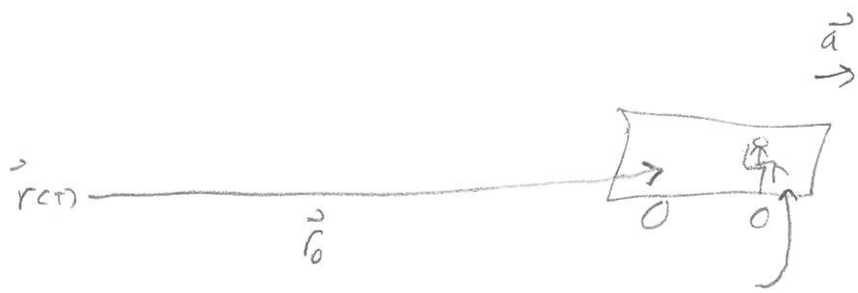
Ch. #10

Motion in a non-inertial reference frame

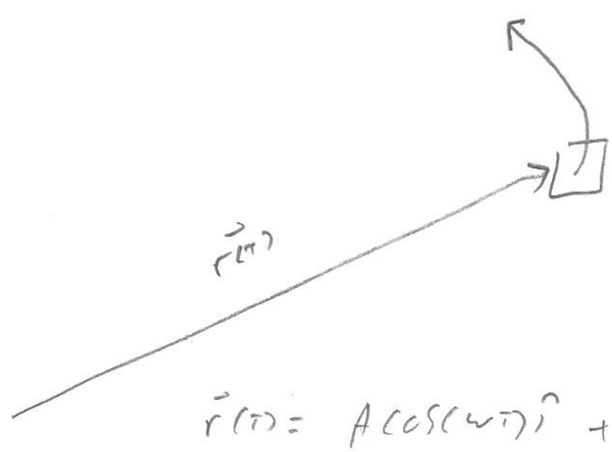
Step #1. draw $r(t)$ in terms of a fixed inertial frame

Ex.



$\vec{r}(t) = \vec{r}_0 + \frac{1}{2}at^2$ $\quad \ddot{r}(t) = a$ $\quad F = m\vec{a}$

Ex.



$\vec{r}(t) = A\cos(\omega t)\hat{i} + B\sin(\omega t)\hat{j}$

$\dot{\vec{r}}(t) = -A\omega\sin(\omega t)\hat{i} + B\omega\cos(\omega t)\hat{j}$

$|\dot{\vec{r}}| = (A^2\omega^2 + B^2\omega^2)^{1/2} = (A^2+B^2)^{1/2}\omega$

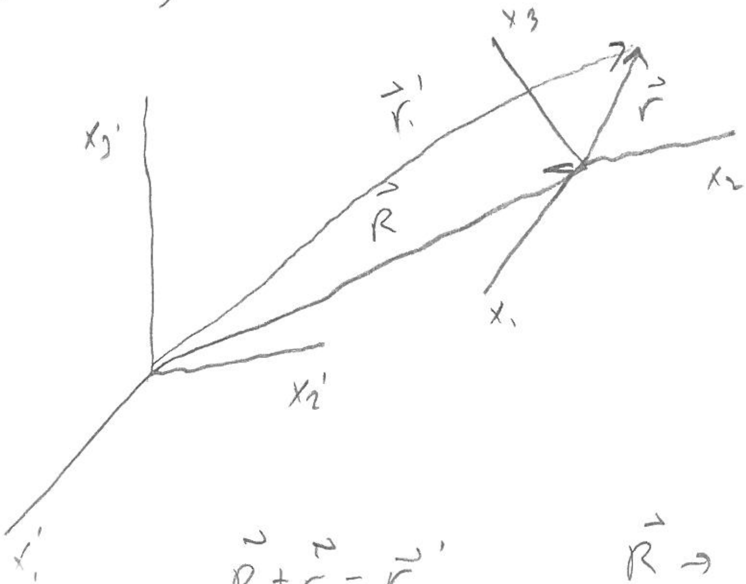
elipse else if $A=B$ circle

$\ddot{\vec{r}} = -A\omega^2\cos(\omega t)\hat{i} - B\omega^2\sin(\omega t)\hat{j}$

$|\ddot{\vec{r}}| = \sqrt{A^2+B^2}\,\omega^2 \rightarrow A=B \Rightarrow A\omega^2$

$\vec{F} = mA\omega^2$

More generally $\qquad x' \to$ inertial



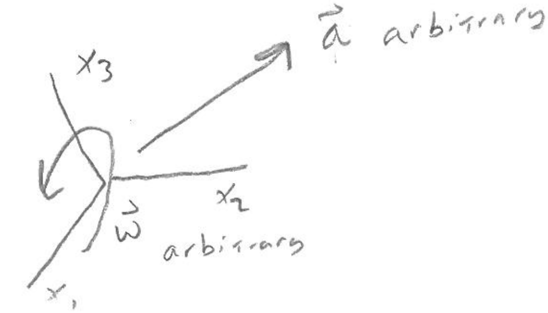
$\vec{R} + \vec{r} = \vec{r}\,'$

$\vec{R} \to$ Fixed origin to origin moving

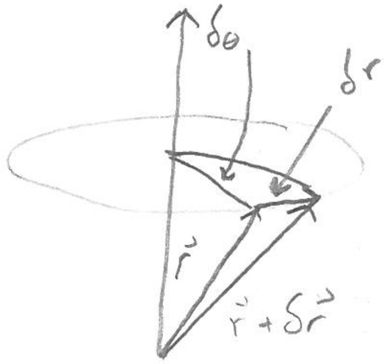
$\vec{r} \to$ wrt origin in Non inertial

$\vec{r}\,' \to$ Fixed to point in Non-Inertial

Now, allow



If non inertial is rotating then an infinitesimal



$\delta\theta$ corresponds to a δr

to $\vec{r}$ to $\vec{r} + d\vec{r}$

where $d\vec{r} = \delta\vec{\theta} \times \vec{r}$

or $\dfrac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$

1: CALL $\vec{r}' = \vec{R} + \vec{r} = \vec{Q}$ Then For $\vec{r} = \vec{0}$ or any Fixed ~~vs~~ non-inertial quantity

