

Laboratory 7: Newton's Second Law: Atwood's Machine

Atwood's machine consists of two blocks connected by a string suspended over a pulley as illustrated in Fig. 1. Typically the two blocks are released from rest. Newton's laws can be used to predict their subsequent acceleration, which can also be determined by measuring the speeds of the blocks as time passes. The aim of this experiment is to verify the accelerations as predicted by Newton's second law.

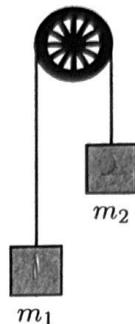


Figure 1: Atwood's machine.
Block 1 is on the left, block 2 is on the right.

1 Theory and Experimental Design

- Suppose that the string cannot stretch. How is the velocity of block 1 at one instant related to that of block 2 at the same instant? How is the acceleration of block 1 related to that of block 2?
- Assume that $m_2 > m_1$ and consider block 1 in Fig. 1. Is the y -component of its acceleration positive, negative or zero? Is the y -component of the acceleration of block 2 positive, negative or zero?

Denote the *magnitude of the acceleration* of each block by a .

- Express the y -component of each block's acceleration in terms of a .
- Draw a free body diagram for *each block*.
- Apply Newton's second law separately to *each block*. This should give two equations with two unknowns: a and the tension in the string. Combine these to determine an expression for the magnitude of the acceleration (i.e. a formula of the form $a = \text{some combination of } m_1, m_2 \text{ and } g$).
- What does your expression predict for the acceleration when the blocks have the same mass? Does your prediction appear to be correct? If it predicts this correctly, then have the instructor check your expression for the acceleration.
- The sensor records the speed of block 1 versus time. How can the acceleration of the this block be determined from this data?

2 Experiment: acceleration

The pulley is equipped with a sensor which determines the *speed* with which the edge of the pulley is moving; this equals the speed with which either of the blocks move. The PASCO interface can record this speed as time passes and DataStudio can graph this data. The acceleration of either block can be inferred from this graph.

- a) Set up DataStudio and connect the smart pulley. Configure it to read speed.
- b) Set up the mass hangers and choose values of m_1 and m_2 (it is best to arrange these so that there is not a huge discrepancy in mass on either side). Predict the acceleration of the mass on the left. Denote this a_{theory} .
- c) Arrange mass with the values of m_1 and m_2 used above. Release them from rest and determine the acceleration of that on the left. Denote this a_{exp} .
- d) Determine the percentage difference:

$$\frac{a_{\text{theory}} - a_{\text{exp}}}{a_{\text{theory}}} \times 100.$$

- e) Repeat the previous steps for two more different arrangements of m_1 and m_2 .
- f) What may have caused the discrepancies? One thing to check is the acceleration when $m_1 = m_2$. What acceleration should result? For two different values of $m_1 = m_2$, set up the masses and record the speed of that on the left vs time after giving the mass on the right a gentle downward pull. Measure and record the acceleration. Do the experimental results match your predictions? If not, how could they explain the discrepancies between measured and calculated accelerations?

We will now consider whether the direction of motion affects the acceleration. Suppose that $m_2 > m_1$ and that the blocks are initially held at rest. Then block 1 is given a brief sharp tug down. It moves down and then reverses direction.

- g) Predict how the magnitude of the acceleration when moving down compares to that when moving up (same, larger smaller, etc, ...)?
- h) Set up the masses as indicated. Run DataStudio, give the lighter mass a brief tug and release it. Graph speed versus time. Save this data.
- i) You should observe a period during which the mass on the left moves down with decreasing speed followed by a period during which it moves up with increasing speed. Identify these on the DataStudio plot and determine the acceleration for each period. Note that the smart pulley cannot distinguish between various directions of motion. This means that it will lose the sign of the velocity while the object moves down. So it will get the incorrect sign for the acceleration while the object moves down. After you have done this correction, how does the acceleration when moving down compare to that when moving up?

3 Tension in the String

It is possible to infer the tension in the string from the motion of the masses.

