2.6 Systems of Differential Equations

Example 4

$dR/dt = R(3-R-2S)$

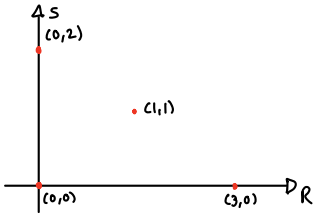
$dS/dt = S(2-S-R)$

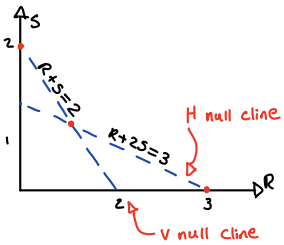
Solve the Algebraic System (R,S) to obtain

$R(3-R-2S)=0$ $\quad (0,0), (3,0), (0,2), (1,1)$

$S(2-S-R)=0$

R	S	(R,S)
0	0	(0,0)
3	0	(3,0)
0	2	(0,2)
1	1	(1,1)





$V - x = 0$

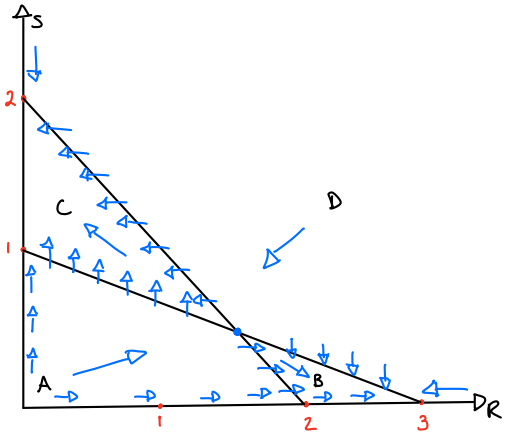
$h - y = 0$

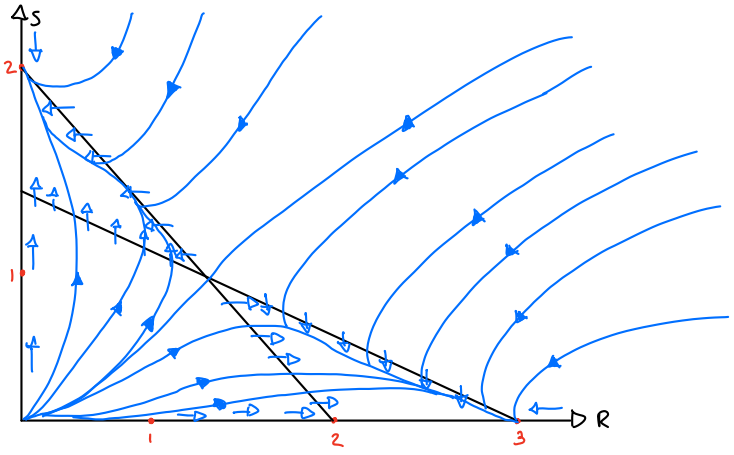
$(R,S) = (0,2)$: the sheep have driven the rabbits to extinction

$(R,S) = (3,0)$: the rabbits have driven the sheep to extinction

$(R,S) = (0,0)$: both sheep and rabbits are extinct

$(R,S) = (1,1)$: both sheep and rabbits are in balance





Region A: $dR/dt > 0$ $\quad ds/dt > 0$

Region B: $dR/dt > 0$ $\quad ds/dt < 0$

Region C: $dr/dt < 0$ $\quad ds/dt > 0$

Region D: $dr/dt < 0$ $\quad ds/dt < 0$

Along the R-nullcline, where $dR/dt = 0$, tangent vectors are vertical

Along the S-nullcline, where $ds/dt = 0$, tangent vectors are horizontal