Lecture 31

Tues: Warm Up 12

Weds: Review III

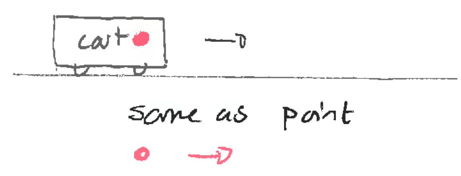
Fri: Exam III Ch 9, 10, 11

Review 2014, 2015 Exam III

Rotational Motion

We have mostly used classical mechanics to address translational motion of pointlike objects. In some cases we have reduced motion of extended objects to that of a pointlike object.

For these cases all points on the extended object moved in the same way.

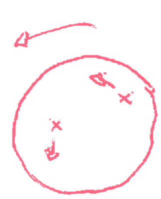


We need to extend the rules of classical physics to describe motion of extended objects where various points on the object move differently at the same time.

Demo: Rotate meter stick

endpoints move differently

Spin ball



points on ball move in different directions.

We will separate out:

1) bulk translational motion

2) rotational motion

We will apply Newton's laws to describe rotational motion in terms of new quantities. Selected applications are:

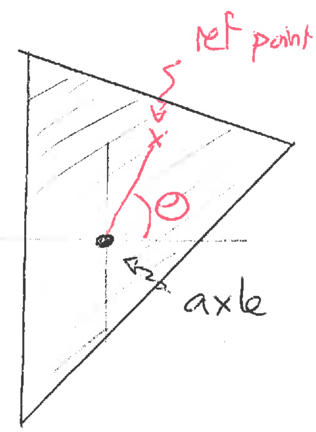
1) rotating wheels, disks, ...
2) pivot rods, beams, e.g. in structures
3) rotating spheres, planets, ..
4) various aspects of molecular + atomic motion.

Although we will apply these to rigid objects, the ideas are more broadly applicable and extend to non-rigid objects, e.g. galaxies

Demo: U Zurich Milky way at 1:20

Rotational kinematics

Consider a rigid object that rotates about an axle or pivot point. We can describe the state of this object using a single reference point of our choice. We then describe the configuration using an angle to this reference point. This leads to the following kinematics:



Angular position θ measured in radians → Angular velocity $\omega = \frac{d\theta}{dt}$ measured in rad/s → Angular acceleration $\alpha = \frac{d\omega}{dt}$ measured in rad/s²

$\omega > 0 \iff$ counterclockwise
$\omega < 0 \iff$ clockwise

ω = slope θ vs t

Sign of α depends on whether ω increases or decreases

α = slope of ω vs t

Quiz 1   70% → 100%   ≥ 30% → 80%

Quiz 2   50% → 50%   ≥ 20% - 70%

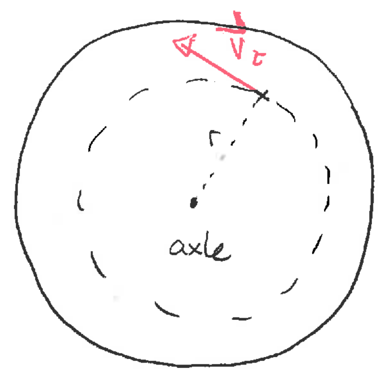
Demo: Lady bug revolution

a) $\alpha = 0$ $\omega_i < 0$

b) $\alpha > 0$ $\omega_i < 0$

c) $\alpha < 0$ $\omega_i < 0$ } observe graph of ω vs t.

Rotational and linear kinematics



Consider a rotating disk. Any point will move around the axis of rotation in a circle. Thus the velocity is purely tangential (to the orbit of the point). Previously we have shown that

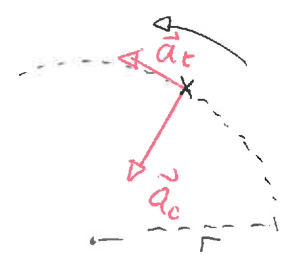
$$\boxed{v_t = \omega r}$$

where ω is the angular velocity and

r is the distance from the axis of rotation to the point.

