3.1 Matrices: Sums and Products

3.1 #1-16, 51-54, 68, 69,

Matrix

$$A = \overset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 4 & 8 \end{bmatrix}}, \quad B = \overset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}},$$

$$\vec{x} = \overset{3\times 1}{\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}}, \quad \vec{f}(x) = \overset{3\times 1}{\begin{bmatrix} x^2 \\ \sin x \\ -e^x \end{bmatrix}}$$

$A_{21} = 4$

Diagonal Matrix

$$D = \begin{bmatrix} d_{1,1} & 0 & 0 & \cdots & 0 \\ 0 & d_{2,2} & & & \\ 0 & & \ddots & & \\ \vdots & & & & \\ 0 & & & & d_{n,n} \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & & & \\ 0 & & 1 & & \\ \vdots & & & \ddots & \\ 0 & & & & 1 \end{bmatrix}$$

Zero matrix, all elements are zero

$A = B$ if all elements are the same

If A, B be $m\times n$ matrices and c, k are scalars.

$A + B = C$ is a $m\times n$ matrix with $C_{iN} = a_{iN} + b_{iN}$

$A + B = B + A$ (commutative under addition)

$(A+B)+C = A + (B+C)$

$cA = C$ is a $m\times n$ matrix with $c_{ij} = ca_{ij}$

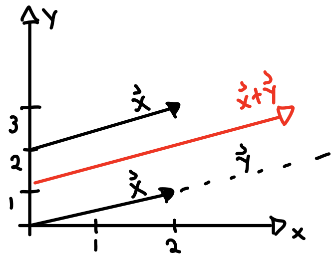
$A + 0 = A$

$A - A = 0$

$c(A+B) = cA + cB$

Vector Addition

$\vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = (2,1) \qquad \vec{y} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$



$$\vec{x} + \vec{y} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Scalar Product (Dot Product)

$\vec{x}, \vec{y}$ are vectors of dimension n.

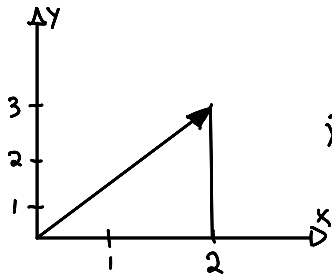
$\vec{x} + \vec{y} = x_1y_1 + x_2y_2 + \ldots\ldots x_ny_n$

$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} \qquad \vec{x}\cdot\vec{y} = 3\cdot 2 + 1\cdot 0 + 2\cdot 4$

$\vec{x}\cdot\vec{y} = 14$

$\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \vec{x}\cdot\vec{y} = 1\cdot 0 + 0\cdot 1 = 0$

Two vectors $\vec{x}, \vec{y}$ are orthogonal iff $\vec{x}\cdot\vec{y} = 0$



$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$= \sqrt{\vec{v}\cdot\vec{v}}$

$= \sqrt{\vec{v}^2}$

Matrix Product

$$C_{ij} = \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix}$$

$$\underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 8 \\ 0 & 5 \end{bmatrix}} = \begin{bmatrix} 2\cdot 1 + 1\cdot 0 & 2\cdot 8 + 1\cdot 5 \\ 3\cdot 1 + 4\cdot 0 & 3\cdot 8 + 4\cdot 5 \end{bmatrix}$$