For $\vec{R}$ fixed $\left.\dfrac{d\vec{Q}}{dt}\right|_{\text{Fixed}} = \left.\dfrac{d\vec{Q}'}{dt}\right|_{\text{rotating}} + \vec{\omega} \times \vec{Q}'$

$\uparrow$

$\vec{V}_r(t)$

Note $\left.\dfrac{d\vec{\omega}}{dt}\right|_{\text{Fixed}} = \left(\dfrac{d\vec{\omega}}{dt}\right)_{\text{rotating}} + \underbrace{\vec{\omega} \times \vec{\omega}}_{0} = \dot{\vec{\omega}}$

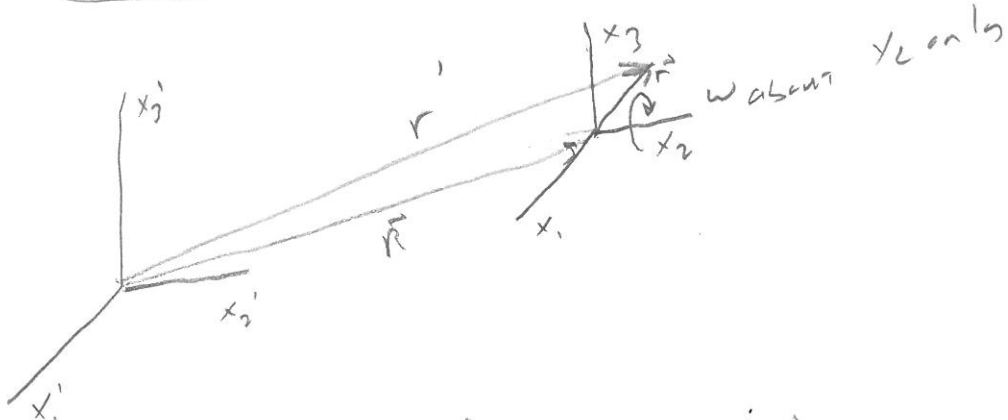
Now $\dfrac{d\vec{r}'}{dt} = \dfrac{d\vec{R}}{dt} + \left.\dfrac{d\vec{r}}{dt}\right| + \vec{\omega} \times \vec{r}$

Fixed

motion of origin of Non inertial Coordinate System

linear wrt noninertial

1: only rotation



$$\frac{d\vec{r}'}{dt} = \frac{d}{dt}\left(\vec{R} + \vec{r}\right) = \left.\frac{d\vec{r}}{dt}\right|_{\text{rotational}} = \vec{\omega} \times \vec{r}$$

2: R(t) + rotation

how about $\vec{R} = \vec{R}(t)$ so some linear velocity WRT inertial frame.

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{R}}{dt} + \left.\frac{d\vec{r}}{dt}\right|_{\text{rotational}} = \underset{\text{Translation}}{V(t)} + \underset{\text{rotation}}{\vec{\omega} \times \vec{r}}$$

3. $\vec{R}(t)$, linear $\vec{r}(t)$, rotational $r(t)$

how about $\vec{r}(t)$, $\vec{R}(t)$, and rotation

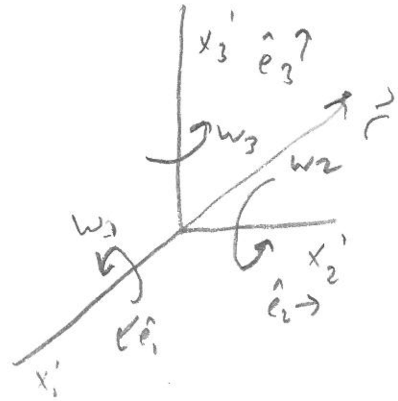
$$\frac{d\vec{r}'}{dt} = \frac{d\vec{R}}{dt} + \left.\frac{d\vec{r}}{dt}\right|_{\text{linear}} + \vec{\omega} \times \vec{r} = \vec{V}(t) + \vec{v}_r(t) + \vec{\omega} \times \vec{r}$$

rotational

relative to moving coordinate

relative to inertial frame

Physically what is $\vec{w} \times \vec{r}$?



Make $\vec{R}$ fixed

but non zero w_1, w_2, w_3

$$\frac{d\vec{r}\,'}{dT} = \frac{d\vec{r}}{dT} = \frac{d}{dT}\sum x_i' e_i = \frac{dx_i'}{dT}\hat{e}_i + x_i'\frac{d\hat{e}_i'}{dT} = \dot{\vec{r}}\,' + \sum x_i'\dot{\hat{e}}_i$$

hmm what is $\frac{d\hat{e}_i'}{dT}$?

$$\frac{d\hat{e}_1'}{dT} = w_3\hat{e}_2' - w_2\hat{e}_3'$$

do you see this?

$$\frac{d\hat{e}_2'}{dT} = w_1\hat{e}_3' - w_3\hat{e}_1'$$

$$\frac{d\hat{e}_3'}{dT} = w_2\hat{e}_1 - w_1\hat{e}_2$$

or $\dot{\hat{e}}_i' = w \times \hat{e}_i$ so $\frac{d\vec{r}\,'}{dT} = \dot{\vec{r}} + \sum \vec{w} \times \hat{e}_i x_i$

$$\frac{d\vec{r}\,'}{dT} = \dot{\vec{r}} + \vec{w} \times \vec{r}$$

How about

$$\left.\frac{d}{dT}\right|_{Fixed}\left(\dot{\vec{r}}'\right) = \left.\frac{d}{dT}\right|_{Fixed}\left[\vec{V}(T) + \vec{V}_r(T) + \vec{\omega}\times\vec{r}\right]$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \left.\frac{d}{dT}V_r(T)\right|_{Fixed} + \left.\frac{d}{dT}\right|_{Fixed}(\vec{\omega}\times\vec{r})$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \vec{a}_r(T) + \vec{\omega}\times\vec{V}_r + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\left.\frac{d}{dT}\right|_{Fixed}\vec{r}$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \vec{a}_r(T) + \vec{\omega}\times\vec{V}_r + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\left[V_r + \vec{\omega}\times\vec{r}\right]$$

