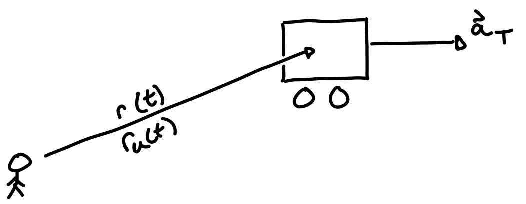
11-13-18

motion in noninertial reference frame ? $\longrightarrow$ go to inertial reference frame

#1



$$\vec{r}(t) = \vec{r}_u + \frac{1}{2}at^2$$

$$\ddot{r} = \vec{a} \quad , \quad \vec{F} = m\vec{a}$$

#2

$$\vec{r}(t) = a\cos(\omega t)\hat{\imath} + B\sin(\omega t)\hat{\jmath}$$

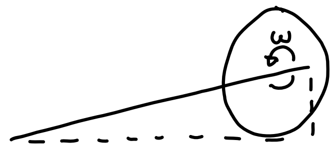
$$\dot{\vec{r}}(t) = -a\omega\sin(\omega t)\hat{\imath} + B\omega\cos(\omega t)\hat{\jmath}$$

$$|\dot{r}(t)| = \left((a^2\omega^2\sin^2(\omega t)) + (B^2\omega^2\cos^2(\omega t))\right)^{\frac{1}{2}}$$

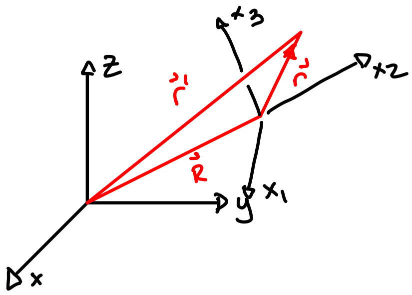
$$= (a^2+b^2)^{\frac{1}{2}} \cdot \omega$$

$$\ddot{r}(t) = -a\omega^2\cos(\omega t)\hat{\imath} - B\omega^2\sin(\omega t)\hat{\jmath}$$

$$|\ddot{r}(t)| = \left((a^2\omega^4\cos^2(\omega t)) + (B^2\omega^4\sin^2(\omega t))\right)^{\frac{1}{2}} = \sqrt{B^2+a^2}\,\omega^2 \quad : \quad \vec{F} = ma\omega^2$$



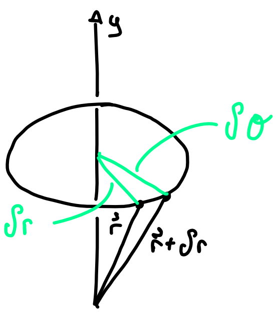
x' inertial



$\vec{r}$ - object in noninertial frame

$\vec{R}$ - vector connecting both frames origins

$\vec{r}' = \vec{r} + \vec{R}$



$$\delta\vec{r} = \delta\vec{\theta} \times \vec{r}$$

$$\frac{\delta\vec{r}}{\delta T} = \frac{\delta\vec{\theta}}{\delta T} \times \vec{r}$$

$$\boxed{\frac{d\vec{r}_{inertial}}{dT} = \vec{\omega} \times \vec{r}}$$

1: $\vec{r}' = \vec{R} + \vec{r} = Q$

For R_{Fixed} $\quad \left.\frac{d\vec{Q}}{dt}\right|_{Fixed} = \left.\frac{d\vec{Q}}{dT}\right|_{rotating} + \vec{\omega} \times \vec{Q} \qquad \left.\frac{d\vec{\omega}}{dt}\right|_{fixed} = \left.\frac{d\vec{\omega}}{dt}\right|_{rotating} + \underbrace{\cancel{\vec{\omega} \times \vec{\omega}}}_{0}$

$\vec{R} = \vec{R}(t)$

$$\left.\frac{d\vec{r}}{dt}\right|_{Fixed} = \underbrace{\frac{d\vec{R}}{dT}}_{\vec{V}_{NI}} + \underbrace{\frac{d\vec{r}}{dt}}_{\vec{O}bject\ NI} + \vec{\omega} \times \vec{r}$$

