Phys 131 2016

Lecture 26

Thurs: Seminar WS 117
12:30

Fri: HW by 5pm

Supp Ex 61

Ch 10 ConcQ 4, 5

Ch 10 Probs 10, 11, 20, 43, 45

Gravitational forces + energy conservation

We saw that

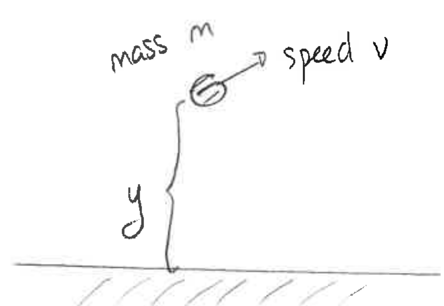
> If gravity is the only force that does non-zero work on an object then the total mechanical energy
>
> $$E_{mech} = K + U_g$$
>
> remains constant.
>
> Energy conservation

Here the kinetic energy is

$$K = \tfrac{1}{2}mv^2$$

and the gravitational potential energy is

$$U_g = mgy$$

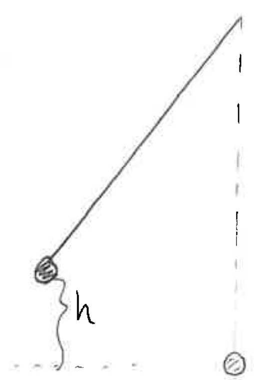


Energy conservation can also be expressed as

$$\Delta E_{mech} = 0 \iff \Delta K + \Delta U_g = 0 \iff K_f + U_{gf} = K_i + U_{gi}$$
$$\iff E_f = E_i$$

This is particularly useful when there are constraining forces present.

<u>Example:</u> A pendulum is an object which swings on a string under the influence of Earth's gravity. Suppose that a pendulum is released from height h above its lowest point. What is its maximum height on the other side of the "swing"?



<u>Answer:</u> Let $y=0$ at lowest point. Then at release the energy is entirely gravitational

↳ Note tension does zero work

$$E_{mech\,i} = \cancel{K_i} + U_{gi} = mgh$$

At the max height on the other side of the swing, the energy is again entirely gravitational

$$E_{mech\,f} = K_f + U_{gf} = mg\,y_{max}$$

But energy is conserved $\Rightarrow$ $E_{mech\,f} = E_{mech\,i} \Rightarrow y_{max} = h$. It reaches the same height.

<u>Demo</u>: U Iowa large pendulum

Quiz 1

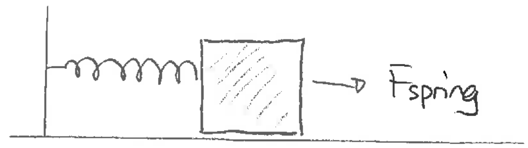
<u>Demo</u> Stopped pendulum

Quiz 2

Demo: Show loop the loop with ball at height R.

Elastic Potential Energy

Suppose that the only force that does non-zero work on an object is that exerted by a spring. Then:



$$\Delta K = W_{net}$$

$$K_f - K_i = W_{spring} = -\tfrac{1}{2}k(\Delta s_f)^2 + \tfrac{1}{2}k(\Delta s_i)^2$$

$$\Rightarrow \quad K_f + \tfrac{1}{2}k(\Delta s_f)^2 = K_i + \tfrac{1}{2}k(\Delta s_i)^2$$

Thus defining:

> The elastic potential energy stored in a spring with spring constant k is:
>
> $$U_{spring} = \frac{1}{2}k\Delta s^2$$
>
> where Δs is the stretch/compression of the spring.

Gives

$$K_f + U_{spf} = K_i + U_{spi}$$

which describes how the total energy $E_{mech} = K + U_{sp}$ is conserved. We can broaden this to include situations where gravity acts on the object.

> If the only forces that do non-zero work are gravity and spring forces, then the total mechanical energy
>
> $$E_{mech} = K + U_{grav} + U_{spring}$$
>
> remains constant, i.e.
>
> $$\Delta E_{mech} = 0$$

64 Bungee jumper

A 100 kg person is attached to a bungee cord and, starting at rest, jumps off a bridge that is 120 m above a river. The bungee cord behaves like a spring and the length of the cord when it is unstretched is 100 m. The spring constant of the cord needs to be such that person stops just above the river.

a) Determine the total energy of the system at the moment that the person jumps.

b) Determine the total energy of the system at the moment that the person stops just above the river and use the result to determine the spring constant of the bungee cord.

c) Determine the maximum force that the bungee cord exerts on this person.

d) Now suppose that a person with mass 70 kg jumps from the same bridge using the same cord. Determine the height above the river at which the person reverses direction and the maximum force exerted on the person.