$$\ddot{\vec{r}}' = \vec{A}(T) + \vec{a}_r(T) + \dot{\vec{\omega}}\times\vec{r} + 2\vec{\omega}\times\vec{V}_r + \vec{\omega}\times\vec{\omega}\times\vec{r}$$

acceleration of origin

acceleration of $\vec{r}$ wrt origin in non-inertial

$\dot{\vec{\omega}}$ Term

Coriolis effect

centripetal acceleration

Now $\vec{F}_{ext} = m\ddot{\vec{r}}'$ what we really want is $\vec{a}_r(T)$

what do <u>you</u> feel?

$$m\vec{a}_r(T) = \vec{F}_{ext} - m\vec{A}(T) - m\dot{\vec{\omega}}\times\vec{r} - 2m\vec{\omega}\times\vec{V}_r - m(\vec{\omega}\times(\vec{\omega}\times\vec{r}))$$

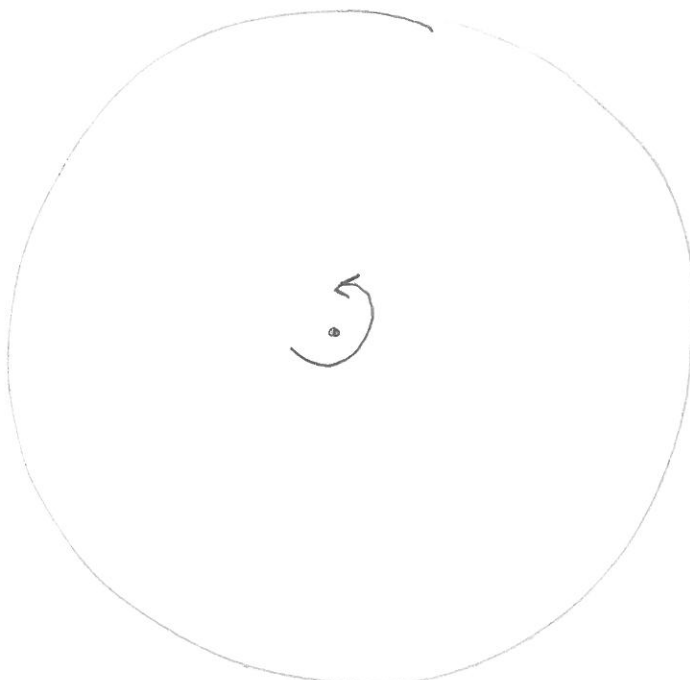
$\uparrow$ Force YOU FEEL!! $\qquad$ $\nwarrow \ddot{R}$

or $\vec{F}_{ext} = m\vec{a}_r(T)$ iff $\vec{\omega} = \vec{A} = 0$ iff your frame doesn't turn or accelerate $\rightarrow$ inertial

2 Important Forces

$$-2\vec{\omega}\times\vec{v} \quad \text{and} \quad -\vec{\omega}\times\vec{\omega}\times\vec{r}$$

1st 2D



$$\vec{\omega}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ v_x & v_y & 0 \end{vmatrix} = -\omega v_y \hat{i} + \omega v_x \hat{j}$$

$$-2\vec{\omega}\times\vec{v} = 2\omega\left[v_y \hat{i} - v_x \hat{j}\right]$$

V_x $\longrightarrow$ goes to $\downarrow$ $\qquad$ $\uparrow$ v_y goes to $\longrightarrow$

or

$$\vec{v} \nearrow = \searrow \vec{a} \quad \rightarrow \text{Turning Force}$$

$$\vec{\omega} \times \vec{\omega} \times \vec{r} = \vec{\omega} \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ x & y & 0 \end{vmatrix} = \vec{\omega} \times \left[-\omega y \hat{i} + \omega_x \hat{j} \right]$$

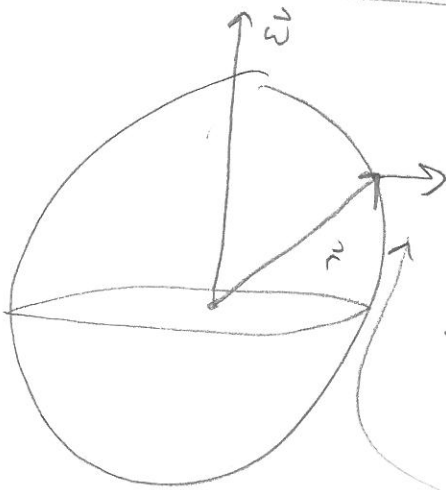
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ -\omega y & \omega_x & 0 \end{vmatrix} = -\omega^2 x \hat{i} - \omega^2 y \hat{j}$$

$$-\vec{\omega} \times \vec{\omega} \times \vec{r} = \omega^2 \left[x\hat{i} + y\hat{j} \right]$$

or $\quad \rightarrow + \uparrow = \nearrow \quad \underline{\underline{\text{OUT}}}$

