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PHYS 321 Ch.1 # 1,2,3,4,9,10
HW 1

Problem 1.1

14, 15, 16, 16, 16, 22, 22, 24, 24, 25, 25, 25, 25, 25 : $N = 14$

a.)

i) $\langle j \rangle^2$: $\langle j \rangle = \sum_{j=0}^{\infty} j P(j)$

$j = 14 : P(14) = 1/14$
$j = 15 : P(15) = 1/14$
$j = 16 : P(16) = 3/14$
$j = 22 : P(22) = 1/7$
$j = 24 : P(24) = 1/7$
$j = 25 : P(25) = 5/14$

$\langle j \rangle = 14(\tfrac{1}{14}) + 15(\tfrac{1}{14}) + 16(\tfrac{3}{14}) + 22(\tfrac{2}{14}) + 24(\tfrac{2}{14}) + 25(\tfrac{5}{14}) = 21$

$\boxed{\langle j \rangle^2 = 441}$

ii) $\langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$

$j = 14 : P(14) = 1/14$
$j = 15 : P(15) = 1/14$
$j = 16 : P(16) = 3/14$
$j = 22 : P(22) = 1/7$
$j = 24 : P(24) = 1/7$
$j = 25 : P(25) = 5/14$

$\langle j^2 \rangle = 14^2(\tfrac{1}{14}) + 15^2(\tfrac{1}{14}) + 16^2(\tfrac{3}{14}) + 22^2(\tfrac{1}{7}) + 24^2(\tfrac{1}{7}) + 25^2(\tfrac{5}{14}) = 459.571$

$\boxed{\langle j^2 \rangle = 460}$

b.) $\Delta j = j - \langle j \rangle$: $\langle j \rangle = \sum_{j=0}^{\infty} j P(j)$

$j = 14$: $\langle j \rangle = 21$: $\Delta j = 14 - 21 = -7$: $\Delta j = -7$: $\Delta j^2 = 49$: $\Delta j^2 \cdot P(j) = 49 \cdot 1/14 = 3.5$
$j = 15$: $\langle j \rangle = 21$: $\Delta j = 15 - 21 = -6$: $\Delta j = -6$: $\Delta j^2 = 36$: $\Delta j^2 \cdot P(j) = 36 \cdot 1/14 = 2.57$
$j = 16$: $\langle j \rangle = 21$: $\Delta j = 16 - 21 = -5$: $\Delta j = -5$: $\Delta j^2 = 25$: $\Delta j^2 \cdot P(j) = 25 \cdot 3/14 = 5.36$
$j = 22$: $\langle j \rangle = 21$: $\Delta j = 22 - 21 = 1$: $\Delta j = 1$: $\Delta j^2 = 1$: $\Delta j^2 \cdot P(j) = 1 \cdot 1/7 = 0.14$
$j = 24$: $\langle j \rangle = 21$: $\Delta j = 24 - 21 = 3$: $\Delta j = 3$: $\Delta j^2 = 9$: $\Delta j^2 \cdot P(j) = 9 \cdot 1/7 = 1.28$
$j = 25$: $\langle j \rangle = 21$: $\Delta j = 25 - 21 = 4$: $\Delta j = 4$ $\Delta j^2 = 16$: $\Delta j^2 \cdot P(j) = 16 \cdot 5/14 = 5.71$

$\Delta j^2 \cdot P(j)_{Tot} = 18.57 \rightarrow \sigma^2 = \langle (\Delta j)^2 \rangle \rightarrow \sigma^2 = 18.57 \rightarrow \sigma = \sqrt{18.57}$

With the use of EQ. (11): $\boxed{\sigma = \sqrt{18.57}}$

Problem 1.1 Continued

c.) $\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$ $\langle j \rangle^2 = 441$, $\langle j^2 \rangle = 459.571$

$$\sigma = \sqrt{459.571 - 441} = \sqrt{18.57}$$

$$\boxed{\sigma = \sqrt{18.57}}$$

Problem 1.2

a.) $\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$: $\langle x\rangle = \int_{-\infty}^{\infty} x\rho(x)\,dx$: $\rho(x) = \frac{1}{2\sqrt{hx}}$, $0 \le x \le h$

$$\langle x\rangle = \int_0^h x \cdot \frac{1}{2\sqrt{hx}}\,dx = \frac{1}{2\sqrt{h}}\int_0^h (x)^{1/2}\,dx = \frac{1}{3\sqrt{h}} \cdot x^{3/2}\Big|_0^h = \frac{h^{3/2}}{3\sqrt{h}} = \frac{h}{3}$$

$$\langle x\rangle^2 = \frac{h^2}{9}$$

$$\langle x^2\rangle = \int_0^h x^2 \cdot \frac{1}{2\sqrt{h}} \cdot \frac{1}{x^{1/2}}\,dx = \frac{1}{2\sqrt{h}}\int_0^h x^{3/2}\,dx = \frac{1}{2\sqrt{h}} \cdot \frac{2}{5}x^{5/2}\Big|_0^h = \frac{1}{5\sqrt{h}} \cdot h^{5/2} = \frac{h^2}{5}$$

