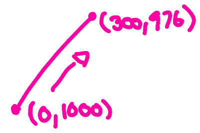
1.)



$\frac{976-1000}{300-0} = \frac{-24}{300} = \frac{-2}{25}$

$\frac{-2}{25}$

$D_u f(x,y) = f_x(x,y)a + f_y(x,y)b$

$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$

2.) $f(x,y) = \sin(3x+4y)$ $P(-12,9)$ $u = \frac{1}{2}(\sqrt{3}\,i - j)$

$\nabla f(x,y) = \sin(3x+4y)$

$\langle 3\cos(3x+4y), 4\cos(3x+4y) \rangle$

$\langle 3\cos(3x+4y), 4\cos(3x+4y) \rangle$

$\nabla f(-12,9) = \langle 3\cos(3(-12)+4(9)), 4\cos(3(-12)+4(9)) \rangle$

$\langle 3\cos(-36+36), 4\cos(-36+36) \rangle$

$\langle 3\cos(0), 4\cos(0) \rangle$

$\langle 3,4 \rangle$

$\langle 3,4 \rangle$

$D_u f(-12,9) = \nabla f(-12,9) \cdot \vec{u}$

$\langle 3,4 \rangle \cdot \langle \frac{\sqrt{3}}{2}, \frac{-1}{2} \rangle$

$\frac{3\sqrt{3}}{2} + (4(\frac{-1}{2}))$

$\frac{3\sqrt{3}}{2} - 2$

$\frac{3\sqrt{3}}{2} - 2$

3.) Find the directional derivative of the function

$f(x,y) = 3e^x \sin y$ $(0, \pi/3)$ $v = \langle -5, 12 \rangle$

$D_u f(x,y,z) = \nabla f(x,y,z) \cdot \vec{u}$

$\nabla f = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z) \rangle$

$f_x = 3e^x \sin y$ $\quad f_y = 3e^x \cos y$

$f_x(0, \frac{\pi}{3}) = 3e^0 \sin\frac{\pi}{3}$ $\quad f_y(0, \frac{\pi}{3}) = 3e^0 \cos(\frac{\pi}{3})$

$3 \cdot (\frac{\sqrt{3}}{2})$ $\quad 3(\frac{1}{2})$

$\nabla f \langle \frac{3\sqrt{3}}{2}, \frac{3}{2} \rangle$

$\vec{v} = \langle -5, 12 \rangle$ $\quad u = \sqrt{25+144}$

$\vec{u} = \frac{1}{13} \langle -5, 12 \rangle$ $\quad u = \sqrt{169}$

$\vec{u} = \langle -\frac{5}{13}, \frac{12}{13} \rangle$ $\quad u = 13$

$\nabla f \cdot u = D_u f$

$\langle \frac{3\sqrt{3}}{2}, \frac{3}{2} \rangle \cdot \langle -\frac{5}{13}, \frac{12}{13} \rangle$

$\frac{-15\sqrt{3}}{26} + \frac{36}{26}$

$\frac{-15\sqrt{3}}{26} + \frac{18}{13}$

$\frac{18}{13} - \frac{15\sqrt{3}}{26}$

4.) Find the directional derivative

$F(x,y,z) = xe^y + ye^z + ze^x$

$P(0,0,0)$ $\quad V = \langle 5,1,-2\rangle$

$$\frac{4}{\sqrt{30}}$$

$f_x = e^y + ze^x \quad f_y = xe^y + e^z$

$f_z = ye^z + e^x$

$\nabla f = \langle 1,1,1\rangle$

$f_x(0,0,0) = e^0 + 0(e)^0 \quad f_y(0,0,0) = 0e^0 + e^0 \quad f_z(0,0,0) = (0)e^0 + e^0$

$1+0 \quad 0+1 \quad 0+1$

$f_x(0,0,0) = 1 \quad f_y(0,0,0) = 1 \quad f_z(0,0,0) = 1$

$V = \langle 5,1,-2\rangle$

$u = \sqrt{25+1+4}$

$u = \sqrt{30}$

$\vec{u} = \langle \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}\rangle$

