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PHYS 230 HW2

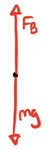
P1. Imagine a ship made of steel that is roughly shaped like a box with length 75.0 m, width 30.0 m, and height 30.0 m. The ship has a total mass of 3.75×10^6 kg. What is the *maximum* cargo load that the ship can carry without sinking?

$F_B = \rho_f V_f g$

$\rho_f = 1000\ kg/m^3$
$V_f = 67{,}500\ m^3$
$g = 9.8\ m/s^2$

$V = (75.0m)(30.0m)(30.0m)$
$V = 67{,}500\ m^3$

$F_B = (1000\ kg/m^3)(67{,}500\ m^3)(9.8\ m/s^2)$
$F_B = 6.615 \times 10^8\ N$



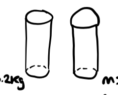
$6.615 \times 10^8\ N = mg$

$m = 6.75 \times 10^7\ kg$

$6.75 \times 10^7\ kg - 3.75 \times 10^6\ kg = 6.375 \times 10^7\ kg$

Load of 6.38×10^7 Kg

P2. Imagine having two identical cylindrical glasses, each containing water. The first glass has 0.200 kg of water and the second glass has 0.185 kg of water with a 25 g ice cube floating in it. How do the heights of the water in the two glasses compare? Your answer should be written as a ratio of h_2/h_1.



$m = 0.2kg$

$m = 0.185kg$
$m_i = 0.025kg$

$V_1 = \pi r^2 h_1$
$V_2 = \pi r^2 h_2$

$r^2 = \frac{V_1}{\pi h_1} = \sqrt{\frac{V_1}{\pi h_1}} = r$

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$\sqrt{\frac{V_1}{\pi h_1}} = \sqrt{\frac{V_2}{\pi h_2}}$

$\frac{V_1}{h_1} = \frac{V_2}{h_2}$

$\frac{h_2}{h_1} = \frac{V_2}{V_1}$

V_1: $V = \frac{m}{\rho}$ $V = 2.0 \times 10^{-4} m^3$
$m = 0.2kg$
$\rho = 1000\ kg/m^3$

$F_B = \rho_f V_f g$
$F_G = mg$

$\rho_f V_f = m$

$m = 0.025\ kg$
$\rho_f = 917\ kg/m^3$

$V_f = \frac{m}{\rho_f} = \frac{0.025\ kg}{917\ kg/m^3} = 2.73 \times 10^{-5}\ kg$

$V_{ice} = 2.73 \times 10^{-5}\ kg$: underwater $V_{ice} = 2.5 \times 10^{-5}\ kg$

$V_{H_2O} = \frac{0.185\ kg}{1000\ kg/m^3} = 1.85 \times 10^{-4}\ kg$

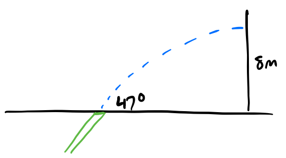
$V_T = V_i + V_{H_2O} = 2.1 \times 10^{-4} m^3$

$\frac{h_2}{h_1} = \frac{2.1 \times 10^{-4} m^3}{2.0 \times 10^{-4} m^3}$

$\frac{h_2}{h_1} = 1.05$

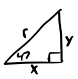
$h_2 : h_1 = 1.05$

P3. A garden hose sprays water from ground level at an angle of 47° above the horizontal towards a building. The initial speed of the water is such that it strikes a building horizontally at a height of 8.0 m above the ground. The diameter of the garden hose is 2.5 cm. Calculate the flow rate of the water leaving the garden hose.



$V_1 = 0$
$a = 9.8 m/s^2$
$\Delta x = 8m$

$V_1^2 = V_0^2 + 2a\Delta x$
$-V_0^2 = 2a(-\Delta x)$
$V_0^2 = 2a(\Delta x)$
$V_0 = \sqrt{2(9.8 m/s^2)(8m)}$
$V_0 = 12.52 m/s$



$r = \frac{y}{\sin 47} = \frac{12.52 m/s}{\sin 47} = 17.11 m/s$

$Q = V_0 A$
$V_0 = 17.12\ m/s$
$A = \pi(1.25 \times 10^{-2} m)^2$
$Q = 17.12 m/s(\pi(1.25 \times 10^{-2} m)^2)$
$Q = 8.4 \times 10^{-3}\ m^3/s$

$8.4 \times 10^{-3}\ m^3/s$

$3.3 \times 10^{-2}\ m^3/s$

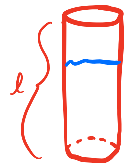
P4. A long cylindrical tube has a length of 3.0 m and a radius of 0.25 m and is filled with water. The cylindrical tube is oriented with its long axis vertical and it is open to the atmosphere at the upper end. A hole with a radius of 0.11 m is drilled in the bottom of the cylindrical tube, which allows the water to drain out vertically downward.

