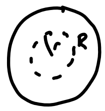
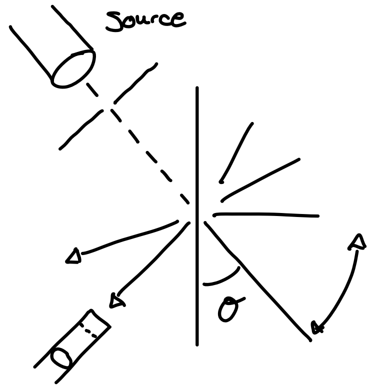
Rutherford Scattering

$e \simeq \frac{4}{3}\pi r^3 \rho$





$F = -\frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi r^3 \rho\right)\frac{e}{r^2}$

$= -\frac{\rho e}{3\varepsilon_0}$ $\quad r \sim -k\nu$

$k \sim \frac{\rho l}{3\varepsilon_0}$

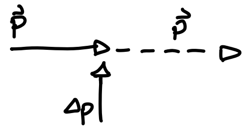
$\frac{1}{\lambda} \sim \left(\frac{1}{n^2} - \frac{1}{k^2}\right)$

Conservation of Momentum:

$\alpha : m_\alpha$
$\quad V_\alpha$
$e : m_e$
$\quad (V_e = 0)$

$m_\alpha >> m_e$

$|V_\alpha'| \approx |V_\alpha|$
$|V_e'| \approx 2|V_\alpha|$



$(\Delta\vec{p} \perp \vec{p}_i)$

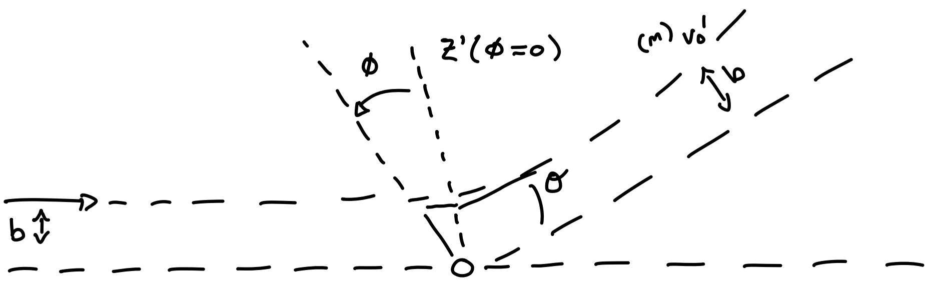
$\theta = \frac{\Delta P_\alpha}{P_\alpha} = \frac{2 m_e}{M_\alpha} \approx 0.016^\circ$

For head-on collision with single e^- (At rest)

$m_\alpha V_\alpha = m_\alpha V_\alpha' + m_e V_e'$

$\Delta \vec{P}_\alpha = m_\alpha \vec{V}_\alpha - m_\alpha \vec{V}_\alpha' = m_e \vec{V}_e'$

$= m_e (2 V_\alpha)$



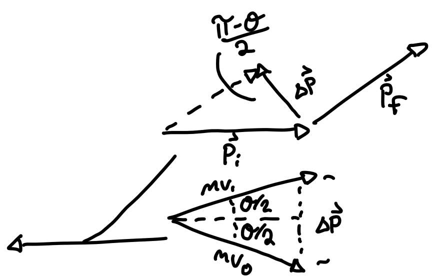
Assumptions

1) Scattering center massive enough to ignore recoil.
2) Targe thin enough so that beam "sees" all atoms
3) Size of target & projectile negligible => point mass / charge
4) The only force acting is Coulombs law

The change in momentum:

$\Delta\vec{p} = \int F_\parallel \, dt$

$\Delta p = 2 m V_0 \sin\left(\frac{\theta}{2}\right)$



For force:

$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\hat{r}$ component along $\uparrow$: $F_\parallel = F\cos\phi = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\cos\phi$

$\Delta p = 2 m v_0 \sin\left(\frac{\theta}{2}\right) = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\int \frac{\cos\phi}{r^2}\, dt$

To change variable of integration, use cons of angular momentum:

