MAT 201
Larson/Edwards – Section 3.2
Rolle's Theorem and the Mean Value Theorem

Recall the Extreme Value Theorem:

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

Both of these values could occur at the endpoints. Rolle's Theorem (named after Michel Rolle (1652-1719), gives conditions that guarantee existence of an extreme value in the *interior* of a closed interval.

<u>Rolle's Theorem</u>:
Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), if $f(a) = f(b)$ then there is at least one number c in (a, b) such that $f'(c) = 0$.

Ex: Determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

a) $f(x) = x^2 - 5x + 4, \quad [1, 4]$

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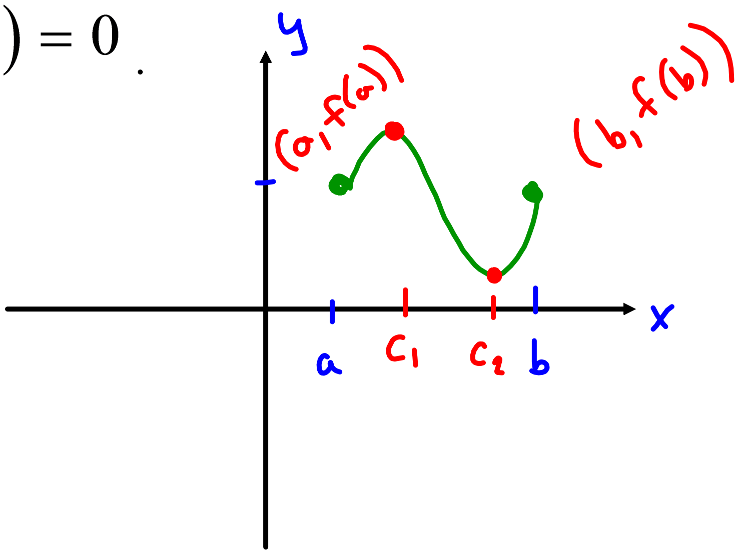

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a) $f(x) = x^2 - 5x + 4,\quad \overset{a,\,b}{[1,\ 4]}$

Is f continuous on $[1,4]$? Yes

Is f differentiable on $(1,4)$? Yes

Does $f(1) = f(4)$? $f(1) = 0$ ✓ Yes

$f(4) = 0$

There exists at least one c such that $f'(c)=0$.

$f'(x) = 2x - 5$

$0 = 2x - 5$

$\frac{5}{2} = x$

$$\boxed{c = \frac{5}{2}}$$

b) $f(x) = \tan x, \quad [0, \pi]$

Rolle's Theorem can be used to prove the Mean Value Theorem.

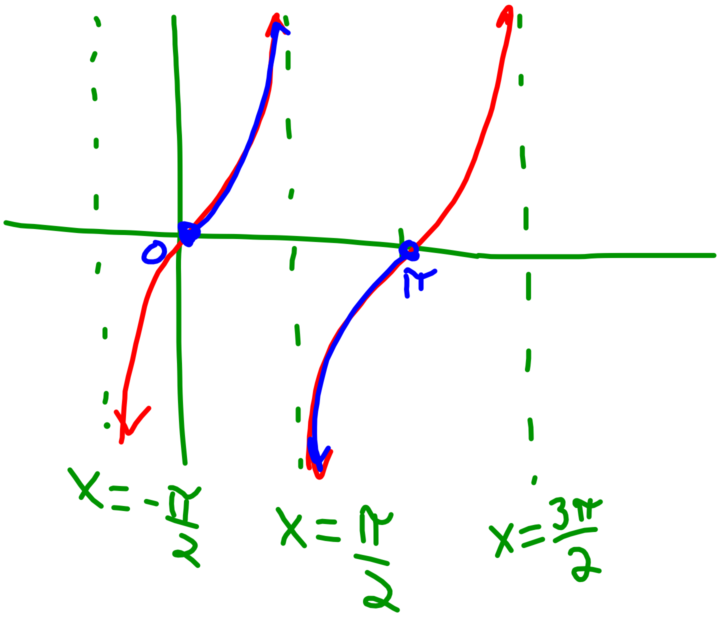
<u>Mean Value Theorem</u>:
If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Note: "Mean" refers to average, as in average rate of change.

