Taylor Larrechea
Dr. Gustafson
MATH 360
HW Ch. 10.2

#3

Problem 10.2.3 $\int_{(\pi/2,\pi)}^{(\pi,0)} \frac{1}{2}\cos(\frac{1}{2}x)\cos(2y)\,dx - 2\sin(\frac{1}{2}x)\sin(2y)\,dy$

$F_1 = \frac{1}{2}\cos(\frac{1}{2}x)\cos(2y)$ $\quad F_2 = -2\sin(\frac{1}{2}x)\sin(2y)$

For exactness : $\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$

$\frac{\partial F_2}{\partial x} = -\cos(\frac{1}{2}x)\sin(2y)$ $\quad \frac{\partial F_1}{\partial y} = -\cos(\frac{1}{2}x)\sin(2y)$

$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$ ✓

∴ we can say that this equation is exact.

$\frac{\pi}{2} \le x \le \pi$
$\pi \le y \le 0$
$\underset{\vec{a}}{[\frac{\pi}{3},\pi]} \to \underset{\vec{b}}{[\pi,0]}$
$\vec{c} = \vec{b} - \vec{a}$
$\vec{c} = [\frac{\pi}{2}, -\pi]$
$[\frac{\pi}{2},\pi] + t[\frac{\pi}{2},-\pi]$
$0 \le t \le 1$
$[\frac{\pi}{2} + t\frac{\pi}{2}, \pi - t\pi]$
$\vec{r}(t) = [\frac{\pi}{2} + t\frac{\pi}{2}, \pi - t\pi]$
$\vec{r}'(t) = [\frac{\pi}{2}, -\pi]$

$\vec{r}(t) = [\frac{\pi}{2} + t\frac{\pi}{2}, \pi - t\pi]$ $\quad 0 \le t \le 1$
$\vec{r}'(t) = [\frac{\pi}{2}, -\pi]\,dt$
$\vec{F} = [\frac{1}{2}\cos(\frac{1}{2}x)\cos(2y), -2\sin(\frac{1}{2}x)\sin(2y)]$

$$\vec{F}(r) = \left[\frac{1}{2}\cos\left(\frac{1}{2}\left(\frac{\pi}{2} + t\frac{\pi}{2}\right)\right)\cos(2(\pi - t\pi)), -2\sin\left(\frac{1}{2}\left(\frac{\pi}{2} + t\frac{\pi}{2}\right)\right)\sin(2(\pi - t\pi))\right]$$

$$\vec{F}(r) = \left[\frac{1}{2}\cos\left(\frac{\pi}{4} + \frac{\pi}{4}t\right)\cos(2\pi - 2\pi t), -2\sin\left(\frac{\pi}{4} + \frac{\pi}{4}t\right)\sin(2\pi - 2\pi t)\right]$$

$$\int_C \vec{F}(r)\cdot d\vec{r} = \int_0^1 \left[\frac{1}{2}\cos\left(\frac{\pi}{4} + \frac{\pi}{4}t\right)\cos(2\pi - 2\pi t), -2\sin\left(\frac{\pi}{4} + \frac{\pi}{4}t\right)\sin(2\pi - 2\pi t)\right]\cdot\left[\frac{\pi}{2}, -\pi\right] dt$$

$$= \int_0^1 \frac{\pi}{4}\cos\left(\frac{\pi}{4}(1+t)\right)\cos(2\pi(1-t)) + 2\pi\sin\left(\frac{\pi}{4}(1+t)\right)\sin(2\pi(1-t))\,dt$$

$$= \frac{\pi}{4}\int_0^1 \cos(\underbrace{\tfrac{\pi}{4}(1+t)}_{\alpha})\cos(\underbrace{2\pi(1-t)}_{\beta})\,dt + 2\pi\int_0^1 \sin(\underbrace{\tfrac{\pi}{4}(1+t)}_{\alpha})\sin(\underbrace{2\pi(1-t)}_{\beta})\,dt$$

$$= \frac{\pi}{4}\int_0^1 \cos(\alpha)\cos(\beta)\,dt + 2\pi\int_0^1 \sin(\alpha)\sin(\beta)\,dt$$

