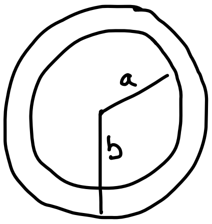
Taylor Larrechea
Dr. Middleton
PHYS 311 HW 6

Problem 1

$\rho(r)=kr^{\tilde{n}}$ $\qquad V(r) = -\int_{\mathcal{O}}^{r} \vec{E}\cdot d\vec{l}$



$s<a$: $\vec{E}=0$

$a<s<b$: $\vec{E}=\frac{K}{\varepsilon_0 r^2(n+3)}\left[r^{(n+3)}-a^{(n+3)}\right]\hat{r}$

$\vec{E}=\frac{Kr^{(n+1)}}{\varepsilon_0(n+3)}-\frac{Ka^{(n+3)}}{\varepsilon_0 r^2(n+3)}$

$s>b$: $\vec{E}=\frac{K}{r^2\varepsilon_0(n+3)}\left[b^{(n+3)}-a^{(n+3)}\right]\hat{r}$

From ∞ to b: $d\vec{l}=dr\,\hat{r}$

$$V(r)=-\int_{\infty}^{b}\vec{E}\cdot d\vec{l}=\frac{-K}{\varepsilon_0(n+3)}\left[b^{(n+3)}-a^{(n+3)}\right]\int_{\infty}^{b}\frac{1}{r^2}\,dr=\alpha\cdot\frac{-1}{r}\Big|_{\infty}^{b}$$

$$V(r)=\frac{-K}{\varepsilon_0(n+3)}\left[b^{(n+3)}-a^{(n+3)}\right]\cdot\left[\frac{-1}{b}-\left(\cancel{\frac{-1}{\infty}}^{\,0}\right)\right]=\frac{K}{\varepsilon_0(n+3)}\left[b^{(n+3)}-a^{(n+3)}\right]\cdot\frac{1}{b}$$

$$V(r)=-\int_{\infty}^{b}\vec{E}\cdot d\vec{l}=\frac{-K}{\varepsilon_0(n+3)}\left[a^{(n+3)}-b^{(n+3)}\right]\cdot\frac{1}{b}$$

$$V(r)_a=-\int_{b}^{a}\vec{E}\cdot d\vec{l}=\frac{-K}{(n+3)\varepsilon_0}\int_{b}^{a}r^{(n+1)}-\frac{a^{(n+3)}}{r^2}\,dr=\frac{-K}{(n+3)\varepsilon_0}\left[\frac{r^{(n+2)}}{(n+2)}+\frac{a^{(n+3)}}{r}\right]_{b}^{a}$$

$$\frac{-K}{(n+3)\varepsilon_0}\left[\left(\frac{a^{(n+2)}}{(n+2)}+\frac{a^{(n+3)}}{a}\right)-\left(\frac{b^{(n+2)}}{(n+2)}+\frac{a^{(n+3)}}{b}\right)\right]$$

$$\frac{-K}{(n+3)\varepsilon_0}\left[\frac{1}{(n+2)}\left[a^{(n+2)}-b^{(n+2)}\right]+a^{(n+3)}\left[\frac{1}{a}-\frac{1}{b}\right]\right]$$

$$V(r)_b=-\int_{b}^{a}\vec{E}\cdot d\vec{l}=\frac{-K}{(n+3)\varepsilon_0}\left[\frac{1}{(n+2)}\left[a^{(n+2)}-b^{(n+2)}\right]+a^{(n+3)}\left[\frac{1}{a}-\frac{1}{b}\right]\right]$$

$$V(r)_c=-\int_{a}^{0}\vec{E}\cdot d\vec{l}=-\int_{a}^{0}0\cdot d\vec{l}=0$$

$$V=V(r)_a+V(r)_b+V(r)_c$$

$$V=\frac{-K}{(n+3)\varepsilon_0}\left[\left[\frac{a^{(n+3)}}{b}-b^{(n+2)}\right]+\left[\frac{1}{(n+2)}\left[a^{(n+2)}-b^{(n+2)}\right]+a^{(n+3)}\left[\frac{1}{a}-\frac{1}{b}\right]\right]\right]$$

$$V=\frac{-K}{(n+3)\varepsilon_0}\left[\cancel{\frac{a^{(n+3)}}{b}}^{\,0}-b^{(n+2)}+\frac{1}{(n+2)}\left[a^{(n+2)}-b^{(n+2)}\right]+a^{(n+2)}-\cancel{\frac{a^{(n+3)}}{b}}^{\,0}\right]$$

$$V=\frac{-K}{(n+3)\varepsilon_0}\left[\frac{1}{(n+2)}\left[a^{(n+2)}-b^{(n+2)}\right]+a^{(n+2)}-b^{(n+2)}\right]$$

Problem 1 | Continued

$$V = \frac{-K}{(\pi+3)\varepsilon_0}\left[\frac{a^{(\pi+2)}}{(\pi+2)} - \frac{b^{(\pi+2)}}{(\pi+2)} + a^{(\pi+2)} - b^{(\pi+2)}\right]$$

$$V = \frac{-K}{(\pi+3)\varepsilon_0}\left[a^{(\pi+2)}\left[\frac{1}{(\pi+2)} + 1\right] - b^{(\pi+2)}\left[\frac{1}{(\pi+2)} + 1\right]\right]$$

