MAT 201
Larson/Edwards – Section 3.6
A Summary of Curve Sketching

In this chapter we will be focusing on sketching polynomial, rational, radical, and trigonometric graphs. In addition to what we already know from College Algebra, tools in calculus such as concavity, relative extrema, inflection points, and limits at infinity (for asymptotes) allow us to sketch more accurate graphs.

<u>Tools for Curve-Sketching</u>:

- What type of function is f?
- What are the x- and y-intercept(s)?
- Symmetry?
- What's the domain and range?
- Is f continuous?
- Is f differentiable?
- Where is f increasing or decreasing?
- What are the relative extrema?
- What are the intervals of concavity?
- What are the points of inflection?
- What is the end behavior, that is, infinite limits at infinity?
- Does f have vertical or horizontal asymptotes?

Ex: Sketch the graph of the following functions:

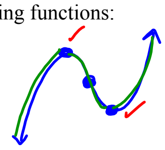
a) $y = 2x^3 + 7x^2 - 5x - 4$

b) $y = \dfrac{9x^2}{x^2 - x - 2}$

Ex: Sketch the graph of the following functions:

a) $y = 2x^3 + 7x^2 - 5x - 4$

Polynomial function



$y' = 6x^2 + 14x - 5$

$0 = 6x^2 + 14x - 5$

$x = \dfrac{-14 \pm \sqrt{(14)^2 - 4(6)(-5)}}{2(6)}$

Crit. #'s.

$x \approx 0.31, -2.65$

Inc.	Dec.	Inc.
$(-\infty, -2.65)$	$(-2.65, 0.31)$	$(0.31, \infty)$
$y'(-3) > 0$	$y'(0) < 0$	$y'(1) > 0$

Rel. Max @ $(-2.65, 21.2)$

Rel. Min. @ $(0.31, -4.8)$

$y'' = 12x + 14$

$0 = 12x + 14$

$-\frac{7}{6} = x$

Conc. ↓	Conc. ↑
$(-\infty, -\frac{7}{6})$	$(-\frac{7}{6}, \infty)$
$y''(-2) < 0$	$y''(0) > 0$

P.O.I. $(-1\frac{1}{6}, 8.2)$

y-int: $(0, -4)$

x-int: $0 = 2x^3 + 7x^2 - 5x - 4$ $(x-1)(2x+1)(x+4)$

$\frac{P}{Q} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{2},$

Synthetic Division

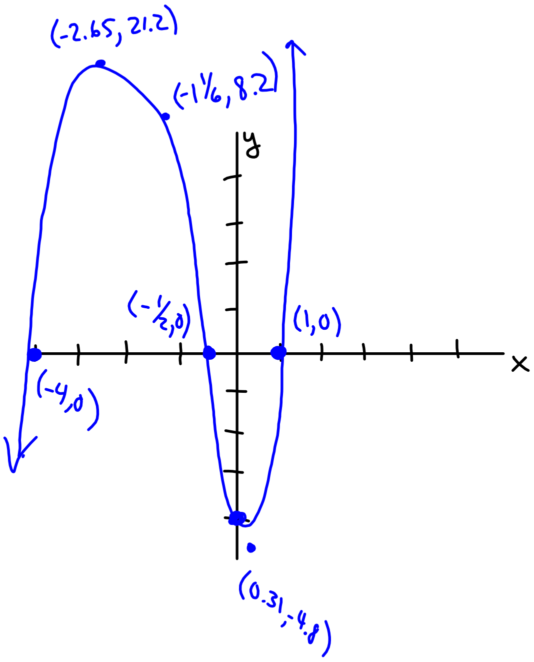
$$\begin{array}{r|rrrr} 1 & 2 & 7 & -5 & -4 \\ & & 2 & 9 & 4 \\ \hline & \multicolumn{3}{l}{2x^2+9x+4} & \boxed{0} \end{array}$$

