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PHYS 362 HW 6

## Problem 1

a.)

i.)

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \quad : \quad K_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$V = \frac{1}{\alpha}\left(\frac{\partial V}{\partial T}\right)_P \quad : \quad V = \frac{-1}{K_T}\left(\frac{\partial V}{\partial P}\right)_T \quad \rightarrow \quad \frac{1}{\alpha}\left(\frac{\partial V}{\partial T}\right)_P = \frac{-1}{K_T}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{-\alpha}{K_T}\left(\frac{\partial V}{\partial P}\right)_T \quad : \quad \left(\frac{\partial z}{\partial x}\right)_y\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial z}{\partial y}\right)_x \quad : \quad z = V,\ y = P,\ x = T$$

$$\left(\frac{\cancel{\partial V}}{\cancel{\partial T}}\right)_P = \frac{\alpha}{K_T}\left(\frac{\cancel{\partial V}}{\cancel{\partial T}}\right)_P\left(\frac{\partial T}{\partial P}\right)_V \quad : \quad 1 = \frac{\alpha}{K_T}\left(\frac{\partial T}{\partial P}\right)_V \quad \therefore \quad \left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{K_T}$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{K_T}}$$

ii.) $PV = NkT$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\frac{Nk}{P} = \frac{Nk}{PV} \quad : \quad K_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{V}\frac{NkT}{P^2} = \frac{NkT}{P^2V}$$

$$\frac{\alpha}{K_T} = \frac{\frac{Nk}{PV}}{\frac{NkT}{P^2V}} = \frac{\cancel{Nk}\cdot P^{\cancel{2}}\cancel{V}}{\cancel{PV}\cdot \cancel{Nk}T} = \frac{P}{T} \quad : \quad \left(\frac{\partial P}{\partial T}\right)_V = \frac{Nk}{V} = \frac{P\cancel{V}}{T\cancel{V}} = \frac{P}{T} \ \checkmark$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha}{K_T}, \ \text{True for ideal gas}}$$

b.) $K_T = 4.52 \times 10^{-10}\ Pa^{-1}$, $\alpha = 2.1 \times 10^{-4}\ K^{-1}$

$$dP = \left(\frac{\partial P}{\partial T}\right)_V dT \quad : \quad dP = \frac{\alpha}{K_T}\,dT \quad \therefore \quad P_f - P_i = \frac{\alpha}{K_T}(T_f - T_i) \quad : \quad \Delta P = \frac{(2.1\times10^{-4}K^{-1})}{(4.52\times10^{-10}Pa^{-1})}(10\,K)$$

$$\boxed{\Delta P = 4.65 \times 10^{6}\ Pa}$$

c.) $K_T = 4.5\times10^{-12}\ Pa^{-1}$, $\alpha = 0.42\times10^{-4}\ K^{-1}$

$$\Delta P = \frac{(0.42\times10^{-4}K^{-1})}{(4.5\times10^{-12}Pa^{-1})}(10\,K)$$

$$\boxed{\Delta P = 9.33\times10^{7}\ Pa}$$

d.) Because at constant pressure there is work done compared to that of constant volume.

Problem 2

a.) $P = P(\rho)$ but $V = V(P) \Rightarrow \frac{dP}{d\rho} = \frac{dP}{dV}\frac{dV}{d\rho}$ : $\rho = \frac{m}{V} \rightarrow V = \frac{m}{\rho}$ : $B = -V\frac{\partial P}{\partial V}$

$\frac{dV}{d\rho} = \frac{-m}{\rho^2}$ : $m = \rho V$ : $\frac{dV}{d\rho} = -\frac{V}{\rho} \rightarrow \frac{dP}{d\rho} = \frac{-V}{\rho}\frac{dV}{d\rho} = \frac{B}{\rho}$

$$\therefore \boxed{V_{sound} = \sqrt{\frac{B}{\rho}}}$$

b.) Determine B for ideal gas that undergoes an isothermal Process

$B = -V\cdot\left(\frac{\partial P}{\partial V}\right)_T \rightarrow P = \frac{NkT}{V} \therefore \left(\frac{\partial P}{\partial V}\right)_T = -\frac{NkT}{V^2}$ : $B = \cancel{-}\cancel{V}\cdot\cancel{-}\frac{NkT}{V^{\cancel{2}}} = \frac{NkT}{V}$

