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PHYS 362 HW 9

Problem 1

a.) $V_i \rightarrow V_f$: $dE = TdS - PdV = 0$ ⟶ Temperature is constant during free expansion

$TdS - PdV = 0$: $TdS = PdV$: $\Delta S = \frac{1}{T}\int_{V_i}^{V_f} P\,dV$

$$\Delta S = \int_{V_i}^{V_f} \frac{1}{T}\frac{NkT}{V}\,dV = Nk\int_{V_i}^{V_f}\frac{1}{V}\,dV = Nk\ln(V)\Big|_{V_i}^{V_f} = Nk\left(\ln(V_f) - \ln(V_i)\right)$$

$$\boxed{\Delta S = Nk\ln\left(\frac{V_f}{V_i}\right)}$$

b.) The change in entropy does not depend on whether or not the gas is monoatomic or diatomic since the change in energy during a free expansion for an ideal gas is 0.

Problem 2

$$S = S(E,V,N) = \frac{3}{2}Nk\ln\left(E + \frac{N^2a}{V}\right) + Nk\ln(V-Nb) + f(N)$$

a.) $dE = TdS - PdV + \mu dN$

$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}$$

$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{3}{2}Nk\left(\frac{\frac{1}{1}}{E + \frac{N^2a}{V}}\right) = \frac{3}{2}Nk\left(\frac{1}{\frac{EV+N^2a}{V}}\right) = \frac{3}{2}Nk\left(\frac{V}{EV+N^2a}\right)$$

$$\frac{1}{T} = \frac{3}{2}Nk\left(\frac{V}{EV+N^2a}\right) \quad \therefore \quad T = \frac{2}{3Nk}\left(\frac{EV+N^2a}{V}\right)$$

$$T\cdot\frac{3}{2}Nk = \left(\frac{EV+N^2a}{V}\right) \quad : \quad \frac{3}{2}NkT\cdot V = EV+N^2a \quad : \quad EV = \frac{3}{2}NkTV - N^2a$$

$$\boxed{E(T,V,N) = \frac{3}{2}NkT - \frac{N^2a}{V}}$$

b.) $$S = S(E,V,N) = \frac{3}{2}Nk\ln\left(\frac{3}{2}NkT - \cancel{\frac{N^2a}{V}} + \cancel{\frac{N^2a}{V}}\right) + Nk\ln(V-Nb) + f(N)$$

$$= \frac{3}{2}Nk\ln\left(\frac{3}{2}Nk\cdot T\right) + Nk\ln(V-Nb) + f(N)$$

$$= \frac{3}{2}Nk\left(\ln\left(\frac{3}{2}Nk\right) + \ln(T)\right) + Nk\ln(V-Nb) + f(N)$$

$$= \frac{3}{2}Nk\ln\left(\frac{3}{2}Nk\right) + \frac{3}{2}Nk\ln(T) + Nk\ln(V-Nb) + f(N)$$

$$g(N) = \frac{3}{2}Nk\ln\left(\frac{3}{2}Nk\right) + f(N)$$

$$= \frac{3}{2}Nk\ln(T) + Nk\ln(V-Nb) + g(N)$$

$$\boxed{S = S(T,V,N) = \frac{3}{2}Nk\ln(T) + Nk\ln(V-Nb) + g(N)}$$

c.) $$\left(\frac{\partial S}{\partial V}\right)_{E,N} = \frac{P}{T} \quad \therefore \quad P = T\left(\frac{\partial S}{\partial V}\right)_{E,N} = T\left(\frac{3}{2}Nk\left(\frac{1}{E+N^2a/V}\right)\left(-\frac{N^2a}{V^2}\right) + Nk\left(\frac{1}{V-Nb}\right)\right)$$

$$P = T\left(-\frac{N^2a}{V^2T} + \frac{Nk}{V-Nb}\right) = \left(\frac{NkT}{V-Nb} - \frac{N^2a}{V^2}\right) \quad : \quad P + \frac{N^2a}{V^2} = \frac{NkT}{V-Nb}$$

$$\boxed{\left(P + \frac{N^2a}{V^2}\right)(V-Nb) = NkT}$$

Problem 3

a.) $V_i \longrightarrow 3V_i$: $PV = NkT$ $\therefore$ $T = \frac{P_iV_i}{Nk}$: $T = \frac{3P_fV_i}{Nk}$

