Phys 396 Homework Set 3 Solutions

1. a) $dt' = \gamma(dt - \frac{v}{c^2}dx)$

$dx' = \gamma(dx - v\,dt)$

$dy' = dy$

$dz' = dz$

NOW, PLUGGING THESE INTO THE LINE ELEMENT YIELDS...

$$ds'^2 = -c^2 dt'^2 + dx'^2 + dy'^2 + dz'^2$$

$$= -c^2\gamma^2(dt - \frac{v}{c^2}dx)^2 + \gamma^2(dx - v\,dt)^2 + dy^2 + dz^2$$

$$= -c^2\gamma^2(dt^2 - \cancel{2\frac{v}{c^2}dx\,dt} + \frac{v^2}{c^4}dx^2)$$

$$+ \gamma^2(dx^2 - \cancel{2v\,dx\,dt} + v^2dt^2) + dy^2 + dz^2$$

GROUPING LIKE TERMS...

$$ds'^2 = \gamma^2(-c^2 + v^2)dt^2 + \gamma^2(1 - \frac{v^2}{c^2})dx^2 + dy^2 + dz^2$$

$$= -c^2\gamma^2(1 - \frac{v^2}{c^2})dt^2 + \gamma^2(1 - \frac{v^2}{c^2})dx^2 + dy^2 + dz^2$$

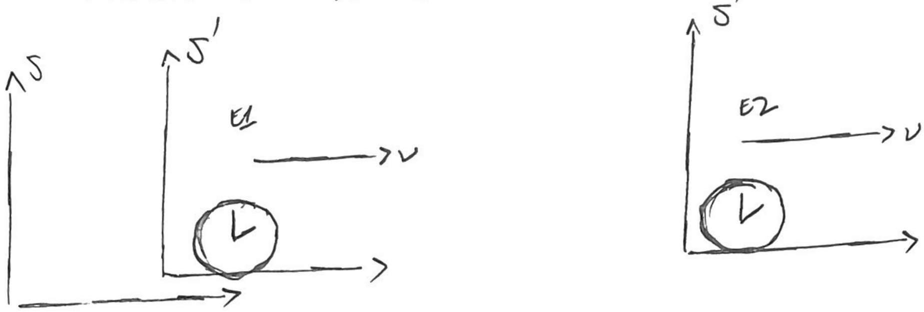
NOW

$$\gamma^2(1 - \frac{v^2}{c^2}) = \frac{1}{(1 - v^2/c^2)} \cdot (1 - v^2/c^2) = 1$$

SO

$$ds'^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = ds^2 \checkmark$$

b) Let S' BE THE REST FRAME OF THE CLOCK, THAT MOVES W/ SPEED v RELATIVE TO THE S FRAME...



Let E1 BE THE EVENT DESCRIBING COORDINATES OF THE CLOCK AT t_1, t_1'

Let E2 BE THE EVENT DESCRIBING COORDINATES OF THE CLOCK AT t_2, t_2'

NOTICE THAT $\Delta x' = 0 = x_2' - x_1'$

NOW

$$\Delta t = \gamma\left(\Delta t' + \frac{v}{c^2}\cancel{\Delta x'}^{\,0}\right)$$

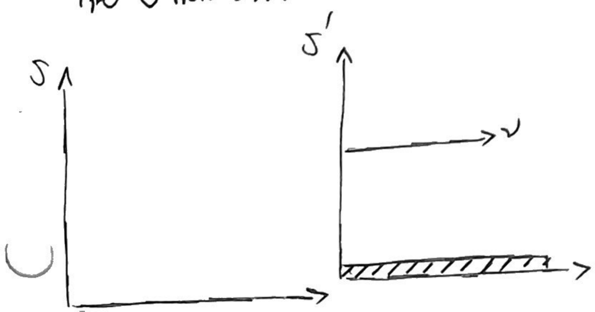
$$\Delta x = \gamma\left(\underbrace{\Delta x'}_{0} + v\Delta t'\right)$$

THE FIRST EQN YIELDS $\Delta t = \gamma \Delta t'$ SO $\boxed{\Delta t' = \frac{1}{\gamma}\Delta t = \sqrt{1 - v^2/c^2}\,\Delta t}$

SO

$\Delta t' < \Delta t$ - THE 'MOVING' CLOCK MEASURES A SHORTER TIME INTERVAL

c) Let S' BE THE REST FRAME OF AN OBJECT OF LENGTH L_*, WHICH MOVES W/ SPEED v RELATIVE TO THE S FRAME...



TO MEASURE THE LENGTH, THE S OBSERVER NOTES THE LEFT AND RIGHT ENDS AT THE SAME MOMENT IN TIME, RELATIVE TO THE S FRAME.

