

4.6 The Discrete Cosine Transform of Type II

DCT II & IDCT II

DCT II

$$C_0^{(2)} = \sqrt{\frac{1}{N}} \sum_{n=0}^{N-1} x_n,$$

$$C_K^{(2)} = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N} \left(n + \frac{1}{2}\right) K\right], \quad K=1, 2, \dots, N-1$$

IDCT II

$$x_n = \sqrt{\frac{1}{N}} C_0^{(2)} + \sqrt{\frac{2}{N}} \sum_{m=1}^{N-1} C_m^{(2)} \cos\left[\frac{\pi}{N} \left(k + \frac{1}{2}\right) m\right], \quad K=0, 1, \dots, N-1$$

DCT II IDCT II Matrices $N \times N$

$$C_N^{\text{II}} = \sqrt{\frac{2}{N}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \dots & \frac{1}{\sqrt{2}} \\ \cos\frac{\pi}{2N} & \cos\frac{3\pi}{2N} & \dots & \cos\frac{(2N-1)\pi}{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \cos\frac{(N-1)\pi}{2N} & \cos\frac{(N-1)3\pi}{2N} & \dots & \cos\frac{(N-1)(2N-1)\pi}{2N} \end{bmatrix}$$

For the $N \times N$ IDCT II matrix,

$$(C_N^{\text{II}})^{-1} = (C_N^{\text{II}})^T$$

DCT II IDCT II Matrices $N=4$

$$C_4^{\text{II}} = \sqrt{\frac{1}{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos(\pi/8) & \cos(3\pi/8) & \cos(5\pi/8) & \cos(7\pi/8) \\ \cos(2\pi/8) & \cos(6\pi/8) & \cos(10\pi/8) & \cos(14\pi/8) \\ \cos(3\pi/8) & \cos(9\pi/8) & \cos(15\pi/8) & \cos(21\pi/8) \end{bmatrix}$$

The DCT II of x as the DFT of y

DFT vector y

$$C_0^2 = \frac{1}{2} \sqrt{\frac{1}{N}} \sum_{m=0}^{4N-1} y_m,$$

$$C_K^2 = \frac{1}{2} \sqrt{\frac{2}{N}} \sum_{m=0}^{4N-1} y_m \cdot e^{\frac{2\pi i}{4N} m K} \quad K=1, 2, \dots, N-1$$