Taylor Larrechea
Dr. Collins
PHYS 362 HW 15

## Problem 1

State A : $\uparrow\uparrow\uparrow\downarrow\downarrow$ , State B : $\downarrow\uparrow\uparrow\downarrow\downarrow$ , State C : $\downarrow\downarrow\uparrow\uparrow\uparrow$ , State D : $\downarrow\downarrow\downarrow\uparrow\uparrow$ : $2^5 = 32$

$N_\uparrow = 3$ , $N_\uparrow = 2$ , $N_\uparrow = 3$ , $N_\uparrow = 2$

$$\binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!(N-N_\uparrow)!} \quad : \quad \binom{5}{3} = \frac{5!}{3!(2!)} = 10 \quad : \quad \binom{5}{2} = \frac{5!}{2!(3!)} = 10$$

| Macrostate ($N_\uparrow$) | # of Microstates | |
|---|---|---|
| 0 | 1 | |
| 1 | 5 | |
| 2 | 10 | State (B,D) : 2/10 |
| 3 | 10 | State (A,C) : 2/10 |
| 4 | 5 | |
| 5 | 1 | |

States (A,C) will occur with the same probability. States (B,D) will also occur with the same probability. But since we do not know P and q, we cannot state if State (A,C) and State (B,D) occur with the same probability.

Problem 2

| | Subsystem A | | Subsystem B | | |
|---|---|---|---|---|---|
| Particle | $a_1$ | $a_2$ | $b_1$ | $b_1$ | $b_3$ |
| Microstate 1 | 4 | 1 | 1 | 1 | 1 |
| Microstate 2 | 2 | 2 | 1 | 2 | 1 |
| Microstate 3 | 3 | 2 | 1 | 1 | 2 |
| Microstate 4 | 3 | 2 | 1 | 0 | 2 |
| Microstate 5 | 0 | 5 | 1 | 1 | 1 |

a.) Microstates 1,4,5 represent the same macrostate. Microstate 2 and 3 represent their own macrostate. This is because for a microstate to represent the same macrostate there has to be the same total number of particles in each subsystem as well as the same total energy.

b.) There are 3 total microstates for all possible macrostates. For a microstate to occur with the same probability, the total energy of the system must be the same. The energy of the system is calculated as $E = E_A + E_B$. Microstates 1,2,4,5 have a total energy of $E = 8$ units. Therefore this means microstates 1,2,4,5 will all occur with the same probability.

## Problem 3

Standard Form: $\ln(n!) \approx n\ln(n) - n + \frac{1}{2}\ln(2\pi n)$ : weaker Form $\ln(n!) \approx n\ln(n) - n$

| n value | Standard form | Weaker Form | Percent Diff |
|---|---|---|---|
| 15 | 27.8987 | 25.6208 | 8.15 % |
| 150 | 605.02 | 601.595 | 0.57 % |

For smaller values of n, the percent difference $\left(\frac{\text{Standard} - \text{Weaker}}{\text{Standard}}\right)$ is greater than that of larger values of n. As n increases the difference between the standard and weaker form decreases thus making the choice between the standard and weaker form less important.

## Problem 4

a.) $\Omega(N,q) = \binom{N+q-1}{q} = \frac{(N+q-1)!}{q!(N-1)!}$ : $N! \approx N^N e^{-N}\sqrt{2\pi N}$

$$(N+q-1)! \approx (N+q-1)^{(N+q-1)} e^{-(N+q-1)}\sqrt{2\pi(N+q-1)}$$
$$(N-1)! \approx (N-1)^{(N-1)} e^{-(N-1)}\sqrt{2\pi(N-1)}$$
$$q! \approx q^q e^{-q}\sqrt{2\pi q}$$

