

Answer the following question in the space provided. **Show all work.** (10 pts. total.)

Consider  $f(x) = 3x^3 + 9x^2 - 5$  for  $x \in [0, 1]$ . Let  $g(x) = \frac{1}{3}\sqrt{5 - 3x^3}$  be an iterating function whose fixed point  $p$  corresponds to the root of  $f(x)$  on  $[0, 1]$ . Define  $p_{n+1} = g(p_n)$  with  $p_n \rightarrow p$ . Beginning with  $p_0 = 0.5$ , use Steffensen's Method to find the iteration values indicated below. Be sure to specify the additional iterating function  $A(x, y, z)$  used, and also be sure to use function notation to indicate how you obtain each iterate, as discussed in class.

$$A(x, y, z) = x - \frac{(y-x)^2}{z - 2y + x}, \quad g(x) = \frac{1}{3}\sqrt{5 - 3x^3}$$

$$p_0^{(0)} = 0.5$$

$$p_1^{(0)} = g(p_0^{(0)}) = 0.7168604389$$

$$p_2^{(0)} = g(p_1^{(0)}) = 0.6578449798$$

$$p_0^{(1)} = A(p_0^{(0)}, p_1^{(0)}, p_2^{(0)}) = 0.6704695854$$

$$p_1^{(1)} = g(p_0^{(1)}) = 0.674603793$$

$$p_2^{(1)} = g(p_1^{(1)}) = 0.6732164187$$

$$p_0^{(2)} = A(p_0^{(1)}, p_1^{(1)}, p_2^{(1)}) = 0.6735650138$$

