Problem 1

$m = 0.550\ kg$ $\quad w_0 = 9.15\ rad/s$

$K = 46\ N/m$ $\quad A_1 = \frac{1}{4} A_0$

a.) calculate γ and b

$$\frac{A_N}{A_{N+1}} = e^{\gamma T/2}$$

$A_0 = 4A_1$

$$\frac{A_1}{4A_1} = e^{\gamma T/2}$$

$$\frac{1}{4} = e^{\gamma T/2}$$

$$\ln(1/4) = \gamma T/2$$

$$2\ln(1/4) = \gamma T$$

$$\frac{2\ln(1/4)}{6.00s} = \gamma$$

$$\gamma = 0.462\ s^{-1}$$

$\gamma = b/m \quad \therefore \quad b = \gamma m$

$\gamma = 0.462\ s^{-1}$

$m = 0.55kg$

$b = (0.462\ s^{-1})(0.55kg)$

$b = 0.254\ kg/s$

$$\boxed{\begin{matrix} \gamma = 0.462\ s^{-1} \\ b = 0.254\ kg/s \end{matrix}}$$

b.) calculate the angular frequency

$w = \sqrt{w_0^2 - (\gamma/2)^2}$

$w_0 = 9.15\ s^{-1}$

$\gamma = 0.462\ s^{-1}$

$w = \sqrt{(9.15\ s^{-1})^2 - (0.462\ s^{-1}/2)^2}$

$w = 9.147\ s^{-1}$

$$\boxed{w = 9.15\ s^{-1}}$$

C.) calculate Q

$Q = w_0/\gamma$

$w_0 = 9.15\ rad/s$

$\gamma = 0.462\ s^{-1}$

$$Q = \frac{9.15\ rad/s}{0.462\ s^{-1}} = 19.8$$

$$\boxed{Q = 19.8}$$

Problem 2

$M = 0.55\ kg \quad K = 46\ N/m$

$W_0 = 9.15\ s^{-1} \quad \Delta x = 5.5 \times 10^{-2}\ m$

$\dot{x} = 0.450\ m/s \quad w = 9.15\ s^{-1}$

a.) Calculate $\quad x(t) = A e^{-\gamma t/2} \cos(wt + \varnothing)$

$x(t=0) = A\cos(\varnothing)$

$\Delta x = A\cos(\varnothing) \qquad \Delta x \equiv x(t=0)$

$A = \dfrac{\Delta x}{\cos(\varnothing)} \qquad \varnothing = -0.743\ rad$

$A = \dfrac{0.055\ m}{\cos(-0.743\ rad)}$

$A = 0.07468\ m$

$\dot{x}(t) = -wAe^{-\gamma t/2}\sin(wt + \varnothing) - \gamma/2\, Ae^{-\gamma t/2}\cos(wt+\varnothing)$

$\dot{x}(t=0) = -wA\sin(\varnothing) - \gamma/2\, A\cos(\varnothing) \qquad \dot{x}(t=0) \equiv \dot{x}$

$\dot{x} = \dfrac{-w\Delta x \sin(\varnothing)}{\cos(\varnothing)} - \dfrac{\gamma/2\, \Delta x \cos(\varnothing)}{\cos(\varnothing)}$

