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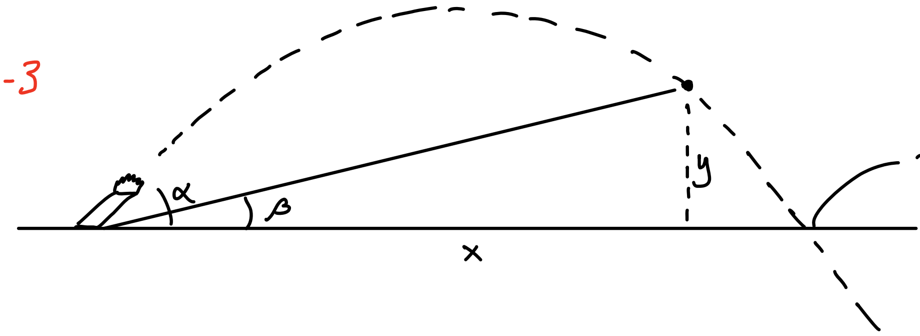
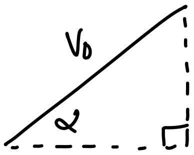
Dr. Workman
PHYS 342
HW 2

Problem 1 | 2-3



$Tan(\beta) = \frac{y}{x}$



$V_{0y} = V_0 \sin\alpha$
$V_{0x} = V_0 \cos\alpha$

x direction

$0 = m\ddot{x}$

no accel in x

y direction

$-mg = m\ddot{y}$

$-g = \ddot{y}$

$\dot{y} = -gt + V_{0y}$

$y = y_0 + V_{0y} t - \frac{gt^2}{2}$ $\quad y_0 = 0$

$y = V_{0y} t - gt^2/2$

$y = V_0 \sin(\alpha) t - gt^2/2$

assuming @ $t=0$, $x=0$ & $y=0$

$\ddot{x} = 0$

$\dot{x} = V_{0x}$ — $V_0 \cos(\alpha)$

$x = \int \dot{x}\, dt = \int V_0 \cos(\alpha)\, dt$

$x = V_0 t \cos(\alpha)$

$$Tan(\beta) = \frac{V_0 \sin(\alpha) t - gt^2/2}{V_0 t \cos(\alpha)}$$

$$Tan(\beta) = \frac{V_0 \sin(\alpha) - gt/2}{V_0 \cos(\alpha)} \equiv Tan(\beta) = \frac{\frac{2 v_0 \sin(\alpha) - gt}{2}}{V_0 \cos(\alpha)}$$

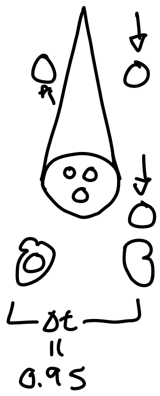
$$Tan(\beta) = \frac{2 v_0 \sin(\alpha) - gt}{2 v_0 \cos(\alpha)} \equiv 2 v_0 \cos(\alpha) Tan(\beta) = 2 v_0 \sin(\alpha) - gt$$

$$gt - 2 v_0 \sin(\alpha) = -2 v_0 \cos(\alpha) Tan(\beta)$$

$$gt = 2 v_0 \sin(\alpha) - 2 v_0 \cos(\alpha) Tan(\beta)$$

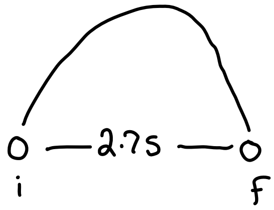
$$\boxed{t = \frac{2 v_0 (\sin(\alpha) - \cos(\alpha) Tan(\beta))}{g}}$$

