Phys 131 2016

Lecture 37

Thurs: Seminar

Fri: Supp Ex 77, 78, 79

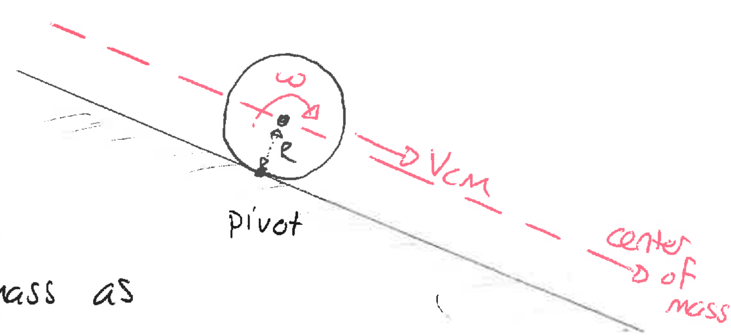
Ch 12 Conc Q: 7

Ch 12 Prob: 35, 40, 69, 74

Rolling motion

In many situations, particularly where an object rolls along a surface, there is both rotational and translational kinetic energy. If an object rolls without slipping, then the rotational and translational velocities are related. Thus the energies are related.

Consider an object rolling down a ramp. If it does not slip, then at any instant it pivots about the contact point. However, when we analyze this motion we will use the center of mass as a reference. We can show (pg 315)

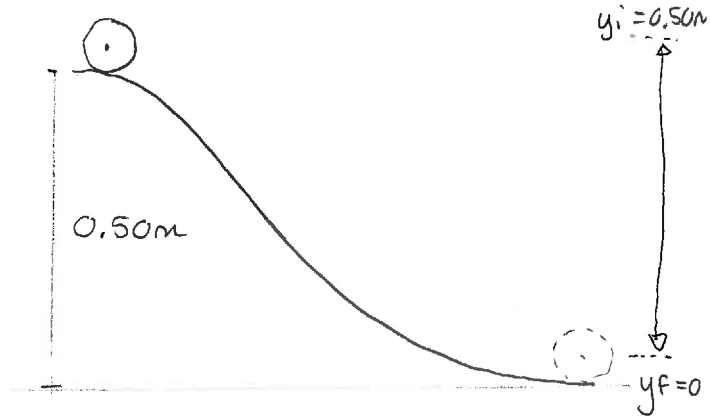


> If an object rolls without slipping then the speed of the center of mass is:
>
> $$V_{cm} = \omega R$$
>
> where R is its radius and ω the angular velocity about the center of mass

In such cases the total kinetic energy is:

$$K = \underbrace{K_{trans}}_{\text{of c.o.m}} + \overbrace{K_{rot}}^{\text{about c.o.m}}$$

Example: A disk with mass M and radius R is released from the top of the illustrated track. It rolls without slipping. Determine its speed at the bottom.



Answer: Energy is conserved

$y_i = 0.50m$ $y_f = 0m$

$v_i = 0m/s$ $v_f =$

$\omega_i = 0 rad/s$ $\omega_f =$

$$E_f = E_i$$

$$K_f + \cancel{U_{gf}} = \cancel{K_i} + U_{gi}$$

$$K_{trans\,f} + K_{rot\,f} = U_{gi}$$

$$\frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 = M g y_i$$

Now $v_f = \omega_f R \Rightarrow \omega_f = v_f / R$ and so

$$\frac{1}{2} M v_f^2 + \frac{1}{2} I \frac{v_f^2}{R^2} = M g y_i$$

$$\Rightarrow \frac{1}{2}\left(M + \frac{I}{R^2}\right) v_f^2 = M g y_i$$

For the disk $I = \frac{1}{2} M R^2 \Rightarrow I/R^2 = \frac{1}{2} M$. Thus:

$$\frac{1}{2}\left(M + \frac{1}{2}M\right) v_f^2 = M g y_i$$

$$\Rightarrow \frac{1}{2}\,\frac{3}{2}\,\cancel{M}\, v_f^2 = \cancel{M} g y_i \Rightarrow v_f^2 = \frac{4 g y_i}{3}$$

$$\Rightarrow v_f = \sqrt{\frac{4 g y_i}{3}} = \sqrt{\frac{4 \times 9.8 m/s^2 \times 0.50m}{3}}$$

$$\Rightarrow v_f = 2.6 m/s$$

☒

Quiz 1 Demo hoop vs disk

Quiz 2 Demo solid vs frozen

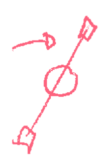
Vector description of angular motion

Although angular velocity is useful for describing the state of rotational motion of an object, it does not describe the orientation of the axis of rotation. Combining the magnitude of angular velocity and axis of rotation into a single angular velocity vector does give a complete description of the rotational state of an object. The rules for constructing the angular velocity vector $\vec{\omega}$ are:

1) magnitude of $\vec{\omega}$ is magnitude of angular velocity, ω, as before
2) direction is along axis of rotation using the right hand rule.

