4.2.29 $V=[5,-5]^T$ $g_1=\begin{bmatrix}-1\\2\end{bmatrix}$ $g_2=\begin{bmatrix}2\\1\end{bmatrix}$

a.) Transform of V, C.

$$c_k=\frac{\langle V,g_k\rangle}{\langle g_k,g_k\rangle} \qquad c_1=\frac{(5(-1)-5(2))}{(-1(-1)+2(2))}=\left(\frac{-15}{5}\right)=-3$$

$$c_2=\frac{(5(2)-5(1))}{(2(2)+1(1))}=\left(\frac{5}{5}\right)=1$$

$C=\begin{bmatrix}-3\\1\end{bmatrix}$

b.) Discrete transform matrix,

$$b=\begin{bmatrix}-1&2\\2&1\end{bmatrix} \quad b^T=\begin{bmatrix}-1&2\\2&1\end{bmatrix} \quad H=\begin{bmatrix}-1/5&2/5\\2/5&1/5\end{bmatrix} \quad H=\frac{1}{5}\begin{bmatrix}-1&2\\2&1\end{bmatrix}\approx\begin{bmatrix}-0.2&0.4\\0.4&0.2\end{bmatrix}$$

$\vec{a}_1\cdot\vec{a}_1=5$

$\vec{a}_2\cdot\vec{a}_2=5$

$$=\begin{bmatrix}-0.2&0.4\\0.4&0.2\end{bmatrix}\begin{bmatrix}5\\-5\end{bmatrix}=\begin{bmatrix}5(-0.2)-5(0.4)\\5(0.4)-5(0.2)\end{bmatrix}=\begin{bmatrix}-3\\1\end{bmatrix}$$

$C=\begin{bmatrix}-3\\1\end{bmatrix}$

c.) use inverse discrete transform to find V from C

$V=HC$ or $V=GC$

$$V=\begin{bmatrix}-1&2\\2&1\end{bmatrix}\begin{bmatrix}-3\\1\end{bmatrix}=\begin{bmatrix}3+2\\-6+1\end{bmatrix}=\begin{bmatrix}5\\-5\end{bmatrix}$$

$V=\begin{bmatrix}5\\-5\end{bmatrix}$

d.) V as an expansion g_1, g_2

$V=g\cdot C$

$$V=-3\begin{bmatrix}-1\\2\end{bmatrix}+1\begin{bmatrix}2\\1\end{bmatrix}$$