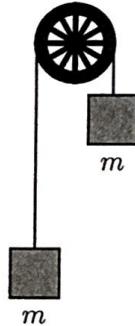
- a) Determine an expression for the tension in the string (on the left) in terms of a, m_1, m_2 and g . Have the instructor check your expression.
- b) Again consider the situation where block 1 is given an initial downward tug. Predict how the tension in the string compares (larger, smaller, same) to the gravitational force on block 1 while it is moving down? How does it compare while moving up?
- c) In part 2 (h) you recorded data for a run of this experiment. Use this data and the formula from part 3 (a) to determine the tension in the string for both periods. How does tension in the string when moving down compare to that when moving up? How do these compare to the force exerted by gravity on block 1?

4 Conclusion

- a) Does Newton's second law predict the accelerations correctly?
- b) What may have caused any discrepancies between measured and predicted accelerations?

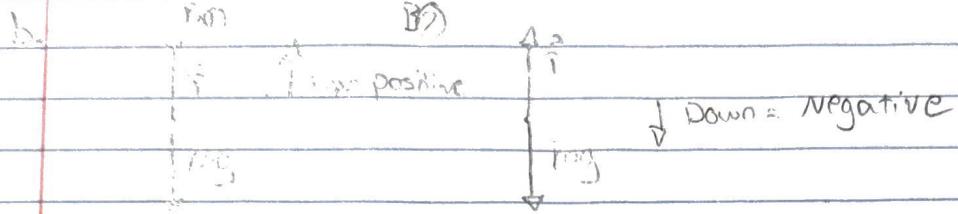
5 Exercises

- a) Two identical masspieces are held in the arrangement as illustrated. The masspieces are then released. Predict their subsequent motions.
- b) Two identical masses are held at rest in the configuration as illustrated. A hand briefly pulls the mass on the right down. Predict the subsequent motion of both masses. In particular, describe whether the velocity remains constant or varies (before either mass hits the floor or pulley and after the hand leaves the mass).



Part 1

- a. The velocities would have equal magnitudes but opposite in direction.
 The same would be true for the acceleration.



Block 1 would have a positive acceleration, and block 2 would be negative.

Block 1	Block 2
$\Sigma y = m_1 a_y$	$\Sigma y = m_2 a_y$
$\vec{T} - m_1 g = m_1 a_y$	$\vec{T} - m_2 g = -m_2 a_y$
$\vec{T} = m_1 a_y + m_1 g$	$\vec{T} = m_2 g - m_2 a_y$
$\vec{T} - m_1 g = m_1 a_y$	$\vec{T} = m_2 g - m_2 a_y$
$\vec{T} - m_2 g = -m_2 a_y$	
$m_2 g - m_2 a_y - m_1 g = m_1 a_y$	
$-m_2 a_y - m_1 a_y = m_1 g - m_2 g$	
$a_y (-m_2 - m_1) = m_1 g - m_2 g$	
$a_y = \frac{m_1 g - m_2 g}{-m_2 - m_1}$	

Block 1	Positive for up	Negative for down
$\vec{T} - m_1 g = m_1 a_y$		$\vec{T} - m_2 g = -m_2 a_y$
$m_1 g$		$m_2 g$

E. Σy : Block 1 = $m_1 a$

$$\begin{cases} \vec{T} \\ m_1 g \end{cases} \quad \vec{T} - m_1 g = m_1 a$$

$$\vec{T} = m_1 a + m_1 g$$

Σy : Block 2 = $m_2 a$

$$\begin{cases} \vec{T} \\ m_2 g \end{cases} \quad \vec{T} - m_2 g = m_2 a$$

$$\vec{T} = m_2 a + m_2 g$$

$$\vec{T} = m_1 g + m_1 a \quad \vec{T} = m_2 a + m_2 g$$

$$-m_2 a + m_2 g - m_1 g = m_1 a$$

$$-m_2 a - m_1 a = -m_2 g + m_1 g$$

$$a(-m_2 - m_1) = g(-m_2 + m_1)$$

$$a = \frac{g(m_2 - m_1)}{(m_2 + m_1)}$$

$$a = \frac{+g(m_2 - m_1)}{(m_2 + m_1)}$$

F when the blocks have the same mass,
the acceleration is zero. This does appear to
be correct.

