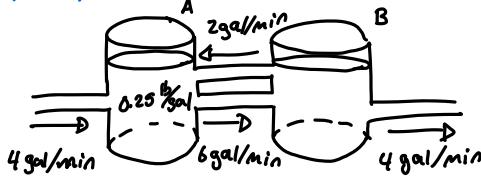


6.2 Linear Systems with Real Eigenvalues

52, 55, 56

5-8, 9-22, 25-34

6.2.52



$$x_1' = (0.1b/gal)(4 \text{ gal/min}) + \left(\frac{x_2 - 1b}{100 \text{ gal}}\right)(2 \text{ gal/min}) - \left(\frac{x_1 - 1b}{100 \text{ gal}}\right)(6 \text{ gal/min})$$

$$x_1' = \frac{1}{50}x_2 - \frac{3}{50}x_1 - 1b/\text{min}$$

$$x_2' = \left(\frac{x_1 - 1b}{100 \text{ gal}}\right)(6 \text{ gal/min}) - \left(\frac{x_2 - 1b}{100 \text{ gal}}\right)(2 \text{ gal/min}) - \left(\frac{x_2 - 1b}{100 \text{ gal}}\right)(4 \text{ gal/min})$$

$$x_2' = \frac{3}{50}x_1 - \frac{3}{50}x_2 - 1b/\text{min}$$

$$x_1' = -\frac{3}{50}x_1 + \frac{1}{50}x_2 \quad x_1(0) = 25$$

$$x_2' = \frac{3}{50}x_1 - \frac{3}{50}x_2 \quad x_2(0) = 0$$

$$A = \begin{bmatrix} -\frac{3}{50} & \frac{1}{50} \\ \frac{3}{50} & -\frac{3}{50} \end{bmatrix}$$

Eigenvalues: $|A - \lambda I|$

vectors: $(A - \lambda I)\vec{v}_1$

$$|A - \lambda I| = \begin{vmatrix} -0.06 - \lambda & 0.02 \\ 0.06 & -0.06 - \lambda \end{vmatrix} = 0$$

$$\vec{v}_1: \lambda_1 \quad rref = \left[\begin{array}{cc|c} 1 & -1/\sqrt{3} & 0 \\ 0 & 0 & 0 \end{array} \right] \quad x = \frac{1}{\sqrt{3}}y \quad \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} = \vec{v}_1$$

$$\lambda^2 + 0.12\lambda + 0.0024$$

$$\lambda_1 = -0.025$$

$$\lambda_2 = -0.095$$

$$\vec{v}_2: \lambda_2 \quad rref = \left[\begin{array}{cc|c} 1 & \sqrt{3} & 0 \\ 0 & 0 & 0 \end{array} \right] \quad x = -\sqrt{3}y \quad \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix} = \vec{v}_2$$

$$x(t) = c_1 e^{\lambda_1 t} [v_1] + c_2 e^{\lambda_2 t} [v_2] \quad x(t) = 12.5 e^{-0.025t} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} + 12.5 e^{-0.095t} \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$$

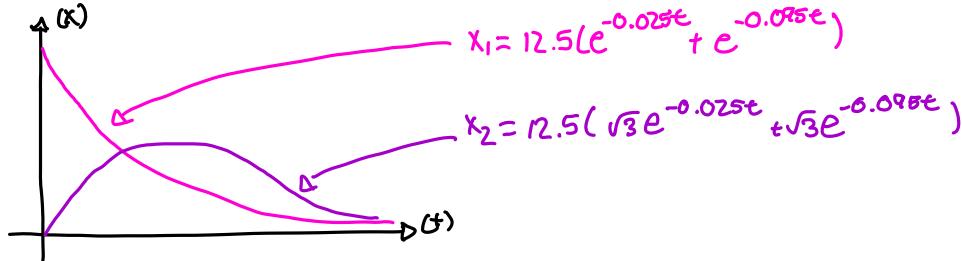
$$a.) \quad x(t) = c_1 e^{-0.025t} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} + c_2 e^{-0.095t} \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$$

$$x(t) = 12.5(e^{-0.025t} + e^{-0.095t}) + 12.5(\sqrt{3}e^{-0.025t} - \sqrt{3}e^{-0.095t})$$

$$x(0) = 25 \quad \left[\begin{array}{cc|c} c_1 e^{-0.025t} & c_2 e^{-0.095t} & 25 \\ \sqrt{3}c_1 e^{-0.025t} & c_2 e^{-0.095t} & 0 \end{array} \right] \quad c_1 = 12.5$$

$$c_2 = 12.5$$

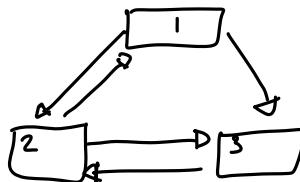
b.)



c.) No it does not

d.) Tank B ends up having more than tank A in the long run, the salt water will eventually reach zero.

