5.4 Coordinates and Diagonalization

5.4 # 12, 14, 40, 42, 53, 54

5.4.12) $S = [\vec{e}_1, \vec{e}_2, \vec{e}_3]$

$B = \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\-1\end{bmatrix}$ $\quad \begin{bmatrix}-1\\-1\\4\end{bmatrix}, \begin{bmatrix}1\\-1\\3\end{bmatrix}, \begin{bmatrix}3\\1\\4\end{bmatrix}$ $\quad \begin{bmatrix}-9\\-4\\3\end{bmatrix}, \begin{bmatrix}7\\3\\-4\end{bmatrix}, \begin{bmatrix}11\\4\\-3\end{bmatrix}$

$\vec{u}_s = M_B \vec{u}_B$

$M_B = \begin{bmatrix}1&0&2\\0&0&1\\0&1&-1\end{bmatrix}$ $\vec{u}_B = \begin{bmatrix}-1\\-1\\-4\end{bmatrix}$ $\quad M_B = \begin{bmatrix}1&0&2\\0&0&1\\0&1&-1\end{bmatrix}$ $\vec{u}_B = \begin{bmatrix}1\\-1\\3\end{bmatrix}$ $\quad M_B = \begin{bmatrix}1&0&2\\0&0&1\\0&1&-1\end{bmatrix}$ $\vec{u}_B = \begin{bmatrix}3\\1\\4\end{bmatrix}$

$M_B(\vec{u}_B) = \begin{bmatrix}-9\\-4\\3\end{bmatrix}$ $\quad M_B(\vec{u}_B) = \begin{bmatrix}7\\3\\-4\end{bmatrix}$ $\quad M_B(\vec{u}_B) = \begin{bmatrix}11\\4\\-3\end{bmatrix}$

5.4.14) $S = [x^2, x, 1]$
$N = [2x^2 - x, x^2, x^2+1]$

$P(x) = x^2 + 2x + 3$
$Q(x) = x^2 - 2$
$r(x) = 4x - 5$

$\begin{bmatrix}-2\\2\\3\end{bmatrix}, \begin{bmatrix}0\\3\\-2\end{bmatrix}, \begin{bmatrix}-4\\13\\-5\end{bmatrix}$

$M_B = \begin{bmatrix}2&1&1\\-1&0&0\\0&0&1\end{bmatrix}$ $\quad \vec{u}_B = (M_B^{-1})(\vec{u}_s)$

$M_B^{-1} \begin{bmatrix}0&-1&0\\1&2&-1\\0&0&1\end{bmatrix}$ $\quad P(x) = \begin{bmatrix}1\\2\\3\end{bmatrix}$ $\quad Q(x) = \begin{bmatrix}1\\0\\-2\end{bmatrix}$ $\quad r(x) = \begin{bmatrix}0\\4\\-5\end{bmatrix}$

$P(x): \quad M_B^{-1}(P(x)) = \begin{bmatrix}-2\\2\\3\end{bmatrix}$ $\quad Q(x): \quad M_B^{-1}(Q(x)) = \begin{bmatrix}0\\3\\-2\end{bmatrix}$ $\quad R(x): \quad M_B^{-1}(R(x)) = \begin{bmatrix}-4\\13\\-5\end{bmatrix}$

5.4.40) $A = \begin{bmatrix}0&0&1\\0&1&2\\0&0&1\end{bmatrix}$ Diagonalization

$\lambda = 0,1,1$

$|A - \lambda I| = \left|\begin{bmatrix}-\lambda&0&1\\0&1-\lambda&2\\0&0&1-\lambda\end{bmatrix}\right| = -\lambda(\lambda-1)^2$

$\lambda = 0$
$\lambda = 1,1$

One eigenvalue is repeated therefore this is not diagonizable.

5.4.42

$A=\begin{bmatrix}4&2&3\\2&1&2\\-1&2&0\end{bmatrix}$ $D=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&5\end{bmatrix}$ $P=\begin{bmatrix}-1&-1&2\\-2&0&1\\3&1&0\end{bmatrix}$ $P^{-1}=\begin{bmatrix}1/6&-1/3&1/6\\-1/2&1&1/2\\1/3&1/3&1/3\end{bmatrix}$

$|A-\lambda I| = -1,1,5$ $P=[\vec{v}_1|\vec{v}_2|\vec{v}_3]$

$$\left|\begin{bmatrix}4&2&3\\2&1&2\\-1&2&0\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\right| = (5-\lambda)(\lambda-1)(\lambda+1)$$
$\lambda=-1,1,5$

$(A-\lambda I)\vec{v}_1$ $\lambda=-1$

$$\begin{bmatrix}5&2&3\\2&2&2\\-1&2&1\end{bmatrix}\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}$$

$rref=\left[\begin{array}{ccc|c}1&0&1/3&0\\0&1&2/3&0\\0&0&0&0\end{array}\right]$ $x=-\frac{1}{3}z$ $y=-\frac{2}{3}z$ $z=3$ $\vec{v}_1=\begin{bmatrix}-1\\-2\\3\end{bmatrix}$

$(A-\lambda_2 I)\vec{v}_2$ $\lambda_2=1$

$\begin{bmatrix}3&2&3\\2&0&2\\-1&2&-1\end{bmatrix}\begin{bmatrix}v_{2x}\\v_{2y}\\v_{2z}\end{bmatrix}$ $\left[\begin{array}{ccc|c}1&0&1&0\\0&1&0&0\\0&0&0&0\end{array}\right]$ $-x=z$ $y=0$ $z=?$ $\vec{v}_2=\begin{bmatrix}-1\\0\\1\end{bmatrix}$

$(A-\lambda_3 I)\vec{v}_3$ $\lambda_3=5$

$P^{-1}AP=D$

$\begin{bmatrix}-1&2&3\\2&-4&2\\-1&2&-5\end{bmatrix}$ $rref=\begin{bmatrix}1&-2&0\\0&0&1\\0&0&0\end{bmatrix}$ $x=2y$ $y=1$ $z=0$ $\begin{bmatrix}2\\1\\0\end{bmatrix}\vec{v}_3$

$P^{-1}AP=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&5\end{bmatrix}$ ✓

5.4.53

Non Invertible
Non Diagonizable

$A=\begin{bmatrix}0&0\\0&0\end{bmatrix}$