Blackbody Radiation

Spectral Emittance

$$M(T) = \int_{\nu_1}^{\nu_2} M_\nu(T)\, d\nu$$

$$I_\nu \Leftrightarrow M_\lambda(T) = \frac{c}{\lambda^2} M_\nu(T) \quad (\nu = c/\lambda)$$

$$M_\nu(T) = \frac{c}{4} \underbrace{u_\nu(T)}_{\text{Energy Density}}$$

$$u_\nu(T) = \frac{N_\nu}{V} \langle \varepsilon \rangle$$

# of modes per ν

Averge energy per mode

Classical
$\langle \varepsilon \rangle = k_B T$

Planck's Result
$$\langle \varepsilon \rangle = \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

because $\varepsilon_n = nh\nu$

Putting Together

$$M_\lambda(T) = \frac{c}{\lambda^2} M_\nu(T)$$

$$= \frac{c}{\lambda^2} \frac{c}{4} u_\nu(T)$$

$$= \frac{c^2}{4\lambda^2} \cdot \frac{N\nu}{V} \langle \varepsilon \rangle$$

$$= \frac{c^2}{4\lambda^2} \frac{8\pi\nu^2 V}{c^3 V} \langle \varepsilon \rangle$$

$$\Rightarrow = \frac{2\pi (c/\lambda)^2}{c^2} \cdot \frac{hc}{e^{hc/\lambda k_B T} - 1} \frac{1}{\lambda}$$

$$= \frac{2\pi hc}{\lambda^3 (e^{hc/\lambda k_B T} - 1)}$$

$$M_\lambda = \frac{c}{\lambda^2} M_\nu = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} = I_\lambda(T)$$

$$I = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \Rightarrow x = \frac{hc}{\lambda k_B T} \Rightarrow I = \frac{2\pi (k_B T)^5}{h^4 c^3} g(x)$$

$$g(x) = \frac{x^5}{e^x - 1} \; : \; g'(x) = \left(\frac{x^4}{e^x - 1}\right)\left(5 - \frac{x}{1 - e^{-x}}\right) = 0 \Rightarrow 5 - \frac{x}{1 - e^{-x}} = 0$$

$$\Rightarrow 1 - e^{-x} = 5/x \longrightarrow x = 4.965$$

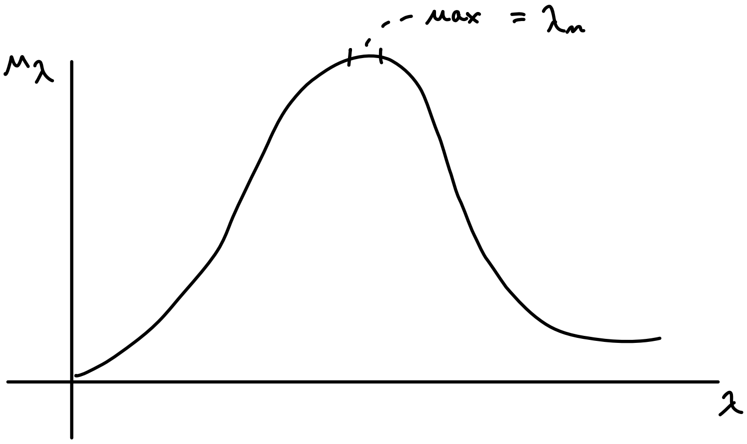
1) Trial and error
2) Graphical method
3) Numerical Solutions

$$x_{max} = \frac{hc}{\lambda_n k_B T}$$

$$\lambda_n T = \frac{hc}{k_B x_m} = 2.9 \times 10^{-3}\ \text{K} \cdot \text{m}$$

Example:

Blackbody radiation in a cavity @ $T = 500\,K$ Passes through a Filter w/ A 2 nm $\Delta\lambda$ (Bandwidth) Centered on λ_m. IF filter is placed in a 1 cm radius aperture, what is the total Transmitted power?



$$M(T) = \int_{\lambda_1}^{\lambda_2} M_\lambda \, d\lambda = M_{\lambda_m} \Delta\lambda$$

units : W/m^2

$\boxed{h = 6.63 \times 10^{-34}\ J \cdot s}$

$$P = \pi r^2 M_\lambda \Delta\lambda$$

$$= \pi r^2 \frac{2\pi h c^2}{\lambda_m^5} \frac{\Delta\lambda}{e^{x_m} - 1}$$

$$= 25.3\ W$$