Taylor Larrechea
Dr. Gustafson
MATH 360
HW 5

Ch. 11.1 #1, 9, 13, 16

Problem 11.1.1

fundamental period: $\cos(x)$, $\sin(x)$, $\cos(2x)$, $\sin(2x)$
$\cos(\pi x)$, $\sin(\pi x)$, $\cos(2\pi x)$, $\sin(2\pi x)$

$A\sin(Bx)$, $A\cos(Bx)$ fundamental period: $p = \frac{2\pi}{B}$

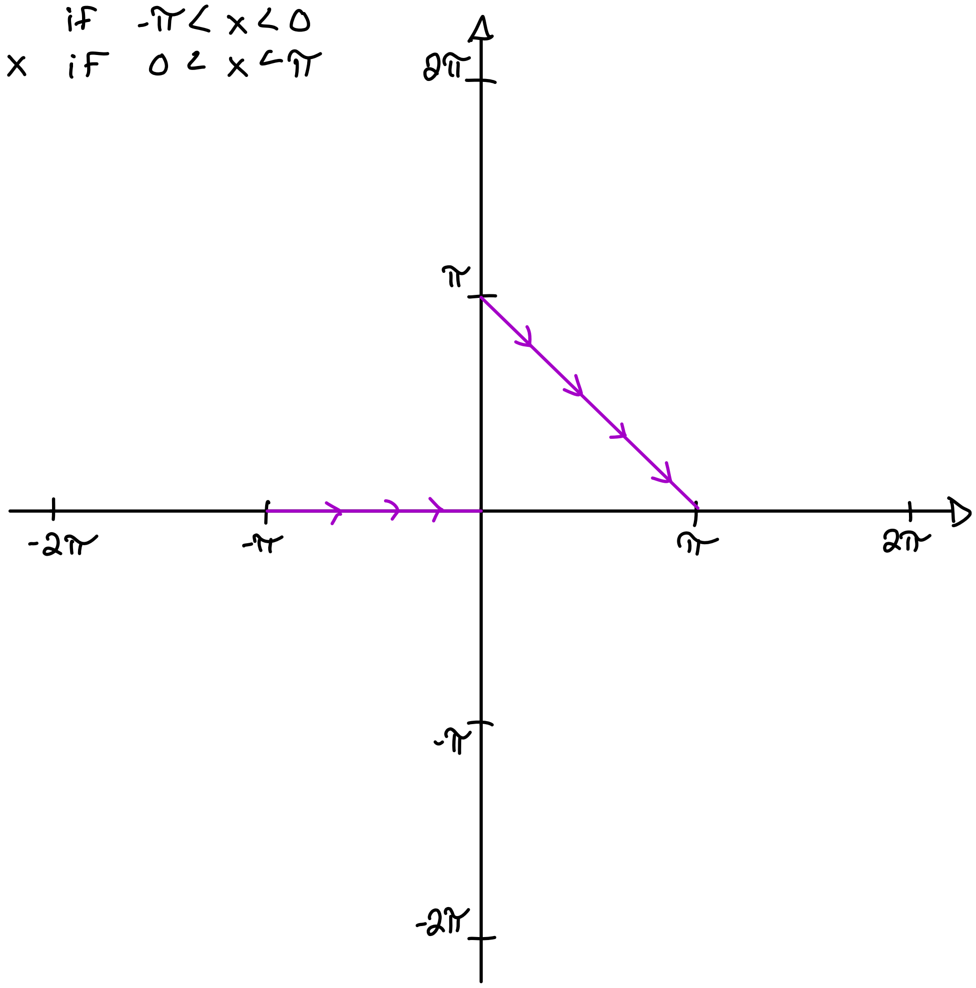
$$\boxed{\begin{array}{lcl} \cos(x) & \to & 2\pi \\ \sin(x) & \to & 2\pi \\ \cos(2x) & \to & \pi \\ \sin(2x) & \to & \pi \\ \cos(\pi x) & \to & 2 \\ \sin(\pi x) & \to & 2 \\ \cos(2\pi x) & \to & 1 \\ \sin(2\pi x) & \to & 1 \end{array}}$$

