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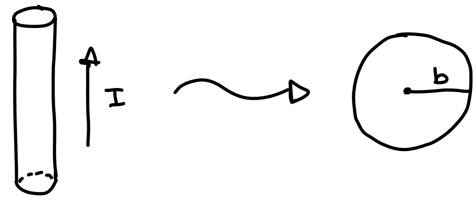
Dr. Middleton
PHYS 311
HW 11

Problem 1)

Ampere's Law : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$

$I_{NC} = \int \vec{J}\cdot d\vec{a}$

a.)

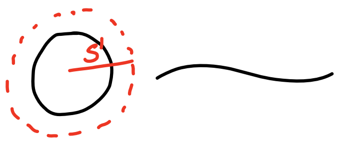


Inside :

no I_{ENC} $\therefore$ $\vec{B}=0$

$$\boxed{\vec{B}_{inside} = 0}$$

Outside:



: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$ $\qquad I_{ENC} = I$, $d\vec{l} = s' d\phi\,\hat{\phi}$

$\vec{B} = B\,\hat{\phi}$

$$B\int_0^{2\pi} s'\,d\phi = \mu_0 I$$

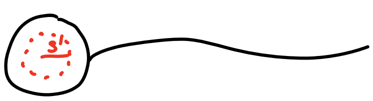
$$B\cdot s'\cdot \phi\Big|_0^{2\pi} = \mu_0 I$$

$$B\cdot 2\pi\cdot s' = \mu_0 I$$

$$\boxed{\vec{B}_{outside} = \frac{\mu_0 I}{2\pi s'}\,\hat{\phi}}$$

b.)

Inside:



: $\int \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$ $\qquad \vec{J} = k\cdot s^2\,\hat{z}$

$d\vec{a} = s\,ds\,d\phi\,\hat{z}$
$\vec{J} = ks^2\,\hat{z}$

$$I_{ENC} = \int \vec{J}\cdot d\vec{a} = k\int_0^{2\pi}\int_0^{s'} s^3\,ds\,d\phi = k\cdot 2\pi\cdot\frac{s'^4}{4} = \frac{k\cdot\pi}{2}\cdot s'^4$$

$\int \vec{B}\cdot d\vec{l}$: $\vec{B} = B\hat{\phi}$, $d\vec{l} = s'\,d\phi\,\hat{\phi}$ $\qquad B\cdot s'\int_0^{2\pi} d\phi = B\cdot s'\cdot 2\pi$

$$Bs'\cdot 2\pi = \mu_0 k\cdot\frac{\pi}{2}\cdot s'^4 \rightarrow \vec{B} = \mu_0\frac{k}{4}\cdot s'^3\,\hat{\phi}$$

$$\boxed{\vec{B}_{inside} = \mu_0\frac{k}{4}\cdot s'^3\,\hat{\phi}}$$

Outside:

: Distance away is

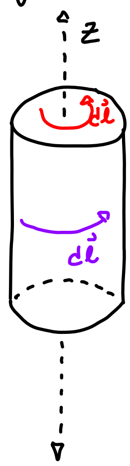
$\vec{J} = k\cdot s^2\,\hat{z}$
$d\vec{a} = s\,ds\,d\phi\,\hat{z}$
ds limits : 0 $\rightarrow$ b
$d\vec{l} \rightarrow s\,d\phi$

$$I_{ENC} = k\int_0^{2\pi}\int_0^{b} s^3\,ds\,d\phi = k\cdot 2\pi\cdot\frac{b^4}{4} = k\cdot\pi\cdot\frac{b^4}{2}$$

$\int \vec{B}\cdot d\vec{l} = B\cdot s'\int_0^{2\pi} d\phi = B\cdot s'\cdot 2\pi$ $\quad\therefore\quad$ $B\cdot 2\pi s' = k\cdot\frac{\pi b^4}{2}$ $\quad\therefore\quad$ $\boxed{\vec{B} = \frac{k\cdot b^4}{4s'}\,\hat{\phi}}$

Problem 1)

a.) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$: $I_{ENC} = \int \vec{J}\cdot d\vec{a}$



Inside : All charge resides on the outter edge of the cylinder. Therefore the magnetic field of the inside region is 0.