NI Frame

ω only $\quad \left.\frac{d\vec{r}}{dt}\right|_F = \left.\frac{d}{dT}(\vec{\omega}\times\vec{r})\right|_{rot} = \vec{\omega}\times\vec{r}$

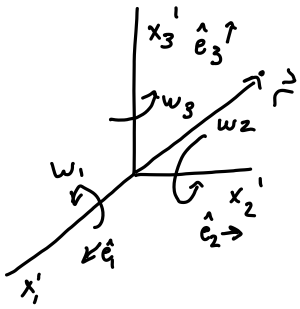
$\vec{R}(T) + \vec{\omega} \quad \frac{d\vec{r}\,'}{dt} = \left.\frac{d}{dt}(\vec{R}+\vec{r})\right|_{Rot} = \frac{d\vec{R}}{dT} + \vec{\omega}\times\vec{r} = \vec{V}_{NIF} + \vec{\omega}\times\vec{r}$

$\vec{R}(t) + \vec{r}(t) + \omega \quad \frac{d\vec{r}\,'}{dt} = \frac{d\vec{R}}{dt} + \frac{d\vec{r}}{dt} + \vec{\omega}\times\vec{r} = \vec{V}(t) + \vec{V}_r(t) + \vec{\omega}\times\vec{r}$

RT inertial frame

RT moving coordinate

rotational



Fix $\vec{R}$

$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\sum x_i\hat{e}_i = \frac{dx_i\,\hat{e}_i}{dt} + x_i\frac{d\hat{e}_i}{dt}$$

$$\frac{dx_i'}{dt}\hat{e}_i' + x_i'\frac{d\hat{e}_i'}{dt} = \dot{\vec{r}}\,' + \sum x_i'\dot{\hat{e}}_i$$

$$x_1\frac{d\hat{e}_1}{dt} = x_1'\left[\omega_3\hat{e}_2' - \omega_2\hat{e}_3'\right]$$
$$x_2\frac{d\hat{e}_2}{dt} = x_2'\left[\omega_1\hat{e}_3' - \omega_3\hat{e}_1'\right]$$
$$x_3\frac{d\hat{e}_3}{dt} = x_3'\left[\omega_2\hat{e}_1' - \omega_1\hat{e}_2'\right]$$

$\vec{V}\,' = \vec{V} + \vec{v} + \vec{\omega}\times\vec{r}$

$$\left.\frac{d}{dt}\dot{\vec{r}}\right|_{fixed} = \frac{d}{dt}\left[\vec{V}(t) + \vec{V}_r(t) + \vec{\omega}\times\vec{r}\right] = \vec{A}(t) + \left.\frac{d}{dt}\vec{V}_r(t)\right|_{fixed} + \left.\frac{d}{dt}\right|_{fixed}(\vec{\omega}\times\vec{r})$$

$$\ddot{\vec{r}}\,'(t) = \vec{A}(t) + \underbrace{\vec{a}_r(t) + \vec{\omega}\times\vec{v}_r}_{\#1} + \underbrace{\dot{\vec{w}}\times\vec{r} + \omega\times\left.\frac{d}{dt}\right|_{fixed}}\vec{r}$$

$$= \vec{A}(t) + \vec{a}_r(T) + \vec{\omega}\times\vec{V}_r + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\left[\vec{V}_r + \vec{\omega}\times\vec{r}\right]$$

$$= \vec{A}(t) + a_r(t) + 2\vec{\omega}\times\vec{V}_r + \vec{\omega}\times\vec{\omega}\times\vec{r} + \dot{\vec{\omega}}\times\vec{r}$$

$$\vec{a}\,' = \vec{A}(t) + \vec{a}_r(t) + 2\vec{\omega}\times\vec{v}_r + \vec{\omega}\times\vec{\omega}\times\vec{r}$$