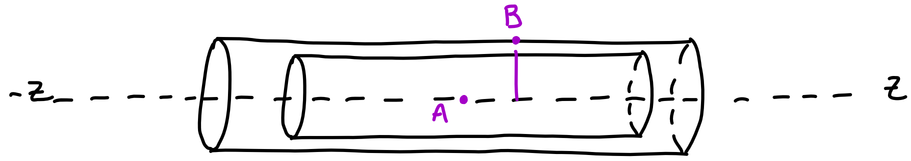
$$V = \frac{-K}{(\pi+3)\varepsilon_0}\left[\left[\frac{1}{(\pi+2)} + 1\right]\left(a^{(\pi+2)} - b^{(\pi+2)}\right)\right]$$

$$\frac{1 + (\pi+2)}{(\pi+2)} = \frac{\pi+3}{\pi+2}$$

$$V = \frac{-K}{(\pi+3)\varepsilon_0}\left[\left(\frac{\pi+3}{\pi+2}\right)\left(a^{(\pi+2)} - b^{(\pi+2)}\right)\right] = \frac{-K}{(\pi+2)\varepsilon_0}\left[a^{(\pi+2)} - b^{(\pi+2)}\right]$$

$$\boxed{V = \frac{-K}{(\pi+2)\varepsilon_0}\left[a^{(\pi+2)} - b^{(\pi+2)}\right]}$$

Problem 2



$$\Delta V = V(B) - V(A) = -\int_b^0 \vec{E}\cdot d\vec{l} = -\int_b^a \vec{E}\cdot d\vec{l} - \int_a^0 \vec{E}\cdot d\vec{l}$$

$s < a$: $\vec{E} = \left[\frac{\rho_0}{3\varepsilon_0}\frac{s^2}{a}\right]\hat{s}$ $\quad a < s < b$: $\vec{E} = \left[\frac{\rho_0}{3\varepsilon_0}\frac{a^2}{s}\right]\hat{s}$

$s > b$: $\vec{E} = \frac{1}{s\cdot\varepsilon_0}\left[\frac{1}{3}\rho_0 a^2 + \sigma b\right]\hat{s}$

V @ $a < s < b$: $\vec{E} = \left[\frac{\rho_0}{3\varepsilon_0}\cdot\frac{a^2}{s}\right]\hat{s}$ $\quad d\vec{l} = ds\,\hat{s}$

$$V_a = -\int_b^a \vec{E}\cdot d\vec{l} = -\int_b^a \frac{\rho_0}{3\varepsilon_0}\cdot\frac{a^2}{s}\,ds = \frac{-\rho_0}{3\varepsilon_0}\cdot a^2\int_b^a \frac{1}{s}\,ds = \frac{-\rho_0}{3\varepsilon_0}\cdot a^2\cdot \ln(s)\Big|_b^a$$

$$V = \frac{-\rho_0}{3\varepsilon_0}\cdot a^2\left[\ln(a) - \ln(b)\right] = \underline{\frac{-\rho_0\cdot a^2}{3\varepsilon_0}\left[\ln(a/b)\right]}$$

V @ $s < a$: $\vec{E} = \frac{\rho_0}{3\varepsilon_0}\cdot\frac{s^2}{a}\hat{s}$, $d\vec{l} = ds\,\hat{s}$

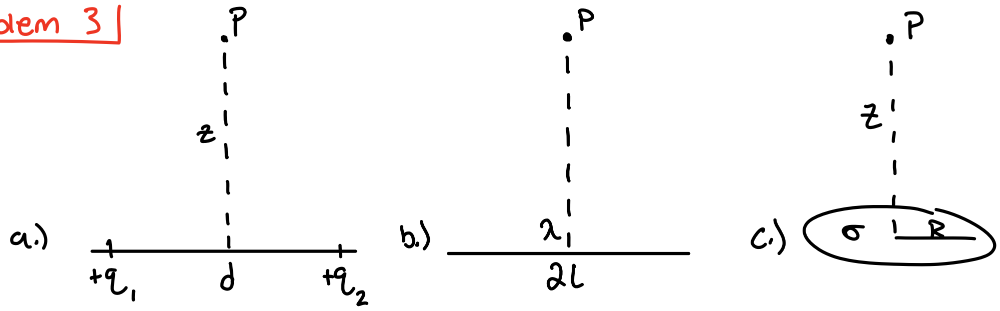
$$V = -\int_a^0 \frac{\rho_0}{3\varepsilon_0}\cdot\frac{s^2}{a}\,ds = \frac{-\rho_0}{3\varepsilon_0 a}\int_a^0 s^2\,ds = \frac{-\rho_0}{3\varepsilon_0 a}\left[\frac{s^3}{3}\right]_a^0 = \frac{-\rho_0}{3\varepsilon_0 a}\left[\frac{0^3}{3} - \frac{a^3}{3}\right]$$

$$V_b = \frac{-\rho_0}{3\varepsilon_0 a}\left[-\frac{a^3}{3}\right] = \underline{\frac{\rho_0}{3\varepsilon_0}\cdot\frac{a^2}{3}}$$

$$V_{tot} = V_a + V_b = \frac{-\rho_0\cdot a^2}{3\varepsilon_0}\left[\ln(a/b)\right] + \frac{\rho_0}{3\varepsilon_0}\cdot\frac{a^2}{3} = \frac{\rho_0\cdot a^2}{3\varepsilon_0}\left[\frac{1}{3} - \ln\left(\frac{a}{b}\right)\right]$$

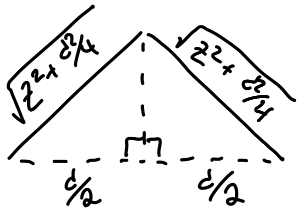
$$\boxed{V = \frac{\rho_0\cdot a^2}{3\varepsilon_0}\left[\frac{1}{3} + \ln\left(\frac{b}{a}\right)\right]}$$

