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MATH 361 HW 2.3

1-3, 5b, 5c, 7, 9, 14, 30

Problem 1 $f(x) = x^2 - 6$, $P_0 = 1$

$$g(x) = x - \frac{f(x)}{f'(x)} \quad : \quad f'(x) = 2x \quad \therefore \quad g(x) = x - \frac{x^2-6}{2x}$$

$$P_1 = g(P_0) = 3.5 \quad : \quad P_2 = g(P_1) = 2.607142857$$

$$\boxed{P_2 = 2.607142857}$$

Problem 2 $f(x) = -x^3 - \cos(x)$, $p_0 = -1$

$$g(x) = x - \frac{f(x)}{f'(x)} \; ; \; f'(x) = -3x^2 + \sin(x) \;\therefore\; g(x) = x - \frac{-x^3 - \cos(x)}{-3x^2 + \sin(x)}$$

$$P_1 = g(P_0) = -0.8803328996 \; : \; P_2 = -0.8656841632$$

$$\boxed{\begin{array}{l} P_2 = -0.8656841632 \\ \text{No, } P_0 \neq 0 \end{array}}$$

Problem 3 $f(x) = x^2 - 6$, $p_0 = 3$, $p_1 = 2$

a.) $$g(x,y) = x - \frac{f(x)(x-y)}{f(x) - f(y)} = x - \frac{(x^2-6)(x-y)}{(x^2-6)-(y^2-6)}$$

$P_2 = g(P_0, P_1) = 2.4$: $P_3 = g(P_1, P_2) = 2.454545455$

$$\boxed{P_3 = 2.454545455}$$

b.) $$g(x,y) = x - \frac{f(x)(x-y)}{f(x) - f(y)} = x - \frac{(x^2-6)(x-y)}{(x^2-6)-(y^2-6)}$$

$f(P_0) \cdot f(P_1) = (3)(-2) = -6 < 0$

$P_2 = g(P_1, P_0) = 2.4$: $f(P_2) \cdot f(P_1) = (-0.24)(-2) = 0.48 > 0$ $\therefore$ $P_3 = g(P_2, P_0)$

$P_3 = g(P_2, P_0) = 2.444444444$

$$\boxed{P_3 = 2.444444444}$$

c.)

$$\boxed{\text{False position is closer to } \sqrt{6}}$$

Problem 5

b.) $f(x) = x^3 + 3x^2 - 1 = 0$, $[-3;2]$

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^3+3x^2-1}{3x^2+6x}$$

$P_1 = g(P_0) = -3.066666667$ ..... $P_5 = g(P_4) = -2.879385242$

$$\boxed{P_5 = -2.87939}$$

c.) $f(x) = x - \cos(x) = 0$, $[0, \pi/2]$

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x - \cos(x)}{1 + \sin(x)}$$

$P_1 = g(P_0) = 0.7395361335$ .... $P_3 = g(P_2) = 0.7390851332$

$$\boxed{P_3 = 0.73909}$$

Problem 7

b.) $f(x) = x^3 + 3x^2 - 1 = 0$, $[-3;2]$: $g(x,y) = x - \dfrac{f(x)(x-y)}{f(x) - f(y)}$

$g(x,y) = x - \dfrac{(x^3+3x^2-1)(x-y)}{(x^3+3x^2-1)-(y^3+3y^2-1)}$: $P_0 = -2.33$, $P_1 = -2.67$

$P_2 = g(P_0, P_1) = -3.027918177$: $P_3 = g(P_1, P_2) = -2.868584735$: $P_4 = g(P_2, P_3) = -2.876936191$

$P_5 = g(P_3, P_4) = -2.879429156$: $P_6 = g(P_4, P_5) = -2.879385162$: $P_7 = g(P_5, P_6) = -2.879385242$

$$\boxed{P_7 = -2.879389}$$

c.) $f(x) = x - \cos(x) = 0$, $[0, \pi/2]$: $g(x,y) = x - \dfrac{f(x)(x-y)}{f(x) - f(y)}$

$g(x,y) = x - \dfrac{(x-\cos(x))(x-y)}{(x-\cos(x)) - (y-\cos(y))}$: $P_0 = \pi/3$, $P_1 = \pi/6$

$P_2 = g(P_0, P_1) = 0.7251379956$: $P_3 = g(P_1, P_2) = 0.7398323421$: $P_4 = g(P_2, P_3) = 0.7390828161$

$P_5 = g(P_3, P_4) = 0.7390851328$

$$\boxed{P_5 = 0.73909}$$

Problem 9

b.) $f(x) = x^3+3x^2-1 = 0$, $[-3,-2]$: $g(x,y) = x - \dfrac{f(x)(x-y)}{f(x)-f(y)}$

$g(x,y) = x - \dfrac{(x^3+3x^2-1)(x-y)}{(x^3+3x^2-1)-(y^3+3y^2-1)}$: $P_0 = -2.9$, $P_1 = -2.1$

$f(p_0) = -0.159$: $f(p_1) = 2.969$: $f(p_0)\cdot f(p_1) = -0.159 \cdot 2.969 = -0.472071 < 0$

$P_2 = g(P_1, P_0) = -2.859338088$: $f(P_2)\cdot f(P_1) = 0.1500\,(2.969) > 0 \therefore P_3 = g(P_2, P_0)$

$P_3 = g(P_2, P_0) = -2.879078571$: $f(P_3)\cdot f(P_2) = 0.0023(0.1500) > 0 \therefore P_4 = g(P_3, P_0)$

$P_4 = g(P_3, P_0) = -2.879380603$: $f(P_4)\cdot f(P_3) = 0.00003(0.0023) > 0 \therefore P_5 = g(P_4, P_0)$

$P_5 = g(P_4, P_0) = -2.879385171$

$\therefore \boxed{P_5 = -2.879389}$

c.) $f(x) = x - \cos(x) = 0$, $[0, \pi/2]$: $g(x,y) = x - \dfrac{f(x)(x-y)}{f(x)-f(y)}$

$g(x,y) = x - \dfrac{(x-\cos(x))\,(x-y)}{(x-\cos(x))-(y-\cos(y))}$: $P_0 = \pi/6$, $P_1 = \pi/4$

$f(P_0) = -0.3424$, $f(P_1) = 0.07829$: $f(p_0)\cdot f(p_1) = -0.3424 \cdot 0.07829 = -0.026809034 < 0$

$P_2 = g(P_0, P_1) = 0.7366799354$: $f(P_2)\cdot f(P_1) = -0.00402\cdot(0.07829) < 0 \therefore P_3 = g(P_1, P_2)$

$P_3 = g(P_1, P_2) = 0.7390610992$: $f(P_3)\cdot f(P_2) = -0.00004\cdot(-0.00402) > 0 \therefore P_4 = g(P_1, P_3)$

$P_4 = g(P_1, P_3) = 0.7390848934$: $f(P_4)\cdot f(P_3) = -0.0000004(-0.00004) > 0 \therefore P_5 = g(P_1, P_4)$

$P_5 = g(P_1, P_4) = 0.7390851308$

$\therefore \boxed{P_5 = 0.73909}$

Problem 14

Secant method will be the best

Problem 30

It will take a lot of iterations