

## Exercise 1

$$|\psi\rangle = \cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle : \quad \text{G} \rightarrow \boxed{\text{ } \quad \text{}} \quad S_n = +\frac{\hbar}{2} \\ S_n = -\frac{\hbar}{2}$$

Problem 1 | SG  $\hat{z}$  :  $|+\hat{n}\rangle = \cos(\alpha/2)|+\hat{z}\rangle + e^{i\phi}\sin(\alpha/2)|-\hat{z}\rangle : |-\hat{n}\rangle = \sin(\alpha/2)|+\hat{z}\rangle - e^{i\phi}\cos(\alpha/2)|-\hat{z}\rangle$   
 $\theta = 0, \phi = 0$

$$|+\hat{z}\rangle = \cos(0)|+\hat{z}\rangle + e^0 \cdot \sin(0)|-\hat{z}\rangle = |+\hat{z}\rangle : \therefore |+\hat{z}\rangle = |+\hat{z}\rangle$$

$$|-\hat{z}\rangle = \sin(0)|+\hat{z}\rangle - e^0 \cdot \cos(0)|-\hat{z}\rangle = |- \hat{z}\rangle : \therefore |-\hat{z}\rangle = |-\hat{z}\rangle$$

$$\langle +\hat{z}|\psi\rangle = (\langle +\hat{z}| + 0 \cdot \langle -\hat{z}|)(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = \cancel{\cos(\alpha/2)}\langle +\hat{z}|+\hat{z}\rangle + 0 \cdot \cancel{\sin(\alpha/2)}\langle -\hat{z}|-\hat{z}\rangle$$

$$\langle +\hat{z}|\psi\rangle = \cos(\alpha/2) : |\langle +\hat{z}|\psi\rangle|^2 = \cos^2(\alpha/2) = \frac{n_+}{n}$$

$$\langle -\hat{z}|\psi\rangle = (0 \cdot \langle +\hat{z}| - \langle -\hat{z}|)(\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = 0 \cdot \cancel{\cos(\alpha/2)}\langle +\hat{z}|+\hat{z}\rangle - \cancel{\sin(\alpha/2)}\langle -\hat{z}|-\hat{z}\rangle$$

$$\langle -\hat{z}|\psi\rangle = -\sin(\alpha/2) : |\langle -\hat{z}|\psi\rangle|^2 = \sin^2(\alpha/2) = \frac{n_-}{n}$$

$$|\langle +\hat{z}|\psi\rangle|^2 + |\langle -\hat{z}|\psi\rangle|^2 = 1 : \cos^2(\alpha/2) + \sin^2(\alpha/2) = 1 \therefore \alpha \text{ can be any value}$$

The  $\alpha$  value can be calculated via

$$\cos^2(\alpha/2) = \frac{n_+}{n}, \sin^2(\alpha/2) = \frac{n_-}{n}$$

Problem 2 | SG  $\hat{x}$  :  $|+\hat{n}\rangle = \cos(\pi/4)|+\hat{x}\rangle + e^{i\phi}\sin(\pi/4)|-\hat{x}\rangle : |-\hat{n}\rangle = \sin(\pi/4)|+\hat{x}\rangle - e^{i\phi}\cos(\pi/4)|-\hat{x}\rangle$   
 $\theta = \pi/4, \phi = 0$

$$|+\hat{x}\rangle = \cos(\pi/4)|+\hat{x}\rangle + e^0 \sin(\pi/4)|-\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\hat{x}\rangle + \frac{1}{\sqrt{2}}|-\hat{x}\rangle \therefore |+\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\hat{x}\rangle + \frac{1}{\sqrt{2}}|-\hat{x}\rangle$$

$$|-\hat{x}\rangle = \sin(\pi/4)|+\hat{x}\rangle - e^0 \cos(\pi/4)|-\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\hat{x}\rangle - \frac{1}{\sqrt{2}}|-\hat{x}\rangle \therefore |-\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\hat{x}\rangle - \frac{1}{\sqrt{2}}|-\hat{x}\rangle$$

$$\langle +\hat{x}|\psi\rangle = \frac{1}{\sqrt{2}}(\langle +\hat{x}| + \langle -\hat{x}|)(\cos(\alpha/2)|+\hat{x}\rangle + \sin(\alpha/2)|-\hat{x}\rangle) = \frac{1}{\sqrt{2}}(\cos(\alpha/2)\langle +\hat{x}|+\hat{x}\rangle + \sin(\alpha/2)\langle -\hat{x}|-\hat{x}\rangle)$$

$$\langle +\hat{x}|\psi\rangle = \frac{1}{\sqrt{2}}(\cos(\alpha/2) + \sin(\alpha/2)) : |\langle +\hat{x}|\psi\rangle|^2 = \frac{1}{2}(\cos^2(\alpha/2) + \sin^2(\alpha/2) + 2\cos(\alpha/2)\sin(\alpha/2)) \\ = \frac{1}{2}(1 + \sin(\alpha)) : |\langle +\hat{x}|\psi\rangle|^2 = \frac{1}{2}(1 + \sin(\alpha)) = \frac{n_+}{n}$$

$$\langle -\hat{x}|\psi\rangle = \frac{1}{\sqrt{2}}(\langle +\hat{x}| - \langle -\hat{x}|)(\cos(\alpha/2)|+\hat{x}\rangle + \sin(\alpha/2)|-\hat{x}\rangle) = \frac{1}{\sqrt{2}}(\cos(\alpha/2)\langle +\hat{x}|+\hat{x}\rangle - \sin(\alpha/2)\langle -\hat{x}|-\hat{x}\rangle)$$

$$\langle -\hat{x}|\psi\rangle = \frac{1}{\sqrt{2}}(\cos(\alpha/2) - \sin(\alpha/2)) : |\langle -\hat{x}|\psi\rangle|^2 = \frac{1}{2}(\cos^2(\alpha/2) + \sin^2(\alpha/2) - 2\cos(\alpha/2)\sin(\alpha/2)) \\ = \frac{1}{2}(1 - \sin(\alpha)) : |\langle -\hat{x}|\psi\rangle|^2 = \frac{1}{2}(1 - \sin(\alpha)) = \frac{n_-}{n}$$