Coriolis effect

Centripetal acceleration

acceleration of object in NIRF

Acceleration of origin

$\vec{F} = m\ddot{\vec{r}}\,'$ — External forces

$m\vec{a}_r(t) = \vec{F}_{Ext} - m\vec{A}(t) - m\dot{\vec{\omega}}\times\vec{r} - 2m\vec{\omega}\times\vec{v}_r + m(\vec{\omega}\times\vec{\omega}\times\vec{r})$

if $\vec{\omega} = \vec{A} = 0$: $\vec{F} = m\vec{a}$

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$$m\vec{a}_r(t) = \vec{F}_{Ext} - m\vec{A}(t) - m\dot{\vec{\omega}}\times\vec{r} - 2m\vec{\omega}\times\vec{V}_r - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

Experienced by thing in rotating frame.

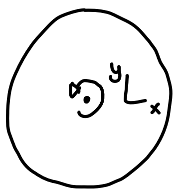
linear $\vec{A}$ of frame

Spin up or down

Coriolis

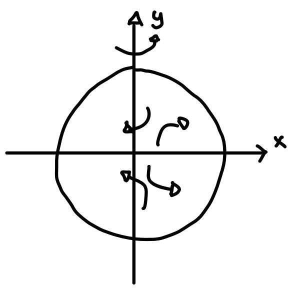
Centripetal

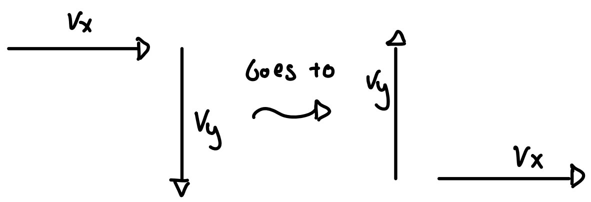
2D Coriolis Effect



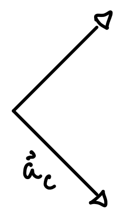
$$\vec{a}_r(t) = -\vec{\omega}\times\vec{V}_r$$

$$= -\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ V_x & V_y & 0 \end{vmatrix} = (-2)\left[-\omega V_y\hat{i} + \omega V_x\hat{j}\right] = 2\omega\left[V_y(\hat{i}) - V_x(\hat{j})\right]$$



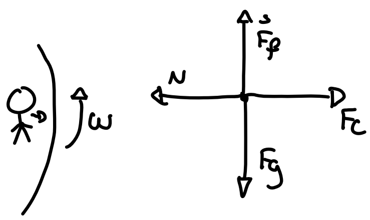


$\vec{V}_1 =$

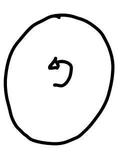


Final Exam Hint

Spinotron

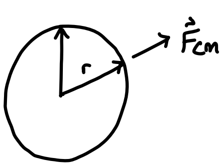


$\vec{\omega}\times\vec{\omega}\times\vec{r}$

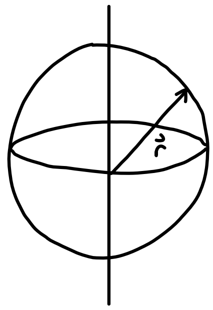


$$\vec{\omega}\times\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ x & y & 0 \end{vmatrix} = \vec{\omega}\times\left[-\omega y\hat{i} + \omega x\hat{j}\right] : = \begin{vmatrix} \hat{i} & \hat{j} & K \\ 0 & 0 & \omega \\ -V_y & V_x & 0 \end{vmatrix} = -\omega^2 x\hat{i} - \omega^2 y\hat{j}$$

$\vec{a}_{centripetal} = \omega^2(x\hat{i} + y\hat{j})$: $m\vec{a}_{cm} = m\omega^2(x\hat{i} + y\hat{j})$



<u>The Earth</u>



$\vec{\omega} \times \vec{r} = \otimes$

$\vec{\omega} \times \vec{\omega} \times \vec{r} = \hat{\uparrow} \times \otimes = \longleftarrow \quad = |\omega_i r| \sin\theta$

$-\vec{\omega} \times \vec{\omega} \times \vec{r} = \longrightarrow$

or zero at poles $\qquad -\vec{\omega} \times \vec{\omega} \times \vec{r} = \longrightarrow$