Problem 3



$$(2.27)\quad V(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i} \qquad (2.30)\quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(\vec{r}\,')}{\mathscr{r}}\,dl' \;\;\&\;\; V = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(r')}{\mathscr{r}}\,da'$$

a.)



$$\mathscr{r}_{1p} = \sqrt{z^2+d^2/4}$$
$$\mathscr{r}_{2p} = \sqrt{z^2+d^2/4}$$

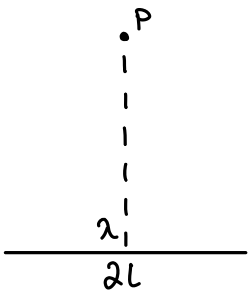
$$\frac{2q}{4\pi\varepsilon_0}\cdot\left(z^2+d^2/4\right)^{-1/2}$$
$$-\left(\frac{2q}{4\pi\varepsilon_0}\cdot\frac{-1}{2}\left(z^2+d^2/4\right)^{-3/2}\cdot 2z\right)$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{z^2+d^2/4}} + \frac{q}{\sqrt{z^2+d^2/4}}\right] = \frac{2q}{4\pi\varepsilon_0\sqrt{z^2+d^2/4}}$$

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2z}{\left(z^2+d^2/4\right)^{3/2}}\,\hat{z}}$$

$$\boxed{V(r) = \frac{2q}{4\pi\varepsilon_0\sqrt{z^2+d^2/4}}}$$

b.)



$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' \;,\quad \vec{\mathscr{r}} = -x\hat{x} + z\hat{z} \;,\quad \mathscr{r} = \sqrt{x^2+z^2} \;,\quad dl' = dx$$

$$\vec{r} = z\hat{z} \;,\quad \vec{r}\,' = x\hat{x}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(r')}{\mathscr{r}}\,dl'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{\lambda}{\sqrt{x^2+z^2}}\,dx = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{1}{\sqrt{x^2+z^2}}\,dx \qquad \frac{\lambda}{4\pi\varepsilon_0} \equiv \alpha$$

$$V = \alpha\int_{-L}^{L}\frac{1}{\sqrt{x^2+z^2}}\,dx$$

$$x = z\tan(u)$$
$$dx = z\cdot\sec^2(u)\,du$$

$$\sin^2(x) + \cos^2(x) = 1$$
$$\sec^2(x) - \tan^2(x) = 1$$
$$\csc^2(x) - \cot^2(x) = 1$$

$$= \alpha\int_{-L}^{L}\frac{z\cdot\sec^2(u)\,du}{\sqrt{z^2\tan^2(u)+z^2}} = \alpha\int_{-L}^{L}\frac{z\cdot\sec^2(u)}{\sqrt{z^2\cdot\sec^2(u)}}\,du = \alpha\int_{-L}^{L}\frac{z\cdot\sec^2(u)}{z\cdot\sec(u)}\,du$$

$$\alpha\int_{-L}^{L}\sec(u)\,du = \alpha\left[\ln\left|\tan(u)+\sec(u)\right|\right]_{-L}^{L}\Bigg|_{u=\tan^{-1}\left(\frac{x}{z}\right)}$$

$$\alpha\left[\ln\left|\tan\left(\tan^{-1}\left(\tfrac{x}{z}\right)\right) + \sec\left(\tan^{-1}\left(\tfrac{x}{z}\right)\right)\right|\right]_{-L}^{L} = \alpha\left[\ln\left|\frac{x}{z} + \sqrt{x^2+z^2}\right|\right]_{-L}^{2}$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left|\frac{x+z\sqrt{x^2+z^2}}{z}\right|\right]_{-L}^{L} = \frac{\lambda}{4\pi\varepsilon_0}\left[\left[\ln\left|\frac{L+z\sqrt{L^2+z^2}}{z}\right|\right] - \ln\left[\frac{-L+\sqrt{L^2+z^2}}{z}\right]\right]$$

Problem 3 | Continued

$$V = \frac{\lambda}{4\pi\varepsilon_0} \cdot 2\ln\left|\frac{L+z\sqrt{L^2+z^2}}{z}\right| = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\left|\frac{L+z\sqrt{L^2+z^2}}{z}\right|$$

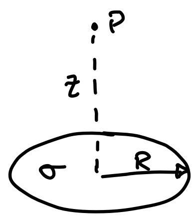
$$\boxed{V = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\left|\frac{L+z\sqrt{L^2+z^2}}{z}\right|}$$

$$\vec{E} = -\vec{\nabla}V : -\frac{\partial}{\partial z} \ln\left|\frac{L+z\sqrt{L^2+z^2}}{z}\right|$$

$$\frac{d}{dx}\ln(x) = \frac{x'}{x} \qquad X = (L + z\sqrt{L^2+z^2})z^{-1} \qquad x' = \left(0 + (L^2+z^2)^{\frac12} + \frac{z}{2}(L^2+z^2)^{-\frac12}\right)z^{-1} - \frac{1}{z^2}(L+z\sqrt{L^2+z^2})$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{(L^2+z^2)^{\frac12} + \frac{z}{2}(L^2+z^2)^{-\frac12}z' - \frac{1}{z^2}(L+z\sqrt{L^2+z^2})}{\frac{L+z\sqrt{L^2+z^2}}{z}}\,\hat{z}}$$

c.)



