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PHYS 321 HW 8

4.24 ✓, 4.25 ✓, 4.27 ✓, 4.29 ✓, 4.32 ✓, 4.33 ✓

Problem 4.24

a.) $E_n = \frac{\hbar^2 n(n+1)}{ma^2}$, for $n = 0, 1, 2 \ldots\ldots$

Rigid rotor: $H = \frac{L^2}{2I} \longrightarrow (1)$

Moment of Inertia: $I = 2m\left(\frac{r}{2}\right)^2 \rightarrow I = 2m\left(\frac{a}{2}\right)^2 = \frac{ma^2}{2}$: $I = \frac{ma^2}{2} \longrightarrow (2)$

Putting (2) into (1):

$H = \frac{L^2}{2\frac{ma^2}{2}} = \frac{L^2}{ma^2} \longrightarrow (3)$: $L^2 = \hbar^2 \ell(\ell+1) f_\ell^m \longrightarrow (4)$

Putting (4) into (3): $H = \frac{\hbar^2 \ell(\ell+1)}{ma^2}$, changing $H \to E$ and $\ell \to n$ we have

$$\boxed{E_n = \frac{\hbar^2 n(n+1)}{ma^2}}$$

b.) The spherical harmonics functions are normalized eigenfunctions for this system.

$$\boxed{Y_n^m(\theta, \varphi)}$$

The degeneracy level can be determined by: $-n \leq m \leq n$

$m = 1$: $-1 \leq m \leq 1 \longrightarrow 2n$ values
$m = 2$: $-2 \leq m \leq 2 \longrightarrow 2n$ again, plus m value
$m = 3$: $-3 \leq m \leq 3$

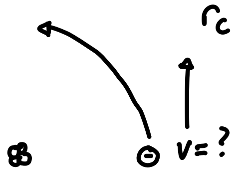
$n + n + 1 = 2n + 1$

$\vdots$

$\therefore$ $\boxed{\text{degeneracy} = 2n+1}$

Problem 4.25

$$r_c = \frac{e^2}{4\pi \varepsilon_0 m c^2}$$



Classically $\quad V = \omega r \longrightarrow$ (5)

Equation (5) is how we relate the tangential velocity of this electron to its radius. We now need to solve for ω.

Angular momentum : $L = I\omega \longrightarrow$ (6)

Moment of Inertia for a solid sphere is : $I = \frac{2}{5} m r^2 \longrightarrow$ (7)

We can put I from (7) into (6) and we have with $L = \hbar/2$

$$\omega = \frac{L}{I} = \frac{\hbar/2}{\frac{2}{5} m r^2} = \frac{5\hbar}{4mr^2} \longrightarrow (8)$$

Putting (8) into (5) we get

$$V = \frac{5\hbar}{4mr^2} \cdot r = \frac{5\hbar}{mr} \longrightarrow (9)$$

:

$e = 1.60218 \times 10^{-19}\ C$
$\hbar = 1.05457 \times 10^{-34}\ Js$
$m = 9.10938 \times 10^{-31}\ kg$
$\varepsilon_0 = 8.85419 \times 10^{-12}\ C^2/Jm$
$c = 3.0 \times 10^{8}\ m/s$

$\therefore r = 2.81 \times 10^{-15}\ m$

$$\frac{J \cdot s}{kg \cdot m} = \frac{N \cancel{m}\, s}{kg \cdot \cancel{m}} = \frac{\cancel{kg} \cdot m/s^2 \cdot s}{\cancel{kg}} = \frac{m}{s^2} \cdot s = \frac{m}{s} \checkmark$$

$$V = \frac{5 \cdot 1.05457 \times 10^{-34}\ Js}{4 \cdot 9.10938 \times 10^{-31}\ kg\ (2.81 \times 10^{-15}\ m)} = 5.15 \times 10^{10}\ \frac{m}{s}$$

With the classical electron radius of $r_c = 2.81 \times 10^{-5}\ m$, its tangential velocity is $V = 5.15 \times 10^{10}\ m/s$. This result is 172 times greater than the speed of light which is the speed limit of the universe. This once again proves that classical mechanics gives shit results at the quantum level.