When the water level is 2/3 the height of the length of the tube,

a) calculate the speed of the water exiting the hole,

b) calculate how fast the top of the water column is falling, and

c) calculate the flow rate of the water exiting the hole.



$r_c = 0.25m$

$r_h = 0.11m$

$P_1 + \frac{1}{2}\rho v_0^2 + \rho g y_0 = P_2 + \frac{1}{2}\rho v_1^2 + \rho g y_1$

$\frac{1}{2}v_0^2 + g(\frac{2}{3}\ell) = \frac{1}{2}v_1^2$

$v_0 A_0 = v_1 A_1$

$v_0 = \frac{v_1 A_1}{A_0} \quad v_1 = \frac{v_0 A_0}{A_1}$

a.)

$\frac{1}{2}v_1^2\left(\frac{A_1}{A_0}\right)^2 + g\frac{2}{3}\ell = \frac{1}{2}v_1^2$

$\frac{1}{2}v_1^2\left(\frac{A_1}{A_0}\right)^2 - \frac{1}{2}v_1^2 = -g\frac{2}{3}\ell$

$\frac{1}{2}v_1^2\left(\left(\frac{A_1}{A_0}\right)^2 - 1\right) = -g\frac{2}{3}\ell$

$\frac{1}{2}v_1^2 = \frac{-g\frac{2}{3}\ell}{\left(\frac{A_1}{A_0}\right)^2 - 1}$

$v_1^2 = \frac{-2g\frac{2}{3}\ell}{\left(\frac{A_1}{A_0}\right)^2 - 1}$

$v_1 = \sqrt{\frac{-4g\ell}{3\left(\frac{A_1}{A_0}\right)^2 - 1}} \quad v_1 = 6.4 m/s$

$= \sqrt{\frac{-4(9.8 m/s^2)(3m)}{3\left(\frac{0.11m^2}{0.25m^2}\right)^2 - 1}} = 6.4 m/s$

$v_1 = 6.4 m/s$

$v_2 = 3.2 m/s$

b.)

$v_0 = \frac{v_1 A_1}{A_0}$

$= \frac{6.4 m/s(0.11m)^2}{(0.25m)^2} = 1.24 m/s$

$\frac{1}{2}v_0^2 + g(\frac{2}{3})\ell = \frac{1}{2}v_0^2\left(\frac{A_0}{A_1}\right)^2$

$\frac{2}{3}g\ell = \frac{1}{2}v_0^2\left(\frac{A_0}{A_1}\right)^2 - \frac{1}{2}v_0^2$

$\frac{2}{3}g\ell = \frac{1}{2}v_0^2\left(\left(\frac{A_0}{A_1}\right)^2 - 1\right)$

$\frac{\frac{4}{3}g\ell}{\left(\frac{A_0}{A_1}\right)^2 - 1} = v_0^2$

$v_0 = \sqrt{\frac{4g\ell}{3\left(\frac{A_0}{A_1}\right)^2 - 1}} = 1.24 m/s$

$= \sqrt{\frac{4(9.8 m/s^2)(3m)}{3\left(\frac{0.25m}{0.11m}\right) - 1}} = 1.24 m/s$

$v_0 = 1.2 m/s$

$v_1 = 0.62 m/s$

c.) $Q = vA$

$A = \pi(0.11m)^2$

$V = 6.4 m/s$

$Q = 6.4 m/s\,(\pi(0.11m)^2)$

$Q = 0.24 m^3/s$

$Q = 0.24 m^3/s$

$Q = 0.12 m^3/s$

Now consider an object freely falling in the earth's uniform gravitational field. Allow the object to fall from a height equal to the height of the water level considered above and allow this falling object to have an initial downward velocity equal to the velocity of the top of the falling water column, which was calculated in part b).

d) Calculate the speed of the freely falling object when it reaches the bottom of the water column. How does this speed compare to the speed of the water exiting the hole? Are your answers reasonable? Explain

$v_1^2 = v_0^2 + 2g\Delta y$

d.) $v_1 = 6.4 m/s$

The speed of the ball falling equals that of the water exiting the hole. This makes sense since the velocity of a liquid increases through a smaller constrained cross sectional area (the hole)

$v_0 = 1.2 m/s$

$\Delta y = 2m$

$v_1 = \sqrt{v_0^2 + 2g\Delta y}$

$v_1 = \sqrt{(1.2 m/s)^2 + 2(9.8 m/s^2)(2m)}$

$v_1 = 6.4 m/s$