Problem 2 | 2-4

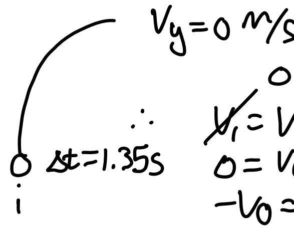


3 Balls in air at one time

Toss to catch time



$V_y = 0$ m/s @ top



$\therefore\ \cancel{V_f} = V_0 + a\Delta t$

$0 = V_0 + a\Delta t$

$-V_0 = a\Delta t$

Time for ball to reach peak is half of total time from toss to catch

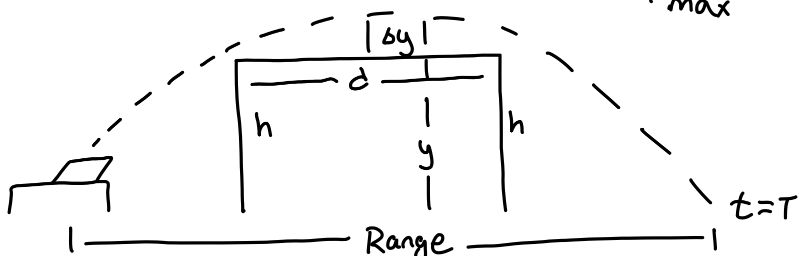
$\therefore\ \Delta t = 1.36S$

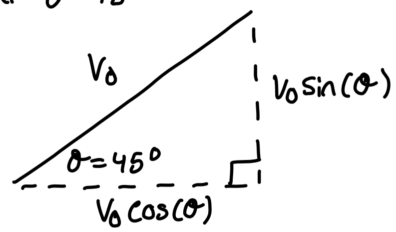
$V_0 = (9.81\ m/s^2)(1.35\ s) = 13.24\ m/s$

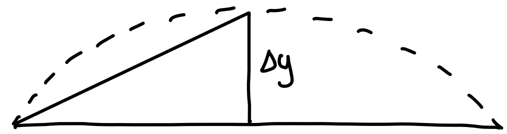
$\boxed{V_0 = 13.2\ m/s}$

Problem 3 | 2-8

R_{max} occurs at $\theta = 45^0$







Range $\equiv r = \sqrt{x^2+y^2}$

x-direction

$0 = m\ddot{x}$

$\dot{x} = \int m\ddot{x}\, dt$

$\dot{x} = V_0\cos(\theta)$

$x = \int \dot{x}\, dt = V_0 t\cos(\theta) + x_0$

$x = x_0 + V_0\cos(\theta)$

y-direction

$-mg = m\ddot{y}$

$-g = \ddot{y}$

$\dot{y} = \int \ddot{y}\, dy = \int -g dt$

$\dot{y} = -gt + V_0\sin(\theta)$

$y = \int \dot{y}\, dy = \int -gt + V_0\sin(\theta)$; $y = y_0 + V_0\sin(\theta)t - \frac{gt^2}{2}$

EQ's of Position

$x = x_0 + V_0\cos(\theta)t$

$y = y_0 + V_0\sin(\theta)t - \frac{gt^2}{2}$

Range:

$x = x_0 + V_0\cos(\theta)t$ $\quad x_0 = 0 \quad x = V_0\cos(\theta)t$

$y = y_0 + V_0\sin(\theta)t - \frac{gt^2}{2}$ $\quad y_0 = 0 \quad y = V_0\sin(\theta)t - \frac{gt^2}{2}$

$y = 0$: @ $t = T$ $\quad \frac{gT^2}{2} = V_0\sin(\theta)T$: $\frac{gT}{2} = V_0\sin(\theta)$: $T = \frac{2V_0\sin(\theta)}{g}$

$X(T) = V_0\cos(\theta)\left(\frac{2V_0\sin(\theta)}{g}\right) = \frac{2V_0^2}{g}\sin(\theta)\cos(\theta)$ $\quad 2\sin(\theta)\cos(\theta) = \sin(2\theta)$

$X(T) = \frac{V_0^2}{g}\sin(2\theta)$ when $\theta = 45^0$

$X(T) = \frac{V_0^2}{g}$ $\quad$ Range of Shot $= \frac{V_0^2}{g}$ $\quad \frac{1}{2}$ Range $= \frac{V_0^2}{2g}$

