4.1 Vector Expansions and the DFT

Inner Product

$x = [x_1, x_2, \ldots x_n]$ $\quad$ $y = [y_1, y_2, \ldots y_n]$

$$\langle x, y \rangle = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n = \sum_{k=1}^{n} x_k y_k$$

Two vectors are orthogonal if $\langle x, y \rangle = 0$

Two vectors are orthonormal if $\langle x, y \rangle = 1$

Matrix Addition

A & B are $M \times N$ matrices

$C = A + B$

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1N} \\ c_{21} & \cdots & c_{2N} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mn} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & \cdots & a_{2n}+b_{2n} \\ \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

Scalar Multiple

Let A be an $M \times N$ matrix, let c be a scalar

$$CA = \begin{bmatrix} ca_{11} & \cdots & ca_{1n} \\ ca_{21} & \cdots & ca_{2n} \\ \vdots & \ddots & \vdots \\ ca_{m1} & \cdots & ca_{mn} \end{bmatrix}$$

Matrix - Matrix Multiplication

Let A be a $m \times r$ matrix

Let B be a $r \times n$ matrix

$m \times n$ product is $C = AB$

$$C_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \ldots a_{ir} b_{rj}$$

Identity Matrix

The $n \times n$ identity matrix I_n is defined as

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Let A^{-1} denote the inverse of A, then...

$$A_n^{-1} \cdot A_n = I_n$$

Matrix-Vector Multiplication

Let A be a $m \times n$ matrix

Let x be a $n \times 1$ vector

$C = Ax$ can be computed by...

$$C = x_1 a_1 + x_2 a_2 + \ldots + x_n a_n$$

Matrix Inner Product

Let A and B be $m \times n$ matrices...

The inner product of A & B is...

$$\langle A, B \rangle = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} b_{ij}$$

A & B are orthogonal if $\langle A, B \rangle = 0$