$V_{sound} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{NkT}{m}}$

$$\boxed{V_{sound} = \sqrt{\frac{NkT}{m}}}$$

c.) $B = \gamma P$ : $B = -V\cdot\left(\frac{\partial P}{\partial V}\right)_{adiabatic}$ : $\left(\frac{\partial P}{\partial V}\right) = -\frac{NkT}{V^2} \therefore B = \frac{NkT}{V} = P$

$dE = \left(\frac{\partial E}{\partial T}\right)_V dT + \cancel{\left(\frac{\partial E}{\partial V}\right)}^{0} dV$ : $dE = \left(\frac{\partial E}{\partial T}\right)_V dT \therefore dE = C_V dT = -P dV$

$C_V dT = -\frac{NkT}{V}dV \therefore C_V\int\frac{dT}{T} = -Nk\int\frac{1}{V}dV$ : $C_V \ln(T) = -Nk\ln(V) + \text{const} \frown \omega$

$T^{C_V} \times V^{Nk} = \omega$ : $\left(\frac{PV}{Nk}\right)^{C_V} V^{Nk} = \omega$ : $\left(\frac{P}{Nk}\right)^{C_V} V^{C_P} = \omega \therefore P^{C_V} = \frac{(Nk)^{C_V}}{V^{C_P}}\omega$

$\frac{\partial}{\partial V}\left(P^{C_V} = \frac{(Nk)^{C_V}}{V^{C_P}}\omega\right) = C_V P^{C_V-1} = -\frac{(Nk)^{C_V}}{V^{C_P+1}}\omega C_P \therefore \frac{\partial P}{\partial V} = -\frac{(Nk)^{C_V}}{V^{C_P+1}}\frac{C_P}{C_V}\frac{\omega}{P^{C_V-1}}$

$\frac{\partial P}{\partial V} = -\gamma\frac{(Nk)^{C_V}}{V^{C_P+1}}\frac{T^{C_V}V^{Nk}}{P^{C_V-1}}$ : $\frac{\partial P}{\partial V} = -\gamma\frac{(Nk)^{C_V}T^{C_V}}{V^{C_P+1-Nk}P^{C_V-1}} = \frac{-\gamma(Nk)^{C_V}T^{C_V}}{V^{C_V+1}P^{C_V-1}}$

$\frac{\partial P}{\partial V} = -\gamma\frac{\cancel{(Nk)}^{C_V}\cancel{P^{C_V}}}{\left(\frac{\cancel{NkT}}{\cancel{P}}\right)^{C_V} V \cancel{P^{C_V-1}}} = -\gamma\frac{P}{V}$ : $\frac{\partial P}{\partial V} = -\gamma\frac{P}{V} \rightarrow \frac{B}{-V} = -\gamma\frac{P}{V} \therefore B = \gamma P$

$$\boxed{V_{sound} = \sqrt{\frac{\gamma P}{\rho}}}$$

Problem 2 Continued

d.) In an adiabatic process the speed of sound will be greater due to $\gamma$ factor that is $\frac{7}{5}$. This means the speed of sound for an adiabatic process is $\sqrt{\frac{7}{5}}$ greater.

The adiabatic process is more likely to occur for sound propagation in air.

$V_{adiabatic} > V_{isothermal}$

e.) $N = 6.02 \times 10^{23}$, $k = 1.38 \times 10^{-23}$ J/K, $T = 290$ K, $m = 0.029$ kg/mol
↳ Avg. Temp @ sea level

