Ch 3.4

To choose models we must consider,

- How well they make predictions
- Their Simplicity
- The reasonability of the predictions

Want

- Lines going through data
- Small oscillations
- High R^2 values

Ch 4.1

A dynamical system is simply a system that changes over time.

A dynamical System measured in discrete units is a discrete dynamical system. $\longrightarrow$ A sequence of numbers

"Easy to model, hard to solve"

Definition 4.1.1: A discrete dynamical system is a sequence of numbers $\{a_n \mid n=0,1,....\}$ defined by a relation of the form

$$a_{n+1} = f(a_n) \qquad (1)$$

$a_n \rightarrow$ State of the system

A discrete dynamical system is determined by previous States

where f is some real-valued function

Definition 4.1.2: A linear discrete dynamical System is a sequence of numbers $\{a_n \mid n=0,1,....\}$ defined by a relation of the form

$$a_{n+1} = ba_n \qquad (2)$$

Where $b \neq 0$ is a constant.

Theorem 4.1.1: The solution of a linear dynamical System $a_{n+1} = ba_n$ for $b \neq 0$ is

$$a_n b^0 a_0 = a_0 \qquad (3)$$

Where a_0 is the intial State.

Ch 4.4

In a forest containing foxes and rabbits, foxes eat rabbits for food.

Assumptions

1. The only source of food for the foxes are rabbits, the only predator of the rabbits are foxes.
2. Without rabbits present, the fox population would die out.
3. Without foxes present, the rabbit population would grow.
4. Presence of rabbits increases rate at which fox population grows.
5. Presence of foxes decrease rate at which rabbit population grows.

Assumptions 2 & 3

2. : $\Delta F_n = -\alpha F_n$
3. : $\Delta R_n = \beta R_n$

Assumptions 4 & 5

4. : $\Delta F_n = (-\alpha + \xi R_n) F_n$: $F_{n+1} = F_n(1-\alpha) + F_n R_n \xi$
5. : $\Delta R_n = (\beta - \delta F_n) R_n$: $R_{n+1} = R_n(1+\beta) - F_n R_n \delta$

$\alpha > 0,\ 0 \leq \beta \leq 1,\ \xi \geq 0,\ \delta \geq 0$

: Because this system has two objects in it, rabbits and foxes, it is considered a two-dimensional discrete dynamical system.

These types of models are called Lokta-Volterra models. Since the final equation for the population increase has a $F_n R_n$ term in it, the model is considered to be nonlinear.

Ch. 4.6

Epidemic Models

Consider the following assumptions:

1. No one enters or leaves the community and no one in the community has contact with anyone outside the community.
2. Each person is either susceptible, S (able to get the flu); infected, I (currently has the flu and able to spread it); or removed, R (already had the flu and is not able to get it again). Initially every person is either susceptible or infected.
3. A susceptible person can get the flu only by contact with an infected person.
4. Once a person gets the flu, he or she cannot get it again.
5. The average duration of the flu is 2 weeks, during which time an infected person can spread the disease to a susceptible person.

$S \rightarrow I \rightarrow R$ Average duration of flu, 2 weeks: $\gamma = 0.5$

Removal Rate: $\Delta R_n = \gamma I_n \rightarrow R_{n+1} = R_n + \gamma I_n$

$\gamma \rightarrow$ Removal rate

$\gamma I_n \rightarrow$ Infected people removed

Infected Rate: $\Delta I_n = \alpha S_n I_n - \gamma I_n \rightarrow I_{n+1} = I_n + \alpha S_n I_n - \gamma I_n$

$\alpha \rightarrow$ Transmission Coefficient (likelihood of a susceptible and infected person interacting)

$\alpha S_n I_n \rightarrow$ Interactions between susceptible and infected people. "Newly infected"

Susceptible: $\Delta S_n = -\alpha S_n I_n \rightarrow S_{n+1} = S_n - \alpha S_n I_n$

All Equations:

$$S_{n+1} = S_n - \alpha S_n I_n$$
$$I_{n+1} = I_n + \alpha S_n I_n - \gamma I_n$$
$$R_{n+1} = R_n + \gamma I_n$$

$$\gamma = \frac{1}{\text{avg. duration of infectious period}}$$

$$\alpha = \frac{\text{\# new cases in week}}{(\text{initial \# of cases})(\text{Population} - \text{initial \# of cases})}$$

Ex: $\# \text{ cases/wk} \rightarrow \lambda$

$\lambda = 5$

initial cases $\rightarrow I_0$

$I_0 = 3$

Population $\rightarrow P_0$

$P_0 = 1000$

$$\alpha = \frac{\lambda}{I_0(P_0 - I_0)} = \frac{5}{3(1000-3)} = \frac{5}{3(997)} = 0.00167$$

$\alpha = 0.00167$, $\gamma = 0.5$

Ch 6.1

Deterministic systems are systems where the behavior is known once its parameters are known. $\rightarrow A = \int_a^b f(x)\,dx$

A probabilistic system is one in which the behavior is determined, in part, by random events.

One very popular type of probabilistic model is a simulation model.

- A simulation model is a model that uses random numbers.
  - usually used to imitate some type of real-world behavior.

Reasons to use a Simulation Model

1. The System is far too complex to model analytically. This means the system has too many variables too model analytically. This system would have to be simplified resulting in a very low-fidelity model.

2. It may be difficult, costly, or dangerous to collect data for creating an empirical model. Some systems may be too risky to try to model.

3. The System may not exist yet. You may be modeling something that has never been constructed yet.

4. System may contain random events that we do not want to oversimplify. Some variables may be drastically over simplified messing up the model.

Monte Carlo Games

The simulations consist of three basic steps:

1.) Construct a model that uses random numbers
2.) Evaluate, or "run", the model many times (Possibly hundreds or thousands) using different random numbers each time.
3.) Statistically analyze the results

Benefits

- Test "what if" scenarios
- Do thousands of tests
- No required data gathering