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Dr. Middleton PHYS 230 HW
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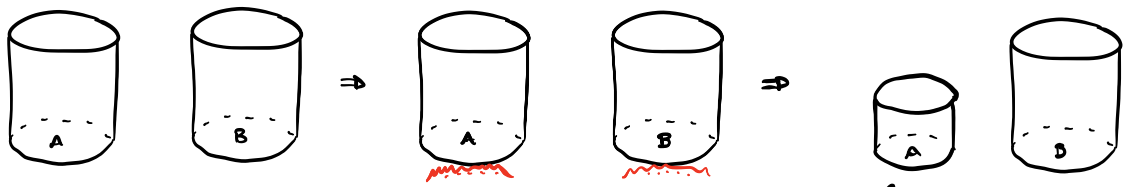
HW 4

CQ1. Imagine you have two containers of equal amounts of oxygen gas at initially equal temperatures. You then supply equal amounts of heat to each container. The first container undergoes an *isobaric contraction* and the second container undergoes an *isochoric cooling*. After each container was heated, which has the higher final temperature? Your explanation needs to include the first law of thermodynamics.



$\Delta E_{th} = W + Q$

$P_0 = P_1$, $V_0 > V_1$ ∴ work is negative

$V_0 = V_1$ ∴ $W = 0$

$P_0 = P_1$, $V_0 > V_1$

$V_0 = V_1$, $T_0 > T_1$, $W = 0$ ∴ $\Delta E_{th} = Q$ ∴ $-\Delta E_{th}$

$\Delta E_{th_A} = W_A + Q_A$
$\Delta E_{th_B} = Q_B$

⇒ The change in thermal energy of the gas in cylinder A is less than that of cylinder B's. This is because of the positive work contribution in cylinder A. This in turn means the change in temperature is less and thus $T_A > T_B$.

$T_A > T_B$

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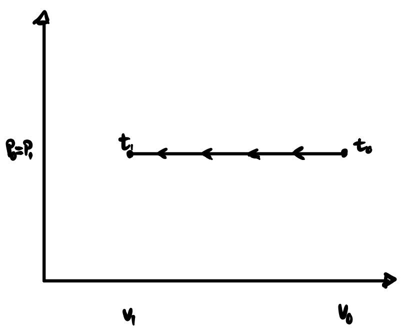
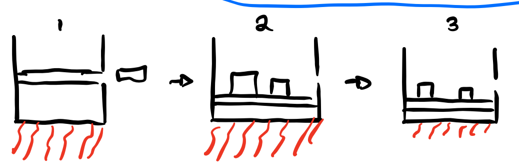


$\Delta E_{th} = W + Q$

To achieve this task you must first heat the cylinder with the pin out so the volume can change. Secondly, put masses on top of the piston and lastly decrease the flame on the heat.



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$\Delta E_{th} = W + Q$

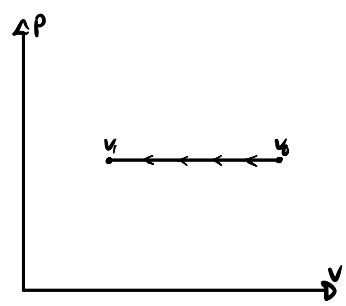
a.)

i.) $\Delta T < 0$ Since the heat from the gas is being transferred outside this system due to the colder ice, $\Delta T < 0$ because the gas temperature will drop due to the ice.

ii.) $W > 0$ Since the piston level will drop thus contracting the volume.

iii.) $Q < 0$ Since heat is being transferred out of this system.

b.)



P1. A cylinder with an initial volume of 3.60×10^3 cm^3 holds 0.100 mol of oxygen gas at 120.°C. The gas is expanded by a piston to a final volume of 5.40×10^3 cm^3. How much work was done on this gas if this process was done at

a) constant pressure and

b) constant temperature?

$V_0 = 3.60 \times 10^3\ cm^3$ $V_1 = 5.40 \times 10^3 cm^3$ $n = 0.1\ mol$ 3 sig figs

$t_0 = 120°C$

a.) w @ P_c ?

