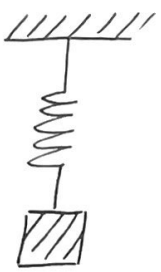
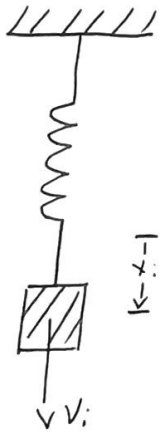
PHYS 230 Homework Set 11

IN EQUILIBRIUM..





. $m = 0.55\,kg$

$k = 46\,N/m$

$x_i = 5.5\times10^{-2}\,m$

$v_i = 0.45\,m/s$

1) $x(t) = A\cos(\omega t + \phi)$ WHERE $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{46\,N/m}{0.55\,kg}}$

$\omega = 9.15\,rad/s$

NOW, APPLYING INITIAL CONDITIONS..

1) $x(t=0) = A\cos(\phi) = 5.5\times10^{-2}\,m$

THE VELOCITY IS GIVEN BY...

$\dot{x}(t) = -\omega A\sin(\omega t + \phi)$ SO AT $t=0$..

2) $\dot{x}(t=0) = -\omega A\sin\phi = +0.45\,m/s$

NOW DIVIDING THE SECOND BY THE FIRST YIELDS..

$$\frac{+0.45\,m/s}{5.5\times10^{-2}\,m} = \frac{-\omega A\sin\phi}{A\cos\phi} = -\omega\tan\phi = 8.18\,s^{-1}$$

SO $\phi = \tan^{-1}\left(\frac{8.18\,s^{-1}}{-9.15\,rad/s}\right) = -41.8° = -0.729\,rad$

PLUGGING THIS BACK INTO THE FIRST INITIAL CONDITION YIELDS..

$$A = \frac{5.5\times10^{-2}\,m}{\cos(-41.8°)} = 7.4\times10^{-2}\,m$$

SO, IN SUMMARY..

$x(t) = A\cos(\omega t + \phi)$ WHERE $A = 7.4\times10^{-2}\,m$, $\omega = 9.15\,rad/s$, AND $\phi = -41.8° = -0.729\,rad$

b) NOW $T = \frac{2\pi}{\omega} = \frac{2\pi}{9.15\ \text{RAD/S}} = 0.69\ \text{S}$

c) THE TOTAL ENERGY OF THE OSCILLATOR IS GIVEN BY..

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(46\ \text{N/m})(7.4\times10^{-2}\ \text{m})^2 = 0.126\ \text{J}$$

$$\boxed{E = 0.126\ \text{J}}$$

d) SINCE $\dot{x}(t) = -\omega A \sin(\omega t + \phi)$ REACHES A MAXIMUM SPEED WHEN $\sin(\omega t + \phi) = -1$

SO

$$\dot{x}_{MAX} = \omega A = (9.15\ \text{RAD/S})(7.4\times10^{-2}\ \text{m}) = 0.68\ \text{m/s}$$