Problem 4.27

$$\chi = A\begin{pmatrix}3i\\4\end{pmatrix}, \quad \chi^* = A(-3i \;\; 4)$$

a.) A ?

$$\chi^*\chi = A(-3i\;4)\cdot A\begin{pmatrix}3i\\4\end{pmatrix} = A^2(-9i^2+16) = A^2(-9(-1)+16) = A^2(25) = 1$$

$$A^2 = \frac{1}{25} \quad \therefore \quad A = \pm\frac{1}{5}$$

$$\boxed{A = \pm\frac{1}{5}}$$

b.) $\langle S_x\rangle, \langle S_y\rangle, \langle S_z\rangle$?

i.) $S_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\langle S_x\rangle = \chi^* S_x \chi \longrightarrow$ (10)

(10) becomes: $\langle S_x\rangle = \frac{1}{5}(-3i\;4)\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix}$

$$= \frac{\hbar}{10}\Big(\big(\cancel{-3i(0)}^{0}+4(1)\big)\;\; -3i+\cancel{4(0)}^{0}\Big)\frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix}$$

$$= \frac{\hbar}{10}(4\;\;-3i)\frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix} = \frac{\hbar}{50}(12i-12i) = \frac{\hbar}{50}(0) = 0$$

$$\boxed{\langle S_x\rangle = 0}$$

ii.) $S_y = \frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$, $\langle S_y\rangle = \chi^* S_y \chi \longrightarrow$ (11)

(11) becomes: $\langle S_y\rangle = \frac{1}{5}(-3i\;4)\frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix}$

$$= \frac{\hbar}{10}\Big(\big(-3i(0)+4(i)\big)+\big(-3i(-i)+4(0)\big)\Big)\frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix}$$

$$= \frac{\hbar}{50}(4i+3i^2)\begin{pmatrix}3i\\4\end{pmatrix} = \frac{\hbar}{50}(4i\;\;-3)\begin{pmatrix}3i\\4\end{pmatrix}$$

$$= \frac{\hbar}{50}(4i(3i)-3(4)) = \frac{\hbar}{50}(12(i)^2-12) = \frac{\hbar}{50}(-12-12)$$

$$= \frac{-24}{50}\hbar = \frac{-12}{25}\hbar$$

$$\boxed{\langle S_y\rangle = \frac{-12}{25}\hbar}$$

Problem 4.27 | Continued

iii.) $S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\langle S_z \rangle = \chi^* S_z \chi \longrightarrow (12)$

(12) becomes :

$$\langle S_z \rangle = \frac{1}{5}(-3i \;\; 4)\,\frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{5}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{10}\big((-3i(1)+4(0)) + (-3i(0)+4(-1))\big)\,\frac{1}{5}\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

$$= \frac{\hbar}{10}(-3i \;\; -4)\,\frac{1}{5}\begin{pmatrix} 3i \\ 4 \end{pmatrix} = \frac{\hbar}{50}\big(-9(i)^2 - 4(4)\big)$$

$$= \frac{\hbar}{50}(9-16) = \frac{-7\hbar}{50}$$

$$\boxed{\langle S_z \rangle = \frac{-7\hbar}{50}}$$

Problem 4.29

a.) $S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

i.) Eigenvalues: $\frac{\hbar}{2}\begin{pmatrix} -\lambda & -i \\ i & -\lambda \end{pmatrix} = \begin{pmatrix} -\lambda & -\frac{i\hbar}{2} \\ \frac{i\hbar}{2} & -\lambda \end{pmatrix} \longrightarrow (13)$

Taking a determinant of (13) $= -\lambda(-\lambda) - -\frac{i\hbar}{2}\left(\frac{i\hbar}{2}\right)$

$\det = \lambda^2 - \frac{\hbar^2}{4} \longrightarrow (14)$

Putting (14) equal to zero: $\lambda^2 = \frac{\hbar^2}{4}$ $\therefore$ $\boxed{\lambda = \pm\frac{\hbar}{2}}$

ii.) Eigenspinors: $\chi_+ = A\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\chi_- = A\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\chi_+^*\chi_+ + \chi_-^*\chi_-$: $A(1\ 0)A\begin{pmatrix} 1 \\ 0 \end{pmatrix} + A(0\ 1)A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$

$A^2(1) + A^2(1) = 1$ $\therefore$ $A^2 = \frac{1}{2}$: $A = \pm\frac{1}{\sqrt{2}}$