$V_0: 3.60 \times 10^3 cm^3 \frac{(0.01m)^3}{(1cm)^3} = 3.6\times10^{-3} m^3$ $t_0: 120 + 273 = 393K$ $V_1: 5.4\times10^3 cm^3 \cdot \frac{(0.01m)^3}{(1cm)^3} = 5.4\times10^{-3} m^3$ $PV = nRT \therefore T = \frac{PV}{nR}$

$V_0 = 3.6\times10^{-3} m^3$ $n = 0.1\ mol$ $P_0V_0 = nRT_0$ $V_1 = 5.4\times10^{-3} m^3$ $n = 0.1\ mol$ $T_1 = \frac{P_1V_1}{nR} = \frac{(9.07\times10^4 Pa)(5.4\times10^{-3} m^3)}{(0.1mol)(8.31 J/mol K)}$

$t_0 = 393K$ $R = 8.31\ J/mol K$ $P_0 = \frac{nRT_0}{V_0} = \frac{(0.1mol)(8.31 J/mol K)(393K)}{(3.6\times10^{-3} m^3)} = 90717.5\ Pa$ $t_1 = 590\ K$ $R = 8.31\ J/mol K$

$P_0 = 9.07\times10^4 Pa$ $P_1 = 9.07\times10^4 Pa$ $T_1 = 589\ K$

work in isobaric : $W = -P\Delta V$

$\Delta V = V_1 - V_0 = 5.4\times10^{-3}m^3 - 3.6\times10^{-3}m^3 = 1.8\times10^{-3} m^3$ $W = -9.07\times10^4 Pa(1.8\times10^{-3} m) = -163.26\ J$

$\Delta V = 1.8\times10^{-3} m^3$

$P = 9.07\times10^4 Pa$

$W = -163\ J$

b.) $V_0 = 3.6\times10^{-3} m^3$ $n = 0.1 mol$ work in isothermal : $W = -nRT \ln\left(\frac{V_f}{V_i}\right)$

$P_0 = 9.07\times10^4 Pa$ $R = 8.31\ J/mol K$ $= -(0.1mol)(8.31 J/mol k)(393K) \ln\left(\frac{5.4\times10^{-3} m^3}{3.6\times10^{-3} m^3}\right)$

$t_0 = 393K$ $W = -132.4 J$

$V_1 = 5.4\times10^{-3} m^3$

$W = -132\ J$

P2. Reconsider the isobaric and isothermal processes outlined in the previous problem.

For the *isobaric* process...

a) *i.* What is the change in thermal energy of the diatomic gas?

ii. What is the heat added or removed from the gas?

For the *isothermal* process...

b) *i.* What is the change in thermal energy of the diatomic gas?

ii. What is the heat added or removed from the gas?

$\Delta E_{th} = W + Q$, $\Delta E_{th} = nc_p\Delta t$
$\Delta E_{th} = nc_v\Delta t$

a.) isobaric : $W_p = -163 J$

i.) ΔE_{th} ? $\Delta T = T_1 - T_0$

$\Delta E_{th} = nc_p\Delta t$

$n = 0.1\ mol$ $T_1 = 589\ K$ $\Delta E_{th} = (0.1mol)(29.2)(196K) = 572\ J$

$C_p = 29.2$ $T_0 = 393\ K$

$\Delta t = 195\ K$

$\Delta E_{th} = 572\ J$

ii.) Q ? : $\Delta E_{th} = W + Q$: $Q = \Delta E_{th} - W$

$W_p = -163\ J$ $Q = 572 J + 163 J$

$\Delta E_{th} = 572 J$

$Q = 735\ J$

b.) isochoric : $\Delta E_{th} = W + Q$, $\Delta E_{th} = nc_p\Delta t$
$\Delta E_{th} = nc_v\Delta t$

$W_T = -132\ J$

i.) ΔE_{th} ? $\Delta T = 0 \therefore \Delta E_{th} = 0$

$\Delta E_{th} = 0\ J$

ii.) Q ? : $\Delta E_{th} = 0 \therefore Q = -W$

$W = -132 J \therefore Q = -(-132 J) = 132 J$

$Q = 132\ J$

P3. Consider the cylinder of Figure 17.13 that contains a gas that undergoes an *isobaric expansion.*

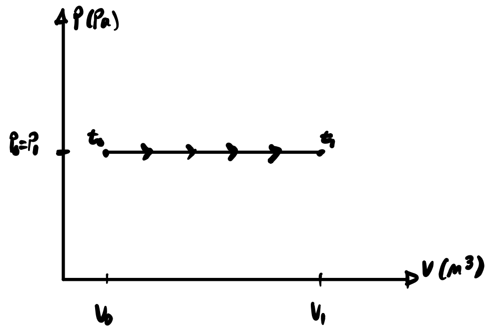
a) Draw a pV diagram showing this process. This diagram needs to include axes labels and an arrow indicating the direction traversed between the initial and final points on the pV curve.

b) Describe a series of steps to implement this ideal-gas process.

c) Show the process as a first-law bar chart.

