Problem 1 $m = 0.55\ kg$ $K = 46\ N/m$

$\Delta x = 5.5 \times 10^{-2} m$ $\dot{x} = 0.45\ m/s$

a.) Solve for $x(t) = A\cos(\omega_0 t + \phi)$

$\omega_0 = \sqrt{K/m} = \sqrt{\frac{46\ N/m}{0.55\ kg}} = 9.15\ rad/s$

$x(t=0) = A\cos(\phi)$ $\quad x(t=0) \equiv \Delta x$

$\Delta x = A\cos(\phi)$

$A = \frac{\Delta x}{\cos(\phi)}$ $\quad \phi = -0.73\ rad$

$A = \frac{0.055m}{\cos(-0.73\ rad)} = 0.074m$

$\dot{x}(t) = -\omega_0 A\sin(\omega_0 t + \phi)$

$\dot{x}(t=0) = -\omega_0 A\sin(\phi)$ $\quad \dot{x}(t=0) \equiv \dot{x}$

$\dot{x} = \frac{-\omega_0 \Delta x \sin(\phi)}{\cos(\phi)}$

$\dot{x} = -\omega_0 \Delta x \tan(\phi)$

$\frac{\dot{x}}{-\omega_0 \Delta x} = \tan(\phi)$

$\phi = \tan^{-1}\left(\frac{\dot{x}}{-\omega_0 \Delta x}\right) = \tan^{-1}\left(\frac{0.45\ m/s}{-(9.15\ rad/s)(0.055m)}\right) = -0.729\ rad \simeq -0.73\ rad$

$$\boxed{\begin{array}{l} x(t) = (0.074m)\cos((9.15)\ rad/s\,(t) - 0.73\ rad) \\ A = 0.074m \quad \omega_0 = 9.15\ rad/s \quad \phi = -0.73\ rad \end{array}}$$

b.) Calculate the period of oscillation.

$b = 0 \therefore \omega_0 = \frac{2\pi}{T} \therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi}{9.15\ rad/s} = 0.69s$

$\omega_0 = 9.15\ rad/s$

$$\boxed{T = 0.69s}$$

C.) Total energy of the oscillator

$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2$

$m = 0.55kg$ $\quad \dot{x}(t=0)$ $\quad A = 0.074m$

$K = 46N/m$ $\quad x(t=0)$

$E = \frac{1}{2}m\omega^2 A^2 \sin^2(\phi) + \frac{1}{2}KA^2\cos^2(\phi)$

$E = \frac{1}{2}KA^2\left[\sin^2(\phi) + \cos^2(\phi)\right]$

$E = \frac{1}{2}KA^2$

$E = \frac{1}{2}(46\ N/m)(0.074m)^2 = 0.1259 \simeq 0.126\ J$

$$\boxed{E = 0.126\ J}$$

D.) Calculate the maximum speed.

