Taylor Larrechea
Dr. Gustafson
MATH 361 HW Ch. 4.1

✓✓✓✓
1,3,5,7

Problem 1

x	f(x)	FWD f'(x)	BWD f'(x)
0.5	0.4794	0.852	NA
0.6	0.5646	0.796	0.852
0.7	0.6442	NA	0.796

x	f(x)	FWD f'(x)	BWD f'(x)
0.0	0.00000	3.707	NA
0.2	0.74140	3.152	3.707
0.4	1.37180	NA	3.152

Problem 3

a.)

$f(x) = \sin(x)$: $f''(x) = -\sin(x)$

x	f(x)	FWD f'(x)	BWD f'(x)	Error
0.5	0.4794	0.862	NA	0.028238
0.6	0.5646	0.796	0.852	0.032210
0.7	0.6442	NA	0.796	0.032210

b.)

$f(x) = e^x - 2x^2 + 3x$: $f''(x) = e^x - 4$

x	f(x)	FWD f'(x)	BWD f'(x)	Error
0.0	0.00000	3.707	NA	0.27785
0.2	0.74140	3.152	3.707	0.25081
0.4	1.37180	NA	3.152	0.28081

Problem 5

a.)

x	f(x)	f'(x) 3pt	f'(x) 2pt
1.1	9.025013	17.769705	
1.2	11.028318	21.70385	28.193685
1.3	13.46374	26.617865	27.1078 5
1.4	16.44465	32.51085	

b.)

x	f(x)	f'(x) 3pt	f'(x) 2pt
8.1	16.944410	3.09205	
8.3	17.86492	3.116425	3.11615
8.5	18.19056	3.14085	3.139975
8.7	18.82091	3.163525	

c.)

x	f(x)	f'(x) 3pt	f'(x) 2pt
2.9	-4.827866	5.101375	
3.0	-4.240058	6.64665	6.684785
3.1	-3.496909	8.208195	8.21683
3.2	-2.596792	9.78601	

d.)

x	f(x)	f'(x) 3pt	f'(x) 2pt
2.0	3.6887983	0.1853315	
2.1	3.6905701	-0.106122	-0.099895 5
2.2	3.6888192	-0.335122 5	-0.329896
2.3	3.6345909	-0.55467	

Problem 7

a.)

x	f(x)	f'(x) 3Pt	f'(x) 3Pt	Error Bound
1.1	9.025013	17.769705		0.359033
1.2	11.023818	21.703885	28.193685	0.179517
1.3	13.463714	26.617665	27.10785	0.219262
1.4	16.44465	32.51085		0.438824

$f(x) = e^{2x}$, $f'(x) = 2e^{2x}$, $f''(x) = 4e^{2x}$, $f'''(x) = 8e^{2x}$, $f^4(x) = 16e^{2x}$

$[1.1, 1.3]$: $Er = \frac{(0.1)^2}{3} \cdot |8e^{2(1.3)}| = 0.359033$

$[1.1, 1.3]$: $Er = \frac{(0.1)^2}{6} |8e^{2(1.3)}| = 0.179517$

$[1.2, 1.4]$: $Er = \frac{(0.1)^2}{6} |8e^{2(1.4)}| = 0.219262$

$[1.2, 1.4]$: $Er = \frac{(0.1)^2}{3} |8e^{2(1.4)}| = 0.438524$

b.)

x	f(x)	f'(x) 3Pt	f'(x) 3Pt	Error Bound
8.1	16.944410	3.09203		2.0322×10^{-4}
8.3	17.886492	3.116425	3.11615	1.0161×10^{-4}
8.5	18.190056	3.14085	3.139975	9.667×10^{-5}
8.7	18.82091	3.163525		1.9355×10^{-4}

$f(x) = x \cdot \ln(x)$, $f'(x) = \ln(x) + 1$, $f''(x) = 1/x$, $f'''(x) = -1/x^2$, $f^4(x) = 2/x^3$