$PV = nRT$ $\quad P = \frac{nRT}{V}$

a.)



b.)

i.) Set a flame to the cylinder.

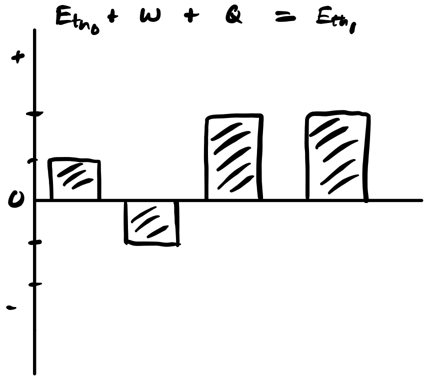
ii.) Raise the temperature of flame

iii.) Remove masses from piston to increase volume.

iv.) Keep new temp constant at desired volume with pin inserted

$E_{th_0} + W + Q = E_{th_1}$

c.)



$\Delta E_{th} = W + Q$

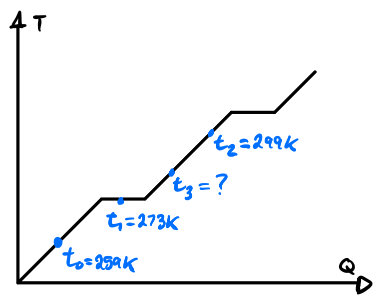
$+ E_{th} = -W + Q$

$Q = \Delta E_{th} + W \quad \therefore \; Q = (+)$

P4. Imagine two ice cubes, each with a mass of 55 grams and an initial temperature of -14.0°C. The ice cubes are dropped into 220 grams of water, which has an initial temperature of 26.0°C and is in a thermally insulated container.

a) What is the final temperature of the beverage once equilibrium is reached?

b) What is the final temperature if only one ice cube is used?



$M_1 = 110 \times 10^{-3}$ kg $\quad$: M_1 is both ice cubes $\quad M_2 = 220 \times 10^{-3}$ kg

$t_{0_1} = 259$ K $\quad Q_P + Q_{\Delta T} = 0 \quad t_{0_2} = 299$ K

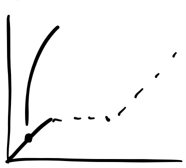
Part a.)

Q_P : $M_1 L_f$

$M_1 = 110 \times 10^{-3}$ kg

$L_f = 3.33 \times 10^5$ J/kgK

$Q_{\Delta T}$: Scene a



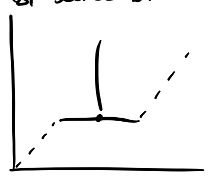
$Q_{\Delta T}$: $M_1 C_a \Delta T_1$

$C_a = 2090$ J/kgK

$\Delta T_1 = 14$ K

$M_1 = 110 \times 10^{-3}$ kg

$Q_{\Delta T}$ Scene b:



$Q_{\Delta T}$: $M_1 C_b \Delta T_2$

$C_b = 4190$ J/kgK

$\Delta T_2 = (T_f - 273K)$

$M_1 = 110 \times 10^{-3}$ kg

$Q_{\Delta T}$: water $\quad Q_{\Delta T_3}$: $M_2 C_c \Delta T_3$

$C_c = 4190$ J/kgK

$M_2 = 220 \times 10^{-3}$ kg

$\Delta T_3 = (T_f - 299K)$

$Q_P + Q_{\Delta T} = M_1 L_f + M_1 C_a \Delta T_1 + M_1 C_b \Delta T_2 + M_2 C_c \Delta T_3 = 0$

