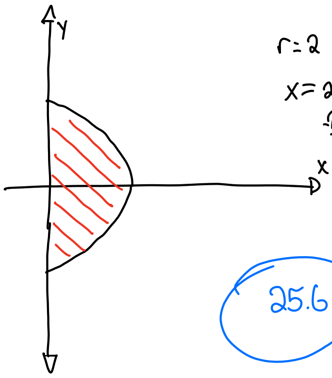
1.) $\int_C xy^4\,ds$ C Right half of $x^2+y^2=4$



$r=2$

$x=2\cos t$, $y=2\sin t$

$-\frac{\pi}{2}\le t\le\frac{\pi}{2}$

25.6

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos t\,(2\sin t)^4\sqrt{(-2\sin t)^2+(2\cos t)^2}\;dt$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2^5\cos t\sin^4 t\sqrt{4\sin^2 t+4\cos^2 t}\;dt$$

$u=\sin t$

$du=\cos t\,dt$

$$2^6\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos t\sin^4 t\;dt$$

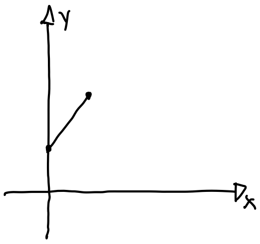
$$2^6\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u^4\;du$$

$$\frac{2^6}{5}\left[u^5\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{125}{8}$$

$$\frac{2^6}{5}\left(1-(-1)\right)$$

$$\frac{2^6}{5}(2) = 25.6$$

2.) $\int_C x\sin y\;ds$ C is line segment from $(0,3)$ to $(3,7)$



$$y-3=\frac{7-3}{3-0}(x-0)$$

$$y-3=\frac{4}{3}x$$

$$y=\frac{4}{3}x+3$$

$$\frac{dy}{dx}=\frac{4}{3}$$

$$\sqrt{\left(\frac{4}{3}\right)^2+1}$$

$$\sqrt{\frac{16}{9}+\frac{9}{9}}$$

$$\sqrt{\frac{25}{9}}$$

$$\frac{5}{3}$$

$$\frac{5}{3}\int_0^3 x\sin\left(\frac{4}{3}x+3\right)dx$$

$u=x$ $\quad dv=\sin\left(\frac{4}{3}x+3\right)$

$du=1$ $\quad v=-\frac{3}{4}\cos\left(\frac{4}{3}x+3\right)$

$u=\frac{4}{3}x+3$

$du=\frac{4}{3}\,dx$

$\frac{3}{4}du=dx$

$$x\left(-\frac{3}{4}\cos\left(\frac{4}{3}x+3\right)\right)-\int -\frac{3}{4}\cos\left(\frac{4}{3}x+3\right)dx$$

$$-\frac{3}{4}x\cos\left(\frac{4}{3}x+3\right)+\frac{3}{4}\int_0^3\cos\left(\frac{4}{3}x+3\right)dx$$

$$-\frac{3}{4}x\cos\left(\frac{4}{3}x+3\right)+\frac{9}{16}\int_0^3\cos(u)\;du$$

$$-\frac{3}{4}x\cos\left(\frac{4}{3}x+3\right)+\frac{9}{16}\left[\sin\left(\frac{4}{3}x+3\right)\right]\Big|_0^3$$

$$-\frac{9}{4}\cos(7)+\frac{9}{16}\left[\sin(7)\right]-0+\frac{9}{16}\sin(3)$$

$$-\frac{9}{4}\cos(7)+\frac{9}{16}\sin(7)-\frac{9}{16}\sin(3)$$

$u=\frac{4}{3}x+3$

$du=\frac{4}{3}\,dx$

$\frac{3}{4}du=dx$

$$\frac{5}{3}\left[-\frac{9}{4}\cos(7)+\frac{9}{16}\left(\sin(7)-\sin(3)\right)\right]$$

