3.3 HW The Inverse of The Matrix

3.3.1

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$$

$$AB = I : \begin{bmatrix} 5(-1)+3(2) & , & 5(3)+3(-5) \\ 2(-1)+1(2) & , & 2(3)+1(-5) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

B is an inverse of A

3.3.3

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ -1/4 & 1/4 & 3/4 \\ 1/4 & -1/4 & 1/4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1(3/4)+0(-1/4)+1(1/4) & , & 1(1/4)+0(1/4)+1(-1/4) & , & 1(-1/4)+0(3/4)+1(1/4) \\ 1(3/4)+1(-1/4)-2(1/4) & , & 1(1/4)+1(1/4)-2(-1/4) & , & 1(-1/4)+1(3/4)-2(1/4) \\ 0(3/4)+1(1/4)+1(-1/4) & , & 0(1/4)+1(1/4)+1(-1/4) & , & 0(-1/4)+1(3/4)+1(1/4) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad \therefore \quad \text{A \& B are inverses}$$

3.3.5

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A$$

$$A^{-1} = rref \begin{bmatrix} 2 & 0 & | & 1 & 0 \\ 1 & 1 & | & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & | & 1/2 & 0 \\ 0 & 1 & | & -1/2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}$$

3.3.13

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 1 & -1 & 3 & 3 \end{bmatrix}$$

$$A^{-1} = rref \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & | & 0 & 0 & 1 & 0 \\ 1 & -1 & 3 & 3 & | & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & | & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & | & 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & | & -1/3 & 1/6 & 1/2 & 1/3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1/2 & -1/2 & 0 \\ -1/3 & 1/6 & 1/2 & 1/3 \end{bmatrix}$$