$p = \frac{2\pi}{1}$ $\quad$ $p = \frac{2\pi}{\pi} = 2$

$p = \frac{2\pi}{2} = \pi$ $\quad$ $p = \frac{2\pi}{2\pi} = 1$

Problem 11.1.9

$$f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$$



Problem 11.1.13

$$f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \quad , \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \quad , \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

a_0 : $\frac{1}{2\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{2\pi}\int_{0}^{\pi} \pi - x\,dx$

$$\frac{1}{2\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} = \frac{1}{2\pi}\left[0 - \left(\frac{\pi^2}{2}\right)\right] = \underline{\frac{-\pi}{4}}$$

$$\frac{1}{2\pi}\int_{0}^{\pi} \pi - x\,dx = \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]_{0}^{\pi} = \frac{1}{2\pi}\left[\left(\pi^2 - \frac{\pi^2}{2}\right) - (0 - 0)\right]$$

$$\frac{1}{2\pi}\left[\frac{2\pi^2 - \pi^2}{2}\right] = \frac{2\pi^2 - \pi^2}{4\pi} = \underline{\frac{\pi}{2} - \frac{\pi}{4}}$$

$$-\frac{\pi}{4} + \frac{\pi}{2} - \frac{\pi}{4} = -\frac{\pi}{2} + \frac{\pi}{2} = 0 \quad \therefore \quad \boxed{a_0 = 0}$$

a_n : $\frac{1}{\pi}\int_{0}^{\pi} (\pi - x)\cos(nx)\,dx$ $\qquad u = \pi - x \quad v' = \cos(nx)$; $\quad du = -dx \quad v = \frac{1}{n}\sin(nx)$

$$\frac{1}{\pi}\left[\cancel{\frac{(\pi - x)\sin(nx)}{n}}^{\,0}\Big|_{0}^{\pi} + \frac{1}{n}\int_{0}^{\pi} \sin(nx)\,dx\right] = \frac{1}{\pi}\left[\frac{-1}{n^2}\left[\cos(nx)\right]\right]_{0}^{\pi}$$

$$\boxed{\frac{-1}{\pi n^2}\left[\cos(\pi n) - 1\right]} \quad \cos(\pi n) = (-1)^n$$

(1)

$\frac{1}{\pi}\int_{-\pi}^{0} x\cos(nx)\,dx$ $\qquad u = x \quad v' = \cos(nx)$; $\quad du = 1\,dx \quad v = \frac{1}{n}\sin(nx)$

$$\frac{1}{\pi}\left[\frac{x}{n}\sin(nx)\Big|_{-\pi}^{0} - \frac{1}{n}\int \sin(nx)\,dx\right] = \frac{1}{\pi}\left[\frac{1}{n^2}\cos(nx)\right]_{-\pi}^{0}$$

$$\frac{1}{\pi}\left[\left(\frac{1}{n^2}\cos(0)\right) - \left(\frac{1}{n^2}\cos(-\pi n)\right)\right] = \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{\cos(\pi n)}{n^2}\right]$$

$$\boxed{\frac{1}{\pi n^2}\left[1 - \cos(\pi n)\right]} \quad (2) \qquad \frac{1}{\pi n^2}\left[1 - (-1)^n\right] + \frac{1}{\pi n^2}\left[1 - (-1)^n\right] = \frac{2}{\pi n^2}\left[1 - (-1)^n\right]$$

$$\boxed{a_n = \frac{2}{\pi n^2}\left[1 - (-1)^n\right]}$$

$n =$ even, $a_n = 0$

put in table of calculator to reduce tedious calcs.

Problem 11.1.13 Continued

b_n: $\frac{1}{\pi}\int_{-\pi}^{0} x\sin(nx)\,dx$ $\quad u = x \quad v' = \sin(nx)$

$du = dx \quad v = -\frac{1}{n}\cos(nx)$

$$\frac{1}{\pi}\left[-\frac{x}{n}\cos(nx) + \int \frac{1}{n}\cos(nx)\,dx \right]$$

$$\frac{1}{\pi}\left[-\frac{x}{n}\cos(nx) + \frac{1}{n^2}\sin(nx) \right]_{-\pi}^{0}$$

$$\frac{1}{\pi}\left[\left(\cancel{-\frac{0}{n}\cos(0)}^{\,0} + \cancel{\frac{1}{n^2}\sin(0)}^{\,0}\right) - \left(+\frac{\pi}{n}\cos(\pi n) - \cancel{\frac{1}{n^2}\sin(\pi n)}^{\,0}\right)\right]$$

$$\frac{1}{\pi}\left[-\left(\frac{\pi}{n}\cos(\pi n)\right)\right] = -\frac{1}{n}\cos(\pi n) = -\frac{1}{n}(-1)^n = \boxed{\frac{(-1)^{n+1}}{n}}$$

$\frac{1}{\pi}\left[\int_{0}^{\pi}(\pi - x)\sin(nx)\,dx\right]$ $\quad u = \pi - x \quad V' = \sin(nx)$

