4.3 HW Complex Characteristic Roots

4.3 # 14, 21-28, 42, 50, 75

4.3.14

$y''-y'+y=0 \quad y(0)=0 \;,\; y'(0)=1$

$c_1=0$
$c_2=\frac{2}{\sqrt{3}}$

$r=\alpha\pm\beta i$
$\alpha=\frac{-b}{2a} \quad \beta=\frac{\sqrt{4ac-b^2}}{2a}$ $\quad y_h=e^{\alpha t}(c_1\cos\beta t+c_2\sin\beta t)$

$A=1$ $\quad \Delta=(-1)^2-4(1)(1)$ $\quad \alpha=\frac{1}{2}$
$B=-1$ $\quad 1-4$ $\quad \beta=\frac{\sqrt{3}}{2}$
$C=1$ $\quad \Delta=-3 \therefore$ complex

$\alpha=\frac{-b}{2a}: \quad \frac{-(-1)}{2(1)}$
$\alpha=\frac{1}{2} \quad \frac{1}{2}$

$\beta=\frac{\sqrt{4ac-b^2}}{2a}$
$\beta=\frac{\sqrt{4(1)(1)-(-1)^2}}{2(1)}$
$=\frac{\sqrt{4-1}}{2}$
$\beta=\frac{\sqrt{3}}{2}$

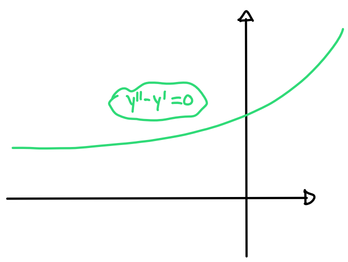
$y=e^{\frac{1}{2}t}\left(\frac{2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

$y(t)=e^{\frac{1}{2}t}\left(c_1\cos\left(\frac{\sqrt{3}}{2}t\right)+c_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$
$y(0)=e^{0}\left(c_1\cos(0)+c_2\sin(0)\right)$
$1(c_1)=0$
$c_1=0$
$y'(t)=\frac{1}{2}e^{\frac{1}{2}t}\left(c_1\cos\left(\frac{\sqrt{3}}{2}t\right)+c_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right)+e^{\frac{1}{2}t}\left(-\frac{\sqrt{3}}{2}c_1\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{\sqrt{3}}{2}c_2\cos\left(\frac{\sqrt{3}}{2}t\right)\right)$
$1=\frac{1}{2}e^{0}\left(c_1\cos(0)+c_2\sin(0)\right)+e^{0}\left(-\frac{\sqrt{3}}{2}c_1\sin(0)+\frac{\sqrt{3}}{2}c_2\cos(0)\right)$
$1=\frac{1}{2}(0+0)+1\left(0+\frac{\sqrt{3}}{2}c_2\right)$
$1=\frac{\sqrt{3}}{2}c_2$
$\frac{2}{\sqrt{3}}=c_2$

$$\boxed{y=\frac{2\sqrt{3}}{3}e^{\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)}$$

4.3.21

$y''-y'=0$

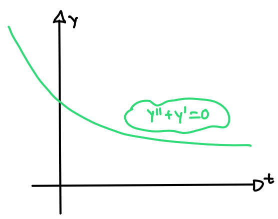


(d)

4.3.22

$y''+y'=0$

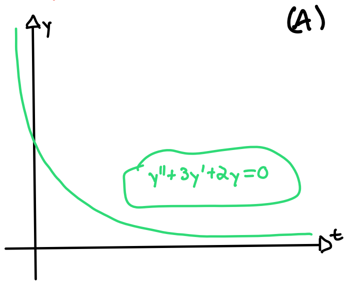
(B)



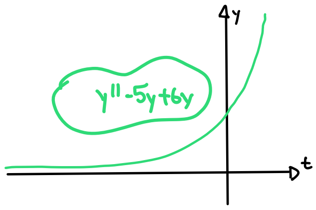
4.3.23

$y''+3y'+2y=0$

(A)



4.3.24) $y''-5y+6y=0$



(c)

b^2-4ac

$25-4(1)(6)$

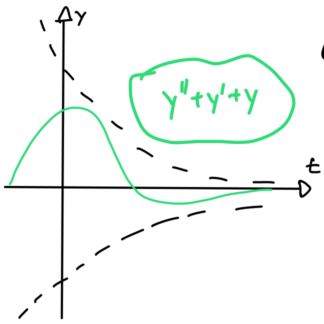
$25-24$

1

$r_1=\frac{5+1}{2}$ $r_2=\frac{5-1}{2}$

$r_1=3$ $r_2=2$

4.3.25) $y''+y'+y=0$



(G)

