Problem 11

$m = 0.55\,kg$ $\quad \gamma = 2.2875\ s^{-1}$
$K = 46\,N/m$ $\quad \omega_0 = 9.15\ s^{-1}$
$F_0 = 2.50\,N$

a.) The Frequency that gives rise to max amp.

$$\omega_{max} = \omega_0 \sqrt{1 - \gamma^2/2\omega_0^2} = (9.15\ s^{-1})\sqrt{1-(2.2875\ s^{-1})^2/2(9.15\ s^{-1})^2}$$

$$\boxed{\omega_{max} = 9.01\ s^{-1}}$$

b.) The max amplitude

$$A_{max} = \frac{F_0\,\omega_0}{K\gamma\sqrt{1-\gamma^2/4\omega_0^2}} = \frac{2.5\,N\,(9.15\ s^{-1})}{(46\,N/m)(2.2875\ s^{-1})\sqrt{1-(2.2875\ s^{-1})^2/4(9.15\ s^{-1})^2}}$$

$$\boxed{A_{max} = 0.219\ m}$$

c.) The quality factor

$$Q = \omega_0/\gamma = 9.15\ s^{-1}/2.2875\ s^{-1} = 4$$

$$\boxed{Q = 4}$$

d.) $A(\omega)$ when $\omega = 2\pi\ s^{-1}$

$$A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2 + \omega^2\gamma^2}} = \frac{2.5\,N/0.55\,kg}{\sqrt{(9.15\ s^{-1\,2} - 2\pi\ s^{-2})^2 + (2\pi\ s^{-1})^2(2.2875\ s^{-1})^2}}$$

$$\boxed{A(\omega) = 0.0977\ m}$$

e.) The phase angle δ

$$\tan(\delta) = \frac{\omega\gamma}{\omega_0^2-\omega^2} \qquad \delta = \tan^{-1}\left(\frac{\omega\gamma}{\omega_0^2-\omega^2}\right) = \tan^{-1}\left(\frac{(2\pi\ s^{-1})(2.2875\ s^{-1})}{(9.15\ s^{-1})^2 - (2\pi\ s^{-1})^2}\right)$$

$$\boxed{\delta = 0.314\ rad}$$

Problem 2

$$x(t) = A(\omega)\cos(\omega t - \delta) + Be^{-\gamma t/2}\cos(\omega_1 t + \phi) \qquad \Delta x = 0.055\,m$$
$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2/4} \qquad \dot{x} = 0.450\,m/s$$

a.) calculate time dependent velocity

$$\dot{x}(t) = -\omega A(\omega)\sin(\omega t - \delta) - \omega_1 Be^{-\gamma t/2}\sin(\omega_1 t + \phi) - \gamma/2\, Be^{-\gamma t/2}\cos(\omega_1 t + \phi)$$

$$\dot{x}(t) = -\omega A(\omega)\sin(\omega t - \delta) - Be^{-\gamma t/2}\left(\omega_1 \sin(\omega_1 t + \phi) + \gamma/2\cos(\omega_1 t + \phi)\right)$$

b.) Solve for ϕ and B algebraically

$$x(t=0) = A(\omega)\cos(\delta) + B\cos(\phi)$$
$$\Delta x - A(\omega)\cos(\delta) = B\cos(\phi) \qquad x(t=0) \equiv \Delta x$$
$$B = \frac{\Delta x - A(\omega)\cos(\delta)}{\cos(\phi)}$$

$$\dot{x}(t) = -\omega A(\omega)\sin(\omega t - \delta) - Be^{-\gamma t/2}\left(\omega_1\sin(\omega_1 t + \phi) + \gamma/2\cos(\omega_1 t + \phi)\right)$$
$$\dot{x}(t=0) = -\omega A(\omega)\sin(\delta) - B(\omega_1\sin(\phi) + \gamma/2\cos(\phi)) \qquad \dot{x}(t=0) \equiv \dot{x}$$
$$\dot{x} = \omega A(\omega)\sin(\delta) - B\omega_1\sin(\phi) - B\gamma/2\cos(\phi)$$
$$\dot{x} = \omega A(\omega)\sin(\delta) - \frac{(\Delta x - A(\omega)\cos(\delta))\omega_1\sin(\phi)}{\cos(\phi)} - \frac{(\Delta x - A(\omega)\cos(\delta))\frac{\gamma}{2}\cos(\phi)}{\cos(\phi)}$$
$$\dot{x} = \omega A(\omega)\sin(\delta) - \frac{\Delta x\,\omega_1\sin\phi + A(\omega)\cos(\delta)\omega_1\sin\phi}{\cos(\phi)} - \frac{\Delta x\frac{\gamma}{2}\cos(\phi) + A(\omega)\cos(\delta)\frac{\gamma}{2}\cos(\phi)}{\cos(\phi)}$$
$$\dot{x} = \omega A(\omega)\sin(\delta) - \Delta x\,\omega_1\tan\phi + A(\omega)\cos(\delta)\omega_1\tan(\phi) - \Delta x\frac{\gamma}{2} + A(\omega)\cos(\delta)\frac{\gamma}{2}$$
$$\dot{x} = \omega A(\omega)\sin(\delta) + \tan(\phi)(\omega_1 A(\omega)\cos(\delta) - \Delta x\,\omega_1) + \frac{\gamma}{2}(A(\omega)\cos(\delta) - \Delta x)$$
$$\dot{x} - \omega A(\omega)\sin(\delta) - \gamma/2(A(\omega)\cos(\delta) - \Delta x) = \tan(\phi)(\omega_1 A(\omega)\cos(\delta) - \Delta x\,\omega_1)$$

