Mon 28 Nov Warm Up 14

Tues 29 Nov Discussion

Ch 12 CQ 3, 11

Prob 10, 27, 66

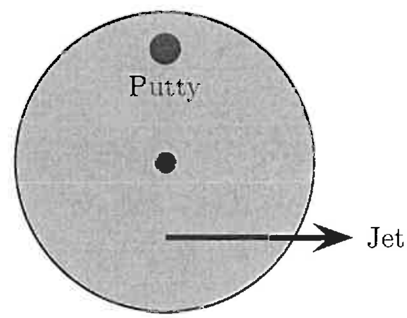
Non-Equilibrium Rotational Dynamics

The rotational version of Newton's 2nd Law is:

> The angular acceleration, α, of any object satisfies
>
> $$\tau_{net} = I\alpha$$
>
> where τ_{net} is the net torque about a pivot and I is the moment of inertia about the same pivot

75 Rotating disk

A 2.0 kg turntable (disk) has radius 0.10 m and can rotate horizontally about a frictionless axle through its center. A 1.2 kg blob of putty is stuck to a point on the disk three quarters of the distance from the center to the edge. A jet attached halfway from the center to the edge of the disk exerts a tangential force 4.0 N as illustrated. The aim of this exercise is to determine the angular acceleration of the disk via the following steps.



a) Write the rotational version of Newton's second law.

b) Determine the moment of inertia of the disk plus putty.

c) Determine the net torque acting on the disk.

d) Determine the angular acceleration of the disk.

e) Suppose that a brake pad presses on the rim of the disk, producing a frictional force with magnitude 1.5 N while the jet is operating. Determine the angular acceleration of the disk in this situation.

Answer:

a) $\tau_{net} = I \alpha$

b) $I = I_{disk} + I_{putty}$

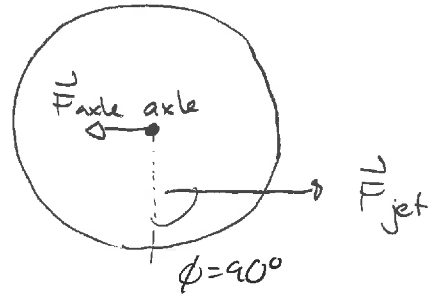
$I_{disk} = \frac{1}{2} M_{disk} r_{disk}^2 = \frac{1}{2} 2.0\,kg \times (0.10\,m)^2 = 0.010\,kg\,m^2$

$I_{putty} = M_{putty} r_{putty}^2 = 1.2\,kg \times \left(\frac{3}{4} \times 0.10\,m\right)^2 = 0.0068\,kg\,m^2$

$I = 0.010\,kg\,m^2 + 0.0068\,kg\,m^2$

$I = 0.017\,kg\,m^2$

c) $\tau_{net} = \tau_{axle} + \tau_{jet} + \tau_{grav}$



$\tau_{axle} = 0$ since $r = 0$

$\tau_{grav} = 0$ since $r = 0$

$\tau_{jet} = rF_{jet} \sin\phi$

$= \frac{0.10m}{2} \times 4.0N \sin 90°$

$= 0.20Nm$

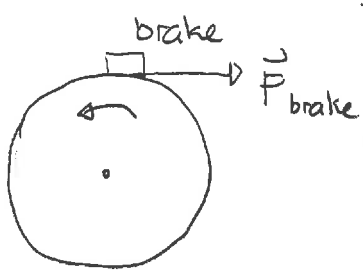
$\Rightarrow \tau_{net} : 0.20Nm$

d) $\tau_{net} = I\alpha \Rightarrow 0.20N \cdot m = 0.017 kg\, m^2\, \alpha$

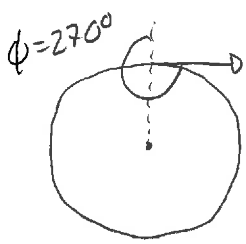
$\Rightarrow \alpha = \frac{0.20\, N \cdot m}{0.017\, kg\, m^2} = 12\, rad/s^2$