For an isothermal process: $V_{sound} = \sqrt{\frac{6.02 \times 10^{23}(1.38 \times 10^{-23} \text{ J/K})(290 \text{ K})}{0.029 \text{ kg/mol}}} = 288 \frac{m}{s}$

$$\boxed{V_{sound} = 288 \text{ m/s}}$$

$\gamma = \frac{7}{5}$, $P = 1.01 \times 10^5$ Pa, $\rho = 1.225$ kg/m$^3$

For an adiabatic process: $V_{sound} = \sqrt{\frac{7}{5} \frac{1.01 \times 10^5 \text{ Pa}}{1.225 \text{ kg/m}^3}} = 340$ m/s

$$\boxed{V_{sound} = 340 \frac{m}{s}}$$

## Problem 3

a.) $H = E + PV$ : $H = \frac{5}{2} NkT$ : $C_p = \left(\frac{\partial H}{\partial T}\right)_p = \frac{5}{2} Nk$ , $C_v = \left(\frac{\partial H}{\partial T}\right)_v - V\left(\frac{\partial P}{\partial T}\right)_v = \frac{5}{2} Nk - V \cdot \left(\frac{Nk}{V}\right) = \frac{3}{2} Nk$

$$\boxed{H = \frac{5}{2} PV, \quad C_v = \frac{3}{2} \frac{PV}{T}, \quad C_p = \frac{5}{2} \frac{PV}{T}}$$

b.) $H = E + PV$ : $H = \frac{5}{2} PV + PV = \frac{7}{2} PV$ : $C_p = \left(\frac{\partial H}{\partial T}\right)_p = \frac{7}{2} Nk$ , $C_v = \left(\frac{\partial H}{\partial T}\right)_v - V\left(\frac{\partial P}{\partial T}\right)_v = \frac{7}{2} Nk - V\left(\frac{Nk}{V}\right) = \frac{5}{2} Nk$

$$\boxed{H = \frac{7}{2} PV, \quad C_v = \frac{5}{2} \frac{PV}{T}, \quad C_p = \frac{7}{2} \frac{PV}{T}}$$

c.) $H = H_0 + AT + \frac{B}{2} T^2 + \frac{C}{3} T^3$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p = A + BT + CT^2$$

$$C_p(300\,k) = 25.97 \frac{J}{mol\,k} + 6.096 \times 10^{-3} \frac{J}{mol\,k^2}(300\,k) + 4.055 \times 10^{-6} \frac{J}{mol\,k^3}(300\,k)^2 = 27.76 \frac{J}{mol\,k}$$

$$C_p(400\,k) = 25.97 \frac{J}{mol\,k} + 6.096 \times 10^{-3} \frac{J}{mol\,k^2}(400\,k) + 4.055 \times 10^{-6} \frac{J}{mol\,k^3}(400\,k)^2 = 28.66 \frac{J}{mol\,k}$$

$$\boxed{C_p = A + BT + CT^2, \quad C_p(300\,k) = 27.76 \frac{J}{mol\,k}, \quad C_p(400\,k) = 28.66 \frac{J}{mol\,k}}$$

These are greater than a diatomic gas

d.) $\Delta H = Q + \int V\,dP$ : $dP = 0$ $\therefore$ $\Delta H = Q$

From $300\,k \rightarrow 350\,k$

$H(350\,k) = 9381\,J$ , $H(300\,k) = 7982\,J$ : $\Delta H = H(350\,k) - H(300\,k) = 9381\,J - 7982\,J = 1399\,J$

$$\boxed{\Delta H_{300\,k \rightarrow 350\,k} = 1399\,J}$$

From $350\,k \rightarrow 400\,k$

$H(400\,k) = 10802\,J$ , $H(350\,k) = 9381\,J$ : $\Delta H = H(400\,k) - H(350\,k) = 10802\,J - 9381\,J = 1421\,J$

$$\boxed{\Delta H_{350\,k \rightarrow 400\,k} = 1421\,J}$$

## Problem 4

a.) $PV^{\gamma} = \text{Constant}$ ; $PV = NkT$ $\quad P = \frac{NkT}{V}$

$\frac{NkT}{V} V^{\gamma} = \text{Constant}$ $\quad T \cdot V^{\gamma-1} = \text{const}$