$$\frac{(N+q-1)^{(N+q-1)} e^{-(N+q-1)}\sqrt{2\pi(N+q-1)}}{q^q e^{-q}\sqrt{2\pi q}\,(N-1)^{(N-1)} e^{-(N-1)} e^{-(N-1)}\sqrt{2\pi(N-1)}} \approx \frac{(N+q-1)^{(N+q-1)} e^{-(N+q-1)}\sqrt{2\pi(N+q-1)}}{q^q (N-1)^{(N-1)} e^{-q} e^{-(N-1)}\sqrt{4\pi^2 q(N-1)}}$$

$$\approx \frac{(N+q-1)^{(N+q-1)}\cancel{e^{-q}}\cancel{e^{-(N-1)}}}{q^q (N-1)^{(N-1)}\cancel{e^{-q}}\cancel{e^{-(N-1)}}}\sqrt{\frac{(N+q-1)}{2\pi q(N-1)}} \approx \frac{(N+q-1)^q (N+q-1)^{N-1}}{q^q (N-1)^{(N-1)}}\sqrt{\frac{(N+q-1)}{2\pi q(N-1)}}$$

$$\approx \left(\frac{N+q-1}{q}\right)^q \left(\frac{N+q-1}{N-1}\right)^{N-1}\sqrt{\frac{(N+q-1)}{2\pi q(N-1)}} \approx \left(1+\frac{N-1}{q}\right)^q \left(1+\frac{q}{N-1}\right)^{N-1}\sqrt{\frac{(N+q-1)}{2\pi q(N-1)}}$$

when $N \gg 1$, $N-1 \approx N$ : when $q \gg 1$, $q = q$ $\therefore$

$$\boxed{\Omega(N,q) \approx \left(1+\frac{q}{N}\right)^N \left(1+\frac{N}{q}\right)^q \sqrt{\frac{N+q}{2\pi Nq}}}$$

b.) when $q \gg N$ $(N+q) \approx q$ : $x = q$, $N = n$ : $\left(1+\frac{q}{N}\right)^N \approx e^q$ : $x = N$, $n = q$ : $\left(1+\frac{N}{q}\right)^q \approx e^N$

$$\left(1+\frac{x}{n}\right)^n \approx e^x : \Omega(N,q) \approx e^q e^N \sqrt{\frac{q}{2\pi N q}} \approx e^q e^N \frac{1}{\sqrt{2\pi N}} \approx \left(1+\frac{q}{N}\right)^N e^N \frac{1}{\sqrt{2\pi N}}$$

when $q \gg N$ $\left(1+\frac{q}{N}\right) \approx \frac{q}{N}$ $\therefore$ $\Omega(N,q) \approx \left(\frac{q}{N}\right)^N e^N \frac{1}{\sqrt{2\pi N}} \approx \left(\frac{eq}{N}\right)^N \frac{1}{\sqrt{2\pi N}}$

$$\boxed{\Omega(N,q) \approx \left(\frac{eq}{N}\right)^N \frac{1}{\sqrt{2\pi N}}}$$

c.) $\Omega = K q_A^{N_A} q_B^{N_B}$ : $\Omega(N_A, q_A, N_B, q_B) = \Omega(N_A, q_A)\,\Omega(N_B, q_B)$

$$\Omega(N_A, q_A) = \left(\frac{eq_A}{N_A}\right)^{N_A}\frac{1}{\sqrt{2\pi N_A}}, \quad \Omega(N_B, q_B) = \left(\frac{eq_B}{N_B}\right)^{N_B}\frac{1}{\sqrt{2\pi N_B}}$$

$$\Omega(N_A, q_A)\,\Omega(N_B, q_B) = \left(\frac{eq_A}{N_A}\right)^{N_A}\frac{1}{\sqrt{2\pi N_A}}\left(\frac{eq_B}{N_B}\right)^{N_B}\frac{1}{\sqrt{2\pi N_B}} = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{N_A N_B}}\left(\frac{eq_A}{N_A}\right)^{N_A}\left(\frac{eq_B}{N_B}\right)^{N_B}$$

