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Phys 131L
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Laboratory 8: Work and Kinetic Energy

Newton's Laws of motion relate forces and accelerations and, in principle, using these suffices for analyzing any classical mechanics situation. However, it is often more feasible and informative to use work and energy; in some cases this offers the only analytical method of describing the situation. These exercises introduces you to the concepts of work and energy. Much of this exercise is based on a similar exercise in *Tutorials in Introductory Physics* by McDermott and Shaffer.

1 Work and Kinetic Energy

Consider an object which moves under the influence of a constant force $\vec{\mathbf{F}}$ while moving in a straight line through displacement $\Delta\vec{\mathbf{r}}$, as illustrated in Fig. 1.

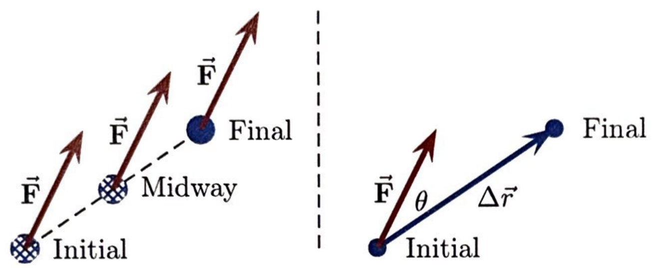


Figure 1: Constant force acting on an object moving in a straight line.

The work done by this force is

$$\boxed{W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = F\Delta r\cos\theta} \tag{1}$$

where θ is as defined in Fig. 1, F is the magnitude of $\vec{\mathbf{F}}$ and Δr is the distance traveled by the object.

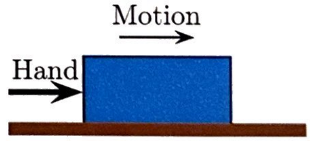


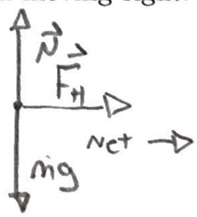
Figure 2: Block moving right.

a) A block on a frictionless surface moves right while a hand pushes horizontally to the right on it. Using Newton's second law, explain whether the block's speed increases, decreases or is constant while the hand pushes on it. $\rightarrow$

$\Sigma_x = ma_x$

$\vec{F}_H = ma_x$

While the hand implements a force on the block, the blocks speed increases due to the net force being to the right.



b) Sketch vectors for the displacement and the force exerted by the hand on the block. Use the sketch to identify the angle between the vectors and then use the definition of work, Eq. (1), to explain whether the work done on the block by the hand is positive, negative or zero.

$W = \vec{F} \cdot \Delta r \cdot \cos\theta$

Hand on Block $\vec{F_H}$ ——▷ Displacement $\vec{\Delta r}$ ——▷ $\theta = 0°$

$\to \vec{F_H} \cdot \Delta r \cdot \cos(0)$

$\to \vec{F_H}\, \Delta r\,(1) = \vec{F_H}\,\Delta r$

The work is positive

c) Use Eq. (1) to explain whether the work done on the block by Earth's gravitational force and also the work done by normal force are positive, negative or zero.

Gravity $W = \vec{F} \Delta r \cdot \cos\theta$

$\theta = 90$

$mg \cdot \Delta r \cdot \cos 90$

$mg\,\Delta r\,(0) = 0$

Δr ——▷, mg ↓

$\boxed{\text{Work by Gravity} = 0}$

Normal $W = F \Delta r \cos\theta$

$\theta = 90$, $F = \vec{N}$

$= \vec{N} \Delta r \cos(90)$

$= \vec{N} \Delta r\,(0)$

$W = 0$

$\vec{N}$ ↑, Δr ——▷

$\boxed{\text{Work by Normal} = 0}$

d) The net work, W_{net}, is the sum of the works done by all forces. Explain whether the net work done on the block is positive, negative or zero.

$W_{net} = W_{Hand} + W_{Gravity} + W_{Normal}$

$W_{net} = (+) + 0 + 0$

$W_{net} = \text{Positive}$

$\boxed{\text{Positive}}$

The net work done on any object must be related to changes in the state of motion of the object. The connection will be provided via the kinetic energy of the object. The kinetic energy of an object with mass m, which moves with speed v is

$$\boxed{K = \frac{1}{2}\, mv^2.} \tag{2}$$

The change in kinetic energy is $\Delta K = K_f - K_i$ where K_i is the kinetic energy at an earlier moment and K_f is the kinetic energy at a later moment.

e) Use your answer from section 1.a) to explain whether the change in kinetic energy, ΔK, of the block is positive, negative or zero.

