

Problem 1 $v_A = v_B = (\frac{4}{5})c$

A.) Sketch what ships look like RT A's frame

$$\sqrt{1 - v^2/c^2} = \sqrt{1 - (\frac{4}{5}c)^2} = (\frac{3}{5}c)$$

$$D_{BR} = 450\text{m}$$

$$D_{AR} = 150\text{ m}$$

B will contract in length

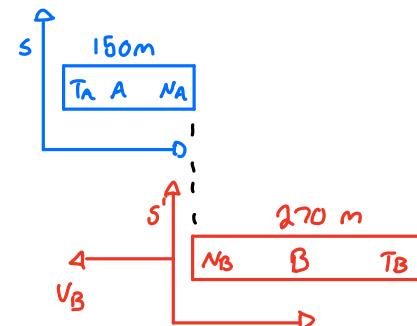
$$d_B = D_{BQ} \cdot \sqrt{1 - v^2/c^2} = 450\text{m} (\frac{3}{5}) = 270\text{ m}$$

$$d_B = 270\text{ m}$$

To A's RF

$$0.5\text{ cm} = 50\text{ m}$$

S - SS A Frame
 S' - SS B Frame



A will see ...

$$D_A = 150\text{ m}$$

$$D_B = 270\text{ m}$$

B.) What do all the clocks read? RT A.

$T_A = N_A = 0$ Because A is not moving and all clocks are synchronized.
 $N_A = N_B$ Because both clocks are synchronized when origins pass.

But Since B is moving, its nose clock will lag and the tail clock will be ahead.

$$\Delta t = \frac{vx}{c^2} = \frac{(\frac{4}{5})c(450)\text{m}}{c^2} = 360\text{ m/c}$$

$$v = (\frac{4}{5})c$$

$$x = (450)\text{m}$$

$$T_A = 0\text{ m/c} \quad N_A = 0\text{ m/c}$$

$$T_B = 360\text{ m/c} \quad N_B = 0\text{ m/c}$$

C.) Time for tail of B to reach nose of A? RT A.

Values RT A

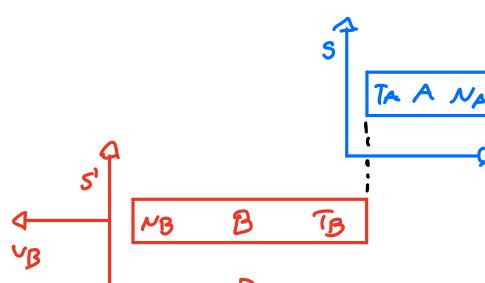
$$D_B = 270\text{ m} \quad \Delta t = \frac{D}{v} \quad \therefore \Delta t = \frac{270\text{ m}}{(\frac{4}{5})c} = 270\text{ m} \cdot \frac{5}{4c} = \frac{1350}{4} = \frac{675}{2}\text{ m/c}$$

$$\Delta t = \frac{675}{2}\text{ m/c}$$

D.) Sketch when tail of B is at nose of A. RT A

$$0.5\text{ cm} = 50\text{ m}$$

S - SS A Frame
 S' - SS B Frame



E.) What are the clock readings in part D? RT A

T_A and N_A will tick by at same rate
 $T_A = N_A = \left(\frac{625}{2}\right) m/c$

B is moving so its clocks will be out of sync with A.

$$\sigma = \sqrt{1 - v^2/c^2}$$
$$\sigma = (\sqrt{3}/5)c$$

$$\Delta t = \frac{\Delta t'}{\sigma}$$

$$\Delta t' = \Delta t \sigma$$

$$\Delta t' = \left(\frac{675}{2}\right) m/c (\sqrt{3}/5) = \frac{405}{2} m/c \equiv N_B$$

$$T_B = N_B + (360 m/c) = \frac{1125}{2} m/c \equiv T_B$$

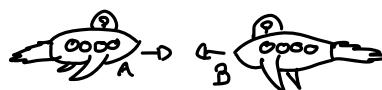
$$T_A = \left(\frac{675}{2}\right) m/c$$

$$T_B = \left(\frac{1125}{2}\right) m/c$$

$$N_A = \left(\frac{675}{2}\right) m/c$$

$$N_B = \left(\frac{405}{2}\right) m/c$$

Problem 2 $v_A \wedge v_B = (4/s)c$



A.) Sketch when noses meet RT B.

