Physics 342, Homework 7

1: Use both a leapfrog and Runge Kutta solver for the following assignment. Numerically integrate the equations for two bodies orbiting each other. You can do the Sun and the Earth and look up initial conditions online. Essentially you need to solve $\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r}$ for both objects. Do this in two dimensions (no z coordinate) so that $\vec{F}_{1,2} = -\frac{GM_1M_2}{[(x_1-x_2)^2+(y_1-y_2)^2]^{3/2}}(x_1-x_2)\hat{\imath}+(y_1-y_2)\hat{\jmath}$ If you are clever, recall that $\vec{F}_{1,2} = -\vec{F}_{2,1}$ and don't do twice the calculations needed. Plot your solution for 10-20 orbits and plot then total energy. Do this using the RK4 method and the leapfrog method and comment on the differences in the results between solvers. Google around for different initial conditions but recall that the angular momentum is conserved and take your initial condition to be one where the initial velocity i purely in the y direction. As no one in this course is required to know programming you may just use the built in solvers which default to RK4 methods. As such, you may just solve the equations of motion using maple or python with the RK4 solvers. Solving with the leap frog method as well will net you a plus 40 extra credit for this homework assignment.

2: (Medium) Integrate equation 8.38 to obtain 8.41. Take the constant to be $-\pi/2$. You may manipulate the integral into a form and then use a table, you may not use an integrator. Hint, let u =1/r also look at appendix E.

3: (Longer) For a force $F(r) = -k/r^3$ set up the equation of motion 8.20. There are three possible cases $l^2 = \mu k$, $l^2 > \mu k$, and $l^2 < \mu k$. Each of these three cases results in a different solution to the differential equation you'll "derive" using equation 8.20. Solve it in terms of u then convert it to r. As this is a second order equation each solution will have two unknown constants. Pick a value for each case, write it out explicitly and produce a plot of the solution for me.

Describe what is physically happening in each case - is the motion unbounded, bounded, or inspiralling? One trick to explore the behavior is to plot the effective potential (equation 8.34, depends on your form for r) and examine behavior based on your effective potential. Specifically, either plot the effective potential using your solution for r in each of the three cases or plot your solution.

Hint - combinations of $A\cos(\alpha\theta) + B\sin(\alpha\theta) = C\cos(\alpha\theta + \delta)$. Another Hint - for the case of sinusoidal motion the effective potential is the easiest way to determine the motion. Similarly linear combinations of exponentials of the form $Ae^{\beta x} + Be^{-\beta x} = C\cosh(\beta x + \delta)$.

4: (easy) Taking the center of the sun as the origin, calculate the center of mass of the solar system 8 times. Once with The Sun and Mercury, then the Sun and Venus, then the Sun and Earth, then the .. keep doing it as if each of the eight planets were the only planets in the solar system. Look up the planet masses and semi-major axis on the internet. Finally, calculate the center of mass of the solar system with all planets assuming they are all aligned in a perfect line on the same side of the sun. The best way to do this problem is with an excel spreadsheet where you can write 1.5×10^{30} as 5E30.

Graph this and label the x axis by the planets with the last entry being the combined center of mass. You will need to plot the log of the center of mass to see differences. Write a comment as to whether or not this is a reasonable estimate for the average center of mass and if not why not. NOTE - You must arrange the planets in order from closest to farthest. Order them this way for the next problem as well.

5: (Tedious only due to looking things up, combine this with 4 on the same spreadsheet.) The spin angular momentum of a solid body about it's own center is $L_{spin} = I\omega_{rotation}$ where you can take the moment of inertia to be for a solid ball and the radius to be the radius of the planet. The orbital angular momentum of a planet around the sun is $L_{orbit} = md^2\omega_{orbit}$ where d is the planet's distance from the sun.

A) Recalling the definition or orbital and rotational ω and using values from the internet, create an excel spreadsheet which calculates the individual orbital and spin angular momentum of each planet about it's own axis and about the sun. Also, calculate the spin orbital angular momentum of the sun. Add each planets spin and orbital momentum together and give each planet's TOTAL angular momentum. Add a tenth entry that has the total spin, orbital, and spin+orbital angular momentum of all planets and the sun. Your 1st column should read SUN. Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Total

In three separate plots plot the spin angular momentum versus planet, the orbital angular momentum of each planets, and the total angular momentum of each planet. Comment on where most of the angular momentum lies. How much of the total angular momentum is in the sun?

Give me the plots as bar plots with axis titles with the appropriate units. Plot the log of the quantity, this can be done by clicking on the y axis and selecting scale. The only plot that you should present as a scatter plot is the last one in the question above. You will be graded on the neatness of the plot, if you need to rest minimum values to neaten up plots then do so. Set the graph of total angular momentum with a minimum y value of 1e36.

B) The binding energy of a spherical mass is given by $E_{binding} = \frac{3GM^2}{5r}$. The kinetic energy of a spinning sphere is $L^2/(2I)$. Determine the angular momentum which would cause the kinetic energy in spin to equal the binding energy. Is the sun in danger of spinning itself out? Is there enough angular momentum in the solar system, that, if it was all put into the sun, would be problematic?

6: 8-6 Hint - use conservation of energy and conservation of momentum.