Phys 131 2016

Lecture 27

Mon: Warm Up 11

Tues: Discussion /quiz
Supp 65

Ch 10 CG 8, 13

Ch 10 Prob 35, 49, 54

Energy conservation

We saw that

> If the only forces acting on a system that do non-zero work are gravity and spring forces, then the total mechanical energy
>
> $$E_{mech} = K + U_{grav} + U_{spring}$$
>
> stays constant as the system evolves

Here the various energies are:

Kinetic energy
$K = \frac{1}{2}mv^2$

Gravitational Potential Energy
$U_g = mgy$

Elastic Potential Energy
$U_{spring} = \frac{1}{2}k(\Delta s)^2$

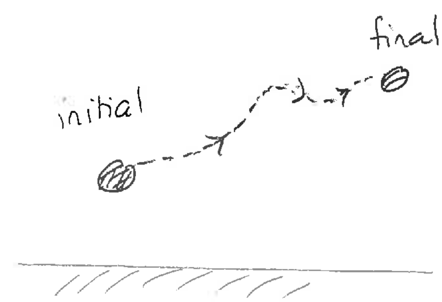
Demo: PhET Springs + masses

- Set friction = 0
- Set time → slowest
- Observe energy bar graphs

What if there are other forces beside the spring forces and gravity that do non-zero work? Can we modify the energy so that it effectively accounts for the work done by these forces? The hope would be to find a potential energy $U_{other\ force}$ so that a redefined mechanical energy $E_{mech} = K + U_{grav} + U_{spring} + U_{other}$ remains constant. Can this be accomplished?

Conservative and non-conservative forces

Consider the gravitational force acting on an object. We showed that the work done by the gravitational force is



Quiz1

$$W_{grav} = -\Delta U_g$$

90%-0

90%-0

and since the gravitational potential energy only depends on the state of the object (i.e. its vertical position) the work done by gravity only depends on the initial + final state of the object. It does not depend on the path (or the history) of the system between these states.

Similarly for a spring force the work done is

$$W_{spring} = -\tfrac{1}{2}k(\Delta s_f)^2 + \tfrac{1}{2}k(\Delta s_i)^2 = -\left[\tfrac{1}{2}k(\Delta s_f)^2 - \tfrac{1}{2}k(\Delta s_i)^2\right]$$

$$= -\left[U_{spring\,f} - U_{spring\,i}\right]$$

So we have

$$W_{spring} = -\Delta U_{spring}$$

and this again only depends on the initial + final states of the system.

This motivates the definition

> A <u>conservative force</u> is a force for which there exists a potential energy U_{force} such that the work done by the force is
>
> $$W_{force} = -\Delta U_{force}$$

Clearly both gravity and spring forces are conservative. A non-conservative force is one for which no such potential energy can be found. We can see that

> A force is conservative $\iff$ the work done by the force does not depend on the trajectory followed by the object. It only depends on the initial + final locations (or system states)

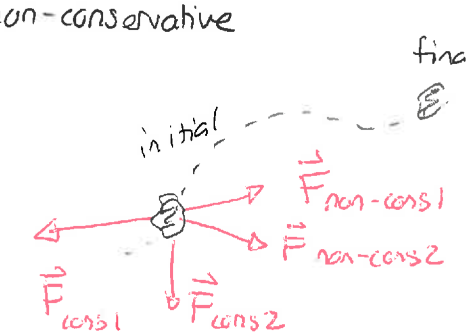
Quiz 2 70% → 95% // 80% – 90%

We can see that not every force is conservative. So we cannot always extend the definition of mechanical energy to all forces.

Mechanical energy in general.

Suppose that various conservative and non-conservative act on an object. The work kinetic energy theorem is still true.

$$\Delta K = W_{net}.$$



Then

$$\Delta K = W_{cons\,1} + W_{cons\,2} + \dots + W_{non\text{-}cons\,1} + \dots$$

But for each conservative force there is a potential energy

$$W_{cons\,i} = -\Delta U_{cons\,1}.$$

Thus

$$\Delta K = -\Delta U_{cons\,1} - \Delta U_{cons\,2} + \dots + W_{non\text{-}cons\,1} + \dots$$

$$\Rightarrow \Delta K + \Delta U_{cons\,1} + \Delta U_{cons\,2} + \dots = W_{non\text{-}cons\,1} + \dots$$

Defining

The total energy of the system:

$$E_{mech} = K + U_{cons\,1} + U_{cons\,2} + \dots$$

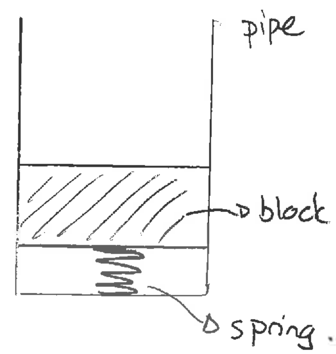
Gives:

$$\boxed{\Delta E_{mech} = W_{nc}}$$

where

$$\boxed{W_{nc} = \text{work done by all non-conservative forces}}$$

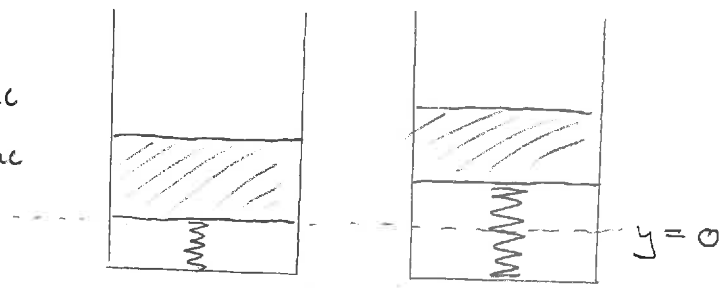
Example: A 0.20 kg block can slide up + down a vertical tube with rough sides. The block is held at rest against a spring (constant 200N/m) compressing it by 0.10m. It is released and as it moves up friction exerts a 1.80N force on the block. Determine the speed of the block as it leaves the spring (spring is relaxed).



Answer: In general

$$\Delta E_{mech} = W_{nc} \Rightarrow E_f - E_i = W_{nc}$$

$$\Rightarrow E_f = E_i + W_{nc}$$



and

$$E_{mech} = K + U_{grav} + U_{spring}$$

initial (release)

$y_i = 0m$
$v_i = 0m/s$
$\Delta s_i = 0.10m$

final (leaves block

$y_f = 0.10m$
$v_f = ??$
$\Delta s_f = 0m$

with

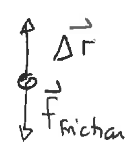
$$W_{nc} = W_{friction}$$

Thus

$$K_f + U_{grav\,f} + \cancel{U_{spring\,f}} = \cancel{K_i} + \cancel{U_{grav\,i}} + U_{spring\,i} + W_{nc}$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}k(\Delta s_i)^2 + W_{friction}$$

Now $\quad W_{friction} = f_{friction}\,\Delta r \cos 180°$



$$= 1.80N \times 0.10m \cos 180°$$

$$= -0.18J$$

$$\Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}k(\Delta s_i)^2 - 0.18J$$

$$\Rightarrow \frac{1}{2}\,0.20kg\,v_f^2 + 0.20kg \times 9.8m/s^2 \times 0.10m = \frac{1}{2}\,200N/m\,(0.10m)^2 - 0.18J$$

$$\Rightarrow 0.10kg\,v_f^2 + 0.196J = 0.82J$$

$$\Rightarrow 0.10kg\,v_f^2 = 0.62J \Rightarrow v_f = \sqrt{\frac{0.62J}{0.10kg}} \Rightarrow v_f = 2.5m/s$$