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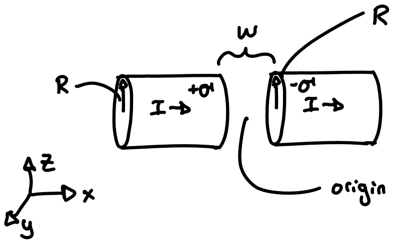
PHYS 311
HW 13

Problem 1



Electric field in parallel plate capacitor: $\vec{E} = \frac{\sigma'}{\varepsilon_0}(\hat{x})$: $J = 0$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \cdot \frac{\partial E}{\partial t}$$

The charge in this wire is dependent upon time since the current is flowing through the capacitor. More and more charge will stack up on the capacitor end.

$$Q = I \cdot t \to \quad \vec{E} = \frac{1}{\varepsilon_0} \cdot \frac{I \cdot t}{A} = \frac{I \cdot t}{\varepsilon_0 \cdot A}(\hat{x}) : \quad \vec{E} = \frac{It}{\varepsilon_0 A}(\hat{x})$$

$$\sigma' = \frac{Q}{A}$$

$$\int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \varepsilon_0 \int_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s} \quad : \quad S \to \text{Surface of plate on capacitor}$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{I}{\varepsilon_0 A} \cdot (\hat{x}) \qquad \int (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} \to \oint_C \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \int_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$$

$d\vec{l} = R\, d\phi\, \hat{\phi}$
$\vec{B} = B\, \hat{\phi}$
$ds \Rightarrow da = R \cdot dr \cdot d\phi$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \cdot \varepsilon_0 \cdot \frac{I}{\varepsilon_0 \cdot A} \int ds$$

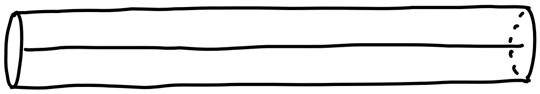
$$B \cdot R \int_0^{2\pi} d\phi = \frac{\mu_0 I}{A} \int_0^{2\pi} \int_0^R r\, dr\, d\phi$$

$$B \cdot R \cdot 2\pi = \frac{\mu_0 I}{A} \cdot \cancel{2\pi} \cdot \frac{R^2}{\cancel{2}}$$

$$B \cdot R \cdot 2\cancel{\pi} = \frac{\mu_0 I \cancel{\pi} R^2}{A} \to \quad \vec{B} = \frac{\mu_0 I}{2A} . R\ (\hat{\phi})$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2A} . R\ (\hat{\phi})} \longrightarrow A \equiv \text{Area of plate}$$

Problem 2



$$\vec{E}(s,t) = -\frac{\mu_0 I_0 \omega}{2\pi} \ln\left(\frac{b}{s}\right) \cos(\omega t)\,(\hat{z})$$

a.) $J_d = \varepsilon_0 \cdot \dfrac{\partial \vec{E}}{\partial t}$

$$\frac{\partial \vec{E}}{\partial t} : \frac{\mu_0 I_0 \cdot \omega^2}{2\pi} \cdot \ln\left(\frac{b}{s}\right) \sin(\omega t)\,(\hat{z})$$

$$\boxed{J_d = \frac{\varepsilon_0 \mu_0 I_0}{2\pi} \cdot \omega^2 \cdot \ln\left(\frac{b}{s}\right) \sin(\omega t)\,(\hat{z})}$$

b.) $I_d = \int \vec{J}_d \cdot d\vec{a}$

$d\vec{a} = s\,ds\,d\phi$

$$I_d = \frac{\varepsilon_0 \mu_0 I_0 \omega^2}{2\pi} \cdot \sin(\omega t) \int_0^{2\pi} \int_0^{b} (\ln(b) - \ln(s))\, s\, ds\, d\phi$$

$$= \varepsilon_0 \mu_0 I_0 \omega^2 \cdot \sin(\omega t) \cdot \left[\int_0^b s \ln(b)\, ds - \int_0^b s \ln(s)\, ds \right]$$

$$\underbrace{\int_0^b s \ln(b)\, ds}_{1} - \underbrace{\int_0^b s \ln(s)\, ds}_{2}$$

$$1 : \int_0^b s \ln(b)\, ds = \frac{b^2 \cdot \ln(b)}{2}$$

$$2 : \int_0^b s \ln(s)\, ds = \frac{b^2 \cdot \ln(b)}{2} - \frac{b^2}{4}$$

used Calculator

$$1 - 2 : \cancel{\frac{b^2 \cdot \ln(b)}{2}} - \cancel{\frac{b^2 \cdot \ln(b)}{2}} + \frac{b^2}{4}$$

$$I_d = \varepsilon_0 \mu_0 I_0 \omega^2 \cdot \sin(\omega t) \cdot \frac{b^2}{4}$$

$$\boxed{I_d = \frac{\varepsilon_0 \mu_0 I_0}{4} \cdot b^2 \omega^2 \sin(\omega t)}$$

c.)

$$\eta = \frac{I_d}{I} \qquad \eta = \frac{1}{4} \cdot \frac{\varepsilon_0 \mu_0 \cancel{I_0} b^2 \omega^2 \cancel{\sin(\omega t)}}{\cancel{I_0} \cancel{\sin(\omega t)}} = \frac{1}{4} \varepsilon_0 \mu_0 b^2 \omega^2$$

$$I_d = \frac{\varepsilon_0 \mu_0 I_0}{4} \cdot b^2 \omega^2 \sin(\omega t)$$

$I = I_0 \cdot \sin(\omega t)$

From Problem 7

$$\boxed{\eta = \frac{\varepsilon_0 \mu_0 b^2 \omega^2}{4}}$$

$\varepsilon_0 \mu_0 \rightarrow \dfrac{1}{c^2}$, $f = \dfrac{\omega}{2\pi}$: $\omega = 2\pi f$

$\eta = 0.01$, $b = 0.003\,m$

$$\eta = \frac{b^2}{c^2 \cdot 4} \cdot (2\pi)^2 \cdot f^2 \; : \; \eta = \frac{b^2}{c^2} \cdot \pi^2 \cdot f^2 \; : \; f = \left(\frac{\eta c^2}{b^2 \cdot \pi^2}\right)^{\frac{1}{2}} = \left(\frac{(0.01)(3.0\times10^{8}\,m/s)^2}{(0.003m)^2\,(\pi)^2}\right)^{\frac{1}{2}} = 1.9\times10^{9}\ Hz$$

$$\boxed{f = 1.9\times10^{9}\ Hz}$$