Section 3.4 HW

1.) Square wave function f on $[0,1]$.

a.) Square Wave

$a_0=0$, $a_k=0$, $b_k=\begin{cases}\frac{4}{\pi k}, & k \text{ odd}\\ 0, & k \text{ even}\end{cases}$, $k=1,2,3\ldots$

Fourier Expansion

$$h_n(t)=\sum_{k=0}^{n} a_k\cos\left(\frac{2\pi k}{T}t\right)+\sum_{k=1}^{n} b_k\sin\left(\frac{2\pi k}{T}t\right)$$

$a_k=0$ ∴ no cosine terms

$b_0=0$, $b_1=\frac{4}{\pi}$, $b_2=0$, $b_3=\frac{4}{3\pi}$, $b_4=0$, $b_5=\frac{4}{5\pi}$, $b_6=0$

k	a_k	b_k	C_k	C_{-k}
0	0	0	0	
1	0	$4/\pi$	$-4i/\pi$	$2/\pi$
2	0	0	0	
3	0	$4/3\pi$	$-4i/3\pi$	$2/3\pi$
4	0	0	0	
5	0	$4/6\pi$	$-4i/6\pi$	$2/5\pi$
6	0	0	0	

$C_0=a_0$

$C_k=\frac{1}{2}(a_k-ib_k)$, $k=1,2,\ldots\infty$

$C_k=\frac{1}{2}(a_k+ib_k)$, $k=-1,-2,\ldots-\infty$

b.) $$h_{(n)}(t)=\frac{2i}{\pi}\left[\frac{e^{-ik\omega t}}{k}+\ldots k_0-\frac{e^{ik\omega t}}{k}\ldots\right]$$

$$h_{(6)}(t)=\frac{4i}{\pi}\left[\frac{e^{-i5\omega t}}{5}+\frac{e^{-i3\omega t}}{3}+\frac{e^{-i\omega t}}{1}-\frac{e^{i\omega t}}{1}-\frac{e^{i3\omega t}}{3}-\frac{e^{i5\omega t}}{5}\right]$$