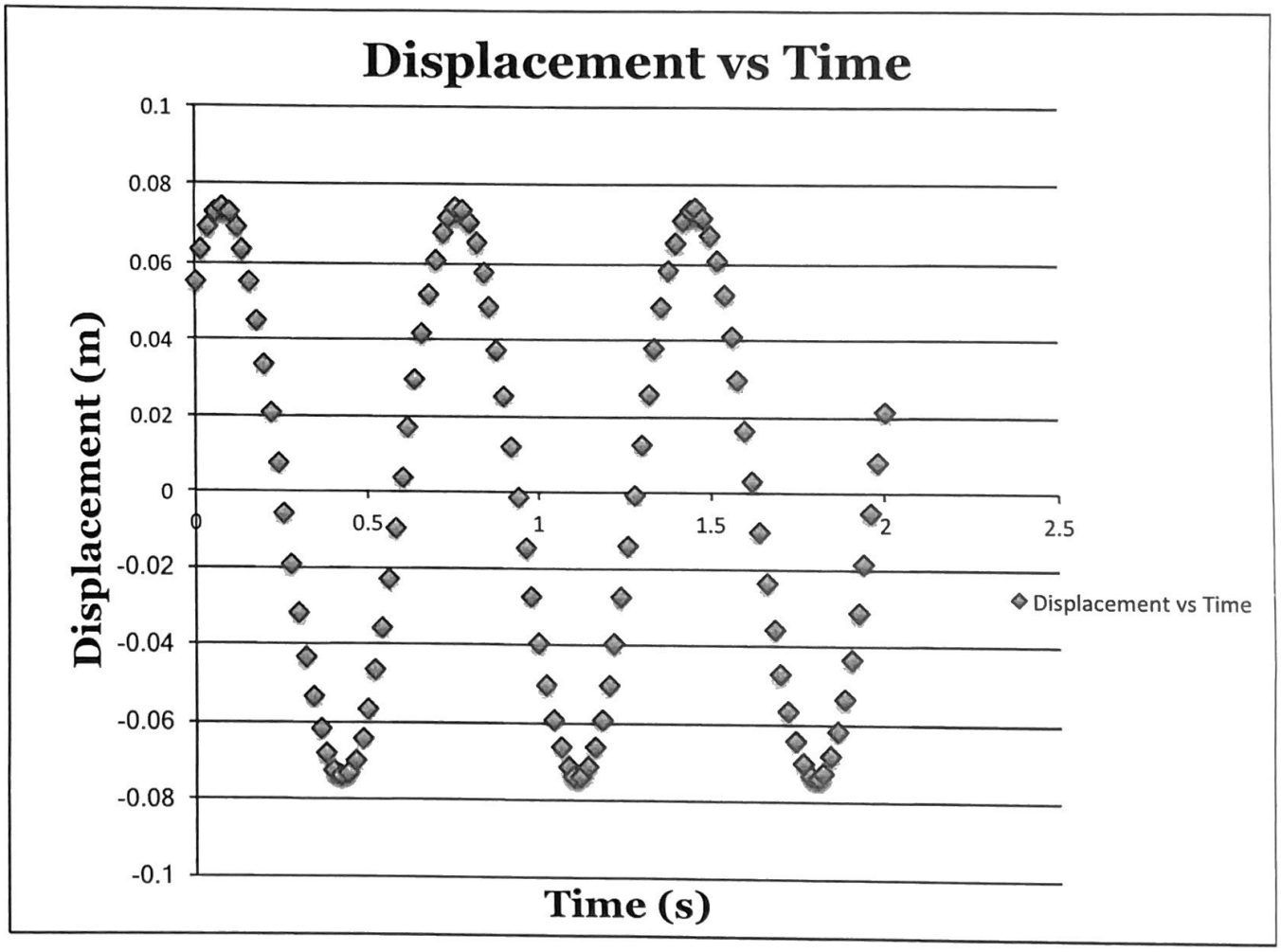
$$\boxed{\dot{x}_{MAX} = 0.68\ \text{m/s}}$$

A = 0.074 m

ω = 9.15 1/s

φ = -0.729

t(s)	Acos(ωt+φ)
0	0.05519223
0.02	0.06324102
0.04	0.06917785
0.06	0.07280443
0.08	0.07399967
0.1	0.07272363
0.12	0.06901895
0.14	0.06300933
0.16	0.05489548
0.18	0.04494835
0.2	0.03350015
0.22	0.02093318
0.24	0.00766714
0.26	-0.0058549
0.28	-0.0191815
0.3	-0.0318675
0.32	-0.0434892
0.34	-0.0536586
0.36	-0.0620361
0.38	-0.0683418
0.4	-0.0723651
0.42	-0.0739718
0.44	-0.0731082
0.46	-0.0698031



3. NOW

a) $x(t) = A\cos(\omega t + \phi)$

NOW, APPLYING INITIAL CONDITIONS

$$x(t=0) = A\cos(\phi) = 0 \quad \therefore \quad \phi = \pi/2 \text{ RAD} = 90^\circ$$

