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CQ1. You get the flu while traveling in Europe. You measure your temperature and find that it is 38.7 °C. What is your corresponding temperature in K and in °F? (Three significant figures, right?)

$T_F = \frac{9}{5} T_C + 32$

$T_K = T_C + 273.15$

$T_C = 5/9(T_F - 32)$

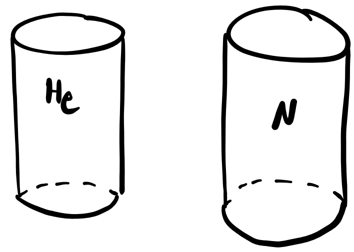
$T_K = 38.7 + 273.15 = 311.85\ K$

$T_F = 9/5(38.7) + 32 = 101.66\ °F$

$T_K = 312\ K$

$T_F = 102°\ F$

CQ2. Imagine that you have two identical tanks, one contains nitrogen gas and the other contains helium gas. Both gases have the same pressure and temperature. How do the *number densities* and *mass densities* compare?



$PV = nRT$

$\frac{PV}{T} = nR \qquad n_{He} R = n_N R$

$n = N/N_A, \quad N = \frac{M}{m}, \quad n = \frac{M}{M_{mol}}$

$N_A = 6.02 \times 10^{23}\ mol^{-1}$

$m_{He} = 4u$

$m_N = 14u$

$u = 1.66 \times 10^{-27}\ kg$

$H_2 : N_2$

$H_2 : M = 8u \qquad N_{H_2} = \frac{8u}{4u} = 2 \qquad N_2 : \frac{28}{14u} = 2$

a.) $n = N/V$

$V = N/n$

$\frac{N_{He}}{n_{He}} = \frac{N_N}{n_N} \qquad n_H = \frac{2}{6.02 \times 10^{23}} \qquad n_N = \frac{2}{6.02 \times 10^{23}}$

$\frac{N_{He}}{N_N} = \frac{n_{He}}{n_N} \qquad n_h = 3.3 \times 10^{-24} \qquad n_N = 3.3 \times 10^{-24}$

Carried Sig Figs in Calculator

$N_{He} : N_N = n_{He} / n_N = \frac{3.3 \times 10^{-24}}{3.3 \times 10^{-24}} = 1$

$N_{He} : N_N = 1:1$

b.) $\rho = m/V \qquad V = m/\rho$

$m_{He} = 2u$

$m_N = 14u$

$\frac{m_{He}}{\rho_{He}} = \frac{m_N}{\rho_N}$

$\frac{m_N}{m_{He}} = \frac{\rho_N}{\rho_{He}}$

$\rho_N : \rho_{He} = 7:1$

CQ3. Imagine a gas in a sealed container. The temperature is doubled and the volume is tripled.

a) By what factor does the pressure of the gas change?

b) By what factor does the number density of the gas in the container change?

$PV = nRT$

a.) P? $P_0 = \frac{nRT}{V}$ $P_1 = \frac{nR(2T)}{3(V)} = \frac{2}{3}\frac{nRT}{V}$

$P_1 = \frac{2}{3}P_0$

b.) $\frac{N}{V}$? $\frac{N}{V_0} = \frac{N}{V_1}$

$\frac{1}{1} : \frac{1}{3}$

$P_1 = \frac{1}{3}P_0$

CQ4. A gas undergoes an isobaric process where the volume of the gas triples. By what factor does the gas temperature change?

$P = \frac{nRT}{V}$ $V_1 = 3V_0$

Isobaric = same Pressure

The temperature triples

$\frac{T_0}{V_0} = \frac{T_1}{V_1}$ $\frac{T_1}{T_0} = \frac{V_1}{V_0} = \frac{3V_0}{V_0} = 3$

$\frac{T_1}{T_0} = 3$ $T_1 = 3T_0$

P1. 1.10 moles of gas occupies a volume of 0.0268 m^3 at a pressure of 1.15 atm.

4 sig figs

a) What is the temperature of the gas in Kelvin?

b) Calculate the number density of the gas.

c) Assume that the gas is nitrogen (N_2). What is the total mass of the gas and the mass density of the gas?

