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PHYS 342 HW 4

Problem 1

$$\int_{x_1,y_1}^{x_2,y_2} ds \qquad ds = \sqrt{dx^2+dy^2} \qquad \frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0 \qquad (6.18)$$

$$f = \sqrt{1+y'^2} \qquad ds = \sqrt{1+y'^2} \qquad y' = \frac{dy}{dx}$$

$$(1+y'^2)^{\frac{1}{2}}$$

$$\frac{\partial F}{\partial y} = 0$$

$$\frac{\partial F}{\partial y'} = \frac{1}{2}(1+y'^2)^{-\frac{1}{2}} \cdot 2y' = \frac{y'}{\sqrt{1+y'^2}}$$

$$\frac{d}{dx}\left(\frac{y'}{\sqrt{1+y'^2}}\right) : \frac{d}{dx}\left(y'(1+y'^2)^{-\frac{1}{2}}\right) = y''(1+y'^2)^{-\frac{1}{2}} - \frac{1}{2}y'(1+y'^2)^{-\frac{3}{2}} \cdot 2y' \cdot y''$$

$$\frac{d}{dx} y'y' = y'y'' + y''y' = 2y'y''$$

$$-\frac{d}{dx} = \frac{1}{2} \cdot y'(1+y'^2)^{-\frac{3}{2}} \cdot 2y'y'' - y''(1+y'^2)^{-\frac{1}{2}}$$

$$y'y'' \cdot y'(1+y'^2)^{-\frac{3}{2}} - y''(1+y'^2)^{-\frac{1}{2}} = 0$$

$$y''\left(y'^2(1+y'^2)^{-\frac{3}{2}} - (1+y'^2)^{-\frac{1}{2}}\right) = 0$$

$$y'' = 0$$

$$\frac{dy'}{dx} = 0 \qquad y'' = \frac{dy'}{dx}$$

$$\int dy' = \int 0$$

$$y' = m \qquad y' \equiv \frac{dy}{dx}$$

$$\frac{dy}{dx} = m$$

$$\int dy = \int m\, dx$$

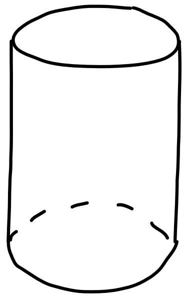
$$y = mx + b$$

The shortest distance is,

$$\boxed{y(x) = mx + b}$$

A strait line!

Problem 2



$$ds=\sqrt{dx^2+dy^2+dz^2}$$
$$x=R\cos(\theta)$$
$$y=R\sin(\theta)$$

$$\frac{\partial F}{\partial y}-\frac{d}{dx}\frac{\partial F}{\partial y'}=0 \quad (6.18)$$

$$dx=-R\sin(\theta)$$
$$dy=R\cos(\theta)$$

$$d\vec{s}=dr\hat{r}+rd\theta\hat{\theta}+r\sin\theta\, d\phi\hat{\phi}$$

$$ds=\sqrt{(-R\sin\theta)^2+(R\cos\theta)^2+dz^2}$$

$$R^2\sin^2\theta+R^2\cos^2\theta$$
$$R^2(\sin^2+\cos^2\theta)=R^2$$

$$0\le\theta\le2\pi$$

$$ds=\sqrt{R^2+dz^2}\equiv\sqrt{R^2+z'^2}$$

$$\frac{\partial F}{\partial z}-\frac{d}{d\theta}\left(\frac{\partial F}{\partial z'}\right)$$

$$ds=(R^2+z'^2)^{\frac{1}{2}}$$

$$\frac{\partial F}{\partial z}=0$$

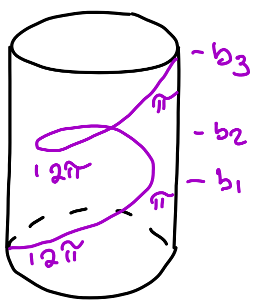
$$\frac{\partial F}{\partial z'}=\frac{1}{2}\cdot(R^2+z'^2)^{-\frac{1}{2}}\cdot 2z' = \frac{z'}{\sqrt{R^2+z'^2}}$$

$$\frac{d}{d\theta}z'z'=z'z''+z''z'$$

$$\frac{d}{d\theta}\left(z'(R^2+z'^2)^{-\frac{1}{2}}\right)=z''(R^2+z'^2)^{-\frac{1}{2}}-\frac{1}{2}\cdot z'(R^2+z'^2)^{-\frac{3}{2}}\cdot 2z'z''$$

$$\frac{\partial F}{\partial z}-\frac{d}{d\theta}\left(\frac{\partial F}{\partial z'}\right)=0$$

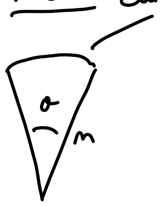
$$-z''(R^2+z'^2)^{-\frac{1}{2}}+z'\cdot z'z''(R^2+z'^2)^{-\frac{3}{2}}=0$$



$$z''\left(z'\cdot z'(R^2+z^2)^{-\frac{3}{2}}-(R^2+z'^2)\right)=0$$

$$z''=0 \qquad z''\equiv\frac{dz'}{d\theta}$$
$$\frac{dz'}{d\theta}=0$$
$$dz'=0$$
$$\int dz'=\int 0$$
$$z'=m \qquad z'\equiv\frac{dz}{d\theta}$$

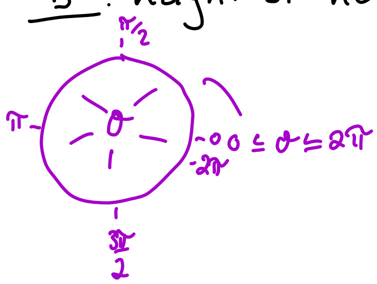
$m\theta$: curvature



$$0\le\theta\le2\pi$$

$$\frac{dz}{d\theta}=m$$
$$dz=m\,d\theta$$
$$\int dz=\int m\,d\theta$$
$$z=m\theta+b$$

b: height of helix



The shortest distance is,

$$\boxed{z(\theta)=m\theta+b}$$

A helix