

## Fundamental Mechanics: Class Exam III

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Total:

**64 /70**

*excellent*

### Instructions

- There are 8 questions on 6 pages.
- Show your reasoning and calculations and always explain your answers.

### Physical constants and useful formulae

$$g = 9.80 \text{ m/s}^2$$

#### Question 1

A 2.0 kg block is held at rest against a spring compressing it by 0.40 m. At this point the base of the block is 0.50 m above the ground. The spring constant of this spring is 800 N/m. These are contained in a cylinder with frictionless walls. The block is released and launches vertically, leaving the spring. Determine the maximum height above the ground that it reaches.

$$m = 2.0 \text{ kg}$$

$$\Delta s = 0.40 \text{ m}$$

$$h_i = 0.50 \text{ m}$$

$$k = 800 \text{ N/m}$$

$$E_i = E_f$$

$$K_i + U_{6i} + U_{5i} = K_f + U_{6f} + U_{5f}$$

$$mgy_i + \frac{1}{2}k(\Delta s)^2 = mgy_f$$

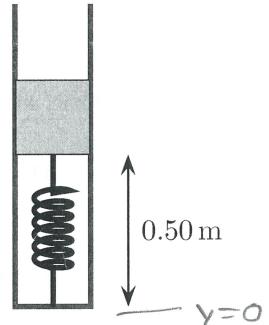
$$(2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.50 \text{ m}) + \frac{1}{2}(800 \text{ N/m})(0.40 \text{ m})^2 = 2.0 \text{ kg}(9.8 \text{ m/s}^2)y_f$$

$$9.8 \text{ kg} \cdot \text{m}^2/\text{s}^2 + 64 \text{ N} \cdot \text{m} = 19.6 \text{ kg m/s}^2(y_f)$$

$$73.8 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 19.6 \text{ kg m/s}^2 y_f$$

$$y_f = \frac{73.8 \text{ kg} \cdot \text{m}^2/\text{s}^2}{19.6 \text{ kg} \cdot \text{m/s}^2}$$

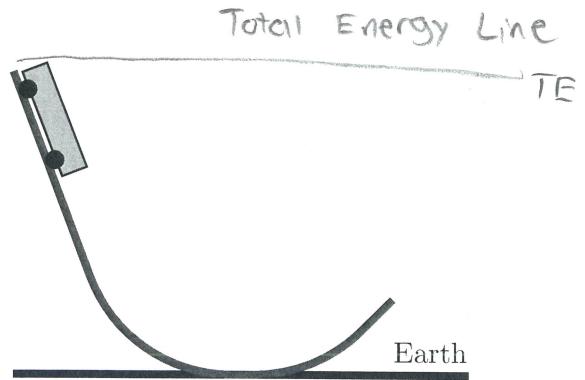
$$y_f = 3.77 \text{ m}$$



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### Question 2

A cart is released from rest from height  $h_i$  above the Earth on the illustrated frictionless track. The cart flies off the ramp on the right and travels through the air. The surface of the Earth is indicated. Consider the maximum height,  $h_f$ , reached above the surface of the Earth after it leaves the ramp. Air resistance is negligible. Which of the following (choose one) is true?

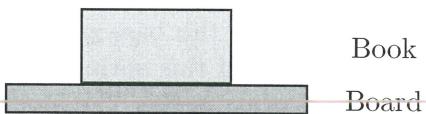


- i) If the cart is light enough then  $h_f > h_i$  but if it is heavy enough then  $h_f < h_i$ .
- ii) If the cart is light enough then  $h_f = h_i$  but if it is heavy enough then  $h_f < h_i$ .
- iii) Regardless of the cart mass,  $h_f < h_i$ .
- iv) Regardless of the cart mass,  $h_f = h_i$ .
- v) The height reached depends on the angle of the ramp compared to that of the slope on the left.

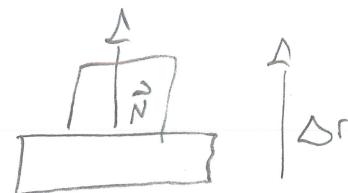
To reach a height greater than the initial, Energy would have to add to the system changing the total Energy; this does not happen.  $h_f = h_i$

### Question 3

A book rests on the top of a board. The board is moved vertically up; the book is always in contact with the board. Which of the following (choose one) is true of the work,  $W$ , done by the normal force acting on the book?



