Factorizing

Ex. 1 : 21

→ Divide By Prime numbers First

$21/1 = 21$: $21/2$ → No : $\boxed{21/3 = 7}$

Ex. 2 : 221

$221/1 = 221$: $221/2$ → NO : $221/3$ → NO : $221/5$ → NO

$221/7$ → NO : $221/11$ → NO : $\boxed{221/13 = 17}$

Ex. 3 : 1147

$1147/1 = 1147$: $1147/2$ → NO : $1147/3$ → NO : $1147/5$ → NO : $1147/7$ → NO

$1147/11$ → NO : $1147/13$ → NO : $1147/17$ → NO : $1147/19$ → NO

$1147/23$ → NO : $1147/29$ → NO : $\boxed{1147/31 = 37}$

Increasing the digits of a number increases the number of operations required to factorize a number. More digits means more arithmetic processes. This requires better hardware, more energy, and longer time for a computer to process.

Binary Numbers

$x = a_0 2^0 + a_1 2^1 + a_2 2^2 + \dots$: $x = \sum_{i=0}^{n-1} a_k 2^k$: x is binary number

$x = a_{n-1} a_{n-2} \cdots a_2 a_1 a_0$

Ex : 8 → $8 = 1\cdot 2^3 + 0\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0 = 1000$: $\boxed{8 \equiv 1000}$

Ex : 13 → $13 = 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 = 1101$: $\boxed{13 = 1101}$

Computers represent numbers with bits : i.e 1000, 1101. Now we wish to add these numbers together.

Ex : 4+2 : $4 = 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$: $4 \equiv 100$

$2 = 0\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0$: $2 \equiv 010$

$$\therefore\ 4+2 = \begin{array}{r} 100 \\ +\ 010 \\ \hline 110 \end{array} \rightarrow \boxed{6 \equiv 110}$$

Ex: 4+4 : $4 \equiv 100$ $\therefore$ 4+4 =

```
  100
+ 100
-----
 1000
```

$\longrightarrow$ 1+1 in binary = 0, carry over 1

$\boxed{4+4 = 8 \equiv 1000}$

Binary Addition

Ex: $13 = 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 \equiv 1101$: $15 = 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 \equiv 1111$

$8 + 4 + 0 + 1 = 13$ ✓ $\qquad$ $8+4+2+1 = 15$ ✓

Ex: 13 + 15 =

```
  111
  1101
  1111
 -----
 11100
```

$\longrightarrow$ $28 \equiv 11100 = 1\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0$

$16+8+4 = 28$ ✓

$\boxed{28 \equiv 11100}$

Binary Multiplication

Ex: $5 \times 3 = 15$: $5 \equiv 101$, $3 \equiv 011$

101 × 011 =

```
    101
  x 011
  -----
    101
   101x
  000xx
  -----
  01111
```

$\longrightarrow$ $15 \equiv 01111 = 0\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

$0+8+4+2+1 = 15$ ✓

$\boxed{15 \equiv 01111}$

Problem : 7×6 : $7 \equiv 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 \equiv 111$: $6 \equiv 1\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0 \equiv 110$

$4+2+1 = 7$ ✓ $\qquad$ $4+2+0 = 6$ ✓

7×6 =

```
     111
   x 110
  ------
   , 000
  , 111x
  111xx
  ------
  101010
```

$101010 = 1\cdot 2^5 + 0\cdot 2^4 + 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0$

$32+0+8+0+2+0 = 42$ ✓

$\boxed{7\times 6 = 42 \equiv 101010}$

Modular addition

$\oplus \longrightarrow$ signifies modular addition, add digits, divide by 2, keep remainder.