In frame of person ON merry go round

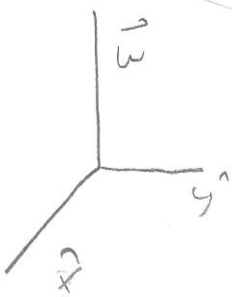
3 D



$\vec{\omega} \times \vec{r} = \otimes$

$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \uparrow \times \otimes$

$-\vec{\omega} \times (\vec{\omega} \times \vec{r}) =$ OUT

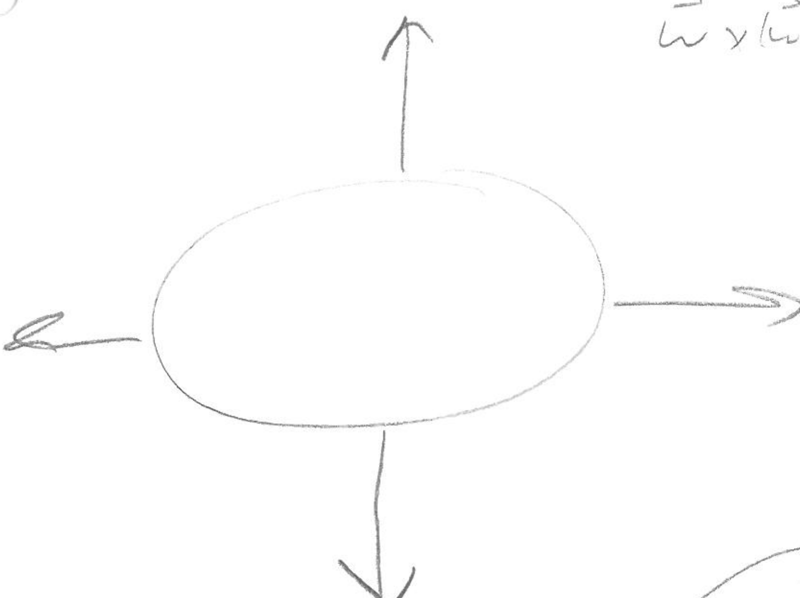


$\vec{\omega} \times \hat{y} = -|\omega| \hat{x}$

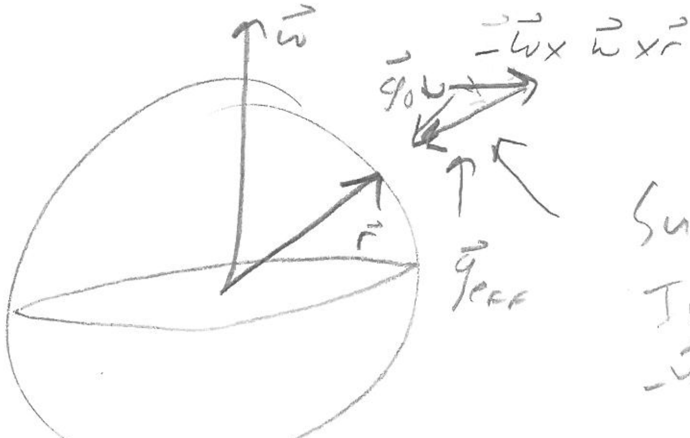
$\vec{\omega} \times (\vec{\omega} \times \hat{y}) = -\hat{y} |\omega|^2$

$\vec{\omega} \times \hat{x} = \hat{y} |\omega|$

$\vec{\omega} \times (\vec{\omega} \times \hat{y}) = -\hat{x} |\omega|^2$

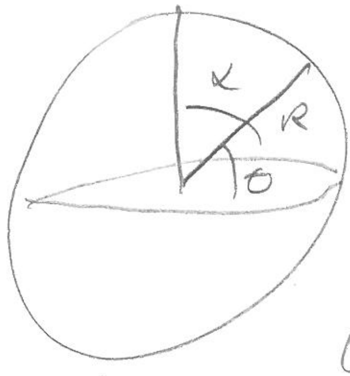


Locally



Super Inflated $-\vec{\omega} \times \vec{\omega} \times \vec{r}$

$\vec{g} = \vec{g}_0 - \vec{\omega} \times \vec{\omega} \times \vec{r}$



$\vec{\omega} \times (\vec{\omega} \times \vec{r}) =$

$\omega^2 r \, \mathrm{SIN}(\alpha)$

Calculate?

Where do you weigh less?

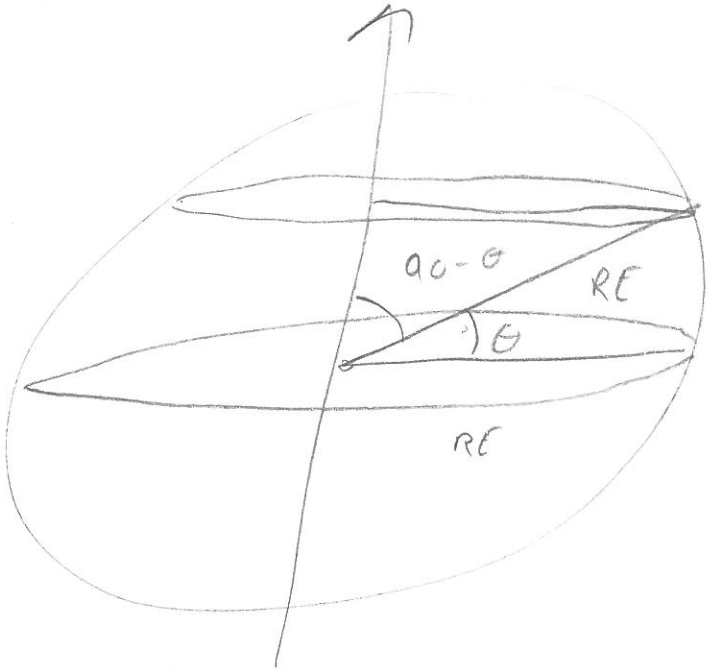
The Earth is BIGGER AT THE EQUATOR

Coriolis Vs Centripetal

$\frac{|\vec{\omega} \times \vec{v}|}{|\vec{\omega} \times \vec{\omega} \times \vec{r}|} = \ldots\ldots$ what $|\vec{V}|$?

Coriolis?