The  $\alpha$  value can be calculated via

$$\frac{1}{2}(1 + \sin(\alpha)) = \frac{n_+}{n}, \frac{1}{2}(1 - \sin(\alpha)) = \frac{n_-}{n}$$

### Exercise 1 Continued

$$|\psi\rangle = \cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle : \quad \text{Spin-up} \rightarrow \boxed{\text{S}_z = +\frac{\hbar}{2}} \quad \text{Spin-down} \rightarrow \boxed{\text{S}_z = -\frac{\hbar}{2}}$$

Problem 3 | SG  $\hat{Y}$  :  $|+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\phi}\sin(\theta/2)|-\hat{z}\rangle$  :  $|-\hat{n}\rangle = \sin(\theta/2)|+\hat{z}\rangle - e^{i\phi}\cos(\theta/2)|-\hat{z}\rangle$   
 $\theta = \pi/2, \phi = \pi/2$

$$|+\hat{y}\rangle = \cos(\pi/4)|+\hat{z}\rangle + e^{i\pi/2}\sin(\pi/4)|-\hat{z}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{i}{\sqrt{2}}|-\hat{z}\rangle \therefore |+\hat{y}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{i}{\sqrt{2}}|-\hat{z}\rangle$$

$$|-\hat{y}\rangle = \sin(\pi/4)|+\hat{z}\rangle - e^{i\pi/2}\cos(\pi/4)|-\hat{z}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle - \frac{i}{\sqrt{2}}|-\hat{z}\rangle \therefore |-\hat{y}\rangle = \frac{1}{\sqrt{2}}|+\hat{z}\rangle - \frac{i}{\sqrt{2}}|-\hat{z}\rangle$$

$$\langle +\hat{y} | \psi \rangle = \frac{1}{\sqrt{2}} (\langle +\hat{z} | -i\langle -\hat{z} |) (\cos(\alpha/2)|+\hat{z}\rangle + \sin(\alpha/2)|-\hat{z}\rangle) = \frac{1}{\sqrt{2}} (\cos(\alpha/2)\langle +\hat{z}| + \sin(\alpha/2)\langle -\hat{z}|)$$

$$\langle +\hat{y} | \psi \rangle = \frac{1}{\sqrt{2}} (\cos(\alpha/2) - i\sin(\alpha/2))$$

$$|\langle +\hat{y} | \psi \rangle|^2 = \frac{1}{2} (\cos(\alpha/2) - i\sin(\alpha/2)) (\cos(\alpha/2) + i\sin(\alpha/2)) = \frac{1}{2} (\cos^2(\alpha/2) + \sin^2(\alpha/2))$$

$$|\langle +\hat{y} | \psi \rangle|^2 = \frac{1}{2} (\cos^2(\alpha/2) + \sin^2(\alpha/2)) \quad \text{This means } \alpha \text{ can be anything}$$

We cannot learn anything about  $\alpha$  by measuring along the  $Y$  direction.

## Exercise 2

$$|\varphi_1\rangle = |0\rangle, |\varphi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, |\varphi_3\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

Problem 1  $|\varphi_1\rangle|\varphi_2\rangle$

$$|\varphi_1\rangle|\varphi_2\rangle = (|0\rangle)\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|1\rangle$$

$$|\varphi_1\rangle|\varphi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |0\rangle|1\rangle)$$

Problem 2  $|\varphi_1\rangle|\varphi_3\rangle$

$$|\varphi_1\rangle|\varphi_3\rangle = (|0\rangle)\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle|0\rangle - \frac{i}{\sqrt{2}}|0\rangle|1\rangle$$

$$|\varphi_1\rangle|\varphi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle - i|0\rangle|1\rangle)$$

Problem 3  $|\varphi_2\rangle|\varphi_3\rangle$

$$|\varphi_2\rangle|\varphi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \cdot \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) = \frac{1}{2}(|0\rangle|0\rangle - i|0\rangle|1\rangle + |1\rangle|0\rangle - i|1\rangle|1\rangle)$$

$$|\varphi_2\rangle|\varphi_3\rangle = \frac{1}{2}(|0\rangle|0\rangle - i|0\rangle|1\rangle + |1\rangle|0\rangle - i|1\rangle|1\rangle)$$

