1.) $\int_0^1 \int_{x^2}^{x} (8+16y)\,dy\,dx$

$\int_0^1 \left[\int_{x^2}^{x} (8+16y)\,dy\right] dx$

$\int_0^1 \left[\, 8y + 8y^2 \right]_{x^2}^{x} dx$

$\boxed{\dfrac{12}{5}}$

$\int_0^1 \left[ (8x + 8(x)^2) - (8(x^2) + 8(x^2)^2) \right] dx$

$\int_0^1 \left[ 8x + 8x^2 - 8x^2 - 8x^4 \right] dx$

$\int_0^1 \left[ 8x - 8x^4 \right] dx$

$4x^2 - \dfrac{8x^5}{5} \Big|_0^1 \quad 4(1)^2 - \dfrac{8(1)^5}{5} = 4 - \dfrac{8}{5} = \dfrac{12}{5}$

2.) $\iint_D 7y^2\,dA$, $D = \{(x,y) \mid -1 \le y \le 1, -y-2 \le x \le y\}$

$\int_{-1}^{1} \int_{-y-2}^{y} 7y^2\,dx\,dy$

$\boxed{\dfrac{28}{3}}$

$\int_{-1}^{1} \left[ 7xy^2 \right]_{-y-2}^{y} dy$

$7(y)y^2 - 7(-y-2)y^2$

$7y^3 - 7(-y^3 - 2y^2)$

$7y^3 + 7y^3 + 14y^2$

$\int_{-1}^{1} 14y^3 + 14y^2\,dy$

$\dfrac{7}{2}y^4 + \dfrac{14}{3}y^3 \Big|_{-1}^{1}$

$\dfrac{7}{2}(1)^4 + \dfrac{14}{3}(1) - \left(\dfrac{7}{2}(-1)^4 + \dfrac{14}{3}(-1)^3\right)$

$\dfrac{7}{2} + \dfrac{14}{3} - \dfrac{7}{2} + \dfrac{14}{3}$

$\dfrac{28}{3}$

3.) $\iint_D \dfrac{6y}{6x^5+1}\,dA \qquad D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le x^2\}$

$\int_{x=0}^{x=1} \int_{0}^{x^2} \dfrac{6y}{6x^5+1}\,dy\,dx = \boxed{\dfrac{1}{10}(\ln(7) - \ln(1))}$

