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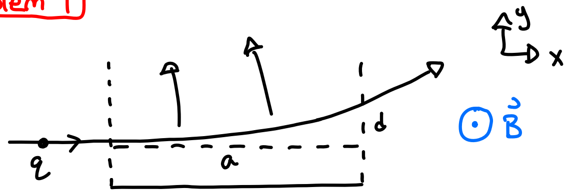
Dr. Middleton
PHYS 311
HW 9 Ch. 5

Problem 1)

a.)



$\vec{F} = q\vec{v}\times\vec{B}$

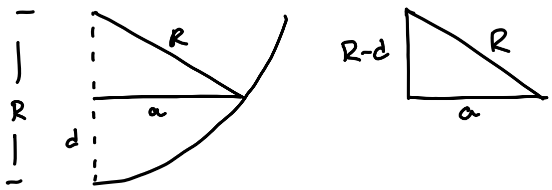
$\odot \vec{B}$

By the RHR, this particle will feel a deflection in the negative y-direction. Therefore it's charge is negative.

$\boxed{q \text{ is negative}}$

b.) $F = QBV$, $F = ma$, $a = \frac{v^2}{r}$

$QVB = \frac{mv^2}{r}$ $\therefore$ $mV = QBr$



$R^2 = (R-d)^2 + a^2$

$R^2 = R^2 - 2Rd + d^2 + a^2$

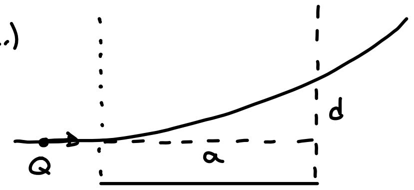
$-d^2 - a^2 = -2Rd$

$R = \frac{a^2+d^2}{2d}$

$$\boxed{V = -\frac{QB}{m}\left(\frac{a^2+d^2}{2d}\right)}$$

Problem 1

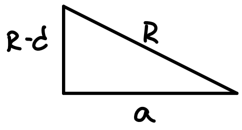
a.)



$F = Q\vec{v}\times\vec{B}$

This charge experiences centripetal acceleration $\therefore$ The charge is negative

b.)



$R^2 = (R-d)^2 + a^2$

$R^2 = R^2 - 2Rd + d^2 + a^2$

$0 = -2Rd + d^2 + a^2$

$2Rd = d^2 + a^2$

$R = \dfrac{d^2 + a^2}{2d}$

$F = QE = QvB$

$v = \dfrac{F}{QB}$ $\quad F = ma = \dfrac{mv^2}{R}$

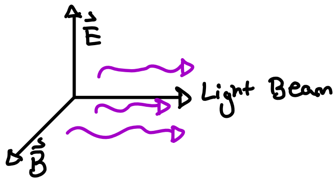
$v = \dfrac{mv^2}{QBR}$

$1 = \dfrac{mv}{QBR}$ $\quad \therefore \quad QBR = mv$

$v = \dfrac{QBR}{m}$

$$\boxed{v = \frac{QB}{m}\left(\frac{d^2 + a^2}{2d}\right)}$$

Problem 2)



no deflection!

a.) $F = Q\vec{E} + Q(\vec{v}\times\vec{B})$: $F=0$ $\quad \vec{v}\times\vec{B} = vB\sin\theta$, $\theta = 90°$

$0 = E + vB\sin(90) = E + vB$: $0 = E + vB$

$-E = vB$

$v = \frac{-E}{B} \qquad v = \left|\frac{E}{B}\right|$

$$\boxed{v = \frac{E}{B}}$$

b.) $F = Q\vec{E} + Q(\vec{v}\times\vec{B})$: $ma = QvB\sin\theta \quad \theta = 90°$

$\frac{mv^2}{R} = QvB$ $\qquad \vec{E}$ field off!

$P\cdot\frac{v}{R} = QvB$

$P = QBR$: $mv = QBR$: $\frac{Q}{m} = \frac{v}{BR}$

$$\boxed{\frac{Q}{m} = \frac{E}{B^2R}}$$

Problem 2

a.) $F = Q(\vec{E} + \vec{V}\times\vec{B})$

$0 = QE + Q\vec{V}\times\vec{B}$

$0 = QE + QVB$

$-QE = QVB$

$V = \dfrac{E}{B}$

$$\boxed{|\vec{V}| = \frac{E}{B}}$$

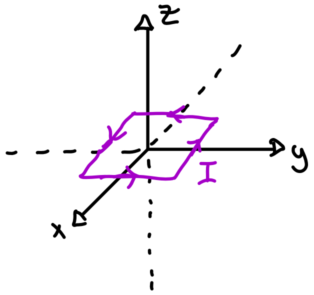
b.) $F = Q(\cancel{\vec{E}}^{\,0} + \vec{V}\times\vec{B}) = Q(\vec{V}\times\vec{B})$