SO

$$\Delta t = 0$$

SO

$$\Delta t' = \gamma\left(\cancel{\Delta t}^{\,0} - \frac{v}{c^2}\Delta x\right)$$

$$\Delta x' = \gamma\left(\Delta x - v\cancel{\Delta t}^{\,0}\right)$$

THE SECOND EQN YIELDS...

$$\Delta x' = \gamma \Delta x$$

IF THE 2 EVENTS CORRESPOND TO THE LOCATIONS OF THE LEFT AND RIGHT ENDS OF THE OBJECT, THEN

$\Delta x' \equiv L_*$ - THE REST LENGTH OF OBJECT

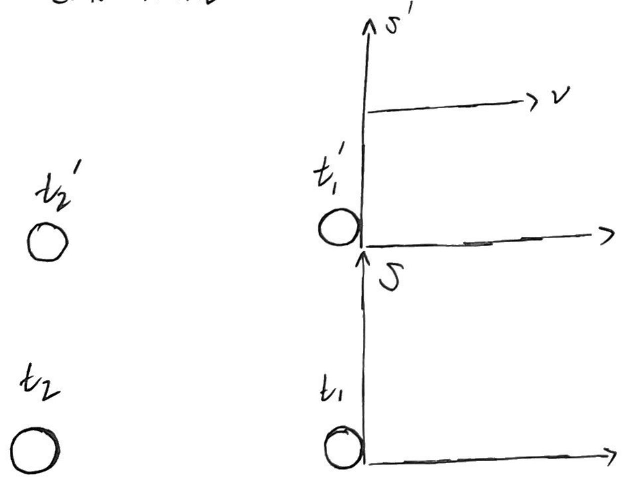
$\Delta x \equiv L$ - THE LENGTH OF THE OBJECT, AS MEASURED BY THE MOVING FRAME SO

$$L_* = \gamma L = \frac{L}{\sqrt{1-v^2/c^2}} \quad \therefore \left[L = \sqrt{1-v^2/c^2}\, L_*\right]$$

NOTICE: $L < L_*$ THE 'MOVING' FRAME MEASURES A SHORTER LENGTH

∴ MOVING OBJECTS ARE CONTRACTED

d) LET S' BE MOVING W/ SPEED v RELATIVE TO THE S FRAME. EACH IRF HAS A SET OF CLOCKS AT DIFFERENT LOCATIONS, BOTH AT REST RELATIVE TO ONE ANOTHER, AND BOTH SYNCHRONIZED IN THEIR OWN FRAME OF REFERENCE.



AT THE ORIGIN OF EACH COORDINATE SYSTEM LIES A CLOCK. BOTH OBSERVERS' SET THEIR ORIGIN CLOCKS EQUAL TO ONE ANOTHER THE MOMENT THEY ALIGN SO

$$t_1 = t_1' = 0$$

LET A SECOND CLOCK IN THE S' FRAME BE TRAILING THE ORIGIN CLOCK BY A REST DISTANCE D. (DISTANCE MEASURED IN S' FRAME)

THE S FRAME HAS A CLOCK AT EACH SPATIAL POSITION AND ONE OF THEM, CLOCK 2, ALIGNS W/ CLOCK 2' WHEN CLOCK 1 : 1' ALIGN.

NOW, ACCORDING TO THE S FRAME...

CLOCK 1 ALIGNS W/ 1' AT PRECISELY THE SAME TIME AS CLOCK 2 ALIGNS W/ 2' ∴ THESE EVENTS ARE SIMULTANEOUS SO

$$\Delta t = t_2 - t_1 = 0$$

$\Delta t' = \gamma(\cancel{\Delta t}_{\to 0} - \frac{v}{c^2}\Delta x) \quad \therefore \quad \Delta t' = -\frac{v}{c^2}\gamma\Delta x$

$\Delta x' = \gamma(\Delta x - v\cancel{\Delta t}_{\to 0}) \qquad \Delta x' = \gamma\Delta x$

PLUGGING THE 2ND INTO THE 1ST YIELDS..

$$\Delta t' = -\frac{v}{c^2}\Delta x' = +\frac{v}{c^2}D = t_2' - \cancel{t_1'}_{\to 0}$$ AS $t_1 = t_1' = 0$

SO

$\Delta x' = x_2' - \cancel{x_1'}^{\to 0} = -D$ SINCE $x_2' < 0$ $\qquad t_2' = +\frac{v}{c^2}D$

THE POSITIVE SIGN INDICATES THAT CLOCK 2' READS A LATER TIME THAN CLOCK 1' ∝

LEADING CLOCKS LAG BY

$$\boxed{\Delta t' = \frac{v}{c^2}D}\ \checkmark$$

2. ACCORDING TO THE S FRAME...

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0 \quad \text{WHERE } \vec{E} = \vec{E}(x,y,z,t)$$

ACCORDING TO THE S' FRAME, WHICH MOVES AT A SPEED V RELATIVE TO THE S FRAME, THIS OBSERVER MEASURES AN $\vec{E}$-FIELD WHERE $\vec{E} = \vec{E}(x',y',z',t')$