$K_i = \frac{1}{2} m v_i^2$ $\quad v_i = 0\ m/s$

$K_i = \frac{1}{2} m (0\ m/s)^2$

$K_i = 0$

$K_f = \frac{1}{2} m v_f^2$ $\quad v_f = +\ m/s$ ← Positive Sign

$K_f = \frac{1}{2} m v (+\ m/s)^2$

$K_f = +$ "Positive"

$\Delta K = + - 0$

$\Delta K = \text{Positive}$

$\boxed{\text{The change in kinetic energy is positive}}$

The *work-kinetic energy theorem* states that

$$\boxed{\Delta K = W_{\text{net}}} \tag{3}$$

and this provides the key relationship between work and kinetic energy in any mechanical situation.

f) For the situation of Fig. 2, consider whether ΔK and W_{net} are positive, negative or zero. Do they agree as the work-kinetic energy theorem predicts?

W_{net} = Positive

ΔK = Positive

Positive = Positive ✓ ✓ They do agree

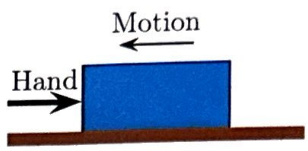


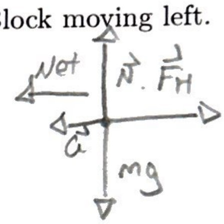
Figure 3: Block moving left.

g) A block on a frictionless surface moves left while a hand pushes horizontally to the right on it. Using Newton's second law explain whether the block's speed increases, decreases or is constant while the hand pushes on it. object moves left

$\Sigma x = m(-a_x)$

$\vec{F}_H = -ma_x$

Since acceleration points to the left, the speed would be decreasing.



Explain whether ΔK positive, negative or zero for this case.

$W_{Hand} = F\Delta r\cos\theta$ $\theta = 180$

$= F\Delta r\cos 180$

$= F\Delta r(-1)$

$W_{Hand} = -F\Delta R$

$W_{net} = -F\Delta R + 0 + 0$

$W_{net} = -F\Delta R$

$W_{net} = \Delta K$

$W_{Gravity} = F\Delta r\cos\theta$

$= \vec{F}_H\Delta r\cos 90$

mg $\theta = 90$ $= \vec{F}_H\Delta r(0)$

$W_{gravity} = 0$

$\boxed{\Delta K = -F\Delta R}$

$W_{normal} = F\Delta r\cos\theta$

$= \vec{N}\Delta r\cos 90$

$\theta = 90$ $= \vec{N}\Delta r(0)$

$W_{normal} = 0$

Using Eq. (1), explain whether W_{net} is positive, negative or zero. Explain whether your answer agrees with what the work-kinetic energy theorem predicts.

$W_{net} = W_{Hand} + W_{Gravity} + W_{Normal}$

$= -F\Delta R + 0 + 0$

$W_{net} = -F\Delta R$

$W_{Hand} = -F\Delta R$

$W_{Gravity} = 0$

$W_{Normal} = 0$

h) In several cases illustrated in Fig. 4, two forces push horizontally on a block moving along a frictionless surface.

Case A: Moves right; $F_1 > F_2$.

Case B: Moves right; $F_1 = F_2$.

Case C: Moves right; $F_1 < F_2$.

Case D: Moves left; $F_1 < F_2$.

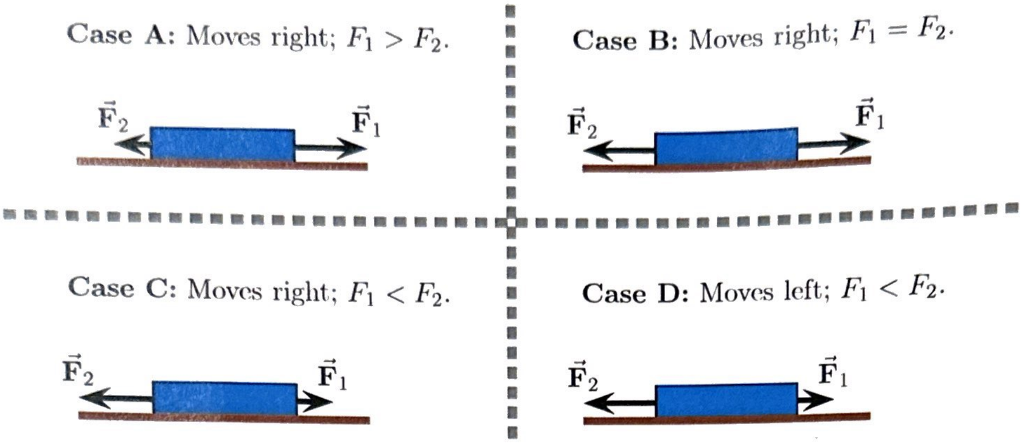


Figure 4: Blocks moving horizontally.

