

9.3 # 11

a.)

$$\vec{a} = [2, 1, 0] \quad \vec{b} = [-3, 2, 0]$$

$$\vec{a} \times \vec{b}: a_1 = 2 \quad b_1 = -3$$

$$a_2 = 1 \quad b_2 = 2$$

$$a_3 = 0 \quad b_3 = 0$$

$$\vec{a} \times \vec{b} = \vec{v}: \vec{v} = [v_1, v_2, v_3] = [0, 0, 7]$$

$$v_1 = 1(0) - 0(2) = 0$$

$$v_2 = 0(3) - 2(0) = 0$$

$$v_3 = 2(2) - 1(-3) = 7 \quad \Longleftrightarrow$$

$$\begin{aligned} \vec{a} \times \vec{b} &= [0, 0, 7] \\ \vec{a} \times \vec{b} &= 0\hat{i} + 0\hat{j} + 7\hat{k} \end{aligned}$$

$$\begin{array}{c|c|c} \hat{i} & \hat{j} & \hat{k} \\ \hline a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array} = \begin{array}{c} \text{Determinant} \\ [a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1] \\ \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \end{array}$$

$$= \hat{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \hat{j} \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} + \hat{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

b.)

$$\vec{a} = [2, 1, 0] \quad \vec{b} = [-3, 2, 0]$$

$$\vec{b} \times \vec{a}: b_1 = -3 \quad a_1 = 2$$

$$b_2 = 2 \quad a_2 = 1$$

$$b_3 = 0 \quad a_3 = 0$$

$$\vec{b} \times \vec{a} = \vec{v}: \vec{v} = [v_1, v_2, v_3] = [0, 0, 7]$$

$$v_1 = 2(0) - 1(0) = 0$$

$$v_2 = 0(2) - (-3)(0) = 0$$

$$v_3 = -3(1) - 2(2) = 7 \quad \Longleftrightarrow$$

$$\begin{aligned} \vec{b} \times \vec{a} &= [0, 0, 7] \\ \vec{b} \times \vec{a} &= 0\hat{i} + 0\hat{j} + 7\hat{k} \end{aligned}$$

$$\begin{array}{c|c|c} \hat{i} & \hat{j} & \hat{k} \\ \hline a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array} = \begin{array}{c} \text{Determinant} \\ [a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1] \\ \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \end{array}$$

$$= \hat{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \hat{j} \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} + \hat{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

c.)

$$\vec{a} = [2, 1, 0] \quad \vec{b} = [-3, 2, 0]$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\vec{a} \cdot \vec{b} = 2(-3) + 1(2) + 0(0) = -6 + 2 + 0 = -4$$

$$\vec{a} \cdot \vec{b} = -4$$

9.3] #32

Volume of parallel piped: $|\vec{a} \cdot (\vec{b} \times \vec{c})|$

$$\vec{a} = 1\hat{i} + 1\hat{j} + 0\hat{k}$$

$$\vec{b} = -2\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\vec{c} = -2\hat{i} + 0\hat{j} - 3\hat{k}$$

$$\begin{array}{c|c|c} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array} = \begin{array}{c} \text{Determinant} \\ [a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1] \\ \begin{array}{ccc} \nearrow & \nearrow & \nearrow \\ v_1 & v_2 & v_3 \end{array} \end{array}$$

$$= \hat{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \hat{j} \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} + \hat{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

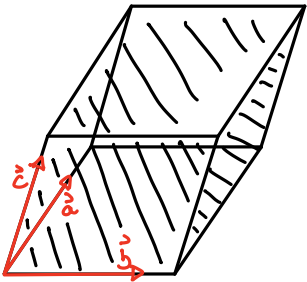
$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 2 \\ -2 & 0 & -3 \end{vmatrix} = [0(-3) - 2(0), 2(-2) + 2(-3), -2(0) - 0(-2)]$$

$$[0, -10, 0]$$

$$\vec{b} \times \vec{c} = [0, -10, 0]$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = [1, 1, 0] \cdot [0, -10, 0] = 0(1) + 1(-10) + 0(0) = -10$$

$$|\vec{a} \cdot (\vec{b} \times \vec{c})| = 10$$



$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})| = 10$$