$$\langle x^2\rangle = \frac{h^2}{5}$$

$$\sigma = \sqrt{\frac{h^2}{5} - \frac{h^2}{9}} = \sqrt{\frac{9h^2 - 5h^2}{45}} = \sqrt{\frac{4h^2}{45}} = \frac{\sqrt{4h^2}}{\sqrt{45}} = \frac{2h}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{(\sqrt{5})} = \frac{2\sqrt{5}h}{15}$$

$$\sigma = \frac{2h}{3\sqrt{5}}$$

$$\boxed{\sigma = \frac{2\sqrt{5}\,h}{15}}$$

b.) $P_{ab} = \int_a^b \rho(x)\,dx$: $\rho(x) = \frac{1}{2\sqrt{hx}}$: $a = \langle x\rangle - \sigma$: $b = \langle x\rangle + \sigma$

$$P_{ab} = \int_{\langle x\rangle - \sigma}^{\langle x\rangle + \sigma} \frac{1}{2\sqrt{hx}}\,dx = \frac{1}{2\sqrt{h}}\int_\alpha^\beta x^{-1/2}\,dx = \frac{1}{2\sqrt{h}} \cdot 2x^{1/2}\Big|_\alpha^\beta = \frac{\sqrt{x}}{\sqrt{h}}\Big|_{\langle x\rangle - \sigma}^{\langle x\rangle + \sigma}$$

$\langle x\rangle = h/3$
$\sigma = \frac{2h}{3\sqrt{5}}$

$\langle x\rangle + \sigma \to \beta$
$\langle x\rangle - \sigma \to \alpha$

$$= \frac{1}{\sqrt{h}}\left[\sqrt{\frac{h}{3} + \frac{2h}{3\sqrt{5}}} - \sqrt{\frac{h}{3} - \frac{2h}{3\sqrt{5}}}\right] = \frac{1}{\sqrt{h}}\left[\sqrt{h}\left\{\sqrt{\frac{1}{3} + \frac{2}{3\sqrt{5}}} - \sqrt{\frac{1}{3} - \frac{2}{3\sqrt{5}}}\right\}\right]$$

$$= \sqrt{\frac{1}{3} + \frac{2}{3\sqrt{5}}} - \sqrt{\frac{1}{3} - \frac{2}{3\sqrt{5}}} = \sqrt{\frac{\sqrt{5}}{3\sqrt{5}} + \frac{2}{3\sqrt{5}}} - \sqrt{\frac{\sqrt{5}}{3\sqrt{5}} - \frac{2}{3\sqrt{5}}} = \sqrt{\frac{\sqrt{5}+2}{3\sqrt{5}}} - \sqrt{\frac{\sqrt{5}-2}{3\sqrt{5}}}$$

$P_{ab} = 0.607$ now compute $1 - P_{ab}$ to find true probability

$$1 - 0.607 = 0.393 \times 100 = 39.3\%$$

$$\boxed{P = 39.3\%}$$

Problem 1.3

$$\rho(x) = Ae^{-\lambda(x-a)^2}$$

a.) $1 = \int_{-\infty}^{\infty} \rho(x)\,dx$

$1 = \int_{-\infty}^{\infty} Ae^{-\lambda(x-a)^2}\,dx \qquad u = x-a,\ du = dx \qquad x=0,\ u=-a;\ x=\infty,\ u=\infty$

$1 = 2\cdot\int_0^{\infty} Ae^{-\lambda u^2}\,du \rightarrow \frac{1}{2} = A\cdot\int_0^{\infty} e^{-\lambda u^2}\,du$

$\int_0^{\infty} x^{2n} e^{-x^2/a^2}\,dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1} : n=0 \;= \sqrt{\pi}\left(\frac{a}{2}\right) \qquad -\lambda\cdot \cancel{u^2} = -\frac{\cancel{x^2}}{a^2}$

$\int_0^{\infty} e^{-\lambda u^2}\,du = \sqrt{\pi}\cdot\left(\frac{1}{\sqrt{\lambda}2}\right) = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}} \qquad \lambda = \frac{1}{a^2} \rightarrow a^2 = \frac{1}{\lambda}$

$a = \frac{1}{\sqrt{\lambda}}\cdot\frac{\sqrt{\lambda}}{\sqrt{\lambda}} = \frac{\sqrt{\lambda}}{\lambda}$

$\frac{1}{2} = A\cdot\frac{1}{2}\sqrt{\frac{\pi}{\lambda}} : 1 = A\cdot\sqrt{\frac{\pi}{\lambda}} : A = \sqrt{\frac{\lambda}{\pi}} \qquad a = \frac{\sqrt{\lambda}}{\lambda} \text{ or } \frac{1}{\sqrt{\lambda}}$

$$\boxed{A = \sqrt{\frac{\lambda}{\pi}}}$$

b.) $\langle x\rangle$, $\langle x^2\rangle$, and $\sigma \qquad u = x-a,\ x = u+a,\ du = dx$