$\dot{x}(t) = -\omega_0 A\sin(\omega t + \phi)$

$\ddot{x}(t) = -\omega_0^2 A\cos(\omega t + \phi)$

$0 = \cos(\omega t + \phi)$

$\pi/2 = \omega t + \phi$

$\pi/2 - \phi = \omega t$

$t = \frac{\pi/2 - \phi}{\omega_0}$

$t = \frac{\frac{\pi}{2} + 0.73\ rad}{9.15\ rad/s}$

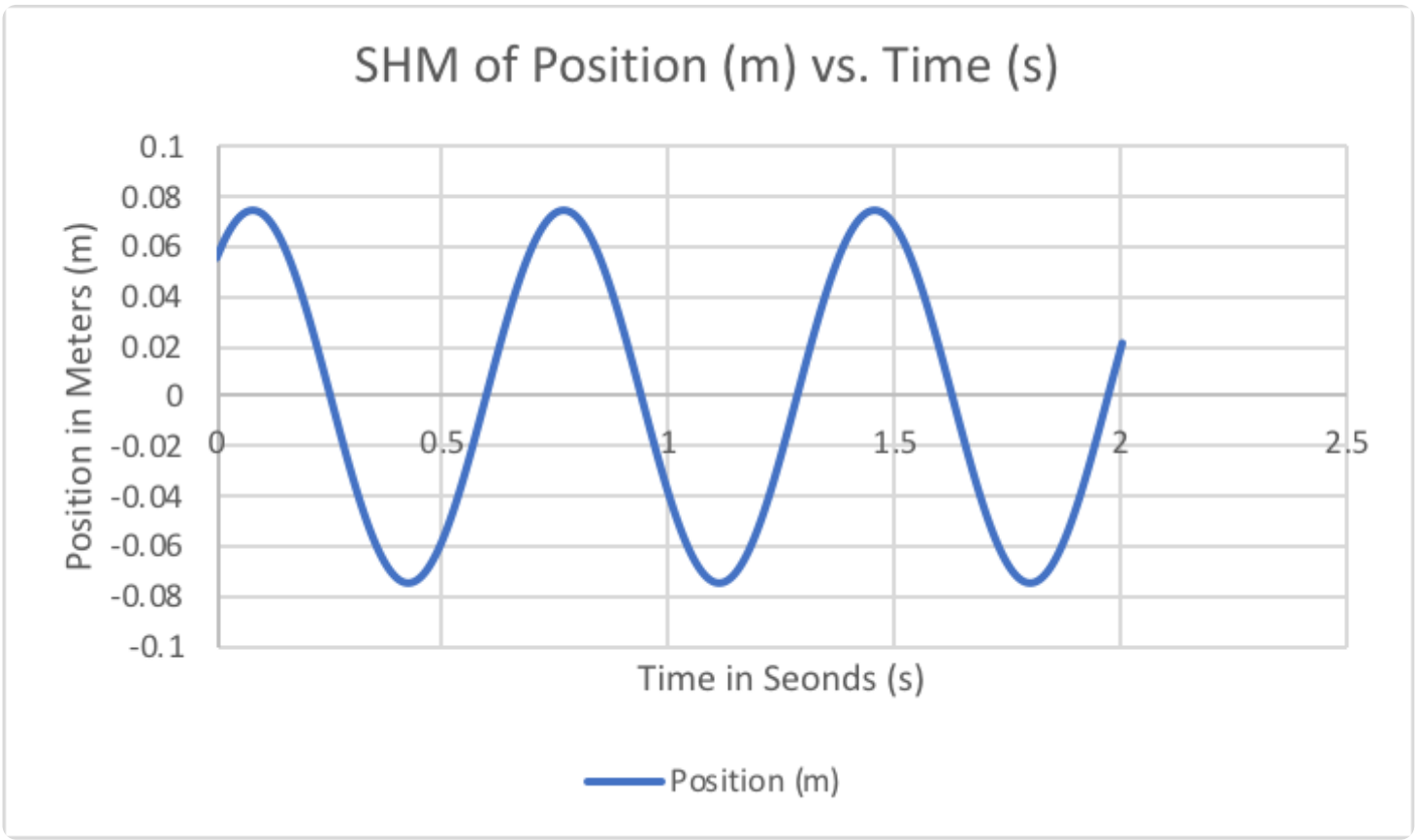
$t = 0.25s$

$\dot{x}(t=0.25s) = -(9.15\ rad/s)(0.074m)\sin((9.15\ rad/s)(0.25s) - 0.73\ rad)$

$\dot{x}(t=0.25s) = -0.68\ m/s(-1) = 0.68\ m/s$

$$\boxed{\text{Max } V = 0.68\ m/s}$$

Problem 2


SHM of Position (m) vs. Time (s)
Position in Meters (m)
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
0
0.5
1
1.5
2
2.5
Time in Seonds (s)
Position (m)