$PV = nRT$

a.) $T = \frac{PV}{nR}$ 1atm = 101,300 Pa $T = \frac{1.15(101{,}300\ Pa)(0.0268\ m^3)}{(1.10\ mol)(8.31\ J/mol\ K)}$

$T = 341.54\ K$

$P = 1.15\ atm$

$V = 0.0268\ m^3$

$n = 1.10\ mol$

$R = 8.31\ J/mol\ K$

$T = 342K$

b.) $n = \frac{N}{N_A}$ $N_D = \frac{N}{V}$ $N_D = \frac{6.622\times10^{23}}{0.0268\ m^3}$

$N_D = 2.4709\times10^{25}\ m^{-3}$

$\frac{N}{V} = 2.47\times10^{25}\ m^{-3}$

$N = n(N_A)$

$= 1.10\ mol(6.02\times10^{23}\ mol^{-1})$

$N = 6.622\times10^{23}$

$V = 0.0268\ m^3$

c.) N_2 : $\mathcal{M}$ & ρ? $N = \frac{\mathcal{M}}{m}$, $\mathcal{M} = N(m)$

$m_{N_2} = 2(14u) = 28(1.66\times10^{-27}\,kg) = 4.648\times10^{-26}\,kg$

$N = 6.622\times10^{23}$ $\mathcal{M} = 6.622\times10^{23}(4.648\times10^{-26}\,kg)$

$V = 2.68\times10^{-2}\,m^3$ $\mathcal{M} = 0.030779\,kg$ $\rho = \frac{\mathcal{M}}{V}$

$\mathcal{M} = 3.08\times10^{-2}\,kg$
$\rho = 1.15\,kg/m^3$

3 sig figs

P2. A car engine contains a cylinder that takes a volume $V = 4.65 \times 10^{-2}$ m^3 of air into the cylinder's chamber at 32.5°C at atmospheric pressure (1.00 atm). The piston then compresses the air to 1/10 of the original volume and to 25.0 times the original pressure. Calculate the temperature of this compressed air.

$PV = nRT$

$V_0 = 4.65\times10^{-2}\,m^3$ $V_1 = \frac{1}{10}(V_0)$
$T_0 = 306\,K$ $T_1 = ?$
$P_0 = 101{,}300\,Pa$ $P_1 = 25P_0$

$nR = \frac{PV}{T}$

$\frac{P_0V_0}{T_0} = \frac{P_1V_1}{T_1}$

$\frac{T_0}{P_0V_0} = \frac{T_1}{P_1V_1}$

$\frac{T_0(P_1V_1)}{P_0V_0} = T_1$

$T_1 = \frac{25P_0(\frac{1}{10}V_0)T_0}{P_0V_0}$

$T_1 = \frac{25T_0}{10}$

$T_1 = 2.5T_0$

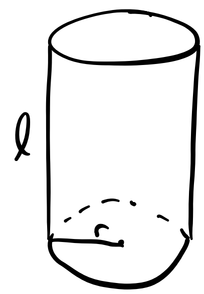
$T_1 = 2.5(306) = 765\,K$

$T_{comp} = 765\,K$

P3. Reconsider the cylinder of the previous problem. Assume the cylinder is *cylindrical* in shape and has an inner radius that is 1/4 the length of the interior of the cylinder during the intake of the gas.

a) How many moles of gas are contained within the occupied cylinder?

b) Assume that the gas is completely nitrogen (N_2). Calculate the mass density of the gas during the intake and after the compression.



$r = \frac{1}{4}\ell$
$V = \pi r^2 h$
$V = \pi\left(\frac{1}{4}\ell\right)^2\ell$
$V = \pi\frac{\ell^3}{4}$
$V = \frac{1}{4}\pi\ell^3$

a.)

$\frac{N}{V} = \frac{P}{k_BT}$, $N = \frac{PV}{k_BT} = \frac{(101{,}300\,N/m^2)(4.65\times10^{-2}\,m^3)}{(1.38\times10^{-23}\,Nm/K)(306\,K)}$

$\frac{\cancel{N}\cancel{m}}{\cancel{N}\cancel{m}}$

$P = 1\,atm$
$V = 4.65\times10^{-2}\,m^3$
$k_B = 1.38\times10^{-23}\,J/K$
$T = 306\,K$

$N = 1.115\times10^{24}$

$n = \frac{N}{N_A} = \frac{1.115\times10^{24}}{6.02\times10^{23}\,mol^{-1}} = 1.852\,mol$

$n = 1.85\,mol$

b.) $\rho = \frac{m}{V}$

$m = \mathcal{M}/N$
$\mathcal{M} = 28u$
$\mathcal{M}_{mol} = 0.028\,kg$
$m = 5.208\times10^{-2}\,kg$