$du = -dx \quad V = -\frac{1}{n}\cos(nx)$

$$\frac{1}{\pi}\left[-\frac{(\pi-x)}{n}\cos(nx) - \frac{1}{n}\int \cos(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[-\frac{(\pi-x)}{n}\cos(nx) - \frac{1}{n^2}\sin(nx)\,dx\right]_{0}^{\pi}$$

$$\frac{1}{\pi}\left[\left(\cancel{-\frac{(\pi-\pi)}{n}\cos(\pi x)}^{\,0} - \cancel{\frac{1}{n^2}\sin(\pi n)}^{\,0}\right) - \left(-\frac{(\pi-0)}{n}\cos(0) - \cancel{\frac{1}{n^2}\sin(0)}^{\,0}\right)\right]$$

$$\frac{1}{\pi}\left[-\left(\frac{-\pi}{n}\right)\right] = \frac{\pi}{\pi n} = \boxed{\frac{1}{n}}$$

$$\boxed{b_n = \frac{(-1)^{n+1}}{n} + \frac{1}{n}}$$

$\longrightarrow$ put in table of calculator to reduce tedious calcs.

$a_0 = 0$

$$a_n = \frac{4}{\pi}\left(\cos(x) + \frac{1}{9}\cos(3x) + \frac{1}{25}\cos(5x)\right)$$

$$b_n = 2\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x)\right)$$

$$\boxed{F(x) = \frac{4}{\pi}\left(\cos(x) + \frac{1}{9}\cos(3x) + \frac{1}{25}\cos(5x)\right) + 2\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x)\right)}$$

Problem 11.1.16

$$f(x) = \begin{cases} 0, & -\pi < x < -\pi/2,\ \pi/2 < x < \pi \\ x, & -\frac{\pi}{2} < x < \frac{\pi}{2} \end{cases}$$ — Fourier coefficients will be zero in this interval

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\,dx = \frac{1}{4\pi}x^2\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{4\pi}\left[\left(\frac{\pi}{2}\right)^2 - \left(-\frac{\pi}{2}\right)^2\right] = \frac{1}{4\pi}\left[\frac{\pi^2}{4} - \frac{\pi^2}{4}\right] = 0$$

$a_0 = 0$

$$a_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos(nx)\,dx \qquad u = x,\ du = dx \qquad v' = \cos(nx),\ v = \frac{\sin(nx)}{n}$$

$$\frac{1}{\pi}\left[\frac{x\sin(nx)}{n} - \frac{1}{n}\int \sin(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[\frac{x\sin(nx)}{n} + \cancel{\frac{1}{n^2}\cos(nx)}^{\,0}\ dx\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\frac{1}{\pi}\left[\left(\frac{\frac{\pi}{2}\sin(\frac{\pi}{2}n)}{n}\right) - \left(\frac{\frac{\pi}{2}\sin(\frac{\pi}{2}n)}{n}\right)\right] = 0 \qquad \boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\sin(nx)\,dx \qquad u = x,\ du = dx \qquad v' = \sin(nx),\ v = -\frac{\cos(nx)}{n}$$

$$\frac{1}{\pi}\left[\frac{-x\cos(nx)}{n} + \int \frac{\cos(nx)}{n}\,dx\right]$$

$$\frac{1}{\pi}\left[\cancel{\frac{-x\cos(nx)}{n}}^{\,0} + \frac{\sin(nx)}{n^2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\frac{1}{\pi}\left[\frac{\sin(n\frac{\pi}{2})}{n^2} + \left(\frac{\sin(n\frac{\pi}{2})}{n^2}\right)\right] = \frac{1}{\pi}\left[\frac{2\sin(n\frac{\pi}{2})}{n^2}\right]$$

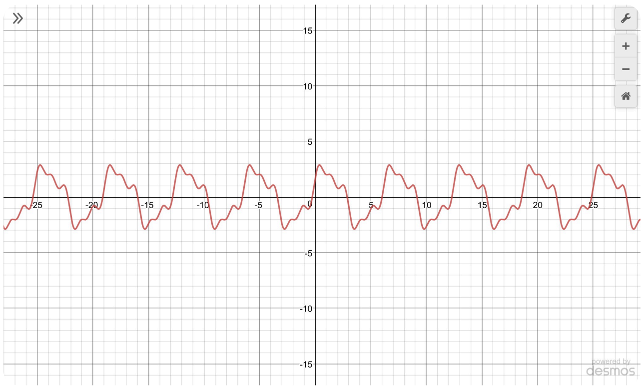
$$\frac{1}{\pi}\left[\frac{2\sin(n\frac{\pi}{2})}{n^2}\right]\ 0 \text{ — } n \text{ is even}$$

$$a_0 = 0,\ a_n = 0,\ b_n = \frac{2}{\pi}\left(\sin(x) - \frac{1}{9}\sin(3x) + \frac{1}{25}\sin(5x)\right)$$