Height: max height occurs @ $\dot{y} = 0$ $\quad h + \Delta y = y$: $\Delta y = y - h$

$\dot{y} = -gt + V_0\sin(\theta)$: $\dot{y} = 0$ $\quad y = \cancel{y_0}^{0} + V_0\sin(\theta)t - \frac{1}{2}gt^2$

$0 = -gt + V_0\sin(\theta)$ $\quad y(t_1) = V_0\sin(\theta)\left(\frac{V_0\sin(\theta)}{g}\right) - \frac{1}{2}g\left(\frac{V_0^2\sin^2(\theta)}{g^2}\right)$

$t_1 = \frac{V_0\sin(\theta)}{g}$ $\quad y(t_1) = \frac{V_0^2\sin^2(\theta)}{g} - \frac{V_0^2\sin^2(\theta)}{2g} = \frac{V_0^2\sin^2(\theta)}{2g}$

$y = \frac{V_0^2}{4g}$ $\quad \Delta y = \frac{V_0^2}{4g} - h$: $\Delta y = \frac{V_0^2 - 4gh}{4g}$

Problem 3) 2-8 Continued

Time when $y=h$

$$h = V_0\sin(\theta)t - \frac{gt^2}{2} \;:\; \frac{gt^2}{2} - V_0\sin(\theta)t + h = 0 \qquad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = g/2$
$b = -V_0\sin(\theta)$
$c = h$

$$t = \frac{V_0\sin(\theta) \pm \sqrt{V_0^2\sin^2(\theta) - 2gh}}{g}$$

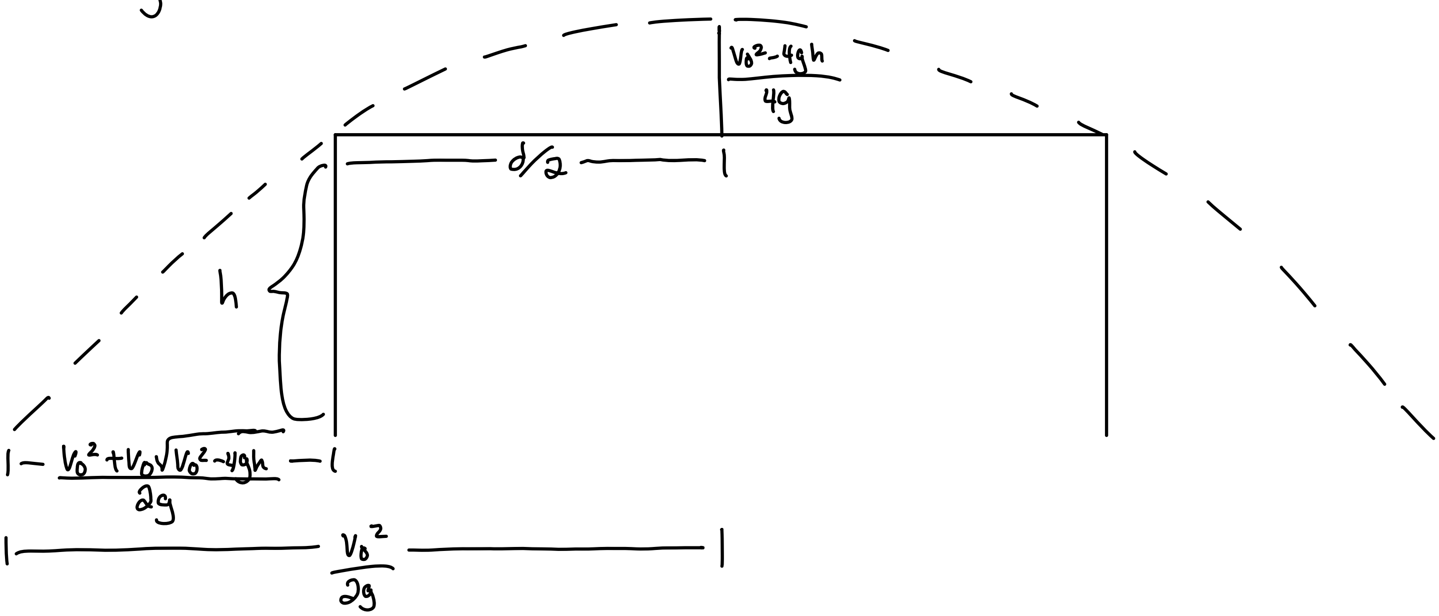
$$x(t) = V_0\cos(\theta)\left(\frac{V_0\sin(\theta) + \sqrt{V_0^2\sin^2(\theta) - 2gh}}{g}\right) = \frac{V_0^2\cos\theta\sin\theta + V_0\cos(\theta)\sqrt{V_0^2\sin^2(\theta) - 2gh}}{g}$$