Outside: $\oint \vec{B}\cdot d\vec{l} = B\cdot s\int_0^{2\pi} d\phi = Bs\cdot\phi\Big|_0^{2\pi} = Bs\cdot 2\pi$

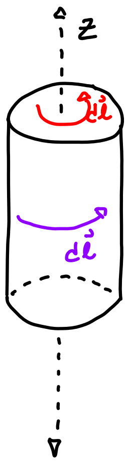
$\vec{B} = B\,\hat{\phi}$

$d\vec{l} = s\,d\phi\,\hat{\phi}$

$Bs\cdot 2\pi = \mu_0 I$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s}\,(\hat{\phi})}$$

b.)



outside : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$

Total wire $\rightarrow$ $I_{ENC} = \int \vec{J}\cdot d\vec{a} = k\int_0^{2\pi}\int_0^{b} s'^3\,ds'\,d\phi$

$\vec{J} = k\cdot s^2\,(\hat{z})$

$d\vec{a} = s\,ds\,d\phi\,(\hat{z})$

$I_{ENC} = k\cdot 2\pi\cdot\frac{s'^4}{4}\Big|_0^{b} = \frac{k\cdot\pi\cdot b^4}{2}$: $\boxed{I_{ENC} = \frac{1}{2}k\cdot\pi\cdot b^4}$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$: $I_{ENC} = I$

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$: $d\vec{l} = s\,d\phi\,(\hat{z})$, $\vec{B} = B(\hat{z})$

$Bs\int_0^{2\pi} d\phi = Bs\cdot 2\pi$: $Bs\cdot 2\pi = \mu_0 I$

$B = \frac{\mu_0}{2\pi}\cdot\frac{I}{s}\,(\hat{z})$

outside : $\boxed{\vec{B} = \frac{\mu_0}{2\pi}\cdot\frac{I}{s}\,(\hat{z})}$

Inside : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$

$I_{ENC} = \int \vec{J}\cdot d\vec{a} = k\int_0^{2\pi}\int_0^{s} s'^3\,ds'\,d\phi = k\cdot 2\pi\cdot\frac{s'^4}{4}\Big|_0^{s} = \frac{k\cdot\pi\cdot s^4}{2} = I\cdot\frac{s^4}{b^4}$

$\vec{J} = k\cdot s^2\,(\hat{z})$

$d\vec{a} = s\cdot ds\,d\phi\,(\hat{z})$

$\oint \vec{B}\cdot d\vec{l} = B\cdot s\int_0^{2\pi} d\phi = B\cdot s\cdot 2\pi$

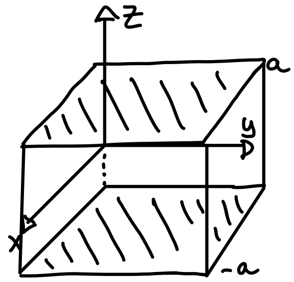
$\vec{B} = B(\hat{\phi})$

$d\vec{l} = s\,d\phi\,(\hat{\phi})$

$B\cdot s\cdot 2\pi = \mu_0 I\cdot\frac{s^4}{b^4}$

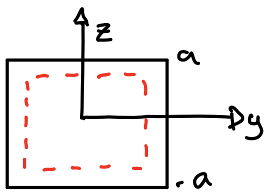
$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi}\cdot\frac{s^3}{b^4}\,(\hat{\phi})}$$

Problem 2



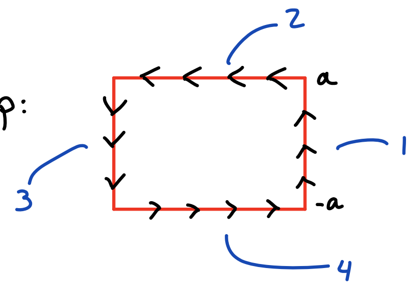
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC} \quad : \quad I_{ENC} = \int \vec{J}\cdot d\vec{a} \quad : \quad \vec{J} = J\hat{x}$$

Inside Slab:



at given x

Amperian loop:



$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC} \quad : \quad I_{ENC} = \int \vec{J}\cdot d\vec{a}$$

$$\oint_c \vec{B}\cdot d\vec{l} = \int_1 \cancelto{0}{\vec{B}\cdot d\vec{l}} + \int_2 \vec{B}\cdot d\vec{l} + \int_3 \cancelto{0}{\vec{B}\cdot d\vec{l}} + \int_4 \vec{B}\cdot d\vec{l} = \int_2 \vec{B}\cdot d\vec{l} + \int_4 \vec{B}\cdot d\vec{l} = 2\int \vec{B}\cdot d\vec{l}$$

$$\oint_c \vec{B}\cdot d\vec{l} = 2\int \vec{B}\cdot d\vec{l}$$

$$\vec{B} = B\hat{y}, \quad d\vec{l} = dy\,\hat{y} \qquad 2\cdot B\int_0^l dy = 2\cdot B\cdot l = 2Bl$$

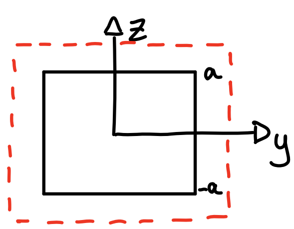
$$I_{ENC} = \int \vec{J}\cdot d\vec{a}$$

$$\vec{J} = J\hat{x}$$
$$d\vec{a} = dy\,dz\,\hat{x} \qquad \int \vec{J}\cdot d\vec{a} = J\int_{-z}^{z}\int_0^l dy\,dz = J\cdot 2z\cdot l$$

$$2\cdot Bl = \mu_0 J\cdot 2zl$$

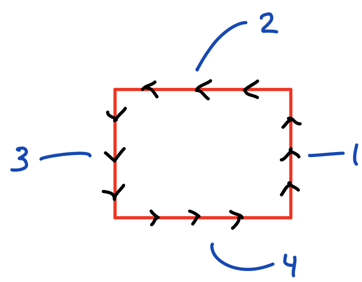
$$\boxed{\vec{B} = \mu_0 J\cdot z\,\hat{y} \quad : \quad -a < z < a}$$

Outside Slab:



at given x

Amperian Loop:



$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC} \quad : \quad I_{ENC} = \int \vec{J}\cdot d\vec{a}$$

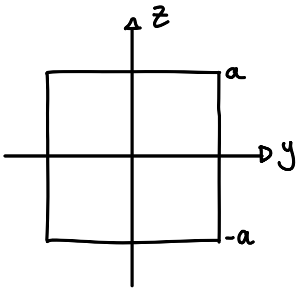
$$\oint_c \vec{B}\cdot d\vec{l} = \int_1 \cancelto{0}{\vec{B}\cdot d\vec{l}} + \int_2 \vec{B}\cdot d\vec{l} + \int_3 \cancelto{0}{\vec{B}\cdot d\vec{l}} + \int_4 \vec{B}\cdot d\vec{l} = \int_2 \vec{B}\cdot d\vec{l} + \int_4 \vec{B}\cdot d\vec{l} = 2B\int dl \ :$$

$$\vec{B} = B\hat{y}$$
$$d\vec{l} = dy\,\hat{y} \qquad 2B\int_0^L dy = 2BL \qquad \int \vec{J}\cdot d\vec{a} = J\int_{-a}^{a}\int_0^L dy\,dz = J\cdot 2a\cdot L$$
$$d\vec{a} = dy\,dz$$

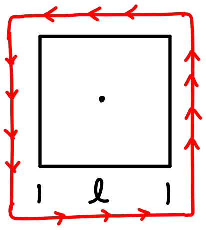
$$2BL = \mu_0 J\cdot 2a\cdot L$$

$$\boxed{\vec{B} = \pm\mu_0 Ja\,\hat{y}}$$

Problem 2



Outside :



$\vec{B} = (\hat{y})$

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{ENC}$: $I_{ENC} = \int \vec{J}\cdot d\vec{a}$

$\vec{J} = J\,(\hat{x})$

$d\vec{a} = dy\,dz\,(\hat{x})$

$$I_{ENC} = J\int_{-a}^{a}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} dy\,dz = J\cdot z\Big|_{-a}^{a}\cdot y\Big|_{-\frac{\ell}{2}}^{\frac{\ell}{2}} = J\cdot 2a\cdot \ell$$

$I_{ENC} = J\cdot 2a\cdot \ell$

$$\vec{B}_{net} = \underbrace{\cancelto{0}{\oint \vec{B}_L\cdot d\vec{\ell}} + \cancelto{0}{\oint \vec{B}_R\cdot d\vec{\ell}}}_{\perp \text{ to one another}} + \oint \vec{B}_b\cdot d\vec{\ell} + \oint \vec{B}_T\cdot d\vec{\ell} = 2\oint \vec{B}\cdot d\vec{\ell}$$

