Methods of Applied Mathematics - 21494 - MATH 360 - 001

Wubben 117 Fall 2018

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Office Hours: 1-1:50 pm MW; 9:25 – 10:50 am TR. Appointments also available (24 hours advanced notice recommended).

Text: Advanced Engineering Mathematics, 10th Edition, by Erwin Kreyszig; Wiley Publishers, 2011.

Course Description: This course provides an exposure to topics common in applications and in mathematical physics. Topics include vector calculus, Fourier analysis, partial differential equations, special functions, complex numbers and analytic functions. A more detailed listing of daily topics is given in the course calendar below.

Student Learning Objectives: The student learning outcomes for this course are to develop the following:

- Independent learning skills, in particular, reading skills
- > Problem-solving skills
- Mathematical language skills
- > Logical and critical thinking skills
- Persistence, skill in exploration, conjecture, generalization
- > Appreciate necessity for rigor and precision in mathematics
- Acquire mathematics background relevant to other subjects

Prerequisite: A grade of C or better in Math 260/Math 236, and a grade of C or better in Math 253.

Required:

- Study skills.
- Time (for reading, thinking, preparing for class, doing homework).
- Regular use of D2L materials.

Grading:

| Attendance & Participation | 5% |
|----------------------------|-----|
| Class Preps | 15% |
| Homework | 20% |
| Exams | 40% |
| Final Exam | 20% |

Grading Scale:

| or adding Scarc. | | |
|------------------|---|--|
| 90 - 100 % | A | |
| 80 - 89% | В | |
| 70 - 79% | С | |
| 60 - 69% | D | |
| 0 - 59% | F | |

Credit Hours Policy: An undergraduate student should expect to spend on this course a minimum of two hours outside of the classroom for every hour in the classroom. The outside hours may vary depending on the number of credit hours or type of course. More details are available from the faculty member or the department office and in CMU's Curriculum Policies and Procedures Manual.

Attendance & Participation: Attendance will be taken daily and converted into a percentage at the end of the semester. Class time will be used to develop and reinforce the following course-related content and activities:

- > Concepts from the reading
- Computational skills and methods
- Collaborations with classmates and instructor

Your attendance for this class will therefore be important. See *Attendance, Course Participation, and Academic Behavior* statements later in this document for further information on the role of attendance and class conduct.

Daily Class Preps: You will be expected to prepare for class each day by reviewing the book and course materials posted on D2L before & after we meet in the classroom. Further, to help develop your independent reading skills, mathematical language skills, and professional planning skills, you will be reading ahead & writing up section material to turn in before class. Additional information on class preps is given later in this document.

Homework: Written homework will be collected weekly at the beginning of class on Mondays; see course calendar for due dates. The homework will be graded on accuracy, completeness and neatness; see rubric given later in this document. Additional expectations regarding homework will be discussed in class. The lowest two homework scores will be dropped.

Exams: There will be two regular semester exams; see course calendar. Make-ups will not normally be given, except possibly for unusual and verifiable circumstances. In these instances you must obtain my approval before the exam date. Work, travel, vacation or any other non-college sanctioned activity is not an acceptable excuse for missing an exam.

Final Exam: The final exam will be cumulative, and all students are expected to take the final during this university-scheduled time. See course calendar.

Cell Phone Policy: At the beginning of class cell phones (and all other portable technology devices) must be silenced and put away out of reach, such as in a backpack. In-class image and video capture is not allowed except in unusual circumstances and permission must be obtained in advance. A few friendly reminders to put your phone away may be granted early in the semester, but for persistent violators each observed unapproved instance of a cell phone or technology violation will result in a 1 point deduction from their semester total (see Course Participation below), with additional consequences possible (see Academic Behavior below). Furthermore, during quizzes and exams, cell phone violations will be interpreted as acts of academic dishonesty.

Attendance: Daily attendance in class is crucial for your learning in this course. If you miss class, it is your responsibility to make up what was covered. The statement on attendance in the Maverick Guide, as part of the CMU Student Code of Conduct found online, indicates that instructors may initiate a drop or withdrawal for any student who fails to attend regularly. Read this attendance statement in your catalog carefully.

Course Participation: Course participation includes coming to class a few minutes early, being prepared for class, asking questions, participating in discussions and activities, and seeking help outside of class when appropriate. If you frequently do not come to class on time, are not prepared, and your participation detracts from the class (including unapproved cell phone and laptop use), then this will adversely affect your grade. Similarly, when you seek help during office hours, bring specific questions along with the work that you have attempted.

Academic Behavior: The overall goals of this class and of the college are for you to learn, to learn how to learn, and to pick up skills needed to be successful in life. Learning in the classroom requires an environment in which each student feels comfortable listening, thinking, concentrating, focusing, and asking questions. When you choose to attend class, you are agreeing to behave in such a way as to not disrupt the learning process of others. Otherwise, you jeopardize your enrollment in the class. So come to class prepared, interested in learning, and respectful of others. See the above paragraph on Course Participation for more guidelines on appropriate behavior in the classroom, and also in the CMU Student Code of Conduct found online.

Academic Dishonesty: Cheating is serious offense and will be treated as such. Cheating is an act of academic dishonesty, which includes using another person's work as though it was your own or knowingly permitting another student to use your work. The consequences of cheating on an assignment or quiz may result in the grade of 0 for all those involved, or in the case of an exam, an F for the course. These penalty scores are not subject to low-grade policies outlined earlier. Other situations involving cheating will be dealt with in a similar way. Further actions may be taken in accordance with the statement on academic dishonesty given in the CMU Student Code of Conduct found online.

Tutorial Learning Center (TLC)

The TLC is a FREE academic service for all Colorado Mesa University students. Tutors are available on a walk-in basis for many courses. Do you have a quick question? Do you need homework clarification or feedback on a paper? Are you reviewing for a test? Help is available at the TLC! Come to Houston Hall 113 to meet with one of our friendly peer tutors. We are open Monday - Thursday from 8am-6pm, and Fridays 8am-5pm. We are also open Sundays 1pm - 6pm. Tutoring at branch campuses and distance tutoring is also available. Check out the website for schedules and locations at www.coloradomesa.edu/tutoring or call 248-1392.

The Writing Center

The Writing Center (Houston Hall 223) serves students across all disciplines and various stages of the writing process. We provide support for students to assimilate into the writing conventions of the university and their specific academic disciplines. Hours of operation are Monday-Friday 10-5 pm. Tutoring in writing will be located at branch campuses also. Workshops for students in APA/MLA and English Language Learner small group tutoring is available.

Educational Access Services (EAS): In coordination with Educational Access Services, reasonable accommodations will be provided for qualified students with disabilities. Students must register with the EAS office in Houston Hall Suite 108 (248-1856) to receive assistance. Please meet with the instructor the first week of class for course information. https://www.coloradomesa.edu/educational-access/

General expectations for my courses:

- We will cover new material every day.
- > The first few minutes of class will be devoted to answering questions from the previous class.
- > If you have difficulties with the material, talk with me outside of class regarding extra help.
- Spend at least two hours each day, working on homework and studying the notes and text. Set aside this time period now in your schedule, preferably at the same time each day.
- Seek help before homework is due, and before quiz or exam
- Ask questions don't delay! Seek help on problems and difficulties as soon as possible. One minute of clarification right after class on a problem may save hours of frustration.
- > Be persistent. Learning math sometimes requires lots of effort before a breakthrough occurs.
- Look over the book before coming to class. Sometimes 5 - 10 minutes of glancing over the text ahead of time can make a big difference in getting a handle on material presented in class.

