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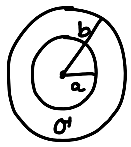
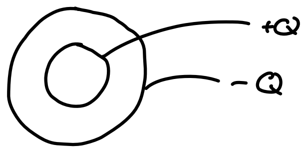
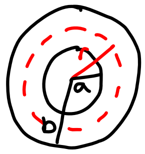
Dr. Middleton
PHYS 311
HW 12

Problem 1





a.) $J = \sigma E$ $\quad \int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$: Electric field in a sphere: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{r}$



$$\Delta V = -\int \vec{E} \cdot d\vec{l} \; , \; d\vec{l} = dr \, \hat{r} \; , \; \Delta V = -\frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r} \right]_a^b$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{b} - \frac{1}{a} \right] \quad : \quad Q = \frac{4\pi\varepsilon_0 \Delta V}{\left(\frac{1}{b} - \frac{1}{a}\right)}$$

$$I = \int \vec{J} \cdot d\vec{a} = \sigma \int \vec{E} \cdot d\vec{a} = \frac{\sigma}{\varepsilon_0} \cdot Q$$

$$I = \frac{\sigma}{\varepsilon_0} \left[\frac{4\pi\varepsilon_0 \Delta V}{\left(\frac{1}{b} - \frac{1}{a}\right)} \right] = 4\pi\sigma \left[\frac{\Delta V}{\left(\frac{1}{b} - \frac{1}{a}\right)} \right]$$

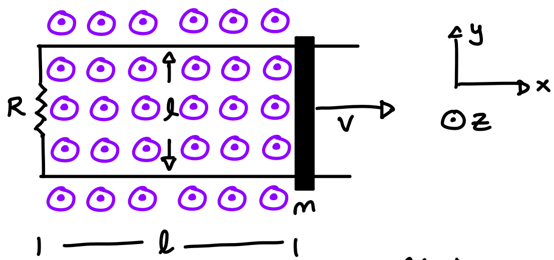
$$\boxed{I = \frac{4\pi\sigma \Delta V}{\left(\frac{1}{b} - \frac{1}{a}\right)}}$$

b.) $V = IR$ so $R = \frac{V}{I}$

$$R = \frac{\cancel{\Delta V}}{\frac{(4\pi\sigma)\cancel{\Delta V}}{\left(\frac{1}{b} - \frac{1}{a}\right)}} = \frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{(4\pi\sigma)}$$

$$\boxed{R = \frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{(4\pi\sigma)}}$$

Problem 2



a.) $I = \frac{\mathcal{E}}{R}$: $\mathcal{E} = -\frac{d\phi}{dt}$: $\Phi = \int \vec{B} \cdot d\vec{a}$: $\vec{B} = B(\hat{z})$, $d\vec{a} = dx\,dy\,(\hat{z})$

$$\Phi = B\int_0^l \int_0^x dx'dy = B \cdot x \cdot l \quad : \quad \Phi = Blx$$

$$-\frac{d\Phi}{dt} = -Bl\frac{dx}{dt} = -BlV \quad \therefore \quad \mathcal{E} = -BlV$$

$$\boxed{I = \frac{-BlV}{R} \text{ In the ccw direction}}$$

b.) $\vec{F} = I\int(d\vec{l} \times \vec{B})$, $d\vec{l} = -dx\,\hat{x}$, $\vec{B} = B(\hat{z})$

$$d\vec{l} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -dx & 0 & 0 \\ 0 & 0 & B \end{vmatrix} = Bdx\,(\hat{y})$$

$$\vec{F} = IB\int_0^l dx\,(\hat{y}) = IBl\,(\hat{y})$$

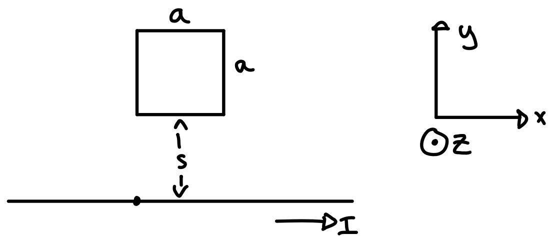
$$\boxed{\vec{F} = \frac{B^2l^2V}{R}\,(\hat{y})}$$