G It is the rate at which the speed changes in a certain direction.

$$a = \frac{g(m_2 - m_1)}{(m_2 + m_1)}$$

Part 2

b) $m_1 = 0.06 \text{ kg}$

$m_2 = 0.16 \text{ kg}$

$$a_{\text{theory}} = \frac{g(0.16 \text{ kg} - 0.06 \text{ kg})}{(0.16 \text{ kg} + 0.06 \text{ kg})}$$

$$a_{\text{theory}} = \frac{g(0.1 \text{ kg})}{(0.22 \text{ kg})}$$

$$a_{\text{theory}} = \frac{9.8 \text{ m/s}^2 (0.1 \text{ kg})}{(0.22 \text{ kg})}$$

$$(a_{\text{theory}} = 2.45 \text{ m/s}^2)$$

c) $m_2 = 0.1 \text{ kg}$ Slope of line is 2.31

$m_1 = 0.06 \text{ kg}$

From graph

$$(a_{\text{exp}} = 2.35 \text{ m/s}^2)$$

d) $\frac{a_{\text{theory}} - a_{\text{Exp}}}{a_{\text{theory}}} \times 100 = \frac{2.45 - 2.35}{2.45} \times 100 = [4.0\%]$

e.) a.) $m_1 = 0.07 \text{ kg}$ $a_{\text{theory}} = \frac{g(0.08 \text{ kg} - 0.07 \text{ kg})}{0.08 \text{ kg} + 0.07 \text{ kg}}$

$m_2 = 0.08 \text{ kg}$

$$(a_{\text{theory}} = 0.65 \text{ m/s}^2)$$

Slope of line is $[0.596 \text{ m/s}^2]$

$$\frac{0.65 - 0.596}{0.65} \times 100 = [8.3\%]$$

b.) $m_1 = 0.05 \text{ kg}$ $a_{\text{theory}} = \frac{g(0.07 \text{ kg} - 0.05 \text{ kg})}{(0.07 \text{ kg} + 0.05 \text{ kg})}$

$m_2 = 0.07 \text{ kg}$

$$(a_{\text{theory}} = 1.63)$$

Slope of line is $[1.55 \text{ m/s}^2]$

$$\frac{1.03 - 1.55}{1.55} \times 100 = [4.9\%]$$

+ when moving up, there should be a zero acceleration. The weights may have never fully stopped swaying when it was released causing a faulty reading.

$$M_1 = 0.06 \text{ kg} \quad m_1 = m_2 \quad a_{\text{theory}} = \frac{g(0.06 - 0.06)}{0.06 + 0.06} = 0$$

$$M_2 = 0.06 \text{ kg}$$

$$a_{\text{theory}} = 0 \text{ m/s}^2 \quad a_{\text{Exp}} = +0.034 \text{ m/s}^2$$

$$M_1 = 0.05 \text{ kg} \quad m_1 = m_2 \quad a_{\text{theory}} = \frac{g(0.05 - 0.05)}{(0.05 + 0.05)} = 0$$

$$M_2 = 0.05 \text{ kg}$$

$$a_{\text{theory}} = 0 \text{ m/s}^2 \quad a_{\text{Exp}} = +0.0369 \text{ m/s}^2$$

They do not match our predictions due to air resistance.