CMU STATEMENT ON SUCCESS

The faculty and staff are glad you have elected to attend Colorado Mesa University or Western Colorado Community College and want you to succeed in achieving your academic goals. The following information is shared with you to enhance the likelihood that you will be successful.

1. Attend class.

Institutional research shows that class attendance and participation are closely linked to your success as a student (i.e., the better your attendance, the better your grade is likely to be). When you are always present, you will understand the course content and how it contributes to your growth as a college student. You are required to attend this class regularly, adhering to the attendance policy established in this course syllabus by your instructor. Additionally, you should review the Attendance Policy of the institution's **Catalog** for further details on expectations. For online courses, check with your instructor and/or class syllabus for expectations delivered in that format.

2. Prepare for and participate in class.

It takes more than showing up for class to succeed. You need to be prepared to actively participate in class. Your instructor has given you a schedule of course topics for the semester, along with readings and/or activities that should be completed prior to coming to class. If you aren't clear about these expectations, talk with your instructor. In general, you should follow the 2:1 rule: two hours of study/homework time for every 1 hour of classroom time. This can vary some from week to week, but on average, most instructors will assume you are putting in the time and keeping pace with the class. So make the effort to stay current and don't leave everything to the end of the term.

By meeting deadlines and managing your time wisely, you will get much more from the class and earn higher grades. Assume that faculty members will not accept late homework and don't offer extra credit assignments. Some may – and by reviewing the syllabus you will know their policies – but instructors have no obligation to do so. A final note. If you need help with study skills, time management, note-taking and the like, consider registering for SUPP 101, a course that helps first-year students with their transition to college life.

3. Use technology to support your success.

All members of this class are expected to show respect to each other and to contribute to a positive academic learning environment of the class. Please turn off cellphones or set them to silent when you are in class. Text messaging, checking email, working on social networking sites, and performing non-class related activities on any electronic device (cell phone, laptop, iPad, etc.) is disruptive and not acceptable behavior during the class session. Check your course syllabus for the consequence of using these devices during class time.

4. Take advantage of campus resources.

We offer numerous academic support resources to help you. The staff of **Tomlinson Library** can assist you with finding information resources either in person or online. The **Tutorial Learning Center** offers *free*, *walk-in* tutoring for a wide variety of subjects. Maybe it's just a math problem that's not making sense, or perhaps having a peer take a look at your assignment is what you need. The TLC can help with the smallest issue or provide you with tutoring if you have a particularly challenging course. Get help before a small problem becomes a big one. Stop by and see the services they offer, most of which are provided by other students. If your semester gets a little overwhelming, contact the **Office of Student Services** for assistance. Need to engage in some activity outside of classes? Stop by the **Maverick Center** for a good workout, or find students with some similar interests by joining a **student club**.

5. Build relationships with your instructors, advisor, and other students.

a. Your best guidance for success will come from your instructors, and research tells us that your interactions with faculty members is the most important determinate in college success. Instructors genuinely want you to be successful and will do what they can to help you reach your goals. Locate their contact information on the syllabus and store that information in your phone. Each instructor keeps office hours that they set aside to meet with students. If you cannot meet during their office hours, schedule an appointment in advance.

b. Plan to meet with your advisor at least once a semester. At a minimum, consult with your advisor on your schedule for the next semester before registration opens. Popular required courses fill quickly, so if you delay registration, you might not get your preferred courses and could possibly delay your graduation. Advisors provide valuable assistance in determining which courses you need to take for your degree and the best order to take courses. Advisors can also direct you to the most appropriate networks when you are in need of assistance.

If you do not know the name of your advisor, log into MAVzone and click on Student Academics tab. Scroll down the Academic Profile column to Advisors; directly email your advisor by clicking on envelope icon.

c. Connect with other students in all your classes. You and your peers have similar goals and will face similar challenges; this can help you feel less alone in solving problems. Being active in a study group can enrich your understanding of course materials and can provide extra motivation and support to succeed. Learn more about the value of creating a study group at *Fight for First Year in College: Form Study Groups* at http://www.academictips.org/acad/first-y-i-c/formstudygroups.html

6. Use financial aid wisely.

Be aware that your decisions about attending class and considering whether to add or drop a class can affect your financial aid. Discuss potential changes with your advisor before making them. You must complete at least 12 credit hours each semester to be considered full-time, often a requirement to receive financial aid. Part-time students should check with the **Office of Financial Aid** for credit hour requirements. Audited classes do not count for enrollment purposes.

To retain your aid for the next term, you are required to make satisfactory academic progress toward your degree and maintain the following minimum grade point averages below. If you receive all F's for one term, you will be suspended from financial aid and must repay all Title IV funds.

| Cumulative Credit Hours Earned | Minimum GPA | |
|---------------------------------------|-------------|--|
| 1 to 15 | 1.70 | |
| 16 to 30 | 1.80 | |
| 31 to 45 | 1.90 | |
| 46+ | 2.00 | |

To remain eligible to receive financial aid, students must be successfully completing 75% of classes attempted. Aid will be suspended until the student successfully increases the completion rate to 75%. Be sure to report any changes in your enrollment, residency status, or receipt of additional resources in writing to the Office of Financial Aid. Financial aid is not available if you have not graduated from your program but exceed the total undergraduate cumulative hours as show below.

Baccalaureate degree: 170 hours Associate degree: 80 hours One-year certificate: 40 hours