Negation

$a \longrightarrow \bar{a}$: i.e :

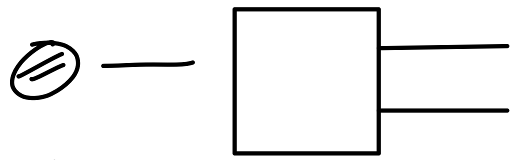
a	$\bar{a}$
0	1
1	0

$\longrightarrow$ $\boxed{\bar{a} = 1 \oplus a}$

$SG\hat{n}$

if $SG\hat{n}$ yields $+\frac{\hbar}{2}$ $\longrightarrow$ State after is $|+\hat{n}\rangle$

if $SG\hat{n}$ yields $-\frac{\hbar}{2}$ $\longrightarrow$ State after is $|-\hat{n}\rangle$



$|+\hat{m}\rangle$

outcome	Probability	State after
$S_n = +\hbar/2$	$\frac{1+\hat{m}\cdot\hat{n}}{2}$	$\|+\hat{n}\rangle$
$S_n = -\hbar/2$	$\frac{1-\hat{m}\cdot\hat{n}}{2}$	$\|-\hat{n}\rangle$

Vector Bases

1: $e_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $e_2 = \begin{pmatrix}0\\1\end{pmatrix}$: $u_1 = 3$, $u_2 = 4i$: $f_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $f_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

a.)

$u = 3e_1 + 4ie_2$: $f_1 + f_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1+1\\1-1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}2\\0\end{pmatrix} = \frac{2}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}\begin{pmatrix}1\\0\end{pmatrix}$

$\longrightarrow \sqrt{2}\begin{pmatrix}1\\0\end{pmatrix} = \sqrt{2}e_1$

$f_1 - f_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1-1\\1-(-1)\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\2\end{pmatrix} = \frac{2}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}\begin{pmatrix}0\\1\end{pmatrix}$

$\longrightarrow \sqrt{2}\begin{pmatrix}0\\1\end{pmatrix} = \sqrt{2}e_2$: $e_1 = \frac{1}{\sqrt{2}}(f_1 + f_2)$: $e_2 = \frac{1}{\sqrt{2}}(f_1 - f_2)$

$$\therefore \quad u = \frac{3}{\sqrt{2}}(f_1 + f_2) + \frac{4i}{\sqrt{2}}(f_1 - f_2) = \underbrace{\left(\frac{3+4i}{\sqrt{2}}\right)}_{u_1} f_1 + \underbrace{\left(\frac{3-4i}{\sqrt{2}}\right)}_{u_2} f_2$$

$$\boxed{u = \left(\frac{3+4i}{\sqrt{2}}\right) f_1 + \left(\frac{3-4i}{\sqrt{2}}\right) f_2}$$

b.)

$u = 3e_1 + 4ie_2$: $u_1 = 3$, $u_2 = 4i$: $f_1 = \begin{pmatrix}2\\0\end{pmatrix}$, $f_2 = \begin{pmatrix}1\\1\end{pmatrix}$

$f_1 = 2e_1$: $f_2 = e_1 + e_2$: $e_1 = \frac{f_1}{2}$, $f_2 = \frac{f_1}{2} + e_2$, $e_2 = f_2 - \frac{f_1}{2}$

$$u = 3\left(\frac{f_1}{2}\right) + 4i\left(f_2 - \frac{f_1}{2}\right) = \frac{3}{2}f_1 - \frac{4i}{2}f_1 + 4if_2 = \underbrace{\left(\frac{3-4i}{2}\right)}_{u_1} f_1 + \underbrace{4i}_{u_2} f_2$$

$$\boxed{u = \left(\frac{3-4i}{2}\right) f_1 + 4if_2}$$

$\longrightarrow$ Got different answer

Inner Products

where are f_1 and f_2 coming from? g_1, g_2?

2: $f = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$, $g = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$ $f_1^* = \frac{-i}{\sqrt{2}}$, $f_1 = \frac{i}{\sqrt{2}}$: $f_2^* = \frac{1}{\sqrt{2}}$, $f_2 = \frac{1}{\sqrt{2}}$

a.) $\langle f,f\rangle = f_1^* f_1 + f_2^* f_2 = \frac{-i}{\sqrt{2}} \cdot \frac{i}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{-i^2}{2} + \frac{1}{2} = \frac{-(-1)}{2} + \frac{1}{2} = 1$

$\boxed{\langle f,f\rangle = 1}$

$g_1^* = \frac{i}{\sqrt{2}}$, $g_1 = \frac{-i}{\sqrt{2}}$: $g_2^* = \frac{1}{\sqrt{2}}$, $g_2 = \frac{1}{\sqrt{2}}$