62.P.55



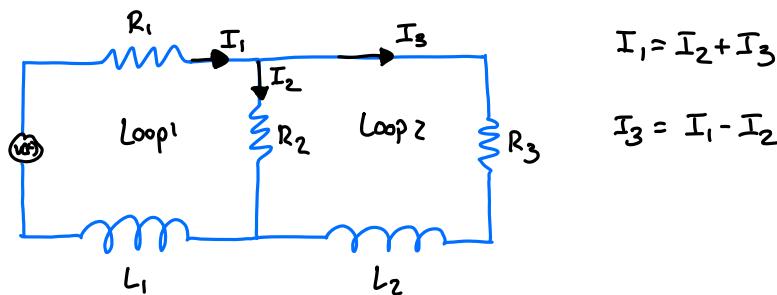
$$\begin{array}{ll} a_{12} = 0.06 & a_{13} = 0.04 \\ a_{21} = 0.10 & a_{23} = 0.01 \\ a_{32} = 0.05 & \end{array}$$

$$\begin{aligned} x_1' &= (x_2)(a_{21}) - (x_1)(a_{12}) - (x_1)(a_{13}) &= 0.1x_2 - 0.06x_1 - 0.04x_1 = 0 \\ x_2' &= (x_1)(a_{12}) + (x_3)(a_{32}) - (x_2)(a_{23}) - (x_2)(a_{21}) &= 0.06x_1 + 0.05x_3 - 0.11x_2 = 0 \\ x_3' &= (x_1)(a_{13}) + (x_2)(a_{23}) - (x_3)(a_{32}) &= 0.04x_1 + 0.01x_2 - 0.05x_3 = 0 \end{aligned}$$

$$\text{rref } \left[\begin{array}{ccc|c} -0.1 & 0.1 & 0 & 0 \\ 0.06 & -0.11 & 0.05 & 0 \\ 0.04 & 0.01 & -0.05 & 0 \end{array} \right] = \begin{array}{l} x_1 = x_3 \\ x_2 = x_3 \\ x_3 = 1 \end{array} \quad \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right]$$

$$\tilde{x}' = \begin{bmatrix} -0.1 & 0.1 & 0 \\ 0.06 & -0.11 & 0.05 \\ 0.04 & 0.01 & -0.05 \end{bmatrix} \vec{x}$$

6.2.56



$$\text{Loop 1: } I_1(R_1) + I_2(R_2) + I_1'L_1 - V(t) = 0$$

$$\text{Loop 2: } I_3(R_3) + L_2 I_3' - I_2 R_2 = 0$$

$$\begin{aligned} I_1' &= V(t) - I_1 R_1 - I_2 R_2 \\ I_3' &= \frac{I_2 R_2}{L_2} + R_3(I_2 - I_1) \\ &\quad - (I_1 - I_2)(R_3) \\ &\quad - I_1 R_3 + I_2 R_3 \end{aligned}$$

$$I_1' = V(t) - I_1(4\Omega) - I_2(4\Omega)$$

$$I_3' = I_2(2) + I_2(4\Omega) - I_1(4\Omega)$$

6.2.5

$$x' = x \\ y' = y$$

$$\frac{dx}{dt} = x \\ x = k e^t \\ y = k e^t$$

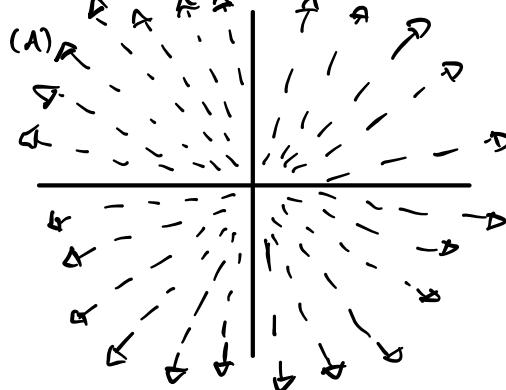
$$\bar{x}' = A\bar{x} \\ \bar{x}' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \bar{x}$$

