MAT 201
Larson/Edwards – Section 3.7
Optimization Problems

In this chapter we get to use what we learned about maximum and minimum points to solve optimization problems!

<u>Tools For Solving Optimization Problems</u>:

1. Identify all given quantities and all quantities to be determined. If possible, make a sketch.
2. Write a primary equation for the quantity that is to be maximized or minimized. Geometry formulas may be found inside the back cover of your text.
3. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.
4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.
5. Determine the desired maximum or minimum value by the calculus techniques discussed earlier.

Ex: Find two positive numbers such that the sum of the first and twice the second is 36 and the product is a maximum.

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$x = 1^{st}$ # (positive)

$y = 2^{nd}$ # (pos.)

$P =$ Product (multiply)

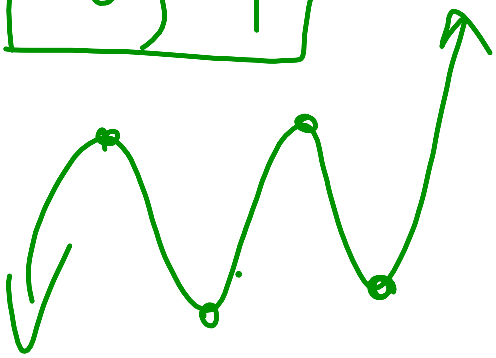
$x + 2y = 36$

$x = 36 - 2y$

$x = 36 - 2(9)$

$x = 18$

$\boxed{18, 9}$



$P = x \cdot y$

$P = (36 - 2y)y$

$P = 36y - 2y^2$

$P = -2y^2 + 36y$

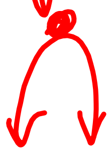
$P' = -4y + 36$

$0 = -4y + 36$

$4y = 36$

$y = 9$

$P'' = -4 < 0$



Ex: Find the length and width of a rectangle with a perimeter of 120 feet and has a maximum area.

Ex: An open box is to be made from a two-foot by three-foot rectangular piece of material by cutting equal squares from the corners and turning up the sides. Find the volume of the largest box that can be made in this manner.

Ex: Find the length and width of a rectangle with a perimeter of 120 feet and has a maximum area.

$L =$ Length $W =$ width $A =$ Area

$120 = 2L + 2W$

$120 - 2L = 2W$

$60 - L = W$

$A = LW$

$A = L(60 - L)$

$A = 60L - L^2$

$A = -L^2 + 60L$

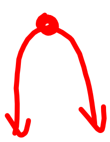
$A' = -2L + 60$

$0 = -2L + 60$

$2L = 60$

$L = 30$

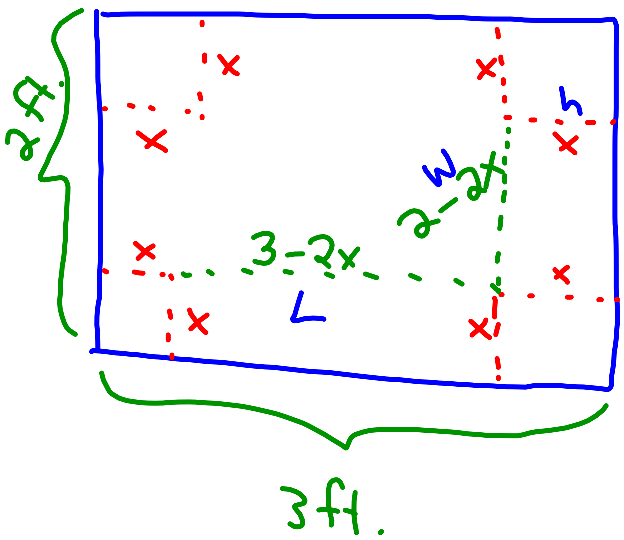
$A'' = -2 < 0$



Length = 30 ft.

Width = 30 ft.

Ex: An open box is to be made from a two-foot by three-foot rectangular piece of material by cutting equal squares from the corners and turning up the sides. Find the volume of the largest box that can be made in this manner.



$$V = LWH$$

$$V = x(2-2x)(3-2x)$$

$$V = (2x-2x^2)(3-2x)$$

$$V = 6x - 4x^2 - 6x^2 + 4x^3$$

$$V = 4x^3 - 10x^2 + 6x$$

$$V' = 12x^2 - 20x + 6$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$V' = 2(6x^2 - 10x + 3)$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(6)(3)}}{2(6)}$$

$$0 = 6x^2 - 10x + 3$$

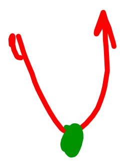
Critical #'s: $x \approx 1.27, 0.39$

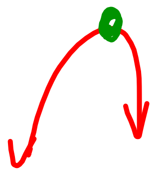
$$V'' = 24x - 20$$

@ $x = 0.39$

Volume is at a Maximum

$V''(1.27) > 0$ $\quad$ $V''(0.39) < 0$





$$\boxed{\text{Volume} \approx 1.06\ \text{ft}^3}$$