$\lambda = \frac{\hbar}{2}$: $\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow -\frac{i\hbar}{2}\beta = \frac{\hbar}{2}\alpha \longrightarrow (15)$: $\frac{\hbar}{2}i\alpha = \frac{\hbar}{2}\beta \longrightarrow (16)$

(15) : $-i\beta = \alpha$ $\therefore$ $\beta = -\frac{\alpha}{i(i)}(i) = -\frac{i(\alpha)}{-1} = i\alpha$: (16) : $i\alpha = \beta \longrightarrow (17)$
$\alpha = -i\beta$

$\lambda = -\frac{\hbar}{2}$: $\cancel{\frac{\hbar}{2}}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = -\cancel{\frac{\hbar}{2}}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow -i\beta = -\alpha \longrightarrow (18)$: $i\alpha = -\beta \longrightarrow (19)$

(18) : $-i\beta = -\alpha$: $\beta = \frac{\alpha}{i(i)}(i) = -i(\alpha)$: (19) : $\beta = -i\alpha \longrightarrow (20)$
$\alpha = i\beta$

Using (17) and (20) : $\boxed{\chi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix},\ \chi_- = \frac{-1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}}$

b.) $\chi = a\chi_+ + b\chi_-$

Eigenvalues: $\lambda = \pm\frac{\hbar}{2}$, $S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$, $|a|^2 + |b|^2 = 1 \longrightarrow (21)$

i.) $S_y\chi = \lambda\chi$: $\lambda = \frac{\hbar}{2}$

$\cancel{\frac{\hbar}{2}}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \cancel{\frac{\hbar}{2}}\begin{pmatrix} a \\ b \end{pmatrix}$: $-ib = a \longrightarrow b = ia$ or $a = -ib$
$ia = b \longrightarrow b = ia$

Using (21) we have : $|a|^2 + |ia|^2 = 1$: $a^2 + a^2 = 1$: $a^2 = \frac{1}{2}$ $\therefore$ $a = \pm\frac{1}{\sqrt{2}}$

Problem 4.29 Continued

$b = \frac{i}{\sqrt{2}}$, $a = \frac{1}{\sqrt{2}}$ $\therefore$ $\chi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$ for $\lambda = \frac{\hbar}{2}$

ii.) $S_y\chi = \lambda\chi$: $\lambda = -\frac{\hbar}{2}$

$\cancel{\frac{\hbar}{2}}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = -\cancel{\frac{\hbar}{2}}\begin{pmatrix}a\\b\end{pmatrix}$: $\begin{matrix}-ib = -a\\ ia = -b\end{matrix}$: $\begin{matrix}a = ib \text{ or } b = -ia\\ a = ib\end{matrix}$

using (21) again we have : $|a|^2 + |-ia|^2 = 1$: $a^2 + a^2 = 1$: $a^2 = \frac{1}{2}$ $\therefore$ $a = \pm\frac{1}{\sqrt{2}}$

$b = \frac{-i}{\sqrt{2}}$, $a = \frac{1}{\sqrt{2}}$ $\therefore$ $\chi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ for $\lambda = -\frac{\hbar}{2}$

The generic spinor χ can be expressed as a linear combination as

$$\chi = \left(\frac{a+b}{\sqrt{2}}\right)\chi_+ + \left(\frac{a-b}{\sqrt{2}}\right)\chi_- \longrightarrow (22)$$: Pg. 175 ([4.152])

With our a and b values of $a = \pm ib$, using (22) we have

$$\chi = \left(\frac{a+ib}{\sqrt{2}}\right)\chi_+ + \left(\frac{a-ib}{\sqrt{2}}\right)\chi_- \longrightarrow (23)$$

If we wish to know the probability of one state, we simply square the term in front of χ_+ and χ_-. Doing this to (23) gives

$$\lambda = -\frac{\hbar}{2} \ \frac{(a+ib)^2}{2} : \lambda = \frac{\hbar}{2} \ \frac{(a-ib)^2}{2} \longrightarrow (24)$$

Adding the results from (24) we have

$$\frac{a^2 + 2aib + i^2b^2}{2} + \frac{a^2 - 2aib + i^2b^2}{2} = \frac{a^2 - b^2}{2} + \frac{a^2 - b^2}{2} = a^2 - b^2 \longrightarrow (25)$$