$\int_0^1 \dfrac{1}{6x^5+1} \int_0^{x^2} 6y\,dy\,dx$

$\int_0^1 \dfrac{1}{6x^5+1} \left[ 3y^2 \right]_0^{x^2} dx$

$\int_0^1 \dfrac{1}{6x^5+1} \left[ 3(x^2)^2 - 3(0)^2 \right] dx$

$\int_0^1 \dfrac{1}{6x^5+1} \left[ 3x^4 \right] dx$

$3\int_0^1 \dfrac{x^4}{6x^5+1}$

$\int_0^1 \dfrac{3x^4}{6x^5+1}\,dx$

$u = 6x^5+1$

$du = 30x^4$

$\dfrac{1}{30}du = x^4\,dx$

$\dfrac{3}{30}\int_0^1 \dfrac{1}{u}\,du$

$\dfrac{3}{30}\left[\ln u\right]_0^1$

$\dfrac{3}{30}\ln(6x^5+1)$

$\dfrac{3}{30}\ln(6(1)^5+1) - \dfrac{3}{30}\ln(6(0)+1)$

$\dfrac{3}{30}\ln(7) - \dfrac{3}{30}\ln(1)$

$\dfrac{3}{30}(\ln(7) - \ln(1))$

4.) $\iint_D y\,dA$ D is bounded by $y = x-56$, $x = y^2$

$y = x-56$ $x = y+56$

$y = \pm\sqrt{x}$ $x = y^2$

$y = y^2 - 56$

$0 = y^2 - 56 - y$

$0 = y^2 - y - 56$

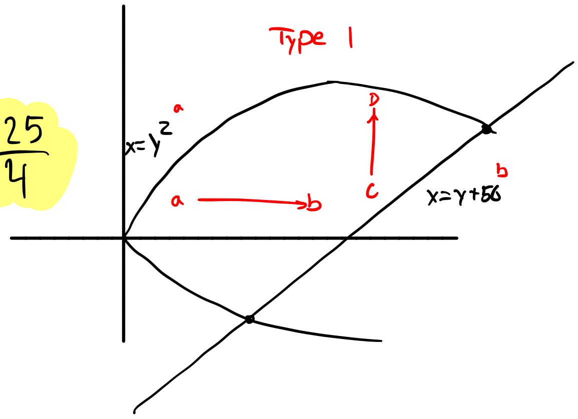
$(y-8)(y+7)$

$y = 8, -7$

$y = -7$ $y = 8$

$x = y^2$ $x = 64$ $x = 49$

$$\int_{-7}^{8}\int_{y^2}^{y+56} y\,dx\,dy = \frac{1125}{4}$$




$$\int_{-7}^{8}\int_{y^2}^{y+56} y\,dx\,dy$$

$$\int_{-7}^{8}\left[ yx \right]_{y^2}^{y+56} dy$$

$$\int_{-7}^{8}\left( y(y+56) - y(y^2) \right) dy$$

$$\int_{-7}^{8}\left( y^2 + 56y - y^3 \right) dy$$

$$\left[ \frac{1}{3}y^3 + 28y - \frac{1}{4}y^4 \right]_{-7}^{8}$$

$$\left( \frac{1}{3}(8^3) + 28(8) - \frac{1}{4}(8^4) \right) - \left( \frac{1}{3}(-7)^3 + 28(-7) - \frac{1}{4}(-7^4) \right)$$

5.) $\iint_D 3y^2\,dA$ (0,1) (1,2) (4,1)

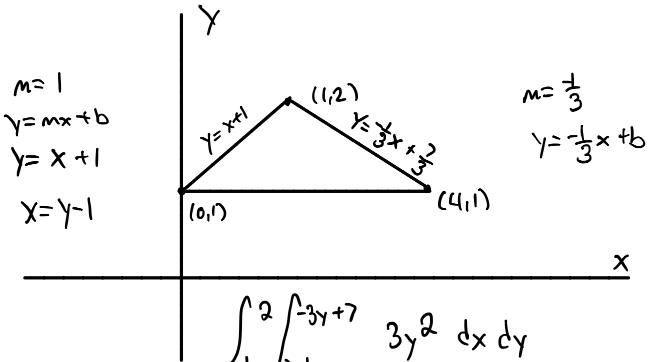
$$\int_1^2\int_{y-1}^{-3y+7} 3y^2\,dx\,dy = 11$$

$m = 1$

$y = mx + b$

$y = x + 1$

$x = y - 1$




$m = \frac{1}{3}$

$y = -\frac{1}{3}x + b$

$2 = -\frac{1}{3}(1) + b$

$2 = \frac{-1}{3} + b$

$\frac{7}{3} = b$

$Y - \frac{7}{3} = -\frac{1}{3}x$

$-3y + 7 = x$

$$\int_1^2\int_{y-1}^{-3y+7} 3y^2\,dx\,dy$$

$$\int_1^2\left[ 3y^2 x \right]_{y-1}^{-3y+7} dy$$

$$\int_1^2\left[ 3y^2(-3y+7) - 3y^2(y-1) \right] dy$$

$$\int_1^2\left[ -9y^3 + 21y^2 - 3y^3 + 3y^2 \right] dy$$

$$\int_1^2\left[ -12y^3 + 24y^2 \right] dy$$

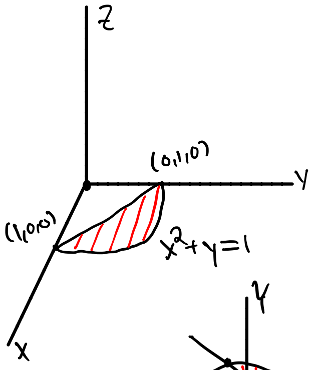
$$-3y^4 + 8y^3 \Big|_1^2$$

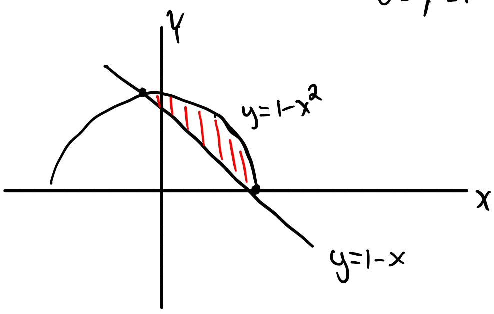
$$-3(2)^4 + 8(2)^3 - (-3(1)^4 + 8(1)^3)$$

$$-48 + 64 + 3 - 8$$

$$11$$

7.) $x-2y+z=3$   bounded by $x+y=1$, $x^2+y=1$




$x=-y+1$
$(-y+1)^2+y=1$
$y^2-2y+1+y=1$
$y^2-y=0$
$y(y-1)=0$
$y=0 \quad y=1$
$y(0)\ x=1 \quad y(1)\ x=0$
$(1,0) \quad (0,1)$
$0 \le x \le 1$
$0 \le y \le 1$