$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$ $\quad \sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$$\alpha - \beta = [\tfrac{\pi}{4} + \tfrac{\pi}{4}t] - [2\pi - 2\pi t] = [9\pi/4\, t - 7\pi/4] = \frac{\pi(9t-7)}{4} = \gamma$$

$$\alpha + \beta = [\tfrac{\pi}{4} + \tfrac{\pi}{4}t] + [2\pi - 2\pi t] = [9\pi/4 - 7\pi/4\, t] = \frac{\pi(9-7t)}{4} = \delta$$

$$= \frac{\pi}{8}\int_0^1 \cos(\gamma)\,dt + \frac{\pi}{8}\int_0^1 \cos(\delta)\,dt + \pi\int_0^1 \cos(\gamma)\,dt - \pi\int_0^1 \cos(\delta)\,dt$$

$$\frac{\pi}{8}\int_0^1 \cos(\gamma)\,dt + \frac{\pi}{8}\int_0^1 \cos(\delta)\,dt + \pi\int_0^1 \cos(\gamma)\,dt - \pi\int_0^1 \cos(\delta)\,dt$$

$$d\gamma = \frac{9\pi}{4}dt \qquad d\delta = \frac{-7\pi}{4}dt \qquad d\gamma = \frac{9\pi}{4}dt \qquad d\delta = \frac{-7\pi}{4}dt$$

$$dt = \frac{4}{9\pi}d\gamma \qquad dt = \frac{-4}{7\pi}d\delta \qquad dt = \frac{4}{9\pi}d\gamma \qquad dt = \frac{-4}{7\pi}d\delta$$

$$\frac{1}{18}\int_0^1 \cos(\gamma)\,d\gamma - \frac{1}{14}\int_0^1 \cos(\delta)\,d\delta + \frac{4}{9}\int_0^1 \cos(\gamma)\,d\gamma + \frac{4}{7}\int_0^1 \cos(\delta)\,d\delta$$

$$\frac{1}{18}\left[\sin\left(\frac{\pi(9t-7)}{4}\right)\Big|_0^1\right] - \frac{1}{14}\left[\sin\left(\frac{\pi(9-7t)}{4}\right)\Big|_0^1\right] + \frac{4}{9}\left[\sin\left(\frac{\pi(9t-7)}{4}\right)\Big|_0^1\right] + \frac{4}{7}\left[\sin\left(\frac{\pi(9-7t)}{4}\right)\Big|_0^1\right]$$

$$\frac{1}{18}\left[\sin(\pi/2) - \sin(-7\pi/4)\right] - \frac{1}{14}\left[\sin(\pi/2) - \sin(9\pi/4)\right] + \frac{4}{9}\left[\sin(\pi/2) - \sin(-7\pi/4)\right] + \frac{4}{7}\left[\sin(\pi/2) - \sin(9\pi/4)\right]$$

$$\frac{1}{18}\left[1 - \frac{\sqrt{2}}{2}\right] - \frac{1}{14}\left[1 - \frac{\sqrt{2}}{2}\right] + \frac{4}{9}\left[1 - \frac{\sqrt{2}}{2}\right] + \frac{4}{7}\left[1 - \frac{\sqrt{2}}{2}\right]$$

$$(1 - \sqrt{2}/2)\left(\frac{1}{18} - \frac{1}{14} + \frac{4}{9} + \frac{4}{7}\right) = (1 - \sqrt{2}/2)(1) = 0.2929 \approx 0.293$$

$$\boxed{0.293 \text{ or } (1 - \sqrt{2}/2)}$$

$$\int_{(\pi/2,\pi)}^{(\pi,0)} \tfrac{1}{2}\cos(\tfrac{1}{2}x)\cos(2y)\,dx - 2\sin(\tfrac{1}{2}x)\sin(2y)\,dy = f(\pi,0) - f(\tfrac{\pi}{2},\pi)$$

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y} \;:\; \frac{\partial F_2}{\partial x} = -\cos(\tfrac{1}{2}x)\sin(2y) \checkmark \qquad \frac{\partial F_1}{\partial y} = -\cos(\tfrac{1}{2}x)\sin(2y) \checkmark$$

$$F_x = \frac{1}{2}\cos(\tfrac{1}{2}x)\cos(2y)$$

$$F = \sin(\tfrac{1}{2}x)\cos(2y) + C(y) = \sin(\tfrac{1}{2}x)\cos(2y) + \sin(2y)$$

$$F(\pi,0) = \sin(\tfrac{1}{2}\cdot\pi)\cdot\cos(2\cdot 0) + \sin(2\cdot 0) = 1$$

$$F(\tfrac{\pi}{2},\pi) = \sin(\tfrac{1}{2}\cdot\tfrac{\pi}{2})\cdot\cos(2\cdot\pi) + \sin(2\cdot\pi) = \sqrt{2}/2$$

$$\therefore \int_{(\pi/2,\pi)}^{(\pi,0)} \tfrac{1}{2}\cos(\tfrac{1}{2}x)\cos(2y)\,dx = F(b) - F(a) = \left[1 - \sqrt{2}/2\right]$$

$$\boxed{0.293 \text{ or } (1 - \sqrt{2}/2)}$$