$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

$$\boxed{\frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma-1}}$$

$\gamma = \frac{7}{5}$

b.) $H_2$ , Hydrogen ignites at 773 K

$\left(\frac{V_f}{V_i}\right)^{\gamma-1} = \frac{T_i}{T_f}$ $\quad \therefore \quad \frac{V_f}{V_i} = \left(\frac{T_i}{T_f}\right)^{\frac{1}{\gamma-1}}$ : $\frac{V_f}{V_i} = \left(\frac{300\text{K}}{773\text{K}}\right)^{\frac{2}{5}} = 0.09$

$$\boxed{\frac{V_f}{V_i} = 0.09}$$

## Problem 5

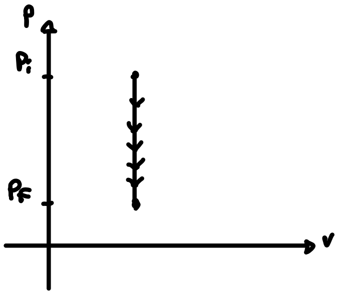
a.) $P_i = 89{,}632 \text{ Pa} + 1.01\times10^5 \text{ Pa}$ : $T_i = 297 \text{ K}$ : $V_i = V_F$ : $T_F = 283 \text{ K}$

$P_i V = NkT_i$ : $P_F V = NkT_F$

$V = \frac{NkT_i}{P_i}$ , $V = \frac{NkT_F}{P_F}$ $\therefore$ $\frac{\cancel{Nk}T_i}{P_i} = \frac{\cancel{Nk}T_F}{P_F}$ $\rightarrow$ $P_F = \frac{P_i T_F}{T_i}$

$$P_F = \frac{190632 \text{ Pa}\cdot(283 \text{ K})}{297 \text{ K}} = 1.8\times10^5 \text{ Pa} = 1.8\times10^5 \text{ Pa}\ \frac{0.000145038 \text{ PSI}}{1 \text{ Pa}} = 26.3 \text{ PSI}$$

$$\boxed{P_F = 7.9\times10^4 \text{ Pa} = 11.5 \text{ PSi, Gauge Pressure}}$$




$W = \int P\,dv$ : $dv = 0$ $\therefore$ $W = 0$ : $\Delta E = Q + \cancel{W}$ $\therefore$ $\Delta E = Q$

$\Delta E = \frac{3}{2}NkT_F - \frac{3}{2}NkT_i = \frac{3}{2}Nk(T_F - T_i)$ Quantity less than 0 $\therefore$ $\Delta E < 0$

There is no work done on the ball and because of the temperature drop heat leaves the ball.

b.) Inside locker room : $T_i = 297 \text{ K}$ : $T_F = ?$ : Adiabatic $\therefore$ $Q = 0$ $\rightarrow$ $\Delta E = W$

$TP^{(1-\gamma)/\gamma} = \text{constant}$ : $T_i P_i^{(1-\gamma)/\gamma} = T_F P_F^{(1-\gamma)/\gamma}$

$P_i = 1.01\times10^5 \text{ Pa}$ , $P_F = 1.9\times10^5 \text{ Pa}$ , $T_i = 297 \text{ K}$ , $\gamma = 7/5 = \frac{c_p}{c_v}$

$$T_F = \frac{T_i P_i^{(1-\gamma)/\gamma}}{P_F^{(1-\gamma)/\gamma}} = (297 \text{ K})\frac{(1.01\times10^5 \text{ Pa})^{(1-\gamma)/\gamma}}{(1.9\times10^5 \text{ Pa})^{(1-\gamma)/\gamma}} = 356 \text{ K}$$

$T_F = 356 \text{ K}$ $\longleftarrow$ Inside room

Outside locker room : $T_i = 356 \text{ K}$ , $T_F = 283 \text{ K}$ , $P_i = 1.9\times10^5 \text{ Pa}$ , $P_F = ?$

$\frac{Nk}{V} = \frac{P}{T}$ $\therefore$ $\frac{P_i}{T_i} = \frac{P_F}{T_F}$ $\rightarrow$ $P_F = \frac{T_F}{T_i}P_i = \frac{(283 \text{ K})}{(356 \text{ K})}(1.9\times10^5 \text{ Pa}) = 1.5\times10^5 \text{ Pa}$

$$\boxed{P_F = 4.9\times10^4 \text{ Pa} = 7.1 \text{ PSI, Gauge Pressure}}$$

c.) If the process of pumping up the balls was truly adiabatic, then the change of inside Temperature to Outside Temperature would explain the drop in PSI.