Math 360 Methods of Applied Mathematics

Fall 2018 Course Calendar

| | Monday | Wednesday | Friday |
|----|---|--|--|
| | Syllabus & Course Intro | - | |
| | 9.1 – 9.3 Review of Vectors, | 9.4 Vector & Scalar Functions | |
| , | Dot Product, Cross Product | 9.5 Curves, Arc Length, Curvature | 9.5 Curves, Arc Length, Curvature |
| 1 | 9.6 Functions of Several Variables | 22-Aug | 24-Aug |
| | 9.7 Gradient & Directional Derivative | | 10.1 Line Integrals |
| | HW 1 Due (9.1-9.4) | 9.8 & 9.9 Divergence & Curl | 10.2 Path Independence |
| 2 | 27-Aug | 29-Aug | 31-Aug |
| | 10.2 Path Independence | <u> </u> | 5 |
| | HW 2 Due (9.5-9.9) | | |
| | Add/Drop Deadline | 10.4 Green's Theorem | 10.5 Surfaces |
| 3 | 3-Sep | 5-Sep | 7-Sep |
| | 10.6 Surface Integrals HW 3 Due (10.1-10.4) | 10.6 Surface Integrals | 10.7 Divergence Theorem 10.8 Applications of Div Thm |
| 4 | 10-Sep | 12-Sep | 14-Sep |
| | 10.9 Stokes' Theorem | 12 306 | 11.2 Arbitrary Period; Even & Odd |
| | HW 4 Due (10.5-10.6) | 11.1 Fourier Series | Functions, Half Range Extensions |
| 5 | 17-Sep | 19-Sep | 21-Sep |
| | 11.5 Sturm-Liouville Problems; | 44.0.0 " | 44.05 |
| | Orthogonal Functions HW 5 Due (10.7-11.1) | 11.6 Orthogonal Series; Generalized Fourier Series | 11.9 Fourier Transform |
| 6 | 24-Sep | 26-Sep | Catch-Up & Review 28-Sep |
| | Review | 20-3eρ | 20-3ер |
| | HW 6 Due (11.2-11.9) | Exam 1, Part 1 | Exam 1, Part 2 |
| 7 | 1-Oct | 3-Oct | 5-Oct |
| | | 12.2 Vibrating String, Wave Equation | |
| | 12.1 Basic Concepts of PDEs | 12.3 Separation of Variables | |
| 8 | 8-Oct | 10-Oct | 13-Oct |
| | 12.3 Wave Eqn: Fourier Series Soln | 12.4 D'Alembert's Solution Wave Eqn | |
| | Withdraw Deadline | 12.4 Method of Characteristics | 12.5 Modeling Heat Flow; Heat Eqn |
| 9 | 15-Oct | 17-Oct | 19-Oct |
| | 12.6 Heat Eqn: Fourier Series Soln | 10.014 | 12.9 Rectangular Membrane and |
| 10 | HW 7 Due (12.1-12.4) 22-Oct | 12.8 Membrane & 2D Wave Eqn | Double Fourier Series |
| 10 | 12.10 Laplacian in Polar Coords; | 24-Oct | 26-Oct |
| | Fourier-Bessel Series | 12.11 Laplace's Equation in Cylindrical | |
| | HW 8 Due (12.5-12.8) | & Spherical Coords | 5.1 Power Series Method |
| 11 | 29-Oct | 31-Oct | 2-Nov |
| | 5.1 Power Series Method | | |
| 40 | HW 9 Due (12.9-12.11) | 5.2 Legendre's Eqn & Polyns | 5.3 Frobenius Method |
| 12 | 5-Nov | 7-Nov | 9-Nov |
| | Review HW 10 Due (5.1-5.2) | Exam 2, Part 1 | Exam 2, Part 2 |
| 13 | 12-Nov | 14-Nov | 16-Nov |
| 10 | 12 1404 | 14 140 | 10 1407 |
| 14 | 20-Nov | 22-Nov | 24-Nov |
| | | | |
| | 5.3 Frobenius Method | 5.3 Frobenius Method | 5.4 Bessel's Eqn & Fcn of 1st Kind |
| 15 | 26-Nov | 28-Nov | 30-Nov |
| | 5.4 Bessel's Eqn & Gamma Fcn | 13 16 Overview of Committee Variable | |
| | 5.5 Bessel Fcn of 2nd Kind HW 11 Due (5.3) | 13 – 16 Overview of Complex Variables Catch-Up & Review | Review |
| 16 | 3-Dec | 5-Dec | 7-Dec |
| | 0.000 | 0-DCC | 7 Dec |
| | Final Exam 3 pm | | |
| 17 | 10-Dec | 12-Dec | 14-Dec |
| | 10 000 | 12 000 | 1.000 |

Class Prep List

MATH 360 Methods of Applied Mathematics

The following list of Class Prep assignments is subject to changes announced in class.

| Section | Class Prep | |
|-------------------------------|---|--|
| 9.4 | Two types of functions introduced in this section: <i>vector functions</i> and <i>scalar functions</i> . The graph of a vector function looks like a field of vectors, so a vector function is often called a vector field, and scalar functions are sometimes called scalar fields. | |
| 9.5 | Parametric representations of curves is an important concept in this class. The first four examples in this section provide important examples that get used often. We will also make use of tangents to curves and lengths of curves. O Write up Examples 1-4 and include figures. O State Equation (8) for the unit tangent vector. Be sure to use vector notation. O State Equation (10) for the formula for length of a curve. Be sure to use vector notation. | |
| 9.6 | The chain rule for multivariate functions requires careful attention to formulas. Pay attention to when the curved partial derivative symbol is used and when the straight derivative symbol is used. | |
| 9.7 | The gradient is a vector function that is obtained from a scalar function <i>f</i> , called the <i>potential function</i> . The shortcut formula for <i>directional derivative</i> uses gradient and unit direction vector. Write up Example 1 on p. 397. Write the differential operator (Eqn 1*) on p. 396. Sketch Figure 216 on p. 399. Write Laplace's Equation (9) on p. 400. Write the Laplace operator (Eqn 11) on p. 401. | |
| 9.8 | The divergence of a vector is a scalar-valued function, and is associated with measuring flux, or outflow minus inflow. Write Eqn 1 on p. 402 for div(v). Write the formula for div(grad(v)), both on the bottom of p. 403 and in Eqn 3 on p. 404. Read Example 2 on pages 404-405 but do not write it up. State the continuity equation of a compressible fluid flow (Eqn 5 on p. 405). State Eqn 6 for steady flow. State Eqn 7, which gives the condition of incompressibility (constant density). | |
| 9.9 | The curl of a vector is a vector valued function, and as associated with rotation. Write the formula for the curl (Eqn 1) on p. 406. Write up Example 1 on p. 407. Read Example 2 on p. 407 but do not write it up. It references an example in 9.3 that we didn't emphasize. State Theorem 1 on p. 407. State Theorem 2 on p. 408. Write up Example 3 on p. 408. (This example is just four sentences long, but it is interesting.) | |
| | | |
| 10.1 Line Integrals | Line integrals are more accurately termed <i>curve integrals</i> , and most integrals in Calculus I and II would be <i>interval integrals</i> . O Write up the formulations (2), (3) and (3') for line integrals. Note that $d\mathbf{r} = \mathbf{r}' \cdot d\mathbf{t}$ in these formulations. O What integration symbol is used for the line integral along a closed curve? O Write up Examples 1 and 2. O Read Examples 3 and 4. The interesting conclusion to Example 4 is that work done is equal to gain in kinetic energy. O Write up Example 5, which illustrates a vector-valued integral. O Read Theorem 2. The proof given is not really a proof; instead it is a comment followed by an example. | |
| 10.2 Path Independen ce | Path independence results when the integration only depends on the starting point and endpoint and not on the path taken. Read introductory comments at beginning of section. Write up the three Theorem results for path independence stated near the bottom of page 419. Write up Examples 1 and 2. Read the top half of page 422. Write up Equations (6) and (6*) for exactness criteria. Read Example 3. | |
| 10.3 Double Integration | Review of double integrals. Skim over this section. No Class Prep. | |
| 10.4 Green's Theorem | Green's Theorem converts measuring circulation on interior of region to circulation on boundary. O Write up Equations (1) and (1') for Green's Theorem. O Write up Example 1. Read Example 4. | |

