Knight: Chapter 18

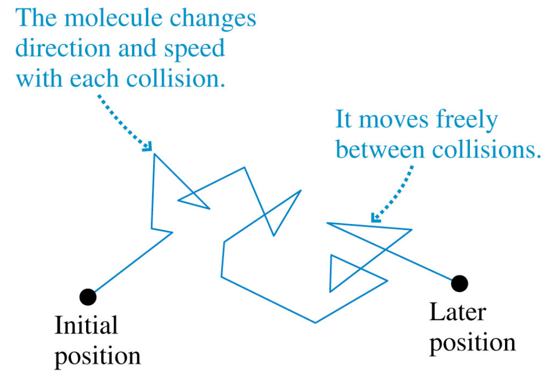
The Micro/Macro Connection

(*Thermal Energy & Specific Heat, Thermal Interactions and Heat*)

Last time...

- *Mean free path..*

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$



- *Pressure in a gas..*

$$p = \frac{F}{A} = \frac{1}{3}\frac{N}{V}mv_{rms}^2$$

- *Root-mean-square speed..* $v_{rms} = \sqrt{(v^2)_{avg}}$

- *Average translational KE..* $\epsilon_{avg} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_B T$

Temperature

- Define ϵ_{ave} to be the *average translational* KE of molecules in a gas.

$$\epsilon_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{1}{2}mv_{rms}^2$$

Using our expression for pressure and the ideal gas law...

$$\epsilon_{avg} = \frac{3}{2}k_B T$$

Equating these expressions and solving for rms speed...

Temperature

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Using our expression for pressure and the ideal gas law…

$$\epsilon_{avg} = \frac{3}{2}k_B T$$

Equating these expressions and solving for rms speed…

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_b T$$

$$v_m = \sqrt{\frac{3k_b T}{m}}$$

i.e. 18.4: Total microscopic KE
i.e. 18.5: Calculating a rms speed

What is the total translational KE of the molecules in 1.0 mol of gas at standard pressure and room temperature (20° C)?

What is the rms speed of nitrogen molecules at room temperature?

a.)

$$\varepsilon_{Avg} = \frac{3}{2} k_b T = \frac{3}{2}(1.38\times10^{-23} J/K)(293K) = 6.1\times10^{-21} J$$

$n = 1.0$ mol $\quad n = \frac{N}{N_A}$

$$N = N_A n = (1.0 mol)(6.02\times10^{23} mol^{-1}) = 6.02\times10^{23}$$

$$K = N\varepsilon_{avg} = 3.7\times10^{3} J$$

b.) $v_{rms} = \sqrt{\frac{2\varepsilon_{Avg}}{m}} = 510\ m/s$

$N_2 = 28u$

$m = 4.6\times10^{-26} kg$

Quiz Question 1

A rigid container holds both hydrogen gas (H_2) and nitrogen gas (N_2) at 100°C.

Which statement describes the *average translational kinetic energies* of the molecules?

1. ϵ_{avg} of H_2 < ϵ_{avg} of N_2.
2. ϵ_{avg} of H_2 = ϵ_{avg} of N_2.
3. ϵ_{avg} of H_2 > ϵ_{avg} of N_2.

Quiz Question 2

A rigid container holds both hydrogen gas (H_2) and nitrogen gas (N_2) at 100°C.

Which statement describes their *rms speeds*?

1. v_{rms} of $H_2 < v_{rms}$ of N_2.
2. v_{rms} of $H_2 = v_{rms}$ of N_2.
3. v_{rms} of $H_2 > v_{rms}$ of N_2.

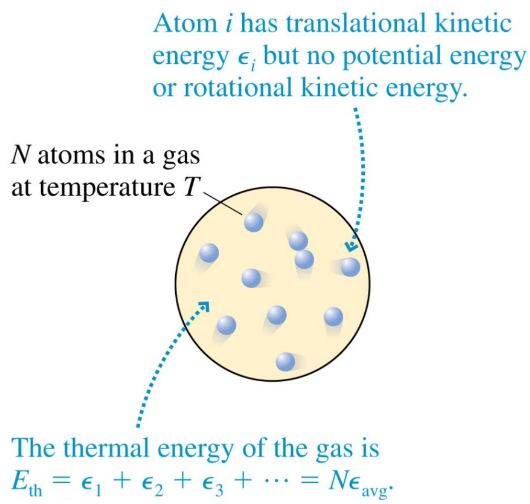
Thermal Energy of a Monatomic Gas..