$\langle x\rangle = \int_{-\infty}^{\infty} x\cdot\rho(x)\,dx,\quad \langle x^2\rangle = \int_{-\infty}^{\infty} x^2\cdot\rho(x)\,dx,\quad \sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$

$\int_0^{\infty} x^{2n+1} e^{-x^2/a^2}\,dx = \frac{n!}{2}a^{2n+2} : \int_0^{\infty} x^{2n} e^{-x^2/a^2}\,dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$

$\langle x\rangle = \sqrt{\frac{\lambda}{\pi}}\cdot 2\cdot\int_0^{\infty} (u+a)e^{-\lambda(u)^2}\,du \rightarrow \langle x\rangle = 2\cdot\sqrt{\frac{\lambda}{\pi}}\int_0^{\infty} ue^{-\lambda(u)^2} + ae^{-\lambda(u)^2}\,du$

$\langle x\rangle : \underbrace{\frac{\sqrt{\lambda}}{\sqrt{\pi}}\int_{-\infty}^{\infty} ue^{-\lambda u^2}\,du}_{1} + \underbrace{\frac{2\sqrt{\lambda}}{\sqrt{\pi}}\int_0^{\infty} ae^{-\lambda(u)^2}\,du}_{2}$

1 : 1 = 0 since $f(u)$ is odd on even interval

2: $n=0 \quad \int_0^{\infty} ae^{-\lambda u^2}\,du = a\int_0^{\infty} e^{-\lambda(u)^2}\,du = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{b}{2}\right)^{2n+1}\Big|_{n=0}\Big|_{b=\frac{1}{\sqrt{\lambda}}}$

2: $a\cdot\sqrt{\pi}\cdot\left(\frac{1}{2\sqrt{\lambda}}\right) = \frac{a}{2}\cdot\frac{\sqrt{\pi}}{\sqrt{\lambda}} : \langle x\rangle = \frac{\cancel{2}\cancel{\sqrt{\lambda}}}{\cancel{\sqrt{\pi}}}\cdot a\cdot\left(\frac{\cancel{\sqrt{\pi}}}{\cancel{2}\cancel{\sqrt{\lambda}}}\right) = a$

$$\boxed{\langle x\rangle = a,\ \langle x\rangle^2 = a^2}$$

$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2\rho(x)\,dx = \sqrt{\frac{\lambda}{\pi}}\cdot\int_{-\infty}^{\infty} x^2 e^{-\lambda(x-a)^2}\,dx \qquad u = x-a,\ du = dx \;\therefore\; x = u+a,\ x=-\infty,\ u=-\infty,\ x=\infty,\ u=\infty$

$\langle x^2\rangle = \sqrt{\frac{\lambda}{\pi}}\cdot\int_{-\infty}^{\infty}(u+a)^2 e^{-\lambda(u)^2}\,du$

Problem 1.3 | Continued

$$= \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{\infty} (u^2 + 2ua + a^2) e^{-\lambda (u)^2} du$$

$$= \sqrt{\frac{\lambda}{\pi}} \left[\int_{-\infty}^{\infty} u^2 e^{-\lambda u^2} du + 2a \cancelto{0}{\int_{-\infty}^{\infty} u e^{-\lambda u^2} du} + a^2 \int_{-\infty}^{\infty} e^{-\lambda u^2} du \right]$$

$$= 2 \cdot \sqrt{\frac{\lambda}{\pi}} \left[\underbrace{\int_0^{\infty} u^2 e^{-\lambda u^2} du}_{①} + \underbrace{a^2 \int_0^{\infty} e^{-\lambda u^2} du}_{②} \right]$$

①: $\int_0^{\infty} u^2 e^{-\lambda u^2} du \rightarrow \int_0^{\infty} x^{2n} e^{-x^2/b^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{b}{2}\right)^{2n+1}$

$b^2 = \frac{1}{\lambda} \quad \therefore \quad b = \frac{1}{\sqrt{\lambda}} \quad ; \ n = 1$

$$① = \int_0^{\infty} u^2 e^{-\lambda u^2} du = \sqrt{\pi} \frac{(2)!}{1!} \left(\frac{1}{2\sqrt{\lambda}} \right)^3 = \sqrt{\pi} \cdot 2 \cdot \frac{1}{8} \cdot \frac{1}{\lambda^{3/2}} = \frac{1}{4} \cdot \sqrt{\frac{\pi}{\lambda^3}}$$

②: $a^2 \int_0^{\infty} e^{-\lambda u^2} du \rightarrow \int_0^{\infty} x^{2n} e^{-x^2/b^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{b}{2}\right)^{2n+1}$

