+15/100

# Physics 396

## Exam 1

1. In Cartesian coordinates $(x, y, z)$, the square of the infinitesimal distance between two neighboring points is of the form

$$ds^2 = dx^2 + dy^2 + dz^2. \tag{1}$$

The *coordinate transformations* linking Cartesian coordinates $(x, y, z)$ to cylindrical coordinates $(r, \phi, z)$ are

$$\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \\ z &= z. \end{aligned} \tag{2}$$

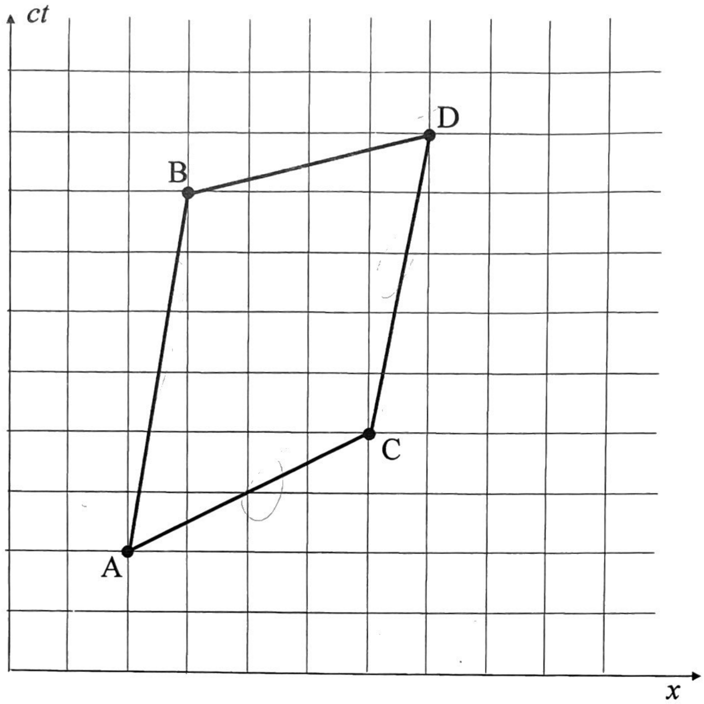
a) Using Eq. (2), calculate the differentials $dx$, $dy$, and $dz$ in terms of $r$, $\phi$, and $z$.

b) Substitute these differentials into Eq. (1) and simplify to arrive at the line element in cylindrical coordinates.

c) Explicitly write the line element for $r = R$, where $R$ is the radius of a cylinder, and $z = H/2$, where $H$ is the height of a cylinder.

d) Calculate the circumference of a circle for $r = R$ and $z = H/2$ on this surface.

2. Consider the spacetime diagram below containing four events labeled **A,B,C,D**. Take spacetime length to be $\sqrt{|\Delta s^2|}$, where the length between two adjacent gridlines is 1 unit.




a) Calculate the spacetime length of $\mathbf{A} \to \mathbf{B} \to \mathbf{D}$ in the units of the grid.

b) Calculate the spacetime length of $\mathbf{A} \to \mathbf{C} \to \mathbf{D}$ in the units of the grid.

c) Which side of the polygon is the longest?

d) Which side of the polygon is the shortest?

e) Which side(s) of the polygon could actually be traversed by a massive particle?

3. A spaceship of proper length $\ell$ passes Earth traveling at a speed of $v = (4/5)c$. Just as the spaceship is passing Earth, a light signal is sent from the back of the spaceship towards the front of the spaceship. At this moment, both the rocket's tail clock and the Earth's clocks read $t = 0$.

   a) Draw a set of two pictures in the rest frame of the Earth for the two important events, that is, the emittance of the light signal from the back of the spaceship and the reception of the light signal at the front of the spaceship.

   b) At the moment when the light signal is emitted from the back of the spaceship, in terms of $\ell/c$, what time does the clock at the front of the spaceship read?

   c) According to the spaceship's clocks, in terms of $\ell/c$, how long does it take the light signal to reach the front of the spaceship.

   d) According to the Earth's clocks, in terms of $\ell/c$, how long does it take the light signal to reach the front of the spaceship?

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Exam 1

Problem 1

+34/34

a.)

$\frac{dx}{dr} = \cos\phi$ , $\frac{dx}{d\phi} = -r\sin\phi$ : $\boxed{dx = \cos\phi\, dr - r\sin\phi\, d\phi}$ ✓

$\frac{dy}{dr} = \sin\phi$ , $\frac{dy}{d\phi} = r\cos\phi$ : $\boxed{dy = \sin\phi\, dr + r\cos\phi\, d\phi}$ ✓

$\frac{dz}{dz} = 1$ , $dz = dz$ : $\boxed{dz = dz}$ ✓

b.)

(1) $dx^2 = (\cos\phi^2 dr^2 - \cancel{2r\cos\phi\sin\phi\, dr d\phi} + r^2\sin^2\phi\, d\phi^2)$ ✓ ✓ ✓

(2) $dy^2 = (\sin\phi^2 dr^2 + \cancel{2r\cos\phi\sin\phi\, dr d\phi} + r^2\cos^2\phi\, d\phi^2)$ ✓ ✓ ✓

(3) $dz^2 = dz^2$

$(1) + (2) + (3) = \cos^2\phi\, dr^2 + r^2\sin^2\phi\, d\phi^2 + \sin^2\phi\, dr^2 + r^2\cos^2\phi\, d\phi^2 + dz^2$

