$\iint_S \vec{F}\cdot\vec{n}$

$\vec{n}$ ← unit vectorfield $\perp$ to S at each point

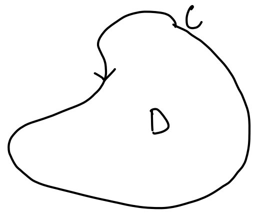
$\iint_S \vec{F}\cdot d\vec{S}$

Stoke's Theorem

$\int_C \vec{F}\cdot dr = \iint_S \text{curl}\vec{F}\cdot d\vec{S}$

$\iint_S \text{curl}\vec{F}\cdot(r_u \times r_v)\, dA$

Greens Theorem

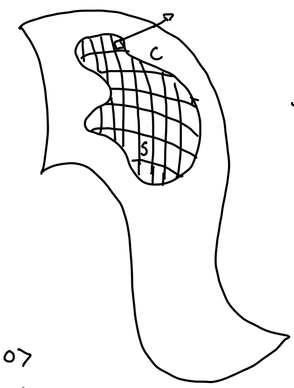


$\iint_D Q_x - P_y \, dA \Rightarrow$ Greens Theorem

$\int_C \vec{F}\cdot dr \Rightarrow$ Line Integral

$\int_C \vec{F}\cdot\vec{T}\, ds$

Stokes Theorem



$\vec{F}(x,y,z) = \langle P, Q, R\rangle$

$\int_C \vec{F}\cdot\vec{T}\, ds$

$\iint_S \text{Curl}\vec{F}\cdot\vec{n}\, ds$

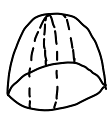
16.8.1 $F = \langle 6y\cos z, e^x \sin z, xe^y\rangle$ oriented out

S $x^2+y^2+z^2=49$ w.r.t $\vec{0}$ $\vec{r}'(t) = \langle -7\sin t, 7\cos t, 0\rangle$

$z \geq 0$, C is $x^2+y^2=49$ $\vec{r}(t) = \langle 7\cos t, 7\sin t, 0\rangle$

$d\vec{S} = \vec{n}\,ds$ $0 \leq t \leq 2\pi$

$\vec{F}(\vec{r}(t)) = \langle 42\sin t, 0, 7\cos t\, e^{7\sin t}\rangle$



$\int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle 42\sin t, 0, 7\cos t\, e^{7\sin t}\rangle \cdot \langle -7\sin t, 7\cos t, 0\rangle$

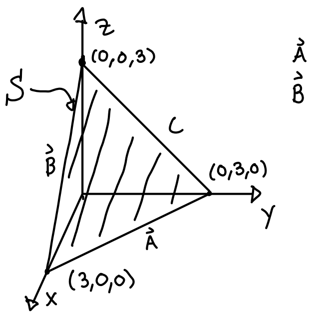
$\int_0^{2\pi} -294\sin^2 t \, dt \Rightarrow -294\int_0^{2\pi} \sin^2 t\, dt$

$\vec{N} = \dfrac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$

$dS = |\vec{r}_u \times \vec{r}_v|\, dA$

$\iint_D \text{curl}\vec{F}\cdot(\vec{r}_u \times \vec{r}_v)\, dA$

16.8.2 $\vec{F} = \langle x+y^2, y+z^2, z+x^2\rangle$



$\vec{A}\ \langle -3,3,0\rangle$

$\vec{B}\ \langle -3,0,3\rangle$ $\langle 3(3)-0(0), 0(-3)+3(3), -3(0)-3(-3)\rangle$

$\langle 6, 6, 6\rangle$

$\vec{N} = \langle 1,1,1\rangle$

Curl $\vec{F}$ $\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y^2 & y+z^2 & z+x^2 \end{vmatrix}$

$1(x-0)+1(y-3)+1(z-0)=0$

$x+y-3+z=0$

$z = 3-x-y$

$\langle \frac{\partial}{\partial y}(z+x^2) - \frac{\partial}{\partial z}(y+z^2), \frac{\partial}{\partial z}(x+y^2) - \frac{\partial}{\partial x}(z+x^2), \frac{\partial}{\partial x}(y+z^2) - \frac{\partial}{\partial y}(x+y^2)\rangle$

$\langle 0-2z, 0-2x, 0-2y\rangle$

$\langle -2z, -2x, -2y\rangle$

$\langle -2z,-2x,-2y\rangle\cdot\langle 1,1,1\rangle$

$-2z-2x-2y$

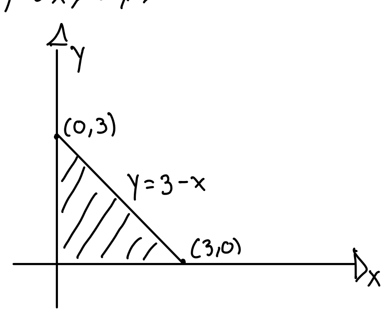
S : $\vec{r}(x,y) = \langle x, y, 3-x-y\rangle$