$b^2 = \frac{1}{\lambda} \quad \therefore \quad b = \frac{1}{\sqrt{\lambda}} \quad ; \ n = 0$

$$a^2 \cdot \sqrt{\pi} \cdot \frac{0!}{0!} \left(\frac{1}{2\sqrt{\lambda}} \right)^1 = a^2 \cdot \frac{\sqrt{\pi}}{\sqrt{\lambda}} \cdot \frac{1}{2} \quad ; \quad ② = \frac{a^2}{2} \sqrt{\frac{\pi}{\lambda}}$$

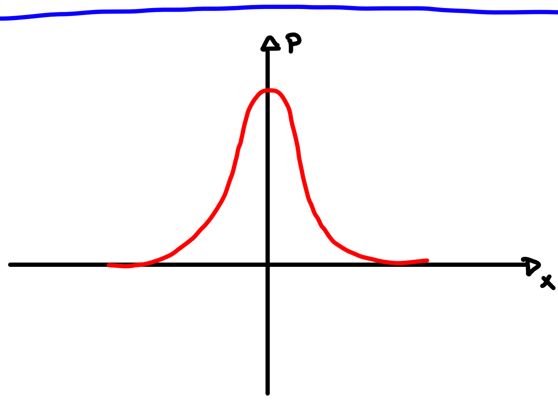
$$= 2\sqrt{\frac{\lambda}{\pi}} \left[\frac{1}{4} \cdot \sqrt{\frac{\pi}{\lambda^3}} + \frac{a^2}{2} \sqrt{\frac{\pi}{\lambda}} \right] = \frac{1}{2} \cdot \frac{1}{\lambda} + a^2 \quad \therefore \quad \boxed{\langle x^2 \rangle = \frac{1}{2\lambda} + a^2}$$

$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \ : \quad \langle x^2 \rangle = \frac{1}{2\lambda} + a^2, \quad \langle x \rangle^2 = a^2$

$$\sigma = \sqrt{\frac{1}{2\lambda} + a^2 - a^2} = \sqrt{\frac{1}{2\lambda}} = \frac{1}{\sqrt{2\lambda}} = \frac{\sqrt{2\lambda}}{2\lambda} \quad \therefore \quad \boxed{\sigma = \frac{\sqrt{2\lambda}}{2\lambda}}$$

$$\boxed{\langle x \rangle = a, \ \langle x^2 \rangle = \frac{1}{2\lambda} + a^2, \ \sigma = \frac{\sqrt{2\lambda}}{2\lambda}}$$

c.)



Problem 1.4

$$\Psi(x,0) = \begin{cases} A\frac{x}{a}, & \text{if } 0 \le x \le a \\ A\frac{(b-x)}{(b-a)}, & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

a.) $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$

$0 \le x \le a$: $\Psi(x,0) = A\cdot\frac{x}{a}$, $|\Psi(x,0)|^2 = A^2\cdot\frac{x^2}{a^2}$

$a \le x \le b$: $\Psi(x,0) = A\frac{(b-x)}{(b-a)}$, $|\Psi(x,0)|^2 = A^2\cdot\frac{(b-x)^2}{(b-a)^2}$

$$\underbrace{\int_0^a A^2\cdot\frac{x^2}{a^2}\,dx}_{①} + \underbrace{\int_a^b A^2\cdot\frac{(b-x)^2}{(b-a)^2}\,dx}_{②} = 1$$

①: $\frac{A^2}{a^2}\cdot\int_0^a x^2 dx = \frac{A^2}{a^2}\cdot\frac{x^3}{3}\Big|_0^a = \frac{A^2}{\cancel{a^2}}\cdot\frac{a^{\cancel{3}}}{3} = A^2\cdot\frac{a}{3}$

②: $\frac{A^2}{(b-a)^2}\int_a^b (b-x)^2 dx$: $u = b-x$, $du = -dx$, $x=a,\ u=b-a$, $x=b,\ u=0$; $-\frac{A^2}{(b-a)^2}\int_{b-a}^0 (u)^2 du = \frac{A^2}{(b-a)^2}\int_0^{b-a}(u)^2 du$

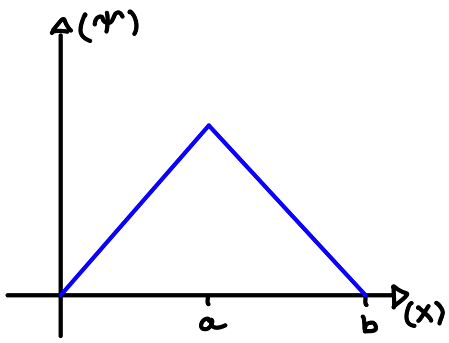
$$\frac{A^2}{(b-a)^2}\cdot\frac{u^3}{3}\Big|_0^{b-a} = \frac{A^2}{\cancel{(b-a)^2}}\cdot\frac{(b-a)^{\cancel{3}}}{3} = A^2\cdot\frac{(b-a)}{3}$$

① + ② $= A^2\cdot\frac{a}{3} + A^2\cdot\frac{b}{3} - A^2\cdot\frac{a}{3} = A^2\cdot\frac{b}{3}$: ① + ② $= A^2\cdot\frac{b}{3}$

① + ② $= 1$: $A^2\cdot\frac{b}{3} = 1 \rightarrow A^2 = \frac{3}{b} \rightarrow A = \frac{\sqrt{3}}{\sqrt{b}}$

$$\boxed{A = \sqrt{\frac{3}{b}}}$$

b.)