Problem 3 $m = 0.55\ kg$ $K = 46\ N/m$

$\Delta x = 0\ m$ $\dot{x} = -1.1\ m/s$

$\omega_0 = 9.15\ rad/s$

a.) Solve for $x(t) = A\cos(\omega_0 t + \phi)$

$x(t=0) = A\cos(\phi)$

$0 = A\cos(\phi)$

$0 = \cos(\phi)$

$\phi = \pi/2\ rad$

$x(t=0) \equiv \Delta x = 0$

$\dot{x}(t) = -\omega_0 A\sin(\omega_0 t + \phi)$

$\dot{x} = -\omega_0 A\sin(\phi)$

$\dfrac{\dot{x}}{-\omega_0\sin(\phi)} = A$

$\dfrac{-1.1\ m/s}{-(9.15\ rad/s)\sin(\pi/2)} = A$

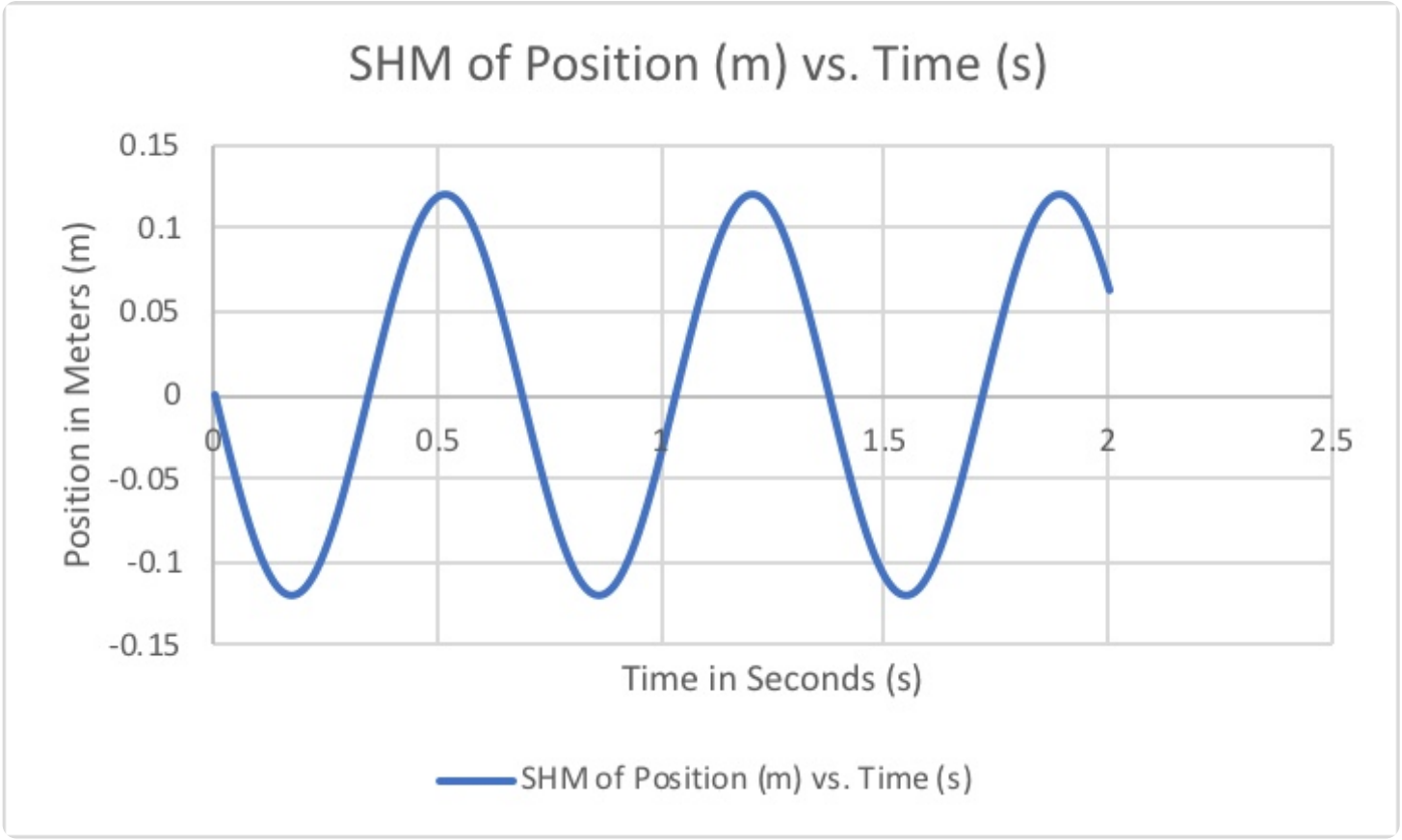
$A = +0.12m$

$\dot{x}(t) \equiv \dot{x}$

$\dot{x} = -1.1\ m/s$

$$\left[\begin{array}{l} x(t) = (0.12m)\cos((9.15\ rad/s)t + \pi/2\ rad) \\ A = 0.12m \quad \omega_0 = 9.15\ rad/s \quad \phi = \frac{\pi}{2}\ rad \end{array}\right]$$

b.)