$$\tan(\phi) = \frac{\dot{x} - \omega A(\omega)\sin(\delta) - \gamma/2(A(\omega)\cos(\delta) - \Delta x)}{(\omega_1 A(\omega)\cos(\delta) - \Delta x\,\omega_1)}$$

$$\phi = \tan^{-1}\left(\frac{\dot{x} - \omega A(\omega)\sin(\delta) - \gamma/2(A(\omega)\cos(\delta) - \Delta x)}{\omega_1(A(\omega)\cos(\delta) - \Delta x)}\right)$$

$$B = \frac{\Delta x - A(\omega)\cos(\delta)}{\cos(\phi)}$$

c.) Calculate ϕ and B

$$\phi = \tan^{-1}\left(\frac{0.450\,m/s - (2\pi s^{-1})(0.0977\,s^{-1})\sin(0.314) - (2.2875\,s^{-1}/2)((0.0977\,s^{-1})\cos(0.314) - 0.055\,m)}{9.08\,s^{-1}((0.0977\,s^{-1})\cos(0.314) - 0.055\,m)}\right)$$

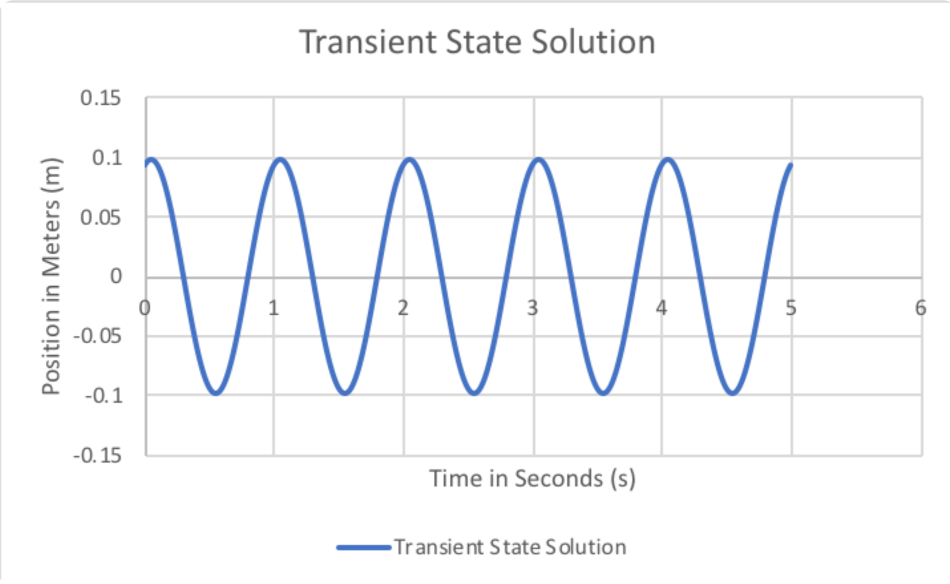
$$\phi = 0.562\text{ rad} \qquad B = \frac{0.055\,m - (0.0977\,s^{-1})\cos(0.314)}{\cos(0.562)}$$

$$B = -0.0448\,m$$

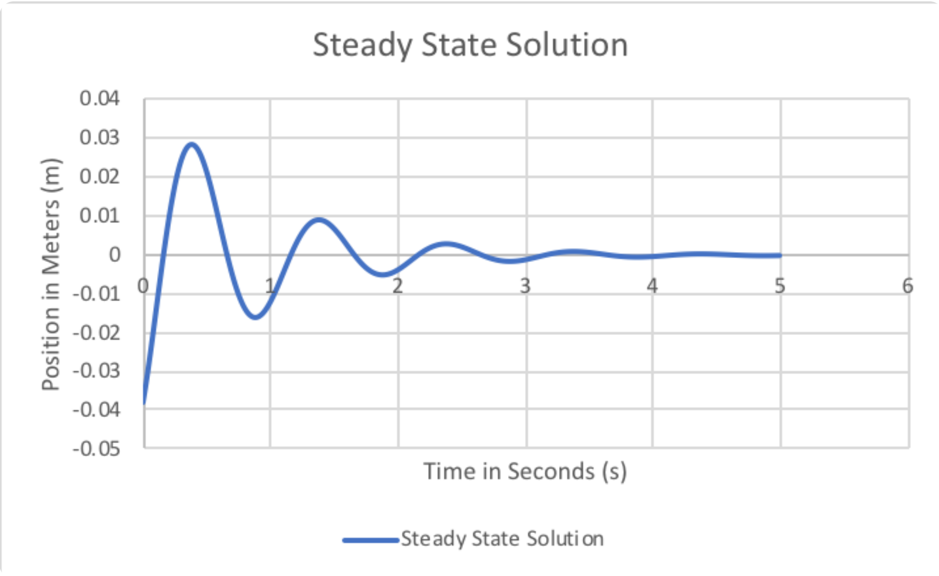
$$\boxed{\begin{aligned}\phi &= 0.562\text{ rad}\\ B &= -0.0448\,m\end{aligned}}$$

