3.7 The Cosine Transform of Type II

Cosine Expansion of Symmetric Extension

Let f satisfy the Dirchlet conditions on $[0,T]$, let g denote the symmetric extension f on $[0,2T]$ given by

$$g(t) = \begin{cases} f(t) & t \in [0,T] \\ f(2-t), & t \in [T,2T] \end{cases}$$

The Fourier Expansion h_n of g on $[0,2T]$ is a cosine expansion with

$$h_n(t) = \sum_{k=0}^{n} a_k \cos(\pi k t/T)$$

Coefficients given by,

$$a_0 = \frac{1}{T}\int_0^T f(t)dt \qquad a_k = \frac{2}{T}\int_0^T f(t)\cdot\cos(\pi k t/T)\,dt$$

CT Coefficient Vector

$$C = [a_0, a_1, a_2, \ldots a_n]$$

Remark 3.7.1

Let f be a function defined on $[0,T]$, and let g denote the symmetric extension of f to $[0,2T]$. Suppose that f is continuous on $[0,T]$ but $f(0) \neq f(T)$.

1. For the Fourier Series of f,
   - (F1) - The periodic extension f_T is discontinuous $(-\infty,\infty)$
   - (F2) - The Fourier series of f does not converge to f at $t=0$ & $t=T$
   - (F3) - Both a_k and b_k typically needed in Fourier Expansion.
   - (F4) - The $|C_k|$ converge to zero on the order of $1/K$.
   - (F5) - The frequency domain plot portrays the spectrum of f.

2. For the Fourier Series of g,
   - (CT1) - The symmetric extension g is continuous on $[0,2T]$ and its periodic extension g_{2T} is continuous on $(-\infty,\infty)$.
   - (CT2) - The Fourier Series of g converges pointwise to f on $[0,T]$, and in particular, at the endpoints $t=0$ & $t=T$.
   - (CT3) - All the sine coefficients are zero ; $b_k=0$ for all K.
   - (CT4) - The $|a_k|$ converge to zero on the order of $1/K^2$ or faster.
   - (CT5) - The frequency domain plot portrays the spectrum of g.