Answer:

initial

$v_i = 0\,m/s$

$\Delta s_i = 0\,m$

$y_i = 120\,m$

a) $E_{mech\,i} = K_i + U_{grav\,i} + U_{spring\,i}$

$$= \frac{1}{2}m\cancel{v_i}^2_{0} + mgy_i + \frac{1}{2}k(\cancel{\Delta s_i})^2_{0} =$$

$$= 100\,kg \times 9.8\,m/s^2 \times 120\,m = 1.18 \times 10^5\,J$$

b) $E_{mech\,f} = E_{mech\,i} = 1.18 \times 10^5\,J$

final

$v_f = 0\,m/s$

$\Delta s_f = 20\,m$

$y_f = 0\,m$

$E_{mech\,f} = K_f + U_{grav\,f} + U_{spring\,f}$

$$= \frac{1}{2}m\cancel{v_f}^2_{0} + mg\cancel{y_f}_{0} + \frac{1}{2}k(\Delta s_f)^2$$

$$\Rightarrow 1.18 \times 10^5\,J = \frac{1}{2}k\,(20\,m)^2 = 200\,m^2\,k$$

$$\Rightarrow k = \frac{1.18 \times 10^5\,J}{200\,m^2} \Rightarrow \boxed{k = 590\,N/m}$$

c) $F_{sp} = -k\Delta s_p \Rightarrow F_{sp} = -590\,N/m \times 20\,m = 12 \times 10^3\,N$

$$\Rightarrow \boxed{F_{sp} = 12\,000\,N}$$

d)

initial

$v_i = 0$

$y_i = 120m$

$\Delta s_i = 0m$

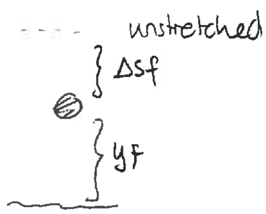
By geometry.

$$\underbrace{L + \Delta s_f}_{>0} + y_f = y_i \quad \Rightarrow \quad \Delta s_f = y_i - y_f - L$$

Then

$$E_{mech\,f} = E_{mech\,i}$$

$$\Rightarrow \frac{1}{2}m\cancel{v_f^2} + mgy_f + \frac{1}{2}k(\Delta s_f)^2 = \frac{1}{2}m\cancel{v_i^2} + mgy_i + \frac{1}{2}\cancel{k(\Delta s_i)^2}$$



$v_f = 0$

$\Delta s_f =$

$y_f = ?$

$$\Rightarrow \quad mg(y_f - y_i) + \frac{1}{2}k(\Delta s_f)^2 = 0$$

$$\Rightarrow \quad -mg(L + \Delta s_f) + \frac{1}{2}k(\Delta s_f)^2 = 0$$

$$\Rightarrow \quad -\frac{2mg}{k}L - \frac{2mg}{k}\Delta s_f + \Delta s_f^2 = 0$$

$$\Rightarrow \quad \Delta s_f = \frac{\frac{2mg}{k} \pm \sqrt{\frac{4m^2g^2}{k^2} + \frac{8mgL}{k}}}{2}$$

$$= \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + \frac{2mgL}{k}}$$

We need $\Delta s_f > 0$ and the only possibility is the + sign.

$$\Rightarrow \quad \Delta s_f = \frac{mg}{k} + \sqrt{\left(\frac{mg}{k}\right)^2\left(1 + \frac{2Lk}{mg}\right)}$$

$$= \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2Lk}{mg}}\right]$$

Here $\Delta s_f = \dfrac{70kg \times 9.8m/s^2}{590N/m}\left[1 + \sqrt{1 + \dfrac{2 \times 100m \times 590N/m}{9.8m/s^2 \times 70kg}}\right]$

$= 16.5m$

The distance above is $120m - 100m - 16.5m = 3.5m$. The max force is

$$F_{sp} = k\Delta s_f = mg\left[1 + \sqrt{1 + \frac{2Lk}{mg}}\right] \quad \Rightarrow \quad F_{sp} = 9700N.$$