$\vec{g}_{eff} = \vec{g} - \vec{\omega} \times \vec{\omega} \times \vec{r} \;:\; |\vec{\omega}| = \frac{2\pi}{T} = 2\pi = 7.27 \times 10^{-5}$ rads/s

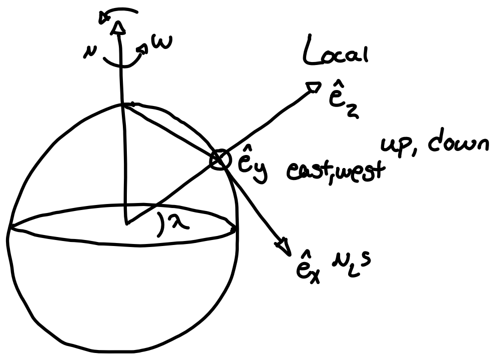
$|r\omega^2| \cong (6.371 \times 10^{6} m \cdot \omega^2) = 0.034 \frac{m}{s^2} \;:\; 0.3\%$ of g

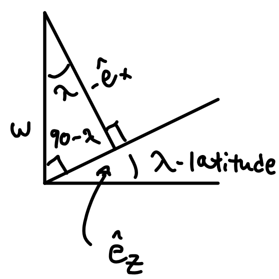
$r\omega^2 = g$: what rotation speed needed for this to be significant?

$$\omega = \left(\frac{g}{r}\right)^{1/2} = 0.001 = \frac{2\pi}{T} \quad : \quad \frac{|\vec{\omega} \times \vec{v}|}{|\vec{\omega} \times \vec{\omega} \times \vec{r}|} = 1$$

$\mathcal{T} \sim 5m \qquad \frac{V_1}{V_2} = \frac{r_1 \omega}{r_2 \omega} = \frac{6,317,000}{6,317,000} = 1$

<u>Local Geometry Approximation</u>





$\vec{\omega} = [-\omega\cos(\lambda), 0, \omega\sin(\lambda)]$

$-\omega\cos(\lambda)\hat{\imath} + \omega\sin(\lambda)\hat{\jmath}$

$$m\vec{a}_r(T) = m\vec{g}_{eff} - 2m\,\vec{\omega} \times \vec{v}_r + \vec{F}_{Ext} - \vec{\omega} \times \vec{\omega} \times \vec{r}$$

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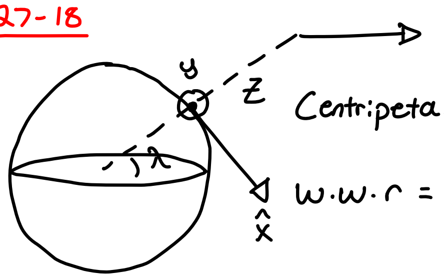


Centripetal $\rightarrow$ Modifies local g, Coriolis turns E-W to N-S

$\omega \cdot \omega \cdot r = r \cdot \omega^2$, $\quad \omega = \frac{2\pi}{24 \cdot 60 \cdot 60}$, $\quad \vec{\omega} = [-\omega\cos(\lambda), 0, \omega\sin(\lambda)]$

Ball being thrown,

$$\vec{F} = m\vec{a} = \left[-m|\vec{g}|\hat{z} - 2m\,\vec{\omega} \times \vec{v}\right]$$

$\ddot{z} = \vec{a}_z = -g\hat{z}$

$\dot{z} = -gt + \dot{z}_0$

$z = z_0 + \dot{z}_0 t - \frac{gt^2}{2}$

$$\vec{\omega} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \omega_x & 0 & \omega_y \\ 0 & 0 & -gt \end{vmatrix} = \omega_x(gt)\hat{\jmath}$$

$\ddot{r} = -|\vec{g}|\hat{z} - 2|\vec{g}|t\omega_x\hat{\jmath}$

$\ddot{z} = -g\hat{z}$, $\quad \ddot{y} = -2|\vec{g}|t\omega_x\hat{y}$, $\quad \dot{y} = -|\vec{g}|t^2\omega_x + \dot{y}_0\hat{y}$, $\quad y = -\frac{|\vec{g}|t^3}{3}\omega_x + \dot{y}_0 t + y_0\hat{y}$

