

Problem 1

a.) Volume doesn't change therefore there is no work done on the gas

b.) $\Delta E = Q + \cancel{W}^0 = 0 : Q = 0 \therefore \Delta E = 0 + 0 = 0$

c.) $E = \frac{3}{2} NkT$: There is no change in energy \therefore no change in temperature

d.) $E = \frac{3}{2} NkT - \frac{N^2 a}{V}$: The change in energy is positive \therefore the change in temperature is negative

e.) $\left(\frac{\partial z}{\partial y}\right)_x = - \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z$

$x = E, y = V, z = T$

$$\left(\frac{\partial T}{\partial V}\right)_E = - \left(\frac{\partial T}{\partial E}\right)_V \left(\frac{\partial E}{\partial V}\right)_T = - \left(\frac{\partial T}{\partial E}\right)_V \left[T \left(\frac{\partial P}{\partial T}\right)_V - P \right] : \left(\frac{\partial E}{\partial T}\right)_V = C_V \therefore \left(\frac{\partial T}{\partial E}\right)_V = \frac{1}{C_V}$$

\therefore

$$\boxed{\left(\frac{\partial T}{\partial V}\right)_E = -\frac{1}{C_V} \left[T \left(\frac{\partial P}{\partial T}\right)_V - P \right]}$$

f.)

i.) $T \left(\frac{\partial P}{\partial T}\right)_V = T \cdot \frac{Nk}{V} = \frac{NkT}{V} = P : T \left(\frac{\partial P}{\partial T}\right)_V - P = P - P = 0$

$\left(\frac{\partial T}{\partial V}\right)_E = -\frac{1}{C_V} (0) = 0 : \boxed{\left(\frac{\partial T}{\partial V}\right)_E = 0 \text{ for ideal gas}} \quad \Delta T = 0$

ii.) $\left(P + \frac{N^2 a}{V^2}\right)(V - Nb) = NkT \therefore P = \frac{NkT}{(V - Nb)} - \frac{N^2 a}{V^2}$

$\left(\frac{\partial P}{\partial T}\right)_V = \frac{Nk}{(V - Nb)} : T \left(\frac{\partial P}{\partial T}\right)_V - P = \frac{NkT}{(V - Nb)} - \frac{NkT}{(V - Nb)} + \frac{N^2 a}{V^2}$

$$\boxed{\left(\frac{\partial T}{\partial V}\right)_E = -\frac{1}{C_V} \left(\frac{N^2 a}{V^2}\right) < 0 \text{ For van der Waals}}$$

ΔT decreases as V increases

Problem 2

a.) $PV = NkT$: $E = \frac{3}{2}PV$ \therefore

$P = p k T$, $E = \frac{3}{2} \frac{P N}{p}$: Volume and Energy are extensive because they can both be added with a two part system

b.) $P = \frac{NkT}{(V-Nb)} - \frac{N^2 a}{V^2}$, $E = \frac{3}{2} NkT - N \frac{Na}{V}$ \therefore

$P = \frac{NkT}{(Vp-Nb)}$ - $p^2 a$, $E = \frac{3}{2} NkT - N p a$: Volume and Energy are extensive because they can both be added with a two part system

Problem 3

$$dS = dS_A + dS_B \quad ; \quad T = \left(\frac{\partial S}{\partial E} \right)^{-1}_{V,N} = \left(\frac{\partial E}{\partial S} \right)_{V,N}, \quad T_A = \left(\frac{\partial S_A}{\partial E_A} \right)^{-1}_{V,N}, \quad T_B = \left(\frac{\partial S_B}{\partial E_B} \right)^{-1}_{V,N}$$

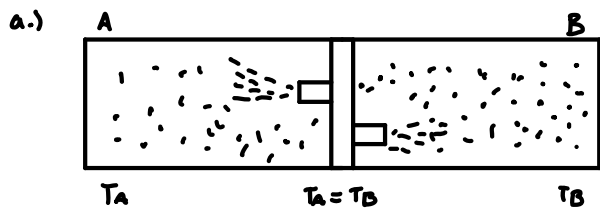
$$dS = \left(\frac{\partial S_A}{\partial E_A} \right)^{-1}_{V,N} dE_A + \left(\frac{\partial S_B}{\partial E_B} \right)^{-1}_{V,N} dE_B \quad ; \quad dE_A = -dE_B$$

$$dS = \left(\frac{\partial S_A}{\partial E_A} \right)^{-1}_{V,N} dE_A - \left(\frac{\partial S_B}{\partial E_B} \right)^{-1}_{V,N} dE_A \quad ; \quad dS = \left[\left(\frac{\partial S_A}{\partial E_A} \right)^{-1}_{V,N} - \left(\frac{\partial S_B}{\partial E_B} \right)^{-1}_{V,N} \right] dE_A$$

$$dS = \left[\frac{1}{T_A} - \frac{1}{T_B} \right] dE_A \rightarrow \left[\frac{1}{T_A} - \frac{1}{T_B} \right] \Delta E_A \geq 0 \quad \therefore \quad \frac{1}{T_A} - \frac{1}{T_B} \geq 0$$

$\frac{1}{T_A} - \frac{1}{T_B} \geq 0 \rightarrow$	$\text{if } T_A > T_B \therefore \Delta E_A < 0$	Thus Energy flows from warmer to colder
	$\text{if } T_A < T_B \therefore \Delta E_A > 0$	

Problem 4



b.) $ds = ds_A + ds_B : P = T \left(\frac{\partial s}{\partial v} \right)_{E,N}$

$$ds = T_A \left(\frac{\partial s_A}{\partial v_A} \right)_{E,N} dv_A + T_B \left(\frac{\partial s_B}{\partial v_B} \right)_{E,N} dv_B : dv_A + dv_B = 0 \therefore dv_A = -dv_B$$

$$ds = T_A \left(\frac{\partial s_A}{\partial v_A} \right)_{E,N} dv_A - T_B \left(\frac{\partial s_B}{\partial v_B} \right)_{E,N} dv_A : ds = \left[T_A \left(\frac{\partial s_A}{\partial v_A} \right)_{E,N} - T_B \left(\frac{\partial s_B}{\partial v_B} \right)_{E,N} \right] dv_A$$

$$ds = (P_A - P_B) dv_A \longrightarrow (P_A - P_B) \Delta v_A \geq 0 \therefore P_A - P_B \geq 0$$

$P_A - P_B \geq 0$: if $P_A > P_B \therefore \Delta v_A > 0$ Thus volume of the higher pressure system must increase as they approach equilibrium.
if $P_A < P_B \therefore \Delta v_A < 0$

Energy could be transferred. At the same temp it does not matter, one gas does work on the other.