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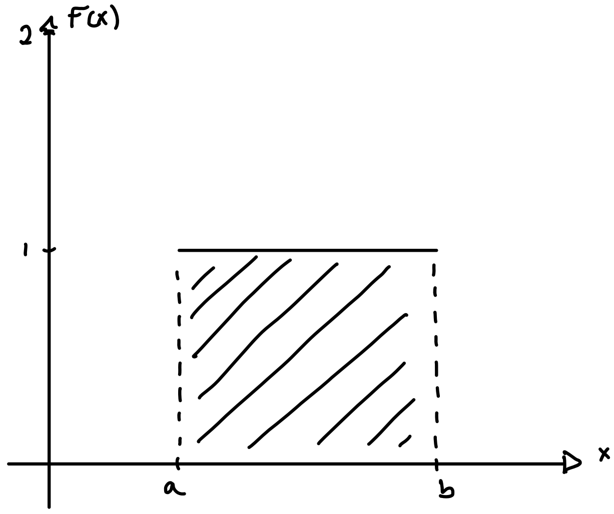
MATH 360
HW Ch. 11.9

11.9 # 3,4

Problem 11.9.3

$$f(x) = \begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$$



$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx \ , \quad -\infty < w < \infty$$

$f(x) = 1 \ : \ a < x < b$

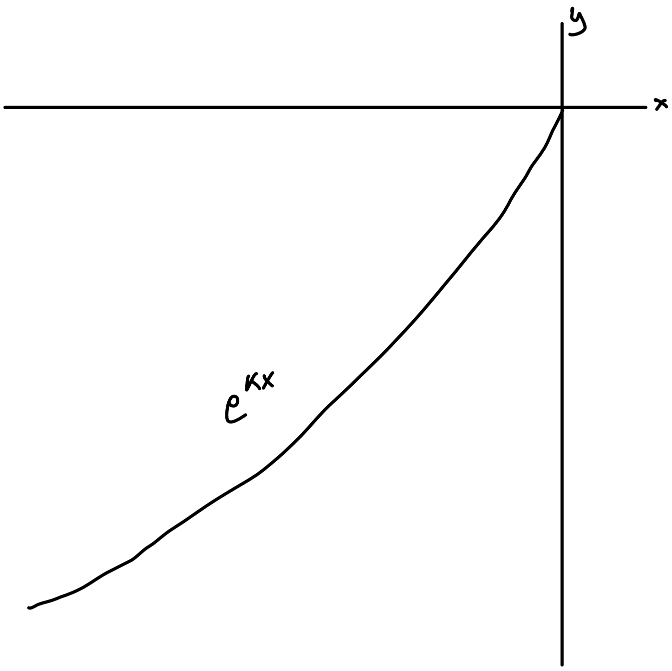
$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-iwx}\, dx$$

$$= \frac{1}{-iw\sqrt{2\pi}} \Big[e^{-iwx} \Big]_a^b = \frac{-1}{iw\sqrt{2\pi}} \left(e^{-iwb} - e^{-iwa} \right)$$

$$\boxed{\hat{f}(w) = \frac{-1}{iw\sqrt{2\pi}} \left[e^{-iwb} - e^{-iwa} \right]}$$

Problem 11.9.4

$$f(x) = \begin{cases} e^{kx} & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}, \quad k > 0$$



$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx, \quad -\infty < w < \infty$$

$$f(x) = e^{kx}, \quad -\infty < x < 0$$

$$e^{kx} \cdot e^{-iwx} = e^{kx - iwx} = e^{(k-iw)x}$$

$$\hat{f}(w) = \lim_{b \to -\infty} \frac{1}{2\sqrt{\pi}} \int_b^0 e^{(k-iw)x}\, dx$$

$$\frac{1}{2\sqrt{\pi}} \cdot \frac{1}{(k-iw)} \cdot e^{(k-iw)x} \Big|_b^0$$

$$\frac{1}{2\sqrt{\pi}} \cdot \frac{1}{(k-iw)} \cdot e^{0} - \overbrace{\cancel{\frac{1}{2\sqrt{\pi}} \cdot \frac{1}{(k-iw)} \lim_{b \to -\infty} e^{(k-iw)b}}}^{0}$$

$$\boxed{\hat{f}(w) = \frac{1}{2\sqrt{\pi}} \cdot \frac{1}{(k-iw)}}$$