| Section | Class Prep |
|--|---|
| | Parametric representations of surfaces $\mathbf{r}(u,v)$ require two input parameters. We will use these for integration later. |
| 10.5 Surfaces | Write up Equation (2). Do the following portions of Examples 1, 2 and 3: Read each one and then write-up the parametric representations (equations) for a cylinder, sphere and cone. Be sure to include the domains specified for the input variables (parameters) u and v. The homework asks about for u = constant and v = constant, so pay attention to that part of the discussion too. Write up Equations (4) and (5) for the normal and unit normal vectors for a tangent plane of a surface. Read Theorem 1. Write up Examples 4 and 5. For Example 4, the author asks if the formula obtained is obvious. This is |
| | probably because for a sphere, we would expect that the outward pointing normal vector to have the direction of the radial vector for the sphere. |
| | Once we parameterize a surface r(u,v), then we can define surface integration. Write up Equations (1) - (3*). The integral in Equation (3) measures flux across surface, since the dot product in the integrand measures how much F is in direction of outward normal vector n to the surface. Write up Example 2. |
| 10.6 Surface Integrals | Write up the statement of Theorem 1. Write up Equation (6). The integral here is not a flux integral, because there is no dot product of F and n. Instead, the integral adds up how much of G there is on the surface. For example, if G is the temperature at each point on the surface, then the integral gives a total temperature across all points on the surface. When this value is divided by the right scalar, then this can be used to compute the average temperature on the surface. The reading mentions the case when G represents mass density of S. Write up Equation (8), which is the result when G = 1. |
| | • Write up the discussion on the bottom of page 449 beginning with the phrase Representations $z = f(x,y)$ and ending with Equation (11); then write up Equation (12). |
| 10.7 Divergence Theorem | Adding up the divergence at each point of a 3D region equals adding up the flux across the 2D boundary surface. |
| 10.8 Applications Divergence Thm | The divergence theorem can be used to the heat equation (diffusion equation) and other flow related results. Write up Equation (5). What are the two names identified with this equation? What first derivative is zero for steady state flow? What does Equation (5) reduce to for steady-state flow? What is a harmonic function? Backtracking to page 453, state Equation (2) for the Divergence Theorem. Then, returning to page 460, write up Example 3. |
| 10.9 Stokes' Thm | Adding up the amount of circulation of F whose axis of rotation is in the direction of n over a surface is equal to adding up the amount of F along the boundary curve <i>C</i> that is in the direction of <i>C</i> . |
| | |
| 11.1 Fourier Series | Periodic functions can be represented as a summation of sines and cosines (Fourier Series). This allows us to obtain frequency information for the function. Fourier series will be helpful when working with PDEs in Ch. 12. O Write up Equations (5) and (6) for the Fourier series and Fourier coefficients. O Write up Example 1, finishing at Equation (8). Include Figure 260 (hand drawn) for the graph of f. |
| 11.2 Arb period. Even & Odd. Half Range Expansions. | Some changes and simplifications to the Fourier formulas depending on the interval or the function. Write up the Orientation on page 483. Write up Equations (5) and (6) on page 484, and then write up Example 1. Write up the Summary on the top of page 487. Read Theorem 1 and then write up Example 5. Hand-draw Figure 270 on page 489 for even and odd periodic extensions of a function. Skim Example 6 and be glad you don't have to write it up. Integration by parts shows up a lot in Ch 11. |
| 11.5 Sturm-Liouville Problems. Orthogonal Functions. | Sturm-Liouville (pronounced Lee-oo-ville) problems provide a framework about which many orthogonal special functions in mathematical physics are developed and studied. Skim section. Write up Examples 1-3. |
| 11.6 Orthogonal Series. Generalized Fourier Series. | Generalized Fourier expansions using orthogonal Sturm-Liouville eigenfunctions are discussed in this section Skim section. Write up Example 1. |
| 11.9 Fourier Transform. DFT & FFT. | The Fourier transform generalizes the Fourier coefficients formulas, and the FFT is a numerically fast and accurate way of computing Fourier coefficients. Skim portions of section. Write up Examples 1,2,4. |

| Section | Class Prep |
|--|--|
| 12.1 Basic Concepts of PDEs | When we study Calculus III, we learn about multivariate calculus. We find that Calculus III is similar but distinct from the single variable Calculus I and Calculus II. Similarly when we learn about PDEs, we study multivariate differential equations, which will be similar but distinct from single variable differential equations. Skim section. Write up first three PDEs in Example 1 and identify each by name. Write up Example 2. |
| 12.2 Modeling: Vibrating String, Wave Eqn | Read section. Sketch Fig. 286. Write up the derivation of the wave equation PDE on page 544. |
| 12.3 Solution by Separating Variables. Fourier Series. 12.4 D'Alembert's Solution of the Wave Equation. | Read section. Identify (state) what the three steps are for the solution method for the PDEs in this section. Read section. Write up page 555, beginning at the section entitled Characteristics; Types and Norm Forms of PDEs, |
| Characteristics 12.5 Modeling: Heat Flow from a Body in Space. Heat Equation. | and continue until the end of Example 1. Read physical assumptions stated on page 557 for derivation of heat equation. Read derivation for heat equation. Note that divergence theorem is used for flux integral of heat flow. |
| 12.6 Heat Equation: Solution by Fourier Series. Steady Two-Dimensional Heat Problems. Dirichlet Problem. | Write up Equations (1), (2) and (3) for the heat equation, boundary conditions, and initial conditions. Starting with Step 1 on the bottom of page 559, write up the material on pages 559 – 561, stopping at Equation (10). Note that the decaying exponential term of the solution in Equation (9) results from the first order time ODE, rather than a second order time ODE as with the wave equation in Ch 12.3. |
| 12.8 Modeling: Membrane, Two Dimensional Wave Equation | Read physical assumptions stated on page 575 for derivation of 2D wave equation. Read derivation for 2D wave equation. |
| 12.9 Rectangular Membrane. Double Fourier Series | Write up Equations (1), (2), (3a) and (3b) for the vibrating rectangular membrane model. Write up the three steps identified on the top of page 378 for solving the 2D vibrating rectangular membrane model. Write up Example 2. |
| 12.10 Laplacian in Polar Coordinates. Circular Membrane. Fourier-Bessel Series | Write Equation (6), the 2D wave equation in polar coordinates, and then write Equations (7)-(9) for the radially symmetric boundary value problem. Identify the three steps for solving this problem, but do not write the analyses provided for each step. Write Equation (17) for the solution. Write up Example 1, but omit writing up the table. |
| 12.11 Laplace's Equation in Cylindrical and Spherical Coordinates. Potential. | Write out Laplace's equation in rectangular, cylindrical, and spherical coordinates: Eqns (1), (3), and (7). Write up the boundary value problem in spherical coordinates: Equations (8) – (10). Write up Legendre's Equation (15) and (15'). Write up Equations (16) – (21), which play a role in solving the Interior and Exterior Potential problems. |
| | |
| 5.1 Power Series Method | Read Chapter 5 Introduction on page 167 and also Section 5.1. Write up Examples 1 – 4. |
| 5.2 Legendre's Equation & Polynomials | Read Section 5.2. Write up pages 175 – 177, stopping at the top of page 177 when you reach the subsection entitled Polynomial Solutions Legendre Polynomials. |
| 5.3 Extended Power Series Method: Frobenius Method | Read Section 5.3.Write up Examples 1 and 2. |
| 5.4 Bessel's Equation & Functions J(v,x) | Read Section 5.4. Write up pages 187-191, stopping when you reach Discovery of Properties from Series. Write up Example 1. |
| 5.5 Bessel Functions Y(υ,x) | o Skim the section |

MATH 360 Methods of Applied Mathematics

Class Prep Grading Rubric

Class Preps play an important role in your learning in this course. Class Preps help you to meet the following subset of the Student Learning Objectives stated in the syllabus:

- 1. Develop independent learning skills, in particular, reading skills
- 2. Develop mathematical language skills
- 3. Develop persistence and skill in exploration.