TAKING THE TIME DERIVATIVE YIELDS THE VELOCITY..

$$\dot{x}(t) = -\omega A \sin(\omega t + \pi/2) \quad \text{so}$$

$$\dot{x}(t=0) = -\omega A \sin(\pi/2) = -\omega A = -1.1 \text{ m/s}$$

$$\text{so} \quad A = +\frac{1.1 \text{ m/s}}{\omega} = +\frac{1.1 \text{ m/s}}{9.15 \text{ RAD/s}} = 0.12 \text{ m}$$

SO

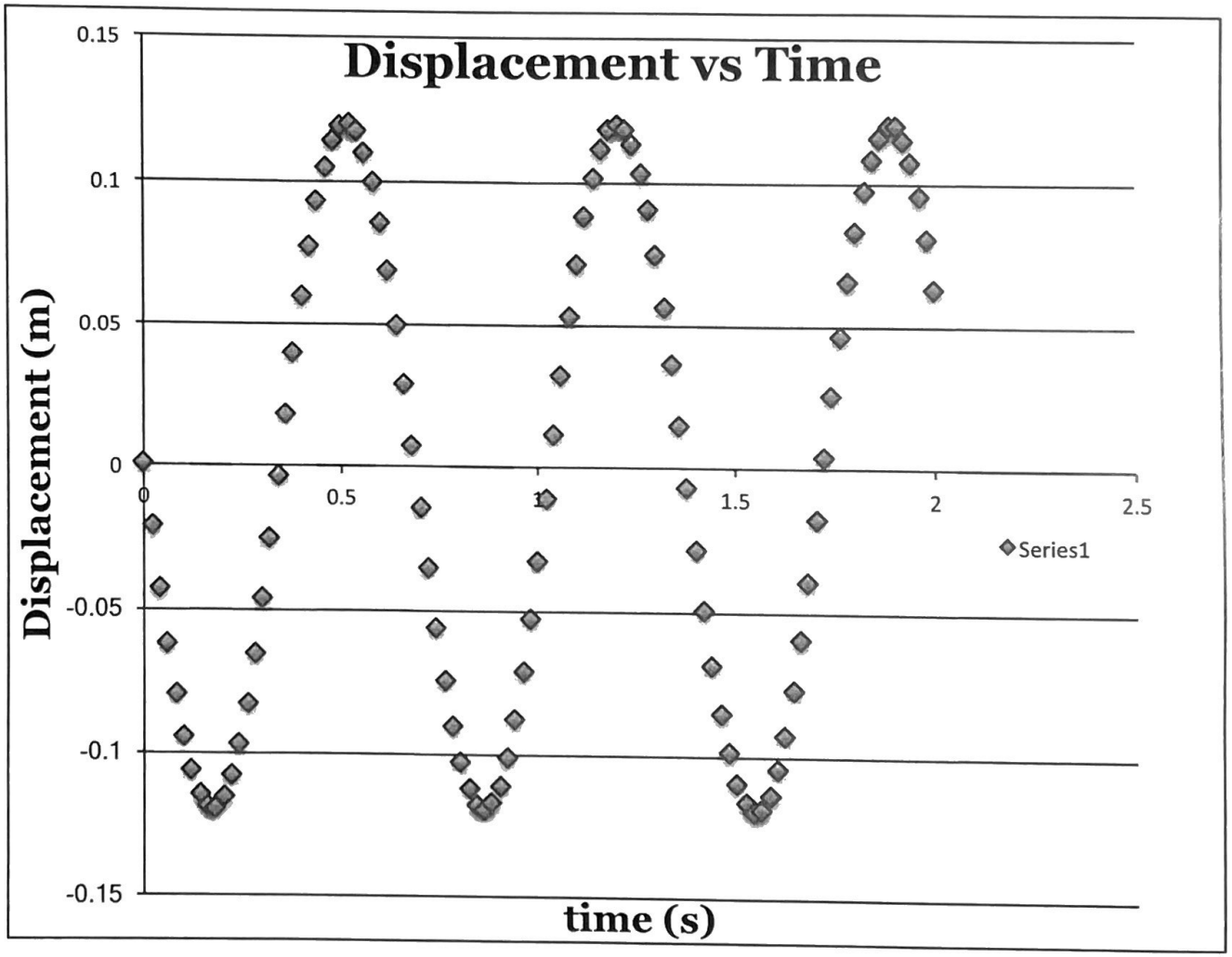
$x(t) = A\cos(\omega t + \phi)$ WHERE $A = 0.12\text{m}$, $\omega = 9.15 \text{ RAD/s}$, $\therefore \phi = \pi/2$ RAD

