Ch 7.3 Gauss-Seidel Iteration

In Ch 2, we learned how to solve $f(x)=0$ using fixed-point iteration for a suitably chosen $g(x)$ and initial guess $x=x_0$.

In these notes we briefly see how fixed point iteration can be used to solve a system of linear equations.

Example Consider the system below, whose solution is $(1,1)$.

$$\begin{aligned} 10x - 2y &= 8 \\ -3x + 12y &= 9 \end{aligned}$$

} Diagonally-dominant system, because $a_{11}=10$ & $a_{22}=12$ are relatively large.

① Solve r_1 for x and r_2 for y:

$$x = g_1(y) = \frac{8+2y}{10}$$

$$y = g_2(x) = \frac{9+3x}{12}$$

② Iterate as follows:

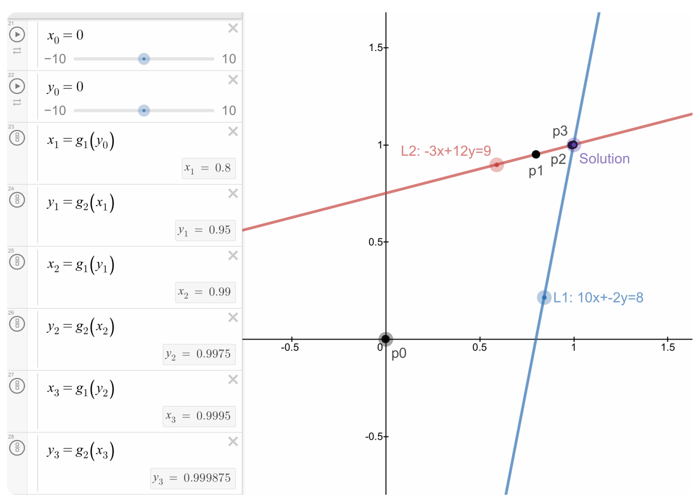
(x_0, y_0) = initial guess

$$(x_1, y_1) = (g_1(y_0), g_2(x_1))$$

$$(x_2, y_2) = (g_1(y_1), g_2(x_2))$$

$$\vdots$$

$$(x_n, y_n) = (g_1(y_{n-1}), g_2(x_n))$$



The iterates $(x_n, y_n) \longrightarrow (x,y)$ = solution of system of equations for diagonally-dominant systems. These systems commonly occur for partial differential equations (multivariate equations with derivatives).

③ Perform iteration out to $n=2$, using $(x_0, y_0) = (0,0)$

$$10x - 2y = 8 \quad \Rightarrow \quad x = g_1(y) = \frac{8+2y}{10}$$

$$-3x + 12y = 9 \quad \Rightarrow \quad y = g_2(x) = \frac{9+3x}{12}$$

$x_0 = 0$

$y_0 = 0$

$x_1 = g_1(y_0) =$

$y_1 = g_2(x_1) =$

$x_2 = g_1(y_1) =$

$y_2 = g_2(x_2) =$