Eigen: $\left| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$$\left| \begin{bmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} \right| = 0 \Rightarrow (1-\lambda)(1-\lambda) \\ \lambda^2 - 2\lambda + 1 \\ \lambda = 1$$

$\lambda > 0$

$\lambda = 1 \therefore$ going away from origin



6.2.6

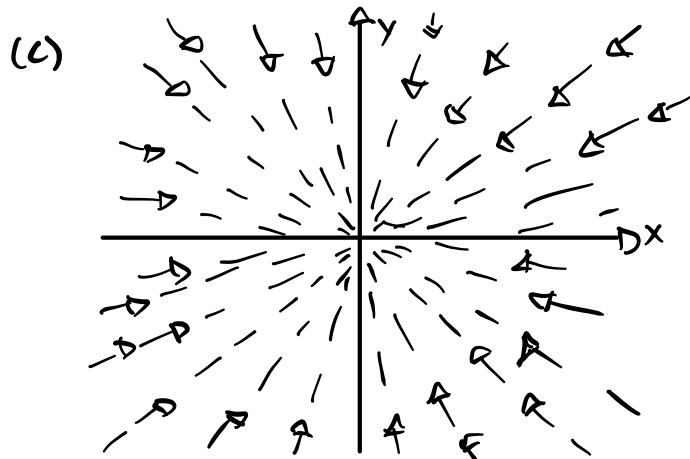
$$x' = -x \\ y' = -y$$

$\lambda < 0 \therefore$ repelling
 $\lambda = -1$

$$\bar{x}' = A\bar{x} \\ \bar{x}' = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \bar{x}$$

$$\left| \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0 \quad \lambda = -1$$

$$\left| \begin{bmatrix} -1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} \right| = 0 : (-\lambda-1)(-\lambda-1) \\ \lambda^2 + 2\lambda + 1$$



6.2.7

$$\bar{x}' = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \bar{x}$$

$$(A - \lambda I) = 0$$

$$|A - \lambda I| = 0$$

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) \bar{v}_1 = 0$$

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \right) \bar{v}_2 = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

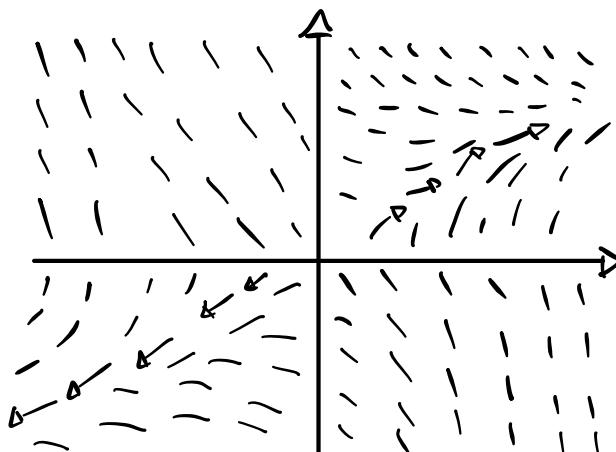
$$\bar{v}_1 = \begin{bmatrix} -1 \\ \frac{2}{(\sqrt{5}-1)} \end{bmatrix} \quad x = -\frac{(\sqrt{5}-1)}{2} y \\ x = -\frac{2}{(\sqrt{5}-1)} y = 1$$

$$\begin{bmatrix} 1 & -\frac{\sqrt{5}}{2} - \frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left| \begin{bmatrix} -2 & 1 \\ 1 & -1-\lambda \end{bmatrix} \right| = 0 : (-\lambda-1)(-\lambda-1) - 1(1) \\ \lambda^2 + 2\lambda - 1$$