Problem 4

a.) Calculate $u(t)$

$$u(t) = \tfrac{1}{2}kx^2$$

$$x(t) = A\cos(\omega t + \phi)$$

$$u(t) = \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) \qquad \left[u(t) = \tfrac{1}{2}(46\,N/m)^2(0.074\,m)^2\cos^2((9.15\,rad/s)(t) - 0.73\,rad)\right]$$

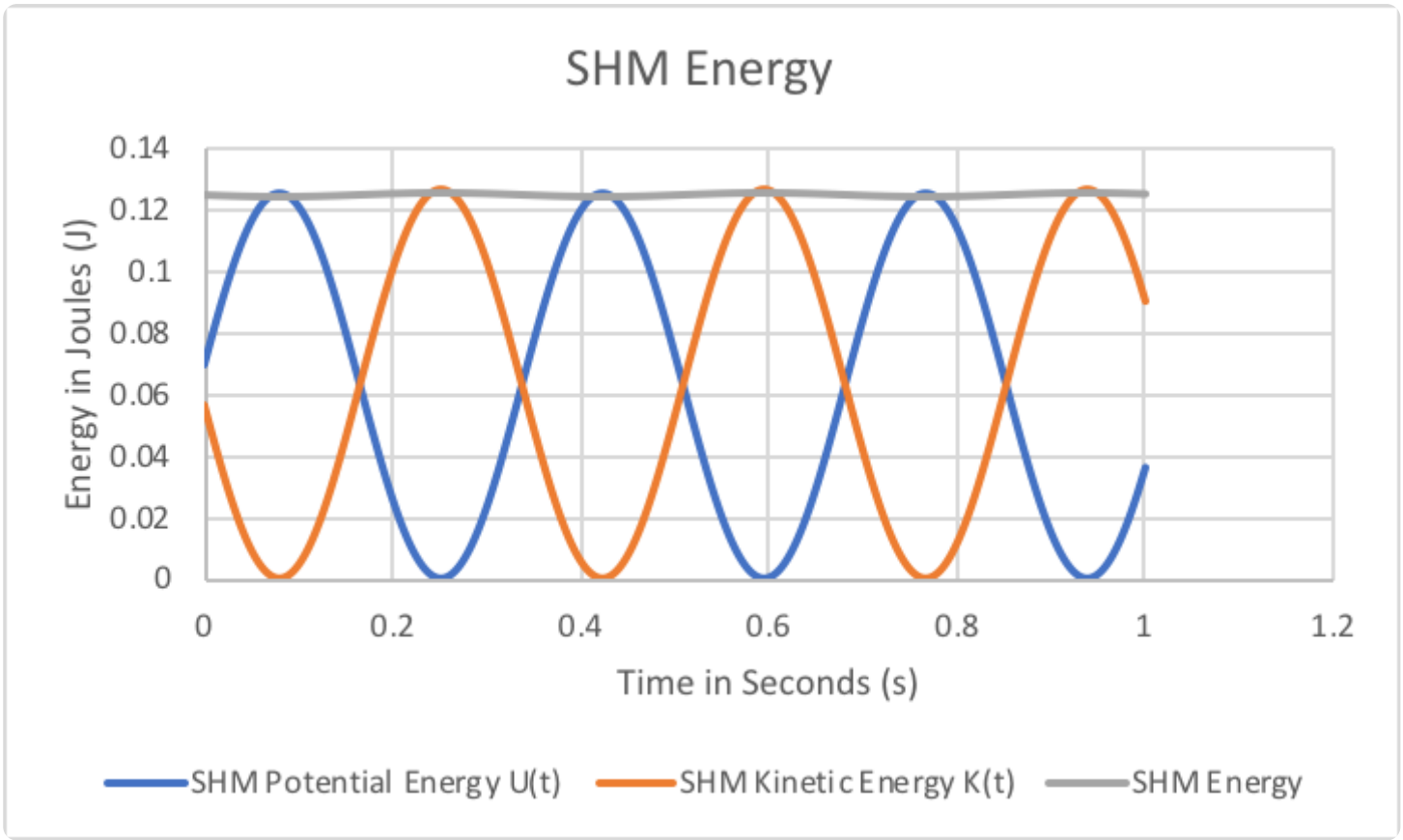
b.) calculate $K(t)$

$$K(t) = \tfrac{1}{2}m\dot{x}^2$$

$$\dot{x}(t) = -\omega_0 A\sin(\omega t + \phi)$$

$$K(t) = \tfrac{1}{2}m\omega_0^2A^2\sin^2(\omega t + \phi) \qquad \left[K(t) = \tfrac{1}{2}(0.55\,kg)(9.15\,rad/s)^2(0.074\,m)^2\sin^2((9.15\,rad/s)(t) - 0.73\,rad)\right]$$

C.)