How do we illustrate the meaning of the mean value theorem?

b) $f(x) = \tan x, \quad [0, \pi]$

Is f continuous on $[0, \pi]$? No



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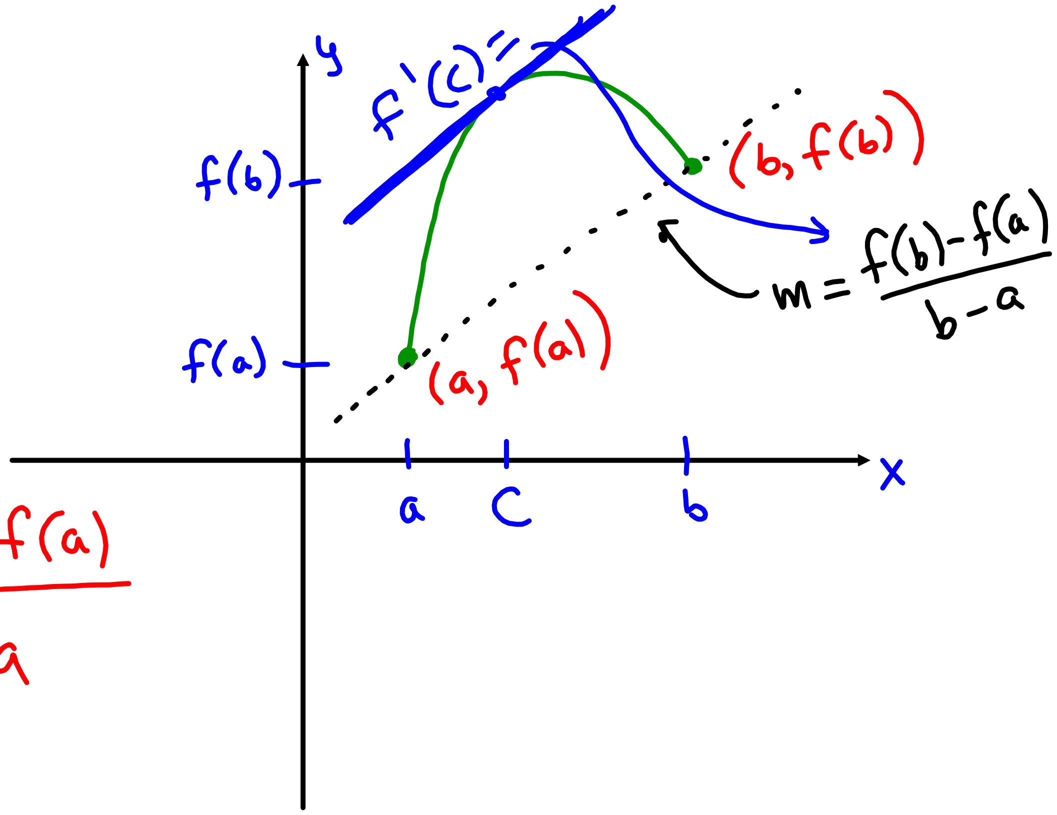

$$\frac{f(b) - f(a)}{b - a}$$

This says that in the interval (a, b), there exists a number c, such that the slope of the tangent line to the function f at c is equal to the slope of the secant line passing through the points $(a, f(a))$ and $(b, f(b))$.

Ex: Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. If the Mean Value Theorem cannot be applied, explain why not.

a) $f(x) = x^3 + 2x, \quad [-2, 0]$

b) $f(x) = |2x + 1|, \quad [-1, 3]$

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If the Mean Value Theorem cannot be applied, explain why not.

a) $f(x) = x^3 + 2x, \quad [\overset{a}{-2}, \overset{b}{0}]$

Is f continuous on [-2,0]? Yes

Is f differentiable on (-2,0)? Yes

$$\frac{f(b)-f(a)}{b-a} = \frac{f(0)-f(-2)}{0-(-2)} = \frac{0-(-12)}{2}$$

$$= 6$$

$f'(c) = 6$

$f'(x) = 3x^2 + 2$

$6 = 3x^2 + 2$

$\pm\sqrt{\frac{4}{3}} = \sqrt{x^2}$

$\pm\frac{2}{\sqrt{3}} = x$

$$\boxed{c = -\frac{2}{\sqrt{3}} \approx -1.15}$$

b) $f(x) = |2x + 1|, \quad [-1, 3]$

Continuous on $[-1, 3]$? Yes

Differentiable on $(-1, 3)$? NO

$f(x) = 2\left|x + \frac{1}{2}\right|$. Not differentiable @ $x = -\frac{1}{2}$

We can't use mean value theorem