Ch.1 The Wave Function

$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$: $\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = 1$: $\langle j\rangle = \frac{\sum jN(j)}{N} = \sum_{j=0}^{\infty} jP(j)$: $\langle j^2\rangle = \sum_{j=0}^{\infty} j^2P(j)$

$\langle f(j)\rangle = \sum_{j=0}^{\infty} f(j)P(j)$: $\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$: $\sigma^2 = \langle(\Delta x)^2\rangle$: $P_{ab} = \int_a^b \rho(x)\,dx$

$\langle x\rangle = \int_{-\infty}^{\infty} x|\Psi(x,t)|^2dx$: $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\Psi(x,t)|^2dx$: $\langle p\rangle = m\frac{d\langle x\rangle}{dt} = -i\hbar\int\left(\Psi^*\frac{\partial\Psi}{\partial x}\right)dx$

$\langle Q(x,p)\rangle = \int\Psi^*Q\left(x,\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\,dx$: $\langle p^2\rangle = \int\Psi^*(-\hbar^2\,\partial^2/\partial x^2)\Psi\,dx$: $\langle T\rangle = \langle p^2/2m\rangle$

$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda}$: $\sigma_x\sigma_p \ge \frac{\hbar}{2}$

Ch.2 Time Independent Schrödinger Equation

$\Psi(x,t) = \psi(x)\phi(t)$: $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$: $H(x,p) = \frac{p^2}{2m} + V(x)$

$\hat H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$: $\hat H\psi = E\psi$: $\langle H\rangle = E$: $\langle H^2\rangle = E^2$: $\Psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)e^{-iE_nt/\hbar}$

∞ Square well $V(x) = \begin{cases}0, & \text{if } 0\le x\le a\\ \infty, & \text{otherwise}\end{cases}$, $E_n = \frac{\hbar^2k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$

$\int\psi_m(x)^*\psi_n(x)\,dx = \delta_{mn}$: $\delta_{mn} = \begin{cases}0, & \text{if } m\ne n\\ 1, & \text{if } m=n\end{cases}$: $c_n = \int\psi_n(x)^*f(x)\,dx \to f(x) = \Psi(x,0)$

$P_n = |c_n|^2$: $\langle H\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$: H.O. $V(x) = \frac{1}{2}m\omega^2x^2$: $a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$: $[A,B] = AB - BA$

$[x,p] = i\hbar$: $\psi_0(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\,e^{-\frac{m\omega}{2\hbar}x^2}$: $\psi_n(x) = A_n(a_+)^n\psi_0(x)$: $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$: $a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$,

$a_-\psi_n = \sqrt{n}\,\psi_{n-1}$, $a_-\psi_0 = 0$: $a_+a_-\psi_n = n\psi_n$ $\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0$: $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$: $p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$

$(a_+)^2\psi_n \propto \psi_{n+2}$: $(a_-)^2\psi_n \propto \psi_{n-2}$: $a_-a_+\psi_n = (n+1)\psi_n$: $\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\phi(k)e^{i(kx - \frac{\hbar k^2}{2m}t)}dk$

$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Psi(x,0)e^{-ikx}dx$: $\int_b^c f(x)\delta(x-a)\,dx = f(a) \sim a\in[b,c]$: $R = \frac{1}{1+(2\hbar^2E/m\alpha^2)}$, $T = \frac{1}{1+(m\alpha^2/2\hbar^2E)}$

$R = \frac{|B|^2}{|A|^2}$, $T = \frac{|F|^2}{|A|^2}$, $R+T=1$ $V(x) = \begin{cases}-V_0, & \text{for } -a<x<a\\ 0, & \text{for } |x|>a\end{cases}$: $\psi(x) = Ce^{-ilx} + De^{ilx}$, for $-a<x<a$

: $\psi(x) = Ae^{-\kappa x} + Be^{\kappa x}$, for $x<-a$

$l = \sqrt{2m(E+V_0)}/\hbar$: $\psi(x) = Ee^{-\kappa x} + Fe^{\kappa x}$, for $x>a$

$\kappa = \sqrt{-2mE}/\hbar$

Ch. 3 Formalism

$|\alpha\rangle \to a = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$, $\langle\alpha| \to a^* = (a_1^*\,a_2^*\,a_3^*)$, wave functions live in Hilbert Space, $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$

