1.) use the chain Rule to find dz/dt

$z = \cos(x+3y)$, $x = 9t^3$ $y = 2/t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = -\sin(x+3y) \qquad \frac{\partial z}{\partial y} = -3\sin(x+3y)$$

$$\frac{dx}{dt} = 27t^2 \qquad \frac{dy}{dt} = \frac{-2}{t^2}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\boxed{\frac{dz}{dt} = -\sin(x+3y)(27t^2) - 3\sin(x+3y)(-2/t^2)}$$

2.) use the chain rule to find $\frac{dw}{dt}$

$w = \ln\sqrt{x^2+y^2+z^2}$ $\quad x = 7\sin t$ $\quad y = 4\cos t$ $\quad z = 2\tan t$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$\ln' = \frac{u'}{u}$

$$\frac{\partial w}{\partial x} = (x^2+y^2+z^2)^{\frac{1}{2}}$$
$$\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\cdot 2x$$
$$\frac{\frac{x}{\sqrt{(x^2+y^2+z^2)}}}{\sqrt{(x^2+y^2+z^2)}}$$
$$\frac{\partial w}{\partial x} = \frac{x}{x^2+y^2+z^2}$$

$x = 7\sin t$

$\frac{dx}{dt} = 7\cos t$

$$\frac{\partial w}{\partial x}\frac{dx}{dt} = \frac{7x\cos t}{x^2+y^2+z^2}$$

$$\frac{\partial w}{\partial y} = (x^2+y^2+z^2)^{\frac{1}{2}}$$
$$\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\cdot 2y$$
$$\frac{y}{\sqrt{x^2+y^2+z^2}}$$
$$\frac{\frac{y}{\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}}$$

$y = 4\cos t$

$\frac{dy}{dt} = -4\sin t$

$$\frac{\partial w}{\partial y} = \frac{y}{(x^2+y^2+z^2)}$$

$$\frac{\partial w}{\partial y}\frac{dy}{dt} = \frac{-4y\sin t}{(x^2+y^2+z^2)}$$

$$\frac{\partial w}{\partial z} = (x^2+y^2+z^2)^{\frac{1}{2}}$$
$$\frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}\cdot 2z$$
$$\frac{z}{\sqrt{x^2+y^2+z^2}}$$
$$\frac{\frac{z}{\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}}$$
$$\frac{\partial w}{\partial z} = \frac{z}{(x^2+y^2+z^2)}$$

$z = 2\tan t$

$\frac{dz}{dt} = 2\sec^2 t$

$$\frac{\partial z}{\partial t}\frac{dz}{dt} = \frac{2z\sec^2 t}{(x^2+y^2+z^2)}$$

$$\boxed{\frac{dw}{dt} = \frac{7x\cos t}{x^2+y^2+z^2} - \frac{4y\sin t}{x^2+y^2+z^2} + \frac{2z\sec^2 t}{x^2+y^2+z^2}}$$

3.) use the chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

$z = \sin\theta\cos\phi$ $\quad \theta = st^3$ $\quad \phi = s^3 t$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial s} + \frac{\partial z}{\partial \phi}\frac{\partial \phi}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial \phi}\frac{\partial \phi}{\partial t}$$

$$\frac{\partial z}{\partial \theta} = \cos\theta\cos\phi \qquad \frac{\partial z}{\partial \phi} = -\sin\phi\sin\theta$$

$$\frac{\partial \theta}{\partial s} = t^3 \qquad \frac{\partial \phi}{\partial s} = 3s^2 t$$

$$\boxed{\frac{\partial z}{\partial s} = \cos\theta\cos\phi(t^3) - \sin\phi\sin\theta(3s^2 t)}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t} + \frac{\partial z}{\partial \phi}\frac{\partial \phi}{\partial t}$$

$z = \sin\theta\cos\phi \qquad \theta = st^3 \qquad \phi = s^3t$

$\frac{\partial z}{\partial \theta} = \cos\theta\cos\phi \qquad \frac{\partial z}{\partial \phi} = -\sin\phi\sin\theta$

$\frac{\partial \theta}{\partial t} = 3st^2 \qquad \frac{\partial \phi}{\partial t} = s^3$

$$\boxed{\frac{\partial z}{\partial s} = \cos\theta\cos\phi\,(3st^2) - \sin\phi\sin\theta\,(s^3)}$$

4.) Use the chain rule to find $\partial z/\partial s$ and $\partial z/\partial t$