$V = R W$



$R < RE$

$\frac{R}{RE} = \mathrm{SIN}(90-\theta)$

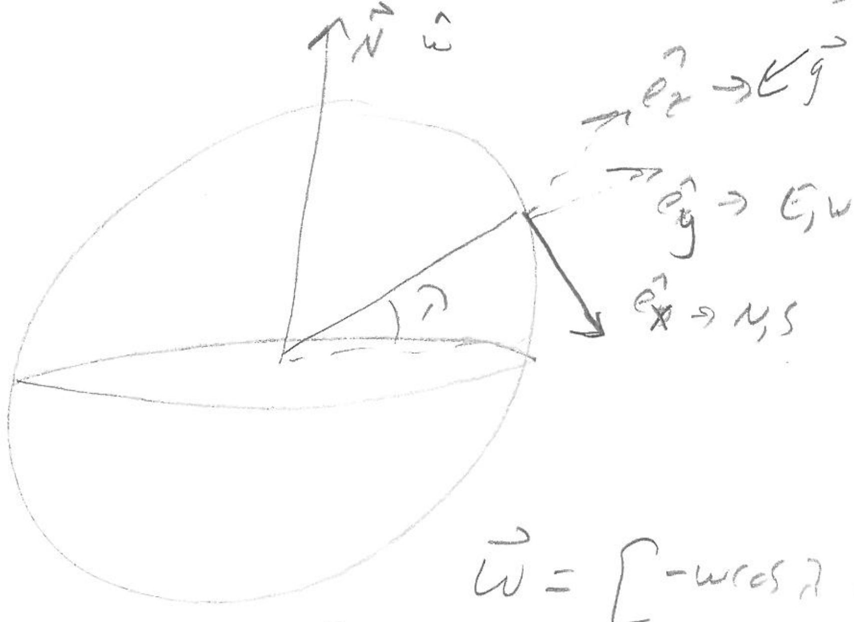
ΔV $\frac{1}{5}$ Second $\sim 5m$ $\frac{V_1}{V_2} = \frac{6317{,}000}{6317{,}005}$ = Small difference

Earth has rotated 8 ten thousandths of a meter

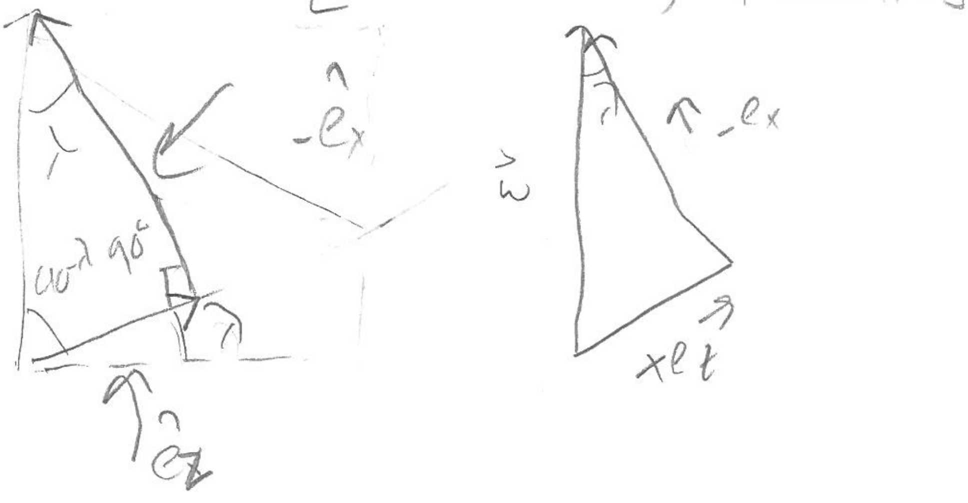
That's your deflection

If you jump due North

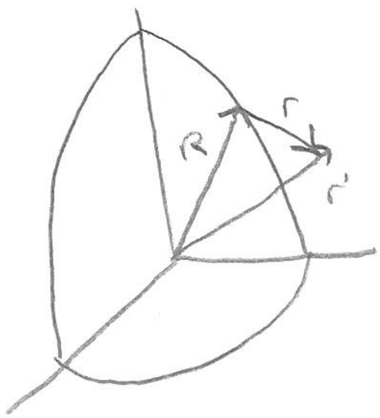
deflection → distance and velocity



$\vec{\omega} = [-\omega\cos\lambda\,\hat{i} + 0\hat{j} + \omega\sin\lambda\,\hat{k}]$



$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m(\vec{\omega}\times\vec{v}) - m\underbrace{\dot{\vec{\omega}}}_{0}\times\vec{r} - m\ddot{\vec{R}}$$



$$\left.\frac{d\dot{R}}{dt}\right|_{Fixed} = \underbrace{\left.\frac{d\dot{R}}{dt}\right|_{moving}}_{\downarrow\, 0} + \vec{\omega}\times\dot{R}$$

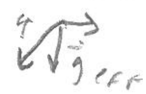
$$\dot{R} = \frac{dR}{dt} \qquad \left.\frac{d\vec{R}}{dt}\right|_{Fixed} = \underbrace{\frac{d\vec{R}}{dt}}_{\downarrow\, 0} + \vec{\omega}\times(\vec{\omega}\times R)$$

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m(\vec{\omega}\times(\vec{\omega}\times[\vec{r}+\vec{R}])) - 2m(\vec{\omega}\times\vec{v})$$

$\vec{R} \gg \vec{r}$

$$\boxed{-m(\vec{\omega}\times(\vec{\omega}\times\vec{R})) + m\vec{g}_0 = m\vec{g}_{eff}}$$

$-m(\vec{\omega}\times(\vec{\omega}\times R))$

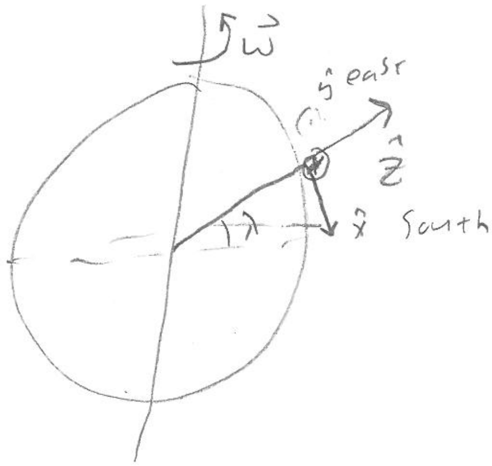


exagerated by 300
or so

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_{eff} - 2m\vec{\omega}\times\vec{v}$$

