

4.4 Undetermined Coefficients

4.4 # 22, 24, 44, 36, 42

4.4.22 Undetermined Coefficients

$$y'' + 4y = 1$$

$$y_b = y_h + y_p$$

$$y_h: \Delta = b^2 - 4ac = 0^2 - 4(1)(4) = -16$$

$$y_h = e^{\alpha t} (C_1 \cos \beta t + C_2 \sin \beta t)$$

$$\alpha = -\frac{b}{2a} = 0$$

$$\beta = \frac{\sqrt{4ac - b^2}}{2a}$$

$$\beta = \frac{\sqrt{4(1)(4) - 0}}{2(1)} = \frac{\sqrt{16}}{2} = \frac{4}{2} = 2$$

$$y_h = C_1 \cos 2t + C_2 \sin 2t$$

$$\beta = 2$$

$$y_p: \text{By inspection } y'' + 4y = 1$$

$$y_p = \frac{1}{4}$$

$$ax^2 + bx + c = d$$

$$y_p = d/c$$

$$y_b = C_1 \cos 2t + C_2 \sin 2t + \frac{1}{4}$$

4.4.24 Undetermined Coefficients

$$y'' + y' - 2y = 3 - 6t$$

$$y_b = y_h + y_p$$

$$y_h: \Delta = b^2 - 4ac = (1)^2 - 4(1)(-2) = 1 + 8 = 9$$

$$y_h = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2}$$

$$r = 1, -2$$

$$y_h = C_1 e^t + C_2 e^{-2t}$$

$$y_p: ay'' + by' + cy = d$$

$$d = 3 - 6t$$

$$y_p = At + B$$

$$y_p' = A$$

$$y_p'' = 0$$

$$0 + A - 2(At + B) = 3 - 6t$$

$$A - 2At - 2B = 3 - 6t$$

$$A - 2B = 3$$

$$-2A = -6, A = 3, B = 0$$

$$y_p = 3t$$

$$y_b = C_1 e^t + C_2 e^{-2t} + 3t$$

4.4.44 $y'' + 2y' + y = 6 \cos t$

$$y(0) = 1, y'(0) = 1$$

$$y_b = y_h + y_p$$

$$y_h: \Delta = b^2 - 4ac = (2)^2 - 4(1)(1) = 4 - 4 = 0$$

$$y_h = C_1 e^{r_1 t} + C_2 t e^{r_2 t}$$

$$y_h = C_1 e^{-t} + C_2 t e^{-t}$$

$$r = -\frac{b}{2a}, \frac{-2}{2(1)} = -1$$

$$y'' + 2y' + y = 6 \cos t$$

$$y_p: 6 \cos t \rightarrow y_p = A \cos t + B \sin t$$

$$y_p = A \cos t + B \sin t = 3 \sin t$$

$$y_p' = -A \sin t + B \cos t$$

$$y_p'' = -A \cos t - B \sin t$$

$$-A \cos t - B \sin t + 2(-A \sin t + B \cos t) + A \cos t + B \sin t = 6 \cos t$$

$$-A \cos t - B \sin t - 2A \sin t + 2B \cos t + A \cos t + B \sin t = 6 \cos t$$

