Exercise 1

Suppose $\hat{\rho}_0 = \frac{1}{2}(\hat{I} + r\hat{\sigma}_x)$ with $0 \le r \le 1$.

a.) $\hat{u} = e^{-i\alpha\hat{\sigma}_x/2}$: $\hat{\rho} = \hat{u}\rho_0\hat{u}^{\dagger}$: $e^{-i\alpha\hat{\sigma}_x/2} = (\cos(\alpha/2)\hat{I} - i\hat{\sigma}_x\sin(\alpha/2))$

$e^{i\alpha\hat{\sigma}_x/2} = (\cos(\alpha/2)\hat{I} + i\hat{\sigma}_x\sin(\alpha/2))$

$$\hat{\rho} = (\cos(\alpha/2)\hat{I} - i\hat{\sigma}_x\sin(\alpha/2)) \cdot \frac{1}{2}(\hat{I} + r\hat{\sigma}_x) \cdot (\cos(\alpha/2)\hat{I} + i\hat{\sigma}_x\sin(\alpha/2))$$

$$= \frac{1}{2}\left(\cos(\alpha/2)\hat{I}^2 + r\cos(\alpha/2)\hat{I}\hat{\sigma}_x - i\hat{\sigma}_x\hat{I}\sin(\alpha/2) - ir\hat{\sigma}_x^2\sin(\alpha/2)\right)(\cos(\alpha/2)\hat{I} + i\hat{\sigma}_x\sin(\alpha/2))$$

$$= \frac{1}{2}\Big(\cos(\alpha/2)^2\hat{I}^3 + r\cos^2(\alpha/2)\hat{I}\hat{\sigma}_x\hat{I} - \cancel{\cos(\alpha/2)\sin(\alpha/2)i\hat{\sigma}_x\hat{I}^2} - \cancel{\sin(\alpha/2)\cos(\alpha/2)ir\hat{\sigma}_x^2\hat{I}}$$

$$+ \cancel{\cos(\alpha/2)\sin(\alpha/2)i\hat{\sigma}_x\hat{I}^2} + \cancel{\cos(\alpha/2)\sin(\alpha/2)ir\hat{I}\sigma_x^2} + \sin^2(\alpha/2)\hat{\sigma}_x^2\hat{I} + \sin^2(\alpha/2)r\hat{\sigma}_x^3\Big)$$

$$= \frac{1}{2}\left(\cos^2(\alpha/2)\overset{\hat{I}}{\cancel{\hat{I}^3}} + r\cos^2(\alpha/2)\overset{\hat{\sigma}_x}{\cancel{\hat{I}^2\hat{\sigma}_x}} + \sin^2(\alpha/2)\overset{\hat{I}}{\cancel{\hat{\sigma}_x^2\hat{I}}} + r\sin^2(\alpha/2)\overset{\hat{\sigma}_x}{\cancel{\hat{\sigma}_x^3}}\right)$$

$$= \frac{1}{2}\left(\cos^2(\alpha/2)\hat{I} + \sin^2(\alpha/2)\hat{I} + r\cos^2(\alpha/2)\hat{\sigma}_x + r\sin^2(\alpha/2)\hat{\sigma}_x\right)$$

$$= \frac{1}{2}\left(\overset{1}{\cancel{(\cos^2(\alpha/2) + \sin^2(\alpha/2))}}\hat{I} + r\overset{1}{\cancel{(\cos^2(\alpha/2) + \sin^2(\alpha/2))}}\hat{\sigma}_x\right) = \frac{1}{2}(\hat{I} + r\hat{\sigma}_x)$$

$$= \frac{1}{2}\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + r\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}1 & r\\ r & 1\end{pmatrix}$$

$$\boxed{\hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & r\\ r & 1\end{pmatrix}}$$

b.) $\hat{u} = e^{-i\alpha\hat{\sigma}_y/2}$: $\hat{\rho} = \hat{u}\hat{\rho}_0\hat{u}$: $e^{-i\alpha\hat{\sigma}_y/2} = (\cos(\alpha/2)\hat{I} - i\hat{\sigma}_y\sin(\alpha/2))$ — u

