3.1 Matrices: Sums and Products

3.1 #1-16, 51-54, 68, 69

3.1.1 2A

$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

$$\boxed{2A = \begin{bmatrix} -2 & 0 & 6 \\ 4 & 2 & 4 \\ -2 & 0 & 2 \end{bmatrix}}$$

3.1.2 A + 2B

$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$2B = \begin{bmatrix} 2 & 6 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$\boxed{A + 2B = \begin{bmatrix} 1 & 6 & 3 \\ 2 & 3 & 2 \\ -1 & 0 & 3 \end{bmatrix}}$$

3.1.3 2C - D

$C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

undefined. Not possible.

3.1.4 AB

$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -1(1)+0(0)+3(0) & -1(3)+0(1)+3(0) & -1(0)+0(0)+3(1) \\ 2(1)+1(0)+2(0) & 2(3)+1(1)+2(0) & 2(0)+1(0)+2(1) \\ -1(1)+0(0)+1(0), & -1(3)+0(1)+1(0), & -1(0)+0(0)+1(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -3 & 3 \\ 2 & 7 & 2 \\ -1 & -3 & 1 \end{bmatrix}$$

3.1.5 BA

$B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

$$BA = \begin{bmatrix} 1(-1)+3(2)+0(-1) & 1(0)+3(1)+0(0) & 1(3)+3(2)+0(1) \\ 0(-1)+1(2)+0(-1) & 0(0)+1(1)+0(0) & 0(3)+1(2)+0(1) \\ 0(-1)+0(2)+1(1), & 0(0)+0(1)+0(0), & 0(3)+0(2)+1(1) \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 3 & 9 \\ 2 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

3.1.6 CD

$$C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

$$CD = \begin{matrix} 1(3)+0(2) & 1(-1)+0(1) & 1(0)+0(2) \\ 2(3)+1(2) & 2(-1)+1(1) & 2(0)+1(2) \\ 1(3)+3(2), & 1(-1)+3(1), & 1(0)+3(2) \end{matrix}$$

$$CD = \begin{bmatrix} 3 & -1 & 0 \\ 8 & -1 & 2 \\ 9 & 2 & 6 \end{bmatrix}$$

3.1.7 DC

$$D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$DC = \begin{matrix} 3(1)-1(2)+0(1) & 3(0)-1(1)+0(3) \\ 2(1)+1(2)+2(1), & 2(0)+1(1)+2(3) \end{matrix}$$

$$\begin{bmatrix} 1 & -1 \\ 6 & 7 \end{bmatrix}$$

3.1.8 $(DC)^T$

$$D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$DC = \begin{matrix} 3(1)-1(2)+0(1), & 3(0)-1(1)+0(3) \\ 2(1)+1(2)+2(1), & 2(0)+1(1)+2(3) \end{matrix}$$

$$\begin{bmatrix} 1 & -1 \\ 6 & 7 \end{bmatrix}$$

$$(DC)^T = \begin{bmatrix} 1 & 6 \\ -1 & 7 \end{bmatrix}$$

3.1.9) $C^T D$

$C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

This is undefined, cannot multiply 3x2 by 2x3
Columns in first have to qual rows in second

3.1.10) $D^T C$

$D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$

The solution is undefined

3.1.11) $A^2 = AA$

$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

$$\begin{matrix} -1(-1)+0(2)+3(-1) & -1(0)+0(1)+3(0) & -1(3)+0(2)+3(1) \\ 2(-1)+1(2)+2(-1) & 2(0)+1(1)+2(0) & 2(3)+1(2)+2(1) \\ -1(-1)+0(2)+1(-1), & -1(0)+0(1)+1(0), & -1(3)+0(2)+1(1) \end{matrix}$$

$$A^2 = \begin{bmatrix} -2 & 0 & 0 \\ -2 & 1 & 10 \\ 0 & 0 & -2 \end{bmatrix}$$

3.1.12) AD

$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

undefined, columns in first must equal
rows in second

3.1.13) $A - I_3$

$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A - I_3 = \begin{matrix} (-1-1) & (0-0) & (3-0) \\ (2-0) & (1-1) & (2-0) \\ (-1-0) & (0-0) & (1-1) \end{matrix}$$

$$A - I_3 = \begin{bmatrix} -2 & 0 & 3 \\ 2 & 0 & 2 \\ -1 & 0 & 0 \end{bmatrix}$$