$$= \frac{1}{2\pi\sqrt{N_A N_B}}\left(\frac{e}{N_A}\right)^{N_A} q_A^{N_A}\left(\frac{e}{N_B}\right)^{N_B} q_B^{N_B} = \frac{1}{2\pi\sqrt{N_A N_B}}\left(\frac{e}{N_A}\right)^{N_A}\left(\frac{e}{N_B}\right)^{N_B} q_A^{N_A} q_{N_B}^{N_B}$$

$$\boxed{\Omega = K q_A^{N_A} q_{N_B}^{N_B}} \quad : \quad K = \frac{1}{2\pi\sqrt{N_A N_B}}\left(\frac{e}{N_A}\right)^{N_A}\left(\frac{e}{N_B}\right)^{N_B}$$

## Problem 4 continued

d.) $S = k\ln(\Omega)$

$$\boxed{S = k\ln\left(K q_A^{N_A} q_B^{N_B}\right)}$$

$$K = \frac{1}{2\pi\sqrt{N_A N_B}}\left(\frac{e}{N_A}\right)^{N_A}\left(\frac{e}{N_B}\right)^{N_B}$$

e.) $S = k\ln(K q_A^{N_A} q_B^{N_B})$ : $q = q_A + q_B$

$$S = k\ln\left(K q_A^{N_A}(q-q_A)^{N_B}\right) = k\ln(K q_A^{N_A}) + k\ln\left(K(q-q_A)^{N_B}\right)$$

$$\frac{\partial S}{\partial q_A} = \frac{\cancel{K}}{\cancel{K} q_A^{N_A}}(N_A)(q_A)^{N_A-1} - \frac{\cancel{K}}{\cancel{K}(q-q_A)^{N_B}}(N_B)(q-q_A)^{N_B-1}$$

$$= \frac{(N_A)\cancel{(q_A)^{N_A}}(q_A)^{-1}}{\cancel{(q_A)^{N_A}}} - \frac{(N_B)\cancel{(q-q_A)^{N_B}}(q-q_A)^{-1}}{\cancel{(q-q_A)^{N_B}}} = \frac{N_A}{q_A} - \frac{N_B}{q-q_A} = \frac{N_A}{q_A} - \frac{N_B}{q_B}$$

$$0 = \frac{N_A}{q_A} - \frac{N_B}{q_B} \quad \therefore \quad \frac{N_A}{q_A} = \frac{N_B}{q_B} \longrightarrow \frac{N_A}{N_B} = \frac{q_A}{q_B}$$

$$\boxed{\frac{N_A}{N_B} = \frac{q_A}{q_B}}$$

This shows that when two subsystems are in thermal equilibrium, the amount of energy shared per particle is the same

## Problem 5

a.) $N = N_A + N_B$ , $E = E_A + E_B$ , $N_A = N_{A+} + N_{A-}$ , $N_B = N_{B+} + N_{B-}$ , $E = -BM$ , $M = \mu B(N_+ - N_-)$ , $\mu B = 1$

$$E_A = -BM = -\mu B(N_{A+} - N_{A-}) = -(N_{A+} - (N_A - N_{A+})) = -(2N_{A+} - N_A) = N_A - 2N_{A+}$$

$$\boxed{E_A = N_A - 2N_{A+}}$$

b.) In order to describe a macrostate of the system we need to know the energies of each subsystem. For instance, if we know $E$, we need to know both $E_A$ and $E_B$. Conversely we need to know the number of particles in each subsystem. These are somewhat redundant because we can find them from knowing how many particles are spin up in each subsystem.

c.) $\Omega(N_A, N_{A+}, N_B, N_{B+}) = \Omega(N_A, N_{A+})\Omega(N_B, N_{B+})$

$\hookrightarrow \Omega$

$E_A = N_A - 2N_{A+} \rightarrow N_{A+} = \dfrac{N_A - E_A}{2}$ ⋮

$E_B = N_B - 2N_{B+}$ : $N_{B+} = \dfrac{N_B - E_B}{2} = \dfrac{N_B - (E - E_A)}{2} = \dfrac{N_B + E_A - E}{2} \rightarrow E_B = \dfrac{N_B + E_A - E}{2}$