$$\sigma = \sqrt{1 - v^2/c^2} = \sqrt{1 - (4/s)^2} = \frac{3}{5}$$

$$\sigma = (\frac{3}{5})$$

$$D_{AR} = 150m$$

$$D_{BR} = 450m$$

From B's perspective A will shrink.

$$d = D\sigma$$

$$d = 150m(\frac{3}{5}) = 90m$$

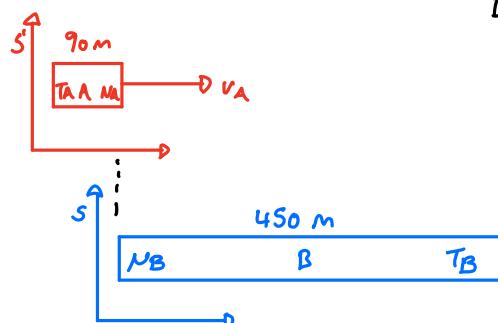
B will see

$$D_A = 90m$$

$$D_B = 450m$$

0.5 cm = 50 m

S - SS B
S' - SS A



B.) What are the clock readings when noses meet? RT B.

on B

$$N_B \wedge T_B \text{ are sync'd since not moving.}$$

$$\therefore N_B = T_B = 0$$

on A

$$N_A \text{ is set to zero when noses meet.}$$

$$T_A \text{ will be ahead of the lagging } N_A \text{ clock.}$$

$$\therefore N_A = 0 \wedge T_A = \frac{vx}{c^2}$$

$$T_A = \frac{vx}{c^2} = \frac{(4/s)c(150)m}{c^2} = 120 \text{ m/c}$$

$$v = (4/s)c$$

$$x = 150m$$

$$T_A = 120 \text{ m/c} \quad N_A = 0 \text{ m/c}$$

$$T_B = 0 \text{ m/c} \quad N_B = 0 \text{ m/c}$$

C.) How long does it take for nose of A to reach tail of B?

B will see

$$D = 450m$$

$$v = (4/s)c$$

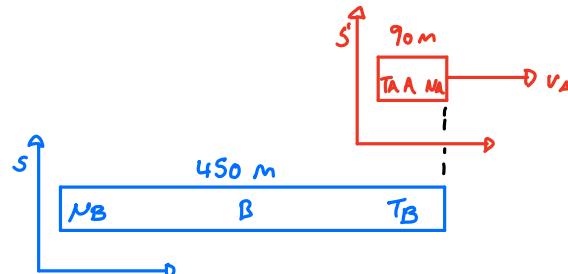
$$\Delta t = \frac{D}{v} \quad \therefore \Delta t = \frac{450m}{(4/s)c} = \frac{112.5}{2} \text{ m/c}$$

$$\Delta t = \frac{112.5}{2} \text{ m/c}$$

D.) Sketch when A reaches tail of B. RT B

0.5 cm = 50 m

S - SS B
S' - SS A



E.) What are the clock readings found in part D? RTB

For B

Their clocks will tick at same rate and be synchronized.

$$\therefore N_B = T_B = \frac{1125}{2} \text{ m/c}$$

$$\boxed{T_A = \left(\frac{915}{2}\right) \text{ m/c} \quad N_A = \left(\frac{675}{2}\right) \text{ m/c}} \\ T_B = \left(\frac{1125}{2}\right) \text{ m/c} \quad N_B = \left(\frac{1125}{2}\right) \text{ m/c}$$

For A

The nose clock will lag the tail

$$\Delta t = \frac{\Delta t'}{\sigma} \quad \sigma = \sqrt{1 - v^2/c^2} = (3/5)$$

$$\Delta t' = \Delta t \cdot \sigma = \left(\frac{1125}{2}\right) \text{ m/c} (3/5) = \frac{675}{2} \text{ m/c}$$

$$\therefore N_A = \frac{675}{2} \text{ m/c}$$

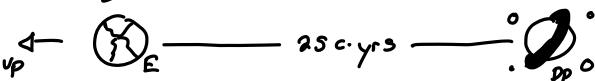
$$\therefore T_A = N_A + 120 \text{ m/c}$$

$$T_A = \frac{675}{2} \text{ m/c} + 120 \text{ m/c} = \frac{915}{2} \text{ m/c}$$

Problem 3

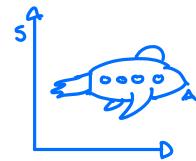
RT SS A @ rest

$$v_p = \left(\frac{5}{13}\right)c$$



A.) Sketch 3 events.

