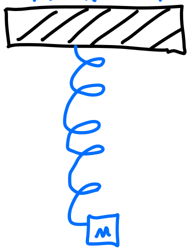
4.1 Harmonic Oscillator

4.1 # 47, 57, 62, 63



Restoring Force $F_R = -Kx$ $\qquad \dot{x} = \frac{dx}{dt}$

Friction $F_F = -b\dot{x}$

External Force $F = f(t)$

Harmonic oscilator : $m\ddot{x} + b\dot{x} + Kx = f(t)$

From Newton $= m \cdot \text{acceleration}$

$= m \cdot \ddot{x}$

$$m\ddot{x} = -Kx - b\dot{x} + f(t)$$

$$m\ddot{x} + b\dot{x} + Kx = f$$

Homogeneous Harmonic Oscillator

$$m\ddot{x} + b\dot{x} + Kx = 0$$

$x(t) = e^{rt}$ $\quad K = 1$

$\dot{x}(t) = re^{rt}$ $\quad b = r$

$\ddot{x}(t) = r^2e^{rt}$ $\quad m = r^2$

1) undamped oscillator

$$m\ddot{x} + Kx = 0$$

$\ddot{x} = -\frac{K}{m}x$

$x(t) = \sin(\omega t)$ $\qquad x(t) = \cos(\omega t)$

$\ddot{x}(t) = -\omega^2 \sin^2(\omega t)$ $\qquad \ddot{x}(t) = -\omega^2\cos(\omega t)$

$\omega = \sqrt{\frac{K}{m}}$

Solutions to the undamped unforced oscillator

$x(t) = C_1\cos(\omega t) + C_2\sin(\omega t)$

$\omega = \sqrt{\frac{K}{m}}$

C_1 and C_2 are determined by the initial conditions

$x(t) = A\cos(\omega t - \delta)$

Ex:3 $\quad \ddot{x} + x = 0$

$x(0) = 0$

$\dot{x}(0) = 1$

$x(t) = C_1\cos(\omega t) + C_2\sin(\omega t)$

$0 = C_1 + 0$

$C_1 = 0$

$1 = -C_1\omega\sin(\omega t) + C_2\omega\cos(\omega t)$

$1 = -C_1\omega(0) + C_2\omega$

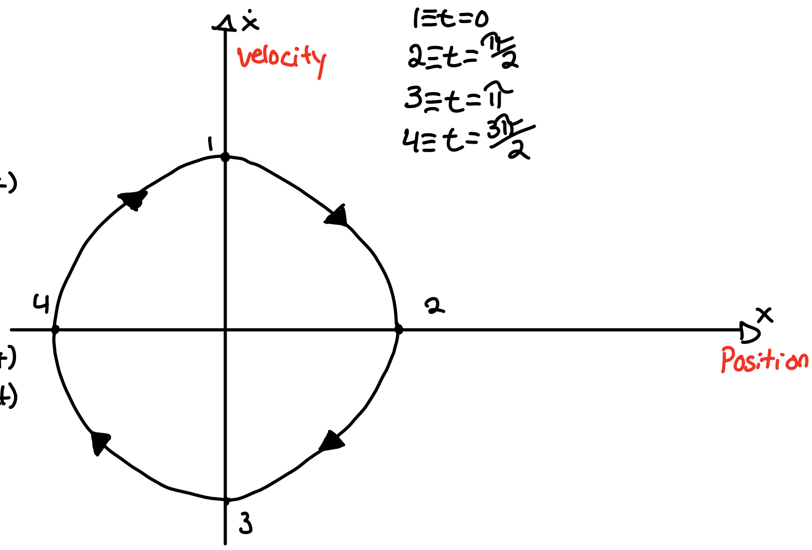
$C_2 = 1/\omega \qquad \omega = 1$

$x(t) = 1/\omega \sin(\omega t)$

$x(t) = \sin(\omega t)$

$x(t) = \sin(t)$

$\dot{x}(t) = \cos(t)$



$1 \equiv t = 0$

$2 \equiv t = \pi/2$

$3 \equiv t = \pi$

$4 \equiv t = 3\pi/2$