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(r')}{\mathscr{r}}\,da'$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' = [0,z]-[s,0] = -s\hat{s} + z\hat{z}$$

$$\vec{r} = z\hat{z}$$

$$\vec{r}\,' = s\hat{s}$$

$$\mathscr{r} = \sqrt{s^2+z^2} \qquad da' = s\,ds\,d\phi$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^R \frac{\sigma \cdot s}{\sqrt{s^2+z^2}}\,ds\,d\phi \qquad u = s^2+z^2, \quad du = 2s\,ds$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_0^{2\pi}\int_0^R u^{-\frac12}\,du\,d\phi = \frac{\sigma}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_0^{2\pi} 2u^{\frac12}\,d\phi\,\Big|_0^R$$

$$\frac{\sigma}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_0^{2\pi} 2\cdot\sqrt{s^2+z^2}\,\Big|_0^R\,d\phi \ : \ \sqrt{s^2+z^2}\,\Big|_0^R = \sqrt{R^2+z^2} - \sqrt{z^2} = \sqrt{R^2+z^2} - z$$

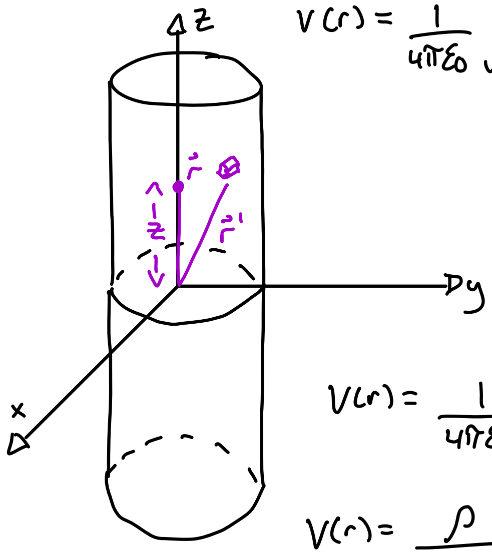
$$\frac{\sigma}{4\pi\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right]\int_0^{2\pi} d\phi = \frac{\sigma}{4\pi\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right]\cdot\phi\,\Big|_0^{2\pi} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right]$$

$$\boxed{V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right]} \qquad E = -\nabla V : \frac{\sigma}{2\varepsilon_0}\left[(R^2+z^2)^{\frac12} - z\right]$$

$$\frac{\partial}{\partial z} \text{ of } \frac{\sigma}{2\varepsilon_0}\left[(R^2+z^2)^{\frac12} - z\right] = \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{2}(R^2+z^2)^{-\frac12}\cdot 2z - 1\right] = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{\sqrt{R^2+z^2}} - 1\right]$$

$$-\nabla V = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]\hat{z} \qquad \boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]\hat{z}}$$

Problem 4



$$V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec r\,')}{\mathscr{r}}\, d\tau' \qquad d\tau' = s\,ds\,d\phi\,dz \qquad \rho(\vec r\,') = \rho$$

$$\vec{\mathscr{r}} = \vec r - \vec r\,' \qquad \vec r = z\hat z \,, \quad \vec r\,' = s\hat s + z'\hat z$$

$$\vec{\mathscr{r}} = z\hat z - s\hat s - z'\hat z = (z-z')\hat z - s\hat s$$

$$\mathscr{r}^2 = \vec{\mathscr{r}}\cdot\vec{\mathscr{r}} = (z-z')^2 + s^2 \,, \quad \mathscr{r} = \sqrt{(z-z')^2+s^2}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi}\int_0^{R} \frac{\rho}{\sqrt{(z-z')^2+s^2}}\, s\,ds\,d\phi\,dz'$$

$$V(r) = \frac{\rho}{4\pi\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi}\int_0^{R} \frac{s}{\sqrt{(z-z')^2+s^2}}\, ds\,d\phi\,dz' \qquad \begin{aligned} u &= (z-z')^2+s^2 \\ du &= 2s\,ds \;\vdots\; ds = \frac{du}{2s}\end{aligned}$$

$$V(r) = \frac{\rho}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi}\int_0^{R} u^{-\frac{1}{2}}\, du\,d\phi\,dz'$$

$$\frac{\rho}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} 2\cdot\sqrt{(z-z')^2+s^2}\,\Big|_0^R\, d\phi\,dz'$$

$$\frac{\rho}{4\pi\varepsilon_0}\cdot\frac{1}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} 2\left[\sqrt{(z-z')^2+R^2} - \sqrt{(z-z')^2}\right] d\phi\,dz'$$

$$\frac{\rho}{4\pi\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_0^{2\pi} \sqrt{(z-z')^2+R^2} - \sqrt{(z-z')^2}\; d\phi\,dz'$$

$$\frac{\rho}{4\pi\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \left[\sqrt{(z-z')^2+R^2} - \sqrt{(z-z')^2}\right]\phi\,\Big|_0^{2\pi} dz'$$

$$\frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{(z-z')^2+R^2} - (z-z')\; dz'$$

$$\frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{(z-z')^2+R^2}\; dz - \frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} (z-z')\; dz' \qquad \begin{aligned} u &= z-z' \\ du &= -dz'\end{aligned}$$

$$-\frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{u^2+R^2}\; du + \frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} u\; du$$

$$+\frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} u\; du \qquad +\frac{\rho}{2\varepsilon_0}\cdot\frac{u^2}{2}\Big|_{u=z-z'}\Big|_{-L/2}^{L/2} = -\frac{\rho}{2\varepsilon_0}\cdot\frac{(z-z')^2}{2}\Big|_{-L/2}^{L/2}$$

$$+\frac{\rho}{2\varepsilon_0}\left[\frac{(z-L/2)^2}{2} - \frac{(z+L/2)^2}{2}\right] \qquad \frac{\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} u\; du = \frac{\rho}{2\varepsilon_0}\left[\frac{(z-L/2)^2}{2} - \frac{(z+L/2)^2}{2}\right]$$

