Taylor Larrechea
Dr. Collins
PHYS 362 HW 13

Problem 1

a.)

Possible V values	Die Combinations	Probabilities of V
2	(1,1)	1/36
3	(2,1),(1,2)	2/36
4	(2,2),(3,1),(1,3)	3/36
5	(2,3),(4,1),(3,2),(1,4)	4/36
6	(1,5),(2,4),(3,3),(5,1),(4,2)	5/36
7	(1,6),(2,5),(3,4),(6,1),(5,2),(4,3)	6/36
8	(2,6),(3,5),(4,4),(6,2),(5,3)	5/36
9	(3,6),(4,5),(6,3),(5,4)	4/36
10	(4,6),(5,5),(6,4)	3/36
11	(5,6),(6,5)	2/36
12	(6,6)	1/36

b.) $\bar{x} = \sum_{i=1}^{n} x_i \cdot P(i)$

$$\bar{x} = \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = 7$$

$$\boxed{\bar{x} = 7}$$

Single Die : $\bar{x} = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2}$: $\bar{x} = 3.5$

Yes this twice that of a single die.

c.) $\sigma = \sqrt{(\bar{x^2}) - (\bar{x})^2}$

$$\bar{x^2} = \sum_{i=1}^{n} x_i^2 \cdot P(i)$$

$$\bar{x^2} = \frac{4+18+48+100+180+294+320+324+300+242+144}{36} = \frac{329}{6}$$

$\sigma = \sqrt{329/6 - 49} = 2.42$ $\boxed{\sigma = 2.42}$

Single Die : $\bar{x^2} = \frac{1+4+9+16+25+36}{6} = \frac{91}{6}$: $\bar{x^2} = \frac{91}{6}$

$\sigma = \sqrt{91/6 - 49/4} = 1.71$, No , the σ for 2 die is not twice as much

Problem 2

a.) I guess that there will be 53 heads

b.) There were 51.5 occurrences of heads. The mean was also 51.5. This was slightly off my guess

c.) There were 685 Trials of this experiment. The most commonly occurring number of heads was 49.5. The average number of heads in this experiment was 50.0. This does not match my estimate any better.

d.)

i.) I predict 5100 heads

ii.) There were 5060.5 occurrences of heads.

This is more accurate regarding my prediction compared to 100 flips

e.) There were 200 trials this time. The average number of heads returned over all trials was 5,012. This method deviated farther from my prediction (I expected more deviation honestly). This however indicates a smaller range of possibilities than that of 100 flips.

f.) This animation indicates there is a higher accuracy of statistical precision when more trials are performed.

Problem 3

a.) Prob(stepping R) = $1/2$ $2^6 = 64$ Possibilities : $R = q$, $L = p$
Prob (stepping L) = $1/2$

Posibilities	Final Pos.	Prob.
LLLLRR	-2	p^4q^2
LLLRRL	-2	p^4q^2
LLRRLL	-2	p^4q^2
LRRLLL	-2	p^4q^2
RRLLLL	-2	p^4q^2
LLLRLR	-2	p^4q^2
LLRLLR	-2	p^4q^2
LRLLLR	-2	p^4q^2
RLLLLR	-2	p^4q^2
LLRLRL	-2	p^4q^2
LRLLRL	-2	p^4q^2
RLLLRL	-2	p^4q^2
LRLRLL	-2	p^4q^2
RLLRLL	-2	p^4q^2
RLRLLL	-2	p^4q^2

→ 15 occurences : $p^4q^2 = (\frac{1}{2})^4(\frac{1}{2})^2 = (\frac{1}{2})^6 = \frac{1}{64}$

$\boxed{\text{Prob (2 steps to left) = } 15/64}$

b.) Prob(stepping R) = $1/3$: $R = q$, $L = p$
Prob (stepping L) = $2/3$

Posibilities	Final Pos.	Prob.
LLLLRR	-2	p^4q^2
LLLRRL	-2	p^4q^2
LLRRLL	-2	p^4q^2
LRRLLL	-2	p^4q^2
RRLLLL	-2	p^4q^2
LLLRLR	-2	p^4q^2
LLRLLR	-2	p^4q^2
LRLLLR	-2	p^4q^2
RLLLLR	-2	p^4q^2
LLRLRL	-2	p^4q^2
LRLLRL	-2	p^4q^2
RLLLRL	-2	p^4q^2
LRLRLL	-2	p^4q^2
RLLRLL	-2	p^4q^2
RLRLLL	-2	p^4q^2

→ 15 occurences : $p^4q^2 = (\frac{2}{3})^4(\frac{1}{3})^2 = \frac{16}{729}$

$\boxed{\text{Prob (2 steps to left) = } \frac{80}{243}}$

Problem 4

$$P(n) = \binom{N}{n} p^n q^{N-n} \quad : \quad p+q=1$$

a.) $$(p+q)^N = \sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n}$$

$$\sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n} = (p+q)^N \rightarrow \sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n} = (1)^N \rightarrow \sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n} = 1$$

$$\sum_{n=0}^{N} \binom{N}{n} p^n q^{N-n} = \sum_{n=0}^{N} P(n) \quad \therefore \quad \boxed{\sum_{n=0}^{N} P(n) = 1}$$

b.) $$\overline{n^2} = N^2p^2 + Npq \quad : \quad \bar{n} = pN \quad \vdots \quad \sigma = \sqrt{(\overline{n^2}) - (\bar{n})^2} = \sqrt{\cancel{N^2p^2} + Npq - \cancel{N^2p^2}} = \sqrt{Npq}$$

$$\boxed{\sigma = \sqrt{Npq}}$$

c.) $$\frac{\sigma}{\bar{n}} = \frac{\sqrt{Npq}}{Np} = \frac{\sqrt{Np(1-p)}}{Np} = \sqrt{\frac{(1-p)}{Np}}$$

$$\boxed{\frac{\sigma}{\bar{n}} = \sqrt{\frac{(1-p)}{Np}} \,, \lim_{N \to \infty} \frac{\sigma}{\bar{n}} = 0}$$

