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MATH 360
HW 5

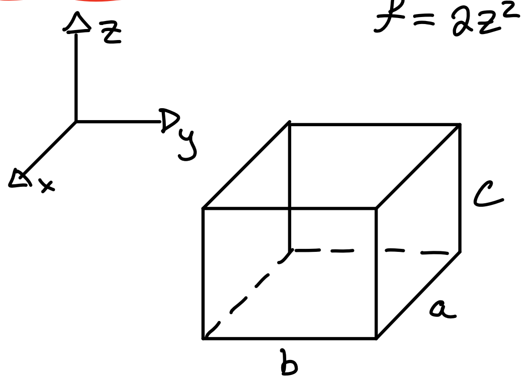
10.8 #1

Problem 10.8.1

$0 \le x \le a, \ 0 \le y \le b, \ 0 \le z \le c$

$f = 2z^2 - x^2 - y^2$

$\vec{\nabla} f = [-2x, -2y, 4z]$



$\int_v \nabla^2 f \, dv$

$\nabla^2 f \equiv \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$\frac{\partial^2 f}{\partial x^2} = -2, \quad \frac{\partial^2 f}{\partial y^2} = -2 \quad \frac{\partial^2 f}{\partial z^2} = 4$

$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = -2 - 2 + 4 = 0 \qquad \iiint \nabla^2 f \, dv = 0$

Verification $\quad \vec{\nabla} f \cdot \vec{n}$

$x = a$, $0 \le y \le b$, $0 \le z \le c$: $\vec{n} = [1,0,0]$, $\vec{\nabla} f \cdot \vec{n} = [-2x,-2y,4z] \cdot [1,0,0] = -2x = -2a$: $x = a$

$x = 0$, $0 \le y \le b$, $0 \le z \le c$: $\vec{n} = [-1,0,0]$, $\vec{\nabla} f \cdot \vec{n} = [-2x,-2y,4z] \cdot [-1,0,0] = 2x = 0$: $x = 0$

$y = b$, $0 \le x \le a$, $0 \le z \le c$: $\vec{n} = [0,1,0]$, $\vec{\nabla} f \cdot \vec{n} = [-2x,-2y,4z] \cdot [0,1,0] = -2y = -2b$: $y = b$

$y = 0$, $0 \le x \le a$, $0 \le z \le c$: $\vec{n} = [0,-1,0]$, $\vec{\nabla} f \cdot \vec{n} = [-2x,-2y,4z] \cdot [0,-1,0] = 2y = 0$: $y = 0$

$z = c$, $0 \le x \le a$, $0 \le y \le b$: $\vec{n} = [0,0,1]$, $\vec{\nabla} f \cdot \vec{n} = [-2x,-2y,4z] \cdot [0,0,1] = 4z = 4c$: $z = c$

$z = 0$, $0 \le x \le a$, $0 \le y \le b$: $\vec{n} = [0,0,-1]$, $\vec{\nabla} f \cdot \vec{n} = [-2x,-2y,4z] \cdot [0,0,-1] = -4z = 0$: $z = 0$

$$\iint_s \frac{\partial f}{\partial n} dA = \int_0^c \int_0^b -2a \, dy dz + \int_0^c \int_0^a -2b \, dx dz + \int_0^b \int_0^a 4c \, dx dy$$

$$= -2abc - 2abc + 4abc = -4abc + 4abc = 0$$

$$\boxed{\int_v \nabla^2 f \, dv = \int_s \frac{\partial f}{\partial n} dA = 0 \checkmark}$$