$$\frac{-1 - \sqrt{1-4(\lambda+1)}}{2} = -\frac{(\sqrt{5}+1)}{2} = \lambda_1$$

$$\frac{-1 + \sqrt{1-4(\lambda+1)}}{2} = \frac{\sqrt{5}-1}{2} = \lambda_2$$



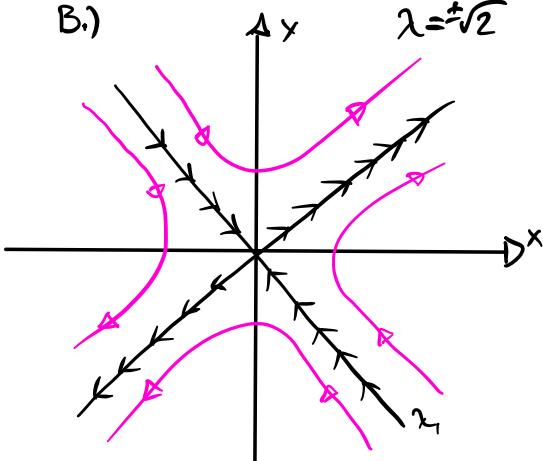
$$6.2.8] \quad \tilde{x} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \bar{x}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1+\lambda \end{bmatrix} \right| = 0 \quad \frac{(-\lambda+1)(\lambda-1)}{\lambda^2 - 1 - 1} = 0$$

B.)



$$(A - \lambda_1 I) v_1 = 0$$

$$\lambda_1 = -1 \quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right) \begin{bmatrix} v_x \\ v_y \end{bmatrix} = 0$$

$$x = (1-1)y$$

$$(A - \lambda_2 I) v_2 = 0$$

$$\lambda_2 = \sqrt{2} \quad v_2 = \begin{bmatrix} \sqrt{2}+1 \\ 1 \end{bmatrix}$$

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \right) \begin{bmatrix} v_x \\ v_y \end{bmatrix} = 0$$

$$\left[\begin{array}{cc|c} \sqrt{2} & 1 & 0 \\ 1 & -1-\sqrt{2} & 0 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & \sqrt{2}-1 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad x = (\sqrt{2}+1)y$$

$$6.2.9] \quad \tilde{x}' = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \bar{x}$$

$$E: |(A - \lambda I)| = 0 \quad (-\lambda-1)(-\lambda-4) - 2(2)$$

$$\lambda^2 + 5\lambda + 4 - 4$$

$$\left| \begin{bmatrix} -4-\lambda & 2 \\ 2 & -1-\lambda \end{bmatrix} \right| = 0 \quad \lambda^2 + 5\lambda = 0 \quad \lambda(\lambda+5) = 0$$

$$\lambda = -5, 0$$

$$\lambda_2 = 0: \quad (A - \lambda_2 I) v_2 = 0$$

$$\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -0.5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad x - 0.5y = 0$$

$$x = 0.5y \quad y = 2x$$

$$\lambda_1 = -5 \quad v_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 0 \quad v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x(t) = c_1 e^{-5t} \bar{v}_1 + c_2 e^{2t} \bar{v}_2$$

$$x(t) = c_1 e^{-5t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

6.2.11

$$\bar{x}' = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \bar{x} \quad x_1' = x_1 - x_2 \\ x_2' = 2x_1 + 4x_2$$

$$|A - \lambda I| = 0$$

$$\lambda = 2 : (A - \lambda I) \bar{v}_1$$

$$\left| \begin{bmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{bmatrix} \right| = 0 : (1-\lambda)(4-\lambda) + 1(2) \\ 4 - 2\lambda - 4\lambda + \lambda^2 + 2$$

$$\frac{\lambda^2 - 5\lambda + 6}{(\lambda-3)(\lambda-2)} \\ \lambda = 2, 3$$

$$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{array}{l} x+y=0 \\ x=-y \end{array} \quad \bar{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = 3 : (A - \lambda I) \bar{v}_2$$

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{array}{l} x + \frac{1}{2}y = 0 \\ x = -\frac{1}{2}y \end{array} \quad \bar{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x(t) = c_1 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