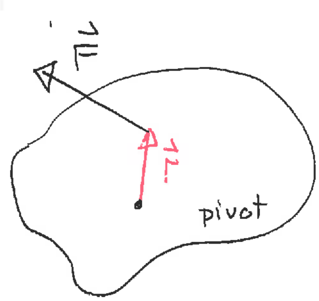
Note, in the following demonstration, this eliminates confusion over the sign of angular velocity (Fig 12.44).

Demo. Use RMS



* point out +ve vs -ve depending on viewpoint
* $\vec{\omega}$ does not depend on viewpoint.

The fact that angular velocity is a vector implies that angular acceleration and torque are also vectors. We will need a rule for constructing any torque vector



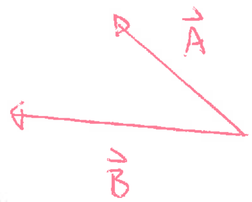
Force $\vec{F}$
Pivot to force $\vec{r}$ → Product gives torque $\vec{\tau}$

The construction will involve the vector cross product

Vector Cross Product

Schematically

Take any two vectors



Cross product

" A cross B" is a new vector, denoted $\vec{A} \times \vec{B}$

The cross product can be defined or calculated by:

Method 1

a) magnitude of $\vec{A} \times \vec{B}$ is: $AB \sin\phi$

positive magnitudes of vectors

shortest angle between vectors

b) direction: – perpendicular to both $\vec{A}$ and $\vec{B}$

– use right hand rule. Pg 318

Quiz 3

Method 2 Use:

a)

$$\hat{i} \times \hat{i} = 0 \qquad \hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{j} = 0 \qquad \hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{k} \times \hat{k} = 0 \qquad \hat{k} \times \hat{i} = \hat{j} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

b)

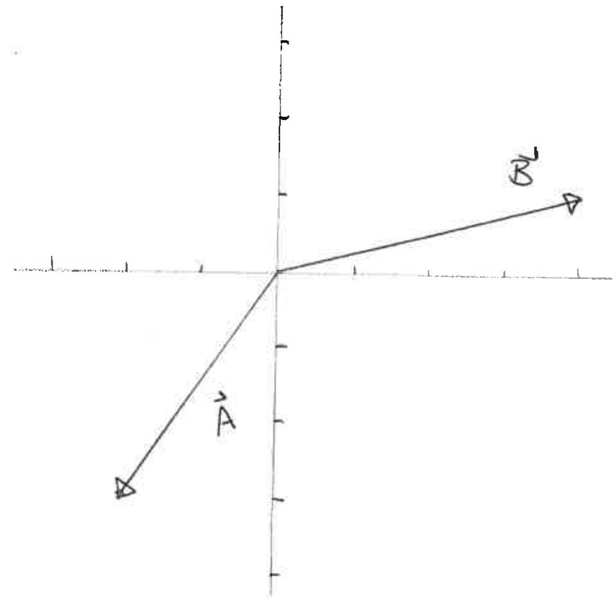
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \qquad (\lambda \vec{A}) \times \vec{B} = \lambda (\vec{A} \times \vec{B})$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Note that regardless of $\vec{A}$:

$$\boxed{\vec{A} \times \vec{A} = 0}$$

Example: Consider the illustrated vectors:
Determine $\vec{A} \times \vec{B}$



Answer: $\vec{A} = -2\hat{\imath} - 3\hat{\jmath}$

$\vec{B} = 4\hat{\imath} + \hat{\jmath}$

Thus:

$$\vec{A} \times \vec{B} = (-2\hat{\imath} - 3\hat{\jmath}) \times (4\hat{\imath} + \hat{\jmath})$$

$$= (-2\hat{\imath} \times 4\hat{\imath}) + (-2\hat{\imath} \times \hat{\jmath})$$
$$(-3\hat{\jmath} \times 4\hat{\imath}) + (-3\hat{\jmath} \times \hat{\jmath})$$

$$= -8 \underbrace{\hat{\imath} \times \hat{\imath}}_{0} - 2 \underbrace{\hat{\imath} \times \hat{\jmath}}_{\hat{k}} - 12 \underbrace{\hat{\jmath} \times \hat{\imath}}_{-\hat{k}} - 3 \underbrace{\hat{\jmath} \times \hat{\jmath}}_{0}$$

$$= -2\hat{k} - 12(-\hat{k}) = 10\hat{k}$$

Quiz 4