We also know that there must be a centripetal acceleration, regardless of how the speed behaves. This always satisfies

$$\boxed{a_c = v^2/r = \omega^2 r}$$



and points radially inward. Additionally if the rate at which the object rotates, there will be a tangential acceleration with magnitude

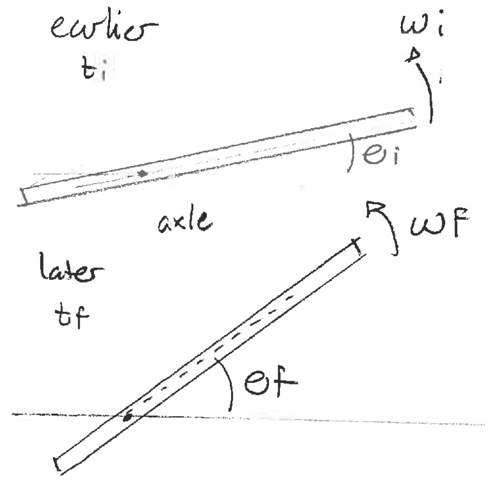
$$\boxed{a_t = \alpha r}$$

This is tangential to the the trajectory. Adding these two acceleration vectors gives the actual acceleration of the object.

Rotational kinematics

When the angular acceleration is constant then mathematics (integration) yields rotational kinematics equations:

$$\boxed{\begin{aligned} \omega_f &= \omega_i + \alpha\Delta t \\ \theta_f &= \theta_i + \omega_i \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2 \\ \omega_f^2 &= \omega_i^2 + 2\alpha\Delta\theta \end{aligned}}$$



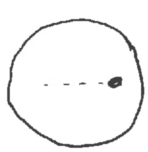
Example: A disk initially rotating clockwise at 120 revolutions per minute slows under friction at a constant rate, stopping in exactly two revolutions. A piece of gum is 0.10 m from the axle of the disk. Determine:

a) the angular acceleration of the gum

b) the tangential and centripetal acceleration of the gum at the initial instant.

Answer: a)

initial



$\theta_i = 0$

$\omega_i =$

final

$\theta_f = 2 \times (-2\pi \text{ rad})$

$= -4\pi \text{ rad}$

$\omega_f = 0 \text{ rad/s}$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \Rightarrow \omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

$$\Rightarrow \alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta}$$

Now $\omega_i = -120 \text{ rev/minute}$

$$= -\frac{120 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = -4\pi \text{ rad/s}$$

$$\Rightarrow \alpha = \frac{(0 \text{ rad/s})^2 - (4\pi \text{ rad/s})^2}{2(-4\pi \text{ rad})}$$

$$\alpha = 2\pi \text{ rad/s}^2 = 6.28 \text{ rad/s}^2$$

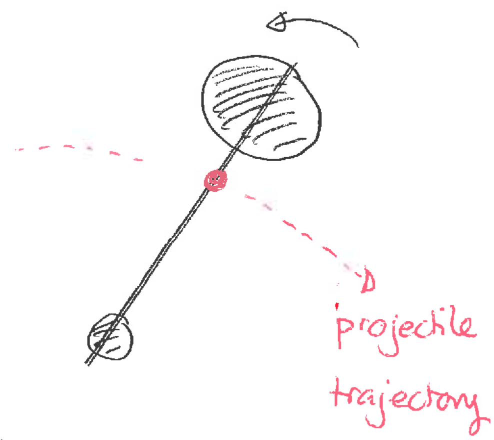
b) $a_c = \omega^2 r = (-4\pi \,\text{rad/s})^2 \times 0.10\,\text{m} = 16.0\,\text{m/s}^2$

$a_t = \alpha r = (6.28\,\text{rad/s}) \times 0.10\,\text{m} = 0.63\,\text{m/s}^2$

Rotational and translational motion

In general objects can display both rotational and translational motion simultaneously. How can we separate these?

Demo: MIT COM video



The videos show:

1) there is a location which moves just as a point particle would with all the forces acting on that particle. In this case it follows the parabolic trajectory of a projectile.
2) this location is not necessarily at the geometric center of the object.

This location is called the center of mass of the object. Intuitively

$\boxed{\text{The center of mass describes the translational motion of the object.}}$

What we will show is:

1) How to determine the center of mass.
2) What Newton's Laws state about the center of mass.