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MATH 366
CP Ch. 21.1

1) "However, in your career as an engineer, applied mathematicians, or physicist you will encounter ODE's and PDE's that cannot be solved by those analytic methods or whose solutions are so difficult that other approaches are needed.

2) $$y' = y + x$$

3) $$y = e^{x} - x - 1$$

4)

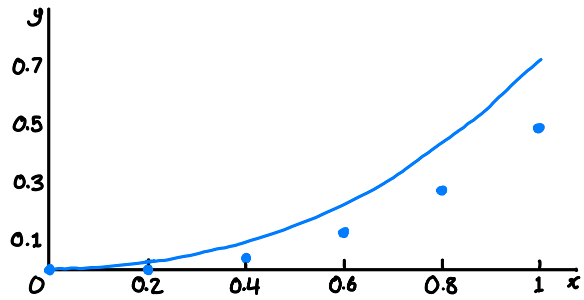

Fig. 9. Euler method: Approximate values in Table 1.1 and solution curve

5)

(1) $$y' = f(x,y), \quad y(x_0) = y_0$$

(2) $$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2} y''(x) + \dots$$

(3) $$y_{n+1} = y_n + hf(x_n, y_n) \qquad (n = 0, 1, \dots)$$

6)

(7a) $$y^*_{n+1} = y_n + hf(x_n, y_n)$$

(7b) $$y_{n+1} = y_n + \frac{1}{2}h[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})]$$

7)

Table 21.1 Improved Euler Method (Heun's Method)

Algorithm Euler (f, x_0, y_0, h, N)

This algorithm computes the solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ at equidistant points $x_1 = x_0 + h$, $x_2 = x_0 + 2h, \dots, x_N = x_0 + Nh$; here f is such that this problem has a unique solution on the interval $[x_0, x_N]$ (see Sec. 1.6)

Input: Initial values x_0, y_0, step size h, number of steps N
Output: Approximation y_{n+1} to the solution $y(x_{n+1})$ at $x_{n+1} = x_0 + (n+1)h$,

For $n = 0, 1, \dots, N-1$ do:
$x_{n+1} = x_n + h$
$k_1 = hf(x_n, y_n)$
$k_2 = hf(x_{n+1}, y_n + k_1)$
$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$
End Stop output x_{n+1}, y_{n+1}
End EULER

9)

Table 21.3 Classical Runge-Kutta Method of Fourth order

Algorithm Runge-Kutta (f, x_0, y_0, h, N)

This algorithm computes the solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ at equidistant points

$$x_1 = x_0 + h,\quad x_2 = x_0 + 2h,\ \dots,\ x_N = x_0 + Nh; \tag{9}$$

here f is such that this problem has a unique solution on the interval $[x_0, x_N]$ (see Sec. 1.7).

Input: Function f, initial values x_0, y_0, step size h, number of steps N
Output: Approximation y_{n+1} to the solution $y(x_{n+1})$ at $x_{n+1} = x_0 + (n+1)h$, where $n = 0, 1, \dots, N-1$

For $n = 0, 1, \dots, N-1$ do:

$k_1 = hf(x_n, y_n)$
$k_2 = hf(x_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}k_1)$
$k_3 = hf(x_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}k_2)$
$k_4 = hf(x_n + h,\ y_n + k_3)$
$x_{n+1} = x_n + h$
$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

End stop output x_{n+1}, y_{n+1}

End Runge-Kutta