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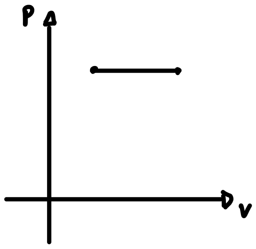
PHYS 362 HW 4

Problem 1

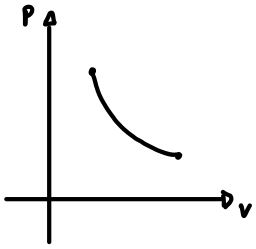
a.) $W = -P\int dV$

The process must be isobaric and the PV diagram would be like so



b.) $W = -\int_{V_i}^{V_f} NkT/V \, dV$

T must remain constant, i.e. isochoric, and this process would look like so



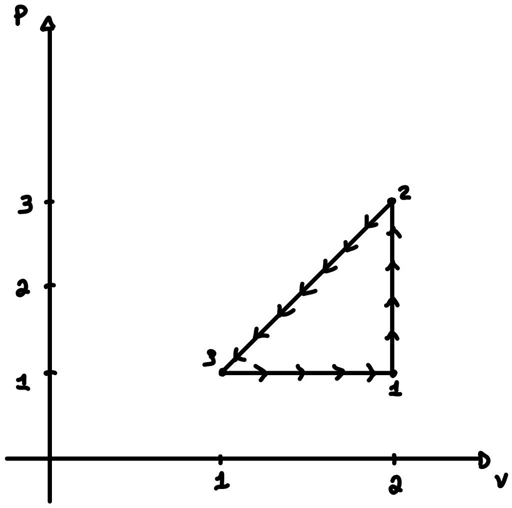
c.) $W = -NkT\int_{V_i}^{V_f} 1/V \, dV = -NkT \ln(V_f/V_i)$?

T has to be dependent on V, if it were independent then there is no way volume or temperature can change.

$V = \frac{NkT}{P}$ independent as $T\uparrow$, V must $\uparrow$
constant $T\downarrow$, V must $\downarrow$
constant

$\frac{PV}{Nk} = T$ independent as $V\uparrow$, T must $\uparrow$
$V\downarrow$, T must $\downarrow$
constant
constant

Problem 2



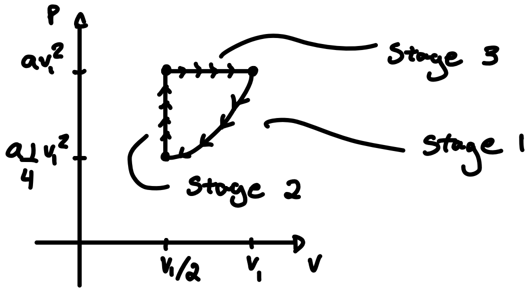
First : $V_i \rightarrow 2V_i$
Second : $P_i \rightarrow 3P_i$
Third : Returns back to P_i, V_i

Area inside triangle, $A = \frac{1}{2}bh$: $b = V_0$, $h = 2P_0$ $\therefore$ $A = \frac{1}{2}V_0 \cdot 2P_0 = V_0 P_0$

$$\boxed{W = V_0 P_0 = NkT_0}$$

Problem 3

a.) $P = aV^2$



b.) $W = -\int_{V_i}^{V_f} P(V)\, dV$

i.) Stage 1 : $V_i \longrightarrow \frac{V_i}{2} = V_f$: $P = aV^2$

$$W = -\int_{V_i}^{V_i/2} aV^2\, dV = -a\int_{V_i}^{V_i/2} V^2\, dV = -\frac{a}{3}\cdot V^3\Big|_{V_i}^{V_i/2} = -\frac{a}{3}\left(\frac{V_i^3}{8} - V_i^3\right) = -\frac{a}{3}\left(\frac{-7V_i^3}{8}\right)$$

$$\boxed{W = \frac{7a}{24} V_i^3}$$

ii.) Stage 2 : $V_i \longrightarrow V_i = V_f$: W = area under curve

$$\therefore \quad \boxed{W = 0}$$

iii.) Stage 3 : $V_i \longrightarrow 2V_i = V_f$: W = - area under curve

$$W = -LW = -\frac{1}{2} V_1 \cdot P_1 = -\frac{P_1 V_1}{2} = \frac{-aV_1^2 \cdot V_1}{2} = -\frac{a}{2} V_1^3$$