$\therefore \; \Omega(N_A, N_{A+}) = \dbinom{N_A}{\frac{N_A - E_A}{2}}$ , $\Omega(N_B, N_{B+}) = \dbinom{N_B}{\frac{N_B + E_A - E}{2}}$

$$\Omega = \Omega(N_A, N_{A+})\Omega(N_B, N_{B+}) = \binom{N_A}{\frac{N_A - E_A}{2}}\binom{N_B}{\frac{N_B + E_A - E}{2}}$$

$$\boxed{\Omega = \binom{N_A}{\frac{N_A - E_A}{2}}\binom{N_B}{\frac{N_B + E_A - E}{2}}}$$

d.) $N_A = 4$ , $N_B = 12$ , $E_A = 4$ , $E_B = 4$ , $E = 8$ : $N_A = N_{A+} + N_{A-}$ , $N_B = N_{B+} + N_{B-}$ , $N_+ = N_{A+} + N_{B+}$

Need $N_{A+}, N_{B+}$ to calculate $\Omega_A$ and $\Omega_B$. Need total $\Omega$ to calculate probability of each macrostate. $\Omega_{Tot} = 1820$ : $P(\text{macro}) = \Omega / \Omega_{Tot}$

| $E_A$ | $E_B$ | $N_{A+}$ | $N_{B+}$ | $\Omega_A$ | $\Omega_B$ | $\Omega = \Omega_A \Omega_B$ | Probability |
|---|---|---|---|---|---|---|---|
| 4 | 4 | 0 | 4 | 1 | 495 | 495 | 0.271978 |
| 2 | 6 | 1 | 3 | 4 | 220 | 880 | 0.483516 |
| 0 | 8 | 2 | 2 | 6 | 66 | 396 | 0.217882 |
| -2 | 10 | 3 | 1 | 4 | 12 | 48 | 0.026374 |
| -4 | 12 | 4 | 0 | 1 | 1 | 1 | 0.000549 |

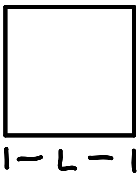
e.) Equilibrium macrostate = most probable state. This occurs when $E_A = 2$, $E_B = 6$, $N_{A+} = 1$, $N_{B+} = 3$.
$\dfrac{E_A}{N_A} = \dfrac{1}{2}$ , $\dfrac{E_B}{N_B} = \dfrac{1}{2}$. Energy flowed from subsystem A to B.

f.) $\bar{E}_A = (4)(0.271978) + (2)(0.483516) - (2)(0.026874) - (4)(0.000549) = 2$
$\bar{E}_B = (4)(0.271978) + (6)(0.483516) + (8)(0.217582) + (10)(0.026874) + (12)(0.000549) = 6$

$$\boxed{\begin{array}{l}\bar{E}_A = 2 \\ \bar{E}_B = 6\end{array}}$$

## Problem 6

a.)



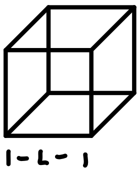

$$E = 110\frac{h^2}{8mL^2}, \quad E = \frac{h^2}{8mL^2}(n_x^2 + \cancel{n_y^2} + \cancel{n_z^2}) = \frac{h^2}{8mL^2} n_x^2$$

$$110\frac{\cancel{h^2}}{\cancel{8mL^2}} = n_x^2 \frac{\cancel{h^2}}{\cancel{8mL^2}} \quad \therefore \quad n_x^2 = 110 \quad \therefore \quad n_x = \sqrt{110} = 10.49 \approx 10$$

| $n_x$ | $n_x^2$ | $E(h^2/8mL^2)$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 4 | 4 |
| 3 | 9 | 9 |
| 4 | 16 | 16 |
| 5 | 25 | 25 |
| 6 | 36 | 36 |
| 7 | 49 | 49 |
| 8 | 64 | 64 |
| 9 | 81 | 81 |
| 10 | 100 | 100 |

