Main Points:

1) Utility of Taylor Expansion

$$T: \quad f(x) = \sum_{n=0} \frac{1}{n!} \frac{\partial^n f}{\partial x^n}\bigg|_{x=x_0} (x-x_0)^n$$

$$M: \quad f(x) = \sum_{n=0} \frac{1}{n!} \frac{\partial^n f}{\partial x^n}\bigg|_{x=0} x^n$$

2) Linearization for small arguments.
3) Many ways to represent FN'S.
4) Complex representation

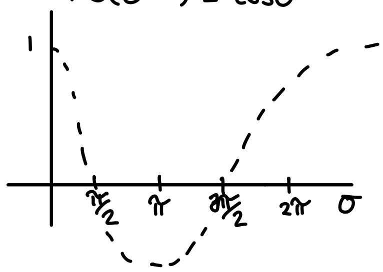
$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\left.\begin{array}{l}\theta = \omega t \\ \omega = 2\pi f\end{array}\right\} \text{Temporal oscillations}$$

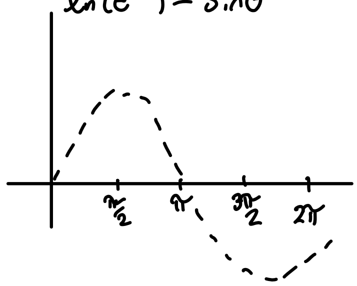
$\theta = Kx$ Spatially periodic systems
$K = 2\pi/\lambda$

$\theta = Kx - \omega t$ Spatio-Temporal waves. (Traveling)

$Re(e^{i\theta}) = \cos\theta$



$Im(e^{i\theta}) = \sin\theta$



Linear Lumped System (n^{th} order Lin O.D.E w/Const. coefs)

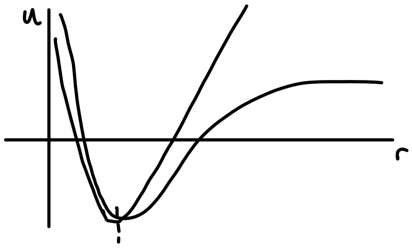
$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_1 \frac{dx}{dt} + a_0 x = F(t)$$

Constants (↑ a_n)

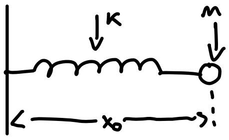
Forcing Function (↑ $F(t)$)

$F = 0$ Homogeneous
$F \neq 0$

1.) Small amplitude mechanical oscillations
2.) RLC Circuits
3.) Control Systems
4.) Bacterial colony growth
5.) Chemical reactions
6.) Atomic electrons



S.H.O



$F = ma$

$-k(x-x_0) = ma = m\ddot{x}$

$\ddot{x} + \frac{K}{m}(x-x_0) = 0 \qquad z = x - x_0$

$\omega_0^2 = k/m$

$$\ddot{z} + \omega_0^2 z = 0 \quad z \sim e^{mt} \Rightarrow z = e^{\pm i\omega_0 t} \Rightarrow \boxed{z = Ae^{i\omega_0 t} + Be^{-i\omega_0 t}}$$

$m^2 e^{mt} + \omega_0^2 e^{mt} = 0$

$m^2 + \omega_0^2 = 0$

$m^2 = -\omega_0^2$

$m = \pm i\omega_0$

or

$z = Ce^{i(\omega_0 t + \phi)}$

"Initial" conditions:

$\begin{cases} z_1 @ t_1 \\ \dot{z}_1 @ t_1 \end{cases}$

$\left| \begin{matrix} z_1 @ t_1 \\ z_2 @ t_2 \end{matrix} \right|$ or $\left| \begin{matrix} \dot{z}_1 @ t_1 \\ \dot{z}_2 @ t_2 \end{matrix} \right|$

$z(0) = 0$

$\dot{z}(0) = v = \sqrt{\frac{2T}{m}}$

$T = \frac{1}{2}mv^2$

$A + B = 0 \Rightarrow A = -B$

$z(t) = Ae^{i\omega_0 t} - Ae^{-i\omega_0 t}$

$(i\omega_0 A + i\omega_0 A) = \sqrt{\frac{2T}{m}}$

$2i\omega_0 A = \sqrt{\frac{2T}{m}}$

$A = \frac{1}{2i\omega_0}\sqrt{\frac{2T}{m}}$

$$z(t) = \frac{1}{2i\omega_0}\sqrt{\frac{2T}{m}}\left(e^{i\omega_0 t} - e^{-i\omega_0 t}\right)$$

$$= \frac{1}{\omega_0}\sqrt{\frac{2T}{m}}\sin\omega_0 t = \sqrt{\frac{2T}{K}}\sin\omega_0 t$$

