Phys 131 2016

Lecture 3

Monday: * Warm Up 2 (D2L) by 8am

* HW due by 5pm

- Ch2 Conc Q 5, 7, 8 Full explanations!

  Ch2 Probs 5, 6, 8

  Supp Ex 4, 7

- No late HW.
- Counts 15pts

* Solutions must be complete - explanations, calculations, definitions

  - show D2L solutions for discussion probs.

Tues Discussion /quiz

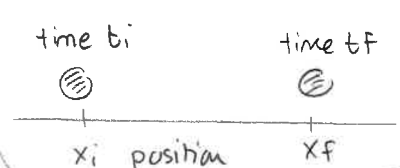
Problems TBA.

Average Velocity

We saw that average velocity quantifies the rate at which position changes.

The schematic definition is:

Observe at two intervals



⇝

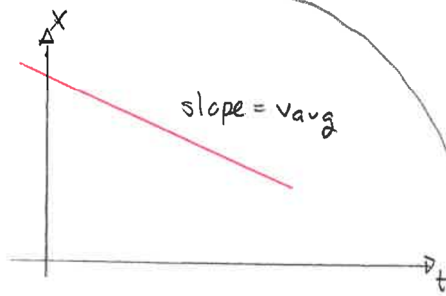
Calculate average velocity over time interval

$$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

If the object moves in the same direction and with constant speed (uniform motion) one can show:

For uniform motion, a graph of position vs. time is a straight line and

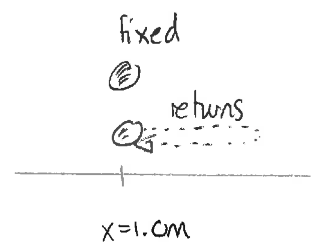
v_{avg} = slope of x vs t



Velocity

We saw that average velocity can be a crude representation of the particle's motion. For example, if over an interval of time:

1) one particle remains fixed at $x = 1.0m$
2) another " moves from $x = 1.0m$ and eventually returns



we would get $v_{avg} = 0m/s$ for both even though their motion is different. We would like a notion of velocity that distinguishes between these and also situations where the speed changes.

Demo: PhET Moving Man

Settings → charts

→ $v = -3.00m/s$, $a = 1.00m/s^2$ } hide all except position

Suppose we want the velocity at 4.0s. We could calculate the average velocity over smaller + smaller intervals starting at 4.0s

t_i	t_f	Δt	x_i	x_f	Δx	v_{avg}
4.0s	5.0s	1.0s	−4.0m	−2.5m	1.5m	1.5m/s
4.0s	4.5s	0.5s	−4.0m	−3.375m	0.625m	1.25m/s
4.0s	4.1s	0.1s	−4.0m	−3.890m	0.110m	1.10m/s

As the time interval decreases (Δt gets smaller) the displacement also decreases (Δx decreases) but the ratio $\Delta x / \Delta t$ appears to approach a fixed value. The limiting value as Δt approaches zero is called the instantaneous velocity or just velocity

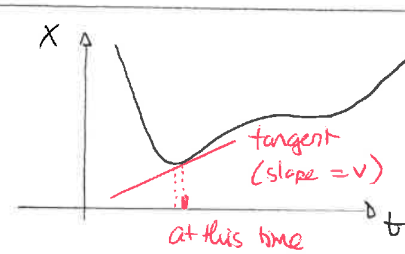
We get:

$$\boxed{\text{The velocity of an object is} \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}$$

units: m/s

There are ways to compute this that do not involve actually calculating a series of average velocities and deducing the limiting value. One of these is graphical:

velocity v = slope of tangent to position vs. time graph.



Warm Up! ~~Quiz~~

The definition of velocity makes the following apparent:

$v > 0$	$\Leftrightarrow$	object is moving right
$v < 0$	$\Leftrightarrow$	" " " left

Then we can give a meaning to speed

$$\boxed{\text{speed} = \text{magnitude of velocity}}$$

We see that speed loses information about the direction of motion. So speed is what a speedometer records - the speedometer has no idea of the direction of motion.