$\Psi: 0 \rightarrow a$: $\underbrace{\sqrt{\frac{3}{b}}\cdot\frac{1}{a}}_{\text{slope}}x$

$\Psi: a \rightarrow b$: $\underbrace{\sqrt{\frac{3}{b}}\cdot\frac{b}{(b-a)}}_{\text{Intercept}} - \underbrace{\sqrt{\frac{3}{b}}\cdot\frac{1}{(b-a)}}_{\text{Slope}}\,x$

c.) When the particle is at $t=0$, the most probable location for the particle to be at is @ $x=a$

$$\boxed{x=a}$$

Problem 1.4 continued

d.)

$$0 \le x \le a : \Psi(x,0) = \sqrt{\frac{3}{b}} \cdot \frac{x}{a} : \Psi(x,0)^2 = \frac{3}{b} \cdot \frac{x^2}{a^2}$$

$$\int_0^a \frac{3}{b} \cdot \frac{x^2}{a^2}\, dx = \frac{\cancel{3}}{b} \cdot \frac{x^3}{\cancel{3} \cdot a^2}\Big|_0^a = \frac{a^{\cancel{3}}}{b\cancel{a^2}} = \frac{a}{b}$$

$$a \le x \le b : \Psi(x,0) = \sqrt{\frac{3}{b}}\, \frac{(b-x)}{(b-a)} : \Psi(x,0)^2 = \frac{3}{b}\, \frac{(b-x)^2}{(b-a)^2}$$

$$\int_a^b \frac{3}{b}\, \frac{(b-x)^2}{(b-a)^2} dx : \begin{array}{l} u = b-x \\ du = -dx \end{array} : \begin{array}{l} x=a,\ u=b-a \\ x=b,\ u=0 \end{array} : -\frac{3}{b} \cdot \frac{1}{(b-a)^2} \int_{b-a}^{0} u^2\, du$$

$$-\frac{\cancel{3}}{b} \cdot \frac{1}{(b-a)^2} \cdot \frac{u^3}{\cancel{3}}\Big|_{b-a}^{0} = \frac{1}{b(b-a)^2} \cdot u^3 \Big|_0^{b-a} = \frac{(b-a)^{\cancel{3}}}{b\cancel{(b-a)^2}} = \frac{b-a}{b}$$

From $0 \to a$: Probability $= \frac{a}{b}$
when $b=a$: Probability $= 1$
when $b=2a$: Probability $= \frac{1}{2}$

e.) $\langle x \rangle = \int_{-\infty}^{+\infty} x|\Psi(x,t)|^2 dx$: $0 \le x \le a \to \Psi(x,0) = 3/b \cdot x^2/a^2$
$a \le x \le b \to \Psi(x,0) = 3/b \cdot (b-x)^2/(b-a)^2$

$$\langle x \rangle = \underbrace{\frac{3}{b \cdot a^2} \int_0^a x^3 dx}_{①} + \underbrace{\frac{3}{b \cdot (b-a)^2} \int_a^b x(b-x)^2 dx}_{②}$$

$$①: \frac{3}{b \cdot a^2} \int_0^a x^3 dx = \frac{3}{b \cdot a^2} \cdot \frac{x^4}{4}\Big|_0^a = \frac{3 \cdot a^{\cancel{4}\,2}}{4b\cancel{a^2}} = \frac{3a^2}{4b}$$

$b^2 - 2bx + x^2$

$$②: \frac{3}{b(b-a)^2} \int_a^b x(b-x)^2 dx = \frac{3}{b(b-a)^2} \int_a^b x^3 - 2bx^2 - b^2x\, dx = \frac{3}{b(b-a)^2}\left[\frac{x^4}{4} - \frac{2bx^3}{3} - \frac{b^2x^2}{2}\right]_a^b$$

$$= \frac{3}{b(b-a)^2}\left[\left(b^4/4 - 2b^4/3 - b^4/2\right) - \left(\frac{a^4}{4} - \frac{2ba^3}{3} - \frac{b^2a^2}{2}\right)\right] = \frac{3}{b(b-a)^2}\left[\frac{-11 \cdot b}{12} - \left(\frac{a^4}{4} - \frac{2ba^3}{3} - \frac{b^2a^2}{2}\right)\right]$$

$$\frac{b^4}{4} - \frac{2b^4}{3} - \frac{b^4}{2} = \frac{-11 \cdot b}{12}$$

$$= \frac{-3a^2 + 2ab + b^2}{4b} = \frac{-3a^2}{4b} + \frac{a}{2} + \frac{b}{4}$$

$$① + ② = \cancel{\frac{3a^2}{4b}} - \cancel{\frac{3a^2}{4b}} + \frac{a}{2} + \frac{b}{4} = \frac{a}{2} + \frac{b}{4}$$

$$\boxed{\langle x \rangle = \frac{a}{2} + \frac{b}{4}}$$

Problem 1.9

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]} \qquad \Psi(x,t)^* = Ae^{-a[(mx^2/\hbar)-it]}$$

a.)

