Growth and Decay

2.2 # 24, 25
2.3 # 6, 10, 30

$\frac{d}{dt} y = Ky$ $\quad y(t) = y_0 e^{Kt} \longleftarrow$ exponential decay

Ex: C-14 is radioactive

Half life of 5,600 years

y is the amount of C14
t is years

$y(t) = y_0 e^{-0.000124t}$

$\frac{1}{2} = 1e^{5600K}$

$\ln \frac{1}{2} = \ln(e^{5600K})$

$\frac{\ln(\frac{1}{2})}{5600} = K$ $\quad K = 1.24 \times 10^{-4}$

Ex: 15% of C-14 is left, how old is the paint?

$0.15 = e^{-0.000124t}$

$\ln 0.15 = -0.000124t$

$t = \frac{\ln(0.15)}{-0.000124}$

$t = 15{,}299.4$

$t = 15{,}299.14$ years old

Compound Interest

$A = Pe^{rt}$ Continuous

$A = P(1 + \frac{r}{n})^{nt}$ n is compounding periods per t.

Ex. $P = 100$ \$
$t = 1$ year
$r = 5\%$

n	A
1	105
2	105.06

Ex.
$P = 24$
$r = 8\%$
$t = 391$
t = time since 1626
Continuous

$A = Pe^{rt}$
$= 24e^{(0.08)(391)}$
$= 9.22 \times 10^{14}$ \$

Ex.

60	100%		$n=-1$
59	50%	$\frac{1}{2}$	$n=0$
58	25%	$\frac{1}{2^2}$	$n=1$
57	12.5%		$n=2$
56	6.25%		
55	3.125%		

$\left(\frac{1}{2^{n+1}}\right)$