For each case use Newton's second law to describe whether the acceleration points left, right or is zero. Use this to describe whether the block's speed increases, decreases or is constant and use that to describe whether, ΔK, is positive, negative or zero. Enter these in the table below.

Case	Acceleration	Speed increases, decreases or constant?	ΔK
A	Right	The speed will increase	+
B	0	The speed is constant	0
C	Left	The speed will decrease	−
D	Left	The speed will increase	+

For each case, describe the signs of the work done by each force acting on the block, and use these to describe the sign of W_{net}. Enter these in the table below. Do your results agree with the work-kinetic energy theorem?

Case	W_1	W_2	W_{net}	Agree with work-KE theorem?
A	+	−	+	Yes
B	+	−	0	Yes
C	+	−	+	Yes
D	−	+	−	Yes

i) Consider two identical blocks that slide rightward along frictionless horizontal surfaces. A rope exerts a force on each block and the magnitudes of these are identical but their directions are different, as illustrated in Fig. 5.

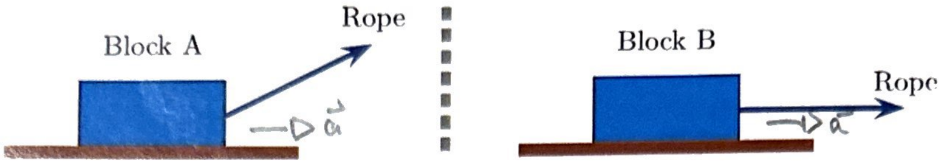
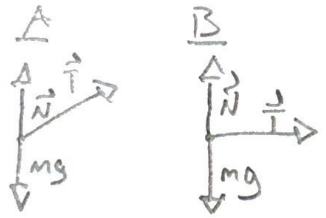


Figure 5: Forces at different angles on two blocks.

Using Newton's second law, explain whether the acceleration of the block A is equal to, larger than or smaller than that of block B.



A: $\Sigma_x = \vec{T}\cos\theta = ma_x$ $\qquad \vec{T}\cos\theta < \vec{T}$

B: $\Sigma_x = \vec{T} = ma_x$

Acceleration of a < b

Each block starts from rest and the two travel the same distance. Using the answer above and kinematics, explain whether ΔK for block A is equal to, larger than or smaller than that of block B.

$V_f^2 = \cancel{V_i^2} + 2a\Delta f$ $\qquad V_i^2 = 0$

$\Delta K = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}$

$V_f^2 = 2a\Delta f$

$\Delta K = \frac{1}{2} m (V_f)^2$

$V_f^2 = 2a$

ΔK of B > A

$\vec{a}$ of A < B

Using Eq. (1) explain whether W_{net} for block A is equal to, larger than or smaller than that for block B. Then use the work-kinetic energy theorem to explain whether ΔK for block A is equal to, larger than or smaller than that of block B. Does this agree with your previous answer?

$W = F \Delta R \cos\theta$

$\Delta K\ B > \Delta K\ A \quad \therefore \quad W_{net}\ B > W_{net}\ A$

2 Work and Vector Dot Products

The general definition of the work done by a force involves the dot product,

$$\boxed{W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}.} \tag{4}$$

The dot product can be computed using components of the vectors, rather than magnitudes and relative angle. Specifically if $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$ then

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z.$$

This computational approach is useful for some forces and situations.

a) A telephone is moved from an initial location to a final location via the indicated path. Two constant forces act on the phone. These are:

$$\vec{F}_1 = 15\,\mathrm{N}\hat{\mathbf{i}} - 10\,\mathrm{N}\hat{\mathbf{j}}$$
$$\vec{F}_2 = 10\,\mathrm{N}\hat{\mathbf{i}} - 15\,\mathrm{N}\hat{\mathbf{j}}$$

King Zog notes that: "The magnitudes of the forces are equal and the distances traveled are equal." He then asks: "Does this mean that the work done by the $\vec{\mathbf{F}}_1$ is the same as that done by $\vec{\mathbf{F}}_2$?"