$D_u F(0,0,0) = \nabla f(0,0,0) \cdot \vec{u}$

$\langle 1,1,1\rangle \cdot \langle \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}\rangle$

$\frac{5}{\sqrt{30}} + \frac{1}{\sqrt{30}} - \frac{2}{\sqrt{30}} = \frac{4}{\sqrt{30}}$

5.) Find the directional derivative of $F(x,y,z)$

$F(x,y,z) = xy + yz + zx \quad P(3,-3,6) \quad Q(2,4,5)$

$f_x = y+z \quad f_y = x+z \quad f_z = y+x$

$f_x(-3,6) = -3+6 \quad f_y(3,6) = 3+6 \quad f_z(3,-3) = -3+3$

$\hookrightarrow 3 \quad \hookrightarrow 9 \quad \to 0$

$$\frac{60}{\sqrt{51}}$$

$\nabla f = \langle 3,9,0\rangle$

$\vec{PQ} = \langle -1,7,-1\rangle$

$\vec{u} = \sqrt{1+49+1}$

$\nabla f \cdot \vec{u}$

$\vec{u} = \frac{1}{\sqrt{51}}$

$\vec{u} = \langle -\frac{1}{\sqrt{51}}, \frac{7}{\sqrt{51}}, \frac{-1}{\sqrt{51}}\rangle$

$\frac{-3}{\sqrt{51}} + \frac{63}{\sqrt{51}} + 0$

$\frac{60}{\sqrt{51}}$

6.) $f(x,y) = 8y\sqrt{x} \quad (16,9)$

Maximum rate of change is $\sqrt{1105}$

Direction vector $= 9\hat{i} + 32\hat{j}$

$f_y = 8y \cdot \frac{1}{2\sqrt{x}} \quad f_y = 8\sqrt{x}$

$f_x = \frac{4y}{\sqrt{x}} \quad \langle \frac{4y}{\sqrt{x}}, 8\sqrt{x}\rangle$

$\langle \frac{4(9)}{\sqrt{16}}, 8\sqrt{16}\rangle$

$\langle \frac{36}{4}, 8(4)\rangle$

$\sqrt{9^2 + 32^2} \quad \langle 9, 32\rangle = 9\hat{i} + 32\hat{j}$

$\sqrt{1105}$

7.) Find the maximum rate of change

$f(x,y,z) = \frac{8x+5y}{z}$ (3,7,-1)

$f_x = \frac{1}{z}(8x+5y)$ $\to \frac{8}{z}$

$f_y = \frac{1}{z}(8x+5y)$ $\to \frac{5}{z}$

$f_z = \frac{1}{z}(8x+5y)$

$8x \cdot z^{-1} + 5y \cdot z^{-1}$

$8x \cdot (-z^{-2}) + 5y \cdot (-z^{-2})$

$-8xz^{-2} - 5yz^{-2}$

$-\frac{8x}{z^2} - \frac{5y}{z^2}$

$\langle 8/z, 5/z, \frac{-8x-5y}{z^2} \rangle$

$\frac{8}{-1}, \frac{5}{-1}, \frac{-8(3)-5(7)}{+1}$

$-8, -5, \frac{-24-35}{+1}$

$\langle -8, -5, 59 \rangle$

$-8\hat{i} - 5\hat{j} + 59\hat{k}$

$\sqrt{3570}$

$\sqrt{64+25+3481}$

$\frac{\langle -8, -5, -59 \rangle}{\sqrt{3570}}$

8.) $z = 1200 - 0.005x^2 - 0.01y^2$

(100, 120, 1006)