Problem 4 continued

$$\frac{-\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{u^2+R^2}\, du \qquad u = R\tan\theta, \quad du = R\cdot\sec^2(\theta)$$

$$u^2 = R^2\tan^2\theta$$

$$\frac{-\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sqrt{R^2\tan^2\theta + R^2}\cdot R\cdot\sec^2(\theta)\, d\theta$$

$$\frac{-\rho}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} R\sec(\theta)\cdot R\cdot\sec^2(\theta)\, d\theta = \frac{-\rho\cdot R^2}{2\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sec^3(\theta)\, d\theta$$

$$\frac{-\rho\cdot R^2}{2\varepsilon_0}\left[\frac{\sec^2(\theta)\sin(\theta)}{2} + \frac{1}{2}\ln\left|\tan(\theta)+\sec(\theta)\right|\right]_{-\frac{L}{2}}^{\frac{L}{2}}\Bigg|_{\theta=\tan^{-1}\left(\frac{u}{R}\right)}$$

$$\frac{-\rho R^2}{2\varepsilon_0}\left[\frac{\sec^2\left(\tan^{-1}\left(\frac{u}{R}\right)\right)\cdot\sin\left(\tan^{-1}\left(\frac{u}{R}\right)\right)}{2} + \frac{1}{2}\ln\left|\tan\left(\tan^{-1}\left(\frac{u}{R}\right)\right)+\sec\left(\tan^{-1}\left(\frac{u}{R}\right)\right)\right|\right]\Bigg|_{u=(z-z')}$$

$$\frac{-\rho R^2}{2\varepsilon_0}\left[\sec^2\left(\tan^{-1}\left(\frac{z-z'}{R}\right)\right)\cdot\sin\left(\tan^{-1}\left(\frac{z-z'}{R}\right)\right) + \frac{1}{2}\ln\left|\tan\left(\tan^{-1}\left(\frac{z-z'}{R}\right)\right)+\sec\left(\tan^{-1}\left(\frac{z-z'}{R}\right)\right)\right|\right]\Bigg|_{-L/2}^{L/2}$$

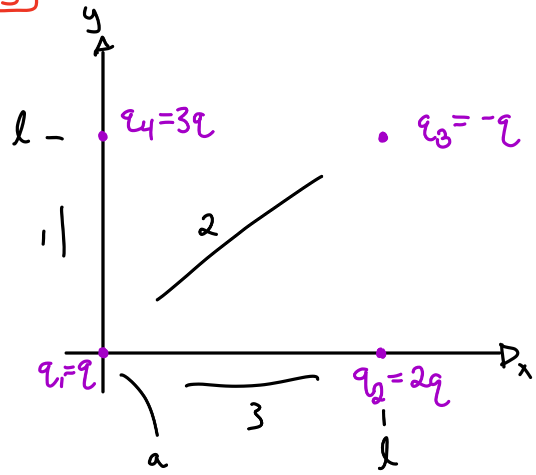
$$\frac{-\rho R^2}{2\varepsilon_0}\left[\frac{-\sqrt{R^2+(z'-z)^2}\,(z'-z)}{R^2} + \frac{1}{2}\ln\left|\left(\frac{z-z'}{R}\right)+\frac{\sqrt{R^2+(z'-z)^2}}{R}\right|\right]\Bigg|_{-L/2}^{L/2}$$

$$\frac{-\rho R^2}{2\varepsilon_0}\left[-\frac{\sqrt{R^2+(L/2-z)^2}\,(L/2-z)}{R^2} + \frac{1}{2}\ln\left|\frac{z-L/2+\sqrt{R^2+(L/2-z)^2}}{R}\right|\right]$$

$$+\frac{\rho R^2}{2\varepsilon_0}\left[\frac{-\sqrt{R^2+(-L/2-z)^2}\,(-L/2-z)}{R^2} + \frac{1}{2}\ln\left|\frac{z+L/2+\sqrt{R^2+(-L/2-z)^2}}{R}\right|\right]$$

$$\boxed{\begin{aligned} V = {} & \frac{\rho}{2\varepsilon_0}\left[\sqrt{R^2+(L/2-z)^2}\,(L/2-z) - \frac{R^2}{2}\ln\left|\frac{z-L/2+\sqrt{R^2+(L/2-z)^2}}{R}\right|\right] \\ & + \frac{\rho}{2\varepsilon_0}\left[-\sqrt{R^2+(-L/2-z)^2}\,(-L/2-z) + \frac{R^2}{2}\ln\left|\frac{z+L/2+\sqrt{R^2+(-L/2-z)^2}}{R}\right|\right] \\ & + \frac{\rho}{2\varepsilon_0}\left[\frac{(z-L/2)^2}{2} - \frac{(z+L/2)^2}{2}\right] \end{aligned}}$$

Problem 5)



$\sqrt{\ell^2+\ell^2} = \sqrt{2}\,\ell$

potential from $V = \frac{1}{4\pi\varepsilon_0}\sum_i^{\infty} \frac{q_i}{r_i}$

$W = Q \cdot V_{@\,spot}$

a.) $W = Q\Delta V_{net}$

$V : \frac{1}{4\pi\varepsilon_0}\sum_i^{3} \frac{q_i}{r_i}$: $V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{\ell}$: $V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{\sqrt{2}\ell}$: $V_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{\ell}$

$\therefore$ $\boxed{W = \frac{Q}{4\pi\varepsilon_0}\left[\frac{5q}{\ell} - \frac{q}{\sqrt{2}\ell}\right]}$

b.)