Example align | z with $\vec{g}_{eff}$

$$\vec{a}_r = \vec{g} - 2\vec{\omega}\times\vec{v}$$



$\omega_x = -\omega \cos\lambda$

$\omega_y = 0$

$\omega_z = \omega \sin\lambda$

Free Fall $\dot{z} = -gT$

$$m\ddot{r} = m\vec{g} - 2m\,\vec{\omega}\times\vec{v} = m\vec{g} - 2m\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & 0 & \omega_y \\ 0 & 0 & -gT \end{vmatrix}$$

$$m\ddot{r} = m\vec{g} - 2m\,[gT\omega_x\,\hat{j}]$$

$$m\ddot{r} = [0\hat{i} + 2mgT\omega\cos\lambda\,\hat{j} - m\vec{g}\,\hat{k}]$$

$m\ddot{x} = 0 \qquad \dot{x} = \dot{x}_0$

$m\ddot{y} = 2mgT\omega\cos\lambda \qquad \dot{y} = gT^2\omega\cos\lambda + \dot{y}_0$

$m\ddot{z} = -m\vec{g} \qquad \dot{z} = -g\vec{T} + \dot{z}_0$

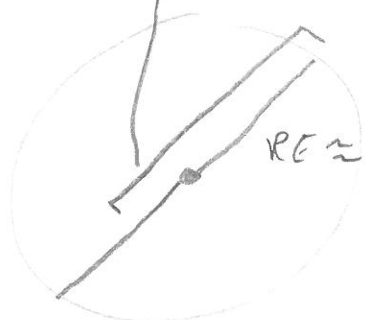
$\dot{y}$? $\quad |gT^2\omega\cos\lambda| = |gT|$

$T = \dfrac{1}{\omega\cos\lambda} \qquad T = \dfrac{24\cdot 3600}{2\pi\,[0.1]} = 13751$ seconds

silly 4 hours

$\frac{1}{10th}$ g ? $1.376 \cdot 10^3$ seconds $\to$ 22 minutes

$(y) = \frac{1}{2} g T^2 = \underbrace{9.26 \cdot 10^6 m}$ in perspective Silly



So $\ddot{x} = 0$ $\ddot{y} = 2\omega g T \cos\lambda$ $\ddot{z} = -g$

Take $\dot{x}_0 = 0 = \dot{y}_0 = \dot{z}_0$

Then $\vec{x}(T) = 0$

$y(T) \cong \frac{1}{3} \omega g T^3 \cos(\lambda)$

$z(T) = z(0) - \frac{1}{2} g T^2$ $z(0) = h$

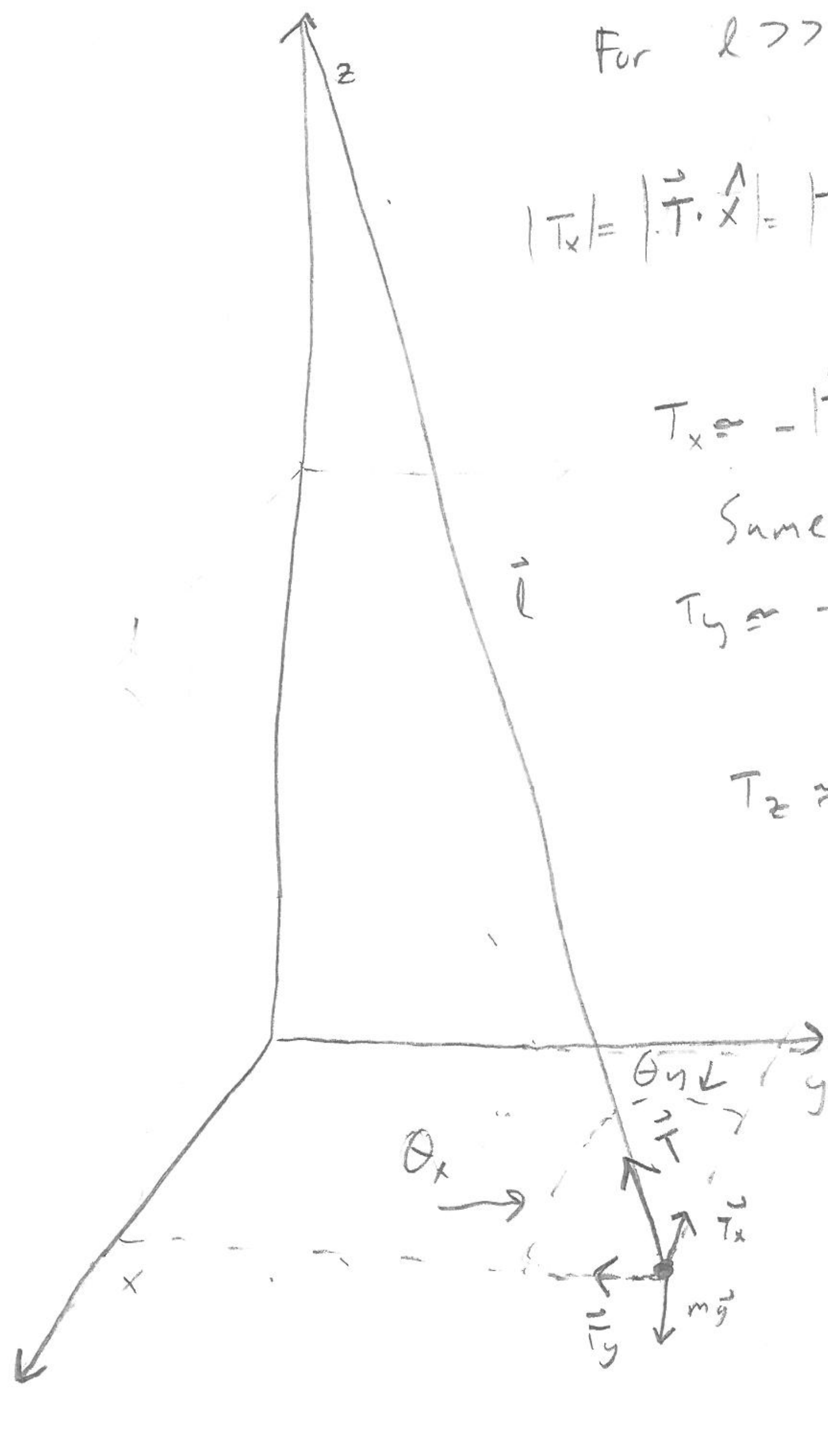
$T_{Fall} \sim \sqrt{\frac{2h}{g}} \to T^3 = \frac{2h}{g}\sqrt{\frac{2h}{g}}$