$e^{i\alpha\hat{\sigma}_y/2} = (\cos(\alpha/2)\hat{I} + i\hat{\sigma}_y\sin(\alpha/2))$ — v

$$\hat{\rho} = (\cos(\alpha/2)\hat{I} - i\hat{\sigma}_y\sin(\alpha/2)) \cdot \frac{1}{2}(\hat{I} + r\hat{\sigma}_x) \cdot (\cos(\alpha/2)\hat{I} + i\hat{\sigma}_y\sin(\alpha/2))$$

$$= \frac{1}{2}\left(\cos(\alpha/2)\hat{I}^2 + r\cos(\alpha/2)\hat{I}\hat{\sigma}_x - i\sin(\alpha/2)\hat{\sigma}_y\hat{I} - ir\sin(\alpha/2)\hat{\sigma}_y\hat{\sigma}_x\right)\left(\cos(\alpha/2)\hat{I} + i\sin(\alpha/2)\hat{\sigma}_y\right)$$

$$= \frac{1}{2}\Big(\cos^2(\alpha/2)\overset{\hat{I}}{\cancel{\hat{I}^3}} + r\cos^2(\alpha/2)\overset{\hat{\sigma}_x}{\cancel{\hat{I}\hat{\sigma}_x\hat{I}}} - \cancel{i\cos(\alpha/2)\sin(\alpha/2)\hat{\sigma}_y\hat{I}^2} - ir\cos(\alpha/2)\sin(\alpha/2)\overset{-i\hat{\sigma}_z}{\cancel{\hat{\sigma}_y\hat{\sigma}_x}}\hat{I}$$

$$+ \cancel{i\cos(\alpha/2)\sin(\alpha/2)\hat{I}^2\hat{\sigma}_y} + ir\cos(\alpha/2)\sin(\alpha/2)\hat{I}\overset{+i\hat{\sigma}_z}{\cancel{\hat{\sigma}_x\hat{\sigma}_y}} + \sin^2(\alpha/2)\overset{\hat{I}}{\cancel{\hat{\sigma}_y\hat{I}\hat{\sigma}_y}} + r\sin^2(\alpha/2)\overset{-\hat{\sigma}_x}{\cancel{\hat{\sigma}_y\hat{\sigma}_x\hat{\sigma}_y}}\Big)$$

$$= \frac{1}{2}\left(\overset{1}{\cancel{(\cos^2(\alpha/2) + \sin^2(\alpha/2))}}\hat{I} + r\overset{\cos(\alpha)}{\cancel{(\cos^2(\alpha/2) - \sin^2(\alpha/2))}}\hat{\sigma}_x - \overset{\sin(\alpha)}{\cancel{2r\cos(\alpha/2)\sin(\alpha/2)}}\hat{\sigma}_z\right)$$

$$= \frac{1}{2}\left(\hat{I} + r\cos(\alpha)\hat{\sigma}_x - r\sin(\alpha)\hat{\sigma}_z\right) = \frac{1}{2}\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}0 & r\cos(\alpha)\\ r\cos(\alpha) & 0\end{pmatrix} - \begin{pmatrix}r\sin(\alpha) & 0\\ 0 & -r\sin(\alpha)\end{pmatrix}\right)$$

$$\boxed{\hat{\rho} = \frac{1}{2}\begin{pmatrix}1 - r\sin(\alpha) & r\cos(\alpha)\\ r\cos(\alpha) & 1 + r\sin(\alpha)\end{pmatrix}}$$

Exercise 2

a.) For $|\nearrow\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ construct density operator $\hat{\rho}$ and show $\hat{\rho}^2 = \hat{\rho}$.

$$\hat{\rho} = |\nearrow\rangle\langle\nearrow| = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} \quad \therefore \quad \hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

$$\hat{\rho}^2 = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}2 & 2\\2 & 2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} = \hat{\rho} \quad \checkmark$$

$$\boxed{\hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix},\ \hat{\rho} \text{ is pure}}$$

b.) For $\hat{\rho} = \frac{1}{2}(\hat{I} + r\hat{\sigma}_y)$ find condition on r s.t. $\hat{\rho}$ is pure.

$$\hat{I} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix},\ \hat{\sigma}_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \quad \therefore \quad \hat{\rho} = \frac{1}{2}\left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} + \begin{pmatrix}0 & -ri\\ri & 0\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}1 & -ir\\ir & 1\end{pmatrix}$$

$$\hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & -ir\\ir & 1\end{pmatrix} : \hat{\rho}^2 = \frac{1}{2}\begin{pmatrix}1 & -ir\\ir & 1\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}1 & -ir\\ir & 1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}1+r^2 & -2ir\\2ir & r^2+1\end{pmatrix}$$

$$\hat{\rho}^2 = \frac{1}{4}\begin{pmatrix}1+r^2 & -2ir\\2ir & 1+r^2\end{pmatrix},\ \text{if } r=1,\ \hat{\rho}^2 = \frac{1}{4}\begin{pmatrix}2 & -2i\\2i & 2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & -i\\i & 1\end{pmatrix} = \hat{\rho}$$

$$\boxed{\hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & -ir\\ir & 1\end{pmatrix},\ r=1,\ \hat{\rho} \text{ is pure}}$$

c.) For $\hat{\rho} = \frac{1}{2}(\hat{I} + r_x\hat{\sigma}_x + r_y\hat{\sigma}_y)$ find condition on r_x, r_y s.t. $\hat{\rho}$ is pure.

