4.3 The Discrete Fourier Transform

orthogonality

$$\langle g_m, g_n \rangle = \begin{cases} N, & m = n \\ 0, & m \neq n \end{cases}$$

orthogonal if $\langle g_m, g_n \rangle = 0$

$$\langle x, y \rangle = \sum_{1}^{N} x_k \bar{y}_k$$

Conjugate Symmetry

$C \equiv$ DFT vector associated w/x

$N \equiv$ Length

Conjugates about $N/2$ entries

$$\mathrm{Im}\left(c_{N/2}\right) = 0$$

$$c_{(N/2)+1} = \bar{c}_{(N/2)-1}$$

$$c_{(N/2)+2} = \bar{c}_{(N/2)-2}$$

$$\vdots$$

$$c_{N-1} = \bar{c}_1$$

Discrete Fourier Transform (DFT) Matrix

$$t_k = kT/N, \quad k = 0, 1, \ldots, N-1$$

$$F_N = \frac{1}{N}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w & \cdots & w^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{N-1} & \cdots & w^{(N-1)(N-1)} \end{bmatrix} \qquad w = e^{-2\pi i \frac{1}{N}}$$