- g The magnitude of the acceleration is the same but opposite in direction.
- i The acceleration moving up is the same when it is moving down.

b $M_2 = 0.05 \text{ kg}$

$M_1 = 0.06 \text{ kg}$

A

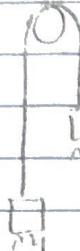
$$a = \frac{g(m_2 - m_1)}{(m_2 + m_1)}$$

Part 3

a)

$$\vec{T} - m_1 g = m_1 a$$

$$\vec{T} - m_2 g = -m_2 a$$



$$m_1 = 0.06 \text{ kg}$$

$$m_2 = 0.05 \text{ kg}$$

$$a = 0.826 \text{ m/s}^2$$

$$(M_2) \quad \vec{T} = m_2 g - m_2 a$$

$$\vec{T} = 0.05 \text{ kg}(9.8 \text{ m/s}^2) - 0.05 \text{ kg}(0.826 \text{ m/s}^2)$$

$$\vec{T} = 0.078 \text{ N}$$

$$(M_1) \quad \vec{T} = m_1 g + m_1 a$$

$$\vec{T} = 0.06 \text{ kg}(9.8 \text{ m/s}^2) + 0.06 \text{ kg}(0.826 \text{ m/s}^2)$$

$$\vec{T} = 0.638 \text{ N}$$

When the mass is moving upward, the tension has to be greater than the force of gravity. When the mass is moving downward, the tension has to be less than the force of gravity.

- b) In the situation where the block is given a gentle push the tension is greater than the force of gravity when it is moving up. The tension is less than the force of gravity when it is moving down.

Part 4

a) Newton's 2nd Law is proved from this equation

$$a = \frac{g(m_2 - m_1)}{(m_2 + m_1)}$$

And states that the magnitudes are equal and opposite in direction.

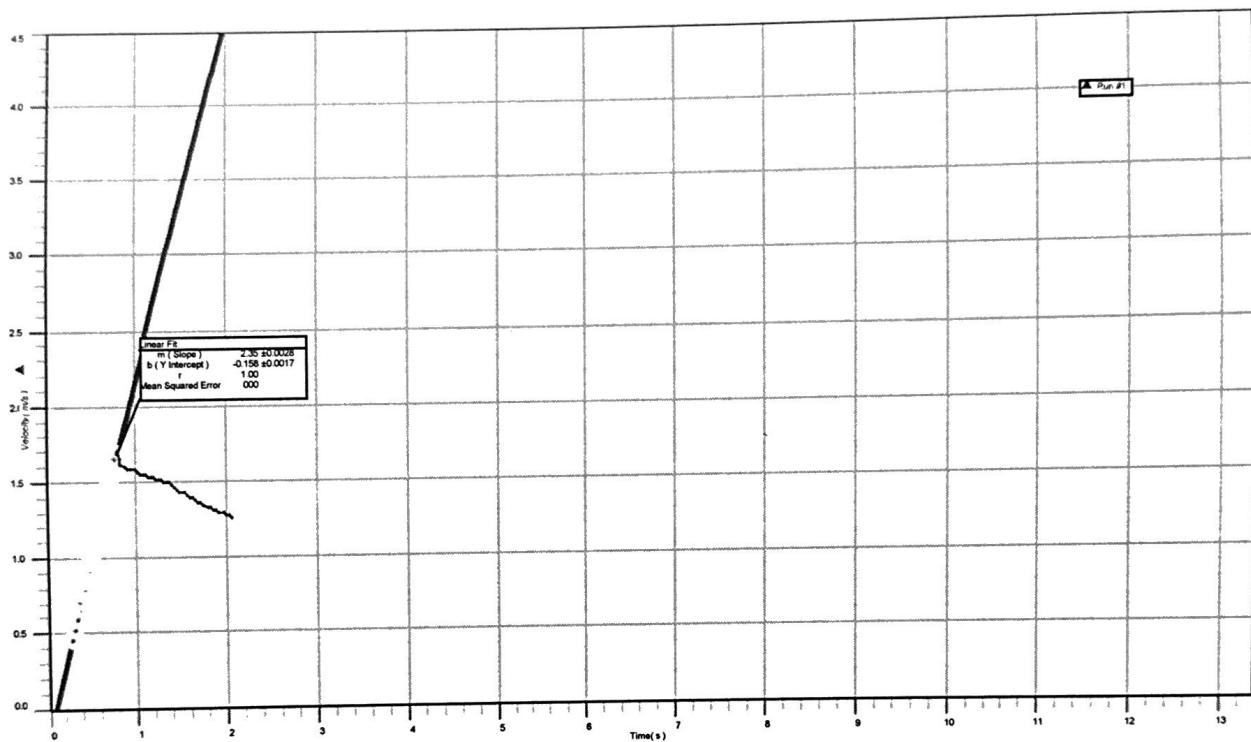
- b) Discrepancies can come from human error; drag, friction from the string and the pulley.