$x-2y+z=3$
$x-2y-3+z=0$
$x-2y-3=-z$
$-x+2y+3=z$

$$\int_0^1\int_{1-x}^{1-x^2} 3-x+2y \; dy\,dx = \frac{37}{60}$$

$$\int_0^1\int_{1-x}^{1-x^2} 3-x+2y \; dy\,dx$$

$(-x^2+1)(-x^2+1)$

$$\int_0^1 \left[3y-xy+y^2\right]_{1-x}^{1-x^2} dx \qquad (1-x^2)(1-x^2) \qquad (-x+1)^2$$

$$\int_0^1 \left(3(1-x^2)-x(1-x^2)+(1-x^2)^2-\left(3(1-x)-x(1-x)+(1-x)^2\right)\right) dx$$

$$\int_0^1 \left(3-3x^2-x+x^3+x^4-2x^2+1\right)-\left(3-3x-x+x^2+x^2-2x+1\right) dx$$

$$\int_0^1 (x^4+x^3-5x^2-x+4)-(2x^2-6x+4)\,dx$$

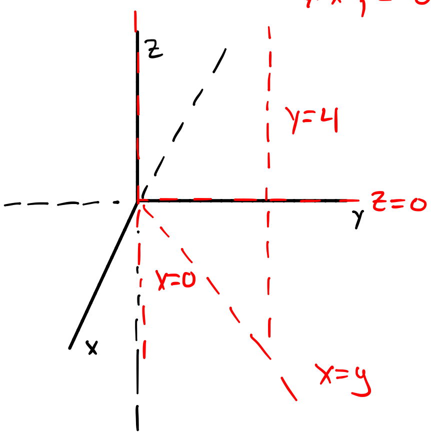
$$\int_0^1 x^4+x^3-7x^2+5x \; dx$$

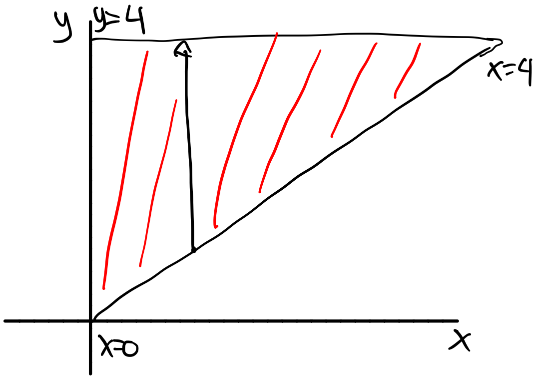
$$\frac{1}{5}x^5+\frac{1}{4}x^4-\frac{7x^3}{3}+\frac{5}{2}x^2$$

$$\frac{1}{5}+\frac{1}{4}-\frac{7}{3}+\frac{5}{2}$$

$\frac{37}{60}$

8.) $z=2x^2+2y^2$   $x=0,\ y=4$, $y=x,\ z=0$




$$\int_0^4\int_x^4 2x^2+2y^2 \; dy\,dx = \frac{512}{3}$$

$$\int_0^4\int_x^4 2x^2+2y^2 \; dy\,dx$$

$$\int_0^4 \left[2x^2y+\frac{2}{3}y^3\right]_x^4 dx$$

$$\int_0^4 2x^2(4)+\frac{2}{3}(4)^3-\left(2x^2(x)+\frac{2}{3}(x)^3\right)dx$$

$$\int_0^4 8x^2+\frac{128}{3}-2x^3-\frac{2}{3}x^3\,dx$$

$$\int_0^4 -\frac{8}{3}x^3+8x^2+\frac{128}{3}\,dx$$

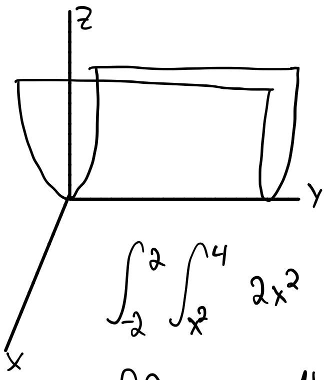
$$\left[-\frac{8}{12}x^4+\frac{8}{3}x^3+\frac{128}{3}x\right]_0^4$$

