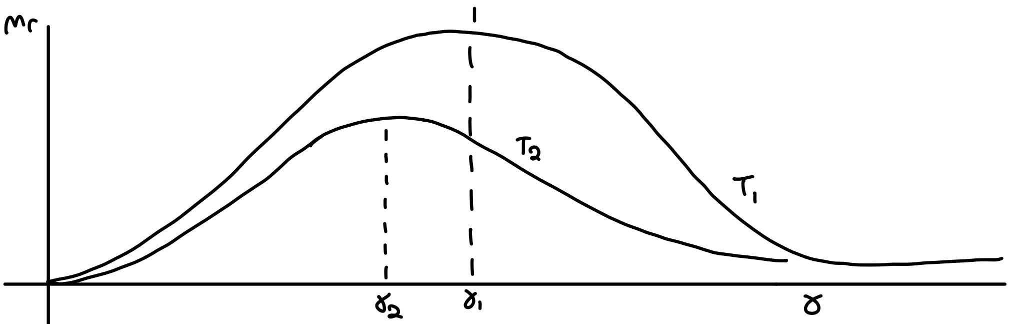
Blackbody Radiation

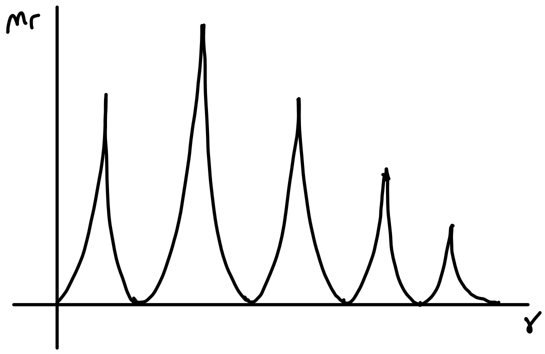
1) Natural Radiation (α, β, γ)
2) Cathode Rays (Artificial)
3) X - Rays
4) Thompson measures e/m
5) Milikan measures e
6) Avogadro, Maxwell & Boltzman, Atomic Hypothesis

Implications
1) Fundamental building blocks of matter involve (Bound) discrete charges
2) Charge & mass appear Quantized (Dynamical Quantities, :F, E, P, assumed Continuous)

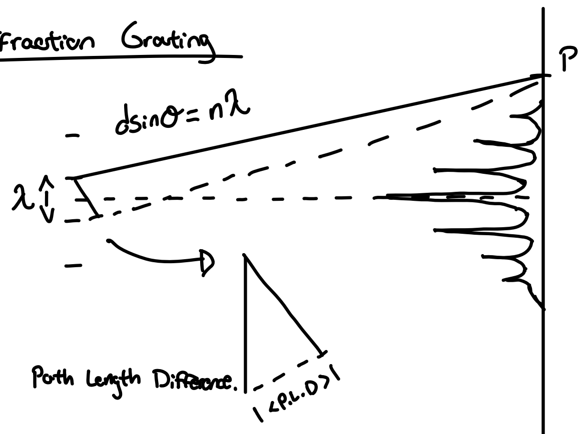
Thermal Radiation (Continuous Spectrum)



Ionization / Excitation



Diffraction Grating



$\lambda = L/2 \Rightarrow$ 1st (Lowest)

$$\frac{n\lambda}{2} = L$$

$$\frac{1}{\lambda} = R_A\left(\frac{1}{n^2} - \frac{1}{k^2}\right)$$ n, k are integers

K ⇒ Peak index > n

n ⇒ series index

$M_\gamma(T)$ Power (Per unit area) is emitted by a radiator ΔT Temp. T.

