1.3 1,4,7,11,15,18,31

1.) $y' = 1+y$

$y = -1$

$y' = f(t)g(y)$ $\quad g(y) = 1+y$

$\frac{dy}{dt} = (1)1+y$ $\quad 0 = 1+y$

$y = -1$

$\frac{dy}{1+y} = dt$

$\ln(1+y) = t$

$(1+y) = e^t$

$y = e^t - 1$

4.) $y' = \ln(ty)$

not seperable

7.) $y' = \frac{e^{t+y}}{y+1}$

$y = -e^{t+y} + ce^{y} - 1$

$\frac{dy}{dt} = \frac{e^t \cdot e^y}{y+1}$

$\frac{dy}{dt}(y+1) = e^t \cdot e^y$

$dy(y+1) = e^t \cdot e^y dt$

$dy \frac{y+1}{e^y} = e^t dt$

$dy\, e^{-y}(y+1) = e^t dt$

$ye^{-y} + e^{-y} = e^t dt$

$u = y \quad dv = e^{-y}$

$du = 1 dy \quad v = -e^{-y}$

$-ye^{-y} - \int -e^{-y} dy$

$-ye^{-y} - e^{-y}$

$-e^{-y}(y+1)$

$-e^{-y}(y+1) = e^t + c$

$y+1 = -e^t e^y + ce^y$

$y = -e^t e^y + ce^y - 1$

$y = -e^{t+y} + ce^y - 1$

11.) $y' = \frac{t^2}{y}$

$y = \sqrt{\frac{2}{3}t^2 + C}$

$\frac{dy}{dt} = \frac{1}{y} \cdot t^2$

$dy \cdot y = dt\, t^2$

$\frac{1}{2}y^2 = \frac{1}{3}t^3 + c$

$y^2 = \frac{2}{3}t^2 + 2c$

$y = \sqrt{\frac{2}{3}t^2 + C}$

15. $y' = y\cos t$

$\frac{dy}{dt} = y\cos t$

$dy \cdot \frac{1}{y} = \cos t \, dt$

$\ln y = \sin t + C$

$y = e^{\sin t + C}$

$y = e^{\sin t} \cdot e^{C}$

$y = Ke^{\sin t}$

$y = Ke^{\sin t}$

18. $y' = y^2 - 4 \qquad y(0) = 0$

$\frac{dy}{dt} = (y+2)(y-2)$

$\frac{1}{(y+2)(y-2)} = \frac{A}{(y+2)} + \frac{B}{(y-2)}$

$dy \frac{1}{(y+2)(y-2)} = dt$

$1 = A(y-2) + B(y+2)$

$1 = Ay - 2A + By + 2B$

$1 = -2A + 2B$

$0 = A + B$

$A = -\frac{1}{4} \quad B = \frac{1}{4}$

$1 = -2A + 2B$

$1 = -2A + \frac{1}{2}$

$\frac{1}{2} = -2A$

$A = \frac{-1}{4}$

$-\int \frac{1}{4(y+2)} dy + \int \frac{1}{4(y-2)} dy = \int dt$

$-\ln(y+2) + \ln(y-2) = 4\int dt$

$1 = -2A + 2B$

$0 = 2A + 2B$

$1 = 4B$

$B = \frac{1}{4}$

$-\ln(y+2) + \ln(y-2) = 4t + C$

$\ln(y-2) - \ln(y+2) = 4t + C$

$\ln \frac{(y-2)}{(y+2)} = 4t + C$

$\frac{|y-2|}{|y+2|} = e^{4t} e^{C}$

$\frac{2}{2} = e^{4(0)} K$

$1 = K$

$\frac{|y-2|}{|y+2|} = ke^{4t}$

$\frac{y-2}{y+2} = ke^{2t}$

$y - 2 = ke^{2t}(y+2)$

$y - 2 = yke^{2t} + 2ke^{2t}$

$y - yke^{2t} = 2ke^{2t} + 2$

$y(1 - ke^{2t}) = 2ke^{2t} + 2$

$y = \frac{2ke^{4t} + 2}{1 - ke^{4t}}$

$y = \frac{2ke^{4t} + 2}{1 - ke^{4t}}$

31. $y' = 1 - y^2$

$\frac{dy}{dt} = (1-y)(1+y)$

$dy \frac{1}{(1-y)(1+y)} = \frac{A}{(1-y)} + \frac{B}{(1+y)}$

$\frac{|1+y|}{|1-y|} = Ke^{2t}$

$y = \pm 1$

$y=1 \quad A(1+y) + B(1-y)$

$1 = A + Ay + B - By$

$1 = A + A$

$1 = 2A$

$A = \frac{1}{2}$

$B = \frac{1}{2}$

$y = -1$

$1 = 2B$

$B = \frac{1}{2}$

$\int \frac{1}{2(1-y)} dy + \int \frac{1}{2(1+y)} dy = \int dt$

$$\int \frac{1}{(1-y)}dy + \int \frac{1}{(1+y)}dy = \int 2\,dt$$

$$-\ln(1-y) + \ln(1+y) = 2t + c$$

$$\ln(1+y) - \ln(1-y) = 2t + c$$

$$\ln\left|\frac{1+y}{1-y}\right| = 2t + c$$

$$\frac{|1+y|}{|1-y|} = e^{2t+c}$$

$$\frac{|1+y|}{|1-y|} = Ke^{2t}$$

$$\int \frac{1}{1-y}dy$$

$u = 1-y$
$du = -1\,dy$

$$-\int \frac{1}{u}du$$

$$-\ln u + c$$

$$-\ln(1-y) + c$$

Equilibrium $y' = 0$

$$y' = 1-y^2$$

$$0 = 1-y^2$$

$$-1 = -y^2$$

$$y = \pm 1$$