Substitute $a = \frac{1}{\sqrt{2}}$ and $b = \frac{i}{\sqrt{2}}$ into (26) : $\frac{1}{2} - \left(\frac{i}{\sqrt{2}}\right)^2 = \frac{1}{2} - \left(\frac{-1}{2}\right) = \frac{1}{2} + \frac{1}{2} = 1$ ✓

Therefore our probabilities do indeed add up to 1!

$$\boxed{P(\lambda = \hbar/2) = \frac{(a-ib)^2}{2}, \quad P(\lambda = -\hbar/2) = \frac{(a+ib)^2}{2}}$$

c.) S_y^2? $S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ $\therefore$ $S_y^2 = \frac{\hbar^2}{4}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$

$$= \frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \longrightarrow (26)$$

Problem 4.29 Continued

Taking a determinant of (26) we get:

$$= \frac{\hbar^2}{4}(1)$$: $\boxed{\text{This gives a probability of 1.}}$

Problem 4.32

a.) $\chi(t) = \begin{pmatrix} a e^{i\gamma B_0 t/2} \\ b e^{-i\gamma B_0 t/2} \end{pmatrix} = \begin{pmatrix} \cos(\alpha/2) e^{i\gamma B_0 t/2} \\ \sin(\alpha/2) e^{-i\gamma B_0 t/2} \end{pmatrix}$: $\chi(t) = \begin{pmatrix} \cos(\alpha/2) e^{i\gamma B_0 t/2} \\ \sin(\alpha/2) e^{-i\gamma B_0 t/2} \end{pmatrix}$

When $\lambda = \frac{\hbar}{2}$: $\chi_+^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \longrightarrow$ [4.151] pg. 175 $P_+^{(x)} = |C_+^{(x)}|^2 : C_+^{(x)} = \chi_+^{(x)*}\chi$

$\chi_+^{(x)*} = \frac{1}{\sqrt{2}}(1\ 1)$: $\chi(t) = \begin{pmatrix} \cos(\alpha/2) e^{i\gamma B_0 t/2} \\ \sin(\alpha/2) e^{-i\gamma B_0 t/2} \end{pmatrix}$

$$C_+^{(x)} = \frac{1}{\sqrt{2}}(1\ 1)\cdot\begin{pmatrix} \cos(\alpha/2) e^{i\gamma B_0 t/2} \\ \sin(\alpha/2) e^{-i\gamma B_0 t/2} \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\cos(\alpha/2)e^{i\gamma B_0 t/2} + \sin(\alpha/2)e^{-i\gamma B_0 t/2}\right)$$

$P_+^{(x)} = |C_+^{(x)}|^2$:

$$\frac{1}{\sqrt{2}}\left(\cos(\alpha/2)e^{-i\gamma B_0 t/2} + \sin(\alpha/2)e^{i\gamma B_0 t/2}\right)\cdot\frac{1}{\sqrt{2}}\left(\cos(\alpha/2)e^{i\gamma B_0 t/2} + \sin(\alpha/2)e^{-i\gamma B_0 t/2}\right)$$

$$= \frac{1}{2}\left(\cos^2(\alpha/2) + \cos(\alpha/2)\sin(\alpha/2)e^{-i\gamma B_0 t} + \cos(\alpha/2)\sin(\alpha/2)e^{i\gamma B_0 t} + \sin^2(\alpha/2)\right)$$

$$= \frac{1}{2}\left(\cos^2(\alpha/2) + \sin^2(\alpha/2) + \sin(\alpha/2)\cos(\alpha/2)\left(e^{i\gamma B_0 t} + e^{-i\gamma B_0 t}\right)\right)$$

$$e^{ix} + e^{-ix} = \cos(x) + \cancel{i\sin(x)} + \cos(x) - \cancel{i\sin(x)} = 2\cos(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$= \frac{1}{2}\left(1 + \sin(\alpha/2)\cos(\alpha/2)\cdot 2\cos(\gamma B_0 t)\right) = \frac{1}{2}\left(1 + 2\sin(\alpha/2)\cos(\alpha/2)\cdot\cos(\gamma B_0 t)\right)$$

$$\boxed{P_+^{(x)} = \frac{1}{2}\left(1 + \sin(\alpha)\cos(\gamma B_0 t)\right)}$$

b.) $\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$: $\chi_+^{(y)*} = \frac{1}{\sqrt{2}}(1\ -i)$: $P_+^{(y)} = |C_+^{(y)}|^2$: $C_+^{(y)} = \chi_+^{(y)*}\chi(t)$