$\vec{r}_x = \langle 1, 0, -1\rangle$

$\vec{r}_y = \langle 0, 1, -1\rangle$

$\langle 0(-1)+1(1), -1(0)-1(-1), 1(1)-0(0)\rangle$

$\langle 1,1,1\rangle$



$z = 3-x-y$

$\int_0^3\int_0^{3-x} -2z-2x-2y \; dy\,dx$

$\int_0^3\int_0^{3-x} -6+2x+2y-2x-2y \; dy\,dx$

$\int_0^3\int_0^{3-x} -6 \; dy\,dx$

$\int_0^3 (-6(3-x))\, dx$

-27

$\int_0^3$

$\int_0 -18+6x \; dx$

$-18x+3x^2 \Big|_0^3$

$-54+27$

-27

16.8.4 $\quad F(x,y,z) = x^2y\hat{i} + \frac{1}{3}x^3\hat{j} + xy\hat{k}$

$z = y^2 - x^2 \qquad x^2+y^2=1 \qquad \iint_S \text{curl}\vec{F}\cdot d\vec{s}$

$\text{curl}\vec{F}: \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & \frac{1}{3}x^3 & xy \end{vmatrix} \qquad \vec{r}(x,y) = \langle x, y, y^2-x^2\rangle \qquad r_x = \langle 1, 0, -2x\rangle$

$r_y = \langle 0, 1, 2y\rangle$

$\langle \frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(\frac{1}{3}x^3), \frac{\partial}{\partial z}(x^2y) - \frac{\partial}{\partial x}(xy), \frac{\partial}{\partial x}(\frac{1}{3}x^3) - \frac{\partial}{\partial y}(x^2y)\rangle \qquad \langle 0(2y)+2x(1), -2x(0)-1(2y), 1(1)-0(0)\rangle$

$\langle x, -y, 0\rangle \qquad \langle 2x, -2y, 1\rangle$

$\iint_S \langle x,-y,0\rangle\cdot\langle 2x,-2y,1\rangle \; dA$

$\iint_S 2x^2+2y^2+0 \; dA$

$2\iint_S x^2+y^2 \; dA$

π

$2\int_0^{2\pi}\int_0^1 r^2 \; r \, dr d\theta$

$2\int_0^{2\pi}\int_0^1 r^3 \, dr d\theta$

$2\int_0^{2\pi} \frac{1}{4}r^4 \Big|_0^1 d\theta$

$2\int_0^{2\pi} \frac{1}{4} \, d\theta$

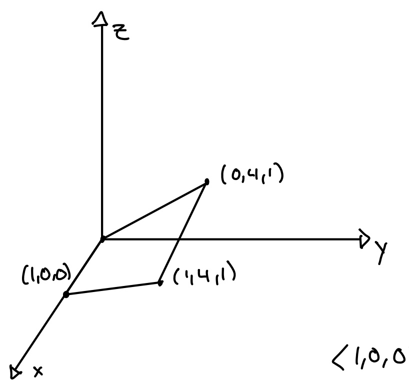
$2\left[\frac{1}{4}\theta\right]_0^{2\pi}$

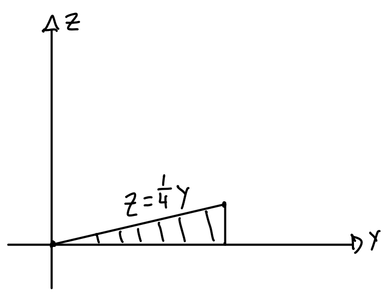
$2\left[\frac{\pi}{2}\right]$

π

16.8.7 $\quad F(x,y,z) = \langle z^2, 3xy, 5y^2\rangle$

$(1,0,0), (1,4,1), (0,4,1)$





23

$\langle 1,0,0\rangle \qquad \langle 0(1)-0(4), 0(0)-1(1), 1(4)-0(0)\rangle$

$\langle 0,4,1\rangle \qquad \langle 0, -1, 4\rangle$

$\text{curl}\vec{F}$

$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & 3xy & 5y^2 \end{vmatrix}$

$0(x-1) - 1(y-0) + 4(z-0) = 0$

$-y+4z=0$

$4z = y$

$z = \frac{1}{4}y \qquad \vec{r}(x,y) = \langle x, y, \frac{1}{4}y\rangle$

$r_x = \langle 1,0,0\rangle$

$r_y = \langle 0,1,\frac{1}{4}\rangle$

$\langle 0(\frac{1}{4})-0(1), 0(0)-1(\frac{1}{4}), 1(1)-0(0)\rangle$

$\langle 0, -\frac{1}{4}, 1\rangle$

$\langle \frac{\partial}{\partial y}(5y^2) - \frac{\partial}{\partial z}(3xy), \frac{\partial}{\partial z}(z^2) - \frac{\partial}{\partial x}(5y^2), \frac{\partial}{\partial x}(3xy) - \frac{\partial}{\partial y}(z^2)\rangle$

$\langle 10y - 0, 2z - 0, 3y - 0\rangle$

$\langle 10y, 2z, 3y\rangle$

$\langle 10y, \frac{1}{2}y, 3y\rangle\cdot\langle 0, -\frac{1}{4}, 1\rangle$

$-\frac{1}{8}y + 3y$

$\frac{23y}{8} \qquad \frac{23}{8}\int_0^4\int_0^1 y \; dz dx$

$\frac{23}{8}\int_0^4 y \; dy$

$$\frac{23}{8}\left[\frac{1}{2}y^2\right]_0^4$$

$$\frac{28}{8}(8)$$

$$2$$