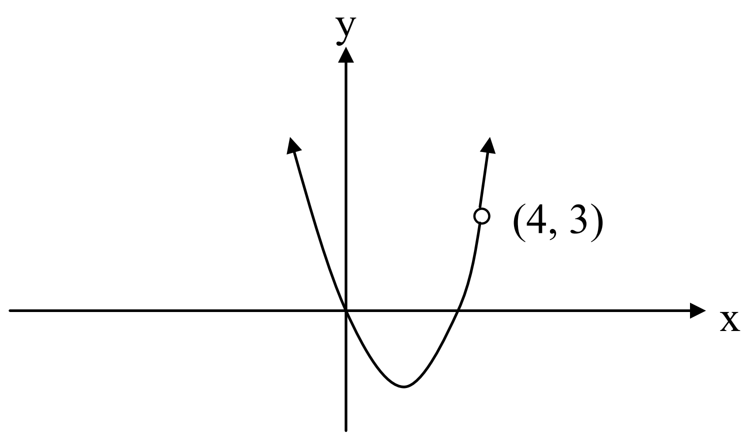
MAT 201
Larson/Edwards – Section 1.2
Finding Limits Graphically and Numerically

Eventually in calculus we will find the slope of a tangent line at a point by means of the derivative. The derivative has many practical applications as we will see in later sections. In order for us to understand the derivative we must understand what is meant by “limit” (mathematically speaking).

Finding Limits Graphically:

Take, for example, the graph of the function $f(x) = \dfrac{(x^2 - 4x + 3)(x - 4)}{(x - 4)}$.



If we wanted to find the “limit of *f(x)* as *x* approaches 4”, we would have to ask ourselves: To what output is the function approaching as the input *x* gets closer and closer (but not equal to) 4 from the right side AND left side? Graphically, we can see that the function *f* approaches 3 as *x* gets closer and closer to 4 (from the right and left).

Finding Limits Numerically:
Using a table of values we can make the same conclusion:

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)	2.61	2.9601	2.996	**?**	3.004	3.0401	3.41

x approaches 4 from the left → ← ***x approaches 4 from the right***

How do we write limits?

Notation: "the limit of $f(x) = x^2 - 4x + 3$ as x approaches 4 is 3"

translates to: $\lim_{x \to 4}\left(x^2 - 4x + 3\right) = 3$

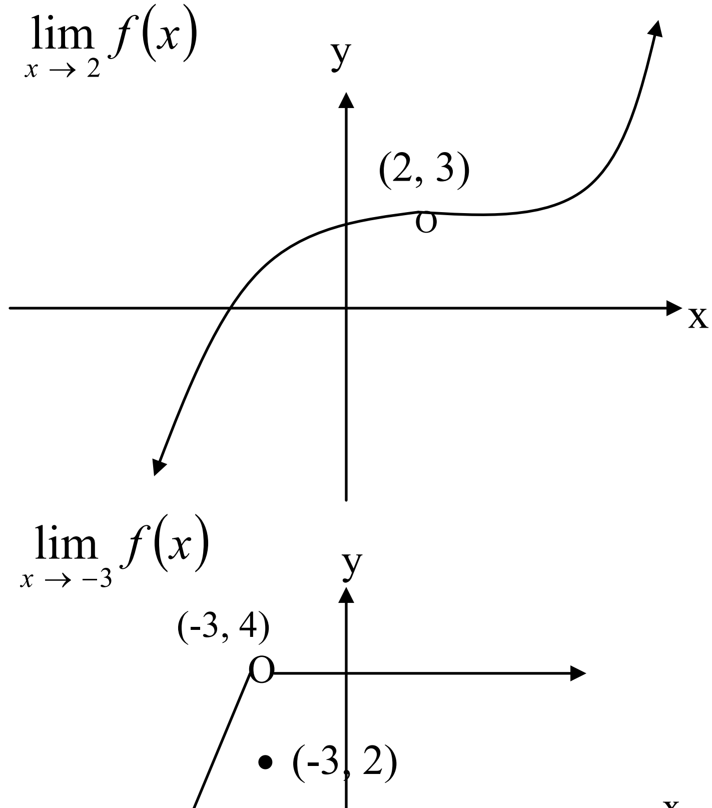
Does every function have a limit for all values in the domain of *f*?
Does the function have to be defined at the *x*-value that it is approaching in order to have a limit?
How do I determine if the limit of a function exists as *x* approaches a certain value *c*?