- i)  $W > 0$  if the book is slowing down,  $W < 0$  if the book is speeding up.
- ii)  $W < 0$  if the book is slowing down,  $W > 0$  if the book is speeding up.
- iii)  $W = 0$  in all circumstances.
- iv)  $W > 0$  in all circumstances.
- v)  $W < 0$  in all circumstances.



$$W = \vec{F} \cdot \vec{\Delta r} \cdot \cos\theta \quad 5/5$$

$\theta = 0$  since direction of movement is in the same direction the force is acting.

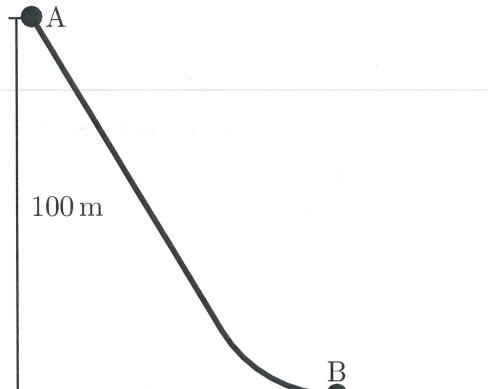
$$\cos(0) = 1$$

$$(1) \cdot (\vec{F} \cdot \vec{\Delta r}) = \text{Positive value}$$

### Question 4

Alice, a 80 kg, skier is at rest at point A on the illustrated slope. She starts to slide down the slope which is frictionless until point B. Just before point B, Alice collides with another, 100 kg, skier, Bob, who is at rest. They stick together and the move along horizontal surface past B. After point B there is a constant 300 N friction force acting on the two skiers.

- a) Determine the speed of the two skiers as they pass point B.



$v_A$  = velocity of Alice

$$v_A \approx \sqrt{2(9.8 \text{ m/s})(100 \text{ m})}$$

$$v_A = \sqrt{1960 \text{ m}^2/\text{s}}$$

$$v_A = 44.27 \text{ m/s}$$

Alice :

$$m = 80 \text{ kg}$$

$$y_i = 100 \text{ m}$$

$$E_i = E_f \quad \checkmark$$

$$K_i + U_{G,i} = K_f + U_{G,f}$$

$$mg y_i = \frac{1}{2} m v_f^2$$

$$2g y_i = v_f^2$$

$$v_f = \sqrt{2g y_i}$$

$P_f = P_i$  Inelastic At rest

$$m_c v_f = m_A v_A + m_B v_B$$

$$180 \text{ kg} (v_f) = 80 \text{ kg} (44.27 \text{ m/s})$$

$$180 \text{ kg} (v_f) = 3541.6 \text{ kg} \cdot \text{m/s}$$

$$v_f = 19.68 \text{ m/s}$$

$m_c$  = Mass of combined

$$m_c = 180 \text{ kg}$$

B = Bob

A = Alice

Question 4 continued ...

$$\vec{f}_k = 300 \text{ N}$$

$$V_f = 19.68 \text{ m/s}$$

$$m_c = 180 \text{ kg}$$

b) Determine the distance that the skiers travel beyond point B.

$$\begin{aligned} E_F &= E_i + W_{nc} \\ K_F + U_{GF} &= K_i + U_{G_i} + W_{nc} \end{aligned}$$

$$0 = K_i + W_{nc}$$

$$-(\frac{1}{2} m v_f^2) = 300 \text{ N} \cdot \Delta r \cdot \cos 180^\circ$$

$$-\frac{1}{2} m v_f^2 = -300 \text{ N} \cdot \Delta r$$

$$\frac{1}{2} (180 \text{ kg}) (19.68 \text{ m/s})^2 = 300 \text{ N} \cdot \Delta r$$

$$34,857 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 300 \text{ N} \cdot \Delta r$$