### Exercise 3

#### Problem 1

$$|\psi\rangle = \frac{1}{2} [ |0\rangle|0\rangle + |0\rangle|1\rangle - |1\rangle|0\rangle - |1\rangle|1\rangle ]$$

a.) SG  $\hat{Z}$  for A, SG  $\hat{Z}$  for B  $\{|0\rangle, |1\rangle\}$

$S6\hat{Z}(A)$	$S6\hat{Z}(B)$	State
+	+	$ 0\rangle 0\rangle$
+	-	$ 0\rangle 1\rangle$
-	+	$ 1\rangle 0\rangle$
-	-	$ 1\rangle 1\rangle$

$$\langle 01|01|\psi\rangle = \frac{1}{2} [\cancel{\langle 01|01|0\rangle} + \cancel{\langle 01|01|1\rangle} - \cancel{\langle 01|10|0\rangle} - \cancel{\langle 01|10|1\rangle}]$$

$$\langle 01|01|\psi\rangle = \frac{1}{2} \therefore |\langle 01|01|\psi\rangle|^2 = \frac{1}{4}$$

$$\langle 01|11|\psi\rangle = \frac{1}{2} [\cancel{\langle 01|11|0\rangle} + \cancel{\langle 01|11|1\rangle} - \cancel{\langle 01|11|0\rangle} - \cancel{\langle 01|11|1\rangle}]$$

$$\langle 01|11|\psi\rangle = \frac{1}{2} \therefore |\langle 01|11|\psi\rangle|^2 = \frac{1}{4}$$

$$\langle 11|01|\psi\rangle = \frac{1}{2} [\cancel{\langle 11|01|0\rangle} + \cancel{\langle 11|01|1\rangle} - \cancel{\langle 11|01|0\rangle} - \cancel{\langle 11|01|1\rangle}]$$

$$\langle 11|01|\psi\rangle = -\frac{1}{2} \therefore |\langle 11|01|\psi\rangle|^2 = \frac{1}{4}$$

$$\langle 11|11|\psi\rangle = \frac{1}{2} [\cancel{\langle 11|11|0\rangle} + \cancel{\langle 11|11|1\rangle} - \cancel{\langle 11|11|0\rangle} - \cancel{\langle 11|11|1\rangle}]$$

$$\langle 11|11|\psi\rangle = -\frac{1}{2} \therefore |\langle 11|11|\psi\rangle|^2 = \frac{1}{4}$$

b.) SG  $\hat{x}$  for A, SG  $\hat{z}$  for B  $\left\{ \frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}} \right\}, \{|0\rangle, |1\rangle\}$

$S6\hat{x}(A)$	$S6\hat{z}(B)$	State
+	+	$\frac{ 0\rangle+ 1\rangle}{\sqrt{2}},  0\rangle = \frac{ 0\rangle 0\rangle+ 1\rangle 0\rangle}{\sqrt{2}}$
+	-	$\frac{ 0\rangle+ 1\rangle}{\sqrt{2}},  1\rangle = \frac{ 0\rangle 1\rangle+ 1\rangle 1\rangle}{\sqrt{2}}$
-	+	$\frac{ 0\rangle- 1\rangle}{\sqrt{2}},  0\rangle = \frac{ 0\rangle 0\rangle- 1\rangle 0\rangle}{\sqrt{2}}$
-	-	$\frac{ 0\rangle- 1\rangle}{\sqrt{2}},  1\rangle = \frac{ 0\rangle 1\rangle- 1\rangle 1\rangle}{\sqrt{2}}$

$$\frac{\langle 01|01+\langle 11|01|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} [\cancel{\langle 01|01|0\rangle} + \cancel{\langle 11|01|0\rangle} + \cancel{\langle 01|01|1\rangle} + \cancel{\langle 11|01|1\rangle} - \cancel{\langle 01|01|0\rangle} - \cancel{\langle 11|01|0\rangle} - \cancel{\langle 01|01|1\rangle} - \cancel{\langle 11|01|1\rangle}]$$

$$= \frac{1}{2\sqrt{2}} (1-1) = 0 \therefore \left| \frac{\langle 01|01+\langle 11|01|\psi\rangle}{\sqrt{2}} \right|^2 = 0$$

### Exercise 3 Continued

$$\frac{\langle 01\langle 11 + \langle -11\langle 11 \rangle \rangle \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \langle 01\langle 110\rangle 10 \rangle + \langle -11\langle 110\rangle 10 \rangle + \langle 01\langle 110\rangle 11 \rangle + \langle -11\langle 110\rangle 11 \rangle - \langle 01\langle 111\rangle 10 \rangle - \langle 11\langle 111\rangle 10 \rangle - \langle 01\langle 111\rangle 11 \rangle - \langle 11\langle 111\rangle 11 \rangle \right]$$

$$= \frac{1}{2\sqrt{2}} (1-1) = 0 \quad \therefore \boxed{\left| \frac{\langle 01\langle 11 + \langle -11\langle 11 \rangle \rangle \rangle}{\sqrt{2}} \right|^2 = 0}$$

$$\frac{\langle 01\langle 01 - \langle -11\langle 01 \rangle \rangle \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \langle 01\langle 010\rangle 10 \rangle - \langle -11\langle 010\rangle 10 \rangle + \langle 01\langle 010\rangle 11 \rangle - \langle -11\langle 010\rangle 11 \rangle - \langle 01\langle 011\rangle 10 \rangle + \langle -11\langle 011\rangle 10 \rangle - \langle 01\langle 011\rangle 11 \rangle + \langle -11\langle 011\rangle 11 \rangle \right]$$

$$= \frac{1}{2\sqrt{2}} (1+1) = \frac{1}{\sqrt{2}} \quad \therefore \boxed{\left| \frac{\langle 01\langle 01 - \langle -11\langle 01 \rangle \rangle \rangle}{\sqrt{2}} \right|^2 = \frac{1}{2}}$$

