

$$(x, y, z) = (-8, 8, 8) \quad \text{to} \quad (r, \theta, z)$$

$$r^2 = x^2 + y^2$$

$$r^2 = -8^2 + 8^2$$

$$r^2 = 128$$

$$r = \sqrt{128}$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{8}{-8}\right)$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = \frac{7\pi}{4}$$

$$\left(\frac{8\sqrt{2}}{2}, \frac{7\pi}{4}, 8\right)$$

15.8.1)  $\iiint_E \sqrt{x^2 + y^2} \, dv$

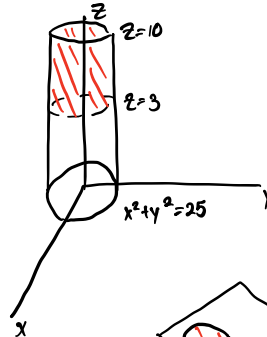
$$\int_3^{10} \int_0^{2\pi} \int_0^5 r^2 \, r \, d\theta \, dz$$

$$\int_3^{10} \int_0^{2\pi} \int_0^5 r^3 \, d\theta \, dz$$

$$x^2 + y^2 = 25$$

$$z = 3 \rightarrow z = 10$$

$$\begin{aligned} 0 &\leq r \leq 5 \\ 0 &\leq \theta \leq 2\pi \\ 3 &\leq z \leq 10 \end{aligned}$$



15.8.2)  $\iiint_E x \, dv$

$$z = 0, \quad z = x + y + 9$$

$$x^2 + y^2 = 9, \quad x^2 + y^2 = 16$$

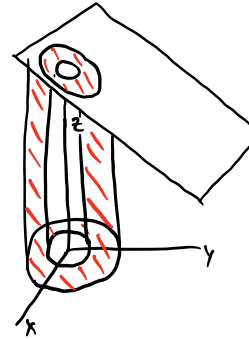
$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\begin{aligned} 3 &\leq r \leq 4 \\ 0 &\leq \theta \leq 2\pi \end{aligned}$$

$$\int_0^{2\pi} \int_3^4 \int_0^{x+y+9} x \, r \, dz \, dr \, d\theta$$

$$\int_0^{2\pi} \int_3^4 \int_0^{r \cos \theta + r \sin \theta + 9} r^2 \cos \theta \, dz \, dr \, d\theta$$



15.8.3)  $\iiint_E x^2 \, dv$

$$\int_0^{2\pi} \int_0^2 \int_0^{3r} r^3 \cos^2 \theta \, dz \, dr \, d\theta$$

$$\text{Inside } x^2 + y^2 = 4$$

$$\text{Above } z = 0$$

$$z^2 = 9x^2 + 9y^2$$

$$z = \sqrt{9x^2 + 9y^2}$$

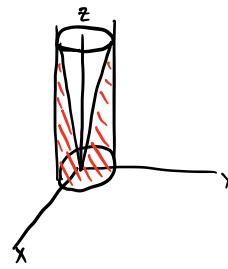
$$z = \sqrt{9(x^2 + y^2)}$$

$$z = \sqrt{r^2}$$

$$z = 3r$$

$$\int_0^{2\pi} \int_0^2 \int_0^{3r} r^2 \cos^2 \theta \, r \, dz \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^2 \int_0^{3r} r^3 \cos^2 \theta \, dz \, dr \, d\theta$$



$$I = \int_0^{2\pi} \int_3^4 \int_0^{r \cos \theta + r \sin \theta + 9} r^2 \cos \theta \, dz \, dr \, d\theta$$

$$\int_0^{2\pi} \int_3^4 r^2 \cos \theta \, z \Big|_0^{r \cos \theta + r \sin \theta + 9} \, dr \, d\theta$$

$$\int_0^{2\pi} r^2 \cos \theta \int_3^4 r \cos \theta + r \sin \theta + 9 \, dr \, d\theta$$

$$\int_0^{2\pi} \int_3^4 r^3 \cos^2 \theta + r^3 \cos \theta \sin \theta + 9r^2 \cos \theta \, dr \, d\theta$$

$$\int_0^{2\pi} \left[ \frac{r^4}{4} \cos^2 \theta + \frac{r^4}{4} \cos \theta \sin \theta + 3r^3 \cos \theta \right]_3^4 \, d\theta$$

$$\frac{1}{2} + \frac{1}{2} \cos 2\theta$$

$$\int_0^{2\pi} (\cos^2 \theta + \cos \theta \sin \theta + \cos \theta) d\theta$$

$$u = 2\theta$$

$$du = 2 d\theta$$

$$\frac{1}{2} du = d\theta$$

$$\frac{175}{4} \int_0^{2\pi} \cos^2 \theta d\theta + \frac{175}{4} \int_0^{2\pi} \cos \theta \sin \theta + 111 \int_0^{2\pi} \cos \theta d\theta$$

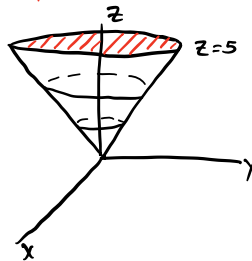
$$\frac{175}{4} \int_0^{2\pi} \frac{1}{2} + \frac{1}{2} \cos 2\theta d\theta + \frac{175}{4} \int_0^{2\pi} \cos \theta \sin \theta + 111 \int_0^{2\pi} \cos \theta d\theta$$

15.8.4)  $\int_{-3}^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^5 xz dz dy dx$

$$0 \leq \theta = 2\pi$$

$$0 \leq r \leq 3$$

$$r \leq z \leq 5$$



$$z = \sqrt{x^2 + y^2}$$

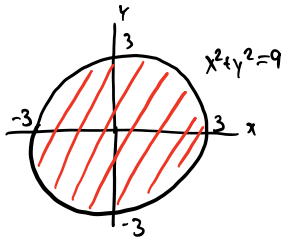
$$z = 5$$

$$x = \sqrt{9 - y^2}$$

$$x = -\sqrt{9 - y^2}$$

$$y = 3$$

$$y = -3$$



$$\int_0^{2\pi} \int_0^3 \int_r^5 xz r dz dr d\theta$$

$$\int_0^{2\pi} \int_0^3 \int_r^5 r^2 \cos \theta z dz dr d\theta$$

$$x = r \cos \theta$$

15.8.5)  $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} \sqrt{x^2+y^2} dz dy dx$

$$\int_0^{2\pi} \int_0^2 \int_0^{4-r^2} r r dz dr d\theta$$

$$\int_0^{2\pi} \int_0^2 \int_0^{4-r^2} r^2 dz dr d\theta$$

$$x^2 + y^2 = r^2$$