$\longrightarrow$ $\boxed{\Gamma(E) = 10 \text{ microstates}}$

b.)




$$E = 15\frac{h^2}{8mL^2}, \quad E = \frac{h^2}{8mL^2}(n^2) \qquad 15\frac{\cancel{h^2}}{\cancel{8mL^2}} = n^2\frac{\cancel{h^2}}{\cancel{8mL^2}} \qquad n^2 = 15, \ (n_x^2 + n_y^2 + n_z^2) = 15$$

| $n_x$ | $n_y$ | $n_z$ | $n^2$ | $E(h^2/8mL^2)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 3 |
| 1 | 1 | 2 | 6 | 6 |
| 1 | 2 | 1 | 6 | 6 |
| 2 | 1 | 1 | 6 | 6 |
| 1 | 2 | 2 | 9 | 9 |
| 2 | 1 | 2 | 9 | 9 |
| 2 | 2 | 1 | 9 | 9 |
| 2 | 2 | 2 | 12 | 12 |
| 3 | 1 | 1 | 11 | 11 |
| 1 | 3 | 1 | 11 | 11 |
| 1 | 1 | 3 | 11 | 11 |
| 3 | 2 | 1 | 14 | 14 |
| 3 | 1 | 2 | 14 | 14 |
| 2 | 3 | 1 | 14 | 14 |
| 2 | 1 | 3 | 14 | 14 |
| 1 | 3 | 2 | 14 | 14 |
| 1 | 2 | 3 | 14 | 14 |