$$Ae^{-a[(mx^2/\hbar)+it]} \cdot Ae^{-a[(mx^2/\hbar)-it]} = A^2 e^{-a[2mx^2/\hbar]}$$

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1 \rightarrow 2\cdot\int_0^{\infty} A^2 e^{-a[2mx^2/\hbar]} dx = 1$$

$$\int_0^{\infty} x^{2n} e^{-x^2/b^2} dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{b}{2}\right)^{2n+1}$$

$$2A^2\int_0^{\infty} e^{-a[2mx^2/\hbar]} dx = 1 \quad : n=0 \quad : 2A^2 \cdot \frac{\sqrt{\pi}}{2}\left(\sqrt{\frac{\hbar}{2ma}}\right) = 1 \quad : \quad A^2\sqrt{\frac{\hbar\pi}{2ma}} = 1$$

$$b^2 = \frac{\hbar}{2ma} \quad \therefore \quad b = \sqrt{\frac{\hbar}{2ma}}$$

$$A^2\sqrt{\frac{\hbar\pi}{2ma}} = 1 \quad : \quad A^2 = \sqrt{\frac{2ma}{\hbar\pi}} \qquad A = \left(\frac{2ma}{\hbar\pi}\right)^{1/4}$$

$$\boxed{A = \left(\frac{2ma}{\hbar\pi}\right)^{1/4}}$$

b.)

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(\Psi) \quad : \quad V(\Psi) = i\hbar\frac{\partial\Psi}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \qquad \Psi = Ae^{-\frac{amx^2}{\hbar} - ait}$$

$$\Psi(x,t) = Ae^{-a[mx^2/\hbar + it]} \quad : \quad \frac{\partial\Psi}{\partial t} = -ia\cdot\Psi \quad : \quad \frac{\partial\Psi}{\partial x} = -\frac{2max}{\hbar}\cdot\Psi$$

$$: \frac{\partial^2\Psi}{\partial x^2} = -\frac{2ma\Psi}{\hbar} + \frac{(2max)^2\Psi}{\hbar^2} \quad : \Psi\left(\frac{(2max)^2}{\hbar^2} - \frac{(2ma)}{\hbar}\right) \quad : \quad -\frac{\hbar^2}{2m}\cdot\Psi\left(\frac{(2max)^2}{\hbar^2} - \frac{(2ma)}{\hbar}\right)$$

$$: \frac{\hbar^2}{2m}\cdot\Psi\left(\frac{(2ma)}{\hbar} - \frac{(2max)^2}{\hbar^2}\right) = \Psi\left(\hbar a - 2ma^2x^2\right)$$

$$i\hbar\left(-ia\cdot\Psi\right) = \hbar a\Psi \quad : \quad \hbar a\cancel{\Psi} = \cancel{\Psi}(\hbar a - 2ma^2x^2) + V(\cancel{\Psi})$$

$$\hbar a = \hbar a - 2ma^2x^2 + V$$

$$2ma^2x^2 = V$$

$$\boxed{V = 2ma^2x^2}$$

Problem 1.9 | Continued

c.) $\langle x\rangle = \int_{-\infty}^{\infty} x|\Psi(x,t)|^2 dx$, $\langle P\rangle = m\frac{d\langle x\rangle}{dt}$, $\Psi(x,t)^2 = A^2 e^{-a(2mx^2/\hbar)}$

$\langle x\rangle = \int_{-\infty}^{\infty} x\cdot A^2 e^{-a(2mx^2/\hbar)} dx = 0$ since odd function

$\langle P\rangle = m\frac{d\langle x\rangle}{dt} = m\cdot 0 = 0$

$\langle x^2\rangle = \int_{-}^{\infty} x^2|\Psi(x,t)|^2 dx = A^2\int_{-\infty}^{\infty} x^2 e^{-a(2mx^2/\hbar)} dx \; : \lambda = \frac{2ma}{\hbar} : = A^2\int_{-\infty}^{\infty} x^2 e^{-\lambda x^2}$

$\int_0^{\infty} x^{2n} e^{-x^2/b^2} dx = \sqrt{\pi}\frac{(2n)!}{n!}\left(\frac{b}{2}\right)^{2n+1} : \quad b^2 = \frac{\hbar}{2ma},\; b = \sqrt{\frac{\hbar}{2ma}},\; b = \frac{1}{\sqrt{\lambda}},\; \lambda = \frac{2ma}{\hbar}$

