Math 361 **Numerical Analysis** Taylor Larrechea
9/19/19 Quiz 5 Name

Answer the following questions in the space provided. **Show all work.** (20 pts. total.)

1. Let $f(x) = x^3 + x^2 - 9$. Suppose we are interested in using the Bisection Method for finding the solution to $f(x) = 0$ on $[1, 2]$.

(a) Use the IVT to demonstrate that there is a solution to $f(x) = 0$ in $(1, 2)$. (4 pts)

$f(x)$ is a polynomial $\therefore$ $f(x) \in C[1,2]$

$f(1) = -7$: $f(2) = 3$: $f(1) \cdot f(2) = -7 \cdot 3 = -21 < 0$ ✓

$\therefore$ $\exists\, c \in [1,2]$ s.t. $f(c) = 0$

(b) Implement the Bisection Method by completing the table below. For this part, you may choose not to show work if desired. However, there is some space below the table for work that you would like me to consider for partial credit if necessary. (10 pts.)

n	a	b	p	sign f(a)	sign f(b)	sign f(p)
1	1	2	1.5	-1	+1	-1
2	1.5	2	1.75	-1	+1	-1
3	1.75	2	1.875	-1	+1	+1
4	1.75	1.875	1.8125	-1	+1	+1

2. Given that $\alpha_n = \dfrac{2n+1}{5n} \to \dfrac{2}{5}$ as $n \to \infty$, find the rate of convergence of $\{\alpha_n\}_{n=1}^{\infty}$, expressing your answer using "big O" notation. Show your work/steps as presented in class. (6 pts)

$$|\alpha_n - \alpha| = \left|\frac{2n+1}{5n} - \frac{2}{5}\right| = \left|\frac{2n+1}{5n} - \frac{2n}{5n}\right| = \frac{1}{5n} = \frac{1}{5}\left|\frac{1}{n}\right|$$

$$\alpha_n = \frac{2}{5} + O\left|\frac{1}{n}\right|$$