$\langle\alpha|\alpha\rangle = \int_a^b|f(x)|^2dx$, $\langle f_m|f_n\rangle = \delta_{mn}$: Finite inner product → In Hilbert Space

$\langle Q\rangle = \int\Psi^*\hat Q\Psi\,dx = \langle\Psi|\hat Q\Psi\rangle$: $Q(x,p)$: $\langle f|\hat Q g\rangle = \langle\hat Q f|g\rangle$:

Observables are represented by hermitian operators : Operators in QM represent physically observable quantities

Determinate States : Every measurement of $\hat Q$ returns the same value q

: Determinate states are eigenfunctions of $\hat Q$

$|c_n|^2$ where $c_n = \langle f_n|\Psi\rangle$: $|c(z)|^2dz = \langle f_z|\Psi\rangle$: $c_n = \langle f_n|\Psi\rangle = \int f_n(x)^*\Psi(x,t)\,dx$: $\langle\hat Q\rangle = \sum_n q_n|c_n|^2$

$\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty}e^{-ipx/\hbar}\cdot\Psi(x,t)\,dx$: $\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty}e^{ipx/\hbar}\Phi(p,t)\,dp$

$\Psi(x,t) = \frac{\sqrt{m\alpha}}{\hbar}e^{-m\alpha|x|/\hbar^2}e^{-iEt/\hbar}$: $E = -\frac{m\alpha^2}{2\hbar^2}$: $\langle\hat A\rangle = \langle\mathcal{S}(0)|\hat A|\mathcal{S}(0)\rangle$: $P = |\langle\lambda_n|\mathcal{S}(t)\rangle|^2$

$|\mathcal{S}(t)\rangle = \varphi(t)\cdot|\mathcal{S}(0)\rangle$: $|\mathcal{S}(0)\rangle = |\lambda_n\rangle$: $\varphi(t) = e^{\frac{-iE_nt}{\hbar}}$

$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$: $\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$: $[\hat{x},\hat{p}] = i\hbar$: $\Delta t \Delta E \geq \frac{\hbar}{2}$: $\frac{d}{dt}\langle Q\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$

$\Psi(x,t) = \langle x|\mathcal{S}(t)\rangle$: $\varphi(p,t) = \langle p|\mathcal{S}(t)\rangle$: $C_n(t) = \langle n|\mathcal{S}(t)\rangle$: $\langle e_m|\hat{Q}|e_n\rangle = Q_{mn}$: $i\hbar\frac{d}{dt}|\mathcal{S}\rangle = H|\mathcal{S}\rangle$

$H|\mathcal{S}\rangle = E|\mathcal{S}\rangle$: $\hat{P} = |\alpha\rangle\langle\alpha|$: $\hat{P}|\beta\rangle = \langle\alpha|\beta\rangle|\alpha\rangle$: $\langle e_m|e_n\rangle = \delta_{mn}$

Discrete Values :

$C_n = \langle\Psi_n|\Psi(x,0)\rangle$

$\Psi(x,t) = \sum C_n e^{-iE_nt/\hbar}$

$\langle\Psi_m|\Psi_n\rangle = \delta_{mn}$

Continuous:

$\varphi(k) = \langle F_{p'}|\Psi(x,t)\rangle$

$\Psi(x,t) = \langle\varphi(k)|F_p e^{-iE_nt/\hbar}\rangle$

$\langle F_{p'}|F_p\rangle = \delta(p-p')$

$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$: $\hat{H}\Psi = \hat{E}\Psi$

$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$: $\hat{E} = i\hbar\frac{\partial}{\partial t}$: $\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x}$: $\hat{T} = \frac{\hat{P}^2}{2m}$

$\int_0^\infty x^n e^{-x/a}dx = n!a^{n+1}$: $\int_0^\infty x^{2n} e^{-x^2/a^2}dx = \sqrt{\pi}\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$: $\int_0^\infty x^{2n+1}e^{-x^2/a^2}dx = \frac{n!}{2}a^{2n+2}$

$[x^2+y, z] = [x^2,z] + [y,z]$: $[x^2,z] = x[x,z] + [x,z]x$

Ch. 4 Quantum Mechanics in Three Dimensions

$p_x \to \frac{\hbar}{i}\frac{\partial}{\partial x}$, $p_y \to \frac{\hbar}{i}\frac{\partial}{\partial y}$, $p_z \to \frac{\hbar}{i}\frac{\partial}{\partial z}$: $p \to \frac{\hbar}{i}\nabla$: $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$: $-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$

