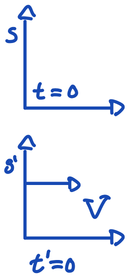
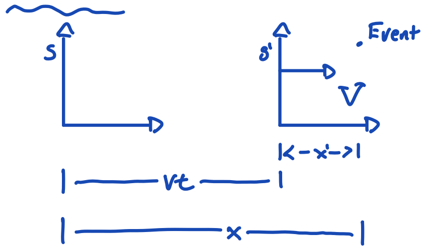
Lorentz Transformation





According to S....

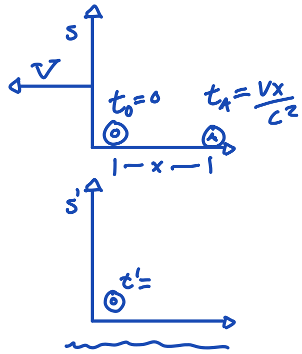
· According to observers in S' measure x' observers in S measure it to be $x'\sqrt{1-v^2/c^2}$

So

$$x = x'\sqrt{1-v^2/c^2} + vt$$

$$x' = \frac{x - vt}{\sqrt{1-v^2/c^2}} \qquad y' = y \qquad z' = z$$

Now place clock A @ rest in S @ (x, y, z)....



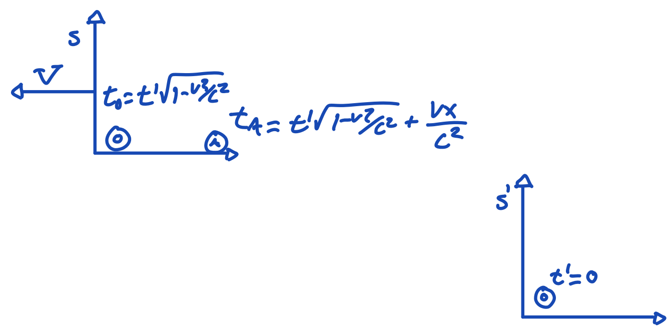
According to S'....

· clock A leads the $t_0 = 0$ by $t_A = \frac{vx}{c^2}$

Later....

· The unprimed clock @ x=0 runs slow so

$$t_0 = t'\sqrt{1-v^2/c^2}$$



$$t_0 = t'\sqrt{1-v^2/c^2} \qquad t' = \frac{t - vx/c^2}{\sqrt{1-v^2/c^2}}$$

$$t = t'\sqrt{1-v^2/c^2} + vx/c^2$$

The Lorentz Transformation

$$\begin{bmatrix} t' = \gamma(t - vx/c^2) \\ x' = \gamma(x - Vt) \\ y' = y \\ z' = z \end{bmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} > 1$$

Inverse Lorentz Transformations

$$\begin{bmatrix} t = \gamma(t' + vx/c^2) \\ x = \gamma(x' + Vt) \\ y = y' \\ z = z' \end{bmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

Notice: $\Delta x' = x_2' - x_1' = \gamma(\Delta x - V\Delta t) \therefore \boxed{\Delta x = v\Delta t}$

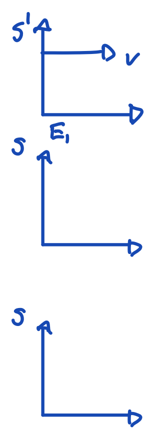
$\Delta t' = \gamma(\Delta t - V/c^2 \Delta x) = \gamma(\Delta t - V/c^2 \cdot v\Delta t) =$

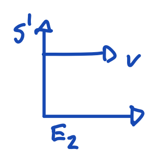
$\gamma(1 - v^2/c^2)\Delta t = \Delta t\sqrt{1-v^2/c^2}$

$\boxed{\Delta t' = \Delta t\sqrt{1-v^2/c^2}}$

↳ moving clocks tic slow

Time Dilation, Length Contraction, Leading clocks lag

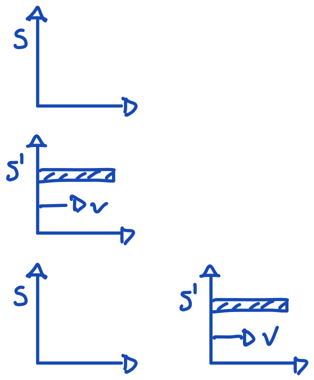




$$x_2' - x_1' = \gamma(x_2 - Vt_2) - \gamma(x_1 - Vt_1)$$
$$= \gamma(x_2 - x_1) - \gamma V(t_2 - t_1)$$
$$\Delta x' = \gamma(\Delta x - V\Delta t)$$

$$t_2' - t_1' = \gamma(t_2 - Vx/c^2) - \gamma(t_1 - vx/c^2)$$
$$= \gamma(t_2 - t_1) - \frac{\gamma V}{c^2}(x_2 - x_1)$$

$$\Delta t' = \gamma(\Delta t - \frac{V}{c^2}\Delta x)$$



The rest length is...