The direction of motion.

Quiz 1

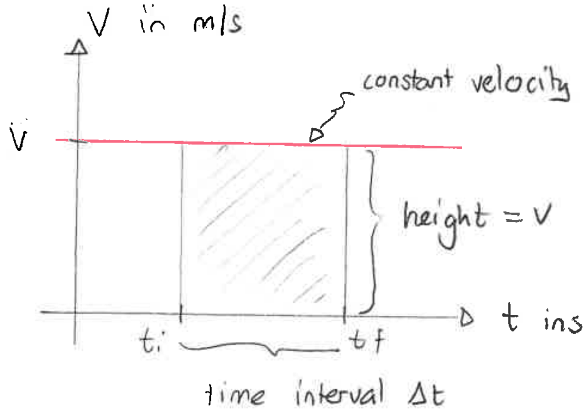
Position information from velocity

We will often have information about the velocity of an object as time passes and would like to use this to determine position information. Consider uniform motion, in which case velocity stays constant as time passes. Here $V = V_{avg}$, and we know that over any time interval

$$V_{avg} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = V_{avg}\,\Delta t$$

$$\Rightarrow \Delta x = V\,\Delta t$$

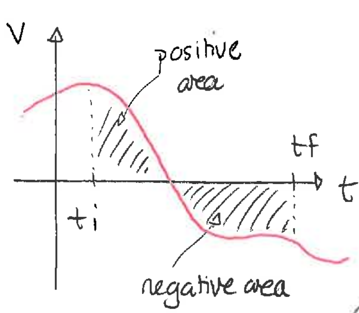
uniform motion only!



We can see that $V\Delta t$ is the area under the graph of velocity versus time. Thus for uniform motion we have proved that Δx is the area under the velocity versus time graph. This is true even when the velocity changes with time. Thus we have the scheme

see Fig 2.15

Given a graph of velocity versus time



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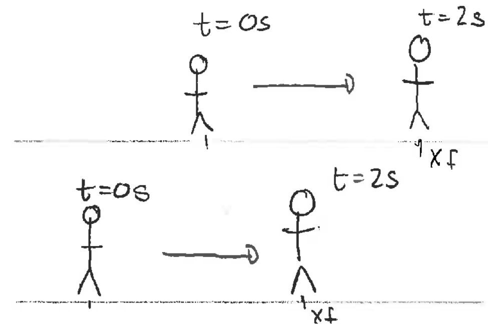
Displacement (change in position) from time t_i to t_f is

Δx = area under graph of V versus t from t_i to t_f

One caveat is that an area beneath the $v=0$ axis is negative. This accounts for the fact that when $V<0$ the object is moving left and position decreases, giving $\Delta x < 0$.

Quiz 2

Note that the graph of velocity versus time only gives information about the change in position. We cannot use it to give information about the actual position - this would require knowing the exact position at one instant.



same $v \Rightarrow$ same Δx

different x_f possible.

Calculating velocity

Can one calculate velocity without using a graph of x vs t or a limiting process? If one knows the function for position vs time, one can use calculus to do an exact analytical calculation. Here

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt} = \text{"derivative of } x \text{ with respect to } t\text{"}}$$

Calculus gives rules for taking derivatives of many types of function. For example:

$$\boxed{\text{If} \quad x = at^n \quad \text{where } a, n \text{ are constants, then} \quad \frac{dx}{dt} = n a t^{n-1}}$$

Example: Suppose $x = 5t^2 + 3t$. Determine velocity at $t = 3s$.

Answer: $v =$ deriv of x w.r.t t

$=$ deriv of $5t^2$ + deriv of $3t$

$= 2 \cdot 5t^{2-1} + 1 \cdot 3t^{1-1}$

$v = 10t + 3$

So at $t = 3s$ $v = 10 \times 3 + 3 = 33 m/s$ ☒