Phys 131 2016

Lecture 21

Tues: Review Exam 2

Weds: Exam 2 Covers Chs 5,6,7,8
Lectures 11-21
HW 4-6

Review 2014 Class Exam 2
2015 Class Exam 2

Nobel Prize → show website.

Dynamics of uniform circular motion

In uniform circular motion:

The acceleration is radially inward with magnitude $a = v^2/r \qquad a = \omega^2 r$ and the net force is radially inward.

Aside from these facts the usual rules of Newton's dynamics apply to such situations

Quiz 1 50% - 70%

60% - 90%

These facts about circular motion also apply in more general cases where the forces align in one straight line

Quiz 2 100% 3 100%

Quiz 3 30% - 50% 3 30% - 60%

55 Banked turn on a road

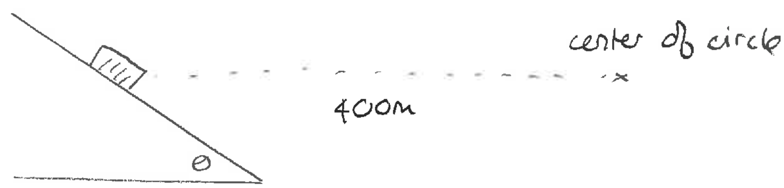
A road is constructed with a turn of radius ~~400~~ 300 m. The surface of the road is banked at an angle in order to assist cars to make the turn. A car travels through this turn with constant speed of ~~30~~ 35 m/s (about 67 mph).

a) Suppose that there is no friction between the tires and road. Determine the angle at which the road must be banked so that the car can complete the turn without slipping.

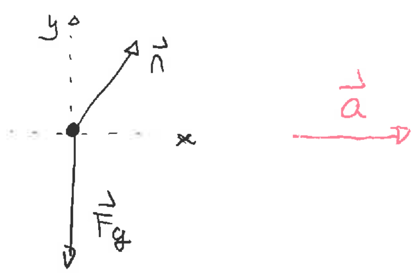
b) There usually is friction between the tires and road. For wet concrete and rubber the coefficient of static friction is ~~1.00~~ 0.30 and the coefficient of kinetic friction is ~~0.80~~ 0.20. Determine the minimum angle at which the road must be banked so that the car completes the turn without slipping.

Answer:

a)



1) FBD



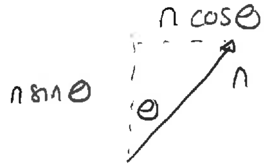
$a_x = a = \frac{v^2}{r}$

$a_y = 0$

2) $\sum F_x = m a_x \quad \Rightarrow \quad \sum F_x = m v^2/r$

$\sum F_y = m a_y = 0 \qquad \sum F_y = 0$

3) components



	x	y
$\vec{n}$	$n\sin\theta$	$n\cos\theta$
$\vec{F}_g$	0	$-mg$

$\Rightarrow$

$$\sum F_x = mv^2/r \Rightarrow n\sin\theta = \frac{mv^2}{r}$$

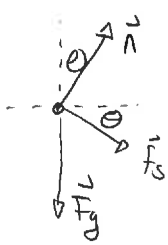
$$\sum F_y = 0 \Rightarrow n\cos\theta - mg = 0$$

$$\Rightarrow n = \frac{mg}{\cos\theta}$$

Combine:
$$\frac{\cancel{m}g}{\cos\theta}\sin\theta = \frac{\cancel{m}v^2}{r} \Rightarrow \tan\theta = \frac{v^2}{gr}$$

Here
$$\tan\theta = \frac{(35\,m/s)^2}{9.8\,m/s^2 \times 300\,m} = 0.42 \Rightarrow \theta = \tan^{-1}(0.42) \Rightarrow \theta = 23°$$

b The friction prevents the car from sliding off the outer edge of the road. It is static.



$$\sum F_x = mv^2/r$$

$$\sum F_y = 0$$

	x	y
$\vec{F}_g$	0	$-mg$
$\vec{n}$	$n\sin\theta$	$n\cos\theta$
$\vec{f}_s$	$f_s\cos\theta$	$-f_s\sin\theta$

We want max friction

$$f_s = \mu_s n$$

Then
$$\sum F_x = mv^2/r \Rightarrow n\sin\theta + \mu_s n\cos\theta = \frac{mv^2}{r} \Rightarrow n(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$$

$$\sum F_y = 0 \Rightarrow -mg + n\cos\theta - \mu_s n\sin\theta = 0$$

$$\Rightarrow n(\cos\theta - \mu_s\sin\theta) = mg$$

$$\Rightarrow n = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

Combine:
$$\cancel{m}g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = \frac{\cancel{m}v^2}{r}$$

$$\Rightarrow \quad (\sin\theta + \mu_s \cos\theta) = \frac{v^2}{gr}(\cos\theta - \mu_s \sin\theta)$$

Divide by $\cos\theta$

$$\tan\theta + \mu_s = \frac{v^2}{gr}(1 - \mu_s \tan\theta)$$

$$\Rightarrow \tan\theta\left[1 + \frac{\mu_s v^2}{gr}\right] = \frac{v^2}{gr} - \mu_s$$

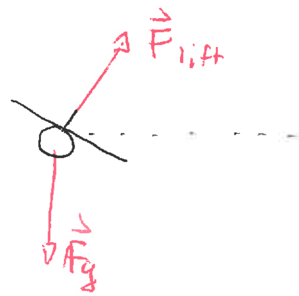
$$\Rightarrow \tan\theta = \frac{\frac{v^2}{gr} - \mu_s}{\frac{\mu_s v^2}{gr} + 1}$$

Then $\frac{v^2}{gr} = 0.42$ $\quad \Rightarrow \tan\theta = \frac{0.42 - 0.30}{0.30 * 0.42 + 1} = \frac{0.12}{1.13}$

$$\Rightarrow \tan\theta = 0.11$$

$$\Rightarrow \theta = \tan^{-1}(0.11) \Rightarrow \theta = 6.1^\circ \quad \blacksquare$$

Analysis of this sort applies to banking aircraft



* vertical components cancel
* horiz component is radially inward + provides acceleration.

Quiz 4] no time

Quiz 5