$$x(t, \theta=\pi/4) = \frac{V_0^2/2 + \frac{\sqrt{2}}{2}V_0\sqrt{V_0^2/2 - 2gh}}{g} = \frac{V_0^2 + V_0\sqrt{V_0^2-4gh}}{2g} = \frac{2V_0^2 + 2V_0\sqrt{V_0^2-4gh}}{4g}$$

$$\tfrac{1}{2}\text{ Range} = \frac{V_0^2}{2g} \;:\; \frac{V_0^2}{2g} - \frac{2V_0^2 + 2V_0\sqrt{V_0^2-4gh}}{4g} = \frac{2V_0^2 - 2V_0^2 + 2V_0\sqrt{V_0^2-4gh}}{4g}$$

$$\frac{V_0\sqrt{V_0^2-4gh}}{2g}$$

So, our diagram now looks like



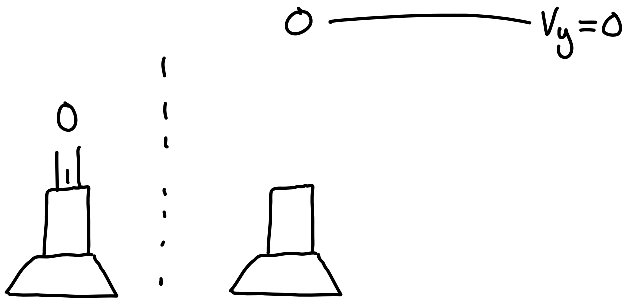
$$\frac{d}{2} = \frac{V_0^2}{2g} - \frac{2V_0^2 + 2V_0\sqrt{V_0^2-4gh}}{4g} = \frac{2V_0^2 - 2V_0^2 + 2V_0\sqrt{V_0^2-4gh}}{4g} = \frac{V_0\sqrt{V_0^2-4gh}}{2g}$$

$$\frac{d}{2} = \frac{V_0\sqrt{V_0^2-4gh}}{2g} \qquad d = \frac{d}{2}\cdot 2 \qquad \therefore\; \frac{V_0\sqrt{V_0^2-4gh}}{2g}\cdot 2 = \frac{V_0\sqrt{V_0^2-4gh}}{g} = d$$

$$\boxed{d = \frac{V_0}{g}\sqrt{V_0^2-4gh}} \quad \checkmark$$

Problem 4 | 2-9

a.) Zero resisting force



y-direction

$m\ddot{y} = -mg$

$\ddot{y} = -g \qquad \dot{y} \equiv \int \ddot{y}\,dy = \int -g\,dt$

$\dot{y} = -gt + V_{0y} \qquad y \equiv \int \dot{y}\,dy = \int -gt + V_{0y}\,dt$