$$\boxed{W = -\frac{a}{2} V_1^3}$$

c.) $E = \frac{3}{2} NkT$: $PV = NkT$: $\Delta E = E_f - E_i$

i.) $E_f = \frac{3}{2} NkT_f$, $E_i = \frac{3}{2} NkT_i$: $T = \frac{PV}{Nk}$, $T_f = \frac{P_f V_f}{Nk}$, $T_i = \frac{P_i V_i}{Nk}$ $\quad$ $V_i = V_i$, $V_f = \frac{1}{2} V_i$, $P_i = aV_i^2$, $P_f = \frac{a}{4} V_i^2$

$$T_i = \frac{a \cdot V_1 \cdot V_1^2}{Nk} = \frac{aV_1^3}{Nk} , \quad T_f = \frac{1}{8} \frac{aV_1^3}{Nk}$$

$$E_f = \frac{3}{2} \cancel{Nk} \cdot \frac{1}{8} \frac{aV_1^3}{\cancel{Nk}} = \frac{3}{16} aV_1^3 , \quad E_f = \frac{3}{16} aV_1^3 \ : \ E_i = \frac{3}{2} \cancel{Nk} \cdot \frac{aV_1^3}{\cancel{Nk}} = \frac{3}{2} aV_1^3 , \quad E_i = \frac{3}{2} aV_1^3$$

$$E_f - E_i = \frac{3}{16} aV_1^3 - \frac{24}{16} aV_1^3 = -\frac{21}{16} aV_1^3$$

$$\boxed{\Delta E = -\frac{21}{16} aV_1^3}$$

Problem 3 continued

ii.) $E_f = \frac{3}{2}Nk T_f$, $E_i = \frac{3}{2}NkT_i$: $T = \frac{PV}{Nk}$, $T_f = \frac{P_f V_f}{Nk}$, $T_i = \frac{P_i V_i}{Nk}$ $\quad P_i = \frac{a}{4}v_1^2$, $P_f = av_1^2$

$T_i = \frac{1}{8}\frac{av_1^3}{Nk}$, $T_f = \frac{1}{2}\frac{av_1^3}{Nk}$ $\quad V_i = \frac{V_1}{2}$, $V_f = \frac{V_1}{2}$

$E_f = \frac{3}{2}\cancel{Nk}\cdot\frac{1}{2}\frac{av_1^3}{\cancel{Nk}} = \frac{3}{4}av_1^3$, $E_f = \frac{3}{4}av_1^3$: $E_i = \frac{3}{2}\cancel{Nk}\cdot\frac{1}{8}\frac{av_1^3}{\cancel{Nk}} = \frac{3}{16}av_1^3$, $E_i = \frac{3}{16}av_1^3$

$E_f - E_i = \frac{12}{16}av_1^3 - \frac{3}{16}av_1^3 = \frac{9}{16}av_1^3$

$$\boxed{\Delta E = \frac{9}{16}av_1^3}$$

iii.) $E_f = \frac{3}{2}NkT_f$, $E_i = \frac{3}{2}NkT_i$: $T = \frac{PV}{Nk}$, $T_f = \frac{P_f V_f}{Nk}$, $T_i = \frac{P_i V_i}{Nk}$ $\quad P_i = av_1^2 \quad P_f = av_1^2$

$T_i = \frac{1}{2}\frac{a\cdot v_1^3}{Nk}$, $T_f = \frac{a\cdot v_1^3}{Nk}$ $\quad V_i = \frac{1}{2}v_1 \quad V_f = v_1$

$E_f = \frac{3}{2}\cancel{Nk}\cdot\frac{av_1^3}{\cancel{Nk}} = \frac{3}{2}av_1^3$, $E_f = \frac{3}{2}av_1^3$: $E_i = \frac{3}{2}\cancel{Nk}\cdot\frac{1}{2}\frac{av_1^3}{\cancel{Nk}} = \frac{3}{4}av_1^3$, $E_i = \frac{3}{4}av_1^3$

$E_f - E_i = \frac{24}{16}av_1^3 - \frac{12}{16}av_1^3 = \frac{12}{16}av_1^3$

$$\boxed{\Delta E = \frac{12}{16}av_1^3}$$

d.) $Q = \Delta E - W$

i.) $\Delta E = \frac{-21}{16}av_1^3$, $W = \frac{7}{24}av_1^3$: $Q = -\frac{21}{16}av_1^3 - \frac{7}{24}av_1^3 = \frac{-77}{48}av_1^3$

$$\boxed{Q = \frac{-77}{48}av_1^3}$$

ii.) $\Delta E = \frac{9}{16}av_1^3$, $W = 0$ $\therefore$ $Q = \Delta E = \frac{9}{16}av_1^3$

$$\boxed{Q = \frac{9}{16}av_1^3}$$

iii.) $\Delta E = \frac{12}{16}av_1^3$, $W = -\frac{a}{2}v_1^3$: $Q = \frac{12}{16}av_1^3 + \frac{8}{16}av_1^3 = \frac{20}{16}av_1^3$

$$\boxed{Q = \frac{20}{16}av_1^3}$$

e.)