$y(T_{Fall}) \approx \sqrt{\frac{8h^3}{g}} \left(\frac{1}{3} \omega \cos(\lambda)\right)$

$z(0) = 100m$ 45° latitude 1.55 cm deflection

Foucalt Pendulum

$$\vec{a}_r = \vec{g} + \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v}_r$$



For $l \gg x, y$ displacements

$$|T_x| = |\vec{T} \cdot \hat{x}| = |T| \cos\theta_x \approx T\theta_x \approx T\frac{x}{l}$$

$$T_x \approx -|\vec{T}|\frac{x}{l}$$

Same l

$$T_y \approx -|\vec{T}|\frac{y}{l}$$

$$T_z \approx |\vec{T}|\frac{z}{l} \approx |\vec{T}|$$

Then Say $\dot{z} < \dot{x}$ and $\dot{y}$, $\dot{z} \approx 0$ $z \approx$ constant

$$\ddot{x} \approx -\frac{T}{m}\frac{x}{e} + 2\dot{y}\omega \sin\lambda$$

$$\ddot{y} \approx -\frac{T}{m}\frac{y}{e} - 2\dot{x}\omega \sin\lambda$$

ONLY CONSIDER OSCILLATIONS IN THE XY PLANE

Let $\vec{T} \approx m\vec{g}$

$$\ddot{x} = -\overset{\alpha^2}{\left(\frac{g}{e}\right)} x + 2\dot{y}\underbrace{\omega \sin\lambda}_{\omega_z} = -\alpha^2 x + 2\dot{y}\omega_z$$

$$\ddot{y} = -\alpha^2 y - 2\dot{x}\omega_z$$

$$\ddot{x} + \alpha^2 x = 2\dot{y}\omega_z \quad \text{and} \quad \ddot{y} + \alpha^2 y = -2\dot{x}\omega_z$$

$\eta = x + iy$ $\ddot{\eta} = \ddot{x} + i\ddot{y}$

$$\begin{array}{l} \ddot{x} + \alpha^2 x = 2\dot{y}\omega_z \\ i\ddot{y} + \alpha^2 i y = -2i\dot{x}\omega_z \end{array} \rightarrow \ddot{\eta} + \alpha^2\eta = 2\omega_z[\dot{y} - i\dot{x}]$$

$$\ddot{\eta} + \alpha^2\eta = 2i\omega_z\left[\frac{\dot{y}}{i} - \dot{x}\right]$$

$$\ddot{\eta} + \alpha^2 \eta = -2i\omega_z \left[i\dot{y} + \dot{x} \right]$$

Now $\alpha^2 = \frac{g}{l}$, $\omega_z = \omega \sin \lambda$

$$\ddot{\eta} + \alpha^2 \eta + 2i\omega_z \dot{\eta} = 0$$

$$\eta \propto e^{\lambda T} \qquad a = 1 \quad b = 2i\omega_z \quad c = \alpha^2$$

$$\lambda = -2i\omega_z \pm \sqrt{-4\omega^2 - 4\alpha^2} \left(\frac{1}{2}\right)$$

$$\lambda = -i\omega_z \pm \sqrt{(\omega_z^2 + \alpha^2)(-1)}$$

$$\eta(T) = A_{1,2} e^{-i\omega_z T} e^{\pm \sqrt{-\omega_z^2 - \alpha^2}\, T}$$

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if $\omega_z = 0$ $\quad \eta(T) = A_{1,2} e^{\pm i\alpha T} = \underbrace{A_{1,2} e^{\pm \sqrt{\frac{g}{l}}\, T}}$

Simple Pendulum

$$\sqrt{\frac{g}{l}}? \qquad \sqrt{\frac{10}{10{,}000}} = 1, .1 = \omega_{\text{Pendulum}} >>> \omega_{z_{\text{earth}}}$$

So $\quad \alpha >> \omega_z$

and $\quad e^{-i\omega_z T}\left[Ae^{i\alpha T} + Be^{-i\alpha T}\right] = \eta(T)$

$$\eta(T) = e^{-i\omega_z T}\eta'(T) = e^{-i\omega_z T}\left[x'(T) + iy'(T)\right]$$

$$\eta(T) = \left[x'(T) + iy'(T)\right]\left[\cos(\omega_z T) - i\sin(\omega_z T)\right]$$

$$x(T) + iy(T) = \left[x'\cos - ix'\sin + iy'\cos + y'\sin\right]$$

or $\quad x = x'\cos(\omega_z T) + y'\sin(\omega_z)T$

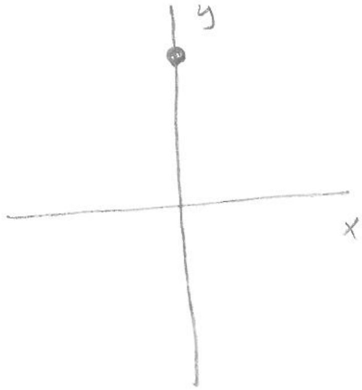
$$y = -x'\sin(\omega_z T) + y'\cos(\omega_z T)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos(\omega_z T) & \sin(\omega_z T) \\ -\sin(\omega_z T) & \cos(\omega_z T) \end{pmatrix}}_{\text{rotation matrix}} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$\theta = \omega_z T \quad$ Period $\omega_z \quad$ Example