Part 5

- a) Since the masses are equal, the objects have the same force of gravity acting on them and since they are connected to one another with a massless string and a frictionless pulley, the tensions are equal. The object does not move. ✓
- b) Newton proved that an object at rest will stay at rest and an object in motion will stay in motion until acted upon by an outside force (The pulley or the ground in this case). When the block is tugged down, it is accelerating at the rate of gravity so the velocity is constantly changing. The block on the left will accelerate X at the rate of gravity until it hits the pulley. The block on the right will accelerate until it hits the ground at the rate of gravity. ✓

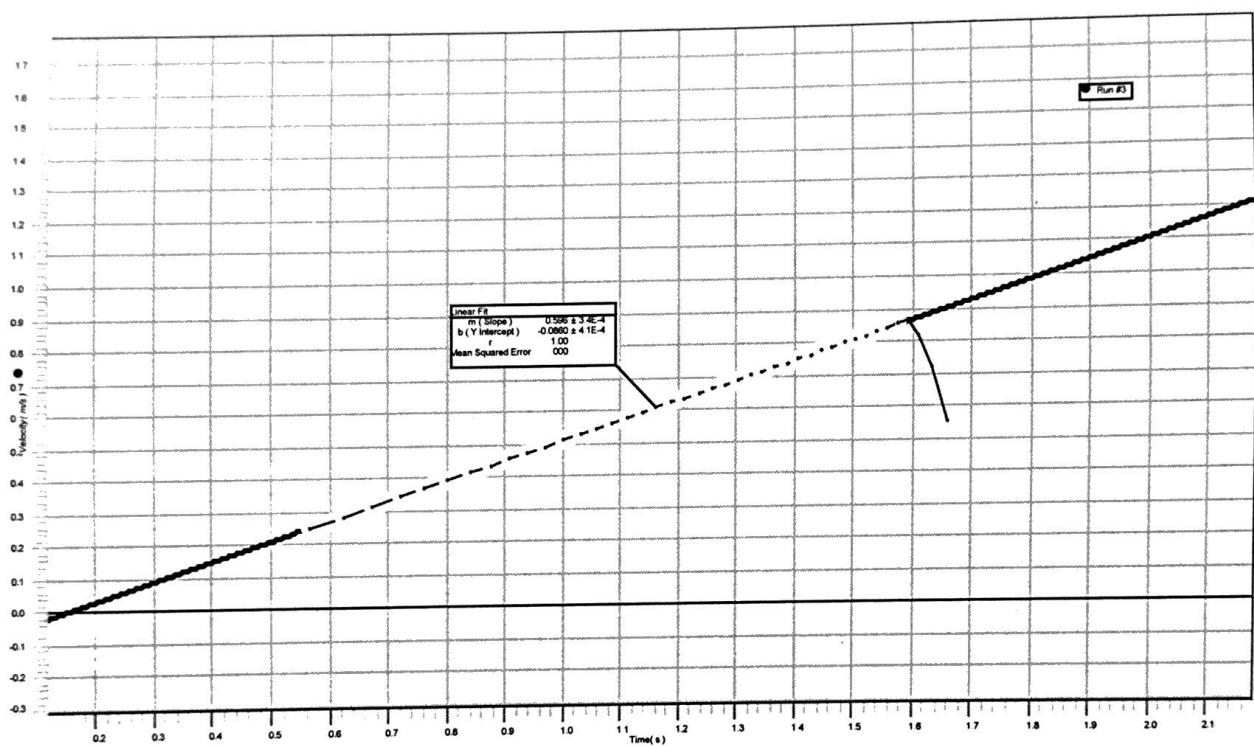
Graph 1

2.c



Graph 1

2e.a)



Graph 1

2e.b)