The *thermal energy of a system* is

$$E_{th} = K_{micro} + U_{micro}.$$

- The atoms in a *monatomic gas* have no molecular bonds with their neighbors, hence $U_{micro} = 0$.
- Since the *average kinetic energy* of a single atom in an ideal gas is
$$\epsilon_{avg} = 3/2\ k_B T..$$
- The *total thermal energy* is:

$$\boxed{E_{th} = \frac{3}{2} N k_B T = \frac{3}{2} nRT}$$



Thermal Energy of a Monatomic Gas..

The *molar specific heat* for a *monatomic gas* is predicted to be...

TABLE 17.4 Molar specific heats of gases (J/mol K)

Gas	C_P	C_V	$C_P - C_V$
Monatomic Gases			
He	20.8	12.5	8.3
Ne	20.8	12.5	8.3
Ar	20.8	12.5	8.3
Diatomic Gases			
H_2	28.7	20.4	8.3
N_2	29.1	20.8	8.3
O_2	29.2	20.9	8.3

Thermal Energy of a Monatomic Gas..

The *molar specific heat* for a *monatomic gas* is predicted to be...

$$C_V = \frac{3}{2}R = 12.5\ \mathrm{J/mol\ K}$$

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Gas	C_P	C_V	$C_P - C_V$
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The Equipartition Theorem…

- Atoms in a monatomic gas carry energy *exclusively* as *translational K*..

$$\epsilon = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) = \epsilon_x + \epsilon_y + \epsilon_z$$

- $\epsilon_x, \epsilon_y, \epsilon_z$ can be thought of as independent *modes* of storing energy (a.k.a. 3 degrees of freedom (d.o.f.)).

Notice:

- Molecules in a gas may have additional modes of energy storage, i.e. the *K* and *U* associated with *vibration*, or *rotational K*.

The Equipartition Theorem…

The thermal energy of a system of particles is *equally divided* among all the possible degrees of freedom.

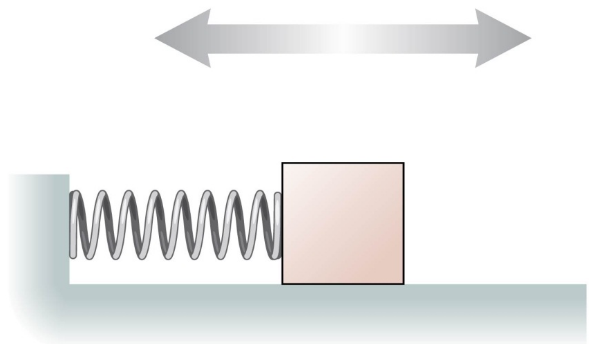
For a system of N particles at temperature T, the energy stored in each mode is…

$$\frac{1}{2}Nk_BT \quad \text{or} \quad \frac{1}{2}nRT$$

Quiz Question 3

A mass on a spring oscillates back and forth on a frictionless surface.

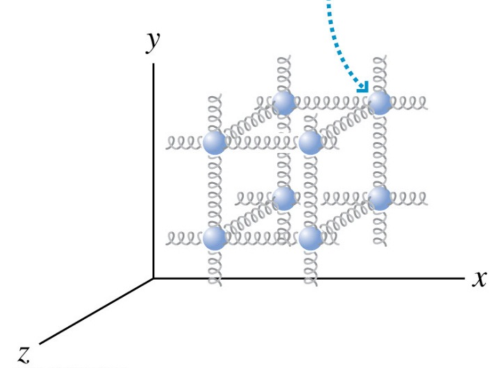
How many degrees of freedom does this system have?



1. 1
2. 2
3. 3
4. 4
5. 6

Thermal Energy of a Solid…

Model a solid as atoms connected by *spring-like* molecular bonds…



- 3 d.o.f. associated with K
- 3 d.o.f. associated with U
- 6 d.o.f. *total*.

- The energy stored in each d.o.f. is ½ Nk_BT…

$$\boxed{E_{th} = 3Nk_BT = 3nRT}$$

Thermal Energy of a Solid..

