3.3.3

$f(t) = \begin{cases} 3, & 0 \le t \le 0.5 \\ 7, & 0.5 \le t \le 1.0 \\ 0, & \text{otherwise} \end{cases}$

a.)

$a_0 = \frac{1}{T}\int_0^T f(t)\,dt \qquad T = 1$

$a_k = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi k}{T}t\right)dt$

$b_k = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi k}{T}t\right)dt$

$k = 1$

a_0: $a = \int_0^{0.5} 3\,dt + \int_{0.5}^{1.0} 7\,dt = 5$

a_k: $a_1 = \int_0^{0.5} 3\cos(2\pi t)\,dt$

$= 3\int_0^{0.5}\cos(2\pi t)\,dt = \frac{3}{2\pi}\sin(2\pi t)\Big|_0^{0.5} = 0$

$a_1 = \int_{0.5}^{1.0} 7\cos(2\pi t)\,dt$

$= 7\int_{0.5}^{1.0}\cos(2\pi t)\,dt = \frac{7}{2\pi}\sin(2\pi t)\Big|_{0.5}^{1.0} = 0$

b_k: $b_1 = 2\int_0^{0.5} 3\sin(2\pi t)\,dt$

$= 6\int_0^{0.5}\sin(2\pi t)\,dt = \frac{-6}{2\pi}\cos(2\pi t)\Big|_0^{0.5} = -1.91$

$b_2 = 2\int_{0.5}^{1.0} 7\sin(2\pi t)\,dt$

$14\int_{0.5}^{1.0}\sin(2\pi t)\,dt = \frac{-14}{2\pi}\cos(2\pi t)\Big|_{0.5}^{1.0} = -4.45$

$a_0 = 5,\ a_k = 0,\ b_k = \frac{-8}{\pi k}$

$a_0 = 5$

$a_1 = 0$

$b_1 = \frac{-8}{\pi k}$

k

a_k: $2\int_0^{0.5} 3\cdot\cos(2\pi k t)\,dt$

$6\left(\frac{1}{2\pi k}\sin(2\pi k t)\Big|_0^{0.5}\right) = 0$

$2\int_{0.5}^{1.0} 7\cdot\cos(2\pi k t)\,dt$

$14\left(\frac{1}{2\pi k}\sin(2\pi k t)\Big|_0^{0.5}\right) = 0$

b_k: $2\int_0^{0.5} 3\cdot\sin(2\pi\cdot k\cdot t)\,dt$

$6\left(-\frac{1}{2\pi k}\cos(2\pi k\cdot t)\Big|_0^{0.5}\right)$

$6\left(-\frac{1}{2\pi k}\cos(\pi k t) + \frac{1}{2\pi k}\cos(0\cdot\pi t)\right)$

$6\left(\frac{1}{2\pi k} + \frac{1}{2\pi k}\right) = 6\left(\frac{1}{k\pi}\right) = \frac{6}{\pi k}$

$2\int_{0.5}^{1.0} 7\cdot\sin(2\pi\cdot k\cdot t)$

$14\left(-\frac{1}{2\pi k}\cos(2\pi\cdot k\cdot t)\Big|_{0.5}^{1.0}\right)$

$14\left(-\frac{1}{2\pi k}\cos(2\pi k) + \frac{1}{2\pi k}\cos(\pi k)\right)$

$14\left(-\frac{1}{\pi k}\right) = \frac{-14}{\pi k} \qquad b_k = \frac{-8}{\pi k}$

b.) $a_0 = 5$ $\qquad b_0 = 0,\ b_1 = \frac{-8}{\pi},\ b_2 = \frac{-4}{\pi},\ b_3 = \frac{-8}{3\pi},\ b_4 = \frac{-2}{\pi}$

$a_k = 0$

$b_k = \frac{-8}{\pi k}$

$h_4(t) = 5 + \frac{1}{\pi}\left(-8\cos(2\pi t) - 4\cos(4\pi t) - \frac{8}{3}\cos(6\pi t) - 2\cos(8\pi t)\right)$