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PHYS 362 HW 10

Problem 1

$$\eta = \left| \frac{W_{net}}{Q_+} \right|$$

Stage 1 → 2 : $PV^{\gamma} = \text{Const} \therefore TV^{\gamma-1} = \text{Const}$: $W = \frac{5}{2}Nk(T_2 - T_1)$, $Q = 0$

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

Stage 2 → 3 : $\frac{5}{2}Nk(T_3 - T_2) = Q$, $W = 0$

Stage 3 → 4 : $PV^{\gamma} = \text{const} \therefore TV^{\gamma-1} = \text{const}$: $W = \frac{5}{2}Nk(T_4 - T_3)$, $Q = 0$

$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} \therefore \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$

Stage 4 → 5 : $\frac{3}{2}Nk(T_4 - T_1) = Q$, $W = 0$ $\quad Q < 0$

$W_{net} = \frac{5}{2}Nk(T_2 - T_1 + T_4 - T_3)$, $Q = \frac{5}{2}Nk(T_3 - T_2)$

$V_4 = V_1$, $V_3 = V_2$, $T_1 = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \cdot T_2$, $T_4 = \left(\frac{V_2}{V_1}\right)^{\gamma-1} T_3$

$$W_{net} = \frac{5}{2}Nk\left(T_2 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}T_2 + \left(\frac{V_2}{V_1}\right)^{\gamma-1}T_3 - T_3\right)$$

$$\frac{W_{net}}{Q} = \frac{\left(T_3\left(\left(\frac{V_2}{V_1}\right)^{\gamma-1} - 1\right) - T_2\left(\left(\frac{V_2}{V_1}\right)^{\gamma-1} - 1\right)\right)}{T_3 - T_2} = \frac{\left(\left(\frac{V_2}{V_1}\right)^{\gamma-1} - 1\right)\cancel{(T_3 - T_2)}}{\cancel{(T_3 - T_2)}}$$

$$\frac{W_{net}}{Q} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} - 1 \quad \therefore \quad \left|\frac{W_{net}}{Q}\right| = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \eta$$

$$\boxed{\eta = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}}$$

For 10:1 ratio

$\eta = 1 - \left(\frac{1}{10}\right)^{2/3} = 0.79$ For monoatomic

$\eta = 1 - \left(\frac{1}{10}\right)^{2/5} = 0.60$ For diatomic

Problem 2

a.) $dS_h = c\frac{dT}{T}$, $Q_h = -c(T_{hf} - T_{hi})$, $Q_c = c(T_{cf} - T_{ci})$, $dS = dS_h + dS_L$

$\int_i^f dS_h = c\int_i^f \frac{dT}{T}$: $S\Big|_{Si}^{Sf} = c\cdot \ln(T)\Big|_{Ti}^{Tf}$: $\Delta S_h = c\cdot \ln(T_{hf}/T_{hi})$

$T_{hi} - \frac{Q_h}{c} = T_{hf}$: $\frac{\frac{cT_{hi} - Q_h}{c}}{T_{hi}} = \frac{cT_{hi} - Q_h}{cT_{hi}} = 1 - \frac{Q_h}{cT_{hi}}$

$$\boxed{\Delta S_h = c\cdot \ln(1 - Q_h/cT_{hi})}$$

$\int_i^f dS_c = c\int_i^f \frac{dT}{T}$: $S\Big|_{Si}^{Sf} = c\cdot \ln(T_c)\Big|_{Ti}^{Tf}$: $\Delta S_c = c\cdot \ln(T_{cf}/T_{ci})$

$T_{ci} + \frac{Q_c}{c} = T_{cF}$: $\frac{\frac{cT_{ci} + Q_c}{c}}{T_{ci}} = \frac{cT_{ci} + Q_c}{cT_{ci}} = 1 + \frac{Q_c}{cT_{ci}}$

$$\boxed{\Delta S_c = c\cdot \ln(1 + Q_c/cT_{ci})}$$

b.) $\frac{Q_c}{Q_h} \geq \frac{T_{ci}}{T_{hi}} + \frac{Q_c}{cT_{hi}} = \frac{T_{cf}}{T_{hi}}$

$$\begin{aligned}\Delta S = \Delta S_h + \Delta S_c &= c\cdot \ln(1 - Q_h/cT_{hi}) + c\ln(1 + Q_c/cT_{ci}) \geq 0\\ &= c\left(\ln(1 - Q_h/cT_{hi}) + \ln(1 + Q_c/cT_{ci})\right)\\ &= c\left(\ln((1 - Q_h/cT_{hi})(1 + Q_c/cT_{ci}))\right)\\ &= c\ln\left(1 + \frac{Q_c}{cT_{ci}} - \frac{Q_h}{cT_{hi}} - \frac{Q_hQ_c}{c^2T_{hi}T_{ci}}\right)\end{aligned}$$

$(1 - Q_h/cT_{hi})(1 + Q_c/cT_{ci})$
$1 + \frac{Q_c}{cT_{ci}} - \frac{Q_h}{cT_{hi}} - \frac{Q_hQ_c}{c^2T_{hi}T_{ci}}$

