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MATH 361 HW 2.2

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1,2,5,7,9,10

Problem 1

$f(x) = x^4 + 2x^2 - x - 3$

a.) $0 = x^4 + 2x^2 - x - 3 \longrightarrow x^4 = 3 + x - 2x^2 \longrightarrow x = (3 + x - 2x^2)^{1/4}$

$x = (3 + x - 2x^2)^{1/4} \quad \therefore \quad \boxed{\exists c \in [a,b] \text{ s.t. } f(c) = 0}$

b.) $0 = x^4 + 2x^2 - x - 3 \longrightarrow 2x^2 = x + 3 - x^4 \longrightarrow x^2 = \dfrac{x+3-x^4}{2} \longrightarrow x = \left(\dfrac{x+3-x^4}{2}\right)^{1/2}$

$x = \left(\dfrac{x+3-x^4}{2}\right)^{1/2} \quad \therefore \quad \boxed{\exists c \in [a,b] \text{ s.t. } f(c) = 0}$

c.) $0 = x^4 + 2x^2 - x - 3 \longrightarrow x + 3 = x^4 + 2x^2 \longrightarrow x + 3 = x^2(x^2+2) \longrightarrow x^2 = \dfrac{x+3}{x^2+2}$

$x = \left(\dfrac{x+3}{x^2+2}\right)^{1/2} \quad \therefore \quad \boxed{\exists c \in [a,b] \text{ s.t. } f(c) = 0}$

d.) $0 = x^4 + 2x^2 - x - 3 \longrightarrow x^4 + 2x^2 - x - 3 + 3x^4 + 2x^2 + 3 = 0 \longrightarrow$

$x = \dfrac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1} \quad \therefore \quad \boxed{\exists c \in [a,b] \text{ s.t. } f(c) = 0}$

Problem 2

a.)

i.) $x = (3+x-2x^2)^{1/4}$: $g(x) = (3+x-2x^2)^{1/4}$

$P_{n+1} = g(P_n)$: $P_0 = 1$

$P_1 = g(P_0) = 1.189207115$: $P_2 = g(P_1) = 1.080057753$: $P_3 = g(P_2) = 1.149671431$: $P_4(P_3) = 1.10782053$

$$\boxed{P_4 = 1.10782053}$$

ii.) $x = \sqrt{\dfrac{x+3-x^4}{2}}$: $g(x) = \sqrt{\dfrac{x+3-x^4}{2}}$

$P_1 = g(P_0) = 1.224744871$: $P_2 = g(P_1) = 0.9936661591$: $P_3 = g(P_2) = 1.228568645$: $P_4 = g(P_3) = 0.9875064292$

$$\boxed{P_4 = 0.9875064292}$$

iii.) $x = \sqrt{\dfrac{(x+3)}{(x^2+2)}}$: $g(x) = \sqrt{\dfrac{x+3}{x^2+2}}$

$P_1 = g(P_0) = 1.154700538$: $P_2 = g(P_1) = 1.11642741$: $P_3 = g(P_2) = 1.126052233$: $P_4 = g(P_3) = 1.123638885$

$$\boxed{P_4 = 1.123638885}$$

iv.) $x = \dfrac{3x^4+2x^2+3}{4x^3+4x-1}$: $g(x) = \dfrac{3x^4+2x^2+3}{4x^3+4x-1}$

$P_1 = g(P_0) = 1.142857143$: $P_2 = g(P_1) = 1.12448169$: $P_3 = g(P_2) = 1.124123164$: $P_4 = g(P_3) = 1.12412303$

$$\boxed{P_4 = 1.12412303}$$

b.)

I would use $g(x) = \dfrac{3x^4+2x^2+3}{4x^3+4x-1}$ Since it converges quickly.