$$C_+^{(y)} = \frac{1}{\sqrt{2}}(1\ -i)\cdot\begin{pmatrix} \cos(\alpha/2) e^{i\gamma B_0 t/2} \\ \sin(\alpha/2) e^{-i\gamma B_0 t/2} \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\cos(\alpha/2)e^{i\gamma B_0 t/2} - i\sin(\alpha/2)e^{-i\gamma B_0 t/2}\right)$$

$$|C_+^{(y)}|^2 = \frac{1}{\sqrt{2}}\left(\cos(\alpha/2)e^{-i\gamma B_0 t/2} + i\sin(\alpha/2)e^{i\gamma B_0 t/2}\right)\frac{1}{\sqrt{2}}\left(\cos(\alpha/2)e^{i\gamma B_0 t/2} - i\sin(\alpha/2)e^{-i\gamma B_0 t/2}\right)$$

$$= \frac{1}{2}\left(\cos^2(\alpha/2) - i\cos(\alpha/2)\sin(\alpha/2)e^{-i\gamma B_0 t} + i\cos(\alpha/2)\sin(\alpha/2)e^{i\gamma B_0 t} - i^2\sin^2(\alpha/2)\right)$$

$$= \frac{1}{2}\left(\cos^2(\alpha/2) + \sin^2(\alpha/2) + i\cos(\alpha/2)\sin(\alpha/2)\left(e^{i\gamma B_0 t} - e^{-i\gamma B_0 t}\right)\right)$$

$$e^{ix} - e^{-ix} = \cancel{\cos(x)} + i\sin(x) - \cancel{\cos(x)} + i\sin(x) = 2i\sin(x)$$

Problem 4.32 | Continued

$$= \frac{1}{2}\left(1 + 2(i)^2\cos(\alpha/2)\sin(\alpha/2)\sin(\gamma B_0 t)\right) = \frac{1}{2}\left(1 - 2\sin(\alpha/2)\cos(\alpha/2)\sin(\gamma B_0 t)\right)$$

$$\therefore \boxed{P_+^{(y)} = \frac{1}{2}\left(1-\sin(\alpha)\sin(\gamma B_0 t)\right)}$$

c.) $\chi_+^{(z)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} : \chi_+^{(z)^*} = (1\ 0) : P_+^{(z)} = |C_+^{(z)}|^2 : \quad C_+^{(z)} = \chi_+^{(z)}\chi$

$$C_+^{(z)} = (1\ 0)\begin{pmatrix} \cos(\alpha/2)e^{i\gamma B_0 t/2} \\ \sin(\alpha/2)e^{-i\gamma B_0 t/2} \end{pmatrix} = \cos(\alpha/2)e^{i\gamma B_0 t/2}$$

$$|C_+^{(z)}|^2 = \left(\cos(\alpha/2)e^{-i\gamma B_0 t/2}\right)\left(\cos(\alpha/2)e^{i\gamma B_0 t/2}\right) = \cos^2(\alpha/2)$$

$$\cos^2(x) = \frac{1}{2}\left(1+\cos(2x)\right) \quad \therefore \quad |C_+^{(z)}|^2 = \frac{1}{2}\left(1+\cos(\alpha)\right)$$

$$\boxed{P_+^{(z)} = \frac{1}{2}\left(1+\cos(\alpha)\right)}$$

Problem 4.33

$$B = B_0 \cos(\omega t)\,\hat{k}$$

a.) $H = -\vec{\mu}\cdot\vec{B}$ $\because$ $H = -\gamma\vec{B}\cdot\vec{S}$: $\vec{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$H = -\gamma\cdot\vec{B}\cdot\vec{S}_z$

$$H = -\gamma\cdot B_0\cos(\omega t)\cdot\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\boxed{H = -\gamma B_0\cos(\omega t)\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}$$

b.) $i\hbar\frac{\partial\chi}{\partial t} = H\chi$ $\quad \chi = \begin{pmatrix}a\\ b\end{pmatrix}$: $H = -\gamma B_0\cos(\omega t)\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$i\hbar\begin{pmatrix}\partial a/\partial t\\ \partial b/\partial t\end{pmatrix} = -\gamma B_0\cos(\omega t)\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}$: $i\hbar\frac{\partial a}{\partial t} = -a\gamma B_0\frac{\hbar}{2}\cos(\omega t) \longrightarrow (27)$

