MAT 202
Larson – Section 8.1
Basic Integration Rules: A Review

In chapter 8 we will focus on many different techniques for integration. In this section we will review what we've already covered.

Basic Integration Rules (You may use your cut-out & "cheat sheet"): *k* and *C* are constants.

1. $\int 0\,du = C$
2. $\int du = u + C$
3. $\int k\,du = ku + C$
4. $\int k \cdot f(u)\,du = k\int f(u)\,du$
5. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
6. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
7. $\int \dfrac{1}{u}\,du = \ln|u| + C$
8. $\int e^u\,du = e^u + C$
9. $\int a^u\,du = \left(\dfrac{1}{\ln a}\right)a^u + C$
10. $\int \sin u\,du = -\cos u + C$
11. $\int \cos u\,du = \sin u + C$
12. $\int \tan u\,du = -\ln|\cos u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\int \csc u \, du = -\ln\left|\csc u + \cot u\right| + C$

16. $\int \sec^2 u \, du = \tan u + C$

17. $\int \csc^2 u \, du = -\cot u + C$

18. $\int \sec u \tan u \, du = \sec u + C$

19. $\int \csc u \cot u \, du = -\csc u + C$

20. $\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$

21. $\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$

22. $\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} arc\sec \frac{|u|}{a} + C$

23. $\int \cosh u \; du = \sinh u + C$

24. $\int \sinh u \; du = \cosh u + C$

25. $\int \operatorname{sech}^2 u \; du = \tanh u + C$

26. $\int \operatorname{csch}^2 u \; du = -\coth u + C$

27. $\int \operatorname{sech} u \tanh u \; du = -\operatorname{sech} u + C$

28. $\int \operatorname{csch} u \coth u \; du = -\operatorname{csch} u + C$

29. $\int \frac{1}{\sqrt{u^2 \pm a^2}} \, du = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

30. $\int \frac{1}{a^2 - u^2} \, du = \frac{1}{2a} \ln\left|\frac{a + u}{a - u}\right| + C$

31. $\int \frac{1}{u\sqrt{a^2 \pm u^2}} \, du = -\frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 \pm u^2}}{|u|}\right) + C$

Ex: For each, determine the integration technique that would best fit the problem. Do not actually integrate.

a) $\int (1 + e^x)^2 \, dx$

b) $\int \frac{1 + x}{x^2 + 1} \, dx$

c) $\int \frac{1}{\sqrt{2x - x^2}} \, dx$

d) $\int \frac{x^2}{x^2 + 1} \, dx$

e) $\int \frac{2x}{x^2 + 2x + 1} \, dx$

f) $\int \cot^2 x \, dx$

g) $\int \frac{1}{1 + \sin x} \, dx$

Ex: For each, determine the integration technique that would best fit the problem. Do not actually integrate.

a) $\int \left(1+e^x\right)^2 dx$ Expand & split the integrals

$$\int 1 + 2e^x + e^{2x}\, dx$$

b) $$\int \frac{1+x}{x^2+1}\,dx$$

Split up the fraction

$$\int \frac{1}{x^2+1}\,dx + \int \frac{x}{x^2+1}\,dx$$

c) $\int \frac{1}{\sqrt{2x - x^2}}\,dx$

Completing the square on the radicand

$$\int \frac{1}{\sqrt{a^2 - u^2}}\,du$$

d) $\int \frac{x^2}{x^2+1}dx$

Long division & split up

$$\int\left(1-\frac{1}{x^2+1}\right)dx$$

$$\begin{array}{r l}
 & \quad 1 \\
x^2+0x+1 \; \big) & \overline{\cancel{x^2}+\cancel{0x}+0} \\
 & -(\cancel{x^2}+\cancel{0x}+1) \\
\hline
 & \qquad -1
\end{array}$$

e) $\int \frac{2x}{x^2 + 2x + 1} dx$

Add/subtract a constant & split up

$$\int \frac{(2x+2) - 2}{x^2+2x+1} dx$$

$$= \int \frac{2x+2}{x^2+2x+1} dx - 2\int \frac{1}{(x+1)^2} dx$$

f) $\int \cot^2 x\,dx$ Use Trig. Identities $1+\cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$\int \csc^2 x - 1 \; dx$

$= \int \csc^2 x - \int 1\,dx$

g) $\int \frac{1}{1+\sin x}\,dx$

Multiply by conjugate to Create a monomial divisor

$$\int \frac{1}{(1+\sin x)}\frac{(1-\sin x)}{(1-\sin x)}\,dx = \int \frac{1-\sin x}{1-\sin^2 x}\,dx$$

$$= \int \frac{1-\sin x}{\cos^2 x}\,dx$$

$$= \int \frac{1}{\cos^2 x}\,dx - \int \frac{\sin x}{\cos^2 x}\,dx$$