6.7 Nonhomogeneous Linear Systems

$\bar{x}' = A\bar{x} + f$

① Solve for $\bar{x}_h$

② $\bar{x}_p = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

③ $\bar{x}_p' = A\bar{x}_p + f$

$0 = A\bar{x}_p + f$

$-f = A\bar{x}_p$

$\bar{x}_p = A^{-1}(-f)$

$x(t) = x_h + x_p$

$x(t) = c_1 e^{3t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} -t \\ 1 \end{bmatrix}$

Ex 2) $\bar{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\bar{x} + \begin{bmatrix} t-2 \\ 4t-1 \end{bmatrix}$

① $\bar{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\bar{x}$

$\bar{x}_h = c_1 e^{3t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

② Particular

$\bar{x}_p = \begin{bmatrix} At+B \\ Ct+D \end{bmatrix} \qquad \bar{x}_p' = \begin{bmatrix} a \\ c \end{bmatrix}$

$\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} at+b \\ ct+d \end{bmatrix} + \begin{bmatrix} t-2 \\ 4t-1 \end{bmatrix}$

$\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} at+b+ct+d \\ 4at+4b+ct+d \end{bmatrix} + \begin{bmatrix} t-2 \\ 4t-1 \end{bmatrix}$

$\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} at+b+ct+d+t-2 \\ 4at+4b+ct+d+4t-1 \end{bmatrix}$

$\begin{bmatrix} a \\ c \end{bmatrix} = t\begin{bmatrix} a+c+1 \\ 4a+c+4 \end{bmatrix} + \begin{bmatrix} b+d-2 \\ 4b+d-1 \end{bmatrix}$

$a = b+d-2$

$c = 4b+d-1$

$0 = a+c+1$

$0 = 4a+c+4$

$$\left[\begin{array}{cccc|c} -1 & 1 & 0 & 1 & 2 \\ 0 & 4 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 & -1 \\ 4 & 0 & 1 & 0 & -4 \end{array}\right] \quad \begin{matrix} a=-1 \\ b=0 \\ c=0 \\ d=1 \end{matrix}$$