WHERE

$x' = x'(x,t)$

$t' = t'(x,t)$ VIA THE LORENTZ BOOSTS WHICH TAKE THE FORM

$$t' = \gamma(t - \frac{v}{c^2}x)$$

$$x' = \gamma(x - vt)$$

WE NEED TO TRANSFORM THE DERIVATIVES BY USING THE CHAIN RULE...

$$\frac{\partial}{\partial t}\vec{E}(x',y',z',t') = \frac{\partial \vec{E}}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial \vec{E}}{\partial t'}\frac{\partial t'}{\partial t}$$

BUT $\frac{\partial x'}{\partial t} = -\gamma v$; $\frac{\partial t'}{\partial t} = \gamma$

SO

$$\frac{\partial \vec{E}}{\partial t} = -\gamma v \frac{\partial \vec{E}}{\partial x'} + \gamma \frac{\partial \vec{E}}{\partial t'} = \gamma\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)\vec{E}$$

A SECOND TIME-DERIVATIVE TAKES THE FORM...

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial \vec{E}}{\partial t}\right) = \gamma^2\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)\vec{E} = \gamma^2\left(\frac{\partial^2}{\partial t'^2} - 2v\frac{\partial^2}{\partial x'\partial t'} + v^2\frac{\partial^2}{\partial x'^2}\right)\vec{E}$$

NOW TRANSFORM THE SPATIAL DERIVATIVES IN AN ANALOGOUS WAY...

$$\frac{\partial \vec{E}(x',y',z',t')}{\partial x} = \frac{\partial \vec{E}}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial \vec{E}}{\partial t'}\frac{\partial t'}{\partial x}$$

BUT $\frac{\partial x'}{\partial x} = \gamma$; $\frac{\partial t'}{\partial x} = -\frac{\gamma v}{c^2}$

SO

$$\frac{\partial \vec{E}}{\partial x} = \gamma\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)\vec{E}$$

A SECOND X-DERIVATIVE TAKES THE FORM..

$$\frac{\partial^2}{\partial x^2}\vec{E} = \frac{\partial}{\partial x}\left(\frac{\partial \vec{E}}{\partial x}\right) = \gamma^2\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)\vec{E}$$

$$= \gamma^2\left(\frac{\partial^2}{\partial x'^2} - 2\frac{v}{c^2}\frac{\partial^2}{\partial t'\partial x'} + \frac{v^2}{c^4}\frac{\partial^2}{\partial t'^2}\right)\vec{E}$$

NOW

$$\frac{\partial}{\partial y}\vec{E} = \frac{\partial \vec{E}}{\partial y'}\frac{\partial y'}{\partial y} = \frac{\partial \vec{E}}{\partial y'} \qquad \text{ALSO} \qquad \frac{\partial^2}{\partial y^2}\vec{E} = \frac{\partial^2}{\partial y'^2}\vec{E}$$

LIKEWISE

$$\frac{\partial}{\partial z}\vec{E} = \frac{\partial \vec{E}}{\partial z'}\frac{\partial z'}{\partial z} = \frac{\partial \vec{E}}{\partial z} \qquad \text{ALSO} \qquad \frac{\partial^2}{\partial z^2}\vec{E} = \frac{\partial^2}{\partial z'^2}\vec{E}$$

NOW, PUTTING IT ALL TOGETHER...

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{E} = 0$$

BECOMES

$$0 = \gamma^2\left(\frac{\partial^2}{\partial x'^2} - 2\frac{v}{c^2}\frac{\partial^2}{\partial t'\partial x'} + \frac{v^2}{c^4}\frac{\partial^2}{\partial t'^2}\right)\vec{E} + \frac{\partial^2}{\partial y'^2}\vec{E} + \frac{\partial^2}{\partial z'^2}\vec{E} - \frac{1}{c^2}\gamma^2\left(\frac{\partial^2}{\partial t'^2} - 2v\frac{\partial^2}{\partial x'\partial t'} + v^2\frac{\partial^2}{\partial x'^2}\right)\vec{E}$$

NOW GROUPING LIKE TERMS,...

$$\underbrace{\gamma^2\left(1 - \frac{v^2}{c^2}\right)}_{1}\frac{\partial^2}{\partial x'^2}\vec{E} - \gamma\underbrace{\left(\cancel{-\frac{2v}{c^2} + \frac{2v}{c^2}}\right)}_{0}\frac{\partial^2}{\partial t'\partial x'}\vec{E} + \gamma^2\underbrace{\left(\frac{v^2}{c^4} - \frac{1}{c^2}\right)}_{-\frac{1}{c^2}}\frac{\partial^2}{\partial t'^2}\vec{E} + \frac{\partial^2}{\partial y'^2}\vec{E} + \frac{\partial^2}{\partial z'^2}\vec{E} = 0$$

