Geogebra

Math 360 Ch 9.4: Vector & Scalar Functions; Vector Derivatives

Two types of functions:

1) **Vector function** $\vec{v}(x,y,z) = v_1(x,y,z)\vec{i} + v_2(x,y,z)\vec{j} + v_3(x,y,z)\vec{k}$

- Output values are vectors
- Graph is field of vectors
- $\vec{v}(x,y,z)$ is often called a **vector field** for this reason.

$$\mathbf{v}(x, y, z) = [v_1(x, y, z), \quad v_2(x, y, z), \quad v_3(x, y, z)].$$

Fig. 195. Field of tangent vectors of a curve

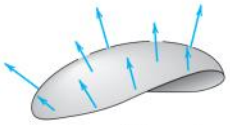


Fig. 196. Field of normal vectors of a surface

2) **Scalar Function** $z = f(x,y)$, $w = f(x,y,z)$, etc

- often called a **scalar field**

Examples

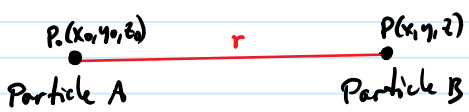
1) Ex 1, p.376: $f(x,y,z) = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$

= distance between (x_0,y_0,z_0) & (x,y,z)

2) Ex 2, p. 377: $\vec{p}(x,y,z) = -\frac{c(x-x_0)}{r^3}\vec{i} - \frac{c(y-y_0)}{r^3}\vec{j} - \frac{c(z-z_0)}{r^3}\vec{k}$

(Newton's law of gravitation)

= gravitational force acting on B from A



$\Leftrightarrow$ r = distance between A & B $= \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$

Vector Field Calculus

1) **Continuity**

$$\lim_{t\to t_0} \vec{v}(t) = \vec{v}(t_0)$$

$$\mathbf{v}(x, y, z) = [v_1(x, y, z), \quad v_2(x, y, z), \quad v_3(x, y, z)].$$

Fig. 195. Field of tangent vectors of a curve

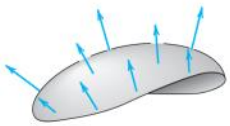
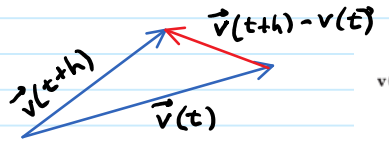
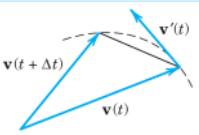


Fig. 196. Field of normal vectors of a surface

2) **Derivative** $\vec{v}'(t) = \lim_{h\to 0} \frac{\vec{v}(t+h) - \vec{v}(t)}{h}$





Shortcut:

$$\mathbf{v}'(t) = [v_1'(t), \quad v_2'(t), \quad v_3'(t)].$$

3) Product Rules, Partial Derivatives: p. 379-380

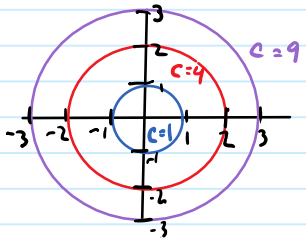
Ch 9.5 Class Prep: - Write up Examples 1-4 & include figures
- State Eqn 10, the formula for curve length

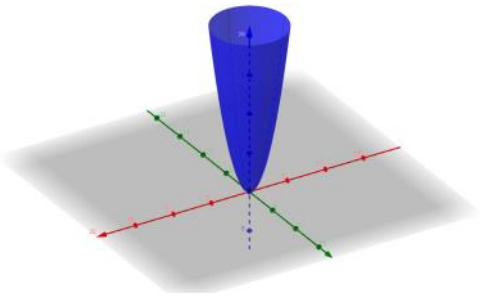
Ch 9.4 Examples

1) Find isotherms $T(x,y) = C$ & sketch some of them.

a) $T(x,y) = x^2 + y^2$

Solution Isotherms: $x^2 + y^2 = C \rightarrow$ circles centered at $(0,0)$ radius $r = \sqrt{C}$





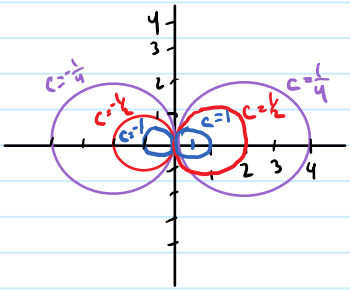
Graph generated online using GeoGebra 3D calculator

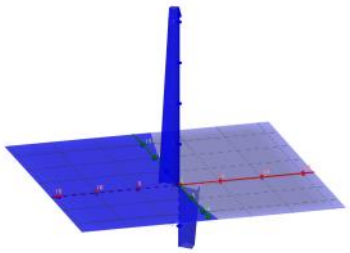
2) $T = \dfrac{x}{x^2+y^2}$

Solution Isotherms: $\dfrac{x}{x^2+y^2} = C \Rightarrow x^2 + y^2 - \frac{1}{C}x = 0$

$\left(x - \frac{1}{2C}\right)^2 + y^2 = \frac{1}{4C^2}$ ← complete square

$\Rightarrow$ circle centered at $\left(\frac{1}{2C}, 0\right)$ with radius $r = \left|\frac{1}{2C}\right|$.

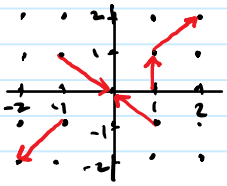




- Take leading linear term and divide by 2
- Add to both sides
- Group and factor

3) Sketch $\vec{v}(x,y) = y\,\vec{i} + x\,\vec{j}$

Solution



x	y	$\vec{v}(x,y)$
1	0	$0\vec{i} + 1\vec{j}$
1	1	$1\vec{i} + 1\vec{j}$
-1	1	$1\vec{i} - 1\vec{j}$
-1	-1	$-\vec{i} - \vec{j}$
1	-1	$-\vec{i} + \vec{j}$

4) $\vec{v} = [xy^2, 3x+2y]$

a) $\dfrac{\partial \vec{v}}{\partial x} = [y^2, 3]$ b) $\dfrac{\partial \vec{v}}{\partial y} = [2xy, 2]$