Phys 131 2016

Lecture 29

~~We~~ Thurs: Physics seminar

Fri: Lecture

Mon: HW by 5pm

Supp Ex 69

Ch 11 CQ 6, 13

Ch 11 Prob 15, 18, 28, 29, 49

Interactions within isolated systems

In some situations a physical system consisting of a collection of objects is effectively isolated from its surroundings.

Demo Two carts on a track + collide

If we consider the two carts as a single system then, ignoring friction + air resistance, the surroundings exert no net force on the system while the carts interact. The carts themselves exert complicated forces on each other and using Newton's 2nd Law to assess their motion could be impossible. However, we will see that certain features of their motion can be analyzed in terms of a new physical quantity, called momentum.

Momentum is convenient for analyzing various "collision" situations. Examples include:

Demo * Ship collision video

* Brookhaven LHC link

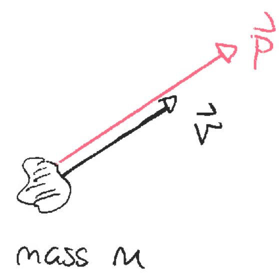
What we will do is rewrite Newton's 3rd Law in terms of the new quantity momentum and produce a new law describing how momentum is conserved

Momentum

Momentum combines information about the mass and velocity of any object.

The momentum of an object with mass m and velocity $\vec{v}$ is:

$$\vec{p} = m\vec{v}$$



Note: 1) momentum is a vector

2) the direction of momentum vector is the same as the direction of velocity

3) units: kg m/s.

4) objects with same mass but different velocities have different momenta

5) objects with same velocity but different mass have different momenta.

Quiz: 30% → 80% ; 50% → 70%

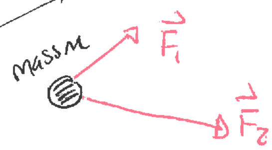
The momentum can be related to the dynamics of an object. Consider an object moving in one dimension, e.g. along the x-axis. Then:

$$F_{net} = ma = m\frac{dv}{dt}$$

and for constant mass this gives

$$F_{net} = \frac{d}{dt}(mv) \Rightarrow F_{net} = \frac{dp}{dt}$$

This can be extended to all dimensions. Thus:



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \dots$$

Rate of change of momentum:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

describes changes in velocity

Conservation of momentum

The reworking of Newton's Second Law can occasionally be convenient for isolated particles. But its true power comes when considering systems of objects. With such a system we define

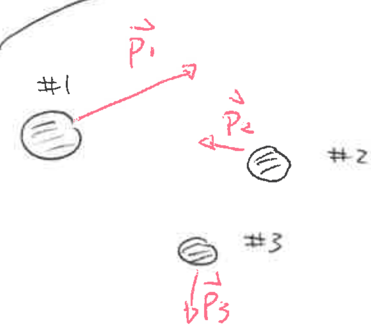
The net (total) momentum of the system is:

$$\vec{P}_{tot} = \vec{p}_1 + \vec{p}_2 + \dots = \sum \vec{p}_i$$

where

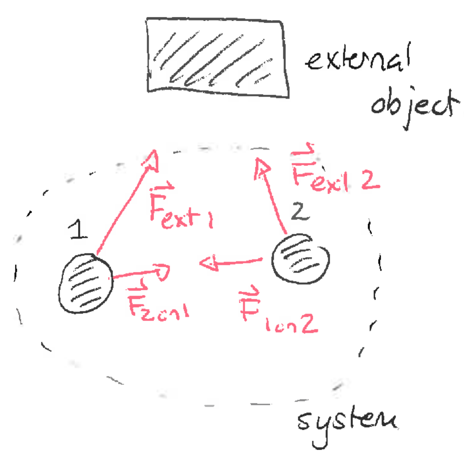
$$\vec{p}_i = m_i \vec{v}_i$$

is the momentum of object i



Quiz 2 90%

We now consider a system of two objects that can also interact with an external object. Then:



$$\frac{d\vec{P}_{tot}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt}$$

$$= \vec{F}_{net\,1} + \vec{F}_{net\,2}$$

But $\vec{F}_{net\,1} = \vec{F}_{ext\,1} + \vec{F}_{2on1}$

$\vec{F}_{net\,2} = \vec{F}_{ext\,2} + \vec{F}_{1on2}$

gives:

$$\frac{d\vec{P}_{tot}}{dt} = \vec{F}_{ext\,1} + \vec{F}_{2on1} + \vec{F}_{ext\,2} + \vec{F}_{1on2}$$

action/reaction pair

But Newton's 3rd Law gives: $\vec{F}_{2on1} = -\vec{F}_{1on2}$. So

$$\frac{d\vec{P}_{tot}}{dt} = \vec{F}_{ext\,net}$$

where the net external force is $\vec{F}_{ext\,net} = \vec{F}_{ext\,1} + \vec{F}_{ext\,2}$. This can be extended to any number of objects giving:

> For any system of objects
>
> $$\frac{d\vec{P}_{tot}}{dt} = \vec{F}_{ext\,net}$$
>
> where $\vec{F}_{ext\,net}$ is the net force exerted on all constituents of the system by any external entities

In the special case where the net external force is zero, the total momentum satisfies $\frac{d\vec{P}_{tot}}{dt} = 0$. Thus we arrive at the law of the <u>conservation of momentum</u>:

> If the net external force on a system is zero then the total momentum of the system stays constant.

Quiz 3

Example: A 100kg astronaut is at rest in space. A 0.0050kg fleck of paint moves toward the astronaut at speed 15×10^3 m/s. It collides with and sticks to the astronaut. Determine their speeds after the collision.

Answer: Net external force = 0 $\Rightarrow$ $\vec{P}_{tot}$ = constant.

Before

$V_{paint\,i} = 15 \times 10^3 \, m/s$

$V_{astro\,i} = 0 \, m/s$

After

move together

$V_{astro\,f} = V_{paint\,f} \equiv V_f$

We only work with horizontal components.

$$P_{tot\,f\,x} = P_{tot\,i\,x} \iff P_{tot\,f} = P_{tot\,i}$$

$$\iff P_{paint\,f} + P_{astro\,f} = P_{paint\,i} + P_{astro\,i}$$

$$\iff M_{paint\,f} V_f + M_{astro} V_f = M_{paint} V_{paint\,i} + \cancel{M_{astro} V_{astro\,i}}$$

$$\iff (M_{paint} + M_{astro}) V_f = M_{paint} V_{paint\,i}$$

$$= 0.0050kg \times 15 \times 10^3 \, m/s$$

$$(100.005 \, kg) \, V_f = 75 \, kg\,m/s$$

$$\iff V_f = \frac{75 \, kg\,m/s}{100.005 \, kg}$$

$$\iff V_f = 0.75 \, m/s$$