4.6 Forced Oscillations

4.6 # 11, 14, 24

4.6.11) $100\ddot{x} + 500\dot{x} + 4900x = 100\cos w_f t$

LRC Circuit equivalent

$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = f(t) \Rightarrow 100\ddot{Q} + 500\dot{Q} + 4900Q = 100\cos(w_f t)$

$L = 100$

$R = 500$

$\frac{1}{C} = 4900$

$\boxed{100\ddot{Q} + 500\dot{Q} + 4900Q = 100\cos(w_f t)}$

4.6.14) $L = 4$ Henries $Q(0) = 0$, $\dot{Q}(0) = 0$

$C = 0.01$ Farads $\Rightarrow L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = f(t)$

$v(t) = 10\cos(4t)$ $4\ddot{Q} + 100Q = 10\cos(4t)$

$t = 0,\ I = 0$

$Y = Y_h + Y_p$

$Y_h:\ 4r^2 + 100 = 0$

$4(r^2 + 25) = 0$

$r = \pm 5i$

$4(4)(100)$

1600

$Y_h:\ e^{\alpha t}(c_1\cos\beta t + c_2\sin\beta t)$ $\alpha = 0, \beta = 5$

$Y_h = c_1\cos 5t + c_2\sin 5t$

$\alpha = \frac{-b}{2a}$ $\alpha = 0$

$\beta = \frac{\sqrt{4ac - b^2}}{2a}$ $\beta = \frac{40}{2(4)} = 5$

$Y = Y_h + Y_6$

$4\ddot{Q} + 100Q = 10\cos(4t)$

$Y_p:\ Y_p = A\cos(4t) + B\sin(4t) = 10\cos(4t)$

$Y_p' = -4A\sin(4t) + 4B\cos(4t)$

$Y_p'' = -16A\cos(4t) - 16B\sin(4t)$ $Y_p = \frac{5}{18}\cos(4t)$

$4(-16A\cos(4t) - 16B\sin(4t)) + 100(A\cos(4t) + B\sin(4t)) = 10\cos(4t)$

$-64A\cos(4t) - 64B\sin(4t) + 100A\cos(4t) + 100B\sin(4t) = 10\cos(4t)$

$36A\cos(4t) + 36B\sin(4t) = 10\cos(4t)$

$36A = 10$ $A = \frac{5}{18}$

$36B = 0$ $B = 0$

$y = c_1\cos 5t + c_2\sin 5t + \frac{5}{18}\cos(4t)$ $c_1 = -\frac{5}{18}$

$0 = c_1\cos(0) + c_2\sin(0) + \frac{5}{18}\cos(0)$ $c_2 = 0$

$0 = c_1 + \frac{5}{18}$

$-\frac{5}{18} = c_1$

$y' = -5c_1\sin(5t) + 5c_2\cos(5t) - \frac{20}{18}\sin(4t)$

$0 = -5c_1\sin(0) + 5c_2\cos(0) - \frac{20}{18}\sin(0)$

$0 = 5c_2$

$0 = c_2$

$\boxed{Q = -\frac{5}{18}\cos(5t) + \frac{5}{18}\cos(4t)}$

4.6.24) $M = 1$ slug $m\ddot{x} + b\dot{x} + Kx = f(t)$

$K = 12$ lb/ft $\ddot{x} + 12x = 16\cos(2\sqrt{3}t)$

$f(t) = 16\cos wt$ $y(0) = 0$

$\dot{y}(0) = 0$

$Y = Y_h + Y_p$

$Y_h:\ \Delta = b^2 - 4ac$

$= 0^2 - 4(1)(12)$

$= -48$

$Y_h = e^{\alpha t}(c_1\cos\beta t + c_2\sin\beta t)$

$\alpha = \frac{-b}{2a}$ $\beta = \frac{\sqrt{4ac - b^2}}{2(a)}$ $\beta = \frac{\sqrt{4(1)(12)}}{2(1)}$ $W = \sqrt{\frac{K}{m} - \frac{b^2}{2m^2}}$

$\alpha = 0$ $\beta = 2\sqrt{3}$ $W = 2\sqrt{3}$

$Y_h = c_1\cos(2\sqrt{3}t) + c_2\sin(2\sqrt{3}t)$

$y = c_1\cos(2\sqrt{3}t) + c_2\sin(2\sqrt{3}t) + \frac{4}{\sqrt{3}}t\sin(2\sqrt{3}t)$

$y_p: \; At\cos(2\sqrt{3}t) + Bt\sin(2\sqrt{3}t) = 16\cos(2\sqrt{3}t)$

$y_p' = A(\cos(2\sqrt{3}t) - 2\sqrt{3}t\sin(2\sqrt{3}t)) + B(2\sqrt{3}t\cos(2\sqrt{3}t) + \sin(2\sqrt{3}t)) = 16\cos(2\sqrt{3}t)$

$y_p'' = A(-2\sqrt{3}\sin(2\sqrt{3}t) - 12t\cos(2\sqrt{3}t) - 2\sqrt{3}\sin(2\sqrt{3}t)) + B(2\sqrt{3}\cos(2\sqrt{3}t) - 12t\sin(2\sqrt{3}t) + 2\sqrt{3}\cos(2\sqrt{3}t)) = 16\cos(2\sqrt{3}t)$

$B(4\sqrt{3})\cos(2\sqrt{3}t) = 16\cos(2\sqrt{3}t)$

$B = \frac{4}{\sqrt{3}}$

$y_p = \frac{4}{\sqrt{3}}t\sin(2\sqrt{3}t)$

$y = C_1\cos(2\sqrt{3}t) + C_2\sin(2\sqrt{3}t) + \frac{4}{\sqrt{3}}t\sin(2\sqrt{3}t)$

$c_1 = 0$

$c_2 = 0$

$\boxed{y = \frac{4}{\sqrt{3}}t\sin(2\sqrt{3}t)}$

$0 = c_1\cos(0) + c_2\sin(0) + \frac{4}{3}(0)\sin(0)$

$0 = c_1$

$y' = 2\sqrt{3}c_2\cos(2\sqrt{3}t) + \frac{4}{\sqrt{3}}\sin(2\sqrt{3}t) + 8t\cos(2\sqrt{3}t)$

$0 = 2\sqrt{3}c_2\cos(0) + \frac{4}{3}\sin(0) + 8(0)\cos(0)$

$0 = c_2$