Physics 396

Homework Set 2

1. Consider a free particle (a particle with no net force acting on it) examined in an inertial reference frame with coordinates (x, y, z). A free particle obeys the equations of motion

$$\frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2z}{dt^2} = 0, \tag{1}$$

as free particles don't accelerate relative to inertial frames. This particular free particle follows a trajectory described by

$$\begin{aligned} x(t) &= x_0 + v_x t \\ y(t) &= y_0 \\ z(t) &= z_0 + v_z t, \end{aligned} \tag{2}$$

where x_0, y_0, z_0, v_x, v_z are constants in time.

a) Explicitly show that this free particle, with a trajectory described by Eq. (2), obeys the three equations of motion given in Eq. (1).

Consider the *non*-inertial reference frame (x', y', z'), which rotates with respect to the inertial frame with a *constant* angular velocity ω about the common $z = z'$ axis. The coordinate transformation from the inertial to non-inertial reference frame is of the form

$$\begin{aligned} x'(t) &= \cos(\omega t)x + \sin(\omega t)y \\ y'(t) &= -\sin(\omega t)x + \cos(\omega t)y \\ z'(t) &= z, \end{aligned} \tag{3}$$

b) What are the equations of motion describing $x'(t)$, $y'(t)$, and $z'(t)$ in the rotating frame?

2. Consider an inertial reference frame with coordinates (x, y, z) and a *non*-inertial reference frame with coordinates (x', y', z'). At $t = t' = 0$, the two coordinate systems align and are momentarily at rest relative to one another. The non-inertial reference frame is *uniformly accelerating* with respect to the inertial frame at a rate $\vec{a} = g\hat{\jmath}$.

a) Construct the coordinate transformations linking these two frames of reference.

Now consider a free particle that follows a trajectory described by

$$\begin{aligned} x &= x_0 + v_x t \\ y &= y_0 \\ z &= z_0 + v_z t, \end{aligned} \tag{4}$$

where x_0, y_0, z_0, v_x, v_z are constants in time.

b) What are the equations of motion describing $x'(t)$, $y'(t)$, and $z'(t)$ in the uniformly accelerating frame?

3. Consider an object of *inertial* mass, m_I, and *gravitational* mass, m_G, in the earth's uniform gravitational field, where the force of gravity acting on the object is given by

$$\vec{F} = -m_G g \hat{\jmath} \tag{5}$$

a) For a freely falling object, under the influence of gravity alone, construct the equations of motion analogous to Eq. (1) for a non-zero net force. For the moment, assume that $m_I \neq m_G$.

b) Now setting $m_I = m_G$, show that these equations reduce to the form given above in Prob. 2b). In one sentence, what does this imply?

4. Consider a spherically-symmetric massive object of radius R, mass M with mass density

$$\mu(r) = \kappa r^n \quad \text{for} \quad r < R, \tag{6}$$

where n is an arbitrary power.

a) Find an expression for κ in terms of M, R, and n.

b) What values can n have if one demands that the total mass of the object is finite?

c) Using Gauss' law in integral form for Newtonian gravitation, calculate the *gravitational field vector* both inside and outside the massive object. Write this expression in terms of G, M, n and the radial distance r.

d) Calculate the *gravitational potential* of the massive object both inside and outside the massive object.