

4.2

9.9, 9.6, 9.5, 9.7, 9.8

$$(a) \frac{9.9 + 9.6 + 9.5 + 9.7 + 9.8}{5} = 9.7$$

$$\text{mean} = 9.7$$

Value x_i	Deviation d_i	Deviation Squared d_i^2
9.5	-0.2	0.04
9.6	-0.1	0.01
9.7	0.0	0.00
9.8	0.1	0.01
9.9	0.2	0.04
$\sum x_i = 48.5$	$\sum d_i = 0$	$\sum d_i^2 = 0.1$

method used:

$$\sigma_s^2 = \frac{1}{N-1} \sum_i d_i^2 = \sqrt{\frac{0.1}{5-1}} = 0.158$$

$$\boxed{\begin{array}{l} \sigma_x = 0.158 \\ \bar{x} = 9.70 \end{array}}$$

4.6]

a.)

x_i	d_i	d_i^2
10	-0.7	0.49
13	2.3	5.29
8	-2.7	7.29
15	4.3	18.49
8	-2.7	7.29
13	2.3	5.29
14	3.3	10.89
13	2.3	5.29
19	8.3	68.89
8	-2.7	7.29
13	2.3	5.29
13	2.3	5.29
7	-3.7	13.69
8	-2.7	7.29
6	-4.7	22.09
8	-2.7	7.29
11	0.3	0.09
12	1.3	1.69
8	-2.7	7.29
7	-3.7	13.69

$$\bar{x} = 10.7 \quad \sum d_i^2 = 220.2$$

method used:

$$\sigma_x^2 = \frac{1}{n} \sum d_i^2$$

$$\sigma_x = \sqrt{\frac{220.2}{20}} = 3.3$$

$$\boxed{\begin{array}{l} \sigma_x = 3.3 \\ \bar{x} = 10.7 \end{array}}$$

b.) Claim 1: $\sigma_x^2 = \bar{x}$ Claim 2: $\sigma_x = \sqrt{\bar{x}}$

$$3.3^2 = 10.89 \quad \sqrt{10.7} = 3.27$$

From this, the difference from the calculated values in part (a) with Claim 1, is about +0.2. This is close enough to say this is true.

Claim 1 is true.

For claim 2, the difference between the calculated values found in part (a) is about -0.03. This is even closer than claim 1 so this claim must be true.

Claim 2 is true.

4.16

9.9, 9.6, 9.5, 9.7, 9.8

a.) $\bar{x} = 9.70$ $\sigma_{\bar{x}} = \sigma_x / \sqrt{N}$
 $\sigma_x = 0.158$ $= 0.158 / \sqrt{5}$
 $\sigma_{\bar{x}} = 0.07066$

$$g = 9.70 \pm 0.07 \text{ m/s}^2$$

b.) From this calculated value, g can be at a value of,

$$g = 9.63 \text{ m/s}^2 \text{ \& } g = 9.77 \text{ m/s}^2 \quad \bar{x} = 9.70 \text{ m/s}^2$$

$$\Delta = \bar{x} - \text{accepted}$$

$$\Delta = 9.70 \text{ m/s}^2 - 9.80 \text{ m/s}^2$$

$$\Delta = -0.1 \text{ m/s}^2 \longrightarrow \sigma_x = 0.07$$

Δ is 1.4 times higher than σ_x .

The difference, " Δ " is more than 1.0 times greater than σ_x so this is not an acceptable measurement.

not acceptable measurement

4.18

$$\sigma_{\mu} = 10 \text{ m/s}$$

a.) uncertainty of $\pm 3 \text{ m/s}$?

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \quad \begin{array}{l} \sigma_x = 10 \text{ m/s} \\ \sigma_{\bar{x}} = \pm 3 \text{ m/s} \end{array}$$

$$\sqrt{N} = \frac{\sigma_x}{\sigma_{\bar{x}}} : N = \left(\frac{\sigma_x}{\sigma_{\bar{x}}} \right)^2 = \left(\frac{10 \text{ m/s}}{3 \text{ m/s}} \right)^2 = \left(\frac{100}{9} \right) \approx 11$$

You would need 11 measurements

b.) uncertainty of $\pm 0.5 \text{ m/s}$?