$z = e^r\cos\theta \qquad r = st \qquad \theta = \sqrt{s^6+T^6}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial s}$$

$\frac{\partial z}{\partial r} = e^r\cos\theta \qquad \frac{\partial z}{\partial \theta} = -e^r\sin\theta$

$\frac{\partial r}{\partial s} = t \qquad \frac{\partial \theta}{\partial s} = (s^6+T^6)^{\frac{1}{2}}$

$\frac{1}{2}(s^6+T^6)^{-\frac{1}{2}} \cdot 6s^5$

$\frac{\partial \theta}{\partial s} = \frac{3s^5}{\sqrt{s^6+T^6}}$

$$\frac{\partial z}{\partial s} = t\cdot e^r\cos\theta - e^r\sin\theta\left(\frac{3s^5}{\sqrt{s^6+T^6}}\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t}$$

$\frac{\partial z}{\partial r} = e^r\cos\theta \qquad \frac{\partial z}{\partial \theta} = -e^r\sin\theta$

$\frac{\partial r}{\partial t} = s \qquad \frac{\partial \theta}{\partial t} = \sqrt{s^6+T^6}$

$(s^6+T^6)^{\frac{1}{2}}$

$\frac{1}{2}(s^6+T^6)^{-\frac{1}{2}} \cdot 6T^5$

$\frac{3T^5}{\sqrt{s^6+T^6}}$

$$\boxed{\frac{\partial z}{\partial t} = se^r\cos\theta - e^r\sin\theta\left(\frac{3T^5}{\sqrt{s^6+T^6}}\right)}$$

6.) Let $w(s,t) = f(u(s,t), v(s,t))$

$u(7,-9) = 4$ $\quad v(7,-9) = -3$

$u_s(7,-9) = -1$ $\quad v_s(7,-9) = 5$

$u_t(7,-9) = -6$ $\quad v_t(7,-9) = 1$

$f_u(4,-3) = 0$ $\quad f_v(4,-3) = -2$

Find $w_s(7,-9)$ $\quad w_t(7,-9)$

$w_s(7,-9) = -10$

$w_t(7,-9) = -2$

$$w_s(s,t) = \frac{\partial w}{\partial s}$$

$$\frac{\partial w}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial s}$$

$$f_u(u(s,t), v(s,t))u_s + f_v(u(s,t), v(s,t))v_s$$

$$w_s(7,-9) = f_u(u(7,-9), v(7,-9))u_s(7,-9) + f_v(u(7,-9), v(7,-9))v_s(7,-9)$$

$$f_u(4,-3)u_s(7,-9) + f_v(4,-3)v_s(7,-9)$$

$$0(-1) + (-2)5$$

$$0 \quad -10$$

$$-10$$

$$w_t(s,t) = \frac{\partial w}{\partial t}$$

$$\frac{\partial w}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial t}$$

$$w_t(7,-9) \quad f_u(u(s,t), v(s,t))u_t + f_v(u(s,t), v(s,t))v_t$$

$$f_u(u(7,-9), v(7,-9))u_t(7,-9) + f_v(u(7,-9), v(7,-9))v_t(7,-9)$$

$$f_u(4,-3)(-6) + f_v(4,-3)(1)$$

$$0(-6) + (-2)(1)$$

$$-2$$

5.) If $z = f(x,y)$, where f is differentiable, and

$x = g(t)$ $\quad y = h(t)$

$g(5) = -7$ $\quad h(5) = 0$

$g'(5) = -2$ $\quad h'(5) = 5$

$f_x(-7,0) = -2$ $\quad f_y(-7,0) = -4$

$t = 5$

Find $\frac{dz}{dt}$

$\boxed{-16}$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$\frac{dx}{dt} = g'(t) \qquad \frac{dy}{dt} = h'(t)$$

$$\frac{\partial f}{\partial x} = f_x(x,y) \qquad \frac{\partial f}{\partial y} = f_y(x,y)$$

$$f_x(g(t),h(t)) \qquad f_y(g(t),h(t))$$

$$\frac{dz}{dt} = f_x(g(5),h(5))g'(5) + f_y(g(5),h(5))h'(5)$$

$$f_x(-7,0)(-2) + f_y(-7,0)(5)$$

$$-2(-2) + (-4)5$$

$$4-20$$

$$-16$$

7.) $g(u,v) = f(e^u+\sin v, e^u+\cos v)$

$g_u(0,0) = 15$

$g_v(0,0) = 6$

$x = e^u + \sin v$

$y = e^u + \cos v$

$$\frac{\partial x}{\partial u} = e^u \qquad \frac{\partial x}{\partial v} = \cos v$$

