

Lecture (3.2)

Think of voice waves as a function f , and when we sample it we get a vector of recorded values \vec{F}_n . (Chapter 1)

$$F \cdot g = c_1 g_1 \cdot g + c_2 g_2 \cdot g + \dots + c_n g_n \cdot g$$
$$\int F \cdot g \, dt = c_1 \int g_1 \cdot g \, dt$$

$$c_1 = \frac{\langle F, g_1 \rangle}{\langle g_1, g_1 \rangle} \quad c_k = \frac{\langle F, g_k \rangle}{\langle g_k, g_k \rangle} = \text{Similarity Coefficients}$$

Ex $f(t) = \begin{cases} 2, & 0 \leq t < \frac{1}{2} \\ 3, & \frac{1}{2} \leq t < 1 \end{cases}, \quad g(t) = 2t-1, \quad 0 \leq t < 1$

Find similarity coefficient

$$C = \frac{\langle f, g \rangle}{\langle g, g \rangle} = \frac{1/4}{1/3} = \frac{3}{4}$$

$$\langle f, g \rangle = \int_0^1 f(t)g(t) \, dt$$

$$\langle f, g \rangle = \int_0^{0.5} 2(2t-1) \, dt + \int_{0.5}^1 3(2t-1) \, dt$$
$$2 \left[t^2 - t \right]_0^{0.5} + 3 \left[t^2 - t \right]_{0.5}^1$$
$$2 \left[\frac{1}{4} - \frac{1}{2} \right] + 3 \left[(1-1) + \left(\frac{1}{4} - \frac{1}{2} \right) \right]$$
$$\frac{1}{4}$$

$$\langle g, g \rangle = \int_0^1 (2t-1)^2 \, dt = \frac{1}{2} \int_{-1}^1 u^2 \, du$$

when $t=0, u=-1$
 $u=2t-1$

$$= \frac{1}{6} \left[(2t-1)^3 \right]_0^1$$
$$= \frac{1}{3}$$

In chapter 3.1 & 3.2, we transform this raw signal by using an orthogonal expansion

$$f(t) = c_1 g_1(t) + c_2 g_2(t) + \dots + c_n g_n(t)$$

where $\{g_k(t)\}_{k=1}^n$ is an orthogonal set ($\langle g_k(t), g_j(t) \rangle = 0$)

and $\langle g_k(t), g_k(t) \rangle \neq 0$

and $\vec{c} = [c_1, c_2, \dots, c_n]^T$ is called the transform of f relative to $\{g_k\}$

Also, $c_k = \frac{\langle f, g_k \rangle}{\langle g_k, g_k \rangle} = \text{Similarity coefficient}$