Phys 131 2016

Lecture 8

~~Alerts~~ Thurs: Seminar 12:30
WS 117

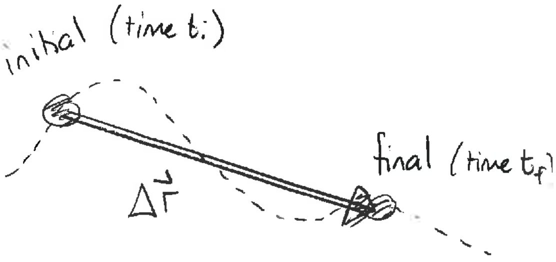
Fri: Lecture

Mon: Warm Up 4.

Motion in Two Dimensions

We have developed a language, kinematics, for describing motion in one dimension and must now extend this to describe motion in two and three dimensions.

Position + Velocity in Two Dimensions



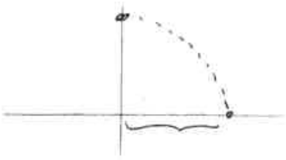
The displacement of an object moving in two dimensions can be described by a displacement vector from the objects location at an earlier time t_i to its location at a later time t_f. This is denoted $\Delta \vec{r}$. Then the average velocity of the object between these moments is:

Average velocity from time t_i to t_f is a vector

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

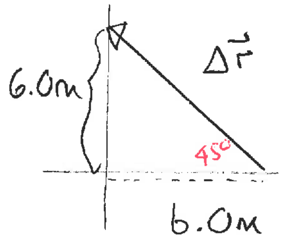
Example: An object moves along a circular path with radius 6.0m taking 3.0s to move from $\theta = 0°$ to $\theta = 90°$.

Determine the displacement vector (using components) and the average velocity



Answer: The displacement is as illustrated. By inspection

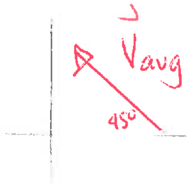
$$\Delta \vec{r} = -6.0\text{m}\,\hat{\imath} + 6.0\text{m}\,\hat{\jmath}$$



The average velocity is

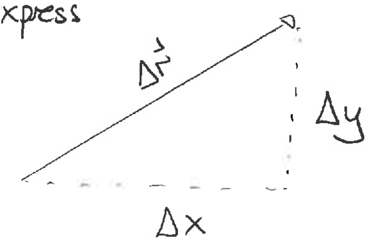
$$\vec{V}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{1}{3.0\text{s}}\left(-6.0\text{m}\,\hat{\imath} + 6.0\text{m}\,\hat{\jmath}\right)$$

$$= -2.0\text{m/s}\,\hat{\imath} + 2.0\text{m/s}\,\hat{\jmath}$$



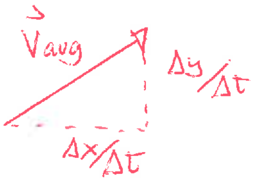
Note that the average velocity vector is in the same direction as the displacement vector. In general one can express

$$\Delta \vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath}$$



and

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$$



Again we will want to consider cases where the velocity changes with time. To describe these effectively we define:

The instantaneous velocity vector is

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \equiv \frac{d\vec{r}}{dt}$$

Again in terms of components:

$$\vec{v} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \hat{\imath} + \frac{\Delta y}{\Delta t} \hat{\jmath} \right) = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \right) \hat{\imath} + \lim_{\Delta t \to 0} \left(\frac{\Delta y}{\Delta t} \right) \hat{\jmath}$$

$$\Rightarrow \quad \vec{v} = \frac{dx}{dt} \hat{\imath} + \frac{dy}{dt} \hat{\jmath}$$

We will typically denote the components of velocity by v_x, v_y. Thus

$$\boxed{\vec{v} = v_x \hat{\imath} + v_y \hat{\jmath} \quad \text{where} \quad v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt}}$$

A geometrical construction (Fig 4.1) shows that.

> The direction of $\vec{v}$ is tangent to the trajectory followed by the object and points along the direction of motion.

Demo: Lady bug Motion 2D

Select - show velocity vector

- trace = line

Use ellipse + observe direction + magnitude of velocity

A further definition is that

> The speed of an object = magnitude of instantaneous velocity

Quiz 1

Acceleration in Two Dimensions

Acceleration again will describe the rate at which velocity changes. Since velocity is a vector, acceleration will also be a vector.

Acceleration is a vector which describes the rate at which the velocity vector changes with time.

Then:

The average acceleration from time t_i to t_f is:

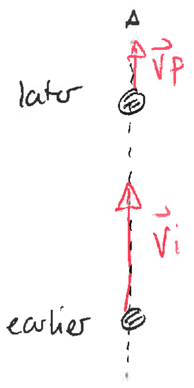
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

where $\vec{v}_i$ = velocity at time t_i and $\vec{v}_f$ = velocity at time t_f

Note that this involves subtracting vectors. For motion in a line- this can be relatively straightforward.

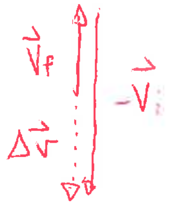
Example: A ball is thrown upwards. After it leaves the hand it slows. During this period determine the direction of the acceleration vector.

Answer: ① Sketch trajectory + velocities



② get $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$= \vec{v}_f + (-\vec{v}_i)$



③ $\Delta\vec{v}$ is down. Since Δt is positive $\frac{\Delta\vec{v}}{\Delta t}$ is down

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

In terms of components

$$\vec{a} = \underset{\substack{\uparrow \\ \text{zero here}}}{a_x}\hat{i} + \underset{\substack{\uparrow \\ \text{negative here}}}{a_y}\hat{j}$$

In two dimensions, one must be careful with vector subtraction.

Quiz 2

Quiz 3

Quiz 4