2.4 Aliasing

Aliasing

$T_N = \{t_k\}_{k=0}^{N-1}$ $[a,b]$

$f(t_k) = g(t_k)$

Division Algorithm

$m = qN + r$

- m = higher frequency
- N = Nodes
- r = remainder
- q = Division #

- $f_{Ny} = N/2$
- if $r > f_{Ny}$ then aliasing $f = j$
- $j = N - r$

Aliasing of Complex Exponentials

$m = qN + r$

a.) $r \in \{0, 1, \dots, N/2\}$

$$e^{i\frac{2\pi m}{T}t_k} = e^{i\frac{2\pi r}{T}t_k}$$

$r = N/2$

$$e^{i\frac{2\pi m}{T}t_k} = (-1)^k$$

b.) $r \in \{(N/2)+1, \dots, N-1\}$

$$e^{i\frac{2\pi m}{T}t_k} = e^{i\frac{2\pi j}{T}t_k}$$

$j = N - r \in \{0, 1, \dots (N/2)-1\}$

Aliasing of Sine and Cosine

$m = qN + r$

a.) $r \in \{0, 1, \dots N/2\}$

$$\sin\left(\frac{2\pi m}{T}t_k\right) = \sin\left(\frac{2\pi r}{T}t_k\right)$$
$$\cos\left(\frac{2\pi m}{T}t_k\right) = \cos\left(\frac{2\pi r}{T}t_k\right)$$

$r = N/2$

$$\sin\left(\frac{2\pi m}{T}t_k\right) = 0$$
$$\cos\left(\frac{2\pi m}{T}t_k\right) = (-1)^k$$

b.) $r \in \{(N/2)+1, \dots, N-1\}$

$$\sin\left(\frac{2\pi m}{T}t_k\right) = -\sin\left(\frac{2\pi j}{T}t_k\right)$$
$$\cos\left(\frac{2\pi m}{T}t_k\right) = \cos\left(\frac{2\pi j}{T}t_k\right)$$

$j = N - r$ $r \in \{0, 1, \dots (N/2)-1\}$

Nyquist Frequency

Nyquist Frequency

$f_{Ny} = N/2$

Nyquist Frequency index is $N/2$