

Fundamental Mechanics: Class Exam III

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Instructions

- There are 8 questions on 5 pages.
- Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

$$g = 9.80 \text{ m/s}$$

Question 1

An ice skater with mass m_s holds a rock of mass $m_r = \frac{1}{4} m_s$. These move together east with speed v_0 . At an instant the skater throws the rock east. After this the skater is at rest. Determine an expression (i.e. formula) for the velocity of the rock after this.

$$P_f = P_i$$

$$m_s v_s + m_r v_r = m_b v_f$$

$$\cancel{m_s v_{sf}} + \cancel{\frac{1}{4} m_s v_{rf}} = \frac{5}{4} m_s v_i$$

$$\begin{aligned} m_s &= 1 \\ m_r &= \frac{1}{4} m_s \\ m_b &= \frac{5}{4} m_s \end{aligned}$$

$$4\left(\frac{5}{4}\right) = \frac{20}{4} = 5$$

$$m_s v_{rf} = 5 m_s v_i$$

$$v_{rf} = 5 v_i$$

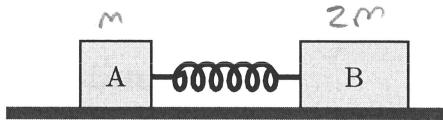
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$$E_i = E_f$$

$$K_i + U_{sp,i} = K_f + U_{sp,f}$$

Question 2

Two boxes on a frictionless surface are separated by a compressed spring. The mass of box B is twice the mass of box A. They are held at rest and then released. Which of the following (choose one) is true *at any instant* while the spring extends as they move apart?



Net force is equal on both, Newton's third law.

- i) The net force on A equals that on B. They have the same speed.
- ii) The net force on A equals that on B. A has a greater speed than B.
- iii) The net force on A is larger than on B. A has a greater speed than B.
- iv) The net force on A is larger than on B. A has a smaller speed than B.
- v) The net force on A is larger than on B. A has the same speed as B.

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Question 3

A box can move along a frictionless track. The box is held at rest against a spring compressing it by 10 cm. It is released, reaching height h_0 up the ramp before it returns. This is repeated but the spring is now initially compressed by 20 cm. Which of the following (choose one) is true? Ignore air resistance.



$$\Delta s = 0.01 \text{ m}$$

- i) The maximum height reached by the block is again h_0 .
- ii) The maximum height reached by the block is between h_0 and $2h_0$.
- iii) The maximum height reached by the block is $2h_0$.
- iv) The maximum height reached by the block is between $2h_0$ and $4h_0$.
- v) The maximum height reached by the block is $4h_0$.

Explain your choice.

~~$$E_i = E_f$$~~
~~$$K_i + U_{sp,i} + U_{g,i} = K_f + U_{sp,f} + U_{g,f}$$~~

$$\frac{1}{2}k(\Delta s)^2 = mg\gamma_f$$

$$k(\Delta s)^2 = 2mg\gamma_f$$

$$\frac{k(\Delta s)^2}{2mg} = \gamma_f$$

$$(10)^2 = 100$$

$$(20)^2 = 400$$

$$\frac{400}{100} = 4$$

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Question 4

A car of mass 500 kg moves along the illustrated road. At point A it is moving with speed 20 m/s. From that moment onwards its engine is not running and it moves along the road. While it continues to move friction and air resistance can be ignored.



- a) Determine the speed of the car when it reaches point B.

$$E_i = E_f$$

$$K_i + U_{G_i} = K_f + U_{G_f}$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}(500\text{kg})(20\text{m/s})^2 + (500\text{kg})(9.8\text{m/s}^2)(20\text{m}) = \frac{1}{2}(500\text{kg})v_f^2 + (500\text{kg})(9.8\text{m/s}^2)(15\text{m})$$

$$100,000 \text{kg}\cdot\text{m}^2/\text{s}^2 + 98,000 \text{kg}\cdot\text{m}^2/\text{s}^2 = 250 \text{kg}\cdot v_f^2 + 73,500 \text{kg}\cdot\text{m}^2/\text{s}^2$$

$$124,500 \text{kg}\cdot\text{m}^2/\text{s}^2 = 250 \text{kg}\cdot v_f^2$$

$$498 \text{m}^2/\text{s}^2 = v_f^2$$

$$v_f = 22.3 \text{ m/s}$$

- b) Determine the maximum speed of the car during the trip from point A to B.

