Chapter 1

$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$: $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$: $\langle j \rangle = \frac{\sum j N(j)}{N} = \sum_{j=0}^{\infty} j P(j)$: $\langle j^2 \rangle = \sum_{j=0}^{\infty} j^2 P(j)$

$\langle f(j) \rangle = \sum_{j=0}^{\infty} f(j) P(j)$: $\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$: $\sigma^2 = \langle (\Delta x)^2 \rangle$: $P_{ab} = \int_a^b \rho(x)\, dx$

$\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi(x,t)|^2 dx$: $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\Psi(x,t)|^2 dx$: $\langle p \rangle = m\frac{d\langle x \rangle}{dt} = -i\hbar \int \left(\Psi^* \frac{\partial \Psi}{\partial x}\right) dx$

$\langle Q(x,p) \rangle = \int \Psi^* Q\left(x, \frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\, dx$: $\langle p^2 \rangle = \int \Psi^* \left(-\hbar^2 \partial^2/\partial x^2\right)\Psi\, dx$: $\langle T \rangle = \langle p^2/2m \rangle$

$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda}$: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$

Chapter 2

$\Psi(x,t) = \psi(x)\phi(t)$: $\Psi(x,t) = \psi(x) e^{-iEt/\hbar}$: $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$: $H(x,p) = \frac{p^2}{2m} + V(x)$

$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$: $\hat{H}\psi = E\psi$: $\langle H \rangle = E$: $\langle H^2 \rangle = E^2$: $\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}$

∞ Square well $V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq a \\ \infty, & \text{otherwise} \end{cases}$, $E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$

$\int \psi_m(x)^* \psi_n(x)\, dx = \delta_{mn}$: $\delta_{mn} = \begin{cases} 0, & \text{if } m \neq n \\ 1, & \text{if } m = n \end{cases}$: $c_n = \int \psi_n(x)^* f(x)\, dx \rightarrow f(x) = \Psi(x,0)$

$P_n = |c_n|^2$: $\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n$: H.O. $V(x) = \frac{1}{2}m\omega^2 x^2$: $a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$: $[A,B] = AB - BA$

$[x,p] = i\hbar$: $\psi_0(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\, e^{-\frac{m\omega}{2\hbar}x^2}$: $\psi_n(x) = A_n (a_+)^n \psi_0(x)$: $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$: $a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$,

$a_-\psi_n = \sqrt{n}\,\psi_{n-1}$, $a_-\psi_0 = 0$: $a_+a_-\psi_n = n\psi_n$ $\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n \psi_0$: $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$: $p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$

$(a_+)^2\psi_n \propto \psi_{n+2}$: $(a_-)^2\psi_n \propto \psi_{n-2}$: $a_-a_+\psi_n = (n+1)\psi_n$

MATH

Trig: $\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

Integrals: $\int_0^{\infty} x^n e^{-x/a} dx = n!\, a^{n+1}$

$\int_0^{\infty} x^{2n} e^{-x^2/a^2} dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$

$\int_0^{\infty} x^{2n+1} e^{-x^2/a^2} dx = \frac{n!}{2} a^{2n+2}$

SOLDES:

$ay'' + by' + cy = 0$ } $\Delta = b^2 - 4ac$

$\Delta > 0$: $r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $y_h = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$\Delta = 0$: $r = -\frac{b}{2a}$: $y_h = c_1 e^{rt} + c_2 t e^{rt}$

$\Delta < 0$: $r_1, r_2 = \alpha \pm \beta i$: $\alpha = \frac{-b}{2a}$, $\beta = \frac{\sqrt{4ac - b^2}}{2a}$: $y_h = e^{\alpha t}(c_1 \cos\beta t + c_2 \sin\beta t)$

(i) k	(i) A_0
(ii) $P_n(t)$	(ii) $A_n(t)$
(iii) Ce^{kt}	(iii) $A_0 e^{kt}$
(iv) $C\cos\omega t + D\sin\omega t$	(iv) $A_0\cos\omega t + B_0\sin\omega t$
(v) $P_n(t)e^{kt}$	(v) $A_n(t)e^{kt}$
(vi) $P_n\cos\omega t + Q_n(t)\sin\omega t$	(vi) $A_n(t)\cos\omega t + B_n(t)\sin\omega t$
(vii) $Ce^{kt}\cos\omega t + De^{kt}\sin\omega t$	(vii) $A_0 e^{kt}\cos\omega t + B_0 e^{kt}\sin\omega t$
(viii) $P_n(t)e^{kt}\cos\omega t + Q_n(t)e^{kt}\sin(\omega t)$	(viii) $A_n(t)e^{kt}\cos\omega t + B_n(t)e^{kt}\sin\omega t$