HOMEWORK SET 5 SOLUTIONS

CQ1 $V_{RMS,He} = V_{RMS,O_2}$

NOW

$$\mathcal{E}_{AVG} = \frac{1}{2} m V_{RMS}^2 = \frac{3}{2} k_B T \quad \text{so} \quad T = \frac{m V_{RMS}^2}{3 k_B}$$

SO, LOOKING AT THE RATIOS

$$\frac{T_{O_2}}{T_{He}} = \frac{m_{O_2} V_{RMS,O_2}^2 / 3k_B}{m_{He} V_{RMS,He}^2 / 3k_B} = \frac{m_{O_2}}{m_{He}} = \frac{32u}{4u} = 8$$

$$\text{so} \quad \boxed{T_{O_2} = 8 T_{He}}$$

CQ2 $\mathcal{E}_{AVG} \uparrow$, W/ V = CONSTANT

NOW

$$\mathcal{E}_{AVG} = \frac{3}{2} k_B T$$ SO IF $\mathcal{E}_{AVG}$ INCREASES, THEN T INCREASES

NOW $pV = N k_B T \;\therefore\; T = \dfrac{pV}{N k_B}$ IF T INCREASES, THEN $\dfrac{p}{N}$ INCREASES IF V STAYS CONSTANT.

IF N STAYS CONSTANT THEN PRESSURE MUST INCREASE

(d)

CQ 3. (b)

CQ4 IF $T \to 2T$, BUT $p \to p$

$pV = nRT$ THEN $V \to 2V$

$$V_{RMS} = \sqrt{\frac{3k_B T}{m}} \quad \text{so AS } T \to 2T, \quad \boxed{V_{RMS} \to \sqrt{2}\, V_{RMS}}$$

(c)

P1 $\lambda = 8.0 \times 10^{-8} m$

$T = 273 K$

$p = 1.01 \times 10^{5} Pa$

NOW $pV = N k_B T$ so $\frac{N}{V} = \frac{p}{k_B T} = \frac{(1.01 \times 10^{5} Pa)}{(1.38 \times 10^{-23} J/K)(273 K)}$

$= 2.68 \times 10^{25} m^{-3}$

NOW

$\lambda = \frac{1}{4\sqrt{2}\pi (N/V) r^2}$ so $r = \left[\frac{1}{4\sqrt{2}\pi (N/V)\lambda} \right]^{1/2} = \left[\frac{1}{4\sqrt{2}\pi (2.68 \times 10^{25} m^{-3})(8.0 \times 10^{-8} m)} \right]^{1/2}$

$= 1.6 \times 10^{-10} m$

SO THE DIAMETER IS..

$$\left[D = 2r = 3.2 \times 10^{-10} m \right]$$

P2 a) $\vec{V}_{AVG} = \frac{1}{5}\sum_{i=1}^{N} \vec{V}_i = \frac{1}{5}\left[(15\hat{i} - 25\hat{j} + 25\hat{k})m/s + (20\hat{i} - 30\hat{j} + 40\hat{k})m/s + (-35\hat{i} + 20\hat{j} - 10\hat{k})m/s + (20\hat{i} - 30\hat{j} - 30\hat{k})m/s + (-20\hat{i} + 65\hat{j} + 25\hat{k})m/s\right]$

$= \frac{1}{5}\left[(15+20-35+20-20)\hat{i} + (-25-30+20-30+65)\hat{j} + (25+40-10-30+25)\hat{k}\right]m/s$

$= (0\hat{i} + 0\hat{j} + 50\hat{k})m/s$

so $\left[\vec{V}_{AVG} = (50\hat{k})m/s\right]$

b) $V_1 = \sqrt{(15)^2 + (-25)^2 + (25)^2}\ m/s = 38.4m/s$

$V_2 = \sqrt{(20)^2 + (-30)^2 + (40)^2}\ m/s = 53.9m/s$

$V_3 = \sqrt{(-35)^2 + (20)^2 + (-10)^2}\ m/s = 41.5m/s$

$V_4 = \sqrt{(20)^2 + (-30)^2 + (-30)^2}\ m/s = 46.9m/s$

$V_5 = \sqrt{(-20)^2 + (65)^2 + (25)^2}\ m/s = 72.5m/s$

$V_{AVG} = \frac{1}{5}\sum_{i=1}^{5} V_i = \frac{1}{5}(38.4m/s + 53.9m/s + 41.5m/s + 46.9m/s + 72.5m/s) \approx 50.6m/s$

so $\left[V_{AVG} = 51m/s\right]$

c) $V_{RMS} = \sqrt{(V^2)_{AVG}}$ so $(V_i^2)_{AVG} = \frac{1}{5}\sum_{i=1}^{5} V_i^2 = \frac{1}{5}(V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2)$