$\rho_1 = \frac{M}{V_1}$
$m = 5.18\times10^{-2}\,kg$
$V_1 = 4.65\times10^{-2}\,m^3$
$\rho_1 = \frac{5.18\times10^{-2}\,kg}{4.65\times10^{-2}\,m^3}$
$\rho_1 = 1.11\,kg/m^3$

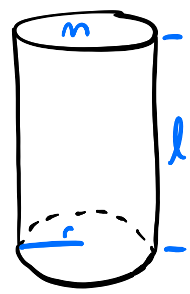
$\rho_2 = \frac{M}{V_2}$
$m = 5.18\times10^{-2}\,kg$
$V_2 = 4.65\times10^{-3}\,m^3$
$\rho_2 = \frac{5.18\times10^{-2}\,kg}{4.65\times10^{-3}\,m^3}$
$\rho_2 = 11.1\,kg/m^3$

$\rho_1 = 1.11\,kg/m^3$
$\rho_2 = 11.1\,kg/m^3$

P4. A 15.0-cm radius vertical cylinder is sealed at the top by a frictionless 28.0 kg piston. The piston is 96.0 cm above the bottom when the gas temperature is 350.°C. The air above the piston is at 1.00 atm pressure.

a) What is the gas pressure inside the cylinder?

b) What will be the height of the piston if the temperature of the gas is lowered to 35.0°C?



$r = 15\times10^{-2}$ m
$l = 96\times10^{-2}$ m
$m = 28$ kg
$P_a = 1$ atm
$T_0 = 623$ K

a.) $PV = nRT$

$m = 28$ kg
$A = \pi r^2$
$r = 15\times10^{-2}$ m

$P = P_{atm} + \frac{mg}{A}$

$P = 101,300 + \frac{28\text{kg}(9.8 \text{m/s}^2)}{\pi(15\times10^{-2}\text{m})^2}$

$P = 106182$ Pa
$P_6 = 1.05\times10^5$ P

$P_{6a3} = 1.05\times10^5$ Pa

b.) $PV = nRT$

$P_0 = 1.06\times10^5$ Pa $\quad P_1 = P_0$
$V_0 = \pi r^2 l_0 \quad V_1 = \pi r^2 l_1$
$T_0 = 623$ K $\quad T_1 = 308$ K

$l_1 = 0.475$ m

$P = \frac{nRT}{V}$

$\frac{nRT_0}{V_0} = \frac{nRT_1}{V_1}$

$\frac{V_0}{T_0} = \frac{V_1}{T_1}$

$\frac{V_0 T_1}{T_0} = \pi r^2 l_1$

$\frac{\pi r^2 l_0 T_1}{\pi r^2 T_0} = l_1$

$l_1 = \frac{l_0 T_1}{T_0}$

$l_0 = 0.96$ m
$T_1 = 308$ K
$T_0 = 623$ K

$l_1 = \frac{0.96\text{m}(308\text{K})}{623\text{K}} = 0.475$ m

P5. 0.35 mol of nitrogen gas (N_2) is admitted to an evacuated 55 cm^3 container at 25°C. The gas then undergoes an isobaric heating to a temperature of 350°C.

a) What is the final volume of the gas?

b) Show the process on a pV diagram. Include a proper scale and units on both axes.

a.)

$n = 0.35$ mol $\quad R = 8.31$ J/mol
$V_0 = 5.5\times10^{-5}$ m^3 $\quad V_1 = ?$
$T_0 = 298$ K $\quad T_1 = 623$ K
$P_0 = P_1$