A = 0.12 m

ω = 9.15 1/s

φ = 1.57

t(s)	Acos(ωt+φ)
0	9.5559E-05
0.02	-0.0217437
0.04	-0.0428567
0.06	-0.0625386
0.08	-0.0801319
0.1	-0.0950492
0.12	-0.1067923
0.14	-0.1149689
0.16	-0.1193061
0.18	-0.119659
0.2	-0.1160158
0.22	-0.1084982
0.24	-0.0973572
0.26	-0.0829649
0.28	-0.0658019
0.3	-0.0464415
0.32	-0.0255301
0.34	-0.0037661
0.36	0.0181237
0.38	0.03940821
0.4	0.05937666
0.42	0.07736219
0.44	0.09276415
0.46	0.1050682

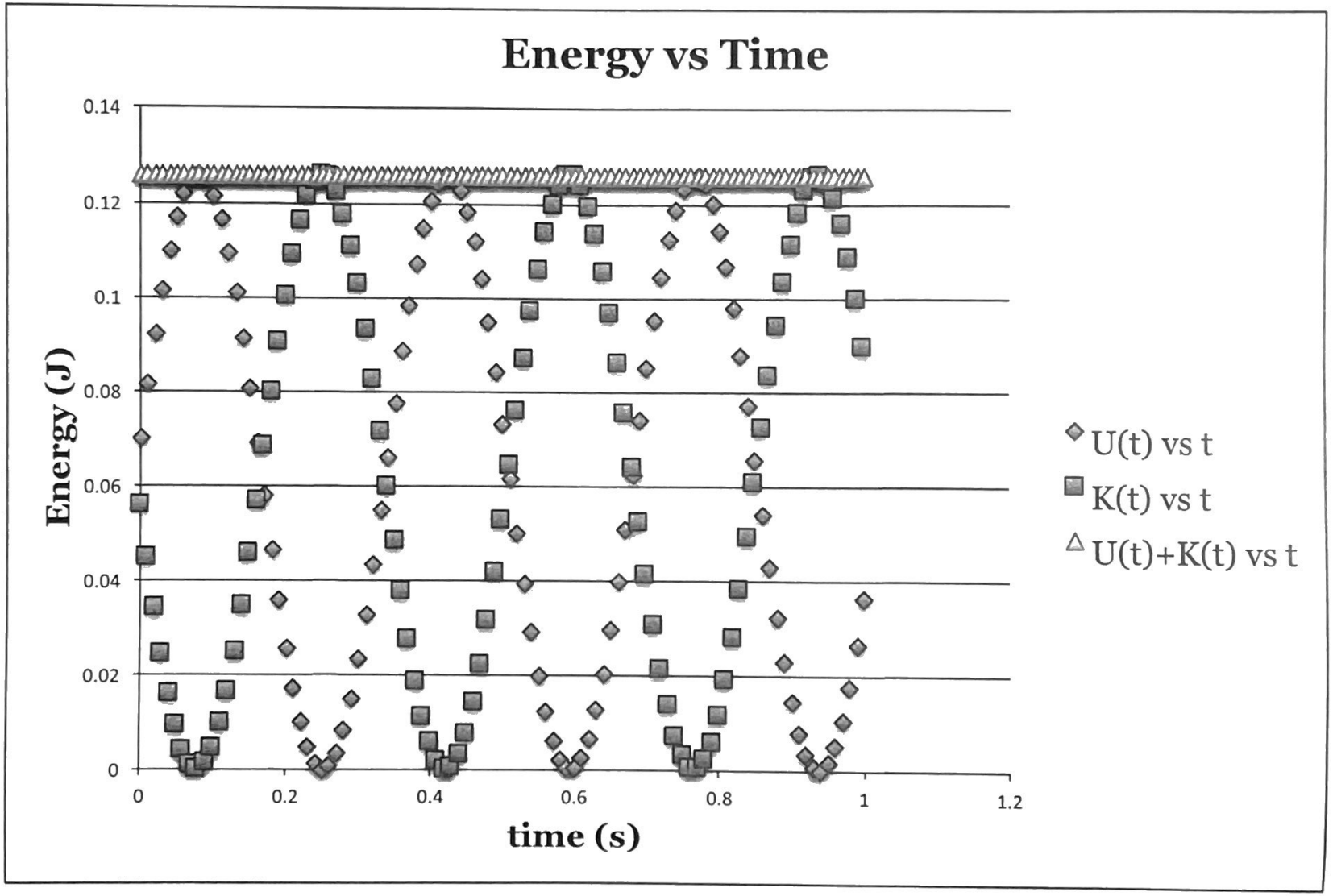


4. a) $U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$

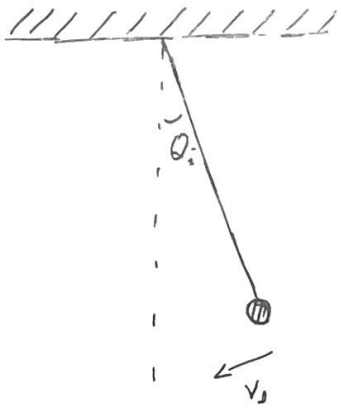
b) $K(t) = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$

A =	0.074	m
ω =	9.15	1/s
φ =	-0.729	
m =	0.55	kg
k =	46	N/m

t(s)	$x(t)=A\cos(\omega t+\varphi)$	$v(t)=-\omega A\sin(\omega t+\varphi)$	$U(t)=kx^2(t)/2$	$K(t)=mv^2(t)/2$	U(t) +K(t)
0	0.055192227	0.451032647	0.070062183	0.055943373	0.12601
0.01	0.059465382	0.403002024	0.081331027	0.044662924	0.12599
0.02	0.063241025	0.35159972	0.091986826	0.03399615	0.12598
0.03	0.066487568	0.297255789	0.101673724	0.024299276	0.12597
