Phys 131 2016

Lecture 38

Fri: HW due by 5pm

Mon: Warm Up 15 D2L

Tues: Discussion/quiz

Supp 80, 81		Ch 13 Conc Q	5
Ch 12 Prob 46, 79		Ch 13 Prob	29, 35

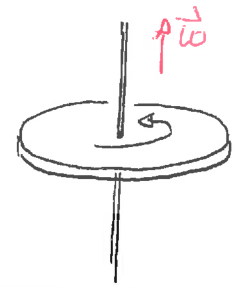
Vector description of rotational motion

We have seen that rotational motion can be described using vectors. First

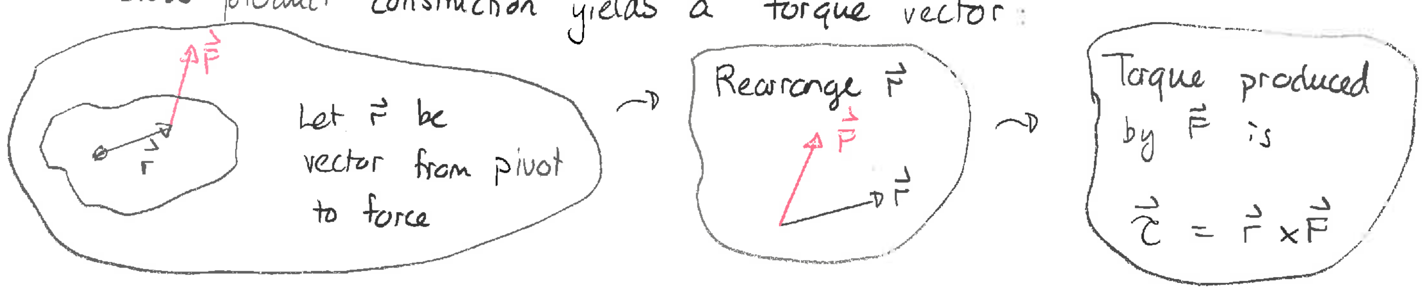
Angular velocity, $\vec{\omega}$, is a vector with

1) magnitude $\omega = \left|\frac{d\theta}{dt}\right|$

2) direction along axis of rotation (r.h. rule)

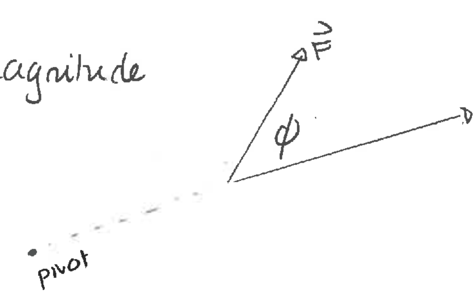


A cross product construction yields a torque vector:



Note that this gives magnitude

$$\tau = rF\sin\phi$$



Quiz 1 $\}$ 50% → 90%

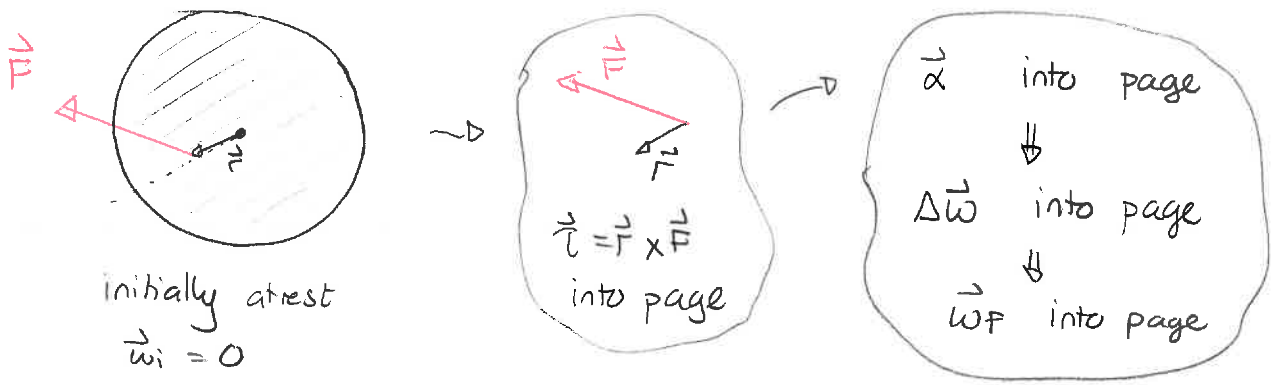
With the vector definition of torque we arrived at a general version of Newton's 2nd Law for rotational motion.