$\langle g,g\rangle = g_1^* g_1 + g_2^* g_2 = \frac{i}{\sqrt{2}} \cdot \frac{-i}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{-i^2}{2} + \frac{1}{2} = \frac{-(-1)}{2} + \frac{1}{2} = 1$

$\boxed{\langle g,g\rangle = 1}$

b.) $\langle f,g\rangle = f_1^* g_1 + f_2^* g_2 = \frac{-i}{\sqrt{2}} \cdot \frac{-i}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{i^2}{2} + \frac{1}{2} = \frac{-1}{2} + \frac{1}{2} = 0$

$\boxed{\langle f,g\rangle = 0}$

c.) These two vectors constitute an orthonomal basis.

orthogonal & normalized

$\langle u,v\rangle = u_1^* v_1 + u_2^* v_2$

Ket Algebra

3: $|\psi\rangle := 1|+\hat{z}\rangle - 2i|-\hat{z}\rangle$, $|\varphi\rangle := \frac{1}{\sqrt{5}}\left(i|+\hat{z}\rangle + 2i|-\hat{z}\rangle\right)$

a.) $\boxed{|\psi\rangle = \begin{pmatrix} 1 \\ -2i \end{pmatrix}, \quad |\varphi\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} i \\ 2i \end{pmatrix}}$

b.) $\langle\psi|\psi\rangle = \psi_1^*\psi_1 + \psi_2^*\psi_2$: $\psi_1^* = 1$, $\psi_1 = 1$: $\psi_2^* = 2i$, $\psi_2 = -2i$

$\langle\psi|\psi\rangle = 1(1) + 2i(-2i) = 1 - 4(i^2) = 1 + 4 = 5 \longrightarrow$ No, $\neq 1$

$\langle\varphi|\varphi\rangle = \varphi_1^*\varphi_1 + \varphi_2^*\varphi_2$: $\varphi_1^* = \frac{-i}{\sqrt{5}}$, $\varphi_1 = \frac{i}{\sqrt{5}}$: $\varphi_2^* = \frac{-2i}{\sqrt{5}}$, $\varphi_2 = \frac{2i}{\sqrt{5}}$

$\langle\varphi|\varphi\rangle = -\frac{i}{\sqrt{5}} \cdot \frac{i}{\sqrt{5}} - \frac{2i}{\sqrt{5}} \cdot \frac{2i}{\sqrt{5}} = \frac{-i^2}{5} - \frac{4i^2}{5} = \frac{1}{5} + \frac{4}{5} = 1 \longrightarrow$ Yes, $= 1$

$\boxed{\langle\varphi|\varphi\rangle = 1}$ ✓

c.) $\langle\psi|\varphi\rangle$: $\langle\psi| = (1 \; 2i)$, $|\varphi\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} i \\ 2i \end{pmatrix}$

$\langle\psi|\varphi\rangle = (1 + 2i)\frac{1}{\sqrt{5}}\begin{pmatrix} i \\ 2i \end{pmatrix} = \frac{1}{\sqrt{5}}\left(i + 4i^2\right) = \frac{1}{\sqrt{5}}\left(i - 4\right) = \frac{i-4}{\sqrt{5}}$

$\boxed{\langle\psi|\varphi\rangle = \frac{i-4}{\sqrt{5}}}$

d.) $\langle\varphi|\psi\rangle$: $\langle\varphi| = \frac{1}{\sqrt{5}}(-i \; -2i)$, $|\psi\rangle = \begin{pmatrix} 1 \\ 2i \end{pmatrix}$

$\langle\varphi|\psi\rangle = \frac{1}{\sqrt{5}}(-i \; -2i)\begin{pmatrix} 1 \\ -2i \end{pmatrix} = \frac{1}{\sqrt{5}}\left(-i + 4i^2\right) = \frac{1}{\sqrt{5}}(-i - 4i) = \frac{-i-4i}{\sqrt{5}}$

$\boxed{\langle\varphi|\psi\rangle = \frac{-i-4i}{\sqrt{5}}}$

$\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$