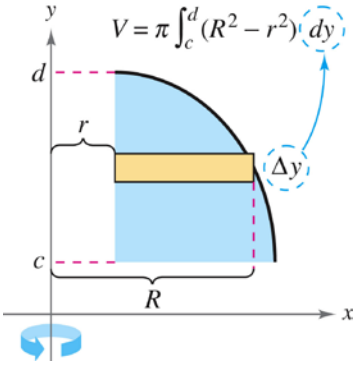
MAT 201
Larson/Edwards – Section 7.3 (Part 2)
Volume: The Shell Method

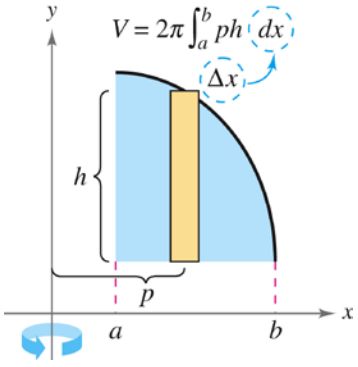
In this section we will compare the shell and the disk method for finding volumes of solids of revolution.

Given the following diagrams and axis of revolution, determine which method (disk or shell) that would be the easiest to evaluate the volume:

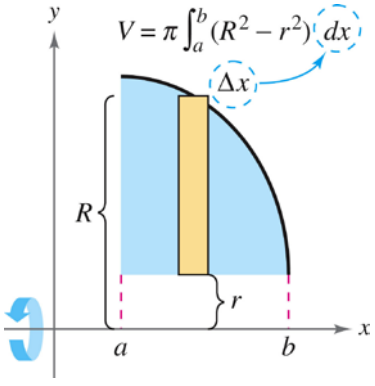
a) Disk Method



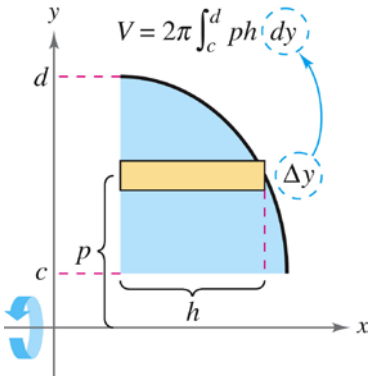
Shell Method



b) Disk Method



Shell Method



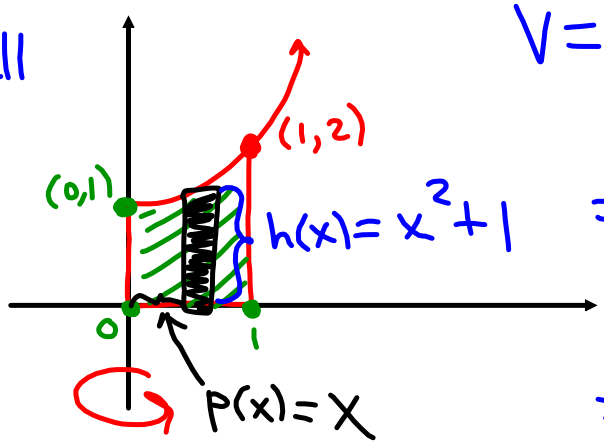
Ex: Use any method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis: $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$.

Ex: Use any method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region given below about the x-axis. $y = 1 - \frac{x^2}{4}$, $-2 \le x \le 2$

Ex: Use any method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis:

$y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$.

Shell

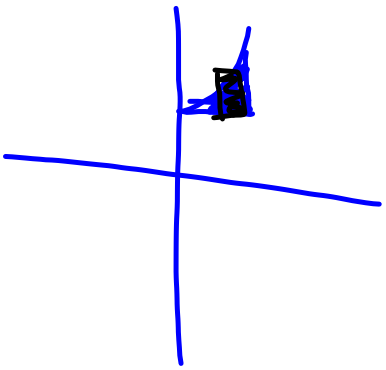


$$V = 2\pi \int_0^1 x(x^2+1)\,dx$$

$$= 2\pi \int_0^1 x^3 + x\,dx$$

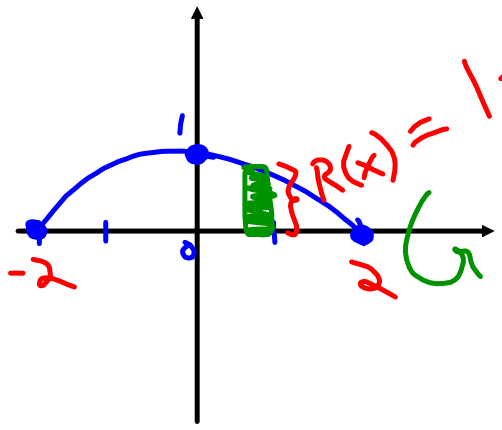
$$= \frac{3\pi}{2} \text{ units}^3$$

$$\approx 4.71$$



Ex: Use any method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region given below about the x-axis.

$$y = 1 - \frac{x^2}{4}, \quad -2 \le x \le 2$$



Disk

$$V = \pi \int_{-2}^{2} \left[1 - \frac{x^2}{4}\right]^2 dx$$

$$= \boxed{\frac{32\pi}{15} \text{ units}^3}$$

$$\approx 6.7$$

Ex: Use any method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region given below about the line $x = 2$. $y = x^3 + x + 1,\ y = 1,\ and\ x = 1$

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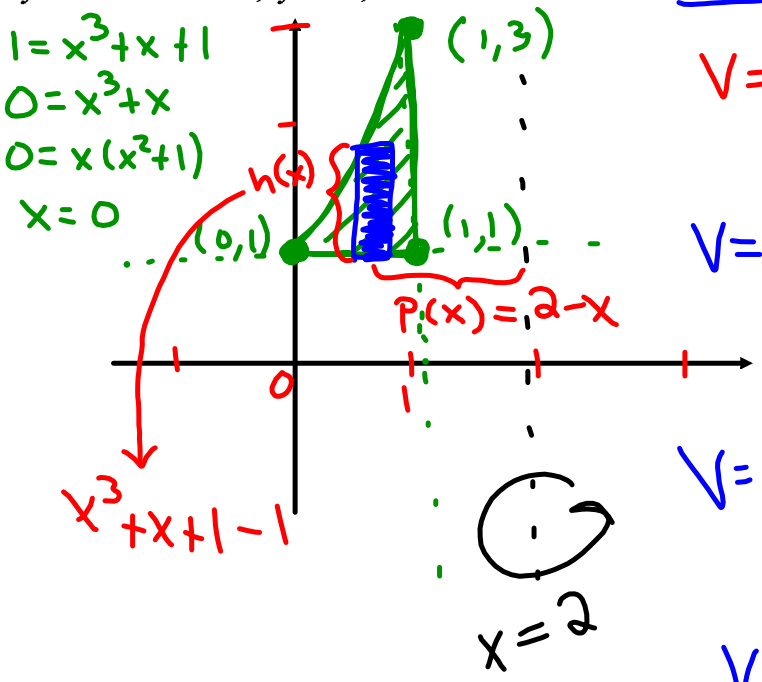

Shell

$$V = 2\pi \int_a^b [p(x)\,h(x)]\,dx$$

$$V = 2\pi \int_0^1 (2-x)(x^3+x)\,dx$$

$$V = 2\pi \int_0^1 2x^3 + 2x - x^4 - x^2\,dx$$

$$V = \frac{29\pi}{15} \approx 6.07 \text{ units}^3$$