$$\hat{I} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix},\ \hat{\sigma}_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix},\ \hat{\sigma}_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$$

$$\hat{\rho} = \frac{1}{2}\left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} + \begin{pmatrix}0 & r_x\\r_x & 0\end{pmatrix} + \begin{pmatrix}0 & -ir_y\\ir_y & 0\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}1 & r_x-ir_y\\r_x+ir_y & 1\end{pmatrix} : \hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & r_x-ir_y\\r_x+ir_y & 1\end{pmatrix}$$

$$\hat{\rho}^2 = \frac{1}{2}\begin{pmatrix}1 & r_x-ir_y\\r_x+ir_y & 1\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}1 & r_x-ir_y\\r_x+ir_y & 1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}1+r_x^2+r_y^2 & 2r_x-2ir_y\\2r_x+2ir_y & 1+r_x^2+r_y^2\end{pmatrix}$$

$$\frac{1+r_x^2+r_y^2}{4} = \frac{1}{2} \qquad 1+r_x^2+r_y^2 = 2 \qquad r_x^2+r_y^2 = 1$$

$$\boxed{\hat{\rho} = \frac{1}{2}\begin{pmatrix}1 & r_x-ir_y\\r_x+ir_y & 1\end{pmatrix},\ r_x^2+r_y^2=1,\ \hat{\rho} \text{ is pure}}$$

Exercise 3

$$\hat{u}(\lambda) = e^{-i\lambda\hat{\sigma}_z/2} = \left(\cos(\lambda/2)\hat{I} - i\hat{\sigma}_z\sin(\lambda/2)\right) = \begin{pmatrix}\cos(\lambda/2) & 0 \\ 0 & \cos(\lambda/2)\end{pmatrix} - \begin{pmatrix} i\sin(\lambda/2) & 0 \\ 0 & -i\sin(\lambda/2)\end{pmatrix}$$

$$\hat{u}(\lambda) = \begin{pmatrix}\cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2)\end{pmatrix}$$

$$H = 4\left[\frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} + \left(\langle\psi|\frac{\partial|\psi\rangle}{\partial\lambda}\right)^2\right]$$

a.) Get QFI for $|\psi\rangle = |0\rangle$

$$|\psi(\lambda)\rangle = \hat{u}(\lambda)|\psi\rangle = \begin{pmatrix}\cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2)\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix}$$

$$|\psi(\lambda)\rangle = \begin{pmatrix}\cos(\lambda/2) - i\sin(\lambda/2) \\ 0\end{pmatrix}, \quad \langle\psi(\lambda)| = \begin{pmatrix}\cos(\lambda/2) + i\sin(\lambda/2) & 0\end{pmatrix}$$

$$\frac{\partial|\psi\rangle}{\partial\lambda} = \begin{pmatrix}-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ 0\end{pmatrix}, \quad \frac{\partial\langle\psi|}{\partial\lambda} = \begin{pmatrix}-\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) & 0\end{pmatrix}$$

$$\frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \begin{pmatrix}-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) & 0\end{pmatrix}\begin{pmatrix}-\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) \\ 0\end{pmatrix}$$

$$= \frac{1}{4}\sin^2(\lambda/2) - \cancel{\frac{i}{4}\sin(\lambda/2)\cos(\lambda/2)} + \cancel{\frac{i}{4}\sin(\lambda/2)\cos(\lambda/2)} + \frac{1}{4}\cos^2(\lambda/2)$$

$$= \frac{1}{4}\left(\sin^2(\lambda/2) + \cos^2(\lambda/2)\right) = \frac{1}{4} \quad \therefore \quad \frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{4}$$

$$\langle\psi|\frac{\partial|\psi\rangle}{\partial\lambda} = \begin{pmatrix}\cos(\lambda/2) + i\sin(\lambda/2) & 0\end{pmatrix}\begin{pmatrix}-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ 0\end{pmatrix}$$

$$= \cancel{-\frac{1}{2}\sin(\lambda/2)\cos(\lambda/2)} - \frac{i}{2}\cos^2(\lambda/2) - \frac{i}{2}\sin^2(\lambda/2) + \cancel{\frac{1}{2}\sin(\lambda/2)\cos(\lambda/2)}$$

$$= \frac{-i}{2}\left(\cancel{\cos^2(\lambda/2) + \sin^2(\lambda/2)}\right) = \frac{-i}{2}$$

$$\left(\langle\psi|\frac{\partial\psi\rangle}{\partial\lambda}\right)^2 = \frac{-i}{2}\cdot\frac{i}{2} = \frac{1}{4}$$

$$\therefore \quad H = 4\left[1/4 - 1/4\right] = 4\cdot 0 = 0$$

$$\boxed{H = 0}$$

Exercise 3 Continued

b.) Get QFI for $|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

$$|+(\lambda)\rangle = \hat{u}(\lambda)|+\rangle = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|+(\lambda)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) \\ \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix}, \quad \langle +(\lambda)| = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\lambda/2) + i\sin(\lambda/2) & \cos(\lambda/2) - i\sin(\lambda/2) \end{pmatrix}$$

