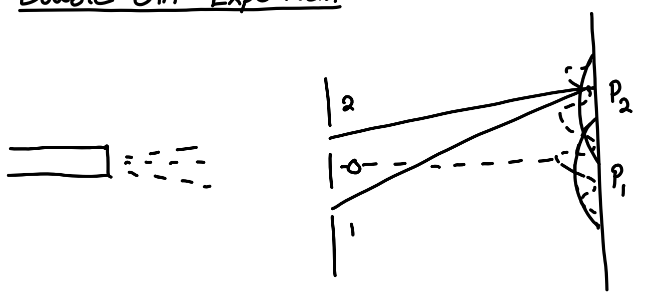
Double Slit Experiment



Phase difference

$$P = P_1 + P_2 + 2\sqrt{P_1P_2}\cos\delta$$

Either hole closed: $P_{12} \neq P_1 + P_2$
Both holes open: $P_{12} = P_1 + P_2$
$\Rightarrow$ Probabilities

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\theta$$

Summary

I. (By comparing intensities of Light ~ # of photons & Knowing E & M relationships)
The probability of an event in an ideal experiment is given by the square of the absolute value of a complex #, ϕ, called the probability amplitude.

$P \equiv$ Probability
$\phi \equiv$ Probability amplitude $\quad (e^{i\theta})^n = \cos n\theta + i\sin n\theta$
$P = |\phi^2|$

II. When an event can occur several ways (one of....), The probability amplitude for the event is the sum of amplitudes for each event.

$$\phi_{12} = \phi_1 + \phi_2$$
$$P_{12} = |\phi_1 + \phi_2|^2$$

III. If an experiment is performed which can determine the event (Path) that actually occurred, the probability is just the sum, $\quad P = P_1 + P_2$

The interference term is lost!

Uncertainty Principle

There is an analagous reaction for every pair of Canonically conjugate variables.

$\Delta x \Delta p \geq \frac{\hbar}{2}$
$\Delta E \Delta t \geq \frac{\hbar}{2}$

Ex 5.8 Uncertainty Principle

Baseball confined to $\Delta x \sim 17.5\,m$
$\Delta p \sim 10^{-36}\ kg \cdot m/s$

Electron $\rightarrow$ $\Delta x \sim a_0$
$\Delta p \sim 10^{-24}\ kg \cdot m/s \rightarrow \Delta x \sim 10^{6}\ m/s$

$$E = \frac{p^2}{2m} + \frac{Kx^2}{2}$$

In Ground State

$\langle x \rangle = 0 \qquad \langle p \rangle = 0$

$$E = \frac{\langle p \rangle^2}{2m} + \frac{K\langle x \rangle^2}{2}$$

$\langle p \rangle^2 = \Delta p^2$

$\langle x \rangle^2 = \Delta x^2$

Invoke uncertainty principle.

$$\langle p \rangle^2 = \Delta p^2 = \left(\frac{\hbar}{2\Delta x}\right)^2 \qquad \Delta x \to \delta$$

Energy is now:

$$E = \frac{\hbar^2}{8m\delta^2} + \frac{K}{2}\delta^2$$

We want a minimum energy $\Rightarrow$ condition for a minimum

$$\frac{\partial E}{\partial \delta} = 0 \Rightarrow \frac{-\hbar^2}{4m\delta^3} + K\delta = 0$$

$$\delta_0^2 = \sqrt{\frac{\hbar^2}{4mK}}$$

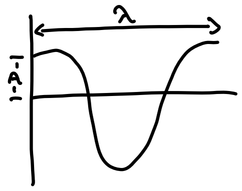
$$E_0 = \frac{\hbar^2}{8m}\sqrt{\frac{4mK}{\hbar^2}} + \frac{K}{2}\sqrt{\frac{\hbar^2}{4mK}} = \frac{\hbar}{2}\sqrt{\frac{K}{m}}$$

$$E_0 = \frac{\hbar}{2}\omega_0$$

Waves

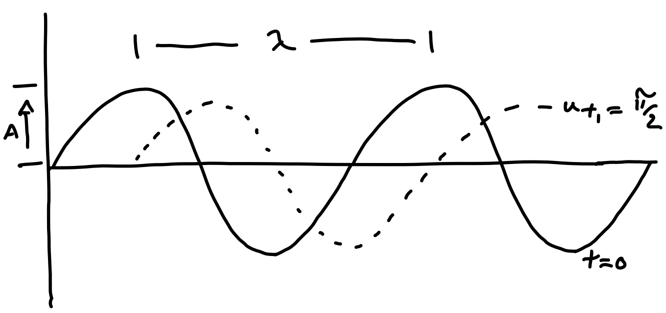
1.) Standing Waves $\qquad \psi(x) = A\cos Kx = Re(Ae^{iKx})$

$K = \frac{2\pi}{\lambda}$: wavenumber



2) Travelling Waves : $\psi(x,t) = A\cos(Kx - \omega t + \phi) = Re(Ae^{i(Kx-\omega t+\phi)})$

$T = \frac{2\pi}{\omega} \qquad \phi = \pi/2 \qquad \omega = cK$



Group velocity

$$V_G = \frac{\partial \omega}{\partial K}$$

Phase velocity

$$v_p = \frac{\omega}{k} = \frac{\lambda}{T}$$

1D wave Equation & Seperability & Linearity

Travelling waves obey:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2} \qquad (1) \qquad V_G \equiv c$$

$$\psi(x,t) \sim e^{i(kx-\omega t)} = e^{ikx}e^{-i\omega t} \equiv \phi(x)T(t)$$

Put into (1)

$$\phi \frac{\partial^2 T}{\partial t^2} = c^2 T \frac{\partial^2 \phi}{\partial x^2}$$

$$\frac{1}{T}\frac{\partial^2 T}{\partial t^2} = \frac{c^2}{\phi}\frac{\partial^2 \phi}{\partial^2 x} \qquad (2)$$