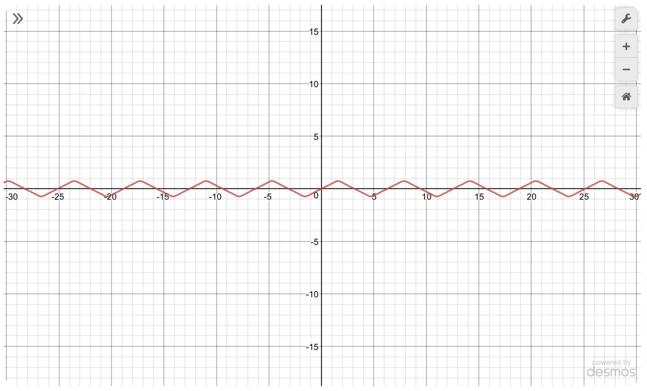
$$\boxed{F(x) = \frac{2}{\pi}\left(\sin(x) - \frac{1}{9}\sin(3x) + \frac{1}{25}\sin(5x)\right)}$$

Plots for # 13,16

Problem 13



Problem 16



17.) $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$, $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx$

$-\pi \leq x \leq 0$, $f(x) = x+\pi$

$0 \leq x \leq \pi$, $f(x) = \pi - x$

$$a_{0_1} = \frac{1}{2\pi}\int_{-\pi}^{0}(x+\pi)\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}+\pi x\right]_{-\pi}^{0} = \frac{1}{2\pi}\left[0+0-\left(\frac{\pi^2}{2}-\pi^2\right)\right]$$

$$= \frac{1}{2\pi}\left[\pi^2-\frac{\pi^2}{2}\right] = \frac{1}{2\pi}\left[\frac{\pi^2}{2}\right] = \frac{\pi}{4}$$

$$a_{0_2} = \frac{1}{2\pi}\int_{0}^{\pi}(\pi-x)\,dx = \frac{1}{2\pi}\left[\pi x-\frac{x^2}{2}\right]_{0}^{\pi} = \frac{1}{2\pi}\left[\pi^2-\frac{\pi^2}{2}\right] = \frac{1}{2\pi}\left[\frac{\pi^2}{2}\right] = \frac{\pi}{4}$$

$$a_0 = a_{0_1}+a_{0_2} = \frac{\pi}{4}+\frac{\pi}{4} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\cos(nx)\,dx \qquad u=(x+\pi) \quad v'=\cos(nx)\,dx$$

$$du = dx \qquad v = \frac{\sin(nx)}{n}$$

$$\frac{1}{\pi}\left[\cancel{\frac{(x+\pi)\sin(nx)}{n}\Bigg|_{-\pi}^{0}}^{\,0} - \frac{1}{n}\int_{-\pi}^{0}\sin(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[\frac{1}{n^2}\cos(nx)\Bigg|_{-\pi}^{0}\right] = \frac{1}{\pi n^2}\left[1-\cos(-\pi n)\right] = \frac{1}{\pi n^2}\left[1-(-1)^n\right]$$

$$\frac{1}{\pi}\int_{0}^{\pi}(\pi-x)\cos(nx)\,dx \qquad u=\pi-x \quad v'=\cos(nx)$$

$$du=-dx \qquad v=\frac{\sin(nx)}{n}$$

$$\frac{1}{\pi}\left[\cancel{\frac{(\pi-x)\sin(nx)}{n}\Bigg|_{0}^{\pi}}^{\,0} + \frac{1}{n}\int_{0}^{\pi}\sin(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[-\frac{1}{n^2}\cos(nx)\Bigg|_{0}^{\pi}\right] = \frac{-1}{\pi n^2}\left[\cos(n\pi)-1\right] = \frac{-1}{\pi n^2}\left[(-1)^n-1\right]$$

$$a_n = \frac{1}{\pi n^2} - \frac{(-1)^n}{\pi n^2} - \frac{(-1)^n}{\pi n^2} + \frac{1}{\pi n^2} = \frac{2}{\pi n^2} - 2\frac{(-1)^n}{\pi n^2}$$

$$a_n = \frac{2}{\pi n^2}\left[1-(-1)^n\right], \quad a_0 = \frac{\pi}{2}, \quad \text{Even } f(x) \therefore \ b_n = 0$$