<u>Informal Definition of the Limit of a Function</u>: If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, then

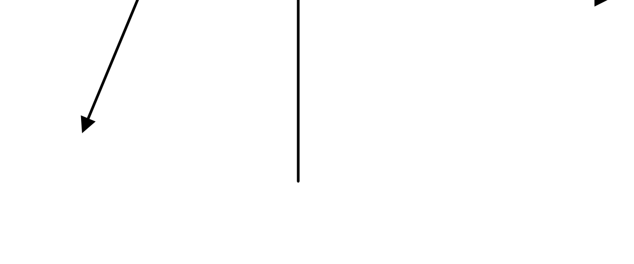
$$\lim_{x \to c} f(x) = L$$

Ex: Use the graph given of the function *f* to determine the limit if it exists:

a) $\lim_{x \to 2} f(x)$



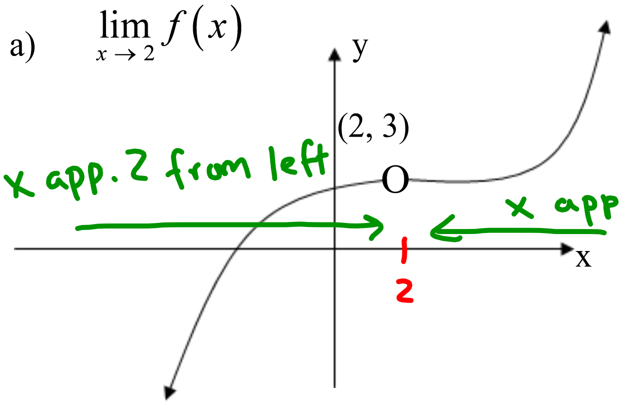
b) $\lim_{x \to -3} f(x)$



Ex: Use the graph given of the function f to determine the limit if it exists:

a) $\lim_{x \to 2} f(x)$

$$\boxed{\lim_{x \to 2} f(x) = 3}$$



x app. 2 from left

x approaches 2 from the right

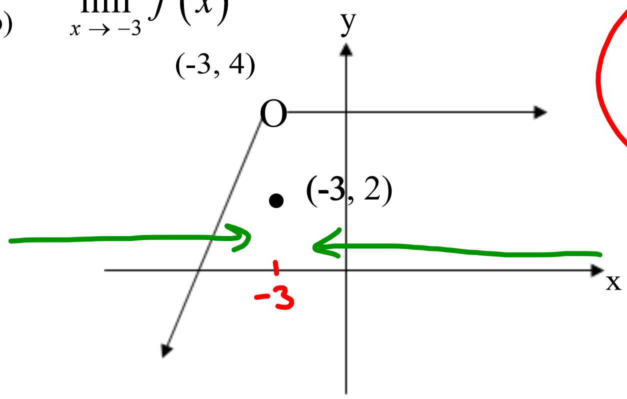
Right Hand Limit:

$$\lim_{x \to 2^+} f(x) = 3$$

Left Hand Limit

$$\lim_{x \to 2^-} f(x) = 3$$

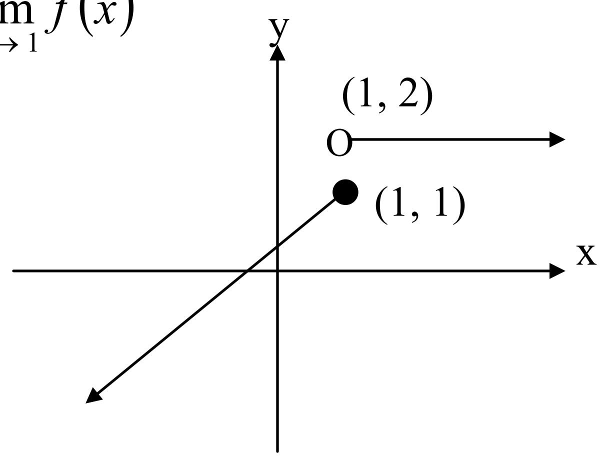
b) $\lim_{x \to -3} f(x)$



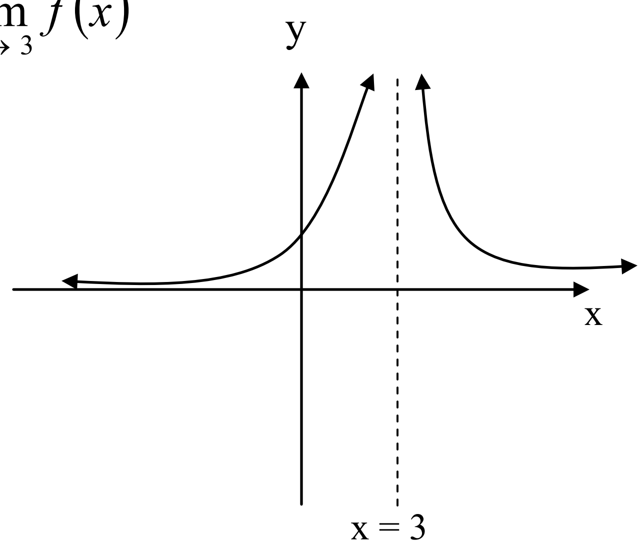
$$\lim_{x \to -3} f(x) = 4$$

Note: The existence or non-existence of *f(x)* at $x = c$ has no bearing on the existence of the limit of *f(x)* as *x* approaches *c*.

c) $\lim_{x \to 1} f(x)$



d) $\lim_{x \to 3} f(x)$

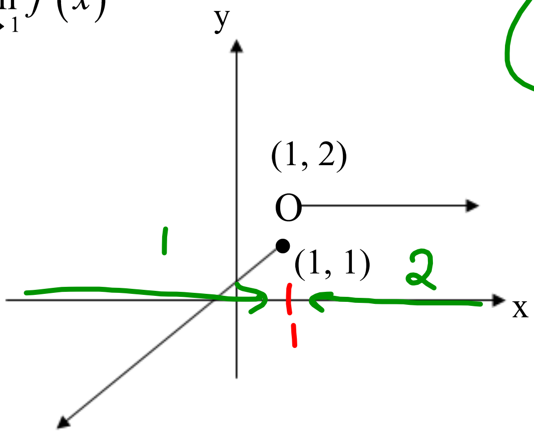


Note: *L* must be a **single** real number (not ∞ or -∞) in order for the limit to exist!

<u>Behavior Associated With Nonexistence of a Limit</u>:

1. *f(x)* approaches a different number from the right side of c than it approaches from the left side.
2. *f(x)* increases or decreases without bound as x approaches c.
3. *f(x)* oscillates between two fixed values as x approaches c.

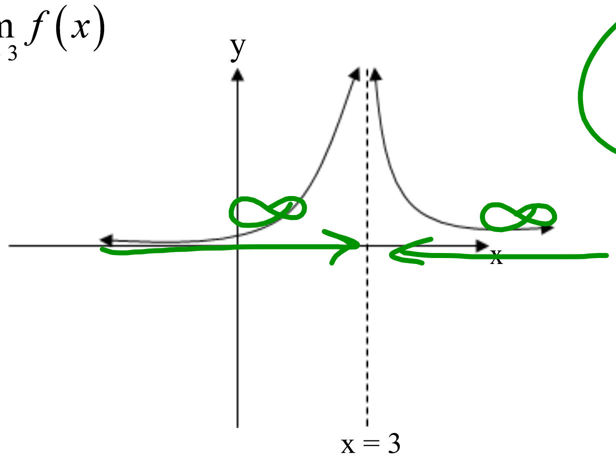
c) $\lim_{x \to 1} f(x)$



$\lim_{x \to 1} f(x)$ D.N.E.

Does Not exist.

d) $\lim_{x \to 3} f(x)$



$\lim_{x \to 3} f(x)$ D.N.E.

<u>Epsilon-Delta (ε – δ) Definition of a Limit</u>: The informal definition of a limit is subjective, that is, the exact meaning of the phrases "*f(x)* becomes arbitrarily close to *L*" and "*x* approaches *c*" is unclear. The first person to assign mathematical rigor to the informal definition was Augustin-Louis Cauchy (1789 – 1857). His epsilon-delta definition of limit is the standard used today.

Let *f* be a function defined on an open interval containing *c* (except possibly at *c*) and let *L* be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

Means that for each ε > 0 (ε is a small number) there exists a δ > 0 such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

What does this mean???

$0 < |x - c| < \delta$ means that *x* lies in either interval $(c - \delta, c)$ or interval $(c, c + \delta)$.

$|f(x) - L| < \varepsilon$ means that *f(x)* lies in the interval $(L - \varepsilon, L + \varepsilon)$.

Does every function have a limit for all values in the domain of f? NO
Does the function have to be defined at the x-value that it is approaching in order to have a limit? NO
How do I determine if the limit of a function exists as x approaches a certain value c?

Right hand limit & Left hand limit have to be defined and the same.