Problem 5

a.) $P_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$, b.) $P_n = P_{n-1} - \dfrac{P_{n-1}^3 - 21}{3P_{n-1}^2}$

c.) $P_n = P_{n-1} - \dfrac{P_{n-1}^4 - 21P_{n-1}}{P_{n-1}^2 - 21}$, d.) $P_n = \left(\dfrac{21}{P_{n-1}}\right)^{1/2}$

Fastest to Slowest : b, d, a. C does not converge

Problem 7

$x^4 - 3x^2 - 3 = 0$, $[1,2]$, $p_0 = 1$

$$x^4 - 3x^2 - 3 = 0 \Leftrightarrow x^4 = 3x^2 + 3 \Leftrightarrow x = (3x^2+3)^{1/4}$$

$g(x) = (3x^2+3)^{1/4}$ $\longrightarrow$ 0.01 TOL

$P_1 = g(P_0) = 1.56508458$, $P_2 = g(P_1) = 1.793572879$, $P_3 = g(P_2) = 1.885943743$

$P_4 = g(P_3) = 1.922847844$, $P_5 = g(P_4) = 1.93750754$, $P_6 = g(P_5) = 1.94331693$

$$\boxed{P_6 = 1.94331693}$$

Problem 9

$g(x) = \pi + 0.5\sin(x/2) \quad [0, 2\pi]$

a.)

i.) $g \in C[0, 2\pi]$ Since there are no discontinuities on $g(x)$

ii.) $g'(x) = \frac{1}{4}\cos(x/2)$: $0 = \frac{1}{4}\cos(x/2)$: $0 = \cos(x/2)$: $\frac{\pi}{2} = \frac{x}{2}$ $\therefore x = \pi$

Using EVT : $g(0) = \pi$, $g(\pi) = \pi + 0.5$, $g(2\pi) = \pi$

Thus by EVT, $g(x) \in [0, 2\pi]$ for all $x \in [0, 2\pi]$

iii.) $|g'(x)| = |0.25 \cdot \cos(x/2)| \leq |0.25| < 1$ for all $x \in [0, 2\pi]$

Choose $K = 0.25$

iv.) Since $g \in C[0, 2\pi]$ and $g(x) \in [0, 2\pi]$ for all $x \in [0, 2\pi]$ and $|g'(x)| \leq 0.25 < 1$ for all $x \in [0, 2\pi]$

$$\boxed{\exists!\, p \in [0, 2\pi] \text{ s.t. } g(p) = p}$$

b.) $g(x) = \pi + 0.5\sin(x/2)$: $P_0 = \pi$

$P_1 = g(P_0) = 3.641592654$: $P_2 = g(P_1) = 3.626048864$: $P_3 = g(P_2) = 3.626995622$

$$\boxed{P_3 = 3.626995622}$$

c.) $|P - P_n| \leq \frac{K^n}{1-K}|P_1 - P_0|$: $K = 0.25$, $P_1 = \pi + 1/2$, $P_0 = \pi$, $|P - P_n| = 0.01$

$\frac{|P - P_n|}{|P_1 - P_0|}(1-K) \leq K^n$ $\therefore$ $n \geq \frac{\ln\left(\frac{P - P_n}{P_1 - P_0} \cdot (1-K)\right)}{\ln(K)} \doteq 3.029446845$

$$\boxed{n \geq 4 \text{ iterations}}$$

Problem 10

$g(x) = 2^{-x}$ $[1/3, 1]$

a.)

i.) No points of discontinuity for $g(x)$ $\therefore$ $g \in C[1/3, 1]$

ii.) $g'(x) = \ln(2) \cdot 2^{-x}$: $0 = \ln(2) \cdot 2^{-x}$: $0 = 2^{-x}$ $\therefore$ no critical points to Function

using EVT : $g(1/3) = 0.793700526$, $g(1) = 0.5$

Thus by EVT, $g(x) \in [1/3, 1]$ for all x $[0, 2\pi]$

iii.) $|g'(x)| = |\ln(2) \cdot 2^{-x}| \leq |0.5501512818| < 1$ for all $[1/3, 1]$

iv.) Since $g \in C[1/3, 1]$ and $g(x) \in [1/3, 1]$ For all $x \in [1/3, 1]$ and $|g'(x)| \leq 0.55 < 1$ for all $x \in [1/3, 1]$

$$\boxed{\exists!\, p \in [1/3, 1] \text{ s.t. } g(p) = p}$$

b.) $g(x) = 2^{-x}$ $P_0 = 2/3$

$P_1 = g(P_0) = 0.6299605249$: $P_9 = g(P_8) = 0.6411686114$

$$\boxed{P_9 = 0.6411686114}$$

c.) $k = 0.5501512818$, $P_1 - P_0 = -0.0367061418$, $P - P_n = 10^{-4}$

$$n \geq \frac{\ln\left(\frac{P - P_n}{P_1 - P_0} \cdot (1-k)\right)}{\ln(k)} \geq 11.2$$

$$\boxed{n \geq 12}$$