

## Electromagnetic Theory II: Homework 14

Due: 30 March 2021

### 1 Potentials for a spherical situation

Suppose that, in spherical coordinates,

$$V = 0$$

and

$$\mathbf{A} = \begin{cases} 0 & \text{if } r < R \\ \frac{\alpha}{r} \cos(\omega t) \hat{\phi} & \text{if } r > R \end{cases}$$

where  $\alpha, \omega$  and  $R$  are positive constants.

- Determine the electric and magnetic fields associated with these potentials.
- Determine the charge and current distributions that give rise to these potentials.
- Determine the Poynting vector associated with these fields. Which way is energy transported by the fields?

### 2 Time-varying electric fields

The potential formulation allows one to construct time-varying electric fields that are not accompanied by induced magnetic fields. Suppose that  $\mathbf{A} = 0$  and that

$$V = \begin{cases} 0 & \text{if } r < R \\ \frac{q}{4\pi\epsilon_0 r} \cos(\omega t) & \text{if } r > R \end{cases}$$

where  $q$  and  $\omega > 0$  are constants.

- Determine the electric and magnetic fields produced by these potentials. Sketch the fields.
- Do these fields radiate electromagnetic energy? Explain your answer.
- Determine the source charge and current densities that produce these fields.
- Verify that these source charges and currents satisfy the continuity equation for  $r > R$ .

### 3 Localized charge and current distribution and time varying fields

A charge and current distribution produces a potential, given in cylindrical coordinates by

$$V = \begin{cases} 0 & \text{if } s < R \\ \frac{\sigma(t)R}{\epsilon_0} \ln s & \text{if } s > R \end{cases}$$

where  $\sigma(t)$  has units of  $\text{C}/\text{m}^2$  and varies with time. Is it possible that this can be produced by a charge and current density which is strictly zero beyond  $R' > R$ ? Explain your answer.