3.3 The Inverse of a Matrix

Matrix Inverse

$$A \cdot A^{-1} = I$$
$$A^{-1} \cdot A = I$$

RREF Method

$M = [A|I]$ Set up as matrix nxn next to Identity all as one matrix

$$\text{RREF}(M) = [I|A^{-1}]$$

$$\text{RREF}\left(\begin{bmatrix} 3 & 2 & 1 & 0 \\ 1 & 8 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 4/11 & -1/11 \\ 0 & 1 & -1/22 & 3/22 \end{bmatrix}$$

Third order Differential Equation

$$y''' - 2y'' - y' + 2y = 0$$

$y(0) = 1$, $y'(0) = 2$ $y''(0) = 3$

$$y(t) = c_1 e^{2t} + c_2 e^{t} + c_3 e^{-t}$$

$$y(0) = c_1 e^0 + c_2 e^0 + c_3 e^{-0} = 1$$
$$c_1 + c_2 + c_3 = 1$$
$$y'(0) = 2c_1 e^{2t} + c_2 e^{t} - c_3 e^{-t} = 2$$
$$2c_1 + c_2 - c_3 = 2$$
$$y''(0) = 4c_1 e^{2t} + c_2 e^{t} + c_3 e^{-t} = 3$$
$$4c_1 + c_2 + c_3 = 3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 2 \\ 4 & 1 & 1 & 3 \end{array}\right]$$

$$\text{rref}\left(\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 1 \end{bmatrix}\right) \cdot \vec{b}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$$

Solutions to Linear Systems

$$A\vec{x} = \vec{b}$$

1) $m = [A|b]$, RREF(m)

2)
$$A\vec{x} = \vec{b}$$
$$A^{-1}A\vec{x} = A^{-1}\vec{b}$$
$$I\vec{x} = A^{-1}\vec{b}$$
$$\vec{x} = A^{-1}\vec{b}$$

Ex.
$$\vec{x} = A^{-1}\vec{b}$$
$$A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 1/7 & 4/7 \\ 1/7 & -3/7 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 + 32/7 \\ 7 - 24/7 \end{bmatrix} = \vec{x}$$

$$\vec{x} = \begin{bmatrix} 39/7 \\ -17/7 \end{bmatrix}$$

Inevertible Matrix

- A is invertible
- A^T is invertible
- A is row equivalent to I_n
- A has n pivot columns
- $A\vec{x} = \vec{b}$ has a unique solution for each $\vec{b}$
- $A\vec{x} = 0$ has only the solution $\vec{x} = \vec{0}$