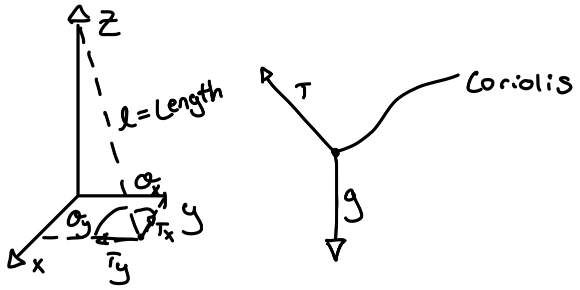
$\dot{x}(T) = 0$

$\dot{y}(T) \cong \frac{1}{3} w |\vec{g}| t^3 \cos(\lambda)$

$\dot{z}(T) = z(h(o)) - \frac{1}{2} g t^2$, $t = \sqrt{\frac{2h}{g}}$, $y(T_{Fall}) = \sqrt{\frac{8h^3}{g}} \left(\frac{1}{3} w \cos(\lambda)\right)$

$h = 100\ m$, $y \approx 1.55\ m$

Foucal Pendulum $\quad z >> xy$



$\vec{a} = \vec{g} + \frac{\vec{T}}{m} - 2\vec{w} \times \vec{v}$, $l >> xy$ displacement

$|\vec{T}_x| \cong \vec{T} \cdot \hat{x} = |\vec{T} \cdot \cos(\theta_x)| = |\vec{T}| \cdot \frac{x}{L}$

$|\vec{T}_y| \cong \vec{T} \cdot \vec{y} = |\vec{T}| \cdot \frac{y}{L}$

$|\vec{T}_z| \approx |\vec{T}| \approx m|\vec{g}|$

$w_x = \vec{w} \cdot \vec{x} = -w\cos(\lambda)$

$w_y = \vec{w} \cdot \vec{y} = 0$

$w_z = \vec{w} \cdot \vec{z} = w\sin(\lambda)$

$\ddot{x} \approx 2\dot{y} w \sin(\lambda) - \frac{|\vec{g}| x}{L}$

$\ddot{y} \approx -2\dot{x} w \sin(\lambda) - \frac{|\vec{g}| y}{L}$

$z = x + iy = re^{i\theta}$

$z(t) = x(t) + i y(t)$

$\ddot{z}(t) = \ddot{x}(t) + i\ddot{y}(t)$

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$\ddot{x} = -\alpha x^2 + 2\dot{y} w_z$ $\qquad \eta = x + iy$, $\ddot{\eta} = \ddot{x} + i\ddot{y}$

$\ddot{y} = -a y^2 + 2\dot{x} w_z$

$$
\begin{array}{rl}
 & \ddot{x} + \alpha^2 x = 2\dot{y} w_z \\
+ & i\ddot{y} + i\alpha^2 y = -2i\dot{x} w_z \\
\hline
\end{array}
$$

$\ddot{\eta} + \alpha^2 \eta = 2w_z(\dot{y} - i\dot{x})$

$= 2w_z i\left(\frac{\dot{y}}{i} - \dot{x}\right) = -2w_z i\left(\dot{x} - \frac{\dot{y}i}{ii}\right) = -2w_z i(\dot{x} + i\dot{y})$

$\ddot{\eta} + 2w_z i \dot{\eta} + \alpha^2 \eta = 0$, $\eta \propto e^{\Omega t}$, $a = 1 \quad b = 2w_z i \quad c = \alpha^2$

$\Omega = -w_z i \pm \sqrt{-4w_z^2 - 4\alpha^2} = -w_z i (\pm)\sqrt{(-1)(w_z^2 + \alpha^2)}$

$\eta(t) = A_{1,2} e^{-iw_z t} e^{\pm(\sqrt{-w_z^2 - \alpha^2})t}$ $\qquad$ if $\quad w_z = 0$