Taylor Lamechea
Dr. Gustafson
MATH 360
CP 9.5

Example 1 Circle, Parametric Representation, Positive Sense

The circle $x^2+y^2=4$, $z=0$ in the xy-plane with center 0 and radius 2 can be represented parametrically by

$$r(t)=[2\cos(t),\ 2\sin(t),\ 0] \quad \text{or} \quad r(t)=[2\cos(t),\ 2\sin(t)]$$

Where $0 \le t \le 2\pi$. Indeed $x^2+y^2=(2\cos(t))^2+(2\sin(t))^2=4(\cos^2(t)+\sin^2(t))=4$, For $t=0$ we have $r(0)=[2,0]$, for $t=\frac{1}{2}\pi$ we get $r(\frac{1}{2}\pi)=[0,2]$, and so on. The positive sense induced by this representation is the counterclockwise sense.

If we replace t with $t^*=-t$, we have $t=-t^*$ and get

$$r^*(t^*)=[2\cos(-t^*),\ 2\sin(-t^*)]=[2\cos(t^*),\ -2\sin(t^*)]$$

This has reversed the orientation, and the circle is now oriented clockwise.

Example 2 Ellipse

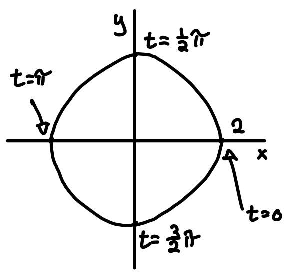
The vector function

$$r(t)=[a\cos(t),\ b\sin(t),\ 0]=a\cos(t)\hat{i}+b\sin(t)\hat{j}$$

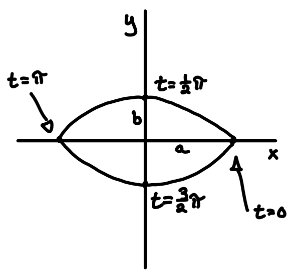
represents an ellipse in the xy-plane with center at the origin and principal axes in the direction of the x- and y-axes. In fact, since $\cos^2(t)+\sin^2(t)=1$, we obtain

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \qquad z=0$$

If $b=a$, then $r(t)$ represents a circle of radius a



Ex : 1



Ex: 2

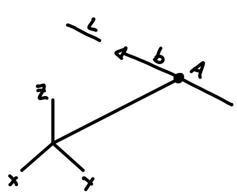
Example 3 Straight Line

A straight line L through a point A with position vector a in the direction of a constant vector b can be represented parametrically in the form

$$r(t)=a+tb=[a_1+tb_1,\ a_2+tb_2,\ a_3+tb_3]$$

If b is a unit vector, its components are the direction cosines of L. In this case, $|t|$ measures the distance of the points of L from A. For instance, the straight line in the xy-plane through $A:(3,2)$ having slope 1 is

$$r(t)=[3,2,0]+t[1,1,0]=[3+t,\ 2+t,\ 0]$$



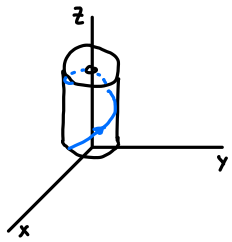
A plane curve is a curve that lies in a plane in space. A curve that is not plane is called a twisted curve. A standard example of a twisted curve is the following.

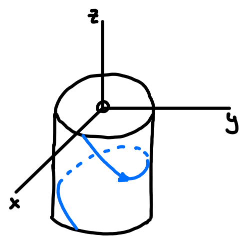
Example 4 Circular Helix

The twisted curve C represented by the vector function

$$r(t) = [a\cos(t), a\sin(t), ct] = a\cos(t)\hat{i} + a\sin(t)\hat{j} + ct\hat{k}$$

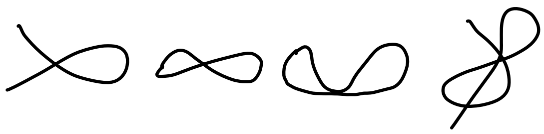
is called a circular helix. It lies on the cylinder $x^2+y^2=a^2$. If $c>0$, the helix is shaped like a right-handed screw. If $c<0$, it looks like a left-handed screw. If $c=0$, then $r(t)$ is a circle.





A simple curve is a curve without multiple points, that is, without points at which the curve intersects or touches itself. Circle and helix are simple curves. Figure 206 shows curves that are not simple. An example is $[\sin(2t), \cos(t), 0]$.

An arc of a curve is the portion between any two points of the curve. For simplicity, we say "curve" for curves as well as for arcs.



Equation 8

$$u = \frac{1}{|r'|} r'$$

Equation 10

$$l = \int_a^b \sqrt{r' \cdot r'}\, dt$$