$(x-1)$

$(2x+1)(x+4) = 0$

$x = \frac{-1}{2}, -4$

x-ints:
$(1,0)$
$(-\frac{1}{2}, 0)$
$(-4, 0)$



b) $y = \dfrac{9x^2}{1x^2 - x - 2} = \dfrac{9x^2}{(x-2)(x+1)}$

V.A.: $x = 2$, $x = -1$ H.A.: $y = 9$

x-int: $0 = 9x^2$ $x = 0$ $(0,0)$

$$y' = \frac{18x(x^2-x-2) - (2x-1)(9x^2)}{(x^2-x-2)^2}$$

$$y' = \frac{\cancel{18x^3} - 18x^2 - 36x - \cancel{18x^3} + 9x^2}{(x^2-x-2)^2}$$

$$y' = \frac{-9x^2 - 36x}{(x^2-x-2)^2}$$

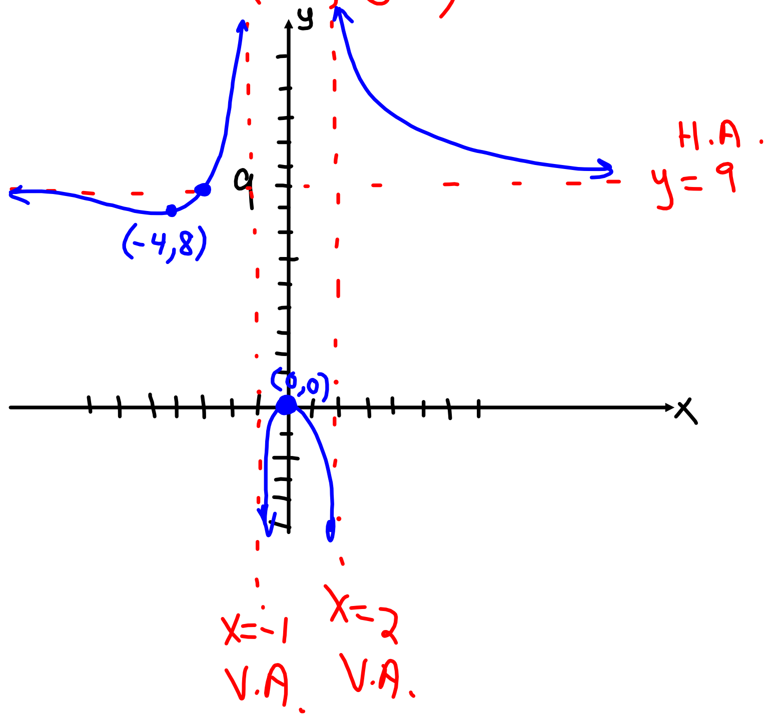
Crit. #'s. $x = 2, -1, 0, -4$

$0 = -9x(x+4)$

dec.	inc.	inc.	dec.	dec.
$(-\infty, -4)$	$(-4, -1)$	$(-1, 0)$	$(0, 2)$	$(2, \infty)$
$y'(-5) < 0$	$y'(-2) > 0$	$y'(-0.5) > 0$	$y'(1) < 0$	$y'(3) < 0$

Rel. min. $(-4, 8)$

Rel. Max $(0, 0)$

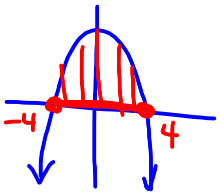


c) $y = x\sqrt{16 - x^2}$

d) $y = x^{1/3}(x + 3)^{2/3}$

c) $y = x\sqrt{16 - x^2} = x(16-x^2)^{1/2}$ Domain: $[-4, 4]$

$16 - x^2 \geq 0$



$x = -4 \Rightarrow y = -4\sqrt{0} = 0$

$(-4, 0)$

$x = 4 \Rightarrow y = 4\sqrt{0} = 0$

$(4, 0)$

y-int: $(0, 0)$

$$y' = \sqrt{16-x^2} + \frac{1}{2}(16-x^2)^{-1/2}(-2x)x$$

$$y' = \left(\frac{\sqrt{16-x^2}}{1}\cdot\frac{\sqrt{16-x^2}}{\sqrt{16-x^2}} - \frac{x^2}{\sqrt{16-x^2}}\right)$$