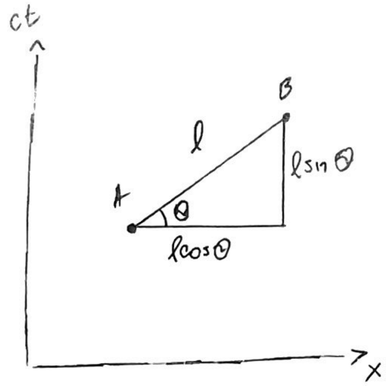
SINCE $\gamma^2\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{1 - v^2/c^2}\left(1 - v^2/c^2\right) = 1$

BECOMES.

$$\frac{\partial^2}{\partial x'^2}\vec{E} + \frac{\partial^2}{\partial y'^2}\vec{E} + \frac{\partial^2}{\partial z'^2}\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\vec{E} = \nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\vec{E} = 0$$

3.

a)



b) $\Delta s^2 = -(c\Delta t)^2 + \Delta x^2$

$$= -(l^2\sin^2\theta) + l^2\cos^2\theta$$

$$\Delta s^2 = l^2(\cos^2\theta - \sin^2\theta)$$

c) SETTING $l=1$,

$$\Delta s^2 = +\cos^2\theta - \sin^2\theta$$

d) $\underline{-1} \leq \Delta s^2 \leq \underline{1}$

SMALLEST MAGNITUDE : $\theta = \pi/4$ OR 45° WHERE $\Delta s^2 = 0$

MAXIMUM VALUE : $\theta = 0$ WHERE $|\Delta s| = 1$.

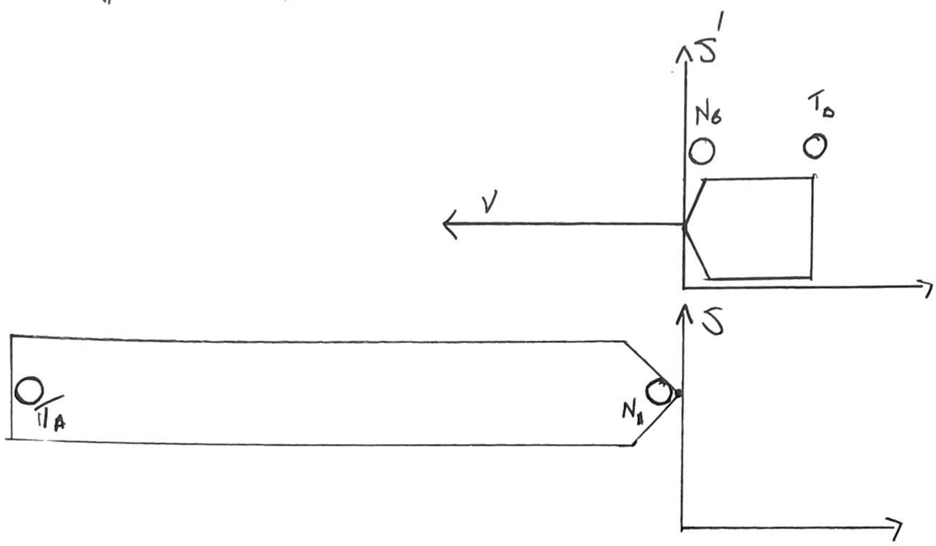
MAXIMUM VALUE : $\theta = \pi/2$ or 90° WHERE $|\Delta s| = 1$.

SCALE: 100m : 1,5cm

4. $v = -\frac{12}{13}c$

$L_{*A} = 598m$, $L_{*B} = 299m$

a)



AS WE ANALYZE THIS PROBLEM FROM THE S FRAME, AND AS BOTH S FRAME CLOCKS ARE SYNCHRONIZED,

BOTH
$$t_{N_A} = t_{T_A} = 0$$

THE NOSE B CLOCK SETS IS SET TO ZERO WHEN PASSING THE NOSE A CLOCK,
$$t'_{N_B} = 0$$

NOW, ALTHOUGH THE N_B : T_B CLOCKS ARE SYNCHRONIZED IN THE S' FRAME OF REFERENCE. TO GET THE TIME ON THE T_B CLOCK, TAKES IT'S READING AS THE 2ND EVENT SO THAT

$$t_2 = t_1 \;,\; x'_1 = 0 \;,\; x'_2 = L_{*B} \;,\; v = \frac{-12}{13}c$$

SO

$$t'_2 = \gamma(\cancel{t_2} - \frac{v}{c^2}x_2) \quad \therefore \quad t'_2 = -\gamma\frac{v}{c^2}x_2 \quad \therefore \quad t'_2 = -\frac{v}{c^2}x'_2$$

