Phys 131 2016

Lecture 24

Mon Warm Up 10

Tues: Discussion / quiz

Supp Ex ~~58~~ 56, 59

Ch 9 Conc Q. 12

Ch 9 Prob 28, 36 (const speed), 39, 59

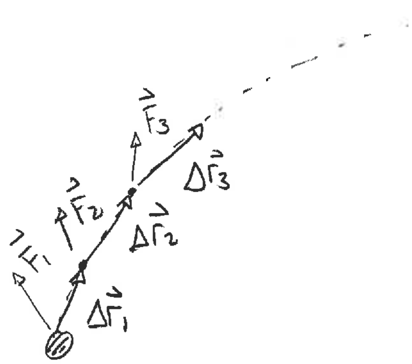
Work done by variable forces

When an object moves in a straight line and a constant force acts on it the work done by the force is

$$W = \vec{F} \cdot \Delta\vec{r}$$

where $\Delta\vec{r}$ is the displacement of the object.
If the object moves in a curved path or the force varies then the strategy is to break the trajectory into infinitesimally small segments, each of which is approximately straight and along each segment the force is approximately constant. Then the work done is

$$W \approx \sum \vec{F}_i \cdot \Delta\vec{r}_i$$

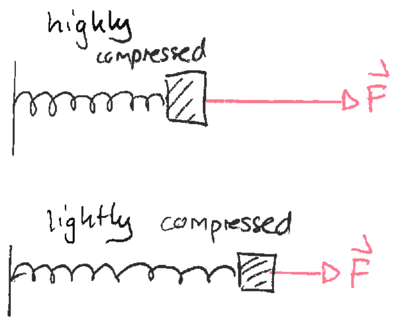


and the approximation becomes exact as $\Delta\vec{r}_i \to 0$ and the number of segments approaches infinity.

Quiz 1 10% → 70% } 30% − 80%

We can see that if the force is always perpendicular to the motion it does no work. It only serves to change the direction of motion, not the speed.

Suppose that the object moves in a straight line while a variable force acts on it. This is the case for an object attached to a spring



As the spring uncompresses the magnitude of the force decreases. We can apply the previous generalization of work to such situations. Consider a general force which acts along the x-axis while the object moves along the x-axis. Then breaking this into small pieces gives

$$W = \sum \vec{F}_i \cdot \Delta\vec{r}_i$$

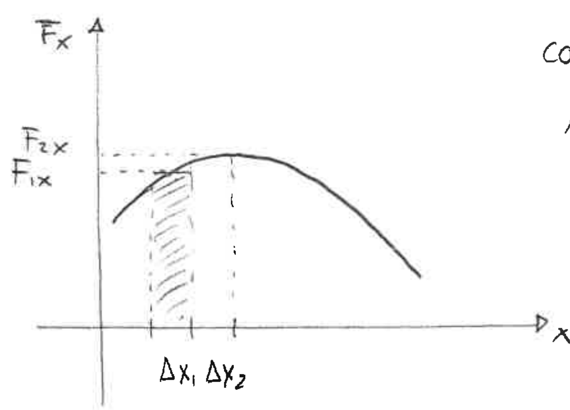
$$W = \sum F_{ix} \Delta x_i$$

For ultimate accuracy we need to break this into infinitely many segments. Then we get in the limit

$$W = \int_{x_i}^{x_f} F_x \, dx$$

horizontal component of $\vec{F}$

We can give an alternative graphical description of this: Suppose we plot F_x vs x. We see that $F_{1x} \Delta x_1$ gives the area of the shaded column. To get the work we add the areas. As the columns narrow this gives:



Work from x_i to x_f is $W =$ area under graph of F_x vs x from x_i to x_f

Spring forces + energy

Springs and spring-like objects are widespread in mechanical situations and provide a manageable model of a system which can exert a variable force.

Demo: PhET Spring + Mass
— produce oscillations

Demo: UCLA molecular vibrations

Demo: PhET normal modes
— two dimensions

Such oscillating phenomena, which are the consequence of spring-like forces, are widespread in the physical sciences.