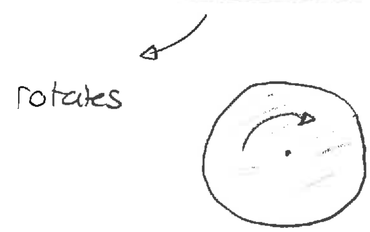
$$\boxed{\vec{\tau}_{net} = I\vec{\alpha}}$$

If the angular acceleration vector is constant, then

$$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} \Rightarrow \Delta\vec{\omega} = \vec{\alpha}\,\Delta t \Rightarrow \vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\,\Delta t$$

gives the change in angular velocity For example

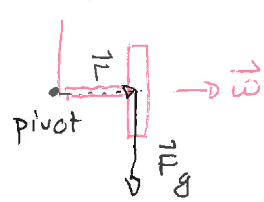




Such vector descriptions of rotational motion are essential in situations where the axis of rotation changes.

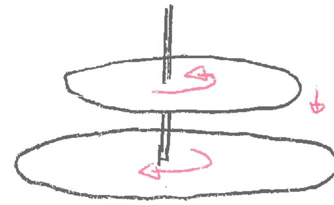
Demo: Gyroscope

Demo Bicycle wheel



$\vec{\tau}$ into page.

Angular momentum



Rotating objects can collide and stick. If we know their rotational states prior to collision could we determine the rotational state after collision?

Using torques will be too complicated. Fortunately a generalization of momentum is useful. We define

> The angular momentum of an object rotating about a fixed axle is
>
> $$\vec{L} = I\vec{\omega} \qquad \text{Units } kg\,m^2/s$$
>
> where $\vec{\omega}$ is the angular velocity vector and I is the moment of inertia.

A general result that can be derived from Newton's Laws is:

> If the net external torque on a system of objects is zero then the (vector) angular momentum of the system is constant.

This is the <u>conservation of angular momentum</u>.

In typical conservation of angular momentum situations either or both of the moments of inertia or angular velocity could change so

$$\underbrace{\vec{L}_f}_{\text{final}} = \underbrace{\vec{L}_i}_{\text{initial}} \iff I_f \vec{\omega}_f = I_i \omega_i$$

expresses the conservation of angular momentum

Quiz 2

82 Rotational collision

A 0.150 kg solid disk with radius 0.050 m rotates at 180 revolutions per minute. A 0.400 kg ring with radius 0.030 m is held at rest and then gently dropped onto the disk so that its center coincides with the center of the disk. It sticks. Determine the angular velocity of the combination after the ring sticks to the disk.

Answer: No net external torque. So angular momentum is conserved. Thus

$$\vec{L}_f = \vec{L}_i$$

or since these are one dimensional,

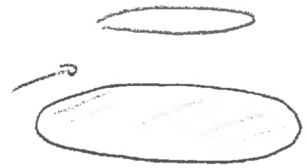
$$L_f = L_i$$

$$L_{ring\,f} + L_{disk\,f} = \cancel{L_{ring\,i}}^{\,0} + L_{disk\,i}$$

$$I_{ring}\,\omega_{ring\,f} + I_{disk}\,\omega_{disk\,f} = I_{disk}\,\omega_{disk\,i}$$

$$(I_{ring} + I_{disk})\,\omega_f = I_{disk}\,\omega_{disk\,i}$$

initial



$\omega_{ring\,i} = 0$

$\omega_{disk\,i} =$

final

$\omega_{ring\,f} = \omega_{disk\,f} = \omega_f$

Now $I_{disk} = \frac{1}{2} M_{disk}\, r_{disk}^2 = \frac{1}{2}\, 0.150\,\text{kg} \times (0.050\,\text{m})^2 = 1.9 \times 10^{-4}\,\text{kg m}^2$

$I_{ring} = M_{ring}\, r_{ring}^2 = 0.400\,\text{kg} \times (0.030\,\text{m})^2 = 3.6 \times 10^{-4}\,\text{kg m}^2$

Then $\omega_{disk\,i} = 3\,\text{rev/s} = 6\pi\,\text{rad/s}$

So

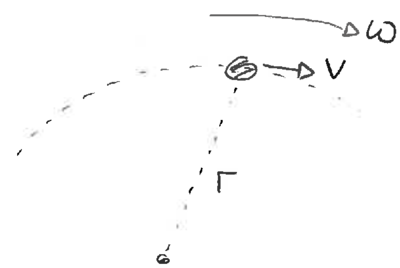
$$(1.9 \times 10^{-4}\,\text{kg m}^2 + 3.6 \times 10^{-4}\,\text{kg m}^2)\,\omega_f = 1.9 \times 10^{-4}\,\text{kg m}^2 \times 6\pi\,\text{rad/s} = 3.5 \times 10^{-3}\,\text{kg m}^2/\text{s}$$

$$\Rightarrow \quad \omega_f = \frac{3.5 \times 10^{-3}\,\text{kg m}^2/\text{s}}{5.5 \times 10^{-4}\,\text{kg m}^2} = 6.4\,\text{rad/s}$$

$$= 61\,\text{rpm}$$

Demo Rotating platform / wheel

Note that for a point particle



$$L = I\omega$$

$$= mr^2\omega$$

$$= mr(\omega r) = rmv$$

$$\Rightarrow L = rp.$$