$\longrightarrow$ $\boxed{\Gamma(E) = 17 \text{ microstates}}$

Problem 7

| n | Exact | Approximate | % Error | n | Exact | Approximate | % Error |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.785398163 | 21.46 | 45 | 1632 | 1590.43128 | 2.55 |
| 2 | 4 | 3.141592654 | 21.46 | 46 | 1701 | 1661.90251 | 2.30 |
| 3 | 9 | 7.068583471 | 21.46 | 47 | 1776 | 1734.94454 | 2.31 |
| 4 | 15 | 12.56637061 | 16.22 | 48 | 1850 | 1809.55737 | 2.19 |
| 5 | 22 | 19.63495408 | 10.75 | 49 | 1929 | 1885.74099 | 2.24 |
| 6 | 33 | 28.27433388 | 14.32 | 50 | 2006 | 1963.49541 | 2.12 |
| 7 | 43 | 38.48451001 | 10.50 | 51 | 2091 | 2042.82062 | 2.30 |
| 8 | 56 | 50.26548246 | 10.24 | 52 | 2173 | 2123.71663 | 2.27 |
| 9 | 71 | 63.61725124 | 10.40 | 53 | 2252 | 2206.18344 | 2.03 |
| 10 | 86 | 78.53981634 | 8.67 | 54 | 2339 | 2290.22104 | 2.09 |
| 11 | 104 | 95.03317777 | 8.62 | 55 | 2421 | 2375.82944 | 1.87 |
| 12 | 121 | 113.0973355 | 6.53 | 56 | 2516 | 2463.00864 | 2.11 |
| 13 | 142 | 132.7322896 | 6.53 | 57 | 2603 | 2551.75863 | 1.97 |
| 14 | 166 | 153.93804 | 7.27 | 58 | 2694 | 2642.07942 | 1.93 |
| 15 | 189 | 176.7145868 | 6.50 | 59 | 2786 | 2733.97101 | 1.87 |
| 16 | 214 | 201.0619298 | 6.05 | 60 | 2879 | 2827.43339 | 1.79 |
| 17 | 239 | 226.9800692 | 5.03 | 61 | 2978 | 2922.46657 | 1.86 |
| 18 | 269 | 254.4690049 | 5.40 | 62 | 3076 | 3019.07054 | 1.85 |
| 19 | 300 | 283.528737 | 5.49 | 63 | 3175 | 3117.24531 | 1.82 |
| 20 | 331 | 314.1592654 | 5.09 | 64 | 3276 | 3216.99088 | 1.80 |
| 21 | 363 | 346.3605901 | 4.58 | 65 | 3374 | 3318.30724 | 1.65 |
| 22 | 400 | 380.1327111 | 4.97 | 66 | 3483 | 3421.1944 | 1.77 |
| 23 | 435 | 415.4756284 | 4.49 | 67 | 3584 | 3525.65236 | 1.63 |
| 24 | 471 | 452.3893421 | 3.95 | 68 | 3691 | 3631.68111 | 1.61 |
| 25 | 510 | 490.8738521 | 3.75 | 69 | 3805 | 3739.28066 | 1.73 |
| 26 | 553 | 530.9291585 | 3.99 | 70 | 3910 | 3848.451 | 1.57 |
| 27 | 598 | 572.5552611 | 4.25 | 71 | 4023 | 3959.19214 | 1.59 |
| 28 | 640 | 615.7521601 | 3.79 | 72 | 4131 | 4071.50408 | 1.44 |
| 29 | 683 | 660.5198554 | 3.29 | 73 | 4252 | 4185.38681 | 1.57 |
| 30 | 732 | 706.8583471 | 3.43 | 74 | 4369 | 4300.84034 | 1.56 |
| 31 | 780 | 754.767635 | 3.23 | 75 | 4486 | 4417.86467 | 1.52 |
| 32 | 833 | 804.2477193 | 3.45 | 76 | 4606 | 4536.45979 | 1.51 |
| 33 | 884 | 855.2985999 | 3.25 | 77 | 4727 | 4656.62571 | 1.49 |
| 34 | 937 | 907.9202769 | 3.10 | 78 | 4852 | 4778.36243 | 1.52 |
| 35 | 995 | 962.1127502 | 3.31 | 79 | 4972 | 4901.66994 | 1.41 |
| 36 | 1048 | 1017.87602 | 2.87 | 80 | 5097 | 5026.54825 | 1.38 |
| 37 | 1107 | 1075.210086 | 2.87 | 81 | 5228 | 5152.99735 | 1.43 |
| 38 | 1165 | 1134.114948 | 2.65 | 82 | 5354 | 5281.01725 | 1.36 |
| 39 | 1230 | 1194.590607 | 2.88 | 83 | 5489 | 5410.60795 | 1.43 |
| 40 | 1293 | 1256.637061 | 2.81 | 84 | 5616 | 5541.76944 | 1.32 |
| 41 | 1353 | 1320.254313 | 2.42 | 85 | 5751 | 5674.50173 | 1.33 |
| 42 | 1422 | 1385.44236 | 2.57 | 86 | 5889 | 5808.80482 | 1.36 |
| 43 | 1489 | 1452.201204 | 2.47 | 87 | 6026 | 5944.6787 | 1.35 |
| 44 | 1562 | 1520.530844 | 2.65 | 88 | 6165 | 6082.12338 | 1.34 |

