Electromagnetic Theory II: Homework 10

Due: 12 March 2021

1 Reflection and transmission of waves

Consider waves traveling in one dimension along a medium with a single discontinuity; to the left the wave speed is v_1 and to the right v_2. Consider a wave that is incident from the left.

a) Express the amplitudes of the reflected and transmitted waves in terms of the amplitude of the incident waves and the velocities of the waves (rather than wavenumbers).

b) Suppose that the wave speed to the left is $100\,\mathrm{m/s}$ and to the right $120\,\mathrm{m/s}$. Determine the ratio of the amplitude of the reflected wave to the incident wave. Repeat this for the transmitted wave.

c) Suppose that the wave speed to the left is $100\,\mathrm{m/s}$ and to the right $80\,\mathrm{m/s}$. Determine the ratio of the amplitude of the reflected wave to the incident wave. Repeat this for the transmitted wave.

d) Determine the ratio of the wave speeds so that the amplitude of the reflected wave is half that of the incident wave.

2 Energy flow and reflection and transmission

Two strings meet at a point. To the left of this point wavespeed is v_1 and to the right it is v_2. Suppose that a wave with angular frequency ω is incident from the left.

a) Show that the rate at which energy is reflected is always less than the rate at which energy is incident.

b) Show that the rate at which energy is transmitted is always less than the rate at which energy is incident.

c) Show that the rate at which energy is incident on the junction between the strings equals the rate at which it leaves the junction.

3 Polarization

Consider complex representations of vector waves,

$$\tilde{\mathbf{f}} = \tilde{\mathbf{A}}e^{i(kz-\omega t)}$$

where

$$\tilde{\mathbf{A}} = \tilde{A}_x\hat{\mathbf{x}} + \tilde{A}_y\hat{\mathbf{y}}$$

and $\tilde{A}_x = A_x e^{i\delta_x}$, $\tilde{A}_y = A_y e^{i\delta_y}$ with A_x, A_y and δ_x, δ_y all real.

a) Suppose that $\tilde{A}_x = Ae^{i\pi/2}$ and $\tilde{A}_y = A$. Determine a real expression for the wave and describe how the vector evolves at $z = 0$.

b) Suppose that $\tilde{A}_x = A$ and $\tilde{A}_y = Ae^{i\pi/2}$. Determine a real expression for the wave and describe how the vector evolves at $z = 0$.

c) Suppose that $\tilde{A}_x = Ae^{i\delta}$ and $\tilde{A}_y = Be^{i\delta}$ where the two phases are identical. Describe the polarization of this wave.