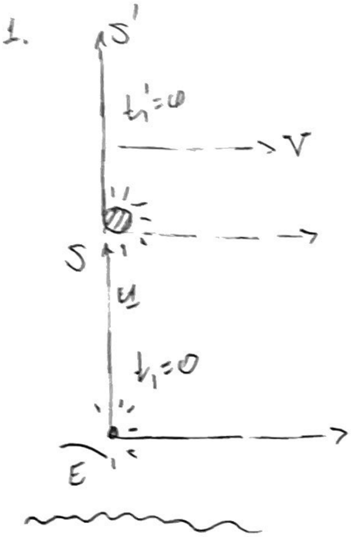
PHYS 230 Homework Set 9 Solutions

1.

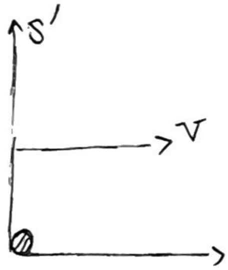


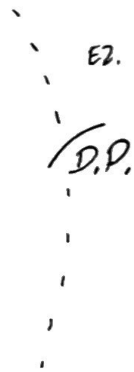
EVENT 1: FLASHBULB EXPLODES @ $x_1 = x_1' = 0$ @ $t_1 = t_1' = 0$

$$V = \frac{5}{13}c$$

$$D_{E,DP} = 25\ c\cdot yrs$$

EVENT 2: FLASH ARRIVES AT DISTANT PLANET @ $x_1 = 25\,c\cdot yrs$





a) $\Delta x = c\Delta t$ AND $\Delta x = D_{E,DP}$ SO

$$\Delta t = \frac{\Delta x}{c} = \frac{25\,c\cdot yrs}{c} = 25\,yrs = t_2 - \cancel{t_1}$$

SO

$$t_2 = 25\,yrs$$

b) NOW

$$x_2' = \gamma(x_2 - Vt_2) = \frac{1}{\sqrt{1-(5/13)^2}}\left(25\,c\cdot yrs - \frac{5}{13}c\cdot 25\,yrs\right) = \frac{1}{12/13}\,25\,c\cdot yrs\left(1-\frac{5}{13}\right)$$

$$= \frac{13}{12}\cdot 25\,c\cdot yrs\cdot\frac{8}{13} = \frac{8}{12}\cdot 25\,c\cdot yrs = \frac{2}{3}\cdot 25\,c\cdot yrs$$

SO

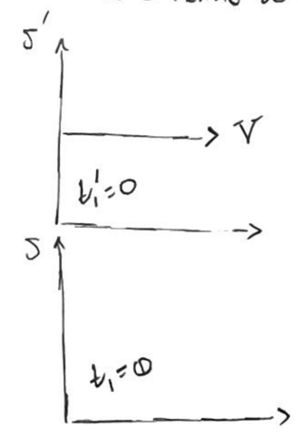
$$x_2' = \frac{50}{3}\,c\cdot yrs \simeq 16.7\,c\cdot yrs$$

c) $t_2' = \gamma\left(t_2 - \frac{V x_2}{c^2}\right) = \frac{13}{12}\left(\frac{25\text{yrs} - \frac{5}{13}c \cdot 25c\text{yrs}}{c^2}\right) = \frac{13}{12} \cdot 25\text{yrs}\left(1 - \frac{5}{13}\right) = \frac{\cancel{13}}{12} \cdot \frac{8}{\cancel{13}} \cdot 25\text{yrs}$

$$t_2' = \frac{2}{3} \cdot 25\text{yrs}$$

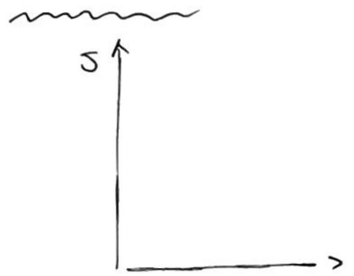
$$\boxed{t_2' = \frac{50}{3}\text{yrs} \simeq 16.7 \text{ years}}$$

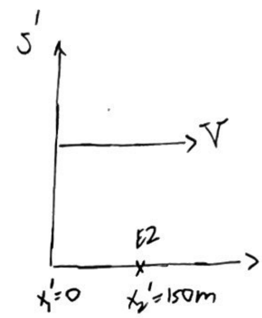
2. Let the S' frame be the rest frame of spaceship A : S be the rest frame of spaceship B.