Problem 7 continued

| n | Exact | Approximate | % Error | n | Exact | Approximate | % Error |
|---|---|---|---|---|---|---|---|
| 89 | 6297 | 6221.138852 | 1.20 | 133 | 14021 | 13892.9081 | 0.91 |
| 90 | 6448 | 6361.725124 | 1.34 | 134 | 14233 | 14102.6094 | 0.92 |
| 91 | 6587 | 6503.882191 | 1.26 | 135 | 14434 | 14313.8815 | 0.83 |
| 92 | 6732 | 6647.610055 | 1.25 | 136 | 14655 | 14526.7244 | 0.88 |
| 93 | 6878 | 6792.908715 | 1.24 | 137 | 14869 | 14741.1381 | 0.86 |
| 94 | 7025 | 6939.778172 | 1.21 | 138 | 15088 | 14957.1226 | 0.87 |
| 95 | 7178 | 7088.218425 | 1.25 | 139 | 15305 | 15174.6779 | 0.85 |
| 96 | 7324 | 7238.229474 | 1.17 | 140 | 15519 | 15393.804 | 0.81 |
| 97 | 7475 | 7389.811319 | 1.14 | 141 | 15748 | 15614.5009 | 0.85 |
| 98 | 7634 | 7542.963961 | 1.19 | 142 | 15967 | 15836.7686 | 0.82 |
| 99 | 7787 | 7697.687399 | 1.15 | 143 | 16192 | 16060.607 | 0.81 |
| 100 | 7949 | 7853.981634 | 1.20 | 144 | 16418 | 16286.0163 | 0.80 |
| 101 | 8102 | 8011.846665 | 1.11 | 145 | 16647 | 16512.9964 | 0.80 |
| 102 | 8269 | 8171.282492 | 1.18 | 146 | 16882 | 16741.5473 | 0.83 |
| 103 | 8431 | 8332.289115 | 1.17 | 147 | 17110 | 16971.6689 | 0.81 |
| 104 | 8588 | 8494.866535 | 1.08 | 148 | 17339 | 17203.3614 | 0.78 |
| 105 | 8757 | 8659.014751 | 1.12 | 149 | 17578 | 17436.6246 | 0.80 |
| 106 | 8919 | 8824.733764 | 1.06 | 150 | 17815 | 17671.4587 | 0.81 |
| 107 | 9094 | 8992.023573 | 1.12 | 151 | 18053 | 17907.8635 | 0.80 |
| 108 | 9263 | 9160.884178 | 1.10 | 152 | 18284 | 18145.8392 | 0.76 |
| 109 | 9430 | 9331.315579 | 1.05 | 153 | 18529 | 18385.3856 | 0.78 |
| 110 | 9602 | 9503.317777 | 1.03 | 154 | 18767 | 18626.5028 | 0.75 |
| 111 | 9775 | 9676.890771 | 1.00 | 155 | 19018 | 18869.1909 | 0.78 |
| 112 | 9956 | 9852.034562 | 1.04 | 156 | 19259 | 19113.4497 | 0.76 |
| 113 | 10132 | 10028.74915 | 1.02 | 157 | 19504 | 19359.2793 | 0.74 |
| 114 | 10311 | 10207.03453 | 1.01 | 158 | 19760 | 19606.6798 | 0.78 |
| 115 | 10498 | 10386.89071 | 1.06 | 159 | 20001 | 19855.651 | 0.73 |
| 116 | 10679 | 10568.31769 | 1.04 | 160 | 20252 | 20106.193 | 0.72 |
| 117 | 10859 | 10751.31546 | 0.99 | 161 | 20504 | 20358.3058 | 0.71 |
| 118 | 11044 | 10935.88403 | 0.98 | 162 | 20769 | 20611.9894 | 0.76 |
| 119 | 11233 | 11122.02339 | 0.99 | 163 | 21026 | 20867.2438 | 0.76 |
| 120 | 11423 | 11309.73355 | 0.99 | 164 | 21276 | 21124.069 | 0.71 |
| 121 | 11612 | 11499.01451 | 0.97 | 165 | 21537 | 21382.465 | 0.72 |
| 122 | 11797 | 11689.86626 | 0.91 | 166 | 21796 | 21642.4318 | 0.70 |
| 123 | 11991 | 11882.28881 | 0.91 | 167 | 22067 | 21903.9694 | 0.74 |
| 124 | 12198 | 12076.28216 | 1.00 | 168 | 22325 | 22167.0778 | 0.71 |
| 125 | 12387 | 12271.8463 | 0.93 | 169 | 22582 | 22431.7569 | 0.67 |
| 126 | 12590 | 12468.98124 | 0.96 | 170 | 22857 | 22698.0069 | 0.70 |
| 127 | 12780 | 12667.68698 | 0.88 | 171 | 23127 | 22965.8277 | 0.70 |
| 128 | 12985 | 12867.96351 | 0.90 | 172 | 23398 | 23235.2193 | 0.70 |
| 129 | 13192 | 13069.81084 | 0.93 | 173 | 23665 | 23506.1816 | 0.67 |
| 130 | 13390 | 13273.22896 | 0.87 | 174 | 23944 | 23778.7148 | 0.69 |
| 131 | 13601 | 13478.21788 | 0.90 | 175 | 24222 | 24052.8188 | 0.70 |
| 132 | 13804 | 13684.7776 | 0.86 | 176 | 24493 | 24328.4935 | 0.67 |