Since this is an upper level course, it is reasonable to expect you to read and understand portions of the material found in the book. You are encouraged to clarify in class any questions you have regarding the reading.

The Class Preps are due before in-class coverage of the section, and will be graded according to the following criteria. Other reasonable considerations may also play a role, depending on the nature of the section being covered or changes announced in class.

| Grade Range | Description | |
|-------------|--|--|
| 90 – 100 | Submitted on time, and it is mostly clear to the reader that you have read and made an | |
| | effort to understand and articulate the ideas of the section. Work is nicely organized | |
| | and information is easy to find. Formatting is reasonably well-thought out. Topic areas, | |
| | important ideas, concepts, results, diagrams and examples are not omitted. Where | |
| | appropriate, equations are formatted in display mode. Spelling and grammar correct. | |
| 80 - 89 | Submitted on time, and it is mostly clear to the reader that you have read and made an | |
| | effort to understand and articulate the ideas of the section. Work is somewhat | |
| | organized but formatting may not be the best for finding information. A few omissions | |
| | of topics, important ideas, concepts, results, diagrams and examples. Spelling and | |
| | grammar incorrect in some places. | |
| 70 - 79 | Submitted on time, and it is somewhat clear to the reader that you have read and r | |
| | an effort to understand and articulate the ideas of the section. Work is somewhat | |
| | sloppy and appears to be put together in a hurry. Work is poorly organized. Multipl | |
| | omissions of topics, important ideas, concepts, results, diagrams and examples. Spelling | |
| | and grammar incorrect throughout. | |
| 60 - 69 | Submitted on time, but is significantly lacking in the grading categories listed above. | |
| 0 - 59 | Submitted on time, but content does not substantially reflect material in reading. | |
| Late Work | Late is better than never, and thus late work may be submitted for up to one week | |
| | (7 days) after the deadline. However, in fairness to those who turned it in on time, a | |
| | letter grade will be deducted for each class day after the deadline. | |

Fall 2018 Homework List

MATH 360 Methods of Applied Mathematics Fall 2018 Howard The following list of homework problems is subject to changes announced in class.