0.04	0.069177849	0.240424894	0.110068219	0.015896136	0.12596
0.05	0.071289359	0.181582506	0.116889972	0.009067357	0.12596
0.06	0.072804433	0.121220924	0.121911167	0.004040991	0.12595
0.07	0.073710396	0.059845158	0.124964116	0.000984897	0.12595
0.08	0.073999667	-0.002031297	0.125946866	1.1347E-06	0.12595
0.09	0.073669827	-0.063890757	0.124826597	0.001122558	0.12595
0.1	0.072723634	-0.125215682	0.12164072	0.004311716	0.12595
0.11	0.071169006	-0.185493	0.11649563	0.009462105	0.12596
0.12	0.069018949	-0.244218408	0.109563151	0.016401723	0.12596
0.13	0.06629145	-0.300900584	0.101074797	0.024898819	0.12597
0.14	0.063009331	-0.355065303	0.091314043	0.034669627	0.12598
0.15	0.059200049	-0.406259399	0.080606854	0.045387842	0.12599
0.16	0.054895476	-0.454054563	0.069310805	0.056695525	0.12601
0.17	0.050131624	-0.49805092	0.057803134	0.068215048	0.12602
0.18	0.044948351	-0.537880378	0.046468148	0.079561708	0.12603
0.19	0.039389021	-0.573209709	0.035684385	0.090356577	0.12604
0.2	0.033500147	-0.603743332	0.025811976	0.100239153	0.12605
0.21	0.027330997	-0.62922579	0.017180618	0.108879401	0.12606