$\vec{B} = B(\hat{y})$

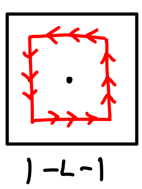
$d\vec{\ell} = dy\,(\hat{y})$

$$2\oint \vec{B}\cdot d\vec{\ell} = 2\cdot B\int_0^{\ell} dy = 2B\cdot \ell$$

$$\therefore \quad 2B\ell = \mu_0\cdot 2Ja\ell$$

$$\boxed{\begin{aligned}\vec{B} &= \mu_0\cdot J\cdot a\,(\hat{y}) \;:\; z<-a \\ \vec{B} &= -\mu_0\cdot J\cdot a\,(\hat{y}) \;:\; z>a\end{aligned}}$$

Inside:



$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{ENC}$: $I_{ENC} = \int \vec{J}\cdot d\vec{a}$: $\vec{J} = J\,(\hat{x})$

$d\vec{a} = dy\,dz\,(\hat{x})$

$$I_{ENC} = J\int_{-z}^{z}\int_0^{\ell} dy\,dz = J\cdot z\Big|_{-z}^{z}\cdot y\Big|_0^{\ell}$$

$I_{ENC} = J\cdot 2z\cdot \ell$

$$\oint \vec{B}\cdot d\vec{\ell} = B\int_0^{\ell} dy = B\cdot y\Big|_0^{\ell} = B\cdot \ell \;:\; B_{Tot} = 2B\cdot \ell$$

$\vec{B} = B(\hat{y})$

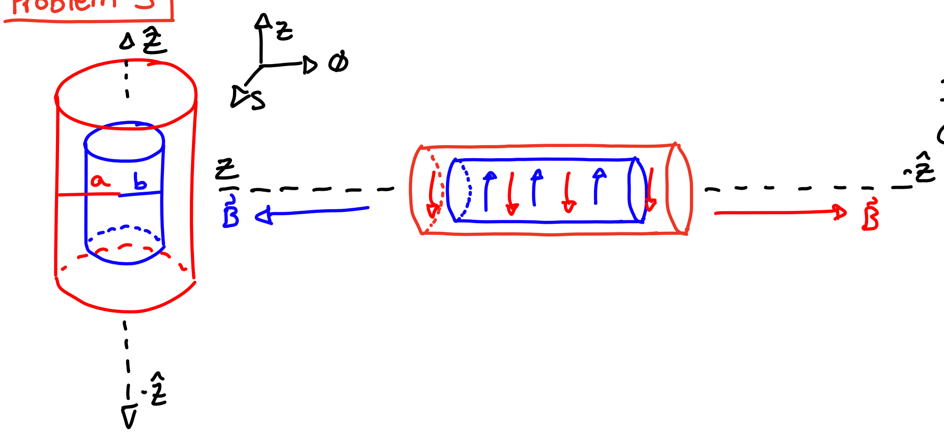
$d\vec{\ell} = dy\,(\hat{y})$

$$2B\cdot \ell = \mu_0\cdot J\cdot 2z\cdot \ell$$

$B = \mu_0\cdot J\cdot z\,(\hat{y}) \rightarrow$ points in negative y $\therefore$

$$\boxed{\vec{B}(z) = \mu_0\cdot J\cdot z\,(-\hat{y})}$$

Problem 3



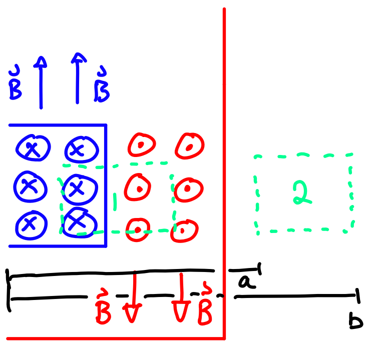
Inner: n turns per length : I_b (1*)

Outter: ℓ turns per length : I_a (2*)

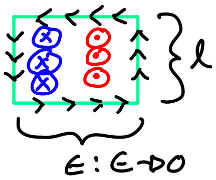
a.) $s < b$

Amp loop 2: $\oint \vec{B}_{out} \cdot d\vec{\ell} = [B(a) - B(b)]L = \mu_0 I_{ENC}$: $I_{ENC} = 0$

$B(a) = B(b)$ ⟵ outside $= 0$!



