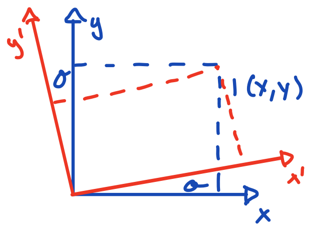
Spacetime

How are the primed coordinates related?



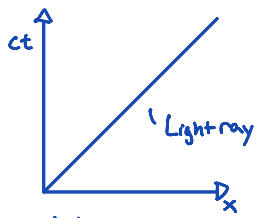
$x' = (\cos\theta)x + (\sin\theta)y$
$y' = -(\sin\theta)x + (\cos\theta)y$

$r'^2 = x^2 + y^2 = r^2$
r^2 is an invariant quantity

$x' = \gamma(x - vt) = \gamma(x) - (\gamma v)t$
$t' = \gamma(t - vx/c^2) = -\left(\frac{v\gamma}{c^2}\right)x + (\gamma)t$ } Somewhat like a rotation

Notice,
- $\gamma, \gamma v, \gamma v/c^2 \geq 1$
- Sign of t term of x' EQN is negative

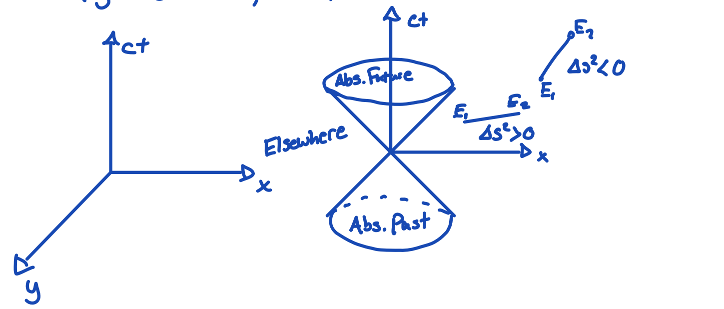
Minkowski Spacetime



A line with a slope of $45°$ is a lightray

$c = \frac{\Delta x}{\Delta t}$
$c\Delta t = \Delta x$
$\Delta(ct) = \Delta x$

Allow For 2 Spacial Dimensions
Flash of light @ $t=0$, $x=y=0$



Time Like, Null, and Spacetime intervals

The position four vector

$x_\mu = (ct, x, y, z) \quad \mu = 0,1,2,3$
$= (x_0, x_1, x_2, x_3)$

Is there an invariant quantity, Like r^2, that we can construct

$s^2 = -c^2t^2 + x^2 + y^2 + z^2$ is invariant
$s^2 = s'^2$

For a flash of light, the sphere has a Radius of
$R^2 = c^2t^2 = x^2 + y^2 + z^2$

.... For Primed observer
$c^2t'^2 = x'^2 + y'^2 + z'^2$
$0 = -c^2t^2 + x^2 + y^2 + z^2$

Define the spacetime interval, Δs, is
$\Delta s^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$
- 4D 'Distance' between 2 events
- An invariant Quantity

Δs^2 can be positive, negative, or zero

when $\Delta s^2 > 0$,
- The spatial seperation is greater than the temporal seperation
- Spacelike Seperation
- True in all I.R.F

Notice:

There exists a Frame where $\Delta t = 0$, so
$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 = \Delta \ell^2 \left(\text{Proper length}\right)^2$

when $\Delta s^2 < 0$
- The temporal Seperation is greates than the Spatial seperation for 2 events
- Timelike seperation
- True in all I.R.F's

Notice:

Two events occur at the same place
$\Delta\tau$
$\Delta r^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 = 0$
$\Delta s^2 = -c^2(PL)^2 = -c^2\Delta\tau^2$

For observer not at rest RT this frame

$\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$
$= -c^2\Delta t^2\left(1 - \frac{1}{c^2}\left(\frac{\Delta x^2}{\Delta t^2} + \frac{\Delta y^2}{\Delta t^2} + \frac{\Delta z^2}{\Delta t^2}\right)\right)$
$= -c^2\Delta t^2(1 - v^2/c^2) = \Delta s'^2$

$\Delta s^2 = 0 = -c^2\Delta t^2 + \Delta x^2 \qquad \Delta y = \Delta z = 0$

– Null Seperation

Interval between 2 events along a light ray

$\Delta x = \pm c\Delta t$

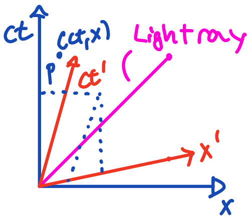
$-c^2\Delta\tau^2 = -c^2\Delta t^2 (1 - v^2/c^2)$