$V = \frac{4}{5}c$

Event 1: Flashbulb explodes @ $x_1 = x_1' = 0$ @ $t_1 = t_1' = 0$





Event 2: Flash arrives at $x_2' = 150m$

a) $\Delta x' = c\Delta t'$ where $\Delta x' = x_2' - x_1' = 150m$

so $\Delta t' = \frac{\Delta x'}{c} = 150m/c = t_2' - \underbrace{\cancel{t_1'}}_{0}$ & $\left[t_2' = 150m/c\right]$

b) $x_2 = \gamma(x_2' + Vt_2') = \frac{1}{\sqrt{1-(4/5)^2}}\left(150m + \frac{4}{5}c \cdot 150\frac{m}{c}\right) = \frac{1}{3/5}\left(1 + \frac{4}{5}\right)(150m) = \frac{5}{3} \cdot \frac{9}{5}(150m)$

$\left[x_2 = 450m\right]$

c) $t_2 = \gamma\left(t_2' + \frac{Vx_2'}{c^2}\right) = \frac{1}{\sqrt{1-(4/5)^2}}\left(150m/c + \frac{\frac{4}{5}c(150m)}{c^2}\right) = \frac{1}{3/5}\left(1 + \frac{4}{5}\right)\left(150\frac{m}{c}\right) = 450m/c$

$\left[t_2 = 450m/c\right]$

3) Let S BE THE REST FRAME OF SPACESHIP B AND S' BE THE REST FRAME OF SPACESHIP A.

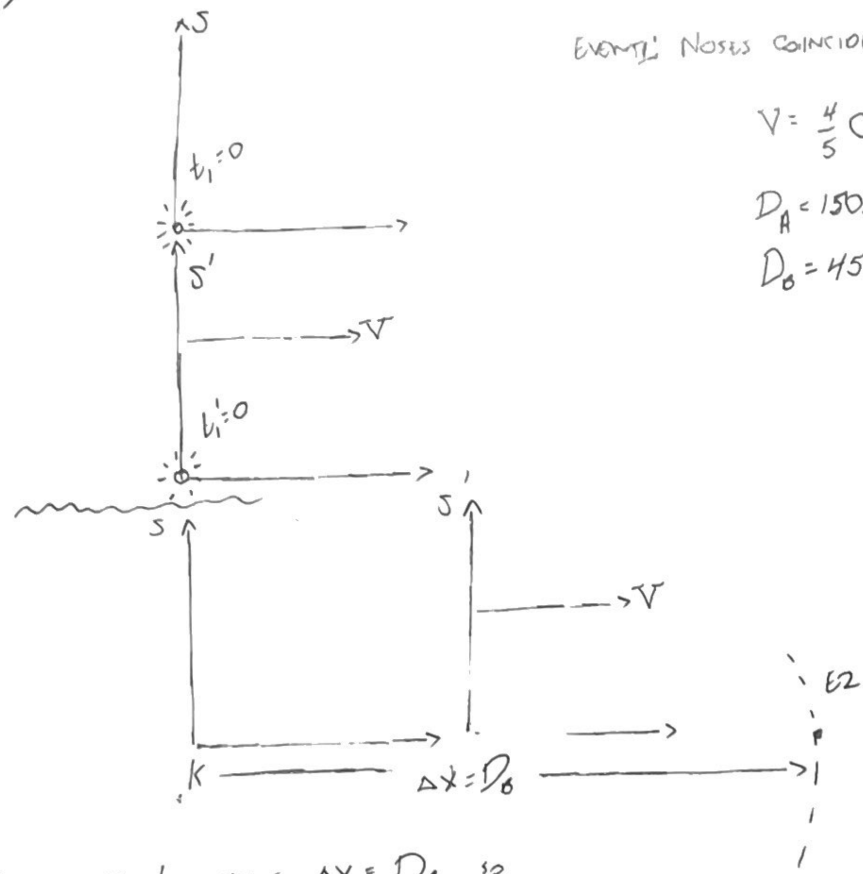
EVENT 1: NOSES COINCIDE @ $x_1 = x_1' = 0$ @ $t_1 = t_1' = 0$

$V = \frac{4}{5}c$

$D_A = 150.m$

$D_B = 450.m$

EVENT 2: T_B RECEIVES THE LIGHT SIGNAL.



a) $\Delta x = c\Delta t$ WHERE $\Delta x = D_B$ SO

$$\Delta t = \frac{\Delta x}{c} = \frac{450.m}{c} = t_2 - \cancel{t_1}$$ so $t_2 = 450m/c$

$x_2 = 450m$

b) $$t_2' = \gamma\left(t_2 - \frac{Vx_2}{c^2}\right) = \frac{1}{\sqrt{1-(4/5)^2}}\left(450m/c - \frac{4}{5}c\,\frac{450m}{c^2}\right)$$

