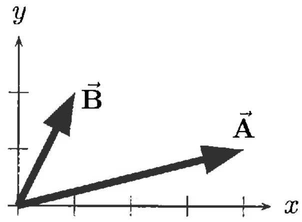
58 Vectors: Dot Products

For each of the following, express $\vec{\mathbf{A}}$ in component form $\vec{\mathbf{B}}$ using unit vectors and determine the dot product, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$. *Note: If your answer for the dot product contains* $\hat{\mathbf{i}}$ *and* $\hat{\mathbf{j}}$ *then it is very incorrect!*

a)

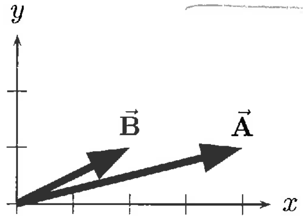


$\vec{A} = 4\hat{i} + \hat{j}$

$\vec{B} = \hat{i} + 2\hat{j}$

$\vec{A} \cdot \vec{B} = 4\times1 + 1\times2 = 6$

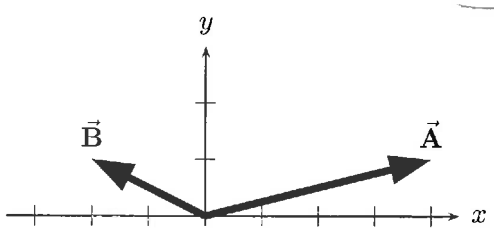
b)



$\vec{A} = 4\hat{i} + \hat{j}$ $\quad \vec{B} = 2\hat{i} + \hat{j}$

$\vec{A} \cdot \vec{B} = 4\times2 + 1\times1 \Rightarrow \vec{A} \cdot \vec{B} = 9$

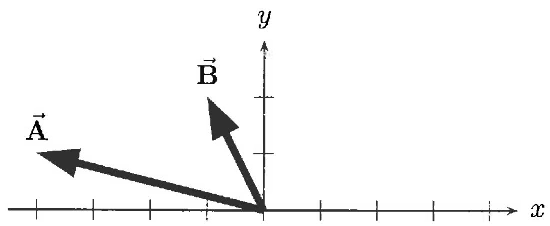
c)



$\vec{A} = 4\hat{i} + \hat{j}$ $\quad \vec{B} = -2\hat{i} + \hat{j}$

$\vec{A} \cdot \vec{B} = 4(-2) + 1.1 \Rightarrow \vec{A} \cdot \vec{B} = -6$

d)

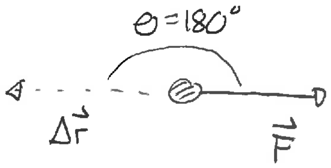


$\vec{A} = -4\hat{i} + \hat{j}$ $\quad \vec{B} = -\hat{i} + 2\hat{j}$

$\vec{A} \cdot \vec{B} = -4\times-1 + 1\times2 \Rightarrow \vec{A} \cdot \vec{B} = 6$

4ed Knight Ch9

Conc Q 6 *



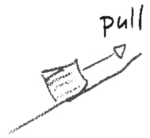
$$W = F\Delta r \cos\theta$$

$$\Rightarrow W = F\Delta r \cos 180°$$

$$W = -F\Delta r$$

4ed Knight Ch9

Conc Q 8



$\Delta K = W_{net} = W_{grav} + W_{pull}$

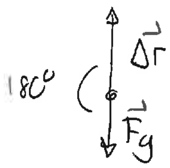
since the block moves at constant speed $\Delta K = 0$. Thus

$$W_{grav} + W_{pull} = 0$$

$$\Rightarrow W_{pull} = -W_{grav}$$

Now consider the work done by gravity

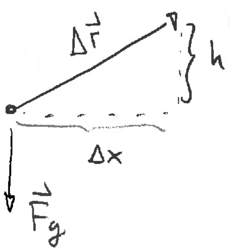
a)



$$W = F_g \Delta r \cos 180° = -mgh$$

$$\Rightarrow W_{grav} = -mgh$$

b)



$$W = \vec{F} \cdot \Delta\vec{r}$$

$$\vec{F} = -mg\hat{z} \qquad \Delta\vec{r} = \Delta x\,\hat{x} + h\,\hat{z}$$

$$\Rightarrow \vec{F} \cdot \Delta\vec{r} = 0 \times \Delta x + (-mg)h = -mgh$$

$$\Rightarrow W = -mgh$$

Thus work done by gravity is the <u>same</u>

So W_{pull} is <u>same</u>

4ed Knight Ch 9

Prob 12

a) $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$

$\vec{B} = 2\hat{\imath} - 6\hat{\jmath}$

$\vec{A} \cdot \vec{B} = 3 \times 2 + 4 \times (-6) = 6 - 24 \Rightarrow \vec{A} \cdot \vec{B} = -18$

b) $\vec{A} = 3\hat{\imath} - 2\hat{\jmath}$

$\vec{B} = 6\hat{\imath} + 4\hat{\jmath}$

$\vec{A} \cdot \vec{B} = 3 \times 6 + (-2) \times 4 = 18 - 8 \Rightarrow \vec{A} \cdot \vec{B} = 10$

4ed Knight Ch 9

Prob 14

In all cases

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

a) $\vec{A}\cdot\vec{B} = 3\times5\times\cos 40^\circ \Rightarrow \vec{A}\cdot\vec{B} = 11$

b) $\vec{C}\cdot\vec{D} = 2\times3\cos 140^\circ \Rightarrow \vec{C}\cdot\vec{D} = -4.6$