$$\frac{\partial |+\rangle}{\partial \lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) \end{pmatrix}, \quad \frac{\partial \langle +|}{\partial \lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) & -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \end{pmatrix}$$

$$\frac{\partial \langle +|}{\partial \lambda} \cdot \frac{\partial |+\rangle}{\partial \lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) & -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) \end{pmatrix}$$

$$= \frac{1}{2}\left(\left(-\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2)\right)\left(-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2)\right) + \left(-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2)\right)\left(-\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2)\right)\right)$$

$$= \frac{1}{2}\left(\frac{1}{4}\sin^2(\lambda/2) + \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} - \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} + \frac{1}{4}\cos^2(\lambda/2) \right.$$
$$\left. + \frac{1}{4}\sin^2(\lambda/2) - \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} + \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} + \frac{1}{4}\cos^2(\lambda/2) \right)$$

$$= \frac{1}{2}\left(\frac{1}{2}\left(\overbrace{\sin^2(\lambda/2) + \cos^2(\lambda/2)}^{1}\right)\right) = \frac{1}{4} \quad \therefore \quad \frac{\partial \langle +|}{\partial \lambda} \cdot \frac{\partial |+\rangle}{\partial \lambda} = \frac{1}{4}$$

$$\langle +| \frac{\partial |+\rangle}{\partial \lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\lambda/2) + i\sin(\lambda/2) & \cos(\lambda/2) - i\sin(\lambda/2) \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) \end{pmatrix}$$

$$= \frac{1}{2}\left(\left(\cos(\lambda/2) + i\sin(\lambda/2)\right)\left(-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2)\right) + \left(\cos(\lambda/2) - i\sin(\lambda/2)\right)\left(-\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2)\right)\right)$$

$$= \frac{1}{2}\left(\cancel{-\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} - \cancel{\frac{i}{2}\cos^2(\lambda/2)} - \cancel{\frac{i}{2}\sin^2(\lambda/2)} + \cancel{\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} \right.$$
$$\left. \cancel{- \frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} + \cancel{\frac{i}{2}\cos^2(\lambda/2)} + \cancel{\frac{i}{2}\sin^2(\lambda/2)} + \cancel{\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} \right) = 0$$

$$\left(\langle +| \frac{\partial |+\rangle}{\partial \lambda}\right)^2 = 0 \qquad \therefore \quad H = 4\left[\frac{1}{4} + 0\right] = 1$$

$$\boxed{H = 1}$$

Exercise 3 continued

c.) Get QFI for $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

$$|\psi(\lambda)\rangle = \hat{U}(\lambda)|\psi\rangle = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$|\psi(\lambda)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) \\ i\cos(\lambda/2) - \sin(\lambda/2) \end{pmatrix}, \quad \langle\psi(\lambda)| = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\lambda/2) + i\sin(\lambda/2) & -i\cos(\lambda/2) - \sin(\lambda/2) \end{pmatrix}$$

$$\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ -\frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2) \end{pmatrix}, \quad \frac{\partial\langle\psi|}{\partial\lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) & \frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2) \end{pmatrix}$$

$$\frac{\partial\langle\psi|}{\partial\lambda} \cdot \frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{2} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2) & \frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2) \end{pmatrix} \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ -\frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2) \end{pmatrix}$$

$$= \frac{1}{2}\left(\left(-\frac{1}{2}\sin(\lambda/2) + \frac{i}{2}\cos(\lambda/2)\right)\left(-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2)\right) + \left(\frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2)\right)\left(-\frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2)\right) \right)$$

$$= \frac{1}{2}\Big(\frac{1}{4}\sin^2(\lambda/2) + \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} - \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} + \frac{1}{4}\sin^2(\lambda/2)$$
$$+ \frac{1}{4}\sin^2(\lambda/2) - \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} + \cancel{\frac{i}{4}\cos(\lambda/2)\sin(\lambda/2)} + \frac{1}{4}\cos^2(\lambda/2) \Big)$$

$$= \frac{1}{2}\left(\frac{1}{2}\left(\cancelto{1}{\sin^2(\lambda/2) + \cos^2(\lambda/2)}\right)\right) = \frac{1}{4} \quad \therefore \quad \frac{\partial\langle\psi|}{\partial\lambda}\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{4}$$

$$\langle\psi| \cdot \frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{2} \begin{pmatrix} \cos(\lambda/2) + i\sin(\lambda/2) & -i\cos(\lambda/2) - \sin(\lambda/2) \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2) \\ -\frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2) \end{pmatrix}$$

