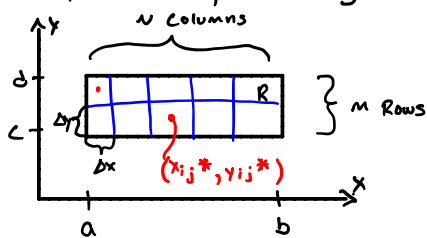


1. Partition I into n subintervals
Each has width $\Delta x = \frac{b-a}{n}$
2. Choose a sample point in each subinterval. Build rectangle over interval $\# i$ with height $f(x_i^*)$
3. Calculate Riemann sum

$$\sum_{i=1}^n f(x_i^*) \Delta x$$

Let R be a rectangle in the xy -plane

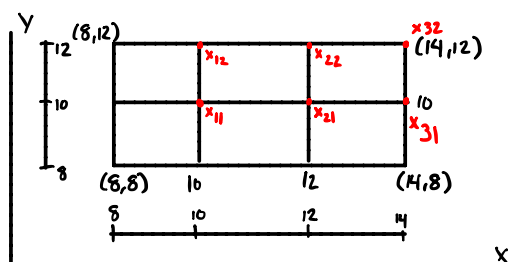
$$R = \{ (x, y) \mid a \leq x \leq b, c \leq y \leq d \}$$



1. Partition R .
2. Choose sample points
 $1 \leq i \leq m$
 $1 \leq j \leq n$
 Build rectangular solid over square ij .
 Its volume is $f(x_{ij}^*, y_{ij}^*) \Delta x \Delta y$

$$\sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta x \Delta y$$

$$\iint_R f(x, y) dA = \lim_{\substack{m \rightarrow \infty \\ n \rightarrow \infty \\ \Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta x \Delta y$$



$$m=3$$

$$n=2$$

$$8 \leq x \leq 14$$

$$8 \leq y \leq 12$$

$$V = \sum_{i=1}^3 \sum_{j=1}^2 f(x_{ij}, y_{ij}) \Delta A$$

$$\Delta A = \Delta x \cdot \Delta y$$

$$= (10(10) + 10(12) + 12(10) + 12(12) + 14(10) + 14(12)) (4)$$

$$(100 + 120 + 120 + 144 + 140 + 168) 4$$

$$V = 3168$$