$\alpha_\gamma(T)$

Kirchoff showed $\frac{M_\gamma(T)}{A_\gamma(T)}$ is a universal Function

Related to Black Body radiation

Stephan - Boltzman:

$$M^b(T) = \int_0^\infty M_\gamma^b(T)\, d\gamma = \sigma T^4$$

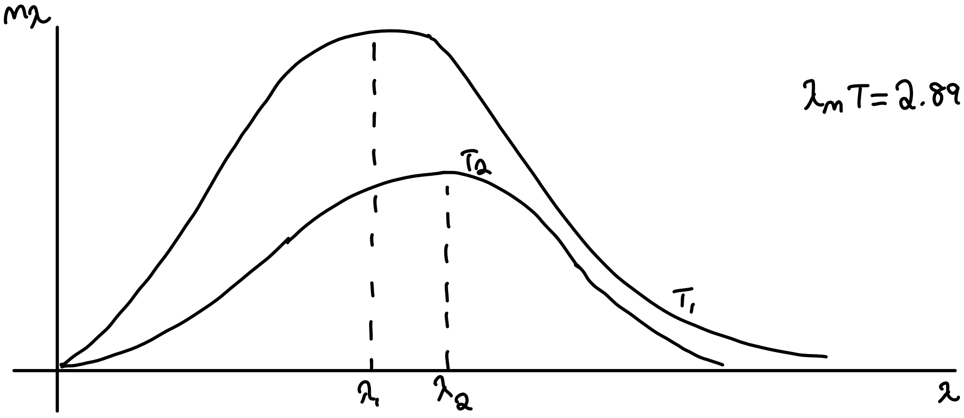
With $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$

$$\int_0^\infty M_\gamma(T)\, d\gamma = \int_\infty^0 M_\gamma \frac{\partial \gamma}{\partial \lambda} d\lambda$$

$$\Downarrow$$

$$M_\lambda(T) = -M_\gamma(T)\frac{\partial \gamma}{\partial \lambda}$$

$$\lambda = c/\gamma \quad = \frac{c}{\lambda^2} M_\gamma(T)$$



$$\lambda_m T = 2.89 \times 10^{-3}\,K \cdot m$$

(1) Object ~ 300K

$$\lambda_m = \frac{2.898 \times 10^{-3}\,K \cdot m}{300K} \sim 1000\,nm$$

(2) Hot stove ~ 500K

$$\lambda_m = \frac{2.898 \times 10^{-3}\,K \cdot m}{500K} \sim 600\,nm$$

(3) Surface of the sun ~ 6000K

$$\lambda_m \sim \frac{2.898 \times 10^{-3}\,K \cdot m}{600K} \sim 500\,nm$$

(4) cosmic background radiation ~ 3K $\Rightarrow \lambda_m = \dfrac{2.898 \times 10^{-3}\,K \cdot m}{3K} \sim 1\,mm$

Energy Density @ γ

$$M_\gamma(T) = \frac{c}{4} u_\gamma(T)$$

Radiation in the cavity is E-m standing waves

Total E. Density:

\# of modes w/ γ

$$u_\gamma(T) = \frac{N_\gamma}{V} \langle \varepsilon \rangle$$

Avg. Energy per mode.

Volume of cavity

N_γ: Requires analysis of 3D Standing Waves (see 2-2)

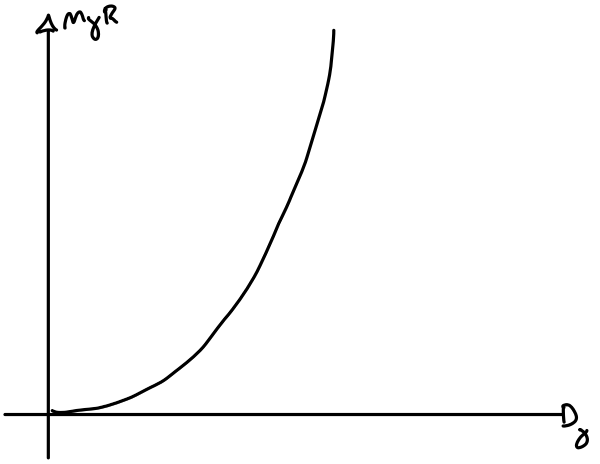
$$\left.\begin{aligned} \gamma &= \frac{c}{2L}\sqrt{n_x^2 + n_y^2 + n_z^2} \\ \lambda &= \frac{2L}{\sqrt{n_x^2 + n_y^2 + n_z^2}} \end{aligned}\right\} N_\gamma = \frac{8\pi \gamma^2}{c^3} V$$