$W = \frac{1}{2}\sum_{i=1}^{4} q_i V(\vec{r}_i) = \frac{1}{2}\left[q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2) + q_3 V(\vec{r}_3) + q_4 V(\vec{r}_4)\right]$

$\vec{r}_{12} = [\ell,0] - [0,0] = [\ell,0]$: $r_{12} = \ell$
$\vec{r}_{13} = [\ell,\ell] - [0,0] = [\ell,\ell]$: $r_{13} = \sqrt{2}\ell$
$\vec{r}_{14} = [0,\ell] - [0,0] = [0,\ell]$: $r_{14} = \ell$

$\vec{r}_{21} = [0,0] - [\ell,0] = [-\ell,0]$: $r_{21} = \ell$
$\vec{r}_{24} = [0,\ell] - [\ell,0] = [-\ell,\ell]$: $r_{24} = \sqrt{2}\ell$
$\vec{r}_{23} = [\ell,\ell] - [\ell,0] = [0,\ell]$: $r_{23} = \ell$

$\vec{r}_{32} = [\ell,0] - [\ell,\ell] = [0,-\ell]$: $r_{32} = \ell$
$\vec{r}_{31} = [0,0] - [\ell,\ell] = [-\ell,-\ell]$: $r_{31} = \sqrt{2}\ell$
$\vec{r}_{34} = [0,\ell] - [\ell,\ell] = [-\ell,0]$: $r_{34} = \ell$

$\vec{r}_{43} = [\ell,\ell] - [0,\ell] = [\ell,0]$: $r_{43} = \ell$
$\vec{r}_{42} = [\ell,0] - [0,\ell] = [\ell,-\ell]$: $r_{43} = \sqrt{2}\ell$
$\vec{r}_{41} = [0,0] - [0,\ell] = [0,-\ell]$: $r_{41} = \ell$

$V(\vec{r}_1) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\ell}\left[5 - \frac{1}{\sqrt{2}}\right]$

$V(\vec{r}_2) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\ell} + \frac{q_3}{\ell} + \frac{q_4}{\sqrt{2}\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\ell} - \frac{q}{\ell} + \frac{3q}{\sqrt{2}\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{3q}{\sqrt{2}\ell}\right]$

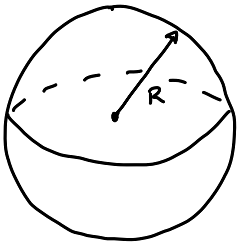
$V(\vec{r}_3) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\sqrt{2}\ell} + \frac{q_2}{\ell} + \frac{q_4}{\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{2}\ell} + \frac{2q}{\ell} + \frac{3q}{\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{5q}{\ell} + \frac{q}{\sqrt{2}\ell}\right]$

$V(\vec{r}_4) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\ell} + \frac{q_2}{\sqrt{2}\ell} + \frac{q_3}{\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\ell} + \frac{2q}{\sqrt{2}\ell} - \frac{q}{\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{\sqrt{2}\ell}\right]$

$W = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\left[\frac{5q}{\ell} - \cancel{\frac{q}{\sqrt{2}\ell}} + \frac{3q}{\sqrt{2}\ell} + \frac{5q}{\ell} + \cancel{\frac{q}{\sqrt{2}\ell}} + \frac{2q}{\sqrt{2}\ell}\right] = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\left[\frac{10q}{\ell} + \frac{5q}{\sqrt{2}\ell}\right]$

$\boxed{W = \frac{1}{4\pi\varepsilon_0}\left[\frac{5q}{\ell} + \frac{5q}{2\sqrt{2}\ell}\right]}$

Problem 61

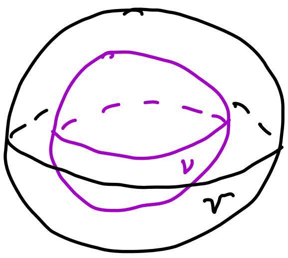


(2.43) $W = \frac{1}{2}\int \rho V \, d\tau$

(2.44) $W = \frac{\varepsilon_0}{2}\left(\int_V E^2 d\tau + \oint_S V\vec{E}\cdot d\vec{a}\right)$

(2.45) $W = \frac{\varepsilon_0}{2}\int E^2 d\tau$ (All space)

Finding $\vec{E}$ & V Charge: q Radius: R ——— of actual sphere $\rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi r^3}$



$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$

$Q_{ENC} = \int_V \rho \, d\tau$, $d\tau = r'^2 \sin\theta \, dr' d\theta d\phi$, $\rho = \frac{q}{\frac{4}{3}\pi R^3}$

$Q_{ENC} = \int_V \rho \, d\tau'$

$$\frac{q}{\frac{4}{3}\pi R^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{r} r'^2 \sin\theta \, dr' d\theta d\phi$$

$$\frac{q}{\frac{4}{3}\pi R^3}\int_0^{2\pi}\int_0^{\pi} \frac{r'^3}{3}\Big|_0^r \sin\theta \, d\theta d\phi$$

$$\frac{q \cdot r^3}{4\pi R^3}\int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta d\phi$$

$$\frac{q\cdot r^3}{4\pi R^3}\int_0^{2\pi} -\cos\theta \Big|_0^{\pi} d\phi$$

$$\frac{q\cdot r^3}{2\pi R^3}\int_0^{2\pi} d\phi = \frac{q\cdot r^3}{R^3}$$

$Q_{ENC} = \frac{q \cdot r^3}{R^3}$

$\oint \vec{E}\cdot d\vec{a}$: $d\vec{a} = r^2 \sin\theta \, d\theta d\phi \, \hat{r}$