$\dot{x} = -w\Delta x \tan(\varnothing) - \gamma/2\, \Delta x$

$\dot{x} = -\Delta x(w\tan(\varnothing) + \gamma/2))$

$\dfrac{\dot{x}}{-\Delta x} = w\tan(\varnothing) + \gamma/2$

$-\dfrac{\dot{x}}{\Delta x} - \dfrac{\gamma}{2} = w\tan(\varnothing)$

$\dfrac{-2\dot{x} - \Delta x\gamma}{2\Delta x} = w\tan(\varnothing)$

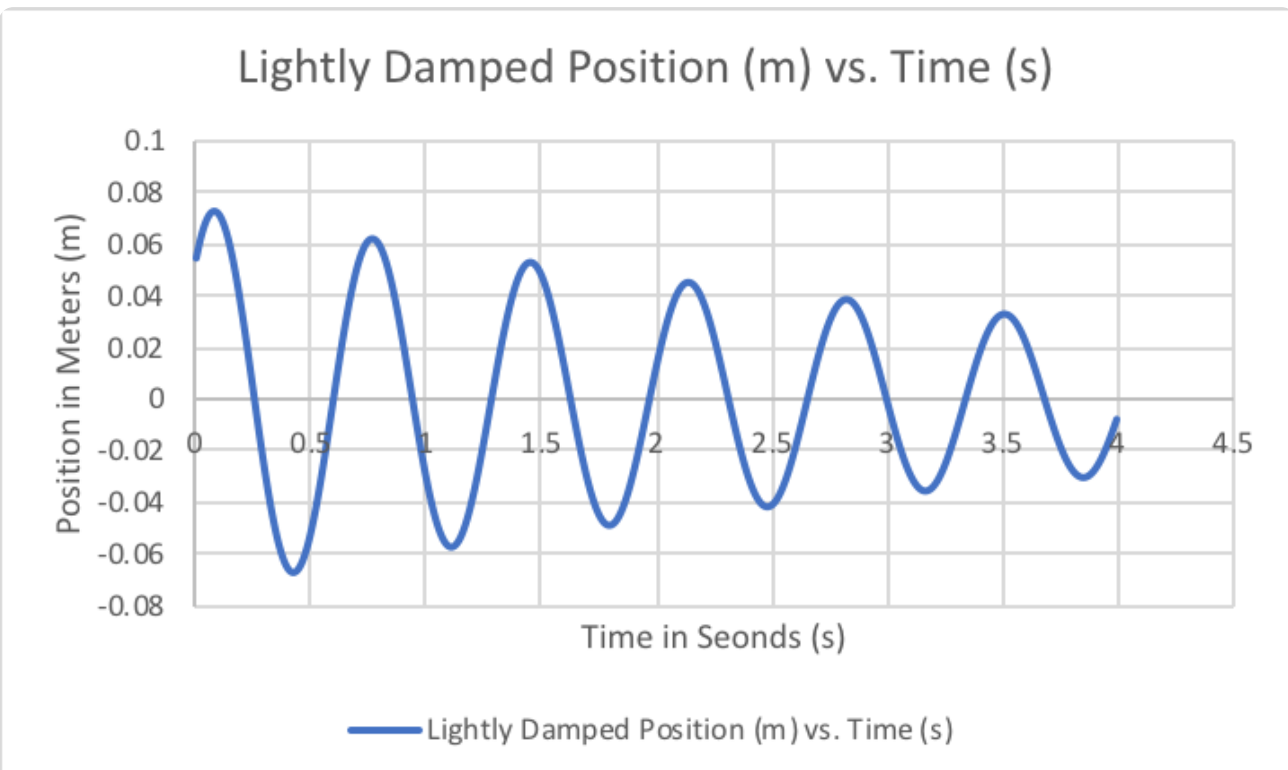
$\dfrac{-2\dot{x} - \Delta x\gamma}{2\Delta x w} = \tan(\varnothing)$

$\varnothing = \tan^{-1}\left(\dfrac{-2\dot{x} - \Delta x\gamma}{2\Delta x w}\right) = \tan^{-1}\left(\dfrac{-2(0.450\ m/s) - (0.055\ m)(0.462\ s^{-1})}{2(0.055\ m)(9.15\ s^{-1})}\right)$

$\varnothing = -0.743\ rad$

$$\boxed{\begin{array}{l} x(t) = (0.0747\ m)e^{-\frac{(0.462)t}{2}}\cos((9.15\ s^{-1})(t) - 0.743\ rad) \\ A = 0.0747\ m \qquad w = 9.15\ s^{-1} \\ \gamma = 0.462\ s^{-1} \qquad \varnothing = -0.743\ rad \end{array}}$$

b.)



Problem 3

$M = 0.850$ kg $K = 46$ N/m
$W_0 = 9.15\ s^{-1}$

a.) Determine γ and b

Critically damped $\therefore w_0 = \gamma/2 \quad \therefore \quad \gamma = 2w_0$ $\gamma = b/m \quad \therefore b = 2mw_0$

$\gamma = 2w_0 = 2(9.15\ rad/s) = 18.3\ s^{-1}$ $b = 2mw_0 = 2(0.55\ kg)(9.15\ s^{-1}) = 10.06\ kg/s$