$z_x = -0.01x$ $z_y = -0.02y$

$z_x(100) = -1$ $z_y = -0.02(120)$

$z_x(100) = -1$ $z_y(120) = -2.4$

$\langle 1, -2.4 \rangle \cdot \langle 0, -1 \rangle$

$0 + 2.4$

2.4 m/s

$N_w = \langle -1, 1 \rangle$

$u = \sqrt{1+1}$

$u = \frac{1}{\sqrt{2}} \langle -1, 1 \rangle$

$u = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\langle 1, -\frac{24}{10} \rangle \cdot \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$+\frac{1}{\sqrt{2}} - \frac{2.4}{\sqrt{2}} = -0.99$

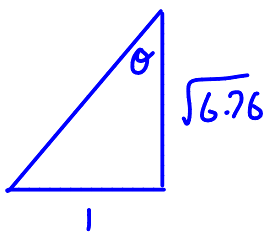
-0.99 m/s

$\sqrt{6.76}$ V/m/s

$\sqrt{1^2 + 2.4^2}$

$\sqrt{1 + 5.76}$

$\sqrt{6.76}$



$\tan\theta = \frac{y}{x}$

$\tan\theta = \sqrt{6.76}$

$\theta = \tan^{-1}(\sqrt{6.76})$

$\theta = 68.96°$

$\theta = 68.96°$

9.) $A(6,4)$ $B(9,4)$ $C(6,13)$ $D(11,16)$ $(x_0, y_0) = 6,4$

$\vec{AB} = 8$ $\vec{AC} = 12$

$\vec{AB} = \langle 3, 0 \rangle$ $\vec{AC} = \langle 0, 9 \rangle$ $u\,\vec{AB} = \langle 1, 0 \rangle$

$|\vec{AB}| = \sqrt{9+0}$ $|\vec{AC}| = \sqrt{81}$ $u\,\vec{AC} = \langle 0, 1 \rangle$

$|\vec{AB}| = 3$ $|\vec{AC}| = 9$ $u\,\vec{AD} = \langle \frac{5}{13}, \frac{12}{13} \rangle$

$\vec{AD} = \langle 5, 12 \rangle$ $\vec{AD} = \langle \frac{5}{13}, \frac{12}{13} \rangle$

$|\vec{AD}| = \sqrt{25+144}$

$|\vec{AD}| = \sqrt{169}$

$|\vec{AD}| = 13$

$$\vec{\nabla} f(6,4) \cdot u_{AB} = \langle f_x(6,4), f_y(6,4) \rangle \cdot \langle 1, 0 \rangle = 8$$

$$f_x(6,4) = 8$$

$$\vec{\nabla} f(6,4) \cdot u_{BC} = \langle f_x(6,4), f_y(6,4) \rangle \cdot \langle 0, 1 \rangle = 12$$

$$f_y(6,4) = 12$$

$$\nabla f(A) = \langle 8, 12 \rangle \cdot \langle \tfrac{5}{13}, \tfrac{12}{13} \rangle$$

14.15

$$\nabla f(A) \cdot \langle \vec{AD} \rangle$$

$$\langle \frac{40}{13}, \frac{144}{13} \rangle = \frac{184}{13}$$

$$\frac{40}{13} + \frac{144}{13} = 14.15$$

10.) Second directional derivative

$$D_u^2 f(x,y) = D_u[D_u f(x,y)]$$

$f(x,y) = x^3 + 5x^2y + y^3$ $\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$ $a = \frac{3}{5}$

