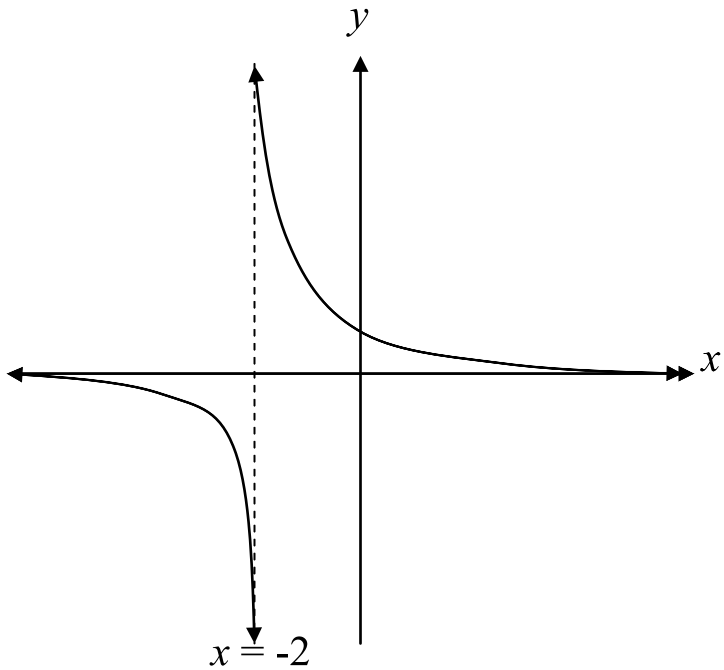
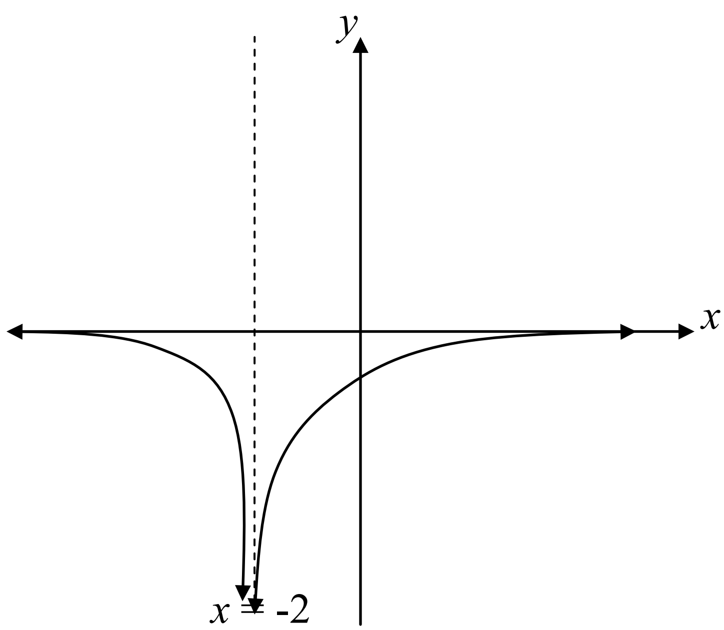
MAT 201
Larson/Edwards – Section 1.5
Infinite Limits

Ex: Determine the limit of each function as x approaches -2 from the left and from the right.

a)



b)



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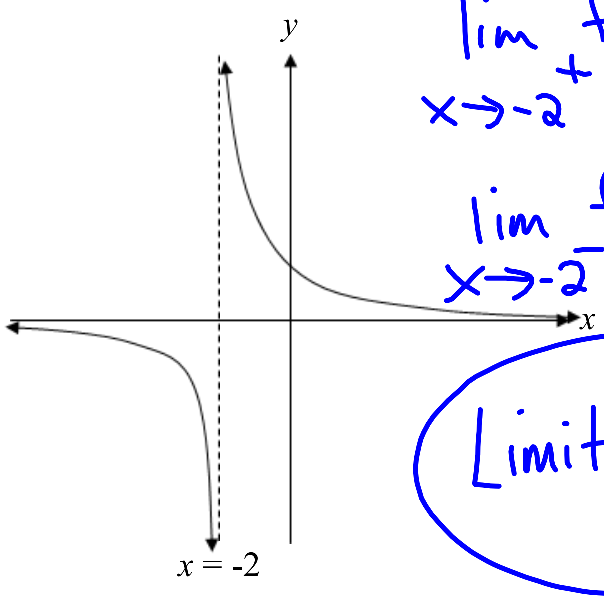
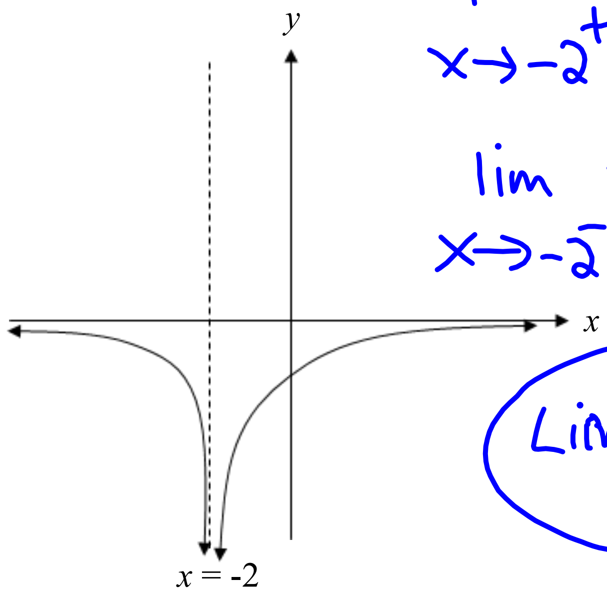


$$\lim_{x \to -2^+} f(x) = \infty$$

$$\lim_{x \to -2^-} f(x) = -\infty$$

Limit D.N.E.

b)



$$\lim_{x \to -2^+} f(x) = -\infty$$

$$\lim_{x \to -2^-} f(x) = -\infty$$

Limit D.N.E.

