MAT 201
Larson/Edwards – Section 4.5
Integration by Substitution (Part 2)

In Part 1 of our integration by substitution notes we neglected to find indefinite integrals of trigonometric functions as well as definite integrals. We will address those types of problems in these notes. We will begin with some examples.

Ex: Find the following indefinite integrals.

a) $\int 4x^3 \sin x^4 \; dx$

b) $\int \frac{\sin x}{\cos^3 x} \, dx$

Ex: Find the following indefinite integrals.

a) $\underbrace{\int 4x^3}_{g'(x)} \underbrace{\sin x^4}_{f(g(x))} \, dx = \boxed{-\cos x^4 + c}$

$u = x^4$

$du = 4x^3 \, dx$

$\int \sin u \, du$

$= -\cos u + C$

$= \boxed{-\cos x^4 + C}$

b) $\int \frac{\sin x}{\cos^3 x}\,dx = \int \frac{\sin x}{(\cos x)^3}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$-\int \frac{1}{u^3}\,du$

$-\int u^{-3}\,du = \frac{u^{-2}}{2} + C$

$= \frac{(\cos x)^{-2}}{2} + C$

$= \boxed{\frac{1}{2\cos^2 x} + C}$

$\int \frac{\sin x}{\cos^3 x}\,dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^2 x}\,dx$

$= \int \tan x \cdot \sec^2 x\,dx$ $\quad u = \tan x$

$du = \sec^2 x\,dx$

$= \int u\,du$

$= \frac{u^2}{2} + C = \frac{\tan^2 x}{2} + C$

How does $\frac{1}{2}\tan^2 x + C = \frac{1}{2\cos^2 x} + C$?

$= \frac{\sec^2 x}{2} + C$

$\frac{1}{2}(\sec^2 x - 1) + C$

$\frac{1}{2}\sec^2 x - \frac{1}{2} + C$

How would we evaluate the definite integral $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$?

How does the method of substitution to evaluate this integral affect the limits of integration?

<u>Change of Variables for Definite Integrals</u>:

If the function $u = g(x)$ has a continuous derivative on the closed interval $[a, b]$ and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du$$

What is this saying?

<u>Tools For Evaluating Definite Integrals with a Change of Variables</u>:

1. Evaluate the ***definite integral*** with the lower limit and upper limit changed according to the u-substitution, or
2. Evaluate the ***indefinite integral*** with the u-substitution, then use the original limits of integration.

Ex: Evaluate the definite integral $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$ by (1) changing the limits of integration according to the u-substitution, and by (2) using the original limits of integration. Confirm that these answers are the same.

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

Lower limit $= 2(0)+1 = 1$

Upper limit $= 2(4)+1 = 9$

$$\int_1^9 \frac{1}{2u^{1/2}}\,du$$

$$\int_1^9 \frac{1}{2}u^{-1/2}\,du = \frac{1}{2}\frac{u^{1/2}}{1/2}\Bigg|_1^9$$

$$= u^{1/2}\Big|_1^9$$

$$= 9^{1/2} - 1^{1/2} = \boxed{2}$$

$$\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx = u^{1/2}$$

$$\int \frac{1}{\sqrt{2x+1}}\,dx = u^{1/2}$$

$$= (2x+1)^{1/2}\Big|_0^4$$

$$= (9)^{1/2} - (1)^{1/2}$$

$$= \boxed{2}$$

Ex: Evaluate the definite integral $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$ by (1) changing the limits of integration according to the *u*-substitution, and by (2) using the original limits of integration. Confirm that these answers are the same.

Integration of Even and Odd Functions:
Let f be integrable on the closed interval $[-a, a]$.

1. If f is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx.$

2. If f is an odd function, then $\int_{-a}^{a} f(x)dx = 0.$

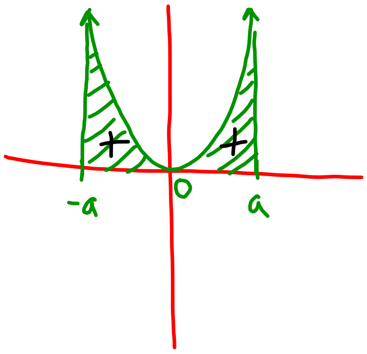
Why do these rules make sense?

ntegration of Even and Odd Functions:
Let f be integrable on the closed interval $[-a, a]$.

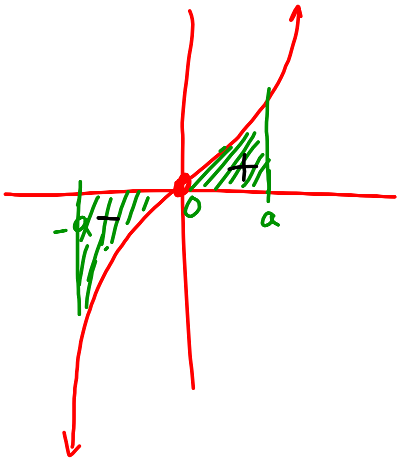
1. If f is an even function, then
$$\int_{-a}^{a} f(x)\,dx = 2\left[\int_{0}^{a} f(x)\,dx.\right]$$
2. If f is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0.$

Why do these rules make sense?

Even function:



Odd Function:



$$\int_{-5}^{5} \sqrt[3]{x}\,dx = 0$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = 2\int_{0}^{\pi/2} \cos x\,dx$$