$$\frac{\langle 01\langle 11 - \langle -11\langle 11 \rangle \rangle \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \langle 01\langle 110\rangle 10 \rangle - \langle -11\langle 110\rangle 10 \rangle + \langle 01\langle 110\rangle 11 \rangle - \langle -11\langle 110\rangle 11 \rangle - \langle 01\langle 111\rangle 10 \rangle + \langle -11\langle 111\rangle 10 \rangle - \langle 01\langle 111\rangle 11 \rangle + \langle -11\langle 111\rangle 11 \rangle \right]$$

$$= \frac{1}{2\sqrt{2}} (1+1) = \frac{1}{\sqrt{2}} \quad \therefore \boxed{\left| \frac{\langle 01\langle 11 - \langle -11\langle 11 \rangle \rangle \rangle}{\sqrt{2}} \right|^2 = \frac{1}{2}}$$

c.) S6 $\hat{x}$  for A, S6 $\hat{x}$  for B  $\left\{ \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right\}$

S6 $\hat{x}$ (A)	S6 $\hat{x}$ (B)	State
+	+	$\frac{ 0\rangle +  1\rangle}{\sqrt{2}}, \frac{ 0\rangle +  1\rangle}{\sqrt{2}}$ $= \frac{1}{2}( 0\rangle 0\rangle +  0\rangle 1\rangle +  1\rangle 0\rangle +  1\rangle 1\rangle)$
+	-	$\frac{ 0\rangle +  1\rangle}{\sqrt{2}}, \frac{ 0\rangle -  1\rangle}{\sqrt{2}}$ $= \frac{1}{2}( 0\rangle 0\rangle -  0\rangle 1\rangle +  1\rangle 0\rangle -  1\rangle 1\rangle)$
-	+	$\frac{ 0\rangle -  1\rangle}{\sqrt{2}}, \frac{ 0\rangle +  1\rangle}{\sqrt{2}}$ $= \frac{1}{2}( 0\rangle 0\rangle +  0\rangle 1\rangle -  1\rangle 0\rangle -  1\rangle 1\rangle)$
-	-	$\frac{ 0\rangle -  1\rangle}{\sqrt{2}}, \frac{ 0\rangle -  1\rangle}{\sqrt{2}}$ $= \frac{1}{2}( 0\rangle 0\rangle -  0\rangle 1\rangle -  1\rangle 0\rangle +  1\rangle 1\rangle)$

$$\frac{\langle 01\langle 01 + \langle 01\langle 11 + \langle -11\langle 01 + \langle -11\langle 11 \rangle \rangle \rangle \rangle \rangle}{\sqrt{2}} = \frac{1}{4} \left[ \langle 01\langle 010\rangle 10 \rangle + \langle 01\langle 010\rangle 11 \rangle - \langle 01\langle 011\rangle 10 \rangle - \langle 01\langle 011\rangle 11 \rangle + \langle 01\langle 110\rangle 10 \rangle + \langle 01\langle 110\rangle 11 \rangle + \langle 01\langle 111\rangle 10 \rangle + \langle 01\langle 111\rangle 11 \rangle + \langle 11\langle 010\rangle 10 \rangle + \langle 11\langle 010\rangle 11 \rangle + \langle 11\langle 011\rangle 10 \rangle + \langle 11\langle 011\rangle 11 \rangle - \langle 11\langle 011\rangle 10 \rangle - \langle 11\langle 011\rangle 11 \rangle + \langle 11\langle 110\rangle 10 \rangle + \langle 11\langle 110\rangle 11 \rangle + \langle 11\langle 111\rangle 10 \rangle - \langle 11\langle 111\rangle 11 \rangle \right]$$

$$\frac{\langle 01\langle 01 + \langle 01\langle 11 + \langle -11\langle 01 + \langle -11\langle 11 \rangle \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} (1+1-1-1) = 0$$

$$\therefore \boxed{\left| \frac{\langle 01\langle 01 + \langle 01\langle 11 + \langle -11\langle 01 + \langle -11\langle 11 \rangle \rangle \rangle \rangle}{2} \right|^2 = 0}$$

### Exercise 3 Continued

$$\frac{\langle 01\langle 01 - \langle 01\langle 11 + \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} \left[ \langle 01\langle 0\langle 0\rangle 10 \rangle + \langle 01\langle 0\langle 0\rangle 11 \rangle - \langle 01\langle 0\langle 1\rangle 10 \rangle - \langle 01\langle 0\langle 1\rangle 11 \rangle - \langle 01\langle 1\langle 0\rangle 10 \rangle - \langle 01\langle 1\langle 0\rangle 11 \rangle + \langle 01\langle 1\langle 1\rangle 10 \rangle + \langle 01\langle 1\langle 1\rangle 11 \rangle + \langle 11\langle 0\langle 0\rangle 10 \rangle + \langle 11\langle 0\langle 0\rangle 11 \rangle - \langle 11\langle 0\langle 1\rangle 10 \rangle - \langle 11\langle 0\langle 1\rangle 11 \rangle - \langle 11\langle 1\langle 0\rangle 10 \rangle - \langle 11\langle 1\langle 0\rangle 11 \rangle + \langle 11\langle 1\langle 1\rangle 10 \rangle + \langle 11\langle 1\langle 1\rangle 11 \rangle \right]$$

$$\frac{\langle 01\langle 01 - \langle 01\langle 11 + \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} (1 - 1 - 1 + 1) = 0$$