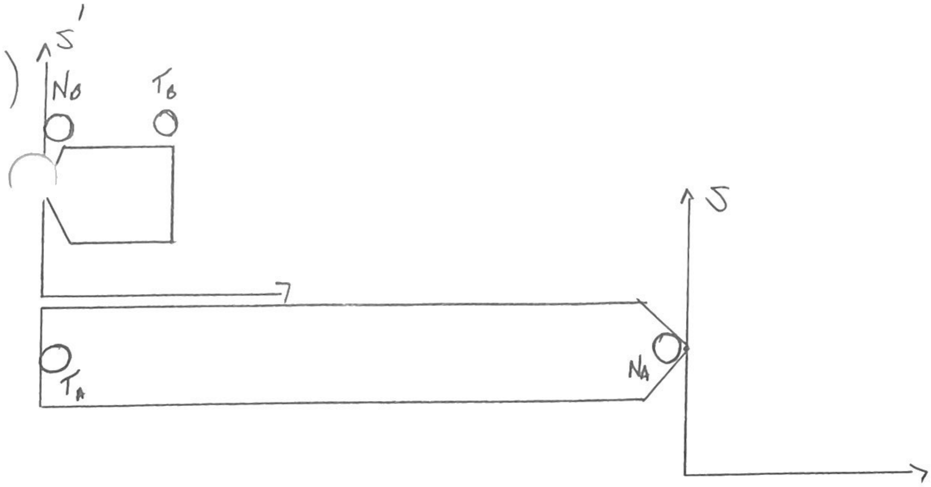
$$x'_2 = \gamma(x_2 - v\cancel{t_2}) \qquad x'_2 = \gamma x_2 \qquad = -\frac{v}{c^2}L_{*B}$$

PLUGGING OUR VALUES IN YIELD..

$$t'_2 = -\frac{(-\frac{12}{13}c)}{c^2}(299m) = 276\frac{m}{c} \times \frac{1c}{3.0\times10^8 m/s} = 9.2\times10^{-7}s$$

$$\therefore \left[t'_2 = 276m/c = 9.2\times10^{-7}s\right] \checkmark$$

)



c) LET E1 BE THE EVENT WHEN THE NOSE OF B REACHES THE NOSE OF A

LET E2 BE THE EVENT WHEN THE NOSE OF B REACHES THE TAIL OF A.

NOW $\Delta x = x_2 - x_1 = -L_{*A} - 0 = -L_{*A}$

$\Delta x' = x_2' - x_1' = 0$ (BOTH EVENTS OCCUR AT THE SAME x' COORDINATE)

SO

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$$

$$\Delta x' = \gamma(\Delta x - v\Delta t) = 0 \qquad \text{SO} \quad \Delta t = \frac{\Delta x}{+v} = \frac{-L_{*A}}{v} = \frac{-(598\,m)}{-\frac{12}{13}c}$$

$$= 648\,m/c \times \left(\frac{1c}{3.0\times 10^8\,m/s}\right)$$

$$\boxed{\Delta t = 648\,m/c = 2.16\times 10^{-6}\,s}$$ — THE TIME INTERVAL THAT TICKS BY ON THE S FRAME'S WATCH

d) PLUGGING $\Delta x = v\Delta t$ INTO THE FIRST EQN YIELDS.

$$\Delta t' = \gamma\left(\Delta t - \frac{v^2}{c^2}\Delta t\right) = \frac{1}{\sqrt{1 - v^2/c^2}}(1 - v^2/c^2)\Delta t = \sqrt{1 - v^2/c^2}\,\Delta t$$

SO

$$\Delta t' = (648\,m/c)\sqrt{1 - (12/13)^2} = (648\,m/c)\cdot\frac{5}{13} = 249\,m/c \times \left(\frac{1c}{3.0\times 10^8\,m/s}\right)$$

$$\boxed{\Delta t' = 249\,m/c = 8.31\times 10^{-7}\,m/c}$$

$$\boxed{\begin{aligned} t_{T_{A2}} &= t_{N_{A2}} = 648\,m/c \\ t'_{N_{B2}} &= t'_{N_{B1}} + \Delta t' = 249\,m/c \\ t'_{T_{B2}} &= t'_{T_{A1}} + \Delta t' = 276\,m/c + 249\,m/c = 525\,m/c \end{aligned}}$$

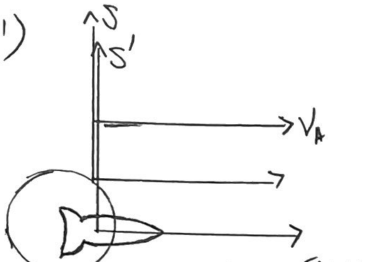
5 Let S be the Earth/star rest frame, let S' be spaceship A's rest frame, which moves at a speed $V_A = \frac{5}{13}c$ relative to S. Let S'' be the rest frame of spaceship B, which moves at speed $V_B = \frac{12}{13}c$ relative to S.