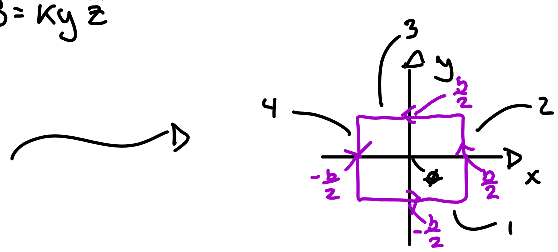
$\dfrac{mV^2}{R} = QVB \quad : \quad \dfrac{mV}{R} = QB \quad : \quad \dfrac{Q}{m} = \dfrac{V}{B\cdot R} = \dfrac{E}{B^2R}$

$$\boxed{\frac{Q}{m} = \frac{E}{B^2R}}$$

Problem 3

$\vec{B} = ky\,\hat{z}$





$$F_{mag} = I\int (d\vec{l}\times\vec{B}) = I(\vec{L}\times\vec{B})$$

① $d\vec{l} = dy\,\hat{y}$, $\vec{B} = ky\,\hat{z}$

$$(dy\,\hat{y})\times(ky)\hat{z} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & dy & 0\\ 0 & 0 & ky\end{vmatrix} = -(ky\,dy)\hat{x} : IK\int_{-\frac{b}{2}}^{\frac{b}{2}} y\,dy = \frac{IK}{2}\cdot y^2\Big|_{-\frac{b}{2}}^{\frac{b}{2}} -\hat{x}$$

$$\vec{F}_1 = \int_1 I(d\vec{l}\times\vec{B}) = \frac{IK}{2}\left(\left(\frac{b}{2}\right)^2 - \left(\frac{-b}{2}\right)^2\right)\hat{x} = 0 \ : \ \vec{F}_1 = 0$$

② $d\vec{l} = dx\,\hat{x}$, $\vec{B} = ky\,\hat{z}$

$$(dx\,\hat{x})\times(ky\,\hat{z}) = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ dx & 0 & 0\\ 0 & 0 & ky\end{vmatrix} = ky\,dx\,(-\hat{y}) \ : \ IKy\int_{\frac{b}{2}}^{-\frac{b}{2}} dx\,(-\hat{y}) = Iky\,x\Big|_{\frac{b}{2}}^{-\frac{b}{2}}(-\hat{y})$$

$$\vec{F}_2 = Iky\cdot\left(-\frac{b}{2}-\frac{b}{2}\right)(-\hat{y}) = Ikyb\,(\hat{y})\Big|_{y=\frac{b}{2}} = \frac{Ikb^2}{2}(\hat{y}) \ : \ \vec{F}_2 = \frac{Ikb^2}{2}(\hat{y})$$

③ $d\vec{l} = dy\,(\hat{y})$, $\vec{B} = ky\,(\hat{z})$

$$(dy\,\hat{y})\times(ky)\hat{z} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & dy & 0\\ 0 & 0 & ky\end{vmatrix} = -(ky\,dy)(\hat{x}) : IK\int_{\frac{b}{2}}^{-\frac{b}{2}} y\,dy\,(-\hat{x})$$

$$\vec{F}_3 = \frac{IK}{2}\cdot y^2\Big|_{\frac{b}{2}}^{(-\frac{b}{2})}(-\hat{x}) = \frac{IK}{2}(-\hat{x})\cdot\left[\left(-\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)\right] = 0 \ : \ \vec{F}_3 = 0$$

④ $d\vec{l} = dx\,(\hat{x})$, $\vec{B} = ky\,(\hat{z})$

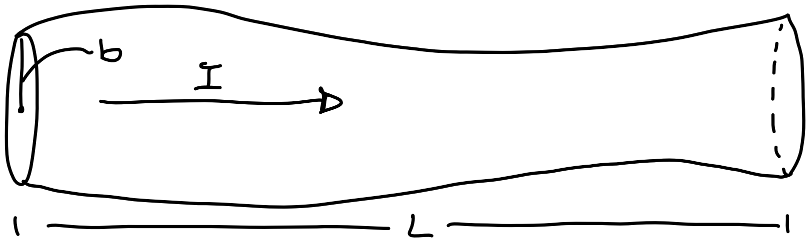
$$(dx\,\hat{x})\times(ky\,\hat{z}) = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ dx & 0 & 0\\ 0 & 0 & ky\end{vmatrix} = (ky\,dx)(-\hat{y}) \ : \ IKy\int_{-\frac{b}{2}}^{\frac{b}{2}} dx\,(-\hat{y}) = Ik\,y\cdot x\Big|_{-\frac{b}{2}}^{\frac{b}{2}}(-\hat{y})$$

$$\vec{F}_4 = Iky\cdot\left(\frac{b}{2}-\left(-\frac{b}{2}\right)\right)\Big|_{y=-\frac{b}{2}}(-\hat{y}) = Ik\cdot -\frac{b}{2}\cdot b\,(-\hat{y}) = \frac{Ikb^2}{2}(\hat{y})$$

$$F_{net} = \sum_{n=1}^{4}\vec{F}_n = \overset{0}{\cancel{F_1}} + F_2 + \overset{0}{\cancel{F_3}} + F_4 = \frac{Ikb^2}{2}(\hat{y}) + \frac{Ikb^2}{2}(\hat{y}) = Ikb^2(\hat{y})$$

$$\boxed{\vec{F} = Ikb^2\,\hat{y}}$$

Problem 4)