∴

$$\boxed{\left| \frac{\langle 01\langle 01 - \langle 01\langle 11 + \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} \right|^2 = 0}$$

$$\frac{\langle 01\langle 01 + \langle 01\langle 11 - \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} \left[ \langle 01\langle 0\langle 0\rangle 10 \rangle + \langle 01\langle 0\langle 0\rangle 11 \rangle - \langle 01\langle 0\langle 1\rangle 10 \rangle - \langle 01\langle 0\langle 1\rangle 11 \rangle + \langle 01\langle 1\langle 0\rangle 10 \rangle + \langle 01\langle 1\langle 0\rangle 11 \rangle - \langle 01\langle 1\langle 1\rangle 10 \rangle - \langle 01\langle 1\langle 1\rangle 11 \rangle - \langle 11\langle 0\langle 0\rangle 10 \rangle - \langle 11\langle 0\langle 0\rangle 11 \rangle + \langle 11\langle 0\langle 1\rangle 10 \rangle + \langle 11\langle 0\langle 1\rangle 11 \rangle - \langle 11\langle 1\langle 0\rangle 10 \rangle - \langle 11\langle 1\langle 0\rangle 11 \rangle + \langle 11\langle 1\langle 1\rangle 10 \rangle + \langle 11\langle 1\langle 1\rangle 11 \rangle \right]$$

$$\frac{\langle 01\langle 01 + \langle 01\langle 11 - \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} (1 + 1 + 1 + 1) = 1$$

∴

$$\boxed{\left| \frac{\langle 01\langle 01 + \langle 01\langle 11 - \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} \right|^2 = 1}$$

$$\frac{\langle 01\langle 01 - \langle 01\langle 11 - \langle 11\langle 01 + \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} \left[ \langle 01\langle 0\langle 0\rangle 10 \rangle + \langle 01\langle 0\langle 0\rangle 11 \rangle - \langle 01\langle 0\langle 1\rangle 10 \rangle - \langle 01\langle 0\langle 1\rangle 11 \rangle - \langle 01\langle 1\langle 0\rangle 10 \rangle - \langle 01\langle 1\langle 0\rangle 11 \rangle + \langle 01\langle 1\langle 1\rangle 10 \rangle + \langle 01\langle 1\langle 1\rangle 11 \rangle + \langle 11\langle 0\langle 0\rangle 10 \rangle + \langle 11\langle 0\langle 0\rangle 11 \rangle - \langle 11\langle 0\langle 1\rangle 10 \rangle - \langle 11\langle 0\langle 1\rangle 11 \rangle + \langle 11\langle 1\langle 0\rangle 10 \rangle + \langle 11\langle 1\langle 0\rangle 11 \rangle + \langle 11\langle 1\langle 1\rangle 10 \rangle + \langle 11\langle 1\langle 1\rangle 11 \rangle \right]$$

$$\frac{\langle 01\langle 01 - \langle 01\langle 11 - \langle 11\langle 01 + \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} = \frac{1}{4} (1 - 1 + 1 - 1) = 0$$

∴

$$\boxed{\left| \frac{\langle 01\langle 01 - \langle 01\langle 11 - \langle 11\langle 01 + \langle 11\langle 11 \rangle \rangle \rangle \rangle}{2} \right|^2 = 0}$$

There is certainty with the S6 $\hat{x}$  for A, S6 $\hat{x}$  for B, pair of measurements. Precisely the (+-) pair for A and B specifically.

### Exercise 3 Continued

Problem 2  $|+\rangle = \frac{1}{2} [|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle]$

a.) SG $\hat{z}$  for A, SG $\hat{z}$  for B  $\{|0\rangle, |1\rangle\}$

SG $\hat{z}$ (A)	SG $\hat{z}$ (B)	State
+	+	$ 0\rangle 0\rangle$
+	-	$ 0\rangle 1\rangle$
-	+	$ 1\rangle 0\rangle$
-	-	$ 1\rangle 1\rangle$

$$\langle 01|01|\psi\rangle = \frac{1}{2} \left[ \cancel{\langle 01|01|0\rangle} + \cancel{\langle 01|01|1\rangle} + \cancel{\langle 01|11|0\rangle} - \cancel{\langle 01|11|1\rangle} \right]$$

$$\langle 01|01|\psi\rangle = \frac{1}{2} \quad \therefore$$

$$|\langle 01|01|\psi\rangle|^2 = \frac{1}{4}$$

$$\langle 01|11|\psi\rangle = \frac{1}{2} \left[ \cancel{\langle 01|11|0\rangle} + \cancel{\langle 01|11|1\rangle} + \cancel{\langle 01|11|0\rangle} - \cancel{\langle 01|11|1\rangle} \right]$$

$$\langle 01|11|\psi\rangle = \frac{1}{2} \quad \therefore$$

$$|\langle 01|11|\psi\rangle|^2 = \frac{1}{4}$$

$$\langle 11|01|\psi\rangle = \frac{1}{2} \left[ \cancel{\langle 11|01|0\rangle} + \cancel{\langle 11|01|1\rangle} + \cancel{\langle 11|01|0\rangle} - \cancel{\langle 11|01|1\rangle} \right]$$

$$\langle 11|01|\psi\rangle = \frac{1}{2} \quad \therefore$$

$$|\langle 11|01|\psi\rangle|^2 = \frac{1}{4}$$

$$\langle 11|11|\psi\rangle = \frac{1}{2} \left[ \cancel{\langle 11|11|0\rangle} + \cancel{\langle 11|11|1\rangle} + \cancel{\langle 11|11|0\rangle} - \cancel{\langle 11|11|1\rangle} \right]$$

$$\langle 11|11|\psi\rangle = -\frac{1}{2} \quad \therefore$$

$$|\langle 11|11|\psi\rangle|^2 = \frac{1}{4}$$

b.) SG $\hat{x}$  for A, SG $\hat{z}$  for B  $\left\{ \frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle-|1\rangle}{\sqrt{2}} \right\}, \{|0\rangle, |1\rangle\}$