$\gamma = 18.3\ s^{-1}$ $b = 10.1\ kg/s$

$$\boxed{\begin{array}{l}\gamma = 18.3\ s^{-1} \\ b = 10.1\ kg/s\end{array}}$$

b.) Solve for $x(t) = (A + Bt)e^{-\gamma t/2}$

$\Delta x = 0.055m$ $\dot{x} = 0.450\ m/s$

$x(t=0) = A$ $x(t=0) \equiv \Delta x$
$\Delta x = A$
$0.055m = A$

$\dot{x}(t) = -\gamma/2(A+Bt)e^{-\gamma t/2} + Be^{-\gamma t/2}$
$\dot{x}(t=0) = -\gamma/2(A) + B$ $\dot{x}(t=0) \equiv \dot{x}$
$\dot{x} = -\gamma/2(A) + B$
$\dot{x} + \gamma/2(A) = B$
$0.450\ m/s + 9.15\ s^{-1}(0.055m) = B$
$B = 0.9532\ m/s$

$$\boxed{\begin{array}{l}x(t) = (0.055\ m + 0.953\ m/s\,(t))\,e^{\frac{-18.3 s^{-1}(t)}{2}} \\ A = 0.055m \quad \gamma = 18.3\ s^{-1} \\ B = 0.953\ m/s\end{array}}$$

c. Find the maximum displacement.

$x(t) = (A+Bt)e^{-\gamma t/2}$
$\dot{x}(t) = -\gamma/2(A+Bt)e^{-\gamma t/2} + Be^{-\gamma t/2}$
$\dot{x}(t) = e^{-\gamma t/2}(-\gamma/2(A+Bt) + B)$
$0 = e^{-\gamma t/2}(-\gamma/2(A+Bt) + B)$
$0 = -\gamma/2(A+Bt) + B$
$-B = -\gamma/2(A+Bt)$
$\frac{2B}{\gamma} = A + Bt$

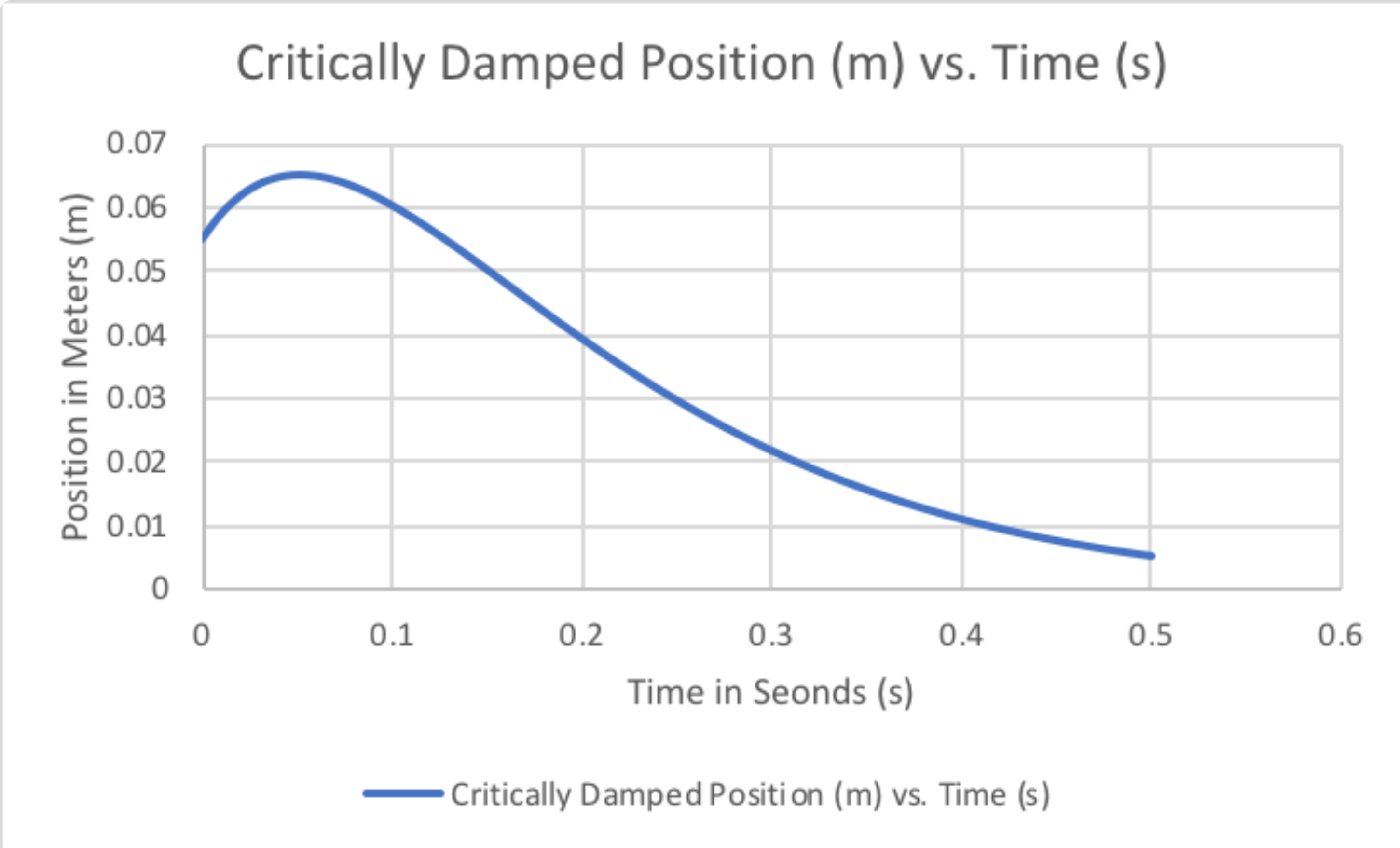
$\frac{2B}{\gamma} - A = Bt$
$\frac{2B}{\gamma B} - \frac{A}{B} = t$
$\frac{2}{\gamma} - \frac{A}{B} = t$
$t = \frac{2B - \gamma A}{\gamma B} = \frac{2(0.953\ m/s) - (18.3\ s^{-1})(0.055m)}{(18.3\ s^{-1})(0.953\ m/s)}$

