1.) $f(x,y) = 18x^2y^3$

a.) $\int_0^4 f(x,y)\,dx = 6x^3y^3 \Big|_0^4 = 6(4)^3y^3 - 6(0)^3y^3$

$384y^3$

$\int_0^4 f(x,y)\,dx = 384y^3$

b.) $\int_0^1 f(x,y)\,dy = \frac{18x^2}{4}y^4 \Big|_0^1 = 4.5x^2y^4 \Big|_0^1 = 4.5x^2(1)^4 - 4.5x^2(0)^4$

$4.5x^2$

$\int_0^1 f(x,y)\,dy = \frac{18}{4}x^2$

2.) $f(x,y) = y + xe^y$

a.) $\int_0^3 f(x,y)\,dx = yx + \frac{x^2}{2}e^y \Big|_0^3 = \left(y(3) + \frac{(3)^2}{2}e^y\right) - \left(y(0) + \frac{0^2}{2}e^y\right)$

$3y + \frac{9}{2}e^y$

$\int_0^3 f(x,y)\,dx = 3y + \frac{9}{2}e^y$

b.) $\int_0^1 f(x,y)\,dy = \frac{y^2}{2} + xe^y \Big|_0^1 = \left(\frac{(1)^2}{2} + xe^1\right) - \left(\frac{0^2}{2} + xe^0\right)$

$\frac{1}{2} + xe - x$

$\int_0^1 f(x,y)\,dy = \frac{1}{2} + xe - x$

3.) $\int_{-4}^{4}\int_0^{\pi/2} (y + y^2\cos x)\,dx\,dy$

$\int_0^{\pi/2} (y + y^2\cos x)\,dx = yx + y^2\sin x \Big|_0^{\pi/2}$

$\left(y\left(\frac{\pi}{2}\right) + y^2\sin\left(\frac{\pi}{2}\right)\right) - \left(y(0) + y^2\sin(0)\right)$

$= \left(\frac{y\pi}{2} + y^2\right)$

$\int_{-4}^{4} \frac{\pi}{2}y + y^2 = \frac{\pi}{4}y^2 + \frac{y^3}{3} \Big|_{-4}^{4}$

$\left(\frac{\pi}{4}(4)^2 + \frac{(4)^3}{3}\right) - \left(\frac{\pi}{4}(-4)^2 + \frac{(-4)^3}{3}\right)$

$\frac{16\pi}{4} + \frac{64}{3} - \left(\frac{16\pi}{4} + \frac{(-64)}{3}\right)$

$\frac{16\pi}{4} + \frac{64}{3} - \frac{16\pi}{4} + \frac{64}{3}$

$\frac{128}{3}$

$\frac{128}{3}$

4.) $\int_0^8 \int_0^{\pi} 3r\sin^2\theta \, d\theta dr$

$= \int_0^{\pi} 3r\sin^2\theta \, d\theta$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$3r\left(\frac{1}{2}(1-\cos 2\theta)\right)$

$3r\left(\frac{1}{2} - \frac{\cos 2\theta}{2}\right)$

$= \int_0^{\pi} \frac{3r}{2} - \int_0^{\pi} \frac{3r\cos 2\theta}{2}$

$\frac{3r}{2} - \frac{3r\cos 2\theta}{2}$

$= \frac{3}{2}\int_0^{\pi} r\,d\theta - \frac{3}{2}\int_0^{\pi} r\cos 2\theta\,d\theta$

$\frac{1}{2}r\sin 2\theta$

$= \frac{3}{2}r\theta\Big|_0^{\pi} - \frac{1}{2}r\sin 2\theta\Big|_0^{\pi}$

48π

$= \frac{3\pi}{2}r\,dr \qquad \int_0^8 \frac{3\pi}{2}r\,dr = \frac{3\pi}{2}\int_0^8 r\,dr = \frac{3\pi}{2}\left(\frac{r^2}{2}\right)\Big|_0^8$