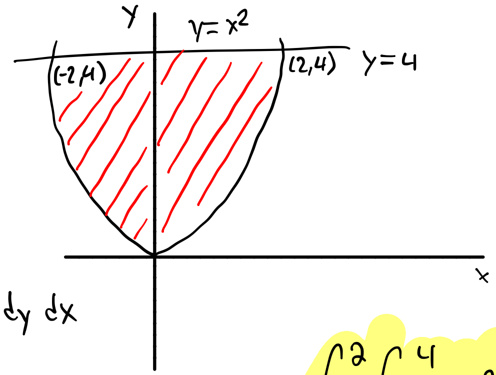
$$-\frac{8}{12}(4)^4+\frac{8}{3}(4)^3+\frac{128}{3}(4)$$

$$-\frac{512}{3}+\frac{512}{3}+\frac{512}{3}$$

$$\frac{512}{3}$$

9.) $z=2x^2$, $y=x^2$   $z=0$, $y=4$




$$\int_{-2}^{2}\int_{x^2}^{4} 2x^2 \, dy \, dx$$

$$\int_{-2}^{2} \left[2x^2 y\right]_{x^2}^{4} dx$$

$$\int_{-2}^{2} \left(2(x^2)(4) - 2x^2(x^2)\right) dx$$

$$\int_{-2}^{2} (8x^2 - 2x^4) \, dx$$

$$\frac{8}{3}x^3 - \frac{2}{5}x^5 \Big|_{-2}^{2}$$

$$\left(\frac{8}{3}(2)^3 - \frac{2}{5}(2)^5\right) - \left(\frac{8}{3}(-2)^3 - \frac{2}{5}(-2)^5\right)$$

$$\left(\frac{64}{3} - \frac{64}{5}\right) - \left(-\frac{64}{3} + \frac{64}{5}\right)$$