$$= \frac{1}{2}\left(\left(\cos(\lambda/2) + i\sin(\lambda/2)\right)\left(-\frac{1}{2}\sin(\lambda/2) - \frac{i}{2}\cos(\lambda/2)\right) + \left(-i\cos(\lambda/2) - \sin(\lambda/2)\right)\left(-\frac{i}{2}\sin(\lambda/2) - \frac{1}{2}\cos(\lambda/2)\right) \right)$$

$$= \frac{1}{2}\Big(\cancel{-\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} - \cancel{\frac{i}{2}\cos^2(\lambda/2)} - \cancel{\frac{i}{2}\sin^2(\lambda/2)} + \cancel{\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)}$$
$$- \cancel{\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} + \cancel{\frac{i}{2}\cos^2(\lambda/2)} + \cancel{\frac{i}{2}\sin^2(\lambda/2)} + \cancel{\frac{1}{2}\cos(\lambda/2)\sin(\lambda/2)} \Big) = 0$$

$$\left(\langle\psi|\frac{\partial|\psi\rangle}{\partial\lambda}\right)^2 = 0 \quad \therefore \quad H = 4\left[1/4 + 0\right] = 1$$

$$\boxed{H = 1}$$

Exercise 3 continued

d.)

$$H = 4\left[\frac{\partial\langle\psi|}{\partial\lambda}\frac{\partial|\psi\rangle}{\partial\lambda} + \left(\langle\psi|\cdot\frac{\partial|\psi\rangle}{\partial\lambda}\right)^2\right]$$

$$|\psi(\lambda)\rangle = \hat{u}(\lambda)|\psi\rangle = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$|\psi(\lambda)\rangle = \begin{pmatrix} a_0\left(\cos(\lambda/2) - i\sin(\lambda/2)\right) \\ a_1\left(\cos(\lambda/2) + i\sin(\lambda/2)\right) \end{pmatrix} : \langle\psi(\lambda)| = \left(a_0\left(\cos(\lambda/2) + i\sin(\lambda/2)\right) + a_1\left(\cos(\lambda/2) - i\sin(\lambda/2)\right)\right)$$

$$\frac{\partial|\psi\rangle}{\partial\lambda} = \begin{pmatrix} a_0/2\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right) \\ a_1/2\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right) \end{pmatrix} : \frac{\partial\langle\psi|}{\partial\lambda} = \left(\frac{a_0}{2}\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right) + \frac{a_1}{2}\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right)\right)$$

$$\frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \left(\frac{a_0}{2}\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right) + \frac{a_1}{2}\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right)\right)\begin{pmatrix} a_0/2\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right) \\ a_1/2\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right) \end{pmatrix}$$

$$= \frac{a_0^2}{4}\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right)\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right) + \frac{a_1^2}{4}\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right)\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right)$$

$$= \frac{a_0^2}{4}\left(\overbrace{\sin^2(\lambda/2) + \cos^2(\lambda/2)}^{1}\right) + \frac{a_1^2}{4}\left(\overbrace{\sin^2(\lambda/2) + \cos^2(\lambda/2)}^{1}\right) = \frac{a_0^2}{4} + \frac{a_1^2}{4} = \frac{1}{4}\left(a_0^2 + a_1^2\right)$$

$$\langle\psi|\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \left(a_0\left(\cos(\lambda/2) + i\sin(\lambda/2)\right) + a_1\left(\cos(\lambda/2) - i\sin(\lambda/2)\right)\right)\begin{pmatrix} a_0/2\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right) \\ a_1/2\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right) \end{pmatrix}$$

$$= \frac{a_0^2}{2}\left(\cos(\lambda/2) + i\sin(\lambda/2)\right)\left(-\sin(\lambda/2) - i\cos(\lambda/2)\right) + \frac{a_1^2}{2}\left(\cos(\lambda/2) - i\sin(\lambda/2)\right)\left(-\sin(\lambda/2) + i\cos(\lambda/2)\right)$$

$$= \frac{a_0^2}{2}\left(-i\left(\overbrace{\cos^2(\lambda/2) + \sin^2(\lambda/2)}^{1}\right)\right) + \frac{a^2}{2}\left(i\left(\overbrace{\cos^2(\lambda/2) + \sin^2(\lambda/2)}^{1}\right)\right) = -\frac{i}{2}a_0^2 + \frac{i}{2}(a_1^2) = \frac{i}{2}\left(a_1^2 - a_0^2\right)$$

$$\left(\langle\psi|\cdot\frac{\partial|\psi\rangle}{\partial\lambda}\right)^2 = \frac{-1}{4}\left(a_1^2 - a_0^2\right)^2$$

$$\therefore\ H = 4\left(\frac{1}{4}\left(a_0^2 + a_1^2\right) - \frac{1}{4}\left(a_1^2 - a_0^2\right)^2\right) = \left(a_0^2 + a_1^2 - \left(a_1^2 - a_0^2\right)^2\right) = a_0^2 + a_1^2 - \left(a_1^4 + a_0^4 - 2a_0^2a_1^2\right)$$