$M_1 C_b \Delta T_2 + M_2 C_c \Delta T_3 = -M_1 L_f - M_1 C_a \Delta T_1$

$M_1 C_b (T_1 - 273K) + M_2 C_c (T_1 - 299K) = -M_1 L_f - M_1 C_a \Delta T_1$

$M_1 C_b T_1 - M_1 C_b (273K) + M_2 C_c T_1 - M_2 C_c (299K) = -M_1 L_f - M_1 C_a \Delta T_1$

$M_1 C_b T_1 + M_2 C_c T_1 = -M_1 L_f - M_1 C_a \Delta T_1 + M_1 C_b (273K) + M_2 C_c (299K)$

$M_1 = 110 \times 10^{-3}$ kg $\quad C_a = 2090$ J/kgK $\quad M_2 = 220 \times 10^{-3}$ kg

$L_f = 3.33 \times 10^5$ kg $\quad C_b = C_c = 4190$ J/kgK $\quad \Delta T_1 = 14$ K

$$T_1 = \frac{-M_1 L_f - M_1 C_a \Delta T_1 + M_1 C_b (273K) + M_2 C_c (299K)}{M_1 C_b + M_2 C_c}$$

$$T_1 = \frac{-(110\times10^{-3}kg)(3.33\times10^5 J/kg\cdot K)-(110\times10^{-3}kg)(2090 J/kg\cdot K)(14K)+(110\times10^{-3}kg)(4190 J/kg\cdot K)(273K)+(220\times10^{-3}kg)(4190 J/kg\cdot K)(299K)}{(4190 J/kg\cdot K)(110\times10^{-3}kg+220\times10^{-3}kg)}$$

$T_1 = 261.5 = 262K = -11°C$

Doesn't make Sense

With Two cubes

$T_1 = 0°C$, This is because not all of the ice cubes melt when they are put into the drink. This in turn brings the temperature down but won't go past 0°C because the water isn't melting all of the ice, thus keeping it at 0°

Part b) $T_1 = \frac{-M_1L_f - M_1C_a\Delta T_1 + M_1C_b(273K) + M_2C_c(299K)}{M_1C_b + M_2C_c}$

$M_1 = 55\times10^{-3}kg$ $C_a = 2090 J/kgK$ $M_2 = 220\times10^{-3}kg$

$L_F = 3.33\times10^5 kg$ $C_b = C_c = 4190 J/kgK$ $\Delta T_1 = -14K$

$$T_1 = \frac{-(55\times10^{-3}kg)(3.33\times10^5 J/kg\cdot K)-(55\times10^{-3}kg)(2090 J/kg\cdot K)(14K)+(55\times10^{-3}kg)(4190 J/kg\cdot K)(273K)+(220\times10^{-3}kg)(4190 J/kg\cdot K)(299K)}{(4190 J/kg\cdot K)(55\times10^{-3}kg+220\times10^{-3}kg)}$$

$T_1 = 276.5K = 277K$

$T_1 = 4°C$, $T_1 = 277K$

All of the ice cube melts in this Scenario

P5. Consider a scenario where an unknown mass of 100°C steam is mixed with 210 g of ice, which is at its melting point. The mixing occurs in a thermally insulated container. After thermal equilibrium is reached, the liquid is found to have a temperature of 55°C. Calculate the unknown mass of the steam.

$T_S = 373K$ $T_i = 273K$ $T_1 = 328K$ $Q_{net} = Q_{\Delta T} + Q_{Phase} = 0$

$m_S = ?$ $m_i = 0.21kg$

$L_S = -22.6\times10^5 J/kg$ $L_i = 3.33\times10^5 J/kg$

$C = 4190 J/kgK$

$Q_{\Delta T}: m_S C\Delta T_S + m_I C\Delta T_I$ $Q_P: m_S(L_S) + m_I(L_I)$ $Q_{\Delta T} + Q_P = 0$

$m_S C\Delta T_S + m_i C\Delta T_i + m_S L_S + m_i L_i = 0$

$m_S C\Delta T_S + m_S L_S = -m_i C\Delta T_i - m_i L_i$

$m_S(C\Delta T_S + L_S) = -m_i C\Delta T_i - m_i L_i$

ΔT_i: $T_0 = 273K$ $T_1 = 328K$

$\Delta T = T_1 - T_0 = 328K - 273K = 55K$

$\Delta T_i = 55K$

ΔT_S: $T_0 = 373K$ $T_1 = 328K$

$\Delta T = T_1 - T_0 = 328K - 373K = -45K$

$\Delta T_S = -45K$

$$m_S = \frac{-m_i C\Delta T_i - m_i L_i}{C\Delta T_S + L_S} = \frac{-(0.21kg)(4190 J/kgK)(55K) - (0.21kg)(3.33\times10^5 J/kg)}{(4190 J/kgK)(-45K) + (-22.6\times10^5 J/kg)} = 0.048924\ kg$$