SG $\hat{x}$ (A)	SG $\hat{z}$ (B)	State
+	+	$\frac{ 0\rangle+ 1\rangle}{\sqrt{2}},  0\rangle = \frac{ 0\rangle 0\rangle +  1\rangle 0\rangle}{\sqrt{2}}$
+	-	$\frac{ 0\rangle+ 1\rangle}{\sqrt{2}},  1\rangle = \frac{ 0\rangle 1\rangle +  1\rangle 1\rangle}{\sqrt{2}}$
-	+	$\frac{ 0\rangle- 1\rangle}{\sqrt{2}},  0\rangle = \frac{ 0\rangle 0\rangle -  1\rangle 0\rangle}{\sqrt{2}}$
-	-	$\frac{ 0\rangle- 1\rangle}{\sqrt{2}},  1\rangle = \frac{ 0\rangle 1\rangle -  1\rangle 1\rangle}{\sqrt{2}}$

$$\frac{\langle 01|01 + \langle 11|01|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \langle 01|01|0\rangle + \langle 11|01|0\rangle + \cancel{\langle 01|01|1\rangle} + \cancel{\langle 11|01|1\rangle} + \cancel{\langle 01|01|0\rangle} + \cancel{\langle 11|01|0\rangle} - \cancel{\langle 01|01|1\rangle} - \cancel{\langle 11|01|1\rangle} \right]$$

$$\frac{\langle 01|01 + \langle 11|01|\psi\rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} (1+1) = \frac{1}{\sqrt{2}} : \quad$$

$$|\frac{\langle 01|01 + \langle 11|01|\psi\rangle}{\sqrt{2}}|^2 = \frac{1}{2}$$

### Exercise 3 Continued

$$\frac{\langle 01\langle 11 + \langle \bar{1}1\langle 11 \rangle \rangle \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \langle 01\langle 1/\cancel{1}\rangle 10 \rangle + \langle \bar{1}1\langle \cancel{1}/0 \rangle 10 \rangle + \langle 01\langle \cancel{1}/1 \rangle 11 \rangle + \langle \bar{1}1\langle \cancel{1}/0 \rangle 11 \rangle \right. \\ \left. + \langle 01\langle 1/\cancel{1} \rangle 10 \rangle + \langle \bar{1}1\langle \cancel{1}/1 \rangle 10 \rangle - \langle 01\langle 1/\cancel{1} \rangle 11 \rangle - \langle \bar{1}1\langle \cancel{1}/1 \rangle 11 \rangle \right]$$

$$\frac{\langle 0| \langle \pm | + \langle \pm | \langle \mp | \gamma \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} (1-1) = 0 \quad : \quad \left| \frac{\langle 0| \langle \pm | + \langle \pm | \langle \mp | \gamma \rangle}{\sqrt{2}} \right|^2 = 0$$

$$\frac{\langle 01\langle 01 - \langle 11\langle 01|\gamma \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \langle 01\langle 0|\overset{0}{0}\rangle 10\rangle - \langle 11\langle 0|\overset{0}{0}\rangle 10\rangle + \langle 01\langle 0|\overset{0}{1}\rangle 11\rangle - \langle 11\langle 0|\overset{0}{1}\rangle 11\rangle \right. \\ \left. + \langle 01\langle 0|\overset{0}{1}\rangle 10\rangle - \langle 11\langle 0|\overset{0}{1}\rangle 10\rangle - \langle 01\langle 0|\overset{0}{1}\rangle 11\rangle + \langle 11\langle 0|\overset{0}{1}\rangle 11\rangle \right]$$

$$\frac{\langle 01|01\rangle - \langle 11|01\rangle \gamma^r}{\sqrt{2}} = \frac{1}{2\sqrt{2}} (1-1) = 0 : \quad \left| \frac{\langle 01|01\rangle - \langle 11|01\rangle \gamma^r}{\sqrt{2}} \right|^2 = 0$$

$$\frac{\langle 01\langle 1| - \langle 1|\langle 1| + \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \left[ \cancel{\langle 01\langle 1| \cancel{0}\cancel{1}0\rangle}^0 - \cancel{\langle 1|\cancel{1}\cancel{0}\cancel{1}0\rangle}^0 + \cancel{\langle 01\langle 1| \cancel{0}\cancel{1}1\rangle}^1 - \cancel{\langle 1|\cancel{1}\cancel{0}\cancel{1}1\rangle}^0 \right. \\ \left. + \cancel{\langle 01\langle 1| \cancel{1}\cancel{1}0\rangle}^0 - \cancel{\langle 1|\cancel{1}\cancel{1}\cancel{1}0\rangle}^0 - \cancel{\langle 01\langle 1| \cancel{1}\cancel{1}1\rangle}^1 + \cancel{\langle 1|\cancel{1}\cancel{1}\cancel{1}1\rangle}^1 \right]$$

$$\frac{\langle 01\langle 11 - \langle 11\langle 11|\gamma\rangle \rangle}{\sqrt{2}} = \frac{1}{2\sqrt{2}} (1+1) = \frac{1}{\sqrt{2}} : \boxed{\left| \frac{\langle 01\langle 11 - \langle 11\langle 11|\gamma\rangle \rangle}{\sqrt{2}} \right|^2 = y_2}$$

$$C.) S6\hat{x} \text{ for A, } S6\hat{x} \text{ for B} \quad \left\{ \frac{10>+12>}{\sqrt{2}}, \frac{10>-12>}{\sqrt{2}} \right\}$$