6.2.13

$$\bar{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \bar{x}$$

$$x_1' = 5x_1 - x_2 \\ x_2' = 3x_1 + x_2$$

$$|A - \lambda I| = 0$$

$$(A - \lambda I) v_1 \\ \lambda = 2$$

$$\left| \begin{bmatrix} 5-\lambda & -1 \\ 3 & 1-\lambda \end{bmatrix} \right| = 0$$

$$(5-\lambda)(1-\lambda) + 1(3)$$

$$5 - 5\lambda - \lambda + \lambda^2 + 3$$

$$\lambda^2 - 6\lambda + 8$$

$$(\lambda-4)(\lambda-2)$$

$$\lambda = 2, 4$$

$$\lambda = 4$$

$$\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & 0 \end{bmatrix} \quad \begin{array}{l} x - \frac{1}{3}y = 0 \\ x = \frac{1}{3}y \end{array} \quad \bar{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = c_1 e^{2t} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

6.2.15

$$\bar{x}' = \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix} \bar{x}$$

$$|A - \lambda I| = 0$$

$$\lambda = 1 \\ (A - \lambda I) v_1$$

$$\left| \begin{bmatrix} 1-\lambda & 0 \\ -2 & 2-\lambda \end{bmatrix} \right| = 0$$

$$(2-\lambda)(1-\lambda) - 0(-2)$$

$$\lambda^2 - 3\lambda + 2$$

$$\lambda^2 - 3\lambda + 2$$

$$(\lambda-2)(\lambda-1)$$

$$\lambda = 1, 2$$

$$\lambda = 2$$

$$\begin{bmatrix} -1 & 0 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{array}{l} x = 0 \\ y = \text{free} \end{array} \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(t) = c_1 e^t \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

6.2.17

$$\bar{x}' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \bar{x}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-2-\lambda) + 2(2)$$

$$-6 - 3\lambda + 2\lambda + 2^2 + 4$$

$$\lambda^2 - \lambda - 2$$

$$(\lambda - 2)(\lambda + 1)$$

$$\lambda = -1, 2$$

$$(A - \lambda I)\bar{v}$$

$$\lambda = -1$$

$$\begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -0.5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad x - 0.5y = 0 \quad \bar{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda = 2$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad x - 2y = 0 \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$x(t) = c_1 e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

6.2.19

$$\bar{x}' = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \bar{x}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 3 & -4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-4-\lambda) + 2(3)$$

$$-4 + 3\lambda + \lambda^2 + 6$$

$$\lambda^2 + 3\lambda + 2$$

$$\lambda = -1, -2 \quad (A - \lambda I)\bar{v}$$

$$\lambda = -1$$

$$\begin{bmatrix} 0 & -2 \\ 3 & -3 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{free}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3 + \sqrt{9 - 8}}{2(1)}$$

$$\frac{-3 + 1}{2} = -1$$

$$\frac{-4}{2} = -2$$

$$\lambda = -2$$

$$\begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{free}} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$x(t) = c_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

6.2.21

$$\bar{x}' = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \bar{x}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & -3 \\ 8 & -6-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(-6-\lambda) + 3(8)$$

$$-24 - 10\lambda + \lambda^2 + 24$$

$$\lambda^2 - 10\lambda = 0$$

$$\lambda(10-\lambda) = 0$$

$$\lambda = 0, 10$$

$$(A - \lambda I)\bar{v}$$

$$\lambda = 0$$

$$\begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -\frac{3}{4} \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{free}} \begin{bmatrix} 1 & -\frac{3}{4} \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\lambda = 10$$

$$\begin{bmatrix} -6 & -3 \\ 8 & -16 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{free}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = c_1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_2 e^{10t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

6.2.25

$$\bar{x}' = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix} \bar{x} \quad \bar{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} -2-\lambda & 1 \\ -5 & 4-\lambda \end{bmatrix} \right| = 0$$

$$(-2-\lambda)(4-\lambda) - (-5) = 0$$

$$-8 - 2\lambda + \lambda^2 + 5 = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$(\lambda-3)(\lambda+1) = 0$$

$$\lambda = -1, 3$$

$$(A - \lambda I)\bar{v}$$

$$\lambda = -1$$

$$\begin{bmatrix} -1 & 1 \\ -5 & 5 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad x - y = 0 \quad \bar{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = 3$$

$$\begin{bmatrix} -5 & 1 \\ -5 & 1 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & -\frac{1}{5} \\ 0 & 0 \end{bmatrix} \quad x - \frac{1}{5}y = 0 \quad \bar{v}_2 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$x(t) = c_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$x(t) = \frac{1}{2} e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{2} e^{3t} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$t=0 \quad \left[\begin{array}{cc|c} c_1 e^{-t} & c_2 e^{3t} & 1 \\ c_1 e^{-t} & 5c_2 e^{3t} & 3 \end{array} \right]$$

$$\left[\begin{array}{cc|c} c_1 & c_2 & 1 \\ c_1 & 5c_2 & 3 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 5 & 3 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \end{array} \right] \quad c_1 = 0.5, c_2 = 0.5$$