$D_u^2 f(1,3)$ $b = \frac{4}{5}$

$$\frac{\partial f}{\partial x} = 3x^2 + 10xy$$

$$\frac{\partial f}{\partial y} = 5x^2 + 3y^2$$

$$D_{\vec{u}} f(x,y) = (3x^2 + 10xy)a + (5x^2 + 3y^2)b$$

$$= 3ax^2 + 10axy + 5bx^2 + 3by^2$$

$$\frac{\partial f}{\partial x} = (6ax + 10ay + 10bx)a$$

$$\frac{\partial f}{\partial y} = (10ax + 6by)b$$

$\frac{852}{25}$

$a = \frac{3}{5}$ $b = \frac{4}{5}$

$x = 1$ $y = 3$

$$D_{\vec{u}}^2 f(x,y) = (6ax + 10ay + 10bx)a + (10ax + 6by)b$$

$$D_{\vec{u}}^2 f(1,3) = \left(6(\tfrac{3}{5})1 + 10(\tfrac{3}{5})3 + 10(\tfrac{4}{5})1\right)\tfrac{3}{5} + \left(10(\tfrac{3}{5})1 + 6(\tfrac{4}{5})3\right)\tfrac{4}{5}$$

$$\left(\frac{18}{5} + \frac{90}{5} + \frac{40}{5}\right)\frac{3}{5} + \left(\frac{30}{5} + \frac{72}{5}\right)\frac{4}{5}$$

$$\frac{54}{25} + \frac{270}{25} + \frac{120}{25} + \frac{120}{25} + \frac{288}{25} = \frac{852}{25}$$

11.) Find the equations of the following $(9,10,3)$

$2(x-8)^2+(y-8)^2+(z-1)^2=10$

$f_x = 4(x-8)$ $\quad f_x(9,10,3) = 4(9-8) = 4(1) = 4$

$f_y = 2(y-8)$ $\quad f_y(9,10,3) = 2(10-8) = 2(2) = 4$

$f_z = 2(z-1)$ $\quad f_z(9,10,3) = 2(3-1) = 2(2) = 4$

$f_x(x,y,z)(x-x_0) + f_y(x,y,z)(y-y_0) + f_z(x,y,z)(z-z_0) = 0$

$4(x-9)+4(y-10)+4(z-3)=0$

$4x-36+4y-40+4z-12=0$

$4x+4y+4z=88$

$x+y+z=22$

Tangent plane $\equiv x+y+z=22$

$(9,10,3)$ $\langle 4,4,4\rangle$

$\frac{x-9}{4} = \frac{y-10}{4} = \frac{z-3}{4} = t$

$x-9=t$ $\quad y-10=t$ $\quad z-3=t$

$x=t+9$ $\quad y=t+10$ $\quad z=t+3$

Normal line
$x=t+9$
$y=t+10$
$z=t+3$

12.) $f(x,y)=xy$ find $\nabla f(7,3)$, use it to find level curve $f(x,y)=21$ @ $(7,3)$

$\nabla f(7,3) =$ $\quad \nabla f = f_x + f_y$

$f_x(x,y) + f_y(x,y)$

$f_x = y$ $\quad f_y = x$

$f_x(7,3)=3$ $\quad f_y(7,3)=7$

$\nabla f = \langle 3,7\rangle$

$(7,3)$ L.C $f(x,y)=21$

$x_0=7$

$y_0=3$

$f_x(x,y)(x-x_0) + f_y(x,y)(y-y_0) = 0$

$3(x-7)+7(y-3)=0$

$3x-21+7y-21=0$

$3x+7y=42$

$3x+7y=42$

13.) At what point on the paraboloid $y=x^2+z^2$ is the tangent plane parallel to the plane $3x+2y+5z=8$

$\langle 3,2,5\rangle$

$$f=y-x^2-z^2$$

$$\frac{\partial f}{\partial x}=-2x \quad \frac{\partial f}{\partial y}=1 \quad \frac{\partial f}{\partial z}=-2z$$

$$\vec{N}=\langle -2x_0, 1, -2z_0\rangle=\langle 3,2,5\rangle$$

$-2x_0c=3$ $\quad c=2$ $\quad y=x^2+z^2$

$1c=2$

$-2z_0c=5$

$y_0=(-3/4)^2+(-5/4)^2$

$-2x_0(2)=3$ $\quad -2z_0(2)=5$ $\quad y_0=\frac{9}{16}+\frac{25}{16}$

$-4x_0=3$ $\quad -4z_0=5$

$x_0=-3/4$ $\quad z_0=-\frac{5}{4}$ $\quad \frac{34}{16}$ $\quad \frac{17}{8}$

$$\left(-\frac{3}{4}, \frac{17}{8}, -\frac{5}{4}\right)$$

14.) Where does the normal line to the paraboloid $z=x^2+y^2$ @ $P(4,4,32)$ intersect a second time.