Let E1 be the departure of SSA from Earth

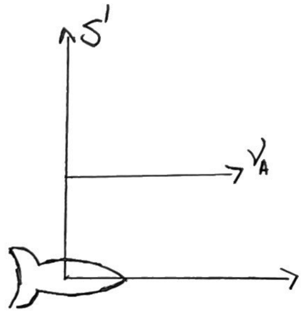
Let E2 be the departure of SSB from Earth

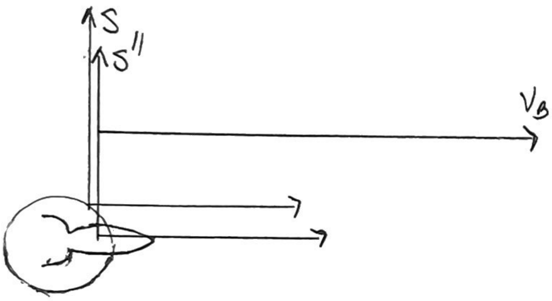
Let E3 be the arrival of SSA & SSB at the distant star

1)



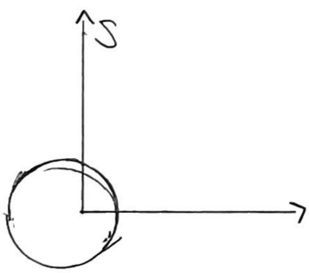
E1: SSA departs Earth

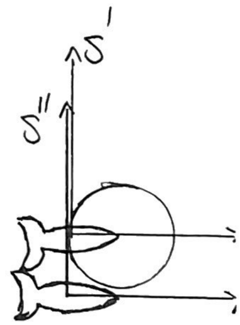




E2: SSB departs Earth

E4: Location of SSA when SSB departs Earth





E3: SSA & SSB arrive at distant star

Notice:

$\Delta t_{12} = t_2 - t_1 = 7\,\mu s$

$\Delta X_{13} = \Delta X_{23}$ (the rest length between the Earth & star)

$\Delta t_{13} = \Delta t_{23} + \Delta t_{12} = \Delta t_{23} + 7\,\mu s$

$\Delta X'_{13} = \Delta X''_{23} = 0$ (as both events occur at the same spatial coordinate)

Now, write down the Lorentz boosts and inverse Lorentz boosts between E1 & E2 and between E2 & E3

Let $\gamma_A \equiv \dfrac{1}{\sqrt{1 - v_A^2/c^2}}$; $\gamma_B \equiv \dfrac{1}{\sqrt{1 - v_B^2/c^2}}$

$$\Delta t_{13} = \gamma_A \left(\Delta t_{13}' + \frac{v_A}{c^2} \cancel{\Delta x_{13}'}\right) \quad \Delta x_{13}' \to 0$$

$$\Delta x_{13} = \gamma_A \left(\cancel{\Delta x_{13}'} + v_A \Delta t_{13}'\right)$$

$$\Delta t_{13}' = \gamma_A \left(\Delta t_{13} - \frac{v_A}{c^2} \Delta x_{13}\right)$$

$$\cancel{\Delta x_{13}'} = \gamma_A \left(\Delta x_{13} - v_A \Delta t_{13}\right)$$

$$\Delta t_{23} = \gamma_B \left(\Delta t_{23}'' + \frac{v_B}{c^2} \cancel{\Delta x_{23}''}\right) \quad \Delta x_{23}'' \to 0$$

$$\Delta x_{23} = \gamma_B \left(\cancel{\Delta x_{23}''} + v_B \Delta t_{23}''\right)$$

$$\Delta t_{23}'' = \gamma_B \left(\Delta t_{23} - \frac{v_B}{c^2} \Delta x_{23}\right)$$

$$\cancel{\Delta x_{23}''} = \gamma_B \left(\Delta x_{23} - v_B \Delta t_{23}\right)$$

WHICH SIMPLIFY TO...

$$\Delta t_{13} = \gamma_A \Delta t_{13}'$$

$$\Delta x_{13} = \gamma_A v_A \Delta t_{13}'$$

$$\Delta t_{13}' = \gamma_A \left(\Delta t_{13} - \frac{v_A}{c^2} \Delta x_{13}\right)$$

$$\Delta x_{13} = v_A \Delta t_{13}$$

$$\Delta t_{23} = \gamma_B \Delta t_{23}''$$

$$\Delta x_{23} = \gamma_B v_B \Delta t_{23}''$$

$$\Delta t_{23}'' = \gamma_B \left(\Delta t_{23} - \frac{v_B}{c^2} \Delta x_{23}\right)$$

$$\Delta x_{23} = v_B \Delta t_{23}$$

---

2) NOW $\Delta x_{13} = \Delta x_{23}$, SO THE LAST SET OF EQNS GIVE..

$$v_A \Delta t_{13} = v_B \Delta t_{23} = v_A (\Delta t_{23} + 7\text{ yrs})$$

USING $\Delta t_{13} = \Delta t_{23} + 7\text{ yrs}$

NOW SOLVE FOR Δt_{23}...

$$(v_B - v_A)\Delta t_{23} = v_A \cdot 7\text{ yrs}$$

$$\Delta t_{23} = \frac{v_A}{(v_B - v_A)} \cdot 7\text{ yrs} = \frac{\frac{5}{13}c}{\left(\frac{12}{13}c - \frac{5}{13}c\right)} \cdot 7\text{ yrs} = \frac{5/13}{7/13} \cdot 7\text{ yrs} = 5\text{ yrs}$$