$$-2A \sin t + 2B \cos t = 6 \cos t$$

$$2B \cos t = 6 \cos t$$

$$A = 0, B = 3$$

$$y_p = 3 \sin t$$

$$y = C_1 e^{-t} + C_2 t e^{-t} + 3 \sin t$$

$$1 = C_1 e^0 + C_2 (0) e^0 + 3 \sin(0)$$

$$1 = C_1$$

$$y' = -C_1 e^{-t} + C_2 e^{-t} - C_2 t e^{-t} + 3 \cos t$$

$$1 = -C_1 e^0 + C_2 e^0 - C_2 (0) e^0 + 3 \cos(0)$$

$$1 = -C_1 + C_2 + 3$$

$$0 = C_2 + 3$$

$$-3 = C_2$$

$$y = e^{-t} - 3te^{-t} + 3 \sin t$$

4.4.36 $y'' + 3y' = \sin(t) + 2\cos(t)$

$$r^2 + 3r = 0$$

$$r(r+3)=0, r=0, -3$$

$$\Delta = b^2 - 4ac$$

$$= (3)^2 - 4(1)(0)$$

$$= 9$$

$$r = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{-3 \pm 3}{2(1)} = 0$$

$$= \frac{-3 - \sqrt{9}}{2(1)} = \frac{-6}{2} = -3$$

$$y = c_1 + c_2 e^{-3t} + \frac{1}{2}(\sin(t) - \cos(t))$$

$$y_h: y = c_1 e^{r_1 t} + c_2 e^{r_2 t} = c_1 + c_2 e^{-3t}$$

$$y_h = c_1 + c_2 e^{-3t}$$

$$y_p: \sin(t) + 2\cos(t)$$

$$y_p = A\cos(t) + B\sin(t)$$

$$y_p = \frac{1}{2}(\sin(t) - \cos(t))$$

$$y_p' = -A\sin(t) + B\cos(t)$$

$$y_p'' = -A\cos(t) - B\sin(t)$$

$$-A\cos(t) - B\sin(t) + 3(-A\sin(t) + B\cos(t)) = \sin(t) + 2\cos(t)$$

$$-A\cos(t) - B\sin(t) - 3A\sin(t) + 3B\cos(t) = \sin(t) + 2\cos(t)$$

$$\cos(t)(-A + 3B) = 2\cos(t)$$

$$A = \begin{bmatrix} -1 & 3 & 2 \\ -3 & -1 & 1 \end{bmatrix} \quad A = -\frac{1}{2}$$

$$\sin(t)(-3A - B) = \sin(t)$$

$$B = \frac{1}{2}$$

4.4.42 $y'' + 4y' + 4y = te^{-t}$ $y(0) = -1$ $y'(0) = 1$

$$y_h: r^2 + 4r + 4$$

$$(r+2)(r+2)$$

$$r = -2$$

$$y_h = c_1 e^{-2t} + c_2 t e^{-2t}$$

$$y_h = c_1 e^{r_1 t} + c_2 t e^{r_2 t}$$

$$r = \frac{-b}{2a}$$

$$r = \frac{-4}{2(1)} = -2$$

$$y_p: y_p = te^{-t}$$

$$y_p = Ate^{-t} + Be^{-t}$$

$$y_p = te^{-t} - 2e^{-t}$$

$$y_p' = Ae^{-t} - Ate^{-t} - Be^{-t}$$

$$y_p'' = -Ae^{-t} - Ae^{-t} + Ate^{-t} + Be^{-t}$$

$$y_p'' = Ate^{-t} + Be^{-t} - 2Ae^{-t}$$

$$Ate^{-t} + Be^{-t} - 2Ae^{-t} + 4(Ate^{-t} - Ate^{-t} - Be^{-t}) + 4(Ate^{-t} + Be^{-t}) = te^{-t}$$

$$Ate^{-t} + Be^{-t} - 2Ae^{-t} + 4Ae^{-t} - 4Ae^{-t} - 4Be^{-t} + 4Ae^{-t} + 4Be^{-t} = te^{-t}$$

$$Ate^{-t} + Be^{-t} + 2Ae^{-t} = te^{-t}$$

$$y = c_1 e^{-2t} + c_2 t e^{-2t} + te^{-t} - 2e^{-t}$$

$$-1 = c_1 - 2$$

$$1 = c_1$$

$$y' = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t} + e^{-t} - te^{-t} + 2e^{-t}$$

$$1 = -2c_1 e^0 + c_2 e^0 - 2c_2(0)e^0 + e^0 - 0e^0 + 2e^0$$

$$1 = -2 + c_2 + 1 + 2$$

$$1 = c_2 + 1$$

$$0 = c_2$$

$$c_1 = 1, c_2 = 0$$

$$Ate^{-t} = te^{-t}$$

$$e^{-t}(B + 2A) = 0$$

$$A = 1$$

$$2A = -B$$

$$B = -2$$

$$B = -2$$

$$y = e^{-2t} + te^{-t} - 2e^{-t}$$