E₁
Earth departs.

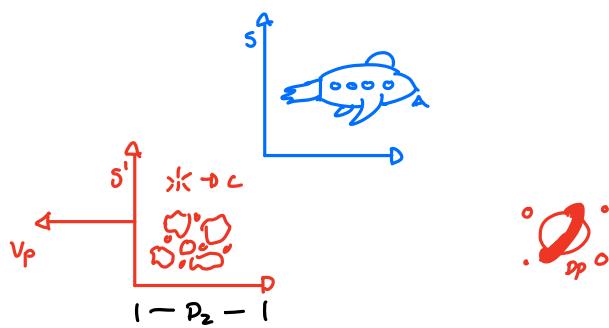


S - SS A
S' - E & DP

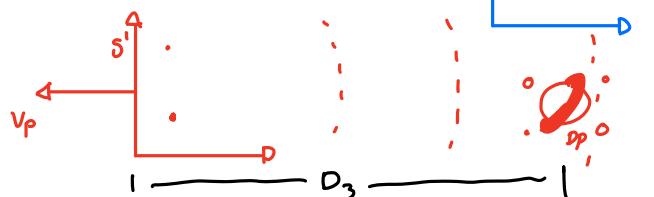
E₂
Earth explodes.



E₃
Signal arrives.



$$D_3 = v \cdot t_3 \quad D_3 = c(t_3 - t_2)$$



B.) Distance of planet's RT SS?

$$\sigma \equiv \sqrt{1 - v^2/c^2}$$

$$d = D\sigma$$

$$D = 25 \text{ c} \cdot \text{yrs}$$

$$\sigma = \sqrt{1 - v^2/c^2} = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \left(\frac{12}{13}\right)$$

$$\sigma = \left(\frac{12}{13}\right)$$

$$d = 25 \text{ c} \cdot \text{yrs} \left(\frac{12}{13}\right) = 23 \text{ c} \cdot \text{yrs}$$

$$D_{ss} = 23 \text{ c} \cdot \text{yrs}$$

C.) When distant planet arrives, what time does SS read?

RT SS

$$D = 23 \text{ c} \cdot \text{yrs}$$

$$v = \left(\frac{5}{13}\right)c$$

$$\Delta t = \frac{D}{v} \quad \therefore \quad \Delta t = \frac{23 \text{ c} \cdot \text{yrs}}{\left(\frac{5}{13}\right)c} = 60 \text{ yrs}$$

$$\Delta t_{ss} = 60 \text{ yrs}$$

D.) When distant planet arrives, what does Earth clock read?

$$\sigma = \sqrt{1-v^2/c^2}$$

$$\Delta t = \frac{\Delta t'}{\sigma} \quad \therefore \quad \Delta t' = \sigma \Delta t = 60 \text{ yrs} (12/13) = 55 \text{ yrs}$$

$$\Delta t = 60 \text{ yrs}$$

$$\sigma = (12/13)$$

$$\Delta t'_E = 55 \text{ yrs}$$

E.) When distant planet arrives, what does distant planet read?

$$D_p \text{ originally off by } \frac{vx}{c^2}$$

$$v = (12/13)c$$

$$x = 25 \text{ c.yrs}$$

$$t_{dp} = \frac{(12/13)c(25 \text{ c.yrs})}{c^2} = 9.6 \text{ yrs}$$

$$t_{pp} = \Delta t_E + 9.6 \text{ yrs}$$

$$= 55 \text{ yrs} + 9.6 \text{ yrs}$$

$$t_{pp} = 65 \text{ yrs}$$

$$\Delta t_{pp} = 65 \text{ yrs}$$

F.) When signal is sent, what does ship clock read?

