Physics 396

Homework Set 4

1. Consider a particle launched in the x, y plane at a speed $V = 3/5$ relative to an inertial reference frame with coordinates (t, x, y, z), with this velocity vector making an angle of $\pi/3$ with the x axis.[1]

   a) Calculate the components of the *velocity four-vector* relative to this frame.

   b) Show that this velocity four-vector is *normalized*, namely, that

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = -1. \tag{1}$$

   Now consider another inertial reference frame with coordinates (t', x', y', z') moving at a constant speed $v = 4/5$ relative to the aforementioned inertial frame.

   c) Calculate the components of the *velocity four-vector* relative to this S' frame.

   d) Show that this velocity four-vector is *normalized*, namely, that

$$\underset{\sim}{u}' \cdot \underset{\sim}{u}' = -1. \tag{2}$$

2. Consider a particle moving along the x-axis whose *velocity four-vector* is described by

$$u^\alpha(t) = \left(\sqrt{1 + f^2(t)}, f(t), 0, 0\right), \tag{3}$$

   where $f(t)$ is a generic function of coordinate time t.

   a) Explicitly show that this velocity four-vector is normalized, namely, that

$$\underset{\sim}{u} \cdot \underset{\sim}{u} = -1. \tag{4}$$

   b) Calculate the components of the *acceleration four-vector*, $a^\alpha(\mathrm{t})$, and write the components in an analogous way to Eq. (3).

   c) Explicitly show that the four-acceleration is *orthogonal* to the four-velocity, namely, that

$$\underset{\sim}{u} \cdot \underset{\sim}{a} = 0. \tag{5}$$

[1] Here we're working in units where $c = 1$

for any generic function $f(t)$.

d) Construct an expression for the particles velocity, $V(t) = dx/dt$ in terms of $f(t)$.

e) Find expressions for $\tau(t)$ and $x(t)$, which here take the form of integrals of $f(t)$.

f) Calculate the components of the four-force and the three-force acting on the particle.

3. The worldline of a massive particle is described parametrically by the spacetime coordinates

$$\begin{aligned} t(\tau) &= \frac{1}{\alpha}\ln\left(\tan(\alpha\tau) + \sec(\alpha\tau)\right) \\ x(\tau) &= -\frac{1}{\alpha}\ln\left(\cos(\alpha\tau)\right). \end{aligned} \tag{6}$$

a) Explicitly show that the parameter τ represents the *proper time* of the particle.

b) Calculate the components of the *four-velocity*, u^α, and *four-acceleration*, a^α, of the particle and display the components as the elements of a row.

Now consider an inertial reference frame with coordinates (t', x') moving at a constant speed v relative to the aforementioned inertial frame with coordinates (t, x).

c) Calculate the components of the *four-velocity*, $u^{\alpha'}$, and *four-acceleration*, $a^{\alpha'}$, of the particle in this S' frame and display the components as the elements of a row.

d) Explicitly show that in this S' frame the velocity four-vector is *normalized* and the four-acceleration is orthogonal to the four-velocity, namely, that

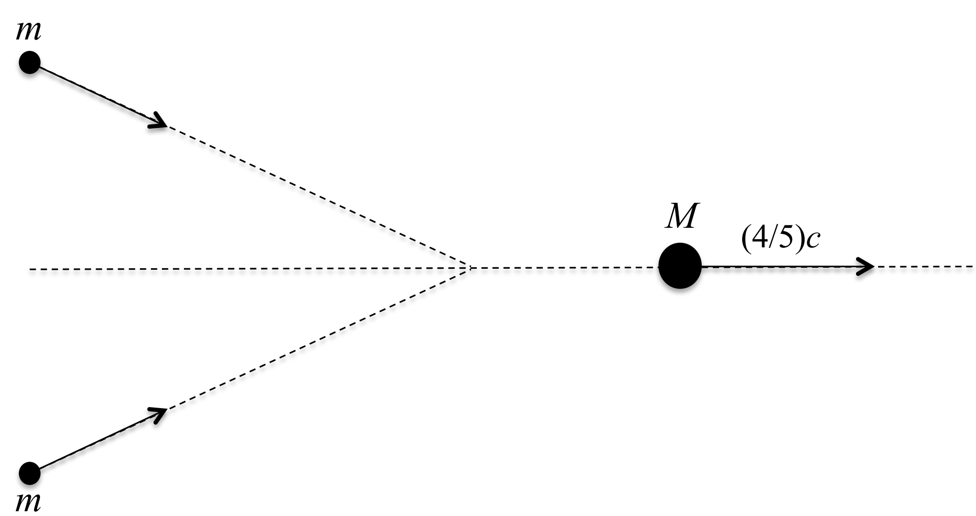
$$\begin{aligned} \underset{\sim}{u}' \cdot \underset{\sim}{u}' &= -1 \\ \underset{\sim}{u}' \cdot \underset{\sim}{a}' &= 0. \end{aligned} \tag{7}$$

4. A particle of mass M is moving in the laboratory with speed $(3/5)c$. It decays into a particle of mass M_0, where M_0 is at rest in the laboratory, and a photon of energy 12,000 MeV. Using MeV units, find

a) the mass M,

b) the mass M_0, and

c) the momentum of M.



5. Two particles of mass $m = (1/3)M$ collide and fuse together, as shown in the accompanying picture. The newly formed particle of mass M moves off to the right at a speed of $V = (4/5)c$. Find the x and y components of the initial velocity of each particle of mass m before the collision occurs.