$$y' = \frac{16-x^2-x^2}{\sqrt{16-x^2}} = \frac{-2x^2+16}{\sqrt{16-x^2}}$$

$0 = -2x^2 + 16$

$-16 = -2x^2$

$8 = x^2$

$\pm 2\sqrt{2} = x$

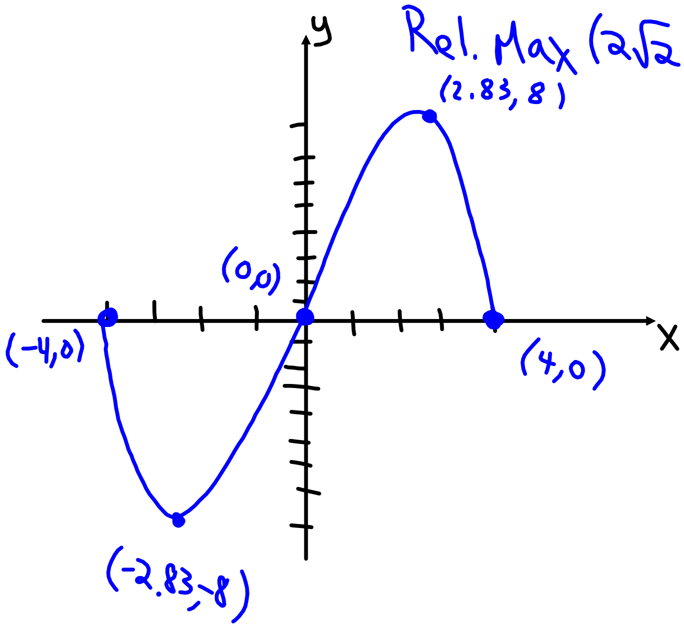
dec. -2.83 inc. dec.

$(-4, -2\sqrt{2})$ $(-2\sqrt{2}, 2\sqrt{2})$ $(2\sqrt{2}, 4)$

$y'(-3) < 0$ $y'(0) > 0$ $y'(3) < 0$

Rel. min. $(-2\sqrt{2}, -8)$

Rel. Max $(2\sqrt{2}, 8)$



d) $y = x^{1/3}(x+3)^{2/3}$ D: $(-\infty,\infty)$

x-int: $(-3,0)$, $(0,0)$

$$y' = \frac{1}{3}x^{-2/3}(x+3)^{2/3} + \frac{2}{3}(x+3)^{-1/3}x^{1/3}$$

$$y' = \frac{(x+3)^{2/3}(x+3)^{1/3}}{3x^{2/3}(x+3)^{1/3}} + \frac{2x^{1/3}x^{2/3}}{3(x+3)^{1/3}x^{2/3}}$$

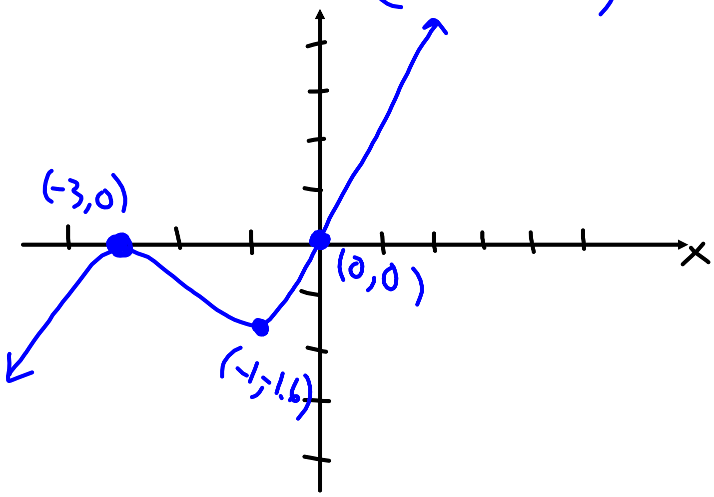
$$y' = \frac{x+3+2x}{3x^{2/3}(x+3)^{1/3}} = \frac{3x+3}{3x^{2/3}(x+3)^{1/3}} = \frac{x+1}{x^{2/3}(x+3)^{1/3}}$$

Crit. #'s: $x = -1, 0, -3$

inc.	dec.	inc.	inc.
$(-\infty,-3)$	$(-3,-1)$	$(-1,0)$	$(0,\infty)$
$y'(-4)>0$	$y'(-2)<0$	$y'(-0.5)>0$	$y'(1)>0$

Rel. Max @ $(-3,0)$

Rel. Min. @ $(-1,-1.6)$



e) $y = x + \cos x$, on the interval $[0, 2\pi]$