Exercise 1

Suppose $\vec{B} = B\hat{z}$ with known γ, t

a.) Determine $\hat{U}(t)$:

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \;:\; |\psi(t)\rangle = \underbrace{e^{-i\hat{H}t/\hbar}}_{\hat{U}(t)}|\psi(0)\rangle$$

$$\hat{H} = \underbrace{-\frac{gq}{2m}}_{\gamma}\frac{\hbar}{2}\vec{B}\cdot\hat{\sigma}_z \;:\; \hat{H} = \frac{\gamma\hbar}{2}\left[B_x\hat{\sigma}_x + B_y\hat{\sigma}_z + B_z\hat{\sigma}_z\right] \;\therefore\; \hat{U}(t) = e^{-i\gamma[B_x\hat{\sigma}_x + B_y\hat{\sigma}_y + B_z\hat{\sigma}_z]t/2}$$

$$B_x = B_y = 0 \quad \therefore \quad \hat{U}(t) = e^{-i\gamma(B_z\hat{\sigma}_z)t/2} \;:\; e^{-i\lambda\hat{\sigma}_z} = \cos(\lambda)\hat{I} - i\sin(\lambda)\hat{\sigma}_z$$

$$\lambda \equiv \gamma\cdot B_z\cdot t/2$$

$$\hat{U}(t) = \cos(\gamma\cdot B_z\cdot t/2)\hat{I} - i\sin(\gamma\cdot B_z\cdot t/2)\hat{\sigma}_z$$

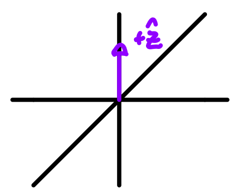
$$= \begin{pmatrix} \cos(\gamma\cdot B_z\cdot t/2) & 0 \\ 0 & \cos(\gamma\cdot B_z\cdot t/2)\end{pmatrix} - \begin{pmatrix} i\sin(\gamma\cdot B_z\cdot t/2) & 0 \\ 0 & -i\sin(\gamma\cdot B_z\cdot t/2)\end{pmatrix}$$

$$= \begin{pmatrix} \cos(\gamma\cdot B_z\cdot t/2) - i\sin(\gamma\cdot B_z\cdot t/2) & 0 \\ 0 & \cos(\gamma\cdot B_z\cdot t/2) + i\sin(\gamma\cdot B_z\cdot t/2)\end{pmatrix}$$

$$\boxed{\hat{U}(t) = \begin{pmatrix} \cos(\gamma\cdot B_z\cdot t/2) - i\sin(\gamma\cdot B_z\cdot t/2) & 0 \\ 0 & \cos(\gamma\cdot B_z\cdot t/2) + i\sin(\gamma\cdot B_z\cdot t/2)\end{pmatrix}}$$

b.)

i.) Suppose $|\psi(0)\rangle = |+\hat{z}\rangle$, get $|\psi(t)\rangle = |+\hat{n}\rangle$



$$|\psi(t)\rangle = \begin{pmatrix} \cos(\gamma\cdot B_z\cdot t/2) - i\sin(\gamma\cdot B_z\cdot t/2) & 0 \\ 0 & \cos(\gamma\cdot B_z\cdot t/2) + i\sin(\gamma\cdot B_z\cdot t/2)\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

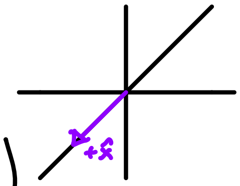
$$|\psi(t)\rangle = \begin{pmatrix} \cos(\gamma\cdot B_z\cdot t/2) - i\sin(\gamma\cdot B_z\cdot t/2) \\ 0 \end{pmatrix} \;:\; |+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\varphi}\sin(\theta/2)|-\hat{z}\rangle$$

$$|\psi(t)\rangle = e^{-i\gamma B_z t/2}\cdot\begin{pmatrix}1\\0\end{pmatrix} = e^{-i\gamma B_z t/2}|+\hat{z}\rangle$$

$$\boxed{|+\hat{n}\rangle = |+\hat{z}\rangle,\ \theta = 0,\ \varphi = 0}$$

Exercise 1 Continued

ii.) Suppose $|\psi(0)\rangle = |+\hat{x}\rangle$, get $|\psi(t)\rangle = |+\hat{n}\rangle$



$$|\psi(t)\rangle = \begin{pmatrix} \cos(\gamma \cdot B_z \cdot t/2) - i\sin(\gamma \cdot B_z \cdot t/2) & 0 \\ 0 & \cos(\gamma \cdot B_z \cdot t/2) + i\sin(\gamma \cdot B_z \cdot t/2) \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\gamma \cdot B_z \cdot t/2) - i\sin(\gamma \cdot B_z \cdot t/2) \\ \cos(\gamma \cdot B_z \cdot t/2) + i\sin(\gamma \cdot B_z \cdot t/2) \end{pmatrix} \quad : \quad |+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\varphi}\sin(\theta/2)|-\hat{z}\rangle$$