$y = y_0 + V_{0y}t - \frac{gt^2}{2}$

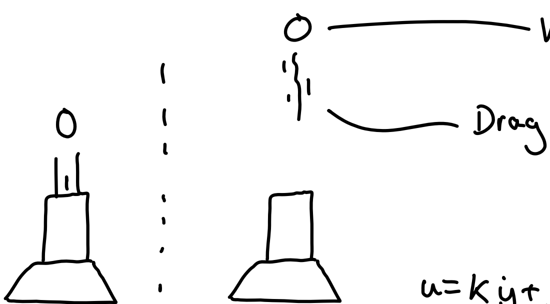
$\dot{y} = 0$: $0 = -gt + V_{0y}$

$gt = V_{0y}$

$t = \frac{V_{0y}}{g}$

$\boxed{\text{No resisting force : } \Delta t = \frac{V_{0y}}{g}}$

b.) Resisting force $\propto V_0$ @ $t=0$, $y=y_0$, $V=V_0$



y-direction

$m\ddot{y} = -mg - km\dot{y}$: $\ddot{y} + k\dot{y} + g = 0$

a b c

$m \cdot \frac{d\dot{y}}{dt} = -mg - km \cdot \dot{y}$

$\frac{d\dot{y}}{dt} = -g - k\dot{y}$

$\frac{d\dot{y}}{k\dot{y}+g} = -dt$: $\int \frac{1}{k\dot{y}+g}\,d\dot{y} = \int -dt$

$u = k\dot{y} + g$

$du = k \cdot d\dot{y}$

$d\dot{y} = \frac{du}{k}$

$\frac{1}{k}\int \frac{1}{u}\,du = \int -dt = \frac{1}{k}\ln(u) = -t + C$

$\frac{1}{k}\ln(k\dot{y}+g) = -t + C$ — $C \equiv y_0$

$\ln(k\dot{y}+g) = -kt + kC$

$k\dot{y} + g = kC \cdot e^{-kt}$

$k\dot{y} = kC \cdot e^{-kt} - g$

$\dot{y} = C \cdot e^{-kt} - g/k$

$\dot{y}(t=0) = V_0$

$V_0 = Ce^{-k(0)} - g/k$

$V_0 = C - g/k$

$C = V_0 + g/k$

$\dot{y} = (V_0 + g/k)e^{-kt} - g/k$:

$\dot{y} = 0$ @ max height

max height is when $\dot{y} = 0$ $\qquad \frac{kV_0 + g}{k} = V_0 + g/k$

$\therefore \quad 0 = (V_0 + g/k)e^{-kt} - g/k$

$g/k = (V_0 + g/k)e^{-kt}$

$\frac{g/k}{V_0 + g/k} = e^{-kt} = \frac{\frac{g}{k}}{\frac{kV_0 + g}{k}} = e^{-kt}$: $\frac{g}{kV_0 + g} = e^{-kt}$

$\ln\left(\frac{g}{kV_0+g}\right) = -kt$ — with resisting force

$\Delta t = \frac{\ln\left(\frac{g}{kV_0+g}\right)}{-k} \rightarrow \boxed{\Delta t = \frac{\ln\left(\frac{kV_0}{g} + 1\right)}{k}}$

Comparison on next page

Problem 4 | 2-9 Continued

$$\ln\left(\frac{Kv_0}{g}+1\right) \rightarrow \ln(\alpha+1) = f(\alpha) \qquad \alpha \equiv \left(\frac{Kv_0}{g}\right)$$

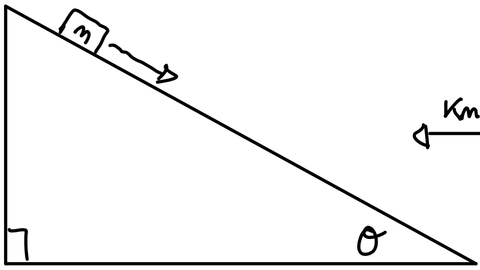
$$\text{Taylor}(f(\alpha)) = \alpha - \frac{\alpha^2}{2} + \frac{\alpha^3}{3} - \frac{\alpha^4}{4} + \frac{\alpha^5}{5} \Rightarrow \frac{KV_0}{g} - \frac{K^2V_0^2}{2g^2} + \frac{K^3V_0^3}{3g^3} - \frac{K^4V_0^4}{4g^4} + \frac{K^5V_0^5}{5g^5}$$