$i\hbar\frac{\partial b}{\partial t} = b\gamma B_0\frac{\hbar}{2}\cos(\omega t) \longrightarrow (28)$

Solving equation (27):

$i\hbar\frac{\partial a}{a} = -\gamma B_0\frac{\hbar}{2}\cos(\omega t)\,\partial t$: $i\hbar\int\frac{da}{a} = -\gamma B_0\frac{\hbar}{2}\int\cos(\omega t)\,dt$

$\int\frac{da}{a} = \frac{i\gamma B_0}{2}\int\cos(\omega t)\,dt$: $\ln(a) + a_0 = \frac{i\gamma B_0}{2\omega}\sin(\omega t)$

$\ln(a) = \frac{i\gamma B_0}{2\omega}\sin(\omega t) + a_0$ $\therefore$ $a(t) = a_0 e^{\frac{i\gamma B_0}{2\omega}\sin(\omega t)} \longrightarrow (29)$

Solving equation (28):

$i\hbar\frac{\partial b}{\partial t} = b\gamma B_0\frac{\hbar}{2}\cos(\omega t)$: $i\int\frac{db}{b} = \int\frac{b\gamma B_0}{2}\cos(\omega t)\,dt$

$\int\frac{db}{b} = \frac{-ib\gamma B_0}{2}\int\cos(\omega t)\,dt$: $\ln(b) + b_0 = -\frac{ib\gamma B_0}{2\omega}\sin(\omega t)$

$\ln(b) = b_0 - \frac{ib\gamma B_0}{2\omega}\sin(\omega t)$ $\therefore$ $b(t) = b_0 e^{-\frac{i\gamma B_0}{2\omega}\sin(\omega t)} \longrightarrow (30)$

With (29) and (30) $\chi(t)$ is : where $a_0 = b_0 = \frac{1}{\sqrt{2}}$

$$\boxed{\chi(t) = \frac{1}{\sqrt{2}}\begin{pmatrix}e^{\frac{i\gamma B_0}{2\omega}\sin(\omega t)}\\ e^{-\frac{i\gamma B_0}{2\omega}\sin(\omega t)}\end{pmatrix}}$$

Problem 4.33 Continued.

c.) $P_+^{(x)} = |C_+^{(x)}|^2$: $C_+^{(x)} = \chi_+^{(x)*}\chi(t)$: $\chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$: $\chi_-^{(x)*} = \frac{1}{\sqrt{2}}(1\ -1)$

$$C_+^{(x)} = \frac{1}{\sqrt{2}}(1\ -1)\cdot\frac{1}{\sqrt{2}}\begin{pmatrix} e^{\frac{i\gamma B_0}{2\omega}\sin(\omega t)} \\ e^{-\frac{i\gamma B_0}{2\omega}\sin(\omega t)}\end{pmatrix}$$

$$= \frac{1}{2}\left(e^{\frac{i\gamma B_0 t}{2\omega}\sin(\omega t)} - e^{\frac{-i\gamma B_0}{2\omega}\sin(\omega t)}\right)$$

$$|C_+^{(x)}|^2 = \frac{1}{2}\left(e^{\frac{i\gamma B_0 t}{2\omega}\sin(\omega t)} - e^{\frac{-i\gamma B_0}{2\omega}\sin(\omega t)}\right)\frac{1}{2}\left(e^{\frac{-i\gamma B_0 t}{2\omega}\sin(\omega t)} - e^{\frac{+i\gamma B_0}{2\omega}\sin(\omega t)}\right)$$

$$e^{ix} - e^{-ix} = 2i\sin(x) \quad \text{where} \quad x = \frac{\gamma B_0 t}{2\omega}\sin(\omega t)$$

$$= \frac{1}{2}\left(2i\sin\left(\frac{\gamma B_0 t}{2\omega}\sin(\omega t)\right)\right)\cdot\frac{1}{2}\left(-2i\sin\left(\frac{\gamma B_0 t}{2\omega}\sin(\omega t)\right)\right) = \frac{-4i^2}{4}\sin^2\left(\frac{\gamma B_0 t}{2\omega}\sin(\omega t)\right)$$

$$\therefore \quad \boxed{P_-^{(x)} = \sin^2\left(\frac{\gamma B_0 t}{2\omega}\sin(\omega t)\right)}$$