$\vec{E} = E\,\hat{r}$ ——— Constant @ given radius

$$E\cdot r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta d\phi$$

$$E\cdot r^2 \int_0^{2\pi} -\cos\theta \Big|_0^{\pi} d\phi = E\cdot 2r^2 \int_0^{2\pi} d\phi$$

when $r<R$

$$E\cdot 2r^2 \cdot \phi \Big|_0^{2\pi} = E\cdot 4\pi r^2$$

$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{ENC}}{\varepsilon_0}$ ⋮ $E \cdot 4\pi r^2 = \frac{Q\cdot r^3}{\varepsilon_0 \cdot R^3}$ ⋮ $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\cdot r}{R^3}\,\hat{r}$

$V = -\int \vec{E}\cdot d\vec{l}$ $d\vec{l} = dr\,\hat{r}$, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q\cdot r}{R^3}\,\hat{r}$ From ∞ to r

∞ to R : R to r

when $r>R$: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\,\hat{r}$

$$V = -\int_{\infty}^{R} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{R}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R} = \frac{1}{4\pi\varepsilon_0}\cdot Q\cdot\left[\frac{1}{R} - \cancelto{0}{\frac{1}{\infty}}\right]$$

$V_1 = \int_{\infty}^{R} \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$ ⋮ $V_1 \equiv V_{outer}$ $V_2 \equiv V_{inner}$

Problem 61 Continued

$$V_2 = -\int_R^r \vec{E}\cdot d\vec{l} = \frac{-Q}{4\pi\varepsilon_0}\int_R^r \frac{r'}{R^3}\,dr' = \frac{-Q}{4\pi\varepsilon_0 R^3}\left[\frac{r'^2}{2}\right]_R^r = \frac{-Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}-\frac{R^2}{2}\right]$$

$$V_2 = -\int_R^r \vec{E}\cdot d\vec{l} = \frac{-Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}-\frac{R^2}{2}\right] \qquad V_{Tot} = V_1 + V_2$$

$$V_{Tot} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr^2}{2R^3} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{QR^2}{2R^3} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{2R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr^2}{2R^3} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}$$

$$V_{TOT} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q}{2R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr^2}{2R}\cdot\frac{1}{R^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}\left[3-\frac{r^2}{R^2}\right] \qquad V_{Tot} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}\left[3-\frac{r^2}{R^2}\right]$$

a.) $W = \frac{1}{2}\int \rho V\,d\tau$

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}\left[3-\frac{r^2}{R^2}\right], \quad \rho = \frac{Q}{\frac{4}{3}\pi R^3}, \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$W = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}\cdot\frac{Q}{\frac{4}{3}\pi R^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{R}(3 - r^2/R^2)\,r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}\cdot\frac{Q}{\frac{4}{3}\pi R^3} \equiv \alpha$$

$$= \alpha\int_0^{2\pi}\int_0^{\pi}\int_0^{R} 3r^2 - \frac{r^4}{R^2}\ \sin\theta\,dr\,d\theta\,d\phi$$

$$= \alpha\int_0^{2\pi}\int_0^{\pi} r^3 - \frac{r^5}{5R^2}\Big|_0^R \sin\theta\,d\theta\,d\phi$$

$$= \alpha\left(R^3 - \frac{R^3}{5}\right)\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi = \alpha\left(\frac{4}{5}\cdot R^3\right)\cdot 2\pi\cdot 2 = \alpha\left(\frac{16\pi}{5}\cdot R^3\right)$$

$$= \left[\frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}\cdot\frac{Q}{\frac{4}{3}\pi R^3}\right]\cdot\left[\frac{16\pi}{5}\cdot R^3\right] = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{R}$$

$$\boxed{W = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{R}}$$

b.) $W = \frac{\varepsilon_0}{2}\int_V E^2\,d\tau$ (All space) $= \frac{\varepsilon_0}{2}\left[\int_{V_2} E_2^2\,d\tau + \int_{V_1} E_1^2\,d\tau\right]$

$$\int_{V_2} E_2^2\,d\tau \quad \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}\,\hat{r} \quad \vdots \quad \int_{V_1} E_1^2\,d\tau \quad \vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$$

$$\frac{\varepsilon_0}{2}\left[\left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot\frac{Q^2}{R^6}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2\cdot r^2\sin\theta\,dr\,d\theta\,d\phi + \left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot Q^2\int_0^{2\pi}\int_0^{\pi}\int_R^{\infty}\frac{r^2}{r^4}\sin\theta\,dr\,d\theta\,d\phi\right]$$

$$\frac{\varepsilon_0}{2}\left[\left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot\frac{Q^2}{R^6}\int_0^{2\pi}\int_0^{\pi}\frac{r^5}{5}\Big|_0^R\sin\theta\,d\theta\,d\phi + \left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot Q^2\int_0^{2\pi}\int_0^{\pi} -\frac{1}{r}\Big|_R^{\infty}\sin\theta\,d\theta\,d\phi\right]$$