$$\frac{kg\left(\frac{J}{kg\cancel{K}}\right)\cancel{K} - kg\left(\frac{J}{kg}\right)}{\left(\frac{J}{kg\cancel{K}}\right)\cancel{K} + \left(\frac{J}{kg}\right)} = \frac{J - J}{\frac{J}{kg} + \frac{J}{kg}} = kg$$

$m_S = 4.80\times10^{-2}kg$ || $m_S = 48g$

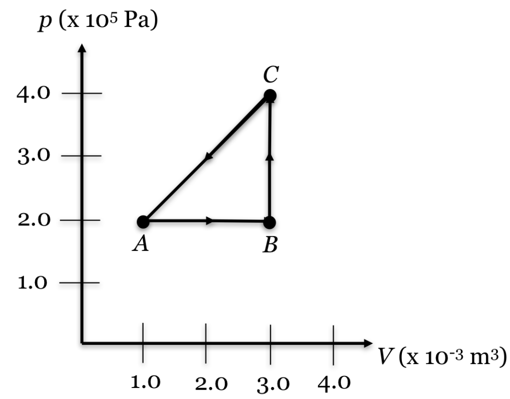
Extra Credit Problem (+12 points):

P6. Consider a sample of oxygen gas, which has a mass of $M = 1.88$ g, that starts at an initial state A as indicated in the attached figure. The gas undergoes an *isobaric expansion* to a state B, then an *isochoric heating*, bringing the gas to state C. Lastly, the gas returns to state A via the path indicated in the attached figure.

a) Calculate the temperature of the gas in Kelvin when the gas is described by states A, B, and C, respectively.

b) Calculate the heat, Q, work done on the gas, W, and thermal energy change, ΔE_{th} for the gas as it goes through *each* of the three aforementioned processes.

Notice: for the state C to state A process, integration must be used!



State A : $PV = nRT$: $T = PV/nR$

$n = \frac{M}{M_{mol}}$ $M = 1.88 \times 10^{-3}$ kg $M_{mol} = 32 \times 10^{-3}$ kg

$n = 0.05875$ mol $\Delta E_{th} = W + Q$ $PV = nRT$

a.) $P_0 = P_1 = 2.0 \times 10^5$ Pa $n = 0.05875$ mol

$V_0 = 1.0 \times 10^{-3}$ m^3 $R = 8.31$ J/mol·K

$V_1 = 3.0 \times 10^{-3}$ m^3

$T_0 = \frac{PV_0}{nR} = \frac{(2.0 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3)}{(0.0588 \text{ mol})(8.31 \text{ J/mol K})} = 409.6 \text{ K} \approx 410 \text{ K}$

$T_1 = \frac{PV_1}{nR} = \frac{(2.0 \times 10^5 \text{ Pa})(3.0 \times 10^{-3} \text{ m}^3)}{(0.0588 \text{ mol})(8.31 \text{ J/mol K})} = 1228.9 \text{ K} \approx 1229 \text{ K}$

State A : isobaric

$T_0 = 410$ K $T_1 = 1229$ K

State B

$P_0 = 2.0 \times 10^5$ Pa $P_1 = 4.0 \times 10^5$ Pa

$V_0 = 3.0 \times 10^{-3}$ m^3 $V_1 = 3.0 \times 10^{-3}$ m^3

$T_0 = 1228.9$ K $T_1 = 2457.8$ K

$T_1 = \frac{P_1 V}{nR} = \frac{(4.0 \times 10^5 \text{ Pa})(3.0 \times 10^{-3} \text{ m}^3)}{(0.0588 \text{ mol})(8.31 \text{ J/mol K})} = 2457.8 \text{ K} \approx 2458 \text{ K}$