$2\cdot A^2\int_0^{\infty} x^2 e^{-\lambda x^2} dx = 2\cdot A^2\left(\sqrt{\pi}\frac{(2)}{1}\cdot\left(\frac{1}{8}\right)\left(\frac{\hbar}{2ma}\right)^{3/2}\right) = \frac{1}{2}A^2\cdot\sqrt{\pi}\left(\frac{\hbar}{2ma}\right)^{3/2}$

$= \frac{1}{2}\sqrt{\frac{2ma}{\hbar\pi}}\cdot\sqrt{\pi}\left(\frac{\hbar}{2ma}\right)^{3/2} = \frac{1}{2}\sqrt{\frac{2ma}{\hbar}}\sqrt{\left(\frac{\hbar}{2ma}\right)^3} = \frac{1}{2}\sqrt{\frac{\hbar^2}{(2ma)^2}} = \frac{1}{2}\cdot\frac{\hbar}{2ma} = \frac{\hbar}{4ma}$

$$\langle x^2\rangle = \frac{\hbar}{4ma}$$

$\langle P^2\rangle = \int \Psi^*\hat{p}^2\Psi \; : \; \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x} \; : \; \hat{p}^2 = \frac{\hbar^2}{i^2}\frac{\partial^2}{\partial x^2}$

$\langle P^2\rangle = \int_{-\infty}^{\infty}\left(Ae^{-a(mx^2/\hbar - it)}\right)\cdot\frac{\hbar^2}{i^2}\cdot\frac{\partial^2}{\partial x^2}\left(Ae^{-a(mx^2/\hbar + it)}\right) dx$

$= -\hbar^2\cdot A^2\int_{-\infty}^{\infty} e^{-a(mx^2/\hbar - it)}\cdot\frac{\partial^2}{\partial x^2} e^{-a(mx^2/\hbar + it)} dx$

$\frac{\partial}{\partial x}\left(e^{-a(mx^2/\hbar + it)}\right) = \frac{-2max}{\hbar} e^{-a(mx^2/\hbar + it)}$

$\frac{\partial^2}{\partial x^2}\left(e^{-a(mx^2/\hbar + it)}\right) = -\frac{2ma}{\hbar} e^{-a(mx^2/\hbar + it)} + \frac{(2max)^2}{\hbar^2} e^{-a(mx^2/\hbar + it)}$

$= e^{-a(mx^2/\hbar + it)}\left[(2amx)^2/\hbar^2 - (2am)/\hbar\right] : \quad \lambda = \frac{2ma}{\hbar}$

$= e^{-a(mx^2/\hbar + it)}\left[\lambda^2 x^2 - \lambda\right]$

$= e^{-a(mx^2/\hbar - it)}\cdot e^{-a(mx^2 + it)}\left[\lambda^2 x^2 - \lambda\right] = e^{\frac{-2amx^2}{\hbar}} = e^{-\lambda x^2}\left[\lambda^2 x^2 - \lambda\right]$

$-\hbar^2\cdot A^2\cdot\int_{-\infty}^{\infty}\lambda^2 x^2 e^{-\lambda x^2} - \lambda e^{-\lambda x^2} dx = -\hbar^2 A^2\left[\underbrace{\int_{-\infty}^{\infty}\lambda^2 x^2 e^{-\lambda x^2} dx}_{1} - \underbrace{\int_{-\infty}^{\infty}\lambda e^{-\lambda x^2} dx}_{2}\right]$

Problem 1.9 Continued

$$① : \int_{-\infty}^{\infty} \lambda^2 x^2 e^{-\lambda x^2} dx = 2\lambda^2 \int_0^{\infty} x^2 e^{-\lambda x^2} dx \; : \; \int_0^{\infty} x^{2n} e^{-x^2/b^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{b}{2}\right)^{2n+1}$$

$$b^2 = \frac{1}{\lambda}, \; b = \frac{1}{\sqrt{\lambda}}$$

$$2\lambda^2 \int_0^{\infty} x^2 e^{-\lambda x^2} dx \underset{n=1}{=} 2\cdot\lambda^2 \left[\sqrt{\pi} \frac{(2)!}{1!} \frac{1}{8} \left(\frac{\hbar}{2ma}\right)^{\frac{3}{2}}\right] = \frac{\lambda^2}{2} \cdot \sqrt{\pi} \sqrt{\left(\frac{\hbar}{2ma}\right)^3} \; : \; ① = \frac{\lambda^2}{2} \sqrt{\pi} \sqrt{\left(\frac{\hbar}{2ma}\right)^3}$$

$$① : \frac{\sqrt{\pi}}{2} \cdot \frac{4m^2a^2}{\hbar^2} \sqrt{\left(\frac{\hbar}{2ma}\right)^2 \left(\frac{\hbar}{2ma}\right)} = \sqrt{\pi} \cdot \frac{\cancel{2}m^{\cancel{2}}a^{\cancel{2}}}{\hbar^{\cancel{2}}} \cdot \frac{\cancel{\hbar}}{\cancel{2ma}} \cdot \sqrt{\frac{\hbar}{2ma}} = \frac{ma}{\hbar} \cdot \sqrt{\frac{\hbar\pi}{2ma}} \; : \; ① = \frac{ma}{\hbar} \sqrt{\frac{\hbar\pi}{2ma}}$$