$\frac{3\pi r^2}{4}\Big|_0^8$

$\frac{192\pi}{4} = 48\pi$

5.) $\iint_R \frac{10(1+x^2)}{1+y^2}\,dA, \quad R = \{(x,y) \mid 0 \le x \le 3,\ 0 \le y \le 1\}$

$\int_0^1 (1+y^2)^{-1} \cdot 10(1+x^2)\,dy$

$10(1+x^2)\int_0^1 (1+y^2)^{-1}$

$10(1+x^2)\int_0^1 \frac{1}{1+y^2}\,dy$

30π

$\int_0^3 10(1+x^2) \cdot \arctan(y)\Big|_0^1$

$\int_0^3 10(1+x^2)\left(\frac{\pi}{4} - 0\right)$

$\frac{\pi}{4}\int_0^3 10 + 10x^2\,dx$

$\frac{\pi}{4}\left(10x + \frac{10}{3}x^3\right)\Big|_0^3$

$\frac{\pi}{4}\left(10(3) + \frac{10}{3}(3)^3\right) - \frac{\pi}{4}\left(10(0) + \frac{10}{3}(0)^3\right)$

$\frac{\pi}{4}(30 + 90)$

$\frac{120\pi}{4} = 30\pi$

6.) $\iint \frac{4x}{1+xy}\,dA$, $R=[0,4]\times[0,1]$

$$\int_0^1 (1+xy)^{-1}(4x)\,dy$$

$$\int_0^1 4x\cdot \ln(1+xy)\cdot\frac{1}{x}$$

$$4\ln(1+xy)\Big|_0^1$$

$$4\ln(1+x)-4\ln(1)$$

$$\int_0^4 4\ln(1+x)\,dx$$

$$4\left((x+1)\cdot\ln(x+1)-x\right)\Big|_0^4 = 4(5\ln(5)-4)$$

$4(5\ln(5)-4)$

8.) $4x+8y-2z+15=0$

$R=\{(x,y)\,|\,-1\le x\le 2,\ -1\le y\le 1\}$

$$4x+8y+15=2z$$

$$z=2x+4y+\tfrac{15}{2}$$

$$\int_{-1}^1 2x+4y+\tfrac{15}{2}\,dy = 2xy+2y^2+\tfrac{15}{2}y\Big|_{-1}^1$$

$$\left(2x(1)+2(1)^2+\tfrac{15}{2}(1)\right)-\left(2x(-1)+2(-1)^2+\tfrac{15}{2}(-1)\right)$$

$$\left(2x+2+\tfrac{15}{2}\right)-\left(-2x+2-\tfrac{15}{2}\right)$$

$$\left(2x+\tfrac{19}{2}\right)-\left(-2x-\tfrac{11}{2}\right)$$

$$2x+\tfrac{19}{2}+2x+\tfrac{11}{2}$$

$$4x+\tfrac{30}{2}$$

$$\int_{-1}^2 4x+15\,dx$$

$$2x^2+15x\Big|_{-1}^2$$

$$\left(2(2)^2+15(2)\right)-\left(2(-1)^2+15(-1)\right)$$

$$(2(4)+30)-(2-15)$$

$$(38)-(-13)$$

$$38+13$$

$$51$$

50 cubic units

9.) Find the volume of the solid that lies under the elliptic paraboloid.

$$\frac{x^2}{9}+\frac{y^2}{16}+z=1 \qquad R=[-1,1]\times[-3,3]$$

$$z=\frac{-x^2}{9}-\frac{y^2}{16}+1 \qquad \begin{matrix}-1\le x\le 1\\ -3\le y\le 3\end{matrix}$$

$$\int_{-3}^{3}-\frac{x^2}{9}-\frac{y^2}{16}+1\,dy = -\frac{x^2y}{9}-\frac{y^3}{48}+y\Big|_{-3}^{3}$$

$$3-\frac{x^2(3)}{9}-\frac{(3)^3}{48}-\left(-3-\frac{x^2(-3)}{9}-\frac{(-3)^3}{48}\right)$$

$$3-\frac{x^2}{3}-\frac{27}{48}-\left(-3-\frac{(-x^2)}{3}-\frac{(-27)}{48}\right)$$