State B Isochoric

$T_0 = 1229$ K $T_1 = 2458$ K

State C

$P_0 = 4.0 \times 10^5$ Pa $P_1 = 2.0 \times 10^5$ Pa

$V_0 = 3.0 \times 10^{-3}$ m^3 $V_1 = 1.0 \times 10^{-3}$ m^3

$T_0 = 2457.8$ K $T_1 = 409.6$ K

$T_1 = \frac{P_1 V_1}{nR} = \frac{(2.0 \times 10^5 \text{ Pa})(1.0 \times 10^{-3} \text{ m}^3)}{(0.0588 \text{ mol})(8.31 \text{ J/mol K})} = 409.6 \text{ K} \approx 410 \text{ K}$

State C

$T_0 = 2458$ K $T_1 = 410$ K

b.) $\Delta E_{th} = W + Q$ Q? W? ΔE_{th}?

State A → B

Isobaric Process

W: $W = -P\Delta V$

$P = 2.0 \times 10^5$ Pa

$\Delta V = V_1 - V_0 = (3.0 \times 10^{-3} \text{ m}^3 - 1.0 \times 10^{-3} \text{ m}^3)$

$\Delta V = 2.0 \times 10^{-3}$ m^3

$W = -2.0 \times 10^5 \text{ Pa}(2.0 \times 10^{-3} \text{ m}^3) = -400$ J

$W = -400$ J

Q: $Q = nC_p\Delta T$

$n = 0.05875$ mol

$C_p = 29.2$ J/molK

$\Delta T = 819$ K

$Q = (0.05875 \text{ mol})(29.2)(819 \text{ K}) = 1404.99$ J

$Q = 1405$ J

ΔE_{th} : $\Delta E_{th} = W + Q$

$W = -400$ J

$Q = 1405$ J

$\Delta E_{th} = -400 \text{ J} + 1405 \text{ J} = 1005$ J

$\Delta E_{th} = 1005$ J

$\Delta E_{th} = 1005$ J

$W = -400$ J

$Q = 1405$ J

State B → C

Isochoric Process $W = 0$

W: $W = 0$

this is because the volume does not change

$W = 0$

Q : $Q = nC_v\Delta T$

$n = 0.05875$ mol

$C_v = 20.9$ J/molK

$\Delta T = 1229$ K

$Q = (0.05875 \text{ mol})(20.9 \text{ J/molK})(1229 \text{ K}) = 1509.06$ J

$Q = 1509$ J

$\Delta E_{th} = Q$ $W = 0$

$\Delta E_{th} = 1509$ J

$Q = 1509$ J

$W = 0$ J

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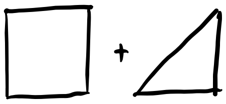
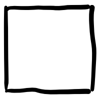
State $C \to D$

$W:$

$W =$ 

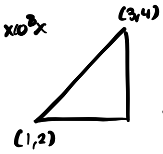


$$\int_{3.0\times10^{-3}}^{1.0\times10^{-3}} \int_{0}^{2.0\times10^{5}Pa} 1 \; dy\,dx$$

$$\int_{3.0\times10^{-3}}^{1.0\times10^{-3}} 2.0\times10^{5}Pa \; dx$$

$$2.0\times10^{5}Pa\, x \Big|_{3.0\times10^{-3}m^3}^{1.0\times10^{-3}m^3} = -400\ J$$

$y = 1.0\times10^{8}x$



$$\int_{3.0\times10^{-3}}^{1.0\times10^{-3}} \int_{0}^{-1.0\times10^{8}x+1.0\times10^{5}} 1 \; dy\,dx$$

$$\int_{3.0\times10^{-3}}^{1.0\times10^{-3}} -1.0\times10^{8}x+1.0\times10^{5} \; dx$$

$$-0.5\times10^{8}x^2+1.0\times10^{5}x \Big|_{3.0\times10^{-3}}^{1.0\times10^{-3}}$$

$A_1 = 400J$ $\quad W = A_1 + A_2$ $\quad A_2 = 200J$

$Q_{net} = 0 \; : \; Q_{AB} + Q_{BC} + Q_{CA} = 0 \; : \; Q_{CA} = -Q_{AB} - Q_{BC}$

$= (-1405J) - (1509J)$

$Q_{CA} = -2914\ J$

$\Delta E_{th} = W + Q$

$= 600J - 2914J$

$\Delta E_{th} = -2314\ J$

$\Delta E_{th} = -2314\ J$
$W = 600\ J$
$Q = -2914\ J$