$= \frac{1}{5}\left[1475 + 2900 + 1725 + 2200 + 5250\right]\frac{m^2}{s^2}$

$= 2710 m^2/s^2$

so

$V_{RMS} = \sqrt{2710 m^2/s^2}$

$\left[V_{RMS} = 52m/s\right]$

P3 $\rho = 1.24\times 10^{-5}\frac{g}{cm^3} \times \frac{1kg}{1000g} \times \left(\frac{100cm}{1m}\right)^3 = 1.24\times10^{-2}\,kg/m^3$

$p = 1.01\times10^{3}Pa$

$T = 273K$

NOTICE:

$\rho = \frac{M}{V} = \frac{Nm}{V} \therefore \frac{N}{V} = \frac{\rho}{m}$ WHERE m IS THE MASS OF ONE PARTICLE

$pV = Nk_BT \therefore \frac{N}{V} = \frac{p}{k_BT} = \frac{\rho}{m}$

so

$m = \frac{\rho k_B T}{p}$

$= \frac{(1.24\times10^{-2}\,kg/m^3)(1.38\times10^{-23}\,J/K)(273K)}{(1.01\times10^{3}\,Pa)}$

$= 4.63\times10^{-26}\,kg \times \frac{1u}{1.66\times10^{-27}\,kg}$

$= 27.9u \cong 28u$

a) $v_{rms} = \sqrt{\frac{3k_BT}{m}}$

$v_{rms} = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(273K)}{4.63\times10^{-26}\,kg}}$

$\boxed{v_{rms} = 494\,m/s}$

b) $n = \frac{M}{M_{mol}} = \frac{N}{N_A} = \frac{Nm}{M_{mol}}$

$M_{mol} = mN_A = (4.63\times10^{-26}\,kg)(6.02\times10^{23}\,\frac{\#}{mol})$

$\boxed{M_{mol} = 0.028\,kg/mol}$ $\therefore$ NITROGEN!

PH. $Q_1 + Q_2 = 0$

so

$$\frac{3}{2}n_2 R\left(T_f - T_{2i}\right) + \frac{5}{2}n_1 R\left(T_f - T_{1i}\right) = 0$$

so

$$\frac{1}{2}R\left(3n_2 + 5n_1\right)T_f = \frac{1}{2}R\left(3n_2 T_{2i} + 5n_1 T_{1i}\right)$$

so

$$T_f = \frac{3n_2 T_{2i} + 5n_1 T_{1i}}{3n_2 + 5n_1} = \frac{3(0.875\,\text{mol})(296\,\text{K}) + 5(0.266\,\text{mol})(598\,\text{K})}{3(0.875\,\text{mol}) + 5(0.266\,\text{mol})}$$

$$= \frac{777\,\text{mol}\cdot\text{K} + 795\,\text{mol}\cdot\text{K}}{3.96\,\text{mol}} = 397\,\text{K}$$

$\therefore \left[T_f = 397\,\text{K}\right]$

a) $E_{2i} = \frac{3}{2}n_2 R T_{2i} = \frac{3}{2}(0.875\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(296\,\text{K}) = 3{,}230\,\text{J}$

$E_{1i} = \frac{5}{2}n_1 R T_{1i} = \frac{5}{2}(0.266\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(598\,\text{K}) = 3{,}300\,\text{J}$

$E_{2f} = \frac{3}{2}n_2 R T_{2f} = \frac{3}{2}(0.875\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(397\,\text{K}) = 4{,}330\,\text{J}$

$E_{1f} = \frac{5}{2}n_1 R T_{1f} = \frac{5}{2}(0.266\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(397\,\text{K}) = 2{,}190\,\text{J}$

b) SINCE $W = 0$, $\Delta E_{th} = Q$ so

$Q_1 = \Delta E_{th,1} = \frac{5}{2}n_1 R\left(T_{1f} - T_{1i}\right) = E_{1f} - E_{1i} = -1{,}110\,\text{J}$

$Q_2 = \Delta E_{th,2} = E_{2f} - E_{2i} = +1{,}100\,\text{J}$

HEAT IS TRANSFERRED FROM THE O_2 TO THE He

c) $\left[T_f = 397\,\text{K}\right]$

P5. MONATOMIC GAS

ISOCHORICALLY HEATED $\Rightarrow V_1 = V_2$

$T_2 = 3T_1$

a) $\varepsilon_{AVG} = \frac{1}{2} m v_{RMS}^2 = \frac{3}{2} k_B T$ so $v_{RMS} = \sqrt{\frac{3 k_B T}{m}}$

