5.) $F(x,y) = x^{3y}$

$F_x(x,y) = 3y \cdot x^{-1+3y}$

$F_y(x,y) =$

$a^t \ln a \cdot t'$

$F_x(x,y)$ x^{3y}

$\frac{\partial}{\partial x} = 3y \cdot x^{-1+3y}$

$F_y(x,y)$ x^{3y}

$\frac{\partial}{\partial y} = x^{3y} \ln x \cdot 3$

$\frac{\partial}{\partial y} = 3x^{3y} \ln x$

7.) $F(x,y,z) = \frac{y}{x+y+z}$; $F_y(-2,1,-4)$

$$F_y = \frac{(x+y+z)(1) - y(1)}{(x+y+z)^2} = \frac{x+\cancel{y}+z-\cancel{y}}{(x+y+z)^2}$$

$$F_y = \frac{x+z}{(x+y+z)^2} \quad F_y(-2,1,-4)$$

$$\frac{-2-4}{(-2+1-4)^2} = \frac{-6}{(-5)^2} = \boxed{\frac{-6}{25}}$$

8.) Find all the Second partial derivatives

$F(x,y) = x^5y^4 + 8x^9y$

$F_{xx}(x,y) =$ $20x^3y^4 + 576x^7y$

$5x^4y^4 + 72x^8y$

$20x^3y^4 + 576x^7y$

$F_{xy}(x,y) =$ $20x^4y^3 + 72x^8$

$5x^4y^4 + 72x^8y$

$20x^4y^3 + 72x^8$

$f_{yy}(x,y) = 12x^5y^2$ $\quad x^5y^4 + 8x^9y$

$4x^5y^3 + 8x^9$

$12x^5y^2$

$f_{yx}(x,y) = 20x^4y^3 + 72x^8$

$4x^5y^3 + 8x^9$

$20x^4y^3 + 72x^8$

10.) Find the indicated partial derivative

F_{xyz} $\quad F(x,y,z) = e^{xyz^3}$

$F(x,y,z) = e^{xyz^3}$

$\frac{\partial f}{\partial x} = yz^3e^{xyz^3}$

$yz^3 \cdot e^{xyz^3}$

$z^3 \cdot e^{xyz^3} + yz^3(xz^3)e^{xyz^3}$

$z^3e^{xyz^3} + xyz^6e^{xyz^3}$

$\frac{\partial f}{\partial y} = e^{xyz^3}(z^3 + xyz^6)$

$3xyz^2e^{xyz^3}(z^3 + xyz^6) + e^{xyz^3}(3z^2 + 6xyz^5)$

$e^{xyz^3}(3xyz^2(z^3 + xyz^6) + 3z^2 + 6xyz^5)$

$e^{xyz^3}(3xyz^5 + 3x^2y^2z^8 + 3z^2 + 6xyz^5)$

$e^{xyz^3}(9xyz^{10} + 3x^2y^2z^8 + 3z^2)$

e^{xyz^3}

$3xyz^2e^{xyz^3}$

$3xz^2e^{xyz^3} + 3xyz^2(xz^3)e^{xyz^3}$

$3xz^2e^{xyz^3} + 3x^2yz^5e^{xyz^3}$

$e^{xyz^3}(3xz^2 + 3x^2yz^5)$

$yz^3e^{xyz^3}(3xz^2 + 3x^2yz^5) + e^{xyz^3}(3z^2 + 6xyz^5)$

$e^{xyz^3}(yz^3(3xz^2 + 3x^2yz^5) + (3z^2 + 6xyz^5))$

$e^{xyz^3}(3xyz^5 + 3x^2y^2z^8 + 3z^2 + 6xyz^5)$

$e^{xyz^3}(9xyz^5 + 3x^2y^2z^8 + 3z^2)$

$e^{xyz^3}(3(3xyz^5 + x^2y^2z^8 + z^2))$

9.) Find all the second partial derivatives

$f(x,y) = \sin^2(mx + ny)$

$u = \sin(mx + ny)$

$u^2 \quad du = 2u \cdot u'$

$u' = m\cos(mx+ny)$ $\quad 2\sin(mx+ny)\cdot\cos(mx+ny)\, m$

$\sin 2(mx+ny)\, m$

$f_x = m\sin 2(mx+ny)$

$2u = mx+ny$

$F_{xx} =$ $2u'_x = 2m$ $\quad \sin 2\theta$

$2u' = 2m$ $\quad \cos 2\theta$

$\boxed{F_{xx} = 2m^2\cos 2(mx+ny)}$

$F_y = n\sin 2(mx+ny)$ $\quad 2u = 2(mx+nx)$

$2u'_y = 2n$ $\quad \sin 2\theta$

$\cos 2\theta$

$\boxed{F_{yy} = 2n^2\cos 2(mx+ny)}$

$f_x = m\sin 2(mx+ny)$

$f_{xy} = 2u' = 2(n)$ $\quad \sin 2\theta$

$2n$ $\quad \cos 2\theta$

$\boxed{f_{xy} = 2mn\cos 2(mx+ny)}$

$f_y = n\sin 2(mx+ny)$ $\quad \sin 2\theta$

$u = mx+ny$ $\quad 2mn$ $\quad \cos 2\theta$

$u/x = m$ $\quad 2mn\cos 2(mx+ny)$

$2u = 2m$

11.) $\boxed{F_{yx} = 2mn\cos 2(mx+ny)}$

Find the indicated partial derivative

$$u = e^{2r\theta}\sin\theta$$

Find $\dfrac{\partial^3 u}{\partial r^2 \partial\theta}$

$$\frac{\partial u}{\partial\theta} = 2re^{2r\theta}\cdot\sin\theta + e^{2r\theta}\cdot\cos\theta$$

