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MATH 366
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1) 1. If the convergence is rapid (Matrices have large main diagonal entries).
2. If a large system is sparse, that is, has many zero coefficients.

2) Example 1 Gauss-Seidel Iteration
We consider the linear system

$$\begin{aligned}
x_1 - 0.25x_2 - 0.25x_3 &= 50 \\
-0.25x_1 + x_2 - 0.25x_4 &= 50 \\
-0.25x_1 + x_3 - 0.25x_4 &= 25 \\
-0.25x_2 - 0.25x_3 + x_4 &= 25
\end{aligned}$$

(Equations of this form arise in the numeric solution of PDEs and in spline interpolation.) We write the system in the form

$$\begin{aligned}
x_1 &= 0.25x_2 + 0.25x_3 + 50 \\
x_2 &= 0.25x_1 + 0.25x_4 + 50 \\
x_3 &= 0.25x_1 + 0.25x_4 + 25 \\
x_4 &= 0.25x_2 + 0.25x_3 + 25.
\end{aligned} \tag{2}$$

These equations are now used for iteration: that is, we start from a (possibly poor) approximation to the solution, say $x_1^{(0)} = 100$, $x_2^{(0)} = 100$, $x_3^{(0)} = 100$, $x_4^{(0)} = 100$, and compute from (2) a perhaps better approximation