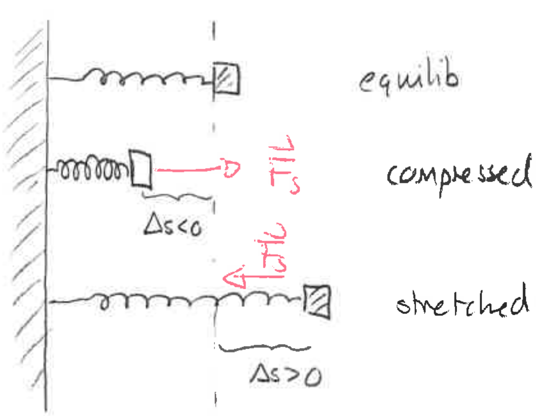
See these in 230/231/342/321/362

Consider a single spring. Observations indicate that

1) the spring has an equilibrium point where it exerts no force
2) when stretched or compressed away from equilibrium the spring exerts a restoring force (pointing toward equilibrium)
3) the component of the spring force along the line of the spring is

$$\boxed{F_{sp} = -k\Delta s}$$

Hooke's Law



where Δs is the displacement away from equilibrium and k is a constant (that depends on the spring). This is called the spring constant and is measured in N/m

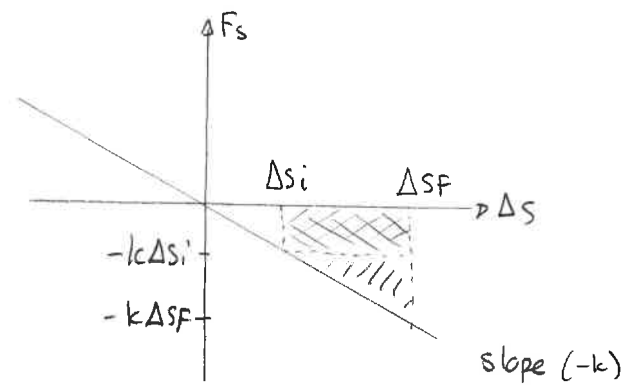
Quiz 2   30% → 60% ? 30%

Using Newton's 2nd Law to assess the motion of an object on which a spring exerts a force is complicated by the fact that the forces changes as the object moves. However we can determine an algebraic expression for the work done by the spring and use work and kinetic energy to answer certain questions about such motion.

To this end.

> The work done by a spring with initial compression /stretch Δs_i and final compression stretch Δs_f is:
>
> $$W = -\frac{1}{2}k(\Delta s_f)^2 + \frac{1}{2}k(\Delta s_i)^2$$

<u>Proof:</u> A graph of F_s vs Δs is



Then

$$W = \text{area under graph}$$

$$= -\left[\text{area rectangle} + \text{area triangle}\right]$$

$$= -\left[k\Delta s_i(\Delta s_f - \Delta s_i) + \frac{1}{2}(\Delta s_f - \Delta s_i)(\Delta s_f - \Delta s_i)k\right]$$

$$= -k(\Delta s_f - \Delta s_i)\left[\Delta s_i + \frac{1}{2}(\Delta s_f - \Delta s_i)\right] = -k(\Delta s_f - \Delta s_i)(\Delta s_f + \Delta s_i)\frac{1}{2}$$

$$= -\frac{k}{2}(\Delta s_f)^2 + \frac{k}{2}(\Delta s_i)^2$$

$\boxed{3}$

Example: A 6.0kg block on a horiz frictionless surface is attached to a spring with spring constant 30N/m. The spring is stretched by 0.20m and released from rest. Determine the speed of the block as it passes equilibrium.

Answer:

initial $\quad v_i = 0 \quad \Delta s_i = 0.20\text{m}$

final $\quad v_f = ? \quad \Delta s_f = 0\text{m}$

$$W_{net} = \Delta K \Rightarrow K_f - \underbrace{\cancel{K_i}}_{0} = W_{net} = \underbrace{\cancel{W_{grav}} + \cancel{W_{normal}}}_{0} + W_{spring}$$

both perpendicular to motion

$$\Rightarrow \tfrac{1}{2} m v_f^2 = W_{spring} = \cancel{-\tfrac{1}{2}k(\Delta s_f)^2} + \tfrac{1}{2}k(\Delta s_i)^2$$

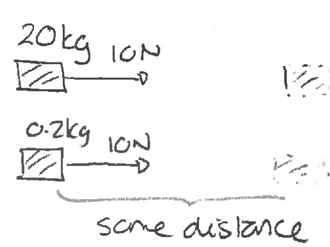
$$\Rightarrow m v_f^2 = k(\Delta s_i)^2$$

$$\Rightarrow v_f^2 = \frac{k}{m}(\Delta s_i)^2 = \frac{30\text{N/m}}{6.0\text{kg}} \times (0.20\text{m})^2 = 0.20\ \text{m}^2/\text{s}^2$$

$$\Rightarrow v_f = \sqrt{0.20\ \text{m}^2/\text{s}^2} = 0.45\text{m/s}$$

Power:

Work + energy do not usually include any information about time. So processes that involve the same work can unfold over different amounts of time. The rate at which work is delivered is described by the power



$$\boxed{P = \frac{W}{\Delta t}}$$

where work W is done in time Δt. This is measured in

Watts $\qquad W = J/s$