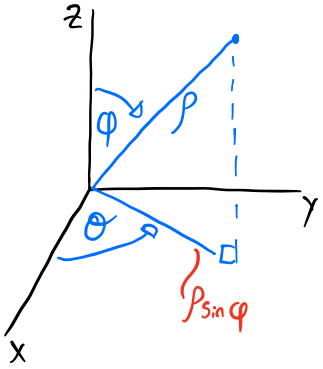


$x = \rho \sin\varphi \cos\theta$

$y = \rho \sin\varphi \sin\theta$

$z = \rho \cos\varphi$

$dv = \rho^2 \sin\varphi \; d\rho d\theta d\varphi$

$\rho^2 = x^2 + y^2 + z^2$

15.9.2

$$\iiint_E f \, dv$$

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{2\pi} \int_1^8 f(\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi)\, \rho^2 \sin\varphi \; d\rho d\theta d\varphi$$

15.9.3

$$\iiint_B (x^2+y^2+z^2)^2 \, dv$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^3 \rho^4 \, \rho^2 \sin\varphi \; d\rho d\theta d\varphi$$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^3 \rho^6 \sin\varphi \; d\rho d\theta d\varphi$$

$$\int_0^{\pi} \int_0^{2\pi} \sin\varphi \left[\tfrac{1}{7}\rho^7\right]_0^3 d\theta d\varphi$$

$$\sin\varphi \int_0^{\pi} \int_0^{2\pi} \frac{2187}{7} \, d\theta d\varphi$$

$$\sin\varphi \int_0^{\pi} \frac{2187}{7}\theta \Big|_0^{2\pi} d\varphi$$

$$\sin\varphi \int_0^{\pi} \frac{4374\pi}{7} \, d\varphi$$

$$\frac{4374\pi}{7} \left[-\cos\varphi\right]_0^{\pi}$$

$$\frac{4374\pi}{7}$$

15.9.4

$$\iiint_H (9 - x^2 - y^2) \, dv$$

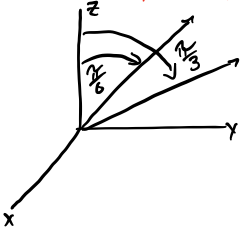
$x^2 + y^2 + z^2 \le 16$, $z \ge 0$

$0 \le \rho \le 4$, $0 \le \theta \le 2\pi$, $0 \le \varphi \le \frac{\pi}{2}$

$x = \rho\sin\varphi\cos\theta$, $y = \rho\sin\varphi\sin\theta$, $z = \rho\cos\varphi$

$$\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^4 (9 - x^2 - y^2)\, \rho^2 \sin\varphi \; d\rho d\varphi d\theta$$

15.9.5 $\rho \le 1$, $\varphi = \frac{\pi}{6}$, $\varphi = \frac{\pi}{3}$



$$\int_0^{3\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \int_0^1 \rho^2 \sin\varphi \; d\rho d\varphi d\theta$$

Volume of Sphere w/ radius r

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^r \rho^2 \sin\varphi \; d\rho d\varphi d\theta$$

$$\int_0^{2\pi} \int_0^{\pi} \frac{\rho^3}{3} \sin\varphi \Big|_0^r d\varphi d\theta$$

$$\int_0^{2\pi} \int_0^{\pi} \frac{r^3}{3} \sin\varphi \; d\varphi d\theta$$

$\frac{4\pi r^3}{3}$

$\int_0^{2\pi} \frac{r^3}{3}\cos\varphi \Big|_0^{\pi} d\theta$

$\frac{1}{3}r^3 \int_0^{2\pi} 2 \, d\theta$

$\frac{2}{3}r^3 \left[\theta\right]_0^{2\pi}$

$\frac{4\pi r^3}{3}$

$V = \int_0^{\frac{\pi}{4}} \int_0^{\pi} \int_0^{8} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$