maximum speed when $h=0$

$$E_i = E_f$$

$$K_i + U_{G_i} = K_f + U_{G_f}$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(500\text{kg})(20\text{m/s})^2 + (500\text{kg})(9.8\text{m/s}^2)(20\text{m}) = \frac{1}{2}(500\text{kg})v_f^2$$

$$100,000 \text{kg}\cdot\text{m}^2/\text{s}^2 + 98,000 \text{kg}\cdot\text{m}^2/\text{s}^2 = 250 \text{kg}\cdot v_f^2$$

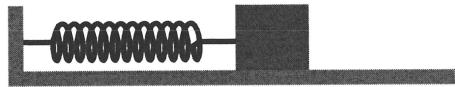
$$198 \text{m}^2/\text{s}^2 = v_f^2$$

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$$v_f = 14 \text{ m/s}$$

Question 5

A box of mass 40.0 kg is pushed against a spring with spring constant 200 N/m. The spring is compressed by 0.50 m. The box is held at rest and then released. As the box moves, a constant kinetic friction force of 30 N acts on the box.



- a) Determine the work done by the friction force from the moment that the box is released until the moment that the spring first reaches its unstretched length.

$$W = \vec{F} \cdot \Delta r \cdot \cos \theta$$

$$= 30N \cdot (0.50m) \cos 180^\circ$$

$$= -1(30 \cdot 0.5)$$

$$W = -15J$$

$$\theta = 180^\circ \text{ opposite to motion}$$

$$\Delta r = 0.50m$$

$$F = 30N$$

$$W = -15J$$

- b) Determine the speed of the box when the spring first reaches its unstretched length.

$$E_f = E_i + W_{nc}$$

$$K_f + U_{SF} = K_i + U_{SF} - 15J$$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}K(\Delta s)^2 - 15J$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}K(\Delta s)^2 - 15J$$

$$\frac{1}{2}(40.0\text{kg})v_f^2 = \frac{1}{2}(200\text{N/m})(0.50\text{m})^2 - 15J$$

$$20\text{kg} \cdot v_f^2 = 25 \text{ NM} - 15J$$

$$20\text{kg} \cdot v_f^2 = 10J$$

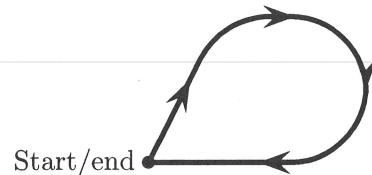
$$v_f^2 = 0.5 \text{ m}^2/\text{s}^2$$

$$v_f = 0.71 \text{ m/s}$$

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Question 6

A heavy wooden block follows the illustrated path (viewed from above) on a rough wooden horizontal table. Which of the following (choose one) is true regarding, W_f , the work done by friction?



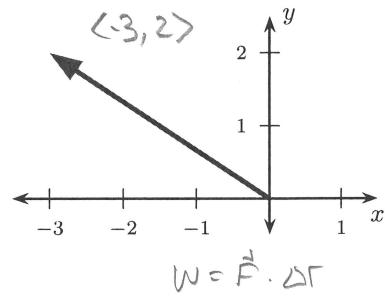
- i) $W_f < 0$.
 ii) $W_f > 0$.
 iii) $W_f = 0$.
 iv) One would need more information about the path to decide between the previous options.
- opposite to direction,
 always negative.

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Question 7

A particle moves along the indicated straight path in the xy plane. While this happens the following two forces act on the particle: $\vec{F}_A = 2\hat{i} + 3\hat{j}$ and $\vec{F}_B = -2\hat{i} + 3\hat{j}$. Which of the following is true (choose one) regarding the works done by these forces?

- i) $W_A = W_B$.
 ii) $W_A > W_B$.
 iii) $W_A < W_B$.
- $F_A = \langle 2, 3 \rangle \cdot \langle -3, 2 \rangle$
 $-6 + 6 = 0$
 $F_B = \langle -2, 3 \rangle \cdot \langle -3, 2 \rangle$
 $6 + 6 = 12$



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Question 8

A rope pulls on a small bead of mass 0.010 kg attached to the end of a spring with spring constant 50 N/m. The bead is initially at rest and the spring is relaxed. The spring is stretched horizontally by 0.40 m and at this point the bead is again at rest. This takes 2.0 s. Determine the power delivered (by the rope).



$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}$$

$$\begin{aligned}
 E_i &= E_f \\
 K_i + U_{S,i} &= K_f + U_{S,f} \\
 \Delta E &= \frac{1}{2}k(\Delta s)^2 \\
 &= \frac{1}{2}(50 \text{ Nm})(0.40)^2 \\
 \Delta E &= 25(0.16)
 \end{aligned}$$

$$P = \frac{\Delta E}{\Delta t} = \frac{4J}{2s} = 2W$$

$$P = 2W$$

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$$\Delta E = 4J$$