Amp loop 1:



$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{ENC}$

$\vec{B}_b \int_0^\ell dz (\hat{z}) = \vec{B}_b \cdot z \Big|_0^\ell (\hat{z}) = \vec{B}_b \cdot \ell \cdot (\hat{z})$

$\vec{B}_a \int_0^\ell dz (-\hat{z}) = \vec{B}_a \cdot z \Big|_0^\ell (-\hat{z}) = \vec{B}_a \cdot \ell \cdot (-\hat{z})$

$\vec{B}_{net} = \vec{B}_b + \vec{B}_a$

$B_b \ell = \mu_0 I_b$

$B_b = \mu_0 \frac{I_b}{\ell} \overset{(1*)}{=} \mu_0 n I_b$: $\vec{B}_b = \mu_0 n I_b (\hat{z})$

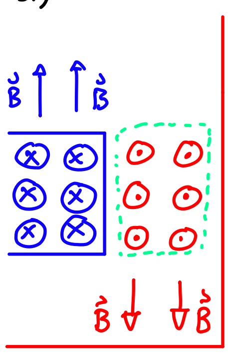
$B_a \ell = \mu_0 I_a$

$B_a = \mu_0 \frac{I_a}{\ell} \overset{(2*)}{=} \mu_0 \ell I_b$: $\vec{B}_a = \mu_0 \ell I_a (-\hat{z})$

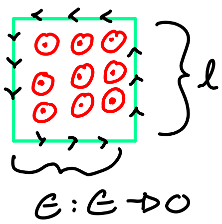
$\therefore$

$$\boxed{\vec{B}_{net} = \mu_0 [n I_b - \ell I_a] (\hat{z})}$$

b.) $b < s < a$



Amp loop:



$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{ENC}$

$\vec{B} \int_0^\ell dz (-\hat{z}) = \vec{B} \ell (-\hat{z})$

$B\ell = \mu_0 I_a$: $\vec{B} = \mu_0 \frac{I_a}{\ell} (-\hat{z}) \overset{(2*)}{=} \mu_0 \ell I_a (-\hat{z})$

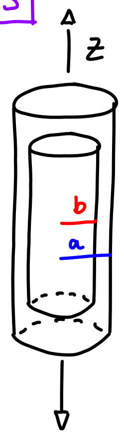
$\therefore$

$$\boxed{\vec{B}_{net} = \mu_0 \ell I_a (-\hat{z})}$$

c.) Referencing amperian loop 2 in part a.), there is no magnetic field outside the solenoids due to no current penetrating the amperian loop. Therefore,

$$\boxed{\vec{B}_{net} = 0}$$

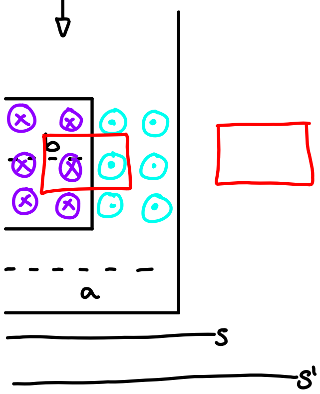
Problem 3



Inner: $r=b,\ I_b,\ n$

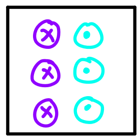
Outter: $r=a,\ I_a,\ \ell$

a.)



$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{ENC}$

Outside loop: no current enclosed $\therefore\ \vec{B}=0$

Inside loop:  $\rightarrow\ \vec{B}_{net} = \vec{B}_a + \vec{B}_b$

For b: $\oint \vec{B}_b\cdot d\vec{\ell} = B_b\int_0^L dz = B_b\cdot L$

$\vec{B}_b = B_b\,(-\hat{z})$
$d\vec{\ell} = dz\,(\hat{z})$

$I_{ENC} = nI_b$

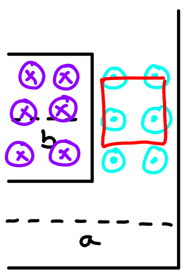
For a: $\oint \vec{B}_a\cdot d\vec{\ell} = B_a\int_0^L dz = B_a\cdot L$

$\vec{B}_a = B_a(\hat{z})$
$d\vec{\ell} = dz(\hat{z})$

$I_{ENC} = \ell I_a$

$\therefore\ \boxed{\vec{B} = \mu_0\left[\ell I_a - nI_b\right](\hat{z})\ :\ s<a}$

b.)