Also $q(T) \approx e^{-i\omega_z T}(Ae^{i\alpha T} + Be^{-i\alpha T})$

pendulum



$x_0 = 0 \quad \dot{x}_0 = 0$

$y_0 = A \quad \dot{y}_0 = 0$

but $A = A_1 + iA_2 \qquad B = B_1 + iB_2$

$$q(T) = (A_1 + iA_2)\left[e^{i(\alpha-\omega)T}\right] + [B_1 + iB_2]e^{-i(\alpha+\omega)T}$$

$$q(T) = (A_1 + iA_2)\left[\cos[(\alpha-\omega)T] + i\sin[(\alpha-\omega)T]\right]$$
$$+ (B_1 + iB_2)\left[\cos[(\alpha+\omega)T] - i\sin[(\alpha+\omega)T]\right]$$

$x \to$ Real $\overset{1}{A_1\cos(\alpha-\omega)T} - \overset{2}{A_2\sin(\alpha-\omega)T} + \overset{3}{B_1\cos(\alpha+\omega)T} + \overset{4}{B_2\sin(\alpha+\omega)T}$

$y \to$ Im $\overset{5}{A_2\cos(\alpha-\omega)T} + \overset{6}{A_1\sin(\alpha-\omega)T} - \overset{7}{B_1\sin(\alpha+\omega)T} + \overset{8}{B_2\cos(\alpha+\omega)T}$

$X(0) = 0 \qquad A_1 + B_1 = 0 \qquad \underline{A_1 = -B_1}$

$\dot{X}(0) = 0 \qquad -A_2(\alpha-\omega) + B_2(\alpha+\omega) = 0$

$A_2 = B_2\dfrac{(\alpha+\omega)}{(\alpha-\omega)} \qquad \alpha \sim \sqrt{\dfrac{g}{L}} \sim \sqrt{\dfrac{10}{100}} \sim \dfrac{1}{\sqrt{10}}$

$\gg \omega$

$\underline{A_2 \approx B_2}$

$y(0) = A \qquad A_2 + B_2 = A \qquad A_2 \approx \frac{A}{2}$

$\dot{y}(0) = 0 \qquad (\alpha - \omega) A_1 - (\alpha + \omega) B_1 = 0$

$(\alpha - \omega) A_1 + (\alpha + \omega) A_1 = 0$

$A_1 \sim 0 \quad \text{So} \quad B_1 \sim 0$

$$X(T) = -\frac{A}{2} \sin(\alpha - \omega)T + \frac{A}{2} \sin(\alpha + \omega)T$$

$$y(T) = \frac{A}{2} \cos(\alpha - \omega)T + \frac{A}{2} \cos(\alpha + \omega)T$$

$$X(T) = \frac{A}{2}\left[S\alpha C\omega + S\omega C\alpha - (S\alpha C\omega - S\omega C\alpha) \right]$$

$$X(T) = A \sin(\omega_2 T) \cos(\alpha T)$$

Similarly

$$y(T) = \frac{A}{2}\left[C\alpha C\omega - \sin\alpha \sin\omega + C\alpha C\omega + S\alpha S\omega \right]$$

$$y(T) = A \cos(\alpha_2 T) \cos(\omega_2 T)$$

$$\vec{r} = X(T)\hat{i} + y(T)\hat{j} = A \cos\left(\frac{\sqrt{5}}{\sqrt{L}} T\right) \left[\sin(\omega_2 T)\hat{i} + \cos(\omega_2 T)\hat{j} \right]$$

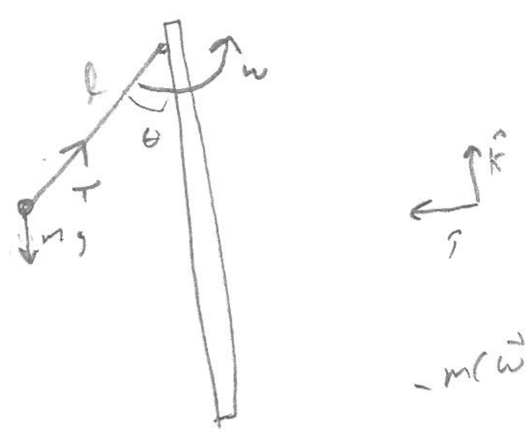
$\vec r = A\cos\sqrt{\frac{g}{l}}t\ \hat n$ $\uparrow\downarrow$ in $\omega = \sqrt{\frac{g}{l}} \to T_p = \frac{2\pi}{\omega} \to$ check

$\hat n = \sin(\omega_z t)\hat i + \cos(\omega_z t)\hat j$

$t = 0$ $\hat n = \hat j$ $T_\omega = \frac{2\pi}{\omega_z}$ $\uparrow \ \rightarrow \ \downarrow \ \leftarrow$

Northern hemisphere, opposite for $\lambda < 0$ Southern

Last Problem then Ch. #11



$-m(\vec\omega\times\vec\omega\times\vec r) = -m(\omega\hat k\times\omega\hat k\times[l\sin\theta\hat i - l\cos\theta\hat k])$

$= -m(\vec\omega\hat k\times\omega l\sin\theta\hat j)$

$= m\omega^2 l\hat i \to$ out

$\vec T + m\vec g - m\vec\omega\times\vec\omega\times\vec r = 0$

$-mg\hat k + m\omega^2 l\sin\theta\hat i + T\cos\theta\hat k - T\sin\theta\hat i = 0$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 l\sin\theta$

$mg\tan\theta = m\omega^2 l\sin\theta$ $T = m\omega^2 l$, $\theta = \cos^{-1}\left(\frac{g}{\omega^2 l}\right)$