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MATH 361 HW 2.1

$\checkmark\checkmark\ \checkmark\ _{x}\ \checkmark\ \checkmark$

1, 2, 3b, 8, 14, 17

Problem 1 $\quad f(x) = \sqrt{x} - \cos(x) \quad$ on $[0, 1]$

use bisection method to find P_3

$f(0) = -1 \quad, \quad f(1) = 0.459 \qquad \therefore \quad f(a) \times f(b) < 0 \quad \checkmark$

$f(0.5) = -0.170 \quad : \quad f(0.5) < 0$

$f(0.75) = 0.134 \quad : \quad f(0.75) > 0$

$f(0.625) = -0.0203 \quad : \quad f(0.625) < 0$

$$\boxed{P_3(x) = 0.625}$$

$$f(x) = 3(x+1)(x-\tfrac{1}{2})(x-1) = 0$$

a.) $[-2, 1.5]$

a	b	P	f(a)	f(b)	f(P)
-2	1.5	-0.25	-1	+1	+1
-2	-0.25	-1.125	-1	+1	-1
-1.125	-0.25	-0.6875	-1	+1	+1

$$\boxed{P_3(x) = -0.6875}$$

b.) $[-1.25, 2.5]$

a	b	P	f(a)	f(b)	f(P)
-1.25	2.5	0.625	~1	+1	-1
0.625	2.5	1.5625	-1	+1	+1
0.625	1.5625	1.09375	~1	+1	+1

$$\boxed{P_3(x) = 1.09375}$$

$$f(x) = x^3 - 7x^2 + 14x - 6$$

b.) [1, 3.2]

a	b	p	f(a)	f(b)	f(p)
1	3.2	2.1	+ 1	- 1	+ 1
2.1	3.2	2.65	+ 1	- 1	+ 1
2.65	3.2	2.925	+ 1	- 1	+ 1
2.925	3.2	3.0625	+ 1	- 1	- 1
2.925	3.0625	2.99375	+ 1	- 1	+ 1
2.99375	3.0625	3.028125	+ 1	- 1	- 1
3.028125	3.0625	3.0453125	+ 1	- 1	- 1
3.028125	3.0453125	3.03671875	+ 1	- 1	- 1

$$P_8(x) = 3.03671875$$

$f(x) = x^2 - 3$ $[1, 2]$ $\sqrt{3} \approx 1.73205$

a	b	P	f(a)	f(b)	f(P)
1	2	1.5	−1	+1	−1
1.5	2	1.75	−1	+1	+1
1.5	1.75	1.625	−1	+1	−1
1.625	1.75	1.6875	−1	+1	−1
1.6875	1.75	1.71875	−1	+1	−1
1.71875	1.75	1.734375	−1	+1	+1
1.71875	1.734375	1.7265625	−1	+1	−1
1.7265625	1.734375	1.73046875	−1	+1	−1
1.73046875	1.734375	1.732421875	−1	+1	+1
1.73046875	1.732421875	1.7314453125	−1	+1	−1
1.7314453125	1.732421875	1.7319335375	−1	+1	−1
1.7319335375	1.732421875	1.73217773	−1	+1	+1

$$\boxed{\sqrt{3} \approx 1.7321}$$

$f(x) = x^3 - x - 1 = 0$, $[1, 2]$

$$\frac{b-a}{2^n} < 10^{-4} \quad \longrightarrow \quad \frac{b-a}{10^{-4}} < 2^n \quad \longrightarrow \quad \log_{10}(2^n) > \log_{10}\left(\frac{b-a}{10^{-4}}\right)$$

$$n \cdot \log_{10}(2) > \log_{10}\left(\frac{b-a}{10^{-4}}\right) \quad \longrightarrow \quad n > \frac{1}{\log_{10}(2)} \cdot \log_{10}\left(\frac{b-a}{10^{-4}}\right)$$

$$n > \frac{1}{\log_{10}(2)} \cdot \log_{10}\left(\frac{2-1}{10^{-4}}\right) \quad \longrightarrow \quad n > 13.2877$$

$$\boxed{n \geq 14 \text{ iterations}}$$