Energy vs Time
Energy (J)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
time (s)
U(t) vs t
K(t) vs t
U(t)+K(t) vs t

5. $l = 16m$



$$\theta_i = 5.0° \times \frac{\pi \text{ RAD}}{180°}$$

$$= 0.0873 \text{ RAD}$$

NOW

$$V_i = l\omega_i = 1.0 \text{ m/s} \quad \text{so}$$

$$\omega_i = \frac{V_i}{l} = \frac{1.0 \text{ m/s}}{16 \text{ m}} = 0.0625 \frac{\text{RAD}}{\text{s}}$$

NOW

$$\theta(t) = \theta_0 \cos(\omega t + \phi)$$

WHERE $\omega = \sqrt{\frac{g}{l}}$ SO

a) $$\omega = \sqrt{\frac{9.8 \text{ m/s}^2}{16 \text{ m}}} = 0.783 \text{ RAD/s}$$

ON APPLYING INITIAL CONDITIONS..

1) $$\theta(t=0) = \theta_0 \cos(\phi) = 0.0873 \text{ RAD}$$

ALSO

$$\dot{\theta} = -\omega \theta_0 \sin(\omega t + \phi)$$

SO

2) $$\dot{\theta}(t=0) = -\omega \theta_0 \sin(\phi) = 0.0625 \text{ RAD/s}$$

DIVIDING THE 2ND BY THE 1ST YIELDS..

$$\frac{0.0625 \text{ RAD/s}}{0.0873 \text{ RAD}} = \frac{-\omega \theta_0 \sin(\phi)}{\theta_0 \cos(\phi)} = -\omega \tan\phi = 0.716 \text{ s}^{-1}$$

SO

$$\tan\phi = \frac{0.716 \text{ s}^{-1}}{-(0.783 \text{ RAD/s})} = -0.914$$

$$[\phi = -42.4° = -0.74 \text{ RAD}]$$

PLUGGING THIS BACK INTO THE 1ST INITIAL CONDITION YIELDS..

$$\theta_0 = \frac{0.0873 \text{ RAD}}{\cos(\phi)} = \frac{0.0873 \text{ RAD}}{\cos(-42.4°)} = 0.118 \text{ RAD} \times \frac{360°}{2\pi \text{ RAD}} = 6.78°$$

So

$$\theta(t) = \theta_0 \cos(\omega t + \phi) \quad \text{where} \quad \theta_0 = 0.12 \text{ RAD}, \ \omega = 0.783 \text{ RAD/S}, \ \phi = -0.74 \text{ RAD}$$

b) $$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.783 \text{ RAD/S}} = 8.0 \text{ S}$$

c) Now

$$\dot{\theta} = -\omega \theta_0 \sin(\omega t + \phi) \quad \text{which is a maximum when} \quad \sin(\omega t' + \phi) = -1$$

So

$$\dot{\theta}_{MAX} = \omega \theta_0 = (0.783 \text{ RAD/S})(0.118 \text{ RAD}) = 0.0924 \text{ RAD/S}$$

So the maximum speed is..

$$V_{MAX} = \ell \dot{\theta}_{MAX} = (16 \text{ M})(0.0924 \text{ RAD/S}) = 1.5 \text{ M/S}$$

$$\boxed{V_{MAX} = 1.5 \text{ M/S}}$$

Now

d) $\theta(t) = \theta_0 \cos(\omega t + \phi)$ so θ_{MAX} when $\cos(\omega t' + \phi) = 1$

So

$$\boxed{\theta_{MAX} = \theta_0 = 0.118 \text{ RAD} = 6.8^\circ}$$