$$② : \int_{-\infty}^{\infty} \lambda e^{-\lambda x^2} dx = 2\lambda \int_0^{\infty} e^{-\lambda x^2} dx \underset{n=0}{=} 2\lambda \cdot \left(\sqrt{\pi} \frac{(0)!}{0!} \cdot \frac{1}{2} \cdot \sqrt{\frac{\hbar}{2ma}}\right) = \lambda\sqrt{\pi} \cdot \sqrt{\frac{\hbar}{2ma}}$$

$$② = \frac{2ma}{\hbar} \sqrt{\frac{\hbar\pi}{2ma}} \; : \; ① - ② = \frac{ma}{\hbar} \sqrt{\frac{\hbar\pi}{2ma}} - \frac{2ma}{\hbar} \sqrt{\frac{\hbar\pi}{2ma}} = -\frac{ma}{\hbar} \sqrt{\frac{\hbar\pi}{2ma}}$$

$$-\hbar^2 \cdot A^2 \cdot -\frac{ma}{\hbar} \sqrt{\frac{\hbar\pi}{2ma}} = A^2 \cdot ma\hbar \cdot \sqrt{\frac{\hbar\pi}{2ma}} = ma\hbar \cdot \sqrt{\frac{\cancel{2ma}}{\cancel{\hbar\pi}}} \sqrt{\frac{\cancel{\hbar\pi}}{\cancel{2ma}}} = ma\hbar$$

$$\langle p^2 \rangle = ma\hbar$$

$$\boxed{\begin{array}{l} \langle x \rangle = 0, \langle p \rangle = 0 \\ \langle x^2 \rangle = \frac{\hbar}{4ma}, \; \langle p^2 \rangle = ma\hbar \end{array}}$$

d.) $\sigma_x \cdot \sigma_p$?

$$\sigma_x = \sqrt{\hbar/4ma}, \; \sigma_p = \sqrt{ma\hbar} \; : \; \sigma_x \cdot \sigma_p = \sqrt{\frac{\hbar}{4ma} \cdot ma\hbar} = \sqrt{\frac{\hbar^2}{4}} = \frac{\hbar}{2}$$

$$\boxed{\sigma_x \sigma_p \geq \frac{\hbar}{2} \; ✓}$$

Problem 1.10

a.) 3.14159265358979323846264 33

$j=1$: $P = 2/25$
$j=2$: $P = 3/25$
$j=3$: $P = 5/25$
$j=4$: $P = 3/25$
$j=5$: $P = 3/25$
$j=6$: $P = 3/25$
$j=7$: $P = 1/25$
$j=8$: $P = 2/25$
$j=9$: $P = 3/25$

#1 : $P = 2/25 = 8\%$
#2 : $P = 3/25 = 12\%$
#3 : $P = 5/25 = 20\%$
#4 : $P = 3/25 = 12\%$
#5 : $P = 3/25 = 12\%$
#6 : $P = 3/25 = 12\%$
#7 : $P = 1/25 = 4\%$
#8 : $P = 2/25 = 8\%$
#9 : $P = 3/25 = 12\%$

b.) 1122233333444555666788999

most probable = # 3
median = # 4
average = 4.72

c.) $\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$

$$\langle x\rangle = \sum_{j=0}^{i} j\cdot P(j) = 1\cdot\tfrac{2}{25} + 2\cdot\tfrac{3}{25} + 3\cdot\tfrac{5}{25} + 4\cdot\tfrac{3}{25} + 5\cdot\tfrac{3}{25} + 6\cdot\tfrac{3}{25} + 7\cdot\tfrac{1}{25} + 8\cdot\tfrac{2}{25} + 9\cdot\tfrac{3}{25}$$

$$\langle x\rangle = \frac{118}{25}, \quad \langle x\rangle^2 = \frac{13924}{625}$$

$$\langle x^2\rangle = \sum_{j=0}^{i} j^2\cdot P(j) = 1^2\cdot\tfrac{2}{25} + 2^2\cdot\tfrac{3}{25} + 3^2\cdot\tfrac{5}{25} + 4^2\cdot\tfrac{3}{25} + 5^2\cdot\tfrac{3}{25} + 6^2\cdot\tfrac{3}{25} + 7^2\cdot\tfrac{1}{25} + 8^2\cdot\tfrac{2}{25} + 9^2\cdot\tfrac{3}{25}$$

$$\langle x^2\rangle = \frac{142}{5}$$

$$\sigma = \sqrt{\frac{142}{5} - \frac{13924}{625}} = 2.47$$

$$\boxed{\sigma = 2.47}$$