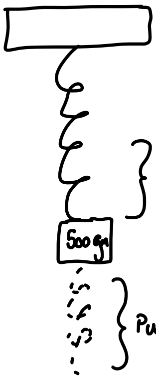
4.1 Harmonic Oscillator

4.1 # 47, 57, 62, 63

4.1.47



50 cm to equilibrium

Pulled down 10 cm

Harmonic Oscillator

$m\ddot{x} + b\dot{x} + Kx = f(t)$

$m\ddot{x} + Kx = f(t)$ <= undamped

$K = \frac{m}{\Delta s} = \frac{500\text{ gm}}{50\text{ cm}}$

$K = 10$

Frictionless = undamped

a.) $m\ddot{x} + Kx = 0$

$K = \frac{m}{\Delta s} = \frac{500}{50} = 10$

$500\ddot{x} + 10x = 0$

b.) $x(t) = C_1 \cos w_0 t + C_2 \sin w_0 t$, $w_0 = \sqrt{\frac{k}{m}}$

$500\ddot{x} + 10x = 0$

$K = 10$

$m = 500$

$w_0 = \sqrt{\frac{10}{500}} = \sqrt{\frac{1}{50}} = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$

$x(t) = C_1 \cos\left(\frac{t}{5\sqrt{2}}\right) + C_2 \sin\left(\frac{t}{5\sqrt{2}}\right)$

$t = 0$

$10 = C_1 \cos(0) + C_2 \sin(0)$

$10 = C_1$

$\dot{x}(t) = -\frac{5}{\sqrt{2}} \sin\left(\frac{t}{5\sqrt{2}}\right) + C_2\left(\frac{1}{5\sqrt{2}}\right)\cos\left(\frac{t}{5\sqrt{2}}\right)$

$t = 0$

$0 = -\frac{5}{\sqrt{2}}\sin(0) + \frac{C_2}{\sqrt{50}}\cos(0)$

$0 = \frac{C_2}{\sqrt{50}}$ $C_2 = 0$

$C_1 = 10$

$C_2 = 0$

$x(t) = 10\cos\left(\frac{t}{5\sqrt{2}}\right)$

c.) Amplitude : $A = \sqrt{C_1^2 + C_2^2}$

Phase Angle : $\tan\delta = C_2/C_1$

Frequency (natural) : $f_0 = \omega_0 / 2\pi$

Period : $T = 2\pi\sqrt{\frac{m}{k}}$

$C_1 = A\cos\delta$

$C_2 = A\sin\delta$

$A = 10$ $f_0 = \frac{1}{10\pi\sqrt{2}}$

$\delta = 0°$ $T = 2\pi\sqrt{50}$

$A = \sqrt{C_1^2 + C_2^2}$

$A = \sqrt{10^2}$

$A = 10$

$C_1 = 10, C_2 = 0$

$\tan\delta = C_2/C_1$

$\delta = \tan^{-1}(0/10)$

$\delta = 0$

$f_0 = \omega_0/2\pi$ $\omega_0 = \frac{1}{5\sqrt{2}}$

$= \frac{1/5\sqrt{2}}{2\pi}$

$= 1/10\sqrt{2}\pi$

$f_0 = \frac{1}{10\sqrt{2}\pi}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{500}{10}}$

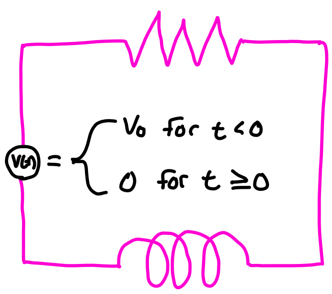
$2\pi\sqrt{50}$

$T = 2\pi\sqrt{50}$

$m = 500$

$K = 10$

4.1.57



a.) The circuit will experience voltage drop,

$V_R(t) = RI(t)$, $V_L(t) = L\dot{I}(t)$

b.) Drop across Resistor : $V_R(t) = RI(t)$

c.) Drop across Inductor : $V_L(t) = L\dot{I}(t)$

$V = IR$, $I = \frac{V}{R}$

$V_{tot} = V_R(t) + V_L(t)$

$V = RI + L\dot{I}$, $V = 0$

$0 = RI + L\dot{I}$

$-L\dot{I} = RI$

$\int \frac{\dot{I}}{I} = \int \frac{R}{-L}$

$\ln I = -(R/L)t + C$

$I = Ke^{-(R/L)t}$

$I = Ke^{-(R/L)t}$

$\frac{V}{R} = Ke^{-(R/L)t}$, $t = 0$

$\frac{V}{R} = Ke^0$

$\frac{V}{R} = K$

$I = \frac{V}{R}e^{-(R/L)t}$

d.) $R = 40\Omega$

$L = 5h$

$V_0 = 10v$

$I = \frac{10}{40}e^{-(40/5)t}$

$I = \frac{1}{4}e^{-8t}$

$I = \frac{1}{4}e^{-8t}$

4.1.62 $5\ddot{q}+15\dot{q}+\frac{1}{10}q=5\cos 3t$

$y=\dot{q}$

$y=f(t)$

$\ddot{q}+3\dot{q}+\frac{1}{50}q=\cos 3t$

$\dot{y}+3y+\frac{1}{50}q=\cos 3t$

$\dot{y}=-3y-\frac{1}{50}q+\cos 3t$

$\dot{y}=\cos 3t-3y-\frac{1}{50}q$

4.1.63 $t^2\ddot{x}+4t\dot{x}+x=t\sin 2t$

$y=\dot{x}$

$t^2\dot{y}+4ty+x=t\sin 2t$

$\dot{y}+\frac{4y}{t}+\frac{x}{t^2}=\frac{\sin 2t}{t}$

$\dot{y}=\frac{\sin 2t}{t}-\frac{4y}{t}-\frac{x}{t^2}$

$\dot{y}=\frac{\sin 2t}{t}-\frac{4y}{t}-\frac{x}{t^2}$