$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{ENC}$

$I_{ENC} = I_a$

$\oint \vec{B}\cdot d\vec{\ell} = B_a\int_0^L dz = B_a\cdot L$

$\vec{B}_a = B_a(\hat{z})$
$d\vec{\ell} = dz\,(\hat{z})$

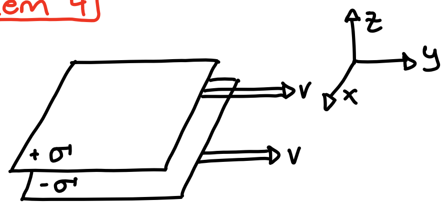
$B_a\cdot L = \mu_0 I_a\ :\ \vec{B}_a = \dfrac{\mu_0 I_a}{L} = \mu_0 \ell I_a\,(\hat{z})$

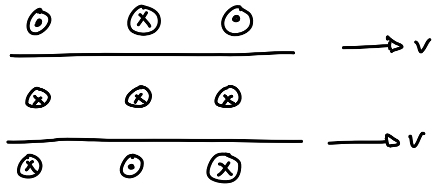
$$\boxed{\vec{B} = \mu_0 \ell I_a\,(\hat{z})}$$

c.) No magnetic field when $s>a$ since there is no current enclosed

$$\boxed{\vec{B}=0}$$

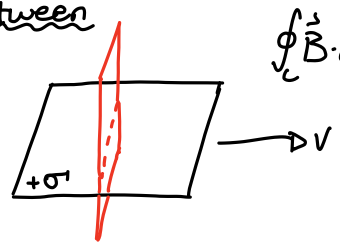
Problem 4





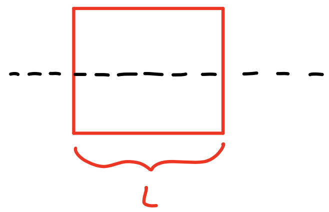
a) $\vec{B}$ between the plates and, above and below

Between



$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{Enc} : \quad \vec{B} = B(-\hat{x}),\ d\vec{l} = dx(-\hat{x}) : \quad I_{Enc} = \int \vec{J}\cdot d\vec{a}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \cancelto{0}{\int_L \vec{B}\cdot d\vec{l}} + \int_T \vec{B}\cdot d\vec{l} + \cancelto{0}{\int_R \vec{B}\cdot d\vec{l}} + \int_B \vec{B}\cdot d\vec{l}$$



$$\oint \vec{B}\cdot d\vec{l} = 2\int_B \vec{B}\cdot d\vec{l} = 2B\int_0^L dx = 2BL$$

$$2BL = \mu_0 \sigma V L$$

$$\vec{B} = \frac{\mu_0 \sigma V}{2}(-\hat{x}) \qquad \sigma V \rightarrow K \qquad \text{Bottom plate has same magnitude}$$

$$B_{net} = \vec{B}_{Top} + \vec{B}_{bot} = \frac{\mu_0\sigma V}{2}(-\hat{x}) + \frac{\mu_0\sigma V}{2}(-\hat{x}) = \mu_0 \sigma V(-\hat{x})$$

$$\boxed{\vec{B} = \mu_0 \sigma V\,(-\hat{x})}$$

b) F/A on top plate:

$$\vec{F}_{mag} = q(\vec{v}\times\vec{B})$$

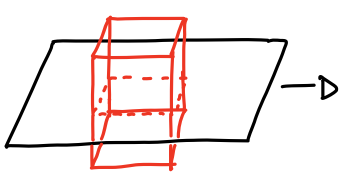
$$\sigma \equiv \frac{Q}{A} \quad \therefore \quad Q = \sigma A \quad : \quad \vec{V}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & V & 0 \\ -\frac{\mu_0\sigma V}{2} & 0 & 0 \end{vmatrix} = \hat{x}(0) + \hat{y}(0) + \hat{z}\left(\frac{\mu_0\sigma V^2}{2}\right)$$

