4.2 HW Real Characteristic Roots

4.2 #'s 18, 20, 30

4.2.18 $y'' - 9y = 0 \quad y(0) = -1, \; y'(0) = 0$

$\Delta = b^2 - 4ac : 0^2 - 4(1)(-9)$

$a = 1 \quad \Delta = 36$

$b = 0 \quad \Delta > 0 \; \therefore \; r_1 = \frac{-b + \sqrt{\Delta}}{2a}, \; r_2 = \frac{-b - \sqrt{\Delta}}{2a}, \; y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$c = -9$

$r_1 = \frac{0 + \sqrt{36}}{2(1)} = 3 \qquad r_2 = \frac{-\sqrt{36}}{2(1)} = -3$

$r_1 = 3$

$r_2 = -3$

$$\boxed{y(t) = -\tfrac{1}{2} e^{3t} - \tfrac{1}{2} e^{-3t}}$$

$y(t) = c_1 e^{3t} + c_2 e^{-3t}$

$-1 = c_1 e^0 + c_2 e^0 \; : \; y(0) = -1$

$-1 = c_1 + c_2$

$y'(t) = 3c_1 e^{3t} - 3c_2 e^{-3t}$

$0 = 3c_1 e^0 - 3c_2 e^0 \; : \; y'(0) = 0$

$0 = 3c_1 - 3c_2$

$3c_2 = 3c_1$

$c_2 = c_1$

$c_1 + c_2 = -1 \qquad x + x = -1$

$c_1 = c_2 \qquad 2x = -1$

$x = -\tfrac{1}{2}$

$c_1 = -\tfrac{1}{2}$

$c_2 = -\tfrac{1}{2}$

4.2.20 $y'' + y' - 6y = 0 \quad y(0) = 1, \; y'(0) = 1$

$\Delta = 25 \; \therefore \; r = \frac{-b \pm \sqrt{\Delta}}{2a} \quad y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$\Delta = b^2 - 4ac$

$A = 1 \quad 1^2 - 4(1)(-6)$

$B = 1 \quad 1 - 4(-6)$

$C = -6 \quad 1 - (-24)$

25

$r_1 = \frac{-1 + \sqrt{25}}{2} = \frac{-1+5}{2}, \quad r_1 = 2$

$r_2 = \frac{-1 - \sqrt{25}}{2} = \frac{-1-5}{2}, \quad r_2 = -3$

$r_1 = 2, \; r_2 = -3$

$c_1 + c_2 = 1$

$2c_1 - 3c_2 = 1$

$A = \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -3 & 1 \end{array}\right] \quad c_1 = 4/5, \; c_2 = 1/5$

$\text{rref}(A) = \left[\begin{array}{cc|c} 1 & 0 & 4/5 \\ 0 & 1 & 1/5 \end{array}\right]$

$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$y(t) = c_1 e^{2t} + c_2 e^{-3t}$

$1 = c_1 e^0 + c_2 e^0 \; : \; y(0) = 1$

$1 = c_1 + c_2$

$y'(t) = 2c_1 e^{2t} - 3c_2 e^{-3t}$

$1 = 2c_1 e^0 - 3c_2 e^0$

$1 = 2c_1 - 3c_2$

$$\boxed{y(t) = 4/5\, e^{2t} + 1/5\, e^{-3t}}$$

4.2.30 $y''' - 10y'' - 15y' = 0 \quad \{te^{-5t}, e^{5t}, 2e^{5t} - 1\}$

$$W = \begin{vmatrix} te^{-5t} & e^{5t} & 2e^{5t} - 1 \\ (1-5t)e^{-5t} & 5e^{5t} & 10e^{5t} \\ (25t-10)e^{-5t} & 25e^{5t} & 50e^{5t} \end{vmatrix}$$

$(10x - 3) = 0$

$10x = 3$

$x = 3/10$

$W \neq 0$ everywhere except $t = \frac{3}{10}$

The set is linearly independent thus creating a basis.

$|W| = 25(10x - 3) \neq 0$

linealy independent