$$\frac{\partial y}{\partial u} = e^u \qquad \frac{\partial y}{\partial v} = -\sin v$$

$$g_u(u,v) = \frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u}$$

$$= f_x(x,y)e^u + f_y(x,y)e^u$$

$$g_u(0,0) = f_x(x(0,0),y(0,0))e^0 + f_y(x(0,0),y(0,0))e^0$$

$$f_x(1,2) + f_y(1,2)$$

$$6+9$$

$$15$$

$$g_v(u,v) = \frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v}$$

$$f_x(x,y)\cos v + f_y(x,y)(-\sin v)$$

$$g_v(0,0) = f_x(x(0,0),y(0,0))\cos(0) + f_y(x(0,0),y(0,0))(-\sin(0))$$

$$f_x(1,2)(1)$$

$$6(1)$$

$$6$$

8.) use the chain Rule to find the indicated partial derivatives.

$z = x^4 + x^2y \qquad x = s+2t-u \qquad y = stu^2 \qquad s=5,\ t=1,\ u=3$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\boxed{\begin{aligned}\frac{\partial z}{\partial s} &= 760\\ \frac{\partial z}{\partial t} &= 1952\\ \frac{\partial z}{\partial u} &= -136\end{aligned}}$$

$$\frac{\partial z}{\partial x} = 4x^3 + 2xy \qquad \frac{\partial z}{\partial y} = x^2$$

$$\frac{\partial x}{\partial s} = 1 \qquad \frac{\partial y}{\partial s} = Tu^2$$

$X = 5 + 2(1) - 3$

$x = 7 - 3$

$x = 4$

$y = STu^2$

$y = 5(1)(3)^2$

$5(9)$

$y = 45$

$4(4)^3 + 2(4)(45) + (4)^2(9)$

$616 + 144$

760

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$Z = x^4 + x^2 y \qquad x = s + 2T - u \qquad y = stu^2$

$x = 4 \qquad y = 45$

$$\frac{\partial z}{\partial x} = 4x^3 + 2xy \qquad \frac{\partial z}{\partial y} = x^2$$

$$\frac{\partial x}{\partial t} = 2 \qquad \frac{\partial y}{\partial t} = su^2$$

$(4x^3 + 2xy)(2) + x^2(su^2)$

$(4(4)^3 + 2(4)(45))2 + (4)^2(5(3)^2)$

$1232 + 16(45)$

$1232 + 720$

1952

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$Z = x^4 + x^2 y \qquad x = s + 2T - u \qquad y = STu^2$

$S = 5$

$T = 1$

$u = 3$

$x = 4$

$y = 45$

$$\frac{\partial z}{\partial x} = 4x^3 + 2xy \qquad \frac{\partial z}{\partial y} = x^2$$

$$\frac{\partial x}{\partial u} = -1 \qquad \frac{\partial y}{\partial u} = 2STu$$

$(4(4)^3 + 2(4)(45))(-1) + (4)^2(2(5)1(3))$

$(256 + 360)(-1) + 16(30)$

$-616 + 480$

-136

9.) Use the equations to find $\partial z/\partial x$ and $\partial z/\partial y$

$x^2 + 4y^2 + 7z^2 = 1$

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}} \qquad \frac{\partial z}{\partial z} = -\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}$$

$$\frac{\partial z}{\partial x} = \frac{-x}{7z} \qquad \frac{\partial z}{\partial y} = \frac{-4y}{7z}$$

$$\frac{\partial f}{\partial x} = 2x \qquad -\frac{2x}{14z} = \frac{-x}{7z} \qquad \frac{\partial f}{\partial y} = 8y \qquad -\frac{8y}{14z} = \frac{-4y}{7z}$$

$$\frac{\partial f}{\partial z} = 14z \qquad \frac{\partial f}{\partial z} = 14z$$

10.) (x,y) $T(x,y)$

$x=\sqrt{2+T}$ $T_x(2,8)=3$ $t=2$ seconds

$y=7+\frac{1}{2}t$ $T_y(2,8)=9$

$$\frac{dT}{dt}=\frac{\partial T}{\partial x}\frac{dx}{dt}+\frac{\partial T}{\partial y}\frac{dy}{dt}$$

$$\frac{dx}{dt}=(2+T)^{\frac{1}{2}}$$
$$\frac{1}{2}(2+T)^{-\frac{1}{2}}$$
$$\frac{dy}{dt}=\frac{1}{2}$$
$$\frac{dx}{dt}=\frac{1}{2\sqrt{2+T}}$$