C.) $F = \frac{d\vec{p}}{dt}$: constant mass : $F = m\ddot{x}$

$$m\ddot{x} = \frac{B^2l^2V}{R} \quad : \quad \ddot{x} = \frac{B^2l^2V}{mR} \quad : \quad \frac{dv}{dt} = \frac{B^2l^2V}{mR} \quad : \quad \frac{dv}{V} = \frac{B^2l^2}{mR}dt \quad : \quad ln(v) = \frac{B^2l^2}{mR}t + V_0$$

$$\boxed{V = V_0 e^{-\frac{B^2l^2}{mR}t}}$$

Problem 3



a.) $\Phi = \int \vec{B} \cdot d\vec{a}$

$\vec{B}$ - Field due to a long strait wire : $\vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{y} \, (\hat{z})$, $d\vec{a} = dx\,dy \, (\hat{z})$

$$\Phi = \frac{\mu_0 I}{2\pi} \int_s^{s+a} \int_0^a \frac{1}{y} \, dx\, dy = \frac{\mu_0 I}{2\pi} a \cdot \ln(y) \Big|_s^{s+a} = \frac{\mu_0 I}{2\pi} \cdot a \cdot \ln(s+a) - \ln(s) = \frac{\mu_0 I}{2\pi} a \ln\left(\frac{s+a}{s}\right)$$

$$\boxed{\Phi = \frac{\mu_0 I}{2\pi} a \ln\left(\frac{s+a}{s}\right)}$$

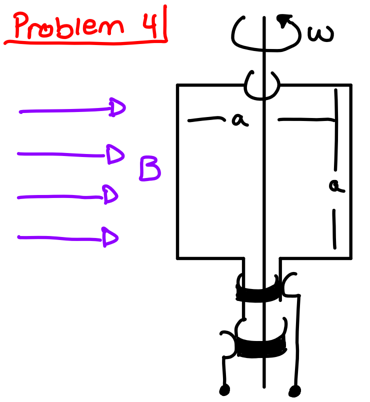
b.) $\mathcal{E} = -\frac{d\phi}{dt}$

$$-\frac{d\phi}{dt} = -\frac{\mu_0 I}{2\pi} a \cdot \frac{d}{dt} \ln\left(\frac{s+a}{s}\right) = -\frac{\mu_0 I}{2\pi} a \cdot \frac{ds}{dt} \ln\left(\frac{s+a}{a}\right)$$

$$-\frac{d\phi}{dt} = -\frac{\mu_0 I}{2\pi} a \cdot \left(\frac{-a}{s(s+a)}\right) = \frac{\mu_0 I}{2\pi} a^2 \cdot \frac{1}{s(s+a)}$$

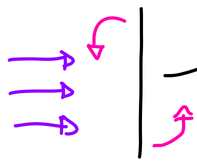
$$\boxed{\mathcal{E} = \frac{\mu_0 I}{2\pi} \cdot \frac{a^2}{s(s+a)}}$$

Problem 4



Dot Product : $\vec{B}\cdot d\vec{a}$ or $B\cdot A\cdot \cos\theta$

Top view



Spins at speed of ω

So, to know position of loop:

$\cos(\theta) \rightarrow \cos(\omega t)$: $\omega = \frac{rad}{\cancel{s}}\cdot t = \cancel{s}$

$$\Phi = \int \vec{B}\cdot d\vec{a} = B\int_0^a\int_0^a dx\,dy = B\cdot a^2$$

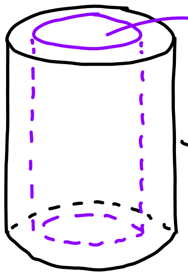
$\Phi = Ba^2$: Rotates @ $\theta(t) = \omega t$, $\cos(\omega t)$

$\Phi = Ba^2\cos(\omega t)$: $\mathcal{E} = \frac{-d\phi}{dt}$: $\frac{-d\phi}{dt} = B\cdot\omega\cdot a^2\sin(\omega t)$

$$\boxed{\mathcal{E}(t) = B\omega a^2\sin(\omega t)}$$

Problem 5

$\vec{B}(t) = B_0 \sin(\omega t)\,\hat{z}$



$I = \frac{\mathcal{E}}{R}$: $\mathcal{E} = -\frac{d\phi}{dt}$: $\Phi = \int \vec{B} \cdot d\vec{a}$

$\vec{B}(t) = B_0 \sin(\omega t)\,\hat{z}$ $\therefore$ $\Phi = B_0 \sin(\omega t) \int_0^{2\pi} \int_0^{\frac{3}{4}a} s\, ds\, d\phi$

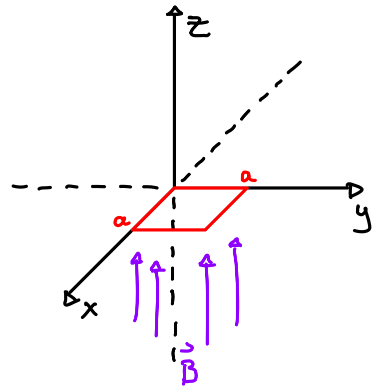
$d\vec{a} = s\, ds\, d\phi\, \hat{z}$

$\Phi = B_0 \sin(\omega t) \cdot 2\pi \cdot \frac{s^2}{2}\Big|_0^{\frac{3}{4}a} = B_0 \sin(\omega t) \cdot \pi \cdot \frac{9}{16} a^2$