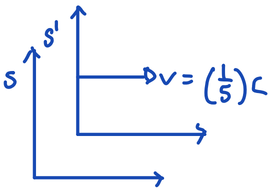
$\Delta\tau^2 = \Delta t^2 (1 - v^2/c^2)$

$\Delta\tau = \Delta t\sqrt{1 - v^2/c^2}$ ∴ PL clock runs slow

Samp 1

According to S observer





Notice: For x-axis, ct = 0 everywhere

For ct-axis, x = 0

For a light ray propigating in the (+x) direction

$\Delta x = c\Delta t$ – line with a slope of 1

L.T's

$x' = \gamma(x - v\Delta t) \Rightarrow x' = \gamma(x - \frac{v}{c}ct)$

$t' = \gamma(t - vx/c^2) \qquad ct' = \gamma(ct - \frac{v}{c}x)$

For the ct' axis, set x' = 0

$x - \frac{v}{c}ct = 0 \quad ct = \frac{1}{(v/c)}x$

For the x'-axis, set ct' = 0

$ct - \frac{v}{c}x = 0 \quad \therefore ct = \frac{v}{c}x$

Samp 2

S = B, S' = C

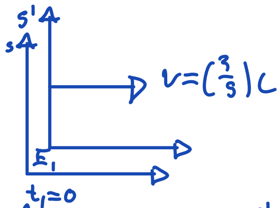
E_1: C leaves B

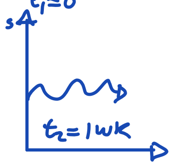
E_2: B fires at C

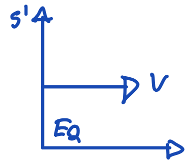
E_3: C fires at B

E_4: C photon reaches B

E_5: B photon reaches C







$t_2 = 1wk$

$t_3' = 1wk$

$D_{13} = V(t_3 - t_1) = C\Delta t_{34}$

$(3/5)c(5/4)wk = Ct_4 - Ct_3$

$\Delta t_{34} = 3/4\, wk$

$\Delta t_{14} = \Delta t_{13} + \Delta t_{34}$

$= \frac{5}{4}wk + \frac{3}{4}wk = 2\,wk$

$\Delta t_{14} = 2wk$

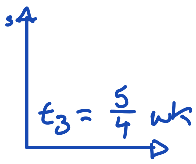
$\Delta x'_{13} = 0$

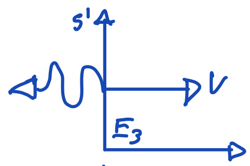
$\Delta t = \gamma(\Delta t' + V\Delta x'/c^2)$

$t_{13} = \gamma(t_{13}' + \frac{V\Delta x_{13}'}{c^2})$

$t_{13} = \gamma(t_{13}')$

$t_{13} = \frac{5}{4}(\text{week}) = \frac{5}{4}wk$





< D13 > $D_{13} = V(\Delta t_{13})$

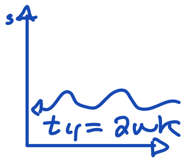
$D_{15} = V(t_5 - t_1) = c(t_5 - t_2) \quad \Delta t_{25} = \Delta t_{15} - \Delta t_{12}$

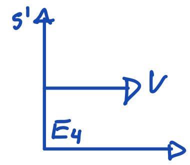
$V\Delta t_{15} = c(\Delta t_{15} - \Delta t_{12})$

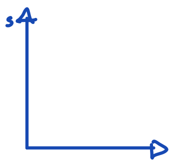
$\frac{1cwk}{c} = \frac{(c - V)\Delta t_{15}}{c}$

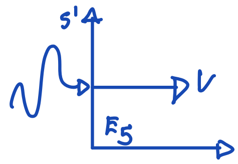
$1wk = (1 - V/c)\Delta t_{15} = \frac{2}{5}\Delta t_{15}$

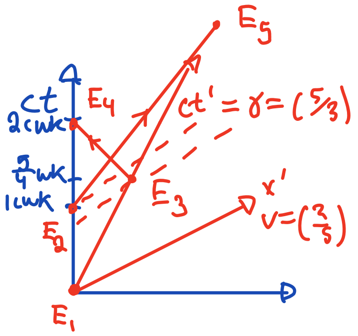
$\Delta t_{15} = \frac{5}{2}wk$











S' - sees E_3 First

S - Sees E_2 First