b^2-4ac

$(1)^2-4(1)(1)$

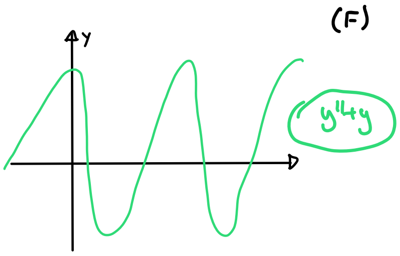
$1-4$

-3

$\Delta=-3$

$\therefore$ complex

4.3.26) $y''+y=0$



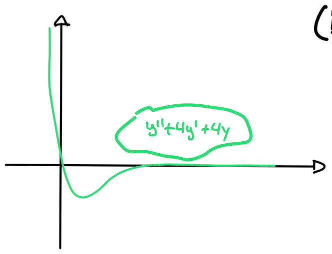
(F)

b^2-4ac

$0-4(1)(1)$

$\Delta=-4$

4.3.27) $y''+4y'+4y=0$

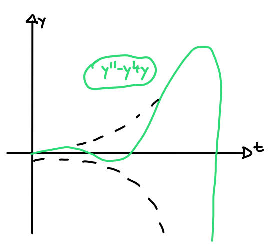


(E)

$9-4(1)(2)$

1

4.3.28) $y''-y'+y=0$

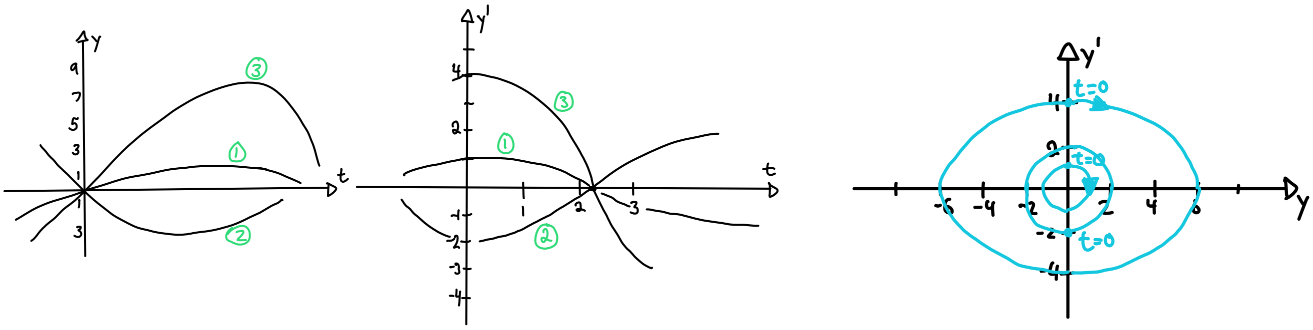


(F)

$(1)-4(1)(1)$

-4

4.3.45)



4.3.50

A) $\ddot{x}+2\dot{x}+3x=0$

$x(0)=1 \quad \dot{x}(0)=0$

$\alpha=-1$

$\beta=\sqrt{2}$

$\Delta=b^2-4ac$
$=4-4(1)(3)$
$=4-12$
$\Delta=-8$

$r=\alpha\pm i\beta$

$\alpha=-\frac{b}{2a} \quad \beta=\frac{\sqrt{4ac-b^2}}{2a}$

$\alpha=\frac{-2}{2(1)}=-\frac{3}{2}=-1 \quad \beta=\frac{\sqrt{4(1)(3)-2^2}}{2(1)}$

$\alpha=-1$

$=\sqrt{\frac{8}{2}}$

$=\frac{2\sqrt{2}}{2}$

$\beta=\sqrt{2}$

$y_h=e^{-t}(c_1\cos\sqrt{2}t+c_2\sin\sqrt{2}t)$

$1=e^0(c_1\cos 0+c_2\sin 0)$

$1=c_1$

$c_1=1$

$c_2=1/\sqrt{2}$

$y_h'=-e^{-t}(c_1\cos\sqrt{2}t+c_2\sin\sqrt{2}t)+e^{-t}(-c_1\sqrt{2}\sin\sqrt{2}t+c_2\sqrt{2}\cos\sqrt{2}t)$