$$\beta = \gamma \cdot B_z \cdot t/2$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\beta}|+\hat{z}\rangle + e^{i\beta}|-\hat{z}\rangle\right) = e^{-i\beta}\left(\frac{1}{\sqrt{2}}|+\hat{z}\rangle + \frac{1}{\sqrt{2}}e^{2i\beta}|-\hat{z}\rangle\right)$$

$$\boxed{|+\hat{n}\rangle = \frac{1}{\sqrt{2}}\left(|+\hat{z}\rangle + e^{2i\beta}|-\hat{z}\rangle\right), \ \theta = \frac{\pi}{2}, \ \varphi = \gamma \cdot B_z \cdot t}$$

Exercise 2

Suppose $\vec{B} = B\hat{y}$ with known γ, t

a.) Determine $\hat{U}(t)$:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H}|\psi(t)\rangle \; : \; |\psi(t)\rangle = \underbrace{e^{-i\hat{H}t/\hbar}}_{\hat{U}(t)} |\psi(0)\rangle$$

$$\hat{H} = -\underbrace{\frac{gq}{2m}}_{\gamma} \frac{\hbar}{2} \vec{B}\cdot\hat{\sigma}_z \; : \; \hat{H} = \frac{\gamma\hbar}{2}\left[B_x\hat{\sigma}_x + B_y\hat{\sigma}_z + B_z\hat{\sigma}_z\right] \;\therefore\; \hat{U}(t) = e^{-i\gamma[B_x\hat{\sigma}_x + B_y\hat{\sigma}_y + B_z\hat{\sigma}_z]t/2}$$

$$B_x = B_z = 0 \;\therefore\; \hat{U}(t) = e^{-i\gamma(B_y\hat{\sigma}_z)t/2} \; : \; e^{-i\lambda\hat{\sigma}_c} = \cos(\lambda)\hat{I} - i\sin(\lambda)\hat{\sigma}_c$$

$$\lambda \equiv \gamma\cdot B_y\cdot t/2$$

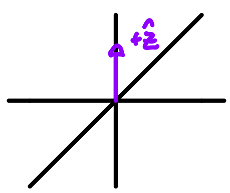
$$\hat{U}(t) = \cos(\gamma\cdot B_y\cdot t/2)\hat{I} - i\sin(\gamma\cdot B_y\cdot t/2)\hat{\sigma}_y$$

$$= \begin{pmatrix} \cos(\gamma\cdot B_y\cdot t/2) & 0 \\ 0 & \cos(\gamma\cdot B_y\cdot t/2) \end{pmatrix} - \begin{pmatrix} 0 & \sin(\gamma\cdot B_y\cdot t/2) \\ -\sin(\gamma\cdot B_y\cdot t/2) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\gamma\cdot B_y\cdot t/2) & -\sin(\gamma\cdot B_y\cdot t/2) \\ \sin(\gamma\cdot B_y\cdot t/2) & \cos(\gamma\cdot B_y\cdot t/2) \end{pmatrix}$$

$$\boxed{\hat{U}(t) = \begin{pmatrix} \cos(\gamma\cdot B_y\cdot t/2) & -\sin(\gamma\cdot B_y\cdot t/2) \\ \sin(\gamma\cdot B_y\cdot t/2) & \cos(\gamma\cdot B_y\cdot t/2) \end{pmatrix}}$$

b.) Suppose $|\psi(0)\rangle = |+\hat{z}\rangle$, get $|\psi(t)\rangle = |+\hat{n}\rangle$



$$|\psi(t)\rangle = \begin{pmatrix} \cos(\gamma\cdot B_y\cdot t/2) & -\sin(\gamma\cdot B_y\cdot t/2) \\ \sin(\gamma\cdot B_y\cdot t/2) & \cos(\gamma\cdot B_y\cdot t/2) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi(t)\rangle = \begin{pmatrix} \cos(\gamma\cdot B_y\cdot t/2) \\ \sin(\gamma\cdot B_y\cdot t/2) \end{pmatrix} \; : \; |+\hat{n}\rangle = \cos(\theta/2)|+\hat{z}\rangle + e^{i\varphi}\sin(\theta/2)|-\hat{z}\rangle$$

$$\boxed{|+\hat{n}\rangle = \cos(\gamma\cdot B_y\cdot t/2)|+\hat{z}\rangle + \sin(\gamma\cdot B_y\cdot t/2)|-\hat{z}\rangle, \; \theta = \gamma\cdot B_y\cdot t, \; \varphi = 0}$$

Exercise 3

How does this relate to estimating λ in $\hat{U}(\lambda) = e^{-i\lambda\hat{\sigma}_z/2}$?

$$\boxed{\lambda = \gamma \cdot \hat{B} \cdot t/2}$$