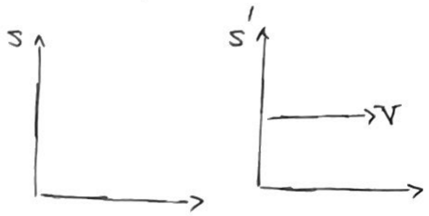
$$= \frac{1}{3/5}\,450m/c\left(1 - \frac{4}{5}\right) = \frac{5}{3}\cdot\frac{1}{5}\left(450m/c\right) = \frac{1}{3}\left(450m/c\right) = 150m/c$$

c) $$x_2' = \gamma(x_2 - Vt_2) = \frac{1}{\sqrt{1-(4/5)^2}}\left(450m - \frac{4}{5}c\cdot 450m/c\right) = \frac{5}{3}\cdot\frac{1}{5}\cdot 450m \quad 150m$$

4. $D_A = 150.m$ LET OUR FRAME BE THE S FRAME AND S' BE THE SPACESHIP FRAME.

$V = \frac{4}{5}c$

$v_x' = +\frac{3}{5}c$



a) $$v_x = \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}} = \frac{+3/5c + (\frac{4}{5}c)}{1 + \frac{(+\frac{3}{5}c)(\frac{4}{5}c)}{c^2}} = \frac{\frac{7}{5}c}{1 + \frac{12}{25}} = \frac{\frac{7}{5}c}{\frac{37}{25}} = \frac{\cancel{25}^{5}}{37} \cdot \frac{7}{\cancel{5}} c = \frac{35}{37}c$$

$$\left[v_x = \frac{35}{37}c \right]$$

b) ACCORDING TO OUR FRAME, THE MOVING SPACESHIP IS LENGTH CONTRACTED TO..

(i) $$d_s = D_A\sqrt{1 - V^2/c^2} = 150.m\sqrt{1-(4/5)^2} = 150.m \cdot \frac{3}{5} = 90m$$

$$[d_s = 90m]$$

(ii) THE SHIP IS AT REST RELATIVE TO ITSELF, SO ...

$$[D = 150m]$$

(iii) THE BULLET MOVES AT A SPEED OF $+\frac{3}{5}c$ RELATIVE TO THE SHIP, SO THE SHIP MOVES AT A SPEED $-\frac{3}{5}c$ RELATIVE TO THE BULLET...

$$d_b = D_A\sqrt{1-(-3/5)^2} = 120m$$

$$[d_b = 120m]$$

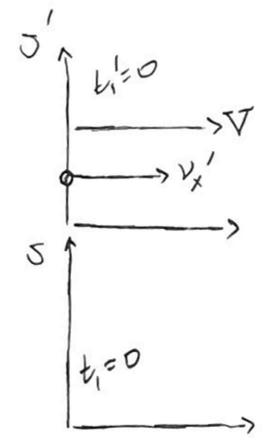
c) THE BULLET IS TRAVELING AT A SPEED $v_x' = \frac{3}{5}c$ RELATIVE TO THE SPACESHIP.

THE DISTANCE TRAVELED, RELATIVE TO THE SPACESHIP, IS $\Delta x' = 150.m$

SO $$v_x' = \frac{\Delta x'}{\Delta t'} \quad \therefore \quad \Delta t' = \frac{\Delta x'}{v_x'} = \frac{150.m}{\frac{3}{5}c} = 250.m/c$$

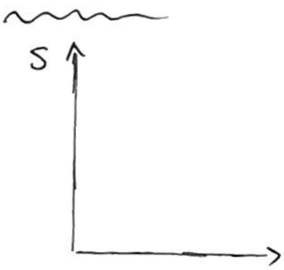
$$[\Delta t' = 250.m/c]$$

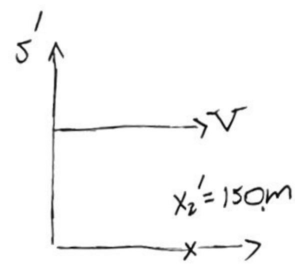
LET THE BULLET BE FIRED AT THE MOMENT WHEN THE ORIGINS PASS ONE ANOTHER.



EVENT 1: ORIGINS PASS : BULLET IS FIRED FROM BACK OF SPACESHIP A

NOTICE THIS ORIGIN IS AT THE BACK OF THE SPACESHIP





EVENT 2: BULLET STRIKES THE FRONT OF THE SHIP.