$f=x^2+y^2-z$ $\quad (4,4,32)$

$f_x=2x \quad f_y=2y \quad f_z=-1$

$$\frac{x-x_0}{f_x(x,y,z)}=\frac{y-y_0}{f_y(x,y,z)}=\frac{z-z_0}{f_z(x,y,z)}$$

$f_x(4,4,32)=2(4)=8$

$f_y(4,4,32)=2(4)=8$

$f_z(4,4,32)=-1$

$$\frac{x-4}{8}=\frac{y-4}{8}=\frac{z-32}{-1}=t$$

$\frac{x-4}{8}=t$ $\quad \frac{y-4}{8}=t$ $\quad \frac{z-32}{-1}=t$

$z=x^2+y^2$ $\quad x-4=8t$ $\quad y-4=8t$ $\quad z-32=-t$

$x=8t+4$ $\quad y=8t+4$ $\quad z=-t+32$

$-t+32=(8t+4)^2+(8t+4)^2$

$-t+32=64t^2+64t+16+64t^2+64t+16$

$-t+32=128t^2+128t+32$

$-t=128t^2+128t$ $\quad t=\frac{-129}{128}$ $\quad t_0=0$

$0=128t^2+129t$ $\quad t=0$ $\quad t_1=\frac{-129}{128}$

$t(128t+129)$

$x=8t+4$ $\quad 8\left(\frac{-129}{128}\right)+4=-\frac{65}{16}$

$y=8t+4$ $\quad 8\left(\frac{-129}{128}\right)+4=-\frac{65}{16}$

$z=-t+32$ $\quad -\left(\frac{-129}{128}\right)+32=\frac{4225}{128}$

$$\left(-\frac{65}{16}, -\frac{65}{16}, \frac{4225}{128}\right)$$

15.) Plane $y+z=13$ $x^2+y^2=25$ in an ellipse $P(3,4,9)$

$\nabla f(x,y,z) = y+z$

$\nabla g(x,y,z) = x^2+y^2$

$\nabla f = f_x(3,4,9), f_y(3,4,9), f_z(3,4,9)$

$f_x=0$ $f_y=1$ $f_z=1$ $= \langle 0,1,1\rangle$

$\nabla g = g_x(3,4,9), g_y(3,4,9), g_z(3,4,9)$ $\nabla g = 2x+2y$

$g_x=2x$ $g_y=2y$ $g_z=0$ $\langle 6,8,0\rangle$

$g(3)=6$ $g(4)=8$

$(\nabla f \times \nabla g)$ $\langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1\rangle$

$\langle 0,1,1\rangle$

$\langle 6,8,0\rangle$

$\langle 1(0)-1(8), 1(6)-0(0), 0(8)-1(6)\rangle$ $64+36+36$

$\langle -8, 6, -6\rangle$ $P(3,4,9)$ $\sqrt{136}$

$x=3-8T$

$y=4+6T$

$z=9-6T$

Cutting plane

$6(x-3)+8(y-4)=0$ $y+z=13$

$6x-18+8y-32=0$ $z=10$

$6x+8y-50=0$ $y=3$

$x=\frac{13}{3}$

$\langle \frac{4}{3}, -1, -1\rangle$

$\langle 3,4,9\rangle + t\langle \frac{4}{3}, -1, 1\rangle$

$x=3+\frac{4}{3}t$

$y=4-t$

$z=9+t$

$$\boxed{\begin{aligned} x&=3+\tfrac{4}{3}t\\ y&=4-t\\ z&=9+t\end{aligned}}$$