Phys 131 2016

Lecture 7

Tues: Discussion / quiz

Ch3 Conc Q: 2 } only time problems are

Ch3 Probs: 6, 12, 22, 25, 30, 32 } explicitly requested from Ch3.

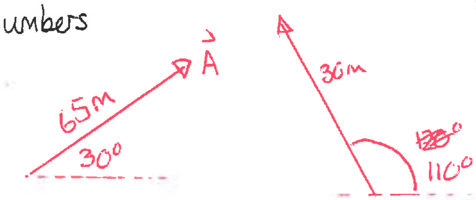
Weds: Lecture

Vector Algebra

Graphical rules give conceptual methods for adding and multiplying vectors but these are not sufficiently precise for actual calculations. We will require a number-based alternative

Represent vectors in terms of numbers

e.g.



Manipulate numbers to form vector sums, products, ..

need to combine

$A = 65m$ $B = 30m$

and angles $30°, 110°$ to get $\vec{A} + \vec{B}$

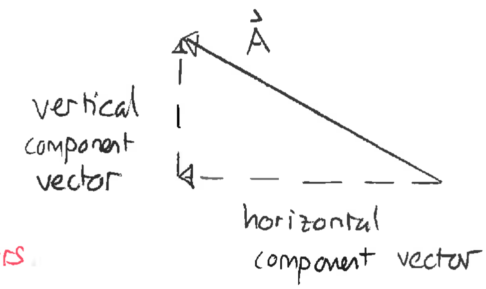
Using lengths + angles is mathematically inconvenient. We will present an alternative method of representing vectors in terms of numbers that yields more efficient calculations.

Components

We can see that any vector in two dimensions can be represented as a sum of a unique horizontal and vertical vector

Demo: PhET Vector Addition

* Settings - grid ✓
  - style 2 ✓



* show vector and pink component vectors.
* show numbers associated with component vectors
* adjust vector ⇒ components adjust.

These two vectors are called component vectors. Since the horizontal component vector either points right or left we can describe it using its length and a + sign or − sign. We can also do this for the vertical component vector. So we can describe the vector with two numbers (either positive or negative). These are called the <u>components of the vector</u>. The rules are:

The components of $\vec{A}$ are two real numbers:

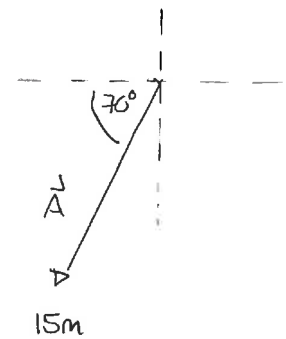
horizontal component	vertical component
* A_x	* A_y
* positive if horiz component is right	* positive if vertical component is up
* negative if horiz component is left	* negative " " " " down
* magnitude equal to magnitude of horiz component vector	* magnitude equal to magnitude of vertical component vector

WARM UP 1

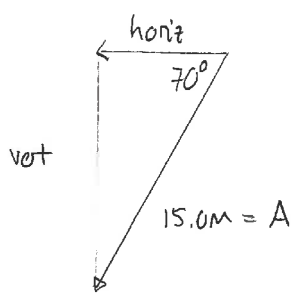
Quiz 1 95% } 95%

We usually compute components using trigonometry.

Example: Determine the components of $\vec{A}$



Answer: Sketch component vectors.



Then $\cos 70° = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{|A_x|}{A} \Rightarrow |A_x| = A\cos 70°$

$$= 15\text{m}\cos 70° = 5.1\text{ m}$$

$\sin 70° = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{|A_y|}{A} \Rightarrow |A_y| = A\sin 70°$

$$= 15\text{m}\sin 70° = 14\text{ m}$$

Inserting signs $\Rightarrow$ $A_x = -5.1\text{m}$ $A_y = -14\text{m}$

Warm Up 2

Vector algebra using components

One can show that vector addition can be done via:

If $\vec{C} = \vec{A} + \vec{B}$ then the components of $\vec{C}$ are:

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$

If $\vec{C} = s\vec{A}$ where s is a number then

$$C_x = sA_x$$

$C_y = sA_y$

So we can now combine vectors mathematically.

Get components of individual vectors → Combine x components → Construct vector combination

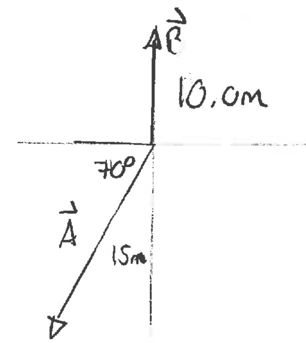
Get components of individual vectors → Combine y components → Construct vector combination

no crossing

Example: For the given $\vec{A}, \vec{B}$, determine

$$\vec{C} = \vec{A} + \vec{B}$$

Determine the magnitude of $\vec{C}$



Answer: Need individual components. Then

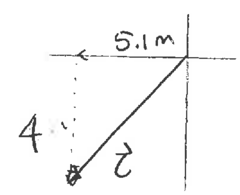
$C_x = A_x + B_x$

$C_y = A_y + B_y$

List components - one can get $\vec{B}$'s by inspection.

vector	x-comp	y-comp
$\vec{A}$	$A_x = -5.1\text{m}$	$A_y = -14\text{m}$
$\vec{B}$	$B_x = 0\text{m}$	$B_y = 10.0\text{m}$
$\vec{C}$	$C_x = -5.1\text{m}$	$C_y = -4.0\text{m}$

$\Rightarrow$



So

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-5.1\text{m})^2 + (4.0\text{m})^2} \Rightarrow C = 6.5\text{m}$$

Unit vector notation

We can use special unit vectors along the axes to represent any vector algebraically. Let

$\hat{\imath}$ = vector along +x-axis with magnitude 1

$\hat{j}$ = " " +y-axis " " 2

Then:

$$\boxed{\vec{A} = A_x \hat{\imath} + A_y \hat{j}}$$

component numbers (positive or negative)

Slides 1, 2, 3

For the previous example:

$$\vec{A} = -5.1\hat{\imath} - 14\hat{j}$$

$$\vec{B} = 10\hat{j}$$

So

$$\vec{C} = \vec{A} + \vec{B} = -5.1\hat{\imath} - 14\hat{j} + 10\hat{j} \Rightarrow \vec{C} = -5.1\hat{\imath} - 4\hat{j}$$

Quiz 2