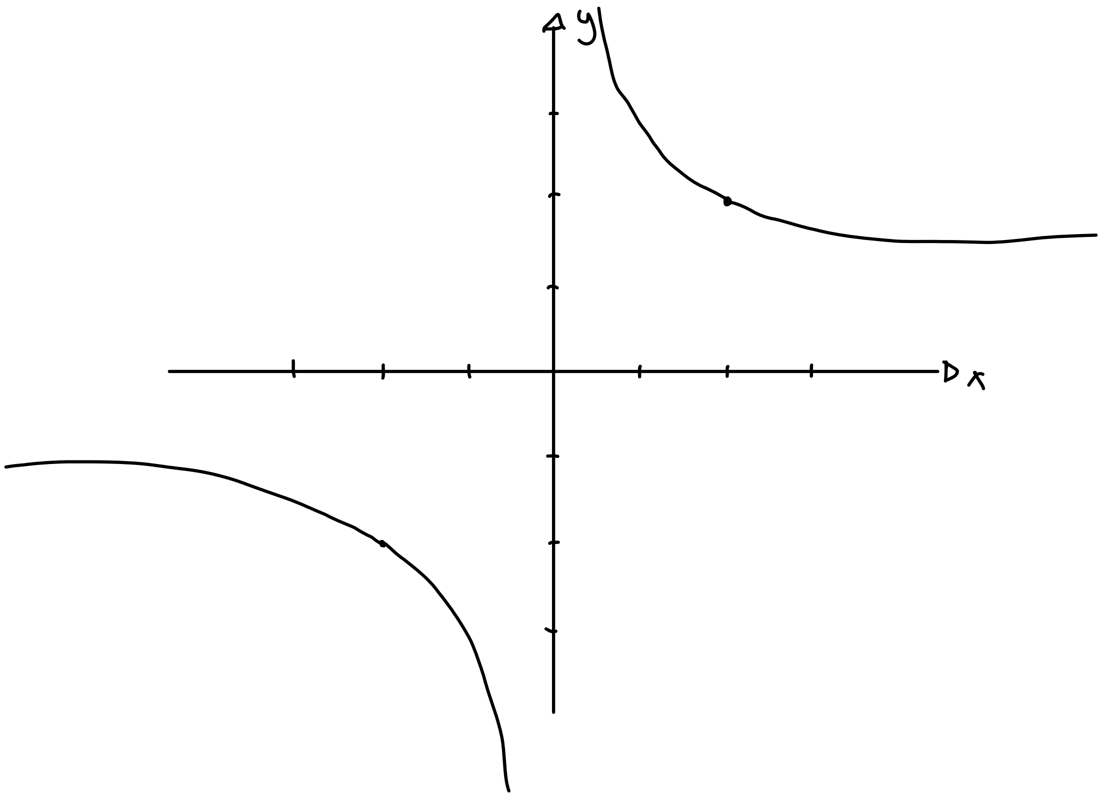
$x = \pm 2$

$u''(x) = 4x^{-3} = \frac{4}{x^3}$: $0 = \frac{4}{x^3} \leftarrow$ no points of inflection

$u(2) = 2$ $\qquad$ asymptote @ $x=0$

$u(-2) = -2$

$\lim_{x \to 0^+} f = \infty$, $\lim_{x \to 0^-} f = -\infty$, $\lim_{x \to -\infty} f = -\infty$ $\quad \lim_{x \to \infty} f = \infty$



The equilibrium points occur at $x = \pm 2$, Since the region we are interested in is $x > 0$,

$\boxed{x=2\text{, is a minimum of the potential}}$

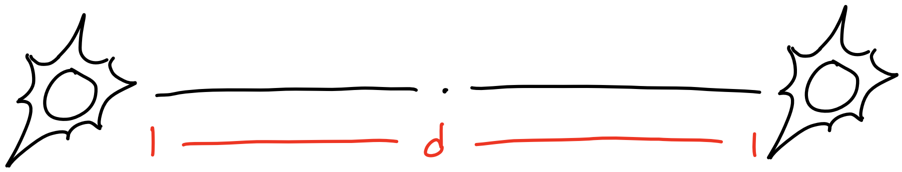
Problem 6

Keplers third law : $\tau^2 \propto d^3$

Square of period is proportional to the cube of the distance

$$(\tau^2)^{\frac{1}{2}} \propto (d^3)^{\frac{1}{2}}$$

$$\tau \propto d^{3/2}$$



$$F = \frac{GMm}{d^2} \quad : \quad F = ma = m\omega^2 d$$

$$m\omega^2 d = \frac{GMm}{d^2}$$

$$\omega^2 = \frac{GM}{d^3} \quad \therefore \quad \omega = \sqrt{\frac{GM}{d^3}} \qquad \tau = \frac{2\pi}{\omega},$$

$$\tau = \frac{2\pi}{\sqrt{\frac{GM}{d^3}}} \quad : \quad \tau^2 = \frac{4\pi^2}{GM} d^3 \quad : \quad \tau = \sqrt{\frac{4\pi^2}{GM}}\, d^{3/2}$$

$$\boxed{\tau = \sqrt{\frac{4\pi^2}{GM}}\, d^{3/2}}$$

Problem 6

$$T = \frac{kv+g}{kg}\left(1-e^{-kT}\right) \quad : \quad 0 = T - \frac{kv+g}{kg}\left(1-e^{-kT}\right)$$

$$f(T) = T - \frac{kv+g}{kg}\left(1-e^{-kT}\right)$$

$$\text{newtons} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(T) = 1 - \frac{kv+g}{kg}\left(ke^{T}\right) = 1 - \frac{kv+g}{kg}\left(ke^{-kT}\right)$$

$v = 10$ m/s , $k = 0.1$, $g = 9.81$

$$T_n - \left[\frac{T_n - \frac{kv+g}{kg}\left(1-e^{-kT}\right)}{1 - \frac{kv+g}{kg}\left(ke^{-kT}\right)}\right]$$