

qubit general state:

$$\hat{\rho} = \frac{1}{2} (\hat{I} + r_x \hat{\sigma}_x + r_y \hat{\sigma}_y + r_z \hat{\sigma}_z)$$

$$\underbrace{r_x^2 + r_y^2 + r_z^2}_{r^2} \leq 1$$

can represent by vector $\vec{r} = r_x \hat{x} + r_y \hat{y} + r_z \hat{z}$

All possible states in sphere

Physical processes represented by vector.

e.g. evolution.

- phase shift about \hat{z} through ψ

$$\hat{U} = e^{-i\psi \hat{\sigma}_z / 2}$$

$$\rightarrow \text{get } \hat{\rho}_F = \hat{U} \hat{\rho}_i \hat{U}^\dagger$$

\rightarrow what vector \vec{r} now represents

e.g. measurement:

- want measure in $|0\rangle, |1\rangle$ basis

$$\begin{array}{l} \text{prob}(0) = ? \\ \text{prob}(1) = ? \end{array} \quad \left. \begin{array}{l} \text{for } \hat{\rho}_i \\ \text{for } \hat{\rho}_F \end{array} \right\}$$

- what about $\langle \sigma_z \rangle = \text{prob}(0) - \text{prob}(1)$

$$\text{or } \langle \sigma_x \rangle = ??$$

