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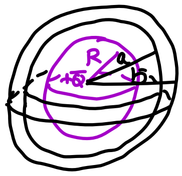
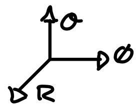
Dr. Middleton
PHYS 311 HW 7

Problem 1

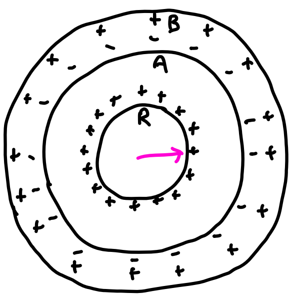


origin at center



a.) $\sigma = -\varepsilon_0 \frac{\partial V}{\partial n}$ $\quad \partial n = \partial r$ $\quad$ normal vector : $\hat{r}$

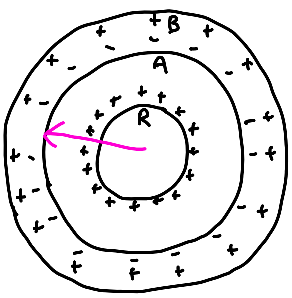
i.) @ R



$\sigma \equiv \frac{\text{Charge}}{\text{Area}}$

$\sigma_R = \frac{Q}{A_R} = \frac{Q}{4\pi R^2} \longrightarrow \boxed{\sigma_R = \frac{Q}{4\pi R^2}}$

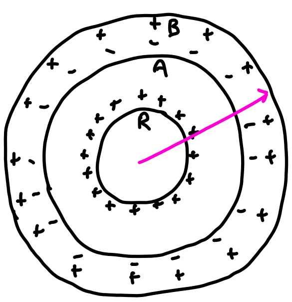
ii.) @ A



$\sigma \equiv \frac{\text{Charge}}{\text{Area}}$

$\sigma_A = \frac{Q_A}{A_A} \quad \therefore \quad \sigma_A = -\frac{Q}{4\pi A^2} \longrightarrow \boxed{\sigma_A = -\frac{Q}{4\pi A^2}}$

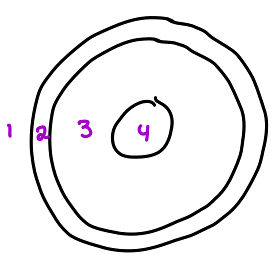
iii.) @ B



$\sigma \equiv \frac{\text{Charge}}{\text{Area}}$

$\sigma_B = \frac{Q_B}{A_B} = \frac{Q}{4\pi B^2} \quad \therefore \longrightarrow \boxed{\sigma_B = \frac{Q}{4\pi B^2}}$

b) $V(\vec{r}) = -\int_{\mathcal{O}}^{0} \vec{E}\cdot d\vec{\ell} = -\int_{\infty}^{B} \vec{E}_1\cdot d\vec{\ell} - \int_{B}^{A} \vec{E}_2\cdot d\vec{\ell} - \int_{A}^{R} \vec{E}_3\cdot d\vec{\ell} - \int_{R}^{0} \vec{E}_4\cdot d\vec{\ell}$



$\vec{E}_2$: There is no Electric field inside a conductor $\therefore$ $\vec{E}_2 = 0$

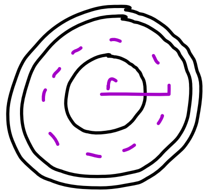
$\vec{E}_4$: There is no Electric field inside the sphere $\therefore$ $\vec{E}_4 = 0$

$\vec{E}_3$: $\oint \vec{E}_3 \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$, $Q_{ENC} = Q$

$\vec{E}_1$: $\oint \vec{E}_1 \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$, $Q_{ENC} = Q$

Problem 1 Continued

$\vec{E}_3$:



$Q_{Enc} = Q$, E constant @ given radius , $d\vec{a} = r^2 \sin\theta \, d\theta \, d\phi$

$$\oint \vec{E} \cdot d\vec{a} = E \cdot r^2 \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi = E \cdot r^2 \cdot 2\pi \cdot 2 = E \cdot 4\pi r^2$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{Enc}}{\varepsilon_0} \quad : \quad E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \therefore \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{r}$$

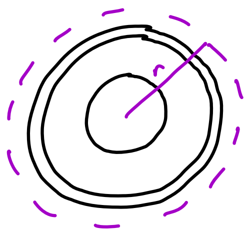
$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$ $\longleftarrow$ point charge $\quad d\vec{l} = dr\,\hat{r}$

$$-\int_\infty^B \vec{E}_1 \cdot d\vec{l} - \int_B^A \cancel{\vec{E}_2}^{\,0} \cdot d\vec{l} - \int_A^R \vec{E}_3 \cdot d\vec{l} - \int_R^0 \cancel{\vec{E}_4}^{\,0} \cdot d\vec{l}$$

$$V(\vec{r}) = -\int_R^0 \vec{E} \cdot d\vec{l} = -\frac{1}{4\pi\varepsilon_0} \cdot Q \int_A^R \frac{1}{r^2} \, dr = \frac{-1}{4\pi\varepsilon_0} \cdot Q \left[-\frac{1}{r} \right]_A^R = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r} \Big|_A^R$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot Q \left[\frac{1}{R} - \frac{1}{A} \right] = V(\vec{r}_3)$$