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right),\ H = a_0^2 + a_1^2 - \left(a_1^2 - a_0^2\right)^2}$$

Exercise 4

a.) Get QFI for $|\psi\rangle = \frac{1}{2}\left(|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle + |1\rangle|1\rangle\right)$

$$\hat{u}(\lambda) = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix}$$

$$|\psi(\lambda)\rangle = \hat{u}(\lambda)|\psi\rangle = \frac{1}{2}\left(\hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|0\rangle + \hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|1\rangle + \hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|0\rangle + \hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|1\rangle\right)$$

$$\hat{u}(\lambda)|0\rangle = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \cos(\lambda/2) - i\sin(\lambda/2)$$

$$\hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|0\rangle = (\cos(\lambda/2) - i\sin(\lambda/2))(\cos(\lambda/2) - i\sin(\lambda/2)) = \cos^2(\lambda/2) - \sin^2(\lambda/2) - 2i\cos(\lambda/2)\sin(\lambda/2)$$

$$\hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|0\rangle = \cos(\lambda) - i\sin(\lambda)$$

$$\hat{u}(\lambda)|1\rangle = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \cos(\lambda/2) + i\sin(\lambda/2)$$

$$\hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|1\rangle = (\cos(\lambda/2) - i\sin(\lambda/2))(\cos(\lambda/2) + i\sin(\lambda/2)) = \cos^2(\lambda/2) + \sin^2(\lambda/2) = 1$$

$$\hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|1\rangle = 1$$

$$\hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|0\rangle = (\cos(\lambda/2) + i\sin(\lambda/2))(\cos(\lambda/2) - i\sin(\lambda/2)) = \cos^2(\lambda/2) + \sin^2(\lambda/2) = 1$$

$$\hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|0\rangle = 1$$

$$\hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|1\rangle = (\cos(\lambda/2) + i\sin(\lambda/2))(\cos(\lambda/2) + i\sin(\lambda/2)) = \cos^2(\lambda/2) - \sin^2(\lambda/2) + 2i\cos(\lambda/2)\sin(\lambda/2)$$

$$\hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|1\rangle = \cos(\lambda) + i\sin(\lambda)$$

$$|\psi(\lambda)\rangle = \frac{1}{2}\left((\cos(\lambda) - i\sin(\lambda))|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle + (\cos(\lambda) + i\sin(\lambda))|1\rangle|1\rangle\right)$$

$$\langle\psi(\lambda)| = \frac{1}{2}\left((\cos(\lambda) + i\sin(\lambda))\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + (\cos(\lambda) - i\sin(\lambda))\langle 1|\langle 1|\right)$$

$$H = 4\left[\frac{\partial\langle\psi|}{\partial\lambda} \cdot \frac{\partial|\psi\rangle}{\partial\lambda} + \left(\langle\psi|\frac{\partial|\psi\rangle}{\partial\lambda}\right)^2\right]$$

$$\frac{\partial\langle\psi|}{\partial\lambda} = \frac{1}{2}\left((-\sin(\lambda) + i\cos(\lambda))\langle 0|\langle 0| + (-\sin(\lambda) - i\cos(\lambda))\langle 1|\langle 1|\right)$$

$$\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{2}\left((-\sin(\lambda) - i\cos(\lambda))|0\rangle|0\rangle + (-\sin(\lambda) + i\cos(\lambda))|1\rangle|1\rangle\right)$$

Exercise 4 continued

$$\frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{4}\Big((-\sin(\lambda)+i\cos(\lambda))\langle 0|\langle 0| + (-\sin(\lambda)-i\cos(\lambda))\langle 1|\langle 1|\Big)$$
$$\cdot\Big((-\sin(\lambda)-i\cos(\lambda))|0\rangle|0\rangle + (-\sin(\lambda)+i\cos(\lambda))|1\rangle|1\rangle\Big)$$

$$= \frac{1}{4}\Big((-\sin(\lambda)+i\cos(\lambda))(-\sin(\lambda)-i\cos(\lambda))\langle 0|\langle 0\overset{1}{\cancel{|0\rangle}}|0\rangle + (-\sin(\lambda)+i\cos(\lambda))(-\sin(\lambda)+i\cos(\lambda))\langle 0|\langle\overset{0}{\cancel{0|1\rangle}}|1\rangle$$
$$+ (-\sin(\lambda)-i\cos(\lambda))(-\sin(\lambda)-i\cos(\lambda))\langle 1|\langle\underset{0}{\cancel{1|0\rangle}}|0\rangle + (-\sin(\lambda)-i\cos(\lambda))(-\sin(\lambda)+i\cos(\lambda))\langle 1|\langle\overset{1}{\cancel{1|1\rangle}}|1\rangle\Big)$$

