Orthogonal Expansions

Function Expansion

f & g are functions on $[a,b]$

$g(t) = c_1 g_1(t) + c_2 g_2(t) + \dots + c_n g_n(t)$

$$c_K = \frac{\langle f, g_K \rangle}{\langle g_K, g_K \rangle} \quad K = 1, 2, \dots, N$$

g is the expansion of f in terms of $g_1, g_2, \dots, g_n$

$f(t) \cong c_1 g_1(t) + c_2 g_2(t) + \dots + c_n g_n(t)$, $t \in [a,b]$

Constants c_K are called the expansion coefficients

Box Function Expansion

box function expansion, $N=4$

$g(t) = c_1 B_1(t) + c_2 B_2(t) + c_3 B_3(t) + c_4 B_4(t)$

$$c_K = \frac{\langle f, B_K \rangle}{\langle B_K, B_K \rangle}$$

Legendre Expansion

f be a function on $[-1,1]$

Legendre Expansion,

$g_n(t) = c_0 P_0(t) + c_1 P_1(t) + \dots c_n P_n(t)$, $t \in [-1,1]$

Expansion Coefficients

$f(t) = g(t)$ for all $t \in [a,b]$

$f(t) = c_1 g_1(t) + c_2 g_2(t) + \dots + c_n g_n(t) \quad t \in [a,b]$

- Expansion Coefficients c_K

$$c_K = \frac{\langle f, g_K \rangle}{\langle g_K, g_K \rangle}, \quad K = 1, 2, \dots, n$$

Signal Transform

$c = [c_1, c_2, \dots, c_n]^T$

- c is known as the signal transform

Haar Wavelet Expansion $N=4$

Wavelet expansion f

$g(t) = c_0 B(t) + c_1 W(t) + c_2 W_0'(t) + c_3 W_1'(t)$, $\therefore t \in [0,1]$

$$c_0 = \frac{\langle f, B \rangle}{\langle B, B \rangle}, \quad c_1 = \frac{\langle f, W \rangle}{\langle W, W \rangle}, \quad c_2 = \frac{\langle f, W_0' \rangle}{\langle W_0', W_0' \rangle}, \quad c_3 = \frac{\langle f, W_1' \rangle}{}$$