IF $T \to 3T$

$$\left[v_{RMS} \to \sqrt{3}\, v_{RMS} \right]$$

b) $\lambda = \frac{1}{4\sqrt{2}\pi \left(\frac{N}{V}\right) r^2}$ so AS $T \to 3T$

$$\left[\lambda \to \lambda \right]$$

c) $E_{TH} = \frac{3}{2} nRT$ so AS $T \to 3T$

$$\left[E_{TH} \to 3 E_{TH} \right]$$

d) $C_V \to C_V$

P6 $n = 1.00\,\text{mol}$, HELIUM $\therefore$ MONATOMIC GAS

STATE A: $p_A = 1.00\,\text{ATM} = 1.01 \times 10^5\,\text{Pa}$, $T_A = 300.\text{K}$

ISOCHORIC HEATING $\therefore$ $p_B = p_A$, $T_B = 600.\text{K}$ ✓

ADIABATIC EXPANSION: $Q_{B \to C} = 0$, $p_C = p_A$

ISOBARIC COMPRESSION: $p_A = p_C$

, NOW $pV = nRT$ so $V_A = \dfrac{nRT_A}{p_A} = \dfrac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300.\text{K})}{(1.01 \times 10^5\,\text{Pa})} = 2.47 \times 10^{-2}\,\text{m}^3$

$$\left[V_A = 2.47 \times 10^{-2}\,\text{m}^3 \right]$$

FROM STATE $A \to B$...

ISOCHORIC HEATING $\therefore V_A = V_B$ so $\dfrac{p_A V_A}{T_A} = \dfrac{p_B V_B}{T_B}$ $\therefore p_B = \dfrac{T_B}{T_A} p_A = \dfrac{(600.\text{K})}{(300.\text{K})}(1.01 \times 10^5\,\text{Pa})$

$$\left[p_B = 2.02 \times 10^5\,\text{Pa} \right]$$

NOW GOING FROM STATE $B \to C$...

ADIABATIC EXPANSION $\therefore p_B V_B^{\gamma} = p_C V_C^{\gamma}$ NOW MONATOMIC GAS so $\gamma = 1.67$

so $V_C = \left(\dfrac{p_B}{p_C}\right)^{1/\gamma} V_B = \left(\dfrac{p_B}{p_A}\right)^{1/\gamma} V_A = \left(\dfrac{2.02 \times 10^5\,\text{Pa}}{1.01 \times 10^5\,\text{Pa}}\right)^{1/1.67} (2.47 \times 10^{-2}\,\text{m}^3)$