$$F = \sigma A\left(\frac{\mu_0 \sigma V^2}{2}\right)$$

$$\frac{F}{A} = \frac{\mu_0\sigma^2 V^2}{2} \equiv f_m$$

$$\boxed{\frac{F}{A} = \frac{\mu_0\sigma^2 V^2}{2}(\hat{z})}$$

c.) $\vec{F}_E$ to balance $\vec{F}_M$? $\quad \vec{F} = Q\vec{E}$



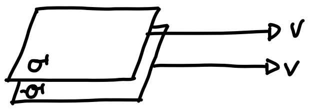
$$\int \vec{E}\cdot d\vec{a} = \frac{Q_{Enc}}{\varepsilon_0} : \quad E\cdot 2A = \frac{\sigma A}{\varepsilon_0} \quad \therefore \quad E = \frac{\sigma}{2\varepsilon_0}$$

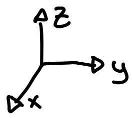
$$F = QE = \sigma A\cdot\frac{\sigma}{2\varepsilon_0} : \quad \frac{F}{A} = \frac{\sigma^2}{2\varepsilon_0} \equiv f_E$$

$$f_E = f_M : \quad \frac{\sigma^2}{2\varepsilon_0} = \frac{\mu_0\sigma^2 V^2}{2} : \quad \frac{1}{\varepsilon_0\mu_0} = V^2 \quad \therefore \quad V = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$$

$$\boxed{V = \sqrt{\frac{1}{\mu_0\varepsilon_0}}}$$

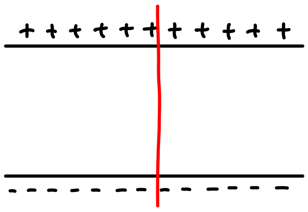
Problem 4

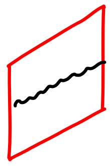




$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

a.)





$\oint \vec{B}\cdot d\vec{l}$: $\vec{B} = B(\hat{x})$, $d\vec{l} = dx\,(\hat{x})$

$\oint \vec{B}\cdot d\vec{l} = B\int_0^L dx = B\cdot L$

I_{enc} : $K = \frac{dI}{dl}$: $dI = K\cdot dl$: $I = K\cdot L$: $B\cdot L = \mu_0 K\cdot L$

$\vec{B} = \mu_0 K\,(-\hat{x})$

Between : $\boxed{\vec{B} = \mu_0 K\,(-\hat{x})}$: $-a < z < a$

There is no current enclosed outside the two plates, so therefore we can say

outside : $\boxed{\vec{B} = 0}$: $z < -a$ & $z > a$

b.) $\frac{F}{A}$: $F = Q[\cancel{\vec{E}}^{\,0} + (\vec{v}\times\vec{B})] = Q(\vec{v}\times\vec{B})$: $(\vec{v}\times\vec{B}) = vB\sin\theta \Big|_{\theta=90^\circ} = vB$

$F = QV\frac{B}{2}$ $\qquad$ $\sigma = \frac{Q}{A}$ $\therefore$ $Q = \sigma A$

$F = \sigma A V\cdot\left(\frac{\mu_0 K}{2}\right)$ $\qquad$ $K = \sigma V$

$\frac{F}{A} = \sigma\left(\frac{\mu_0\cdot\sigma V}{2}\right) = \frac{\mu_0\cdot\sigma^2\cdot V^2}{2}$

$$\boxed{\frac{F}{A} = \frac{\mu_0\cdot\sigma^2\cdot V^2}{2}}$$

c.) $F = QE$

$E = \frac{\sigma}{2\varepsilon_0}$: $Q = \sigma A$ $\therefore$ $F = \frac{\sigma^2 A}{2\varepsilon_0}$: $\frac{F}{A} = \frac{\sigma^2}{2\varepsilon_0}$