3.) $\int_C (x^2y^3-\sqrt{x})\;dy$ C $y=\sqrt{x}$ from $(0,0)$ to $(9,3)$

$y=t$ $\quad x=t^2$

$0\le t\le 3$

$$\int_0^3 t^4(t^3)-\sqrt{t^2}\;dt$$

$$\int_0^3 t^7-t\;dt$$

$$\frac{1}{8}t^8-\frac{1}{2}t^2\Big|_0^3$$

$$\frac{6561}{8}-\frac{9}{2}=\frac{6525}{8}$$

$\frac{6525}{8}$

4.) $\int_C xyz\;ds$ $\quad x=4\sin(t)$ $\quad y=t$ $\quad z=-4\cos t$

$0\le t\le\pi$

$dx=4\cos(t)$ $\quad dy=1$ $\quad dz=4\sin t$

$$\sqrt{17}\int_0^{\pi}-16\sin(t)\cos(t)\cdot t\;dt$$

$4\sqrt{17}\pi$

$$\sqrt{4\cos(t)^2+1+4\sin(t)^2}$$

$$\sqrt{16(\sin^2 t+\cos^2 t)+1}$$

$$\sqrt{17}$$

$4\sqrt{17}\pi$

$u=t$ $\quad dv=\sin 2t$

$du=1\,dt$ $\quad v=-\frac{1}{2}\cos 2t$

5.) $\int_C z^2dx + x^2dy + y^2dz$ $(1,0,0)$ to $(4,1,2)$

$\langle 3,1,2\rangle$

$\langle 1,0,0\rangle + t\langle 3,1,2\rangle$ $0\le t\le 1$

$\int_0^1 4t^2(3) + (1+3t)^2(1) + t^2(2)\,dt$ $x=1+3t$ $dx=3$

$y=t$ $dy=1$

$\int_0^1 12t^2 + (1+3t)^2 + 2t^2\,dt$ $z=2t$ $dz=2$

$\int_0^1 12t^2 + 9t^2+6t+1 + 2t^2\,dt$

$\int_0^1 23t^2+6t+1\,dt$

$\frac{23}{3}t^3 + 3t^2 + t \Big|_0^1$

$\frac{23}{3} + 3 + 1$

$\frac{23}{3} + \frac{9}{3} + \frac{3}{3} = \frac{35}{3}$

$\frac{35}{3}$

7.) $\int_C F\cdot dr$

$F(x,y) = xy\hat{\imath} + 6y^2$

$\vec{r}(t) = 14t^4\hat{\imath} + t^6\hat{\jmath}$ $\vec{r}'(t) = \langle 56t^3, 6t^5\rangle$

$\int_0^1 \vec{F}\cdot\vec{r}(t)\,dt$

$0\le t\le 1$

$\vec{F}(t) = 14t^4(t^6) + 6(t^6)^2$

$\vec{F}(t) = 14t^{10} + 6t^{12}$ $\langle 14t^{10}, 6t^{12}\rangle \cdot \langle 56t^3, 6t^5\rangle$

$784t^{13} + 36t^{17}$

$\int_0^1 784t^{13} + 36t^{17}\,dt$

$56t^{14} + 2t^{18}\Big|_0^1$

$56+2 = 58$

58

8.) $\int_C F\cdot \vec{r}'(t)\,dt$

$F(x,y,z) = x\hat{\imath} + y\hat{\jmath} + xy\hat{k}$ $0\le t\le \pi$

$r(t) = \sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + t\hat{k}$

$\vec{r}'(t) = \langle \cos t, -\sin t, 1\rangle$

$F(\vec{r}(t)) = \sin(t)\hat{\imath} + \cos(t)\hat{\jmath} + \sin(t)\cos(t)\hat{k}$

$\cos(t)\sin(t) - \cos(t)\sin(t) + \sin(t)\cos(t)$

$\int_0^\pi \sin(t)\cos(t)\,dt$ $u=\sin(t)$

$du = \cos(t)\,dt$

$\int_0^\pi u\,du$

$\frac{1}{2}\sin^2(t)\Big|_0^\pi = 0$

$\emptyset$

9.) $F(x,y) = x\hat{\imath} + (y+5)\hat{\jmath}$

$r(t) = (t-\sin t)\hat{\imath} + (1-\cos t)\hat{\jmath}$ $0\le t\le 2\pi$

$F(t) = (t-\sin t)\hat{\imath} + (6-\cos t)\hat{\jmath}$ $\langle t-\sin t, 6-\cos t\rangle \cdot \langle 1-\cos t, \sin t\rangle$ $\vec{F}(t)\cdot\vec{r}(t)$

$\vec{r}'(t) = (1-\cos t)\hat{\imath} + \sin t\,\hat{\jmath}$ $t - t\cos t - \sin t + \cancel{\sin t\cos t} + 6\sin t - \cancel{\cos t\sin t}$