How can we interpret the results of (a) and (b)?

<u>Infinite Limit</u>: A limit in which *f(x)* increases or decreases without bound as *x* approaches *c* is called an infinite limit. A formal definition is given on page 83.

If a function output approaches infinity as *x* approaches *c*, then we may interpret this as: $\lim_{x \to c} f(x) = \infty$

If a function output approaches negative infinity as *x* approaches *c*, then we may interpret this as: $\lim_{x \to c} f(x) = -\infty$

Either way, $\lim_{x \to c} f(x) = \infty$ or $\lim_{x \to c} f(x) = -\infty$ still means that the limit DOES NOT EXIST as *x* approaches *c*!

In problems (a) and (b) on the previous page, what can we call the line x = -2?

<u>Connection Between Vertical Asymptotes and Infinite Limits</u>: If *f(x)* approaches infinity or negative infinity as *x* approaches *c* form the right or the left, then the line *x* = *c* is a vertical asymptote of the graph of *f*.

We can see that in both examples (a) and (b) that there must be a vertical asymptote at *x* = -2 since $\lim_{x \to -2} f(x) = \infty \; or \; -\infty$.

How can we interpret the results of (a) and (b)?

This tells us that there exists a vertical asymptote @ $x = -2$.

<u>Finding Vertical Asymptotes</u>: Given the rational function $h(x) = \frac{f(x)}{g(x)}$ in which *f(x)* and *g(x)* have no variable common factors, then *h(x)* has a vertical asymptote at x = c for all values *c* such that *g(c)* = 0.

Ex: Determine all vertical asymptotes of the graph of the following functions:

a) $f(x) = \dfrac{3}{x^2 + x - 2}$

b) $s(t) = \dfrac{t}{\sin t}$

Ex: Determine all vertical asymptotes of the graph of the following functions:

a) $f(x) = \dfrac{3}{x^2 + x - 2}$

$f(x) = \dfrac{3}{(x+2)(x-1)}$

$x+2=0$ or $x-1=0$

$\boxed{x=-2, \; x=1}$

b) $s(t) = \dfrac{t}{\sin t}$

$\sin t = 0$

$$\left.\begin{aligned} t &= 0 + 2\pi n \\ t &= \pi + 2\pi n \end{aligned}\right\} n \in \mathbb{Z}$$

$$\boxed{t = n\pi \quad n \in \mathbb{Z}}$$

What happens to vertical asymptotes if in the function $h(x) = \frac{f(x)}{g(x)}$ there are variable common factors between *f(x)* and *g(x)*?

Ex: Determine all vertical asymptotes of the graph of the following function: $f(x) = \frac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

<u>Properties of Infinite Limits</u>: Let *c* and *L* be real numbers and let *f* and *g* be functions such that:

$\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L.$

a) Sum or Difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty$

b) Product: $\lim_{x \to c} [f(x) \cdot g(x)] = \infty,\ L > 0$

$\lim_{x \to c} [f(x) \cdot g(x)] = -\infty,\ L < 0$

c) Quotient: $\lim_{x \to c} \left[\frac{g(x)}{f(x)}\right] = 0$

Similar properties hold for one-sided limits and for functions for which the limit of f(x) as x approaches c is -∞.

Ex: Determine all vertical asymptotes of the graph of the following function: $f(x) = \dfrac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

$$f(x) = \frac{4(x^2+x-6)}{x^3(x-2)-9x(x-2)}$$

$$f(x) = \frac{4(x+3)(x-2)}{(x-2)(x^3-9x)}$$

$$f(x) = \frac{4\cancel{(x+3)}\cancel{(x-2)}}{x\cancel{(x+3)}(x-3)\cancel{(x-2)}}$$

Holes:

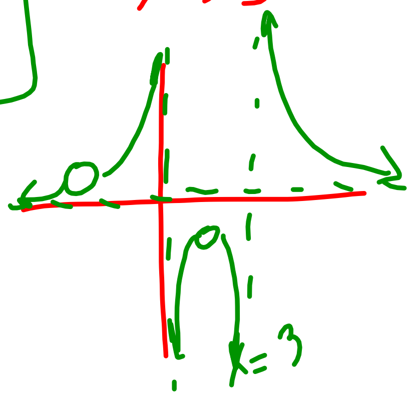
$(2, -2)$

$\left(-3, \frac{2}{9}\right)$

$$f(x) = \frac{4}{x(x-3)}$$

$$\boxed{\begin{aligned} V.A.'s:\ x &= 0 \\ x &= 3 \end{aligned}}$$

$$\lim_{x \to 3^+} \frac{4}{x(x-3)} = \infty$$



Rational Functions: $R(x) = \frac{P(x)}{Q(x)}$

$Q(x) \neq 0$

Vertical Asymptotes: $Q(x) = 0$

Values for x s.t.

Holes: Common factors for $P(x)$ & $Q(x)$ create holes in the graph.

Horizontal Asymptotes: Depends on degree of $P(x)$ & $Q(x)$.