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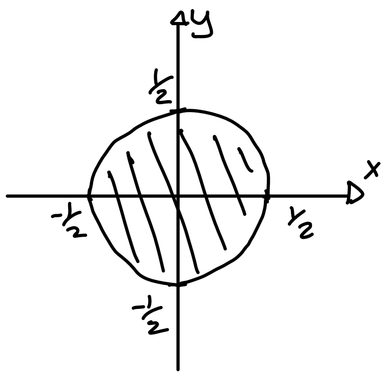
MATH 360
HW Ch. 10.4

#1,4,13

Problem 10.4.1 $\vec{F} = [y, -x]$,
C the circle $x^2+y^2=1/4$
center $(0,0)$, $r=1/2$

$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$



Circle: $(x-h)^2+(y-k)^2=r^2$

$r^2 = 1/4 \;\therefore\; r = \pm 1/2$

Circle dimensions, $0 \leq \theta \leq 2\pi$, $0 \leq r \leq 1/2$

$F_{2x} = -1$, $F_{1y} = 1$

$$\iint_R -1-1\,dxdy = \iint_R -2\,dxy = -2\int_0^{2\pi}\int_0^{1/2} r\,dr\,d\theta$$

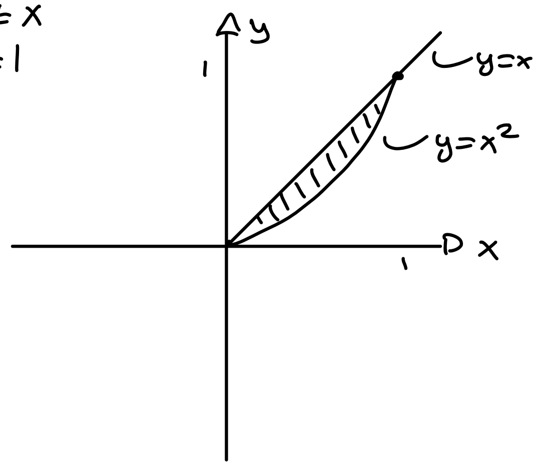
$$-2\int_0^{2\pi}\left[\frac{r^2}{2}\Big|_0^{1/2}\right]d\theta = -2\int_0^{2\pi}\left[\frac{1}{8}-0\right]d\theta$$

$$-2\int_0^{2\pi}\frac{1}{8}\,d\theta = -\frac{1}{4}\int_0^{2\pi}d\theta = \frac{-1}{4}\left[\theta\Big|_0^{2\pi}\right] = -\pi/2$$

From this, Green's theorem is $-\pi/2$ for this $\vec{F}$

$$\boxed{\iint_R (F_{2x}-F_{1y})\,dxdy = -\pi/2}$$

Problem 10.4.4 $\vec{F} = [x\cosh(2y),\ 2x^2\sinh(2y)]$ $R: x^2 \le y \le x$, $0 \le x \le 1$



$$\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

$$\frac{\partial F_2}{\partial x} = 4x\sinh(2y) \quad ; \quad \frac{\partial F_1}{\partial y} = 2x\sinh(2y)$$

So,

$$\int_0^1\int_{x^2}^{x} 4x\sinh(2y) - 2x\sinh(2y)\,dy\,dx$$

$$\int_0^1\int_{x^2}^{x} 2x\sinh(2y)\,dy\,dx$$

$$\int_0^1 \left[x\cosh(2y)\Big|_{x^2}^{x}\right] dx$$

$$\int_0^1 \left[x\cosh(2x) - x\cosh(2x^2)\right] dx$$

$u = 2x^2$
$du = 4x\,dx$; $dx = \frac{du}{4x}$

$u = x$ $dv = \cosh(2x)$
$du = 1$ $v = \frac{\sinh(2x)}{2}$

$$\left[\frac{x\sinh(2x)}{2} - \frac{1}{2}\int_0^1 \sinh(2x)\,dx\right] - \frac{1}{4}\int_0^2 \cosh(u)\,du$$

$$\left(\frac{x\sinh(2x)}{2} - \frac{1}{2}\left[\frac{1}{2}\cosh(2x)\right]\Big|_0^1\right) - \frac{1}{4}\left[\sinh(u)\Big|_0^2\right]$$

$$\left(\frac{\sinh(2)}{2} - \frac{\cosh(2)}{4}\right) - \left(0 - \frac{1}{4}\right) - \frac{1}{4}\left(\sinh(2) - 0\right)$$

$$\frac{\sinh(2)}{2} - \frac{\cosh(2)}{4} + \frac{1}{4} - \frac{1}{4}\sinh(2) = \frac{\sinh(2)}{4} - \frac{\cosh(2)}{4} + \frac{1}{4}$$

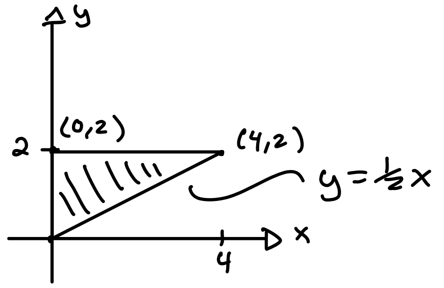
$$\tfrac{1}{4}(\sinh(2) - \cosh(2) + 1)$$

So,

$$\boxed{\iint_R (F_{2x} - F_{1y})\,dy\,dx = \tfrac{1}{4}(\sinh(2) - \cosh(2) + 1)}$$

Problem 10.4.13

$$\oint_C \frac{\partial w}{\partial n}\, ds = \iint_R \nabla^2 w \, dx\, dy$$



$w = \cosh(x)$

$$\nabla^2 w : \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \cosh(x) + 0 = \cosh(x)$$

$$\frac{\partial w}{\partial x} = \sinh(x) \qquad \frac{\partial w}{\partial y} = 0$$

$$\frac{\partial^2 w}{\partial x^2} = \cosh(x) \qquad \frac{\partial^2 w}{\partial y^2} = 0$$

$0 \le y \le 2$
$0 \le x \le 2y$

$y = mx + b$
$m = \frac{1}{2} : y = \frac{1}{2}x : b = 0$

So,

$$\iint_R \nabla^2 w \, dx\, dy = \int_0^2 \int_0^{2y} \cosh(x)\, dx\, dy$$

$$= \int_0^2 \left[\sin(x) \Big|_0^{2y} \right] dy$$

$$= \int_0^2 \sin(2y)\, dy = \left[\frac{\cosh(2y)}{2} \Big|_0^2 \right]$$

$$= \frac{\cosh(4)}{2} - \frac{1}{2} = \frac{1}{2}\left[\cosh(4) - 1\right]$$

$\therefore$ $$\boxed{\int_C \frac{\partial w}{\partial n}\, ds = \frac{1}{2}(\cosh(4) - 1)}$$