1.) $\iint_D e^{-x^2-y^2}\,dA$ $\quad x=\sqrt{49-y^2}$

y axis

$x^2 = 49 - y^2$ $\quad r = 7$

$x^2+y^2 = 49$ $\quad 0 \le r \le 7$

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^7 e^{-r^2}\, r\, dr\,d\theta$ $\quad u = r^2$, $du = 2r\,dr$, $\frac{1}{2}du = r\,dr$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}\int_0^7 e^{-u}\,du\,d\theta$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}\left[-e^{-r^2}\right]_0^7 d\theta$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{1}{2}e^{-49}+\frac{1}{2}e^{0}\,d\theta$ $\quad \frac{\pi}{2}(1-e^{-49})$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\frac{1}{2}e^{-49}-\left(-\frac{1}{2}e^{0}\right)d\theta$

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{2}(1-e^{-49})\,d\theta$

$\frac{1}{2}(1-e^{-49})\theta\Big|_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$\frac{1}{2}(1-e^{-49})\left(\frac{3\pi}{2}\right) - \left(\frac{1}{2}(1-e^{-49})\left(-\frac{\pi}{2}\right)\right)$

$\frac{1}{2}(1-e^{-49})\left(\frac{3\pi}{2}\right) + \frac{1}{2}(1-e^{-49})\left(\frac{\pi}{2}\right)$

$\frac{1}{2}(1-e^{-49})\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)$

$\frac{1}{2}(1-e^{-49})(\pi)$

$\frac{\pi}{2}(1-e^{-49})$

2.) $\iint_D x\,dA$ First Quadrant

$x=r\cos\theta$, $y=r\sin\theta$ $\quad x^2+y^2=4$, $x^2+y^2=2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$ $\quad r = 2$

$2\cos\theta \le r \le 2$

$0 \le \theta \le \frac{\pi}{2}$

$z = 2\cos\theta$

$\int_0^{\frac{\pi}{2}}\int_{2\cos\theta}^{2} (r\cos\theta)\, r\,dr\,d\theta$

$\int_0^{\frac{\pi}{2}}\int_{\ }^{2} r^2\cos\theta\,dr\,d\theta$

$\int_0^{\frac{\pi}{2}}\left[\frac{1}{3}r^3\cos\theta\right]_{2\cos\theta}^{2} d\theta$

$\int_0^{\frac{\pi}{2}}\left[\frac{8}{3}\cos\theta - \frac{8\cos^3\theta}{3}\cos\theta\right]d\theta$

$\int_0^{\frac{\pi}{2}} \frac{8}{3}\left[\cos\theta-\cos^4\theta\right]d\theta$

$\frac{8}{3}\int_0^{\frac{\pi}{2}} \cos\theta-\cos^4\theta\,d\theta = \frac{-(3\pi-16)}{6}$

3.) $r = 4\cos 3\theta$ $\quad$ one loop?

$\iint_D r\,dr\,d\theta$

$0 \le r \le 4\cos\theta$

$-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_0^{4\cos 3\theta} r\,dr\,d\theta$

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\left[\frac{1}{2}r^2\right]_0^{4\cos 3\theta} d\theta$

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 8\cos^2 3\theta\,d\theta$

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 4+4\cos 6\theta\,d\theta$

$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 4(1+\cos 6\theta)$

$4\left(\theta+\frac{1}{6}\sin 6\theta\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4\pi}{3}$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$8\left(\frac{1}{2}(1+\cos 2\theta)\right)$ $\quad \theta = 3\theta$

$4(1+\cos 2\theta)$

$4+4\cos 2\theta$

$4+4\cos 6\theta$

$\theta + \frac{1}{6}\sin 6\theta$

4.) $z = \sqrt{x^2+y^2}$ $\quad x^2+y^2 \le 64$ $\quad r = \sqrt{x^2+y^2}$

$x = 8\cos\theta$, $y = 8\sin\theta$ $\quad z = f(x,y) = \sqrt{x^2+y^2} = \sqrt{r^2} = r$

