Physics 230

Homework Set 13

1. Once again, reconsider the oscillator studied extensively in Homework Sets 11 and 12 involving the mass $m = 0.550$ kg. The mass is attached to the spring and hung vertically, where the oscillator is submerged in a resistive medium. The mass is subjected to a harmonic force of the form $F(t) = F_0 \cos(\omega t)$ where $F_0 = 2.50$ N. It is found that the damping parameter, γ, is one-eighth the critical value. Find
   a) the frequency that gives rise to the maximum amplitude,
   b) the maximum amplitude,
   c) the quality factor.

   Now reconsider this damped, driven harmonic oscillator subject to a driving frequency that differs from that found in a). Find
   d) the amplitude, $A(\omega)$, at this frequency.
   e) the phase angle, δ, when the system is driven at a frequency $\omega = 2\pi$ rad/s, and

2. For a damped, driven harmonic oscillator, the displacement of the mass from equilibrium is described by the solution to the equation of motion, given by Eqn. (3.10), where both the *steady-state* solution and the *transient response* need to be included. As presented in class, the general solution to Eqn. (3.10) is of the form

$$x(t) = x_1(t) + x_2(t) = A(\omega)\cos(\omega t - \delta) + Be^{-\gamma t/2}\cos(\omega_1 t + \phi), \tag{1}$$

   where x_1 corresponds to the steady-state solution whereas x_2 is the transient solution, where $\omega_1 \equiv (\omega_0^2 - (\gamma/2)^2)^{1/2}$ is the frequency of the transient response.

   Reconsider the oscillator of the previous problem where the system is driven at frequency $\omega = 2\pi$ rad/s. At time $t = 0$, the mass is displaced from the equilibrium position by 5.50 cm in the *downward* direction and is given an initial shove, imparting a speed of 0.450 m/s on the mass in the *downward* direction toward the floor.
   a) Construct an expression for the time-dependent velocity of the mass, which is attached to the spring.
   b) Using the aforementioned initial conditions imposed on the oscillator, construct a

system of algebraic equations involving B and ϕ.

c) Show that $B = -0.0448$ m and $\phi = 32.2° = 0.562$ rad are solutions to the above system of algebraic equations.

3. Reconsider the steady state solution and the transient response of the oscillator of the previous problem. Using a spreadsheet program (i.e. Excel),

   a) generate a plot of $x_1(t) = A(\omega)\cos(\omega t - \delta)$ vs t,

   b) generate a plot of $x_2(t) = Be^{-\gamma t/2}\cos(\omega_1 t + \phi)$ vs t, and

   c) generate a plot of x_1 vs t, x_2 vs t, and $x(t) = x_1 + x_2$ vs. t, *all* on the the same plot.

   For all three of these distinct plots, the functions should be plotted over the time period $t = 0$ to 5.00s, with a time step no greater than 0.0250s. Each plot needs to include a title and the axes should be labeled and include units.

4. A transverse wave traveling along a string is described by the function

$$z(x,t) = (0.21\text{ m})\sin\Big((0.36\text{m}^{-1})x + (0.52\text{s}^{-1})t\Big) \tag{2}$$

   a) Calculate the amplitude, wavelength, frequency, and velocity of the wave.

   b) In what direction is the wave traveling?

   c) Show that the aforementioned expression is a solution to the 1D wave equation.

   d) Calculate the maximum speed and acceleration of the transverse displacement of a point on the string.