$S_6 \hat{x} (A)$	$S_6 \hat{x} (B)$	State
+	+	$\frac{10\gamma + i\zeta}{\sqrt{2}}, \frac{10\gamma + i\zeta}{\sqrt{2}} = \frac{1}{2} ( 0\rangle 0\rangle +  0\rangle i\rangle +  i\rangle 0\rangle +  i\rangle i\rangle)$
+	-	$\frac{10\gamma + i\zeta}{\sqrt{2}}, \frac{10\gamma - i\zeta}{\sqrt{2}} = \frac{1}{2} ( 0\rangle 0\rangle -  0\rangle i\rangle +  i\rangle 0\rangle -  i\rangle i\rangle)$
-	+	$\frac{10\gamma - i\zeta}{\sqrt{2}}, \frac{10\gamma + i\zeta}{\sqrt{2}} = \frac{1}{2} ( 0\rangle 0\rangle +  0\rangle i\rangle -  i\rangle 0\rangle -  i\rangle i\rangle)$
-	-	$\frac{10\gamma - i\zeta}{\sqrt{2}}, \frac{10\gamma - i\zeta}{\sqrt{2}} = \frac{1}{2} ( 0\rangle 0\rangle -  0\rangle i\rangle -  i\rangle 0\rangle +  i\rangle i\rangle)$

$$\begin{aligned} & \cancel{\langle 01\langle 01 + \langle 01\langle 11 + \langle \cancel{11}\langle 01 + \langle \cancel{11}\langle \cancel{11} \rangle \rangle \rangle} = \frac{1}{4} \left[ \cancel{\langle 01\langle 010\rangle 10} + \cancel{\langle 01\langle 010\rangle 11} + \cancel{\langle 01\langle 011\rangle 10} - \cancel{\langle 01\langle 011\rangle 11} + \right. \\ & \cancel{\langle 01\langle 110\rangle 10} + \cancel{\langle 01\langle 110\rangle 11} + \cancel{\langle 01\langle 111\rangle 10} - \cancel{\langle 01\langle 111\rangle 11} + \cancel{\langle 11\langle 010\rangle 10} + \cancel{\langle 11\langle 010\rangle 11} + \\ & \left. \cancel{\langle 11\langle 110\rangle 10} - \cancel{\langle 11\langle 110\rangle 11} + \cancel{\langle 11\langle 111\rangle 10} + \cancel{\langle 11\langle 111\rangle 11} \right] \end{aligned}$$

$$\frac{\langle 01\langle 01 + \langle 01\langle 11 + \langle \pm 1\langle 01 + \langle \pm 1\langle 11 \rangle^+ \rangle}{2} = \frac{1}{4} (1+1+1-1) = \frac{1}{2}$$

$$\left| \frac{\langle 0| \langle 0| + \langle 0| \langle 1| + \langle -1| \langle 0| + \langle -1| \langle 1| + \rangle}{2} \right|^2 = 1/4$$

## Exercise 3 Continued

$$\begin{aligned} & \frac{\langle 01\langle 01 - \langle 01\langle 11 + \langle 11\langle 01 - \langle 21\langle 11 \rangle \rangle \rangle \rangle \rangle}{0} = \frac{1}{4} \left[ \begin{array}{c} \langle 01\langle 01\langle 0\rangle 10 \rangle + \langle 01\langle 01\langle 0\rangle 11 \rangle + \langle 01\langle 01\langle 2 \rangle 10 \rangle - \langle 01\langle 01\langle 2 \rangle 11 \rangle \\ \langle 01\langle 1\rangle 0\rangle 10 \rangle - \langle 01\langle 1\rangle 0\rangle 11 \rangle - \langle 01\langle 1\rangle 1\rangle 10 \rangle + \langle 01\langle 1\rangle 1\rangle 11 \rangle + \\ \langle 21\langle 0\rangle 1\rangle 10 \rangle - \langle 21\langle 0\rangle 1\rangle 11 \rangle - \langle 21\langle 1\rangle 0\rangle 10 \rangle + \langle 21\langle 1\rangle 0\rangle 11 \rangle - \langle 21\langle 1\rangle 1\rangle 0 \rangle + \langle 21\langle 1\rangle 1\rangle 1 \rangle \end{array} \right] \end{aligned}$$

$$\frac{\langle 01 \langle 01 - \langle 01 \langle 11 + \langle 11 \langle 01 - \langle 11 \langle 11 \gamma \rangle \rangle}{2} = \frac{1}{4}(1-1+1+1) = k_2$$

$$\left| \frac{\langle 01|01 - \langle 01|11 + \langle 11|01 - \langle 11|11 \rangle}{2} \right|^2 = \frac{1}{4}$$