$\omega_0 = \sqrt{\frac{K}{m}}$

$\ddot{z} + \omega_0^2 z = \frac{F(t)}{m} \quad : \quad F(t) = F_0\cos(\omega t)$

$Re(F_0 e^{i\omega t}) \Rightarrow z(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)}\cos(\omega t)$

$$\frac{-\omega^2 F_0}{m(\omega_0^2-\omega^2)}\cos(\omega t) + \frac{\omega_0^2 F_0}{m(\omega_0^2-\omega^2)}\cos(\omega t) = \frac{F_0}{m}\cos(\omega t)$$

$$\frac{-\omega^2}{(\omega_0^2-\omega^2)} + \frac{\omega_0^2}{(\omega_0^2-\omega^2)} = 1$$

The Driven, damped Oscillator:

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F(t)}{m} \qquad z(t) = \frac{F(t)}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

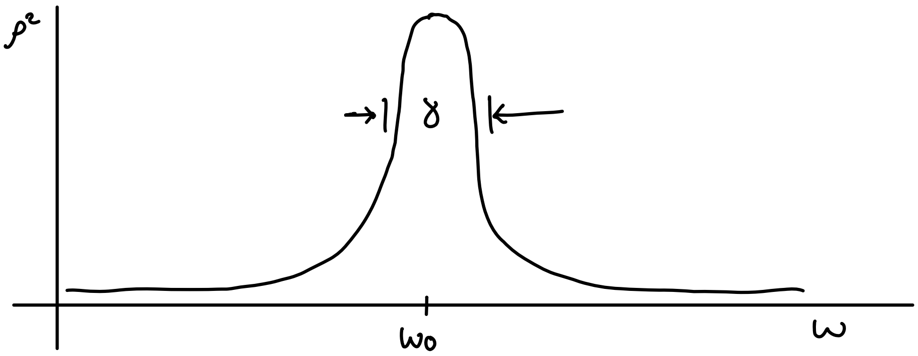
$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = V(t) \qquad R = \frac{1}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} \qquad R = \rho e^{i\phi}$$

$$z(t) = R\,F(t) = \rho e^{i\phi} F_0' e^{i(\omega t + \Delta)}$$

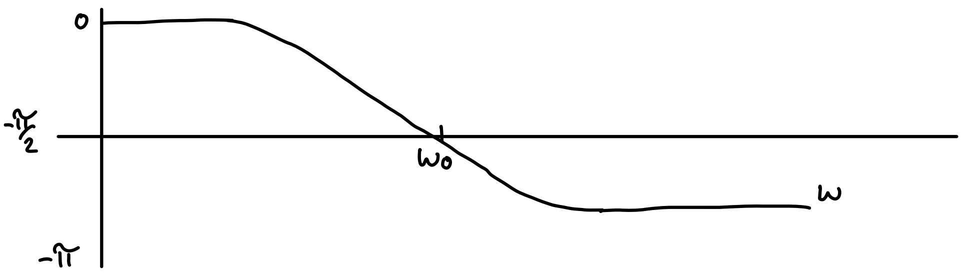
$$z = \rho F e^{i(\omega t + \Delta + \phi)}$$

$$\mathrm{Re}(z) = \rho F \cos(\omega t + \Delta + \phi) \qquad \rho^2 = \rho^* \rho = \frac{1}{m(\omega_0^2 - \omega^2 - i\gamma\omega)} - \frac{1}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

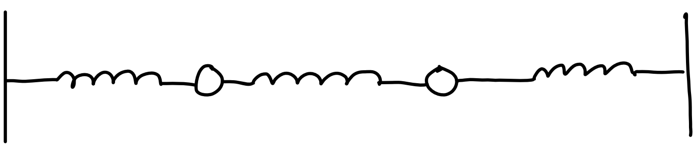
complex conjugate



$$Q = \frac{\omega_0}{\gamma} \qquad \tan\phi = \frac{-\gamma\omega}{\omega_0^2 - \omega^2}$$



Two object oscillator



In-Phase

O –▷   O –▷

◁– O ◁– O

Out of Phase

O –▷ ◁– O

◁– O O –▷