@ $x_2' = 150m$ @ $t_2' = 250.m$

so

$$t_2 = \gamma\left(t_2' + V\frac{x_2'}{c^2}\right) = \frac{1}{\sqrt{1-(4/5)^2}}\left(250.\frac{m}{c} + \frac{\left(\frac{4}{5}c\right)(150m)}{c^2}\right)$$

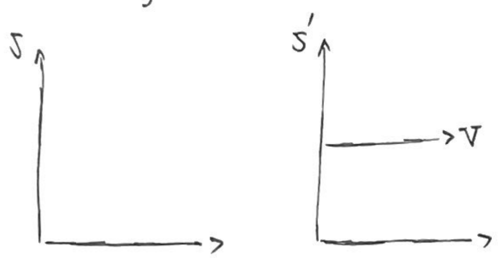
$$= \frac{1}{3/5}\left(250.\frac{m}{c} + 120\frac{m}{c}\right) = \frac{5}{3}\left(370\frac{m}{c}\right)$$

$$\left[t_2 = 617 m/c\right]$$

5. $D_A = 150.m$ LET OUR FRAME BE THE S FRAME AND S' BE THE SPACESHIP FRAME.

$V = \frac{4}{5}c$

$V_x = -\frac{3}{5}c$



a) $$v_x' = \frac{v_x - V}{1 - \frac{v_x V}{c^2}} = \frac{-\frac{3}{5}c - \frac{4}{5}c}{1 - \left(-\frac{3}{5}c\right)\left(\frac{4}{5}c\right)/c^2} = \frac{-\frac{7}{5}c}{1 + \frac{12}{25}} = \frac{-\frac{7}{5}c}{\frac{37}{25}} = \frac{25}{37} \cdot -\frac{7}{5}c = -\frac{35}{37}c$$

$$\left[v_x' = -\frac{35}{37}c\right]$$

b) THE BULLET MOVES AT A SPEED OF $-\frac{35}{37}c$ RELATIVE TO THE SHIP, SO THE SHIP MOVES AT A SPEED OF $\frac{35}{37}c$ RELATIVE TO THE BULLET ..

$$d_B = D_A\sqrt{1 - (35/37)^2} = 48.6m$$

$$[d_B = 48.6m]$$

c) THE BULLET IS TRAVELING AT A SPEED $V_x' = -\frac{35}{37}c$ RELATIVE TO THE SPACESHIP.

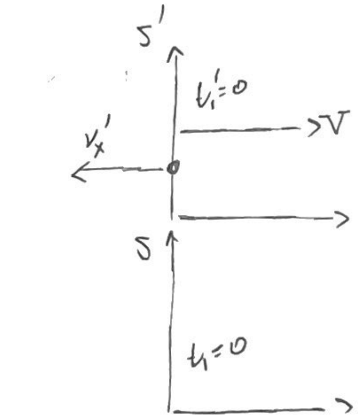
THE DISTANCE TRAVELED, RELATIVE TO THE SPACESHIP, IS $\Delta x' = -150m$

$$\text{so} \quad v_x' = \frac{\Delta x'}{\Delta t'} \therefore \Delta t' = \frac{\Delta x'}{v_x'} = \frac{-150m}{-\frac{35}{37}c} = 159m/c$$

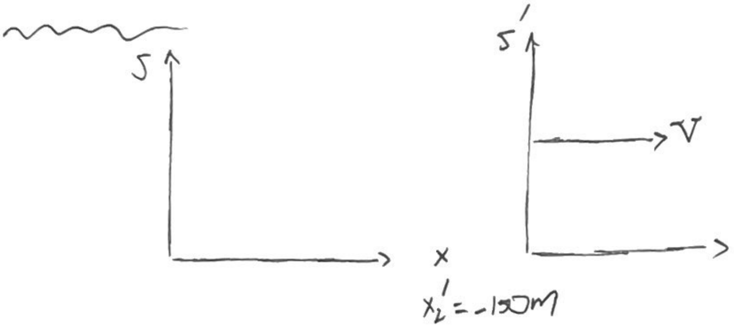
$$\text{so} \quad [t_2' = 159m/c]$$

LET THE BULLET BE FIRED FROM THE NOSE OF THE SHIP AT THE MOMENT WHEN THE ORIGINS PASS ONE ANOTHER...

HERE WE'LL CHOOSE THE ORIGIN OF THE SHIP TO RESIDE AT THE FRONT OF THE SHIP.



EVENT 1: ORIGINS PASS: BULLET IS FIRED FROM FRONT OF SPACESHIP A



EVENT 2: BULLET STRIKES THE BACK OF THE SHIP @ $x_2' = -150m$ @ $t_2' = 159\, m/c$

so

$$t_2 = \gamma\left(t_2' + \frac{V x_2'}{c^2}\right) = \frac{1}{\sqrt{1-(4/5)^2}}\left(159\, m/c + \frac{\left(\frac{4}{5}c\right)(-150m)}{c^2}\right)$$

$$= \frac{5}{3}\left(159\frac{m}{c} - 120\frac{m}{c}\right) = 65\, m/c$$

so

$$\boxed{t_2 = 65\, m/c}$$