$\mathcal{E} = -\frac{d\phi}{dt} = -B_0 \omega \cos(\omega t) \cdot \frac{9\pi}{16} a^2$: $\mathcal{E} = -B_0 \omega \cos(\omega t) \cdot \frac{9\pi}{16} a^2$

$$\boxed{I(t) = -\frac{B_0 \omega \cos(\omega t) \cdot 9\pi a^2}{16R}}$$

Problem 6



$$\vec{B}(y,t) = Ky^2t^3\ \hat{z}$$

$$\Phi = \int \vec{B}\cdot d\vec{a}\ , \quad \mathcal{E} = \frac{-d\phi}{dt}$$

$$\Phi = \int \vec{B}\cdot d\vec{a}$$

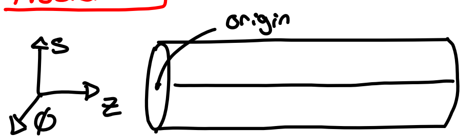
$$\vec{B} = Ky^2t^3(\hat{z}),\ d\vec{a} = dx\,dy\ \hat{z}$$

$$\Phi = Kt^3\int_0^a\int_0^a y^2\,dx\,dy = Kt^3\cdot \frac{y^3}{3}\Big|_0^a \cdot x\Big|_0^a = \frac{K\cdot t^3\cdot a^4}{3}$$

$$\mathcal{E} = \frac{-d\phi}{dt} : \quad \frac{-d\phi}{dt} = -K\cdot t^2\cdot a^4$$

$$\boxed{\mathcal{E} = -K\cdot t^2\cdot a^4}$$

Problem 7



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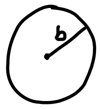


: $I(t) = I_0 \sin(\omega t)$

a.) The magnetic field is circumferential, where as the Electric field is longitudinal

$$\boxed{\text{Longitudinal}}$$

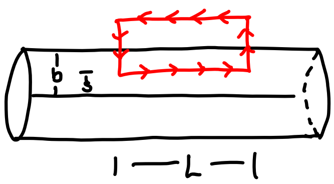
b.) $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$: $\Phi = \int \vec{B}\cdot d\vec{a}$: $\vec{B}$ due to wire : $\vec{B} = \frac{\mu_0 I}{2\pi}\cdot\frac{1}{S}\,\hat{\phi}$

Outside the wire there will be no $\vec{E}$ due to no $Q_{enc.}$

When $s > b$, $\vec{E} = 0$

$\Phi = \int \vec{B}\cdot d\vec{a}$: $\vec{B} = \frac{\mu_0 I}{2\pi}\cdot\frac{1}{S}\,\hat{\phi}$, $d\vec{a} = ds\,dz\,\hat{\phi}$

Inside the wire,



$$\Phi = \frac{\mu_0 I_0 \sin(\omega t)}{2\pi}\int_0^L\int_s^b \frac{1}{S'}\,ds'dz$$

$$= \frac{\mu_0 I_0 \sin(\omega t)}{2\pi}\cdot\left[z\Big|_0^L\right]\cdot \ln(S')\Big|_s^b$$

$$\Phi = \frac{\mu_0 I_0 \sin(\omega t)}{2\pi}\cdot L\cdot \ln(b/s)$$

$$\Phi = \frac{\mu_0 I_0 \sin(\omega t)}{2\pi}\cdot L\cdot \ln(b/s) \qquad \oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$$

$\oint \vec{E}\cdot d\vec{l}$: $d\vec{l} = dz\,(\hat{z})$: $\vec{E} = E\,(\hat{z})$: $E\cdot\int_0^L dz = E\cdot L$: $\oint \vec{E}\cdot d\vec{l} = EL$

$$-\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I_0 \sin(\omega t)\cdot L\cdot \ln(b/s)}{2\pi}\right) = -\frac{\mu_0 I_0 \omega \cos(\omega t)\cdot L\cdot \ln(b/s)}{2\pi}$$

$$\therefore \qquad EL = -\frac{\mu_0 I_0 \omega \cos(\omega t)\cdot L\cdot \ln(b/s)}{2\pi}$$

$$\boxed{\vec{E} = \frac{\mu_0 I_0 \omega \cos(\omega t)\ln(b/s)}{2\pi}\,(-\hat{z})}$$