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \quad \begin{array}{l} \sigma_x = 10 \text{ m/s} \\ \sigma_{\bar{x}} = 0.5 \text{ m/s} \end{array}$$

$$\sqrt{N} = \frac{\sigma_x}{\sigma_{\bar{x}}} : N = \left(\frac{\sigma_x}{\sigma_{\bar{x}}} \right)^2 = \left(\frac{10 \text{ m/s}}{0.5 \text{ m/s}} \right)^2 = \left(\frac{100}{0.25} \right) = 400$$

You would need 400 measurements

4.20

Mass m (kg)	0.513	0.581	0.634	0.691	0.762	0.834	0.901	0.950
Period T (s)	1.24	1.33	1.36	1.44	1.50	1.59	1.65	1.69
$k = 4\pi^2 m / T^2$	13.18	12.98	13.54	13.17	13.21	13.03	13.08	13.14
d_i	0.01	-0.19	0.37	0	0.04	-0.14	-0.09	-0.03
d_i^2	0.0001	0.0361	0.1369	0	0.0016	0.0196	0.0081	0.0009

$$\sum d_i^2 = 0.2033 \quad \bar{k} = 13.17 \text{ N/m}$$

Method used:

$$\sigma_k = \sqrt{\frac{d_i^2}{N-1}} = \sqrt{\frac{0.2033}{7}} = 0.17$$

$$\bar{\sigma}_k = \frac{\sigma_k}{\sqrt{N}} = \frac{0.17}{\sqrt{8}} = 0.06$$

\therefore

$$k = 13.17 \pm 0.06 \text{ N/m}$$

4.24

a.)

Distance (m)	15.43	17.37	19.62	21.68
Time (s)	1.804	1.915	2.043	2.149
$g (m/s^2)$	9.483	9.473	9.401	9.389
d_i	0.047	0.037	-0.035	-0.047
d_i^2	0.002209	0.001369	0.001225	0.002209

$$\bar{g} = 9.436 m/s^2 \text{ accepted: } g = 9.80 m/s^2$$

b.)

$$\sigma_x^2 = \frac{1}{N-1} \sum d_i^2 = \frac{1}{3} (0.005019) \quad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} = \frac{0.040903}{\sqrt{4}} = 0.020451$$

$$\sigma_x = \sqrt{\frac{0.005019}{3}} = 0.040903$$

$$\bar{g} = 9.44 m/s^2$$

From the data, we can say the best estimate for g with its uncertainty is,

$$g = 9.44 \pm 0.02 m/s^2$$

This value is not consistent with the accepted value of $9.80 m/s^2$ because the range from the experiment is,

$$g = 9.42 m/s^2 \text{ or } g = 9.46 m/s^2.$$

Therefore the measured value is not acceptable.

c.) Why air resistance is a believable reason for this experiments discrepancies.

- Since air resistance is being neglected, the time that is being recorded is theoretically shorter than the time if air resistance is accounted for.
- There also might be errors in the electric timer, giving faulty times that are not accurate.
- There may be an error in the measurement of distance. This can come from reading a measurement from the meter measuring device. (i.e.) The measurement could be off from physical defects in the measuring device.

d.) One way to improve this experiment is to modify the timing mechanism. If instead of a Stopwatch, an apparatus that would start timing the instant the ball were to be dropped and stopped immediately once it made contact with the ground, this would improve the timing. If this timing apparatus were to be implemented on a shorter distance, say 5m, the experiment could also be run more to up the number of total trials while also giving more accurate measurements.

Problem 7

a.) For one column values,

Mean: 0.573

SD: 0.279

SDOM: 0.062

i.) The standard deviation (SD) is found to be about 4.5 times greater than the standard deviation of mean (SDOM).

$$SD \simeq 4.5(SDOM)$$

The true mean ($\bar{\mu}$) is found to be 9.2 times greater than the (SDOM).

$$\bar{\mu} = 9.2(SDOM)$$

ii.)

$$\bar{\mu} \simeq 2(SD) : 0.573 \simeq 2(0.279)$$

Proof: $2(0.279) = 0.558$

$$0.558 + (SDOM) = 0.558 + 0.062 = 0.62$$

With the (SDOM) we can verify that the true mean lies within the range given by the (SD).

Yes it does

└ within range of true mean

b.)

Mean of All: $\bar{\mu} = 0.498$

SD of All: $SD = 0.279$

SDOM of All: $SDOM = 0.062$

i.) The averages amongst the twenty data sets do fluctuate throughout the 15 individual data sets.

ii.) The (SD's) all fluctuate throughout the individual data sets. The lowest (SD) of these data sets is; 0.248. The highest value is; 0.370. Where the average SD is 0.279.

iii.) Compared to the (SDOM), the individual (SD)'s of the data sets are roughly 4.5 times greater than the (SDOM)'s. The averages of all the data sets better represent the fluctuations of the individual fluctuations.

c.) Mean of 300 data points: $\bar{x} = 0.505$

(SD) of 300 data points: $SD = 0.291$

(SDOM) of 300 data points: $SDOM = 0.016$

i.) There is about a 0.02 discrepancy in the (SD) from a data set of 300 compared to the averages of (SD)'s for the 15 individual sets.

d.)

i.) The (SDOM) better reflects the uncertainty in a sample average because it represents the uncertainty in how the mean is different within a population.