1.) $F(x,y) = (2x-6y)\hat{i} + (-6x+10y-7)\hat{j}$

$P = (2x-6y)$ $\int_x P = (x^2-6xy+g(y)) \Rightarrow f_y = -6x + g'(x)$

$Q = (-6x+10y-7)$ $g(x) = 10y-7$ $G(y) = 5y^2-7y+K$

$x^2-6xy+5y^2-7y+K$

$P = (2x-6y)$ $\int P\,dx = x^2-6xy+g(y) \Rightarrow f_y = -6x+g'(y)$

$Q = (-6x+10y-7)$ $-6x+10y-7 = -6x+g'(y)$

$g'(y) = 10y-7$

$G(y) = 5y^2-7y+K$

$x^2-6xy+5y^2-7y+K$

2.) $F(x,y) = e^x\sin y\,\hat{i} + e^x\cos y\,\hat{j}$

$P = e^x\sin y$ $Q = e^x\cos y$

$e^x\sin y + K$

$\frac{\partial Q}{\partial x} = e^x\cos y$ $f_y(e^x\sin y + g(y)) = e^x\cos y + g'(y)$

$\frac{\partial P}{\partial y} = e^x\cos y$ $g'(y) = 0$

$f\ e^x\sin y + 0$

$P = e^x\sin y$ $\int P\,dx = e^x\sin y + g(y)$ $P_y = e^x\cos y + g'(y)$

$Q = e^x\cos y$ $e^x\cos y = e^x\cos y + g'(y)$

$g'(y) = 0$

$g(y) = K$ $e^x\sin y + K$

3.) $F(x,y) = e^x\cos y\,\hat{i} + e^x\sin y\,\hat{j}$

$P = e^x\cos y$ $\frac{\partial P}{\partial y} = -e^x\sin y$

$Q = e^x\sin y$ $\frac{\partial Q}{\partial x} = e^x\sin y$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ $\therefore$ None

4.) $F(x,y) = (\ln y + 16xy^3)\hat{i} + (24x^2y^2 + \frac{x}{y})\hat{j}$

$P = \ln y + 16xy^3$ $\frac{\partial P}{\partial y} = \frac{1}{y} + 48xy^2$

$Q = 24x^2y^2 + \frac{x}{y}$ $\frac{\partial Q}{\partial x} = 48xy^2 + \frac{1}{y}$

$P^x = x\ln y + 8x^2y^3 + g(y)$

$P^x_y = \frac{x}{y} + 24x^2y^2 + g'(y)$

$24x^2y^2 + \frac{x}{y} = \frac{x}{y} + 24x^2y^2 + g'(y)$

$\downarrow$

0

$g(y) = K$

$x\ln y + 8x^2y^3 + K$

$P = (\ln y + 16xy^3)$ $\int P\,dx = x\ln y + 8x^2y^3 + g(y)$ $P_y = \frac{x}{y} + 24x^2y^2 + g'(y)$

$Q = (24x^2y^2 + \frac{x}{y})$ $24x^2y^2 + \frac{x}{y} = \frac{x}{y} + 24x^2y^2 + g'(y)$

$g'(y) = 0$

$x\ln y + 8x^2y^3 + K$ $g(y) = K$

5.) $F(x,y) = \langle 2xy, x^2 \rangle$ Start $(1,2)$ End $(3,2)$

$P = 2xy$ $P_y = 2x$ $P^x = x^2y + g(y)$

$Q = x^2$ $Q_x = 2x$ $P^x_y = x^2 + g'(x)$

Q P^x_y

$x^2 = x^2 + g'(y)$ $g'(y) = 0$

Common value $= 0$

$\int_c \nabla\vec{f}\cdot d\vec{r} = f(\text{End}) - f(\text{initial})$

$f(3,2) - f(1,2)$

$P^x(f(3,2) - f(1,2))$

$9(2) - 1(2)$

$18 - 2 + K - K$

16

6.) $F(x,y) = x^2\hat{i} + y^2\hat{j}$

C is $y = 4x^2$ from $(-1,4)$ to $(2,16)$

a.) $P = x^2 \quad Q = y^2$

$P^x = \frac{1}{3}x^3 \quad Q^y = \frac{1}{3}y^3$

$\frac{1}{3}x^3 + \frac{1}{3}y^3 + C$

$\frac{x^3+y^3}{3} + K$

$f = (\frac{1}{3}x^3 + \frac{1}{3}y^3)$

$f(2,16) - f(-1,4)$

$(\frac{8}{3} + \frac{4096}{3}) - (-\frac{1}{3} + \frac{64}{3})$

$(\frac{4104}{3}) - (\frac{63}{3}) = \frac{4041}{3}$

$\frac{4041}{3}$

$P = x^2 \quad \int P\,dx = \frac{1}{3}x^3 + g(y) \quad P_y = g'(y)$

$Q = y^2 \quad y^2 = g'(y)$

$\frac{1}{3}y^3 = g(y)$

$\frac{x^3+y^3}{3} + K$

7.) $F(x,y,z) = yze^{xz}\hat{i} + e^{xz}\hat{j} + xye^{xz}\hat{k}$

C: $r(t) = (t^2+3)\hat{i} + (t^2-3)\hat{j} + (t^2-5t)\,k \quad 0 \le t \le 5$

$f_x = yze^{xz}$

$f_y = e^{xz}$

$f_z = xye^{xz}$

$F_x = ye^{xz} + p(y,z)$

$e^{xz} + P_y(y,z)$

$f_y = e^{xz} + P_y(y,z) \quad P_y(y,z) = 0$

$f_{xz} = xyze^{xz} + P_z$

$f(x,y,z) = ye^{xz} + K$

$P = yze^{xz} \quad \int P\,dx = ye^{xz} + h(y,z) \quad P_y = e^{xz} + h_y(y,z)$

$Q = e^{xz} \quad e^{xz} = e^{xz} + h_y(y,z)$

$R = xye^{xz} \quad h_y(y,z) = 0$

$h(y,z) = K$

$ye^{xz} + K$

$(t^2-3)e^{(t^2+3)(t^2-5t)} \Big|_0^5$

$(25-3)e^{(28)(0)} - ((-3)e^{(3)(0)})$

$22e^0 - (-3e^0)$

$22 + 3 = 25$

25

8.) $F(x,y) = 2y^{3/2}\hat{i} + 3x\sqrt{y}\,\hat{j}$

$A(1,1) \quad B(2,4)$

$P = 2y^{\frac{3}{2}} \quad Q = 3xy^{\frac{1}{2}}$

$\frac{\partial P}{\partial y} = 3y^{\frac{1}{2}} \quad \frac{\partial Q}{\partial x} = 3y^{\frac{1}{2}}$

$P^x = 2xy^{\frac{3}{2}} + g(y) \quad 3xy^{\frac{1}{2}} = 3xy^{\frac{1}{2}} + g'(y)$

$P^x_y = 3xy^{\frac{1}{2}} + g'(y) \quad g'(y) = 0$

$g(y) = K$

$f(x,y) = 2xy^{\frac{3}{2}} + K$

30

$2xy^{\frac{3}{2}} \Big|_{(1,1)}^{(2,4)}$

$2(2)(8) - 2(1)(1)$

$32 - 2 = 30$