Problem 6| Continued

$$\frac{\varepsilon_0}{2}\left[\left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot\frac{Q^2}{R^6}\cdot\frac{R^5}{5}\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi + \left(\frac{1}{4\pi\varepsilon_0}\right)^2\cdot\frac{Q^2}{R}\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi\right]$$

$$\frac{\varepsilon_0}{2}\left[\frac{1}{16\pi^2\varepsilon_0^2}\cdot\frac{Q^2}{5R}\cdot 2\pi\cdot 2 + \frac{1}{16\pi^2\varepsilon_0^2}\cdot\frac{Q^2}{R}\cdot 2\pi\cdot 2\right]$$

$$\frac{\varepsilon_0}{2}\left[\frac{1}{4\pi\varepsilon_0^2}\cdot\frac{Q^2}{5R}+\frac{1}{4\pi\varepsilon_0^2}\cdot\frac{Q^2}{R}\right] = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{5R}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{R}\right]$$

$$\frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{6Q^2}{5R}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q^2}{5R} \quad \therefore \quad \boxed{W = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{R}}$$

on surface

c.) $$W = \frac{\varepsilon_0}{2}\left(\int_V E^2\,d\tau + \oint_S VE\cdot da\right) \Rightarrow \frac{\varepsilon_0}{2}\left(\int_{\text{Inner}} E_2^2\,d\tau + \int_{\text{outer}} E_1^2\,d\tau + \oint_S V\vec{E}\cdot d\vec{a}\right)$$

$\int_{\text{Inner}} E_2^2\,d\tau$: $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r}$ $\quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

$r<R$

$$\frac{Q^2}{(4\pi\varepsilon_0 R^3)^2}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2\cdot r^2\sin\theta\,dr\,d\theta\,d\phi = \frac{Q^2}{16\pi^2\varepsilon_0^2R^6}\int_0^{2\pi}\int_0^{\pi}\frac{r^5}{5}\Big|_0^R\sin\theta\,d\theta\,d\phi$$

$$\frac{Q^2}{16\pi^2\varepsilon_0^2}\cdot\frac{5}{R}\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi = \frac{Q^2}{16\pi^2\varepsilon_0^2}\cdot\frac{1}{5R}\cdot 4\pi = \frac{Q^2}{4\pi\varepsilon_0^2}\cdot\frac{1}{5R}$$

$\int_{\text{outer}} E_1^2\,d\tau$: $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\hat{r}$: $d\tau = r'\sin\theta\,dr'\,d\theta\,d\phi$

$$\left(\frac{Q}{4\pi\varepsilon_0}\right)^2\int_0^{2\pi}\int_0^{\pi}\int_R^{r}\frac{1}{r'^4}\cdot r'^2\sin\theta\,dr'\,d\theta\,d\phi = \frac{Q^2}{16\pi^2\varepsilon_0^2}\int_0^{2\pi}\int_0^{\pi}-\frac{1}{r'}\Big|_R^r\sin\theta\,d\theta\,d\phi$$

$$\frac{Q^2}{16\pi^2\varepsilon_0^2}\left(-\frac{1}{r}+\frac{1}{R}\right)\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi = \frac{Q^2}{16\pi^2\varepsilon_0^2}\cdot 4\pi\left(-\frac{1}{r}+\frac{1}{R}\right) = \frac{Q^2}{4\pi\varepsilon_0^2}\left(\frac{1}{R}-\frac{1}{r}\right)$$

$\oint_S V\vec{E}\cdot d\vec{a}$ $\quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ $\quad r \to \infty$, $d\vec{a} = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$

□ — $r\sin\theta\,d\phi$

$r\,d\theta$ — $d\vec{a}\ \hat{r}$

$$V\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{r}\cdot\left(\frac{1}{4\pi\varepsilon_0}\right)\cdot\frac{Q}{r^2}\hat{r} = \left(\frac{Q}{4\pi\varepsilon_0}\right)^2\cdot\frac{1}{r^3}\hat{r}$$

$$\oint V\vec{E}\cdot d\vec{a} = \left(\frac{Q}{4\pi\varepsilon_0}\right)^2\int_0^{2\pi}\int_0^{\pi}\frac{1}{r^3}\cdot r^2\sin\theta\,d\theta\,d\phi$$

$$= \left(\frac{Q}{4\pi\varepsilon_0}\right)^2\cdot\frac{1}{r}\int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta\,d\phi = \frac{Q^2}{16\pi^2\varepsilon_0^2}\cdot\frac{1}{r}\cdot 4\pi = \frac{1}{4\pi\varepsilon_0^2}\cdot\frac{Q^2}{r}$$

$$\oint V\vec{E}\cdot d\vec{a} = \frac{1}{4\pi\varepsilon_0^2}\frac{Q^2}{r}$$

Problem 6 | Continued

Now,

$$\int_{\text{Inner}} E_2^2 \, d\tau + \int_{\text{outer}} E_1^2 \, d\tau + \oint_S V\vec{E}\cdot d\vec{a} : \frac{Q^2}{4\pi\varepsilon_0^2}\cdot\frac{1}{5R} + \frac{Q^2}{4\pi\varepsilon_0^2}\cdot\frac{1}{R} - \cancel{\frac{Q^2}{4\pi\varepsilon_0^2}\frac{1}{r}} + \cancel{\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r}}$$

$$\frac{Q^2}{4\pi\varepsilon_0^2}\frac{1}{5R} + \frac{Q^2}{4\pi\varepsilon_0^2}\cdot\frac{5}{5R} = \frac{Q^2}{4\pi\varepsilon_0^2}\frac{6}{5R}$$

remember $\frac{\varepsilon_0}{2}\left(\frac{Q^2}{4\pi\varepsilon_0^2}\frac{6}{5R}\right)$

$$\frac{Q^2}{4\pi\varepsilon_0}\cdot\frac{3}{5R} = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{R} \quad \therefore \quad \boxed{W = \frac{3}{5}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{R}}$$