Phys 312
Spring 2021

Electromagnetic Theory II: Homework 18

Due: 16 April 2021

1 Fields and potentials produced by a charge with constant velocity

The term $\boldsymbol{\mathscr{r}} \cdot \mathbf{u}$, evaluated at the retarded time, appears in both the potentials and fields for moving charges. Consider a particle moving with constant velocity.

a) Show that

$$\boldsymbol{\mathscr{r}} \cdot \mathbf{u} = \sqrt{(c^2 t - \mathbf{r} \cdot \mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2 t^2)}.$$

b) Let

$$\mathbf{R} := \mathbf{r} - \mathbf{v}t$$

and show that

$$\boldsymbol{\mathscr{r}} \cdot \mathbf{u} = Rc\sqrt{1 - \frac{v^2}{c^2} \sin^2 \theta}$$

where θ is the angle between $\mathbf{R}$ and $\mathbf{v}$.

2 Charged particle in circular orbit

A particle with charge q follows a circular orbit at constant speed in the xy plane. The radius of the orbit is R and it is centered on the origin. Let ω be the angular velocity.

a) Show that the electric field at the center of the orbit is

$$\mathbf{E} = -\frac{1}{4\pi\epsilon_0 R^2} \left\{ \left[\left(1 - \frac{v^2}{c^2}\right) \cos\left(\omega t_r\right) - \frac{v}{c} \sin\left(\omega t_r\right) \right] \hat{\mathbf{x}} + \left[\left(1 - \frac{v^2}{c^2}\right) \sin\left(\omega t_r\right) + \frac{v}{c} \cos\left(\omega t_r\right) \right] \hat{\mathbf{y}} \right\}$$

b) Determine an expression for the magnetic field at the center of the orbit.

c) Show that the magnetic field at the center of the orbit is exactly the same as that obtained from a loop with uniform current $I = q/T$ where T is the period of orbit of the charge.