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Ch. 1: Vector Analysis

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\hat{r} = \dfrac{\vec{r}}{r}$

$\vec{\mathscr{r}} \equiv \vec{r} - \vec{r}\,'$

$\hat{\mathscr{r}} = \dfrac{\vec{\mathscr{r}}}{|\vec{\mathscr{r}}|}$

$\vec{\nabla} T = (\hat{x})\dfrac{\partial}{\partial x} T + (\hat{y})\dfrac{\partial}{\partial y} T + (\hat{z})\dfrac{\partial}{\partial z} T$

$\vec{\nabla} \cdot \vec{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}$

$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$

$\vec{\nabla} \cdot (\vec{\nabla} T) = \nabla^2 T = \left[\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}\right] T$

$\vec{\nabla} \times (\vec{\nabla} T) = 0$

$\vec{\nabla}(\vec{\nabla} \cdot \vec{v})$

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{v}) = 0$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) - \nabla^2 v$

$\int_{\vec{a}}^{\vec{b}} \vec{v} \cdot d\vec{l} \qquad \int_{\vec{a}}^{\vec{b}} (\vec{\nabla} T) \cdot d\vec{l} = T(\vec{b}) - T(\vec{a})$

$\int v \cdot d\vec{a} \qquad \int_V (\vec{\nabla} \cdot \vec{v})\, d\tau = \oint_S \vec{v} \cdot d\vec{a}$

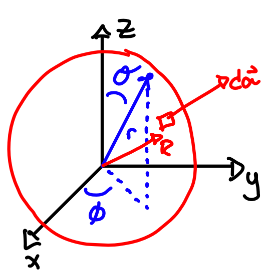
$\int T\, dt \qquad \int_S (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_C \vec{v} \cdot d\vec{l}$

Curvilinear Coordinates

Spherical polar Coordinates (r, θ, ϕ)

Cylindrical Coordinates (s, ϕ, z)

Spherical Coordinates (r, θ, ϕ)

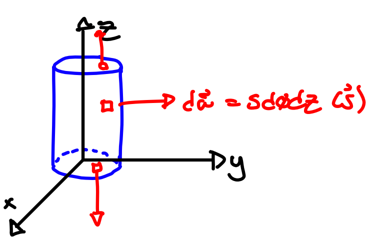


$d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$d\vec{a} = R^2 \sin\theta\, d\theta\, d\phi \ (\hat{r})$

$d\vec{l} = dr\,\hat{r} + r\sin\theta\, d\phi\,\hat{\phi} + r\, d\theta\,\hat{\theta}$

Cylindrical Coordinates (s, ϕ, z)



$d\vec{l} = ds\,\hat{s} + s\, d\phi\,\hat{\phi} + dz\,\hat{z}$

$d\tau = s\, ds\, d\phi\, dz$

$d\vec{a} = s\, d\phi\, dz\ (\hat{s}) \ \| \ s\, ds\, d\phi\ (\pm\hat{z})$

Theorem of Vector Fields

$$\vec{\nabla}\times\vec{F}=0 \Rightarrow \vec{F}=-\vec{\nabla}V$$
$$\vec{\nabla}\cdot\vec{F}=0 \Rightarrow \vec{F}=\vec{\nabla}\times\vec{A}$$

Ch. 2 Electrostatics

$$\vec{E}(\vec{r})=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i \quad \text{where } \vec{F}=Q\vec{E}$$

$$\vec{E}(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}\,')}{\mathscr{r}^2}\,d\tau'\cdot\hat{\mathscr{r}}$$

$$\Phi_E=\oint\vec{E}\cdot d\vec{a}=\frac{Q_{enc}}{\varepsilon_0}$$

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$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$

$$\oint\vec{E}\cdot d\vec{l}=0 \quad \text{or} \quad \vec{\nabla}\times\vec{E}=0$$

$$V(\vec{r})\equiv-\int_{\mathcal{O}}^{\vec{r}}\vec{E}\cdot d\vec{l}$$

$$\vec{E}=\vec{\nabla}\cdot V$$

$$\nabla^2V=-\frac{\rho}{\varepsilon_0}$$

$$\nabla^2V=0$$

$$V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i}$$

$$V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}\,')\,d\tau'}{\mathscr{r}}$$

$$\vec{E}_{Above}-\vec{E}_{Below}=\frac{\sigma}{\varepsilon_0}\hat{n}$$

$$W=Q\cdot V(\vec{r})$$

$$W=\frac{1}{2}\sum_{i=1}^{n}q_i V(\vec{r}_i)$$

$$W=\frac{1}{2}\int\rho V\,d\tau$$

$$W=\frac{1}{2}\varepsilon_0\int_{\text{All space}}E^2\,d\tau$$

Ch. 3 Laplaces Equation

$$\nabla^2 V = 0$$

In 1D......

$$\frac{d^2V}{dx^2} = 0 \quad \therefore \quad V(x) = mx + b \longrightarrow \text{Linear}$$

Two Feauters :

1. $V(x) = \frac{1}{2}\left[V(x+a) + V(x-a)\right]$

2. NO local maxima or minima. The extrema occur at the boundaries

In 2D....

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \longrightarrow \text{Assume a product Solution}$$

In 3D....

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \longrightarrow \text{Assume a product solution}$$

ch. 5 Magnetostatics

$$\vec{F} = Q\left[\vec{E} + (\vec{v} \times \vec{B})\right]$$

$$\vec{I} = \lambda \vec{v}$$

$$\vec{F}_{mag} = \int I (d\vec{l} \times \vec{B})$$

$$\vec{k} = \frac{d\vec{I}}{dl_\perp}$$

$$\vec{k} = \sigma \vec{v}$$

$$\vec{F}_{mag} = \int da (\vec{k} \times \vec{B})$$

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Ch. 5 Magnetostatics

$$\vec{J} = \frac{d\vec{I}}{da_\perp}$$

$$\vec{J} = \rho \vec{v}$$

$$\vec{F}_{mag} = \int d\tau (\vec{J} \times \vec{B})$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \vec{\mathscr{r}}}{\mathscr{r}^2}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{Enc}$$

$$I_{Enc} = \int \vec{J} \cdot d\vec{a}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 A = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\mathscr{r}} \, d\tau'$$

$$B^{\perp}_{Above} = B^{\perp}_{Below}$$

$$B^{\parallel}_{Above} - B^{\parallel}_{Below} = \mu_0 k$$

<u>Ch. 7</u> <u>Electrodynamics</u>

$$\vec{J} = \sigma \vec{E}$$

$$\rightarrow \Delta V = IR$$

$$\mathcal{E} = \int \vec{f} \cdot d\vec{l} = \oint \vec{E} \cdot d\vec{l}$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

where $\Phi = \int \vec{B} \cdot d\vec{a}$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \vec{J}_D$$

$$\vec{J}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$