Notice for distances...

$$D_3 = v \cdot t_3 \quad D_3 = c(t_3 - t_2)$$

$$vt_3 = c(t_3 - t_2)$$

$$t_2 = t_3 (1 - v/c)$$

$$vt_3 = ct_3 - ct_2$$

$$t_2 = (60 \text{ yrs}) (1 - (12/13))$$

$$ct_2 = ct_3 - vt_3$$

$$t_2 = 36.9 \text{ yrs}$$

$$t_2 = t_3 - \frac{v}{c} t_3$$

$$t_2 = t_3 (1 - v/c)$$

$$\Delta t_{ds} = 37 \text{ yrs}$$

G.) When signal is sent, what does Earth clock read?

$$\sigma = \sqrt{1-v^2/c^2} = 12/13$$

$$\Delta t = \frac{\Delta t'}{\sigma} \quad \therefore \quad \Delta t' = \Delta t \sigma = 37 \text{ yrs} (12/13) = 34.1 \text{ yrs}$$

$$\Delta t = 37 \text{ yrs}$$

$$\Delta t_E = 34 \text{ yrs}$$

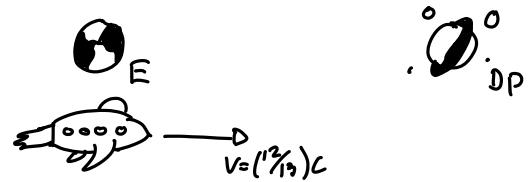
H.) When signal is sent, what does distant planet clock read?

D_p originally off by 9.6 yrs

$$\therefore \Delta t_{pp} = \Delta t_E + 9.6 \text{ yrs} = 34 \text{ yrs} + 9.6 \text{ yrs} = 43.6 \text{ yrs}$$

$$\Delta t_{pp} = 43.6 \text{ yrs}$$

Problem 4]



A.) Sketch diagrams from Albert's frame.

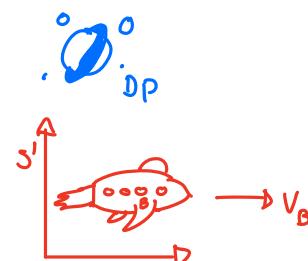
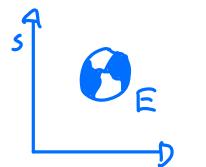
E_1
Billy leaves Albert



S - Albert and Dp
 S' - Billy and SS

E_2
Billy arrives at Dp

Billy measures $\Delta x' = 0$



B.) How old was Albert?

$$\sigma \equiv \sqrt{1 - v^2/c^2} \quad \sigma = (5/13)$$

$$\Delta t = \frac{\Delta t'}{\sigma} \quad \Delta t' = 25 \text{ c.yrs}$$

$$\Delta t = \frac{25 \text{ c.yrs}}{(5/13)} = 65 \text{ yrs}$$

$$\Delta t_A = 65 \text{ yrs}$$

c.) According to Albert what is the distance to Dp?

Acc. To Albert

$$v = (12/13)c$$

$$\Delta t = (65) \text{ yrs}$$

$$D = v \cdot \Delta t = \left(\frac{12}{13}\right)c \cdot (65 \text{ yrs}) = \frac{60c \cdot \text{yrs}}{2} = 30 \text{ c.yrs}$$

$$D_A = 30 \text{ c.yrs}$$

D.) According to Billy what is the distance to Dp?

Acc. To Billy

$$v = (12/13)c$$

$$\Delta t = (25) \text{ yrs}$$

$$D = v \cdot \Delta t = \left(\frac{12}{13}\right)c \cdot (25) \text{ yrs} = \frac{300}{13} \text{ c.yrs} = \frac{150}{13} \text{ c.yrs}$$

$$D = \frac{150}{13} \text{ c.yrs}$$

E.) When Billy reaches D_P, what do clocks read?

