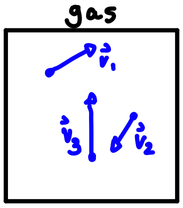
3-3-20

Thermodynamics relates bulk quantities.
Connected to some microscopic physics ?

gas



molecules

for molecule i : kinetic energy $K_i = \frac{1}{2} m v_i^2$

Total Energy is $E = K_1 + K_2 + \ldots.$
will fluctuate from one box with N molecules to another with N molecules

Average to get typical energy of one box N particles

PROBABILITIES !

Probability

Example : Coin toss
toss coin once :

outcome	event	Probability
gives heads	H	P(H)
gives tails	T	P(T)

→ one non-negative number per event

One way of assigning probabilities is to do :

Do N independent trials (coin toss)
⇒ N coin tosses

→ Let N_H = Number of times heads occurs
N_T = ... tails occurs

$P(H) = \frac{N_H}{N}$, $P(T) = \frac{N_T}{N}$

in limit as $N \to \infty$

This implies :
1) $0 \le p(H) \le 1$
$0 \le p(T) \le 1$
2) $p(H) + p(T) = 1$

General Situation

Event (label)	Prob
1	P(1)
2	P(2)
3	P(3)
4	P(4)
5	P(5)
6	P(6)
⋮	⋮

Satisfy : $0 \le p(i) \le 1 \quad i = 1, 2, 3$

$$\sum_{\text{all } i} p(i) = 1$$

Can combine to give : Prob (Either "i" or "j") = $p(i) + p(j)$

If events are continuously distributed with continuous label "x". Can describe :

Probability that event is in range $x \to x + dx = P(x)\,dx$

$P(x) \equiv$ Probability density distribution

Prob (event is in range $a \le x \le b$) = $\int_a^b P(x)\,dx$: $\int_{-\infty}^{\infty} P(x)\,dx = 1$

More than one trial? E.g. toss two coins

Quarter	Dime	Prob
H	H	$P(H,H)$
H	T	$P(H,T)$
T	H	$P(T,H)$
T	T	$P(T,T)$

$\longrightarrow$ Satisfy : $p(HH) + p(HT) + p(TH) + p(TT) = 1$

If tosses were independent :

$$p(HH) = \overset{Q}{p(H)}\,\overset{D}{p(H)}$$
$$p(HT) = p(H)\,p(T)$$
$$\vdots$$

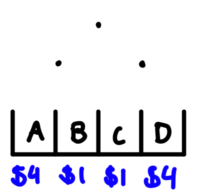
In general two events are independent if

$$P(i \text{ and } j) = p(i)\,p(j)$$

3-10-20

Means, Variance, Standard deviation

Example - Galton board game



- Need typical payout
- Suppose game is run N times
  - n_A = number of occurences of A
  - n_B = " .... " B

Payout per trial : $\dfrac{\$4 \cdot n_A + \$1 \cdot n_B + \$1 \cdot n_C + \$4 \cdot n_D}{N} = \$4 \dfrac{n_A}{N} + \$1 \dfrac{n_B}{N} + \ldots\ldots$

$= \$4 \cdot P_A + \$1 \cdot P_B + \$1 \cdot P_C + \$4 \cdot P_D$

$= \$4 \cdot \frac{1}{8} + \$1 \cdot \frac{3}{8} + \ldots. = \$1.75$

Scheme :

event	Prob	Value
1	$P(1)$	x_1
2	$P(2)$	x_2
3	$P(3)$	x_3
⋮	⋮	⋮

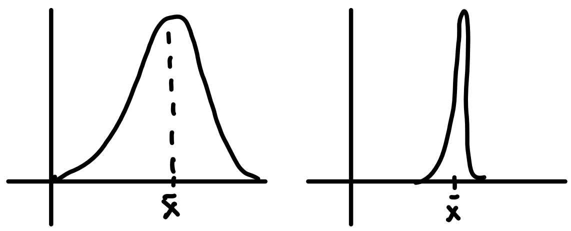
mean of x is

$$\bar{x} = \sum_{\text{all events } i} x_i \, p(i)$$

For any function of x : $f(x)$, we have mean of f is

$$\bar{f} = \sum_{\text{all events } i} f(x_i) \, p(i)$$

For some distribution mean gives some information e.g. center of distribution or most likely event



Can quantify spread by

1) Variance
2) Standard deviation

Variance of x is :

$$\overline{(\Delta x)^2} = \sum_{\text{all events } i} (x_i - \bar{x})^2 p(i)$$

Algebra $\rightarrow$

$$\overline{(\Delta x)^2} = \overline{(x^2)} - (\bar{x})^2 \quad : \quad \overline{x^2} = \sum_{\text{all events } i} x_i^2 \, p(i)$$

Standard Deviation

$$\sigma = \sqrt{\overline{(\Delta x)^2}} = \sqrt{\overline{x^2} - (\bar{x})^2}$$

Binary random variables

Each trial results in one of two possible events :

e.g.

1) Coin toss
2) Random walk in one dimension
3) Z-component of Spin - ½ particle outcomes are

$$\underset{\text{Spin up}}{S_z = +\frac{\hbar}{2}} \quad \text{or} \quad \underset{\text{Spin down}}{S_z = -\frac{\hbar}{2}}$$

- Can extend to multiple trials and consider cumulative outcomes
- e.g. random walk with rules :
  1) Start at $x = 0$
  2) Each step is 1m either left (L) or right (R)
  3) For any step probabilities :

event	Prob
R	p
L	q

Suppose one take N steps. Possible questions

1) Basic "Compound" events
   e.g. $N = 4$
   Prob of RRLR
   Prob of RLRR

2) "Aggregate" events
   e.g. $N = 4$
   - Probability that after 4 steps $x = 2$

3) Average Quantities : After N steps what is mean location
   These Questions Require
   - Rules for basic events
   - Assign "joint probability" to "compound" events
   - Counting & arguments to generate probability of aggregate events

$$P(k) = \binom{4}{k} p^k q^{4-k}$$

$$x = \#\text{right} - \#\text{left}$$
$$= k - (N-k)$$
$$= 2k - N$$
$$\bar{x} = 2\bar{k} - N$$

Mean of k

$$\bar{k} = \sum_{k=0}^{4} k\,p(k)$$
$$= \sum_{k=0}^{4} k \binom{4}{k} p^k q^{4-k}$$
$$= \sum_{k=0}^{4} \binom{4}{k} k p^k q^{4-k}$$
$$= 4P(\cancel{q}^{1} + P)^N$$
$$= 4P$$
$$\bar{x} = 8P - 4$$

These processes are called Bernoulli processes N trials each yielding one of two outcomes (e.g. +1 -1) probability of attaining k outcomes of +1 is

$$P(k) = \binom{N}{k} p^k q^{N-k}$$

p = Prob single trial gives +1 : $p + q = 1$
q = " " -1