$[8.1, 8.5]$: $Er = \frac{(0.2)^2}{3} \cdot |1/(8.1)^2| = 2.0322 \times 10^{-4}$

$[8.1, 8.5]$: $Er = \frac{(0.2)^2}{6} |1/(8.1)^2| = 1.0161 \times 10^{-4}$

$[8.3, 8.7]$: $Er = \frac{(0.2)^2}{6} |1/(8.3)^2| = 9.677 \times 10^{-5}$

$[8.3, 8.7]$: $Er = \frac{(0.2)^2}{3} |1/(8.3)^2| = 1.9355 \times 10^{-4}$

c.)

x	f(x)	f'(x) 3Pt	f'(x) 3Pt	Error Bound
2.9	-4.827866	5.101375		0.018099
3.0	-4.240058	6.646685	6.684785	0.009049
3.1	-3.496909	8.208195	8.21683	0.004939
3.2	-2.596792	9.78601		0.009878

$f(x) = x\cos(x) - x^2\sin(x)$, $f'(x) = (1-x^2)\cos(x) - 3x\sin(x)$

$f''(x) = (x^2-4)\sin(x) - 5x\cos(x)$, $f'''(x) = (x^2-9)\cos(x) + 7x\sin(x)$

$f^4(x) = 9x\cos(x) + (16-x^2)\sin(x)$

$[2.9, 3.1]$: $Er = \frac{(0.1)^2}{3} |(3.1^2-9)\cos(3.1) + 7(3.1)\sin(3.1)| = 0.018099$

$[2.9, 3.1]$: $Er = \frac{(0.1)^2}{6} |(3.1^2-9)\cos(3.1) + 7(3.1)\sin(3.1)| = 0.009049$

$[3.0, 3.2]$: $Er = \frac{(0.1)^2}{6} |(3^2-9)\cos(3) + 7(3)\sin(3)| = 0.004939$

$[3.0, 3.2]$: $Er = \frac{(0.1)^2}{3} |(3^2-9)\cos(3) + 7(3)\sin(3)| = 0.009878$

Problem 5) Continued

d.)

x	f(x)	f'(x) 3pt	f'(x) 2pt	Error
2.0	3.6887983	0.1353315		0.004103
2.1	3.6905701	-0.108122	-0.0998955	0.002052
2.2	3.6888192	-0.3351225	-0.329896	0.0026
2.3	3.6245909	-0.55467		0.003201

$f(x) = 2\cdot\ln x^2 + 3\sin(x)$, $f'(x) = 3\cos(x) + \frac{4\ln(x)}{x}$

$f''(x) = -3\sin(x) - \frac{4\ln(x)}{x^2} + \frac{4}{x^2}$,

$f'''(x) = -3\cos(x) + \frac{8\ln(x)}{x^3} - \frac{12}{x^3}$

$f^4(x) = 3\sin(x) - \frac{24\ln(x)}{x^4} + \frac{44}{x^3}$

$[2.0,2.2]$: $Er = \frac{(0.1)^2}{3}\left|-3\cos(2.2) + \frac{8\cdot\ln(2.2)}{2.2^3} - \frac{12}{2.2^3}\right| = 0.004103$

$[2.0,2.2]$: $Er = \frac{(0.1)^2}{6}\left|-3\cos(2.2) + \frac{8\cdot\ln(2.2)}{2.2^3} - \frac{12}{2.2^3}\right| = 0.002052$

$[2.1,2.3]$: $Er = \frac{(0.1)^2}{6}\left|-3\cos(2.3) + \frac{8\cdot\ln(2.3)}{2.3^3} - \frac{12}{2.3^3}\right| = 0.0026$

$[2.1,2.3]$: $Er = \frac{(0.1)^2}{3}\left|-3\cos(2.3) + \frac{8\cdot\ln(2.3)}{2.3^3} - \frac{12}{2.3^3}\right| = 0.003201$