| 9.2 Inner Product (Dot Product) 9.3 Vector Product (Cross Product) 9.3 Vector Product (Cross Product) 11,32 9.3 Vector Product (Cross Product) 11,32 9.3 Vector Product (Cross Product) 9.4 Vector and Scalar Functions Vector Derivatives 9.5 Curves. Arc Length. Curvature 9.6 Chain Rule 9.6 Chain Rule 9.7 Gradient of Scalar Field. Directional Derivative. 9.8 Divergence of a Vector Field 9.9 Curl of Vector Field 1-3,515-17 2,100,101,15 9.9 Curl of Vector Field 4-6,911,12 5,9.11 10.1 Line Integrals 10.2 Path Independence of Line Integrals 10.3 Calculus Review: Double Integrals 10.3 Calculus Review: Double Integrals 10.4 Green's Theorem in the Plane 10.5 Surfaces for Surface Integrals 10.5 Surfaces for Surface Integrals 11.5 Firm Journal Product Product 1-1,712-15 10.7 Divergence Theorem 10.8 Applications of Divergence Thm 10.9 Stokes Theorem 11.9 Stowns of Divergence Thm 12.1 In Fourier Series 11.1 Authitrary period. Even & Odd Functions. Half 11.1 Fourier Series 11.2 Vibrating period. Even & Odd Functions. Read Section 12.3 Read Section 13.9 Stokes Theorem 14.5 Sturm-Liouville Probs. Orthogonal Functions. 14.9 Stowns Theorem 15.1 Sturm-Liouville Probs. Orthogonal Functions. 14.9 Fourier Transform. DFT & FFT. 2.5 3,4 12.1 Basic Concepts of PDEs 12.2 Warating String, Wave Eqn 12.3 Separating Variables. Fourier Series. 13.9 Fourier Transform. DFT & FFT. 2.5 3,4 12.1 Basic Concepts of PDEs 12.4 Diversing Variables. Fourier Series. 13.9 Compute Inferiors. 14.9 Fourier Transform. DFT & FFT. 2.5 3.4 Separating Variables. Fourier Series. 3.9 Page 12.1 Separating Variables. Fourier Series. 3.1 J. 5,712,13,15,21; Read 16,17 7" (use (k)=1 for #7), 21 12.8 Modeling: Heat Flow. 12.9 Double Fourier Series. 13.9 Compute Inferiors of Acusing handout as guide, and then do 13, 14. 12.9 Double Fourier Series. 13.9 Double Fourier Series. 14.10 Laplacian in Opin Coordinates. Circular Membrane. Bessel Series 13.0 Compute Inferiors and From Series. 14.10 Laplacian in Opin Coordinates. Circular Membrane. Bessel Series 13.0 Compute Inferior | Section | Long List | Short List |
|--|---|---|-----------------------------|
| 9.3 Vector Product (Cross Product) 9.4 Vector Or Product (Cross Product) 9.4 Vector and Scalar Functions. Vector Derivatives 9.5 Curres. Arc Legift. Curvature 9.5 Curres. Arc Legift. Curvature 9.6 Chain Rule 9.7 Gradient of Scalar Field. Directional Derivative, 9.6 Chain Rule 9.7 Gradient of Scalar Field. Directional Derivative, 9.7 Gradient of Scalar Field. Directional Derivative, 9.8 Divergence of a Vector Field 9.9 Suri of a Vector Field 9.1 Sp. 11,12 9.1 Sp. 1 | 9.1 Vectors in 2-Space and 3-Space | 1, 14a | 1, 14a |
| 9.4 Vector and Scalar Functions. Vector Derivatives 1,5,9,11,12,15,19,22,24 1,5,11,19,24 1,3,7,14,25,28,29 9.5 Curves. Arc Length. Curvature 1,3,7,8,11,13,14,25,28,30 1,3,7,14,25,28,29 9.5 Chain Rule Read Section Read Section Read Section 9.7 Gradient of Scalar Field. Directional Derivative. 14,11-15,18,19,21,24,25,30,33,34,36,37,41 34,21,25,30 3.9 Divergence of a Vector Field 1-35,15-17 2,100,10/15 5.9,11 2.100,10/15 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,14 3.9,13,16 3.9,13 | 9.2 Inner Product (Dot Product) | 7,17,37 | 7,37 |
| 9.4 Vector and Scalar Functions. Vector Derivatives 9.5 Gurees. Arc Length. Curvature 1.3,7,8,11,13,14,25,28.30 1,3,7,14,75,28,29 9.5 Curves. Arc Length. Curvature 1.3,7,8,11,13,14,75,28.30 1,3,7,14,75,28,29 9.5 Chain Rule Read Section Read Section 9.7 Gradient of Scalar Field. Directional Derivative. 1.4,11-15,18,19,21,24,25,30,33,34,36,37,41 3,4,21,25,30 9.5 Divergence of a Vector Field 1.35,15-17 2,100,10f,15 5,9,11 2,100,10f,15 5,9,11 1,12 1,12 1,12 1,12 1,12 1,12 1,12 | 9.3 Vector Product (Cross Product) | 11,32 | 11,32 |
| 9.5 Curves. Arc Length. Curvature 1.3,7.8,11,13,14,25,28-30 1.3,7,14,25,28,29 9.6 Chain Rule Read Section 9.6 Chain Rule Read Section 9.7 Gradient of Scalar Field. Directional Derivative. 1.4,11-15,18,19,21,24,25,30,33,34,36,37,41 2,10c,10f,15 9.8 Divergence of a Vector Field 4.6,9,11,12 5,9,11 10.1 Line Integrals 2.5,9,10 4.5,10 10.2 Path Independence of Line Integrals 3.6 3.3 10.3 Calculus Review: Double integrals Read Section 10.4 Green's Theorem in the Plane 10.5 Surfaces for Surface Integrals 10.5 Surfaces for Surface Integrals 11.5 Surfaces for Surface Integrals 12.7,14-16 13.6 Surface Integrals 13.7,14-16 14.9-13 10.7 Divergence Theorem 14.9-13 10.8 Applications of Divergence Thm 12.10.9 Stokes' Theorem 14.9-13 11.1 Fourier Series 1-2,9,13,16,17 11.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.9 Fourier Transform. DFT & FF. 2.5 3,4 12.1 Basic Concepts of PDEs Read Section Read Se | 9.4 Vector and Scalar Functions. Vector Derivatives | 1,5,9,11,12,15,19,22,24 | 1,5,11,19,24 |
| 9.6 Chain Rule Read Section Read Section Read Section 9.7 Gradient of Scalar Field. Directional Derivative. 1.4,11-15,18,19,21,24,25,30, 33,34,36,37,41 3,421,25,30 3,80 Divergence of a Vector Field 1.3,51,51-17 2,10,0,115 9.9 Curl of a Vector Field 4.6,9,11,12 5,9,11 | 9.5 Curves. Arc Length. Curvature | | |
| 9.8 Divergence of a Vector Field 4-6,9,11,12 5,9,11 10.1 Line Integrals 5,9,10 4,5,10 10.2 Path Independence of Line Integrals 10.3 Calculus Review: Double Integrals 10.4 Green's Theorem in the Plane 10.5 Surfaces for Surface Integrals 10.5 Surface Integrals 10.5 Surface Integrals 10.6 Surface Integrals 10.7 Divergence Theorem 10.8 Applications of Divergence Thm 1,2 10.9 Stokes' Theorem 1,4,9-43 10.9 Stokes' Theorem 1,5,13-15 1,2,4,14 11.1 Fourier Series 11.2 Aphtrary period. Even & Odd Functions. Half Range Expansions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.6 Orthogonal Series. Generalized Fourier Series. 11.9 Fourier Transform. DFT & FFT. 2.5 3,4 12.1 Basic Concepts of PDEs Read Section Read Section Read Section 12.2 Vibrating String. Wave Eqn 12.3 Separating Variables. Fourier Series. 12.5 Modeling; Heat Flow. Read Section Read Section Read Section Read Section Read Section 12.9 Double Fourier Series 13 13 Use f(x,y) = xy 14.10 Pourier Series 13 14.10 Laplacian in Polar Coordinates. Circular Membrane. Besels Series 13 14.10 Laplacian in Cylindrical and Spherical Coords 15.1 Power Series Method: Frobenius 15.1 Power Series Method: Frobenius 15.2 Legendre's Equation & Polynomials 15.3 Extended Power Series Method: Frobenius 15.4 Bessel's Equation & Fourtons I(u,x) 15.4 Bessel's Equation & Fourtons I(u,x) 15.5 Again find y1 & y2 using recursion relation for the two values of r, and choose a0 = 1. For #5, can use reduction of order to find y2. For #8, use Theorem 2 to find y2. 15.4 Bessel's Equation & Functions I(u,x) 15.5 Again find y1 & y2 using recursion relation for the two values of r, and choose a0 = 1. For #5, can use reduction of order to find y2. For #8, use Theorem 2 to find y2. | 9.6 Chain Rule | | |
| 9.8 Divergence of a Vector Field 4-6,9,11,12 5,9,11 10.1 Line Integrals 5,9,10 4,5,10 10.2 Path Independence of Line Integrals 10.3 Calculus Review: Double Integrals 10.4 Green's Theorem in the Plane 10.5 Surfaces for Surface Integrals 10.5 Surface Integrals 10.5 Surface Integrals 10.6 Surface Integrals 10.7 Divergence Theorem 10.8 Applications of Divergence Thm 1,2 10.9 Stokes' Theorem 1,4,9-43 10.9 Stokes' Theorem 1,5,13-15 1,2,4,14 11.1 Fourier Series 11.2 Aphtrary period. Even & Odd Functions. Half Range Expansions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.6 Orthogonal Series. Generalized Fourier Series. 11.9 Fourier Transform. DFT & FFT. 2.5 3,4 12.1 Basic Concepts of PDEs Read Section Read Section Read Section 12.2 Vibrating String. Wave Eqn 12.3 Separating Variables. Fourier Series. 12.5 Modeling; Heat Flow. Read Section Read Section Read Section Read Section Read Section 12.9 Double Fourier Series 13 13 Use f(x,y) = xy 14.10 Pourier Series 13 14.10 Laplacian in Polar Coordinates. Circular Membrane. Besels Series 13 14.10 Laplacian in Cylindrical and Spherical Coords 15.1 Power Series Method: Frobenius 15.1 Power Series Method: Frobenius 15.2 Legendre's Equation & Polynomials 15.3 Extended Power Series Method: Frobenius 15.4 Bessel's Equation & Fourtons I(u,x) 15.4 Bessel's Equation & Fourtons I(u,x) 15.5 Again find y1 & y2 using recursion relation for the two values of r, and choose a0 = 1. For #5, can use reduction of order to find y2. For #8, use Theorem 2 to find y2. 15.4 Bessel's Equation & Functions I(u,x) 15.5 Again find y1 & y2 using recursion relation for the two values of r, and choose a0 = 1. For #5, can use reduction of order to find y2. For #8, use Theorem 2 to find y2. | 9.7 Gradient of Scalar Field. Directional Derivative. | 1-4,11-15,18,19,21,24,25,30, 33,34,36,37,41 | 3,4,21,25,30 |