F(t) only F(x) only

into (2) $\qquad T(t) = e^{-i\omega T}$

$$\frac{1}{T}\frac{\partial^2 T}{\partial t^2} = -\omega^2$$

$$\frac{c^2}{\phi}\frac{\partial^2 \phi}{\partial x^2} = -\omega^2 = -k^2c^2$$

$\psi_1 = A_1 \cos(k_1 x - \omega_1 t)$ & $\psi_2 = A_2 \cos(k_2 x - \omega_2 t)$

$\psi = \psi_1 + \psi_2 + \psi_n \ldots\ldots$

$$\left.\begin{aligned}
&\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2} \\
&\frac{\partial^2 \psi_1}{\partial t^2} + \frac{\partial^2 \psi_2}{\partial t^2} = c^2\left[\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial x^2}\right] \\
&\left[\frac{\partial^2 \psi_1}{\partial t^2} - c^2\frac{\partial^2 \psi_1}{\partial x^2}\right] = \left[-\frac{\partial^2 \psi_2}{\partial t^2} + c^2\frac{\partial^2 \psi_2}{\partial x^2}\right]
\end{aligned}\right\}$$

wave Eqn. is linear

A sum of solutions is also a solution

Intensity is proportional to $|A|^2$

$$I \sim \psi^* \psi = (A^* e^{-ikx})(A e^{ikx}) = A^* A$$

$$\psi \sim Re[Ae^{ikx}]$$

Lets examine the intensity of a sum of waves :

$$\psi_1(x,t) = A_1 e^{i(k_1 x - \omega_1 t + \phi_1)}$$
$$\psi_2(x,t) = A_2 e^{i(k_2 x - \omega_2 t + \phi_2)}$$

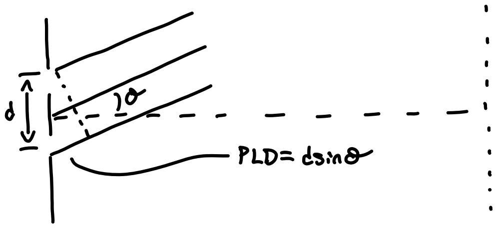
$$\psi = \psi_1 + \psi_2 = A_1 e^{i(kx_1 - \omega_1 t + \phi_1)} + A_2 e^{i(kx_2 - \omega_2 t + \phi_2)}$$

$$\psi^*\psi = (\psi_1^* + \psi_2^*)(\psi_1 + \psi_2) = \psi_1^*\psi_1 + \psi_2^*\psi_2 + 2\psi_1^*\psi_2$$
$$= I_1 + I_2 + (\text{Interference term})$$

First : Interference Term

$$\psi_1^*\psi_2 = A_1^* A_2 \exp\{i[(k_2-k_1)x - (\omega_2-\omega_1)t + (\phi_1-\phi_2)]\}$$

Specialize to double-Slit diffraction



$k_1 = k_2$

$w_1 = w_2$

$\phi_1 = \phi_2$

$A_1 = A_2 = A$ → In this case:

$$\psi_1^* \psi_2 = A^2 e^{i\delta}$$

$$I = I_0 + I_0 + 2I_0 \cos\delta$$
$$= 2I_0(1+\cos\delta)$$

$$\phi_2 - \phi_1 \equiv \delta = \frac{2\pi d\sin\theta}{\lambda},$$

$$d\sin\theta = n\lambda$$

if $d\sin\theta = (n+\tfrac{1}{2})\lambda \Rightarrow$ min

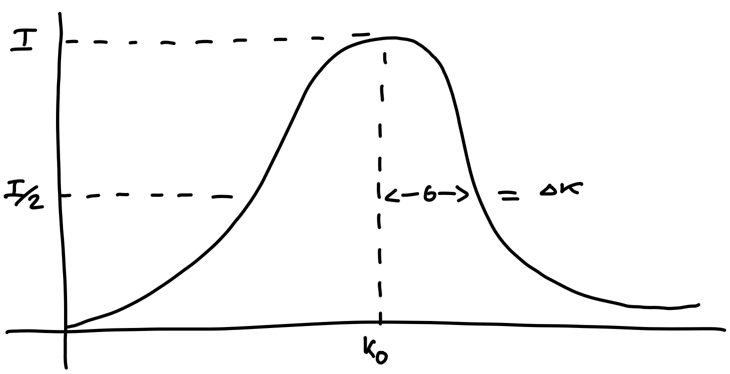
Any Sufficiently Smooth / Regular (+ Bounded) can be presented as a series of harmonic functions of different w, k w/ appropriate weights. ↳ $\sin, \cos, e^{i\theta}$

(Fouriers Theorem)

$$F(x,t) = \sum_n A_n \cos(k_n x - w_n t) \quad \Leftarrow \text{ if we have a discrete Spectrum}$$

$$F(x,t) = \int A(k)\cos(kx - wt)\, dt \quad \Leftarrow \text{ if a continuous Spectrum}$$

Gaussian (Normal Distribution)



$$\psi_b(x,t) = \int A(k)\cos(kx - wt)\, dk$$
$$\psi_b(x,0) = A_0 e^{-\Delta k^2 x^2} \cos(k_0 x)$$