$PV = nRT$

$P = \frac{nRT}{V}$

$\frac{nRT_0}{V_0} = \frac{nRT_1}{V_1}$

$\frac{V_0}{T_0} = \frac{V_1}{T_1}$

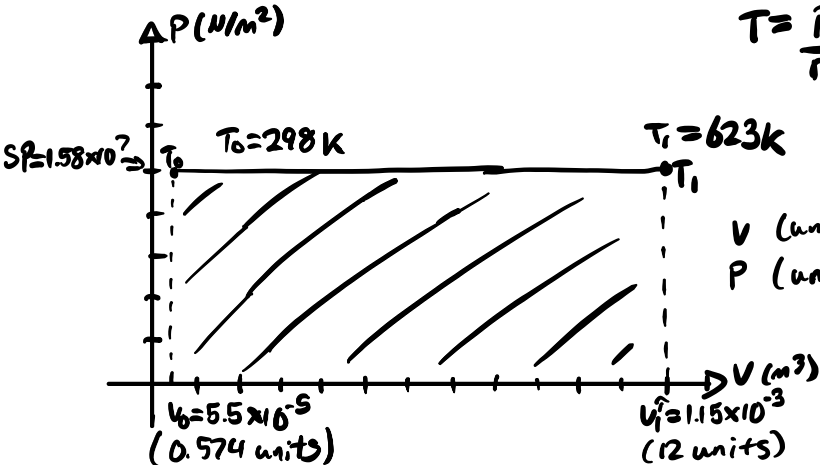
$\frac{T_1 V_0}{T_0} = V_1$

$V_1 = \frac{623\text{K}(5.5\times10^{-5}\text{m}^3)}{298\text{K}}$

$V_1 = 1.15\times10^{-4}$ m^3

$V_1 = 1.15\times10^{-4}$ m^3

b.)



$T = \frac{PV}{nR}$

V (units) $\quad$ 1V = 9.6×10^{-5} m^3
P (units) $\quad$ 1P = 3.16×10^{6} N/m^2

P6. 7.5 grams of oxygen gas at an initial pressure of 3.5 atm and at 36°C undergo an isochoric process until the pressure has tripled.

a) How many moles of gas does our cylinder contain?

b) What is the new pressure of the gas?

c) What is the gas temperature after this process?

The gas volume is then decreased isobarically until the original temperature is reached.

c) What is the volume of the cylinder after the decrease?

Finally, the gas is isothermally expanded until it returns to its initial volume.

d) What is the final gas pressure?

e) Show the full three-step process on a pV diagram. Use appropriate scales and units on both axes.

3 Sig Figs

a.) $n = \frac{N}{N_A}$, $N = \frac{M}{m}$ $O_2 : 32u$

$M = 7.5 \times 10^{-3}$ kg

$m = 32\,u = 5.312 \times 10^{-26}$ kg

$u = 1.66 \times 10^{-27}$ kg

$N_a = 6.02 \times 10^{23}\,mol^{-1}$

$N = \frac{7.5 \times 10^{-3}\,kg}{5.312 \times 10^{-26}\,kg} = 1.41 \times 10^{23}$

$n = \frac{1.41 \times 10^{23}}{6.02 \times 10^{23}\,mol^{-1}} = 0.234$ mol

$n = 0.23$ mol

V_0?

$V_0 = \frac{nRT}{P} = \frac{0.23\,mol(8.31\,J/mol)(309\,K)}{3.5(101.3 \times 10^3\,Pa)}$

$V_0 = 1.7 \times 10^{-3}\,m^3$

b.) $P_0 = 3.5$ atm

$P_1 = 3P_0$: $3(3.5\,atm) = 10.5$ atm

1 atm = 101.3 KPa

$P_1 = 10.5(101.3 \times 10^3\,Pa) = 1.1 \times 10^6$ Pa

$P_1 = 1.1 \times 10^6$ Pa

c.) $V = \frac{nRT}{P}$ $\frac{T_0}{P_0} = \frac{T_1}{P_1}$

$P_1 = 3P_0$

$T_1 = \frac{P_1 T_0}{P_0} = \frac{3P_0 T_0}{P_0} = 3T_0 = 3(309K) = 927K$

$T_1 = 927$ K

d.) $P_1 = P_2$: $P_1 = 1.063 \times 10^6$ Pa

$T_1 = 927K$ $T_2 = 309\,K$

$V_1 = 1.7 \times 10^{-3}\,m^3$ $V_2 = 5.681 \times 10^{-4}\,m^3$

$P = \frac{nRT}{V}$

$\frac{T_1}{V_1} = \frac{T_2}{V_2}$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$V_2 = \frac{T_2(V_1)}{T_1}$