$0 \le r \le 8$

$0 \le \theta \le 2\pi$

$\int_0^{2\pi}\int_0^{8} (r)\, r\,dr\,d\theta$

$\int_0^{2\pi}\int_0^{8} r^2\,dr\,d\theta$

$\int_0^{2\pi} \frac{1}{3}r^3\Big|_0^8\,d\theta$

$\int_0^{2\pi} \frac{512}{3}\,d\theta$

$\frac{512}{3}\theta\Big|_0^{2\pi}$

$\frac{1024}{3}\pi$ $\quad \frac{1024\pi}{3}$

5.) $z = 50 - 2x^2 - 2y^2$ $\quad r^2 = x^2 + y^2$

$z = 50 - 2(x^2+y^2)$

$z = 50 - 2(r^2)$

$0 = 50 - 2r^2$

$-50 = -2r^2$

$25 = r^2$

$r = 5$

xy plane $z = 0$ $\quad 0 \le r \le 5$ $\quad 0 \le \theta \le 2\pi$

$\int_0^{2\pi}\int_0^5 (50 - 2r^2) r \, dr\, d\theta$

$\int_0^{2\pi}\int_0^5 50r - 2r^3 \, dr\, d\theta$

$\int_0^{2\pi} \left[25r^2 - \frac{r^4}{2}\right]_0^5 d\theta$

$\int_0^{2\pi} 625 - \frac{625}{2} \, d\theta$

$\int_0^{2\pi} \frac{625}{2} \, d\theta$

$\frac{625}{2}\theta \Big|_0^{2\pi}$

625π

625π

6.) $x^2 + y^2 + z^2 = 36$ $\quad r = 6$ $\quad$ outside the cylinder $x^2 + y^2 = 4$ $\quad r = 2$

$z^2 = -x^2 - y^2 + 36$

$z = \pm\sqrt{-x^2 - y^2 + 36}$

$z = \pm\sqrt{36 - x^2 - y^2}$

$v = 2$

$0 \le \theta \le 2\pi$

$2 \le r \le 6$

$2\int_0^{2\pi}\int_2^6 \left(\sqrt{36 - x^2 - y^2}\right) r \, dr\, d\theta$

$2\int_0^{2\pi}\int_2^6 r\sqrt{36 - x^2 - y^2} \, dr\, d\theta$

$2\int_0^{2\pi}\int_2^6 r\sqrt{36 - r^2} \, dr\, d\theta$

$u = 36 - r^2$

$du = -2r\, dr$

$-\frac{1}{2}du = r\, dr$

$2\int_0^{2\pi} -\frac{1}{2}\int_2^6 \sqrt{u} \, du\, d\theta$

$2\int_0^{2\pi} -\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_2^6 d\theta$

$2\int_0^{2\pi} -\frac{1}{3}(36 - r^2)^{\frac{3}{2}} \Big|_2^6 \, d\theta$

$2\int_0^{2\pi} \frac{1}{3}(32)^{\frac{3}{2}} \, d\theta$

$2\left(\frac{128\sqrt{2}}{3}\theta\right)\Big|_0^{2\pi} = \frac{512\sqrt{2}}{3}$

7.) $\int_{-4}^{4}\int_0^{\sqrt{16 - x^2}} \sin(x^2 + y^2) \, dy\, dx$

$x^2 + y^2 = 16$

$r = 4$

$0 \le r \le 4$

$0 \le \theta \le \pi$

$x = r\cos\theta$

$y = r\sin\theta$

$\sin(r^2\cos^2\theta + r^2\sin^2\theta)$

$\sin(r^2(\cos^2\theta + \sin^2\theta))$

$\sin(r^2)$

$\int_0^{\pi}\int_0^4 (\sin(r^2)) r \, dr\, d\theta$

$u = r^2$

$du = 2r\, dr$

$\frac{1}{2}du = r\, dr$

$\int_0^{\pi}\int_0^4 r\sin(r^2) \, dr\, d\theta$

$\int_0^{\pi} \frac{1}{2}\int_0^4 \sin(u) \, du\, d\theta$

$\int_0^{\pi} -\frac{1}{2}\cos(r^2)\Big|_0^4 \, d\theta$

$\int_0^{\pi} -\frac{1}{2}\cos(16) + \frac{1}{2}\cos(0) \, d\theta$

$\int_0^{\pi} \frac{1}{2}(-\cos(16) + 1) \, d\theta$

$\int_0^{\pi} \frac{1}{2}(1 - \cos(16)) \, d\theta$

$\frac{1}{2}(1 - \cos 16)\theta \Big|_0^{\pi}$

$\frac{\pi}{2}(1 - \cos 16)$

$\frac{\pi}{2}(1 - \cos 16)$