$\Rightarrow \alpha = 12 rad/s^2$

e)



There is an additional torque, due to friction



$\tau_{brake} = r F_{brake} \sin\phi$

$= 0.10m \times 1.5N \sin 270°$

$= -0.15Nm.$

Then $\tau_{net} = \tau_{jet} + \tau_{brake} = 0.20Nm - 0.15Nm$

$= 0.05Nm$

So $\alpha = \frac{\tau_{net}}{I} = 2.9\, rad/s^2$

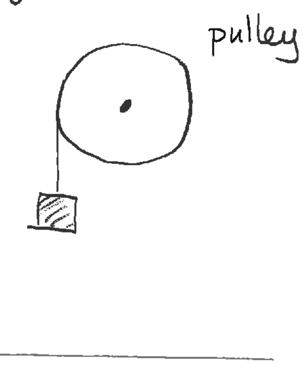
Quiz 1 60-100% $\}$ 80% - 100%

With connected objects like these, we apply Newton's laws to each object individually:

Example: A box with mass, m, is suspended by a string that is wrapped around a pulley with moment of inertia I. The box is released.

a) Determine an expression for the acceleration of the box.

b) If the box has mass 3.0kg and the pulley has moment of inertia 0.030 $kg\,m^2$, and radius 0.20m, determine the time take for the box to reach the ground if it were released from rest 1.5m above the ground.



Answer: Strategy

Rotational dynamics for pulley $\Rightarrow$ α for pulley

linear dynamics for pulley $\Rightarrow$ a for pulley

If the string does not slip, then

$$a_{box} = a_{rim\ of\ pulley} = a_{t\ pulley} \Rightarrow \boxed{a_{box} = \alpha R}$$

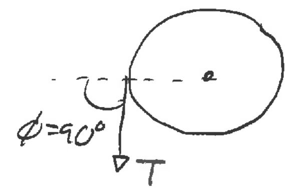
where R is the radius of the pulley. Here a_{box} is the magnitude of the box's acceleration.

a) Pulley $\quad \tau_{net} = I\alpha$. Only the string produces $\tau \neq 0$

Here

$$\tau_{string} = rF\sin\phi$$

$$= RT\sin 90^\circ$$



$$\tau_{string} = RT.$$

So $\quad \tau_{net} = I\alpha \Rightarrow \tau_{string} = I\alpha$

$$\Rightarrow RT = I\alpha$$

Box

$$\vec{F}_{net} = m\cdot\vec{a}$$

$$\Rightarrow T - mg = ma_y$$

$$T - mg = -m\,a_{box}.$$

Combine: $\quad RT = I\alpha \Rightarrow T = \frac{I\alpha}{R}$

and $\quad a_{box} = \alpha R \Rightarrow \alpha = \frac{a_{box}}{R}$ gives: $\quad T = \frac{I\,a_{box}}{R^2}$

$$\Rightarrow \frac{I\,a_{box}}{R^2} - mg = -m\,a_{box} \Rightarrow \frac{I}{R^2}a_{box} + m\,a_{box} = mg$$

$$\Rightarrow \left(\frac{I}{R^2} + m\right)a_{box} = mg$$

$$\Rightarrow a_{box} = \frac{mg}{(m + I/R^2)}$$

b) $$a_{box} = \frac{3.0kg \times 9.8m/s^2}{\left[3.0kg + 0.030kg/m^2 / (0.20m)^2\right]} = 7.8m/s^2 \quad \text{down} \downarrow$$

This is constant. So

$$y_f = y_i + v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\Rightarrow 0m = 1.5m + 0m/s\,\Delta t + \frac{1}{2}(-7.8m/s^2)(\Delta t)^2$$

$$\Rightarrow \frac{1.5m \times 2}{7.8m/s^2} = \Delta t^2 \Rightarrow \Delta t = \sqrt{0.38s^2} = 0.62s$$