3.2 Systems of Linear Equations

3.2 # 32, 40, 52

3.2.32

$x_1 + x_2 + 2x_3 = 0$
$2x_1 - x_2 + x_3 = 0$
$4x_1 + x_2 + 5x_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & 5 & 0 \end{array}\right] = A \qquad \text{rref}(A) = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2^* = (-2)R_1 + R_2$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right]$ $R_2^* = -\frac{1}{3}R_2$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$R_3^* = R_3 - R_2$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $R_1^* = R_1 - R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

3.2.40

$2x + 4y - 2z = 0$
$5x + 3y + 0z = 0$

$$\left[\begin{array}{ccc|c} 2 & 4 & -2 & 0 \\ 5 & 3 & 0 & 0 \end{array}\right] = A \qquad \text{rref}(A) = \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{7} & 0 \\ 0 & 1 & -\frac{5}{7} & 0 \end{array}\right]$$

$x = \frac{2}{7}$
$y = -\frac{5}{7}$

$\frac{15}{7} - \frac{15}{7} + 0(-1) = 0$
$0 + 0 = 0$
0

$\frac{6}{7} - \frac{20}{7} - 2z = 0$
$-\frac{14}{7} - 2z = 0$
$-2 - 2z = 0$
$-2z = 2$
$z = -1$

$$\vec{x} = r\begin{bmatrix} \frac{2}{7} \\ -\frac{5}{7} \\ 1 \end{bmatrix} + \vec{0} \quad r \in \mathbb{R}$$

3.2.52 $A\vec{x} = \vec{0}$ Inconsistency ?

The system could be inconsistent if one of it's rows has zero values in the matrix equal to a constant. In this case, since the matrix is equal to zero it will have solutions and thus be consistent.