Problem 3)

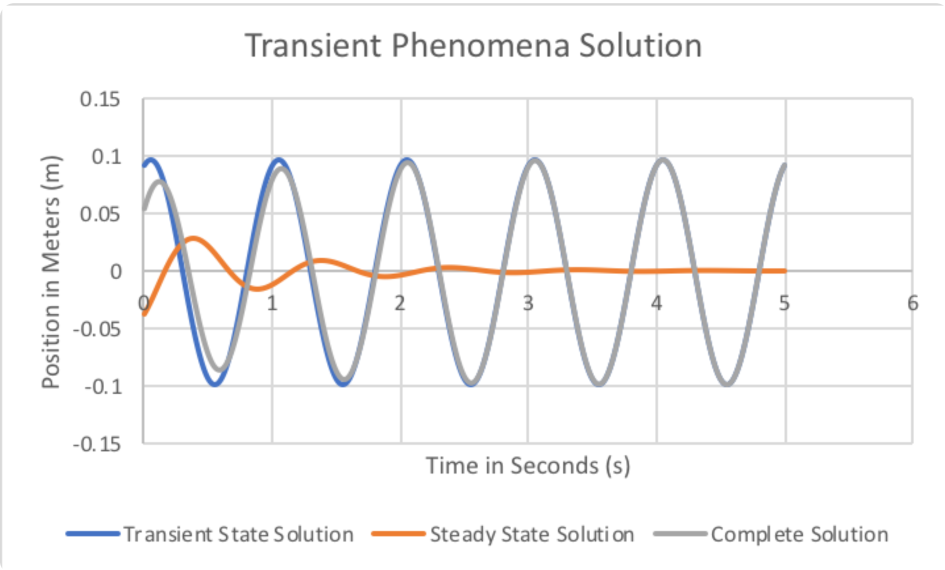
a.) $x_1(t)$



b.) $x_2(t)$



c.) $x_1(t) + x_2(t)$



Problem 4) $z(x,t) = (0.21\,m)\sin((0.36\,m^{-1})x + (0.52\,s^{-1})t)$

a.) calculate the amplitude, wavelength, frequency and velocity of the wave

$z(x,t) = A\sin(kx + \omega t)$

$A = 0.21\,m$

$\lambda = 2\pi/k = 2\pi/0.36\,m^{-1} = 17.4\,m \simeq 17\,m$

$k = 0.36\,m^{-1}$

$\bar{v} = \omega/k = 0.52\,s^{-1}/0.36\,m^{-1} = 1.44\,m/s \simeq 1.4\,m/s$

$\omega = 0.52\,s^{-1}$

$k = 0.36\,m^{-1}$

$$\left[\begin{array}{l} A = 0.21\,m \\ \lambda = 17\,m \\ v = 1.4\,m/s \end{array}\right]$$

b.) The wave is traveling in the [negative x-direction.]

c.)

$\frac{\partial z}{\partial x} = kA\cos(kx + \omega t)$ $\frac{\partial z}{\partial t} = \omega A\cos(kx + \omega t)$

$\frac{\partial^2 z}{\partial x^2} = -k^2 A\sin(kx + \omega t)$ $\frac{\partial^2 z}{\partial t^2} = -\omega^2 A\sin(kx + \omega t)$

$\frac{\partial^2 z}{\partial x^2} = -k^2 A\sin(kx + \omega t)$ $\frac{\partial^2 z}{\partial t^2} = -k^2 v^2 A\sin(kx + \omega t)$

$$\left[\frac{\partial^2 z}{\partial t^2} = v^2 \frac{\partial^2 z}{\partial x^2}\right] \checkmark$$

d.) what is the maximum speed and acceleration?

$\frac{\partial z}{\partial t} = \omega A\cos(kx + \omega t)$

$\cos(x) = 1$

$\frac{\partial z}{\partial t} = \omega A = (0.52\,s^{-1})(0.21\,m) = 0.1092\,m/s$

$$\left[\begin{array}{l} v = 0.11\,m/s \\ a = 0.057\,m/s^2 \end{array}\right]$$

$\frac{\partial^2 z}{\partial t^2} = -\omega^2 A\sin(kx + \omega t)$ $\sin(x) = 1$

$\frac{\partial^2 z}{\partial t^2} = -(0.52\,s^{-1})^2(0.21\,m) = 0.0567\,m/s^2$