Temperature is rising at $\frac{21}{4}$°C/s

$$\frac{dx}{dt}=T_x(x,y)\frac{dx}{dt}+T_y(x,y)\frac{dy}{dt}$$
$$=T_x(x,y)\frac{1}{2\sqrt{2+T}}+T_y(x,y)\frac{1}{2}$$
$$=T_x(x(2),y(2))\frac{1}{2\sqrt{2+2}}+T_y(x(2),y(2))\frac{1}{2}$$
$$=T_x(2,8)\frac{1}{2\sqrt{4}}+T_y(2,8)\frac{1}{2}$$
$$=3\cdot\frac{1}{2\cdot2}+9(\tfrac{1}{2})$$
$$=\frac{3}{4}+\frac{9}{2}$$
$$=\frac{3}{4}+\frac{18}{4}$$
$$=\frac{21}{4}$$

11.) $C=1449.2+4.6T-0.055T^2+0.00029T^3+0.016D$

$$\frac{\partial c}{\partial T}=\frac{\partial c}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial c}{\partial D}\frac{\partial D}{\partial T}$$

$$\frac{\partial c}{\partial T}=(0+4.6+2(0.055T)+3(0.00029T^2)+0$$
$$4.6+0.11T+0.00087T^2$$
$$4.6+0.11T+0.00087T^2$$
$$\frac{\partial c}{\partial T}=4.6+0.11T+0.00087T^2$$

$$\frac{dT}{dt}=\frac{-1}{10}$$
$$\frac{dD}{dt}=\frac{1}{2}$$

when $t=20s$

$T=12.5C^{o}$

$$\frac{\partial c}{\partial T}(12.5°C)=4.6+0.11(12.5)+0.00087(12.5)^2$$
$$4.6+1.375+0.135938$$
$$6.11094$$

$$\frac{\partial c}{\partial D}=(0+0-0+0+0.016)$$
$$\frac{\partial c}{\partial D}=0.016$$

$$6.11094(-\tfrac{1}{10})+0.016(\tfrac{1}{2})$$
$$-0.611094+0.08=-0.531094\ m/s$$

-0.531094 m/s per minute

12.) $\frac{dx}{dt} = 1.4$ in/s $\quad \frac{dy}{dt} = -2.7$ in/s $\quad V = \frac{\pi r^2 h}{3}$

$x = 150$ in $\quad$ 168 in

$$\frac{\partial V}{\partial r} = \frac{\cancel{9}(2\pi r h) - (\cancel{\pi r^2 h})(0)}{\cancel{9}\ 3}$$

$$V = \frac{\pi r^2 h}{3}$$

$$\frac{\partial V}{\partial r} = \frac{2\pi r h}{3}$$

$$\frac{\partial V}{\partial h} = \frac{\pi r^2}{3}$$

$$dV = \frac{2\pi r h}{3}(1.4) + \frac{\pi r^2}{3}(-2.7)$$

$r = 150$ in

$h = 168$ in

$$\frac{2\pi(150)(168)}{3}(1.4) + \frac{\pi(150)^2}{3}(-2.7)$$

$$23520\pi - 20250\pi$$

$$\boxed{3270\pi}$$

13.) ohms Law, $V = IR$

$R = 413\,\Omega \quad I = 0.08A$

$\frac{dR}{dt} = 0.09\,\Omega/s \quad \frac{dV}{dt} = -0.05$ V/s

$$\frac{\partial V}{\partial R} = I \quad \frac{\partial V}{\partial I} = R$$

$$\frac{dV}{dt} = R\left(\frac{dI}{dt}\right) + I\left(\frac{dR}{dt}\right)$$

$$-0.05 = 413\,\Omega\left(\frac{dI}{dt}\right) + 0.08A(0.09\,\Omega/s)$$

$$-0.05 = 413\left(\frac{dI}{dt}\right) + 0.0072$$

$$-0.0572 = 413\left(\frac{dI}{dt}\right)$$

$$\frac{dI}{dt} = -0.000138$$

$\boxed{-0.000138 \text{ A/s}}$

14.) $z = f(x,y) \qquad x = r^2 + s^2 \quad y = 7rs$