5.1 Matrix Expansions and JPEG Compression

Matrix Expansion and Transform

$A \equiv M \times N$ matrix $\quad d_{mn}$ are expansion coefficients

$$A = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} d_{mn} G_{mn} \qquad d_{mn} = \frac{\langle A, G_{mn}\rangle}{\langle G_{mn}, G_{mn}\rangle}, \quad m = 0, 1, \ldots M-1, \quad n = 0, 1, \ldots N-1$$

$$D = \begin{bmatrix} d_{00} & d_{01} & d_{02} & \cdots & d_{0n} \\ d_{10} & d_{11} & d_{12} & \cdots & d_{1n} \\ d_{20} & d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{m0} & d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}$$

Fourier Matrices and Expansions

$$\langle G_{mn}, G_{mn}\rangle = MN, \quad 0 \le m \le M-1, \quad 0 \le n \le N-1$$

$$A = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} d_{mn} G_{mn}, \quad d_{mn} = \frac{\langle A, G_{mn}\rangle}{\langle G_{mn}, G_{mn}\rangle}$$

$$A = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \langle A, G_{mn}\rangle G_{mn}$$

The Two-Dimensional DFT and Inverse DFT

$$d_{mn} = \frac{\langle A, G_{mn}\rangle}{\langle G_{mn}, G_{mn}\rangle} = \frac{\langle A, G_{mn}\rangle}{MN} \quad 0 \le m \le M-1, \quad 0 \le n \le N-1$$

$$A = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} d_{mn} G_{mn}$$

$$A = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \langle A, G_{mn}\rangle G_{mn}$$

Matrix Form for Two-Dimensional DFT

$$D = F_M A F_N^T$$

$F_M \wedge F_N$ are DFT matrices