Problem 5

a.) $P(n) = \binom{N}{n} p^n q^{N-n}$: $\binom{N}{n} = \dfrac{N!}{n!(N-n)!}$

$$P(0) = \binom{6}{0} p^0 q^{6-0} = \frac{6!}{0!(6-0)!} \cdot \left(\frac{1}{2}\right)^6 = \frac{\cancel{6!}}{\cancel{6!}}\left(\frac{1}{64}\right) = \left(\frac{1}{64}\right) \approx 0.016$$

$$P(1) = \binom{6}{1} p^1 q^{6-1} = \frac{6!}{1!(6-1)!} \cdot \left(\frac{1}{2}\right)^1 \cdot \left(\frac{1}{2}\right)^5 = \frac{6!}{5!}\left(\frac{1}{2}\right)^6 = \frac{6 \cdot \cancel{5!}}{\cancel{5!}} \cdot \frac{1}{64} = 6 \cdot \frac{1}{64} = \frac{3}{32} \approx 0.094$$

$$P(2) = \binom{6}{2} p^2 q^{6-2} = \frac{6!}{2!(6-2)!} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^4 = \frac{6!}{2! \cdot 4!}\left(\frac{1}{2}\right)^6 = \frac{6 \cdot 5 \cdot \cancel{4!}}{2! \cdot \cancel{4!}} \cdot \frac{1}{64} = 15 \cdot \frac{1}{64} = \frac{15}{64} \approx 0.234$$

$$P(3) = \binom{6}{3} p^3 q^{6-3} = \frac{6!}{3!(6-3)!} \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^3 = \frac{6!}{3!3!}\left(\frac{1}{2}\right)^6 = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}\,3!} \cdot \frac{1}{64} = 20 \cdot \frac{1}{64} = \frac{5}{16} \approx 0.313$$

$$P(4) = \binom{6}{4} p^4 q^{6-4} = \frac{6!}{4!(6-4)!} \cdot \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^2 = \frac{6!}{4! \cdot 2!}\left(\frac{1}{2}\right)^6 = \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!} \cdot 2!} \cdot \frac{1}{64} = 15 \cdot \frac{1}{64} = \frac{15}{64} \approx 0.234$$

$$P(5) = \binom{6}{5} p^5 q^{6-5} = \frac{6!}{5!(6-5)!} \cdot \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^1 = \frac{6!}{5!1!}\left(\frac{1}{2}\right)^6 = \frac{6 \cdot \cancel{5!}}{\cancel{5!}\,1!} \cdot \frac{1}{64} = 6 \cdot \frac{1}{64} = \frac{3}{32} \approx 0.094$$

$$P(6) = \binom{6}{6} p^6 q^0 = \frac{6!}{6!(6-6)!} \cdot \left(\frac{1}{2}\right)^6 = \frac{\cancel{6!}}{\cancel{6!}\,0!} \cdot \frac{1}{64} = \frac{1}{64} \approx 0.016$$

n	Prob(n)
0	0.016
1	0.094
2	0.234
3	0.313
4	0.234
5	0.094
6	0.016

$\bar{n} = \sum n(P(n)) = 0 \cdot 0.016 + 1 \cdot 0.094 + 2 \cdot 0.234 + 3 \cdot 0.313 + 4 \cdot 0.234 + 5 \cdot 0.094 + 6 \cdot 0.016$

$\bar{n} = 3.003$: mean $= 3.0$

$\overline{n^2} = 0^2 \cdot 0.016 + 1^2 \cdot 0.094 + 2^2 \cdot 0.234 + 3^2 \cdot 0.313 + 4^2 \cdot 0.234 + 5^2 \cdot 0.094 + 6^2 \cdot 0.016$

$\overline{n^2} = 10.817$ $\quad \sigma = \sqrt{(\overline{n^2}) - (\bar{n})^2} = \sqrt{10.817 - (3.003)^2} = 1.224$, $\bar{n} = 3.003$, $\sigma = 1.224$

b.) Max $P(n) = 0.246$, $n_1 = 3$, $n_2 = 7$ FWHM $= |7 - 3| = 4$

$\boxed{\text{FWHM} = 4}$

c.) $N = 50$: Max $P(N) = 0.113$, $n_1 = 21$, $n_2 = 29$ FWHM $= |29 - 21| = 8$
$N = 200$: Max $P(N) = 0.056$, $n_1 = 92$, $n_2 = 108$ FWHM $= |108 - 92| = 16$
$N = 1000$: Max $P(N) = 0.02525$, $n_1 = 481$, $n_2 = 519$ FWHM $= |519 - 481| = 38$

$\boxed{\text{The FWHM increases with more } N\text{, but not linearly}}$

d.) As N increases, $\bar{n}$ increases. As N increases, FWHM increases as well.

$\therefore$ $\boxed{\text{FWHM}/\bar{n} \text{ is decreasing}}$