$\nu \equiv \gamma$

$\gamma \equiv$ Frequency

Classical $\langle \varepsilon \rangle = k_B T$

$$u_\gamma(T) = \frac{8\pi \gamma^2}{c^3} k_B T \qquad M_\gamma(T) = \frac{2\pi \gamma^2}{c^2} k_B T$$

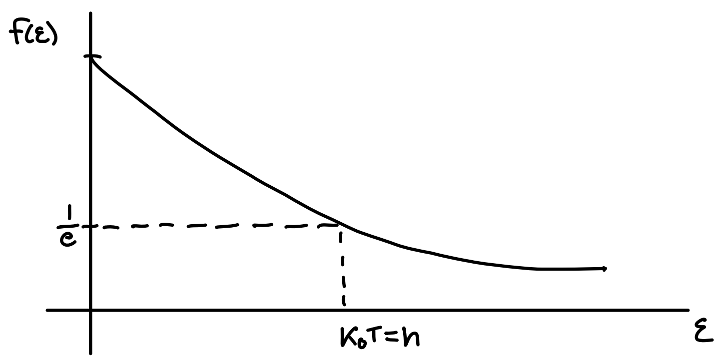


Energy is exchanged only in multiples of some unit that is proportional to γ.

$$\varepsilon_n = nh\gamma$$

The probability of having energy ε occupied

$$f(\varepsilon) = e^{-\varepsilon/k_BT}$$



$$\langle\varepsilon\rangle = \frac{\sum_{n=0}^{\infty} \varepsilon f(\varepsilon)}{\sum_{n=0}^{\infty} f(\varepsilon)} \qquad \varepsilon = nhr \qquad \langle\varepsilon\rangle = \frac{\sum^{\infty} nhr\, e^{-nhr/k_BT}}{\sum_{n}^{\infty} e^{-nhr/k_BT}} \qquad x = hr/k_BT$$

$$= \frac{\sum (kT)\, n x e^{-nx}}{\sum e^{-nx}} = kT \frac{\sum nxe^{-nx}}{\sum e^{-nx}} = -k_BTx \frac{1}{Z(x)} \frac{dZ}{dx}(x)$$

$$= \frac{d}{dx}(\ln Z) = \frac{1}{Z}\frac{dZ}{dx}$$

$$Z(x) = \sum_{n=0}^{\infty} e^{-nx} = 1 + e^{-x} + e^{-2x} + e^{-3x} + \dots = \frac{1}{1-e^{-x}}$$

$$\frac{1}{1-y} = 1 + y + y^2 + y^3 + \dots$$

$$y \rightarrow e^{-x}$$

$$\frac{d}{dx} Z(x) = \frac{d}{dx} \sum_n e^{-nx} = \sum -ne^{-nx}$$

$$-x\frac{dZ}{dx} = -x\left(\sum_n -ne^{-nx}\right) = \sum_n xne^{-nx}$$

$$\frac{d}{dx} \ln(Z(x)) = \frac{d}{dx} \ln\left(\frac{1}{1-e^{-x}}\right) = \frac{-e^{-x}}{1-e^{-x}}$$

$$\langle\varepsilon\rangle = -k_BTx\left(-e^{-x}/1-e^{-x}\right)$$

$$= k_B T \left(\frac{h\nu}{k_B T}\right)\left(\frac{e^{-h\nu/kT}}{1 - e^{-h\nu/kT}}\right)\frac{e^{h\nu/kT}}{e^{h\nu/kT}}$$

$$\langle \varepsilon \rangle = \frac{h\nu}{e^{h\nu} - 1}$$ $\Leftarrow$ Planck's result for average oscillator ε.