$$= \frac{1}{4}\Big(\sin^2(\lambda) + \cancel{i\sin(\lambda)\cos(\lambda)} - \cancel{i\sin(\lambda)\cos(\lambda)} + \sin^2(\lambda) + \sin^2(\lambda) - \cancel{i\sin(\lambda)\cos(\lambda)} + \cancel{i\sin(\lambda)\cos(\lambda)} + \cos^2(\lambda)\Big)$$

$$= \frac{1}{4}\Big(2(\overset{1}{\cancel{\sin^2(\lambda)+\cos^2(\lambda)}})\Big) = \frac{1}{2} \quad \therefore \quad \frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{2}$$

$$\langle\psi|\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{4}\Big((\cos(\lambda)+i\sin(\lambda))\langle 0|\langle 0| + \langle 0|\langle 1| + \langle 1|\langle 0| + (\cos(\lambda)-i\sin(\lambda))\langle 1|\langle 1|\Big)$$
$$\cdot\Big((-\sin(\lambda)-i\cos(\lambda))|0\rangle|0\rangle + (-\sin(\lambda)+i\cos(\lambda))|1\rangle|1\rangle\Big)$$

$$= \frac{1}{4}\Big((\cos(\lambda)+i\sin(\lambda))(-\sin(\lambda)-i\cos(\lambda))\langle 0|\langle\overset{1}{\cancel{0|0\rangle}}|0\rangle + (\cos(\lambda)+i\sin(\lambda))(-\sin(\lambda)+i\cos(\lambda))\langle 0|\langle\overset{0}{\cancel{0|1\rangle}}|1\rangle$$
$$+ (-\sin(\lambda)-i\cos(\lambda))\langle 0|\langle\overset{0}{\cancel{1|0\rangle}}|0\rangle + (-\sin(\lambda)+i\cos(\lambda))\langle 0|\langle\overset{0}{\cancel{1|1\rangle}}|1\rangle + (-\sin(\lambda)-i\cos(\lambda))\langle 1|\langle\overset{0}{\cancel{0|0\rangle}}|0\rangle$$
$$+ (-\sin(\lambda)+i\cos(\lambda))\langle 1|\langle\overset{0}{\cancel{0|1\rangle}}|1\rangle + (\cos(\lambda)-i\sin(\lambda))(-\sin(\lambda)-i\cos(\lambda))\langle 1|\langle\overset{0}{\cancel{1|0\rangle}}|0\rangle$$
$$+ (\cos(\lambda)-i\sin(\lambda))(-\sin(\lambda)+i\cos(\lambda))\langle 1|\langle\overset{1}{\cancel{1|1\rangle}}|1\rangle\Big)$$

$$= \frac{1}{4}\Big(-\cancel{\sin(\lambda)\cos(\lambda)} - \cancel{i\cos^2(\lambda)} - \cancel{i\sin^2(\lambda)} + \cancel{\sin(\lambda)\cos(\lambda)} - \cancel{\sin(\lambda)\cos(\lambda)} + \cancel{i\cos^2(\lambda)} + \cancel{i\sin^2(\lambda)} + \cancel{\sin(\lambda)\cos(\lambda)}\Big) = 0$$

$$\therefore \quad H = 4\left(\frac{1}{2} + 0\right) = 2$$

$$\boxed{H = 2}$$

b.) Get QFI for $\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$

$$\hat{u}(\lambda) = \begin{pmatrix} \cos(\lambda/2) - i\sin(\lambda/2) & 0 \\ 0 & \cos(\lambda/2) + i\sin(\lambda/2) \end{pmatrix}$$

$$|\psi(\lambda)\rangle = \hat{u}(\lambda)|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\hat{u}(\lambda)|0\rangle\,\hat{u}(\lambda)|0\rangle + \hat{u}(\lambda)|1\rangle\,\hat{u}(\lambda)|1\rangle\Big)$$

$$\hat{u}(\lambda)|0\rangle\hat{u}(\lambda)|0\rangle = \cos(\lambda) - i\sin(\lambda)$$

$$\hat{u}(\lambda)|1\rangle\hat{u}(\lambda)|1\rangle = \cos(\lambda) + i\sin(\lambda)$$