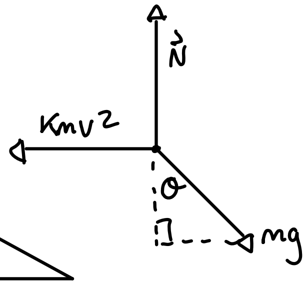
Now since $\ln\left(\frac{Kv_0}{g}+1\right)$ is over K, Taylor $(f(\alpha))$ becomes

$$\left[\frac{V_0}{g} - \frac{KV_0^2}{2g^2} + \frac{K^2V_0^3}{3g^3} - \frac{K^3V_0^4}{4g^4} + \frac{K^4V_0^5}{5g^5}\right]_{K=0} \rightarrow \text{No drag}$$

$\boxed{\Delta t = \frac{V_{0y}}{g}, \text{ which is the same as part a}}$

Problem 5 | 2-15





x-direction

$$m\ddot{x} = mg\sin\theta - Km\dot{x}^2$$

$$\ddot{x} = g\sin\theta - K\dot{x}^2$$

$$\ddot{x} = g\sin\theta - K\dot{x}^2$$

$$\frac{d\dot{x}}{dt} = g\sin\theta - K(\dot{x})^2$$

$$\frac{d\dot{x}}{dt} = K\dot{x}^2 - g\sin(\theta)$$

y-direction

$$0\ddot{y} = \vec{N} - mg\cos\theta$$

$$\ddot{x} = g\sin\theta - k\dot{x}^2$$

$$\frac{d\dot{x}}{dt} = K\dot{x}^2 - g\sin(\theta)$$

$$\int \frac{d\dot{x}}{K\dot{x}^2 - g\sin(\theta)} = \int -dt = -t + C$$

$$\int \frac{d\dot{x}}{K\dot{x}^2 - g\sin\theta} = \int \frac{dx}{a^2x^2 - b^2} = -\frac{1}{ab}\tanh^{-1}\left(\frac{ax}{b}\right), \quad a^2x^2 < b^2$$

$a = \sqrt{K}$
$b = \sqrt{g\sin(\theta)}$
$x = \dot{x}$

$$\int \frac{d\dot{x}}{K\dot{x}^2 - g\sin\theta} = -\frac{1}{\sqrt{K}\sqrt{g\sin\theta}}\tanh^{-1}\left(\frac{\sqrt{K}\dot{x}}{\sqrt{g\sin\theta}}\right)$$

$$-\frac{1}{\sqrt{Kg\sin\theta}}\tanh^{-1}\left(\frac{\sqrt{K}\dot{x}}{\sqrt{g\sin\theta}}\right) = -t + C \quad : \quad C = 0 \text{ Since } V(0) = 0$$

$$\frac{1}{\sqrt{Kg\sin\theta}}\tanh^{-1}\left(\frac{\sqrt{K}\dot{x}}{\sqrt{g\sin\theta}}\right) = t \quad : \quad \tanh^{-1}\left(\frac{\sqrt{K}\dot{x}}{\sqrt{g\sin\theta}}\right) = \sqrt{Kg\sin\theta}\cdot t$$

$$\frac{\sqrt{K}\dot{x}}{\sqrt{g\sin\theta}} = \tanh\left(\sqrt{Kg\sin\theta}\cdot t\right)$$

$$\dot{x} = \frac{\sqrt{g\sin\theta}}{\sqrt{K}}\tanh\left(\sqrt{Kg\sin\theta}\cdot t\right)$$

Looked up in table

$$\frac{dx}{dt} = \frac{\sqrt{g\sin\theta}}{\sqrt{K}}\tanh\left(\sqrt{Kg\sin\theta}\cdot t\right) \quad : \quad \int dx = \frac{\sqrt{g\sin\theta}}{\sqrt{K}}\int \tanh\left(\sqrt{Kg\sin\theta}\cdot t\right)dt$$