$ds^2 = dr^2(\cos^2\phi + \sin^2\phi) + r^2 d\phi^2(\sin^2\phi + \cos^2\phi) + dz^2$

$$\boxed{ds^2 = dr^2 + r^2 d\phi^2 + dz^2}$$ ✓

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Exam 1

Problem 1 continued

c.) $r = R$, $z = H/2$

$dz = 0$

$dr^2 = 0$

$r$ is constant : $r = R$   $z$ is constant : $z = \frac{H}{2}$

$$\boxed{ds^2 = dR^2 d\phi^2}$$

d.) $ds = R\,d\phi$

$$S = R\int_0^{2\pi} d\phi = 2\pi R \quad \therefore \quad S \equiv C$$

$$\boxed{C = 2\pi R}$$

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Exam 1 Corrections

$+\frac{25}{5} = +5$

+80/100

Problem 2

$$\Delta s^2 = -(c\Delta t)^2 + (\Delta x)^2 + \cancel{(\Delta y)^2} + \cancel{(\Delta z)^2}$$

a.) A→B→D :

A→B : $\Delta x = 1$, $c\Delta t = 6$ : $(\Delta x)^2 = 1$ : $-(c\Delta t)^2 = -36$

+18

$$\Delta s^2 = -36 + 1 = -35 \rightarrow \sqrt{|\Delta s^2|} = \sqrt{35}$$ ✓

B→D : $\Delta x = 4$, $c\Delta t = 1$ : $(\Delta x)^2 = 16$ : $-(c\Delta t)^2 = -1$

$$\Delta s^2 = -1 + 16 = 15 \rightarrow \sqrt{|\Delta s^2|} = \sqrt{15}$$ ✓

$$\Delta s = \sqrt{35} + \sqrt{15}$$

$$\boxed{\Delta s_{TOT} = \sqrt{35} + \sqrt{15}}$$ ✓

b.) A→C→D

A→C : $\Delta x = 4$, $c\Delta t = 2$ : $(\Delta x)^2 = 16$ : $-(c\Delta t)^2 = -4$

$$\Delta s^2 = -4 + 16 = 12 \rightarrow \sqrt{|\Delta s^2|} = \sqrt{12}$$ ✓

C→D : $\Delta x = 1$, $c\Delta t = 5$ : $(\Delta x)^2 = 1$ : $-(c\Delta t)^2 = -25$

$$\Delta s^2 = -25 + 1 = -24 \rightarrow \sqrt{|\Delta s^2|} = \sqrt{24}$$ ✓

$$\boxed{\Delta s_{TOT} = \sqrt{12} + \sqrt{24}}$$ ✓

c.) A→B is longest ✓

d.) A→C is shortest ✓

e.) A→B & C→D : slope ≥ 45° here ✓ ✓

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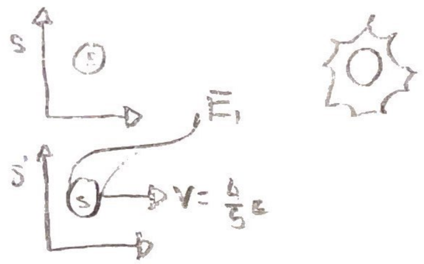
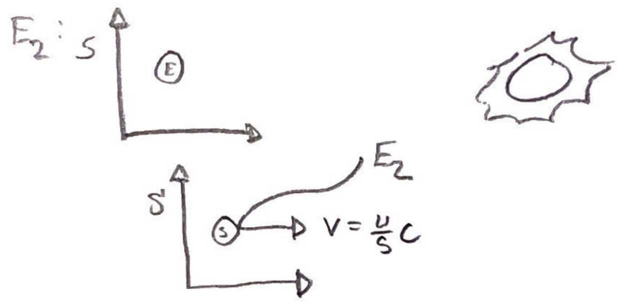
Exam 1 Corrections

Problem 3 +7

a.) E1:




E2:




b.) $\Delta t = 0$ : $\Delta t_{12} = \gamma(\Delta t'_{12} + V \cdot \Delta x'_{12}/c^2)$ : $0 = \gamma(\Delta t'_{12} + V\Delta x'_{12}/c^2)$

$0 = \Delta t'_{12} + V\Delta x'_{12}/c^2 \rightarrow \Delta t'_{12} = -\dfrac{V \cdot \Delta x'_{12}}{c^2}$

$V = \frac{4}{5}c$, $\Delta x'_{12} = l$  $\Delta t'_{12} = -\dfrac{(4/5 c) \cdot l}{c^2} = \dfrac{-4}{5}\dfrac{l}{c}$ ✓

$\boxed{\Delta t'_{12} = \frac{-4}{5} \cdot \frac{l}{c}}$ ✓

c.) $\Delta t'_{12}$ : speed of c is invariant $\therefore$ $D = c \cdot \Delta t'$ : $D = l$

$\Delta t' = \frac{D}{c} = \frac{l}{c}$ : $\boxed{\Delta t'_{12} = \frac{l}{c}}$ ✓

d.) $\Delta t_{12} = \gamma(\Delta t'_{12} + V \cdot \Delta x'_{12}/c^2)$ $\therefore$ $c = \frac{D}{\Delta t'}$ $\therefore$ $D = c \cdot \Delta t' = c \cdot \frac{l}{c} = l$ : $D = l$

$= \gamma\left(\frac{l}{c} + \frac{4}{5}c \cdot \frac{l}{c^2}\right) = \gamma\left(\frac{l}{c} + \frac{4}{5}\frac{l}{c}\right) = \gamma\left(\frac{9}{5}\frac{l}{c}\right)$ : $\gamma = \left(\sqrt{1-(V/c)^2}\right)^{-1}$

$= \frac{5}{3}\left(\frac{9}{5}\frac{l}{c}\right) = \frac{9}{3} \cdot \frac{l}{c} = 3\frac{l}{c}$  $\gamma = 5/3$

$\boxed{\Delta t_{12} = 3\frac{l}{c}}$ ✓