$V_2 = \frac{309K(1.7 \times 10^{-3}\,m^3)}{(927\,K)}$

$V_2 = 5.67 \times 10^{-4}\,m^3$

$V = 5.7 \times 10^{-4}\,m^3$

e.) $T_2 = T_3$ $T_2 = 309\ K$

$V_2 = 5.69 \times 10^{-4} m^3$ $V_3 = 1.7 \times 10^{-3} m^3$

$P_2 = 1.063 \times 10^{6} Pa$ $P_3 = 3.559 \times 10^{5} N/m^2$

$$T = \frac{PV}{nr}$$

$$\frac{P_2 V_2}{nR} = \frac{P_3 V_3}{nR}$$

$$\frac{P_2 V_2}{V_3} = P_3$$

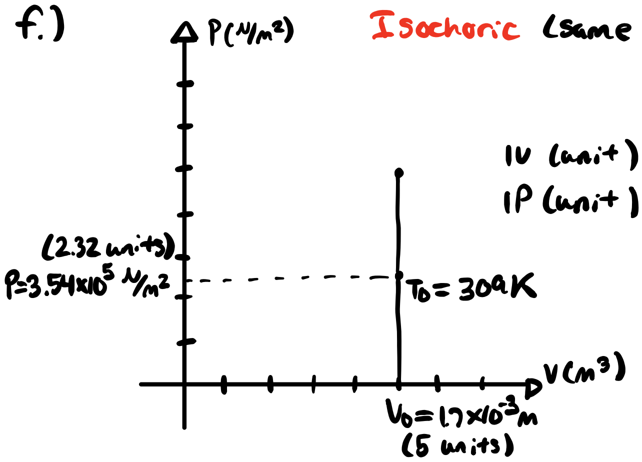
$$\boxed{P_3 = 3.6 \times 10^5\ Pa}$$

$$P_3 = \frac{(1.063 \times 10^6 Pa)(5.691 \times 10^{-4} m^3)}{1.7 \times 10^{-3} m^3}$$

$$P_3 = 3.559 \times 10^5\ N/m^2$$

f.)

Isochoric (same volume)



1V (unit) division $= 3.4 \times 10^{-4} m^3$

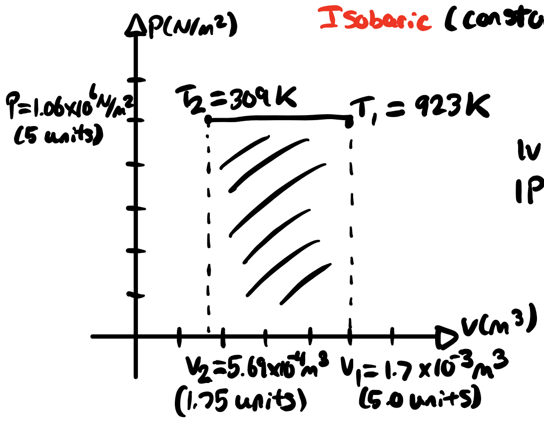
1P (unit) division $= 2.12 \times 10^5$

$V_0 = V_1$: $V_0 = 1.7 \times 10^{-3} m^3$

$P_0 = 3.55 \times 10^5 Pa$ $P_1 = 1.06 \times 10^6 Pa$

$T_0 = 309\ K$ $T_1 = 923\ K$

Isobaric (constant Pressure)



1V (unit) division $= 3.4 \times 10^{-4} m^3$

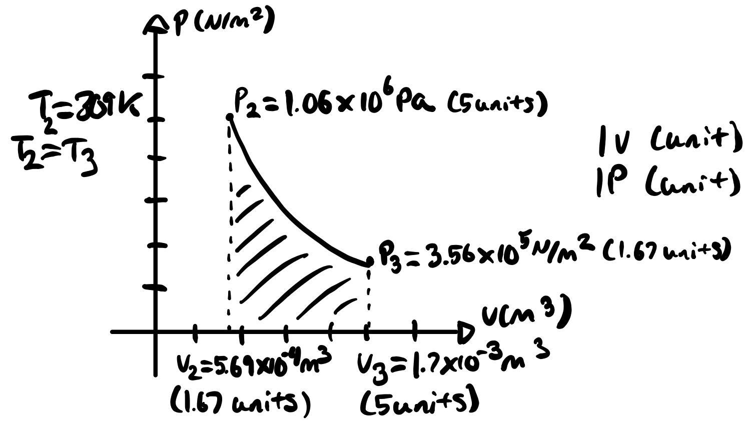
1P (unit) division $= 2.12 \times 10^5 N/m^2$

$P_1 = P_2$: $P_1 = 1.063 \times 10^6 Pa$

$T_1 = 923 K$ $T_2 = 309 K$

$V_1 = 1.7 \times 10^{-3} m^3$ $V_2 = 5.691 \times 10^{-4} m^3$

Isothermal (constant temperature)



1V (unit) division $= 3.4 \times 10^{-4} m^3$

1P (unit) division $= 2.12 \times 10^5 N/m^2$

$T_2 = T_3$ $T_2 = 309\ K$

$V_2 = 5.69 \times 10^{-4} m^3$ $V_3 = 1.7 \times 10^{-3} m^3$

$P_2 = 1.063 \times 10^6 Pa$ $P_3 = 3.559 \times 10^5 N/m^2$