Problem 5

$v = \omega r \qquad \omega = \frac{v}{r} = \frac{1.0\ m/s}{16\ m} = \frac{1}{16}\ s^{-1}$

$l = 16m \qquad \theta_i = 5° = \frac{\pi}{36}\ rad$

$\dot{\theta} = 1/16\ rad/s$

a.) Solve for $\theta(t) = \theta_0 \cos(\omega_0 t + \phi)$

$\omega_0 = \sqrt{g/l} = \sqrt{\frac{9.8 m/s^2}{16 m}} = 0.782\ s^{-1}$

$\theta(t=0) = \theta_0 \cos(\phi)$

$\theta_i = \theta_0 \cos(\phi) \qquad \theta(t=0) = \theta_i$

$\theta_0 = \frac{\theta_i}{\cos(\phi)} \qquad \phi = -0.74\ rad$

$\theta_0 = \frac{\pi/36}{\cos(-0.74\ rad)} = 0.118\ rad$

$\dot{\theta}(t) = -\omega_0 \theta_0 \sin(\omega_0 t + \phi)$

$\dot{\theta}(t=0) = -\omega_0 \theta_0 \sin(\phi) \qquad \dot{\theta}(t=0) \equiv \dot{\theta}$

$\dot{\theta} = \frac{-\omega_0 \theta_i \sin(\phi)}{\cos(\phi)}$

$\dot{\theta} = -\omega_0 \theta_i \tan(\phi)$

$\frac{\dot{\theta}}{-\omega_0 \theta_i} = \tan(\phi)$

$\phi = \tan^{-1}\left(\frac{\dot{\theta}}{-\omega_0 \theta_i}\right) = \tan^{-1}\left(\frac{1/16\ rad/s}{-(0.785^{-1})(\pi/36\ rad)}\right)$

$\phi = -0.742\ rad$

$\left[\theta(t) = (0.12\ rad)\cos((0.785^{-1})t - 0.74\ rad)\right]$

$\left[\theta_0 = 0.12\ rad \quad \omega_0 = 0.78\ s^{-1} \quad \phi = 0.74\ rad\right]$

b.) Calculate the period of oscillation.

Since $\gamma = 0 \quad \omega_0 = \omega \quad \therefore \quad \omega_0 = \frac{2\pi}{T} \quad \therefore \quad T = \frac{2\pi}{\omega_0}$

$\omega_0 = 0.78\ rad \qquad T = \frac{2\pi}{0.78 rad} = 8.03\ s$

$\left[T = 8\ s\right]$

c.) What is the maximum speed?

$\dot{\theta}(t) = -\omega_0 \theta_0 \sin(\omega_0 t + \phi)$

$\ddot{\theta}(t) = -\omega_0^2 \theta_0 \cos(\omega_0 t + \phi)$

$0 = -\omega_0^2 \theta_0 \cos(\omega_0 t + \phi)$

$0 = \cos(\omega_0 t + \phi)$

$\pi/2 = \omega_0 t + \phi$

$\pi/2 - \phi = \omega_0 t$

$t = \frac{\pi/2 - \phi}{\omega_0} = \frac{\pi/2 + 0.74\ rad}{0.782\ s^{-1}} = 2.95\ s$

$\dot{\theta}(t=2.95) = -(0.782 s^{-1})(0.12\ rad)\sin((0.782\ rad)(2.95) - 0.74\ rad)$

$\dot{\theta} = -0.0938\ rad/s$

$V = \omega r = -(0.0938\ rad/s)(16 m) = -1.5\ m/s$

Speed $\therefore \quad S = 1.5\ m/s$

$\left[V = 1.5\ m/s\right]$

d.) Maximum angle

$\dot{\theta}(t) = -\omega_0 \theta_0 \sin(\omega_0 t + \phi)$

$0 = -\omega_0 \theta_0 \sin(\omega_0 t + \phi)$

$0 = \sin(\omega_0 t + \phi)$

$0 = \omega_0 t + \phi$

$-\phi = \omega_0 t$

$t = -\phi/\omega_0$

$t = +0.74\ rad / 0.782\ rad/s$

$t = 0.946\ s$

$\theta(0.95) = (0.12\ rad)\cos((0.7825^{-1})(0.95) - 0.74\ rad)$

$\theta = 0.12\ rad = 6.87°$

$\left[\theta = 6.9°\right]$