Electromagnetic Theory II: Homework 12

Due: 23 March 2021

1 Electromagnetic waves

Write the real and complex representations of the electric and magnetic fields for the following sinusoidal plane electromagnetic waves in a vacuum.

a) Traveling along the $+y$ direction with electric field amplitude E_0, frequency ω, phase $\delta = 0$ and polarization along the $+x$ direction.

b) Traveling along the $+y$ direction with electric field amplitude E_0, frequency ω, phase $\delta = 0$ and polarization along the $+z$ direction.

c) Traveling along in the xy plane at an angle of $60°$ counterclockwise from the $+x$ axis and with electric field amplitude E_0, frequency ω, phase $\delta = 0$ and polarization along the $+z$ direction.

2 Time averaging and complex representations of sinusoidal waves

Consider the sinusoidal electric field with complex representation

$$\tilde{\mathbf{E}}(\mathbf{r}, t) = \tilde{\mathbf{E}}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}.$$

where $\tilde{\mathbf{E}}_0$ is real.

a) Show that the time average using the real representation of the wave is:

$$\langle \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t) \rangle = \frac{\tilde{\mathbf{E}}_0 \cdot \tilde{\mathbf{E}}_0}{2}$$

b) Show that time average can be computed from the complex representation as:

$$\langle \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t) \rangle = \frac{1}{2} \tilde{\mathbf{E}}(\mathbf{r}, t)^* \cdot \tilde{\mathbf{E}}(\mathbf{r}, t).$$

3 Griffiths, *Introduction to Electrodynamics, 4ed*, 9.10, page 400.

4 Radiation pressure on a spacecraft

The intensity of the electromagnetic radiation from the Sun varies as

$$I = \frac{I_0}{r^2}$$

where r is the distance of the Earth from the Sun.

a) Use data from a previous problem to determine I_0.

b) Consider sunlight hitting the Pioneer 10 spacecraft, when it was a distance of 50 AU (AU is a unit of length called equal to the distance from the Sun to the Earth.) from the Sun. Determine the radiation pressure exerted by the Sunlight on Pioneer 10.

c) The bulk of Pioneer 10's area comes from the parabolic dish receiver on the craft. This has a diameter of 2.74 m. Determine the force exerted by the sunlight on Pioneer 10 when it was 50 AU from the Sun. How does this compare to the gravitational force exerted by the Sun on Pioneer 10? The mass of Pioneer 10 is about 220 kg.

d) Determine the acceleration that this force exerted by the sunlight would provide, if it were the only force acting on Pioneer 10.

You may have to look up data elsewhere to answer this question. This was of interest in the attempts to explain the "Pioneer anomaly." This refers to the deviation from expected acceleration, calculated using all known forces acting on the Pioneer spacecraft. The radiation pressure force is one of the known forces. The deviation from expected acceleration was about $8.7 \times 10^{-10}\,\mathrm{m/s^2}$ although this has apparently since been explained.

5 Electromagnetic waves in a medium with no free charge or current

In a general medium Maxwell's equations are

$$\begin{aligned}\boldsymbol{\nabla} \cdot \mathbf{D} &= \rho_f \\ \boldsymbol{\nabla} \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \boldsymbol{\nabla} \cdot \mathbf{B} &= 0 \\ \boldsymbol{\nabla} \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}$$

In a linear medium $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$. Consider a linear medium in which there is no free charge and free current.

a) Rewrite Maxwell's equations in terms of $\mathbf{E}$ and $\mathbf{H}$. Manipulate these so as to obtain wave equations for $\mathbf{E}$ and $\mathbf{H}$.

b) Consider sinusoidal plane wave solutions for which the wavenumber vector is $\mathbf{k}$. Show that $\mathbf{E}, \mathbf{H}$ and $\mathbf{k}$ are all perpendicular to each other.

For certain materials (e.g. turpentine) the magnetization is related to the polarization by

$$\mathbf{M} = \alpha \frac{\partial \mathbf{P}}{\partial t}$$

where α is constant.

c) Use this relationship to show that for such waves $\frac{\partial \mathbf{E}}{\partial t}$ is proportional to $\mathbf{H}$ and use this result to describe how the direction of $\frac{\partial \mathbf{E}}{\partial t}$ is related to the direction of $\mathbf{E}$ for sinusoidal electromagnetic waves.

d) Use the previous result to describe whether linearly polarized waves can exist in this medium.

e) Explain what types of polarized light could exist in this material.

6 Reflection and transmission at a boundary with free surface charges

Consider a boundary between two dielectric materials that lies in the $z = 0$ plane. Suppose that there is a constant and uniform surface charge density on the boundary. A sinusoidal wave traveling in the $+\hat{\mathbf{z}}$ direction is incident from $z < 0$. This wave has wavenumber $\mathbf{k}_I$. Denote the wavenumbers of the reflected and transmitted waves by $\mathbf{k}_R$ and $\mathbf{k}_T$. The purpose of this exercise will be to determine whether the presence of the surface charge affects the rules regarding the frequencies of the three waves and the "plane of incidence" rule for the wavenumbers. *The solutions to these questions do not require complicated calculations. Rather they require you to identify which boundary conditions and which components of the electric field can be used to establish the conclusions. You need to describe these explicitly.*

a) Suppose that the incident wave is perpendicular to the surface. How are the frequencies of the three waves related? Do the three wavenumber vectors all lie in the same plane? Explain the reasoning behind your answer in detail.

b) Suppose that the incident wave is not perpendicular to the surface. How are the frequencies of the three waves related? Do the three wavenumber vectors all lie in the same plane? Explain the reasoning behind your answer in detail.