Statistical and Thermal Physics: Homework 9

Due: 21 February 2020

1 Entropy and free expansion

An isolated ideal gas expands freely from volume V_i to volume V_f.

a) Determine the change in entropy of the gas.

b) Does the change in entropy depend on whether the gas is monoatomic or diatomic?

2 Entropy and equations of state

For a particular gas the entropy is

$$S = S(E,V,N) = \frac{3}{2} Nk \ln\left(E + \frac{N^2 a}{V}\right) + Nk\ln\left(V - Nb\right) + f(N)$$

where a and b are constants and $f(N)$ is some function of N only.

a) Determine an expression for T in terms of E, V, N and use this to determine $E = E(T, V, N)$.

b) Use the previous result to show that

$$S = S(T,V,N) = \frac{3}{2} Nk \ln T + Nk\ln\left(V - Nb\right) + g(N)$$

where $g(N)$ is a function of N only.

c) Determine an expression for P in terms of T, V, N.

3 Entropy changes in isothermal processes: ideal gas

An ideal gas (could be monoatomic, diatomic, etc.) with a fixed number of particles undergoes an isothermal process in which its volume increases by a factor of three.

a) Determine an expression for the factor by which the pressure changes.

b) Determine an expression for the change in entropy of the system; use the expression for the entropy of an ideal gas. Is this consistent with what the heat flow would predict for this process?

4 Entropy changes in adiabatic processes: ideal gas

An ideal gas (could be monoatomic, diatomic, etc.) with a fixed number of particles undergoes an adiabatic process in which its volume increases by a factor of three.

a) Determine an expression for the factor by which the pressure changes.

b) Determine an expression for the factor by which the temperature changes.

c) Determine an expression for the change in entropy of the system; use the expression for the entropy of an ideal gas. Is this consistent with what the heat flow would predict for this process?

a) $PV = NkT$

In isothermal T is constant. So $PV = \text{const}$

$$P_f V_f = P_i V_i$$

$$P_f = P_i \left(\frac{V_i}{V_f}\right) = P_i \frac{1}{3} \Rightarrow \boxed{P_f = \frac{1}{3} P_i}$$

b) $S = fNk \ln(T) + Nk \ln V + \tilde{h}(N)$

$$\Delta S = S_f - S_i$$

$$= fNk \ln T_f + Nk \ln V_f + \tilde{h}(N) - \left[fNk \ln T_i + Nk \ln V_i + \tilde{h}(N)\right]$$

$$= fNk \ln\left(\frac{T_f}{T_i}\right) + Nk \ln\left(\frac{V_f}{V_i}\right)$$

$$= fNk \ln 1 + Nk \ln 3 \Rightarrow \boxed{\Delta S = Nk \ln 3}$$

In this process $\underbrace{\Delta E}_{=0} = Q + \underbrace{W}_{\text{negative}} \Rightarrow Q > 0$

Yes since $\Delta S > 0$

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a) $PV^{\gamma} = \text{const}$

$$P_f V_f^{\gamma} = P_i V_i^{\gamma}$$

$$\Rightarrow P_f = P_i \left(\frac{V_i}{V_f}\right)^{\gamma} \Rightarrow \boxed{P_f = P_i \left(\frac{1}{3}\right)^{\gamma}}$$

b) $P = \frac{NkT}{V}$

$$\Rightarrow \frac{NkT}{V} V^{\gamma} = \text{const} \Rightarrow TV^{\gamma-1} = \text{const}$$

$$\Rightarrow T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} \Rightarrow T_f = T_i \left(\frac{1}{3}\right)^{\gamma-1}$$

Note $T_f < T_i$

c) $S = fNk \ln T + Nk \ln V + \tilde{h}(N)$

$$\Rightarrow \Delta S = fNk \ln\left(\frac{T_f}{T_i}\right) + Nk \ln\left(\frac{V_f}{V_i}\right)$$

$$= fNk \ln\left(\frac{V_i}{V_f}\right)^{\gamma-1} + Nk \ln\left(\frac{V_f}{V_i}\right)$$

$$= f(\gamma-1)Nk \ln\left(\frac{V_i}{V_f}\right) + Nk \ln\left(\frac{V_f}{V_i}\right)$$

$$= -f(\gamma-1) Nk \ln\frac{V_i}{V_f} + Nk \ln\left(\frac{V_f}{V_i}\right) = Nk \ln\left(\frac{V_f}{V_i}\right)\left[1 - f(\gamma-1)\right]$$

$$\gamma = \frac{C_P}{C_V} = \frac{(f+1)\cancel{Nk}}{f\cancel{Nk}} = 1 + \frac{1}{f} \Rightarrow \gamma - 1 = \frac{1}{f} \Rightarrow 1 - f(\gamma-1) = 0$$

So $\Delta S = 0$. Yes no heat enters $\Rightarrow \Delta S = 0$.

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a) The gas expands and its volume increases from V_i to V_f
The temperature stays constant. Then

$$S = fNk\ln(T) + Nk\ln(V) + \hat{h}(N)$$

$$\Rightarrow \quad \Delta S = fNk\left[\ln(T_f) - \ln(T_i)\right] + Nk\left[\ln(V_f) - \ln(V_i)\right]$$

Now $T_f = T_i \Rightarrow$ $\Delta S = Nk\left[\ln(V_f) - \ln(V_i)\right]$

$$\Rightarrow \quad \Delta S = Nk\ln\left(\frac{V_f}{V_i}\right)$$

b) No $\rightarrow$ equation shows

a) $\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N}$

$$= \frac{3}{2} Nk \frac{1}{\left(E + \frac{N^2 a}{V}\right)}$$

$$\Rightarrow T = \frac{2}{3}\frac{1}{Nk}\left(E + \frac{N^2 a}{V}\right) \Rightarrow \frac{3}{2} NkT = E + \frac{N^2 a}{V}$$

$$\Rightarrow E = \frac{3}{2}NkT - \frac{N^2 a}{V}$$

b) Here $E + \frac{N^2 a}{V} = \frac{3}{2} NkT$

gives:

$$S = \frac{3}{2} Nk \ln\left(\frac{3}{2}NkT\right) + Nk\ln(V-Nb) + f(N)$$

$$= \frac{3}{2}Nk\left[\ln T + \ln\left(\frac{3}{2}Nk\right)\right] + \dots$$

$$= \frac{3}{2}Nk\ln T + Nk\ln(V-Nb) + \underbrace{f(N) + \frac{3}{2}Nk\ln\left(\frac{3}{2}Nk\right)}_{g(N)}$$

c) $P = T\left(\frac{\partial S}{\partial V}\right)_{E,N}$

$$= T\left\{\frac{3}{2}Nk\left(\frac{1}{E + \frac{N^2 a}{V}}\right)\left(-\frac{N^2 a}{V^2}\right) + \frac{Nk}{V-Nb}\right\}$$

But $\frac{3}{2}Nk\frac{1}{E + \frac{N^2 a}{V}} = \frac{1}{T}$ from part a) Thus

$$P = T\left\{-\frac{N^2 a}{V^2 T} + \frac{Nk}{V-Nb}\right\}$$

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$$\Rightarrow \quad P = -\frac{N^2 a}{V^2} + \frac{NkT}{V-Nb}$$

$$\Rightarrow \quad \left(P + \frac{N^2 a}{V^2}\right)(V - Nb) = NkT$$