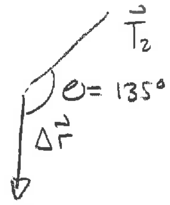
c) $\vec{E}\cdot\vec{F} = 3\times4\cos 90^\circ \Rightarrow \vec{E}\cdot\vec{F} = 0$

4ed Knight Ch 9
Prob 18
~~Knight Ch 4~~ *

$$\Delta \vec{r} = -5.00\text{m}\ \hat{j}$$

$$\vec{F}_G = -mg\,\hat{j} = -2.5\text{kN}\ \hat{j}$$

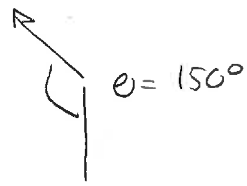
1) $W_G = \vec{F}_G \cdot \Delta\vec{r} = 12.5\text{kJ}$



2) $W_2 = \vec{T}_2 \cdot \Delta\vec{r}$

$$= T_2\,\Delta r\,\cos\theta = 1295\text{N} \times 5.00\text{m} \cos 135° = -4.6\text{kJ}$$

3) $W_3 = \vec{T}_3 \cdot \Delta\vec{r} = T_3\,\Delta r\cos\theta$



$$= 1830\text{N} \times 5.0\text{m}\ \cos 150°$$

$$= -7.9\text{kJ}$$

4ed Knight Ch9

~~Knight Ch11~~

Prob 43

a) $W_g = \int F_s\, ds$ Here $s = y$ $F_y = -mg$

$\Rightarrow W_g = -\int_{0m}^{10m} mg\, dy = -10_m mg = -10 \times 1000\,kg\,m\; 9.8\,m/s^2$

$= -9.8 \times 10^4\,J$

b) $W_T = \int F_s\, ds$ $s = y$ $F_y = T$

$W_T = \int T\, dy = T\int_0^{10m} dy = 10\,m \times T$

But $\vec{F}_{net} = m\vec{a}$

$\Rightarrow T - mg = ma \Rightarrow T = m(g+a)$

$= 1000\,kg\,(10.8\,m/s^2)$

$= 10.8 \times 10^3\,N$

$\Rightarrow W_T = 10.8 \times 10^4\,J$

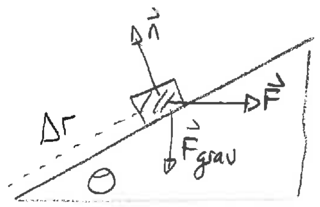
c) $W_{net} = \Delta K \Rightarrow K_f - \cancel{K_i} = W_T + W_G = 1.0 \times 10^4\,J$.

$\Rightarrow K_f = 1.0 \times 10^4\,J$

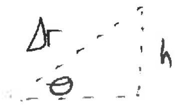
d) $K_f = \frac{1}{2} m v_f^2 \Rightarrow \sqrt{\frac{2K_f}{m}} = v_f \Rightarrow v_f = 4.5\,m/s$

a) $\Delta K = K_f - K_i = W_{net}$

$K_i = 0$



$\Rightarrow \quad K_F = W_{net}$



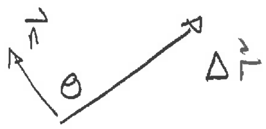
$\frac{1}{2}mv^2 = W_{grav} + W_n + W_F$

$\frac{h}{\Delta r} = \sin\theta$

$\Rightarrow \quad \Delta r = \frac{h}{\sin\theta}$

We need the three works.

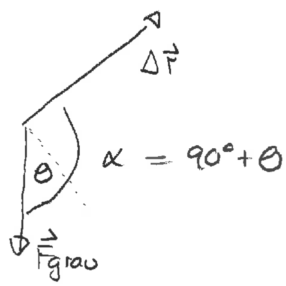
normal



$W_n = \vec{n}\cdot\Delta\vec{r} = 0$

since $\theta = 0$

gravitational

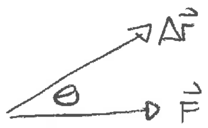


$W_{grav} = F_{grav}\,\Delta r \cos\alpha$

$= F_{grav}\,\Delta r \cos(90° + \theta)$

$= -mg\,\Delta r \sin\theta$

Hand



$W_{hand} = F\,\Delta r \cos\theta$

$\Rightarrow \quad \frac{1}{2}mv^2 = -mg\,\Delta r\sin\theta + F\,\Delta r\cos\theta$

$= -mg\frac{h}{\cancel{\sin\theta}}\cancel{\sin\theta} + Fh\frac{\cos\theta}{\sin\theta} = \left(F\frac{\cos\theta}{\sin\theta} - mg\right)h$

$$\Rightarrow \quad v = \sqrt{\left(F\frac{\cos\theta}{\sin\theta} - mg\right)\frac{2h}{m}}$$

Knight Ch11
Prob 44

b) $$v = \sqrt{\left(F\frac{\cos\theta}{\sin\theta} - mg\right)\frac{2h}{m}}$$

$$= \sqrt{\left(25N\,\frac{\cos 20^\circ}{\sin 20^\circ} - 5.0kg \times 9.8m/s^2\right)\frac{2\times 2.0m}{5.0kg}}$$

$$= 3.97\ m/s$$