$\vec{E}_1$:



$Q_{Enc} = Q$, $\oint \vec{E} \cdot d\vec{a} = E \cdot r^2 \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$

$\vec{E} = E\,\hat{r}$ $\qquad = E \cdot r^2 \cdot 2\pi \cdot 2 = E \cdot 4\pi r^2$

$d\vec{a} = r^2 \sin\theta \, d\theta \, d\phi$

$$\oint \vec{E} \cdot d\vec{a} = E \cdot 4\pi r^2 \quad : \quad \vec{E} \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \hat{r} \quad : \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{r}$$

$d\vec{l} = dr\,\hat{r}$

$$-\int_\infty^B \vec{E}_1 \cdot d\vec{l} = \frac{-1}{4\pi\varepsilon_0} \cdot Q \int_\infty^B \frac{1}{r^2} \, dr = \frac{-1}{4\pi\varepsilon_0} \cdot Q \left[-\frac{1}{r} \right]_\infty^B = \frac{-1}{4\pi\varepsilon_0} Q \left[-\frac{1}{B} - \left(\cancel{-\frac{1}{\infty}}^{\,0} \right) \right]$$

$$-\int_\infty^B \vec{E}_1 \cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{B} = V(\vec{r}_1)$$

$$V_{Tot} = V(\vec{r}_1) + V(\vec{r}_3) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{B} + \frac{1}{4\pi\varepsilon_0} \cdot Q \left[\frac{1}{R} - \frac{1}{A} \right] = \frac{1}{4\pi\varepsilon_0} \cdot Q \left[\frac{1}{B} + \frac{1}{R} - \frac{1}{A} \right]$$

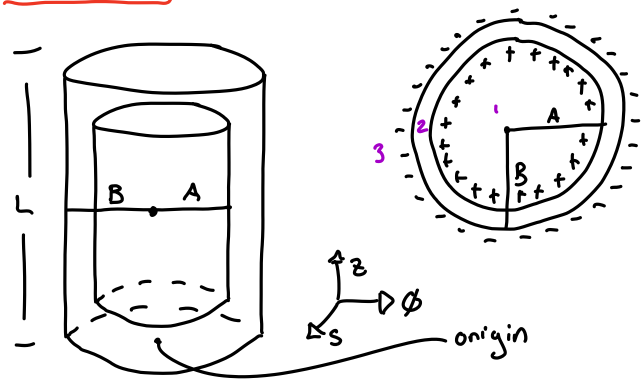
$$\boxed{V_{Tot} = \frac{1}{4\pi\varepsilon_0} \cdot Q \left[\frac{1}{B} + \frac{1}{R} - \frac{1}{A} \right]}$$

c.)

a*) $\sigma_R = \frac{Q}{4\pi R^2}$, $\sigma_A = -\frac{Q}{4\pi A^2}$, $\sigma_B = 0$

b*) $V = \frac{1}{4\pi\varepsilon_0} \cdot Q \left[\frac{1}{R} - \frac{1}{A} \right]$

Problem 2)



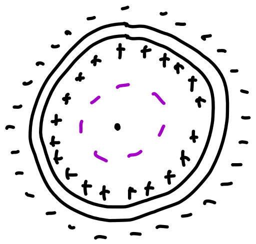
$C = \frac{Q}{V}$

No flux through end caps.

$\vec{E} \perp d\vec{a}$

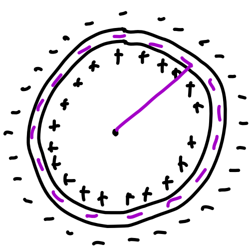
Find $\vec{E}$, Then V

Region 1 $s < a$, there is no charge enclosed in this Gaussian surface



$$\oint E \cdot d\vec{a} = \frac{0}{\varepsilon_0} \quad \therefore \quad \vec{E} = 0$$

Region 2 $a < s < b$



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}, \quad Q_{ENC} = Q, \quad d\vec{a} = s\, d\phi\, dz\, \hat{s}$$

$$\vec{E} = E\hat{s}$$

$$\oint \vec{E} \cdot d\vec{a} = E \cdot s \int_0^L \int_0^{2\pi} d\phi\, dz = E \cdot 2\pi s L$$

$$E \cdot 2\pi s L = \frac{Q}{\varepsilon_0} \quad \therefore \quad \vec{E} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{Q}{sL} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{sL}\, \hat{s}$$

$$V(\vec{r}) = -\int_{O}^{a} \vec{E} \cdot d\vec{l} = -\int_b^a \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{sL}\, ds = -\frac{1}{4\pi\varepsilon_0} \frac{2Q}{L} \int_b^a \frac{1}{s}\, ds$$

$d\vec{l} = ds\,\hat{s}$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{L} \cdot \ln(s) \Big|_b^a = -\frac{1}{4\pi\varepsilon_0} \frac{2Q}{L} \left[\ln(a) - \ln(b) \right]$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{L} \left[\ln(b) - \ln(a) \right] \qquad C \equiv \frac{Q}{V}$$

$$C = \frac{Q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{L} \left[\ln(b) - \ln(a) \right]} = \frac{1}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{2}{L} \left[\ln(b/a) \right]} = \frac{4\pi\varepsilon_0 \cdot L}{2\ln(b/a)}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$