6.2.27

$$\bar{x}' = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad \bar{x}(0) = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$(A - \lambda I)\bar{v}$$

$$\lambda = 2$$

$$\left| \begin{bmatrix} (2-\lambda) & 0 \\ 0 & (3-\lambda) \end{bmatrix} \right| = 0$$

$$(2-\lambda)(3-\lambda) = 0$$

$$6 - 5\lambda + \lambda^2 = 0$$

$$(\lambda-3)(\lambda-2) = 0$$

$$\lambda = 2, 3$$

$$\lambda = 3$$

$$(A - \lambda I)$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad x = 0 \quad \bar{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x = \text{free}$$

$$y = 0$$

$$x(t) = c_1 e^{2t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$t=0 \quad \left[\begin{array}{cc|c} c_1 e^{2t} & 0 & 5 \\ 0 & c_2 e^{3t} & 4 \end{array} \right]$$

$$x(t) = 5e^{2t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 4e^{3t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} c_1 & 0 & 5 \\ 0 & c_2 & 4 \end{array} \right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 4 \end{array} \right] \quad c_1 = 5, c_2 = 4$$

6.2.29]

$$\bar{x}' = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \bar{x} \quad \bar{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} (1-\lambda) & 1 \\ 1 & (1-\lambda) \end{bmatrix} \right| = 0$$

$$(1-\lambda)(1-\lambda) - 1(1) = 0$$

$$1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda - 2) = 0$$

$$\lambda = 0, 2$$

$$(A - \lambda I)\bar{v}$$

$$\lambda = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\xrightarrow{\text{ref}} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$x+y=0 \quad \bar{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = 2$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\xrightarrow{\text{ref}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x=0 \text{ free} \quad \bar{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$t=0 \quad \begin{bmatrix} c_1 & c_2 e^{2t} & | & 2 \\ -c_1 & c_2 e^{2t} & | & 3 \end{bmatrix} \rightarrow \begin{bmatrix} c_1 & c_2 & | & 2 \\ -c_1 & c_2 & | & 3 \end{bmatrix} \xrightarrow{\text{ref}} \begin{bmatrix} 1 & 1 & | & 2 \\ -1 & 1 & | & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & | & -0.5 \\ 0 & 1 & | & 2.5 \end{bmatrix}$$

$$x(t) = -\frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{5}{2} e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$c_1 = -\frac{1}{2}$$

$$c_2 = \frac{5}{2}$$

6.2.31)

$$\bar{x}' = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \bar{x} \quad \bar{x}(0) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} (-2-\lambda) & 1 \\ 4 & (-2-\lambda) \end{bmatrix} \right| = 0$$

$$(-2-\lambda)(-2-\lambda) - 1(4) = 0$$

$$4 + 4\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 + 4\lambda = 0$$

$$\lambda(\lambda + 4) = 0$$

$$\lambda = -4, 0$$

$$(A - \lambda I)\bar{v}$$

$$\lambda = -4$$

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\xrightarrow{\text{ref}} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix}$$

$$x + \frac{1}{2}y = 0 \quad \bar{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\lambda = 0$$

$$\begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}$$

$$\xrightarrow{\text{ref}} \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix}$$

$$x - \frac{1}{2}y = 0 \quad \bar{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x(t) = c_1 e^{-4t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x(t) = \frac{2}{3} e^{-4t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \frac{8}{3} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$t=0 \quad \begin{bmatrix} -c_1 e^{-4t} & c_2 & | & 2 \\ 2c_1 e^{-4t} & c_2 & | & 4 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 & | & 2 \\ 2 & 1 & | & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & | & \frac{2}{3} \\ 0 & 1 & | & \frac{8}{3} \end{bmatrix}$$

$$c_1 = \frac{2}{3}$$

$$c_2 = \frac{8}{3}$$