$$3-\frac{x^2}{3}-\frac{27}{48}-\left(-3+\frac{x^2}{3}+\frac{27}{48}\right)$$

$$3-\frac{x^2}{3}-\frac{27}{48}+3-\frac{x^2}{3}-\frac{27}{48}$$

$$6-\frac{2x^2}{3}-\frac{54}{48}$$

$$6-\frac{2x^2}{3}-\frac{9}{8}$$

$$\int_{-1}^{1}-\frac{2x^2}{3}+\frac{39}{8}\,dx = -\frac{2x^3}{9}+\frac{39}{8}x\Big|_{-1}^{1}$$

$$\frac{-2(1)^3}{9}-\frac{39}{8}(1)-\left(\frac{-2(-1)^3}{9}+\frac{39}{8}(-1)\right)$$

$$-\frac{2}{9}+\frac{39}{8}-\left(\frac{2}{9}-\frac{39}{8}\right)$$

$$-\frac{4}{9}+\frac{78}{8}$$

$$\frac{335}{36}$$

$$\frac{335}{36}$$

10.) $z=5+x^2+(y-2)^2$ planes $z=1$
$x=-2 : x=2$
$y=0 : y=4$

$$V=\int_0^4\int_{-2}^{2}\left(5+x^2+(y-2)^2\right)dx\,dy - \int_0^4\int_{-2}^{2}\underset{z}{(1)}\,dx\,dy$$

$$\int_0^4\left(5x+\frac{x^3}{3}+x(y-2)^2\right)\Big|_{-2}^{2}dy - \int_{-2}^{2}dx\int_0^4 dy$$

$$\int_0^4\left(\left(5(2)+\frac{(2)^3}{3}+2(y-2)^2\right)-\left(5(-2)+\frac{(-2)^3}{3}-2(y-2)^2\right)\right)dy-(x)\Big|_{-2}^{2}(y)\Big|_0^4$$

$$\int_0^4\left((10+8/3+2(y-2)^2)-(-10-8/3-2(y-2)^2)\right)dy-(2-(-2))(4)$$

$$\int_0^4\left(2(y-2)^2+38/3\right)-\left(-2(y-2)^2-38/3\right)dy-(4)(4)$$

$$\int_0^4\left(4(y-2)^2+76/3\right)dy - 16$$

$$\frac{4}{3}(y-2)^3+\frac{76}{3}y\Big|_0^4-16$$

$$\left(\frac{4}{3}(4-2)^3+\frac{76}{3}(4)\right)-\left(\frac{4}{3}(0-2)^3+\frac{76}{3}(0)\right)-16$$

$$\frac{4}{3}(8)+\frac{304}{3}-\left(-\frac{32}{3}\right)-16$$

$$\frac{32}{3}+\frac{304}{3}+\frac{32}{3}-16$$

$$\frac{368}{3}-16=\frac{368}{3}-\frac{48}{3}=\frac{320}{3}$$

$$\frac{320}{3}$$

11.) $f(x,y) = 3x^2y$, R $(-5,0),(-5,5),(5,5),(5,0)$

$-5 \le x \le 5$ $x=10$
$0 \le y \le 5$ $y=5$
$5-(-5)=10$

$A(R) = x \cdot y$
$A(R) = 10 \cdot 5$
$A(R) = 50$

$f_{ave} = \frac{125}{2}$

$A(R) \int_0^5 \int_{-5}^5 3x^2y \; dx\,dy$

$A(R) \int_0^5 (x^3y) \Big|_{-5}^5 \; dy$

$A(R) \int_0^5 \left((5)^3y - ((-5)^3y)\right) dy$

$A(R) \int_0^5 (125y - (-125y)) \; dy$

$A(R) \int_0^5 (250y) \; dy$

$A(R) \left(125y^2\right) \Big|_0^5$

$A(R) \left(125(5)^2\right)$

$\frac{1}{50}(3125)$

$\frac{125}{2}$

$$\frac{125}{2}$$