$m r^2 \frac{d\phi}{dt} = m v_0 b$

Transforming the integral:

$$-\phi_i + \phi_f + \theta = \pi$$

$$\phi_i = \frac{-(\pi-\theta)}{2} \quad \text{to} \quad \phi_f = \frac{\pi-\theta}{2}$$

$$2mv_0 \sin\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 v_0 b} \int \cos\phi \, \frac{d\phi}{dt} \cancel{dt}$$

So that $$\int_{\phi_i}^{\phi_f} \cos\phi \, d\phi = 2\cos\frac{\theta}{2}$$

$$2mv_0 \sin\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 v_0 b}\left(2\cos\frac{\theta}{2}\right)$$

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 (mv_0)^2}\cot\left(\frac{\theta}{2}\right) = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K}\cot\left(\frac{\theta}{2}\right)$$

Scattering Cross-Section (Target Area)

$\sigma = \pi b^2$

~ Probability of hitting a nucleus.

About the target:

n: # atoms/vol.

τ: Target thickness

$\frac{\#}{\text{Area}} = n\tau$

atoms in target

ρ: Density

N_A: # molecules/mol

N_m: # atoms/molecule

M_m: molar mass (g/mol)

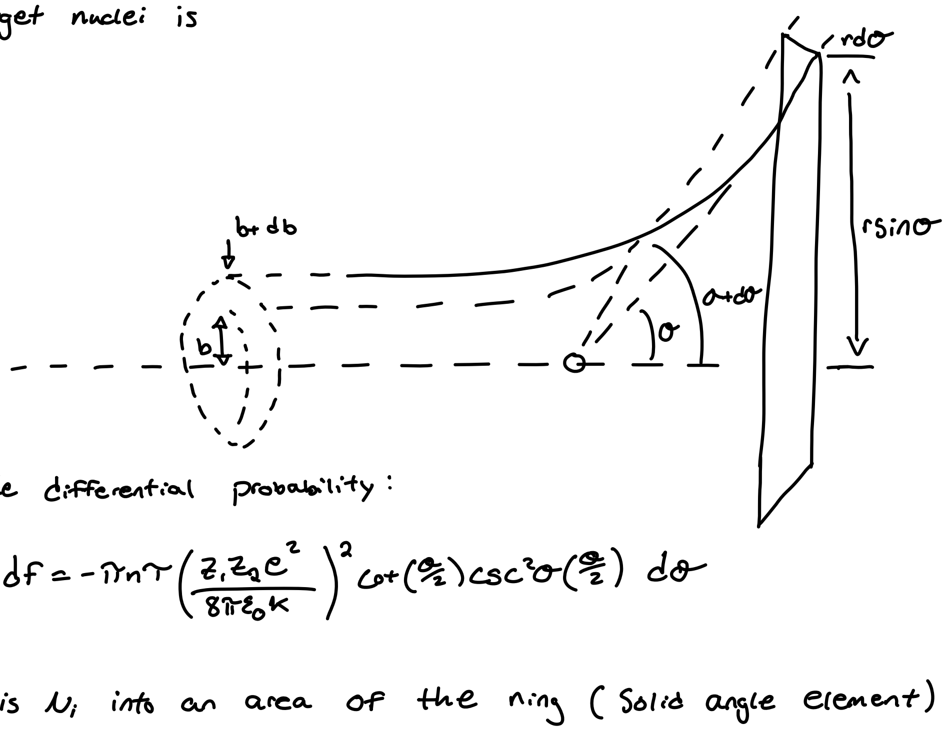
For foil of area A the # of target nuclei is

$$N_s = n\tau A = \frac{\rho N_A N_m \tau A}{M_m}$$

Probability of Scatter f:

$$f = \frac{\text{total area of Scatterers}}{\text{total target area}}$$

$$f = n\tau\sigma = n\tau\pi b^2$$



The differential probability:

$$df = -\pi n\tau\left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K}\right)^2 \cot\left(\frac{\theta}{2}\right)\csc^2\theta\left(\frac{\theta}{2}\right) d\theta$$

If total # of incident particles is N_i into an area of the ring (Solid angle element)

$$dA = 2\pi r^2 \sin\theta \, d\theta$$

Then the # of Scattered into a ring $d\theta$

$$\sim N_i |df|$$

Total # Scattered into Angular ring :

$$N(\theta) = \frac{N_i |df|}{dA} = \frac{N_i n \tau}{16} \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4(\theta/2)}$$