The *molar specific heat* for a *solid* is predicted to be…

TABLE 17.2 Specific heats and molar specific heats of solids and liquids

Substance	c (J/kg K)	C (J/mol K)
Solids		
Aluminum	900	24.3
Copper	385	24.4
Iron	449	25.1
Gold	129	25.4
Lead	128	26.5
Ice	2090	37.6
Liquids		
Ethyl alcohol	2400	110.4
Mercury	140	28.1
Water	4190	75.4

Thermal Energy of a Solid..

The *molar specific heat* for a *solid* is predicted to be...

$$\boxed{C = 3R = 24.9\ \mathrm{J/mol\ K}}$$

TABLE 17.2 Specific heats and molar specific heats of solids and liquids

Substance	***c*** **(J/kg K)**	***C*** **(J/mol K)**
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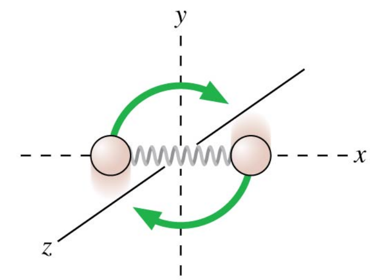
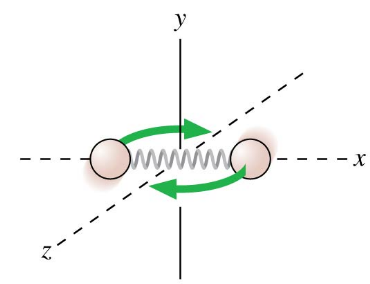
Thermal Energy of a Diatomic Gas…

Model a diatomic molecule as atoms connected by *spring-like* molecular bonds…

- 3 d.o.f. associated with *translational K*
- 2 d.o.f. associated with *rotational K*
- 5 d.o.f. *total**



Rotation end-over-end about the z-axis



Rotation end-over-end about the y-axis

- The energy stored in each d.o.f. is ½ Nk_BT…

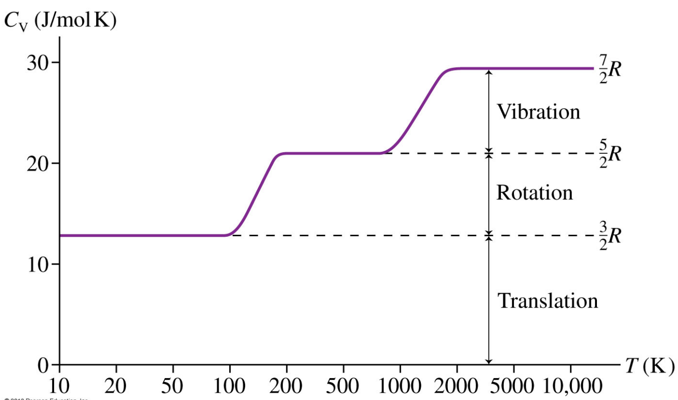
$$\boxed{E_{th} = \frac{5}{2}Nk_BT = \frac{5}{2}nRT}$$

* at commonly used temps

Thermal Energy of a Diatomic Gas..

The *molar specific heat* for a *diatomic gas* is predicted to be…

$$C_V = \frac{5}{2}R = 20.8 \text{ J/mol K}$$



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Quiz Question 4

Systems A and B are both monatomic gases. At this instant,

A	B
$N = 1000$	$N = 2000$
$E_{th} = 2 \times 10^{-17}$ J	$E_{th} = 3 \times 10^{-17}$ J

1. $T_A > T_B$.
2. $T_A = T_B$.
3. $T_A < T_B$.
4. There's not enough info to compare their temps.

i.e. 18.7: *The rotational frequency of a molecule*

The nitrogen molecule N_2 has a bond length of 0.12 nm. Estimate the rotational frequency of N_2 at 20° C.

$l = 1.2 \times 10^{-10}$ m

$T = 293$ K

$$\varepsilon_{ROT} = \frac{1}{2} I \omega^2$$

$$I = m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{2} m l^2$$

$$\varepsilon_{ROT} = \frac{1}{2} \cdot \frac{1}{2} m l^2 \omega^2 = \frac{1}{2} k_B T$$

$$\omega = \sqrt{\frac{2 k_B T}{m l^2}} = 4.9 \times 10^{12}\ s^{-1}$$