$\frac{\sigma^2}{2\varepsilon_0} = \frac{\mu_0\cdot\sigma^2\cdot V^2}{2}$

$V^2 = \frac{1}{\varepsilon_0\mu_0}$

$\therefore$

$$\boxed{V = \sqrt{\frac{1}{\varepsilon_0\mu_0}}}$$

Problem 5) $A = K\hat{\phi}$

$\vec{\nabla}\times\vec{A} = \vec{B}$, $\vec{\nabla}\times\vec{B} = \mu_0 J$

Cylindrical cross product: $\left[\frac{1}{s}\frac{\partial V_z}{\partial\phi} - \frac{\partial V_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sV_\phi) - \frac{\partial V_s}{\partial\phi}\right]\hat{z}$

$$\vec{\nabla}\times\vec{A} = \left[\cancelto{0}{\frac{1}{s}\frac{\partial V_z}{\partial\phi}} - \cancelto{0}{\frac{\partial V_\phi}{\partial z}}\right]\hat{s} + \left[\cancelto{0}{\frac{\partial V_s}{\partial z}} - \cancelto{0}{\frac{\partial V_z}{\partial s}}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sV_\phi) - \cancelto{0}{\frac{\partial V_s}{\partial\phi}}\right]\hat{z}$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{s}\left[\frac{\partial}{\partial s}(sV_\phi)\right]\hat{z} = \frac{1}{s}\left[\frac{\partial}{\partial s}(sK)\right]\hat{z} = \left[\frac{K}{s}\right]\hat{z} = \vec{B} \quad \vdots \quad \vec{B} = \frac{K}{s}\hat{z}$$

$$\vec{\nabla}\times\vec{B} = \left[\cancelto{0}{\frac{1}{s}\frac{\partial V_z}{\partial\phi}} - \cancelto{0}{\frac{\partial V_\phi}{\partial z}}\right]\hat{s} + \left[\cancelto{0}{\frac{\partial V_s}{\partial z}} - \frac{\partial V_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\cancelto{0}{\frac{\partial}{\partial s}(sV_\phi)} - \cancelto{0}{\frac{\partial V_s}{\partial\phi}}\right]\hat{z}$$

$$\vec{\nabla}\times\vec{B} = -\frac{\partial V_z}{\partial s}\hat{\phi} = -\frac{\partial}{\partial s}\left(\frac{K}{s}\right)\hat{\phi} = \frac{K}{s^2}\hat{\phi}$$

$$\frac{d}{ds}(s^{-1}) = -s^{-2}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \quad \therefore \quad \vec{J} = \frac{\vec{\nabla}\times\vec{B}}{\mu_0} = \frac{K}{s^2\mu_0}\hat{\phi}$$

$$\boxed{\vec{J} = \frac{K}{s^2\mu_0}\hat{\phi}}$$

Problem 5

$\vec{\nabla} \times \vec{A} = \vec{B}$, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\quad \vec{A} = k\hat{\phi}$

$$\vec{\nabla} \times \vec{A} = \left[\frac{1}{s} \cancelto{0}{\frac{\partial V_z}{\partial \phi}} - \cancelto{0}{\frac{\partial V_\phi}{\partial z}} \right] \hat{s} + \left[\cancelto{0}{\frac{\partial V_s}{\partial z}} - \cancelto{0}{\frac{\partial V_z}{\partial s}} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial}{\partial s}(s V_\phi) - \cancelto{0}{\frac{\partial V_s}{\partial \phi}} \right] \hat{z}$$

$$\frac{1}{s}\left[\frac{\partial}{\partial s}(s V_\phi) \right] = \frac{1}{s} \cdot \frac{\partial}{\partial s}(s \cdot k) = \frac{k}{s}(\hat{z})$$

$$\vec{\nabla} \times \vec{B} = \left[\frac{1}{s} \cancelto{0}{\frac{\partial V_z}{\partial \phi}} - \cancelto{0}{\frac{\partial V_\phi}{\partial z}} \right] \hat{s} + \left[\cancelto{0}{\frac{\partial V_s}{\partial z}} - \frac{\partial V_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\cancelto{0}{\frac{\partial}{\partial s}(s V_\phi)} - \cancelto{0}{\frac{\partial V_s}{\partial \phi}} \right] \hat{z}$$

$$\vec{\nabla} \times \vec{B} = -\frac{\partial V_z}{\partial s} \hat{\phi} = -\frac{\partial}{\partial s}\left(\frac{k}{s}\right) \hat{\phi} = \frac{k}{s^2} \hat{\phi}$$

$$\frac{k}{s^2} \hat{\phi} = \mu_0 \vec{J} \quad \therefore \quad \boxed{\vec{J} = \frac{k}{\mu_0 s^2} \hat{\phi}}$$