SO

$$\Delta t_{13} = \Delta t_{23} + 7\text{ yrs} = 12\text{ yrs}$$

$\therefore \left[\Delta t_{13} = 12\text{ yrs},\ \Delta t_{23} = 5\text{ yrs}\right]$

IT TAKES SSA 12 YRS TO COMPLETE THE JOURNEY, IT TAKES SSB 5 YRS TO COMPLETE THE JOURNEY, RELATIVE TO THE EARTH-STAR FRAME.

c) $\Delta X_{13} = V_A \Delta t_{13} = \left(\frac{5}{13} c\right)\left(12 \mu s\right) = \frac{60}{13} c \cdot \mu s$

$\Delta X_{23} = V_B \Delta t_{23} = \left(\frac{12}{13} c\right)\left(5 \mu s\right) = \frac{60}{13} c \cdot \mu s$ ✓

so

$$\left[\Delta X_{13} = \Delta X_{23} = \frac{60}{13} c \cdot \mu s\right]$$

d. NOW, let E4 BE THE SPACETIME COORDINATES OF SSA WHEN SSB DEPARTS EARTH

$\Delta t_{14}' = \gamma_A \left(\Delta t_{14} - \frac{V_A}{c^2} \Delta X_{14}\right)$

$\Delta X_{14}' = \gamma_A \left(\Delta X_{14} - V_A \Delta t_{14}\right)$, $\Delta X_{14}' \to 0$

$\Delta t_{14} = \gamma_A \left(\Delta t_{14}' + \frac{V_A}{c^2} \cancel{\Delta X_{14}'}\right)$, $\Delta X_{14}' \to 0$

$\Delta X_{14} = \gamma_A \left(\cancel{\Delta X_{14}'} + V_A \Delta t_{14}'\right)$, $\Delta X_{14}' \to 0$

NOW • $\Delta X_{14}' = 0$ (E1 : E4 OCCUR AT THE SAME X COORDINATE)

• $\Delta t_{14} = \Delta t_{12}$ (SAME TIME INTERVAL ACCORDING TO THE S FRAME)

USING OUR 2 BULLETS THE LORENTZ BOOSTS : INVERSE LORENTZ BOOSTS BECOME ...

$\Delta t_{14}' = \gamma_A \left(\Delta t_{12} - \frac{V_A}{c^2} \Delta X_{14}\right)$

$\Delta X_{14} = V_A \Delta t_{12}$

$\Delta t_{12} = \gamma_A \Delta t_{14}'$

$\Delta X_{14} = \gamma_A V_A \Delta t_{14}'$

so, THE 3RD EQN YIELDS..

$\Delta t_{14}' = \frac{1}{\gamma_A} \Delta t_{12} = \sqrt{1 - V_A^2/c^2}\, \Delta t_{12} = \sqrt{1-(5/13)^2}\left(7 \mu s\right) = \frac{12}{13} \cdot 7 \mu s$

$$\left[\Delta t_{14}' = \frac{84}{13} \mu s\right]$$

e) WE NEED $\Delta t_{13}'$ ..

FROM EARLIER ...

$$\Delta t_{13}' = \frac{1}{\gamma_A} \Delta t_{13} = \sqrt{1 - v_A^2/c^2}\, \Delta t_{13} = \sqrt{1 - (5/13)^2}\,(12 \text{ yrs}) = \frac{12}{13} \cdot 12 \text{ yrs}$$

$$\boxed{\Delta t_{13}' = \frac{144}{13} \text{ yrs}}$$

f) WE NEED $\Delta t_{23}''$ ...

FROM EARLIER ..

$$\Delta t_{23}'' = \frac{1}{\gamma_B} \Delta t_{23} = \sqrt{1 - v_B^2/c^2}\, \Delta t_{23} = \sqrt{1 - (12/13)^2}\,(5 \text{ yrs}) = \frac{5}{13} \cdot 5 \text{ yrs}$$