$$\begin{aligned} & \frac{\langle 01\langle 01 + \langle 01\langle 11 - \langle 11\langle 01 - \langle 11\langle 11 \rangle \rangle \rangle \rangle \rangle}{\langle 01\langle 01\rangle 10 \rangle + \langle 01\langle 01\rangle 11 \rangle + \langle 01\langle 11\rangle 10 \rangle - \langle 01\langle 11\rangle 11 \rangle} = \frac{1}{4} \left[ \right. \\ & + \langle 01\langle 11\rangle 10 \rangle + \langle 01\langle 11\rangle 11 \rangle + \langle 01\langle 11\rangle 10 \rangle - \langle 01\langle 11\rangle 11 \rangle - \langle 11\langle 01\rangle 10 \rangle - \langle 11\langle 01\rangle 11 \rangle \\ & - \langle 11\langle 11\rangle 10 \rangle + \langle 11\langle 11\rangle 11 \rangle - \langle 11\langle 11\rangle 10 \rangle - \langle 11\langle 11\rangle 11 \rangle - \langle 11\langle 11\rangle 10 \rangle + \langle 11\langle 11\rangle 11 \rangle \left. \right] \end{aligned}$$

$$\frac{\langle 01|01\rangle + \langle 01|11\rangle - \langle 11|01\rangle - \langle 11|11\rangle}{2} = \frac{1}{4} (1+1-1+1) = \frac{1}{2}$$

$$\therefore \frac{\langle 01 \langle 01 + \langle 01 \langle 11 - \langle 11 \langle 01 - \langle 11 \langle 11 \rangle \rangle}{2} = 1/4$$

$$\begin{aligned} & \cancel{\langle 01\langle 01 - \langle 01\langle 21 - \langle 21\langle 01 + \langle 21\langle 21 \rangle \rangle} = \left[ \begin{array}{l} \cancel{\langle 01\langle 01\langle 0\rangle 10\rangle} + \cancel{\langle 01\langle 01\langle 0\rangle 11\rangle} + \cancel{\langle 01\langle 01\langle 1\rangle 10\rangle} - \cancel{\langle 01\langle 01\langle 1\rangle 11\rangle} - \\ \cancel{\langle 01\langle 21\langle 0\rangle 10\rangle} - \cancel{\langle 01\langle 21\langle 0\rangle 11\rangle} - \cancel{\langle 01\langle 21\langle 1\rangle 10\rangle} + \cancel{\langle 01\langle 21\langle 1\rangle 11\rangle} - \cancel{\langle 21\langle 01\langle 0\rangle 10\rangle} - \cancel{\langle 21\langle 01\langle 0\rangle 11\rangle} - \\ \cancel{\langle 21\langle 21\langle 0\rangle 10\rangle} + \cancel{\langle 21\langle 21\langle 0\rangle 11\rangle} + \cancel{\langle 21\langle 21\langle 1\rangle 10\rangle} + \cancel{\langle 21\langle 21\langle 1\rangle 11\rangle} + \cancel{\langle 21\langle 21\langle 2\rangle 10\rangle} - \cancel{\langle 21\langle 21\langle 2\rangle 11\rangle} \end{array} \right] \end{aligned}$$

$$\frac{\langle 01\langle 01 - \langle 01\langle 11 - \langle 11\langle 01 + \langle 11\langle 11 \rangle \rangle}{2} = \frac{1}{4}(1-1-1-1) = \frac{-1}{2}$$

$$\therefore \left| \frac{\langle 01 \rangle \langle 01 - \langle 01 \rangle \langle 11 - \langle 11 \rangle \langle 01 + \langle 11 \rangle \langle 11 \rangle \rangle \rangle}{2} \right|^2 = 1/4$$

### Exercise 3 Continued

Problem 3 | Find  $|\psi_A\rangle, |\psi_B\rangle$  s.t.  $|\Psi\rangle = |\psi_A\rangle|\psi_B\rangle$

$$|\psi_A\rangle = a_0|0\rangle + a_1|1\rangle, \quad |\psi_B\rangle = b_0|0\rangle + b_1|1\rangle$$

$$|\psi_A\rangle|\psi_B\rangle = a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle$$

$$\text{for, } |\Psi\rangle = \frac{1}{2} [ |0\rangle|0\rangle + |0\rangle|1\rangle - |1\rangle|0\rangle - |1\rangle|1\rangle ]$$

$$a_0 = \frac{-1}{\sqrt{2}}, \quad b_0 = \frac{-1}{\sqrt{2}}, \quad a_1 = \frac{1}{\sqrt{2}}, \quad b_1 = \frac{1}{\sqrt{2}} \quad \xrightarrow{\text{These satisfy } |\Psi\rangle} \quad \therefore$$

$$|\psi_A\rangle = \frac{-1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, \quad |\psi_B\rangle = \frac{-1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$|\psi_A\rangle = a_0|0\rangle + a_1|1\rangle, \quad |\psi_B\rangle = b_0|0\rangle + b_1|1\rangle$$

$$|\psi_A\rangle|\psi_B\rangle = a_0b_0|0\rangle|0\rangle + a_0b_1|0\rangle|1\rangle + a_1b_0|1\rangle|0\rangle + a_1b_1|1\rangle|1\rangle$$

$$\text{for, } |\Psi\rangle = \frac{1}{2} [ |0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle ]$$

There is no combination of  $a_0, b_0, a_1, b_1$  that will satisfy  $|\Psi\rangle$ .

This is an entangled state