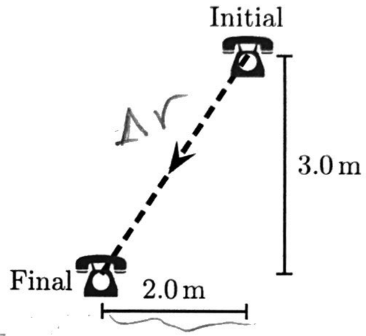


Figure 6: A phone moves between the same points under different forces.

Help answer this question by determining the work done by each force. *Hint: this is easiest done using the dot product.*

$\langle -2\hat{i}, 0\hat{j} \rangle \quad \langle 0\hat{i}, -3\hat{j} \rangle$

$\Delta r = \langle -2\hat{i}, -3\hat{j} \rangle$

$\vec{F}_1 = \langle -2, -3 \rangle \cdot \langle 15, -10 \rangle$

$-30 + 30 = \boxed{0\ \mathrm{J}}$

$\vec{F}_2 = \langle -2, -3 \rangle \cdot \langle 10, -15 \rangle$

$-20 + 45 = \boxed{25\ \mathrm{J}}$

b) A telephone is moved from an initial location to a final location via the paths illustrated in Fig. 7. The final location is located a height h above and horizontal distance d to the right of the initial location.
Use Eq. (4) to determine an expression for the work done by the gravitational force on the telephone for path A.

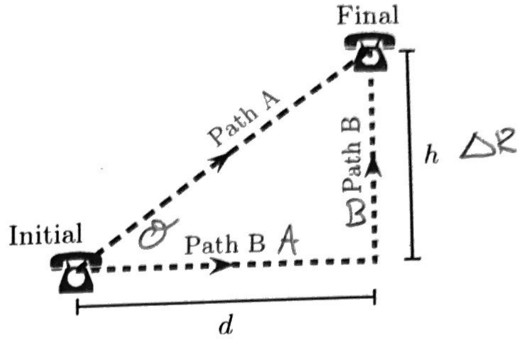


Figure 7: A phone moves between the same points via two different paths.

mg

$\Delta \vec{R} = d\hat{i} + h\hat{j}$

$\vec{F_g} = 0\hat{i} - mg\hat{j}$

$W = \Delta R \cdot \vec{F}$

$W = \langle d\hat{i}, h\hat{j} \rangle \cdot \langle 0\hat{i}, -mg\hat{j} \rangle$

$W = -hmg$

Use Eq. (4) to determine an expression for the work done by the gravitational force on the telephone for path B.

$W = \vec{F} \cdot \Delta R$

Part A

$\Delta R = \langle d\hat{i}, 0\hat{j} \rangle$

$\vec{F_g} = \langle 0\hat{i}, -mg\hat{j} \rangle$

$W = \langle 0\hat{i}, -mg\hat{j} \rangle \cdot \langle d\hat{i}, 0\hat{j} \rangle = 0$

Part B

$\Delta R = \langle 0\hat{i}, h\hat{j} \rangle$

$\vec{F_g} = \langle 0\hat{i}, -mg\hat{j} \rangle$

$W = \vec{F} \cdot \Delta R$

$W = -mhg$

In this example, does the work done by the gravitational force depend on the particular path taken?

It is not dependent on path

3 Applying the Work-Energy Theorem: Ascending and Descending Objects

A phone sits on the floor of an elevator as illustrated. The elevator moves vertically with rope always taut (i.e. not in free fall). Consider the following circumstances where the elevator moves through the same distance:

Case A: The elevator moves up at constant speed.

Case B: The elevator moves up with decreasing speed.

Case C: The elevator moves up with increasing speed.

Case D: The elevator moves down with decreasing speed.

Case E: The elevator moves down with increasing speed.

Figure 8: Phone in an elevator.

a) In each case, explain whether the work done by gravity, W_{grav}, is positive, negative or zero. Explain whether the work done by the normal force exerted by the floor on the elevator, W_{n}, is positive, negative or zero. Use the work-kinetic energy theorem to explain whether the net work, W_{net}, is positive, negative or zero.