$5\sin t - t\cos t + t$

$\int_0^{2\pi} 5\sin t - t\cos t + t\,dt$ $u=t$ $dv=\cos t$

$du=1$ $v=\sin t$ $t\sin t - \int \sin t\,dt$

$2\pi^2$

$t\sin t - (-\cos t)$

$\int_0^{2\pi} 5\sin t\,dt - \int_0^{2\pi} t\cos t\,dt + \int_0^{2\pi} t\,dt$ $t\sin t + \cos t$

$-5\cos t\Big|_0^{2\pi} - (t\sin t + \cos t)\Big|_0^{2\pi} + \frac{t^2}{2}\Big|_0^{2\pi}$

$-5-(-5) - (0+1-(0+1)) + \frac{4\pi^2}{2}$

$-5+5 - (1-1) + 2\pi^2$

$0 + 0 + 2\pi^2$

10.)

$F(r) = \frac{K\vec{r}(t)}{|\vec{r}(t)|^3}$ $(4,0,0)$ to $(4,3,4)$

$\vec{r}(t) = \langle 4,0,0\rangle + t\langle 0,3,4\rangle$

$x=4$ $\vec{F}(\vec{r}(t)) = \langle 4\hat{i}, (3t)\hat{j}, (4t)\hat{k}\rangle$

$y=3t$ $\vec{r}(t) = \langle 4, 3t, 4t\rangle$

$z=4t$ $\vec{r}\,'(t) = \langle 0,3,4\rangle$

$0 \le t \le 1$

$\frac{\sqrt{16+9t^2+16t^2}}{\sqrt{16+25t^2}}$ $= \frac{K\langle 4, 3t, 4t\rangle}{(16+25t^2)^{3/2}} \cdot \langle 0,3,4\rangle$

$K\left(\frac{\langle 0, 9t, 16t\rangle}{(16+25t^2)^{3/2}}\right)$ $0+3t+16t$ $9t$

$25K\int_0^1 \left(\frac{t}{(16+25t^2)^{3/2}}\right) dt$

$u = 16+25t^2$
$du = 50t\,dt$
$\frac{du}{50} = t\,dt$

$\frac{1}{50}\int_0^1 \frac{1}{(u)^{3/2}}$

$\frac{1}{50}\int_0^1 u^{-3/2}$

$\frac{1}{50}\left[-2u^{-1/2}\right]_0^1$

$\frac{-1}{25}\left[\frac{1}{\sqrt{16+25t^2}}\right]_0^1$

$\frac{-1}{25}\left(\frac{1}{\sqrt{41}} - \frac{1}{4}\right)$

$25K\left(\frac{1}{100} - \frac{\sqrt{41}}{1025}\right)$

$25K\left(\frac{1}{100} - \frac{\sqrt{41}}{1025}\right)$

11.) 140 lb man
25 lb can of paint
16 lb leaks out

40ft

Two Revolutions

$r = 25$ ft
$h = 40$ ft

man + can

140 lb + 25 lb = 165 lb $0 \le t \le 4\pi$ $x = 25\cos t$ $z = Kt$

$y = 25\sin t$ $40\text{ft} = K(4\pi)$

$\frac{10\text{ft}}{\pi} = K$

$m = \left(165 - \frac{16\text{lb}}{4\pi}t\right)$

$\frac{40}{4\pi} = \frac{16}{\pi}$

$\vec{F}(t) = \langle 0, 0, 165 - \frac{4}{\pi}t\rangle$

$\vec{r}(t) = \langle 25\cos t, 25\sin t, \frac{10}{\pi}t\rangle$

$\vec{r}\,'(t) = \langle -25\sin t, 25\cos t, \frac{10}{\pi}\rangle$

$\langle 0,0,165-\frac{4}{\pi}t\rangle \cdot \langle -25\sin t, 25\cos t, \frac{10}{\pi}\rangle$

$(165 - \frac{4}{\pi}t)\frac{10}{\pi}$

$\frac{10}{\pi}\int_0^{4\pi} 165 - \frac{4}{\pi}t \; dt$

$\frac{10}{\pi}\left[165t - \frac{2}{\pi}t^2\right]_0^{4\pi}$

$\frac{10}{\pi}\left[165(4\pi) - \frac{2}{\pi}(16\pi^2)\right]$

$\frac{10}{\pi}\left[660\pi - 32\pi\right]$

$\frac{10}{\pi}\left[628\pi\right]$

6,280

6,280