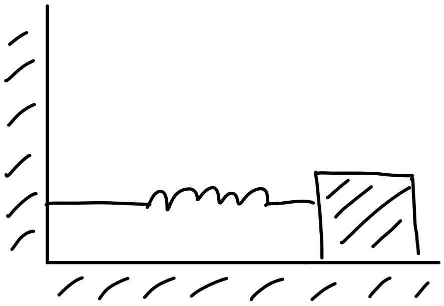
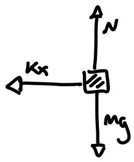
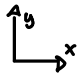
Ch. 1 Simple Harmonic Motion







$$F_{net} = m\vec{a}_x$$
$$-Kx = m\frac{d^2x}{dt^2}$$
$$\left[\frac{d^2x}{dt^2} = -\omega_0^2 x\right] \quad \omega_0 = \sqrt{k/m}$$

Has the solution of the form....

$$\left[x(t) = A\cos(\omega_0 t + \phi)\right]$$

Phase angle

Notice :

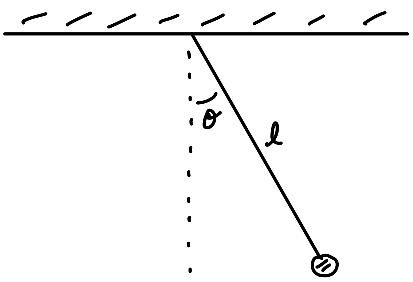
- A : ϕ are determined by $x(t=0)$: $v(t=0)$

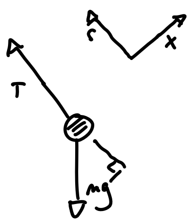
$\nu = \frac{1}{T} \equiv$ The linear frequency

$\omega_0 = 2\pi\nu = \frac{2\pi}{T}$

- $E = \frac{1}{2}mx^2 + \frac{1}{2}kx^2$

The Pendulum





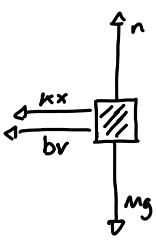
$$\vec{F}_{net_t} = m\vec{a}_t$$
$$-mg\sin\theta = ml\ddot{\theta}$$
$$\frac{d^2\theta}{dt^2} = \frac{-g}{l}\sin\theta \simeq -\frac{g}{l}\theta$$

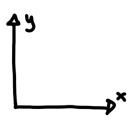
Has a solution of the form......

$$\left[\theta(t) = \theta_0\cos(\omega_0 t + \phi)\right] \quad \omega_0 = \sqrt{\frac{g}{l}}$$

Ch. 2 Damped Harmonic Motion

Free-Body diagram





$$\vec{F}_{net} = m\vec{a}_x$$
$$-Kx - bv = ma_x$$
$$-Kx - b\dot{x} = m\ddot{x}$$
$$\left[\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0\right]$$

Light Damping : $\omega_0 > \gamma/2$

$$\left[x(t) = Ae^{-\frac{\gamma t}{2}}\cos(\omega t + \phi)\right] \quad \omega = \sqrt{\omega_0^2 - \left(\gamma/2\right)^2}$$

$$\frac{A_N}{A_{N+1}} = e^{\gamma T/2} \quad T = \frac{2\pi}{\omega}$$

- A : ϕ found with initial conditions

$$\left[\begin{array}{l} E(t) = E_0 e^{-\gamma t} \\ \frac{dE}{dt} = -bv^2 \end{array}\right]$$

$$Q = \frac{\omega_0}{\gamma}$$

Heavy Damping $w_0 < \gamma/2$

$$\left[x(t) = e^{-\gamma t/2} \left[Ae^{\alpha t} + Be^{-\alpha t} \right] \right] \qquad \alpha = \sqrt{(\gamma/2)^2 - w_0^2}$$

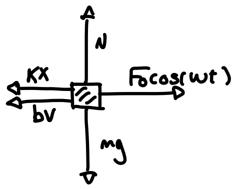
- A & B determined by initial conditions

Critical Damping $w_0 = \gamma/2$

$$x(t) = (A + Bt)e^{-\gamma t/2}$$

Ch. 3 Forced oscillations

FBD



$$\vec{F}_{net_x} = m\vec{a}_x$$
$$-kx - bx + F_0\cos(wt) = m\ddot{x}$$
$$\ddot{x} + \gamma\dot{x} + w_0^2 x = \frac{F_0}{m}\cos(wt)$$

Has a solution of the form...

$$\left[x(t) = A(w)\cos(wt - \delta) \right] \quad \text{where} \quad A(w) = \frac{F_0/m}{\left[(w_0^2 - w^2)^2 + w^2\gamma^2 \right]^{1/2}}$$

$$\tan(\delta) = \frac{w\gamma}{(w_0^2 - w^2)} \quad \text{where} \quad 0 \le \delta \le \pi \quad \| \quad 0 \le \delta \le 180^\circ$$

A_{max} when $\frac{dA}{dw} = 0$: $w = w_{max}$

$$A_{max} = A(w_{max})$$

Transient Phenomena

$$\left[x(t) = x_1(t) + x_2(t) \right]$$

where

$$\left[\begin{array}{l} x_1(t) = A(w)\cos(wt - \delta) \text{ - Steady state solution} \\ x_2(t) = Be^{-\frac{\gamma t}{2}}\cos(w_1 t + \phi) \text{ - Transient solution} \end{array} \right]$$

$$w_1 = \sqrt{(w_0^2 - (\gamma/2)^2)}$$

Notice B & ϕ are from initial conditions

Ch. 5 Traveling waves

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$ - 1D wave equation $\qquad y(x,t)$

with solutions

$$y(x,t) = f(x - vt) + g(x + vt)$$

Traveling Sinusoidal waves

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt) \right] = A\sin(kx - wt)$$

where $K = \frac{2\pi}{\lambda}$ - wave number

$V = \frac{\omega}{K} = \nu\lambda$ - wave speed

$\omega = 2\pi\nu = \frac{2\pi}{T}$

For a taut string ....

$V = \sqrt{\frac{T}{\mu}}$ $\mu = \frac{m}{l}$