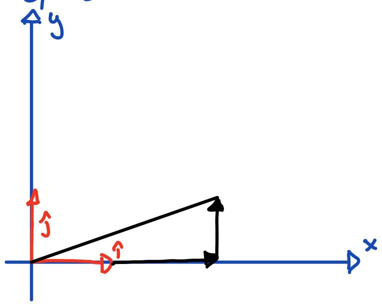
# 3.5 Vector Spaces and Subspaces

3.5 #'s 15, 16, 18, 22, 38, 39, 45, 46, 57

Vector Space




A vector space $V$ is a now empty collection of objects called vectors for which are defined the operations:

- Vector addition $\vec{x}+\vec{y}$
- Scalar multiplication $c\vec{x}$ that satisy the following properties for all $\vec{x}, \vec{y}, \vec{z}$ in $V$ and $c, d \in R$

**Closure Identities**

1. $\vec{x}+\vec{y} \in V$ ⇒ Necessary for a sub
2. $c\vec{x} \in V$ ⇒ space

**Addition Properties**

3. $\vec{x}+\vec{0}=\vec{x}$
4. $\vec{x}+(-\vec{x})=\vec{0}$
5. $(\vec{x}+\vec{y})+\vec{z}=\vec{x}+(\vec{y}+\vec{z})$
6. $\vec{x}+\vec{y}=\vec{y}+\vec{x}$

**Scalar Multiplication Properties**

7. $1\vec{x}=\vec{x}$
8. $c(\vec{x}+\vec{y})=c\vec{x}+c\vec{y}$
9. $(c+d)\vec{x}=c\vec{x}+d\vec{x}$
10. $c(d\vec{x})=(cd)\vec{x}$

**Ex.)** Let $M \times N$ be the space of all $m \times n$ matrices

Closure under linear combination

$A, B \equiv M \times N$ matrices

$cA+dB$, for each $a_{ij}$ we know

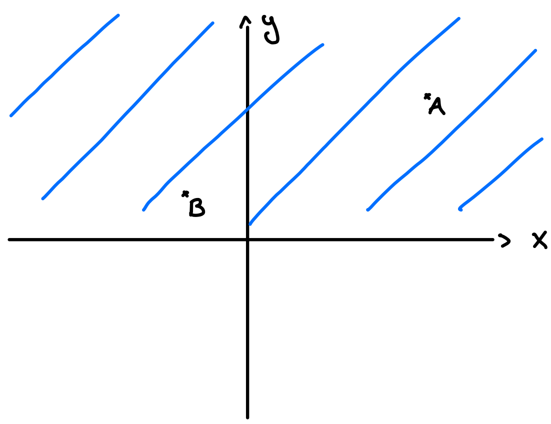
$c[a_{ij}]=[ca_{ij}]$

$d[b_{ij}]=[db_{ij}]$

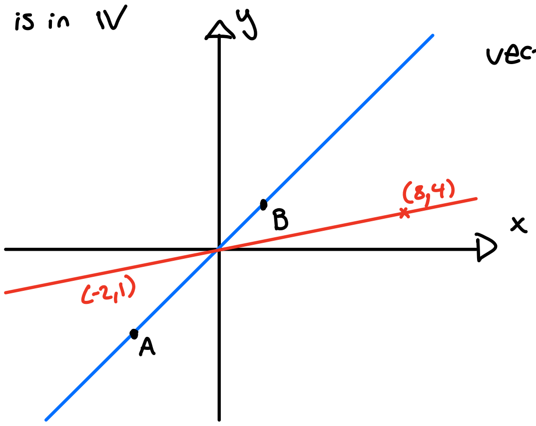
$[ca_{ij}]+[db_{ij}]=[ca_{ij}, db_{ij}]$

$c\vec{x}+d\vec{y}$ is in $W$

Not vector space



$c\vec{x} + d\vec{y}$ is in $V$




vector space - must pass through origin

The set of all Functions which are continuous on the closed interval [0,1]