Electromagnetic Theory II: Homework 20

Due: 23 April 2021

1 Radiation resistance

Consider a point dipole connected by a wire and suppose that the dipole moment oscillates as $\mathbf{p} = p_o \cos(\omega t)\hat{\mathbf{z}}$.

a) Determine an expression of the time averaged current, $\langle I^2 \rangle$, between the point charges.

The radiation resistance, R, is defined in terms of the total power radiated $\langle P \rangle$ via

$$\langle P \rangle = \langle I^2 \rangle R.$$

b) Show that the radiation resistance (in Ohms) for an oscillating point dipole is

$$R = 790 \left(\frac{d}{\lambda}\right)^2.$$

c) Suppose that you designed an electric dipole transmitter for a cellphone that transmits in the usual frequency range. Determine the radiation resistance for this transmitter. State the parameters that you used. *Note: true cellphone transmitters are likely more complicated than this.*

2 Lifetime of a classical hydrogen atom

A classical single electron atom consists of an electron in circular orbit around a proton. The force exerted by the proton's electric field provides the necessary centripetal acceleration. Denote that the radius of orbit by r and suppose that its initial value is 5×10^{-10} m.

a) The electron spirals inwards. Determine the radius of orbit at which the electron moves at the speed of light. Use your result to show that the Larmor formula applies for most of the electron's motion.

b) Determine an expression for the total energy of the electron as a function of r. Use this and the Larmor formula to determine a differential equation for r as a function of time.

c) Use your result to determine the lifetime of the atom.

3 Brehmsstrahlung

Brehmsstrahlung is the radiation produced when a particle decelerates. Suppose that the particle acceleration and velocity are all along the z-axis. Then according to a derivation in the text, the power radiated into a spherical section at angle θ and with area $r^2 \sin\theta d\theta d\phi$ is

$$dP = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^3\theta}{(1-\beta\cos\theta)^5} d\theta d\phi$$

where $\beta = v/c$. Thus the power radiated per angular area $d\Omega = \sin\theta d\theta d\phi$ is

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2\theta}{(1-\beta\cos\theta)^5}.$$

a) Show that the angle at which the power radiated is a maximum satisfies

$$3\beta\cos^2\theta + 2\cos\theta - 5\beta = 0.$$

b) Determine the angle at which maximum power is radiated as $v \to 0$.

c) Determine the angle at which maximum power is radiated as $v \to c$.

d) Determine the speed at which the angle of maximum radiation is 45°.

e) Determine the speed at which the angle of maximum radiation is 5°.

f) Graph the speed of the particle vs. the angle of maximum radiation.