4.2 Real Characteristic Roots

4.2 #'s 18, 20, 30

Second order DE

$ay'' + by' + cy = 0 \Rightarrow$ Second order

$y(t) = e^{rt}$

$y'(t) = re^{rt}$

$y''(t) = r^2e^{rt}$

$ar^2e^{rt} + bre^{rt} + ce^{rt} = 0$

$e^{rt}(ar^2 + br + c) = 0$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Delta = b^2 - 4ac$

$\Delta > 0 \Rightarrow$ Two real solutions

$\Delta = 0 \Rightarrow$ one repeated real solution

$\Delta < 0 \Rightarrow$ Two complex solutions

Ex: $y'' + 5y' + 6y = 0$

characteristic: $(r^2 + 5r + 6)e^{rt} = 0$

$y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2e^{rt}$

$r^2 + 5r + 6 = 0$

$(r+3)(r+2)$

$r_2 = -3$

$r_1 = -2$

$\Delta > 0$ $\quad y(t) = c_1e^{r_1t} + c_2e^{r_2t}$

if $\Delta = 0$ $\quad y(t) = c_1e^{rt} + tc_2e^{rt}$

$y(t) = c_1e^{-2t} + c_2e^{-3t}$

Ex: $y'' - 4y' + 4y = 0$

$e^{rt}(r^2 - 4r + 4) = 0$

$(r-2)(r-2)$

$r = 2$

$c_1 = 1$

$c_2 = -1$

$y(t) = c_1e^{2t} + c_2te^{2t} \Rightarrow$ repeated root

$1 = c_1e^0 + c_2(0)e^0$

$1 = c_1$

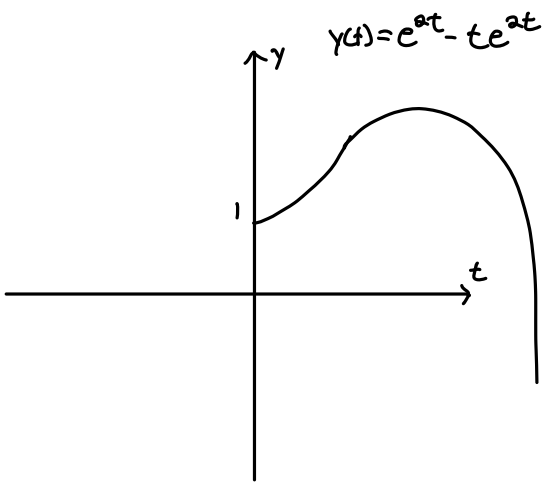
$y(t) = c_1e^{2t} + c_2te^{2t}$

$y'(t) = 2c_1e^{2t} + c_2e^{2t} + 2c_2te^{2t}$

$y'(0) = 2e^0 + c_2 + 0 = 1$

$c_2 = -1$

$y(t) = e^{2t} - te^{2t}$



Δ = Determinant

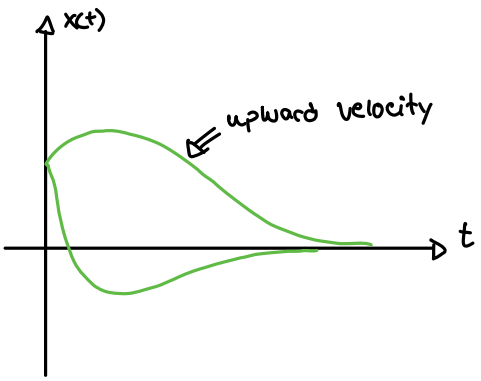
OverDamped Mass Spring System

$\Delta > 0$

$\Delta = b^2 - 4mk > 0$

Solutions tend to 0 crossing the t-axis at most once.

$x(t) = c_1e^{r_1t} + c_2e^{r_2t}$

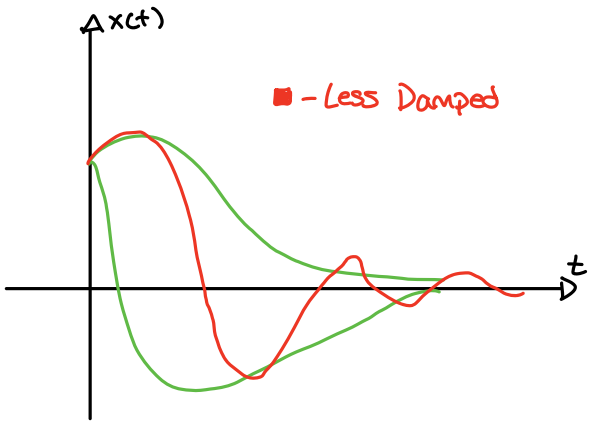


Critically Damped

$\Delta = b^2 - 4mk = 0$

the solutions still tend to 0

crosses the t-axis at most once



A mass spring system with

$m=3$ mass one meter down, no initial velocity

$b=3$ Find $x(t)$

$k=2$

$y'' + y' + y = 0$ $\quad y = \dot{x}$

$m\ddot{x} + b\dot{x} + kx = 0$

$\ddot{x} + 3\dot{x} + 2x = 0$ $\quad (x+1)(x+2)$

$e^{rt}(r^2+3r+2)=0$

$y(t) = c_1 e^{r_1 t} + t c_2 e^{r_2 t}$

$x(t) = c_1 e^{-t} + c_2 e^{-2t}$ $\quad 1 = c_1 e^0 + c_2 e^0$

$x(0) = 1$ $\quad -1 = c_1 + c_2$

$\dot{x}(0) =$ $\quad \dot{x}(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$

$-c_1 - 2c_2 = 0$ $\quad 0 = -c_1 - 2c_2$

$c_1 + c_2 = -1$ $\quad c_1 = -2$

$c_2 = 1$

$x(t) = -2e^{-t} + e^{-2t}$