$t = 0.05157\ s$

$x(t = 0.0516s) = (0.055m + 0.953\ m/s(0.0516s))\,e^{\frac{-(18.3 s^{-1})(0.0516s)}{2}}$
$x = 0.06497\ m$
$x = 0.065m$

$$\boxed{\begin{array}{l}x = 0.065\ m \\ t = 0.0516\ s\end{array}}$$

Problem 4)



Critically Damped Position (m) vs. Time (s)
Position in Meters (m)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
0.6
Time in Seonds (s)
Critically Damped Position (m) vs. Time (s)

Problem 5

$\gamma = 54.9\ s^{-1}$ $\quad m = 0.65\ kg$

$\omega_0 = 9.15\ rad/s$ $\quad \Delta x = 0.055\ m$

$\dot{x} = 0.450\ m/s$

a.) Solve For $x(t) = e^{-\gamma t/2}\left[Ae^{\alpha t} + Be^{-\alpha t}\right]$ $\quad \alpha = \sqrt{(\gamma/2)^2 - \omega_0^2}$

$x(t=0) = A + B$ $\qquad x(t=0) \equiv \Delta x$

$\Delta x = A + B$

$A = \Delta x - B$

$\dot{x}(t) = e^{-\gamma t/2}\left[\alpha A e^{\alpha t} - \alpha B e^{-\alpha t}\right] - \gamma/2\, e^{-\gamma t/2}\left[Ae^{\alpha t} + Be^{-\alpha t}\right]$

$\dot{x}(t=0) = \alpha A - \alpha B - \gamma/2(A+B)$ $\qquad \dot{x}(t=0) \equiv \dot{x}$

$\dot{x} = \alpha(\Delta x - B) - \alpha B - \gamma/2 A - \gamma/2 B$

$\dot{x} = \alpha\Delta x - \alpha B - \alpha B - \gamma/2(\Delta x - B) - \gamma/2 B$

$\dot{x} = \alpha\Delta x - 2\alpha B - \gamma/2\Delta x + \gamma/2 B - \gamma/2 B$

$\dot{x} = \alpha\Delta x - \gamma/2\Delta x - 2\alpha B$

$\dot{x} + \gamma/2\Delta x - \alpha\Delta x = -2\alpha B$

$$B = \frac{\dot{x} + \gamma/2\Delta x - \alpha\Delta x}{-2\alpha} = \frac{0.450 m/s + \left(\frac{54.9 s^{-1}}{2}\right)(0.055m) - (25.88 s^{-1})(0.055m)}{-2(25.88 s^{-1})}$$

$B = -0.01036\ m$

$A = x - B = 0.055m - (-0.01036m)$

$A = 0.0654\ m$

$$\left[\begin{array}{l} x(t) = e^{\frac{-(54.9 s^{-1})t}{2}}\left[(0.0654m)e^{(25.9 s^{-1})t} - (0.0104m)e^{-(25.9 s^{-1})t}\right] \\ A = 0.0654m \qquad B = -0.0104m \\ \alpha = 25.9\ s^{-1} \qquad \gamma = 54.9 s^{-1} \end{array}\right]$$

b.)