Backgrand spin-½ in mag. field physics

mag. dipole moment $\vec{\mu}$

Rules: * torque $\qquad \vec{\tau} = \vec{\mu} \times \vec{B}$

* Newton's 2^{nd} Law $\qquad \frac{d\vec{L}}{dt} = \vec{\tau}$

* connection between $\vec{L}$ and $\vec{\mu}$ (proportional)

$$\vec{L} = \frac{1}{\gamma}\vec{\mu}$$

γ constant that depends on situation.

$$\Rightarrow \boxed{\frac{d\vec{\mu}}{dt} = \gamma(\vec{\mu} \times \vec{B})}$$ eqn of motion

Bulk system many spin-½ objects. Magnetization

$$\vec{M} = \sum \vec{\mu}$$

$$\Rightarrow \boxed{\frac{d\vec{M}}{dt} = \gamma(\vec{M} \times \vec{B})}$$

Can measure magnetization

e.g. in NMR - get $\vec{M}$ to rotate → produces time varying mag field → can detected ...

Standard situation - have constant uniform mag. field

$$\vec{B} = B\hat{z}$$

$\underbrace{B}_{\text{const.}}$

- want how does $\vec{M}$ vary with time.

Task: Solve

$$\frac{d\vec{M}}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)$$

by - eqns for components of $\vec{M}$

- differentials eqns ...
- should get rotating $\vec{M}$ → angular frequency of rotation?

→ depends on B

Next situation - relaxation to equilibrium

- spins interact with surroundings - odd $\vec{E}/\vec{B}$ fields, ..
- relax into lowest energy state $U = -\vec{M}\cdot\vec{B}$

- eqns:

$$\frac{dM_x}{dt} = \gamma(\vec{M}\times\vec{B})_x - \frac{M_x}{T_2}$$

→ fixed parameter for given situation

$$\frac{dM_y}{dt} = \gamma(\vec{M}\times\vec{B})_y - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = 0 + \frac{M_0 - M_z}{T_1}$$

→ different parameter for given situation

Task: Solve for M_z

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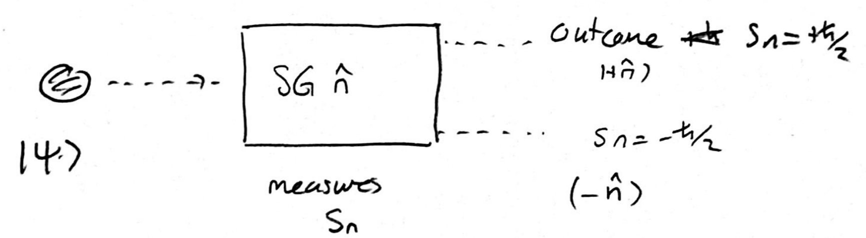
Quantum: spin-$1/2$ systems

- meaning of $|+\hat{n}\rangle$, $|-\hat{n}\rangle$

measure $\hat{n}$ $\to$ $+\hbar/2$ certainty, $-\hbar/2$ certainty

- $|\psi\rangle = a_+|+\hat{z}\rangle + a_-|-\hat{z}\rangle$

e.g. $\frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{1}{\sqrt{2}}|-\hat{z}\rangle$ $\rightsquigarrow$?? mean.



prob $= |\langle +\hat{n}|\psi\rangle|^2$

prob $= |\langle -\hat{n}|\psi\rangle|^2$

- general

$$|+\hat{n}\rangle = \cos\frac{\theta}{2}|+\hat{z}\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\hat{z}\rangle$$

$$|-\hat{n}\rangle = \sin\theta/2\,|+\hat{z}\rangle - e^{i\phi}\cos\theta/2\,|-\hat{z}\rangle$$

where θ, ϕ are spherical co-ords for $\hat{n}$

Task: 1) Exercises Lecture 5

2) Suppose $|\psi\rangle$ corresponds to direction $\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$

want

a) $|\psi\rangle$ in terms of $|+\hat{z}\rangle, |-\hat{z}\rangle$

b) measure SG $\hat{z}$ probs

c) " SG $\hat{x}$ "

d) " SG $\hat{y}$ "