$$\frac{P_iV_i}{\cancel{Nk}} = \frac{3P_fV_i}{\cancel{Nk}} \quad \therefore \quad P_i\cancel{V_i} = 3P_f\cancel{V_i} \longrightarrow P_f = \frac{1}{3}P_i$$

$$\boxed{P_f = \frac{1}{3}P_i \quad \text{or} \quad P_i = 3P_f}$$

b.) $\Delta S = \frac{Q}{T}$: Since $\Delta T = 0$: $\cancel{dE}^{\,0} = \delta Q - PdV$ $\therefore$ $\delta Q = PdV$

$$W = \int_{V_i}^{3V_i} \frac{NkT}{V}\,dV = NkT \cdot \ln(V)\Big|_{V_i}^{3V_i} = NkT \cdot (\ln(3V_i) - \ln(V_i)) = NkT\ln(3V_i/V_i)$$

$W = NkT\ln(3)$: $\Delta S = \frac{Nk\cancel{T}\ln(3)}{\cancel{T}} = Nk\ln(3)$

$$\boxed{\Delta S = Nk\ln(3)}$$

This is consistent with what the heat flow would predict for this process

Problem 4

a.) $V_i \rightarrow 3V_i$: $PV^{\gamma} = \text{const}$ $\quad P_i V_i^{\gamma} = P_f V_f^{\gamma}$ $\quad \frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma} = \left(\frac{\cancel{V_i}}{3\cancel{V_i}}\right)^{\gamma} = \left(\frac{1}{3}\right)^{\gamma}$

$$\boxed{\frac{P_f}{P_i} = \left(\frac{1}{3}\right)^{\gamma}}$$

b.) $V_i \rightarrow 3V_i$: $TV^{\gamma-1} = \text{const}$ $\quad T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$ $\quad \frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{\cancel{V_i}}{3\cancel{V_i}}\right)^{\gamma-1} = \left(\frac{1}{3}\right)^{\gamma-1}$

$$\boxed{\frac{T_f}{T_i} = \left(\frac{1}{3}\right)^{\gamma-1}}$$

c.) $\Delta S = \frac{Q}{T}$ $\therefore$ $Q = 0$ $\quad S(T,V,N) = fNk\ln(T) + Nk\ln(V) + \tilde{h}(N)$

$$S_i(T_i,V_i,N) = fNk\ln(T_i) + Nk\ln(V_i) + \tilde{h}(N)$$
$$S_f(T_f,V_f,N) = fNk\ln(T_f) + Nk\ln(V_f) + \tilde{h}(N)$$

$$S_f(T_f,V_f,N) - S_i(T_i,V_i,N) = fNk\ln(T_f) + Nk\ln(V_f) + \cancel{\tilde{h}(N)} - fNk\ln(T_i) - Nk\ln(V_i) - \cancel{\tilde{h}(N)}$$

$$= fNk\ln(T_f) - fNk(T_i) + Nk\ln(V_f) - Nk\ln(V_i)$$

$$= fNk\ln\left(\frac{T_f}{T_i}\right) + Nk\ln\left(\frac{V_f}{V_i}\right) = fNk\ln\left(\frac{V_i}{V_f}\right)^{\gamma-1} + Nk\ln\left(\frac{V_f}{V_i}\right)$$

$$= f(\gamma-1)Nk\ln\left(\frac{V_i}{V_f}\right) + Nk\ln\left(\frac{V_f}{V_i}\right) = -f(\gamma-1)Nk\ln\left(\frac{V_f}{V_i}\right) + Nk\ln\left(\frac{V_f}{V_i}\right)$$

$\gamma = \frac{C_p}{C_v} = \frac{(f+1)\cancel{Nk}}{f\cancel{Nk}}$

$$= Nk\ln\left(\frac{V_f}{V_i}\right)\left(1 - f(\gamma-1)\right) = Nk\ln\left(\frac{V_f}{V_i}\right)\left(1 - f\left(\frac{(f+1)}{f} - 1\right)\right)$$

$$= Nk\ln\left(\frac{V_f}{V_i}\right)\left(1 - \cancel{f}\,\frac{(f+1) - f}{\cancel{f}}\right) = Nk\ln\left(\frac{V_f}{V_i}\right)\left(1 - ((f+1) - f)\right)$$

$$= Nk\ln\left(\frac{V_f}{V_i}\right)(\overset{0}{\cancel{1 - f - 1 + f}}) = 0$$

$$\boxed{\Delta S = 0}$$

Yes this is consistent, no heat enters.