Case	Direction of motion	Gravity	Normal	Net
A	↑ $\Delta R = +$	$\theta = 180$ $mg\cos\theta\Delta R = -mg$	$\theta = 0$ $\vec{N}\Delta R\cos\theta = N\Delta R$	0
B	↑ $\Delta R = +$	$\theta = 180$ $mg\cos\theta\Delta R = -mg$	$\theta = 0$ $\vec{N}\Delta R\cos\theta = N\Delta R$	−
C	↑ $\Delta R = +$	$\theta = 180$ $mg\cos\theta\Delta R = -mg$	$\theta = 0$ $\vec{N}\Delta R\cos\theta = N\Delta R$	+
D	↓ $\Delta R = -$	$\theta = 0$ $mg\cos\theta\Delta R = -mg$	$\theta = 180$ $\vec{N}\Delta R\cos\theta = -N\Delta R$	−
E	↓ $\Delta R = -$	$\theta = 0$ $mg\cos\theta\Delta R = mg$	$\theta = 180$ $\vec{N}\Delta R\cos\theta = -N\Delta R$	+

Acceleration points up → ∴ Gravity has to be doing negative work

Enter the results into the table below.

Case	W_{grav}	W_{n}	W_{net}
A	−	+	0
B	−	+	−
C	−	+	+
D	−	−	−
E	+	−	+

b) In each case, use your results to explain whether the magnitude of the normal force is equal to, larger than or smaller than the magnitude of the gravitational force. In each case does this agree with what Newton's second law predicts? Explain your answers.

Case A — Bigger Yes

Case B — Bigger Yes

Case C — Bigger Yes

Case D — Equal Yes

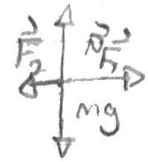
Case E — Smaller Yes

$$W\vec{F}_1 = W_1$$
$$W\vec{F}_2 = W_2$$

1.H case A moves right $\therefore \Sigma_x = max$

$\Sigma_x = \vec{F}_1 - F_2 = max$

$F_1 > F_2$



$\theta = 90$

$W_G = 0$

$W_N = 0$

$W = F \Delta R \cos\theta$

$W_{F_1} = \vec{F}_1 \Delta R \cos 0$ $\quad \theta = 0$

$WF_1 = \vec{F}_1 \Delta R$

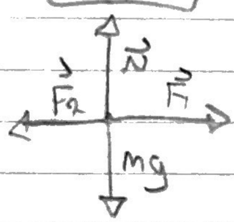
$W_{F_2} = \vec{F}_2 \Delta R \cos(180)$ $\quad \theta = 180$

$= \vec{F}_2 \Delta R (-1)$

$W\vec{F}_2 = -\vec{F}_2 \Delta R$

case B moves Right $= max$

$F_1 = F_2$



$\theta = 180$

$WF_2 = \vec{F}_2 \Delta R \cos(180)$

$W\vec{F}_2 = \vec{F}_2 \Delta R (-1)$

$W\vec{F}_2 = -\vec{F}_2 \Delta R$

$W_{F_1} = \vec{F}_1 \Delta R \cos\theta$ $\quad \theta = 0$

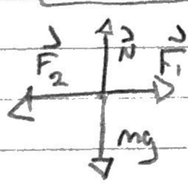
$W\vec{F}_1 = \vec{F}_1 \Delta R \cos(0)$

$= \vec{F}_1 \Delta R (1)$

$WF_1 = \vec{F}_1 \Delta R$

case C moves Right $= max$

$\vec{F}_1 < \vec{F}_2$



$\theta = 180$

$W\vec{F}_2 = \vec{F}_2 \Delta R \cos 180$

$W\vec{F}_2 = \vec{F}_2 \Delta R (-1)$

$W\vec{F}_2 = -\vec{F}_2 \Delta R$

$\theta = 0$

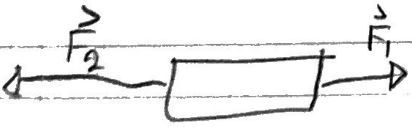
$W\vec{F}_1 = \vec{F}_1 \Delta R \cos\theta$

$= \vec{F}_1 \Delta R \cos(0)$

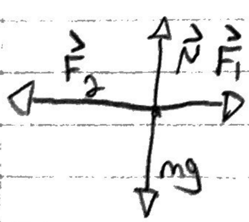
$= \vec{F}_1 \Delta R (1)$

$W\vec{F}_1 = \vec{F}_1 \Delta R$

case D moves left $= -m(ax)$



$\vec{F}_1 < \vec{F}_2$



$\theta = 0$

$W\vec{F}_2 = \vec{F}_2 \Delta R \cos 0$

$= \vec{F}_2 \Delta R (1)$

$W\vec{F}_2 = \vec{F}_2 \Delta R$

$\theta = 180$

$W\vec{F}_1 = \vec{F}_1 \Delta R \cos 180$

$= \vec{F}_1 \Delta R (-1)$

$W\vec{F}_1 = -\vec{F}_1 \Delta R$