$0=-e^0(c_1\cos(0)+c_2\sin(0))+e^0(-c_1\sqrt{2}\sin(0)+c_2\sqrt{2}\cos(0))$

$0=-1(c_1)+1(c_2\sqrt{2})$

$0=-1+c_2\sqrt{2}$

$1=c_2\sqrt{2}$

$1/\sqrt{2}=c_2$

$x(t)=e^{-t}(\cos\sqrt{2}t+\frac{1}{\sqrt{2}}\sin\sqrt{2}t)$

$x(\frac{\pi}{\sqrt{2}})=-e^{-\pi/\sqrt{2}}$

$y_h=e^{-t}(\cos(\sqrt{2}t)+\frac{1}{\sqrt{2}}\sin\sqrt{2}t)$

$y'_h=\frac{-3\sqrt{2}e^{-t}\cdot\sin(\sqrt{2}t)}{2}$

$0=\sin\sqrt{2}t$

$\pi=\sqrt{2}t$

$t=\frac{\pi}{\sqrt{2}}$

$\sin=0$ when $0,\pi,2\pi$

$e^{-\frac{\pi}{\sqrt{2}}}(\cos(\pi)+\frac{1}{\sqrt{2}}\cancel{\sin(\pi)})$ 0

$-e^{-\frac{\pi}{\sqrt{2}}}$

B) $\ddot{x}+2\dot{x}+10x=0$

m b k

$x(0)=0$

$\dot{x}(0)=2$

$\Delta=b^2-4mk$
$4-4(1)(10)$
$4-40$
$\Delta=-36$

$x(t)=e^{\alpha t}(c_1\cos\beta t+c_2\sin\beta t)$

$\alpha=\frac{-b}{2m} \quad \beta=\frac{\sqrt{4mk-b^2}}{2m}$

$\alpha=\frac{-2}{2(1)}=-1 \quad \beta=\frac{\sqrt{4(1)(10)-2^2}}{2(1)}=\sqrt{\frac{36}{2}}=\frac{6}{2}=3$

$x(t)=e^{-t}(c_1\cos(3t)+c_2\sin(3t))$

$x(0)=e^0(c_1\cos(0)+c_2\sin(0))$

$0=1(c_1)$

$c_1=0$

$x(t)=e^{-t}(\frac{2}{3})\sin(3t)$

$\dot{x}(t)=-e^{-t}(c_1\cos(3t)+c_2\sin(3t))+e^{-t}(-3c_1\sin(3t)+3c_2\cos(3t))$

$x(0)=-e^0(c_1\cos(0)+c_2\sin(0))+e^0(-3c_1\sin(0)+3c_2\cos(0))$

$2=-1(0)+1(3c_2)$

$2=3c_2$

$c_2=2/3$

$x(t)=e^{-t}(\frac{2}{3})\sin(3t)$

$x(max)=0.417$

$\dot{x}(t)=-e^{-t}(\frac{2}{3}\sin(3t))+e^{-t}(2\cos(3t))$

$0=e^{-t}(-(\frac{2}{3}\sin(3t))+2\cos(3t))$

$0=-\frac{2}{3}\sin(3t)+2\cos(3t)$

$\frac{2}{3}\sin(3t)=2\cos(3t)$

$\frac{1}{3}\tan(3t)=1$

$\tan(3t)=3$

$(3t)=\tan^{-1}(3)$

$t=\frac{\tan^{-1}(3)}{3}$

$\frac{\frac{2}{3}}{\frac{2}{1}} \quad \frac{2}{6}=\frac{1}{3}$

C) $\ddot{x}+4\dot{x}+4x=0$ $\quad x(0)=0$ $\quad r=-\frac{b}{2a}$ $\quad x(t)=c_1e^{rt}+c_2te^{rt}$

$m \quad B \quad K$ $\quad \dot{x}(0)=2$

$\Delta=b^2-4mk$
$=(4)^2-4(1)(4)$
$=16-16$
$\Delta=0$

$r=\frac{-4}{2(1)}=-\frac{4}{2}=-2$
$r=-2$

$c_1=0 \quad c_2=-2$
$x(t)=c_1e^{-2t}+c_2te^{-2t}$
$x(0)=c_1e^0+c_2(0)e^0$
$0=c_1$

$\dot{x}(t)=c_2e^{-2t}-2c_2te^{-2t}$
$2=e^{-2t}(c_2-2c_2t)$
$2=e^0(c_2-0)$
$2=c_2$

$x(\frac{1}{2})=+2(\frac{1}{2})e^{-(2\cdot\frac{1}{2})}$
$=+1e^{-1}$
$=+e^{-1}$

$x(t)=+2te^{-2t}$
$\dot{x}(t)=+2e^{-2t}-4te^{-2t}$
$0=e^{-2t}(+2-4t)$
$0=+2-4t$
$2=4t$
$t=\frac{1}{2}$

$x(t)=2te^{-2t}$
$x(max)=e^{-1}$

4.3.25) $\quad y''+y=0$ $\quad y(0)=0$
$y(\pi/2)=1$

$x(t)=e^{\alpha t}(c_1\cos\beta t+c_2\sin\beta t)$

$c_1=0$
$c_2=1$

$\Delta=b^2-4ac$
$=0-4(1)(1)$
$\Delta=-4$

$r=\alpha\pm i\beta$

$\alpha=\frac{-b}{2a} \quad \beta=\frac{\sqrt{4ac-b^2}}{2a}$
$\alpha=0 \quad \beta=\frac{\sqrt{4(1)(1)-0}}{2(1)}$
$=\frac{2}{2}$
$\beta=1$

$\alpha=0$
$\beta=1$

$x(t)=c_1\cos t+c_2\sin t$
$0=c_1\cos(0)+c_2\sin(0)$
$0=c_1$
$1=c_2\sin(\frac{\pi}{2})$
$1=c_2$

$y(t)=\sin(t)$

$x(t)=\sin(t)$