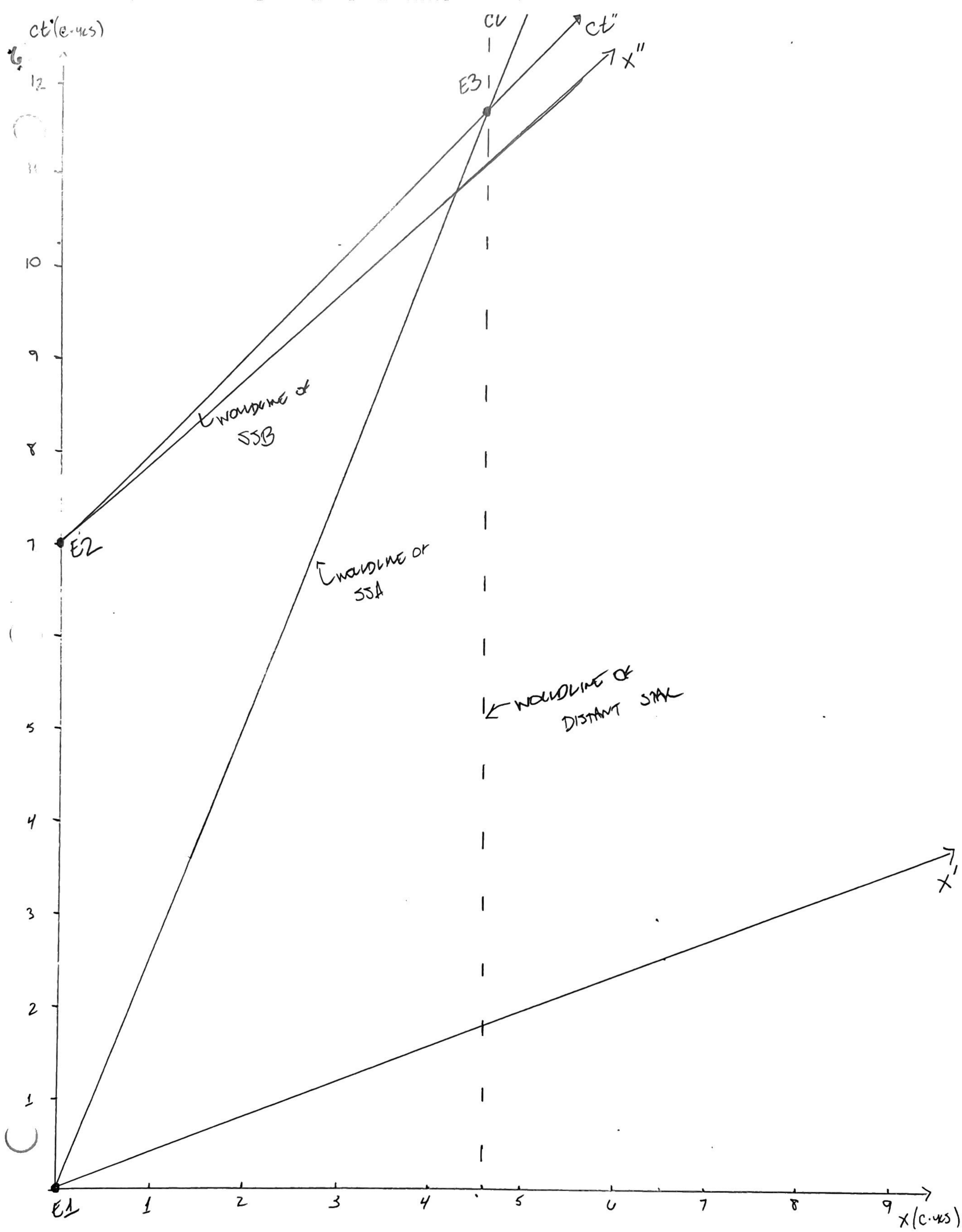
$$\boxed{\Delta t_{23}'' = \frac{25}{13} \text{ yrs}}$$

g) $\ell' = v_A \Delta t_{13}' = \left(\frac{5}{13} c\right)\left(\frac{144}{13} \text{ yrs}\right) = \frac{720}{169}\ c \cdot \text{yrs}$

$$\boxed{\ell' = \frac{720}{169}\ c \cdot \text{yrs}}$$

h) $\ell'' = v_B \Delta t_{23}'' = \left(\frac{12}{13} c\right)\left(\frac{25}{13} \text{ yrs}\right) = \frac{300}{169}\ c \cdot \text{yrs}$

$$\boxed{\ell'' = \frac{300}{169}\ c \cdot \text{yrs}}$$



ct'(c·yrs)
ct''
x''
x'
x(c·yrs)
E1
E2
E3
WORLDLINE OF SSB
WORLDLINE OF SSA
WORLDLINE OF DISTANT STAR

b) SINCE SPACESHIP A TRAVELS AT A SPEED

$V_A = \frac{5}{13}c = \frac{\Delta x}{\Delta t}$ SO $\frac{c\Delta t}{\Delta x} = \frac{13}{5}$ - THE WORLDLINE OF SSA HAS A SLOPE OF $\frac{13}{5}$ RELATIVE TO THE S FRAME.

SINCE SPACESHIP B TRAVELS AT A SPEED

$V_B = \frac{12}{13}c = \frac{\Delta x}{\Delta t}$ THEN $\frac{c\Delta t}{\Delta x} = \frac{13}{12}$ - THE WORLDLINE OF SSB HAS A SLOPE OF $\frac{13}{12}$ RELATIVE TO THE S FRAME, w/ A ct INTERCEPT AT $ct = 7$ c-yrs

THE WORLDLINES OF SSA & SSB CORRESPOND TO THE ct' & ct'' AXES.

• THE x' AXIS HAS A SLOPE OF $\frac{5}{13}$; THE x'' AXIS HAS A SLOPE OF $\frac{12}{13}$