Exercise 4 continued

$$|\psi(\lambda)\rangle = \frac{1}{\sqrt{2}}\Big((\cos(\lambda) - i\sin(\lambda))|0\rangle|0\rangle + (\cos(\lambda) + i\sin(\lambda))|1\rangle|1\rangle \Big)$$

$$\langle\psi(\lambda)| = \frac{1}{\sqrt{2}}\Big((\cos(\lambda) + i\sin(\lambda))\langle 0|\langle 0| + (\cos(\lambda) - i\sin(\lambda))\langle 1|\langle 1| \Big)$$

$$H = 4\left[\frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} + \left(\langle\psi|\frac{\partial|\psi\rangle}{\partial\lambda}\right)^2 \right]$$

$$\frac{\partial\langle\psi|}{\partial\lambda} = \frac{1}{\sqrt{2}}\Big((-\sin(\lambda) + i\cos(\lambda))\langle 0|\langle 0| + (-\sin(\lambda) - i\cos(\lambda))\langle 1|\langle 1| \Big)$$

$$\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{\sqrt{2}}\Big((-\sin(\lambda) - i\cos(\lambda))|0\rangle|0\rangle + (-\sin(\lambda) + i\cos(\lambda))|1\rangle|1\rangle \Big)$$

$$\frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{2}\Big((-\sin(\lambda) + i\cos(\lambda))\langle 0|\langle 0| + (-\sin(\lambda) - i\cos(\lambda))\langle 1|\langle 1| \Big) \cdot \Big((-\sin(\lambda) - i\cos(\lambda))|0\rangle|0\rangle + (-\sin(\lambda) + i\cos(\lambda))|1\rangle|1\rangle \Big)$$

$$= \frac{1}{2}\Big((-\sin(\lambda) + i\cos(\lambda))(-\sin(\lambda) - i\cos(\lambda))\langle 0|\overset{1}{\cancel{\langle 0|0\rangle}}|0\rangle + (-\sin(\lambda) + i\cos(\lambda))(-\sin(\lambda) + i\cos(\lambda))\langle 0|\overset{0}{\cancel{\langle 0|1\rangle}}|1\rangle$$
$$+ (-\sin(\lambda) - i\cos(\lambda))(-\sin(\lambda) - i\cos(\lambda))\langle 1|\overset{0}{\cancel{\langle 1|0\rangle}}|0\rangle + (-\sin(\lambda) - i\cos(\lambda))(-\sin(\lambda) + i\cos(\lambda))\langle 1|\overset{1}{\cancel{\langle 1|1\rangle}}|1\rangle \Big)$$

$$= \frac{1}{2}\Big(\sin^2(\lambda) + \cancel{i\sin(\lambda)\cos(\lambda)} - \cancel{i\sin(\lambda)\cos(\lambda)} + \cos^2(\lambda) + \sin^2(\lambda) - \cancel{i\sin(\lambda)\cos(\lambda)} + \cancel{i\sin(\lambda)\cos(\lambda)} + \cos^2(\lambda) \Big)$$

$$= \frac{1}{2}\Big(2\,(\overset{1}{\cancel{\sin^2(\lambda) + \cos^2(\lambda)}}) \Big) = 1 \quad \therefore \quad \frac{\partial\langle\psi|}{\partial\lambda}\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = 1$$

$$\langle\psi|\cdot\frac{\partial|\psi\rangle}{\partial\lambda} = \frac{1}{2}\Big((\cos(\lambda) + i\sin(\lambda))\langle 0|\langle 0| + (\cos(\lambda) - i\sin(\lambda))\langle 1|\langle 1| \Big) \cdot \Big((-\sin(\lambda) - i\cos(\lambda))|0\rangle|0\rangle + (-\sin(\lambda) + i\cos(\lambda))|1\rangle|1\rangle \Big)$$

$$= \frac{1}{2}\Big((\cos(\lambda) + i\sin(\lambda))(-\sin(\lambda) - i\cos(\lambda))\langle 0|\overset{1}{\cancel{\langle 0|0\rangle}}|0\rangle + (\cos(\lambda) + i\sin(\lambda))(-\sin(\lambda) + i\cos(\lambda))\langle 0|\overset{0}{\cancel{\langle 0|1\rangle}}|1\rangle$$
$$+ (\cos(\lambda) - i\sin(\lambda))(-\sin(\lambda) - i\cos(\lambda))\langle 1|\overset{0}{\cancel{\langle 1|0\rangle}}|0\rangle + (\cos(\lambda) - i\sin(\lambda))(-\sin(\lambda) + i\cos(\lambda))\langle 1|\overset{1}{\cancel{\langle 1|1\rangle}}|1\rangle \Big)$$

$$= \frac{1}{2}\Big(-\cancel{\sin(\lambda)\cos(\lambda)} - \cancel{i\cos^2(\lambda)} - \cancel{i\sin^2(\lambda)} + \cancel{\sin(\lambda)\cos(\lambda)} - \cancel{\sin(\lambda)\cos(\lambda)} + \cancel{i\cos^2(\lambda)} + \cancel{i\sin^2(\lambda)} + \cancel{\sin(\lambda)\cos(\lambda)} \Big) = 0$$

$$\therefore \quad H = 4\,(1 + 0) = 4$$

$$\boxed{H = 4}$$