RT E and D_P

Clocks are synched
will tick at same
rate

$$t_E \wedge t_{DP} = \frac{65}{2} \text{ yrs}$$

$$\Delta t_E = \frac{65}{2} \text{ yrs}$$

$$\Delta t_{DP} = \frac{65}{2} \text{ yrs}$$

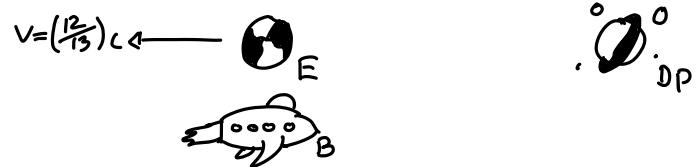
RT Billy

He sees his time as

$$\Delta t_B = \frac{150}{13} \text{ yrs}$$

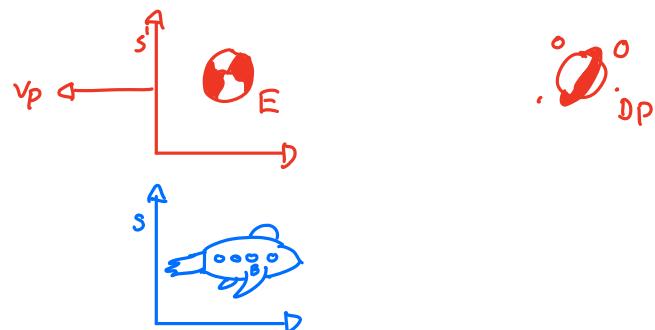
$$\boxed{\Delta t_E = \frac{65}{2} \text{ yrs} \quad \Delta t_{DP} = \frac{65}{2} \text{ yrs}}$$
$$\Delta t_B = \frac{150}{13} \text{ yrs}$$

Problem 5]

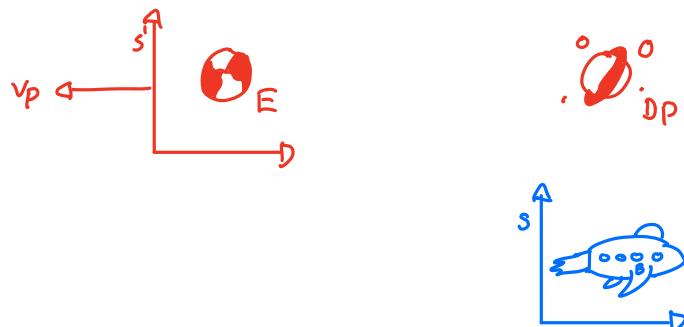


A.) Sketch two events from rest frame of Billy.

E₁
Earth departs Billy



E₂
Dp arrives at Billy



B.) What are the clock readings when Earth leaves?

Billy and Albert synched clocks originally
 $\therefore t_B \wedge t_E = 0 \text{ yrs}$

$$\boxed{t_B = 0 \text{ yrs} \quad t_E = 0 \text{ yrs}} \\ t_{DP} = \frac{360}{13} \text{ yrs}$$

DP will be ahead by $\frac{vx}{c^2}$

$$v = \left(\frac{12}{13}\right)c$$

$$x = 30 \text{ c} \cdot \gamma \text{ c s}$$

$$\frac{\left(\frac{12}{13}\right)c (30) \text{ yrs}}{c^2} = \frac{360}{13} \text{ yrs}$$

C.) What are the clock readings when Dp arrives?

$$\sigma = \left(\frac{5}{13}\right)$$

RT Billy

$$d = D \sigma = 30 \text{ yrs} \left(\frac{5}{13}\right)c = \frac{150}{13} \text{ c yrs}$$

$$v = \left(\frac{12}{13}\right) \quad \Delta t = \frac{D}{v} = \frac{\left(\frac{150}{13}\right) \text{ c yrs}}{\left(\frac{12}{13}\right) c} = \frac{150}{12} \text{ yrs}$$

$$\Delta t_B = \frac{150}{12} \text{ yrs}$$

$$\boxed{\Delta t_B = \frac{150}{12} \text{ yrs} \quad \Delta t_{DP} = \frac{65}{2} \text{ yrs}} \\ \Delta t_E = \frac{125}{26} \text{ yrs}$$

RT Planets

$$\Delta t_E = \Delta t \sigma = \left(\frac{150}{13}\right) \text{ yrs} \left(\frac{5}{13}\right) = \frac{125}{26} \text{ yrs}$$

$$\Delta t_{DP} = \Delta t_E + \frac{360}{13} = \frac{65}{2} \text{ yrs}$$

$$\Delta t_{DP} = \frac{65}{2} \text{ yrs}$$