$\cancel{1} + \frac{Q_c}{cT_{ci}} - \frac{Q_h}{cT_{hi}} - \frac{Q_hQ_c}{c^2T_{hi}T_{ci}} \geq \cancel{1}$: $-\frac{Q_h}{cT_{hi}} - \frac{Q_hQ_c}{c^2T_{hi}T_{ci}} \geq -\frac{Q_c}{cT_{ci}}$: $\frac{Q_c}{cT_{ci}} \geq \frac{Q_hQ_c}{c^2T_{hi}T_{ci}} + \frac{Q_h}{cT_{hi}}$

$\frac{Q_c}{cT_{ci}} \geq \frac{Q_hQ_c}{c^2T_{hi}T_{ci}} + \frac{cQ_hT_{ci}}{c^2T_{hi}T_{ci}}$: $Q_c \geq \frac{Q_hQ_c}{cT_{hi}} + \frac{Q_hT_{ci}}{T_{hi}}$: $\frac{Q_c}{Q_h} \geq \frac{Q_c}{cT_{hi}} + \frac{T_{ci}}{T_{hi}}$

$\frac{Q_c}{cT_{hi}} + \frac{cT_{ci}}{cT_{hi}} = \frac{Q_c + cT_{ci}}{cT_{hi}}$: $T_{cf} = \frac{Q_c}{c} + T_{ci}$, $T_{hi} = \frac{Q_h}{c} + T_{hf}$: $T_{cf} = \frac{Q_c + cT_{ci}}{c}$, $T_{hi} = \frac{Q_h + cT_{hf}}{c}$

$\frac{T_{cF}}{T_{hi}} = \frac{Q_c + cT_{ci}}{Q_h + cT_{hf}} = \frac{Q_c + cT_{ci}}{cT_{hi}}$: $\boxed{\frac{Q_c}{Q_h} \geq \frac{Q_c}{cT_{hi}} + \frac{T_{ci}}{T_{hi}} = \frac{T_{cf}}{T_{hi}}}$

c.) $\eta = 1 - \frac{Q_c}{Q_h}$: $\lim_{c\to\infty}\left(\frac{Q_c}{Q_h} \geq \frac{Q_c}{cT_{hi}} + \frac{T_{ci}}{T_{hi}}\right)$: $\frac{Q_c}{Q_h} \geq \frac{T_{ci}}{T_{hi}}$ $\therefore$ $\eta \leq 1 - \frac{T_{ci}}{T_{hi}} \leq 1 - \frac{T_L}{T_h}$

$\boxed{\eta \leq 1 - \frac{T_L}{T_h}}$ $\qquad$ $\boxed{c \text{ Finite} : \eta \leq 1 - \frac{T_{cf}}{T_{hi}}}$

d.) The efficiency will decrease because T_h decreases and T_L increases

Problem 3

a.) $\eta \leq 1 - \frac{T_L}{T_h}$

Since T_h is held constant, when the difference between T_L and T_h is increased this means T_L must decrease and thus the fraction $\frac{T_L}{T_h}$ decreases and in turn η will increase. Therefore i.) is the correct answer.

b.) $\eta \leq 1 - \frac{T_L}{T_h}$

i.e: $T_L = 300k$, $T_h = 500k$: $\eta \leq 1 - \frac{300k}{500k} \leq 1 - \frac{3}{5} \leq \frac{2}{5}$

$T_L = 400k$, $T_h = 600k$: $\eta \leq 1 - \frac{400k}{600k} \leq 1 - \frac{2}{3} \leq \frac{1}{3}$

The difference can be the same but the efficiencies of the two will not be the same. Therefore ii.) is the correct answer.

Problem 4

Since both of these Carnot cycles reside on the same isothermal lines the efficiencies of each Carnot cycle are the same.

$$\eta = 1 - \frac{T_L}{T_h}$$

$T_{L_i} = T_{L_{ii}}$, $T_{h_i} = T_{h_{ii}}$ so therefore $\eta_i = \eta_{ii}$

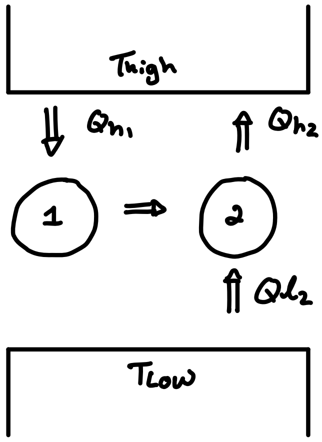
Problem 5

$T ds = \delta Q \quad \therefore \quad \Delta S = \int \frac{1}{T} \delta Q \quad : \quad \Delta S = \Delta S_H + \Delta S_L + \Delta S_E \geq 0$

$\Delta S_E = 0$ since this is a perfect cycle $\therefore \quad \Delta S_H + \Delta S_L \geq 0 \longrightarrow \Delta S_H \geq -\Delta S_L$

This means that the entropy change of the higher temperature bath has to be greater than or equal to the lower temperature bath.

Problem 6



$$Q_{h_1} = W \qquad \therefore \quad Q_{h_1} = Q_{h_2} + Q_{l_2}$$
$$Q_{h_2} = W - Q_{l_2} \qquad Q_{h_2} = Q_{h_1} - Q_{l_2}$$

$$Q_{h_2} = Q_{h_1} - Q_{l_2}$$

The existence of the First system is not possible because heat cannot move from low to high