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MATH 361 HW 1.3

✓ ✓ ✓
2a, 2b, 6

Problem 2

$e = \sum_{n=0}^{\infty} 1/n! \qquad n! = n(n-1)$

a.) $e \approx \sum_{n=0}^{5} 1/n! \approx 1/0! + 1/1! + 1/2! + 1/3! + 1/4! + 1/5!$

$\approx 1 + 1 + 1/2 + 1/6 + 1/24 + 1/120$

$\approx 2 + 0.5000 + 0.1666 + 0.04166 + 0.008333$

$\approx 2.500 + 0.1666 + 0.04166 + 0.008333$

$\approx 2.666 + 0.04166 + 0.008333$

$\approx 2.707 + 0.008333$

≈ 2.715

$$\boxed{e \approx 2.715}$$

b.) $e \approx \sum_{j=0}^{5} 1/(5-j)! \approx \frac{1}{5!} + \frac{1}{4!} + \frac{1}{3!} + \frac{1}{2!} + \frac{1}{1!} + \frac{1}{0!}$

$\approx 1/120 + 1/24 + 1/6 + 1/2 + 1 + 1$

$\approx 0.008333 + 0.04166 + 0.1666 + 0.5000 + 2$

$\approx 0.04999 + 0.1666 + 0.5000 + 2$

$\approx 0.2165 + 0.5000 + 2$

$\approx 0.7165 + 2$

≈ 2.716

$$\boxed{e \approx 2.716}$$

Problem 6

a.) $\lim_{n\to\infty} \sin(1/n) = 0 \quad : \quad \lim_{n\to\infty} \sin(1/n) = 0$

$|\alpha_n - \alpha| = |\sin(1/n) - 0| = |\sin(1/n)|$ what from here

$\sin(1/n) \le 1/n$

$$\boxed{|\alpha_n - \alpha| = 0 + O(1/n)}$$

b.) $\lim_{n\to\infty} \sin(1/n^2) = 0 \quad : \quad \lim_{n\to\infty} \sin(1/n^2) = 0$

$|\alpha_n - \alpha| = |\sin(1/n^2) - 0| = |\sin(1/n^2)|$

$\sin(1/n^2) \le 1/n^2$

$$\boxed{|\alpha_n - \alpha| = 0 + O(1/n^2)}$$

c.) $\lim_{n\to\infty} (\sin(1/n))^2 = 0 \quad : \quad \lim_{n\to\infty} (\sin(1/n))^2 = 0$

$|\alpha_n - \alpha| = |(\sin(1/n))^2 - 0| = |(\sin(1/n))^2|$?

$\sin(1/n) \le 1/n \quad \therefore \; (\sin(1/n))^2 \le (1/n)(1/n) = (1/n^2)$

$$\boxed{|\alpha_n - \alpha| = 0 + O(1/n^2)}$$

d.) $\lim_{n\to\infty} [\ln(n+1) - \ln(n)] = 0$

$|\alpha_n - \alpha| = |[\ln(n+1) - \ln(n)] - 0| = |[\ln(n+1) - \ln(n)]| = \left|\ln\left(\frac{n+1}{n}\right)\right| = |\ln(1+1/n)| \le \frac{1}{n}$

Let $x = 1/n \quad \therefore \; f(x) = x - \ln(1+x)$, $x \ge 0$: Show $f(x) > 0$ on $(0,\infty)$

$f'(x) = 1 - \frac{1}{1+x} > 0 \; (0,\infty)$: Now $f(0) = 0 - \ln(1+0) = -\ln(1) = 0$

$\therefore \; f(x) > 0$ on $(0,\infty)$

$$\therefore \quad \boxed{|\alpha_n - \alpha| = 0 + O(1/n)}$$