$K \equiv \frac{dI}{d\ell_\perp} = \frac{I}{dr \cdot 2\pi}$

$d\ell_\perp = b \cdot 2\pi$

a.)

$$\boxed{K = \frac{I}{2\pi b}}$$

b.) distance from axis $= b$, inverse prop. $\equiv J \propto \frac{1}{s} = \frac{K}{s}$

$J \equiv \frac{dI}{da_\perp} = \frac{I}{A_\perp}$

what is I ?

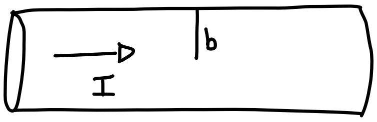
$\int dI = \int J \, da_\perp \rightarrow I = J \int da \; \vdots \; da = s \, ds d\phi$

$$I = K \int_0^{2\pi} \int_0^{s} \frac{1}{s} \cdot s \, ds d\phi$$

$$= K \int_0^{2\pi} \int_0^{b} ds d\phi = K \cdot 2\pi \cdot b$$

$$\boxed{I = 2\pi K b}$$

Problem 4



a.) $\vec{K} \equiv \frac{d\vec{I}}{dl_\perp}$: $K = \frac{I}{2\pi b}$

$dl_\perp = 2\pi b$

$$\boxed{K = \frac{I}{2\pi b}}$$

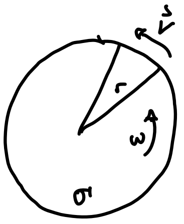
b.) $\vec{J} \equiv \frac{d\vec{I}}{da_\perp}$: $\vec{J} = \frac{I}{da_\perp}$: $I = J \cdot da_\perp$

$$I = \frac{K}{s} \cdot \int_0^{2\pi}\int_0^{b} s\, ds\, d\phi$$

$$= K \int_0^{2\pi}\int_0^{b} ds\, d\phi = K \cdot 2\pi \cdot b$$

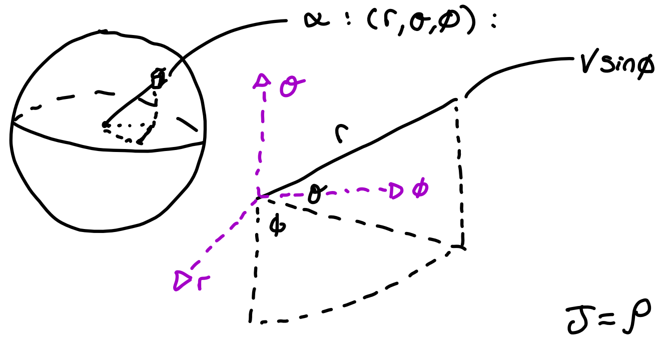
$$\boxed{I = K \cdot 2\pi \cdot b}$$

Problem 5



$\omega = \frac{v}{r}$

$K = \sigma V$



$J = \rho V$

a.) $K = \sigma V$

$\sigma' = \sigma'$

$V = \omega r$

$$\boxed{K = \sigma' \omega r}$$

b.) $J = \rho V$

$\rho \equiv \frac{Q}{V} = \frac{q}{\frac{4}{3}\pi R^3}$: $\rho = \frac{q}{\frac{4}{3}\pi R^3}$

$V = \omega r$: Point on Sphere with velocity in it $= V\sin\theta \equiv \omega r \sin\theta$

$J = \rho V = \frac{q}{\frac{4}{3}\pi R^3} V\sin\theta = \frac{q}{\frac{4}{3}\pi R^3}\omega r \sin\theta$

$$\boxed{J = \frac{q}{\frac{4}{3}\pi R^3}\omega r \sin\theta}$$

Problem 5

a.) $K = \frac{dI}{dl_\perp} = \sigma' v$

$\omega = \frac{v}{R} \therefore v = \omega R$

$K = \sigma' \cdot \omega \cdot R\, \hat{\phi}$

$$\boxed{K(R) = \sigma' \omega R\, \hat{\phi}}$$

b.) $\vec{J} = \rho \vec{v} = \frac{Q \cdot \vec{v}}{V} = \frac{Q \cdot \vec{v}}{\frac{4}{3}\pi R^3} = \frac{Q}{\frac{4}{3}\pi R^3} \omega r \cdot \sin\theta$ Point on Sphere

$\rho = \frac{Q}{V}$

$\vec{v} = \omega r \cdot \sin\theta$

$$\boxed{\vec{J} = \frac{Q}{\frac{4}{3}\pi R^3} \cdot \omega r \sin\theta\, \hat{\phi}}$$