$$\Delta r = 116.2 \text{ m}$$

$$W_{nc} = \vec{F} \cdot \vec{\Delta r} \cdot \cos \theta$$

$$\theta = 180^\circ$$

opposite to direction  
of motion

$$\Delta r = 116.2 \text{ m}$$

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### Question 5

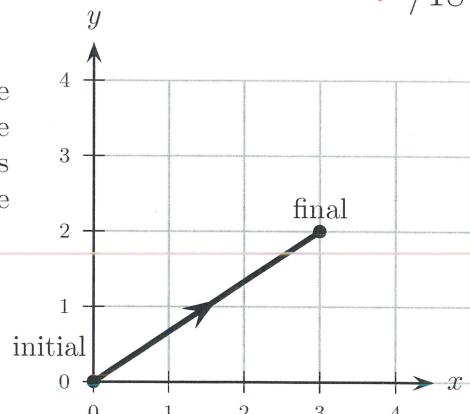
A ball moves along the indicated path. Throughout the motion a hidden object exerts a force  $\vec{F} = 10 \text{ N} \hat{i}$  on the ball. Determine the work done by this force on the ball as it moves from the indicated initial to final location. The distances on the graph are in meters.

$$W = \vec{F} \cdot \vec{\Delta r}$$

$$\vec{F} = \langle 10, 0 \rangle$$

$$\vec{\Delta r} = \langle 3, 2 \rangle$$

$$30 + 0 = 30$$



$$\vec{\Delta r} = \langle 3, 2 \rangle$$

$$W = 30 \text{ J/m}$$

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### Question 6

A particle can move horizontally along the x axis. The potential energy of the particle is  $U(x) = 4x^2 - 2x$  where  $x$  is the horizontal location of the particle. Determine the location where the force on the particle is zero.

$$U'(x) = 8x - 2$$

$$-\frac{du}{dx} = -(8x - 2)$$

$$-\frac{du}{dx} = -8x + 2$$

$$0 = -8x + 2$$

$$-2 = -8x$$

$$x = \frac{1}{4}$$

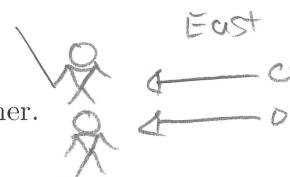
$$x = 0.25 \text{ J}$$

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### Question 7

Two people, Alice and Bob, of equal mass, sit at rest on a sheet of ice. Identical balls are thrown east with the same speed to each skater. Alice catches and holds the ball thrown to her. Bob bats the ball thrown to him directly back so that it reverses direction. Which of the following (choose one) is true after these events?

- i) Alice and Bob remain at rest.
- ii) Alice and Bob move east with the same speed as each other.
- iii) Alice moves east with a speed greater than that of Bob.
- iv) Alice moves east with a speed smaller than that of Bob.



C=combined

b=ball

B=Bob

A=Alice

Alice (ball and her)

$$P_f = P_i \text{ moving East}$$

$$m_C v_f = m_A v_A + m_B v_B$$

$$v_f = \frac{-m_B v_B}{m_C}$$

one final velocity, both negative

to left or east

Bob

$$P_f = P_i \text{ } \textcircled{O}$$

$$m_B v_B + m_B v_B = m_B v_B - m_B v_B$$

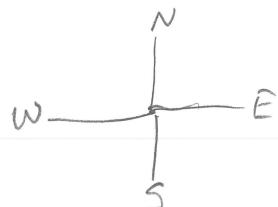
Elastic collision

two final velocities

one positive, one negative

one west, one East

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### Question 8

Zog, with mass 60 kg skates east with speed 10 m/s. Geraldine, with mass 40 kg skates west with the same speed. They collide and subsequently hold each other. Determine their velocity after they collide.

$$m_Z = 60 \text{ kg}$$

$$v_Z = 10 \text{ m/s}$$

$$m_G = 40 \text{ kg}$$

$$v_G = -10 \text{ m/s}$$

$m_C$  = combined mass

$$m_C = 100 \text{ kg}$$

$$P_F = P_i$$

$$m_C v_F = m_Z v_Z + m_G v_G$$

$$(100 \text{ kg}) v_F = (60 \text{ kg})(10 \text{ m/s}) + (40 \text{ kg})(-10 \text{ m/s})$$

$$100 \text{ kg} v_F = 600 \text{ kg} \cdot \text{m/s} - 400 \text{ kg} \cdot \text{m/s}$$

$$100 \text{ kg} v_F = 200 \text{ kg} \cdot \text{m/s}$$

$$v_F = 2 \text{ m/s}$$

rightward

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