| 9.9 Curl of a Vector Field 4-6,9,11,12 5,9,11 10.1 Line Integrals 2-5,9,10 4,5,10 10.2 Path Independence of Line Integrals 3-6 3 10.3 Calculus Review: Double Integrals Read Section Read Section 10.4 Green's Theorem in the Plane 1-5,13,14 1,4,13 10.5 Surfaces for Surface Integrals 1-5,13,14 1,4,13 10.5 Surfaces for Surface Integrals 1-5,13,14 1,4,13 10.5 Surfaces for Surface Integrals 1-7,12-15 2,4,12,15 10.6 Surface Integrals 1-7,12-15 2,4,12,15 10.7 Divergence Theorem 1,4,9-13 9,12 10.9 Stokes' Theorem 1,2 11.9 Storn-Louville Probs. Orthogonal Functions. 11,5,13-15 1,2,4,14 11.1 Fourier Series 1-3,9,13,16,17 1,9,13,16 11.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 11,9 Fourier Transform. DFT & FFT. 2-5 3,4 11.6 Orthogonal Series. Generalized Fourier Series. 12,6-10,12,13 7,10,13 11.6 Orthogonal Series. Generalized Fourier Series. 12.5 Surm-Louville Probs. Orthogonal Functions. 11.9 Fourier Transform. DFT & FFT. 2-5 3,4 12.1 Basic Concepts of PDEs Read Section Read Sec | 9.8 Divergence of a Vector Field | | |
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| 10.4 Green's Theorem in the Plane 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,15 1.5,14,15 1.6.6 Surface Integrals 1.7,12-15 2.4,12,15 1.7,12-15 2.4,12,15 1.7,12-13-15 1.7,12-13-13-15 1.7,12-13-13-13-13 1.7,12-13-13-13-13 1.7,12-13-13-13-13 1.7,12-13-13-13-13 1.7,12-13-13-13-13-13-13-13-13-13-13-13-13-13- | 10.2 Path Independence of Line Integrals | | 3 |
| 10.4 Green's Theorem in the Plane 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,14 1.5,13,15 1.5,14,15 1.6.6 Surface Integrals 1.7,12-15 2.4,12,15 1.7,12-15 2.4,12,15 1.7,12-13-15 1.7,12-13-13-15 1.7,12-13-13-13-13 1.7,12-13-13-13-13 1.7,12-13-13-13-13 1.7,12-13-13-13-13 1.7,12-13-13-13-13-13-13-13-13-13-13-13-13-13- | 10.3 Calculus Review: Double integrals | Read Section | Read Section |
| 10.5 Surfaces for Surface Integrals 1.5,7,14-16 3.5,14,15 10.6 Surface Integrals 1.7,12-15 2.4,12,15 10.7 Divergence Theorem 1.4,9-13 9,12 10.8 Applications of Divergence Thm 1.2 10.9 Stokes' Theorem 1.5,13-15 1.2,4,14 11.1 Fourier Series 1.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 1.5 Sturm-Liouville Probs. Orthogonal Functions. 1.1.6 Orthogonal Series. Generalized Fourier Series. 1.1.9 Fourier Transform. DFT & FFT. 2.5 1.2.1 Basic Concepts of PDEs Read Section Read Section 1.2.2 Vibrating String, Wave Eqn 1.2.3 Separating Variables. Fourier Series. 7.9 1.2.4 D'Alembert's Soln. Characteristics 9-12 1.2.5 Modeling: Heat Flow. 1.2.6 Modeling: Heat Flow. 1.2.6 Modeling: Membrane, 2D Wave Eqn 1.2.9 Double Fourier Series 1.3 1.2.10 Laplacian in Polar Coordinates. Circular Membrane. Bessel Series 1.3 1.3 Laplacian in Cylindrical and Spherical Coords 1.3 Separating Warier Series 1.3 Compute left-sum approximation of Az using handout as guide, and then do 13, 14. Read Section | - | 1-5,13,14 | 1,4,13 |
| 1.7. Divergence Theorem 1.4,9-13 1.0.9 Stokes' Theorem 1.5,13-15 1.1. Applications of Divergence Thm 1.2 Interpretable 1.3,5-7,8,9,23,24,27,28 1.1.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 1.1.5 Sturm-Liouville Probs. Orthogonal Functions. 1.3,5-7,8,9,23,24,27,28 1.3,5-7,8,9,23,24,27,28 1.3,6-10,12,13 1.4.6 Orthogonal Series. Generalized Fourier Series. 1.5 Faced Section 1.6 Orthogonal Series. Generalized Fourier Series. 1.7.9 Read Section 1.8 Read Section 1.9 Fourier Transform. DFT & FFT. 1.9 Fourier Transform. DFT & FFT. 1.9 Fourier Transform. DFT & FFT. 1.1 Basic Concepts of PDEs 1.2 Vibrating String, Wave Eqn 1.3 Separating Variables. Fourier Series. 1.4 Read Section 1.5 Modeling: Heat Flow. 1.5 Modeling: Heat Flow. 1.5 Modeling: Membrane, 2D Wave Eqn 1.5 Read Section 1.5 | 10.5 Surfaces for Surface Integrals | 1-5,7,14-16 | |
| 1.7. Divergence Theorem 1.4,9-13 1.0.9 Stokes' Theorem 1.5,13-15 1.1. Applications of Divergence Thm 1.2 Interpretable 1.3,5-7,8,9,23,24,27,28 1.1.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 1.1.5 Sturm-Liouville Probs. Orthogonal Functions. 1.3,5-7,8,9,23,24,27,28 1.3,5-7,8,9,23,24,27,28 1.3,6-10,12,13 1.4.6 Orthogonal Series. Generalized Fourier Series. 1.5 Faced Section 1.6 Orthogonal Series. Generalized Fourier Series. 1.7.9 Read Section 1.8 Read Section 1.9 Fourier Transform. DFT & FFT. 1.9 Fourier Transform. DFT & FFT. 1.9 Fourier Transform. DFT & FFT. 1.1 Basic Concepts of PDEs 1.2 Vibrating String, Wave Eqn 1.3 Separating Variables. Fourier Series. 1.4 Read Section 1.5 Modeling: Heat Flow. 1.5 Modeling: Heat Flow. 1.5 Modeling: Membrane, 2D Wave Eqn 1.5 Read Section 1.5 | 10.6 Surface Integrals | 1-7,12-15 | 2,4,12,15 |
| 10.8 Applications of Divergence Thm 1.2 10.9 Stokes' Theorem 1-5,13-15 1,2,4,14 1.1.1 Fourier Series 1.2,9,13,16,17 1,9,13,16 1.1.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.6 Orthogonal Series. Generalized Fourier Series. 11.9 Fourier Transform. DFT & FFT. 2-5 2-5 3,4 12.1 Basic Concepts of PDEs Read Section Read Section Read Section 12.2 Vibrating String, Wave Eqn Read Section Read | | | |
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| 11.1 Fourier Series 1.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 1.5 Sturm-Liouville Probs. Orthogonal Functions. 1.6 Orthogonal Series. Generalized Fourier Series. 1.7 Journal Series. Generalized Fourier Series. 1.9 Fourier Transform. DFT & FFT. 2.5 Read Section 1.1.2 Prourier Transform. DFT & FFT. 3.4 1.2.1 Basic Concepts of PDEs Read Section 1.2.2 Vibrating String, Wave Eqn Read Section 1.3.2 Separating Variables. Fourier Series. 7,9 9 1.4 D'Alembert's Soln. Characteristics 9-12 1.5 Modeling: Heat Flow. 1.6 Heat Equation. Fourier Series. 1.7,5,7,12,13,15,21; Read 16, 17 1.2.8 Modeling: Membrane, 2D Wave Eqn Read Section 1.3 *Use f(x) = 1 for #7), 21 1.3 *Use f(x) = 1 for #7), 21 1.4 Boulde Fourier Series 1.5 Compute left-sum approximation of A2 using handout as guide, and then do 13, 14. 1.3 *Lia Laplacian in Cylindrical and Spherical Coords 1.4 Read Section 1.5 *Lia Laplacian in Cylindrical and Spherical Coords 1.5 *Lia Laplacian in Cylindrical Sphe | 10.9 Stokes' Theorem | | 1,2,4,14 |
| 11.2 Arbitrary period. Even & Odd Functions. Half Range Expansions. 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.6 Orthogonal Series. Generalized Fourier Series. 11.9 Fourier Transform. DFT & FFT. 2-5 3,4 12.1 Basic Concepts of PDEs Read Section Read Secti | | | |
| Range Expansions. 1.5,5-7,8,9,2,3,24,27,28 3.9,27a 11.5 Sturm-Liouville Probs. Orthogonal Functions. 2,6-10,12,13 7,10,13 11.6 Orthogonal Series. Generalized Fourier Series. Read Section 1.9 Fourier Transform. DFT & FFT. 2-5 Read Section Read Section Read Section 1.2 Vibrating String, Wave Eqn 1.2 Vibrating String, Wave Eqn 1.3 Separating Variables. Fourier Series. 7,9 9 1.4 D'Alembert's Soln. Characteristics 9-12 1.5 Modeling: Heat Flow. Read Section Read Section Read Section 1.6 Heat Equation. Fourier Series. 1,5,7,12,13,15,21; Read 16, 17 7* (use f(x)= 1 for #7), 21 12.8 Modeling: Membrane, 2D Wave Eqn Read Section 1.9 Double Fourier Series 13 13* Use f(x,y)= xy 1.10 Laplacian in Polar Coordinates. Circular Membrane. Bessel Series 12.11 Laplacian in Cylindrical and Spherical Coords Read Section Read Section Read Section Same as Long List 5.1 Power Series Method 2,3,5,10-13 Same as Long List 5.2 Legendre's Equation & Polynomials Read problem statements for 10 – 15. Same as Long List 5.3 Extended Power Series Method: Frobenius 5.4 Bessel's Equation & Functions J(v,x) Read #13-15 and #21 but do not turn in. Same as Long List | 11.1 Fourier Series | 1-3,9,13,16,17 | 1,9,13,16 |
| 11.5 Sturm-Liouville Probs. Orthogonal Functions. 11.6 Orthogonal Series. Generalized Fourier Series. 11.9 Fourier Transform. DFT & FFT. 2-5 2-5 3,4 12.1 Basic Concepts of PDEs Read Section Read Section Read Section 12.2 Vibrating String, Wave Eqn Read Section Same as Long List Read Section Read Section Read Section Read Section Read Section Read Section Same as Long List Read Section Read Section Read Section Read Section Read Section Read Section Same as Long List Read Section Same as Long List Read Section Read Section Same as Long List | 11.2 Arbitrary period. Even & Odd Functions. Half | 1,3,5-7,8,9,23,24,27,28 | 8,9,27a |
| 11.6 Orthogonal Series. Generalized Fourier Series. 11.9 Fourier Transform. DFT & FFT. 2-5 3,4 12.1 Basic Concepts of PDEs Read Section 12.2 Vibrating String, Wave Eqn Read Section Read Section Read Section Read Section 12.3 Separating Variables. Fourier Series. 7,9 9 12.4 D'Alembert's Soln. Characteristics 9-12 Read Section Read Section Read Section Read Section Read Section 12.5 Modeling: Heat Flow. Read Section Read Section Read Section 12.6 Modeling: Membrane, 2D Wave Eqn Read Section Read Section 12.9 Double Fourier Series 13 Compute left-sum approximation of A₂ using handout as guide, and then do 13, 14. Same as Long List 12.11 Laplacian in Cylindrical and Spherical Coords Read Section Read Section Read Section Read Section 3.5 Power Series Method 2,3,5,10-13 8.2 Agend Froblem statements for 10 − 15. Same as Long List 3,5,8 For #3, can find y1 & y2 using recursion relation for the two values of r, and choose a0 = 1. For #5, can use reduction of order to find y2. For #8, use Theorem 2 to find y2. Same as Long List Same as Long List | | | |
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| | 5.5 Bessel Functions Y(ບ,x) | Read #11 but do not turn in. | Same as Long List |

