Physics 311

Homework Set 1

1. By writing vectors in terms of the components, explicitly show that the cross product is distributive, i.e. Show that

$$\vec{\alpha} \times (\vec{\beta} + \vec{\gamma}) = (\vec{\alpha} \times \vec{\beta}) + (\vec{\alpha} \times \vec{\gamma}) \tag{1}$$

2. Find the angle between the face diagonal and a body diagonal of a cube.

3. Use the cross product to find the components of the unit vector $\hat{n}$ perpendicular to the shaded plane in Fig. 1.11 in your textbook.

4. By writing out both sides of the equation in component form, explicitly show that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \tag{2}$$

5. Find the separation vector $\vec{\imath}$ from the *source point* (1,4,5) m to the *field point* (1,2,3) m. Determine its magnitude $|\imath|$, and construct the unit vector $\hat{\imath}$.

6. Find the gradients of the following functions:
   a) $f(x, y, z) = x^2 + y^5 + z^3$.
   b) $f(x, y, z) = x^2 y^5 z^3$.
   c) $f(x, y, z) = \ln(2x)e^z \sin(3y)$.

7. The height of a certain hill (in feet) is given by

$$h(x, y) = 3(2xy - 3x^2 - 4y^2 - 8x + 8y + 5) \tag{3}$$

   where y is the distance (in miles) north, x the distance east of Topeka, Kansas.
   a) Where is the top of the hill located?
   b) How high is the hill?
   c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of Topeka? In what direction is the slope steepest, at that point?