3.1.14) $4B-3I_3$

$$B=\begin{bmatrix}1&3&0\\0&1&0\\0&0&1\end{bmatrix}\quad I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$4B=\begin{bmatrix}4&12&0\\0&4&0\\0&0&4\end{bmatrix}\quad 3I_3=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$$

$$4B-3I_3=\begin{matrix}(4-3)&(12-0)&(0-0)\\(0-0)&(4-3)&(0-0)\\(0-0)&(0-0)&(4-3)\end{matrix}$$

$$4B-3I_3=\begin{bmatrix}1&12&0\\0&1&0\\0&0&1\end{bmatrix}$$

3.1.15) $C-I_3$

$$C=\begin{bmatrix}1&0\\2&1\\1&3\end{bmatrix}\quad I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

undefined, not compatible

3.1.16) AC

$$A=\begin{bmatrix}-1&0&3\\2&1&2\\-1&0&1\end{bmatrix}\quad C=\begin{bmatrix}1&0\\2&1\\1&3\end{bmatrix}$$

$$AC=\begin{bmatrix}2&9\\6&7\\0&3\end{bmatrix}$$

$$AC=\begin{matrix}-1(1)+0(2)+3(1) & -1(0)+0(1)+3(3)\\2(1)+1(2)+2(1) & 2(0)+1(1)+2(3)\\-1(1)+0(2)+1(1), & -1(0)+0(1)+1(3)\end{matrix}$$

3.1.51) $A+2B$

$$A=\begin{bmatrix}1+i&2i\\2&2-3i\end{bmatrix}\quad B=\begin{bmatrix}1&-i\\2i&1+i\end{bmatrix}$$

$$2B=\begin{bmatrix}2&-2i\\4i&2+2i\end{bmatrix}\quad A+2B=\begin{bmatrix}3+i&0\\2+4i&4-i\end{bmatrix}$$

$$A+2B=\begin{bmatrix}3+i&0\\2+4i&4-i\end{bmatrix}$$

3.1.52) AB

$$A=\begin{bmatrix}1+i&2i\\2&2-3i\end{bmatrix}\quad B=\begin{bmatrix}1&-i\\2i&1+i\end{bmatrix}$$

$$AB=\begin{matrix}(1+i)(1)+2i(2i), & 1+i(-i)+2i(1+i)\\2(1)+(2-3i)(2i), & 2(2i)+(2-3i)(1+i)\end{matrix}$$

$$\begin{matrix}1+i+4i^2, & -i-i^2+2i+2i^2\\2+4i-6i^2, & 4i+2-i-3i^2\end{matrix}$$

$$AB=\begin{bmatrix}i-3&-1+i\\4i+8&5-3i\end{bmatrix}$$

3.1.53) BA

$B=\begin{bmatrix} 1 & -i \\ 2i & 1+i \end{bmatrix}$ $A=\begin{bmatrix} 1+i & 2i \\ 2 & 2-3i \end{bmatrix}$

$BA = \begin{matrix} 1(1+i)-i(2) & 1(2i)-i(2-3i) \\ 2i(1+i)+(1+i)(2) & 2i(2i)+(1+i)(2-3i) \end{matrix}$

$\begin{matrix} 1+i-2i & 2i-2i+3i^2 \\ 2i+2i^2+2+2i & 4i^2+2-i-3i^2 \end{matrix}$

$BA=\begin{bmatrix} 1-i & -3 \\ 4i & 1-i \end{bmatrix}$

3.1.54) A^2

$A=\begin{bmatrix} 1+i & 2i \\ 2 & 2-3i \end{bmatrix}$ $A=\begin{bmatrix} 1+i & 2i \\ 2 & 2-3i \end{bmatrix}$

$A^2 = \begin{matrix} (1+i)(1+i)+(2i)(2) & (1+i)(2i)+2i(2-3i) \\ 2(1+i)+(2-3i)(2) & 2(2i)+(2-3i)(2-3i) \end{matrix}$

$i^2=-1$ $\begin{matrix} 1+2i+i^2+4i, & 2i+2i^2+4i-6i^2 \\ 2+2i+4-6i, & 4i+4-12i+9i^2 \end{matrix}$

$A^2 = \begin{matrix} i^2+6i+1, & -4i^2+6i \\ -4i+6, & 9i^2-8i+4 \end{matrix}$

$A^2=\begin{bmatrix} 6i & 6i+4 \\ -4i+6 & -8i-5 \end{bmatrix}$

$A^2=\begin{bmatrix} 6i & 6i+4 \\ -4i+6 & -8i-5 \end{bmatrix}$

3.1.68) $\begin{bmatrix} 1 \\ K \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$K=-\frac{1}{2}$

$1(1)+K(2)+0(3)=0$

$1+2K=0$

$1=-2K$

$K=-\frac{1}{2}$

3.1.69) $\begin{bmatrix} K \\ 2 \\ K \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$

$K=0$

$K+0+4K=0$

$5K=0$

$K=0$