Goal

Develop your professional content knowledge and communication skills.

General Comments about Written Work

- ➤ Math 360 is an upper level course. Thus it is reasonable to expect you to read and persist in your learning efforts. Plan ahead and ask questions as needed.
- ➤ When writing up solutions, do not assume your reader understands the problem and solution ahead of time. Show you understand concepts, methods, and steps. Neatness, organization and style all count, as does accuracy.
- > Be careful in using answers in the back of the book, as they may often be incomplete.
- ➤ Use scratch paper for preliminary work. If you get stuck, then ask about the problem in class. Once you know how to solve the problem, then write your solutions up neatly on a clean sheet of paper.

Read Lecture Notes outside of Class

- The lecture notes will typically include examples that illustrate how to work various problems in the homework and how to use notation and methods properly.
- ➤ The examples in the notes may be included the classroom discussion or they may be left for you to read. Either way you will likely need to review the steps and notation shown, and then apply this knowledge to you homework.

Grading Rubric for Homework Packet

- A. You must demonstrate competence with the methods shown in class. Here is a partial checklist:
 - 1. Your methods and notation must match what is covered in this class for that section of the book. Carefully review handouts given in class and be sure to demonstrate competence with these methods.
 - If you have questions regarding the methods, be sure you ask at least one class day in advance of the
 due date. As a recipe for your success, for material covered in Monday's class, I expect you to have
 attempted one or more of the homework problems by Monday evening or Tuesday evening, and similarly
 for Wednesday and Friday.
 - 3. Alternative methods used for similar material in a different class can be written up separately for me to review and comment on; however, you must first demonstrate that you have learned the methods and the notation of this class.
 - 4. If you do not show supporting steps in a manner consistent to what is demonstrated in class, for example, u-substitution, integration by parts, etc., then you will lose points for that problem. Remember, this is a math methods class, and you are required to show your methods.
 - 5. Use of internet resources and unapproved computational aids are not allowed for use as a substitute for demonstrating competence with the methods developed in this class.
- B. Part of your homework grade will be based on professional presentation. Here is a partial checklist:
 - 1. Problems must be worked on loose-leaf paper with clean (not ragged) edges.
 - 2. The beginning of each section must be clearly labeled and begin on a new page.
 - 3. The beginning of each section must include a list of the assigned problems for that section.
 - 4. All homework problems must be numbered, worked in order and must be legible.
 - 5. Each homework packet will consist of sections placed in order & attached with a staple for entire packet.
 - 6. This rubric is stapled together with your homework as the last page of your packet.
 - 7. Show steps in order, not scattered around page. Your writing should be legible and not crowded. If your ideas and intent are hard to find or ascertain, then you may not get credit for them.
 - 8. Use correct mathematical notation.
 - 9. Write on one side of page only. For lengthy solutions, each problem should begin on new page.
 - 10. When graphing, all graphs must be clearly labeled and the scale used on each axis must be labeled.