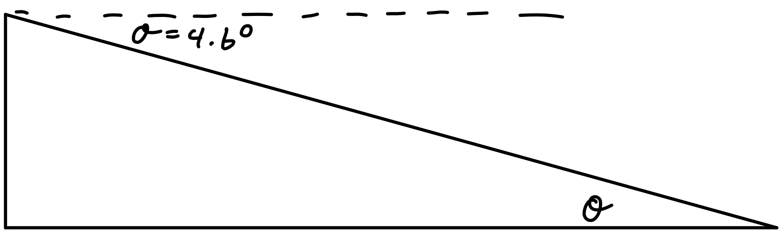
$$x = \frac{\sqrt{g\sin\theta}}{\sqrt{K}}\left(\frac{\ln\left(\cosh\left(\sqrt{Kg\sin\theta}\cdot t\right)\right)}{\sqrt{Kg\sin\theta}}\right) + C \quad : \quad C = 0 \text{ since } x(0) = 0$$

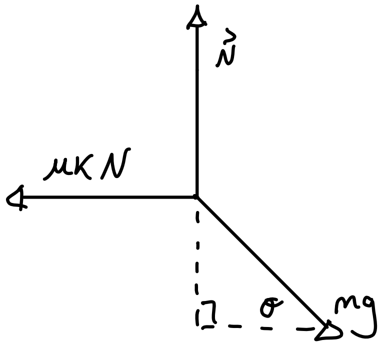
$$d = \frac{\sqrt{g\sin(\theta)}\cdot\ln\left(\cosh\left(\sqrt{Kg\sin\theta}\cdot t\right)\right)}{K\sqrt{g\sin\theta}} = \frac{\ln\left(\cosh\left(\sqrt{Kg\sin\theta}\cdot t\right)\right)}{K}$$

$$Kd = \ln\left(\cosh\left(\sqrt{Kg\sin\theta}\cdot t\right)\right) \quad : \quad e^{Kd} = \cosh\left(\sqrt{Kg\sin\theta}\cdot t\right)$$

$$\cosh^{-1}\left(e^{Kd}\right) = \sqrt{Kg\sin\theta}\cdot t \quad \therefore \quad \boxed{t = \frac{\cosh^{-1}\left(e^{Kd}\right)}{\sqrt{Kg\sin\theta}}} \checkmark$$

Problem 6 | 2-29





y-direction $m\ddot{y} = 0$

$0 = N - mg\sin(\theta)$
$N = mg\sin(\theta)$

x-direction

$x = 30\ m$

$m\ddot{x} = mg\cos(\theta) - \mu k \cdot mg\sin(\theta)$

$\ddot{x} = g\cos(\theta) - \mu k \cdot g\sin(\theta)$
$\ddot{x} = g(\cos(\theta) - \mu k\sin(\theta))$ $\qquad \ddot{x} = \alpha g$

$$\frac{d\dot{x}}{dt} = g(\cos(\theta) - \mu k\sin(\theta))$$

$$\int d\dot{x} = \int g(\cos(\theta) - \mu k\sin(\theta))\,dt \quad \vdots \quad \underline{\dot{x} = g\alpha t + \dot{x}_0}$$

$$\frac{dx}{dt} = g(\cos(\theta) - \mu k\sin(\theta)) \cdot t + \dot{x}_0 \qquad \alpha = (\cos(\theta) - \mu k\sin(\theta)) \qquad \mu k = 0.45$$

$$\int dx = \int \alpha g t + \dot{x}_0 \quad \vdots \quad \underline{x_1 = x_0 + \dot{x}_0 t + \frac{\alpha g t^2}{2}}$$