The gas does work on the system , $W < 0$

Problem 3 continued

f.)

Second Stage : Heat is added at a constant rate

Third Stage : Heat is added at a constant rate

6.) Only when $Q > 0$: $W_{net} = \frac{-5}{24}$, $+Q_{net} = \frac{29}{16}$ $\therefore$ $\frac{W_{net}}{+Q_{net}} = \frac{-10}{87}$

$$\boxed{\frac{-10}{87}}$$

Problem 4

$$\left(P + a\frac{N^2}{V^2}\right)(V - Nb) = NkT, \quad E = \frac{3}{2}NkT - a\frac{N^2}{V^2}$$

$$V_i \rightarrow V_i = V_f, \quad P_i \rightarrow 2P_i = P_f$$

$$PV - NbP + \frac{aN^2}{V} + \frac{aN^3b}{V^2} = NkT \quad \therefore \quad T = \frac{PV}{Nk} - \frac{bP}{k} + \frac{aN}{kV} + \frac{aN^2b}{kV^2}$$

a.) Because volume remains constant, $W = -\int_{V_i}^{V_f} P(V)\,dV = 0$

$$\boxed{W = 0}$$

b.) $\Delta E = E_f - E_i = \left(\frac{3}{2}NkT_f - a\frac{N^2}{V_f^2}\right) - \left(\frac{3}{2}NkT_i - a\frac{N^2}{V_i^2}\right)$

$$T_f = \frac{2P_iV}{Nk} - \frac{2bP_i}{k} + \frac{aN}{kV_i} + \frac{aN^2b}{kV_i^2}, \quad T_i = \frac{P_iV}{Nk} - \frac{bP_i}{k} + \frac{aN}{kV_i} + \frac{aN^2b}{kV_i^2}$$

$$E_f = \frac{3}{2}Nk\left(\frac{2P_iV_i}{Nk} - \frac{2bP_i}{k} + \frac{aN}{kV_i} + \frac{aN^2b}{kV_i^2}\right) - \frac{aN^2}{V_i^2}, \quad E_i = \frac{3}{2}Nk\left(\frac{P_iV_i}{Nk} - \frac{bP_i}{k} + \frac{aN}{kV_i} + \frac{aN^2b}{kV_i^2}\right) - \frac{aN^2}{V_i^2}$$

$$E_f - E_i = \frac{3}{2}Nk\left(\frac{2P_iV_i}{Nk} - \frac{2bP_i}{k} + \frac{aN}{kV_i} + \frac{aN^2b}{kV_i^2}\right) - \cancel{\frac{aN^2}{V_i^2}} - \frac{3}{2}Nk\left(\frac{P_iV_i}{Nk} - \frac{bP_i}{k} + \frac{aN}{kV_i} + \frac{aN^2b}{kV_i^2}\right) + \cancel{\frac{aN^2}{V_i^2}}$$

$$= 3P_iV_i - 3NbP_i + \cancel{\frac{3}{2}\frac{aN^2}{V_i}} + \cancel{\frac{3}{2}\frac{aN^3b}{V_i^2}} - \frac{3}{2}P_iV_i - \frac{3}{2}NbP_i - \cancel{\frac{3}{2}\frac{aN^2}{V_i}} - \cancel{\frac{3}{2}\frac{aN^3b}{V_i^2}}$$

$$= \frac{3}{2}P_iV_i - \frac{3}{2}NbP_i = \frac{3}{2}P_i(V_i - Nb) = \Delta E$$

$$\boxed{\Delta E = \frac{3}{2}P_i(V_i - Nb)}$$

c.) $Q = \Delta E - W$, $W = 0$ $\therefore$ $Q = \Delta E$

$$\boxed{Q = \frac{3}{2}P_i(V_i - Nb)}$$