HERE

$p_C = p_A$

$$\left[V_C = 3.74 \times 10^{-2}\,\text{m}^3 \right]$$

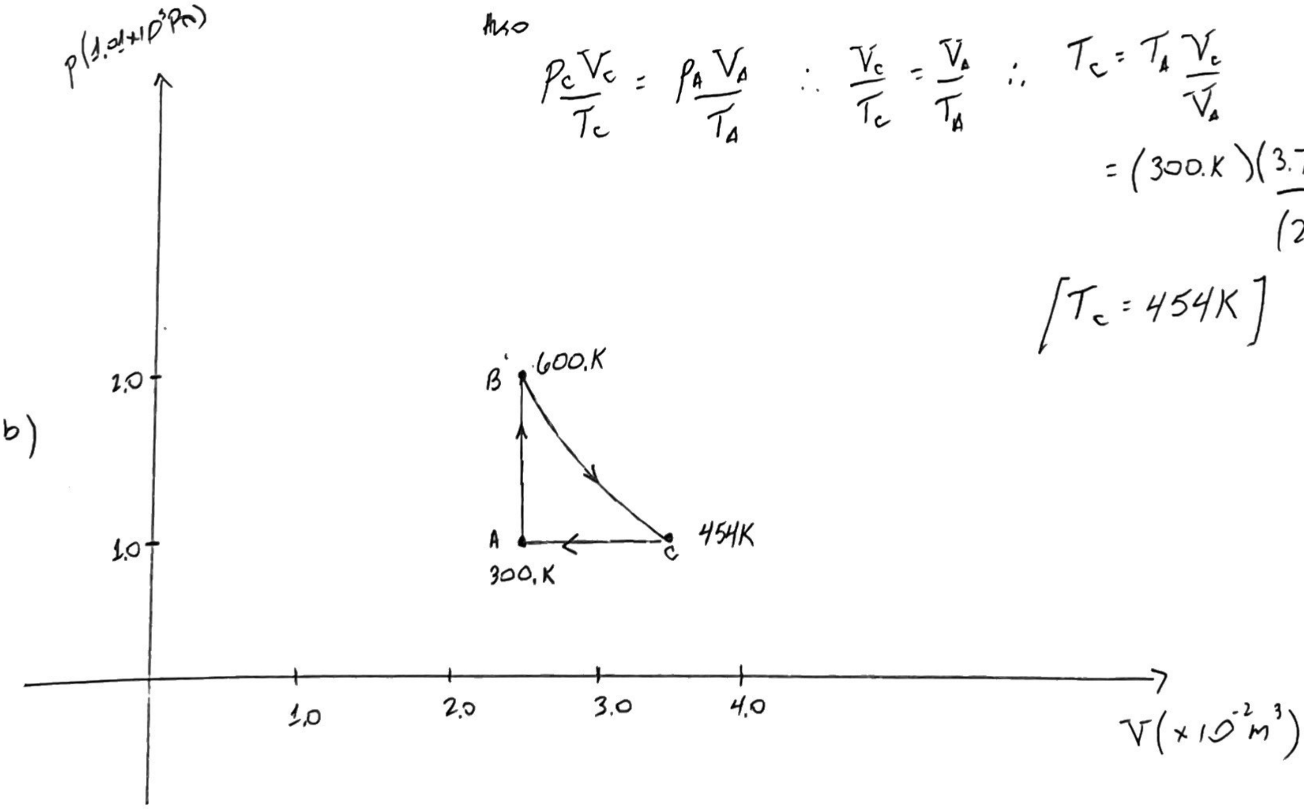
ALSO

$\dfrac{p_C V_C}{T_C} = \dfrac{p_A V_A}{T_A}$ $\therefore \dfrac{V_C}{T_C} = \dfrac{V_A}{T_A}$ $\therefore T_C = T_A \dfrac{V_C}{V_A}$

$= (300.\text{K}) \dfrac{(3.74 \times 10^{-2}\,\text{m}^3)}{(2.47 \times 10^{-2}\,\text{m}^3)}$

$$\left[T_C = 454\,\text{K} \right]$$

b)



c) • FOR PROCESS $A \to B$, $W = 0$ so $\Delta E_{TH} = nC_V\Delta T = Q$

$$\Delta E_{TH} = nC_V\Delta T = (1.00\,mol)(12.5\,J/mol\cdot K)(600.K - 300.K) = 3.75\times10^3\,J$$

$C_V = 12.5\,J/mol\cdot K$ for HELIUM

so

$$\Delta E_{TH,A\to B} = 3.75\times10^3\,J$$

$$Q_{A\to B} = 3.75\times10^3\,J$$

$$W_{A\to B} = 0\,J$$

• FOR PROCESS $B \to C$, ADIABATIC EXPANSION $\therefore$ $Q = 0$ so $\Delta E_{TH} = nC_V\Delta T = W$

$$\Delta E_{TH} = nC_V\Delta T = (1.00\,mol)(12.5\,J/mol\cdot K)(454K - 600.K) = -1825\,J$$

so

$$\Delta E_{TH,B\to C} = -1.83\times10^3\,J$$

$$Q_{B\to C} = 0$$

$$W_{B\to C} = -1.83\times10^3\,J$$

• FOR PROCESS $C \to A$, ISOBARIC COMPRESSION so

$$W = -p\Delta V = -p_c(V_A - V_c) = -(1.01\times10^5\,Pa)(2.47\times10^{-2}\,m^3 - 3.74\times10^{-2}\,m^3)$$

$$W = 1.28\times10^3\,J$$

ALSO

$$\Delta E_{TH,C\to A} = nC_V\Delta T = (1.00\,mol)(12.5\,J/mol\cdot K)(300K - 454K) = -1.93\times10^3\,J$$

LAST...

$$\Delta E_{TH} = W + Q \quad \text{so} \quad Q = \Delta E_{TH} - W$$

$$= -1.93\times10^3\,J - (1.28\times10^3\,J)$$

$$= -3.21\times10^3\,J$$

so

$$\Delta E_{TH,C\to A} = -1.93\times10^3\,J$$

$$Q_{C\to A} = -3.21\times10^3\,J$$

$$W_{C\to A} = 1.28\times10^3\,J$$