Problem 7 continued

| n | Exact | Approximate | % Error |
|---|---|---|---|
| 177 | 24772 | 24605.73906 | 0.67 |
| 178 | 25044 | 24884.55541 | 0.64 |
| 179 | 25333 | 25164.94255 | 0.66 |
| 180 | 25618 | 25446.90049 | 0.67 |
| 181 | 25896 | 25730.42923 | 0.64 |
| 182 | 26183 | 26015.52876 | 0.64 |
| 183 | 26476 | 26302.19909 | 0.66 |
| 184 | 26771 | 26590.44022 | 0.67 |
| 185 | 27051 | 26880.25214 | 0.63 |
| 186 | 27344 | 27171.63486 | 0.63 |
| 187 | 27645 | 27464.58838 | 0.65 |
| 188 | 27939 | 27759.11269 | 0.64 |
| 189 | 28236 | 28055.20779 | 0.64 |
| 190 | 28529 | 28352.8737 | 0.62 |
| 191 | 28828 | 28652.1104 | 0.61 |
| 192 | 29136 | 28952.9179 | 0.63 |
| 193 | 29431 | 29255.29619 | 0.60 |
| 194 | 29744 | 29559.24528 | 0.62 |
| 195 | 30044 | 29864.76516 | 0.60 |
| 196 | 30357 | 30171.85585 | 0.61 |
| 197 | 30670 | 30480.51732 | 0.62 |
| 198 | 30977 | 30790.7496 | 0.60 |
| 199 | 31293 | 31102.55267 | 0.61 |
| 200 | 31602 | 31415.92654 | 0.59 |

The value of $N$ that is within less than 1% is $N = 117$.

## Problem 8

$$V_n(R) = \frac{2\pi^{n/2}}{n\Gamma(n/2)} R^n \quad , \quad \Gamma(n) = (n-1)!$$

$n=1$, This is a one dimensional picture. A straight line between two points.

$\Gamma(n/2) = (n/2 - 1)! = (1/2 - 1)! = (-1/2)! = \sqrt{\pi}$

$V_1(R) = \frac{2\cancel{\sqrt{\pi}}}{\cancel{\sqrt{\pi}}} R = 2R$ $\boxed{V_1(R) = 2R}$ ✓

$n=2$, This is a two dimensional surface, i.e. a circle

$\Gamma(n/2 - 1)! = (2/2 - 1)! = (0)! = 1$

$V_2(R) = \frac{\cancel{2}\pi}{\cancel{2} \cdot 1} R^2 = \pi R^2$ $\boxed{V_2(R) = \pi R^2}$ ✓

$n=3$, This is a three dimensional surface, i.e. a sphere

$\Gamma(n/2 - 1)! = \Gamma(3/2 - 1) = \Gamma(1/2) = \frac{\sqrt{\pi}}{2}$ $\longrightarrow$ Found this from google

$V_3(R) = \frac{2\pi^{3/2}}{3\frac{\pi^{1/2}}{2}} R^3 = \frac{4}{3}\pi R^3$ $\boxed{V_3(R) = \frac{4}{3}\pi R^3}$ ✓