$$\frac{\partial u}{\partial\theta} = 2re^{2r\theta}\sin\theta + e^{2r\theta}\cos\theta$$

$$\frac{\partial u}{\partial\theta} = e^{2r\theta}(2r\sin\theta + \cos\theta)$$

$$\frac{\partial u}{\partial r} = 2\theta e^{2r\theta}(2r\sin\theta + \cos\theta) + e^{2r\theta}(2\sin\theta)$$

$$e^{2r\theta}(2\theta(2r\sin\theta + \cos\theta) + 2\sin\theta)$$

$$e^{2r\theta}(4r\theta\sin\theta + 2\theta\cos\theta + 2\sin\theta)$$

$$2\theta e^{2r\theta}(4r\theta\sin\theta + 2\theta\cos\theta + 2\sin\theta) + e^{2r\theta}(4\theta\sin\theta)$$

$$\boxed{\frac{\partial^3 u}{\partial r^2 \partial \theta} = 2\theta e^{2r\theta}(4r\theta\sin\theta + 2\theta\cos\theta + 2\sin\theta) + e^{2r\theta}(4\theta\sin\theta)}$$

12.) Find the indicated partial derivative

$\frac{\partial^3 z}{\partial u \partial v \partial w}$ $\quad z = u\sqrt{v-w}$ $\quad \frac{\partial^3 z}{\partial u \partial v \partial w}$

$$\frac{\partial z}{\partial w} \quad u \cdot \frac{1}{\sqrt{v-w}} \cdot -1$$

$$\frac{-u}{2\sqrt{v-w}}$$

$$(v-w)^{-\frac{1}{2}}$$

$$\frac{-u}{2} \cdot -\frac{1}{2}(v-w)^{-\frac{3}{2}}(1)$$

$$\frac{u}{4}(v-w)^{-\frac{3}{2}}$$

$$\frac{\partial^2 z}{\partial v \partial w} = \frac{u}{4(v-w)^{3/2}}$$

$$\frac{u}{4(v-w)^{3/2}}$$

$$\frac{\partial z}{\partial u \partial v \partial w} = \frac{1}{4(v-w)^{3/2}}$$

$$\frac{x}{6} = x \cdot \frac{1}{6}$$

$$f'(x) = \frac{1}{6}$$

$$\boxed{\frac{\partial^3 z}{\partial u \partial v \partial u} = \frac{1}{4(v-w)^{3/2}}}$$

13) Given the following function $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Find ∂R

$$\overline{\partial R_1}$$

$$R^{-1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{\partial R^{-1}}{\partial R}\frac{\partial R}{\partial R_1} = \frac{\partial\left(1/R_1 + 1/R_2 + 1/R_3\right)}{R_1}$$

$$-R^{-2}\frac{\partial R}{\partial R_1} = -R_1^{-2}$$

$$\frac{\partial R}{\partial R_1} = \frac{-R_1^{-2}}{-R^{-2}}$$

$$\frac{\partial R}{\partial R_1} = \frac{R^2}{R_1^2}$$

$$\boxed{\frac{\partial R}{\partial R_1} = \frac{R^2}{R_1^2}}$$

14.) $W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}$

$T = 11°C$

$V = 11\ Km/h$

$$\partial w/\partial T = 0.6215 + 0.3965v^{0.16}$$

$$\partial w/\partial v = \underset{-1.8192}{-11.37(0.16)} + \underset{0.06344T}{0.3965(0.16)T}$$

$$\partial w/\partial T\,(11\ Km/h) = 0.6215 + 0.3965(11)^{0.16} \approx \boxed{1.20°c}$$

$$\partial w/\partial v\,(11°C) = (-1.8192 + 0.06347\,(11))$$

15.) $z = 8 - x - x^2 - 6y^2$ $\quad x = 3 \quad (3, 2, -28)$

$x = 3$

$y = 2 + T$

$z = 8 - 3 - 3^2 - 6y^2$

$z = 5 + 9 - 6y^2$

$z = 14 - 6y^2$

$\frac{\partial z}{\partial y} = -12y$

$\frac{\partial z}{\partial y}(2) = -24$

$z = -24y + b$

$-28 = -24(2) + b$

$-28 = -48 + b$

$20 = b$

$z = -24y + 20$

$z = -24(2+T) + 20$

$z = -48 - 24T + 20$

$z = -28 - 24T$

$$\boxed{\begin{aligned} x &= 3 \\ y &= 2 + T \\ z &= -28 - 24T \end{aligned}}$$