$\ddot{x} = \alpha g \quad \vdots \quad 2\dot{x}\ddot{x} = 2\dot{x}\alpha g$

$$\frac{d}{dt}(\dot{x}^2) = 2\alpha g \frac{dx}{dt}$$

$$\int_0^{v_0} d(\dot{x}^2) = 2\alpha g \int_0^x dx$$

$$\underline{V_0^2 = 2\alpha g x}$$

$V_0^2 = 2\alpha g \cdot x \qquad x = 30m \qquad \alpha = (\cos(4.57) - 0.45 \cdot \sin(4.57))$

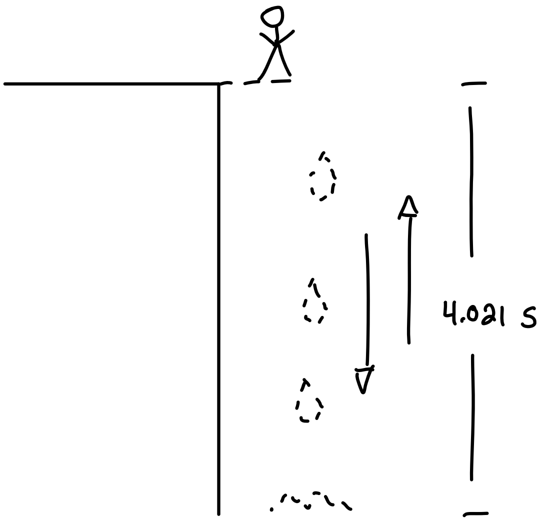
$$V_0 = \sqrt{2(\cos(4.57) - 0.45\sin(4.57)) \cdot 9.81\,m/s^2 \cdot 30m} = 9.28\ m/s$$

Speed limit: $25\,\frac{mi}{hr} \cdot \frac{1hr}{3600S} \cdot \frac{5280ft}{1mi} \cdot \frac{1M}{3.28ft} = 11.18\ m/s$

$9.28\ m/s < 11.18\ m/s \quad \therefore \quad V_0 <$ Speed limit.

The lawyer is not justified in his Claim.
The driver was not Speeding.

Problem 7| 2-30



$v_{Sound} = 331\ m/s$

$\dot{y}(0) = 0$, $\Delta t = 4.021\ s$

y-Direction

$$\int dy = \int -gt + \dot{y}_0\, dt$$

$$y = y_0 + \dot{y}_0 t - \frac{gt^2}{2}$$

$$\dot{y}_0 = 0 \quad y_0 = 0 \longrightarrow y = -\frac{gt^2}{2}, \quad t = \sqrt{\frac{2y}{-g}} \quad g = -9.81\ m/s^2$$

$$T = T_1 + T_2$$

$T_1 \equiv$ Time for balloon to drop

$T_2 \equiv$ Time for sound to drawel back up

$$T_1 = \sqrt{\frac{2y_0}{g}}, \quad T_2 = \frac{y_0}{v_s}$$

$$T = T_1 + T_2 \quad \therefore \quad T_1 + T_2 - T = 0$$

$$a = 1/v_s, \quad b = \sqrt{\tfrac{2}{g}}, \quad c = 4.021s, \quad \frac{1}{v_s} \cdot y_0 + \sqrt{\frac{2}{g}}\sqrt{y_0} - T = 0$$

$$\sqrt{y_0} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\sqrt{2/g} \pm \sqrt{\frac{2}{g} - 4\left(\frac{1}{v_s}\right)(-4.021\ s)}}{2 \cdot 1/v_s}$$

$$\sqrt{y_0} = \frac{-\sqrt{\frac{2}{9.81\ m/s^2}} \pm \sqrt{\frac{2}{9.81\ m/s^2} - 4\left(\frac{1}{331\ m/s}\right)(-4.021\ s)}}{2 \cdot \frac{1}{331\ m/s}} = 8.43\ m$$

$$\sqrt{y_0} = 8.43\ m \quad \therefore \quad y_0 = 71.06\ m$$

$$\boxed{y_0 = 71.0\ m}$$