$\Psi_n(r,t) = \Psi_n(r)e^{-iE_nt/\hbar}$: $\Psi(r,t) = \sum C_n\Psi_n(r)e^{-iE_nt/\hbar}$: $P_l^m(x) \equiv (1-x^2)^{|m|/2}\left(d/dx\right)^{|m|}P_l(x)$

$P_l(x) \equiv \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l(x^2-1)^l$: $Y_l^m(\theta,\varphi) = \epsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}e^{im\varphi}P_l^m(\cos\theta)$: $j_l(x) = (-x)^l\left(\frac{1}{x}\frac{d}{dx}\right)\frac{\sin x}{x}$

$n_l(x) = -(-x)^l\left(\frac{1}{x}\frac{d}{dx}\right)^l\frac{\cos x}{x}$: $E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}$: $E_1 = -13.6\text{ eV}$: $\Psi_{100}(r,\theta,\varphi) = \frac{e^{-r/a}}{\sqrt{\pi a^3}}$

$\Psi_{nlm} = \sqrt{\left(\frac{2}{na}\right)^3\frac{(n-l-1)!}{2n[(n+l)!]^3}}e^{-r/na}\left(\frac{2r}{na}\right)^l\left[L_{n-l-1}^{2l+1}(2r/na)\right]Y_l^m(\theta,\varphi)$: $\Delta E = E_1\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$: $R = 1.097\times10^7\text{ m}^{-1}$: $P = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}(f(r))^2r^2\sin^2\theta\, dr\,d\theta\,d\varphi$: $L = r\times p$

$L_x = yp_z - zp_y$, $L_y = zp_x - xp_z$, $L_z = xp_y - yp_x$: $[L_x,L_y] = i\hbar L_z$, $[L_y,L_z] = i\hbar L_x$, $[L_z,L_x] = i\hbar L_y$

$[L^2,L] = 0$: $L_\pm = L_x \pm iL_y$: $L^2 f_l^m = \hbar^2 l(l+1)f_l^m$, $L_z f_l^m = \hbar m f_l^m$: $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}$

$L_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi}$: $L_\pm = \pm\hbar e^{\pm i\varphi}(\partial/\partial\theta \pm i\cot\theta\,\partial/\partial\varphi)$: $L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$

$[S_x,S_y] = i\hbar S_z$, $[S_y,S_z] = i\hbar S_x$, $[S_z,S_x] = i\hbar S_y$: $\chi = \begin{pmatrix}a\\b\end{pmatrix} = a\chi_+ + b\chi_-$

$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$: $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$: $S^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1&0\\0&1\end{pmatrix}$: $S_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$: $S_y = \frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$: $S_z = \frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}, \lambda = \frac{\hbar}{2}$: $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}, \lambda = -\frac{\hbar}{2}$: $\chi_+^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \lambda = \frac{\hbar}{2}$: $\chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}, \lambda = -\frac{\hbar}{2}$

$\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}, \lambda = \frac{\hbar}{2}$: $\chi_-^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}, \lambda = -\frac{\hbar}{2}$: $\chi_+^{(z)} = \begin{pmatrix}1\\0\end{pmatrix}, \lambda = \frac{\hbar}{2}$: $\chi_-^{(z)} = \begin{pmatrix}0\\1\end{pmatrix}, \lambda = -\frac{\hbar}{2}$

$\chi = \left(\frac{a+b}{\sqrt{2}}\right)\chi_+^{(i)} + \left(\frac{a-b}{\sqrt{2}}\right)\chi_-^{(i)}$: $\langle S\rangle = \chi^* S\chi$: $H = -\mu\cdot B = -\gamma B\cdot S$: $\chi(t) = \begin{pmatrix}\cos(\alpha/2)e^{i\gamma B_0t/2}\\ \sin(\alpha/2)e^{-i\gamma B_0t/2}\end{pmatrix}$

$P_\pm^{(i)} = |C_\pm^{(i)}|^2$: $C_\pm^{(i)} = \chi_\pm^{(i)*}\chi_\pm$: $i\hbar\frac{\partial\chi}{\partial t} = H\chi$