$D = x_2' - x_1' = \Delta x' = \gamma(\Delta x - V\Delta t)$

for S to measure the length of the stick, two observers measure $x_2 : x_1$ @ $t_2 = t_1$ $\therefore$ $\Delta t = 0$

$$\left[D = \Delta x' = \gamma \Delta x \therefore \Delta x = \frac{D}{\gamma} \quad \therefore \quad \Delta x = D\sqrt{1 - v^2/c^2} \right]$$ - Moving objects are Contracted

using same pics above

$\Delta t' = \gamma\left(\Delta t - \frac{V\Delta x}{c^2}\right) \qquad \Delta t = 0$

$t_2' - t_1' = -\frac{\gamma V \Delta x}{c^2} = \frac{-VD}{c^2}$

$$\left[t_2' = t_1' - \frac{VD}{c^2} \right]$$

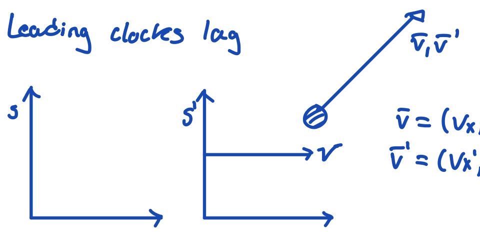
Leading clocks lag

L.T

$$\left[\begin{aligned} x' &= \gamma(x - Vt) \\ t' &= \gamma(t - Vx/c^2) \\ y' &= y \\ z' &= z \end{aligned}\right]$$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Now



$\bar{v} = (v_x, v_y, v_z)$ - velocity of particle RT S

$\bar{v}' = (v_x', v_y', v_z')$ - . . . . . . S'

$v_x' = \frac{dx'}{dt'} = \frac{dx'}{dt}\frac{dt}{dt'} = \frac{dx'/dt}{dt'/dt}$

$\frac{dx'}{dt} = \frac{d}{dt}\gamma(x - vt) = \gamma\left(\frac{dx}{dt} - V\right) = \gamma(v_x - v)$

$\frac{dt'}{dt} = \gamma\left(1 - \frac{V}{c^2}\frac{dx}{dt}\right) = \gamma\left(1 - \frac{Vv_x}{c^2}\right)$

$v_y' = \frac{dy'}{dt} = \frac{dy'}{dt}\frac{dt}{dt'} = \frac{dy'/dt}{dt'/dt} = \frac{v_y}{\gamma(1 - v_x V/c^2)}$

$\frac{dy'}{dt} = \frac{dy}{dt} = v_y$

Notice: (i) v_x appears in all equations for transformations

(ii) In the limit $c \to \infty$

$$\left[\begin{aligned} v_x' &= v_x - V \\ v_y' &= v_y \\ v_z' &= v_z \end{aligned}\right] \longrightarrow \text{Galilean Transformation}$$

L.V.T

$$\left[\begin{aligned} v_x' &= \frac{v_x - V}{1 - v_x V/c^2} \\ v_y' &= \frac{v_y\sqrt{1 - V^2/c^2}}{(1 - \frac{v_x V}{c^2})} \\ v_z' &= \frac{v_x\sqrt{1 - V^2/c^2}}{(1 - \frac{v_x V}{c^2})} \end{aligned}\right]$$

I.L.V.T

$$\left[\begin{aligned} v_x &= \frac{v_x + V}{1 - v_x V/c^2} \\ v_y &= \frac{v_y\sqrt{1 - V^2/c^2}}{(1 + \frac{v_x' V}{c^2})} \\ v_z &= \frac{v_x\sqrt{1 - V^2/c^2}}{(1 + \frac{v_x' V}{c^2})} \end{aligned}\right]$$

Ex $\quad v_x = c, \quad v_y = 0, \quad v_z = 0$ "Light shined moving to right"

$v_x' = \frac{c - V}{1 - cV/c^2} = \frac{c(1 - V/c)}{(1 - V/c)} = c$

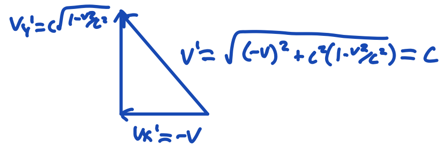
$v_y' = 0$

$v_z' = 0$

"Now moving in y direction" - Light beam

According to S Frame

According to S' Frame



$V_x = 0 \quad V_y = C \quad V_z = 0$

$V_x' = \dfrac{0 - V}{1} = -V$

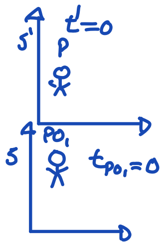
$V_y' = C\sqrt{1 - V^2/c^2}$

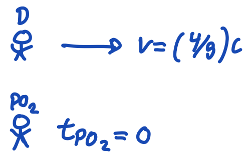
$V_z' = 0$

S.P.2

$D_B = 15.0m$

$V = (4/5)c$





E_1 - P passes P_{O_1} $\quad \Delta t = t_2 - t_1$

E_2 - D passes P_{O_2} $\quad \Delta t = 0$

$\Delta x' = \gamma(\Delta x - V\Delta t) = \gamma \Delta x$

$\Delta t' = \gamma(\Delta t - \frac{V\Delta x}{c^2}) = -\frac{\gamma V \Delta x}{c^2}$

a.) $D_B = \gamma \Delta x$

$\therefore \Delta x = \dfrac{1}{\gamma} D_B = D_B\sqrt{1 - V^2/c^2}$

$= 15.0m(3/5) = 9.0m$

$\Delta x = 9.0\ m$

b.) $\Delta t' = \dfrac{-VD_B}{c^2}$

$\Delta t' = t_2' - t_1' = -12.0 m/c = \dfrac{-(4/5)c(15.0m)}{c^2} = -12.0\ m/c$

$\Delta t' = -12.0\ m/c$

S.P.3

Passenger moving at $V = (3/5)c$

$V = (4/5)c$

$V_x' = (3/5)c$

$V_x = ?$

$V_x = \dfrac{V_x' + V}{1 + V_x'V/c^2} = \dfrac{\frac{3}{5}c + \frac{4}{5}c}{1 + \frac{(3/5)(4/5)}{c^2}} = \dfrac{35}{37}c$