$$\frac{128}{3} - \frac{128}{5} = \frac{256}{15}$$

$$\int_{-2}^{2}\int_{x^2}^{4} 2x^2 \, dy \, dx = \frac{256}{15}$$

12.) Reverse the order of integration

$$\int_0^4 \int_{3y}^{12} 7e^{x^2} \, dx \, dy$$

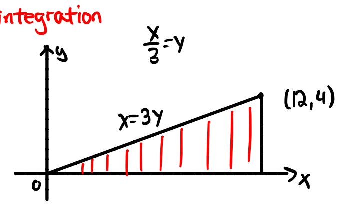
$x=3y, 12$
$y=0,4$

$x=12$
$x=3y$
$\frac{x}{3}=y$

$y=0$   $y=\frac{x}{3}$

$0 \le x \le 12$

$0 \le y \le \frac{x}{3}$




$$\int_0^4 \int_{3y}^{12} 7e^{x^2} \, dx \, dy = \frac{7}{6}(e^{144}-1)$$

$$\int_0^{12}\int_0^{\frac{x}{3}} 7e^{x^2} \, dy \, dx$$

x   y

$$\int_0^{12} \left[7e^{x^2} y\right]_0^{\frac{x}{3}} dx$$

$uv - \int v \, du$

$$7e^{x^2}\left(\frac{x}{3}\right) - 7e^{x^2}(0)$$

$u = x^2$
$du = 2x \, dx$
$\frac{1}{2}du = x \, dx$

$$\int_0^{12} 7e^{x^2}\left(\frac{x}{3}\right) dx$$

$$\frac{7}{3}\int_0^{12} xe^{x^2} \, dx$$

$$\frac{7}{3}\left(\frac{1}{2}\right)\int_0^{12} e^u \, du$$

$$\frac{7}{6}\left[e^u\right]$$

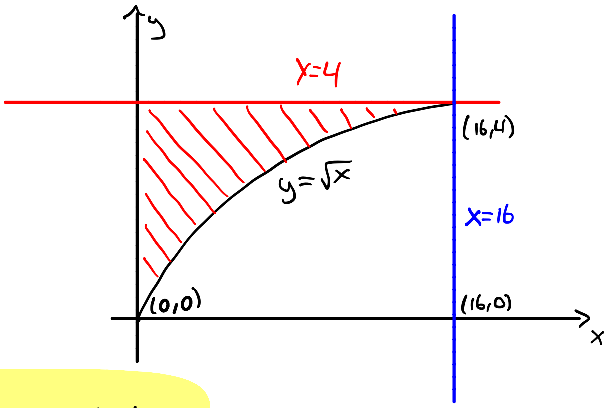
$$\frac{7}{6}e^{x^2}\Big|_0^{12} \qquad \frac{7}{6}(e^{144}-1)$$

13.) $\int_0^{16}\int_{\sqrt{x}}^{4} \frac{4}{y^3+1}\,dy\,dx$

$y=\sqrt{x}, 4$
$x=0, 16$

$x=y^2$ $\quad y=4$
$x=0$ $\quad y=16$

$0 \le x \le y^2$
$4 \le y \le 16$




$$\int_0^{4}\int_0^{y^2} \frac{4}{y^3+1}\,dx\,dy$$

$$\int_0^4 \left[\frac{4x}{y^3+1}\right]_0^{y^2} dy$$

$$\int_0^4 \left[\frac{4y^2}{y^3+1}\right] dy$$

$$4\int_0^4 \left(\frac{y^2}{y^3+1}\right) dy$$

$$\frac{4}{3}\int_0^4 \frac{1}{u}\,du$$

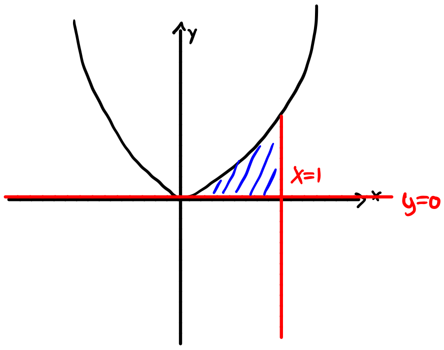
$$\frac{4}{3}\left[\ln(y^3+1)\right]_0^4$$

$$\frac{4}{3}\left[\ln(65)-\ln(1)\right]$$

$$\frac{4}{3}\ln(65)$$

$$\int_0^{16}\int_{\sqrt{x}}^{4} \frac{4}{y^3+1}\,dy\,dx = \frac{4}{3}\ln(65)$$

14.) $f(x,y)=6x\sin(y)$
D is enclosed by the curves;
$y=0$, $y=x^2$, $x=1$




$$A=\iint_D dA$$

$$A=\int_0^1\int_0^{x^2} dy\,dx$$

$$\int_0^1 (y)\Big|_0^{x^2} dx$$

$$\int_0^1 (x^2)\,dx$$

$$\frac{x^3}{3}\Big|_0^1$$

$$\frac{1}{3}=A$$

$$\frac{1}{A}\int_0^1\int_0^{x^2} 6x\sin(y)\,dy\,dx$$

$$\frac{1}{A}\int_0^1 6x\int_0^{x^2}\sin(y)\,dy\,dx$$

$$3\int_0^1 6x\int_0^{x^2}\sin(y)\,dy\,dx$$

$$3\int_0^1 -6x\cos(y)\Big|_0^{x^2} dx$$

$$3\int_0^1 -6x\cos(x^2)+6x\cos(0)\,dx$$

$$3\int_0^1 -6x\cos(x^2)+6x\,dx$$

$$3\int_0^1 6x(-\cos(x^2)+1)\,dx$$

$$3\int_0^1 6x\left(1-\cos(x^2)\right)dx$$

$u=x^2$
$du=2x$
$\frac{1}{2}du = x\,dx$

$$\frac{3}{2}\int_0^1 6(1-\cos(u))\,du$$

$$\frac{3}{2}\int_0^1 6-6\cos(u)\,du$$

$$\frac{3}{2}\left[6u-6\sin(u)\right]_0^1$$

$$\frac{3(6-6\sin(1))}{2}$$

$$\frac{1}{A}\int_0^1\int_0^{x^2} 6x\sin(y)\,dy\,dx = \frac{3(6-6\sin(1))}{2}$$